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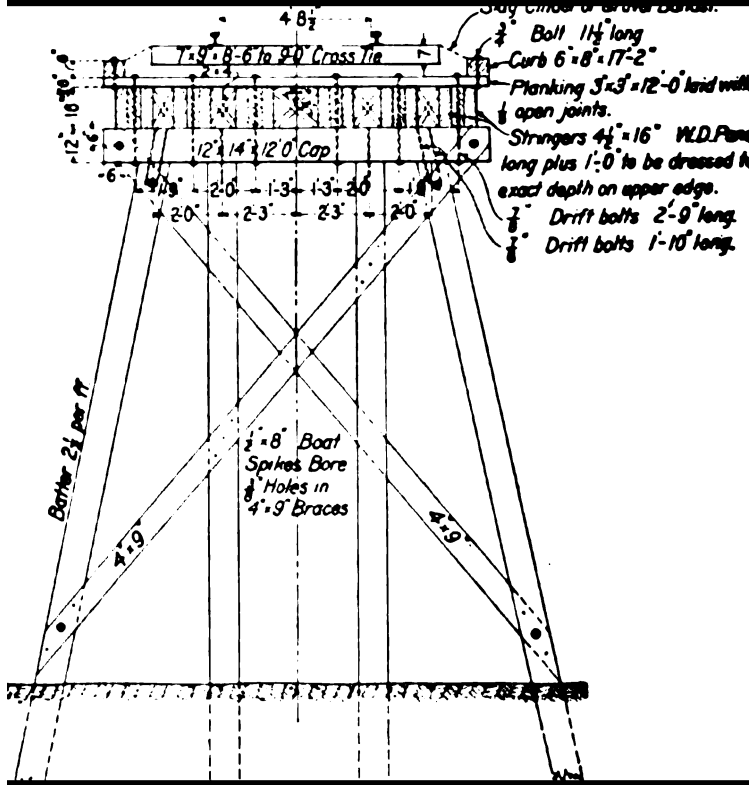

Civil engineers' pocket book

Albert Irvin Frye

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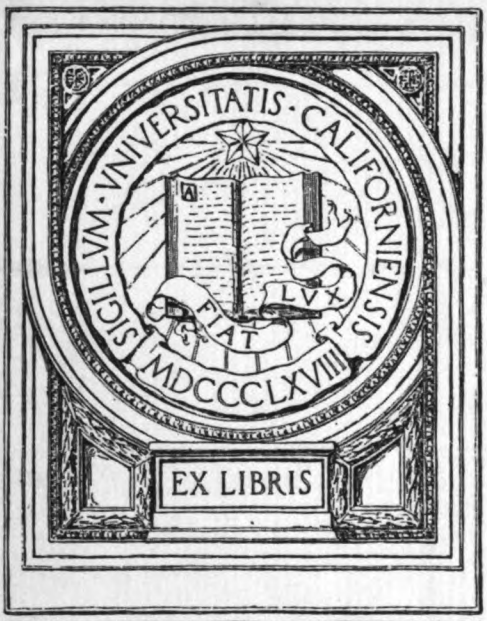

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CIVIL ENGINEERS' POCKET-BOOK

A REFERENCE-BOOK FOR ENGINEERS
CONTRACTORS AND STUDENTS
CONTAINING RULES, DATA
METHODS, FORMULAS
AND TABLES

BY

ALBERT I. FRYE, S. B.
Member American Society of Civil Engineers

NEW YORK:
D. VAN NOSTRAND COMPANY
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PREFACE.

The preparation of notes for this volume was begun several years ago, while chief engineer of the Hoffman & Bates Bridge and Construction Company, and has been continued systematically up to the present time.

Three separate systems of note-keeping have been employed, namely: (1) Two blank-books, size eight by twelve and one-half inches, with quadrangular ruling and marginal index, were started, one for mathematical and structural data and the other for general construction notes, each book containing five hundred pages in flexible leather binding. (2) Note books of the same general style, but of pocket size, were classified—for bridges, buildings, water-works, sewers, surveys, etc.—and kept at hand for general reference and memoranda. (3) The loose-leaf method was inaugurated and found to be specially useful in consulting practice. The last named is explained in an article prepared for the "Engineering Record" and published in the issue of January 21, 1911.

In 1904, the present system was outlined—arranging the matter logically in numbered Sections for convenient reference. From that time up to the present, the work has gradually been crystallized and brought up to date.

Special attention is invited to the vast number of tables, their completeness and arrangement. Although most of them have never before appeared in print—at least in their present form—yet nearly all of them have been subjected to the test of more or less constant use for a number of years, in connection with practical work. All of them have been thoroughly checked—the proofs of all were checked twice before electrotyping, and the proofs of the most important ones were checked again after electrotyping.

The logarithmic tables comprise both the common and hyperbolic systems, side by side, the latter being useful in certain bridge calculations and in steam engineering. Both the logarithmic and trigonometric tables are carried out to five decimal places, a sufficient refinement for most engineering opera-

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tions, being much more exact than actual measurements in shop or field.

The tables of cubes and squares, in Section 33, will be found useful to structural detailers. They were calculated by the incremental additive method, which is self-checking.

The text is so arranged that all general data may be found readily from the Table of Contents, which should be consulted as frequently as the Index, the latter being necessary for more specific reference. The number and title of each Section appear as even-page captions throughout the work. The tables and illustrations are numbered from one upward for each Section. It has been the aim to begin each subject and paragraph with the leading or key word, as a supplementary page index.

At the end of most of the Sections will be found references to valuable data in leading technical publications, which should be in the library of every engineer. The reader is advised to supplement this data with his own lists, perhaps in a separate note book, under the respective Section numbers.

Acknowledgments are due to Mr. Paul D. Ford, for his kindness in volunteering to read over portions of the manuscript while in preparation, and making many valuable suggestions; also to Professor C. H. Peabody, for permission to use the steam tables and other material in Section 69; and to many others whose names appear in the body of the work, where specific credit is given.

The Publishers, Electrotypers and Printers are to be congratulated on the neat appearance of the book, and the author desires to thank Mr. Jacob Wernli for his excellent work in preparing the illustrations.

ALBERT I. FRYE.

New York, December, 1912.

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INTRODUCTION.

General Discussion.—The young engineer or student should realize that, although engineering is not an exact science itself, it is, in its entire field, founded on the exact sciences, supplemented with experimental data. As he becomes more proficient in his work he will discern certain broad, underlying *principles* which he will begin to use instead of mere *methods* or rules, formerly employed. It should be his aim to master these principles thoroughly.

MATHEMATICS.

The various branches of mathematics as taught in our common schools are based on methods of operation rather than on mathematical principles, for when viewed from the latter standpoint they overlap each other to such an extent that the dividing lines are not always clearly marked. For instance, the science of *number*, *space*, *quantity*, *position*, *motion*, *mass*, *force* and *inertia* cannot be taught in logical sequence from out text-books as now arranged.

In view of the above, and of the fact that an attempt has been made in the body of this work to classify the subject matter of mathematics under the usual headings—Arithmetic, Algebra, Geometry, etc.,—it is deemed pertinent here to introduce a discussion of the more abstract science of *number*, *space*, *quantity*, etc., as preliminary to the main text, recognizing here none of the divisions, as such, which are contained in the latter.

NUMBER.*

The science of Arithmetic involves the principles of Algebra; these principles in turn having been deduced from the theory of number, and so on.

Number is independent of the order of counting.—In counting any number of things, the counting of the last one contains the numeral word which designates the total number, no matter in what order they are counted. In the illustration, for example, there are five dots in each case.

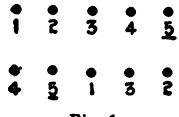

Fig. 1.

A Sum is independent of the order of adding.—In finding the sum of two or more groups of things, it matters not in what order they are added; as 3 added to 5 equals 5 added to 3, equals 8; or, $3+5=5+3=8$. 3 plus 5 plus $4=5+3+4=4+5+3$; etc., $=12$.

Using *shorthand* characters or letters to represent numbers the same law holds true: as, $a+b=b+a$; $a+b+c=b+a+c$; etc.

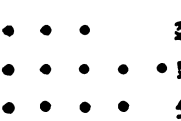

Fig. 2.

A Product is independent of the order of multiplying.—

4 3's = 4 times $3=4\times3=12$; or, $a\times b=ab=12$.

3 4's = 3 times $4=3\times4=12$; or, $b\times a=ba=12$.

Similarly, $4\times3\times5=(4\times3)\times5=4\times(3\times5)=3\times(4\times5)=60$. And using shorthand, in which the letter $a=4$, $b=3$, and $c=5$: $a\times b\times c=abc=acb=bac=bca=cab=cba=60$.

Note the 6 $(1\times2\times3)$ different ways of arranging the 3 letters.

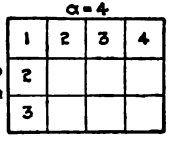

Fig. 3.

* See, also, Clifford's "Common Sense of the Exact Sciences."

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A Product is independent of the parting of its factors.—Thus, $7\times5=(7\times3)+(7\times2)=21+14=35$, or, by shorthand, $ab=ae+af=a(e+f)$. Again, $7\times5=(6+1)(3+2)=6(3+2)+1\times(3+2)=30+5=35$ or, $ab=(c+d)\times(e+f)=c(e+f)+d(e+f)=e(c+d)+f(c+d)$.

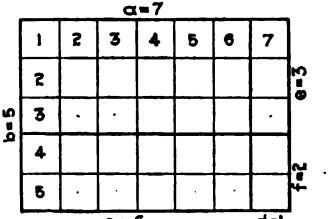

Fig. 4.

A Power is independent of the parting of the index.—

The *first power* of a number is the number itself; as $a^1=a$.

The *square* of $5=5^2=5\times5$; of $a=a^2=aa$; in which 2 is the *index*.

The *cube* of $7=7^3=7^1\times7^2=7^{1+2}=7\times7^2=7^{1+1+1}=7\times7\times7$; of $b=b^3=bb^2=bbb$.

The *fourth power* of $a=a^4=a^{3+1}=a^{2+2}=a^3a=aa^3=a^2a^2=a^{\frac{8}{2}}=\sqrt{a^8}$.

The *fifth power* of $2=2^5=2\times2\times2\times2\times2=2(2\times2\times2\times2)=(2\times2)(2\times2\times2)$; or, $2^5=2^{1+4}=2^{2+3}=2\times2^4=2^2\times2^3$.

The *square root* of the *fifth power* of $4=\sqrt{4^5}=\sqrt{4^{4+1}}=\sqrt{4^4\times4}=\sqrt{4^4}\times\sqrt{4}=4^2\sqrt{4}=16\sqrt{4}=16\times2=32$; or $\sqrt{4^5}=4^{\frac{5}{2}}=4^{2\frac{1}{2}}=4^2\times4^{\frac{1}{2}}=4\times4^{1\frac{1}{2}}=4\times4\times4^{\frac{1}{2}}=32$; or, $\sqrt{4^5}=\sqrt{4^2\times4^3}=\sqrt{4^2\times4^3\times4}=\sqrt{4^2}\times\sqrt{4^3}\times\sqrt{4}=4\times4\times2=32$.

From the above it will be seen that $a^0=1$, for $a=a^{n-n}=\frac{a^n}{a^n}=1$.

Square of $(a+1)$ is a special case of $(a+b)^2$.

$(4+1)^2=(4+1)(4+1)=4(4+1)+1(4+1)=16+4+4+1=25$.

$(a+1)^2=(a+1)(a+1)=a(a+1)+1(a+1)=a^2+2a+1$.

$(a+b)^2=(a+b)(a+b)=a(a+b)+b(a+b)=a^2+2ab+b^2$.

Again: $(a-1)^2=a^2-2a+1$; and $(a-b)^2=a^2-2ab+b^2$.

Practical application: $(21)^2=(20+1)^2=(20)^2+2\times20+1=400+40+1=441$:

also, $(19)^2=(20-1)^2=(20)^2-(2\times20)+1=400-40+1=361$.

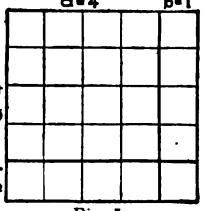

Fig. 5.

Note.—By making b *extremely small* compared with a, in the case of $(a+b)^2$, it will be seen that the third term b^2, as b approach zero, can be omitted, as it will be the square of an infinitely small decimal or fraction. Hence, the limit of $(a+b)^2$ as b approaches zero, is a^2+2ab, or, in other words, the actual *increase* of $(a+b)^2$ over a^2, where b is infinitely small, is $(a^2+2ab)-a^2=2ab$.

This is an elementary principle of the *Differential Calculus*—the method of limits.

$(a^2-1)=(a+1)(a-1)$ **is a special case of** $(a^2-b^2)=(a+b)(a-b)$.

$(4^2-1)=(4+1)(4-1)=4(4-1)+1(4-1)=4\times3+1\times3=12+3=15$.

$a^2-1=(a+1)(a-1)=a(a-1)+1(a-1)=a^2-a+a-1=a^2-1$.

$a^2-b^2=(a+b)(a-b)=a(a-b)+b(a-b)=a^2-ab+ba-b^2=a^2-b^2$.

Practical application: $41\times39=(40+1)(40-1)=40^2-1=1600-1=1599$.

From $a^2=(a+1)(a-1)+1$, we have $39^2=40\times38+1=1520+1=1521$.

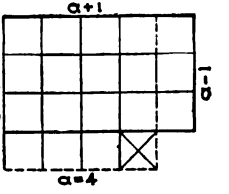

Fig. 6.

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SPACE.

An Object is composed of layers and bounded by a surface.—The *surface* of an object has a definite area, but as it has no thickness, in itself, it occupies no space. On the other hand, the layers composing the object, no matter how thin we may assume them theoretically, must occupy space and have thickness. In theory we assume such shapes as cones, cylinders, spheres, etc., to be composed of an *infinite* number of layers of an infinitesimal thickness, *t*.

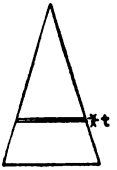

Fig. 7.

A Surface is an area with or without lineal boundary.—The surface of the Earth, of a chair, of a cube, or of any *whole* object can have no lineal boundary; but the surface of *part* of an object, as of a continent, of one face of a cube, or of the top of a table is bounded by lines along the edges of the surface. Neither the surface nor its bounding lines can have thickness nor occupy space.

The surface of an object is its *shape*.

A Line is a boundary, division, or an intersection, of surfaces.—The *boundary* of a surface is a curved or broken line of definite length, and *continuous*; that is, without terminal points. A good illustration of this is a traverse survey, as around a farm. If the field notes do not "close" they are made to close by "adjustment."

A *plane* surface can be divided by a *straight* line, of definite length only, that is, with terminal points at its boundaries; but it can be divided also by a *curved* or broken continuous line, as for instance by a circle, entirely within its boundaries.

Fig. 8.

Fig. 9.

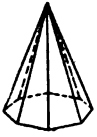
Fig. 10.

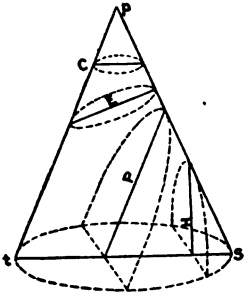

Fig. 11.

The intersections of the surfaces of the *cube* furnish the *right angle* and the *square*; the intersections of the surfaces of the *wedge* furnish the *triangle* and the *rectangle*; and, similarly, the *pyramid*, in its various forms, furnishes the numerous *polygons*.

By *cutting surfaces*, the *cone* furnishes the *point*, *p*, the *straight line*, *p s*, the *triangle*, *p s t*, the *circle*, *C*, the *ellipse*, *E*, the *parabola*, *P*, the *hyperbola*, *H*, etc. Its properties embrace such a wide range in Analytic Geometry that the subject is often termed *Conic Sections*.

The boundary of a surface is its *shape*.

QUANTITY.

Quantity is a summation of units, of one, two or three dimensions.—The *first* involves continuity, length, or number; as time, angles, weights, lengths, money and numerals.

The *second* involves length and breadth, or area; as surface measures.

The *third* involves length, breadth and depth, or contents; as measures of volume.

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Area is a summation of one or more products of two factors each.—If we divide the surface of the right triangle, Fig. 12, into *extremely thin* strips of length y, and thickness t, and then move the apex to one side, as in Fig. 13, allowing the thin strips to slide on one another, their ends will still form straight lines from the apex, a, to the base of the triangle, and its height and area will remain the same. The area of the triangle is the summation of the areas ($y \times t$) of all the thin strips. The above illustrates two principles, namely, simple proportion and the equation of the straight line, $a\,e$; thus—

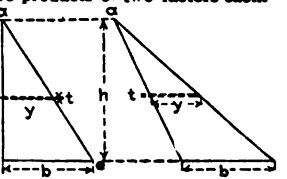

Fig. 12.

Fig. 13.

Simple proportion: $\frac{y}{x} = \frac{b}{h}$.

Equation of the straight line, ae: $y = \frac{b}{h}x$.*

The straight line is sometimes called a curve of the first degree.

The same law holds true in the case of any figure, as Fig. 14 where the boundary line ae is a curve of the second degree; for it is necessary only to find the equation of the curve in terms of x and y so that for each value of x we may know the value of y. In either case we derive the area from the summation of all the infinitesimal strips yt between the limits $x=h$ and $x=0$. This may be performed accurately by the method of the Calculus, by assuming an infinite number of infinitesimal strips.

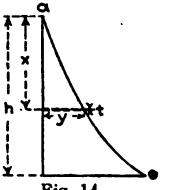

Fig. 14.

If we consider Fig. 12 to be an elastic layer of definite thickness, having the surface as shown, and apply a lateral pressure at the apex, a, to change its shape to Fig. 13, we conceive in Mechanics the terms *force*, *elasticity*, *stress*, *strain* and *shear*.

Length is a summation of the square roots of the sums of squares.—It is an elementary problem to prove that the square H (hypothenuse²) is equal to the square B (base²) plus the square P (perpendicular²), by the proposition that H is composed of two rectangles B' and P' equal in area to B and P, respectively. From this principle the length of any curve may be obtained by the summation of its infinitesimal parts.

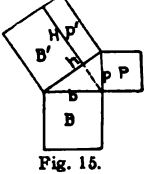

Fig. 15.

Thus, required (Fig. 16) to find the length S of the curve ac between the limits $x=c'$ and $x=a'$. We have here to find the summation of the infinitesimal tangent lengths ds in terms of dy and dx for every change of position of the tangent as the values of x and y change. It is evident that each infinitesimal length $ds = \sqrt{dx^2+dy^2}$, hence if we find the values of dx^2 and dy^2 from the equation of the curve, by the Calculus, we can get the value of ds in general terms. It only remains to find the summation of all the values of ds, that is, S, and this may be obtained by integration, subtracting the result obtained by making $x=a'$, from the result obtained by making $x=c'$.

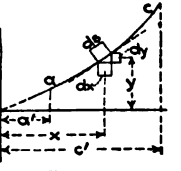

Fig. 16.

The ratio of the circumference of a circle to its diameter = 3.141592+, called π.

Volume is a summation of layers; or area times thickness.—If a beam of homogeneous material and of rectangular cross-section be loaded so

* The general equation of the straight line is $y = mx + c$, in which m is the tangent of the angle, and c is a constant. In the above, $c=0$.

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that it deflects, as in Fig. 17, there will be a plane $s-s$ through the middle of the beam which will not change from its original length; that is, there will be no stretching or shortening of the fibers along this plane, which may be called the *neutral axis* of the beam. If now the beam be turned

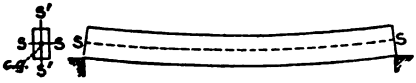

Fig. 17.

on its side and again deflected, a new plane, $s'-s'$, at right angle to the first, will become a neutral axis. In either case the fibres at the top of the beam will be compressed and the fibers at the bottom of the beam stretched, equally. Also, the stretching or compression of any fibre in the beam will be proportional to its distance from the neutral axis. In other words, while the beam loses in volume above the neutral axis it gains an equal amount below. The intersection of the neutral axes will also be found to be at the center of gravity, *c. g.*, of the section. The center of gravity may be defined as the point of intersection of all possible neutral axes, for no matter which way the beam may rest, on its corner or otherwise, the neutral axis will pass through the center of gravity of the section.

Fig. 18.

The center of gravity of any *regular* section, as a square, rectangle, parallelogram, equilateral triangle, circle or regular polygon is in the center of the figure. (Fig. 18.)

The center of gravity of a triangular section is one-third the height from either base. (Fig. 19.)

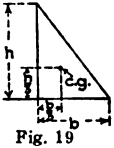

Fig. 19

These principles furnish the rules for finding the contents of the cylinder, circular ring, cone, paraboloid, ellipsoid, or any similarly curved bodies.*

Consider any section lying wholly on one side of the axis of revolution, or center of curvature; multiply the area of the section by the distance traversed by its center of gravity. The product will be the volume of revolution of that section.

For example, the volume of a cone is equal to the area of the triangle of revolution $\times \frac{1}{3}$ the base $\times 2\pi$

$$= \frac{bh}{2} \times \frac{b}{3} \times 2\pi = \frac{\pi b^2 h}{3} = \frac{h}{3} \times \text{area of base.} \quad \text{(Fig. 20.)}$$

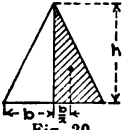

Fig. 20.

Position.

The Relative Position of an object may be determined by *one*, *two* or *three* quantities, according as it is on a known *line*, *surface* or *in space*. The third dimension problem can be reduced to the second by passing a known surface or plane through the point in space; and this can be reduced to the first by passing a known line through the point on the surface or plane. A distance along the line, measured to the object in question, will fix its exact position.

The Position of a Point on a Plane.—

The *polar co-ordinates*, angle α and distance D, locate the point P from A. Polar co-ordinates may be reduced to *rectangular co-ordinates*, x and y, Fig. 21.

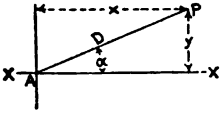

Fig. 21.

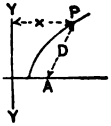

Fig. 22.

* See also Pappus's Theorems for surfaces and solids, page 243.

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A point P, Fig. 22, may be located from a fixed point A by a distance D, and from a fixed line $Y-Y$ by a distance x. If D and x are allowed to increase and decrease with constant ratio, the point P will trace (1) the parabola, if the ratio of D to x is unity; (2) the hyperbola, if the ratio of D to x is greater than unity; and (3) the ellipse, if the ratio of D to x is less than unity.

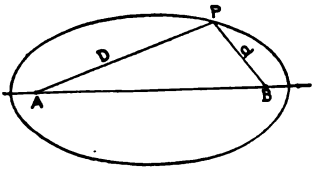

Fig. 23.

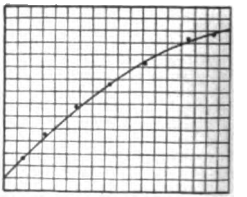

Fig. 24.

A point P, Fig. 23, may be located from two fixed points: A, by distance D, and B, by distance d. If the ratio of D to d is allowed to increase and decrease, but so their sum will remain constant, the point P will trace an ellipse.

The results of experiments are conveniently platted on cross-section paper, and a curve drawn as nearly regular as possible through the average position of the points. The curve then serves as a formula for future use.

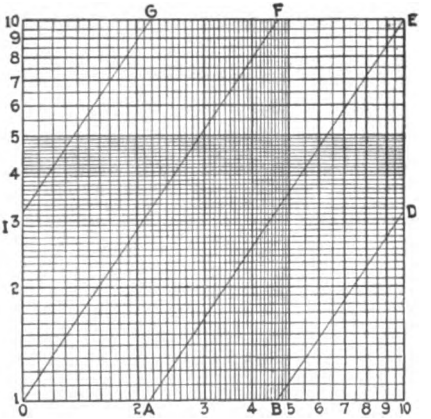

Fig. 25.

If the curve is of the second degree it can be represented by a straight line if logarithmic cross-section paper, Fig. 25, is used. Hydraulic formulas and experiments are often platted on this kind of paper, for simplicity.

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MOTION.

A man starts to row a boat, at a constant rate of speed, directly across the stream from A to B, not allowing for any current. On reaching the east bank he finds he has landed at D. If the current was uniform in all parts of the stream his course was the straight line AoD, and his motion down stream uniform in passing from the west to the east bank. If the current was very strong at the west bank and gradually lessened toward the east bank his course was the line AcD and his motion down stream a retarded one—uniformly retarded if the current at any point was directly proportional to its distance from the east bank. If the current gradually increased toward the east bank his course was the line AbD and his motion down stream an accelerated one—uniformly accelerated if the current at any point was directional proportional to its distance from the west bank.

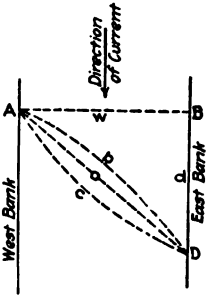

Fig. 26.

The *average velocity* of the (surface of the) stream is the same for all three cases above cited if the boat lands at the same point, D.

Uniform velocity or motion may be represented by a straight line, i. e., curve of the first degree; uniformly accelerated or retarded velocity or motion, by a curve of the second degree.

The *acceleration* per second of time due to the gravity of the earth on any body falling in vacuo is practically constant. It is designated by the letter g, and its value is about 32.16 ft. per sec. It varies somewhat with the elevation above sea level and with the lattitude of the place.

MASS (MATTER), FORCE AND INERTIA.

These are convenient terms used in Applied Mechanics to express certain co-relative ideas.

We may consider *mass* as matter, or that property which cannot be destroyed, because matter is indestructible, although the body of which it is composed may change its form or apparently disappear. Mass is proportional to weight, and the unit of weight is one pound. The mass of a body is its weight, W, divided by gravity acceleration, g, as $M=\frac{W}{g}$; or, the mass of a body is measured by a force, F, which will produce an acceleration, a, as $M=\frac{F}{a}$. The acceleration in either case is the velocity attained at the end of the first second of time.

Force is a shorthand expression of stating the value of one mass acting on another, by pressure or impact. Force = mass × acceleration, $F=Ma$. Its unit is one pound.

Inertia is a negative term implying inherent inactivity in a relative manner: the tendency which a body has to continue doing what it is already doing—if at rest to *remain* at rest, and if in motion to *continue* in motion in the same direction and at the same velocity.

EXPERIMENTATION.

Several years ago the late Professor Joseph L. LeConte, in a very elaborate scientific discussion, "proved conclusively" that the heavier-than-air flying-machine was an absolute impossibility; and yet, today, aerial flights in such machines are attracting but passing interest.

Our college professors are daily teaching the maxim: "There is no conflict between theory and practice." This should be restated as follows: "There can be no conflict between correct theory and perfect practice, but neither can always be attained." The theory of Professor LeConte was incorrect—but nevertheless a theory—and the first experiments in flying, by Octave Chanute, were simply imperfect ones.

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Joseph H. Choate, the able jurist, in an address before a society of engineers, a few years ago, exposed himself to criticism in stating that engineering was an exact science—a statement that is very far from the truth.

The great advance in the whole field of engineering—quite phenomenal during the past decade, especially—is due almost wholly to experimental work, scientifically conducted.

Management.

Scientific management is becoming a potent factor in the industrial world, and is destined to possess an increasing sphere of influence in the future. It simply means *intelligent cooperation*: distinctly opposed to the old system of bossism, which is more or less militant and invites toadyism—a qualification distasteful to every honest workman.

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SIGNS AND ABBREVIATIONS.

Common Abbreviations.—The following abbreviations, and others not listed here, are used constantly thoughout this volume in order to conserve space. In general, they appear in the text after the full word has been used so there can be no doubt as to the meaning:

abut	= abutment.
accel	= acceleration.
ans	= answer.
approx	= approximate.
cen	= center.
circum	= circumference.
diag	= diagonal.
diam	= diameter.
dist	= distance.
grav	= gravity.
hd	= head.
hor	= horizontal.
ht	= height.
hyp, hypoth	= hypothenuse.
hyp log	= hyperbolic log.
log	= logarithm.
perp	= perpendicular.
pres	= pressure.
pt	= point.
rad	= radius.
tang	= tangent.
vel, veloc	= velocity.
vert	= vertical.
vol	= volume.
wrt	= wrought.
wt	= weight.
sin	= sine = perp + hyp.
cos	= cosine = base + hyp.
tan	= tangent = perp + base.
cot, cotan	= cotangent.
	= base + perp.
sec	= secant.
	= hyp + base.
csc, cosec	= cosecant.
	= hyp + perp.
vers	= versed sine.
	= (hyp − base) + hyp.
covers	= co-versed sine.
	= (hyp − perp) + hyp.
exsec	= exsecant.
	= (hyp − base) + base.
coexsec	= co-exsecant.
	= (hyp − perp) + perp
sec, s.	= seconds.
min, m.	= minutes.
hrs, h.	= hours.
d.	= days.
mos.	= months.
yrs.	= years.
e. g.	= for instance.
i. e.	= that is.
etc.	= and so forth.
	= et. cetera.
c. g.	= center of gravity.
ins.	= inches.
ft.	= feet.
yds.	= yards.
lbs.	= pounds.
pts.	= pints.
qts.	= quarts.
gals.	= gallons.
bu.	= bushels.
bbls.	= barrels.
lin.	= lineal.
sq.	= square.
cu.	= cubic.
B. M.	= board measure.
M., B. M.	= thousand ft. B. M.
H. P.	= horse-power.
A. W. G.	= American wire gauge.
B. W. G.	= Birmingham W. G.
r. p. s.	= revolutions per sec.
r. p. m.	= revolutions per min.

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Greek Alphabet.—Greek letters are used in the sciences to designate certain properties, as angles, latitude, temperature, etc.

English Equivalents.	Greek Letters.			Used to designate.
	Name.	Capital	Small	
A	Alpha	A	α	Angles and coefficients.
B	Beta	B	β	Angles and coefficients.
G	Gamma	Γ	γ	Angles, coefficients, and specific gravity.
D	Delta	Δ	δ	Angles, coefficients, deflection, variation and density.
Ĕ	Epsilon	E	ϵ	Eccentricity; base of natural logarithms = 2.7182818; strain.
Z	Zeta	Z	ζ	Coefficients, co-ordinates.
Ē	Eta	H	η	Coefficients.
Th	Theta	Θ	θ, ϑ	Angles and coefficients.
I	Iota	I	ι	
K	Kappa	K	κ	Coefficients.
L	Lambda	Λ	λ	Latitude, angles and coefficients.
M	Mu	M	μ	Angles and co-efficients.
N	Nu	N	ν	
X	Xi	Ξ	ξ	Coefficients, co-ordinates.
Ŏ	Omicron	O	o	
P	Pi	Π	π	Π signifies continued product, as $\Pi\, 3 = 1 \times 2 \times 3$. $\pi = 3.14159+$ = ratio of circum to dia. = 180° of arc.
R	Rho	P	ρ	Rad of gyration; rad of curvature; ratio.
S	Sigma	Σ	σ, s	Σ signifies summation; as ΣW means the summation of all the weights W', W'', W''', etc., in any system. In the Calculus it is replaced by the symbol $\int$. σ is used as a coefficient; stress.
T	Tau	T	τ	Coefficients; temperature, time.
U, Y	Upsilon	Υ	υ	
Ph	Phi	Φ	ϕ	Angles and coefficients.
Ch	Chi	X	χ	
Ps	Psi	Ψ	ψ	
Ō	Omega	Ω	ω	Coefficients; angular velocities.

MATHEMATICAL SYMBOLS.

$a, b, c, ---$ Represent known or constant quantities.

$--- x, y, z,$ Represent unknown or variable quantities.

$=$ Equals, is equal to.

$+$ Plus, as $3+2=5$; positive, as $+\frac{1}{2}=+.5$; extension, as $\frac{1}{7}=.14285+$.

$-$ Minus, as $5-3=2$; negative, as $-\frac{1}{2}=-.5$; contraction, as $\frac{1}{6}=.17-$.

$\pm$ Plus or minus, as $\sqrt{4}=\pm 2$.

$\mp$ Minus or plus.

$\times$ Times, multiplied by; as $3\times 2=6$; $a\times b=a.b=ab$.

$a.b$ $=a\times b=ab$.

$\div$ Divided by; as $8\div 2=4$; $8:2=4$; $8/2=4$; $\frac{8}{2}=4$.

$\frac{a}{b}$ Divided by.

a/b Divided by.

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: :: : Proportion; as 2:3::4:6; means $\frac{2}{3}=\frac{4}{6}$; reads "as 2 is to 3 so is 4 to 6."

4.3 $=4\frac{3}{10}=4\frac{30}{100}$, etc.

> Is greater than, as $4>3$; reads "4 is greater than 3."

< Is less than, as $3<4$; reads "3 is less than 4."

⩾ Is equal to or greater than; as, $a \geqslant 4$.

⩽ Is equal to or less than; as, $4 \leqslant a$.

∴ Therefore, hence.

∵ Because.

∞ Infinity; as, $\frac{a}{0}=\infty$.

∝ Is proportional to, varies directly with; also, angle alpha.

| Bar
— Vinculum
() Parenthesis
[ ] Bracket
{ } Brace

Abbreviation; denote that the quantities covered or enclosed must be "taken together."

$\overline{a+b}\times c=c\,(a+b)=ac+ab.$

$2\overline{[a(b+c)+x]}=2a(b+c)+2x.$

$a\left\{\begin{array}{l}2\overline{[a\,(b+c)+x]}+d\\ =2a\,[a(b+c)+x]+ad\\ =2a^2(b+c)+2ax+ad\\ =2a^2\,b+2a^2c+2ax+ad.\end{array}\right.$

√ Radical sign, square root. Thus $\sqrt{a}=\sqrt[2]{a}=a^{\frac{1}{2}}=\frac{1}{a^{-\frac{1}{2}}}$.

∛ Cube root.

$\sqrt[n]{\ }$ n^{th} root.

a^2 The square or second power of a, as $a\times a$, or aa.

x^n The nth power of x. $x^n=\frac{1}{x^{-n}}$; $x^1=x$; $x^0=1$.

a——d Continuation. Thus, a, b, c, d. (Either dots or dashes may be used.)

a', h'', x''' Primes, to distinguish letters; as $a-$prime, $h-2-$prime or $h-$second, $x-3-$prime or $x-$third.

x_1, a_2, h_4 Subs, to distinguish letters; x sub 1, a sub 2, h sub 4.

˘ Inverted caret indicates repeating decimal; as, $0.016\breve{6}=0.0166666$——.

♯— Number, as ♯2 = number 2.

—♯ Pound, or pounds, as 4♯ = 4 pounds or 4 lbs. in weight.

6′ – 3″ Feet and inches (linear measure); as 6 feet 3 inches, or 6 ft. 3 ins.

0°–18′–15″ Degrees, minutes and seconds (of arc); as 0 deg., 18 min., 15 sec. or 18m. 15s.

○ Round, diameter. 6′° = 6 ft. diameter = 6′ dia. 3″° = 3 ins. diameter = 3″ dia.

□ Square. 2′□ = 2 ft. sq.; 3″□ = 3 ins. sq. 4□′ = 4 sq. ft.; 9□″ = 9 sq. ins.

⊠ Cube. 2′⊠ = 2 ft. cubed; 4″⊠ = 4 ins. cubed. 27⊠′ = 27 cubic ft.

∡ ∠ Angle; ∡ angles.

∟ Right angle.

⊥ Perpendicular to.

|| Parallel with.

◉ Circle.

△ Triangle.

◺ Right angle triangle.

□ Square.

▭ Rectangle.

▱ Parallelogram.

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π Pi $= 3.1416 - = \frac{\text{circumference}}{\text{diameter}}$ of circle; or, $= 180°$ of arc.

$\frac{180°}{\pi} = 57.2958°$ nearly.

e = Base of Naperian, hyperbolic, or natural logarithms = 2.7182818+.

$\sin a$ Sine of the angle a.

$\sin^{-1} a$ Inverse on anti-sine of a; the angle whose sine is a. It is *not* $= \frac{1}{\sin a}$; see below.

$\sin a^{-1} = (\sin a)^{-1} = \frac{1}{\sin a}$.

d Differential (in Calculus); $d(x^2) = 2xdx$.

Σ Sigma, summation.

Δ Delta, difference; usually considered as a very small quantity.

$\int$ Integral (in Calculus): the reverse of differentiation; $\int 2xdx = x^2$

$\int_k^h$ Intergral between limits h and k; $\int_{x=k}^{x=h} 2xdx = h^2 - k^2$.

MECHANICAL SYMBOLS.

L or l = length.
M or m = mass.
T or t = time.
v = velocity.
a = acceleration, $a = \frac{F}{M}$.
g = gravity acceleration, $= \frac{W}{M} = 32.16$ ft. per sec. per sec.
W or w = work, or weight.
P = power.
F = force.
ft.-lbs. = foot-pounds.
I = moment of inertia.
E = modulus of elasticity.
D = diameter.
r = radius.
H. P. = horse-power.
B.H.P. = brake horse-power.
I. H. P. = indicated horse-power.
M. E. P. = mean effective pressure.
r. p m. = revolutions per minute.
C. G. S. = centimeter-gram-seconds (system).
A.W.G. = American wire gauge.
B.W.G. = Birmingham wire gauge.
B. T. U. = British thermal unit = pound-degree-Fahr.
cal. = calorie (French). = kilogram-degree-Cent.
lb.-cal. = pound-calorie, = pound-degree-Cent.

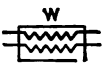
ELECTRICAL SYMBOLS.

E. E.M.F. = electro-motive force.
P. D. = potential difference.
C = current.
R = resistance.
ρ = specific resistance.
Q = quantity.
K = capacity.
L = inductance.
A. M. = ampere meter.
V. M. = volt meter.
F. M. = field magnet.
+ = positive pole.
− = negative pole.
V = volt, potential.
Amp. = ampere.
Ω = megohm.
B. A. U. = British Association units.
mfd = microfared.
h., hy. = Henry.
J = Joule.
K. W. = kilowatt.
⌒ = complete period (alt. current).
—○— = dynamo.
-■| ■|- = battery.

Galvanometer.

Ammeter.

Voltmeter.



Wattmeter.

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ELECTRICAL UNITS

Name.	Symbol.	Unit of	How Obtained.	CGS*	Equivalent.
Ohm.....	R	Resistance...	The electrical resistance of a column of mercury 106 centimeters long and of 1 square millimeter section.	10^9	1 true ohm = 1.0112 British Association ohms.
Ampere.....	C	Current.....	Is that current of electricity that decomposes .00009324 gramme of water per second.	10^1	Deposits 1.118 milligrams of silver per second.
Volt.....	E	Electromotive force..	One ampere of current passing through a substance having 1 ohm of resistance = 1 volt.	10^8	.926 of a standard Daniel Cell.
Coulomb.....	Q	Quantity....	A current of 1 ampere during 1 second of time.	10^1	Deposits 1.118 milligrams of silver.
Farad.....	K	Capacity....	The capacity that a current of 1 ampere for 1 second (= 1 coulomb) charges it to potential of 1 volt.	10^9	2.5 knots of D. U. S. cable.
Microfarad...	"	.....	1 millionth of farad.	10^{15}	
Watt.....	Pw.	Power.....	Power of 1 ampere current passing through resistance of 1 ohm.	10^7	.0013405 (or $\frac{1}{746}$) of a horse power.
Joule.....	W. j.	Work.....	Is the work done by 1 watt of electrical power in 1 second.	10^7	.238 unit of heat (Therm.)

* C. G. S. = Electro--magnetic units.

MAGNETIC AND ELECTROMAGNETIC UNITS.

	Magnetic Units.		Dimensions.
m	Strength of pole / Quantity of magnetism	$= \sqrt{\text{force} \times (\text{distance})^2} =$	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}$
V	Magnetic potential	= work ÷ strength of pole =	$M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1}$
H	Intensity of field	= force ÷ strength of pole =	$M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1}$
N	Magnetic flux	= intensity × area =	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}$
Z	Reluctance	= flux ÷ mag. potential =	L

	Electromagnetic Units.		Dimensions.
C	Current (strength)	= {intensity of field × length} =	$M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1}$
Q	Quantity	= current × time =	$M^{\frac{1}{2}} L^{\frac{1}{2}}$
V / E	Potential / Electromotive-force	= work ÷ quantity =	$M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-2}$
R	Resistance	= E.M.F. ÷ current =	LT^{-1}
K	Capacity	= quantity ÷ potential =	$L^{-1} T^{2}$
W	Power	= current × potential =	$ML^{2} T^{-3}$
L / M	Self-induction / Mutual induction	= {E. M. F. ÷ current per sec} =	L

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MAGNETIC SYMBOLS.

N = north pole.
S = south pole.
m = strength of pole.
H = magnetic force (C. G. S.).
B = magnetic inductance (C. G. S.).
I = intensity of magnetization.
μ = magnetic permeability.
k = magnetic susceptibility.
H = horizontal intensity of earth's magnetism.
Z = reluctance.

MEDICAL SIGNS AND ABBREVIATIONS.

℞ (Lat. Recipe), take; āā, of each; ℔, pound; ℥, ounce; ʒ, drachm; ℈, scruple; ♏, minim, or drop; O or o, pint; f ℥, fluid ounce; f ʒ, fluid drachm; as, ℥ ss, half an ounce; ℥i, one ounce; ℥iss, one ounce and a half; ℥ij, two ounces; gr., grain; Q. S., as much as sufficient; Ft. Mist., let a mixture be made; Ft. Haust., let a draught be made; Ad., add to; Ad. lib., at pleasure; Aq., water; M., mix; Mac., macerate; Pulv., powder; Pil., pill; Solv., dissolve; St., let it stand; Sum., to be taken; D., dose; Dil., dilute; Filt., filter; Lot., a wash; Garg., a gargle; Hor., Decub., at bed time: Inject., injection; Gtt., drops; ss, one-half; Ess., essence.

SURVEYING SYMBOLS.

(U. S. Public Lands Surveys.)

The following contractions are authorized to be used in the preparation of field notes, transcripts, inspection reports and similar records, and no others should be introduced:

A.	for acres.	mag.	for magnetic
a. m.	" forenoon.	M. C.	" meander corder.
A. M. C.	" aux. meander corner.	mer.	" meridian.
asc.	" ascend.	mkd.	" marked.
astron.	" astronomical.	N.	" north.
bdy.	" boundary.	NE.	" northeast.
bdrs.	" boundaries.	NW.	" northwest.
bet.	" between.	obs.	" observe.
B. O.	" bearing object.	obsn.	" observation.
B. T.	" bearing tree.	p. m.	" afternoon.
C. C.	" closing corner.	Pol.	" Polaris.
chs.	" chains.	Pr. Mer.	" principal meridian.
cor., cors.	" corner, corners.	Pt. of Tr.	" point of triangulation.
corr.	" correction.	¼ sec.	" quarter section.
decl.	" declination.	R., Rs.	" range, ranges.
dep.	" departure.	red.	" reduce, reduction.
desc.	" descend.	S.	" south.
dia.	" diameter.	S. C.	" standard corner.
diff.	" difference.	SE.	" southeast.
dist.	" distance.	sec., secs.	" section, sections.
D. S.	" deputy surveyor.	S. M. C.	" special meander corner.
E.	" east.	sq.	" square.
elong.	" elongation.	St. Par.	" standard parallel.
frac.	" fractional.	SW.	" southwest.
ft.	" foot, feet.	T., or Tp.	" township.
G. M.	" guide meridian.	Ts. or Tps.	" townships.
h., hrs.	" hour, hours.	temp.	" temporary.
ins.	" inches.	U. C.	" upper culmination.
lat.	" latitude.	var.	" variation.
L. C.	" lower culmination.	W.	" west.
lks.	" links.	W. C.	" witness corner.
l. m. t.	" local mean time.	w. corr.	" watch correction.
long.	" longitude.	W. P.	" witness point.
m.	" minutes.	w. t.	" watch time.

For ordinary surveying abbrevations see general text on Surveying. Section 58.

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1.—ELEMENTARY ARITHMETIC.

NUMBERS.

Roman System.—This system is used in the arts, for dates, for numbering chapters in literature, and for distinctive numbering where the Arabic numerals will not suffice. It employs seven letters, corresponding with the Arabic numbers, as follows:

Roman....	I	V	X	L	C	D	M
Arabic....	1	5	10	50	100	500	1000

Higher basic denominations are sometimes represented by the letters C, D and M, *inverted*, but they are not in general use. The following table will be found useful in expressing any number by a combination of these letters.

1.—ROMAN NUMERALS.

Roman				Arabic Units.
Thousands.	Hundreds.	Tens.	Units.	
M	C	X	I	1
	CC	XX	II	2
	CCC	XXX	III	3
	CD	XL	IV	4
	D	L	V	5
	DC	LX	VI	6
	DCC	LXX	VII	7
	DCCC	LXXX	VIII	8
	DCCCC	XC	IX	9

To write any number in Roman system: Begin with the highest denomination of the number—tens, hundreds, thousands, etc.—and pick out the letter or letters in the Roman column, corresponding with that denomination and opposite the proper figure in the Arabic column; then proceed in the same manner with each figure of lower denomination. Thus, 3 = III; 34 = XXX IV = XXXIV; 648 = DC XL VIII = DCXLVIII; 1799 = M DCC XC IX = MDCCXCIX; 1900 = MDCCCC (preferred) = MCM.

Arabic System.

Trillions.	Hundred billions.	Ten billions.	*Billions.*	Hundred millions.	Ten millions.	*Millions.*	Hundred thousands.	Ten thousands.	*Thousands.*	Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thousandth.	Ten thousandths.	Hundred thousandth.	Millionths.
0,	0	0	0 ,	9	8	7 ,	6	5	4 ,	3	2	1 .	1	2	3 ,	4	5	6 , 0 0 0

This number is read thus: "Nine hundred eighty-seven million, six hundred fifty-four thousand, three hundred twenty-one . . . *and* one hundred twenty-three thousand, four hundred fifty-six, millionths." By moving the decimal point to the *right* or *left*, the number is respectively *multiplied* or *divided* by *ten*, times the number of *places* moved.

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Primes, Multiples and Factors.—A prime number differs from a multiple in that it cannot be factored; that is, it is not *exactly* divisible by any *other* number, except 1. There is no known positive rule for detecting *all* prime numbers; tentative rules have been framed from time to time only to fail high up in the scale. But negative rules, universal in their application, may be stated as follows:

(*a.*) No even number (2 excepted) is a prime, because divisible by 2.

(*b*). No number (3 excepted), the sum of whose digits is divisible by 3, is a prime, because itself divisible by 3. Example: 171 $(1+7+1=9)$ is not a prime number, because 9 is divisible by 3.

(*c.*) No number (5 excepted), ending in 5 or 0, is a prime, because divisible by 5. Examples: 15, 30, 125, are divisible by 5.

(*d.*) No number composed of prime factors can have more than *one set* of prime factors. Example: $1001=7\times11\times13$; and 7, 11 and 13 is the *only set* of prime factors.

From the above we rightly conclude that all numbers, excepting 2 and 5, which end in 2, 4, 5, 6, 8 and 0, are multiple or composite; that a multiple may end in any figure; and that the ending of prime numbers is limited strictly to the digits 1, 3, 7 and 9. Furthermore, we may examine any number ending in 1, 3, 7 or 9 to see if the sum of the digits is a multiple of 3; if so, the number is composite and divisible by 3; but if not, it *may be either prime or composite.*

Table 2, following, contains a list of numbers up to 9600, which, by the preceding rules and analysis, cannot be detected as prime or composite. The numbers are composed of *hundreds* at the left of the *lines*, and *tens* and *units* at the top of the *columns*. At the intersection of the respective line and column of a number will be found the *smallest prime factor* (above unity) of that number. If the number is prime, the intersection will be represented by two dots.

The elimination, by Rule (*b*), of numbers ending in 1, 3, 7 and 9 which may be factored by 3, makes it convenient to separate the table into three parts by steps of 300, in order to condense it. The arrangement is as follows:

$3n+0$. This part (1) contains hundreds beginning with *0*, as 000, 300, 600, 900, etc. Example: $2400=(3\times8+0)$ hundreds.

$3n+1$. This part (2) contains hundreds beginning with *100*, as 100, 400, 700, 1000, etc. Example: $5200=(3\times17+1)$ hundreds.

$3n+2$. This part (3) contains hundreds beginning with *200*, as 200, 500, 800, 1100, etc. Example: $3500=(3\times11+2)$ hundreds.

Example.—What kind of a number is 27,489?

Solution.—The sum of its digits is 30, hence 3 is a factor. 3) 27489

From part 2 $(91=3n+1)$ of table, the smallest factor of 9163 is 7. 7) 9163

From part 2 $(13=3n+1)$ of table, the smallest factor of 1309 is 7. 7) 1309

From part 2 $(1=3n+1)$ of table, the smallest factor of 187 is 11. 11) 187

From part 1 $(0=3n+0)$ of table, 17 is found to be a prime. 17

Answer.—27489 is found to be a multiple or composite number whose prime factors are 3, 7, 7, 11 and 17; and from Rule (*d*) we learn that these are its *only* prime factors.

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2.—Primes,* Multiples and Factors.

Part 1.—(*3n+0*) *Hundreds.*

(3n+0) Hundreds.	Tens and Units.																							
N	01	07	11	13	17	19	23	29	31	37	41	43	47	49	53	59	61	67	71	73	77	79	83	89	91	97
000	..	..	..	..	..	..	..	..	..	..	..	..	..	7	..	..	..	..	..	..	7	..	..	..	7	..
300	7	..	..	..	..	11	17	7	..	..	11	7	..	..	..	..	19	..	7	..	13	..	..	..	17	..
600	..	..	13	..	..	..	7	17	..	7	..	..	..	11	..	..	..	23	11	..	..	7	..	13	..	17
900	17	..	..	11	7	..	13	..	7	..	..	23	..	13	..	7	31	..	..	7	..	11	..	23	..	..
1200	..	17	7	..	..	23	..	..	..	..	17	11	29	..	7	..	13	7	31	19	..	..	..	..	..	..
1500	19	11	..	17	37	7	..	11	..	29	23	..	7	..	..	..	7	..	..	11	19	..	..	7	37	..
1800	..	13	..	7	23	17	..	31	..	11	7	19	..	43	17	11	..	..	..	..	..	..	7	..	31	7
2100	11	7	..	..	29	13	11	..	..	..	..	..	19	7	..	17	..	11	13	41	7	..	37	11	7	13
2400	7	29	..	19	..	41	..	7	11	..	..	7	..	31	11	..	23	..	7	..	..	37	13	19	47	11
2700	37	..	..	..	11	..	7	..	..	7	..	13	41	..	..	31	11	..	17	47	..	7	11	..	..	..
3000	..	31	..	23	7		..	13	7	..	..	17	11	..	43	7	..	..	37	7	17	..	..	..	11	19
3300	..	..	7	..	31	..	.	..	..	47	13	..	..	17	7	..	..	7	..	..	11	31	17	..	..	43
3600	13	..	23	..	..	7	..	19	..	..	11	..	7	41	13	..	7	19	..	..	..	13	29	7	..	..
3900	47	..	..	7	..	..	..	..	..	31	7	..	..	11	59	37	17	..	11	29	41	23	7	..	13	7
4200	..	7	..	11	..	..	41	..	..	19	..	..	31	7	..	..	..	17	..	..	7	11	..	..	7	..
4500	7	..	13	..	..	..	..	7	23	13	19	7	..	..	29	47	..	..	7	17	23	19	..	13	..	..
4800	..	11	17	..	..	61	7	11	..	7	47	29	37	13	23	43	..	31	..	11	..	7	19	..	67	59
5100	.	..	19	..	7	..	47	23	7	11	53	37	..	19	..	7	13	..	..	7	31	..	71	..	29	..
5400	11	..	7	..	..	..	11	61	..	..	..	..	13	..	7	53	43	7	..	13	..	..	..	11	17	23
5700	..	13	..	29	..	7	59	17	11	..	..	..	7	..	11	13	7	73	29	23	53	..	..	7	.	11
6000	17	.	..	7	11	13	19	..	37	..	7	..	..	23	..	73	11	..	13	..	59	..	7	..	..	7
6300	..	7	..	59	..	71	..	..	13	..	17	..	11	7	..	..	..	..	23	..	7	..	13	..	7	..
6600	7	..	11	17	13	..	37	7	19	..	29	7	17	61	..	..	..	59	7	..	11	..	41	..	..	37
6900	67	..	.	31	..	11	7	13	29	7	11	53	..	..	17	..	..	..	..	19	..	7	..	29	..	..
7200	19	..	..	..	7	..	31	..	7	..	13	..	..	11	..	7	53	13	11	7	19	29	..	37	23	..
7500	13	..	7	11	..	73	..	..	17	..	..	19	..	..	7	..	..	7	67	..	..	11	..	..	..	71
7800	29	37	73	13	..	7	..	..	41	17	..	11	7	47	..	29	7	..	17	..	..	..	..	7	13	53
8100	..	11	..	7	..	23	..	11	47	79	7	17	..	29	31	41	..	..	..	11	13	..	7	19	..	7
8400	31	7	13	47	19	..	..	..	..	11	23	..	..	7	79	11	..	..	43	37	7	61	17	13	7	29
8700	7	..	31	..	23	..	11	7	..	..	..	7	..	13	..	19	..	11	7	31	67	..	..	11	59	19
9000	..	..	..	..	..	71	29	7	..	11	7	..	..	83	..	11	..	13	..	47	43	29	7	31	61	..
9300	71	41	..	67	7	..	..	19	7	..	..	..	13	..	47	7	11	17	..	7	..	83	11	41	..	..

*The *smallest* prime factors of multiple numbers are given. Prime numbers are indicated by dots.

Example: To find the prime factors of 2413: The smallest prime factor, from above table, is 19; then, 2413÷19=127. Now, from Part 2, on the following page, 127 is found to be a prime number. Hence, the prime factors of 2413 are 19 and 127.

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2.—Primes,* Multiples and Factors.—Continued.

Part 2.—($3n+1$) *Hundreds.*

($3n+1$) Hundreds.	Tens and Units.																													
N	01	03	07	09	13	19	21	27	31	33	37	39	43	49	51	57	61	63	67	69	73	79	81	87	91	93	97	99
100	..	..	..	..	..	7	11	..	..	7	..	..	11	.	..	..	7	..	..	13	..	..	..	11	..	..	..	..
400	..	13	11	..	7	..	..	7	..	..	19	..	..	..	11	..	..	..	..	7	11	..	13	..	..	17	7	..
700	..	19	7	..	23	..	7	..	17	..	11	..	..	7	..	..	..	7	13	..	..	19	11	..	7	13	..	17
1000	7	17	19	..	..	..	..	13	..	..	17	..	7	..	..	7	..	..	11	..	29	13	23	..	..	..	..	7
1300	..	..	..	7	13	..	..	..	11	31	7	13	17	19	7	23	..	29	..	37	..	7	..	19	13	7	11	..
1600	..	7	..	..	..	..	..	..	7	23	..	11	31	17	13	..	11	..	..	..	7	23	41	7	19	..	..	..
1900	..	11	..	23	..	19	17	41	..	..	13	7	29	..	..	19	37	13	7	11	..	..	7	..	11	..	..	..
2200	31	..	..	47	..	7	..	17	23	7	..	..	..	13	..	37	7	31	..	..	..	43	..	..	29	..	..	11
2500	41	..	23	13	13	11	..	7	..	17	43	..	..	..	..	..	13	11	17	7	31	..	29	13	..	..	7	23
2800	..	..	7	53	29	..	7	11	19	..	..	17	..	7	..	..	..	7	47	19	13	..	43	..	7	11	..	13
3100	7	29	13	..	11	..	..	53	31	13	..	43	7	47	23	7	29	..	..	..	19	11	..	..	..	31	23	7
3400	19	41	..	7	..	13	11	23	47	..	7	19	11	..	7	..	..	..	..	..	23	7	59	11	..	7	13	..
3700	..	7	11	..	47	..	61	..	7	..	37	..	19	23	11	13	..	53	..	..	7	..	19	7	17	..	..	29
4000	..	..	..	19	..	..	..	..	29	37	11	7	13	..	..	..	31	17	7	13	..	..	7	61	..	..	17	..
4300	11	13	59	31	19	7	29	..	61	7	..	..	43	..	19	..	7	..	11	17	..	29	13	41	..	23	..	53
4600	43	..	17	11	7	31	..	7	11	41	..	..	..	..	..	..	59	..	13	7	..	..	31	43	..	13	7	37
4900	13	..	7	..	17	..	7	13	..	..	..	11	..	7	..	..	11	7	..	..	..	13	17	..	7	..	19	..
5200	7	11	41	..	13	17	23	..	..	..	..	13	7	29	59	7	..	19	23	11	..	..	..	17	11	67	..	7
5500	..	..	..	7	37	..	..	..	..	11	7	29	23	31	7	..	67	..	19	..	..	7	..	37	..	7	29	11
5800	..	7	..	37	..	11	..	..	7	19	13	..	..	..	..	..	..	11	..	..	7	..	..	7	43	71	..	17
6100	..	17	31	41	..	29	..	11	..	..	17	7	..	11	..	47	61	..	7	31	..	37	7	23	41	11	..	..
6400	37	19	43	13	11	7	..	..	59	7	41	47	17	..	..	11	7	23	29	..	..	11	..	13	..	43	73	67
6700	..	..	19	..	7	..	11	7	53	..	..	23	11	17	43	29	..	..	67	7	13	..	..	11	..	..	7	13
7000	..	47	7	43	..	..	7	..	79	13	31	..	..	7	11	..	23	7	37	..	11	..	73	19	7	41	47	31
7300	7	67	..	..	71	13	..	17	..	..	11	41	7	..	..	7	17	37	53	..	73	47	11	83	19	..	13	7
7600	11	..	..	7	23	19	..	29	13	17	7	..	..	..	7	13	47	79	11	..	..	7	..	..	..	7	43	..
7900	..	7	..	11	41	..	89	..	7	..	..	17	13	..	..	73	19	..	31	13	7	79	23	7	61	..	11	19
8200	59	13	29	..	43	..	..	19	..	..	..	7	..	73	37	23	11	..	7	..	..	17	7	..	..	..	..	43
8500	..	11	47	67	..	7	..	..	19	7	..	..	..	83	17	43	7	.	13	11	..	23	..	31	11	13	..	..
8800	13	..	..	23	7	..	..	7	..	11	..	..	37	..	53	17	..	..	..	7	19	13	83	..	17	..	7	11
9100	19	..	7	..	13	11	7	..	23	..	..	13	41	7	..	..	..	7	89	53	..	67	..	..	7	29	17	..
9400	7	..	23	97	..	..	..	11	..	..	..	..	7	11	13	7	..	..	..	17	..	..	19	53	..	11	..	7

*The *smallest* prime factors of multiple numbers are given. Prime numbers are indicated by dots.

Example: To find the prime factors of 1001: The smallest prime factor, from above table, is 7; then, $1001 \div 7 = 143$, the smallest prime factor of which is 11; then, $143 \div 11 = 13$, a prime number. Hence, the prime factors of 1001 are 7, 11 and 13.

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2.—Primes,* Multiples and Factors.—Concluded.

Part 3.—$(3n+2)$ *Hundreds.*

$(3n+2)$ Hundreds.	Tens and Units.																							
N	03	09	11	17	21	23	27	29	33	39	41	47	51	53	57	59	63	69	71	77	81	83	87	89	93	99
200	7	11	..	7	13	..	..	..	.	..	..	13	..	11	..	7	..	..	..	..	..	..	7	17	..	13
500	..	..	7	11	..	..	17	23	13	7	..	..	19	7	..	13	..	..	..	..	7	11	..	19	..	..
800	11	..	..	19	..	..	..	..	7	..	29	7	23	..	..	..	..	11	13	..	..	..	..	7	19	29
1100	..	..	11	..	19	..	7	..	11	17	7	31	..	..	13	19	..	7	..	11	..	7	..	29	..	11
1400	23	..	17	13	7	..	..	..	..	..	14	..	..	..	31	..	7	13	..	7	..	..	..	..	..	..
1700	13	..	29	17	..	..	11	7	..	37	..	..	17	..	7	..	41	29	7	..	13	..	..	..	11	7
2000	..	7	..	..	43	7	..	..	19	..	13	23	7	..	11	29	..	..	19	31	..	..	..	..	7	..
2300	7	..	..	7	11	23	13	17	..	..	..	..	..	13	..	7	17	23	..	..	..	..	7	..	..	..
2600	19	..	7	..	..	43	37	11	..	7	19	..	11	7	..	..	..	17	..	..	7	..	..	..	..	..
2900	..	..	41	..	23	37	..	29	7	..	17	7	13	..	..	11	..	..	..	13	11	19	29	7	41	..
3200	..	..	13	..	..	11	7	..	53	41	7	17	..	..	..	..	13	7	..	29	17	7	19	11	37	..
3500	31	11	..	..	7	13	..	..	..	..	..	..	53	11	..	..	7	43	..	7	..	..	17	37	..	59
3800	..	13	37	11	..	..	43	7	..	11	23	..	..	..	7	17	..	53	7	..	..	11	13	..	17	7
4100	11	7	..	23	13	7	..	..	..	..	41	11	7	..	..	..	23	11	43	..	37	47	53	59	7	13
4400	7	..	11	7	..	..	19	43	11	23	..	..	..	61	..	7	..	41	17	41	..	..	7	67	..	11
4700	..	17	7	53	..	..	29	..	..	7	11	47	..	7	67	..	11	19	13	17	7	..	..	..	..	..
5000	..	..	..	29	..	..	11	47	7	..	71	7	..	31	13	..	61	37	11	..	..	13	..	7	11	..
5300	..	..	47	13	17	..	7	73	..	19	7	..	..	53	11	23	31	7	41	19	..	7	..	17	..	..
5600	13	71	31	41	7	..	17	13	43	..	..	..	..	..	..	..	7	..	53	7	13	..	11	..	..	41
5900	..	19	23	61	31	..	..	7	17	..	13	19	11	..	7	59	67	47	7	43	..	31	..	53	13	7
6200	..	7	..	..	..	7	13	..	23	17	79	..	7	13	..	11	..	..	..	..	11	61	..	19	7	..
6500	7	23	17	7	..	11	61	..	47	13	31	..	..	..	79	7	..	..	..	..	..	29	7	11	19	..
6800	..	11	7	17	19	..	.	..	..	7	..	41	13	7	..	19	..	..	..	13	7	..	71	83	61	..
7100	..	..	13	11	..	17	..	..	7	11	37	7	..	23	17	..	13	67	71	..	43	11	..	7	..	23
7400	11	31	..	..	41	13	7	17	..	43	7	11	..	29	..	..	17	7	31	..	..	7	..	..	59	..
7700	..	13	11	..	7	..	..	59	11	71	..	61	23	..	..	..	7	17	19	7	31	43	13	..	..	11
8000	53	..	..	..	13	71	23	7	29	..	11	13	83	..	7	..	11	..	7	41	..	59	..	..	..	7
8300	19	7	..	..	53	7	11	..	13	31	19	17	7	..	61	13	..	..	11	..	17	83	..	..	7	37
8600	7	..	79	7	37	..	..	..	89	53	..	..	41	17	11	7	..	..	13	..	..	19	7	..	..	..
8900	29	59	7	37	11	..	79	..	.	7	..	23	..	7	13	17	..	..	..	47	7	13	11	89	17	..
9200	..	..	61	13	..	23	..	11	7	..	..	7	11	19	..	47	59	13	73	..	..	..	37	7	..	17
9500	13	37	..	31	..	89	7	13	..	..	7	..	..	41	19	11	73	7	17	61	11	7	..	43	53	29

*The *smallest* prime factors of multiple numbers are given. Prime numbers are indicated by dots.

Example: To find the prime factors of 3211: The smallest prime factor, from above table, is 13; then, 3211÷13=247, the smallest prime factor of which is 13; then, 247÷13=19, a prime number. Hence, the prime factors of 3211 are 13, 13 and 19.

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Greatest Common Factor.—The *G. C. F.* of two or more numbers is the greatest factor common to all of them. It will be:

1st. A factor of all the numbers, therefore no greater than the least number;

2nd. A factor of all the differences and successive differences except 0, therefore no greater than the least difference;

And can be obtained (1) by differences, (2) by factoring, and (3) by division.

Example: Find the *G. C. F.* of 84, 126, 210 and 231.

1st Method.—By differences.

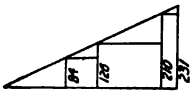

Fig. 1.

Numbers.	1st diff.
231	
	21
210	
	84
126	
	42
84	

Ans.—The least difference, 21. is obviously the *G. C. F.* as it is the greatest common factor of the least number and of all the differences.

2nd Method.—By factoring.

(*a.*)—Finding all the prime factors.

2)84	2)126	2)210	3)231
2)42	3)63	3)105	7)77
3)21	3)21	5)35	11
7	7	7	

3 and 7 are common to all—21.

(*b.*)—Finding all the common prime factors.

3)84	126	210	231
7)28	42	70	77
4	6	10	11

3 and 7 are common to all—21.

(*c.*)—Finding the common factors.

21)84	126	210	231
4	6	10	11

By inspection—21.

3rd Method.—By division (this is simply another form of *differences*).

84 and 126

```
84)126(1 .
   84
   42)84(2
      84
```

```
210
42)210(5
   210
```

```
231
42)231(5
   210
   21)42(2
      42
```

Rule.—Start with *any two* of the numbers. Divide the greater by the less and if there is a remainder use this remainder as a divisor of the previous divisor, and continue until there is no remainder. The last divisor will be the greatest common divisor or *G. C. F.* of the two numbers. Use this *G. C. F.* so obtained with the next number for a new *G. C. F.*, and so on until all the numbers are exhausted. The last divisor without a remainder will be the *G. C. F.* of all the numbers.

Least Common Multiple.—The *L. C. M.* of two or more numbers is the least number that can be divided exactly by each of them.

Rule for finding the L. C. M.—Divide the given numbers by the greatest (or any) factor common to most of them, and if divisible, set down the quotients in the line below; but if not divisible, bring down the numbers themselves. Divide the new line of numbers in a similar manner, and also each successive line, until no two numbers have a common factor except 1. The *L. C. M.* will be the product of all the factors and the last line of numbers. During the process, if any number is a factor of any other number in the same line it can be cancelled.

Example 1.

Find the *L. C. M.* of 7, 24, 168 and 264.

4	7	24	168	264
6	7	6	42	66
7	7		7	11
				11

Ans.—$4 \times 6 \times 7 \times 11 = 1848$.

Example 2.

Find the *L. C. M.* of 13, 28, 52 and 84.

4	13	28	52	84
	13	7	13	21

NOTE.—13 is a factor of 13, and 7 of 21

Ans.—$4 \times 13 \times 21 = 1092$.

Note that the first line in each example above could have been divided by 2, and then again by 2, instead of by 4; and that the second line in Example 1 could have been divided by 2 and then by 3, instead of by 6,

The *L. C. M.* of *two* numbers is obviously a special case of the above in which the product of the factors is the *G. C. F.*, and the last line of numbers are quotients obtained by dividing the respective numbers by their *G. C. F.* Thus, in Example 3, the *L. C. M.* of 126 and 462 is their *G. C. F.* (= 42) multiplied by the product of their respective quotients, $\frac{126}{42} = 3$, and $\frac{462}{42} = 11$.

Example 3.
Find the *L. C. M.* of 126 and 462.

6	126	462
7	21	77
	3	11

Ans.—$(6 \times 7) \times 3 \times 11 = 1386$.

FRACTIONS.

Kinds of Fractions.	*Equivalent values.*
Common Fractions. Simple: Proper.—$\frac{3}{8}$, $\frac{7}{8}$.	$\frac{3}{5} = \frac{3n}{5n} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10} = .6$. Both *terms* of a fraction may be multiplied *or* divided by the same number without affecting its value.
Simple: Improper.—$\frac{5}{3}$, $\frac{8}{5}$.	$\frac{5}{3} = \frac{3+2}{3} = \frac{3}{3} + \frac{2}{3} = 1\frac{2}{3}$.
Mixed.—$2\frac{3}{4}$, $5\frac{6}{7}$.	$2\frac{3}{4} = \frac{8}{4} + \frac{3}{4} = \frac{11}{4}$.
Compound.—$\frac{3}{4}$ of 4, $\frac{2}{3}$ of $3\frac{1}{2}$.	$\frac{2}{3}$ of $3\frac{1}{2} = \frac{2}{3} \times 3\frac{1}{2} = \frac{2}{3} \times \frac{6+1}{2} = \frac{2}{3} \times \frac{7}{2} = \frac{7}{3} = 2\frac{1}{3}$.
Complex.—$\frac{\frac{1}{2}}{3}$, $\frac{\frac{3}{4}}{\frac{1}{4}}$, $\frac{2\frac{1}{2}}{3\frac{1}{2}}$, $\frac{5}{4\frac{1}{4}}$.	$\frac{2\frac{1}{2}}{3\frac{1}{2}} = 2\frac{1}{2} \div 3\frac{1}{2} = \frac{5}{2} \div \frac{7}{2} = \frac{5}{2} \times \frac{2}{7} = \frac{5}{7}$.
Decimals. Pure.—.25, .625.	$.25 = \frac{.25}{1} = \frac{2.5}{10} = \frac{25}{100} = \frac{100}{400} = \frac{10}{40} = \frac{1}{4}$.
Mixed.—1.25, 4.875	$4.875 = 4\frac{875}{1000} = 4\frac{7000}{8000} = 4\frac{7}{8}$.

To reduce to lowest terms.—Divide both terms of the fraction by their greatest common factor; or factor them as in finding their *G. C. F.* Thus,

$$\frac{12}{30} = \frac{12+6}{30+6} = \frac{2}{5}, \text{ or } \frac{12}{30} = \frac{6}{15} = \frac{2}{5}.$$

To reduce to a common denominator.—Reduce the fractions (generally) to their lowest terms and find the least common multiple of their denominators for a common denominator. Then expand both terms of each fraction proportionately so that their denominators will be equal to the common denominator. Thus, $\frac{3}{8}$, $\frac{7}{9}$, $\frac{5}{6}$ and $\frac{4}{12} = \frac{1}{3}$, $\frac{7}{9}$, $\frac{5}{6}$ and $\frac{1}{3}$. The *L. C. M.* of their denominators = 72. Hence, $\frac{1}{3} = \frac{1 \times 36}{72} = \frac{36}{72}$, $\frac{7}{9} = \frac{8 \times 7}{72} = \frac{56}{72}$, $\frac{5}{6} = \frac{9 \times 5}{72} = \frac{45}{72}$, and $\frac{1}{3} = \frac{24 \times 1}{72} = \frac{24}{72}$.

To Change—

(a). *An improper fraction to a mixed number.* $\frac{177}{32} = ?$ Ans. $5\frac{17}{32}$.

Rule.—Divide the numerator by the denominator. The quotient is the whole number, and the remainder the new numerator.

Process:
32)177(5
160
17

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(*b.*) *A mixed number to an improper fraction.* $14\frac{5}{9} = ?$ Ans. $\frac{131}{9}$.

Rule.—Multiply the integral part by the denominator, add the numerator, and place the sum over the denominator.

Process.

$$\begin{array}{r} 14 \\ 9 \\ \hline 126+5=131. \end{array}$$

(*c.*) *A whole number to an improper fraction.* $6 = \frac{?}{15}$ Ans. $\frac{90}{15}$.

Rule.—Multiply the whole number by the given denominator for the required numerator.

Process.

$6 \times 15 = 90.$

(*d.*) *A compound to a simple fraction.* $\frac{2}{3}$ of $\frac{5}{8} = ?$ Ans. $\frac{5}{12}$.

Rule.—Proceed as in multiplication. ("Of" means "times.") Place the product of the numerators over the product of the denominators, and reduce to lowest terms. Cancel, generally, before multiplying.

Process.

$$\frac{\cancel{2}}{3} \times \frac{5}{\underset{4}{\cancel{8}}} = \frac{5}{12}.$$

(*e.*) *A complex to a simple fraction.* $\frac{2\frac{1}{3}}{3\frac{1}{2}} = ?$ Ans. $\frac{2}{3}$.

Rule.—Reduce the numerator and denominator each to simple fractions; then multiply the numerator by the denominator inverted.

Process.

$$\frac{2\frac{1}{3}}{3\frac{1}{2}} = \frac{\frac{7}{3}}{\frac{7}{2}} = \frac{7}{3} \times \frac{2}{7} = \frac{2}{3}.$$

Cancellation.—In practical operations where the multiplication of various kinds of numbers, including fractions, is expressed, the process may often be shortened by cancellation. Thus, let it be required to find the product of $\frac{315}{400}$, $\frac{4}{25}$ and $\frac{5}{9}$. Clearly, we may place the product of the numerators over the product of the denominators for a simple fraction expressing the result, or we may go to the other extreme and cancel as long as factors will cancel each other. There is, however, a medium point where cancellation may sometimes cease for simplicity of operation.

Thus, $\frac{\overset{35}{\cancel{315}} \times 4 \times 5}{400 \times 25 \times \cancel{9}} = \frac{7\cancel{00}}{100\cancel{00}}$. (The denominator is a power of 10.)

Addition of Fractions.—Reduce to the smallest common denominator and add the new numerators. Thus, $\frac{1}{4} + \frac{7}{8} = \frac{2}{8} + \frac{7}{8} = \frac{9}{8} = 1\frac{1}{8}$; $1\frac{1}{8} + \frac{3}{4} = \frac{9}{8} + \frac{6}{8} = \frac{15}{8} = 1\frac{7}{8}$; $1\frac{7}{8} + 15\frac{2}{3} = \frac{15}{8} + \frac{47}{3} = \frac{45}{24} + \frac{376}{24} = \frac{421}{24} = 17\frac{13}{24}$; or, $1\frac{7}{8} + 15\frac{2}{3} = 16 + \frac{7}{8} + \frac{2}{3} = 16 + \frac{21}{24} + \frac{16}{24} = 16\frac{37}{24} = 17\frac{13}{24}$.

Subtraction of Fractions.—Reduce to the smallest common denominator and find the difference of the new numerators. Thus, $\frac{15}{16} - \frac{3}{8} = \frac{15}{16} - \frac{6}{16} = \frac{9}{16}$; $8\frac{1}{4} - 3\frac{7}{16} = 8\frac{4}{16} - 3\frac{7}{16} = 7\frac{20}{16} - 3\frac{7}{16} = 4\frac{13}{16}$.

Multiplication of Fractions.—If necessary, change to simple fractions; cancel where advisable, and place the product of the numerators over the product of the denominators; reduce to required form. Thus, $\frac{1}{3} \times \frac{3}{5} = \frac{3}{15}$;

$$2\frac{1}{4} \times \frac{3}{7} \times 1\frac{17}{18} = \frac{\cancel{9}}{4} \times \frac{\cancel{2}}{\cancel{7}} \times \frac{\overset{5}{\cancel{35}}}{\cancel{18}} = \frac{5}{4} = 1\frac{1}{4}.$$

Division of Fractions.—If necessary, change to simple fractions; invert the divisor and multiply. Thus, $32\frac{5}{8} \div 6\frac{3}{4} = \frac{261}{8} \div \frac{27}{4} = \frac{\overset{29}{\cancel{261}}}{\underset{2}{\cancel{8}}} \times \frac{\cancel{4}}{\underset{3}{\cancel{27}}} = \frac{29}{6} = 4\frac{5}{6}$.

To reduce a fraction to a reciprocal.—Divide the denominator by the numerator, writing the quotient in decimal form. Thus, the reciprocal of $\frac{5}{16} = \frac{16}{5} = \frac{5)16.0}{3.2} = 3.2$. Hence, instead of dividing a number by $\frac{5}{16}$ we may multiply it by 3.2.

To reduce a common fraction to a decimal.—Divide the numerator by the denominator, writing the quotient in decimal form. Thus,

$$\frac{3}{8} = \frac{8)3.000}{.375} = .375.$$

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3.—Fractions (7ths) Reduced to Decimals.

Note.—These decimals are all *repeating* decimals, that is, they contain figures or groups of figures that may be repeated or annexed indefinitely. Thus, $\frac{1}{7}$ = .142857˘142857˘142857˘1.....=.142857˘1, the inverted caret (˘) signifying that the figure following is the first of the repeating group. Examples: $\frac{1}{9}$ = .1˘1 = .1111˘1 = .11111111...; $\frac{1}{3}$ = .3˘3 = 33˘3 = .33333˘3, etc.

Fractions.	Equivalent Decimals.	Fractions.	Equivalent Decimals.	Fractions.	Equivalent Decimals.
$\frac{1}{7}$	.142857˘1	$\frac{3}{7}$	.428571˘4	$\frac{5}{7}$	.714285˘7
$\frac{2}{7}$	.285714˘2	$\frac{4}{7}$	.571428˘5	$\frac{6}{7}$	.857142˘8

4.—Fractions (9ths) Reduced to Decimals.

Fractions.	Equiv. Decimals.	Fractions.	Equiv. Decimals.	Fractions.	Equiv. Decimals.	Fractions.	Equiv. Decimals.
$\frac{1}{9}$	.1˘1	$\frac{3}{9}$	.3˘3	$\frac{5}{9}$	5˘5	$\frac{7}{9}$	.7˘7
$\frac{2}{9}$	.2˘2	$\frac{4}{9}$	.4˘4	$\frac{6}{9}$	.6˘6	$\frac{8}{9}$	.8˘8

Note.—$\frac{2}{9}$ = .222222...; $\frac{5}{9}$ = .5555....; etc.

5.—Fractions (11ths) Reduced to Decimals.

Fractions.	Equiv. Decimals.	Fractions.	Equiv. Decimals.	Fractions.	Equiv. Decimals.	Fractions.	Equiv. Decimals.	Fractions.	Equiv. Decimals.
$\frac{1}{11}$	.09˘0	$\frac{3}{11}$	.27˘2	$\frac{5}{11}$	.45˘4	$\frac{7}{11}$	.63˘6	$\frac{9}{11}$	.81˘8
$\frac{2}{11}$	.18˘1	$\frac{4}{11}$	.36˘3	$\frac{6}{11}$	.54˘5	$\frac{8}{11}$	.72˘7	$\frac{10}{11}$	.90˘9

Note.—$\frac{3}{11}$ = .272727....; $\frac{7}{11}$ = .636363....; etc.

6.—Fractions (12ths) Reduced to Decimals.

Fractions.	Equiv. Decimals.	Fractions.	Equiv. Decimals.	Fractions.	Equiv. Decimals.	Fractions.	Equiv. Decimals.
$\frac{1}{12}$	.083˘3	$\frac{4}{12}$	.333˘3	$\frac{7}{12}$	.583˘3	$\frac{10}{12}$	.833˘3
$\frac{2}{12}$	.166˘6	$\frac{5}{12}$	.416˘6	$\frac{8}{12}$	.666˘6	$\frac{11}{12}$	.916˘6
$\frac{3}{12}$	.25	$\frac{6}{12}$	.50	$\frac{9}{12}$	.75		

Note.—$\frac{1}{12}$ = .083333...; $\frac{5}{12}$ = .416666....; etc.

7.—Fractions (13ths) Reduced to Decimals.

Fractions.	Equivalent Decimals.	Fractions.	Equivalent Decimals.	Fractions.	Equivalent Decimals.
$\frac{1}{13}$	.076923˘0	$\frac{4}{13}$	.304615˘3	$\frac{9}{13}$	.692307˘6
$\frac{2}{13}$	.153846˘1	$\frac{5}{13}$	.461538˘4	$\frac{10}{13}$	.769230˘7
$\frac{3}{13}$	.230769˘2	$\frac{6}{13}$	.538461˘5	$\frac{11}{13}$	.846153˘8
$\frac{4}{13}$	.307692˘3	$\frac{7}{13}$	.615384˘6	$\frac{12}{13}$	.923076˘9

Note.—These are repeating decimals, as above.

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8.—Fractions (64ths) Reduced to Decimals.

64ths.	Fractions.	Decimals.	64ths.	Fractions	Decimals.	64ths.	Fractions.	Decimals.	64ths.	Fractions.	Decimals.
1		.015625	17		.265625	33		.515625	49		.765625
2	$\frac{1}{32}$	.03125	18	$\frac{9}{32}$	.28125	34	$\frac{17}{32}$	.53125	50	$\frac{25}{32}$	.78125
3		.046875	19		.296875	35		.546875	51		.796875
4	1-16	.0625	20	5-16	.3125	36	9-16	.5625	52	13-16	.8125
5		.078125	21		.328125	37		.578125	53		.828125
6	$\frac{3}{32}$	.09375	22	$\frac{11}{32}$	.34375	38	$\frac{19}{32}$	.59375	54	$\frac{27}{32}$	.84375
7		.109375	23		.359375	39		.609375	55		.859375
8	*1-8*	.125	24	*3-8*	.375	40	*5-8*	.625	56	*7-8*	.875
9		.140625	25		.390625	41		.640625	57		.890625
10	$\frac{5}{32}$	.15625	26	$\frac{13}{32}$	.40625	42	$\frac{21}{32}$	.65625	58	$\frac{29}{32}$	.90625
11		.171875	27		.421875	43		.671875	59		.921875
12	3-16	.1875	28	7-16	.4375	44	11-16	.6875	60	15-16	.9375
13		.203125	29		.453125	45		.703125	61		.953125
14	$\frac{7}{32}$	.21875	30	$\frac{15}{32}$	.46875	46	$\frac{23}{32}$	.71875	62	$\frac{31}{32}$	.96875
15		.234375	31		.484375	47		.734375	63		.984375
16	*1-4*	.25	32	*1-2*	.5	48	*3-4*	.75	64	*1*	1.

DECIMALS.

Addition of Decimals.—Have the decimal points "in column" before adding.

Example:
```
  2.0625
 315.25
   .0375
 317.3500
```

Subtraction of Decimals.—Have the decimal points "in column" before subtracting.

Example:
```
 15.125
  3.71825
 11.40675
```

Multiplication of Decimals.—Multiply as with whole numbers; the product to have as many decimal places as the factors combined. Thus, 2.3 × .05 × 7.22 = .83030 = .8303.

Example:
```
  6.75
   2.4
  2700
 1350
 16.200
```

Division of Decimals.—Multiply or divide the divisor and dividend by some power of 10 that will make the divisor a whole number of significant figures, marking the new decimal point in the dividend by a caret (^). The quotient will then contain as many decimal places as the new dividend.

Example: Divide 17.34 by 600?

```
600)^17.34
     .0289  Ans.
```

Example: Divide 17.34 by .006?

```
.006) 17.340^
       2890. Ans.
```

Example:
Divide 3573.2 by 2.375.

```
Ans.      1504.505+
          1504.505+
2.375)3573.200^
      2375
      11982
      11875
        10700
         9500
         12000
         11875
          12500
          11875
```

To reduce a decimal to a common fraction.

a.) *Exact decimals.*—For the numerator of the fraction, use the significant figures of the decimal, the denominator being 1 with as many ciphers annexed as there are decimal places in the decimal; reduce to lowest terms.

Thus, $.75 = \frac{75}{100} = \frac{3}{4}$; $.0072 = \frac{72}{10000} = \frac{9}{1250}$.

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(*b.*) ***Repeating decimals.***—Treat the ***non-repeating*** and the first ***repeating*** groups as a whole number; subtract from this the non-repeating group treated as a whole number; the difference will be the numerator of the fraction. The denominator will be composed of as many 9's as there are repeating figures in a group, followed by as many 0's as there are non-repeating figures. Reduce to lowest terms.

Example: Reduce $.3\overset{\vee}{3}$ to a fraction.

$$\begin{array}{r} -\ 0 \\ \hline \text{numer. } 3 \\ \text{denom. } 9 \end{array} = \tfrac{1}{3}. \text{ Ans.}$$

Example: Reduce $467\overset{\vee}{6}$ to a fraction.

$$\begin{array}{r} -\ 4 \\ \hline \text{num. } 463 \\ \text{demon. } 990 \end{array} \text{ Ans.}$$

ADDITION.

The following examples serve to illustrate approved methods:

Add:	Same by columns:	Add:	Reduced to eighths:	Reduced to decimals:
257.83	24			
986.97	33	$153\frac{7}{8}$	7	.875
456.73	29	$25\frac{3}{4}$	6	.75
523.82	25	$16\frac{1}{2}$	4	.5
357.19	23			
Ans. 2582 54	2582.54	Ans. $196\frac{1}{8}$	$\frac{17}{8}$	2.125

Carried 23 32

Fractions: $\frac{3}{4}+\frac{1}{2}=\frac{3}{4}+\frac{2}{4}=\frac{5}{4}=1\frac{1}{4}$. $31\frac{1}{4}+1\frac{1}{8}=32\frac{3}{8}$.

Decimals: $.75+.5 = 1.25$. $31.25+1.125=32.375$.

Mechanical adding machines are used by accountants.

SUBTRACTION.

Subtract:	Reduced to eighths:	Reduced to decimals:
$153\frac{3}{8}$	8+6	1+.75
$25\frac{7}{8}$	7	.875
Ans. $127\frac{7}{8}$	$\frac{7}{8}$	.875

$\frac{3}{4}-\frac{1}{2}=\frac{3}{4}-\frac{2}{4}=\frac{1}{4}$.

$.75-.5 = .25$.

Reduce the fraction to a common denominator, for addition or subtraction.

MULTIPLICATION.

Accurate:	Sufficiently Accurate:
12.64	12.64
6.74	6.74
5056	50 ..
8848	885 .
7584	7584
Ans. 85.1936	85.19

$\frac{1}{2}\times\frac{3}{4}=\frac{1\times 3}{2\times 4}=\frac{3}{8}$.

$3\frac{3}{4}\times 2\frac{1}{8}=\frac{15}{4}\times\frac{7}{3}=8\frac{3}{4}$.

$3.75\times 2\frac{1}{3}=7.5+1.25=8.75$.

$.2\times .3=.06$

$$\begin{array}{r} .111 \\ .3 \\ \hline .0333 \end{array}$$

A few **short methods** of multiplication will be found useful:

(1). *By reducing the multiplier* to an improper fraction with a simple numerator and denominator.* Where the numerator becomes 1, 10, 100, 1000, etc., it is known as the reciprocal method. In the following, let n represent the number to be multiplied:

$n\times 1\frac{1}{4}=\frac{10n}{8}$. $n\times 8\frac{1}{3}=\frac{100n}{12}$. $n\times 66\frac{2}{3}=\frac{200n}{3}$.

$\times 1\frac{2}{3}=\frac{10n}{6}$. $\times 12\frac{1}{2}=\frac{100n}{8}$. $\times 83\frac{1}{3}=\frac{1000n}{12}$.

$\times 2\frac{1}{2}=\frac{10n}{4}$. $\times 16\frac{2}{3}=\frac{100n}{6}$. $\times 166\frac{2}{3}=\frac{1000n}{6}$.

$\times 3\frac{1}{3}=\frac{10n}{3}$. $\times 25=\frac{100n}{4}$. $\times 333\frac{1}{3}=\frac{1000n}{3}$.

$\times 6\frac{1}{4}=\frac{100n}{4\times 4}$. $\times 33\frac{1}{3}=\frac{100n}{3}$.

$\times 6\frac{2}{3}=\frac{20n}{3}$ $\times 50=\frac{100n}{2}$.

See also Reciprocals, page 30.

* If the multiplier is $1\frac{1}{3}$, add $\frac{1}{3}$.
" " " $13\frac{1}{3}$, " $\frac{1}{3}$ and multiply by 10.
" " " $133\frac{1}{3}$, " $\frac{1}{3}$ " " " 100.

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(2). *When the sum of the unit figures equals* 10; *and balance of numbers are identical*:

11	12	15	24	48	96	124	243
19	18	15	26	42	94	126	247
209	216	225	624	2016	9024	15624	60021

Rule.—Multiply the unit figures together, occupying two places (see first example above); and prefix the product of the *balance* of either number by (itself + 1).

(*a*.) Special Case.—When the last figure is *5* the last two figures of the product will be *25*; thus, $25^2 = 625$, $95^2 = 9025$, $145^2 = 21025$, $245^2 = 60025$; all performed mentally.

(*b*.) Special Case.—When the last figure in each number is a decimal, and their sum is unity, the whole numbers being identical; thus,

$$1.1 \times 1.9 = 2.09, \quad 2.4 \times 2.6 = 6.24, \quad 24.3 \times 24.7 = 600.21$$

(*c*.) Special Case.—When the whole numbers are identical and the numbers contain fractions (instead of decimals, as above) whose sum is unity; thus,

$$1\tfrac{1}{10} \times 1\tfrac{9}{10} = 2\tfrac{19}{100}, \quad 1\tfrac{1}{2} \times 1\tfrac{1}{2} = 2\tfrac{1}{4}, \quad 12\tfrac{3}{4} \times 12\tfrac{3}{4} = 156\tfrac{1}{16}.$$

(3). *The product of any two numbers each ending with the fraction* $\frac{1}{2}$, *or with the decimal* .5:

$$9\tfrac{1}{2} \times 3\tfrac{1}{2} = 9 \times 3 + \frac{9+3}{2} + \tfrac{1}{4} = 33\tfrac{1}{4}.$$

$$7.5 \times 11.5 = 77 + 9 + .25 = 86.25.$$

Rule.—*The product of the whole numbers + half their sum + $\frac{1}{4}$.

(*a*.) Special Case.—When the numbers are whole numbers ending with 5:

$$95 \times 35 = 100\,(27 + 6 + .25) = 3325.$$
$$75 \times 115 = 100\,(77 + 9 + .25) = 8625.$$
$$185 \times 305 = 100\,(540 + 24 + .25) = 56425.$$

Rule.—Apply the general rule above, considering each number divided by 10, and multiply the result by 100.

(4.) †*The product of any two numbers is equal to the square of their mean, minus the square of half their difference*:

$$24 \times 26 = 25^2 - 1 = 624. \qquad 136 \times 144 = 140^2 - 16 = 19584.$$
$$87 \times 93 = 90^2 - 9 = 8091. \qquad 244 \times 256 = 250^2 - 36 = 62464.$$

(5.) †*The converse of* (4), of course, holds true and may be applied readily in finding the squares of numbers—considering them as *means*:

$$39^2 = 38 \times 40 + 1 = 1521. \qquad 68^2 = 66 \times 70 + 2^2 = 4624.$$

Note.—Use for a *base*, any multiple of 10 nearest the number to be squared.

(6.) ‡*The square of any number* is equal to the square of (itself ± 1), called the *base*, ∓ the sum of the base and the number:

$$39^2 = 40^2 - 79 = 1521. \qquad 71^2 = 70^2 + 141 = 5041.$$

(7.) *Miscellaneous Methods.*¶

a.) To multiply any number *n* by 11.

Note.—*Imagine* the number, and its product by 10, arranged thus: 879687
879687 for addition

$n =$ 879,687
Ans. 9,676,557

(*b*.) To multiply a number *n* by any number from 12 to 19, inclusive:

Example: Multilpy 7364 by 17.

Note.—When the *unit* figure of the multiplier is 1, the same principle of course holds true; as, for instance, 7364 × 71

$n =$ 73 64
$n \times 7 =$ + 51 548
Ans. 125,188.
7 364
+ 515 48
= 522,844.

(*c*.) To multiply a number *n* by any number from 92 to 99, inclusive:

Example: Multiply 7364 by 98.

$100n =$ 736 400
$-2n$ 14 728
Ans. 721,672

d.) When the multiplier contains simple factors, it may be quicker to use the factors:

Example: Multiply 987 by 64.

987
$n \times 8 =$ 7896
$n \times 8 \times 8 =$ 63168. Ans.

---

* From Algebra: $(a + c)(b + c) = ab + c(a + b) + c^2$.
† From Algebra: $(x + y)(x - y) = x^2 - y^2$.
‡ From Algebra: $(x \pm 1)^2 = x^2 \pm 2x + 1$.
¶ These illustrations may be expanded almost indefinitely.

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DIVISION.

Accurate:

```
                2.64
674.82 ) 1783.16^88
         1349 64
          433 528
          404 892
           28 6368
           26 9928
```

Sufficiently accurate:

```
                2.64
674.82 ) 1783.17^
         1349 64
          433 53
          404 88
           28 65
```

There are short special methods of division, but they are usually not broad enough in their application to record for general use.

(*a.*) Factor the divisor:

Example: Divide 972 by 16.

```
4 ) 972
4 ) 243
    60¾ Ans.
```

(*b.*) Multiply by the reciprocal:

Example: Divide 43.2 by 2.5

```
  43.2
    .4
17.28 Ans.
```

(*c.*) To divide by a fraction, invert it and multiply, but reducing it to a simple decimal or reciprocal if possible:

$\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times \frac{4}{3} = \frac{2}{3}$. $56 \div \frac{5}{4} = 56 \times .8 = 44.8$.

(*d.*) Cancellation: $\frac{\cancel{8} \times \cancel{9} \times 16}{\cancel{2} \times \underset{3}{\cancel{12}} \times \underset{2}{\cancel{18}}} = \frac{8}{3} = 2\frac{2}{3}$. Ans.

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2.—POWERS, ROOTS AND RECIPROCALS.

The processes of multiplication and division, and of finding the powers, roots and reciprocals of numbers may be performed by arithmetic (and algebra); by the use of tables; by logarithms;* or by the logarithmic slide rule.†

A. ENGINEERS' TABLES.

Under the present heading will be found Engineers' Tables of powers, roots and *reciprocals* of numbers arranged in form similar to logarithmic tables, so that the above properties of any number can be obtained by manipulating the decimal point, and by the use of the proportional parts (P. P.) columns, as in finding the *logarithms* of numbers. These tables may also be used inversely in finding the *squares*, *cubes*, and *inverse reciprocals*, corresponding respectively to the square roots, cube roots and reciprocals—the process being similar to finding the *numbers corresponding* to given logarithms. For a more academic arrangement see Arithmetical Tables 9, 10, and 11, following.

Square Root.—Tables 1 and 2, following, comprise a 4-page table of the square roots of numbers. The *first two pages* (Table 1) are for numbers with an *odd number of figures to the left* of the decimal point; or, if it is a pure decimal the first significant figure must be an *even number of places to the right* of the decimal point. The *last two pages* (Table 2) are for numbers just the *reverse—even to the left* and *odd to the right*. The range of the tables may be extended by remembering that "changing two decimal points in the square = one decimal point in the root—in the same direction." The tables are logarithmic in form, containing P. P. columns for extension or interpolation.

Example.—Required the square root of 907.6.

Solution.—Three figures at the left of the decimal point calls for the "odd" table (Table 1), from which the sq. rt. of 907 = 30.116, and the proportional part of the difference, 17, between it and the next higher number, 908, is found to be 10 (10.2) which, added to 30.116 = 30.126. Ans.

The following methods are given for extracting the square root when tables are not accessible, or when great accuracy is desired, for large numbers:

By Algebra.

Square root of $a^2+2ax+x^2$?

$$\begin{array}{ll} & a^2+2ax+x^2\ (a+x\ .\ \text{Ans.} \\ & a^2 \\ \hline 2a+x\,) & 2ax+x^2 \\ & 2ax+x^2 \end{array}$$

By Arithmetic.

Square root of 576 ?

$$\begin{array}{lll} 2a=40 & 576 & (20+4=24.\ \text{Ans.} \\ \text{Assume}\quad x=\ \ 4 & 400 & \\ \hline 2a+x=44\ ) & 176 & a=20, \\ & 176 & x=\ \ 4. \end{array}$$

Example.—Extract the square root of 46,354.87 ?

Remarks.—Beginning with the unit figure, point off two places each way, forming groups of two figures to bring down each time. Proceed as indicated.

Process:

$$\begin{array}{lll} & & \dot{4}6\dot{3}5\dot{4}.8\dot{7}(215.301+.\ \text{Ans.} \\ & & 4 \\ \left.\begin{array}{l} a_1=\ 20 \\ \text{Assume}\ x_1=\ \ 1 \end{array}\right\} & \therefore\ 2a_1+x_1=41) & 63 \qquad a_0=\ \ 2, \\ & & 41 \qquad a_1=\ 20,\ x_1=1 \\ \left.\begin{array}{l} a_2=210 \\ \text{Assume}\ x_2=\ \ 5 \end{array}\right\} & \therefore\ 2a_2+x_2=425) & 2254 \qquad a_2=210,\ x_2=5 \\ & & 2125 \\ \text{Or,}\quad 215\times 2=430; & 4303) & 12987 \\ & & 12909 \\ 2153\times 2=4306; & 430601) & 780000 \end{array}$$

* See table of logarithms, page 108.

† For description and use of the slide rule, see page 126.

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Example.—Extract the square root of 4,635.4 ?

Note that the position of the decimal point affects the pointing off and, consequently, the entire character of the result.

Process:

```
        . . .
        4635.40 (68.08+  Ans.
        36
  128 )1035
       1024
13608 )  114000
         108864
```

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Odd. Even.

1.—Square Roots (and Squares*) of Numbers—

50301.020406
ODD EVEN

Note.—The *square* must contain an *odd* number of figures to the left of the decimal point. If the square is a *pure decimal*—less than unity—the first significant figure must be an *even* number of places to the right of the decimal point. See page 14 for explanation of table.

Changing 2 decimal points in the square = 1 decimal point in the root.

Sq.	√0	1	2	3	4	5	6	7	8	9
1.0	1.0000	0050	0100	0149	0198	0247	0296	0344	0392	0140
.1	0488	0536	0583	0630	0677	0724	0770	0817	0863	0909
.2	0954	1000	1045	1091	1136	1180	1225	1269	1314	1358
.3	1402	1446	1489	1533	1576	1619	1662	1705	1747	1790
.4	1832	1874	1916	1958	2000	2042	2083	2124	2166	2207
1.5	2247	2288	2329	2369	2410	2450	2490	2530	2570	2610
.6	2649	2689	2728	2767	2806	2845	2884	2923	2961	3000
.7	3038	3077	3115	3153	3191	3229	3266	3304	3342	3379
.8	3416	3454	3491	3528	3565	3601	3638	3675	3711	3748
.9	3784	3820	3856	3892	3928	3964	4000	4036	4071	4107
2.0	4142	4177	4213	4248	4283	4318	4353	4387	4422	4457
.1	4491	4526	4560	4595	4629	4663	4697	4731	4765	4799
.2	4832	4866	4900	4933	4967	5000	5033	5067	5100	5133
.3	5166	5199	5232	5264	5297	5330	5362	5395	5427	5460
.4	5492	5524	5556	5588	5620	5652	5684	5716	5748	5780
2.5	5811	5843	5875	5906	5937	5969	6000	6031	6062	6093
.6	6125	6155	6186	6217	6248	6279	6310	6340	6371	6401
.7	6432	6462	6492	6523	6553	6583	6613	6643	6673	6703
.8	6733	6763	6793	6823	6852	6882	6912	6941	6971	7000
.9	7029	7059	7088	7117	7146	7176	7205	7234	7263	7292
3.0	7321	7349	7378	7407	7436	7464	7493	7521	7550	7578
.1	7607	7635	7664	7692	7720	7748	7776	7804	7833	7861
.2	7889	7916	7944	7972	8000	8028	8055	8083	8111	8138
.3	8166	8193	8221	8248	8276	8303	8330	8358	8385	8412
.4	8439	8466	8493	8520	8547	8574	8601	8628	8655	8682
3.5	8708	8735	8762	8788	8815	8841	8868	8894	8921	8947
.6	8974	9000	9026	9053	9079	9105	9131	9157	9183	9209
.7	9235	9261	9287	9313	9339	9365	9391	9416	9442	9468
.8	9494	9519	9545	9570	9596	9621	9647	9673	9698	9723
.9	9748	9774	9799	9824	9849	9875	9900	9925	9950	9975
4.0	2.0000	0025	0050	0075	0100	0125	0149	0174	0199	0224
.1	0248	0273	0298	0322	0347	0372	0396	0421	0445	0469
.2	0494	0518	0543	0567	0591	0616	0640	0664	0688	0712
.3	0736	0761	0785	0809	0833	0857	0881	0905	0928	0952
.4	0976	1000	1024	1048	1071	1095	1119	1142	1166	1190
4.5	1213	1237	1260	1284	1307	1331	1354	1378	1401	1424
.6	1448	1471	1494	1517	1541	1564	1587	1610	1633	1656
.7	1679	1703	1726	1749	1772	1794	1817	1840	1863	1886
.8	1909	1932	1954	1977	2000	2023	2045	2068	2091	2113
.9	2136	2159	2181	2204	2226	2249	2271	2293	2316	2338

P. P.

	50	48	46	44	42
1	5	5	5	4	4
2	10	10	9	9	8
3	15	14	14	13	13
4	20	19	18	18	17
5	25	24	23	22	21
6	30	29	28	26	25
7	35	34	32	31	29
8	40	38	37	35	34
9	45	43	41	40	38
	41	40	39	38	37
1	4	4	4	4	4
2	8	8	8	8	7
3	12	12	12	11	11
4	16	16	16	15	15
5	21	20	20	19	19
6	25	24	23	23	22
7	29	28	27	27	26
8	33	32	31	30	30
9	37	36	35	34	33
	36	35	34	33	32
1	4	4	3	3	3
2	7	7	7	7	6
3	11	11	10	10	10
4	14	14	14	13	13
5	18	18	17	17	16
6	22	21	20	20	19
7	25	25	24	23	22
8	29	28	27	26	26
9	32	32	31	30	29
	31	30	29	28	27
1	3	3	3	3	3
2	6	6	6	6	5
3	9	9	9	8	8
4	12	12	12	11	11
5	16	15	15	14	14
6	19	18	17	17	16
7	22	21	20	20	19
8	25	24	23	22	22
9	28	27	26	25	24
	26	25	24	23	22
1	3	3	2	2	2
2	5	5	5	5	4
3	8	8	7	7	7
4	10	10	10	9	9
5	13	13	12	12	11
6	16	15	14	14	13
7	18	18	17	16	15
8	21	20	19	18	18
9	23	23	22	21	20

**Square Roots* are obtained directly as in the logarithmic tables, using the Proportional Parts tables for interpolation.

Squares may be obtained by inverse interpolation.

Example.—Square root of $\dot{3}74.9 = 19.339$
$+ \quad 23$
$= \bar{1}9.362$ Ans.

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Odd. Even.

—†FROM 1 TO 10; 100 TO 1,000; 10,000 TO 100,000; ETC.

Sq.	√ 0	1	2	3	4	5	6	7	8	9		P. P.			
5.0	2.2361	2383	2405	2428	2450	2472	2494	2517	2539	2561		23	22	21	20
.1	2583	2605	2627	2650	2672	2694	2716	2738	2760	2782	1	2	2	2	2
.2	2804	2825	2847	2869	2891	2913	2935	2956	2978	3000	2	5	4	4	4
.3	3022	3043	3065	3087	3108	3130	3152	3173	3195	3216	3	7	7	6	6
.4	3238	3259	3281	3302	3324	3345	3367	3388	3409	3431	4	9	9	8	8
5.5	3452	3473	3495	3516	3537	3558	3580	3601	3622	3643	5	12	11	11	10
.6	3664	3685	3707	3728	3749	3770	3791	3812	3833	3854	6	14	13	13	12
.7	3875	3896	3917	3937	3958	3979	4000	4021	4042	4062	7	16	15	15	14
.8	4083	4104	4125	4145	4166	4187	4207	4228	4249	4269	8	18	18	17	16
.9	4290	4310	4331	4352	4372	4393	4413	4434	4454	4474	9	21	20	19	18
6.0	4495	4515	4536	4556	4576	4597	4617	4637	4658	4678		21	20	19	
.1	4698	4718	4739	4759	4779	4799	4819	4839	4860	4880	1	2.1	2.0	1.9	
.2	4900	4920	4940	4960	4980	5000	5020	5040	5060	5080	2	4.2	4.0	3.8	
.3	5100	5120	5140	5159	5179	5199	5219	5239	5259	5278	3	6.3	6.0	5.7	
.4	5298	5318	5338	5357	5377	5397	5417	5436	5456	5475	4	8.4	8.0	7.6	
6.5	5495	5515	5534	5554	5573	5593	5612	5632	5652	5671	5	10.5	10.0	9.5	
.6	5690	5710	5729	5749	5768	5788	5807	5826	5846	5865	6	12.6	12.0	11.4	
.7	5884	5904	5923	5942	5962	5981	6000	6019	6038	6058	7	14.7	14.0	13.3	
.8	6077	6096	6115	6134	6153	6173	6192	6211	6230	6249	8	16.8	16.0	15.2	
.9	6268	6287	6306	6325	6344	6363	6382	6401	6420	6439	9	18.9	18.0	17.1	
7.0	6458	6476	6495	6514	6533	6552	6571	6589	6608	6627		19	18	17	
.1	6646	6665	6683	6702	6721	6739	6758	6777	6796	6814	1	1.9	1.8	1.7	
.2	6833	6851	6870	6889	6907	6926	6944	6963	6981	7000	2	3.8	3.6	3.4	
.3	7019	7037	7055	7074	7092	7111	7129	7148	7166	7185	3	5.7	5.4	5.1	
.4	7203	7221	7240	7258	7276	7295	7313	7331	7350	7368	4	7.6	7.2	6.8	
7.5	7386	7404	7423	7441	7459	7477	7495	7514	7532	7550	5	9.5	9.0	8.5	
.6	7568	7586	7604	7622	7641	7659	7677	7695	7713	7731	6	11.4	10.8	10.2	
.7	7749	7767	7785	7803	7821	7839	7857	7875	7893	7911	7	13.3	12.6	11.9	
.8	7928	7946	7964	7982	8000	8018	8036	8054	8071	8089	8	15.2	14.4	13.6	
.9	8107	8125	8142	8160	8178	8196	8213	8231	8249	8267	9	17.1	16.2	15.3	
8.0	8284	8302	8320	8337	8355	8373	8390	8408	8425	8443		18	17	16	
.1	8460	8478	8496	8513	8531	8548	8566	8583	8601	8618	1	1.8	1.7	1.6	
.2	8636	8653	8671	8688	8705	8723	8740	8758	8775	8792	2	3.6	3.4	3.2	
.3	8810	8827	8844	8862	8879	8896	8914	8931	8948	8965	3	5.4	5.1	4.8	
.4	8983	9000	9017	9034	9052	9069	9086	9103	9120	9138	4	7.2	6.8	6.4	
8.5	9155	9172	9189	9206	9223	9240	9257	9275	9292	9309	5	9.0	8.5	8.0	
.6	9326	9343	9360	9377	9394	9411	9428	9445	9462	9479	6	10.8	10.2	9.6	
.7	9496	9513	9530	9547	9563	9580	9597	9614	9631	9648	7	12.6	11.9	11.2	
.8	9665	9682	9698	9715	9732	9749	9766	9783	9799	9816	8	14.4	13.6	12.8	
.9	9833	9850	9866	9883	9900	9917	9933	9950	9967	9983	9	16.2	15.3	14.4	
9.0	3.0000	0017	0033	0050	0067	0083	0100	0116	0133	0150		17	16	15	
.1	0166	0183	0199	0216	0232	0249	0265	0282	0299	0315	1	1.7	1.6	1.5	
.2	0332	0348	0364	0381	0397	0414	0430	0447	0463	0480	2	3.4	3.2	3.0	
.3	0496	0512	0529	0545	0561	0578	0594	0610	0627	0643	3	5.1	4.8	4.5	
.4	0659	0676	0692	0708	0725	0741	0757	0773	0790	0806	4	6.8	6.4	6.0	
9.5	0822	0838	0854	0871	0887	0903	0919	0935	0952	0968	5	8.5	8.0	7.5	
.6	0984	1000	1016	1032	1048	1064	1081	1097	1113	1129	6	10.2	9.6	9.0	
.7	1145	1161	1177	1193	1209	1225	1241	1257	1273	1289	7	11.9	11.2	10.5	
.8	1305	1321	1337	1353	1369	1385	1401	1417	1432	1448	8	13.6	12.8	12.0	
.9	1464	1480	1496	1512	1528	1544	1559	1575	1591	1607	9	15.3	14.4	13.5	

Changing a decimal point in the square = a decimal point in the root.

†For squares 10—100, 1,000—10,000, etc., see following table.

Example.—Square of 24.64 = 607.1 Ans.

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Even. Odd.

2.—Square Roots (and Squares*) of Numbers—

604020.10305
EVEN ODD

Note.—The *square* must contain an *even* number of figures to the left of the decimal point. If the square is a pure decimal—less than unity—the first significant figure must be an *odd* number of places to the right of the decimal point. See page 14 for explanation of table.

Changing 2 decimal points in the square = 1 decimal point in the root.

Sq.	√.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
10.	3.1623	1780	1937	2094	2249	2404	2558	2711	2863	3015
1.	3166	3317	3466	3615	3764	3912	4059	4205	4351	4496
2.	4641	4785	4928	5071	5214	5355	5496	5637	5777	5917
3.	6056	6194	6332	6469	6606	6742	6878	7014	7148	7283
4.	7417	7550	7683	7815	7947	8079	8210	8341	8471	8601
15.	8730	8859	8987	9115	9243	9370	9497	9623	9749	9875
6.	4.0000	0125	0249	0373	0497	0620	0743	0866	0988	1110
7.	1231	1352	1473	1593	1713	1833	1952	2071	2190	2308
8.	2426	2544	2661	2778	2895	3012	3128	3243	3359	3474
9.	3589	3704	3818	3932	4045	4159	4272	4385	4497	4609
20.	4721	4833	4944	6055	5166	5277	5387	5497	5607	5717
1.	5826	5935	6043	6152	6260	6368	6476	6583	6690	6797
2.	6904	7011	7117	7223	7329	7434	7539	7645	7749	7854
3.	7958	8062	8166	8270	8374	8477	8580	8683	8785	8888
4.	8990	9092	9193	9295	9396	9497	9598	9699	9800	9900
25.	5.0000	0100	0200	0299	0398	0498	0596	0695	0794	0892
6.	0990	1088	1186	1284	1381	1478	1575	1672	1769	1865
7.	1962	2058	2154	2249	2345	2440	2536	2631	2726	2820
8.	2915	3009	3104	3198	3292	3385	3479	3572	3666	3759
9.	3852	3944	4037	4129	4222	4314	4406	4498	4589	4681
30.	4772	4863	4955	5045	5136	5227	5317	5408	5498	5588
1.	5678	5767	5857	5946	6036	6125	6214	6303	6391	6480
2.	6569	6657	6745	6833	6921	7009	7096	7184	7271	7359
3.	7446	7533	7619	7706	7793	7879	7966	8052	8138	8224
4.	8310	8395	8481	8566	8652	8737	8822	8907	8992	9076
35.	9161	9245	9330	9414	9498	9582	9666	9749	9833	9917
6.	6.0000	0083	0166	0249	0332	0415	0498	0581	0663	0745
7.	0828	0910	0992	1074	1156	1237	1319	1400	1482	1563
8.	1644	1725	1806	1887	1968	2048	2129	2209	2290	2370
9.	2450	2530	2610	2690	2769	2849	2929	3008	3087	3166
40.	3246	3325	3403	3482	3561	3640	3718	3797	3875	3953
1.	4031	4109	4187	4265	4343	4420	4498	4576	4653	4730
2.	4807	4885	4962	5038	5115	5192	5269	5345	5422	5498
3.	5574	5651	5727	5803	5879	5955	6030	6106	6182	6257
4.	6332	6408	6483	6558	6633	6708	6783	6858	6933	7007
45.	7082	7157	7231	7305	7380	7454	7528	7602	7676	7750
6.	7823	7897	7971	8044	8118	8191	8264	8337	8411	8484
7.	8557	8629	8702	8775	8848	8920	8993	9065	9138	9210
8.	9282	9354	9426	9498	9570	9642	9714	9785	9857	9929
9.	7.0000	0071	0143	0214	0285	0356	0427	0498	0569	0640

P. P.

	155	150	145	140	135
1	16	15	15	14	14
2	31	30	29	28	27
3	47	45	44	42	41
4	62	60	58	56	54
5	78	75	73	70	68
6	93	90	87	84	81
7	109	105	102	98	95
8	124	120	116	112	108
9	130	135	131	126	122
	130	126	122	118	114
1	13	13	12	12	11
2	26	25	24	24	23
3	39	38	37	35	34
4	52	50	49	47	46
5	65	63	61	59	57
6	78	76	73	70	68
7	91	88	85	83	80
8	104	101	98	94	91
9	117	113	110	106	103
	110	106	102	98	94
1	11	11	10	10	9
2	22	21	20	20	19
3	33	32	31	29	28
4	44	42	41	39	38
5	55	53	51	49	47
6	66	64	61	59	56
7	77	74	71	69	66
8	88	85	82	78	75
9	99	95	92	88	85
	90	88	86	84	82
1	9	9	9	8	8
2	18	18	17	17	16
3	27	26	26	25	25
4	36	35	34	34	33
5	45	44	43	42	41
6	54	53	52	50	49
7	63	62	60	59	57
8	72	70	69	67	66
9	81	79	77	76	74
	80	78	76	74	72
1	8	8	8	7	7
2	16	16	15	15	14
3	24	23	23	22	22
4	32	31	30	30	29
5	40	39	38	37	36
6	48	47	46	44	43
7	56	55	53	52	50
8	64	62	61	59	58
9	72	70	68	67	65

**Square Roots* are obtained directly as in the logarithmic tables, using the Proportional Parts column for interpolation.

Squares may be obtained by inverse interpolation.

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Even. Odd.

—†From 10 to 100; 1,000 to 10,000; etc.

Changing 2 decimal points in the square—1 decimal point in the root.

Sq.	√.0	.1	.2	.3	.4	.5	.6	.7	.8	.9		P. P.				
50.	7.0711	0781	0852	0922	0993	1063	1134	1204	1274	1344		71	70	69	68	67
1.	1414	1484	1551	1624	1694	1764	1833	1903	1972	2042	1	7	7	7	7	7
2.	2111	2180	2250	2319	2388	2457	2526	2595	2664	2732	2	14	14	14	14	13
3.	2801	2870	2938	3007	3075	3144	3212	3280	3348	3417	3	21	21	21	20	20
4.	3485	3553	3621	3689	3756	3824	3892	3959	4027	4095	4	28	28	28	27	27
55	4162	4229	4297	4364	4431	4498	4565	4632	4699	4766	5	36	35	35	34	34
6.	4833	4900	4967	5033	5100	5166	5233	5299	5366	5432	6	43	42	41	41	40
7.	5498	5565	5631	5697	5763	5829	5895	5961	6026	6092	7	50	49	48	48	47
8.	6158	6223	6289	6354	6420	6485	6551	6616	6681	6746	8	57	56	55	54	54
9.	6811	6877	6942	7006	7071	7136	7201	7266	7330	7395	9	64	63	62	61	60
60.	7460	7524	7589	7653	7717	7782	7846	7910	7974	8038		66	65	64	63	62
1	8102	8166	8230	8294	8358	8422	8486	8549	8613	8677	1	7	7	6	6	6
2.	8740	8804	8867	8930	8994	9057	9120	9183	9246	9310	2	13	13	13	13	12
3.	9373	9436	9498	9561	9624	9687	9750	9812	9875	9937	3	20	20	19	19	19
4	8.0000	0062	0125	0187	0250	0312	0374	0436	0498	0561	4	26	26	26	25	25
65.	0623	0685	0747	0808	0870	0932	0994	1056	1117	1179	5	33	33	32	32	31
6.	1240	1302	1363	1425	1486	1548	1609	1670	1731	1792	6	40	39	38	38	37
7.	1854	1915	1976	2037	2098	2158	2219	2280	2341	2401	7	46	46	45	44	43
8.	2462	2523	2583	2644	2704	2765	2825	2885	2946	3006	8	53	52	51	50	50
9.	3066	3126	3187	3247	3307	3367	3427	3487	3546	3606	9	59	59	58	57	56
70.	3666	3726	3785	3845	3905	3964	4024	4083	4143	4202		61	60	59	58	
1.	4261	4321	4380	4439	4499	4558	4617	4676	4735	4794	1	6	6	6	6	
2.	4853	4912	4971	5029	5088	5147	5206	5264	5323	5381	2	12	12	12	12	
3.	5440	5499	5557	5615	5674	5732	5790	5849	5907	5965	3	18	18	18	17	
4.	6023	6081	6139	6197	6255	6313	6371	6429	6487	6545	4	24	24	24	23	
75.	6603	6660	6718	6776	6833	6891	6948	7006	7063	7121	5	31	30	30	29	
6.	7178	7235	7293	7350	7407	7464	7521	7579	7636	7693	6	37	36	35	35	
7.	7750	7807	7864	7920	7977	8034	8091	8148	8204	8261	7	43	42	41	41	
8.	8318	8374	8431	8487	8544	8600	8657	8713	8769	8826	8	49	48	47	46	
9.	8882	8938	8994	9051	9107	9163	9219	9275	9331	9387	9	55	54	53	52	
80.	9443	9499	9554	9610	9666	9722	9778	9833	9889	9944		57	56	55	54	
1.	9.0000	0056	0111	0167	0222	0277	0333	0388	0443	0499	1	6	6	6	5	
2.	0554	0609	0664	0719	0774	0830	0885	0940	0995	1049	2	11	11	11	11	
3	1104	1159	1214	1269	1324	1378	1433	1488	1542	1597	3	17	17	17	16	
4.	1652	1706	1761	1815	1869	1924	1978	2033	2087	2141	4	23	22	22	22	
85.	2195	2250	2304	2358	2412	2466	2520	2574	2628	2682	5	29	28	28	27	
6.	2736	2790	2844	2898	2952	3005	3059	3113	3167	3220	6	34	34	33	32	
7.	3274	3327	3381	3434	3488	3541	3595	3648	3702	3755	7	40	39	39	38	
8.	3808	3862	3915	3968	4021	4074	4128	4181	4234	4287	8	46	45	44	43	
9.	4340	4393	4446	4499	4552	4604	4657	4710	4763	4816	9	51	50	50	49	
90.	4868	4921	4974	5026	5079	5131	5184	5237	5289	5341		53	52	51	50	
1.	5394	5446	5499	5551	5603	5656	5708	5760	5812	5864	1	5	5	5	5	
2.	5917	5969	6021	6073	6125	6177	6229	6281	6333	6385	2	11	10	10	10	
3.	6437	6488	6540	6592	6644	6695	6747	6799	6850	6902	3	16	16	15	15	
4.	6954	7005	7057	7108	7160	7211	7263	7314	7365	7417	4	21	21	20	20	
95.	7468	7519	7570	7622	7673	7724	7775	7826	7877	7929	5	27	26	26	25	
6.	7980	8031	8082	8133	8184	8234	8285	8336	8387	8438	6	32	31	31	30	
7.	8489	8539	8590	8641	8691	8742	8793	8843	8894	8944	7	37	36	36	35	
8.	8995	9045	9096	9146	9197	9247	9298	9348	9398	9448	8	42	42	41	40	
9.	9499	9549	9599	9649	9700	9750	9800	9850	9900	9950	9	48	47	46	45	

†For squares 1—10, 100—1,000, etc., see preceding table.

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Cube Root.—The cube roots of numbers may be obtained from Tables 3, 4 and 5, as follows:

Table.	Whole Numbers (cubes).	Decimals (cubes).	Remarks.
3	1 to 50,	.001 to .050,	Special
	1000 to 50000, etc.	.000001 to .000050, etc.	table.
4	1 to 1000, etc.	.001 to 1, etc.	General
	Any number.	Any decimal.	table.
5	1 to 1.5,	.001 to .0015.	Special
	1000 to 1500, etc.	.000001 to .0000015, etc.	table.

Note that the *special tables*, 3 and 5, give results more accurately, within their limits, than does the *general table*, 4. The least accuracy from these tables is for numbers just above 1.5, or just above 1500; and for the cube roots of such numbers Table 3 may be used, or Tables 9 and 10, remembering that "changing three decimal points in the cube = one decimal point in the root—in the same direction."

Table 3 gives the cube roots of numbers from 1 to 50: directly, advancing by tenths; and by one interpolation of the P. P. table, advancing by hundreths. Thus, the cube root of 24.4 is 2.9004; of 24.45 is 2.9004 + 20 = 2.9024, the 20 being obtained from the P. P. table opposite 5 and under the difference 40 (= 9044 − 9004).

By manipulating the decimal point, the cube root of 0.02445 is 0.29024; of 24450 is 29.024, etc. Note that if the decimal point is changed only *one* or *two* places in the cube the cube root comprises *another set* of significant figures, as, cube root of 24.4 = 2.9004; of 2.44 = 1.3463; of 0.244 = 0.6249.

Table 5 is especially useful in finding the cube roots of numbers from 1 to 1.5, or from 1000 to 1500. The cubes of numbers with four significant figures may be obtained directly from the table, and of numbers with five significant figures may be obtained by one interpolation of the P. P. table. Thus, cube root of 1.145 = 1.04617; of 1.1452 = 1.04623, the increment being $^2/_{10}$ of the difference 30 (= 4647 − 4617). Likewise, the cube root of 1145.2 = 10.4623.

Table 4 is a general table, with numbers from 1 to 1000. Its special range, however, is for numbers from 50 to 1000.

Cubes of numbers may be obtained by inverse interpolation.

For numbers beyond the accurate range of the tables the following methods are given for extracting the cube root:

By Algebra.
Cube root of $a^3 + 3a^2x + 3ax^2 + x^3$?

$$
\begin{array}{lll}
 & a^3+3a^2x+3ax^2+x^3 & (a+x. \text{ Ans.} \\
 & a^3 & \text{Assume } x-3: \\
3a^2+3ax+x^2) & 3a^2x+3ax^2+x^3 & \\
 & 3a^2x+3ax^2+x^3 & 3a^2+3ax+x^2 = 1389)
\end{array}
$$

$3a^2 = 1200$
$3ax = 180$
$x^2 = 9$

By Arithmetic.
Cube root of 12,167 ?

$$
\begin{array}{ll}
12\dot{1}6\dot{7} & (20+3=23. \text{ Ans.} \\
8000 & a+x \\
\hline
4167 & \\
4167 &
\end{array}
$$

Example.—Extract the cube root of 46,354.87 ?

Process:

Remarks.—Beginning with the unit figure, point off 3 places each way, forming groups of 3 figures to be brought down each time. Proceed as indicated.

$$
\begin{array}{lll|l}
 & a_1 = 30. & 3a_1^2 = 3(30^2) = 2700 & 46\dot{3}54.8\dot{7}0(35.9+ \text{ Ans.} \\
\text{Assume } x_1 = 5: & & 3a_1x_1 = 3.30.5 = 450 & 27 \\
 & & x_1^2 = 5^2 = 25 & \\
\hline
 & & 3175 &)19354 \\
 & & & 15875 \\
\hline
 & a_2 = 350. & 3a_2^2 = 3(350^2) = 367500 & 3479870 \\
\text{Assume } x_2 = 9: & & 3a_2x_2 = 3.350.9 = 9450 & \\
 & & x_2^2 = 9^2 = 81 & \\
\hline
 & & 377031 & 3393279 \\
\hline
 & & & 86591000
\end{array}
$$

$a_0 = 3$
$a_1 = 30 \quad x_1 = 5$
$a_2 = 350 \quad x_2 = 9$
$a_3 = 3590 \quad x_3 = ?$

$a_3 = 3590.$ $\quad 3a_3^2 =$
Assume $x_3 = ?$ $\quad 3a_3x_3 =$ } Ready for another operation.
$x^2 =$

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1 to 50 .001 to .050
1,000 to 50,000 .000,001 to .000,050

3.—Cube Roots (and Cubes*) of Numbers 1 to 50.

Changing 3 decimal points in the cube = 1 decimal point in the root.

Cube.	∛.0	.1	.2	.3	.4	.5	.6	.7	.8	.9		P. P.				
0.	0.0000	4642	5848	6694	7368	7937	8434	8879	9283	9655		140	130	120	110	100
1.	1.0000	0323	0627	0914	1187	1447	1696	1935	2164	2386	1	14	13	12	11	10
2.	2599	2806	3006	3200	3389	3572	3751	3925	4095	4260	2	28	26	24	22	20
3.	4422	4581	4736	4888	5037	5183	5326	5467	5605	5741	3	42	39	36	33	30
4.	5874	6005	6134	6261	6386	6510	6631	6751	6869	6985	4	56	52	48	44	40
5.	7100	7213	7325	7435	7544	7652	7758	7863	7967	8070	5	70	65	60	55	50
6.	8171	8272	8371	8469	8566	8663	8758	8852	8945	9038	6	84	78	72	66	60
7.	9129	9220	9310	9399	9487	9574	9661	9747	9832	9916	7	98	91	84	77	70
8.	2.0000	0083	0165	0247	0328	0408	0488	0567	0646	0724	8	112	104	96	88	80
9.	0801	0878	0954	1029	1105	1179	1253	1327	1400	1472	9	126	117	108	99	90
10.	1544	1616	1687	1758	1828	1898	1967	2036	2104	2172		95	90	85	80	76
11.	2240	2307	2374	2440	2506	2572	2637	2702	2766	2831	1	10	9	9	8	8
12.	2894	2958	3021	3084	3146	3208	3270	3331	3392	3453	2	19	18	17	16	15
13.	3513	3573	3633	3693	3752	3811	3870	3928	3986	4044	3	29	27	26	24	23
14.	4101	4159	4216	4272	4329	4385	4441	4497	4552	4607	4	38	36	34	32	30
15.	4662	4717	4771	4825	4879	4933	4987	5040	5093	5146	5	48	45	43	40	38
16.	5198	5251	5303	5355	5407	5458	5510	5561	5612	5662	6	57	54	51	48	46
17.	5713	5763	5813	5863	5913	5963	6012	6061	6110	6159	7	67	63	60	56	53
18.	6207	6256	6304	6352	6400	6448	6495	6543	6590	6637	8	76	72	68	64	61
19.	6684	6730	6777	6824	6870	6916	6962	7008	7053	7099	9	86	81	77	72	68
20.	7144	7189	7234	7279	7324	7369	7413	7457	7501	7545		72	68	64	60	56
21.	7589	7633	7677	7720	7763	7807	7850	7893	7935	7978	1	7	7	6	6	6
22.	8020	8063	8105	8147	8189	8231	8273	8314	8356	8397	2	14	14	13	12	11
23.	8439	8480	8521	8562	8602	8643	8684	8724	8765	8805	3	22	20	19	18	17
24.	8845	8885	8925	8965	9004	9044	9083	9123	9162	9201	4	29	27	26	24	22
25.	9240	9279	9318	9357	9395	9434	9472	9511	9549	9587	5	36	34	32	30	28
26.	9625	9663	9701	9738	9776	9814	9851	9888	9926	9963	6	43	41	38	36	34
27.	3.0000	0037	0074	0111	0148	0184	0221	0257	0293	0330	7	50	48	45	42	39
28.	0366	0402	0438	0474	0510	0546	0581	0617	0652	0688	8	58	54	51	48	45
29.	0723	0759	0794	0829	0864	0899	0934	0968	1003	1038	9	65	61	58	54	50
30.	1072	1107	1141	1176	1210	1244	1278	1312	1346	1380		52	48	44	40	36
31.	1414	1448	1481	1515	1548	1582	1615	1648	1682	1715	1	5	5	4	4	4
32.	1748	1781	1814	1847	1880	1913	1945	1978	2010	2043	2	10	10	9	8	7
33.	2075	2108	2140	2172	2204	2237	2269	2301	2332	2364	3	16	14	13	12	11
34.	2396	2428	2460	2491	2523	2554	2586	2617	2648	2680	4	21	19	18	16	14
35.	2711	2742	2773	2804	2835	2866	2897	2927	2958	2989	5	26	24	22	20	18
36.	3019	3050	3080	3111	3141	3171	3202	3232	3262	3292	6	31	29	26	24	22
37.	3322	3352	3382	3412	3442	3472	3501	3531	3561	3590	7	36	34	31	28	25
38.	3620	3649	3679	3708	3737	3767	3796	3825	3854	3883	8	42	38	35	32	29
39.	3912	3941	3970	3999	4028	4056	4085	4114	4142	4171	9	47	43	40	36	32
40.	4200	4228	4256	4285	4313	4341	4370	4398	4426	4454		32	30	28	26	24
41.	4482	4510	4538	4566	4594	4622	4650	4677	4705	4733	1	3	3	3	3	2
42.	4760	4788	4815	4843	4870	4898	4925	4952	4980	5007	2	6	6	6	5	5
43.	5034	5061	5088	5115	5142	5169	5196	5223	5250	5277	3	10	9	8	8	7
44.	5303	5330	5357	5384	5410	5437	5463	5490	5516	5543	4	13	12	11	10	10
45.	5569	5595	5622	5648	5674	5700	5726	5752	5778	5805	5	16	15	14	13	12
46.	5830	5856	5882	5908	5934	5960	5986	6011	6037	6063	6	19	18	17	16	14
47.	6088	6114	6139	6165	6190	6216	6241	6267	6292	6317	7	22	21	20	18	17
48.	6342	6368	6393	6418	6443	6468	6493	6518	6543	6568	8	26	24	22	21	19
49.	6593	6618	6643	6668	6692	6717	6742	6767	6791	6816	9	29	27	25	23	22
50.	6840	6865	6889	6914	6938	6963	6987	7011	7036	7060						

**Cube Roots* are obtained directly as in the logarithmic tables, using the Proportional Parts tables for interpolation.

Cubes may be obtained by inverse interpolation.

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1 to 500 .001 to .5
1,000 to 500,000 .000,001 to .0005

4.—Cube Roots (and Cubes*) of Numbers 1 to 1,000—

Changing 3 decimal points in the cube = 1 decimal point in the root.

Cube.	∛0.	1.	2.	3.	4.	5.	6.	7.	8.	9.		P.	P.			
See prev. page 0																
0-.	zero	1.0000	2599	4422	5874	7100	8171	9129	0000	0801		240	220	200	190	180
1-.	2.1544	2240	2894	3513	4101	4662	5198	5713	6207	6684	1	24	22	20	19	18
2-.	7144	7589	8020	8439	8845	9240	9625	0000	0366	0723	2	48	44	40	38	36
3-.	3.1072	1414	1748	2075	2396	2711	3019	3322	3620	3912	3	72	66	60	57	54
4-.	4200	4482	4760	5034	5303	5569	5830	6088	6342	6593	4	96	88	80	76	72
50																
5-.	6840	7084	7325	7563	7798	8030	8259	8485	8709	8930	5	120	110	100	95	90
6-.	9149	9365	9579	9791	0000	0207	0412	0615	0817	1016	6	144	132	120	114	108
7-.	4.1213	1418	1602	1793	1983	2172	2358	2543	2727	2908	7	168	154	140	133	126
8-.	3089	3267	3445	3621	3795	3968	4140	4310	4480	4647	8	192	176	160	152	144
9-.	4814	4979	5144	5307	5468	5629	5789	5947	6104	6261	9	216	198	180	171	162
100																
10-.	6416	6570	6723	6875	7027	7177	7326	7475	7622	7769		170	160	150	140	130
11-.	7914	8059	8203	8346	8488	8629	8770	8910	9049	9187	1	17	16	15	14	13
12-.	9324	9461	9597	9732	9866	0000	0133	0265	0397	0528	2	34	32	30	28	26
13-.	5.0658	0788	0916	1045	1172	1299	1426	1551	1676	1801	3	51	48	45	42	39
14-.	1925	2048	2171	2293	2415	2536	2656	2776	2896	3015	4	68	64	60	56	52
150																
15-.	3133	3251	3368	3485	3601	3717	3832	3947	4061	4175	5	85	80	75	70	65
16-.	4288	4401	4514	4626	4737	4848	4959	5069	5178	5288	6	102	96	90	84	78
17-.	5397	5505	5613	5721	5828	5934	6041	6147	6252	6357	7	119	112	105	98	91
18-.	6462	6567	6671	6774	6877	6980	7083	7185	7287	7388	8	136	128	120	112	104
19-.	7489	7590	7690	7790	7890	7989	8088	8186	8285	8383	9	153	144	135	126	117
200																
20-.	8480	8578	8675	8771	8868	8964	9059	9155	9250	9345		120	110	100	90	80
21-.	9439	9533	9627	9721	9814	9907	0000	0092	0185	0277	1	12	11	10	9	8
22-.	6.0368	0459	0550	0641	0732	0822	0912	1002	1091	1180	2	24	22	20	18	16
23-.	1269	1358	1446	1534	1622	1710	1797	1885	1972	2058	3	36	33	30	27	24
24-.	2145	2231	2317	2403	2488	2573	2658	2743	2828	2912	4	48	44	40	36	32
250																
25-.	2996	3080	3146	3247	3330	3413	3496	3579	3661	3743	5	60	55	50	45	40
26-.	3825	3907	3988	4070	4151	4232	4312	4393	4473	4553	6	72	66	60	54	48
27-.	4633	4713	4792	4872	4951	5030	5108	5187	5265	5343	7	84	77	70	63	56
28-.	5421	5499	5577	5654	5731	5808	5885	5962	6039	6115	8	96	88	80	72	64
29-.	6191	6267	6343	6419	6494	6569	6644	6719	6794	6869	9	108	99	90	81	72
300																
30-.	6943	7018	7092	7166	7240	7313	7387	7460	7533	7606		70	66	64	62	60
31-	7679	7752	7824	7897	7969	8041	8113	8185	8256	8328	1	7	7	6	6	6
32-	8399	8470	8541	8612	8683	8753	8824	8894	8964	9034	2	14	13	13	12	12
33-	9104	9174	9214	9313	9382	9451	9521	9589	9658	9727	3	21	20	19	19	18
34-	9795	9864	9932	0000	0068	0136	0203	0271	0338	0406	4	28	26	26	25	24
350																
35-	7.0473	0540	0607	0674	0740	0807	0873	0940	1006	1072	5	35	33	32	31	30
36-	1138	1204	1269	1335	1400	1466	1531	1596	1661	1726	6	42	40	38	37	36
37-	1791	1855	1920	1984	2048	2112	2177	2240	2304	2368	7	49	46	45	43	42
38-	2432	2495	2558	2622	2685	2748	2811	2874	2936	2999	8	56	53	51	50	48
39-	3061	3124	3186	3248	3310	3372	3434	3496	3558	3619	9	63	59	58	56	54
400																
40-.	3681	3742	3803	3864	3925	3986	4047	4108	4169	4229		58	56	55	54	53
41-.	4290	4350	4410	4470	4530	4590	4650	4710	4770	4829	1	6	6	6	5	5
42-.	4889	4948	5007	5067	5126	5185	5244	5302	5361	5420	2	12	11	11	11	11
43-.	5478	5537	5595	5654	5712	5570	5828	5886	5944	6001	3	17	17	17	16	16
44-.	6059	6117	6174	6232	6289	6346	6403	6460	6517	6574	4	23	22	22	22	21
450																
45-.	6631	6688	6744	6801	6857	6914	6970	7026	7082	7138	5	29	28	28	27	27
46-.	7194	7250	7306	7362	7418	7473	7529	7584	7639	7695	6	35	34	33	32	32
47-.	7750	7805	7860	7915	7970	8025	8079	8134	8188	8243	7	41	39	39	38	37
48-.	8297	8352	8406	8460	8514	8568	8622	8676	8730	8784	8	46	45	44	43	42
49-.	8837	8891	8944	8998	9051	9105	9158	9211	9264	9317	9	52	50	50	49	48
500																

***Cube Roots** are obtained directly as in the logarithmic tables, using the Proportional Parts tables for interpolation.

Cubes may be obtained by inverse interpolation.

The dash (-) is to be supplied by figs. 0 to 9 at the head of the respective column.

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500 to 1,000 **.5 to 1**
500,000 to 1,000,000 **.0005 to .001**

—And Any Other Numbers; or Decimals.

Cube.	∛0.	1.	2.	3.	4.	5.	6.	7.	8.	9.		P. P.
500												
50-.	7.9370	9423	9476	9528	9581	9634	9686	9739	9791	9843		53 52 51 50 49
51-.	9896	9948	0000	0052	0104	0156	0208	0260	0311	0363	1	5 5 5 5 5
52-.	8.0415	0466	0517	0569	0620	0671	0723	0774	0825	0876	2	11 10 10 10 10
53-.	0927	0978	1028	1079	1130	1180	1231	1281	1332	1382	3	16 16 15 15 15
54-.	1433	1483	1533	1583	1633	1683	1733	1783	1833	1882	4	21 21 20 20 20
550												
55-.	1932	1982	2031	2081	2130	2180	2229	2278	2327	2377	5	27 26 26 25 25
56-.	2426	2475	2524	2573	2621	2670	2719	2768	2816	2865	6	32 31 31 30 29
57-.	2913	2962	3010	3059	3107	3155	3203	3251	3300	3348	7	37 36 36 35 34
58-.	3396	3443	3491	3539	3587	3634	3682	3730	3777	3825	8	42 42 41 40 39
59-.	3872	3919	3967	4014	4061	4108	4155	4202	4249	4296	9	48 47 46 45 44
600												
60-.	4343	4390	4437	4484	4530	4577	4623	4670	4716	4763		48 47 46 45 44
61-.	4809	4856	4902	4948	4994	5040	5086	5132	5178	5224	1	5 5 5 5 4
62-.	5270	5316	5362	5408	5453	5499	5544	5590	5635	5681	2	10 9 9 9 9
63-.	5726	5772	5817	5862	5907	5952	5997	6043	6088	6132	3	14 14 14 14 13
64-.	6177	6222	6267	6312	6357	6401	6446	6490	6535	6579	4	19 19 18 18 18
650												
65-.	6624	6668	6713	6757	6801	6845	6890	6934	6978	7022	5	24 24 23 23 22
66-.	7066	7110	7154	7198	7241	7285	7329	7373	7416	7460	6	29 28 28 27 26
67-.	7503	7547	7590	7634	7677	7721	7764	7807	7850	7893	7	34 33 32 32 31
68-.	7937	7980	8023	8066	8109	8152	8194	8237	8280	8323	8	38 38 37 36 35
69-.	8366	8408	4451	8493	8536	8578	8621	8663	8706	8748	9	43 42 41 41 40
700												
70-.	8790	8833	8875	8917	8959	9001	9043	9085	9127	9169		43 42 41 40
71-.	9211	9253	9295	9337	9378	9420	9462	9503	9545	9587	1	4 4 4 4
72-	9628	9670	9711	9752	9794	9835	9876	9918	9959	0000	2	9 8 8 8
73-.	9.0041	0082	0123	0164	0205	0246	0287	0328	0369	0410	3	13 13 12 12
74-.	0450	0491	0532	0572	0613	0654	0694	0735	0775	0816	4	17 17 16 16
750												
75-.	0856	0896	0937	0977	1017	1057	1098	1138	1178	1218	5	22 21 21 20
76-.	1258	1298	1338	1378	4118	1458	1498	1537	1577	1617	6	26 25 25 24
77-.	1657	1696	1736	1775	1815	1855	1894	1933	1973	2012	7	30 29 29 28
78-.	2052	2091	2130	2170	2209	2248	2287	2326	2365	2404	8	34 34 33 32
79-.	2443	2482	2521	2560	2599	2638	2677	2716	2754	2793	9	39 38 37 36
800												
80-.	2832	2870	2909	2948	2986	3025	3063	3102	3140	3179		39 38 37 36
81-.	3217	3255	3294	3332	3370	3408	3447	3485	3523	3561	1	4 4 4 4
82-.	3599	3637	3675	3713	3751	3789	3827	3865	3902	3940	2	8 8 7 7
83-.	3978	4016	4053	4091	4129	4166	4204	4241	4279	4316	3	12 11 11 11
84-.	4354	4391	4429	4466	4503	4541	4578	4615	4652	4690	4	16 15 15 14
850												
85-.	4727	4764	4801	4838	4875	4912	4949	4986	5023	5060	5	20 19 19 18
86-.	5097	5134	5171	5207	5244	5281	5317	5354	5391	5427	6	23 23 22 22
87-.	5464	5501	5537	5574	5610	5647	5683	5719	5756	5792	7	27 27 26 25
88-.	5828	5865	5901	5937	5973	6010	6046	6082	6118	6154	8	31 30 30 29
89-.	6190	6226	6262	6298	6334	6370	6406	6442	6477	6513	9	35 34 33 32
900												
90-.	6549	6585	6620	6656	6692	6727	6763	6799	6834	6870		35 34 33
91-.	6905	6941	6976	7012	7047	7082	7118	7153	7188	7224	1	4 3 3
92-.	7259	7294	7329	7364	7400	7435	7470	7505	7540	7575	2	7 7 7
93-.	7610	7645	7680	7715	7750	7785	7819	7854	7889	7924	3	11 10 10
94-.	7959	7993	8028	8063	8097	8132	8167	8201	8236	8270	4	14 14 13
950												
95-.	8305	8339	8374	8408	8443	8477	8511	8546	8580	8614	5	18 17 17
96-.	8648	8683	8717	8751	8785	8819	8854	8888	8922	8956	6	21 20 20
97-.	8990	9024	9058	9092	9126	9160	9194	9227	9261	9295	7	25 24 23
98-.	9329	9363	9396	9430	9464	9497	9531	9565	9598	9632	8	28 27 26
99-.	9666	9699	9733	9766	9800	9833	9866	9900	9933	9967	9	32 31 30
1000												

Ex.—Cube root of 887.2? **Solution—9.6082 for 887**
7 for .2
Ans. 9.6089 for 887.2

The dash (-) is to be supplied by figs. 0 to 9 at the head of the respective column.

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1 to 1.5 .001 to .0015
1,000 to 1,500 .000001 to .0000015
1,000,000 to 1,500,000

5.—Cube Roots (and Cubes*) of Numbers 1,000 to 1,500.

Changing 3 decimal points in the cube = 1 decimal point in the root.

Cube	∛ 0.	1.	2.	3.	4.	5.	6.	7.	8.	9.		P.	P.		
1000															
1,00-.	10.0000	0033	0067	0100	0133	0166	0200	0233	0266	0299		34	33	32	31
1,01-.	0332	0365	0398	0431	0465	0498	0531	0563	0596	0629	1	3	3	3	3
1,02-.	0662	0695	0728	0761	0794	0826	0859	0892	0925	0957	2	7	7	6	6
1,03-.	0990	1023	1055	1088	1121	1153	1186	1218	1251	1283	3	10	10	10	9
1,04-.	1316	1348	1381	1413	1446	1478	1510	1543	1575	1607	4	14	13	13	12
1050															
1,05-.	1640	1672	1704	1736	1769	1801	1833	1865	1897	1929	5	17	17	16	16
1,06-.	1961	1993	2025	2057	2089	2121	2153	2185	2217	2249	6	20	20	19	19
1,07-.	2281	2313	2345	2376	2408	2440	2472	2503	2535	2567	7	24	23	22	22
1,08-.	2599	2630	2662	2693	2725	2757	2788	2820	2851	2883	8	27	26	26	25
1,09-.	2914	2946	2977	3009	3040	3071	3103	3134	3165	3197	9	31	30	29	28
1100															
1,10-.	3228	3259	3291	3322	3353	3384	3415	3447	3478	3509		32	31	30	29
1,11-.	3540	3571	3602	3633	3664	3695	3726	3757	3788	3819	1	3	3	3	3
1,12-.	3850	3881	3912	3943	3973	4004	4035	4066	4097	4127	2	6	6	6	6
1,13-.	4158	4189	4219	4250	4281	4311	4342	4373	4404	4434	3	10	9	9	9
1,14-.	4464	4495	4525	4556	4586	4617	4647	4678	4708	4739	4	13	12	12	12
1150															
1,15-.	4769	4799	4830	4860	4890	4921	4951	4981	5011	5042	5	16	16	15	15
1,16-.	5072	5102	5132	5162	5192	5223	5253	5283	5313	5343	6	19	19	18	17
1,17-.	5373	5403	5433	5463	5493	5523	5553	5583	5612	5642	7	22	22	21	20
1,18-.	5672	5702	5732	5762	5791	5821	5851	5881	5910	5940	8	26	25	24	23
1,19-	5970	6000	6029	6059	6088	6118	6148	6177	6207	6236	9	29	28	27	26
1200															
1,20-.	6266	6295	6325	6354	6384	6413	6443	6472	6501	6531		30	29	28	27
1,21-.	6560	6590	6619	6648	6678	6707	6736	6765	6795	6824	1	3	3	3	3
1,22-.	6853	6882	6911	6940	6970	6999	7028	7057	7086	7115	2	6	6	6	5
1,23-.	7144	7173	7202	7231	7260	7289	7318	7347	7376	7405	3	9	9	8	8
1,24-.	7434	7463	7491	7520	7549	7580	7607	7635	7664	7693	4	12	12	11	11
1250															
1,25-.	7722	7750	7779	7808	7837	7865	7894	7922	7951	7980	5	15	15	14	14
1,26-.	8008	8037	8065	8094	8122	8051	8179	8208	8236	8265	6	18	17	17	16
1,27-.	8293	8322	8350	8378	8407	8435	8463	8492	8520	8548	7	21	20	20	19
1,28-.	8577	8605	8633	8661	8690	8718	8746	8774	8802	8831	8	24	23	22	22
1,29-.	8859	8887	8915	8943	8971	8999	9027	9055	9083	9111	9	27	26	25	24
1300															
1,30-.	9139	9167	9195	9223	9251	9279	9307	9335	9363	9391		29	28	27	26
1,31-.	9418	9446	9474	9502	9530	9757	9585	9613	9641	9668	1	3	3	3	3
1,32-.	9696	9724	9752	9779	9807	9834	9862	9890	9917	9945	2	6	6	5	5
1,33-.	9972	0000	0028	0055	0083	0110	0138	0165	0193	0220	3	9	8	8	8
1,34-.	11.0247	0275	0302	0330	0357	0384	0412	0439	0466	0494	4	12	11	11	10
1350															
1,35-.	0521	0548	0575	0603	0630	0657	0684	0712	0739	0766	5	15	14	14	13
1,36-.	0793	0820	0847	0875	0902	0929	0956	0983	1010	1037	6	17	17	16	16
1,37-.	1064	1091	1118	1145	1172	1199	1226	1253	1280	1307	7	20	20	19	18
1,38-.	1334	1361	1387	1414	1441	1468	1495	1522	1548	1575	8	23	22	22	21
1,39-.	1602	1629	1655	1682	1709	1736	1762	1789	1816	1842	9	26	25	24	23
1400															
1,40-.	1869	1896	1922	1949	1975	2002	2028	2055	2082	2108		27	26	25	
1,41-.	2135	2161	2188	2214	2241	2267	2293	2320	2346	2373	1	3	3	3	
1,42-.	2399	2425	2452	2478	2505	2531	2557	2583	2610	2636	2	5	5	5	
1,43-.	2662	2689	2715	2741	2767	2793	2820	2846	2872	2898	3	8	8	8	
1,44-.	2924	2950	2977	3003	3029	3055	3081	3107	3133	3159	4	11	10	10	
1450															
1,45-.	3185	3211	3237	3263	3289	3315	3341	3367	3393	3419	5	14	13	13	
1,46-.	3445	3471	3496	3522	3548	3574	3600	3626	3652	3677	6	16	16	15	
1,47-.	3703	3729	3755	3780	3806	3832	3858	3883	3909	3935	7	19	18	18	
1,48-.	3960	3986	4012	4037	4063	4089	4114	4140	4165	4191	8	22	21	20	
1 49-.	4206	4242	4268	4293	4319	4344	4370	4395	4421	4446	9	24	23	23	
1500															

*Ex.—Cube of 1.07275? Solution—1.234 for 1.07260
.0005 for 15
Ans. 1.2345 for 1.07275

The dash (-) is to be supplied by figs. 0 to 9 at the head of the respective column.

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6.—Square Roots of Fifth Powers of Numbers, Advancing by 0.25.

Changing 2 decimal places in N—5 decimal places in $\sqrt{N^5}$, in the same direction.

N	$\sqrt{N^5}$.00	.25	.50	.75	N.	$\sqrt{N^5}$.00	.25	.50	.75
0.	Zero.	.03125	.17678	.48714	50.	17678.	17899.	18123.	18348.
1.	1.0000	1.7469	2.7557	4.0513	51.	18575.	18803.	19033.	19265.
2.	5.6569	7.5037	9.8821	12.541	52.	19499.	19734.	19971.	20210.
3.	15.588	19.042	22.918	27.232	53.	20450.	20692.	20936.	21181.
4.	32.000	37.237	42.957	49.174	54.	21428.	21677.	21928.	22180.
5.	55.902	63.154	70.943	79.281	55.	22434.	22690.	22947.	23207.
6.	88.182	97.656	107.72	118.38	56.	23468.	23731.	23995.	24261.
7.	129.64	141.53	154.05	167.21	57.	24529.	24799.	25071.	25344.
8.	181.02	195.50	210.64	226.48	58.	25620.	25896.	26175.	26456.
9.	243.00	260.23	278.17	296.83	59.	26738.	27022.	27308.	27596.
10.	316.23	336.36	357.25	378.90	60.	27886.	28177.	28470.	28765.
11.	401.31	424.50	448.48	473.25	61.	29062.	29361.	29661.	29963.
12.	498.83	525.22	552.43	580.46	62.	30268.	30574.	30882.	31191.
13.	609.34	639.06	669.63	701.06	63.	31503.	31816.	32132.	32449.
14.	733.26	766.54	800.61	835.56	64.	32768.	33089.	33412.	33736.
15.	871.42	908.19	945.87	984.47	65.	34063.	34392.	34722.	35054.
16.	1024.0	1064.5	1105.9	1148.2	66.	35388.	35724.	36062.	36402.
17.	1191.6	1235.9	1281.1	1327.4	67.	36744.	37088.	37433.	37781.
18.	1374.6	1422.8	1472.1	1522.3	68.	38131.	38482.	38835.	39191.
19.	1573.6	1625.8	1679.1	1733.5	69.	39548.	39907.	40268.	40631.
20.	1788.9	1845.3	1902.8	1961.3	70.	40996.	41363.	41733.	42103.
21.	2020.9	2081.6	2143.4	2206.2	71.	42476.	42851.	43228.	43607.
22.	2270.2	2335.2	2401.4	2468.7	72.	43988.	44371.	44755.	45142.
23.	2537.0	2606.5	2677.1	2748.9	73.	45531.	45922.	46315.	46709.
24.	2821.8	2895.9	2971.1	3047.5	74.	47106.	47505.	47906.	48309.
25.	3125.0	3203.7	3283.6	3364.7	75.	48714.	49121.	49530.	49941.
26.	3446.9	3530.4	3615.1	3700.9	76.	50354.	50769.	51186.	51606.
27.	3788.0	3876.3	3965.8	4056.6	77.	52027.	52450.	52875.	53303.
28.	4148.5	4241.8	4336.2	4431.9	78.	53732.	54164.	54598.	55033.
29.	4528.9	4627.2	4726.7	4827.4	79.	55471.	55911.	56353.	56797.
30.	4929.5	5032.8	5137.5	5243.4	80.	57243.	57692.	58142.	58594.
31.	5350.6	5459.2	5569.0	5680.1	81.	59049.	59506.	59964.	60425.
32.	5792.6	5906.4	6021.6	6138.0	82.	60888.	61354.	61821.	62290.
33.	6255.8	6375.0	6495.5	6617.4	83.	62762.	63235.	63711.	64189.
34.	6740.6	6865.2	6991.1	7118.5	84.	64669.	65151.	65636.	66122.
35.	7247.2	7377.3	7508.8	7641.7	85.	66611.	67102.	67595.	68090.
36.	7776.0	7911.7	8048.8	8187.3	86.	68588.	69087.	69589.	70093.
37.	8327.3	8468.7	8611.5	8755.7	87.	70599.	71107.	71618.	72130.
38.	8901.4	9048.5	9197.1	9347.2	88.	72646.	73162.	73681.	74203
39.	9498.6	9651.6	9806.0	9961.9	89.	74727.	75252.	75781.	76311.
40.	10119.	10278.	10438.	10600.	90.	76843.	77378.	77915.	78454.
41.	10764.	10929.	11095.	11263.	91.	78996.	79539.	80085.	80633.
42.	11432.	11603.	11775.	11949.	92.	81184.	81737.	82291.	82849.
43.	12125.	12302.	12480.	12660.	93.	83408.	83970.	84534.	85100.
44.	12842.	13025.	13210.	13396.	94.	85668.	86239.	86812.	87387.
45.	13584.	13774	13965.	14157.	95.	87965.	88545.	89127.	89711.
46.	14351.	14547.	14745.	14944.	96.	90298.	90887.	91478.	92073.
47.	15144.	15346.	15550.	15756.	97.	92668.	93266.	93867.	94470.
48.	15963.	16171.	16382.	16593.	98.	95075.	95682.	96292	96904.
49.	16807.	17022.	17239.	17458.	99.	97519.	98136.	98755.	99376.
50.	17678.	17899.	18123.	18348.	100.	100000.	100626.	101255.	101886.

Note that $\sqrt{N^5} = N^{\frac{5}{2}} = N^{2+\frac{1}{2}} = N^2 \times N^{\frac{1}{2}} = N^2 \times \sqrt{N}$. Hence, the above values may be obtained by multiplying together the square and the square root of the respective numbers.

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7.—Fifth Powers* (and Fifth Roots) of Numbers 1 to 10—

Changing 1 decimal place in the root or $N = 5$ decimal places in the power or N^5.

N	N^5 0	2	4	5	6	8
1.00	Unity	1.01004	1.02016	1.02525	1.03036	1.04064
.01	1.05101	1.06146	1.07199	1.07728	1.08260	1.09330
.02	1.10408	1.11495	1.12590	1.13141	1.13694	1.14806
.03	1.15927	1.17057	1.18196	1.18769	1.19344	1.20500
.04	1.21665	1.22840	1.24023	1.24618	1.25216	1.26417
1.05	1.27628	1.28848	1.30078	1.30696	1.31317	1.32565
.06	1.33823	1.35090	1.36367	1.37009	1.37653	1.38949
.07	1.40255	1.41571	1.42896	1.43563	1.44232	1.45577
.08	1.46933	1.48298	1.49674	1.50366	1.51060	1.52456
.09	1.53862	1.55279	1.56706	1.57424	1.58144	1.59592
1.0	Unity	1.10408	1.21665	1.27628	1.33823	1.46933
.1	1.61051	1.76234	1.92541	2.00135	2.11034	2.28775
.2	2.48832	2.70271	2.93163	3.05176	3.17580	3.43597
.3	3.71293	4.00746	4.32040	4.48403	4.65259	5.00490
.4	5.37824	5.77353	6.19174	6 40973	6.63383	7.10082
1.5	7.59375	8.11368	8.66171	8.94661	9.23896	9.84658
.6	10.4858	11.1577	11.8637	12.2298	12.6049	13.3828
.7	14.1986	15.0537	15.9495	16.4131	16.8874	17.8690
.8	18.8957	19.9690	21.0906	21.6700	22.2620	23.4849
.9	24.7610	26.0919	27.4795	28.1951	28.9255	30.4317
2.0	32.0000	33.6323	35.3306	36.2051	37.0968	38.9329
.1	40.8410	42.8232	44.8817	45.9401	47.0185	49.2360
.2	51.5363	53.9219	56.3949	57.6650	58.9579	61.6133
.3	64.3634	67.2109	70.1583	71.6703	73.2082	76.3633
.4	79.6262	82.9998	86.4867	88.2735	90.0898	93.8120
2.5	97.6563	101.626	105.723	107.820	109.951	114.314
.6	118.814	123.454	128.239	130.686	133.171	138.253
.7	143.489	148.883	154.438	157.276	160.157	166.044
.8	172.104	178.339	184.753	188.029	191.351	198.136
.9	205.111	212.283	219.653	223.414	227.226	235.007
3.0	243.000	251.209	259.638	263.936	268.292	277.175
.1	286.292	295.647	305.245	310.136	315.091	325.189
.2	335.544	346.162	357.047	362.591	368.204	379.637
.3	391.354	403.358	415.655	421.914	428.249	441.147
.4	454.354	467.876	481.717	488.760	495.884	510.383
3.5	525.219	540.398	555.925	563.822	571.808	588.051
.6	604.662	621.646	639.009	647.835	656.758	674.899
.7	693.440	712.385	731.742	741.577	751.517	771.719
.8	792.352	813.424	834.941	845.870	856.913	879.343
.9	902.242	925.615	949.470	961.580	973.814	998.655
4.0	1024.00	1049.86	1076 23	1089.62	1103.14	1130.58
.1	1158.56	1187.10	1216.19	1230.95	1245.85	1276.09
.2	1306.91	1338.33	1370.34	1386.58	1402.97	1436.21
.3	1470.08	1504.59	1539.74	1557.57	1575.55	1612.02
.4	1649.16	1686.99	1725.50	1745.02	1764.71	1804.64
4.5	1845.28	1886.65	1928.77	1950.10	1971.62	2015.24
.6	2059.63	2104.80	2150.75	2174.03	2197.50	2245.07
.7	2293.45	2342.66	2392.72	2418.07	2443.63	2495.40
.8	2548.04	2601.57	2655.99	2683.54	2711.32	2767.57
.9	2824.75	2882.87	2941.94	2971.84	3001.98	3063.00
5.0	3125.00	3188.00	3252.01	3284.41	3317.05	3383.13

*Powers (N^5) are obtained directly as in the logarithmic tables; roots (N), by inverse interpolation.

Note that $N^5 = N^{2+3} = N^2 \times N^3$. Hence, the above values may be obtained by multiplying together the square and the cube of the respective numbers.

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—And Any Whole Number; or Decimal.

Changing 1 decimal place in the root or $N-5$ decimal places in the power or N^5.

N	N⁵ 0	2	4	5	6	8
5.0	3125.00	3188.00	3252.01	3284.41	3317.05	3383.13
.1	3450.25	3518.44	3587.70	3622.73	3658.04	3729.49
.2	3802.04	3875.72	3950.54	3988.38	4026.51	4103.64
.3	4181.95	4261.46	4342.17	4382.97	4424.09	4507.25
.4	4591.65	4677.31	4764.25	4808.20	4852.47	4942.00
5.5	5032.84	5125.02	5218.54	5265.81	5313.42	5409.67
.6	5507.32	5606.37	5706.84	5757.61	5808.74	5912.10
.7	6016.92	6123.22	6231.02	6285.49	6340.34	6451.18
.8	6563.57	6677.52	6793.04	6851.40	6910.16	7028.89
.9	7149.24	7271.24	7394.89	7457.36	7520.24	7647.26
6.0	7776.00	7906.47	8038.68	8105.45	8172.65	8308.41
.1	8445.96	8585.33	8726.54	8797.83	8869.59	9014.52
.2	9161.33	9310.05	9460.69	9536.74	9613.28	9767.83
.3	9924.37	10082.9	10243.5	10324.5	10406.0	10570.7
.4	10737.4	10906.2	11077.2	11163.5	11250.3	11425.5
6.5	11602.9	11782.5	11964.3	12056.1	12148.4	12334.7
.6	12523.3	12714.2	12907.5	13004.9	13103.0	13300.9
.7	13501.3	13704.0	13909.1	14012.6	14116.7	14326.8
.8	14539.3	14754.4	14972.0	15081.8	15192.2	15414.9
.9	15640.3	15868.3	16098.9	16215.3	16332.2	16568.3
7.0	16807.0	17048.5	17292.7	17415.9	17539.8	17789.6
.1	18042.3	18297.8	18556.3	18686.6	18817.6	19081.9
.2	19349.2	19619.4	19892.7	20030.4	20168.9	20448.3
.3	20730.7	21016.3	21305.0	21450.5	21596.8	21891.8
.4	22190.1	22491.5	22796.3	22949.9	23104.4	23415.7
7.5	23730.5	24048.6	24370.1	24532.1	24695.0	25023.4
.6	25355.3	25690.6	26029.6	26200.3	26372.1	26718.1
.7	27067.8	27421.2	27778.3	27958.2	28139.0	28503.5
.8	28871.7	29243.8	29619.7	29809.1	29999.4	30383.0
.9	30770.6	31162.1	31557.5	31756.7	31957.0	32360.4
8.0	32768.0	33179.7	33595.4	33804.9	34015.4	34439.5
.1	34867.8	35300.4	35737.3	35957.4	36178.5	36624.1
.2	37074.0	37528.3	37987.1	38218.1	38450.3	38918.1
.3	39390.4	39867.3	40348.8	40591.3	40834.9	41325.7
.4	41821.2	42321.4	42826.4	43080.8	43336.3	43851.0
8.5	44370.5	44895.0	45424.4	45691.0	45958.8	46498.2
.6	47042.7	47592.3	48146.9	48426.2	48706.8	49271.8
.7	49842.1	50417.6	50998.5	51290.9	51584.7	52176.2
.8	52773.2	53375.6	53983.6	54289.6	54597.0	55216.0
.9	55840.6	56470.9	57106.8	57426.9	57748.4	58395.8
9.0	59049.0	59708.0	60372.9	60707.6	61043.7	61720.4
.1	62403.2	63092.0	63786.8	64136.5	64487.8	65194.9
.2	65908.2	66627.6	67353.5	67718.7	68085.6	68824.0
.3	69568.8	70320.1	71077.9	71459.2	71842.1	72612.9
.4	73390.4	74174.5	74965.3	75363.1	75762.7	76567.0
9.5	77378.1	78196.0	79020.9	79435.9	79852.7	80691.4
.6	81537.3	82390.2	83250.1	83682.9	84117.3	84991.8
.7	85873.4	86762.4	87658.7	88109.6	88562.3	89473.5
.8	90392.1	91318.2	92251.9	92721.6	93193.3	94142.2
.9	95099.0	96063.5	97035.8	97524.9	98015.9	99004.0
10.0	100000					

Examples:—5th root of 7500 = 5.95 + .007 = 5.957.
5th root of 0.75 = .944.

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1 to 5.

8.—Reciprocals of Numbers—

Changing the decimal point N places in the number = N places in the *opposite direction* in the reciprocal.

N	1* N^0	1	2	3	4	5	6	7	8	9	(Subtract the diff.) P. P.				
1.00	Unity	.9990	.9980	.9970	.9960	.9950	.9940	.9930	.9921	.9911		-85	-75	-65	-55
.01	0.9901	9891	9881	9872	9862	9852	9843	9833	9823	9814	1	9	8	7	6
.02	9804	9794	9785	9775	9766	9756	9747	9737	9728	9718	2	17	15	13	11
.03	9709	9699	9690	9681	9671	9662	9653	9643	9634	9625	3	26	23	20	17
.04	9615	9606	9597	9588	9579	9569	9560	9551	9542	9533	4	34	30	26	22
1.05	0.9524	.9515	.9506	.9497	.9488	.9479	.9470	.9461	.9452	.9443	5	43	38	33	28
.06	9434	9425	9416	9407	9398	9390	9381	9372	9363	9355	6	51	45	39	33
.07	9346	9337	9328	9320	9311	9302	9294	9285	9276	9268	7	60	53	46	39
.08	9259	9251	9242	9234	9225	9217	9208	9200	9191	9183	8	68	60	52	44
.09	9174	9166	9158	9149	9141	9132	9124	9116	9107	9099	9	77	68	59	50
1.0	Unity	.9901	.9804	.9709	.9615	.9524	.9434	.9346	.9259	.9174		-45	-35	-25	-2
.1	0.9091	9009	8929	8850	8772	8696	8621	8547	8475	8403	1	5	4	3	2
.2	8333	8264	8197	8130	8065	8000	7937	7874	7813	7752	2	9	7	5	4
.3	7692	7634	7576	7519	7463	7407	7353	7299	7246	7194	3	14	11	8	7
.4	7143	7092	7042	6993	6944	6897	6849	6803	6757	6711	4	18	14	10	8
1.5	0.6667	.6623	.6579	.6536	.6494	.6452	.6410	.6369	.6329	.6289	5	23	18	13	11
.6	6250	6211	6173	6135	6098	6061	6024	5988	5952	5917	6	27	21	15	12
.7	5882	5848	5814	5780	5747	5714	5682	5650	5618	5587	7	32	25	18	11
.8	5556	5525	5495	5464	5435	5405	5376	5348	5319	5291	8	36	28	20	12
.9	5263	5236	5208	5181	5155	5128	5102	5076	5051	5025	9	41	32	23	14
2.0	0.5000	.4975	.4950	.4926	.4902	.4878	.4854	.4831	.4808	.4785		-18	-16	-14	-12
.1	4762	4739	4717	4695	4673	4651	4630	4608	4587	4566	1	2	2	1	1
.2	4545	4525	4505	4484	4464	4444	4425	4405	4386	4367	2	4	3	3	1
.3	4348	4329	4310	4292	4274	4255	4237	4219	4202	4184	3	5	5	4	1
.4	4167	4149	4132	4115	4098	4082	4065	4049	4032	4016	4	7	6	6	1
2.5	0.4000	.3984	.3968	.3953	.3937	.3922	.3906	.3891	.3876	.3861	5	9	8	7	6
.6	3846	3831	3817	3802	3788	3774	3759	3745	3731	3717	6	11	10	8	1
.7	3704	3690	3676	3663	3650	3636	3623	3610	3597	3584	7	13	11	10	1
.8	3571	3559	3546	3534	3521	3509	3497	3484	3472	3460	8	14	13	11	1
.9	3448	3436	3425	3413	3401	3390	3378	3367	3356	3344	9	16	14	13	1
3.0	0.3333	.3322	.3311	.3300	.3289	.3279	.3268	.3257	.3247	.3236		-11	-9	-8	-7
.1	3226	3215	3205	3195	3185	3175	3165	3155	3145	3135	1	1	1	1	1
.2	3125	3115	3106	3096	3086	3077	3067	3058	3049	3040	2	2	2	2	1
.3	3030	3021	3012	3003	2994	2985	2976	2967	2959	2950	3	3	3	2	1
.4	2941	2933	2924	2915	2907	2899	2890	2882	2874	2865	4	4	4	3	1
3.5	0.2857	.2849	.2841	.2833	.2825	.2817	.2809	.2801	.2793	.2786	5	6	5	4	
.6	2778	2770	2762	2755	2747	2740	2732	2725	2717	2710	6	7	5	5	
.7	2703	2695	2688	2681	2674	2667	2660	2653	2646	2639	7	8	6	6	
.8	2632	2625	2618	2611	2604	2597	2591	2584	2577	2571	8	9	7	6	
.9	2564	2558	2551	2545	2538	2532	2525	2519	2513	2506	9	10	8	7	
4.0	0.2500	.2494	.2488	.2481	.2475	.2469	.2463	.2457	.2451	.2445		-6	-5		
.1	2439	2433	2427	2421	2415	2410	2404	2398	2392	2387	1	0.6	0.5		
.2	2381	2375	2370	2364	2358	2353	2347	2342	2336	2331	2	1.2	1.0		
.3	2326	2320	2315	2309	2304	2299	2294	2288	2283	2278	3	1.8	1.5		
.4	2273	2268	2262	2257	2252	2247	2242	2237	2232	2227	4	2.4	2.0		
4.5	0.2222	.2217	.2212	.2208	.2203	.2198	.2193	.2188	.2183	.2179	5	3.0	2.5		
.6	2174	2169	2165	2160	2155	2151	2146	2141	2137	2132	6	3.6	3.0		
.7	2128	2123	2119	2114	2110	2105	2101	2096	2092	2088	7	4.2	3.5		
.8	2083	2079	2075	2070	2066	2062	2058	2053	2049	2045	8	4.8	4.0		
.9	2041	2037	2033	2028	2024	2020	2016	2012	2008	2004	9	5.4	4.5		
5.0	0.2000	.1996	.1992	.1998	.1984	.1980	.1976	.1972	.1969	.1965					

*Reciprocals to *four* decimal places on this page.

Ex.—Find reciprocal of 2.743? Solution—0.3650
−4
0.3646 Ans.

Reciprocal of 27.43 = 0.03646; of 0.2743 = 3.646; etc.

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5 to 10.

—WHOLE AND DECIMAL.

Changing the decimal point N places in the number = N places in the *opposite direction* in the reciprocal.

N		1† / N 0	1	2	3	4	5	6	7	8	9		(Subtract the dif.) P. P.			
5.0		0.1	9960	9920	9881	9841	9802	9763	9724	9685	9646		-38	-36	-34	-32
.1	0.1	9608	9569	9531	9493	9455	9417	9380	9342	9305	9268	1	4	4	3	3
.2		9231	9194	9157	9120	9084	9048	9011	8975	8939	8904	2	8	7	7	6
.3		8868	8832	8797	8762	8727	8692	8657	8622	8587	8553	3	11	11	10	10
.4		8519	8484	8450	8416	8382	8349	8315	8282	8248	8215	4	15	14	14	13
5.5	0.1	8182	8149	8116	8083	8051	8018	7986	7953	7921	7889	5	19	18	17	16
.6		7857	7825	7794	7762	7731	7699	7668	7637	7606	7525	6	23	22	20	19
.7		7544	7513	7483	7453	7422	7391	7361	7331	7301	7271	7	27	25	24	22
.8		7241	7212	7182	7153	7123	7094	7065	7036	7007	6978	8	30	29	27	26
.9		6949	6920	6892	6863	6835	6807	6779	6750	6722	6694	9	34	32	31	29
6.0	0.1	6667	6639	6611	6584	6556	6529	6502	6474	6447	6420		-28	-26	-24	-22
.1		6393	6367	6340	6313	6287	6260	6234	6207	6181	6155	1	3	3	2	2
.2		6129	6103	6077	6051	6026	6000	5974	5949	5924	5898	2	6	5	5	4
.3		5873	5848	5823	5798	5773	5748	5723	5699	5674	5649	3	8	8	7	7
.4		5625	5601	5576	5552	5528	5504	5480	5456	5432	5408	4	11	10	10	9
6.5	0.1	5385	5361	5337	5314	5291	5267	5244	5221	5198	5175	5	14	13	12	11
.6		5152	5129	5106	5083	5060	5038	5015	4993	4970	4948	6	17	16	14	13
.7		4925	4903	4881	4859	4837	4815	4793	4771	4749	4728	7	20	18	17	15
.8		4706	4684	4663	4641	4620	4599	4577	4556	4535	4514	8	22	21	19	18
.9		4493	4472	4451	4430	4409	4388	4368	4347	4327	4306	9	25	23	22	20
7.0	0.1	4286	4265	4245	4225	4205	4184	4164	4144	4124	4104		-21	-19	-17	-16
.1		4085	4065	4045	4025	4006	3986	3966	3947	3928	3908	1	2	2	2	2
.2		3889	3870	3850	3831	3812	3793	3774	3755	3736	3717	2	4	4	3	3
.3		3699	3680	3661	3643	3624	3605	3587	3569	3550	3532	3	6	6	5	5
.4		3514	3495	3477	3459	3441	3423	3405	3387	3369	3351	4	8	8	7	6
7.5	0.1	3333	3316	3298	3280	3263	3245	3228	3210	3193	3175	5	11	10	9	8
.6		3158	3141	3123	3106	3089	3072	3055	3038	3021	3004	6	13	11	10	10
.7		2987	2970	2953	2937	2920	2903	2887	2870	2853	2837	7	15	13	12	11
.8		2821	2804	2788	2771	2755	2739	2723	2706	2690	2674	8	17	15	14	13
.9		2658	2642	2626	2610	2594	2579	2563	2547	2531	2516	9	19	17	15	14
8.0	0.1	2500	2484	2469	2453	2438	2422	2407	2392	2376	2361		-15	-14	-13	-12
.1		2346	2330	2315	2300	2285	2270	2255	2240	2225	2210	1	2	1	1	1
.2		2195	2180	2165	2151	2136	2121	2107	2092	2077	2063	2	3	3	3	2
.3		2048	2034	2019	2005	1990	1976	1962	1947	1933	1919	3	5	4	4	4
.4		1905	1891	1876	1862	1848	1834	1820	1806	1792	1779	4	6	6	5	5
8.5	0.1	1765	1751	1737	1723	1710	1696	1682	1669	1655	1641	5	8	7	7	6
.6		1628	1614	1601	1587	1574	1561	1547	1534	1521	1507	6	9	8	8	7
.7		1494	1481	1468	1455	1442	1429	1416	1403	1390	1377	7	11	10	9	8
.8		1364	1351	1338	1325	1312	1299	1287	1274	1261	1249	8	12	11	11	10
.9		1236	1223	1211	1198	1186	1173	1161	1148	1136	1123	9	14	13	12	11
9.0	0.1	1111	1099	1086	1074	1062	1050	1038	1025	1013	1001		-13	-12	-11	-10
.1		0989	0977	0965	0953	0941	0929	0917	0905	0893	0881	1	1	1	1	1
.2		0870	0858	0846	0834	0823	0811	0799	0787	0776	0764	2	3	2	2	2
.3		0753	0741	0730	0718	0707	0695	0684	0672	0661	0650	3	4	4	3	3
.4		0638	0627	0616	0604	0593	0582	0571	0560	0549	0537	4	5	5	4	4
9.5	0.1	0526	0515	0504	0493	0482	0471	0460	0449	0438	0428	5	7	6	6	5
.6		0417	0406	0395	0384	0373	0363	0352	0341	0331	0320	6	8	7	7	6
.7		0309	0299	0288	0277	0267	0256	0246	0235	0225	0215	7	9	8	8	7
.8		0204	0194	0183	0173	0163	0152	0142	0132	0121	0111	8	11	10	9	8
.9		0101	0091	0081	0070	0060	0050	0040	0030	0020	0010	9	12	11	10	9
10.0	0.1	0000														

†Reciprocals to *five* decimal places on this page.

Ex.—Find reciprocal of 1.2368 by the inverse method.

Solution—8.08 for 0.12376
.005 —8
8.085 for 0.12368
∴ Ans. = 0.8085 for 1.2368

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Reciprocals.—The reciprocal of a number is 1 divided by that number. Thus, the reciprocal of $2.77 = .361$, and likewise the reciprocal of .361 is 2.77; or $2.77 = 1/.361$ and $.361 = 1/2.77$. Hence, to divide any quantity by a number, as 2.77, is equivalent to multiplying it by its reciprocal, $1/2.77$ or .361.

As multiplication is usually performed more readily than division, it is convenient to multiply by the reciprocal of a number rather than to divide by the number itself.

Table 8* comprises the reciprocals of numbers from 1 to 10, advancing by decimals as in logarithmic tables, so as to include a wide range of numbers. Note that changing the decimal point *n* places in the number equals a change of *n* places in the *opposite direction* in the reciprocal; also that the differences between any reciprocal and the following one is a *minus difference*, hence the proportional part must be *subtracted*.

Example.—What is the reciprocal of 79.15?

Solution.—From the table, the reciprocal of $7.91 = .12642$; the proportional part for 5 (under the difference, -16) is -8. Hence, the reciprocal of 7.915 is .12634, and of 79.15 is 0.012634. Ans.

B. ARITHMETICAL OR COMMON TABLES.

The preceding tables are arranged in decimal form, and are called Engineers' Tables; while the arithmetical tables are in the form most commonly used, and are arranged as follows:—

Table 9—Squares, cubes, square roots, cube roots, of numbers, 1 to 1000, page 31:

Nos.	0—130,	page 31.	Nos.	910—1040,	page 38.
"	130—260,	" 32.	"	1040—1170,	" 39.
"	260—390,	" 33.	"	1170—1300,	" 40.
"	390—520,	" 34.	"	1300—1430,	" 41.
"	520—650,	" 35.	"	1430—1560,	" 42.
"	650—780,	" 36.	"	1560—1600,	" 43.
"	780—910,	" 37.			

Table 10—Square roots and cube roots of numbers 1600 to 3200, page 44:

Nos.	1600—1860,	page 44.	Nos.	2640—2900,	page 48.
"	1860—2120,	" 45.	"	2900—3160,	" 49.
"	2120—2380,	" 46.	"	3160—3200,	" 50.
"	2380—2640,	" 47.			

Table 11—Reciprocals of numbers 1 to 1000, page 51:

Nos.	1— 325,	page 51.	Nos.	650— 975,	page 53.
"	325— 650,	" 52.	"	975—1000,	" 54.

* Pages 28 and 29.

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9.—Squares, Cubes, Square Roots, Cube Roots, of Numbers 1 to 1600.

No.	Square	Cube.	Sq. Rt.	Cu. Rt.	No.	Square	Cube.	Sq. Rt.	Cu. Rt.
0	0	0	0.0000000	0.0000000	65	42 25	274 625	8.0622577	4.0207256
1	1	1	1.0000000	1.0000000	6	43 56	287 496	.1240384	.0412401
2	4	8	.4142136	.2599210	7	44 89	300 763	.1853528	.0615480
3	9	27	.7320508	.4422496	8	46 24	314 432	.2462113	.0816551
4	16	64	2.0000000	.5874011	9	47 61	328 509	.3066239	.1015661
5	25	125	2.2360680	1.7099759	70	49 00	343 000	8.3666003	4.1212853
6	36	216	.4494897	.8171206	1	50 41	357 911	.4261498	.1408178
7	49	343	.6457513	.9129312	2	51 84	373 248	.4852814	.1601676
8	64	512	.8284271	2.0000000	3	53 29	389 017	.5440037	.1793392
9	81	729	3.0000000	.0800837	4	54 76	405 224	.6023253	.1983364
10	1 00	1 000	3.1622777	2.1544347	75	56 25	421 875	8.6602540	4.2171633
11	1 21	1 331	.3166248	.2239801	6	57 76	438 976	.7177979	.2358236
12	1 44	1 728	.4641016	.2894286	7	59 29	456 533	.7749644	.2543210
13	1 69	2 197	.6055513	.3513347	8	60 84	474 552	.8317609	.2726586
14	1 96	2 744	.7416574	.4101422	9	62 41	493 039	.8881944	2908404
15	2 25	3 375	3.8729833	2.4662121	80	64 00	512 000	8.9442719	4.3088695
16	2 56	4 096	4.0000000	.5198421	1	65 61	531 441	9.0000000	.3267487
17	2 89	4 913	.1231056	.5712816	2	67 24	551 368	.0553851	.3444815
18	3 24	5 832	.2426407	.6207414	3	68 89	571 787	.1104336	.3620707
19	3 61	6 859	.3588989	.6684016	4	70 56	592 704	.1651514	.3795191
20	4 00	8 000	4.4721360	2.7144177	85	72 25	614 125	9.2195445	4.3968296
1	4 41	9 261	.5825757	2.7589243	6	73 96	636 056	.2736185	.4140049
2	4 84	10 648	.6904158	.8020393	7	75 69	658 503	.3273791	.4310476
3	5 29	12 167	.7958315	.8438670	8	77 44	681 472	.3808315	.4479602
4	5 76	13 824	.8989795	.8844991	9	79 21	704 969	.4339811	.4647451
25	6 25	15 625	5.0000000	2.9240177	90	81 00	729 000	9.4868330	4.4814047
6	6 76	17 576	.0990195	.9624960	1	82 81	753 571	.5393920	.4979414
7	7 29	19 683	.1961524	3.0000000	2	84 64	778 688	.5916630	.5143574
8	7 84	21 952	.2915026	.0365889	3	86 49	804 357	.6436508	.5306549
9	8 41	24 389	.3851648	.0723168	4	88 36	830 584	.6953597	.5468359
30	9 00	27 000	5.4772256	3.1072325	95	90 25	857 375	9.7467943	4.5629026
1	9 61	29 791	.5677644	.1413806	6	92 16	884 736	.7979590	.5788570
2	10 24	32 768	.6568542	.1748021	7	94 09	912 673	.8488578	.5947009
3	10 89	35 937	.7445626	.2075343	8	96 04	941 192	.8994949	.6104363
4	11 56	39 304	.8309519	.2396118	9	98 01	970 299	.9498744	.6260650
35	12 25	42 875	5.9160798	3.2710663	100	1 00 00	1 000 000	10.0000000	4.6415888
6	12 96	46 656	6.0000000	.3019272	1	1 02 01	1 030 301	.0498756	.6570095
7	13 69	50 653	.0827625	.3322218	2	1 04 04	1 061 208	.0995049	.6723287
8	14 44	54 872	.1644140	.3619754	3	1 06 09	1 092 727	.1488916	.6875482
9	15 21	59 319	.2449980	.3912114	4	1 08 16	1 124 864	.1980390	.7026694
40	16 00	64 000	6.3245553	3.4199519	105	1 10 25	1 157 625	10.2469508	4.7176940
1	16 81	68 921	.4031242	.4482172	6	1 12 36	1 191 016	.2956301	.7326235
2	17 64	74 088	.4807407	.4760266	7	1 14 49	1 225 043	.3440804	.7474594
3	18 49	79 507	.5574385	.5033981	8	1 16 64	1 259 712	.3923048	.7622032
4	19 36	85 184	.6332495	.5303483	9	1 18 81	1 295 029	.4403065	.7768562
45	20 25	91 125	6.7082039	3.5568933	110	1 21 00	1 331 000	10.4880885	4.7914199
6	21 16	97 336	.7823300	.5830479	11	1 23 21	1 367 631	.5356538	.8058955
7	22 09	103 823	.8556546	.6088261	12	1 25 44	1 404 928	.5830052	.8202845
8	23 04	110 592	.9282032	.6342411	13	1 27 69	1 442 897	.6301458	.8345881
9	24 01	117 649	7.0000000	.6593057	14	1 29 96	1 481 544	.6770783	.8488076
50	25 00	125 000	7.0710678	3.6840314	115	1 32 25	1 520 875	10.7238053	4.8629442
1	26 01	132 651	.1414284	.7084298	16	1 34 56	1 560 896	.7703296	.8769990
2	27 04	140 608	.2111026	.7325111	17	1 36 89	1 601 613	8166538	.8909732
3	28 09	148 877	.2801099	.7562858	18	1 39 24	1 643 032	.8627805	.9048681
4	29 16	157 464	.3484692	.7797631	19	1 41 61	1 685 159	.9087121	.9186847
55	30 25	166 375	7.4161985	3.8029525	120	1 44 00	1 728 000	10.9544512	4.9324242
6	31 36	175 616	.4833148	.8258624	1	1 46 41	1 771 561	11.0000000	.9460874
7	32 49	185 193	.5498344	.8485011	2	1 48 84	1 815 848	.0453610	.9596757
8	33 64	195 112	.6157731	.8708766	3	1 51 29	1 860 867	.0905365	.9731898
9	34 81	205 379	.6811457	.8929965	4	1 53 76	1 906 624	.1355287	.9866310
60	36 00	216 000	7.7459667	3.9148676	125	1 56 25	1 953 125	11.1803399	5.0000000
1	37 21	226 981	.8102497	.9364972	6	1 58 76	2 000 376	.2249722	.0132979
2	38 44	238 328	.8740079	.9578915	7	1 61 29	2 048 383	.2694277	.0265257
3	39 69	250 047	9372539	.9790571	8	1 63 84	2 097 152	3137085	.0396842
4	40 96	262 144	8.0000000	4.0000000	9	1 66 41	2 146 689	.3578167	.0527743
65	42 25	274 625	8.0622577	4.0207256	130	1 69 00	2 197 000	11.4017543	5.0657970

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9.—Squares, Cubes, Square Roots, Cube Roots, of Numbers 1 to 1600—Continued.

No.	Square	Cube.	Sq. Rt.	Cu. Rt.	No.	Square	Cube.	Sq. Rt.	Cu. Rt.
130	1 69 00	2 197 000	11.4017543	5.0657970	195	3 80 25	7 414 875	13.9642400	5.7988900
1	1 71 61	2 248 091	.4455231	.0787531	6	3 84 16	7 529 536	14.0000000	.8087857
2	1 74 24	2 299 968	.4891253	.0916434	7	3 88 09	7 645 373	.0356688	.8186479
3	1 76 89	2 352 637	.5325626	.1044687	8	3 92 04	7 762 392	.0712473	.8284767
4	1 79 56	2 406 104	.5758369	.1172299	9	3 96 01	7 880 599	.1067360	.8382725
135	1 82 25	2 460 375	11.6189500	5.1299278	200	4 00 00	8 000 000	14.1421356	5.8480355
6	1 84 96	2 515 456	.6619038	.1425632	1	4 04 01	8 120 601	.1774469	.8577660
7	1 87 69	2 571 353	.7046999	.1551367	2	4 08 04	8 242 408	.2126704	.8674643
8	1 90 44	2 628 072	.7473401	.1676493	3	4 12 09	8 365 427	.2478068	.8771307
9	1 93 21	2 685 619	.7898261	.1801015	4	4 16 16	8 489 664	.2828569	.8867653
140	1 96 00	2 744 000	11.8321596	5.1924941	205	4 20 25	8 615 125	14.3178211	5.8963685
1	1 98 81	2 803 221	.8743422	.2048279	6	4 24 36	8 741 816	.3527001	.9059406
2	2 01 64	2 863 288	.9163753	.2171034	7	4 28 49	8 869 743	.3874946	.9154817
3	2 04 49	2 924 207	.9582607	.2293215	8	4 32 64	8 998 912	.4222051	.9249921
4	2 07 36	2 985 984	12.0000000	.2414828	9	4 36 81	9 129 329	.4568323	.9344721
145	2 10 25	3 048 625	12.0415946	5.2535879	210	4 41 00	9 261 000	14.4913767	5.9439220
6	2 13 16	3 112 136	.0830460	.2656374	11	4 45 21	9 393 931	.5258390	.9533418
7	2 16 09	3 176 523	.1243557	.2776321	12	4 49 44	9 528 128	.5602198	.9627320
8	2 19 04	3 241 792	.1655251	.2895725	13	4 53 69	9 663 597	.5945195	.9720926
9	2 22 01	3 307 949	.2065556	.3014592	14	4 57 96	9 800 344	.6287388	.9814240
150	2 25 00	3 375 000	12.2474487	5.3132928	215	4 62 25	9 938 375	14.6628783	5.9907264
1	2 28 01	3 442 951	.2882057	.3250740	16	4 66 56	10 077 696	.6969385	6.0000000
2	2 31 04	3 511 808	.3288280	.3368033	17	4 70 89	10 218 313	.7309199	.0092450
3	2 34 09	3 581 577	.3693169	.3484812	18	4 75 24	10 360 232	.7648231	.0184617
4	2 37 16	3 652 264	.4096736	.3601084	19	4 79 61	10 503 459	.7986486	.0276502
155	2 40 25	3 723 875	12.4498996	5.3716854	220	4 84 00	10 648 000	14.8323970	6.0368107
6	2 43 36	3 796 416	.4899960	.3832126	1	4 88 41	10 793 861	.8660687	.0459435
7	2 46 49	3 869 893	.5299641	.3946907	2	4 92 84	10 941 048	.8996644	.0550489
8	2 49 64	3 944 312	.5698051	.4061202	3	4 97 29	11 089 567	.9331845	.0641270
9	2 52 81	4 019 679	.6095202	.4175015	4	5 01 76	11 239 424	.9666295	.0731779
160	2 56 00	4 096 000	12.6491106	5.4288352	225	5 06 25	11 390 625	15.0000000	6.0822020
1	2 59 21	4 173 281	.6885775	.4401218	6	5 10 76	11 543 176	.0332964	.0911994
2	2 62 44	4 251 528	.7279221	.4513618	7	5 15 29	11 697 083	.0665192	.1001702
3	2 65 69	4 330 747	.7671453	.4625556	8	5 19 84	11 852 352	.0996689	.1091147
4	2 68 96	4 410 944	.8062485	.4737037	9	5 24 41	12 008 989	.1327460	.1180332
165	2 72 25	4 492 125	12.8452326	5.4848066	230	5 29 00	12 167 000	15.1657509	6.1269257
6	2 75 56	4 574 296	.8840987	.4958647	1	5 33 61	12 326 391	.1986842	.1357924
7	2 78 89	4 657 463	.9228480	.5068784	2	5 38 24	12 487 168	.2315462	.1446337
8	2 82 24	4 741 632	.9614814	.5178484	3	5 42 89	12 649 337	.2643375	.1534495
9	2 85 61	4 826 809	13.0000000	.5287748	4	5 47 56	12 812 904	.2970585	.1622401
170	2 89 00	4 913 000	13.0384048	5.5396583	235	5 52 25	12 977 875	15.3297097	6.1710058
1	2 92 41	5 000 211	.0766968	.5504991	6	5 56 96	13 144 256	.3622915	.1797466
2	2 95 84	5 088 448	.1148770	.5612978	7	5 61 69	13 312 053	.3948043	.1884628
3	2 99 29	5 177 717	.1529464	.5720546	8	5 66 44	13 481 272	.4272486	.1971544
4	3 02 76	5 268 024	.1909060	.5827702	9	5 71 21	13 651 919	.4596248	.2058218
175	3 06 25	5 359 375	13.2287566	5.5934447	240	5 76 00	13 824 000	15.4919334	6.2144650
6	3 09 76	5 451 776	.2664992	.6040787	1	5 80 81	13 997 521	.5241747	.2230843
7	3 13 29	5 545 233	.3041347	.6146724	2	5 85 64	14 172 488	.5563492	.2316797
8	3 16 84	5 639 752	.3416641	.6252263	3	5 90 49	14 348 907	.5884573	.2402515
9	3 20 41	5 735 339	.3790882	.6357408	4	5 95 36	14 526 784	.6204994	.2487998
180	3 24 00	5 832 000	13.4164079	5.6462162	245	6 00 25	14 706 125	15.6524758	6.2573248
1	3 27 61	5 929 741	.4536240	.6566528	6	6 05 16	14 886 936	.6843871	.2658266
2	3 31 24	6 028 568	.4907376	.6670511	7	6 10 09	15 069 223	.7162336	.2743054
3	3 34 89	6 128 487	.5277493	.6774114	8	6 15 04	15 252 992	.7480157	.2827613
4	3 38 56	6 229 504	.5646600	.6877340	9	6 20 01	15 438 249	.7797338	.2911946
185	3 42 25	6 331 625	13.6014705	5.6980192	250	6 25 00	15 625 000	15.8113883	6.2996053
6	3 45 96	6 434 856	.6381817	.7082675	1	6 30 01	15 813 251	.8429795	.3079935
7	3 49 69	6 539 203	.6747943	.7184791	2	6 35 04	16 003 008	.8745079	.3163596
8	3 53 44	6 644 672	.7113092	.7286543	3	6 40 09	16 194 277	.9059737	.3247035
9	3 57 21	6 751 269	.7477271	.7387936	4	6 45 16	16 387 064	.9373775	.3330256
190	3 61 00	6 859 000	13.7840488	5.7488971	255	6 50 25	16 581 375	15.9687194	6.3413257
1	3 64 81	6 967 871	.8202750	.7589652	6	6 55 36	16 777 216	16.0000000	.3496042
2	3 68 64	7 077 888	.8564065	.7689982	7	6 60 49	16 974 593	.0312195	.3578611
3	3 72 49	7 189 057	.8924440	.7789966	8	6 65 64	17 173 512	.0623784	.3660968
4	3 76 36	7 301 384	.9283883	.7889604	9	6 70 81	17 373 979	.0934769	.3743111
195	3 80 25	7 414 875	13.9642400	5.7988900	260	6 76 00	17 576 000	16.1245155	6.3825043

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9.—Squares, Cubes, Square Roots, Cube Roots, of Numbers 1 to 1600—Continued.

No.	Square	Cube.	Sq. Rt.	Cu. Rt.	No.	Square	Cube.	Sq. Rt.	Cu. Rt.
260	6 76 00	17 576 000	16.1245155	6.3825043	325	10 56 25	34 328 125	18.0277564	6.8753443
1	6 81 21	17 779 581	.1554944	.3906765	6	10 62 76	34 645 976	.0554701	.8823888
2	6 86 44	17 984 728	.1864141	.3988279	7	10 69 29	34 965 783	.0831413	.8894188
3	6 91 69	18 191 447	.2172747	.4069585	8	10 75 84	35 287 552	.1107703	.8964345
4	6 96 96	18 399 744	.2480768	.4150687	9	10 82 41	35 611 289	.1383571	.9034359
265	7 02 25	18 609 625	16.2788206	6.4231583	330	10 89 00	35 937 000	18.1659021	6.9104232
6	7 07 56	18 821 096	.3095064	.4312276	1	10 95 61	36 264 691	.1934054	.9173964
7	7 12 89	19 034 163	.3401346	.4392767	2	11 02 24	36 594 368	.2208672	.9243556
8	7 18 24	19 248 832	.3707055	.4473057	3	11 08 89	36 926 037	.2482876	.9313008
9	7 23 61	19 465 109	.4012195	.4553148	4	11 15 56	37 259 704	.2756669	.9382321
270	7 29 00	19 683 000	16.4316767	6.4633041	335	11 22 25	37 595 375	18.3030052	6.9451496
1	7 34 41	19 902 511	.4620776	.4712736	6	11 28 96	37 933 056	.3303028	.9520533
2	7 39 84	20 123 648	.4924225	.4792236	7	11 35 69	38 272 753	.3575598	.9589434
3	7 45 29	20 346 417	.5227116	.4871541	8	11 42 44	38 614 472	.3847763	.9658198
4	7 50 76	20 570 824	.5529454	.4950653	9	11 49 21	38 958 219	.4119526	.9726826
275	7 56 25	20 796 875	16.5831240	6.5029572	340	11 56 00	39 304 000	18.4390889	6.9795321
6	7 61 76	21 024 576	.6132477	.5108300	1	11 62 81	39 651 821	.4661853	.9863681
7	7 67 29	21 253 933	.6433170	.5186839	2	11 69 64	40 001 688	.4932420	.9931906
8	7 72 84	21 484 952	.6733320	.5265189	3	11 76 49	40 353 607	.5202592	7.0000000
9	7 78 41	21 717 639	.7032931	.5343351	4	11 83 36	40 707 584	.5472370	.0067962
280	7 84 00	21 952 000	16.7332005	6.5421326	345	11 90 25	41 063 625	18.5741756	7.0135791
1	7 89 61	22 188 041	.7630546	.5499116	6	11 97 16	41 421 736	.6010752	.0203490
2	7 95 24	22 425 768	.7928556	.5576722	7	12 04 09	41 781 923	.6279360	.0271058
3	8 00 89	22 665 187	.8226038	.5654144	8	12 11 04	42 144 192	.6547581	.0338497
4	8 06 56	22 906 304	.8522995	.5731385	9	12 18 01	42 508 549	.6815417	.0405806
285	8 12 25	23 149 125	16.8819430	6.5808443	350	12 25 00	42 875 000	18.7082869	7.0472987
6	8 17 96	23 393 656	.9115345	.5885323	1	12 32 01	43 243 551	.7349940	.0540041
7	8 23 69	23 639 903	.9410743	.5962023	2	12 39 04	43 614 208	.7616630	.0606967
8	8 29 44	23 887 872	.9705627	.6038545	3	12 46 09	43 986 977	.7882942	.0673767
9	8 35 21	24 137 569	17.0000000	.6114890	4	12 53 16	44 361 864	.8148877	.0740440
290	8 41 00	24 389 000	17.0293864	6.6191060	355	12 60 25	44 738 875	18.8414437	7.0806988
1	8 46 81	24 642 171	.0587221	.6267054	6	12 67 36	45 118 016	.8679623	.0873411
2	8 52 64	24 897 088	.0880075	.6342874	7	12 74 49	45 499 293	.8944436	.0939709
3	8 58 49	25 153 757	.1172428	.6418522	8	12 81 64	45 882 712	.9208879	.1005885
4	8 64 36	25 412 184	.1464282	.6493998	9	12 88 81	46 268 279	.9472953	.1071937
295	8 70 25	25 672 375	17.1755640	6.6569302	360	12 96 00	46 656 000	18.9736660	7.1137866
6	8 76 16	25 934 336	.2046505	.6644437	1	13 03 21	47 045 881	19.0000000	.1203674
7	8 82 09	26 198 073	.2336879	.6719403	2	13 10 44	47 437 928	.0262976	.1269369
8	8 88 04	26 463 592	.2626765	.6794200	3	13 17 69	47 832 147	.0525589	.1334925
9	8 94 01	26 730 899	.2916165	.6868831	4	13 24 96	48 228 544	.0787840	.1400370
300	9 00 00	27 000 000	17.3205081	6.6943295	365	13 32 25	48 627 125	19.1049732	7.0465695
1	9 06 01	27 270 901	.3493516	.7017593	6	13 39 56	49 027 896	.1311265	.1530901
2	9 12 04	27 543 608	.3781472	.7091729	7	13 46 89	49 430 863	.1572441	.1595988
3	9 18 09	27 818 127	.4068952	.7165700	8	13 54 24	49 836 032	.1833261	.1660957
4	9 24 16	28 094 464	.4355958	.7239508	9	13 61 61	50 243 409	.2093727	.1725809
305	9 30 25	28 372 625	17.4642492	6.7313155	370	13 69 00	50 653 000	19.2353841	7.1790544
6	9 36 36	28 652 616	.4928557	.7386641	1	13 76 41	51 064 811	.2613603	.1855162
7	9 42 49	28 934 443	.5214155	.7459967	2	13 83 84	51 478 848	.2873015	.1919663
8	9 48 64	29 218 112	.5499288	.7533134	3	13 91 29	51 895 117	.3132079	.1984050
9	9 54 81	29 503 629	.5783958	.7606143	4	13 98 76	52 313 624	.3390796	.2048322
310	9 61 00	29 791 000	17.6068169	6.7678995	375	14 06 25	52 734 375	19.3649167	7.2112479
11	9 67 21	30 080 231	.6351921	.7751690	6	14 13 76	53 157 376	.3907194	.2176522
12	9 73 44	30 371 328	.6635217	.7824229	7	14 21 29	53 582 633	.4164878	.2240450
13	9 79 69	30 664 297	.6918060	.7896613	8	14 28 84	54 010 152	.4422221	.2304268
14	9 85 96	30 959 144	.7200451	.7968844	9	14 36 41	54 439 939	.4679223	.2367972
315	9 92 25	31 255 875	17.7482393	6.8040921	380	14 44 00	54 872 000	19.4935887	7.2431565
16	9 98 56	31 554 496	.7763888	.8112847	1	14 51 61	55 306 341	.5192213	.2495045
17	10 04 89	31 855 013	.8044938	.8184620	2	14 59 24	55 742 968	.5448203	.2558415
18	10 11 24	32 157 432	.8325545	.8256242	3	14 66 89	56 181 887	.5703858	.2621675
19	10 17 61	32 461 759	.8605711	.8327714	4	14 74 56	56 623 104	.5959179	.2684824
320	10 24 00	32 768 000	17.8885438	6.8399037	385	14 82 25	57 066 625	19 6214169	7.2747864
1	10 30 41	33 076 161	.9164729	.8470213	6	14 89 96	57 512 456	.6468827	.2810794
2	10 36 84	33 386 248	.9443584	.8541240	7	14 97 69	57 960 603	.6723156	.2873617
3	10 43 29	33 698 267	.9722008	.8612120	8	15 05 44	58 411 072	.6977156	.2936330
4	10 49 76	34 012 224	18.0000000	.8682855	9	15 13 21	58 863 869	.7230829	.2998936
325	10 56 25	34 328 125	18.0277564	6.8753443	390	15 21 00	59 319 000	19.7484177	7.3061436

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9.—Squares, Cubes, Square Roots, Cube Roots, of Numbers 1 to 1600—Continued.

No.	Square	Cube.	Sq. Rt.	Cu. Rt.	No.	Square	Cube.	Sq. Rt.	Cu. Rt.
390	15 21 00	59 319 000	19.7484177	7.3061436	455	20 70 25	94 196 375	21.3307290	7.6913717
1	15 28 81	59 776 471	.7737199	.3123828	6	20 79 36	94 818 816	.3541565	.6970863
2	15 36 64	60 236 288	.7989899	.3186114	7	20 88 49	95 443 993	.3775583	.7026246
3	15 44 49	60 698 457	.8242276	.3248295	8	20 97 64	96 071 912	.4009346	.7082288
4	15 52 36	61 162 984	.8494332	.3310369	9	21 06 81	96 703 579	.4242853	.7138448
395	15 60 25	61 629 875	19.8746069	7.3372339	460	21 16 00	97 336 000	21.4476106	7.7194426
6	15 68 16	62 099 136	.8997487	.3434205	1	21 25 21	97 972 181	.4709106	.7250325
7	15 76 09	62 570 773	.9248588	.3495966	2	21 34 44	98 611 128	.4941853	.7306141
8	15 84 04	63 044 792	.9499373	.3557624	3	21 43 69	99 252 847	.5174348	.7361877
9	15 92 01	63 521 199	.9749844	.3619178	4	21 52 96	99 897 344	.5406592	.7417532
400	16 00 00	64 000 000	20.0000000	7.3680630	465	21 62 25	100 544 625	21.5638587	7.7473109
1	16 08 01	64 481 201	.0249844	.3741979	6	21 71 56	101 194 696	.5870331	.7528606
2	16 16 04	64 964 808	.0499377	.3803227	7	21 80 89	101 847 563	.6101828	.7584023
3	16 24 09	65 450 827	.0748599	.3864373	8	21 90 24	102 503 232	.6333077	.7639361
4	16 32 16	65 939 264	.0997512	.3925418	9	21 99 61	103 161 709	.6564078	.7694620
405	16 40 25	66 430 125	20.1246118	7.3986363	470	22 09 00	103 823 000	21.6794834	7.7749801
6	16 48 36	66 923 416	.1494417	.4047206	1	22 18 41	104 487 111	.7025344	.7804904
7	16 56 49	67 419 143	.1742410	.4107950	2	22 27 84	105 154 048	.7255610	.7859928
8	16 64 64	67 917 312	.1990099	.4168595	3	22 37 29	105 823 817	.7485632	.7914875
9	16 72 81	68 417 929	.2237484	.4229142	4	22 46 76	106 496 424	.7715411	.7969745
410	16 81 00	68 921 000	20.2484567	7.4289589	475	22 56 25	107 171 875	21.7944947	7.8024538
11	16 89 21	69 426 531	.2731349	.4349938	6	22 65 76	107 850 176	.8174242	.8079254
12	16 97 44	69 934 528	.2977831	.4410189	7	22 75 29	108 531 333	.8403297	.8133892
13	17 05 69	70 444 997	.3224014	.4470342	8	22 84 84	109 215 352	.8632111	.8188454
14	17 13 96	70 957 944	.3469899	.4530399	9	22 94 41	109 902 239	.8860686	.8242942
415	17 22 25	71 473 375	20.3715488	7.4590359	480	23 04 00	110 592 000	21.9089023	7.8297353
16	17 30 56	71 991 296	.3960781	.4650223	1	23 13 61	111 284 641	.9317122	.8351688
17	17 38 89	72 511 713	.4206779	.4709991	2	23 23 24	111 980 168	.9544984	.8406949
18	17 47 24	73 034 632	.4450483	.4769664	3	23 32 89	112 678 587	.9772610	.8460134
19	17 55 61	73 560 059	.4694895	.4829242	4	23 42 56	113 379 904	22.0000000	.8514244
420	17 64 00	74 088 000	20.4939015	7.4888724	485	23 52 25	114 084 125	22.0227155	7.8568281
1	17 72 41	74 618 461	.5182845	.4948113	6	23 61 96	114 791 256	.0454077	.8622242
2	17 80 84	75 151 448	.5426386	.5007406	7	23 71 69	115 501 303	.0680765	.8676130
3	17 89 29	75 686 967	.5669638	.5066607	8	23 81 44	116 214 272	.0907220	.8729944
4	17 97 76	76 225 024	.5912603	.5125715	9	23 91 21	116 930 169	.1133444	.8783684
425	18 06 25	76 765 625	20.6155281	7.5184730	490	24 01 00	117 649 000	22.1359436	7.8837352
6	18 14 76	77 308 776	.6397674	.5243652	1	24 10 81	118 370 771	.1585198	.8890946
7	18 23 29	77 854 483	.6639783	.5302482	2	24 20 64	119 095 488	.1810730	.8944468
8	18 31 84	78 402 752	.6881609	.5361231	3	24 30 49	119 823 157	.2036033	.8997917
9	18 40 41	78 953 589	.7123152	.5419867	4	24 40 36	120 553 784	.2261108	.9051294
430	18 49 00	79 507 000	20.7364414	7.5478423	495	24 50 25	121 287 375	22.2485955	7.9104599
1	18 57 61	80 062 991	.7605395	.5536888	6	24 60 16	122 023 936	.2710575	.9157832
2	18 66 24	80 621 568	.7846097	.5595263	7	24 70 09	122 763 473	.2934968	.9210994
3	18 74 89	81 182 737	.8086520	.5653548	8	24 80 04	123 505 992	.3159136	.9264085
4	18 83 56	81 746 504	.8326667	.5711743	9	24 90 01	124 251 499	.3383079	.9317104
435	18 92 25	82 312 875	20.8566536	7.5769849	500	25 00 00	125 000 000	22.3606798	7.9370053
6	19 00 96	82 881 856	.8806130	.5827865	1	25 10 01	125 751 501	.3830293	.9422931
7	19 09 69	83 453 453	.9045450	.5885793	2	25 20 04	126 506 008	.4053565	.9475739
8	19 18 44	84 027 672	.9284495	.5943633	3	25 30 09	127 263 527	.4276615	.9528477
9	19 27 21	84 604 519	.9523268	.6001385	4	25 40 16	128 024 064	.4499443	.9581144
440	19 36 00	85 184 000	20.9761770	7.6059049	505	25 50 25	128 787 625	22.4722051	7.9633743
1	19 44 81	85 766 121	21.0000000	.6116626	6	25 60 36	129 554 216	.4944438	.9686271
2	19 53 64	86 350 888	.0237960	.6174116	7	25 70 49	130 323 843	.5166605	.9738731
3	19 62 49	86 938 307	.0475652	.6231519	8	25 80 64	131 096 512	.5388553	.9791121
4	19 71 36	87 528 384	.0713075	.6288837	9	25 90 81	131 872 229	.5610283	.9843444
445	19 80 25	88 121 125	21.0950231	7.6346067	510	26 01 00	132 651 000	22.5831796	7.7896697
6	19 89 16	88 716 536	.1187121	.6403213	11	26 11 21	133 432 831	.6053091	.9947883
7	19 98 09	89 314 623	.1423745	.6460272	12	26 21 44	134 217 728	.6274170	8.0000000
8	20 07 04	89 915 392	.1660105	.6517247	13	26 31 69	135 005 697	.6495033	.0052049
9	20 16 01	90 518 849	.1896201	.6574138	14	26 41 96	135 796 744	.6715681	.0104032
450	20 25 00	91 125 000	21.2132034	7.6630943	515	26 52 25	136 590 875	22.6936114	8.0155946
1	20 34 01	91 733 851	.2367606	.6687665	16	26 62 56	137 388 096	.7156334	.0207794
2	20 43 04	92 345 408	.2602916	.6744303	17	26 72 89	138 188 413	.7376340	.0259574
3	20 52 09	92 959 677	.2837967	.6800857	18	26 83 24	138 991 832	.7596134	.0311287
4	20 61 16	93 576 664	.3072758	.6857328	19	26 93 61	139 798 359	.7815715	.0362935
455	20 70 25	94 196 375	21.3307290	.6913717	520	27 04 00	140 608 000	22.8035085	8.0414515

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9.—Squares, Cubes, Square Roots, Cube Roots, of Numbers 1 to 1600—Continued.

No.	Square	Cube.	Sq. Rt.	Cu. Rt.	No.	Square	Cube.	Sq. Rt.	Cu. Rt.
520	27 04 00	140 608 000	22.8035085	8.0414515	585	34 22 25	200 201 625	24.1867732	8.3634466
1	27 14 41	141 420 761	.8284244	.0466030	6	34 33 96	201 230 056	.2074369	.3682095
2	27 24 84	142 236 648	.8473193	.0517479	7	34 45 69	202 262 003	2280829	.3729668
3	27 35 29	143 055 667	.8691933	.0568862	8	34 57 44	203 297 472	.2487113	.3777188
4	27 45 76	143 877 824	.8910463	.0620180	9	34 69 21	204 336 469	.2693222	.3824653
525	27 56 25	144 703 125	22.9128785	8.0671432	590	34 81 00	205 379 000	24.2899156	8.3872065
6	27 66 76	145 531 576	.9346899	.0722620	1	34 92 81	206 425 071	.3104916	.3919423
7	27 77 29	146 363 183	.9564806	.0773743	2	35 04 64	207 474 688	.3310501	.3966729
8	27 87 84	147 197 952	.9782506	.0824800	3	35 16 49	208 527 857	.3515913	.4013981
9	27 98 41	148 035 889	23.0000000	.0875794	4	35 28 36	209 584 584	.3721152	.4061180
530	28 09 00	148 877 000	23.0217289	8.0926723	595	35 40 25	210 644 875	24.3926218	8.4108326
1	28 19 61	149 721 291	.0434372	0977589	6	35 52 16	211 708 736	.4131112	.4155419
2	28 30 24	150 568 768	.0651252	.1028390	7	35 64 09	212 776 173	.4335834	.4202460
3	28 40 89	151 419 437	.0867928	.1079128	8	35 76 04	213 847 192	.4540385	.4249448
4	28 51 56	152 273 304	.1084400	.1129803	9	35 88 01	214 921 799	.4744765	.4296383
535	28 62 25	153 130 375	23.1300670	8.1180414	600	36 00 00	216 000 000	24.4948974	8.4343267
6	28 72 96	153 990 656	.1516738	.1230962	1	36 12 01	217 081 801	.5153013	.4390098
7	28 83 69	154 854 153	.1732605	.1281447	2	36 24 04	218 167 208	.5356883	.4436877
8	28 94 44	155 720 872	.1948270	.1331870	3	36 36 09	219 256 227	.5560583	.4483605
9	29 05 21	156 590 819	.2163735	.1382230	4	36 48 16	220 348 864	.5764115	.4530281
540	29 16 00	157 464 000	23.2379001	8.1432529	605	36 60 25	221 445 125	24.5967478	8.4576906
1	29 26 81	158 340 421	.2594067	.1482765	6	36 72 36	222 545 016	.6170673	.4623479
2	29 37 64	159 220 088	.2808935	.1532939	7	36 84 49	223 648 543	.6373700	.4670000
3	29 48 49	160 103 007	.3023604	.1583051	8	36 96 64	224 755 712	.6576560	.4716471
4	29 59 36	160 989 184	.3238076	.1633102	9	37 08 81	225 866 529	.6779254	.4762892
545	29 70 25	161 878 625	23.3452351	8.1683092	610	37 21 00	226 981 000	24.6981781	8.4809261
6	29 81 16	162 771 336	.3666429	.1733020	11	37 33 21	228 099 131	.7184142	.4855579
7	29 92 09	163 667 323	.3880311	.1782888	12	37 45 44	229 220 928	.7386338	.4901848
8	30 03 04	164 566 592	.4093998	.1832695	13	37 57 69	230 346 397	.7588368	.4948065
9	30 14 01	165 469 149	.4307490	.1882441	14	37 69 96	231 475 544	.7790234	.4994233
550	30 25 00	166 375 000	23.4520788	8.1932127	615	37 82 25	232 608 375	24.7991935	8.5040350
1	30 36 01	167 284 151	.4733892	.1981753	16	37 94 56	233 744 896	.8193473	.5086417
2	30 47 04	168 196 608	.4946802	.2031319	17	38 06 89	234 885 113	.8394847	.5132435
3	30 58 09	169 112 377	.5159520	.2080825	18	38 19 24	236 029 032	.8596058	.5178403
4	30 69 16	170 031 464	.5372046	.2130271	19	38 31 61	237 176 659	.8797106	.5224321
555	30 80 25	170 953 875	23.5584380	8.2179657	620	38 44 00	238 328 000	24.8997992	8.5270189
6	30 91 36	171 879 616	.5796522	2228985	1	38 56 41	239 483 061	.9198716	.5316009
7	31 02 49	172 808 693	.6008474	.2278254	2	38 68 84	240 641 848	.9399278	.5361780
8	31 13 64	173 741 112	.6220236	.2327463	3	38 81 29	241 804 367	.9599679	5407501
9	31 24 81	174 676 879	.6431808	.2376614	4	38 93 76	242 970 624	.9799920	.5453173
560	31 36 00	175 616 000	23.6643191	8.2425706	625	39 06 25	244 140 625	25.0000000	8.5498797
1	31 47 21	176 558 481	.6854386	.2474740	6	39 18 76	245 314 376	.0199920	.5544372
2	31 58 44	177 504 328	.7065392	.2523715	7	39 31 29	246 491 883	.0399681	.5589899
3	31 69 69	178 453 547	.7276210	.2572633	8	39 43 84	247 673 152	.0599282	.5635377
4	31 80 96	179 406 144	.7486842	.2621492	9	39 56 41	248 858 189	.0798724	.5680807
565	31 92 25	180 363 125	23.7697286	8.2670294	630	39 69 00	250 047 000	25.0998008	8.5726189
6	32 03 56	181 321 496	.7907545	.2719039	1	39 81 61	251 239 591	.1197134	.5771523
7	32 14 89	182 284 263	.8117618	.2767726	2	39 94 24	252 435 968	.1396102	.5816809
8	32 26 24	183 250 432	.8327506	.2816355	3	40 06 89	253 636 137	.1594913	.5862047
9	32 37 61	184 220 009	.8537209	.2864928	4	40 19 56	254 840 104	.1793566	.5907238
570	32 49 00	185 193 000	23.8746728	8.2913444	635	40 32 25	256 047 875	25.1992063	8.5952380
1	32 60 41	186 169 411	.8956063	.2961903	6	40 44 96	257 259 456	.2190404	.5997476
2	32 71 84	187 149 248	.9165215	.3010304	7	40 57 69	258 474 853	.2388589	.6042525
3	32 83 29	188 132 517	.9374184	.3058651	8	40 70 44	259 694 072	.2586619	.6087526
4	32 94 76	189 119 224	.9582971	.3106941	9	40 83 21	260 917 119	.2784493	.6132480
575	33 06 25	190 109 375	23.9791576	8.3155175	640	40 96 00	262 144 000	25.2982213	8.6177388
6	33 17 76	191 102 976	24.0000000	.3203353	1	41 08 81	263 374 721	.3179778	.6222248
7	33 29 29	192 100 033	.0208243	.3251475	2	41 21 64	264 609 288	.3377189	.6267063
8	33 40 84	193 100 552	.0416306	.3299542	3	41 34 49	265 847 707	.3574447	.6311830
9	33 52 41	194 104 539	.0624188	.3347553	4	41 47 36	267 089 984	.3771551	.6356551
580	33 64 00	195 112 000	24.0831891	8.3395509	645	41 60 25	268 336 125	25.3968502	8.6401226
1	33 75 61	196 123 941	.1039416	.3443410	6	41 73 16	269 586 136	.4165301	.6445855
2	33 87 24	197 137 368	.1246762	.3491256	7	41 86 09	270 840 023	.4361947	.6490437
3	33 98 89	198 155 287	.1453929	.3539047	8	41 99 04	272 097 792	.4558441	.6534974
4	34 10 56	199 176 704	.1660919	.3586784	9	42 12 01	273 359 449	.4754784	.6579465
585	34 22 25	200 201 625	24.1867732	8.3634466	650	42 25 00	274 625 000	25.4950976	8.6623911

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9.—Squares, Cubes, Square Roots, Cube Roots, of Numbers 1 to 1600—Continued.

No.	Square	Cube.	Sq. Rt.	Cu. Rt.	No.	Square	Cube.	Sq. Rt.	Cu. Rt.
650	42 25 00	274 625 000	25.4950976	8.6623911	715	51 12 25	365 525 875	26.7394839	8.9420140
1	42 38 01	275 894 451	.5147016	.6668310	16	51 26 56	367 061 696	.7581763	.9461809
2	42 51 04	277 167 808	.5342907	.6712665	17	51 40 89	368 601 813	.7768557	.9503438
3	42 64 09	278 445 077	.5538647	.6756974	18	51 55 24	370 146 232	.7955220	.9545029
4	42 77 16	279 726 264	.5734237	.6801237	19	51 69 61	371 694 959	.8141754	.9586581
655	42 90 25	281 011 375	25.5929678	8.6845456	720	51 84 00	373 248 000	26.8328157	8.9628095
6	43 03 36	282 300 416	.6124969	.6889630	1	51 98 41	374 805 361	.8514432	.9669570
7	43 16 49	283 593 393	.6320112	.6933759	2	52 12 84	376 367 048	.8700577	.9711007
8	43 29 64	284 890 312	.6515107	.6977843	3	52 27 29	377 933 067	.8886593	.9752406
9	43 42 81	286 191 179	.6709953	.7021882	4	52 41 76	379 503 424	.9072481	.9793766
660	43 56 00	287 496 000	25.6904652	8.7065877	725	52 56 25	381 078 125	26.9258240	8.9835089
1	43 69 21	288 804 781	.7099203	.7109827	6	52 70 76	382 657 176	.9443872	.9876373
2	43 82 44	290 117 528	.7293607	.7153734	7	52 85 29	384 240 583	.9629375	.9917620
3	43 95 69	291 434 247	.7487864	.7197596	8	52 99 84	385 828 352	.9814751	.9958829
4	44 08 96	292 754 944	.7681975	.7241414	9	53 14 41	387 420 489	27.0000000	9.0000000
665	44 22 25	294 079 625	25.7875939	8.7285187	730	53 29 00	389 017 000	27.0185122	9.0041134
6	44 35 56	295 408 296	.8069758	.7328918	1	53 43 61	390 617 891	.0370117	.0082229
7	44 48 89	296 740 963	.8263431	.7372604	2	53 58 24	392 223 168	.0554985	.0123288
8	44 62 24	298 077 632	.8456960	.7416246	3	53 72 89	393 832 837	.0739727	.0164309
9	44 75 61	299 418 309	.8650343	.7459846	4	53 87 56	395 446 904	.0924344	.0205293
670	44 89 00	300 763 000	25.8843582	8.7503401	735	54 02 25	397 065 375	27.1108834	9.0246239
1	45 02 41	303 111 711	.9036677	.7546913	6	54 16 96	398 688 256	.1293199	.0287149
2	45 15 84	303 464 448	.9229628	.7590383	7	54 31 69	400 315 553	.1477439	.0328021
3	45 29 29	304 821 217	.9422435	.7633809	8	54 46 44	401 947 272	.1661554	.0368857
4	45 42 76	306 182 024	.9615100	.7677192	9	54 61 21	403 583 419	.1845544	.0409655
675	45 56 25	307 546 875	25.9807621	8.7720532	740	54 76 00	405 224 000	27.2029410	9.0450417
6	45 69 76	308 915 776	26.0000000	.7763830	1	54 90 81	406 869 021	.2213152	.0491142
7	45 83 29	310 288 733	.0192237	.7807084	2	55 05 64	408 518 488	.2396769	.0531831
8	45 96 84	311 665 752	.0384331	.7850296	3	55 20 49	410 172 407	.2580263	.0572482
9	46 10 41	313 046 839	.0576284	.7893466	4	55 35 36	411 830 784	.2763634	.0613096
680	46 24 00	314 432 000	26.0768096	8.7936593	745	55 50 25	413 493 625	27.2946881	9.0653677
1	46 37 61	315 821 241	.0959767	.7979679	6	55 65 16	415 160 936	.3130006	.0694220
2	46 51 24	317 214 568	.1151297	.8022721	7	55 80 09	416 832 723	.3313007	.0734726
3	46 64 89	318 611 987	.1342687	.8065722	8	55 95 04	418 508 992	.3495887	.0775197
4	46 78 56	320 013 504	.1533937	.8108681	9	56 10 01	420 189 749	.3678644	.0815631
685	46 92 25	321 419 125	26.1725047	8.8151598	750	56 25 00	421 875 000	27.3861279	9.0856030
6	47 05 96	322 828 856	.1916017	.8194474	1	56 40 01	423 564 751	.4043792	.0896392
7	47 19 69	324 242 703	.2106848	.8237307	2	56 55 04	425 259 008	.4226184	.0936719
8	47 33 44	325 660 672	.2297541	.8280099	3	56 70 09	426 957 777	.4408455	.0977010
9	47 47 21	327 082 769	.2488095	.8322850	4	56 85 16	428 661 064	.4590604	.1017265
690	47 61 00	328 509 000	26.2678511	8.8365559	755	57 00 25	430 368 875	27.4772633	9.1057485
1	47 74 81	329 939 371	.2868789	.8408227	6	57 15 36	432 081 216	.4954542	.1097669
2	47 88 64	331 373 888	.3058929	.8450854	7	57 30 49	433 798 093	.5136330	.1137818
3	48 02 49	332 812 557	.3248932	.8493440	8	57 45 64	435 519 512	.5317998	.1177931
4	48 16 36	334 255 384	.3438797	.8535985	9	57 60 81	437 245 479	.5499546	.1218010
695	48 30 25	335 702 375	26.3628527	8.8578489	760	57 76 00	438 976 000	27.5680975	9.1258053
6	48 44 16	337 153 536	.3818119	.8620952	1	57 91 21	440 711 081	.5862284	.1298061
7	48 58 09	338 608 873	.4007576	.8663375	2	58 06 44	442 450 728	.6043475	.1338034
8	48 72 04	340 068 392	.4196895	.8705757	3	58 21 69	444 194 947	.6224546	.1377971
9	48 86 01	341 532 099	.4386081	.8748099	4	58 36 96	445 943 744	.6405499	.1417874
700	49 00 00	343 000 000	26.4575131	8.8790400	765	58 52 25	447 697 125	27.6586334	9.1457742
1	49 14 01	344 472 101	.4764046	.8832661	6	58 67 56	449 455 096	.6767050	.1497576
2	49 28 04	345 948 408	.4952826	.8874882	7	58 82 89	451 217 663	.6947648	.1537375
3	49 42 09	347 428 927	.5141472	.8917063	8	58 98 24	452 984 832	.7128129	.1577139
4	49 56 16	348 913 664	.5329983	.8959204	9	59 13 61	454 756 609	.7308492	.1616869
705	49 70 25	350 402 625	26.5518361	8.9001304	770	59 29 00	456 533 000	27.7488739	9.1656565
6	49 84 36	351 895 816	.5706605	.9043366	1	59 44 41	458 314 011	.7668868	.1696225
7	49 98 49	353 393 243	.5894716	.9085387	2	59 59 84	460 099 648	.7848880	.1735852
8	50 12 64	354 894 912	.6082694	.9127369	3	59 75 29	461 889 917	.8028775	.1775445
9	50 26 81	356 400 829	6270539	.9169311	4	59 90 76	463 684 824	.8208555	.1815003
710	50 41 00	357 911 000	26.6458252	8.9211214	775	60 06 25	465 484 375	27.8388218	9.1854527
11	50 55 21	359 425 431	.6645833	.9253078	6	60 21 76	467 288 576	.8567766	.1894018
12	50 69 44	360 944 128	.6833281	.9294902	7	60 37 29	469 097 433	.8747197	.1933474
13	50 83 69	362 467 097	.7020598	.9336687	8	60 52 84	470 910 952	.8926514	.1972897
14	50 97 96	363 994 344	.7207784	.9378433	9	60 68 41	472 729 139	.9105715	.2012286
715	51 12 25	365 525 875	26.7394839	8.9420140	780	60 84 00	474 552 000	27.9284801	9.2051641

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9.—Squares, Cubes, Square Roots, Cube Roots, of Numbers 1 to 1600—Continued.

No.	Square	Cube.	Sq. Rt.	Cu. Rt.	No.	Square	Cube.	Sq. Rt.	Cu. Rt.
780	60 84 00	474 552 000	27.9284801	9.2051641	845	71 40 25	603 351 125	29.0688837	9.4540719
1	60 99 61	476 379 541	.9463772	.2090962	6	71 57 16	605 495 736	.0860791	.4577999
2	61 15 24	478 211 768	.9642629	.2130250	7	71 74 09	607 645 423	.1032644	.4615249
3	61 30 89	480 048 687	.9821372	.2169505	8	71 91 04	609 800 192	.1204396	.4652470
4	61 46 56	481 890 304	28.0000000	.2208726	9	72 08 01	611 960 049	.1376046	.4689661
785	61 62 25	483 736 625	28.0178515	9.2247914	850	72 25 00	614 125 000	29.1547595	9.4726824
6	61 77 96	485 587 656	.0356915	.2287068	1	72 42 01	616 295 051	.1719043	.4763957
7	61 93 69	487 443 403	.0535203	.2326189	2	72 59 04	618 470 208	.1890390	.4801061
8	62 09 44	489 303 872	.0713377	.2365277	3	72 76 09	620 650 477	.2061637	.4838136
9	62 25 21	491 169 069	.0891438	.2404333	4	72 93 16	622 835 864	.2232784	.4875182
790	62 41 00	493 039 000	28.1069386	9.2443355	855	73 10 25	625 026 375	29.2403830	9.4912200
1	62 56 81	494 913 671	.1247222	.2482344	6	73 27 36	627 222 016	.2574777	.4949188
2	62 72 64	496 793 088	.1424946	.2521300	7	73 44 49	629 422 793	.2745623	.4986147
3	62 88 49	498 677 257	.1602557	.2560224	8	73 61 64	631 628 712	.2916370	.5023078
4	63 04 36	500 566 184	.1780056	.2599114	9	73 78 81	633 839 779	.3087018	.5059980
795	63 20 25	502 459 875	28.1957444	9.2637973	860	73 96 00	636 056 000	29.3257566	9.5096854
6	63 36 16	504 358 336	.2134720	.2676798	1	74 13 21	638 277 381	.3428015	.5133699
7	63 52 09	506 261 573	.2311884	.2715592	2	74 30 44	640 503 928	.3598365	.5170515
8	63 68 04	508 169 592	.2488938	.2754352	3	74 47 69	642 735 647	.3768616	.5207303
9	63 84 01	510 082 399	.2665881	.2793081	4	74 64 96	644 972 544	.3938769	.5244063
800	64 00 00	512 000 000	28.2842712	9.2831777	865	74 82 25	647 214 625	29.4108823	9.5280794
1	64 16 01	513 922 401	.3019434	.2870440	6	74 99 56	649 461 896	.4278779	.5317497
2	64 32 04	515 849 608	.3196045	.2909072	7	75 16 89	651 714 363	.4448637	.5354172
3	64 48 09	517 781 627	.3372546	.2947671	8	75 34 24	653 972 032	.4618397	.5390818
4	64 64 16	519 718 464	.3548938	.2986239	9	75 51 61	656 234 909	.4788059	.5427437
805	64 80 25	521 660 125	28.3725219	9.3024775	870	75 69 00	658 503 000	29.4957624	9.5464027
6	64 96 36	523 606 616	.3901391	.3063278	1	75 86 41	660 776 311	.5127091	.5500589
7	65 12 49	525 557 943	.4077454	.3101750	2	76 03 84	663 054 848	.5296461	.5537123
8	65 28 64	527 514 112	.4253408	.3140190	3	76 21 29	665 338 617	.5465734	.5573630
9	65 44 81	529 475 129	.4429253	.3178599	4	76 38 76	667 627 624	.5634910	.5610108
810	65 61 00	531 441 000	28.4604989	9.3216975	875	76 56 25	669 921 875	29.5803989	9.5646559
11	65 77 21	533 411 731	.4780617	.3255320	6	76 73 76	672 221 376	.5972972	.5682982
12	65 93 44	535 387 328	.4956137	.3293634	7	76 91 29	674 526 133	.6141858	.5719377
13	66 09 69	537 367 797	.5131549	.3331916	8	77 08 84	676 836 152	.6310648	.5755745
14	66 25 96	539 353 144	.5306852	.3370167	9	77 26 41	679 151 439	.6479342	.5792085
815	66 42 25	541 343 375	28.5482048	9.3408386	880	77 44 00	681 472 000	29.6647939	9.5828397
16	66 58 56	543 338 496	.5657137	.3446575	1	77 61 61	683 797 841	.6816442	.5864682
17	66 74 89	545 338 513	.5832119	.3484731	2	77 79 24	686 128 968	.6984848	.5900939
18	66 91 24	547 343 432	.6006993	.3522857	3	77 96 89	688 465 387	.7153159	.5937169
19	67 07 61	549 353 259	.6181760	.3560952	4	78 14 56	690 807 104	.7321375	.5973373
820	67 24 00	551 368 000	28.6356421	9.3599016	885	78 32 25	693 154 125	29.7489496	9.6009548
1	67 40 41	553 387 661	.6530976	.3637049	6	78 49 96	695 506 456	.7657521	.6045696
2	67 56 84	555 412 248	.6705424	.3675051	7	78 67 69	697 864 103	.7825452	.6081817
3	67 73 29	557 441 767	.6879766	.3713022	8	78 85 44	700 227 072	.7993289	.6117911
4	67 89 76	559 476 224	.7054002	.3750963	9	79 03 21	702 595 369	.8161030	.6153977
825	68 06 25	561 515 625	28.7228132	9.3788873	890	79 21 00	704 969 000	29.8328678	9.6190017
6	68 22 76	563 559 976	.7402157	.3826752	1	79 38 81	707 347 971	.8496231	.6226030
7	68 39 29	565 609 283	.7576077	.3864600	2	79 56 64	709 732 288	.8663690	.6262016
8	68 55 84	567 663 552	.7749891	.3902419	3	79 74 49	712 121 957	.8831056	.6297975
9	68 72 41	569 722 789	.7923601	.3940206	4	79 92 36	714 516 984	.8998328	.6333907
830	68 89 00	571 787 000	28.8097206	9.3977964	895	80 10 25	716 917 375	29.9165506	9.6369812
1	69 05 61	573 856 191	.8270706	.4015691	6	80 28 16	719 323 136	.9332591	.6405690
2	69 22 24	575 930 368	.8444102	.4053387	7	80 46 09	721 734 273	.9499583	.6441542
3	69 38 89	578 009 537	.8617394	.4091054	8	80 64 04	724 150 792	.9666481	.6477367
4	69 55 56	580 093 704	.8790582	.4128690	9	80 82 01	726 572 699	.9833287	.6513166
835	69 72 25	582 182 875	28.8963666	9 4166297	900	81 00 00	729 000 000	30.0000000	9.6548938
6	69 88 96	584 277 056	.9136646	.4203873	1	81 18 01	731 432 701	.0166620	.6584684
7	70 05 69	586 376 253	.9309523	.4241420	2	81 36 04	733 870 808	.0333148	.6620403
8	70 22 44	588 480 472	.9482297	.4278936	3	81 54 09	736 314 327	.0499584	.6656096
9	70 39 21	590 589 719	.9654967	.4316423	4	81 72 16	738 763 264	.0665928	.6691762
840	70 56 00	592 704 000	28.9827535	9.4353880	905	81 90 25	741 217 625	30.0832179	9.6727403
1	70 72 81	594 823 321	29.0000000	.4391307	6	82 08 36	743 677 416	.0998339	.6763017
2	70 89 64	596 947 688	.0172363	.4428704	7	82 26 49	746 142 643	.1164407	.6798604
3	71 06 49	599 077 107	.0344623	.4466072	8	82 44 64	748 613 312	.1330383	.6834166
4	71 23 36	601 211 584	.0516781	.4503410	9	82 62 81	751 089 429	.1496269	.6869701
845	71 40 25	603 351 125	29.0688837	9.4540719	910	82 81 00	753 571 000	30.1662063	9.6905211

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9.—Squares, Cubes, Square Roots, Cube Roots, of Numbers 1 to 1600—Continued.

No.	Square	Cube.	Sq. Rt.	Cu. Rt.	No.	Square.	Cube.	Sq. Rt.	Cu. Rt.
910	82 81 00	753 571 000	30.1662063	9.6905211	975	95 06 25	926 859 375	31.2249900	9.9159624
11	82 99 21	756 058 031	.1827765	.6940694	6	95 25 76	929 714 176	.2409987	.9193513
12	83 17 44	758 550 528	.1993377	.6976151	7	95 45 29	932 574 833	.2569992	.9227379
13	83 35 69	761 048 497	.2158899	.7011583	8	95 64 84	935 441 352	.2729915	.9261222
14	83 53 96	763 551 944	.2324329	.7046989	9	95 84 41	938 313 739	.2889757	.9295042
915	83 72 25	766 060 875	30.2489669	9.7082369	980	96 04 00	941 192 000	31.3049517	9.9328839
16	83 90 56	768 575 296	.2654919	.7117723	1	96 23 61	944 076 141	.3209195	.9362613
17	84 08 89	771 095 213	.2820079	.7153051	2	96 43 24	946 966 168	.3368792	.9396363
18	84 27 24	773 620 632	.2985148	.7188354	3	96 62 89	949 862 087	.3528308	.9430089
19	84 45 61	776 151 559	.3150128	.7223631	4	96 82 56	952 763 904	.3687743	.9463797
920	84 64 00	778 688 000	30.3315018	9.7258883	985	97 02 25	955 671 625	31.3847097	9.9497479
1	84 82 41	781 229 961	.3479818	.7294109	6	97 21 96	958 585 256	.4006369	.9531138
2	85 00 84	783 777 448	.3644529	.7329309	7	97 41 69	961 504 803	.4165561	.9564775
3	85 19 29	786 330 467	.3809151	.7364484	8	97 61 44	964 430 272	.4324673	.9598389
4	85 37 76	788 889 024	.3973683	.7399634	9	97 81 21	967 361 669	.4483704	.9631981
925	85 56 25	791 453 125	30.4138127	9.7434758	990	98 01 00	970 299 000	31.4642654	9.9665549
6	85 74 76	794 022 776	.4302481	.7469857	1	98 20 81	973 242 271	.4801525	.9699095
7	85 93 29	796 597 983	.4466747	.7504930	2	98 40 64	976 191 488	.4960315	.9732619
8	86 11 84	799 178 752	.4630924	.7539979	3	98 60 49	979 146 657	.5119025	.9766120
9	86 30 41	801 765 089	.4795013	.7575002	4	98 80 36	982 107 784	.5277655	.9799599
930	86 49 00	804 357 000	30.4959014	9.7610001	995	99 00 25	985 074 875	31.5436206	9.9833055
1	86 67 61	806 954 491	.5122926	.7644974	6	99 20 16	988 047 936	.5594677	.9866488
2	86 86 24	809 557 568	.5286750	.7679922	7	99 40 09	991 026 973	.5753068	.9899900
3	87 04 89	812 166 237	.5450487	.7714845	8	99 60 04	994 011 992	.5911380	.9933289
4	87 23 56	814 780 504	.5614136	.7749743	9	99 80 01	997 002 999	.6069613	.9966654
935	87 42 25	817 400 375	30.5777697	9.7784616	1000	1 00 00 00	1 000 000 000	31.6227766	10.0000000
6	87 60 96	820 025 856	.5941171	.7819466	1	1 00 20 01	1 003 003 001	.6385840	.0033322
7	87 79 69	822 656 953	.6104557	.7854288	2	1 00 40 04	1 006 012 008	.6543836	.0066622
8	87 98 44	825 293 672	.6267857	.7889087	3	1 00 60 09	1 009 027 027	.6701752	.0099896
9	88 17 21	827 936 019	.6431069	.7923861	4	1 00 80 16	1 012 048 064	.6859590	.0133155
940	88 36 00	830 584 000	30.6594194	9.7958611	1005	1 01 00 25	1 015 075 125	31.7017349	10.0166389
1	88 54 81	833 237 621	.6757233	.7993336	6	1 01 20 36	1 018 108 216	.7175030	.0199601
2	88 73 64	835 896 888	.6920185	.8028036	7	1 01 40 49	1 021 147 343	.7332633	.0232791
3	88 92 49	838 561 807	.7083051	.8062711	8	1 01 60 64	1 024 192 512	.7490157	.0265958
4	89 11 36	841 232 384	.7245830	.8097362	9	1 01 80 81	1 027 243 729	.7647603	.0299104
945	89 30 25	843 908 625	30.7408523	9.8131989	1010	1 02 01 00	1 030 301 000	31.7804972	10.0332228
6	89 49 16	846 590 536	.7571130	.8166591	11	1 02 21 21	1 033 364 331	.7962262	.0365334
7	89 68 09	849 278 123	.7733651	.8201169	12	1 02 41 44	1 036 433 728	.8119474	.0398416
8	89 87 04	851 971 392	.7896086	.8235723	13	1 02 61 69	1 039 509 197	.8276609	.0431469
9	90 06 01	854 670 349	.8058436	.8270252	14	1 02 81 96	1 042 590 744	.8433666	.0464500
950	90 25 00	857 375 000	30.8220700	9.8304757	1015	1 03 02 25	1 045 678 375	31.8590646	10.0497512
1	90 44 01	860 085 351	.8382879	.8339238	16	1 03 22 56	1 048 772 096	.8747549	.0530514
2	90 63 04	862 801 408	.8544972	.8373695	17	1 03 42 89	1 051 871 913	.8904374	.0563485
3	90 82 09	865 523 177	.8706981	.8408127	18	1 03 63 24	1 054 977 832	.9061123	.0596435
4	91 01 16	868 250 664	.8868904	.8442536	19	1 03 83 61	1 058 089 859	.9217794	.0629362
955	91 20 25	870 983 875	30.9030743	9.8476920	1020	1 04 04 00	1 061 208 000	31.9374388	10.0662277
6	91 39 36	873 722 816	.9192497	.8511280	21	1 04 24 41	1 064 332 261	.9530906	.0695159
7	91 58 49	876 467 493	.9354166	.8545617	22	1 04 44 84	1 067 462 648	.9687347	.0728020
8	91 77 64	879 217 912	.9515751	.8579929	23	1 04 65 29	1 070 599 167	.9843712	.0760856
9	91 96 81	881 974 079	.9677251	.8614218	24	1 04 85 76	1 073 741 824	32.0000000	.0793668
960	92 16 00	884 736 000	30.9838668	9.8648483	1025	1 05 06 25	1 076 890 625	32.0156212	10.0826458
1	92 35 21	887 503 681	31.0000000	.8682724	26	1 05 26 76	1 080 045 576	.0312348	.0859226
2	92 54 44	890 277 128	.0161248	.8716941	27	1 05 47 29	1 083 206 683	.0468407	.0892011
3	92 73 69	893 056 347	.0322413	.8751135	28	1 05 67 84	1 086 373 952	.0624391	.0924751
4	92 92 96	895 841 344	.0483494	.8785305	29	1 05 88 41	1 089 547 389	.0780298	.0957466
965	93 12 25	898 632 125	31.0644491	9.8819451	1030	1 06 09 00	1 092 727 000	32.0936131	10.0990166
6	93 31 56	901 428 696	.0805405	.8853574	31	1 06 29 61	1 095 912 791	.1091887	.1022833
7	93 50 89	904 231 063	.0966236	.8887673	32	1 06 50 24	1 099 104 768	.1247568	.1055488
8	93 70 24	907 039 232	.1126984	.8921749	33	1 06 70 89	1 102 302 937	.1403173	.1088111
9	93 89 61	909 853 209	.1287648	.8955801	34	1 06 91 56	1 105 507 304	.1558704	.1120722
970	94 09 00	912 673 000	31.1448230	9.8989830	1035	1 07 12 25	1 108 717 875	32.1714159	10.1153318
1	94 28 41	915 498 611	.1608729	.9023835	36	1 07 32 96	1 111 934 656	.1869539	.1185888
2	94 47 84	918 330 048	.1769145	.9057817	37	1 07 53 69	1 115 157 653	.2024844	.1218442
3	94 67 29	921 167 317	.1929479	.9091776	38	1 07 74 44	1 118 386 872	.2180074	.1250975
4	94 86 76	924 010 424	.2089731	.9125712	39	1 07 95 21	1 121 622 319	.2335229	.1283485
975	95 06 25	926 859 375	31.2249900	9.9159624	1040	1 08 16 00	1 124 864 000	32.2490310	10.1315944

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9.—Squares, Cubes, Square Roots, Cube Roots, of Numbers 1 to 1600—Continued.

No.	Square	Cube.	Sq. Rt.	Cu. Rt.	No.	Square	Cube.	Sq. Rt.	Cu. Rt.
1040	1 08 16 00	1 124 864 000	32.2490310	10.1315941	1105	1 22 10 25	1 349 232 625	33.2415403	10.3384181
41	1 08 36 81	1 128 111 921	.2645316	.1348403	6	1 22 32 36	1 352 899 016	.2565783	.3415358
42	1 08 57 64	1 131 366 088	.2800248	.1380845	7	1 22 54 49	1 356 572 043	.2716095	.3446517
43	1 08 78 49	1 134 626 507	.2955105	.1413266	8	1 22 76 64	1 360 251 712	.2866339	.3477657
44	1 08 99 36	1 137 893 184	.3109888	.1445667	9	1 22 98 81	1 363 938 029	.3016516	.3508778
1045	1 09 20 25	1 141 166 125	32.3264598	10.1478047	1110	1 23 21 00	1 367 631 000	33.3166625	10.3539880
46	1 09 41 16	1 144 445 336	.3419233	.1510406	11	1 23 43 21	1 371 330 631	.3316666	.3570964
47	1 09 62 09	1 147 730 823	.3573794	.1542744	12	1 23 65 44	1 375 036 928	.3466640	.3602029
48	1 10 83 04	1 151 022 592	.3728281	.1575062	13	1 23 87 69	1 378 749 897	.3616546	.3633076
49	1 10 04 01	1 154 320 649	.3882695	.1607359	14	1 24 09 96	1 382 469 544	.3766385	.3664103
1050	1 10 25 00	1 157 625 000	32.4037035	10.1639636	1115	1 24 32 25	1 386 195 875	33.3916157	10.3695113
51	1 10 46 01	1 160 935 651	.4191301	.1671893	16	1 24 54 56	1 389 928 896	.4065862	.3726103
52	1 10 67 04	1 164 252 608	.4345495	.1704129	17	1 24 76 89	1 393 668 613	.4215499	.3757076
53	1 10 88 09	1 167 575 877	.4499615	.1736344	18	1 24 99 24	1 397 415 032	.4365070	.3788030
54	1 11 09 16	1 170 905 464	.4653662	.1768539	19	1 25 21 61	1 401 168 159	.4514573	.3818965
1055	1 11 30 25	1 174 241 375	32.4807635	10.1800714	1120	1 25 44 00	1 404 928 000	33.4664011	10.3849882
56	1 11 51 36	1 177 583 616	.4961536	.1832868	21	1 25 66 41	1 408 694 561	.4813381	.3880781
57	1 11 72 49	1 180 932 193	.5115364	.1865002	22	1 25 88 84	1 412 467 848	.4962684	.3911661
58	1 11 93 64	1 184 287 112	.5269119	.1897116	23	1 26 11 29	1 416 247 867	.5111921	.3942523
59	1 12 14 81	1 187 648 379	.5422802	.1929209	24	1 26 33 76	1 420 034 624	.5261092	.3973366
1060	1 12 36 00	1 191 016 000	32.5576412	10.1961283	1125	1 26 56 25	1 423 828 125	33.5410196	10.4004192
61	1 12 57 21	1 194 389 981	.5729949	.1993336	26	1 26 78 76	1 427 628 376	.5559234	.4034999
62	1 12 78 44	1 197 770 328	.5883415	.2025369	27	1 27 01 29	1 431 435 383	.5708206	.4065787
63	1 12 99 69	1 201 157 047	.6036807	.2057382	28	1 27 23 84	1 435 249 152	.5857112	.4096557
64	1 13 20 96	1 204 550 144	.6190129	.2089375	29	1 27 46 41	1 439 069 689	.6005952	.4127310
1065	1 13 42 25	1 207 949 625	32.6343377	10.2121347	1130	1 27 69 00	1 442 897 000	33.6154726	10.4158044
66	1 13 63 56	1 211 355 496	.6496554	.2153300	31	1 27 91 61	1 446 731 091	.6303434	.4188760
67	1 13 84 89	1 214 767 763	.6649659	.2185233	32	1 28 14 24	1 450 571 968	.6452077	.4219458
68	1 14 06 24	1 218 186 432	.6802693	.2217146	33	1 28 36 89	1 454 419 637	.6600653	.4250138
69	1 14 27 61	1 221 611 509	.6955654	.2249039	34	1 28 59 56	1 458 274 104	.6749165	.4280800
1070	1 14 49 00	1 225 043 000	32.7108544	10.2280912	1135	1 28 82 25	1 462 135 375	33.6897610	10.4311443
71	1 14 70 41	1 228 480 911	.7261363	.2312766	36	1 29 04 96	1 466 003 456	.7045991	.4342069
72	1 14 91 84	1 231 925 248	.7414111	.2344599	37	1 29 27 69	1 469 878 353	.7194306	.4372677
73	1 15 13 29	1 235 376 017	.7566787	.2376413	38	1 29 50 44	1 473 760 072	.7342556	.4403267
74	1 15 34 76	1 238 833 224	.7719392	.2408207	39	1 29 73 21	1 477 648 619	.7490741	.4433839
1075	1 15 56 25	1 242 296 875	32.7871926	10.2439981	1140	1 29 96 00	1 481 544 000	33.7638860	10.4464393
76	1 15 77 76	1 245 766 976	.8024389	.2471735	41	1 30 18 81	1 485 446 221	.7786915	.4494929
77	1 15 99 29	1 249 243 533	.8176782	.2503470	42	1 30 41 64	1 489 355 288	.7934905	.4525448
78	1 16 20 84	1 252 726 552	.8329103	.2535186	43	1 30 64 49	1 493 271 207	.8082830	.4555948
79	1 16 42 41	1 256 216 039	.8481354	.2566881	44	1 30 87 36	1 497 193 984	.8230691	.4586431
1080	1 16 64 00	1 259 712 000	32.8633535	10.2598557	1145	1 31 10 25	1 501 123 625	33.8378486	10.4616896
81	1 16 85 61	1 263 214 441	.8785644	.2630213	46	1 31 33 16	1 505 060 136	.8526218	.4647343
82	1 17 07 24	1 266 723 368	.8937684	.2661850	47	1 31 56 09	1 509 003 523	.8673884	.4677773
83	1 17 28 89	1 270 238 787	.9089653	.2693467	48	1 31 79 04	1 512 953 792	.8821487	.4708185
84	1 17 50 56	1 273 760 704	.9241553	.2725065	49	1 32 02 01	1 516 910 949	.8969025	.4738579
1085	1 17 72 25	1 277 289 125	32.9393382	10.2756644	1150	1 32 25 00	1 520 875 000	33.9116499	10.4768955
86	1 17 93 96	1 280 824 056	.9545141	.2788203	51	1 32 48 01	1 524 845 951	.9263909	.4799314
87	1 18 15 69	1 284 365 503	.9696830	.2819743	52	1 32 71 04	1 528 823 808	.9411255	.4829656
88	1 18 37 44	1 287 913 472	.9848450	.2851264	53	1 32 94 09	1 532 808 577	.9558537	.4859980
89	1 18 59 21	1 291 467 969	33.0000000	.2882765	54	1 33 17 16	1 536 800 264	.9705755	.4890286
1090	1 18 81 00	1 295 029 000	33.0151480	10.2914247	1155	1 33 40 25	1 540 798 875	33.9852910	10.4920575
91	1 19 02 81	1 298 596 571	.0302891	.2945709	56	1 33 63 36	1 544 804 416	34.0000000	.4950847
92	1 19 24 64	1 302 170 688	.0454233	.2977153	57	1 33 86 49	1 548 816 893	.0147027	.4981101
93	1 19 46 49	1 305 751 357	.0605505	.3008577	58	1 34 09 64	1 552 836 312	.0293990	.5011337
94	1 19 68 36	1 309 338 584	.0756708	.3039982	59	1 34 32 81	1 556 862 679	.0440890	.5041556
1095	1 19 90 25	1 312 932 375	33.0907842	10.3071368	1160	1 34 56 00	1 560 896 000	34.0587727	10.5071757
96	1 20 12 16	1 316 532 736	.1058907	.3102735	61	1 34 79 21	1 564 936 281	.0734501	.5101942
97	1 20 34 09	1 320 139 673	.1209903	.3134083	62	1 35 02 44	1 568 983 528	.0881211	.5132109
98	1 20 56 04	1 323 753 192	.1360830	.3165411	63	1 35 25 69	1 573 037 747	.1027858	.5162259
99	1 20 78 01	1 327 373 299	.1511689	.3196721	64	1 35 48 96	1 577 098 944	.1174442	.5192391
1100	1 21 00 00	1 331 000 000	33.1662479	10.3228012	1165	1 35 72 25	1 581 167 125	34.1320963	10.5222506
1	1 21 42 01	1 334 633 301	.1813200	.3259284	66	1 35 95 56	1 585 242 296	.1467422	.5252604
2	1 21 44 04	1 338 273 208	.1963853	.3290537	67	1 36 18 89	1 589 324 463	.1613817	.5282685
3	1 21 66 09	1 341 919 727	.2114438	.3321770	68	1 36 42 24	1 593 413 632	.1760150	.5312749
4	1 21 88 16	1 345 572 864	.2264955	.3352985	69	1 36 65 61	1 597 509 809	.1906420	.5342795
1105	1 22 10 25	1 349 232 625	33.2415403	10.3384181	1170	1 36 89 00	1 601 613 000	34.2052627	10.5372825

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9.—Squares, Cubes, Square Roots, Cube Roots, or Numbers 1 to 1600—Continued.

No.	Square	Cube.	Sq. Rt.	Cu. Rt.	No.	Square	Cube.	Sq. Rt.	Cu. Rt.
1170	1 36 89 00	1 601 613 000	34.205267	10.537285	1235	1 52 52 25	1 883 652 875	35.142566	10.728911
71	1 37 12 41	1 605 723 211	.2198773	.5402837	36	1 52 76 96	1 888 232 256	.1567917	.7318062
72	1 37 35 84	1 609 840 448	.2344855	.5432832	37	1 53 01 69	1 892 819 053	.1710108	.7346997
73	1 37 59 29	1 613 964 717	.2490875	.5462810	38	1 53 26 44	1 897 413 272	.1852242	.7375916
74	1 37 82 76	1 618 096 024	.2636834	.5492771	39	1 53 51 21	1 902 014 919	.1994318	.7404819
1175	1 38 06 25	1 622 234 375	34.2782730	10.5522715	1240	1 53 76 00	1 906 624 000	35.2136337	10.7433707
76	1 38 29 76	1 626 379 776	.2928564	.5552642	41	1 54 00 81	1 911 240 521	.2278299	.7462579
77	1 38 53 29	1 630 532 233	.3074336	.5582552	42	1 54 25 64	1 915 864 488	.2420204	.7491436
78	1 38 76 84	1 634 691 752	.3220046	.5612445	43	1 54 50 49	1 920 495 907	.2562051	.7520277
79	1 39 00 41	1 638 858 339	.3365694	.5642322	44	1 54 75 36	1 925 134 784	.2703842	.7549103
1180	1 39 24 00	1 643 032 000	34.3511281	10.5672181	1245	1 55 00 25	1 929 781 125	35.2845575	10.7577913
81	1 39 47 61	1 647 212 741	.3656805	.5702024	46	1 55 25 16	1 934 434 936	.2987252	.7606708
82	1 39 71 24	1 651 400 568	.3802268	.5731849	47	1 55 50 09	1 939 096 223	.3128872	.7635488
83	1 39 94 89	1 655 595 487	.3947670	.5761658	48	1 55 75 04	1 943 764 992	.3270435	.7664252
84	1 40 18 56	1 659 797 504	.4093011	.5791449	49	1 56 00 01	1 948 441 249	.3411941	.7693001
1185	1 40 42 25	1 664 006 625	34.4238289	10.5821225	1250	1 56 25 00	1 953 125 000	35.3553391	10.7721735
86	1 40 65 96	1 668 222 856	.4383507	.5850983	51	1 56 50 01	1 957 816 251	.3694784	.7750453
87	1 40 89 69	1 672 446 203	.4528663	.5880725	52	1 56 75 04	1 962 515 008	.3836120	.7779156
88	1 41 13 44	1 676 676 672	.4673759	.5910450	53	1 57 00 09	1 967 221 277	.3977400	.7807843
89	1 41 37 21	1 680 914 269	.4818793	.5940158	54	1 57 25 16	1 971 935 064	.4118624	.7836516
1190	1 41 61 00	1 685 159 000	34.4963766	10.5969850	1255	1 57 50 25	1 976 656 375	35.4259792	10.7865173
91	1 41 84 81	1 689 410 871	.5108678	.5999525	56	1 57 75 36	1 981 385 216	.4400903	.7893815
92	1 42 08 64	1 693 669 888	.5253530	.6029184	57	1 58 00 49	1 986 121 593	.4541958	.7922441
93	1 42 32 49	1 697 936 057	.5398321	.6058826	58	1 58 25 64	1 990 865 512	.4682957	.7951053
94	1 42 56 36	1 702 209 384	.5543051	.6088451	59	1 58 50 81	1 995 616 979	.4823900	.7979649
1195	1 42 80 25	1 706 489 875	34.5687720	10.6118060	1260	1 58 76 00	2 000 376 000	35.4964787	10.8008230
96	1 43 04 16	1 710 777 536	.5832329	.6147652	61	1 59 01 21	2 005 142 581	.5105618	.8036797
97	1 43 28 09	1 715 072 373	.5976879	.6177228	62	1 59 26 44	2 009 916 728	.5246393	.8065348
98	1 43 52 04	1 719 374 392	.6121366	.6206788	63	1 59 51 69	2 014 698 447	.5387113	.8093884
99	1 43 76 01	1 723 683 599	.6265794	.6236331	64	1 59 76 96	2 019 487 744	.5527777	.8122404
1200	1 44 00 00	1 728 000 000	34.6410162	10.6265857	1265	1 60 02 25	2 024 284 625	35.5668385	10.8150909
1	1 44 24 01	1 732 323 601	.6554469	.6295367	66	1 60 27 56	2 029 089 096	.5808937	.8179400
2	1 44 48 04	1 736 654 408	.6698716	.6324860	67	1 60 52 89	2 033 901 163	.5949434	.8207876
3	1 44 72 09	1 740 992 427	.6842904	.6354338	68	1 60 78 24	2 038 720 832	.6089876	.8236336
4	1 44 96 16	1 745 337 664	.6987031	.6383799	69	1 61 03 61	2 043 548 109	.6230262	.8264782
1205	1 45 20 25	1 749 690 125	34.7131099	10.6413244	1270	1 61 29 00	2 048 383 000	35.6370593	10.8293213
6	1 45 44 36	1 754 049 816	.7275107	.6442672	71	1 61 54 41	2 053 225 511	.6510869	.8321629
7	1 45 68 49	1 758 416 743	.7419055	.6472085	72	1 61 79 84	2 058 075 648	.6651090	.8350030
8	1 45 92 64	1 762 790 912	.7562944	.6501480	73	1 62 05 29	2 062 933 417	.6791255	.8378416
9	1 46 16 81	1 767 172 329	.7706773	.6530860	74	1 62 30 76	2 067 798 824	.6931366	.8406788
1210	1 46 41 00	1 771 561 000	34.7850543	10.6560223	1275	1 62 56 25	2 072 671 875	35.7071421	10.8435144
11	1 46 65 21	1 775 956 931	.7994253	.6589570	76	1 62 81 76	2 077 552 576	.7211422	.8463485
12	1 46 89 44	1 780 360 128	.8137904	.6618902	77	1 63 07 29	2 082 440 933	.7351367	.8491812
13	1 47 13 69	1 784 770 597	.8281495	.6648217	78	1 63 32 84	2 087 336 952	.7491258	.8520125
14	1 47 37 96	1 789 188 344	.8425028	.6677516	79	1 63 58 41	2 092 240 639	.7631095	.8548422
1215	1 47 62 25	1 793 613 375	34.8568501	10.6706799	1280	1 63 84 00	2 097 152 000	35.7770876	10.8576704
16	1 47 86 56	1 798 045 696	.8711915	.6736066	81	1 64 09 61	2 102 071 041	.7910603	.8604972
17	1 48 10 89	1 802 485 313	.8855271	.6765317	82	1 64 35 24	2 106 997 768	.8050276	.8633225
18	1 48 35 24	1 806 932 232	.8998567	.6794552	83	1 64 60 89	2 111 932 187	.8189894	.8661464
19	1 48 59 61	1 811 386 459	.9141805	.6823771	84	1 64 86 56	2 116 874 304	.8329457	.8689687
1220	1 48 84 00	1 815 848 000	34.9284984	10.6852973	1285	1 65 12 25	2 121 824 125	35.8468966	10.8717897
21	1 49 08 41	1 820 316 861	.9428104	.6882160	86	1 65 37 96	2 126 781 656	.8608421	.8746091
22	1 49 32 84	1 824 793 048	.9571166	.6911331	87	1 65 63 69	2 131 746 903	.8747822	.8774271
23	1 49 57 29	1 829 276 567	.9714169	.6940486	88	1 65 89 44	2 136 719 872	.8887169	.8803435
24	1 49 81 76	1 833 767 424	.9857114	.6969625	89	1 66 15 21	2 141 700 569	.9026461	.8832587
1225	1 50 06 25	1 838 265 625	35.0000000	10.6998748	1290	1 66 41 00	2 146 689 000	35.9165699	10.8861723
26	1 50 30 76	1 842 771 176	.0142828	.7027855	91	1 66 66 81	2 151 685 171	.9304884	.8889845
27	1 50 55 29	1 847 284 083	.0285598	.7056947	92	1 66 92 64	2 156 689 088	.9444015	.8914952
28	1 50 79 84	1 851 804 352	.0428309	.7086023	93	1 67 18 49	2 161 700 757	.9583092	.8943044
29	1 51 04 41	1 856 331 989	.0570963	.7115083	94	1 67 44 36	2 166 720 184	.9722115	.8971123
1230	1 51 29 00	1 860 867 000	35.0713558	10.7144127	1295	1 67 70 25	2 171 747 375	35.9861084	10.8999186
31	1 51 53 61	1 865 409 391	.0856096	.7173155	96	1 67 96 16	2 176 782 336	36.0000000	.9027235
32	1 51 78 24	1 869 959 168	.0998575	.7202168	97	1 68 22 09	2 181 825 073	.0138862	.9055269
33	1 52 02 89	1 874 516 337	.1140997	.7231165	98	1 68 48 04	2 186 875 592	.0277671	.9083290
34	1 52 27 56	1 879 080 904	.1283361	.7260146	99	1 68 74 01	2 191 933 899	.0416426	.9111296
1235	1 52 52 25	1 883 652 875	35.1425668	10.7289112	1300	1 69 00 00	2 197 000 000	36.0555128	10.9139287

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9.—Squares, Cubes, Square Roots, Cube Roots, of Numbers 1 to 1600—Continued.

No.	Square	Cube.	Sq. Rt.	Cu. Rt.	No.	Square	Cube.	Sq. Rt.	Cu. Rt.
1300	69 00 00 2	197 000 000	36.0555128	10.9139287	1365	1 86 32 25 2	543 302 125	36.9459064	11.0928775
1	69 26 01 2	202 073 901	.0693776	.9167265	66	1 86 59 56 2	548 895 896	.9594372	.0955857
2	69 52 04 2	207 155 608	.0832371	.9195228	67	1 86 86 89 2	554 497 863	.9729631	.0982926
3	69 78 09 2	212 245 127	.0970913	.9223177	68	1 87 14 24 2	560 108 032	.9864840	.1009982
4	70 04 16 2	217 342 464	.1109402	.9251111	69	1 87 41 61 2	565 726 409	37.0000000	.1037025
1305	70 30 25 2	222 447 625	36.1247837	10.9279031	1370	1 87 69 00 2	571 353 000	37.0135110	11.1064054
6	70 56 36 2	227 560 616	.1386220	.9306937	71	1 87 96 41 2	576 987 811	.0270172	.1091070
7	70 82 49 2	232 681 443	.1524550	.9334829	72	1 88 23 84 2	582 630 848	.0405184	.1118073
8	71 08 64 2	237 810 112	.1662826	.9362706	73	1 88 51 29 2	588 282 117	.0540146	.1145064
9	71 34 81 2	242 946 629	.1801050	.9390569	74	1 88 78 76 2	593 941 624	.0675060	.1172041
1310	71 61 00 2	248 091 000	36.1939221	10.9418418	1375	1 89 06 25 2	599 609 375	37.0809924	11.1199004
1	71 87 21 2	253 243 231	.2077340	.9446253	76	1 89 33 76 2	605 285 376	.0944740	.1225955
2	72 13 44 2	258 403 328	.2215406	.9474074	77	1 89 61 29 2	610 969 633	.1079506	.1252893
3	72 39 69 2	263 571 297	.2353419	.9501880	78	1 89 88 84 2	616 662 152	.1214224	.1279817
4	72 65 96 2	268 747 144	.2491379	.9529673	79	1 90 16 41 2	622 362 939	.1348893	.1306729
1315	72 92 25 2	273 930 875	36.2629287	10.9557451	1380	1 90 44 00 2	628 072 000	37.1483512	11.1333628
6	73 18 56 2	279 122 496	.2767143	.9585215	81	1 90 71 61 2	633 789 341	.1618084	.1360514
7	73 44 89 2	284 322 013	.2904946	.9612965	82	1 90 99 24 2	639 514 968	.1752606	.1387386
8	73 71 24 2	289 529 432	.3042697	.9640701	83	1 91 26 89 2	645 248 887	.1887079	.1414246
9	73 97 61 2	294 744 759	.3180396	.9668423	84	1 91 54 56 2	650 991 104	.2021505	.1441093
1320	74 24 00 2	299 968 000	36.3318042	10.9696131	1385	1 91 82 25 2	656 741 625	37.2155881	11.1467926
1	74 50 41 2	305 199 161	.3455637	.9723825	86	1 92 09 96 2	662 500 456	.2290209	.1494747
2	74 76 84 2	310 438 248	.3593179	.9751505	87	1 92 37 69 2	668 267 603	.2424489	.1521555
3	75 03 29 2	315 685 267	.3730670	.9779171	88	1 92 65 44 2	674 043 072	.2558720	.1548350
4	75 29 76 2	320 940 224	.3868108	.9806823	89	1 92 93 21 2	679 826 869	.2692903	.1575133
1325	75 56 25 2	326 203 125	36.4005494	10.9834462	1390	1 93 21 00 2	685 619 000	37.2827037	11.1601903
6	75 82 76 2	331 473 976	.4142829	.9862086	91	1 93 48 81 2	691 419 471	.2961124	.1628659
7	76 09 29 2	336 752 783	.4280112	.9889696	92	1 93 76 64 2	697 228 288	.3095162	.1655403
8	76 35 84 2	342 039 552	.4417343	.9917293	93	1 94 04 49 2	703 045 457	.3229152	.1682134
9	76 62 41 2	347 334 289	.4554523	.9944876	94	1 94 32 36 2	708 870 984	.3363094	.1708852
1330	76 89 00 2	352 637 000	36.4691650	10.9972445	1395	1 94 60 25 2	714 704 875	37.3496988	11.1735558
1	77 15 61 2	357 947 691	.4828727	11.0000000	96	1 94 88 16 2	720 547 136	.3630834	.1762250
2	77 42 24 2	363 266 368	.4965752	.0027541	97	1 95 16 09 2	726 397 773	.3764632	.1788930
3	77 68 89 2	368 593 037	.5102725	.0055069	98	1 95 44 04 2	732 256 792	.3898382	.1815598
4	77 95 56 2	373 927 704	.5239647	.0082583	99	1 95 72 01 2	738 124 199	.4032084	.1842252
1335	78 22 25 2	379 270 375	36.5376518	11.0110082	1400	1 96 00 00 2	744 000 000	37.4165738	11.1868894
6	78 48 96 2	384 621 056	.5513338	.0137569	1	1 96 28 01 2	749 884 201	.4299345	.1895523
7	78 75 69 2	389 979 753	.5650106	.0165041	2	1 96 56 04 2	755 776 808	.4432904	.1922139
8	79 02 44 2	395 346 472	.5786823	.0192500	3	1 96 84 09 2	761 677 827	.4566416	.1948743
9	79 29 21 2	400 721 219	.5923489	.0219945	4	1 97 12 16 2	767 587 264	.4699880	.1975334
1340	79 56 00 2	406 104 000	36.6060104	11.0247377	1405	1 97 40 25 2	773 505 125	37.4833296	11.2001913
1	79 82 81 2	411 494 821	.6196668	.0274795	6	1 97 68 36 2	779 431 416	.4966665	.2028479
2	80 09 64 2	416 893 688	.6333181	.0302199	7	1 97 96 49 2	785 366 143	.5099987	.2055032
3	80 36 49 2	422 300 607	.6469644	.0329590	8	1 98 24 64 2	791 309 312	.5233261	.2081573
4	80 63 36 2	427 715 584	.6606056	.0356967	9	1 98 52 81 2	797 260 929	.5366487	.2108101
1345	80 90 25 2	433 138 625	36.6742416	11.0384330	1410	1 98 81 00 2	803 221 000	37.5499667	11.2134617
6	81 17 16 2	438 569 736	.6878726	.0411680	11	1 99 09 21 2	809 189 531	.5632799	.2161120
7	81 44 09 2	444 008 923	.7014986	.0439017	12	1 99 37 44 2	815 166 528	.5765885	.2187611
8	81 71 04 2	449 456 192	.7151195	.0466339	13	1 99 65 69 2	821 151 997	.5898922	.2214089
9	81 98 01 2	454 911 549	.7287353	.0493649	14	1 99 93 96 2	827 145 944	.6031913	.2240554
1350	82 25 00 2	460 375 000	36.7423461	11.0520945	1415	2 00 22 25 2	833 148 375	37.6164857	11.2267007
51	82 52 01 2	465 846 551	.7559519	.0548227	16	2 00 50 56 2	839 159 296	.6297754	.2293448
52	82 79 04 2	471 326 208	.7695526	.0575497	17	2 00 78 89 2	845 178 713	.6430604	.2319876
53	83 06 09 2	476 813 977	.7831483	.0602752	18	2 01 07 24 2	851 206 632	.6563407	.2346292
54	83 33 16 2	482 309 864	.7967390	.0629994	19	2 01 35 61 2	857 243 059	.6696164	.2372696
1355	83 60 25 2	487 813 875	36.8103246	11.0657222	1420	2 01 64 00 2	863 288 000	37.6828874	11.2399087
56	83 87 36 2	493 326 016	.8239053	.0684437	21	2 01 92 41 2	869 341 461	.6961536	.2425465
57	84 14 49 2	498 846 293	.8374809	.0711639	22	2 02 20 84 2	875 403 448	.7094153	.2451831
58	84 41 64 2	504 374 712	.8510515	.0738828	23	2 02 49 29 2	881 473 967	.7226722	.2478185
59	84 68 81 2	509 911 279	.8646172	.0766003	24	2 02 77 76 2	887 553 024	.7359245	.2504527
1360	84 96 00 2	515 456 000	36.8781778	11.0793165	1425	2 03 06 25 2	893 640 625	37.7491722	11.2530856
61	85 23 21 2	521 008 881	.8917335	.0820314	26	2 03 34 76 2	899 736 776	.7624152	.2557173
62	85 50 44 2	526 569 928	.9052842	.0847449	27	2 03 63 29 2	905 841 483	.7756535	.2583478
63	85 77 69 2	532 139 147	.9188299	.0874571	28	2 03 91 84 2	911 954 752	.7888873	.2609770
64	86 04 96 2	537 716 544	.9323706	.0901679	29	2 04 20 41 2	918 076 589	.8021163	.2636050
1365	86 32 25 2	543 302 125	36.9459064	11.0928775	1430	2 04 49 00 2	924 207 000	37.8153408	11.2662318

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9.—Squares, Cubes, Square Roots, Cube Roots, of Numbers 1 to 1600—Continued.

No.	Square	Cube.	Sq. Rt.	Cu. Rt.	No.	Square	Cube.	Sq. Rt.	Cu. Rt.
1430	2 04 49 00	2 924 207 000	37.8153408	11.2662318	1490	2 23 50 25	3 341 362 375	38.6652299	11.4344092
31	2 04 77 61	2 930 345 991	.8285606	.2688573	91	2 23 80 16	3 348 071 936	.6781593	.4369581
32	2 05 06 24	2 936 493 568	.8417759	.2714816	92	2 24 10 09	3 354 790 473	.6910843	.4395059
33	2 05 34 89	2 942 649 737	.8549864	.2741047	93	2 24 40 04	3 361 517 992	.7040050	.4420525
34	2 05 63 56	2 948 814 504	.8681924	.2767266	94	2 24 70 01	3 368 254 499	.7169214	.4445980
1435	2 05 92 25	2 954 987 875	37.8813938	11.2793472	1500	2 25 00 00	3 375 000 000	38.7298335	11.4471424
36	2 06 20 96	2 961 169 856	.8945906	.2819666	01	2 25 30 01	3 381 754 501	.7427412	.4496857
37	2 06 49 69	2 967 360 453	.9077828	.2845849	02	2 25 60 04	3 388 518 008	.7556447	.4523278
38	2 06 78 44	2 973 559 672	.9209704	.2872019	03	2 25 90 09	3 395 290 527	.7685439	.4547688
39	2 07 07 21	2 979 767 519	.9341535	.2898177	04	2 26 20 16	3 402 072 064	.7814389	.4573087
1440	2 07 36 00	2 985 984 000	37.9473319	11.2924322	1505	2 26 50 25	3 408 862 625	38.7943294	11.4598474
41	2 07 64 81	2 992 209 121	.9605058	.2950457	06	2 26 80 36	3 415 662 216	.8072158	.4623850
42	2 07 93 64	2 998 442 888	.9736751	.2976579	07	2 27 10 49	3 422 470 843	.8200978	.4649215
43	2 08 22 49	3 004 685 307	.9868398	.3002688	08	2 27 40 64	3 429 288 512	.8329757	.4674568
44	2 08 51 36	3 010 936 384	38.0000000	.3028786	09	2 27 70 81	3 436 115 229	.8458491	.4699911
1445	2 08 80 25	3 017 196 125	38.0131556	11.3054871	1510	2 28 01 00	3 442 951 000	38.8587184	11.4725242
46	2 09 09 16	3 023 464 536	.0263067	.3080945	11	2 28 31 21	3 449 795 831	.8715834	.4750562
47	2 09 38 09	3 029 741 623	.0394532	.3107006	12	2 28 61 44	3 456 649 728	.8844442	.4775871
48	2 09 67 04	3 036 027 392	.0525952	.3133056	13	2 28 91 69	3 463 512 697	.8973006	.4801169
49	2 09 96 01	3 042 321 849	.0657326	.3159094	14	2 29 21 96	3 470 384 744	.9101529	.4826455
1450	2 10 25 00	3 048 625 000	38.0788655	11.3185119	1515	2 29 52 25	3 477 265 875	38.9230009	11.4851731
51	2 10 54 01	3 054 936 851	.0919939	.3211132	16	2 29 82 56	3 484 156 096	.9358447	.4876995
52	2 10 83 04	3 061 257 408	.1051178	.3237134	17	2 30 12 89	3 491 055 413	.9486841	.4902249
53	2 11 12 09	3 067 586 677	.1182371	.3263124	18	2 30 43 24	3 497 963 832	.9615194	.4927491
54	2 11 41 16	3 073 924 664	.1313519	.3289102	19	2 30 73 61	3 504 881 359	.9743505	.4952722
1455	2 11 70 25	3 080 271 375	38.1444622	11.3315067	1520	2 31 04 00	3 511 808 000	38.9871774	11.4977942
56	2 11 99 36	3 086 626 816	.1575681	.3341022	21	2 31 34 41	3 518 743 761	39.0000000	.5003151
57	2 12 28 49	3 092 990 993	.1706693	.3366964	22	2 31 64 84	3 525 688 648	.0128184	.5028348
58	2 12 57 64	3 099 363 912	.1837662	.3392894	23	2 31 95 29	3 532 642 667	.0256326	.5053535
59	2 12 86 81	3 105 745 579	.1968585	.3418813	24	2 32 25 76	3 539 605 824	.0384426	.5078711
1460	2 13 16 00	3 112 136 000	38.2099463	11.3444719	1525	2 32 56 25	3 546 578 125	39.0512483	11.5103876
61	2 13 45 21	3 118 535 181	.2230297	.3470614	26	2 32 86 76	3 553 559 576	.0640499	.5129030
62	2 13 74 44	3 124 943 128	.2361085	.3496497	27	2 33 17 29	3 560 550 183	.0768473	.5154173
63	2 14 03 69	3 131 359 847	.2491829	.3522368	28	2 33 47 84	3 567 549 952	.0896406	.5179305
64	2 14 32 96	3 137 785 344	.2622529	.3548227	29	2 33 78 41	3 574 558 889	.1024296	.5204425
1465	2 14 62 25	3 144 219 625	38.2753184	11.3574075	1530	2 34 08 90	3 581 577 000	39.1152144	11.5229535
66	2 14 91 56	3 150 662 696	.2883794	.3599911	31	2 34 39 61	3 588 604 291	.1279951	.5254634
67	2 15 20 89	3 157 114 563	.3014360	.3625735	32	2 34 70 24	3 595 640 768	.1407716	.5279722
68	2 15 50 24	3 163 575 232	.3144881	.3651547	33	2 35 00 89	3 602 686 437	.1535439	.5304799
69	2 15 79 61	3 170 044 709	.3275358	.3677347	34	2 35 31 56	3 609 741 304	.1663120	.5329865
1470	2 16 09 00	3 176 523 000	38.3405790	11.3703136	1535	2 35 62 25	3 616 805 375	39.1790760	11.5354920
71	2 16 38 41	3 183 010 111	.3536178	.3728914	36	2 35 92 96	3 623 878 656	.1918359	.5379965
72	2 16 67 84	3 189 506 048	.3666522	.3754679	37	2 36 23 69	3 630 961 153	.2045915	.5404998
73	2 16 97 29	3 196 010 817	.3796821	.3780433	38	2 36 54 44	3 638 052 872	.2173431	.5430021
74	2 17 26 76	3 202 524 424	.3927076	.3806175	39	2 36 85 21	3 645 153 819	.2300905	.5455033
1475	2 17 56 25	3 209 046 875	38.4057287	11.3831906	1540	2 37 16 00	3 652 264 000	39.2428337	11.5480034
76	2 17 85 76	3 215 578 176	.4187454	.3857625	41	2 37 46 81	3 659 383 421	.2555728	.5505025
77	2 18 15 29	3 222 118 333	.4317577	.3883332	42	2 37 77 64	3 666 512 088	.2683078	.5530004
78	2 18 44 84	3 228 667 352	.4447656	.3909028	43	2 38 08 49	3 673 650 007	.2810387	.5554973
79	2 18 74 41	3 235 225 239	.4577691	.3934712	44	2 38 39 36	3 680 797 184	.2937654	.5579931
1480	2 19 04 00	3 241 792 000	38.4707681	11.3960384	1545	2 38 70 25	3 687 953 625	39.3064880	11.5604878
81	2 19 33 61	3 248 367 641	.4837627	.3986045	46	2 39 01 16	3 695 119 336	.3192065	.5629815
82	2 19 63 24	3 254 952 168	.4967530	.4011695	47	2 39 32 09	3 702 294 323	.3319208	.5654740
83	2 19 92 89	3 261 545 587	.5097390	.4037332	48	2 39 63 04	3 709 478 592	.3446311	.5679655
84	2 20 22 56	3 268 147 904	.5227206	.4062959	49	2 39 94 01	3 716 672 149	.3573373	.5704559
1485	2 20 52 25	3 274 759 125	38.5356977	11.4088574	1550	2 40 25 00	3 723 875 000	39.3700394	11.5729453
86	2 20 81 96	3 281 379 256	.5486705	.4114177	51	2 40 56 01	3 731 087 151	.3827373	.5754334
87	2 21 11 69	3 288 008 303	.5616389	.4139769	52	2 40 87 04	3 738 308 608	.3954312	.5779208
88	2 21 41 44	3 294 646 272	.5746030	.4165349	53	2 41 18 09	3 745 539 377	.4081210	.5804064
89	2 21 71 21	3 301 293 169	.5875627	.4190918	54	2 41 49 16	3 752 779 464	.4208067	.5828919
1490	2 22 01 00	3 307 949 000	38.6005181	11.4216476	1555	2 41 80 25	3 760 028 875	39.4334883	11.5853759
91	2 22 30 81	3 314 613 771	.6134691	.4242022	56	2 42 11 36	3 767 287 616	.4461658	.5878588
92	2 22 60 64	3 321 287 488	.6264158	.4267556	57	2 42 42 49	3 774 555 693	.4588393	.5903407
93	2 22 90 49	3 327 970 157	.6393582	.4293079	58	2 42 73 64	3 781 833 112	.4715087	.5928215
94	2 23 20 36	3 334 661 784	.6522962	.4318591	59	2 43 04 81	3 789 119 879	.4841740	.5953013
1495	2 23 50 25	3 341 362 375	38.6652299	11.4344092	1560	2 43 36 00	3 796 416 000	39.4968353	11.5977799

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9.—Squares, Cubes, Square Roots, Cube Roots, of Numbers 1 to 1600—Concluded.

No.	Square	Cube.	Sq. Rt	Cu. Rt	No.	Square	Cube.	Sq. Rt.	Cu. Rt.
1500	2 43 36 00	3 796 416 000	39.4968353	11.5977799	1580	2 49 64 00	3 944 312 000	39.7492138	11.6471329
510	2 43 67 21	3 803 721 481	.5094925	.6002576	810	2 49 95 61	3 951 805 941	.7617907	.6495895
520	2 43 98 44	3 811 036 328	.5221457	.6027342	820	2 50 27 24	3 959 309 368	.7743636	.6520452
530	2 44 29 69	3 818 360 547	.5347948	.6052097	830	2 50 58 89	3 966 822 287	.7869325	.6544998
540	2 44 60 96	3 825 694 144	.5474399	.6076841	840	2 50 90 56	3 974 344 704	.7994975	.6569534
1550	2 44 92 25	3 833 037 125	39.5600809	11.6101575	1580	2 51 22 25	3 981 876 625	39.8120585	11.6594059
560	2 45 23 56	3 840 389 496	.5727179	.6126299	860	2 51 53 96	3 989 418 056	.8246155	.6618574
570	2 45 54 89	3 847 751 263	.5853506	.6151012	870	2 51 85 69	3 996 969 003	.8371686	.6643079
580	2 45 86 24	3 855 122 432	.5979797	.6175715	880	2 52 17 44	4 004 529 472	.8497177	.6667574
590	2 46 17 61	3 862 503 009	.6106046	.6200407	890	2 52 49 21	4 012 099 469	.8622628	.6692058
1600	2 46 49 00	3 869 893 000	39.6232255	11.6225088	1600	2 52 81 00	4 019 679 000	39.8748040	11.6716532
710	2 46 80 41	3 877 292 411	.6358424	.6249759	910	2 53 12 81	4 027 268 071	.8873413	.6740996
720	2 47 11 84	3 884 701 248	.6484552	.6274420	920	2 53 44 64	4 034 866 688	.8998747	.6765449
730	2 47 43 29	3 892 119 517	.6610640	.6299070	930	2 53 76 49	4 042 474 857	.9124041	.6789892
740	2 47 74 76	3 899 547 224	.6736688	.6323710	940	2 54 08 36	4 050 092 584	.9249295	.6814325
1750	2 48 06 25	3 906 984 375	39.6862696	11.6348339	1590	2 54 40 25	4 057 719 875	39.9374511	11.6838748
760	2 48 37 76	3 914 430 976	.6988665	.6372957	960	2 54 72 16	4 065 356 736	.9499687	.6863161
770	2 48 69 29	3 921 887 033	.7114593	.6397566	970	2 55 04 09	4 073 003 173	.9624824	.6887563
780	2 49 00 84	3 929 352 552	.7240481	.6422164	980	2 55 36 04	4 080 659 192	.9749922	.6911955
790	2 49 32 41	3 936 827 539	.7366329	.6446751	990	2 55 68 01	4 088 324 799	.9874980	.6936337
1800	2 49 64 00	3 944 312 000	39.7492138	11.6471329	1600	2 56 00 00	4 096 000 000	40.0000000	11.6960709

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10.—Square Roots and Cube Roots of Numbers 1600 to 3200.

No.	Sq. Rt.	Cu. Rt	No.	Sq. Rt.	Cu. Rt.	No.	Sq. Rt.	Cu. Rt.	No.	Sq. Rt.	Cu. Rt.
1600	40.0000	11.6961	1665	40.8044	11.8524	1730	41.5933	12.0046	1795	42.3674	12.1531
1	.0125	.6985	66	.8167	.8547	31	.6053	.0069	96	.3792	.1554
2	.0250	.7009	67	.8289	.8571	32	.6173	.0093	97	.3910	.1576
3	.0375	.7034	68	.8412	.8595	33	.6293	.0116	98	.4028	.1599
4	.0500	.7058	69	.8534	.8618	34	.6413	.0139	99	.4146	.1622
1605	40.0625	11.7082	1670	40.8656	11.8642	1735	41.6533	12.0162	1800	42.4264	12.1644
6	.0749	.7107	71	.8779	.8666	36	.6653	.0185	1	.4382	.1667
7	.0874	.7131	72	.8901	.8689	37	.6773	.0208	2	.4500	.1689
8	.0999	.7155	73	.9023	.8713	38	.6893	.0231	3	.4617	.1712
9	.1123	.7180	74	.9145	.8737	39	.7013	.0254	4	.4735	.1734
1610	40.1248	11.7204	1675	40.9268	11.8760	1740	41.7133	12.0277	1805	42.4853	12.1757
11	.1373	.7228	76	.9390	.8784	41	.7253	.0300	6	.4971	.1779
12	.1497	.7252	77	.9512	.8808	42	.7373	.0323	7	.5088	.1802
13	.1622	.7277	78	.9634	.8831	43	.7493	.0346	8	.5206	.1824
14	.1746	.7301	79	.9756	.8855	44	.7612	.0369	9	.5323	.1846
1615	40.1871	11.7325	1680	40.9878	11.8878	1745	41.7732	12.0392	1810	42.5441	12.1869
16	.1995	.7350	81	41.0000	.8902	46	.7852	.0415	11	.5558	.1891
17	.2119	.7373	82	.0122	.8926	47	.7971	.0438	12	.5676	.1914
18	.2244	.7398	83	.0244	.8949	48	.8091	.0461	13	.5793	.1936
19	.2368	.7422	84	.0366	.8973	49	.8210	.0484	14	.5911	.1959
1620	40.2492	11.7446	1685	41.0488	11.8996	1750	41.8330	12.0507	1815	42.6028	12.1981
21	.2616	.7470	86	.0609	.9020	51	.8450	.0530	16	.6146	.2003
22	.2741	.7494	87	.0731	.9043	52	.8569	.0553	17	.6263	.2026
23	.2865	.7518	88	.0853	.9067	53	.8688	.0576	18	.6380	.2048
24	.2989	.7543	89	.0974	.9090	54	.8808	.0599	19	.6497	.2071
1625	40.3113	11.7567	1690	41.1096	11.9114	1755	41.8927	12.0622	1820	42.6615	12.2093
26	.3237	.7591	91	.1218	.9137	56	.9047	.0645	21	.6732	.2115
27	.3361	.7615	92	.1339	.9161	57	.9166	.0668	22	.6849	.2138
28	.3485	.7639	93	.1461	.9184	58	.9285	.0690	23	.6966	.2160
29	.3609	.7663	94	.1582	.9208	59	.9404	.0713	24	.7083	.2182
1630	40.3733	11.7687	1695	41.1704	11.9231	1760	41.9524	12.0736	1825	42.7200	12.2205
31	.3856	.7711	96	.1825	.9255	61	.9643	.0759	26	.7317	.2227
32	.3980	.7735	97	.1947	.9278	62	.9762	.0782	27	.7434	.2249
33	.4104	.7759	98	.2068	.9301	63	.9881	.0805	28	.7551	.2272
34	.4228	.7783	99	.2189	.9325	64	42.0000	.0828	29	.7668	.2294
1635	40.4351	11.7807	1700	41.2311	11.9348	1765	42.0119	12.0850	1830	42.7785	12.2316
36	.4475	.7831	1	.2432	.9372	66	.0238	.0873	31	.7902	.2338
37	.4599	.7855	2	.2553	.9395	67	.0357	.0896	32	.8019	.2361
38	.4722	.7879	3	.2674	9418	68	.0476	.0919	33	.8135	.2383
39	4846	.7903	4	.2795	.9442	69	.0595	.0942	34	.8252	.2405
1640	40.4969	11.7927	1705	41.2916	11.9465	1770	42.0714	12.0964	1835	42.8369	12.2427
41	.5093	.7951	6	.3038	.9489	71	.0833	.0987	36	.8486	.2450
42	.5216	.7975	7	.3159	.9512	72	.0951	.1010	37	.8602	.2472
43	.5339	.7999	8	.3280	.9535	73	.1070	.1033	38	.8719	.2494
44	.5463	.8023	9	.3401	.9559	74	.1189	.1056	39	.8836	.2516
1645	40.5586	11.8047	1710	41.3521	11.9582	1775	42.1307	12.1078	1840	42.8952	12.2539
46	.5709	.8071	11	.3642	.9605	76	.1426	.1101	41	.9069	.2561
47	.5832	.8095	12	.3763	.9628	77	.1545	.1124	42	.9185	.2583
48	.5956	.8119	13	.3884	.9652	78	.1663	.1146	43	.9302	.2605
49	.6079	.8143	14	.4005	.9675	79	.1782	.1169	44	.9418	.2627
1650	40.6202	11.8167	1715	41.4126	11.9698	1780	42.1900	12.1192	1845	42.9535	12.2649
51	.6325	.8190	16	.4246	.9722	81	.2019	.1215	46	.9651	.2672
52	.6448	.8214	17	.4367	.9745	82	.2137	.1237	47	.9767	.2694
53	.6571	.8238	18	.4488	.9768	83	.2256	.1260	48	.9884	.2716
54	.6694	.8262	19	.4608	.9791	84	.2374	.1283	49	43.0000	.2738
1655	40.6817	11.8286	1720	41.4729	11.9815	1785	42.2493	12.1305	1850	43.0116	12.2760
56	.6940	.8310	21	.4849	.9838	86	.2611	.1328	51	.0232	.2782
57	.7063	.8333	22	.4970	.9861	87	.2729	.1350	52	.0349	.2804
58	.7185	.8357	23	.5090	.9884	88	.2847	.1373	53	.0465	.2826
59	.7308	.8381	24	.5211	.9907	89	.2966	.1396	54	.0581	.2849
1660	40.7431	11.8405	1725	41.5331	11.9931	1790	42.3084	12.1418	1855	43.0697	12.2871
61	.7554	.8429	26	.5452	.9954	91	.3202	.1441	56	.0813	.2893
62	.7676	.8452	27	.5572	.9977	92	3320	.1464	57	.0929	.2915
63	.7799	.8476	28	.5692	12.0000	93	.3438	.1486	58	.1045	.2937
64	.7922	.8500	29	.5812	.0023	94	.3556	.1509	59	.1161	.2959
1665	40.8044	11.8524	1730	41.5933	12.0046	1795	42.3674	12.1531	1860	43.1277	12.2981

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10.—Square Roots and Cube Roots of Numbers 1600 to 3200—Continued.

No.	Sq. Rt.	Cu. Rt.	No.	Sq. Rt	Cu. Rt.	No.	Sq. Rt.	Cu. Rt.	No.	Sq. Rt.	Cu Rt.
1860	43.1277	12.2981	1925	43.8748	12.4397	1990	44.6094	12.5782	2055	45.3321	12.7137
61	.1393	.3003	26	.8862	.4419	91	.6206	.5803	56	.3431	.7157
62	.1509	.3025	27	.8976	.4440	92	.6318	.5824	57	.3542	.7178
63	.1625	.3047	28	.9090	.4462	93	.6430	.5845	58	.3652	.7198
64	.1741	.3069	29	.9204	.4483	94	.6542	.5866	59	.3762	.7219
1865	43.1856	12.3091	1930	43.9318	12.4505	1995	44.6654	12.5887	2060	45.3872	12.7240
66	.1972	.3113	31	.9431	.4526	96	.6766	.5908	61	.3982	.7260
67	.2088	.3135	32	.9545	.4548	97	.6878	.5929	62	.4093	.7281
68	.2204	.3157	33	.9659	.4569	98	.6990	.5950	63	.4203	.7301
69	.2319	.3179	34	.9773	.4591	99	.7102	.5971	64	.4313	.7322
1870	43.2435	12.3201	1935	43.9886	12.4612	2000	44.7214	12.5992	2065	45.4423	12.7342
71	.2551	.3223	36	44.0000	.4634	1	.7325	.6013	66	.4533	.7363
72	.2666	.3245	37	.0114	.4655	2	.7437	.6034	67	.4643	.7384
73	.2782	.3267	38	.0227	.4676	3	.7549	.6055	68	.4753	.7404
74	.2897	.3289	39	.0341	.4698	4	.7661	.6076	69	.4863	.7425
1875	43.3013	12.2311	1940	44.0454	12.4719	2005	44.7772	12.6097	2070	45.4973	12.7445
76	.3128	.3333	41	.0568	.4741	6	.7884	.6118	71	.5082	.7466
77	.3244	.3354	42	.0681	.4762	7	.7996	.6139	72	.5192	.7486
78	.3359	.3376	43	.0795	.4784	8	.8107	.6160	73	.5302	.7507
79	.3474	.3398	44	.0908	.4805	9	.8219	.6181	74	.5412	.7527
1880	43.3590	12 3420	1945	44.1022	12.4826	2010	44.8330	12.6202	2075	45.5522	12.7548
81	.3705	.3442	46	.1135	.4848	11	.8442	.6223	76	.5631	.7568
82	.3820	.3464	47	.1248	.4869	12	.8553	.6244	77	.5741	.7589
83	.3935	.3486	48	.1362	.4891	13	.8665	.6264	78	.5851	.7609
84	.4051	.3508	49	.1475	.4912	14	.8776	6285	79	.5961	.7630
1885	43.4166	12.3529	1950	44.1588	12.4933	2015	44.8888	12.6306	2080	45.6070	12.7650
86	.4281	.3551	51	.1701	.4955	16	.8999	.6327	81	.6180	.7671
87	.4396	.3573	52	.1814	.4976	17	.9110	.6348	82	.6289	.7691
88	.4511	.3595	53	.1928	.4997	18	.9222	.6369	83	.6399	.7711
89	.4626	.3617	54	.2041	.5019	19	.9333	.6390	84	.6508	.7732
1890	43.4741	12.3639	1955	44.2154	12.5040	2020	44.9444	12.6411	2085	45.6618	12.7752
91	.4856	.3660	56	.2267	.5061	21	.9555	.6432	86	.6727	.7773
92	.4971	.3682	57	.2380	.5083	22	.9667	.6452	87	.6837	.7793
93	.5086	.3704	58	.2493	.5104	23	.9778	.6473	88	.6946	.7814
94	.5201	.3726	59	.2606	.5125	24	.9889	.6494	89	.7056	.7834
1895	43.5316	12.3747	1960	44.2719	12.5146	2025	45.0000	12.6515	2090	45.7165	12.7854
96	.5431	.3769	61	.2832	.5168	26	.0111	.6536	91	.7275	.7875
97	.5546	.3791	62	.2945	.5189	27	.0222	.6557	92	.7384	.7895
98	.5660	.3813	63	.3058	.5210	28	.0333	.6577	93	.7493	.7915
99	.5775	.3834	64	.3170	.5232	29	.0444	.6598	94	.7602	.7936
1900	43.5890	12.3856	1965	44.3283	12.5253	2030	45.0555	12.6619	2095	45.7712	12.7956
1	.6005	.3878	66	.3396	.5274	31	.0666	.6640	96	.7821	.7977
2	.6119	.3900	67	.3509	.5295	32	.0777	.6661	97	.7930	.7997
3	.6234	.3921	68	.3621	.5317	33	.0888	.6681	98	.8039	.8017
4	.6348	.3943	69	.3734	.5338	34	.0999	.6702	99	.8148	.8038
1905	43.6463	12.3965	1970	44.3847	12.5359	2035	45.1110	12.6723	2100	45.8258	12.8058
6	.6578	.3986	71	.3959	.5380	36	.1221	.6744	1	.8367	.8078
7	.6692	.4008	72	.4072	.5401	37	.1331	.6764	2	.8476	.8099
8	.6807	.4030	73	.4185	.5423	38	.1442	.6785	3	.8585	.8119
9	.6921	.4051	74	.4297	.5444	39	.1553	.6806	4	.8694	.8139
1910	43.7035	12.4073	1975	44.4410	12.5465	2040	45.1664	12.6827	2105	45.8803	12.8159
11	.7150	.4095	76	.4522	.5486	41	.1774	.6847	6	.8912	.8180
12	.7264	.4116	77	.4635	.5507	42	.1885	.6868	7	.9021	.8200
13	.7379	.4138	78	.4747	.5528	43	.1996	.6889	8	.9130	.8220
14	.7493	.4160	79	.4860	.5550	44	.2106	.6909	9	.9238	.8241
1915	43.7607	12.4181	1980	44.4972	12.5571	2045	45.2217	12.6930	2110	45.9347	12.8261
16	.7721	.4203	81	.5084	.5592	46	.2327	.6951	11	.9456	.8281
17	.7836	.4225	82	.5197	.5613	47	.2438	.6971	12	.9565	.8301
18	.7950	.4246	83	.5309	.5634	48	.2548	.6992	13	.9674	.8322
19	.8064	.4268	84	.5421	.5655	49	.2659	.7013	14	.9783	.8342
1920	43.8178	12.4289	1985	44.5533	12.5676	2050	45.2769	12.7033	2115	45.9891	12.8362
21	.8292	.4311	86	.5646	.5697	51	.2880	.7054	16	46.0000	.8382
22	.8406	.4332	87	.5758	.5719	52	.2990	.7075	17	.0109	.8403
23	.8520	.4354	88	.5870	.5740	53	.3100	.7095	18	.0217	.8423
24	.8634	.4376	89	.5982	.5761	54	.3211	.7116	19	.0326	.8443
1925	43.8748	12.4397	1990	44.6094	12.5782	2055	45.3321	12.7137	2120	46.0435	12.8463

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10.—Square Roots and Cube Roots of Numbers

1600 to 3200—Continued.

No.	Sq. Rt.	Cu. Rt.	No.	Sq. Rt.	Cu. Rt.	No.	Sq. Rt.	Cu. Rt.	No.	Sq. Rt.	Cu. Rt.
2120	46.0435	12.8463	2185	46.7440	12.9763	2250	47.4342	13.1037	2315	48.1144	13.2287
21	.0543	.8483	86	.7547	.9783	51	.4447	.1056	16	.1248	.2306
22	.0652	.8504	87	.7654	.9802	52	.4552	.1076	17	.1352	.2325
23	.0760	.8524	88	.7761	.9822	53	.4658	.1095	18	.1456	.2344
24	.0869	.8544	89	.7868	.9842	54	.4763	.1115	19	.1560	.2363
2125	46.0977	12.8564	2190	46.7974	12.9862	2255	47.4868	13.1134	2320	48.1664	13.2382
26	.1086	.8584	91	.8081	.9882	56	.4974	.1153	21	.1768	.2401
27	.1194	.8604	92	.8188	.9901	57	.5079	.1173	22	.1871	.2420
28	.1303	.8625	93	.8295	.9921	58	.5184	.1192	23	.1975	.2439
29	.1411	.8645	94	.8402	.9941	59	.5289	.1212	24	.2079	.2458
2130	46.1519	12.8665	2195	46.8508	12.9961	2260	47.5395	13.1231	2325	48.2183	13.2477
31	.1628	.8685	96	.8615	.9980	61	.5500	.1250	26	.2286	.2496
32	.1736	.8705	97	.8722	13.0000	62	.5605	.1270	27	.2390	.2515
33	.1844	.8725	98	.8828	.0020	63	.5710	.1289	28	.2494	.2534
34	.1952	.8745	99	.8935	.0039	64	.5815	.1308	29	.2597	.2553
2135	46.2061	12.8765	2200	46.9042	13.0059	2265	47.5920	13.1328	2330	48.2701	13.2572
36	.2169	.8786	1	.9148	.0079	66	.6025	.1347	31	.2804	.2591
37	.2277	.8806	2	.9255	.0099	67	.6130	.1366	32	.2908	.2610
38	.2385	.8826	3	.9361	.0118	68	.6235	.1386	33	.3011	.2629
39	.2493	.8846	4	.9468	.0138	69	.6340	.1405	34	.3115	.2648
2140	46.2601	12.8866	2205	46.9574	13.0158	2270	47.6445	13.1424	2335	48.3218	13.2667
41	.2709	.8886	6	.9681	.0177	71	.6550	.1443	36	.3322	.2686
42	.2817	.8906	7	.9787	.0197	72	.6655	.1463	37	.3425	.2705
43	.2925	.8926	8	.9894	.0217	73	.6760	.1482	38	.3529	.2724
44	.3033	.8946	9	47.0000	.0236	74	.6865	.1501	39	.3632	.2743
2145	46.3141	12.8966	2210	47.0106	13.0256	2275	47.6970	13.1521	2340	48.3735	13.2761
46	.3249	.8986	11	.0213	.0276	76	.7074	.1540	41	.3839	.2780
47	.3357	.9006	12	.0319	.0295	77	.7179	.1559	42	.3942	.2799
48	.3465	.9026	13	.0425	.0315	78	.7284	.1578	43	.4045	.2818
49	.3573	.9046	14	.0532	.0334	79	.7389	.1598	44	.4149	.2837
2150	46.3681	12.9066	2215	47.0638	13.0354	2280	47.7493	13.1617	2345	48.4252	13.2856
51	.3789	.9086	16	.0744	.0374	81	.7598	.1636	46	.4355	.2875
52	.3897	.9106	17	.0850	.0393	82	.7703	.1655	47	.4458	.2894
53	.4004	.9126	18	.0956	.0413	83	.7807	.1675	48	.4562	.2913
54	.4112	.9146	19	.1063	.0433	84	.7912	.1694	49	.4665	.2931
2155	46.4220	12.9166	2220	47.1169	13.0453	2285	47.8017	13.1713	2350	48.4768	13.2950
56	.4327	.9186	21	.1275	.0473	86	.8121	.1732	51	.4871	.2969
57	.4435	.9206	22	.1381	.0491	87	.8226	.1751	52	.4974	.2988
58	.4543	.9226	23	.1487	.0511	88	.8330	.1771	53	.5077	.3007
59	.4650	.9246	24	.1593	.0530	89	.8435	.1790	54	.5180	.3026
2160	46.4758	12.9266	2225	47.1699	13.0550	2290	47.8539	13.1809	2355	48.5283	13.3045
61	.4866	.9286	26	.1805	.0569	91	.8644	.1828	56	.5386	.3063
62	.4973	.9306	27	.1911	.0589	92	.8748	.1847	57	.5489	.3082
63	.5081	.9326	28	.2017	.0609	93	.8853	.1867	58	.5592	.3101
64	.5188	.9346	29	.2123	.0628	94	.8957	.1886	59	.5695	.3120
2165	46.5296	12.9366	2230	47.2229	13.0648	2295	47.9062	13.1905	2360	48.5798	13.3139
66	.5403	.9386	31	.2335	.0667	96	.9166	.1924	61	.5901	.3157
67	.5510	.9406	32	.2440	.0687	97	.9270	.1943	62	.6004	.3176
68	.5618	.9425	33	.2546	.0706	98	.9375	.1962	63	.6107	.3195
69	.5725	.9445	34	.2652	.0726	99	.9479	.1981	64	.6210	.3214
2170	46.5833	12.9465	2235	47.2758	13.0745	2300	47.9583	13.2001	2365	48.6313	13.3233
71	.5940	.9485	36	.2864	.0765	1	.9687	.2020	66	.6415	.3251
72	.6047	.9505	37	.2969	.0784	2	.9792	.2039	67	.6518	.3270
73	.6154	.9525	38	.3075	.0804	3	.9896	.2058	68	.6621	.3289
74	.6262	.9545	39	.3181	.0823	4	48.0000	.2077	69	.6724	.3308
2175	46.6369	12.9565	2240	47.3286	13.0843	2305	48.0104	13.2096	2370	48.6826	13.3326
76	.6476	.9584	41	.3392	.0862	6	.0208	.2115	71	.6929	.3345
77	.6583	.9604	42	.3498	.0882	7	.0312	.2134	72	.7032	.3364
78	.6690	.9624	43	.3603	.0901	8	.0416	.2153	73	.7134	.3383
79	.6798	.9644	44	.3709	.0920	9	.0521	.2173	74	.7237	.3401
2180	46.6905	12.9664	2245	47.3814	13.0940	2310	48.0625	13.2192	2375	48.7340	13.3420
81	.7012	.9684	46	.3920	.0959	11	.0729	.2211	76	.7442	.3439
82	.7119	.9703	47	.4025	.0979	12	.0833	.2230	77	.7545	.3458
83	.7226	.9723	48	.4131	.0998	13	.0937	.2249	78	.7647	.3476
84	.7333	.9743	49	.4236	.1018	14	.1041	.2268	79	.7750	.3495
2185	46.7440	12.9763	2250	47.4342	13.1037	2315	48.1144	13.2287	2380	48.7852	13.3514

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10.—Square Roots and Cube Roots of Numbers 1600 to 3200—Continued.

No.	Sq. Rt.	Cu. Rt.	No.	Sq. Rt.	Cu. Rt.	No.	Sq. Rt.	Cu. Rt.	No.	Sq. Rt.	Cu. Rt.
2280	48.7852	13.3514	2445	49.4469	13.4718	2510	50.0999	13.5902	2575	50.7445	13.7065
81	.7955	.3532	46	.4571	.4737	11	.1099	.5920	76	.7543	.7082
82	.8057	.3551	47	.4672	.4755	12	.1199	.5938	77	.7642	.7100
83	.8160	.3570	48	.4773	.4773	13	.1298	.5956	78	.7740	.7118
84	.8262	.3588	49	.4874	.4792	14	.1398	.5974	79	.7839	.7136
2285	48.8365	13.3607	2450	49.4975	13.4810	2515	50.1498	13.5992	2580	50.7937	13.7153
86	.8467	.3626	51	.5076	.4828	16	.1597	.6010	81	.8035	.7171
87	.8569	.3644	52	.5177	.4847	17	.1697	.6028	82	.8134	.7189
88	.8672	.3663	53	.5278	.4865	18	.1797	.6046	83	.8232	.7207
89	.8774	.3682	54	.5379	.4883	19	.1896	.6064	84	.8331	.7224
2290	48.8876	13.3700	2455	49.5480	13.4902	3520	50.1996	13.6082	2585	50.8429	13.7242
91	.8979	.3719	56	.5580	.4920	21	.2096	.6100	86	.8527	.7260
92	.9081	.3738	57	.5681	.4938	22	.2195	.6118	87	.8626	.7277
93	.9183	.3756	58	.5782	.4957	23	.2295	.6136	88	.8724	.7295
94	.9285	.3775	59	.5883	.4975	24	.2394	.6154	89	.8822	.7313
2295	48.9387	13.3794	2460	49.5984	13.4993	2525	50.2494	13.6172	2590	50.8920	13.7330
96	.9490	.3812	61	.6085	.5011	26	.2593	.6190	91	.9019	.7348
97	.9592	.3831	62	.6185	.5030	27	.2693	.6208	92	.9117	.7366
98	.9694	.3849	63	.6286	.5048	28	.2792	.6226	93	.9215	.7383
99	.9796	.3868	64	.6387	.5066	29	.2892	.6244	94	.9313	.7401
2400	48.9898	13.3887	2465	49.6488	13.5085	2530	50.2991	13.6262	2595	50.9411	13.7419
1	49.0000	.3905	66	.6588	.5103	31	.3090	.6280	96	.9510	.7436
2	.0102	.3924	67	.6689	.5121	32	.3190	.6298	97	.9608	.7454
3	.0204	.3942	68	.6790	.5139	33	.3289	.6315	98	.9706	.7472
4	.0306	.3961	69	.6890	.5158	34	.3389	.6333	99	.9804	.7489
2405	49.0408	13.3980	2470	49.6991	13.5176	2535	50.3488	13.6351	2600	50.9902	13.7507
6	.0510	.3998	71	.7092	.5194	36	.3587	.6369	1	51.0000	.7525
7	.0612	.4017	72	.7192	.5212	37	.3686	.6387	2	.0098	.7542
8	.0714	.4035	73	.7293	.5231	38	.3786	.6405	3	.0196	.7560
9	.0816	.4054	74	.7393	.5249	39	.3885	.6423	4	.0294	.7577
2410	49.0918	13.4072	2475	49.7494	13.5267	2540	50.3984	13.6441	2605	51.0392	13.7595
11	.1019	.4091	76	.7594	.5285	41	.4083	.6459	6	.0490	.7613
12	.1121	.4109	77	.7695	.5303	42	.4183	.6477	7	.0588	.7630
13	.1223	.4128	78	.7795	.5322	43	.4282	.6495	8	.0686	.7648
14	.1325	.4146	79	.7896	.5340	44	.4381	.6512	9	.0784	.7665
2415	49.1426	13.4165	2480	49.7996	13.5358	2545	50.4480	13.6530	2610	51.0882	13.7683
16	.1528	.4183	81	.8096	.5376	46	.4579	.6548	11	.0979	.7701
17	.1630	.4201	82	.8197	.5394	47	.4678	.6566	12	.1077	.7718
18	.1732	.4220	83	.8297	.5413	48	.4777	.6584	13	.1175	.7736
19	.1833	.4239	84	.8397	.5431	49	.4876	.6602	14	.1273	.7753
2420	49.1935	13.4257	2485	49.8498	13.5449	2550	50.4975	13.6620	2615	51.1371	13.7771
21	.2037	.4276	86	.8598	.5467	51	.5074	.6638	16	.1468	.7788
22	.2138	.4294	87	.8698	.5485	52	.5173	.6655	17	.1566	.7806
23	.2240	.4313	88	.8799	.5503	53	.5272	.6673	18	.1664	.7823
24	.2341	.4331	89	.8899	.5522	54	.5371	.6691	19	.1762	.7841
2425	49.2443	13.4350	2490	49.8999	13.5540	2555	50.5470	13.6709	2620	51.1859	13.7859
26	.2544	.4368	91	.9099	.5558	56	.5569	.6727	21	.1957	.7876
27	.2646	.4387	92	.9199	.5576	57	.5668	.6745	22	.2055	.7894
28	.2747	.4405	93	.9300	.5594	58	.5767	.6762	23	.2152	.7911
29	.2849	.4424	94	.9400	.5612	59	.5866	.6780	24	.2250	.7929
2430	49.2950	13.4442	2495	49.9500	13.5630	2560	50.5964	13.6798	2625	51.2348	13.7946
31	.3052	.4461	96	.9600	.5648	61	.6063	.6816	26	.2445	.7964
32	.3153	.4479	97	.9700	.5667	62	.6162	.6834	27	.2543	.7981
33	.3254	.4497	98	.9800	.5685	63	.6261	.6851	28	.2640	.7999
34	.3356	.4516	99	.9900	.5703	64	.6360	.6869	29	.2738	.8016
2435	49.3457	13.4534	2500	50.0000	13.5721	2565	50.6458	13.6887	2630	51.2835	13.8034
36	.3559	.4553	1	.0100	.5739	66	.6557	.6905	31	.2933	.8051
37	.3660	.4571	2	.0200	.5757	67	.6656	.6923	32	.3030	.8069
38	.3761	.4590	3	.0300	.5775	68	.6754	.6940	33	.3128	.8086
39	.3862	.4608	4	.0400	.5793	69	.6853	.6958	34	.3225	.8104
2440	49.3964	13.4626	2505	50.0500	13.5811	2570	50.6952	13.6976	2635	51.3323	13.8121
41	.4065	.4645	6	.0600	.5829	71	.7050	.6994	36	.3420	.8139
42	.4166	.4663	7	.0700	.5847	72	.7149	.7011	37	.3517	.8156
43	.4267	.4681	8	.0799	.5865	73	.7247	.7029	38	.3615	.8174
44	.4368	.4700	9	.0899	.5884	74	.7346	.7047	39	.3712	.8191
2445	49.4469	13.4718	2510	50.0999	13.5902	2575	50.7445	13.7065	2640	51.3809	13.8208

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10.—Square Roots and Cube Roots of Numbers 1600 to 3200—Continued.

No.	Sq. Rt.	Cu. Rt.	No.	Sq. Rt.	Cu. Rt.	No.	Sq. Rt.	Cu. Rt.	No.	Sq. Rt.	Cu. Rt.
2640	51.3809	13.8208	2705	52.0096	13.9334	2770	52.6308	14.0441	2835	53.2447	14.153
41	.3907	.8226	6	.0192	.9351	71	.6403	.0458	36	.2541	.154
42	.4004	.8243	7	.0288	.9368	72	.6498	.0475	37	.2635	.156
43	.4101	.8261	8	.0384	.9385	73	.6593	.0491	38	.2729	.158
44	.4198	.8278	9	.0481	.9402	74	.6688	.0508	39	.2823	.159
2645	51.4296	13.8296	2710	52.0577	13.9419	2775	52.6783	14.0525	2840	53.2917	14.161
46	.4393	.8313	11	.0673	.9437	76	.6878	.0542	41	.3010	.163
47	.4490	.8331	12	.0769	.9454	77	.6972	.0559	42	.3104	.164
48	.4587	.8348	13	.0865	.9471	78	.7067	.0576	43	.3198	.166
49	.4684	.8365	14	.0961	.9488	79	.7162	.0593	44	.3292	.168
2650	51.4782	13.8383	2715	52.1057	13.9505	2780	52.7257	14.0610	2845	53.3385	14.169
51	.4879	.8400	16	.1153	.9522	81	.7352	.0626	46	.3479	.171
52	.4976	.8418	17	.1249	.9539	82	.7447	.0643	47	.3573	.173
53	.5073	.8435	18	.1344	.9556	83	.7541	.0660	48	.3667	.174
54	.5170	.8452	19	.1440	.9574	84	.7636	.0677	49	.3760	.176
2655	51.5267	13.8470	2720	52.1536	13.9591	2785	52.7731	14.0694	2850	53.3854	14.178
56	.5364	.8487	21	.1632	.9608	86	.7826	.0711	51	.3948	.179
57	.5461	.8504	22	.1728	.9625	87	.7920	.0728	52	.4041	.181
58	.5558	.8522	23	.1824	.9642	88	.8015	.0744	53	.4135	.183
59	.5655	.8539	24	.1920	.9659	89	.8110	.0761	54	.4228	.184
2660	51.5752	13.8557	2725	52.2015	13.9676	2790	52.8205	14.0778	2855	53.4322	14.186
61	.5849	.8574	26	.2111	.9693	91	.8299	.0795	56	.4416	.187
62	.5946	.8591	27	.2207	.9710	92	.8394	.0812	57	.4509	.189
63	.6043	.8609	28	.2303	.9727	93	.8488	.0828	58	.4603	.191
64	.6140	.8626	29	.2398	.9744	94	.8583	.0845	59	.4696	.192
2665	51.6236	13.8643	2730	52.2494	13.9761	2795	52.8678	14.0862	2860	53.4790	14.194
66	.6333	.8661	31	.2590	.9779	96	.8772	.0879	61	.4883	.196
67	.6430	.8678	32	.2685	.9796	97	.8867	.0896	62	.4977	.197
68	.6527	.8695	33	.2781	.9813	98	.8961	.0912	63	.5070	.199
69	.6624	.8713	34	.2877	.9830	99	.9056	.0929	64	.5164	.201
2670	51.6720	13.8730	2735	52.2972	13.9847	2800	52.9150	14.0946	2865	53.5257	14.202
71	.6817	.8747	36	.3068	.9864	1	.9245	.0963	66	.5350	.204
72	.6914	.8765	37	.3163	.9881	2	.9339	.0980	67	.5444	.206
73	.7011	.8782	38	.3259	.9898	3	.9434	.0996	68	.5537	.207
74	.7107	.8799	39	.3355	.9915	4	.9528	.1013	69	.5630	.209
2675	51.7204	13.8817	2740	52.3450	13.9932	2805	52.9623	14.1030	2870	53.5724	14.211
76	.7301	.8834	41	.3546	.9949	6	.9717	.1047	71	.5817	.212
77	.7397	.8851	42	.3641	.9966	7	.9811	.1063	72	.5910	.214
78	.7494	.8868	43	.3737	.9983	8	.9906	.1080	73	.6004	.216
79	.7591	.8886	44	.3832	14.0000	9	53.0000	.1097	74	.6097	.217
2680	51.7687	13.8903	2745	52.3927	14.0017	2810	53.0094	14.1114	2875	53.6190	14.219
81	.7784	.8920	46	.4023	.0034	11	.0189	.1130	76	.6284	.221
82	.7880	.8938	47	.4118	.0051	12	.0283	.1147	77	.6377	.222
83	.7977	.8955	48	.4214	.0068	13	.0377	.1164	78	.6470	.224
84	.8073	.8972	49	.4309	.0085	14	.0471	.1180	79	.6563	.225
2685	51.8170	13.8989	2750	52.4404	14.0102	2815	53.0566	14.1197	2880	53.6656	14.227
86	.8266	.9007	51	.4500	.0119	16	.0660	.1214	81	.6749	.229
87	.8363	.9024	52	.4595	.0136	17	.0754	.1231	82	.6843	.230
88	.8459	.9041	53	.4690	.0153	18	.0848	.1247	83	.6936	.232
89	.8556	.9058	54	.4786	.0170	19	.0943	.1264	84	.7029	.234
2690	51.8652	13.9076	2755	52.4881	14.0187	2820	53.1037	14.1281	2885	53.7122	14.235
91	.8748	.9093	56	.4976	.0204	21	.1131	.1297	86	.7215	.237
92	.8845	.9110	57	.5071	.0221	22	.1225	.1314	87	.7308	.239
93	.8941	.9127	58	.5167	.0238	23	.1319	.1331	88	.7401	.240
94	.9038	.9144	59	.5262	.0255	24	.1413	.1348	89	.7494	.242
2695	51.9134	13.9162	2760	52.5357	14.0272	2825	53.1507	14.1364	2890	53.7587	14.244
96	.9230	.9179	61	.5452	.0289	26	.1601	.1381	91	.7680	.245
97	.9326	.9196	62	.5547	.0305	27	.1695	.1398	92	.7773	.247
98	.9423	.9213	63	.5642	.0322	28	.1789	.1414	93	.7866	.249
99	.9519	.9230	64	.5738	.0339	29	.1883	.1431	94	.7959	.250
2700	51.9615	13.9248	2765	52.5833	14.0356	2830	53.1977	14.1448	2895	53.8052	14.252
1	.9711	.9265	66	.5928	.0373	31	.2071	.1464	96	.8145	.253
2	.9808	.9282	67	.6023	.0390	32	.2165	.1481	97	.8238	.255
3	.9904	.9299	68	.6118	.0407	33	.2259	.1498	98	.8331	.257
4	52.0000	.9316	69	.6213	.0424	34	.2353	.1514	99	.8424	.258
2705	52.0096	13.9334	2770	52.6308	14.0441	2835	53.2447	14.1531	2900	53.8516	14.260

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10.—Square Roots and Cube Roots of Numbers 1600 to 3200—Continued.

No.	Sq. Rt.	Cu. Rt.	No.	Sq. Rt.	Cu. Rt.	No.	Sq. Rt.	Cu. Rt.	No.	Sq. Rt.	Cu. Rt.
2900	53.8516	14.2604	2965	54.4518	14.3662	3030	55.0454	14.4704	3095	55.6327	14.5732
1	.8609	.2621	66	.4610	.3678	31	.0545	.4720	96	.6417	.5747
2	.8702	.2637	67	.4702	.3694	32	.0636	.4736	97	.6507	.5763
3	.8795	.2653	68	.4794	.3710	33	.0727	.4752	98	.6597	.5779
4	.8888	.2670	69	.4885	.3726	34	.0818	.4768	99	.6687	.5794
2905	53.8981	14.2686	2970	54.4977	14.3742	3035	55.0908	14.4784	3100	55.6776	14.5810
6	.9073	.2703	71	.5069	.3759	36	.0999	.4800	1	.6866	.5826
7	.9166	.2719	72	.5161	.3775	37	.1090	.4815	2	.6956	.5841
8	.9259	.2735	73	.5252	.3791	38	.1181	.4831	3	.7046	.5857
9	.9351	.2752	74	.5344	.3807	39	.1271	.4847	4	.7136	.5873
2910	53.9444	14.2768	2975	54.5436	14.3823	3040	55.1362	14.4863	3105	55.7225	14.5888
11	.9537	.2784	76	.5527	.3839	41	.1453	.4879	6	.7315	.5904
12	.9630	.2801	77	.5619	.3855	42	.1543	.4895	7	.7405	.5920
13	.9722	.2817	78	.5711	.3872	43	.1634	.4911	8	.7494	.5935
14	.9815	.2833	79	.5802	.3888	44	.1725	.4927	9	.7584	.5951
2915	53.9907	14.2850	2980	54.5894	14.3904	3045	55.1815	14.4943	3110	55.7674	14.5967
16	54.0000	.2866	81	.5985	.3920	46	.1906	.4958	11	.7763	.5982
17	.0092	.2882	82	.6077	.3936	47	.1996	.4974	12	.7853	.5998
18	.0185	.2899	83	.6168	.3952	48	.2087	.4990	13	.7943	.6014
19	.0278	.2915	84	.6260	.3968	49	.2178	.5006	14	.8032	.6029
2920	54.0370	14.2931	2985	54.6352	14.3984	3050	55.2268	14.5022	3115	55.8122	14.6045
21	.0463	.2948	86	.6443	.4000	51	.2359	.5038	16	.8211	.6060
22	.0555	.2964	87	.6535	.4016	52	.2449	.5053	17	.8301	.6076
23	.0648	.2980	88	.6626	.4032	53	.2540	.5069	18	.8391	.6092
24	.0740	.2999	89	.6717	.4048	54	.2630	.5085	19	.8480	.6107
2925	54.0833	14.3013	2990	54.6809	14.4065	3055	55.2721	14.5101	3120	55.8570	14.6123
26	.0925	.3029	91	.6900	.4081	56	.2811	.5117	21	.8659	.6138
27	.1018	.3046	92	.6992	.4097	57	.2901	.5133	22	.8749	.6154
28	.1110	.3062	93	.7083	.4113	58	.2992	.5148	23	.8838	.6170
29	.1202	.3078	94	.7175	.4129	59	.3082	.5164	24	.8928	.6185
2930	54.1295	14.3094	2995	54.7266	14.4145	3060	55.3173	14.5180	3125	55.9017	14.6201
31	.1387	.3111	96	.7357	.4161	61	.3263	.5196	26	.9106	.6216
32	.1479	.3127	97	.7449	.4177	62	.3353	.5212	27	.9196	.6232
33	.1572	.3143	98	.7540	.4193	63	.3444	.5228	28	.9285	.6248
34	.1664	.3159	99	.7631	.4209	64	.3534	.5243	29	.9375	.6263
2935	54.1756	14.3176	3000	54.7723	14.4225	3065	55.3624	14.5259	3130	55.9464	14.6279
36	.1849	.3192	1	.7814	.4241	66	.3715	.5275	31	.9553	.6294
37	.1941	.3208	2	.7905	.4257	67	.3805	.5291	32	.9643	.6310
38	.2033	.3224	3	.7996	.4273	68	.3895	.5307	33	9732	.6326
39	.2125	.3241	4	.8088	.4289	69	.3986	.5322	34	.9821	.6341
2940	54.2218	14.3257	3005	54.8179	14.4305	3070	55.4076	14.5338	3135	55.9911	14.6357
41	.2310	.3273	6	.8270	.4321	71	.4166	.5354	36	56.0000	.6372
42	.2402	.3289	7	.8361	.4337	72	.4256	.5370	37	.0089	.6388
43	.2494	.3306	8	.8452	.4353	73	.4346	.5385	38	.0179	.6403
44	.2586	.3322	9	.8544	.4369	74	.4437	.5401	39	.0268	.6419
2945	54.2679	14.3338	3010	54.8635	14.4385	3075	55.4527	14.5417	3140	56.0357	14.6434
46	.2771	.3354	11	.8726	.4401	76	.4617	.5433	41	.0446	.6450
47	.2863	.3371	12	.8817	.4417	77	.4707	.5448	42	.0535	.6466
48	.2955	.3387	13	.8908	.4433	78	.4797	.5464	43	.0625	.6481
49	.3047	.3403	14	.8999	.4449	79	.4887	.5480	44	.0714	.6497
2950	54.3139	14.3419	3015	54.9090	14.4465	3080	55.4977	14.5496	3145	56.0803	14.6512
51	.3231	.3435	16	.9181	.4481	81	.5068	.5511	46	.0892	.6528
52	.3322	.3452	17	.9272	.4497	82	5158	.5527	47	.0981	.6543
53	.3415	.3468	18	.9363	.4513	83	.5248	.5543	48	.1070	.6559
54	.3507	.3484	19	.9454	.4529	84	.5338	.5559	49	.1160	.6574
2955	54.3599	14.3500	3020	54.9545	14.4545	3085	55.5428	14.5574	3150	56.1249	14.6590
56	.3691	.3516	21	.9636	.4561	86	.5518	.5590	51	.1338	.6605
57	.3783	.3533	22	.9727	.4577	87	.5608	.5606	52	.1427	.6621
58	.3875	.3549	23	.9818	.4593	88	.5698	.5622	53	.1516	.6636
59	.3967	.3565	24	.9909	.4609	89	.5788	.5637	54	.1605	.6652
2960	54.4059	14.3581	3025	55.0000	14.4624	3090	55.5878	14.5653	3155	56.1694	14.6667
61	.4151	.3597	26	.0091	.4640	91	.5968	.5669	56	.1783	.6683
62	.4243	.3613	27	.0182	.4656	92	.6058	.5684	57	.1872	.6698
63	.4334	.3630	28	.0273	.4672	93	.6147	.5700	58	.1961	.6714
64	.4426	.3646	29	.0364	.4688	94	.6237	.5716	59	.2050	.6729
2965	54.4518	14.3662	3030	55.0454	14.4704	3095	55.6327	14.5732	3160	56.2139	14.6745

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10.—Square Roots and Cube Roots of Numbers 1600 to 3200—Concluded.

No.	Sq. Rt.	Cu. Rt.	No.	Sq. Rt.	Cu. Rt.	No.	Sq. Rt.	Cu. Rt.	No.	Sq. Rt.	Cu. Rt.
3160	56.2139	14.6745	3170	56.3028	14.6899	3180	56.3915	14.7054	3190	56.4801	14.7208
61	.2228	.6760	71	.3116	.6915	81	.4004	.7069	91	.4889	.7223
62	.2317	.6776	72	.3205	.6930	82	.4092	.7084	92	.4978	.7238
63	.2406	.6791	73	.3294	.6946	83	.4181	.7100	93	.5066	.7254
64	.2494	.6807	74	.3383	.6961	84	.4269	.7115	94	.5155	.7269
3165	56.2583	14.6822	3175	56.3471	14.6977	3185	56.4358	14.7131	3195	56.5243	14.7284
66	.2672	.6837	76	.3560	.6992	86	.4447	.7146	96	.5332	.7300
67	.2761	.6853	77	.3649	.7007	87	.4535	.7161	97	.5420	.7315
68	.2850	.6868	78	.3738	.7023	88	.4624	.7177	98	.5509	.7331
69	.2939	.6884	79	.3826	.7038	89	.4712	.7192	99	.5597	.7346
3170	56.3028	14.6899	3180	56.3915	14.7054	3190	56.4801	14.7208	3200	56.5685	14.7361

Note.—For square roots and cube roots of numbers above 3200, see Engineers' Tables, preceding.

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11.—Reciprocals of Numbers 1 to 1000.

No.	Reciprocal	No.	Reciprocal	No.	Reciprocal	No.	Reciprocal	No.	Reciprocal
0	Infinite.	65	.01538 4615	130	.00769 2308	195	.00512 8205	260	.00384 6154
1	.00000 0000	6	.01515 1515	1	.00763 3588	6	.00510 2041	1	.00383 1418
2	.50000 0000	7	.01492 5373	2	.00757 5758	7	.00507 6142	2	.00381 6794
3	.33333 3333	8	.01470 5882	3	.00751 8797	8	.00505 0505	3	.00380 2281
4	.25000 0000	9	.01449 2754	4	.00746 2687	9	.00502 5126	4	.00378 7879
5	.20000 0000	70	.01428 5714	135	.00740 7407	200	.00500 0000	265	.00377 3585
6	.16666 6667	1	.01408 4507	6	.00735 2941	1	.00497 5124	6	.00375 9398
7	.14285 7143	2	.01388 8889	7	.00729 9270	2	.00495 0495	7	.00374 5318
8	.12500 0000	3	.01369 8630	8	.00724 6377	3	.00492 6108	8	.00373 1343
9	.11111 1111	4	.01351 3514	9	.00719 4245	4	.00490 1961	9	.00371 7472
10	.10000 0000	75	.01333 3333	140	.00714 2857	205	.00487 8049	270	.00370 3704
11	.09090 9091	6	.01315 7895	1	.00709 2199	6	.00485 4369	1	.00369 0037
12	.08333 3333	7	.01298 7013	2	.00704 2254	7	.00483 0918	2	.00367 6471
13	.07692 3077	8	.01282 0513	3	.00699 3007	8	.00480 7692	3	.00366 3004
14	.07142 8571	9	.01265 8228	4	.00694 4444	9	.00478 4689	4	.00364 9635
15	.06666 6667	80	.01250 0000	145	.00689 6552	210	.00476 1905	275	.00363 6364
16	.06250 0000	1	.01234 5679	6	.00684 9315	11	.00473 9336	6	.00362 3188
17	.05882 3529	2	.01219 5122	7	.00680 2721	12	.00471 6981	7	.00361 0108
18	.05555 5556	3	.01204 8193	8	.00675 6757	13	.00469 4836	8	.00359 7122
19	.05263 1579	4	.01190 4762	9	.00671 1409	14	.00467 2897	9	.00358 4229
20	.05000 0000	85	.01176 4706	150	.00666 6667	215	.00465 1163	280	.00357 1429
1	.04761 9048	6	.01162 7907	1	.00662 2517	16	.00462 9630	1	.00355 8719
2	.04545 4545	7	.01149 4253	2	.00657 8947	17	.00460 8295	2	.00354 6099
3	.04347 8261	8	.01136 3636	3	.00653 5948	18	.00458 7156	3	.00353 3569
4	.04166 6667	9	.01123 5955	4	.00649 3506	19	.00456 6210	4	.00352 1127
25	.04000 0000	90	.01111 1111	155	.00645 1613	220	.00454 5455	285	.00350 8772
6	.03846 1538	1	.01098 9011	6	.00641 0256	1	.00452 4887	6	.00349 6503
7	.03703 7037	2	.01086 9565	7	.00636 9427	2	.00450 4505	7	.00348 4321
8	.03571 4286	3	.01075 2688	8	.00632 9114	3	.00448 4305	8	.00347 2222
9	.03448 2759	4	.01063 8298	9	.00628 9308	4	.00446 4286	9	.00346 0208
30	.03333 3333	95	.01052 6316	160	.00625 0000	225	.00444 4444	290	.00344 8276
1	.03225 8065	6	.01041 6667	1	.00621 1180	6	.00442 4779	1	.00343 6426
2	.03125 0000	7	.01030 9278	2	.00617 2840	7	.00440 5286	2	.00342 4658
3	.03030 3030	8	.01020 4082	3	.00613 4969	8	.00438 5965	3	.00341 2969
4	.02941 1765	9	.01010 1010	4	.00609 7561	9	.00436 6812	4	.00340 1361
35	.02857 1429	100	.01000 0000	165	.00606 0606	230	.00434 7826	295	.00338 9831
6	.02777 7778	1	.00990 0990	6	.00602 4096	1	.00432 9004	6	.00337 8378
7	.02702 7027	2	.00980 3922	7	.00598 8024	2	.00431 0345	7	.00336 7003
8	.02631 5789	3	.00970 8738	8	.00595 2381	3	.00429 1845	8	.00335 5705
9	.02564 1026	4	.00961 5385	9	.00591 7160	4	.00427 3504	9	.00334 4482
40	.02500 0000	105	.00952 3810	170	.00588 2353	235	.00425 5319	300	.00333 3333
1	.02439 0244	6	.00943 3962	1	.00584 7953	6	.00423 7288	1	.00332 2259
2	.02380 9524	7	.00934 5794	2	.00581 3953	7	.00421 9409	2	.00331 1258
3	.02325 5814	8	.00925 9259	3	.00578 0347	8	.00420 1681	3	.00330 0330
4	.02272 7273	9	.00917 4312	4	.00574 7126	9	.00418 4100	4	.00328 9474
45	.02222 2222	110	.00909 0909	175	.00571 4286	240	.00416 6667	305	.00327 8689
6	.02173 9130	11	.00900 9009	6	.00568 1818	1	.00414 9378	6	.00326 7974
7	.02127 6600	12	.00892 8571	7	.00564 9718	2	.00413 2231	7	.00325 7329
8	.02083 3333	13	.00884 9558	8	.00561 7978	3	.00411 5226	8	.00324 6753
9	.02040 8163	14	.00877 1930	9	.00558 6592	4	.00409 8361	9	.00323 6246
50	.02000 0000	115	.00869 5652	180	.00555 5556	245	.00408 1633	310	.00322 5806
1	.01960 7843	16	.00862 0690	1	.00552 4862	6	.00406 5041	11	.00321 5434
2	.01923 0769	17	.00854 7009	2	.00549 4505	7	.00404 8583	12	.00320 5128
3	.01886 7925	18	.00847 4576	3	.00546 4481	8	.00403 2258	13	.00319 4888
4	.01851 8519	19	.00840 3361	4	.00543 4783	9	.00401 6064	14	.00318 4713
55	.01818 1818	120	.00833 3333	185	.00540 5405	250	.00400 0000	315	.00317 4603
6	.01785 7143	1	.00826 4463	6	.00537 6344	1	.00398 4064	16	.00316 4557
7	.01754 3860	2	.00819 6721	7	.00534 7594	2	.00396 8254	17	.00315 4574
8	.01724 1379	3	.00813 0081	8	.00531 9149	3	.00395 2569	18	.00314 4654
9	.01694 9153	4	.00806 4516	9	.00529 1005	4	.00393 7008	19	.00313 4796
60	.01666 6667	125	.00800 0000	190	.00526 3158	255	.00392 1569	320	.00312 5000
1	.01639 3443	6	.00793 6508	1	.00523 5602	6	.00390 6250	1	.00311 5265
2	.01612 9032	7	.00787 4016	2	.00520 8333	7	.00389 1051	2	.00310 5590
3	.01587 3016	8	.00781 2500	3	.00518 1347	8	.00387 5969	3	.00309 5975
4	.01562 5000	9	.00775 1938	4	.00515 4639	9	.00386 1004	4	.00308 6420
65	.01538 4615	130	.00769 2308	195	.00512 8205	260	.00384 6154	325	.00307 6923

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11.—Reciprocals of Numbers 1 to 1000—Continued.

No.	Reciprocal	No.	Reciprocal	No.	Reciprocal	No.	Reciprocal	No.	Reciprocal
325	.00307 6923	390	.00256 4103	455	.00219 7802	520	.00192 3077	585	.00170 940
6	.00306 7485	1	.00255 7545	6	.00219 2982	1	.00191 9386	6	.00170 648
7	.00305 8104	2	.00255 1020	7	.00218 8184	2	.00191 5709	7	.00170 357
8	.00304 8780	3	.00254 4529	8	.00218 3406	3	.00191 2046	8	.00170 068
9	.00303 9514	4	.00253 8071	9	.00217 8649	4	.00190 8397	9	.00169 779
330	.00303 0303	395	.00253 1646	460	.00217 3913	525	.00190 4762	590	.00169 491
1	.00302 1148	6	.00252 5253	1	.00216 9197	6	.00190 1141	1	.00169 204
2	.00301 2048	7	.00251 8892	2	.00216 4502	7	.00189 7533	2	.00168 918
3	.00300 3003	8	.00251 2563	3	.00215 9827	8	.00189 3939	3	.00168 634
4	.00299 4012	9	.00250 6266	4	.00215 5172	9	.00189 0359	4	.00168 350
335	.00298 5075	400	.00250 0000	465	.00215 0538	530	.00188 6792	595	.00168 067
6	.00297 6190	1	.00249 3766	6	.00214 5923	1	.00188 3239	6	.00167 785
7	.00296 7359	2	.00248 7562	7	.00214 1328	2	.00187 9699	7	.00167 504
8	.00295 8580	3	.00248 1390	8	.00213 6752	3	.00187 6173	8	.00167 224
9	.00294 9853	4	.00247 5248	9	.00213 2196	4	.00187 2659	9	.00166 944
340	.00294 1176	405	.00246 9136	470	.00212 7660	535	.00186 9159	600	.00166 666
1	.00293 2551	6	.00246 3054	1	.00212 3142	6	.00186 5672	1	.00166 389
2	.00292 3977	7	.00245 7002	2	.00211 8644	7	.00186 2197	2	.00166 113
3	.00291 5452	8	.00245 0980	3	.00211 4165	8	.00185 8736	3	.00165 837
4	.00290 6977	9	.00244 4988	4	.00210 9705	9	.00185 5288	4	.00165 562
345	.00289 8551	410	.00243 9024	475	.00210 5263	540	.00185 1852	605	.00165 289
6	.00289 0173	11	.00243 3090	6	.00210 0840	1	.00184 8429	6	.00165 016
7	.00288 1844	12	.00242 7184	7	.00209 6436	2	.00184 5018	7	.00164 744
8	.00287 3563	13	.00242 1308	8	.00209 2050	3	.00184 1621	8	.00164 473
9	.00286 5330	14	.00241 5459	9	.00208 7683	4	.00183 8235	9	.00164 203
350	.00285 7143	415	.00240 9639	480	.00208 3333	545	.00183 4862	610	.00163 934
1	.00284 9003	16	.00240 3846	1	.00207 9002	6	.00183 1502	11	.00163 666
2	.00284 0909	17	.00239 8082	2	.00207 4689	7	.00182 8154	12	.00163 398
3	.00283 2861	18	.00239 2344	3	.00207 0393	8	.00182 4818	13	.00163 131
4	.00282 4859	19	.00238 6635	4	.00206 6116	9	.00182 1494	14	.00162 864
355	.00281 6901	420	.00238 0952	485	.00206 1856	550	.00181 8182	615	.00162 601
6	.00280 8989	1	.00237 5297	6	.00205 7613	1	.00181 4882	16	.00162 337
7	.00280 1120	2	.00236 9668	7	.00205 3388	2	.00181 1594	17	.00162 074
8	.00279 3296	3	.00236 4066	8	.00204 9180	3	.00180 8318	18	.00161 812
9	.00278 5515	4	.00235 8491	9	.00204 4990	4	.00180 5054	19	.00161 550
360	.00277 7778	425	.00235 2941	490	.00204 0816	555	.00180 1802	620	.00161 290
1	.00277 0083	6	.00234 7418	1	.00203 6660	6	.00179 8561	1	.00161 030
2	.00276 2431	7	.00234 1920	2	.00203 2520	7	.00179 5332	2	.00160 771
3	.00275 4821	8	.00233 6449	3	.00202 8398	8	.00179 2115	3	.00160 513
4	.00274 7253	9	.00233 1002	4	.00202 4291	9	.00178 8909	4	.00160 256
365	.00273 9726	430	.00232 5581	495	.00202 0202	560	.00178 5714	625	.00160 000
6	.00273 2240	1	.00232 0186	6	.00201 6129	1	.00178 2531	6	.00159 744
7	.00272 4796	2	.00231 4815	7	.00201 2072	2	.00177 9359	7	.00159 489
8	.00271 7391	3	.00230 9469	8	.00200 8032	3	.00177 6199	8	.00159 233
9	.00271 0027	4	.00230 4147	9	.00200 4008	4	.00177 3050	9	.00158 983
370	.00270 2703	435	.00229 8851	500	.00200 0000	565	.00176 9912	630	.00158 730
1	.00269 5418	6	.00229 3578	1	.00199 6008	6	.00176 6784	1	.00158 478
2	.00268 8172	7	.00228 8330	2	.00199 2032	7	.00176 3668	2	.00158 227
3	.00268 0965	8	.00228 3105	3	.00198 8072	8	.00176 0563	3	.00157 977
4	.00267 3797	9	.00227 7904	4	.00198 4127	9	.00175 7469	4	.00157 728
375	.00266 6667	440	.00227 2727	505	.00198 0198	570	.00175 4386	635	.00157 480
6	.00265 9574	1	.00226 7574	6	.00197 6285	1	.00175 1313	6	.00157 233
7	.00265 2520	2	.00226 2443	7	.00197 2387	2	.00174 8252	7	.00156 985
8	.00264 5503	3	.00225 7336	8	.00196 8504	3	.00174 5201	8	.00156 739
9	.00263 8522	4	.00225 2252	9	.00196 4637	4	.00174 2160	9	.00156 494
380	.00263 1579	445	.00224 7191	510	.00196 0784	575	.00173 9130	640	.00156 250
1	.00262 4672	6	.00224 2152	11	.00195 6947	6	.00173 6111	1	.00156 006
2	.00261 7801	7	.00223 7136	12	.00195 3125	7	.00173 3102	2	.00155 763
3	.00261 0966	8	.00223 2143	13	.00194 9318	8	.00173 0104	3	.00155 521
4	.00260 4167	9	.00222 7171	14	.00194 5525	9	.00172 7116	4	.00155 279
385	.00259 7403	450	.00222 2222	515	.00194 1748	580	.00172 4138	645	.00155 038
6	.00259 0674	1	.00221 7295	16	.00193 7984	1	.00172 1170	6	.00154 799
7	.00258 3979	2	.00221 2389	17	.00193 4236	2	.00171 8213	7	.00154 559
8	.00257 7320	3	.00220 7506	18	.00193 0502	3	.00171 5266	8	.00154 322
9	.00257 0694	4	.00220 2643	19	.00192 6782	4	.00171 2329	9	.00154 085
390	.00256 4103	455	.00219 7802	520	.00192 3077	585	.00170 9402	650	.00153 846

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11.—RECIPROCALS OF NUMBERS 1 TO 1000.—Continued.

No.	Reciprocal	No.	Reciprocal	No.	Reciprocal	No.	Reciprocal	No.	Reciprocal
650	.00153 8462	715	.00139 8601	780	.00128 2051	845	.00118 3432	910	.00109 8901
1	.00153 6088	16	.00139 6648	1	.00128 0410	6	.00118 2033	11	.00109 7695
2	.00153 3742	17	.00139 4700	2	.00127 8772	7	.00118 0638	12	.00109 6491
3	.00152 1394	18	.00139 2758	3	.00127 7139	8	.00117 9245	13	.00109 5290
4	.00152 9052	19	.00139 0821	4	.00127 5510	9	.00117 7856	14	.00109 4092
655	.00152 6718	720	.00138 8889	785	.00127 3885	850	.00117 6471	915	.00109 2896
6	.00152 4390	1	.00138 6963	6	.00127 2265	1	.00117 5088	16	.00109 1703
7	.00152 2070	2	.00138 5042	7	.00127 0648	2	.00117 3709	17	.00109 0513
8	.00151 9757	3	.00138 3126	8	.00126 9036	3	.00117 2333	18	.00108 9325
9	.00151 7451	4	.00138 1215	9	.00126 7427	4	.00117 0960	19	.00108 8139
660	.00151 5152	725	.00137 9310	790	.00126 5823	855	.00116 9591	920	.00108 6957
1	.00151 2859	6	.00137 7410	1	.00126 4223	6	.00116 8224	1	.00108 5776
2	.00151 0574	7	.00137 5516	2	.00126 2626	7	.00116 6861	2	.00108 4599
3	.00150 8296	8	.00137 3626	3	.00126 1034	8	.00116 5501	3	.00108 3424
4	.00150 6024	9	.00137 1742	4	.00125 9446	9	.00116 4144	4	.00108 2251
665	.00150 3759	730	.00136 9863	795	.00125 7862	860	.00116 2791	925	.00108 1081
6	.00150 1502	1	.00136 7989	6	.00125 6281	1	.00116 1440	6	.00107 9914
7	.00149 9250	2	.00136 6120	7	.00125 4705	2	.00116 0093	7	.00107 8749
8	.00149 7006	3	.00136 4256	8	.00125 3133	3	.00115 8749	8	.00107 7586
9	.00149 4768	4	.00136 2398	9	.00125 1564	4	.00115 7407	9	.00107 6426
670	.00149 2537	735	.00136 0544	800	.00125 0000	865	.00115 6069	930	.00107 5269
1	.00149 0313	6	.00135 8696	1	.00124 8439	6	.00115 4734	1	.00107 4114
2	.00148 8095	7	.00135 6852	2	.00124 6883	7	.00115 3403	2	.00107 2961
3	.00148 5884	8	.00135 5014	3	.00124 5330	8	.00115 2074	3	.00107 1811
4	.00148 3680	9	.00135 3180	4	.00124 3781	9	.00115 0748	4	.00107 0664
675	.00148 1481	740	.00135 1351	805	.00124 2236	870	.00114 9425	935	.00106 9519
6	.00147 9290	1	.00134 9528	6	.00124 0695	1	.00114 8106	6	.00106 8376
7	.00147 7105	2	.00134 7709	7	.00123 9157	2	.00114 6789	7	.00106 7236
8	.00147 4926	3	.00134 5895	8	.00123 7624	3	.00114 5475	8	.00106 6098
9	.00147 2754	4	.00134 4086	9	.00123 6094	4	.00114 4165	9	.00106 4963
680	.00147 0588	745	.00134 2282	810	.00123 4568	875	.00114 2857	940	.00106 3830
1	.00146 8429	6	.00134 0483	11	.00123 3046	6	.00114 1553	1	.00106 2699
2	.00146 6276	7	.00133 8688	12	.00123 1527	7	.00114 0251	2	.00106 1571
3	.00146 4129	8	.00133 6898	13	.00123 0012	8	.00113 8952	3	.00106 0445
4	.00146 1988	9	.00133 5113	14	.00122 8501	9	.00113 7656	4	.00105 9322
685	.00145 9854	750	.00133 3333	815	.00122 6994	880	.00113 6364	945	.00105 8201
6	.00145 7726	1	.00133 1558	16	.00122 5490	1	.00113 5074	6	.00105 7082
7	.00145 5604	2	.00132 9787	17	.00122 3990	2	.00113 3787	7	.00105 5966
8	.00145 3488	3	.00132 8021	18	.00122 2494	3	.00113 2503	8	.00105 4852
9	.00145 1379	4	.00132 6260	19	.00122 1001	4	.00113 1222	9	.00105 3741
690	.00144 9275	755	.00132 4503	820	.00121 9512	885	.00112 9944	950	.00105 2632
1	.00144 7178	6	.00132 2751	1	.00121 8027	6	.00112 8668	1	.00105 1525
2	.00144 5087	7	.00132 1004	2	.00121 6545	7	.00112 7396	2	.00105 0420
3	.00144 3001	8	.00131 9261	3	.00121 5067	8	.00112 6126	3	.00104 9318
4	.00144 0922	9	.00131 7523	4	.00121 3592	9	.00112 4859	4	.00104 8218
695	.00143 8849	760	.00131 5789	825	.00121 2121	890	.00112 3596	955	.00104 7120
6	.00143 6782	1	.00131 4060	6	.00121 0654	1	.00112 2334	6	.00104 6025
7	.00143 4720	2	.00131 2336	7	.00120 9190	2	.00112 1076	7	.00104 4932
8	.00143 2665	3	.00131 0616	8	.00120 7729	3	.00111 9821	8	.00104 3841
9	.00143 0615	4	.00130 8901	9	.00120 6273	4	.00111 8568	9	.00104 2753
700	.00142 8571	765	.00130 7190	830	.00120 4819	895	.00111 7318	960	.00104 1667
1	.00142 6534	6	.00130 5483	1	.00120 3369	6	.00111 6071	1	.00104 0583
2	.00142 4501	7	.00130 3781	2	.00120 1923	7	.00111 4827	2	.00103 9501
3	.00142 2475	8	.00130 2083	3	.00120 0480	8	.00111 3586	3	.00103 8422
4	.00142 0455	9	.00130 0390	4	.00119 9041	9	.00111 2347	4	.00103 7344
705	.00141 8440	770	.00129 8701	835	.00119 7605	900	.00111 1111	965	.00103 6269
6	.00141 6431	1	.00129 7017	6	.00119 6172	1	.00110 9878	6	.00103 5197
7	.00141 4427	2	.00129 5337	7	.00119 4743	2	.00110 8647	7	.00103 4126
8	.00141 2429	3	.00129 3661	8	.00119 3317	3	.00110 7420	8	.00103 3058
9	.00141 0437	4	.00129 1990	9	.00119 1895	4	.00110 6195	9	.00103 1992
710	.00140 8451	775	.00129 0323	840	.00119 0476	905	.00110 4972	970	.00103 0928
11	.00140 6470	6	.00128 8660	1	.00118 9061	6	.00110 3753	1	.00102 9866
12	.00140 4494	7	.00128 7001	2	.00118 7648	7	.00110 2536	2	.00102 8807
13	.00140 2525	8	.00128 5347	3	.00118 6240	8	.00110 1322	3	.00102 7749
14	.00140 0560	9	.00128 3697	4	.00118 4834	9	.00110 0110	4	.00102 6694
15	.00139 8601	780	.00128 2051	845	.00118 3432	910	.00109 8901	975	.00102 5641

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11.—Reciprocals of Numbers 1 to 1000.—Concluded.

No.	Reciprocal	No.	Reciprocal	No.	Reciprocal	No.	Reciprocal	No.	Reciprocal
975	.00102 5641	980	.00102 0408	985	.00101 5228	990	.00101 0101	995	.00100 5025
6	.00102 4590	1	.00101 9368	6	.00101 4199	1	.00100 9082	6	.00100 4016
7	.00102 3541	2	.00101 8330	7	.00101 3171	2	.00100 8065	7	.00100 3009
8	.00102 2495	3	.00101 7294	8	.00101 2146	3	.00100 7049	8	.00100 2004
9	.00102 1450	4	.00101 6260	9	.00101 1122	4	.00100 6036	9	.00100 1001
980	.00102 0408	985	.00101 5228	990	.00101 0101	995	.00100 5025	1000	.00100 0000

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3.—PRACTICAL ARITHMETIC.

PROPORTION.

As Algebra is the *shorthand* of Mathematics so is Proportion the *key* to many of its operations. Indeed, all mathematical problems may be expressed in proportion.

Ratio is an expression of the relative magnitude of two quantities in the form of a fraction, either term of which may be considered the unit of measure. Thus, the ratio of the yard to the foot may be expressed as $\frac{Y}{F}$ or $Y : F$, in which case, however, Y and F must be considered in the same unit of measure, yard or foot. Every ratio *implies* proportion, as $\frac{Y}{F} = \frac{x}{1} = \frac{3}{1}$; $F : 1 = Y : x$; etc.

Proportion is the equality of two or more ratios, and it may be represented graphically by similar triangles, as in Fig. 1, from which may be obtained a variety of expressions, as follows:

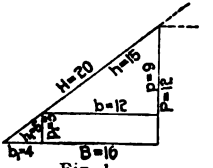

Fig. 1.

By proportional lengths (also illustrating *continued proportion*):

$$\left\{\begin{array}{l} {}^{*}\frac{P}{B} = \frac{p}{b} = \frac{p_1}{b_1} = \frac{p+p_1}{b+b_1} = \frac{P+p}{B+b} = \frac{P+p_1}{B+b_1} = \frac{P+p+p_1}{B+b+b_1}, \text{ etc. (or inverted).} \\ \frac{12}{16} = \frac{9}{12} = \frac{3}{4} = \frac{9+3}{12+4} = \frac{12+9}{16+12} = \frac{12+3}{16+4} = \frac{12+9+3}{16+12+4}, \text{ etc. (or inverted).} \end{array}\right.$$

$$\left\{\begin{array}{l} \frac{P}{B} = \frac{p-p_1}{b-b_1} = \frac{P-p}{B-b} = \frac{P-p_1}{B-b} = \frac{P-p-p_1}{B-b-b_1}, \text{ etc. (or inverted).} \\ \frac{12}{16} = \frac{9-3}{12-4} = \frac{12-9}{16-12} = \frac{12-3}{16-4} = \frac{12-9-3}{16-12-4}, \text{ etc. (or inverted).} \end{array}\right.$$

$\frac{P}{H} = \frac{p}{h} = \frac{p_1}{h_1}$; and $\frac{B}{H} = \frac{b}{h} = \frac{b_1}{h_1}$ may likewise be extended (or inverted).

By proportional squares (also illustrating *compound proportion*):

$H^2 : h^2 : h_1^2 :: P^2+B^2 : p^2+b^2 : p_1^2+b_1^2$; or $\frac{H^2}{h^2} : h_1^2 = \frac{P^2+B^2}{p^2+b^2} : p_1^2+b_1^2$.

$20^2 : 15^2 : 5^2 :: 12^2+16^2 : 9^2+12^2 : 3^2+4^2$; or $\frac{20^2}{15^2} : 5^2 = \frac{12^2+16^2}{9^2+12^2} : 3^2+4^2$.

In the above some of the ratios are compound ratios, hence the term *compound proportion.*

Simple Proportion or single rule of three deals with two simple, equal ratios. Thus,

$$\begin{array}{ccccccc} \text{Extreme} & & \text{mean} & & \text{mean} & & \text{extreme} \\ P & : & B & = & p & : & b \end{array}$$

reads, "As P is to B so is p to b." The first and last terms are called the *extremes*, and the middle terms the *means*. In any simple problem there will

* From $\frac{P}{B} = \frac{p}{b}$ we have $Pb = pB$, whence $\frac{P}{p} = \frac{B}{b}$, or $\frac{b}{p} = \frac{B}{P}$ or $\frac{b}{B} = \frac{p}{P}$, etc.

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be one unknown term which can be solved by applying the rule: *The product of the means is equal to the product of the extremes.*

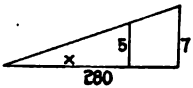

Fig. 2.

Problem.—If a train travels 280 miles in 7 hours, how far will it travel in 5 hours?

Solution.—From similar triangles there is obtained, $7 : 5 = 280 : x$

from which, applying the above rule,

$$7x = 5 \times 280$$

$$\text{or } x = \frac{5 \times 280}{7} = 200 \text{ miles. Ans.}$$

Mean Proportional.—In any proportion where the two means are equal to each other, they are each said to be a mean proportional between the extremes. In the accompanying geometrical figure, b, an ordinate to the circumference, is a mean proportional between a and c, composing the diameter, as, from similar triangles there will obtain the proportion

Fig. 3.

$$a : b = b : c$$

$$\text{or } 4 : 6 = 6 : 9$$

whence $6 = \sqrt{4 \times 9}$; or, the *mean* $= \sqrt{\text{product of extremes}}$.

Inverse Proportion.—This term is used in such problems as those where the rate of speed or velocity are compared, when the total time, distance, amount of work, etc., are given; as, the rate of work is *inversely proportional* to the time occupied in doing it; or, the speed of a train is inversely proportional to the time used in traveling a certain distance.

Compound Proportion or double rule of three.—Compound proportion is merely simple proportion in which two or more of the ratios are compounded.

Problem.—If a man saws 18 cords of wood in 12 days of 9 hours each, how many cords of wood can he saw in 15 days of 8 hours each?

$$\text{Solution.—} \frac{18}{12 \times 9} = \frac{x}{15 \times 8}; \text{ whence } x = \frac{\overset{2}{\cancel{18}} \times \overset{5}{\cancel{15}} \times \overset{2}{\cancel{8}}}{\cancel{12} \times \cancel{9}} = 20 \text{ days. Ans.}$$

A little thought coupled with a graphical conception of the problem will always point to a correct grouping of the terms.

PERMUTATION AND COMBINATION.

Permutation.—The number of different ways in which any number, N, of objects may be arranged in line or counted, is equal to the product of all the numbers from 1 to N. Thus, 3 objects as a, b and c may be arranged $1 \times 2 \times 3 = 6$ different ways in line—*abc, acb, bac, bca, cab, cba*; 7 objects may be arranged $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 = 5040$ different ways, and so on.

Combination.—The number of different groups each composed of n objects (no two groups to be composed of the same objects) which can be formed separately from any number N, of objects, is equal to the product of all the numbers from $(N - n + 1)$ to N, divided by the product of all the numbers from 1 to n. (It must be remembered that the objects in each group are *not* permuted.) Thus, 6 objects as a, b, c, d, e and f may be arranged in groups of 2, $\frac{5 \times 6}{1 \times 2} = 15$ different ways; thus.

ab,	*ac*,	*ad*,	*ae*,	*af*,
	bc,	*bd*,	*be*,	*bf*,
		cd,	*ce*,	*cf*,
			de,	*df*,
				ef,

Similarly, 8 objects may be arranged in groups of 4,

$$\frac{5 \times 6 \times 7 \times 8}{1 \times 2 \times 3 \times 4} = 70 \text{ different ways.}$$

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ALLIGATION.

The average cost of a mixture containing various quantities of ingredients at different unit prices may be obtained by dividing the total cost of the mixture by the total quantity. Let it be required to find the average cost of a mixture as follows:

10 bbls. cement	@	$2.00	=	$ 20.
20 " "		2.25	=	45.
30 " "		2.50	=	75.
∴60			=	$140.

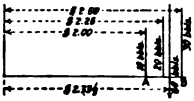

Fig. 4.

Average price per bbl. $= \frac{140}{60} = \$2.33\frac{1}{3}$. Ans.

This problem is analogous to finding the position of the center of gravity and resultant of a system of concentrated loads, in Mechanics.

PROGRESSION.

Arithmetical Progression.—An arithmetical series or progression consists of a series of terms which uniformly increase or decrease by a common constant *difference*, d. Thus,

Ascending:

$(d=d).-\quad a,\ a+d,\ a+2d,\ a+3d,\ a+4d,\ \ldots;$
$(d=3).-\quad 1,\ 4,\ 7,\ 10,\ 13,\ \ldots;$
$(d=\frac{1}{2}).-\quad -2.5,\ -2,\ -1.5,\ -1,\ -.5,\ \ldots;$

Descending:

$(d=-d).-\quad a,\ a-d,\ a-2d,\ a-3d,\ a-4d,\ \ldots;$
$(d=-2).-\quad 7,\ 5,\ 3,\ 1,\ -1,\ \ldots;$
$(d=-\frac{1}{2}).-\quad \frac{3}{2},\ \frac{1}{2},\ 0,\ -\frac{1}{2},\ -\frac{3}{2},\ \ldots;$

are arithmetical series.

As the difference is constant, any arithmetical series may be represented by the equation of a straight line—

$y = mx + c$, in the language of Analytic Geometry;
or $s = d(n-1) + a$, in the language of Arithmetical Progression;
in which $c = a$ = the value of the first term (placed at axis $Y-Y$);
$m = d$ = the common difference, + or −, between adjacent terms;
$x = n-1$ = the number of any term considered *from* (not counting) the first term;
$y = s$ = the value of any term $x = n-1$;
n = the number of any term, counting the first term.

Fig. 5 illustrates an ascending series, and when the first term is a positive quantity. The equation, however, holds good for any case by using the quantities algebraically.

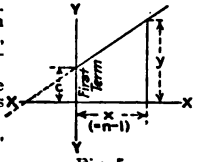

Fig. 5.

Problem 1.—Find the number of the term whose value is 10, in a series whose common difference is $\frac{1}{2}$ and whose first term is 2.

Solution.—From $y = mx + c$ there is obtained,
$10 = \frac{1}{2}x + 2.$
$\therefore x = 24 = n - 1,$
or $n = 25$. Ans.

Problem 2.—Insert 4 terms between the numbers 3 and 28 and find the common difference.

Solution.—From $y = mx + c$
$28 = m\,5 + 3$ (If 4 terms are inserted, 28 = 6th term $\therefore x = 5$.)
whence $m = \frac{28-3}{5} = 5$ = common difference. Ans.

An arithmetical mean between two quantities = one-half their sum.

Geometrical Progression.—A geometrical series or progression consists of a series of terms which increase or decrease by a common constant factor, f. Thus,

Ascending:

$(f=r).-\quad a,\ ar,\ ar^2,\ ar^3,\ ar^4,\ \ldots;$
$(f=2).-\quad 3,\ 6,\ 12,\ 24,\ 48,\ \ldots;$

Decending:

$\left(f=\frac{1}{r}\right).-\quad a,\ \frac{a}{r},\ \frac{a}{r^2},\ \frac{a}{r^3},\ \frac{a}{r^4},\ \ldots;$
$(f=\frac{1}{2}).-\quad 4,\ 2,\ 1,\ \frac{1}{2},\ \frac{1}{4},\ \ldots;$

are geometrical series.

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Any geometrical progression may be represented by a curve whose equation is—

$y = c f^x$, in the language of Analytic Geometry;

or $s = a f^{(n-1)}$, in the language of Geometrical Progression;

in which, $c = a$ = the value of the first term (placed at axis $Y - Y$);

f = the common factor or ratio between the adjacent terms;

$x = n - 1$ = number of any term considered *from* (not including) the first term;

$y = s$ = the value of any term $x = n - 1$;

n = the number of any term, counting the first term.

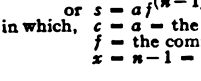

Fig. 6.

Fig. 6 illustrates an increasing series, and when the first term is a positive quantity; but by using the quantities algebraically, the equation holds good for any case.

Problem 1.—Find the value of the 6th term of an increasing series whose 1st term is 3, and the common factor 2.

Solution.—From

$y = c f^x$, there is obtained,

$y (= s) = 3 \times 2^5 = 3 \times 32.$ $(x = n - 1 = 5.)$

$\therefore s = 96.$ Ans.

Problem 2.—Insert 3 terms between the numbers 2 and $10\frac{1}{2}$, and find the common factor.

Solution.—From $y = c f^x$ there is obtained.

$10\frac{1}{2} = 2 f^4$ (If 3 terms are inserted, $10\frac{1}{2}$ = 4th term $\therefore x = 4.$)

$$f^4 = \frac{10\frac{1}{2}}{2} = 5\frac{1}{16} = \frac{81}{16}$$

$\therefore f = \frac{3}{2} = 1\frac{1}{2}.$ Ans.

Note.—The latter part of the above solution may also be performed by logarithms. Thus,

$f^4 = \frac{10\frac{1}{2}}{2} = 5.0625$ log 5.0625 = 0.704365 (Divide this by 4.)

Ans. $f = 1.5$ 0.176091 (Find number corresponding.)

A geometrical mean between two numbers = the square root of their product (see Fig. 3, page 56).

Compound interest is a good illustration of geometrical progression.

PERCENTAGE, INTEREST AND DISCOUNT.

Percentage.—*Per cent* means *hundredths*, and *rate per cent* means any *given number of hundredths*. Thus, 5 per cent, or 5%, means .05, or $^5/_{100}$, in which 5 is the rate. It may also be expressed in true ratio, 5 : 100, meaning 5 *parts* of the 100, both terms being of the same denomination.

To reduce any rate, expressed in two denominations, to a rate per cent, one term must first be reduced to the denomination of the other so that a true ratio may be expressed. Thus, a rate of grade in feet per mile, as $52\frac{8}{10}$ feet per mile, may be expressed in ratio $52\frac{8}{10}$: 5280, as there are 5280 feet in a mile. Clearly, this is equal to a $^1/_{100}$, or 1% grade.

Simple Interest is percentage in which the element of *time* has to be considered. The sum placed at interest is called the *principal*, and the principal plus the interest is called the *amount*. Simple interest differs from compound interest in that the principal is not allowed to increase from time to time by its own interest.

The rate of interest is the rate per cent for one year. There are two methods used in calculating interest:

(*a*.) The common method, in which a year is considered to be divided into 12 months of 30 days each.

(*b*.) The exact method, in which the year is divided into 365 (or 366 if leap year) days.

In either case,

Simple Interest = Principal ($) × Rate % × Time (years).

The Common Method.—Table 1, page 60, shows the simple interest on various principals ($1.-- $1,000.) at various rates ($3\frac{1}{2}$% -- 7%) for time in days and months up to one year. Clearly, by combination and factoring, the interest on any principal, at any rate and for any time, may readily be obtained. Thus, the interest on $1253. at $4\frac{1}{2}$% for 5 months, 11 days, is calculated as follows:

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	5 mos.	10d.	1d.
Interest on $1000 (1000 × 1)	= 18.75	1.25	$\frac{1}{10}$ of 10 days.
200 (200 × 1)	= 3.75	.25	
50 (50 × 1)	= .938	.062	
3	= .056	.004	
Therefore, total interest	= 23.494 +	1.566 +	.157 = $25.22. Ans.

This may be calculated in the ordinary manner, as follows:

$$\text{Interest} = \underset{\text{(Principal)}}{1253} \times \underset{\text{(Rate \%)}}{.045} \times \underset{\text{(Time)}}{\frac{5\frac{11}{20}}{12}} = \$25.22. \quad \text{Ans.}$$

As the interest on any amount is directly proportional to the rate, some prefer to use a round rate like 6%, 6 being a factor of 12 (months) and of 30 (days), for a first result, and then factor this result for the required rate. Thus, in the above,

Interest on $1253 at 6% for 5 mos.	= 1253 × 2½%	=	12.53 12.53 6.265
" " " " 10 days	= 1253 × ⅙%	=	2.089
" " " " 1 "	= $\frac{1}{10}$ above	=	.209
	At 6%		33.62
	Deduct ⅙		8.40
	At 4½%.		$25.22

Ans.

The Exact Method.—This is used by various large financial concerns in dealing with each other and where large amounts are involved. The time is the ratio of the number of days for which the principal has been loaned, to the number of days in the year. Thus, if the loan is for 217 days the time will be $^{217}/_{365}$ of a year; or, if the 29th of February is included, it will be $^{217}/_{366}$ of a year.

Table 2, page 61, gives the number of days from one date to another covering a period of two ordinary years of 365 days each. If one of the years is a leap year add one day to each number of days after February 28th. Thus, if the interest is to be calculated from February 15th of one year to January 10th of the following year, the time would be $\frac{375 - 46}{365} = \frac{329}{365}$ of a year. If the first year is a leap year the time would be $^{330}/_{366}$ of a year, adding *one* to each term of the ratio. For example, the interest on $450. from Feb. 15, 1908, (leap year) to Jan. 10, 1909, at 5% = $450. × .05 × $^{330}/_{366}$ = $20.29. Ans.

Discount.—The fundamental principle of discount is "money off for cash," but most mercantile houses will discount their trade catalogue prices to their regular customers on short time payments.

A certain customer may be favored with a discount of 20%, which means that the cost to him will be 80% of the catalogue price; another customer, more favored or on account of heavier purchases, may receive a discount of "20 and 10"%, making the cost .80 × .90 = 72% of the list price; while a third may receive a discount of "20, 10 and 5" which would make the cost .80 × .90 × .95 = 68 $^{4}/_{10}$ %.

Bank discount is the equivalent interest on the *face* of a note up to the time of its maturity.

True discount is the equivalent interest on the present worth of a note, or principal which, at maturity, will amount to the face of the note. Thus,

$$\frac{\text{Present worth + interest on same (= true discount)}}{\text{Amount (= face of note)}} = \frac{\$1 + \text{interest on same}}{\text{Amount of \$1 and int.}};$$

$$\text{Or, present worth} = \frac{\text{Face of note}}{\text{Amount of \$1 and interest, to maturity}}.$$

Whence the true discount = face of note − present worth.

Compound Interest.—The simple interest on any principal loaned is due annually. But by a special agreement it may become due semi-annually, quarterly, or for any other period. If not paid, it is added periodically to

the principal for a new principal drawing interest; hence, compound interest. (See Table 3, page 62.)

The amount (principal and interest) due at the end of any number of years, *n*, interest payable annually, may be expressed as follows:

$$\text{Amount} = \text{Principal}\,(1 + \text{Rate}\,\%)^n.$$

If the number of years is large the result is easily obtained by the use of logarithms. Further, the above formula may represent any general case by considering *n* the number of periods (as semi-annual or quarterly periods) of compounding — the proportionate rate per cent to be used for that period.

1.—Simple Interest Table.*

Rate	Principal	Time.									
		1 Year	6 Mo.	5 Mo.	4 Mo.	3 Mo.	2 Mo.	1 Mo.	20 d.	10 d.	1 d.
	$1	.040	.0200	.01666˘6	.013˘3	.01000	.0066˘6	.00333˘3	.0022˘2	.00111˘1	.000111˘1
	2	.080	.0400	.03333˘3	.026˘6	.02000	.0133˘3	.00666˘6	.0044˘4	.00222˘2	.000222˘2
	3	.120	.0600	.05000	.040	.03000	.0200	.01000	.0066˘6	.00333˘3	.000333˘3
	4	.160	.0800	.06666˘6	.053˘3	.04000	.0266˘6	.01333˘3	.0088˘8	.00444˘4	.000444˘4
4%	5	.200	.1000	.08333˘3	.066˘6	.05000	.0333˘3	.01666˘6	.0111˘1	.00555˘5	.000555˘5
	6	.240	.1200	.10000	.080	.06000	.0400	.02000	.0133˘3	.00666˘6	.000666˘6
	7	.280	.1400	.11666˘6	.093˘3	.07000	.0466˘6	.02333˘3	.0155˘5	.00777˘7	.000777˘7
	8	.320	.1600	.13333˘3	.106˘6	.08000	.0533˘3	.02666˘6	.0177˘7	.00888˘8	.000888˘8
	9	.360	.1800	.15000	.120	.09000	.0600	.03000	.0200	.01000	.001000
	$1	.045	.0225	.01875	015	.01125	.0075	.00375	.0025	.00125	.000125
	2	.090	.0450	.03750	.030	.02250	.0150	.00750	.0050	.00250	.000250
	3	.135	.0675	.05625	.045	.03375	.0225	01125	.0075	.00375	.000375
	4	.180	.0900	.07500	.060	.04500	.0300	.01500	.0100	.00500	.000500
4½%	5	.225	.1125	.09375	.075	.05625	.0375	.01875	.0125	.00625	.000625
	6	.270	.1350	.11250	.090	.06750	.0450	.02250	.0150	.00750	.000750
	7	.315	.1575	.13125	.105	.07875	.0525	.02625	.0175	.00875	.000875
	8	.360	.1800	.15000	.120	.09000	.0600	.03000	.0200	.01000	.001000
	9	.405	2025	.16875	.135	.10125	0675	.03375	.0225	01125	.001125
	$1	.050	.0250	.02083˘3	.016˘6	.01250	.0083˘3	.00416˘6	.0027˘7	.00138˘8	.000138˘8
	2	.100	.0500	.04166˘6	.033˘3	.02500	.0166˘6	.00833˘3	.0055˘5	.00277˘7	.000277˘7
	3	.150	.0750	.06250	.050	.03750	.0250	.01250	.0083˘3	.00416˘6	.000416˘6
	4	.200	.1000	.08333˘3	.066˘6	.05000	.0333˘3	.01666˘6	.0111˘1	.00555˘5	.000555˘5
5%	5	.250	.1250	.10416˘6	.083˘3	.06250	.0416˘6	.02083˘3	.0138˘8	.00694˘4	.000694˘4
	6	.300	.1500	.12500	.100	.07500	.0500	.02500	.0166˘6	.00833˘3	.000833˘3
	7	.350	.1750	.14583˘3	.116˘6	.08750	.0583˘3	.02916˘6	.0194˘4	.00972˘2	.000972˘2
	8	.400	.2000	.16666˘6	.133˘3	.10000	.0666˘6	.03333˘3	.0222˘2	.01111˘1	.001111˘1
	9	.450	.2250	.18750	.150	.11250	.0750	.03750	.0250	.01250	.001250
	$1	.060	.0300	.02500	.010	.01500	.0100	.00500	.0033˘3	.00166˘6	.000166˘6
	2	.120	.0600	.05000	.040	.03000	.0200	.01000	.0066˘6	.00333˘3	.000333˘3
	3	.180	.0900	07500	.060	.04500	.0300	.01500	.0100	.00500	.000500
	4	.240	.1200	.10000	.080	.06000	.0400	.02000	.0133˘3	.00666˘6	.000666˘6
6%	5	.300	.1500	.12500	.100	.07500	.0500	.02500	.0166˘6	.00833˘3	.000833˘3
	6	.360	.1800	15000	.120	.09000	.0600	.03000	.0200	.01000	.001000
	7	.420	.2100	.17500	.140	.10500	.0700	.03500	.0233˘3	.01166˘6	.001166˘6
	8	.480	.2400	.20000	.160	.12000	.0800	.04000	.0266˘6	.01333˘3	.001333˘3
	9	.540	.2700	22500	.180	.13500	.0900	.04500	.0300	.01500	.001500
	$1	.070	.0350	.02916˘6	.023˘3	.01750	.0116˘6	.00583˘3	.0038˘8	.00194˘4	.000194˘4
	2	.140	.0700	.05833˘3	.046˘6	.03750	.0233˘3	.01166˘6	.0077˘7	.00388˘8	.000388˘8
	3	.210	.1050	.08750	.070	.05250	.0350	.01750	.0116˘6	.00583˘3	.000583˘3
	4	.280	.1400	.11666˘6	.093˘3	.07000	.0466˘6	.02333˘3	.0155˘5	.00777˘7	.000777˘7
7%	5	.350	.1750	14583˘3	.116˘6	.08750	.0583˘3	.02916˘6	.0194˘4	.00972˘2	.000972˘2
	6	.420	.2100	.17500	.140	.10500	.0700	.03500	.0233˘3	.01166˘6	.001166˘6
	7	.490	.2450	20416˘6	.163˘3	.12250	.0816˘6	.04083˘3	.0272˘2	.01361˘1	.001361˘1
	8	560	.2800	.23333˘3	.186˘6	.14000	.0933˘3	.04666˘6	.0311˘1	.01555˘5	.001555˘5
	9	.630	.3150	26250	.210	.15750	.1050	.05250	0350	.01750	.001750

* Note that *all* repeating decimals may be extended indefinitely. Thus, the interest on $1.00 at 4% for 4 months is given as .013˘3 or 1⅓ cents, because the decimal .013˘3 = .01333333...; hence the interest on $1,000,000, at the same rate and for the same time, is $13,333.33⅓. Decimals which are not repeating decimals are *exact*.

Example.—Find the amount of $600. compounded at 2% semi-annually for 6 years (12 periods).

Solution.

log. 600. = 2.7781513
12 × log. 1.02 = 0.1032024

Ans. $760.95 . . . 2.8813537

Clearly, the process of calculating compound interest is simply geometrical progression, in which 1 + Rate % is a constant factor between successive terms.

EQUATION OF PAYMENTS.

Like Alligation, this is a "center of gravity" problem. It consists in finding the average time when a single payment can be made to cancel several notes, bearing the same rate of interest, which fall due on different dates.

Problem.—A holds B's notes as follows: $600 due in 1 month; $700 due in 3 months; $400 due in 4 months; and $300 due in 5 months—all bearing the same rate of interest. At what time can B make a single payment of the whole amount, $2000, to cancel the obligation equitably?

Solution.—

$$600 \times 1 = 600.$$
$$700 \times 3 = 2100.$$
$$400 \times 4 = 1600.$$
$$300 \times 5 = 1500.$$
$$2000 \times x = 5800.$$

$$\therefore \text{Average time } x = \frac{5800}{2000} = 2.9 \text{ months} = 2 \text{ m., } 27 \text{ d. Ans.}$$

2.—Table For Finding Number of Days Between Any Two Dates in Two Consecutive Years.*

First Year.

Day Mo.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1	1	32	60	91	121	152	182	213	244	274	305	335
2	2	33	61	92	122	153	183	214	245	275	306	336
3	3	34	62	93	123	154	184	215	246	276	307	337
4	4	35	63	94	124	155	185	216	247	277	308	338
5	5	36	64	95	125	156	186	217	248	278	309	339
6	6	37	65	96	126	157	187	218	249	279	310	340
7	7	38	66	97	127	158	188	219	250	280	311	341
8	8	39	67	98	128	159	189	220	251	281	312	342
9	9	40	68	99	129	160	190	221	252	282	313	343
10	10	41	69	100	130	161	191	222	253	283	314	344
11	11	42	70	101	131	162	192	223	254	284	315	345
12	12	43	71	102	132	163	193	224	255	285	316	346
13	13	44	72	103	133	164	194	225	256	286	317	347
14	14	45	73	104	134	165	195	226	257	287	318	348
15	15	46	74	105	135	166	196	227	258	288	319	349
16	16	47	75	106	136	167	197	228	259	289	320	350
17	17	48	76	107	137	168	198	229	260	290	321	351
18	18	49	77	108	138	169	199	230	261	291	322	352
19	19	50	78	109	139	170	200	231	262	292	323	353
20	20	51	79	110	140	171	201	232	263	293	324	354
21	21	52	80	111	141	172	202	233	264	294	325	355
22	22	53	81	112	142	173	203	234	265	295	326	356
23	23	54	82	113	143	174	204	235	266	296	327	357
24	24	55	83	114	144	175	205	236	267	297	328	358
25	25	56	84	115	145	176	206	237	268	298	329	359
26	26	57	85	116	146	177	207	238	269	299	330	360
27	27	58	86	117	147	178	208	239	270	300	331	361
28	28	59	87	118	148	179	209	240	271	301	332	362
29	29	...	88	119	149	180	210	241	272	302	333	363
30	30	...	89	120	150	181	211	242	273	303	334	364
31	31	...	90	...	151	...	212	243	...	304	...	365

Second Year.

Day Mo.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1	366	397	425	456	486	517	547	578	609	639	670	700
2	367	398	426	457	487	518	548	579	610	640	671	701
3	368	399	427	458	488	519	549	580	611	641	672	702
4	369	400	428	459	489	520	550	581	612	642	673	703
5	370	401	429	460	490	521	551	582	613	643	674	704
6	371	402	430	461	491	522	552	583	614	644	675	705
7	372	403	431	462	492	523	553	584	615	645	676	706
8	373	404	432	463	493	524	554	585	616	646	677	707
9	374	405	433	464	494	525	555	586	617	647	678	708
10	375	406	434	465	495	526	556	587	618	648	679	709
11	376	407	435	466	496	527	557	588	619	649	680	710
12	377	408	436	467	497	528	558	589	620	650	681	711
13	378	409	437	468	498	529	559	590	621	651	682	712
14	379	410	438	469	499	530	560	591	622	652	683	713
15	380	411	439	470	500	531	561	592	623	653	684	714
16	381	412	440	471	501	532	562	593	624	654	685	715
17	382	413	441	472	502	533	563	594	625	655	686	716
18	383	414	442	473	503	534	564	595	626	656	687	717
19	384	415	443	474	504	535	565	596	627	657	688	718
20	385	416	444	475	505	536	566	597	628	658	689	719
21	386	417	445	476	506	537	567	598	629	659	690	720
22	387	418	446	477	507	538	568	599	630	660	691	721
23	388	419	447	478	508	539	569	600	631	661	692	722
24	389	420	448	479	509	540	570	601	632	662	693	723
25	390	421	449	480	510	541	571	602	633	663	694	724
26	391	422	450	481	511	542	572	603	634	664	695	725
27	392	423	451	482	512	543	573	604	635	665	696	726
28	393	424	452	483	513	544	574	605	636	666	697	727
29	394	...	453	484	514	545	575	606	637	667	698	728
30	395	...	454	485	515	546	576	607	638	668	699	729
31	396	...	455	...	516	...	577	608	...	669	...	730

* Subtract the number opposite the first date from the number opposite the last. If the 29th of February is included, add one day.

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3.—Compound Interest Table.

Amount of $1. at compound interest for periods 1 to 50 at various *periodic rates.

Periods. n.	*Periodic Rates.							
	2%	3%	3½%	4%	4½%	5%	6%	7%
1	1.02000	1.03000	1.03500	1.04000	1.04500	1.05000	1.06000	1.07000
2	1.04040	1.06090	1.07123	1.08160	1.09203	1.10250	1.12360	1.14490
3	1.06121	1.09273	1.10872	1.12486	1.14117	1.15763	1.19102	1.22504
4	1.08243	1.12551	1.14752	1.16986	1.19252	1.21551	1.26248	1.31080
5	1.10408	1.15927	1.18769	1.21665	1.24618	1.27628	1.33823	1.40255
6	1.12616	1.19405	1.22926	1.26532	1.30226	1.34010	1.41852	1.50073
7	1.14869	1.22987	1.27228	1.31593	1.36086	1.40710	1.50363	1.60578
8	1.17166	1.26677	1.31681	1.36857	1.42210	1.47746	1.59385	1.71819
9	1.19509	1.30477	1.36290	1.42331	1.48610	1.55133	1.68948	1.83846
10	1.21899	1.34392	1.41060	1.48024	1.55297	1.62889	1.79085	1.96715
11	1.24337	1.38423	1.45997	1.53945	1.62285	1.71034	1.89830	2.10485
12	1.26824	1.42576	1.51107	1.60103	1.69588	1.79586	2.01220	2.25219
13	1.29361	1.46853	1.56396	1.66507	1.77220	1.88565	2.13293	2.40985
14	1.31948	1.51259	1.61870	1.73168	1.85194	1.97993	2.26090	2.57853
15	1.34587	1.55797	1.67535	1.80094	1.93528	2.07893	2.39656	2.75903
16	1.37279	1.60471	1.73399	1.87298	2.02237	2.18287	2.54035	2.95216
17	1.40024	1.65285	1.79468	1.94790	2.11338	2.29202	2.69277	3.15882
18	1.42825	1.70243	1.85749	2.02582	2.20848	2.40662	2.85434	3.37993
19	1.45681	1.75351	1.92250	2.10685	2.30786	2.52695	3.02560	3.61653
20	1.48595	1.80611	1.98979	2.19112	2.41171	2.65330	3.20714	3.86968
21	1.51567	1.86029	2.05943	2.27876	2.52024	2.78596	3.39957	4.14057
22	1.54598	1.91610	2.13151	2.36991	2.63365	2.92523	3.60354	4.43041
23	1.57690	1.97358	2.20611	2.46471	2.75217	3.07152	3.81976	4.74054
24	1.60844	2.03279	2.28332	2.56330	2.87602	3.22510	4.04894	5.07237
25	1.64061	2.09378	2.36324	2.66583	3.00544	3.38635	4.29188	5.42744
26	1.67342	2.15659	2.44595	2.77246	3.14068	3.55567	4.54939	5.80736
27	1.70689	2.22129	2.53156	2.88336	3.28201	3.73346	4.82224	6.21388
28	1.74103	2.28792	2.62016	2.99870	3.42970	3.92013	5.11170	6.64885
29	1.77585	2.35656	2.71187	3.11864	3.58406	4.11614	5.41840	7.11427
30	1.81134	2.42726	2.80672	3.24339	3.74532	4.32194	5.74351	7.61227
31	1.84759	2.50008	2.90501	3.37312	3.91386	4.53804	6.08812	8.14513
32	1.88454	2.57508	3.00670	3.50805	4.08998	4.76494	6.45340	8.71529
33	1.92224	2.65233	3.11193	3.64837	4.27403	5.00319	6.84061	9.32536
34	1.96068	2.73190	3.22085	3.79430	4.46637	5.25335	7.25115	9.97813
35	1.99989	2.81386	3.33358	3.94608	4.66735	5.51600	7.68611	10.6766
36	2.03989	2.89827	3.45025	4.10392	4.87738	5.79182	8.14728	11.4240
37	2.08069	2.98518	3.57101	4.26806	5.09686	6.08141	8.63611	12.2236
38	2.12230	3.07478	3.69599	4.43880	5.32618	6.38548	9.15428	13.0793
39	2.16475	3.16702	3.82535	4.61635	5.56590	6.70475	9.70354	13.9948
40	2.20801	3.26203	3.95924	4.80100	5.81637	7.03999	10.2855	14.9745
41	2.25221	3.35989	4.09781	4.99306	6.07811	7.39199	10.9029	16.0227
42	2.29725	3.46069	4.24124	5.19276	6.35162	7.76159	11.5571	17.1443
43	2.34320	3.56451	4.38968	5.40047	6.63744	8.14967	12.2505	18.3444
44	2.39006	3.67144	4.54332	5.61649	6.93613	8.55715	12.9855	19.6285
45	2.43786	3.78159	4.70233	5.84115	7.24826	8.98504	13.7647	21.0025
46	2.48662	3.89503	4.86692	6.07480	7.57443	9.43426	14.5906	22.4727
47	2.53635	4.01188	5.03726	6.31779	7.91528	9.90597	15.4660	24.0458
48	2.58708	4.13224	5.21356	6.57050	8.27146	10.4013	16.3939	25.7290
49	2.63882	4.25621	5.39604	6.83330	8.64368	10.9213	17.3776	27.5300
50	2.69160	4.38389	5.58491	7.10665	9.03265	11.4674	18.4202	29.4571

* Periods may be annual, semi-annual or quarterly, etc. Periodic rates are proportioned to the length of the period. Thus, 4% annual = 2% semi-annual rate. For explanation of table, see top of page 60.

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PARTIAL PAYMENTS.

In cancelling notes by partial payments the following is the rule in United States law:

Whenever the payment or payments *equal or exceed* the interest a new principal shall be formed by adding the interest and deducting the payment or payments. That is, the principal cannot be reduced without first cancelling the interest.

A common method, however, is to consider the debt as the original principal together with its accumulated interest down to the date of settlement; this to be cancelled by the various payments (considered as separate principals) together with their accumulated interest, down to the same date.

ANNUITIES.

Specified payment of money amounting to a fixed sum each year is called an annuity:—*Certain annuity*, when payments extend over certain definite periods; *Contingent annuity*, when payments are contingent on certain events; *Life annuity*, when payments are for life of one or more persons; *Deferred annuity*, when payments begin at some future time or event.

The value of an annuity may be reduced to its—

Final Value, or amount of all payments at compound interest to the end of the annuity;

Initial value, or equivalent principal which, at compound interest for the life of the annuity, would amount to its final value;

Present value, or equivalent principal which, at compound interest for the balance of the life of the annuity, would amount to the value of future payments at compound interest to the end of the annuity.

The fundamental equations which enter into the conversion of annuity in one form to its equivalent in another form, are those of geometrical progression, in general, and compound interest in particular.

Final Value of Annuity.—The final value of an annuity is directly proportional to the value of the annual payment. Hence, from a table calculated on the basis of an annuity of $1, at different rates of interest and for any number of years, the final value of any annuity at the same rate and for the same time may be obtained by multiplying the value in the table by the annuity.

Table 4, following, gives the final values of an annuity of $1 at compound interest rates of 2, 3, 3½, 4, 4½, 5, 6 and 7 per cent up to 50 years. It is calculated from the formula:

$$\text{Final value} = \frac{(1 + \text{Rate }\%)^x - 1}{\text{Rate }\%};$$

in which x = the number of years = the number of payments, the final value being calculated up to the time of each payment, and also includes that payment.

Present or Initial Value of Annuity.—The present value of an annuity may be considered the *initial* value of that part of the annuity comprising future payments, the date beginning with the first of these payments. Hence the term "Present value" will be used in the broadest sense. Further, it is the amount of capital or the principal which, if placed at interest, would exactly furnish the annuities for the specified time; that is, the principal would be depleted at the end of that time.

For the same time and at the same rate of interest, the present value is directly proportional to the annuity (and to the final value). Hence, in Table 5, page 65, to find the present value of any annuity, as $1000, multiply the tabular number by that annuity (1000). The table is calculated from the formula:

$$\text{Present value} = \frac{1 - \dfrac{1}{(1 + \text{Rate }\%)^x}}{\text{Rate }\%};$$

in which x = the number of years to run = the number of payments to be made, payments being made at the end of each year.

Sinking Fund.—An annuity may be applied as a sinking fund to cancel a debt payable at some future time, as in the case of bonds maturing in, say, 20, 30 or 50 years. A plant may thus be made to pay for itself by

4.—FINAL VALUE OF ANNUITY OF $1 IN FROM 1 TO 50 YEARS OR PERIODS.

Periods. x	Yearly or Periodic Rates.							
	2%	3%	3½%	4%	4½%	5%	6%	7%
1	1.	1.	1.	1.	1.	1.	1.	1.
2	2.02000	2.03000	2.03500	2.04000	2.04500	2.05000	2.06000	2.07000
3	3.06040	3.09090	3.10623	3.12160	3.13703	3.15250	3.18360	3.21490
4	4.12161	4.18363	4.21495	4.24646	4.27820	4.31012	4.37462	4.43994
5	5.20404	5.30913	5.36247	5.41632	5.47072	5.52563	5.63710	5.75074
6	6.30812	6.46840	6.55016	6.63297	6.71690	6.80191	6.97532	7.15329
7	7.43428	7.66245	7.77942	7.89829	8.01916	8.14201	8.39385	8.65402
8	8.58297	8.89232	9.05170	9.21422	9.38002	9.54911	9.89748	10.2598
9	9.75463	10.1591	10.3685	10.5828	10.8021	11.0266	11.4913	11.9780
10	10.9497	11.4638	11.7313	12.0061	12.2882	12.5779	13.1808	13.8165
11	12.1687	12.8078	13.1419	13.4863	13.8412	14.2068	14.9716	15.7837
12	13.4121	14.1920	14.6019	15.0258	15.4640	15.9171	16.8700	17.8885
13	14.6803	15.6178	16.1129	16.6268	17.1599	17.7130	18.8822	20.1407
14	15.9739	17.0863	17.6769	18.2919	18.9321	19.5986	21.0151	22.5505
15	17.2934	18.5989	19.2956	20.0236	20.7840	21.5786	23.2760	25.1291
16	18.6393	20.1569	20.9709	21.8245	22.7193	23.6576	25.6725	27.8881
17	20.0121	21.7616	22.7049	23.6975	24.7417	25.8405	28.2129	30.8403
18	21.4123	23.4144	24.4996	25.6454	26.8551	28.1326	30.9057	33.9991
19	22.8406	25.1168	26.3571	27.6712	29.0636	30.5389	33.7601	37.3790
20	24.2974	26.8703	28.2795	29.7780	31.3714	33.0660	36.7857	40.9956
21	25.7833	28.6765	30.2693	31.9691	33.7831	35.7193	39.9927	44.8652
22	27.2990	30.5368	32.3287	34.2479	36.3034	38.5052	43.3923	49.0058
23	28.8450	32.4529	34.4602	36.6178	38.9370	41.4305	46.9960	53.4362
24	30.4219	34.4265	36.6663	39.0825	41.6892	44.5020	50.8157	58.1767
25	32.0303	36.4592	38.9497	41.6458	44.5652	47.7271	54.8647	63.2491
26	33.6709	38.5530	41.3129	44.3116	47.5706	51.1134	59.1565	68.6766
27	35.3443	40.7096	43.7589	47.0841	50.7113	54.6691	63.7060	74.4839
28	37.0513	42.9309	46.2904	49.9674	53.9933	60.4025	68.5282	80.6978
29	38.7923	45.2188	48.9106	52.9661	57.4230	64.3226	73.6400	87.3466
30	40.5682	47.5753	51.6224	56.0847	61.0071	66.4389	79.0584	94.4610
31	42.3796	50.0026	54.4291	59.3281	64.7524	70.7608	84.8019	102.073
32	44.2272	52.5026	57.3341	62.7012	68.6663	75.2989	90.8900	110.218
33	46.1117	55.0777	60.3408	66.2093	72.7563	80.0638	97.3434	118.934
34	48.0340	57.7300	63.4528	69.8576	77.0303	85.0670	104.184	128.259
35	49.9947	60.4619	66.6736	73.6519	81.4967	90.3203	111.435	138.237
36	51.9945	63.2758	70.0072	77.5980	86.1641	95.8364	119.121	148.914
37	54.0344	66.1740	73.4574	81.7019	91.0414	101.628	127.269	160.338
38	56.1151	69.1592	77.0284	85.9700	96.1383	107.710	135.905	172.561
39	58.2374	72.2340	80.7244	90.4088	101.464	114.095	145.059	185.641
40	60.4023	75.4010	84.5498	95.0251	107.030	120.800	154.762	199.636
41	62.6103	78.6630	88.5090	99.8261	112.846	127.840	165.048	214.611
42	64.8625	82.0229	92.6069	104.819	118.924	135.232	175.950	230.633
43	67.1598	85.4836	96.8481	110.012	125.276	142.994	187.508	247.778
44	69.5030	89.0481	101.238	115.412	131.914	151.143	199.758	266.122
45	71.8930	92.7195	105.781	121.029	138.850	159.700	212.743	285.750
46	74.3309	96.5012	110.483	126.870	146.098	168.685	226.509	306.753
47	76.8176	100.396	115.350	132.945	153.672	178.119	241.099	329.226
48	79.3540	104.408	120.387	139.263	161.588	188.025	256.565	353.271
49	81.9411	108.540	125.601	145.833	169.859	198.426	273.959	379.000
50	84.5800	112.796	130.997	152.666	178.503	209.348	290.337	406.530

annually setting aside a certain amount of its earnings to be placed at compound interest.

Table 6, page 65, shows the annual amount to be set aside in order to accumulate $1000 at various rates of interest and for various terms of years, compounded annually. It is calculated from the formula:

$$\text{Annuity} = 1000 \left(\frac{\text{Rate } \%}{(1 + \text{Rate } \%)^x - 1} \right);$$

in which x = the number of years = the number of annuities, the payments being made at the end of each year.

Problem.—What semi-annual payments shall be made to a sinking fund to create $1,000,000 in 20 years, interest at 4%?

Solution.—In this case there will be 40 periods compounded at 2%. From the table, 40 payments of $16.556 will create $1,000. But, in order to create $1,000,000 this must be multiplied by 1000 $\left(= \frac{1,000,000}{1000} \right)$. Hence the semi-annual payments would be $16,556 each.

5.—Present Worth, Present Value, or Capitalization of Annuity of $1.

Years.	Rate of Interest.							
	2%	3%	3½%	4%	4½%	5%	6%	7%
5	4.7134	4.5796	4.5150	4.4518	4.3899	4.3294	4.2124	4.1002
10	8.9824	8.5301	8.3165	8.1108	7.9127	7.7217	7.3602	7.0236
15	12.849	11.938	11.517	11.118	10.739	10.380	9.7123	9.1079
20	16.351	14.877	14.212	13.590	13.008	12.462	11.470	10.594
25	19.524	17.413	16.482	15.622	14.828	14.094	12.783	11.654
30	22.396	19.600	18.392	17.292	16.289	15.372	13.765	12.409
35	24.999	21.487	20.000	18.664	17.461	16.374	14.498	12.948
40	27.355	23.115	21.355	19.793	18.401	17.159	15.046	13.332
45	29.490	24.519	22.495	20.720	19.156	17.774	15.456	13.606
50	31.424	25.730	23.456	21.482	19.762	18.256	15.762	13.801
60	34.761	27.676	24.988	22.623	20.638	18.929	16.161	14.039
70	37.499	29.123	26.000	23.395	21.202	19.343	16.385	14.160
80	39.745	30.201	26.749	23.915	21.565	19.596	16.509	14.222
90	41.587	31.002	27.279	24.267	21.799	19.752	16.579	14.253
100	43.098	31.599	27.655	24.505	21.949	19.848	16.613	14.269

6.—Sinking Fund Table.

Annuities (or Annual Saving) Which Will Create $1,000 in Given Number of Years, x, at Various Rates of Interest Compounded Annually.

Years to Run. x	Rate of Interest.							
	2%	3%	3½%	4%	4½%	5%	6%	7%
2	$495.05	$492.61	$491.42	$490.20	$489.00	$487.80	$485.44	$483.09
3	326.72	323.56	321.94	320.36	318.77	317.21	314.10	311.05
4	242.63	239.02	237.26	235.50	233.74	232.01	228.60	225.26
5	192.16	188.35	186.49	184.63	182.79	180.98	177.39	173.89
6	158.53	154.61	152.67	150.79	148.88	147.02	143.36	139.80
7	134.52	130.51	128.57	126.61	124.67	122.82	119.13	115.55
8	116.51	112.46	110.48	108.53	106.60	104.72	101.03	97.468
9	102.52	98.434	96.446	94.493	92.575	90.690	87.022	83.487
10	91.327	87.231	85.242	83.291	81.360	79.505	75.868	72.377
12	74.560	70.462	68.484	66.552	64.666	62.826	59.277	55.902
15	57.826	53.767	51.825	49.941	48.114	46.342	42.963	39.795
20	41.157	37.216	35.361	33.582	31.876	30.243	27.184	24.393
25	31.220	27.428	25.674	24.012	22.439	20.952	18.227	15.811
30	24.650	21.019	19.371	17.830	16.392	15.051	12.649	10.586
35	20.002	16.539	14.998	13.577	12.270	11.072	8.9738	7.2340
40	16.556	13.262	11.827	10.524	9.3432	8.2781	6.4615	4.9976
45	13.910	10.785	9.4535	8.2625	7.2020	6.2617	4.7005	3.4996
50	11.823	8.8656	7.6338	6.5502	5.6021	4.7767	3.4443	2.4598
60	8.7679	6.1330	5.0687	4.2019	3.4543	2.2207	1.8757	1.2292
70	6.6676	4.3366	3.4610	2.7451	2.1651	1.6992	1.0331	.61952
80	5.1607	3.1118	2.3849	1.8141	1.3708	1.0296	.57254	.31357
90	4.0460	2.2556	1.6578	1.2078	.87316	.62711	.31836	.15905
100	3.2027	1.6467	1.1593	.80801	.55839	.38314	.17735	.08076

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4.—MEASURES, WEIGHTS AND MONEY.

FUNDAMENTAL UNITS.

The Metric System, on account of its simplicity, is destined in all probability to become the international standard of weights and measures. It has been legalized by Great Britain, Russia and the United States; and has been adopted by all other European nations, by Mexico, and by many South American States.

Meter.—*Length, Area, Volume.* The international standard Meter, the unit of length, is the distance between two lines on a platinum-iridium bar, at 0° centigrade, deposited at the International Bureau of Weights and Measures, in Paris. The legalized ratio of the standard meter to the standard yard by the United States* is $\frac{39.37}{36.00} = 1.09361111\text{——} = \dagger 1.09361^{\vee}1$; hence, the following equivalents:

Length.

1 meter = $1.09361^{\vee}1$ yards = $3.28083^{\vee}3$ feet = 39.37 inches.

1 yard = 0.914 401 828 80 meter.	Log	= 9.9611371.	Co-log	= 0.0388629
1 foot = 0.304 800 609 60 meter.	"	= 9.4840158.	"	= 0.5159842
1 inch = 0.025 400 050 80 meter.	"	= 8.4048346.	"	= 1.5951654

Area.

1 square meter = 1.195 985 262 $3\bar{5}$ sq. yd. = 10.763 867 $361^{\vee}1$ sq. ft. = 1549.9969 sq. ins.

1 sq. yd. = 0.836 130 704 52 sq.met.	Log	= 9.9222742.	Co-log	= 0.0777258
1 sq. ft. = 0.092 903 411 61 sq.met.	"	= 8.9680316.	"	= 1.0319684
1 sq. in. = 0.000 645 162 58 sq.met.	"	= 6.8096692.	"	= 3.1903308

Volume.

1 cubic meter = 1.307 942 771 $6\bar{3}$ cu. yd. = 35.314 454 833 91 cu. ft. = 61023.377953 cu. ins.

1 cu. yd. = 0.764 559 445 33 cu.met.	Log	= 9.8834113.	Co-log	= 0.1165887
1 cu. ft. = 0.028 317 016 49 cu.met.	"	= 8.4520475.	"	= 1.5479525
1 cu. in. = 0.000 016 387 16 cu.met.	"	= 5.2145038.	"	= 4.7854962

Liter.—*Capacity (Liquid and Dry).* The liter, the unit of capacity, is the volume of one kilogram of pure water at its maximum density; and this is equal to a cubic decimeter, or a cube whose edge is one-tenth of a meter = 3.937 inches. The capacity of one liter is therefore $\frac{1}{1000}$ of the volume of a cubic meter. The following are the United States‡ equivalents:

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*In Great Britain the meter has been legalized at 39.37079 inches, but the length of 39.370432 inches, as adopted by France, Germany, Belgium and Russia, is frequently used.

† Logarithm $1.09361^{\vee}1 = 0.0388629$; co-logarithm = 9.9611371.

‡ The Imperial gallon, the British unit of *liquid* capacity, contains 277.274 cubic inches, or 1.200320 $\left(=\frac{277.274}{231}\right)$ United States gallons. The Imperial bushel, the British unit of *dry* capacity, contains 8 Imperial gallons or 2218.192 cubic inches = 1.03151570 $\left(=\frac{2218.192}{2150.42}\right)$ United States (struck) bushels. British measures of the same denomination as those of the United States are hence greater than the latter by these ratios.

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VOLUME.

1 liter = 1 cubic decimeter = 0.001 cubic meter.		
= 0.001 307 942 77 cu. yd.	Log =	7.1165887
= 0.035 314 454 83 cu. ft.	" =	8.5479525
= 61.023 377 953 cu. ins. (exact.)	" =	1.7854962
1 cu. yd. = 764.559 445 33 liters.	" =	2.8834113
1 cu. ft. = 28.317 016 49 liters.	" =	1.4520475
1 cu. in. = 0.016 387 16 liter.	" =	8.2145038

LIQUID.

1 liter = 0.264 170 467 $3\bar{3}$ U. S. gallon.	Log =	9.4218842
= 1.056 681 869 $3\bar{2}$ quarts.	" =	0.0239442
= 2.113 363 738 $6\bar{4}$ pints.	" =	0.3249742
1 gallon = 231 cu. ins = 0.133 680 5˘5 cu. ft.	" =	9.1260682
= 0.004 951 131 $6\bar{9}$ cu. yd.	" =	7.6947045
= 3.785 434 496 $5\bar{6}$ liters or cubic dm.	" =	0.5781158
1 quart = 0.946 358 624 $1\bar{4}$ liter.	" =	9.9760558

DRY.

1 liter = 0.028 377 422 99 U. S. bushel.	Log =	8.4529730
= 0.113 509 691 97 peck.	" =	9.0550330
= 0.908 077 535 78 quart.	" =	9.9581229
1 bushel = 2150.42 cu.ins. = 1.244 456 018 5˘1 cu.ft.	" =	0.0949796
= 0.046 090 963 $6\bar{5}$ cu. yd.	" =	8.6636158
= 0.035 239 281 60 cubic meter.	" =	8.5470270
= 35.239 281 602 $1\bar{5}$ liters or cubic dm.	" =	1.5470270
1 peck = 8.809 820 400 $5\bar{4}$ liters.	" =	0.9449670
1 quart = 1.101 227 550 $0\bar{7}$ liters.	" =	0.0418771

MASS.

1 liter (= 1 cubic decimeter) of pure water at maximum density weighs 1 kilogram (kilo).

1 millimeter (= 1 cubic centimeter) of pure water at maximum density weighs 1 gram.

Gram.—*Mass* (*Weight*). The Gram, the unit of weight, is the weight of a cubic centimeter (1 millimeter) of pure water at its maximum density = $\frac{1}{1000}$ of the international kilogram. The bureau of Standards, Washington, D. C., gives the fundamental equivalents:

1 avoirdupois pound = 453.592 4277 grams, and	Log =	2.6566658
1 troy pound = $\frac{5760}{7000}$ avoir. pound = 373.241 769 078 857 142 8˘57 grams, from which are derived the following values:	" =	2.5719903

1 kilogram	= 2.204 622 341 406 avoir. pounds.	Log =	0.3433342
	= 2.679 228 539 903 troy pounds.	" =	0.4280097
1 gram	= 0.035 273 957 462 $\bar{5}$ avoir. ounce.	" =	8.5474542
	= 0.032 150 742 478 8 troy ounce.	" =	8.5071910
	= 15.432 356 389 84 grains (troy).	" =	1.1884323
1 avoirdupois ounce	= 28.349 526 731 25 grams.	" =	1.4525458
1 troy ounce	= 31.103 480 756 65 grams.	" =	1.4928090
1 grain (troy)	= 0.064 798 918 243 gram.	" =	8.8115677

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GENERAL TABLES.

1.—Approximate Equivalents—Metric and English.

1 acre	= .40	hectar	.4047
1 bushel	= 35	liters	35.24
1 centimeter	= .39	inch	.3937
1 cubic centimeter	= .061	cubic inch	.0610
1 cubic foot	= .028	cubic meter	.0283
1 cubic inch	= 16	cubic centimet.	16.39
1 cubic meter	= 35	cubic feet	35.31
1 cubic meter	= 1.3	cubic yards	1.308
1 cubic yard	= .76	cubic meter	.7645
1 foot	= 30	centimeters	30.48
1 gallon	= 3.8	liters	3.785
1 grain	= .065	gram	.0648
1 gram	= 15	grains	15.43
1 hectar	= 2.5	acres	2.471
1 inch	= 25	millimeters	25.40
1 kilo	= 2.2	pounds	2.205
1 kilometer	= .62	mile	.6214
1 liter	= .91	quart (dry)	.9081
1 liter	= 1.1	quarts (liquid)	1.057
1 meter	= 3.3	feet	3.281
1 mile	= 1.6	kilometers	1.609
1 millimeter	= .039	inch	.0394
• 1 ounce (avoirdupois)	= 28	grams	28.35
1 ounce (Troy)	= 31	grams	31.10
1 peck	= 8.8	liters	8.809
1 pint	= .47	liter	.4732
1 pound	= .45	kilo	.4536
1 quart (dry)	= 1.1	liters	1.101
• 1 quart (liquid)	= .95	liter	.9464
1 sq. centimeter	= .15	sq. inch	.1550
1 sq. foot	= .093	sq. meter	.0929
1 sq. inch	= 6.5	sq. centimeters	6.452
1 sq. meter	= 1.2	sq. yards	1.196
1 sq. meter	= 11	sq. feet	10.76
1 sq. yard	= .84	sq. meter	.8361
1 ton (2,000 lbs.)	= .91	metric ton	.9072
1 ton (2,240 lbs.)	= 1	metric ton	1.017
1 ton (metric)	= 1.1	ton (2,000 lbs.)	1.102
1 ton (metric)	= .98	ton (2,240 lbs.)	.9842
1 yard	= .91	meter	.9144

2.—Long Measure, English.

	= 1 *inch* (*in.*)	= 0.025400 Meters
12 inches	= 1 *foot* (*ft.*)	= 0.304801 "
3 feet	= 1 *yard* (*yd.*)	= 0.914402 "
5½ yards (= 16½ ft.)	= 1 *rod* (*rd.*)	= 5.029210 "
40 rods (= 220 yds. = 660 ft.)	= 1 *furlong* (*fur.*)	= 201.1684 "
8 furlongs (= 320 rods = 1760 yds = 5280 ft.)	= 1 *statute mile*	= 1609.347 "
3 miles (= 24 furs. = 960 rds. = 5280 yds. = 15840 ft.)	= 1 *league*	= 4828.042 "

3.—Surveyors' Measure (lineal).

7.92 inches	= 1 *link*	= .2011684 Meters
100 links (= 4 rds. = 22 yds. = 66 ft.)	= 1 *chain*	= 20.11684 "
80 chains (320 rds. = 1760 yds = 5280 ft.)	= 1 *statute mile*	= 1609.347 "

4.—*Mariners' Measure.

6 feet	= 1 *fathom*	= 1.828804 Meters
120 fathoms (= 720 ft.)	= 1 *cable length*	= 219.4564 "
[7½ cable lengths (= 880 fathoms = 5280 ft.)	= 1 *statute mile*	= 1609.347 "]
1.15246 statute miles (= 6085 ft.)	= 1 *nautical mile*	= 1854.712 "

60 nautical miles = 1 degree. 360 degrees = circumference of the earth.

* The old nautical mile was given as 7½ cable lengths = 5400 feet, but it is now obsolete. The present nautical mile is not an exact term, being equal to *about* one minute of longitude at the equator. The British Admiralty knot is 6080 ft. The nautical mile of the U. S. Coast Survey is 6086.07 ft. = 1.152664 statute or land miles. Three nautical miles = 1 league.

5.—Lengths—Inches and Millimeters.—Equivalents of Decimal and Common Fractions of an Inch in Millimeters.
From $\frac{1}{64}$ to 1 Inch.

1/2's.	1/4's.	8ths.	16ths.	32nds.	64ths.	Millimeters.	Decimals of an Inch.	Inch.	1/2's.	1/4's.	8ths.	16ths.	32nds.	64ths.	Millimeters.	Decimals of an Inch.
					1	= .397	.015625							33	= 13.097	.515625
				1	2	= .794	.03125						17	34	= 13.494	.53125
					3	= 1.191	.046875							35	= 13.891	.546875
			1	2	4	= 1.588	.0625					9	18	36	= 14.288	.5625
					5	= 1.984	.078125							37	= 14.684	.578125
				3	6	= 2.381	.09375						19	38	= 15.081	.59375
					7	= 2.778	.109375							39	= 15.478	.609375
		1	2	4	8	= 3.175	.1250				5	10	20	40	= 15.875	.625
					9	= 3.572	.140625							41	= 16.272	.640625
				5	10	= 3.969	.15625						21	42	= 16.669	.65625
					11	= 4.366	.171875							43	= 17.066	.671875
			3	6	12	= 4.763	.1875					11	22	44	= 17.463	.6875
					13	= 5.159	.203125							45	= 17.859	.703125
				7	14	= 5.556	.21875						23	46	= 18.256	.71875
					15	= 5.953	.234375							47	= 18.653	.734375
	1	2	4	8	16	= 6.350	.2500				3	6	12	24	48	= 19.050	.75
					17	= 6.747	.265625							49	= 19.447	.765625
				9	18	= 7.144	.28125						25	50	= 19.844	.78125
					19	= 7.541	.296875							51	= 20.241	.796875
			5	10	20	= 7.938	.3125					13	26	52	= 20.638	.8125
					21	= 8.334	.328125							53	= 21.034	.828125
				11	22	= 8.731	.34375						27	54	= 21.431	.84375
					23	= 9.128	.359375							55	= 21.828	.859375
		3	6	12	24	= 9.525	.3750				7	14	28	56	= 22.225	.875
					25	= 9.922	.390625							57	= 22.622	.890625
				13	26	= 10.319	.40625						29	58	= 23.019	.90625
					27	= 10.716	.421875							59	= 23.416	.921875
			7	14	28	= 11.113	.4375					15	30	60	= 23.813	.9375
					29	= 11.509	.453125							61	= 24.209	.953125
				15	30	= 11.906	.46875						31	62	= 24.606	.96875
					31	= 12.303	.484375							63	= 25.003	.984375
1	2	4	8	16	32	= 12.700	.5	1	2	4	8	16	32	64	= 25.400	1.000

6.—Lengths—Hundredths of an Inch to Millimeters.
From 1 to 100 Hundredths.

Hundredths of an Inch.	0	1	2	3	4	5	6	7	8	9
0	0	.254	.508	.762	1.016	1.270	1.524	1.778	2.032	2.286
10	2.540	2.794	3.048	3.302	3.556	3.810	4.064	4.318	4.572	4.826
20	5.080	5.334	5.588	5.842	6.096	6.350	6.604	6.858	7.112	7.366
30	7.620	7.874	8.128	8.382	8.636	8.890	9.144	9.398	9.652	9.906
40	10.160	10.414	10.668	10.922	11.176	11.430	11.684	11.938	12.192	12.446
50	12.700	12.954	13.208	13.462	13.716	13.970	14.224	14.478	14.732	14.986
60	15.240	15.494	15.748	16.002	16.256	16.510	16.764	17.018	17.272	17.526
70	17.780	18.034	18.288	18.542	18.796	19.050	19.304	19.558	19.812	20.066
80	20.320	20.574	20.828	21.082	21.336	21.590	21.844	22.098	22.352	22.606
90	22.860	23.114	23.368	23.622	23.876	24.130	24.384	24.638	24.892	25.146

Example.—21 hundredths of an inch = 5.334 millimeters.

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7.—Lengths—Millimeters to Decimals of an Inch. From 1 to 100 Units.

Millimeters.	0	1	2	3	4	5	6	7	8	9
0	0	.03937	.07874	.11811	.15748	.19685	.23622	.27559	.31496	.35433
10	.39370	.43307	.47244	.51181	.55118	.59055	.62992	.66929	.70866	.74803
20	.78740	.82677	.86614	.90551	.94488	.98425	1.02362	1.06299	1.10236	1.14173
30	1.18110	1.22047	1.25984	1.29921	1.33858	1.37795	1.41732	1.45669	1.49606	1.53543
40	1.57480	1.61417	1.65354	1.69291	1.73228	1.77165	1.81102	1.85039	1.88976	1.92913
50	1.96850	2.00787	2.04724	2.08661	2.12598	2.16535	2.20472	2.24409	2.28346	2.32283
60	2.36220	2.40157	2.44094	2.48031	2.51968	2.55905	2.59842	2.63779	2.67716	2.71653
70	2.75590	2.79527	2.83464	2.87401	2.91338	2.95275	2.99212	3.03149	3.07086	3.11023
80	3.14960	3.18897	3.22834	3.26771	3.30708	3.34645	3.38582	3.42519	3.46456	3.50393
90	3.54330	3.58267	3.62204	3.66131	3.70078	3.74015	3.77952	3.81889	3.85826	3.89763

Example.—21 millimeters = 0.82677 inch.

8.—Lengths.

	Inches	Feet.	Yards.	Miles.
1 *millimeter* (m m) =	.03937	.0032808	.0010936	.....
10 millimeters (= $\frac{1}{100}$ meter) = 1 *centimeter* (c m) =	.3937	.0328083	.0109361	.....
10 centimeters (= $\frac{1}{10}$ meter) = 1 *decimeter* (d m) =	3.937	.328083ˇ3	.109361ˇ1	.....
10 decimeters = 1 *meter* =	39.37	3.28083ˇ3	1.09361ˇ1	.0006213
10 meters = 1 *dekameter* (Dm) =	393.7	32.8083ˇ3	10.9361ˇ1	.0062137
10 dekameters (= 100 meters) = 1 *hectometer* (Hm) =	3937	328.083ˇ3	109.361ˇ1	.062137
10 hectometers (= 1000 meters) = 1 *kilometer* (Km) =	.....	3280.83ˇ3	1093.61ˇ1	.621370
10 kilometers (= 10000 meters) = 1 *myriameter* (Mm) =	.....	32808.3ˇ3	10936.1ˇ1	6.21370

9.—Lengths, Equivalents. 1–10.

Inches.	Millimeters.	Inches.	Centimeters.	Feet.	Meters.	U. S. Yards.	Meters.	U. S. Miles.	Kilometers.
0.03937 =	1	0.3937 =	1	1	= 0.304801	1	= 0.914402	0.62137 =	1
0.07874 =	2	0.7874 =	2	2	= 0.609601	1.093611 =	1	1	= 1.60935
0.11811 =	3	1	= 2.54001	3	= 0.914402	2	= 1.828804	1.24274 =	2
0.15748 =	4	1.1811 =	3	3.28083 =	1	2.187222 =	2	1.86411 =	3
0.19685 =	5	1.5748 =	4	4	= 1.219202	3	= 2.743205	2	= 3.21869
0.23622 =	6	1.9685 =	5	5	= 1.524003	3.280833 =	3	2.48548 =	4
0.27559 =	7	2	= 5.08001	6	= 1.828804	4	= 3.657607	3	= 4.82804
0.31496 =	8	2.3622 =	6	6.56167 =	2	4.374444 =	4	3.10685 =	5
0.25433 =	9	2.7559 =	7	7	= 2.133604	5	= 4.572009	3.72822 =	6
1	= 25.4001	3	= 7.62002	8	= 2.438405	5.468056 =	5	4	= 6.43739
2	= 50.8001	3.1496 =	8	9	= 2.743205	6	= 5.486411	4.34959 =	7
3	= 76.2002	3.5433 =	9	9.84250 =	3	6.561667 =	6	4.97096 =	8
4	= 101.6002	4	= 10.16002	13.12333 =	4	7	= 6.400813	5	= 8.04674
5	= 127.0003	5	= 12.70003	16.40417 =	5	7.655278 =	7	5.59233 =	9
6	= 152.4003	6	= 15.24003	19.68500 =	6	8	= 7.315215	6	= 9.65608
7	= 177.8004	7	= 17.78004	22.96583 =	7	8.748889 =	8	7	= 11.26543
8	= 203.2004	8	= 20.32004	26.24667 =	8	9	= 8.229616	8	= 12.87478
9	= 228.6005	9	= 22.86005	29.52750 =	9	9.842500 =	9	9	= 14.48413

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10.—Lengths—Feet to Meters.

1-250. From 1 to 1.000 Units.

Feet.	Meters.	Feet.	Meters.	Feet.	Meters.	Feet.	Meters.	Feet.	Meters.
0		50	15.24003	100	30.48006	150	45.72009	200	60.96012
1	.30480	1	15.54483	1	30.78486	1	46.02489	1	61.26492
2	.60960	2	15.84963	2	31.08966	2	46.32969	2	61.56972
3	.91440	3	16.15443	3	31.39446	3	46.63449	3	61.87452
4	1.21920	4	16.45923	4	31.69926	4	46.93929	4	62.17932
5	1.52400	5	16.76403	5	32.00406	5	47.24409	5	62.48412
6	1.82880	6	17.06883	6	32.30886	6	47.54890	6	62.78893
7	2.13360	7	17.37363	7	32.61367	7	47.85370	7	63.09373
8	2.43840	8	17.67844	8	32.91847	8	48.15850	8	63.39853
9	2.74321	9	17.98324	9	33.22327	9	48.46330	9	63.70333
10	3.04801	60	18.28804	110	33.52807	160	48.76810	210	64.00813
1	3.35281	1	18.59284	1	33.83287	1	49.07290	1	64.31293
2	3.65761	2	18.89764	2	34.13767	2	49.37770	2	64.61773
3	3.96241	3	19.20244	3	34.44247	3	49.68250	3	64.92253
4	4.26721	4	19.50724	4	34.74727	4	49.98730	4	65.22733
5	4.57201	5	19.81204	5	35.05207	5	50.29210	5	65.53213
6	4.87681	6	20.11684	6	35.35687	6	50.59690	6	65.83693
7	5.18161	7	20.42164	7	35.66167	7	50.90170	7	66.14173
8	5.48641	8	20.72644	8	35.96647	8	51.20650	8	66.44653
9	5.79121	9	21.03124	9	36.27127	9	51.51130	9	66.75133
20	6.09601	70	21.33604	120	36.57607	170	51.81610	220	67.05613
1	6.40081	1	21.64084	1	36.88087	1	52.12090	1	67.36093
2	6.70561	2	21.94564	2	37.18567	2	52.42570	2	67.66574
3	7.01041	3	22.25044	3	37.49047	3	52.73051	3	67.97054
4	7.31521	4	22.55525	4	37.79528	4	53.03531	4	68.27534
5	7.62002	5	22.86005	5	38.10008	5	53.34011	5	68.58014
6	7.92482	6	23.16485	6	38.40488	6	53.64491	6	68.88494
7	8.22962	7	23.46965	7	38.70968	7	53.94971	7	69.18974
8	8.53442	8	23.77445	8	39.01448	8	54.25451	8	69.49454
9	8.83922	9	24.07925	9	39.31928	9	54.55931	9	69.79934
30	9.14402	80	24.38405	130	39.62408	180	54.86411	230	70.10414
1	9.44882	1	24.68885	1	39.92888	1	55.16891	1	70.40894
2	9.75362	2	24.99365	2	40.23368	2	55.47371	2	70.71374
3	10.05842	3	25.29845	3	40.53848	3	55.77851	3	71.01854
4	10.36322	4	25.60325	4	40.84328	4	56.08331	4	71.32334
5	10.66802	5	25.90805	5	41.14808	5	56.38811	5	71.62814
6	10.97282	6	26.21285	6	41.45288	6	56.69291	6	71.93294
7	11.27762	7	26.51765	7	41.75768	7	56.99771	7	72.23774
8	11.58242	8	26.82245	8	42.06248	8	57.30251	8	72.54255
9	11.88722	9	27.12725	9	42.36728	9	57.60732	9	72.84735
40	12.19202	90	27.43205	140	42.67209	190	57.91212	240	73.15215
1	12.49682	1	27.73686	1	42.97689	1	58.21692	1	73.45695
2	12.80163	2	28.04166	2	43.28169	2	58.52172	2	73.76175
3	13.10643	3	28.34646	3	43.58649	3	58.82652	3	74.06655
4	13.41123	4	28.65126	4	43.89129	4	59.13132	4	74.37135
5	13.71603	5	28.95606	5	44.19609	5	59.43612	5	74.67615
6	14.02083	6	29.26086	6	44.50089	6	59.74092	6	74.98095
7	14.32563	7	29.56566	7	44.80569	7	60.04572	7	75.28575
8	14.63043	8	29.87046	8	45.11049	8	60.35052	8	75.59055
9	14.93523	9	30.17526	9	45.41529	9	60.65532	9	75.89535
								250	76.20015

1 inch = .02540 meter.	5 inches = .12700 meter.	9 inches = .22860 meter.
2 inches = .05080 meter.	6 inches = .15240 meter.	10 inches = .25400 meter.
3 inches = .07620 meter.	7 inches = .17780 meter.	11 inches = .27940 meter.
4 inches = .10160 meter.	8 inches = .20320 meter.	12 inches = .30480 meter.

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10.—Lengths—Feet to Meters (Continued).

250-500.

Feet.	Meters.	Feet.	Meters.	Feet.	Meters.	Feet.	Meters.	Feet.	Meters.
250	76.20015	**300**	91.44018	**350**	106.68021	**400**	121.92024	**450**	137.16027
1	76.50495	1	91.74498	1	106.98501	1	122.22504	1	137.46507
2	76.80975	2	92.04978	2	107.28981	2	122.52985	2	137.76988
3	77.11455	3	92.35458	3	107.59462	3	122.83465	3	138.07468
4	77.41935	4	92.65939	4	107.89942	4	123.13945	4	138.37948
5	77.72416	5	92.96419	5	108.20422	5	123.44425	5	138.68428
6	78.02896	6	93.26899	6	108.50902	6	123.74905	6	138.98908
7	78.33376	7	93.57379	7	108.81382	7	124.05385	7	139.29388
8	78.63856	8	93.87859	8	109.11862	8	124.35865	8	139.59868
9	78.94336	9	94.18339	9	109.42342	9	124.66345	9	139.90348
260	79.24816	**310**	94.48819	**360**	109.72822	**410**	124.96825	**460**	140.20828
1	79.55296	1	94.79299	1	110.03302	1	125.27305	1	140.51308
2	79.85776	2	95.09779	2	110.33782	2	125.57785	2	140.81788
3	80.16256	3	95.40259	3	110.64262	3	125.88265	3	141.12268
4	80.46736	4	95.70739	4	110.94742	4	126.18745	4	141.42748
5	80.77216	5	96.01219	5	111.25222	5	126.49225	5	141.73228
6	81.07696	6	96.31699	6	111.55702	6	126.79705	6	142.03708
7	81.38176	7	96.62179	7	111.86182	7	127.10185	7	142.34188
8	81.68656	8	96.92659	8	112.16662	8	127.40665	8	142.64669
9	81.99136	9	97.23139	9	112.47142	9	127.71146	9	142.95149
270	82.29616	**320**	97.53620	**370**	112.77623	**420**	128.01626	**470**	143.25629
1	82.60097	1	97.84100	1	113.08103	1	128.32106	1	143.56109
2	82.90577	2	98.14580	2	113.38583	2	128.62586	2	143.86589
3	83.21057	3	98.45060	3	113.69063	3	128.93066	3	144.17069
4	83.51537	4	98.75540	4	113.99543	4	129.23546	4	144.47549
5	83.82017	5	99.06020	5	114.30023	5	129.54026	5	144.78029
6	84.12497	6	99.36500	6	114.60503	6	129.84506	6	145.08509
7	84.42977	7	99.66980	7	114.90983	7	130.14986	7	145.38989
8	84.73457	8	99.97460	8	115.21463	8	130.45466	8	145.69469
9	85.03937	9	100.27940	9	115.51943	9	130.75946	9	145.99949
280	85.34417	**330**	100.58420	**380**	115.82423	**430**	131.06426	**480**	146.30429
1	85.64897	1	100.88900	1	116.12903	1	131.36906	1	146.60909
2	85.95377	2	101.19380	2	116.43383	2	131.67386	2	146.91389
3	86.25857	3	101.49860	3	116.73863	3	131.97866	3	147.21869
4	86.56337	4	101.80340	4	117.04343	4	132.28346	4	147.52350
5	86.86817	5	102.10820	5	117.34823	5	132.58827	5	147.82830
6	87.17297	6	102.41300	6	117.65304	6	132.89307	6	148.13310
7	87.47777	7	102.71781	7	117.95784	7	133.19787	7	148.43790
8	87.78258	8	103.02261	8	118.26264	8	133.50267	8	148.74270
9	88.08738	9	103.32741	9	118.56744	9	133.80747	9	149.04750
290	88.39218	**340**	103.63221	**390**	118.87224	**440**	134.11227	**490**	149.35230
1	88.69698	1	103.93701	1	119.17704	1	134.41707	1	149.65710
2	89.00178	2	104.24181	2	119.48184	2	134.72187	2	149.96190
3	89.30658	3	104.54661	3	119.78664	3	135.02667	3	150.26670
4	89.61138	4	104.85141	4	120.09144	4	135.33147	4	150.57150
5	89.91618	5	105.15621	5	120.39624	5	135.63627	5	150.87630
6	90.22098	6	105.46101	6	120.70104	6	135.94107	6	151.18110
7	90.52578	7	105.76581	7	121.00584	7	136.24587	7	151.48590
8	90.83058	8	106.07061	8	121.31064	8	136.55067	8	151.79070
9	91.13538	9	106.37541	9	121.61544	9	136.85547	9	152.09550
								500	152.40030

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10.—Lengths—Feet to Meters (Continued).

500-750.

Feet.	Meters.	Feet.	Meters.	Feet.	Meters.	Feet.	Meters.	Feet.	Meters.
500	152.40030	550	167.64034	600	182.88037	650	198.12040	700	213.36043
1	152.70511	1	167.94514	1	183.18517	1	198.42520	1	213.66523
2	153.00991	2	168.24994	2	183.48997	2	198.73000	2	213.97003
3	153.31471	3	168.55474	3	183.79477	3	199.03480	3	214.27483
4	153.61951	4	168.85954	4	184.09957	4	199.33960	4	214.57963
5	153.92431	5	169.16434	5	184.40437	5	199.64440	5	214.88443
6	154.22911	6	169.46914	6	184.70917	6	199.94920	6	215.18923
7	154.53391	7	169.77394	7	185.01397	7	200.25400	7	215.49403
8	154.83871	8	170.07874	8	185.31877	8	200.55880	8	215.79883
9	155.14351	9	170.38354	9	185.62357	9	200.86360	9	216.10363
510	155.44831	560	170.68834	610	185.92837	660	201.16840	710	216.40843
1	155.75311	1	170.99314	1	186.23317	1	201.47320	1	216.71323
2	156.05791	2	171.29794	2	186.53797	2	201.77800	2	217.01803
3	156.36271	3	171.60274	3	186.84277	3	202.08280	3	217.32283
4	156.66751	4	171.90754	4	187.14757	4	202.38760	4	217.62764
5	156.97231	5	172.21234	5	187.45237	5	202.69241	5	217.93244
6	157.27711	6	172.51714	6	187.75718	6	202.99721	6	218.23724
7	157.58192	7	172.82195	7	188.06198	7	203.30201	7	218.54204
8	157.88672	8	173.12675	8	188.36678	8	203.60681	8	218.84684
9	158.19152	9	173.43155	9	188.67158	9	203.91161	9	219.15164
520	158.49632	570	173.73635	620	188.97638	670	204.21641	720	219.45644
1	158.80112	1	174.04115	1	189.28118	1	204.52121	1	219.76124
2	159.10592	2	174.34595	2	189.58598	2	204.82601	2	220.06604
3	159.41072	3	174.65075	3	189.89078	3	205.13081	3	220.37084
4	159.71552	4	174.95555	4	190.19558	4	205.43561	4	220.67564
5	160.02032	5	175.26035	5	190.50038	5	205.74041	5	220.98044
6	160.32512	6	175.56515	6	190.80518	6	206.04521	6	221.28524
7	160.62992	7	175.86995	7	191.10998	7	206.35001	7	221.59004
8	160.93472	8	176.17475	8	191.41478	8	206.65481	8	221.89484
9	161.23952	9	176.47955	9	191.71958	9	206.95961	9	222.19964
530	161.54432	580	176.78435	630	192.02438	680	207.26441	730	222.50445
1	161.84912	1	177.08915	1	192.32918	1	207.56922	1	222.80925
2	162.15392	2	177.39395	2	192.63399	2	207.87402	2	223.11405
3	162.45872	3	177.69876	3	192.93879	3	208.17882	3	223.41885
4	162.76353	4	178.00356	4	193.24359	4	208.48362	4	223.72365
5	163.06833	5	178.30836	5	193.54839	5	208.78842	5	224.02845
6	163.37313	6	178.61316	6	193.85319	6	209.09322	6	224.33325
7	163.67793	7	178.91796	7	194.15799	7	209.39802	7	224.63805
8	163.98273	8	179.22276	8	194.46279	8	209.70282	8	224.94285
9	164.28753	9	179.52756	9	194.76759	9	210.00762	9	225.24765
540	164.59233	590	179.83236	640	195.07239	690	210.31242	740	225.55245
1	164.89713	1	180.13716	1	195.37719	1	210.61722	1	225.85725
2	165.20193	2	180.44196	2	195.68199	2	210.92202	2	226.16205
3	165.50673	3	180.74676	3	195.98679	3	211.22682	3	226.46685
4	165.81153	4	181.05156	4	196.29159	4	211.53162	4	226.77165
5	166.11633	5	181.35636	5	196.59639	5	211.83642	5	227.07645
6	166.42113	6	181.66116	6	196.90119	6	212.14122	6	227.38125
7	166.72593	7	181.96596	7	197.20599	7	212.44602	7	227.68606
8	167.03073	8	182.27076	8	197.51080	8	212.75083	8	227.99086
9	167.33553	9	182.57557	9	197.81560	9	213.05563	9	228.29566
								750	228.60046

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10.—Lengths—Feet to Meters (Concluded).

750-1000.

Feet.	Meters.	Feet.	Meters.	Feet.	Meters.	Feet.	Meters.	Feet.	Meters.
750	228.60046	800	243.84049	850	259.08052	900	274.32055	950	289.56058
1	228.90526	1	244.14529	1	259.38532	1	274.62535	1	289.86538
2	229.21006	2	244.45009	2	259.69012	2	274.93015	2	290.17018
3	229.51486	3	244.75489	3	259.99492	3	275.23495	3	290.47498
4	229.81966	4	245.05969	4	260.29972	4	275.53975	4	290.77978
5	230.12446	5	245.36449	5	260.60452	5	275.84455	5	291.08458
6	230.42926	6	245.66929	6	260.90932	6	276.14935	6	291.38938
7	230.73406	7	245.97409	7	261.21412	7	276.45415	7	291.69418
8	231.03886	8	246.27889	8	261.51892	8	276.75895	8	291.99898
9	231.34366	9	246.58369	9	261.82372	9	277.06375	9	292.30378
760	231.64846	810	246.88849	860	262.12852	910	277.36855	960	292.60859
1	231.95326	1	247.19329	1	262.43332	1	277.67336	1	292.91339
2	232.25806	2	247.49810	2	262.73813	2	277.97816	2	293.21819
3	232.56287	3	247.80290	3	263.04293	3	278.28296	3	293.52299
4	232.86767	4	248.10770	4	263.34773	4	278.58776	4	293.82779
5	233.17247	5	248.41250	5	263.65253	5	278.89256	5	294.13259
6	233.47727	6	248.71730	6	263.95733	6	279.19736	6	294.43739
7	233.78207	7	249.02210	7	264.26213	7	279.50216	7	294.74219
8	234.08687	8	249.32690	8	264.56693	8	279.80696	8	295.04699
9	234.39167	9	249.63170	9	264.87173	9	280.11176	9	295.35179
770	234.69647	820	249.93650	870	265.17653	920	280.41656	970	295.65659
1	235.00127	1	250.24130	1	265.48133	1	280.72136	1	295.96139
2	235.30607	2	250.54610	2	265.78613	2	281.02616	2	296.26619
3	235.61087	3	250.85090	3	266.09093	3	281.33096	3	296.57099
4	235.91567	4	251.15570	4	266.39573	4	281.63576	4	296.87579
5	236.22047	5	251.46050	5	266.70053	5	281.94056	5	297.18059
6	236.52527	6	251.76530	6	267.00533	6	282.24536	6	297.48539
7	236.83007	7	252.07010	7	267.31013	7	282.55017	7	297.79020
8	237.13487	8	252.37490	8	267.61494	8	282.85497	8	298.09500
9	237.43967	9	252.67971	9	267.91974	9	283.15977	9	298.39980
780	237.74448	830	252.98451	880	268.22454	930	283.46457	980	298.70460
1	238.04928	1	253.28931	1	268.52934	1	283.76937	1	299.00940
2	238.35408	2	253.59411	2	268.83414	2	284.07417	2	299.31420
3	238.65888	3	253.89891	3	269.13894	3	284.37897	3	299.61900
4	238.96368	4	254.20371	4	269.44374	4	284.68377	4	299.92380
5	239.26848	5	254.50851	5	269.74854	5	284.98857	5	300.22860
6	239.57328	6	254.81331	6	270.05334	6	285.29337	6	300.53340
7	239.87808	7	255.11811	7	270.35814	7	285.59817	7	300.83820
8	240.18288	8	225.42291	8	270.66294	8	285.90297	8	301.14300
9	240.48768	9	255.73771	9	270.96774	9	286.20777	9	301.44780
790	240.79248	840	256.03251	890	271.27254	940	286.51257	990	301.75360
1	241.09728	1	256.33731	1	271.57734	1	286.81737	1	302.06740
2	241.40208	2	256.64211	2	271.88214	2	287.12217	2	302.36220
3	241.70688	3	256.94691	3	272.18694	3	287.42697	3	302.66701
4	242.01168	4	257.25171	4	272.49174	4	287.73178	4	302.97181
5	242.31648	5	257.55652	5	272.79655	5	288.03658	5	303.27661
6	242.62129	6	257.86132	6	273.10135	6	288.34138	6	303.58141
7	242.92609	7	258.16612	7	273.40615	7	288.64618	7	303.88621
8	243.23089	8	258.47092	8	273.71095	8	288.95098	8	304.19101
9	243.53569	9	258.77572	9	274.01575	9	289.25578	9	304.49581
								1000	304.80061

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11.—Lengths—Meters to Feet.

1-250. From 1 to 1,000 Units.

Meters.	Feet.	Meters.	Feet.	Meters.	Feet.	Meters.	Feet.	Meters.	Feet.
0		50	164.04167	100	328.08333	150	492.12500	200	656.16667
1	3.28083	1	167.32250	1	331.36417	1	495.40583	1	659.44750
2	6.56167	2	170.60333	2	334.64500	2	498.68667	2	662.72833
3	9.84250	3	173.88417	3	337.92583	3	501.96750	3	666.00917
4	13.12333	4	177.16500	4	341.20667	4	505.24833	4	669.29000
5	16.40417	5	180.44583	5	344.48750	5	508.52917	5	672.57083
6	19.68500	6	183.72667	6	347.76833	6	511.81000	6	675.85167
7	22.96583	7	187.00750	7	351.04917	7	515.09083	7	679.13250
8	26.24667	8	190.28833	8	354.33000	8	518.37167	8	682.41333
9	29.52750	9	193.56917	9	357.61083	9	521.65250	9	685.69417
10	32.80833	60	196.85000	110	360.89167	160	524.93333	210	688.97500
1	36.08917	1	200.13083	1	364.17250	1	528.21417	1	692.25583
2	39.37000	2	203.41167	2	367.45333	2	531.49500	2	695.53667
3	42.65083	3	206.69250	3	370.73417	3	534.77583	3	698.81750
4	45.93167	4	209.97333	4	374.01500	4	538.05667	4	702.09833
5	49.21250	5	213.25417	5	377.29583	5	541.33750	5	705.37917
6	52.49333	6	216.53500	6	380.57667	6	544.61833	6	708.66000
7	55.77417	7	219.81583	7	383.85750	7	547.89917	7	711.94083
8	59.05500	8	223.09667	8	387.13833	8	551.18000	8	715.22167
9	62.33583	9	226.37750	9	390.41917	9	554.46083	9	718.50250
20	65.61667	70	229.65833	120	393.70000	170	557.74167	220	721.78333
1	68.89750	1	232.93917	1	396.98083	1	561.02250	1	725.06417
2	72.17833	2	236.22000	2	400.26167	2	564.30333	2	728.34500
3	75.45917	3	239.50083	3	403.54250	3	567.58417	3	731.62583
4	78.74000	4	242.78167	4	406.82333	4	570.86500	4	734.90667
5	82.02083	5	246.06250	5	410.10417	5	574.14583	5	738.18750
6	85.30167	6	249.34333	6	413.38500	6	577.42667	6	741.46833
7	88.58250	7	252.62417	7	416.66583	7	580.70750	7	744.74917
8	91.86333	8	255.90500	8	419.94667	8	583.98833	8	748.03000
9	95.14417	9	259.18583	9	423.22750	9	587.26917	9	751.31083
30	98.42500	80	262.46667	130	426.50833	180	590.55000	230	754.59167
1	101.70583	1	265.74750	1	429.78917	1	593.83083	1	757.87250
2	104.98667	2	269.02833	2	433.07000	2	597.11167	2	761.15333
3	108.26750	3	272.30917	3	436.35083	3	600.39250	3	764.43417
4	111.54833	4	275.59000	4	439.63167	4	603.67333	4	767.71500
5	114.82917	5	278.87083	5	442.91250	5	606.95417	5	770.99583
6	118.11000	6	282.15167	6	446.19333	6	610.23500	6	774.27667
7	121.39083	7	285.43250	7	449.47417	7	613.51583	7	777.55750
8	124.67167	8	288.71333	8	452.75500	8	616.79667	8	780.83833
9	127.95250	9	291.99417	9	456.03583	9	620.07750	9	784.11917
40	131.23333	90	295.27500	140	459.31667	190	623.35833	240	787.40000
1	134.51417	1	298.55583	1	462.59750	1	626.63917	1	790.68083
2	137.79500	2	301.83667	2	465.87833	2	629.92000	2	793.96167
3	141.07583	3	305.11750	3	469.15917	3	633.20083	3	797.24250
4	144.35667	4	308.39833	4	472.44000	4	636.48167	4	800.52333
5	147.63750	5	311.67917	5	475.72083	5	639.76250	5	803.80417
6	150.91833	6	314.96000	6	479.00167	6	643.04333	6	807.08500
7	154.19917	7	318.24083	7	482.28250	7	646.32417	7	810.36583
8	157.48000	8	321.52167	8	485.56333	8	649.60500	8	813.64667
9	160.76083	9	324.80250	9	488.84417	9	652.88583	9	816.92750
								250	820.20833

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11.—Lengths—Meters to Feet (Continued).

250-500

Meters.	Feet.	Meters.	Feet.	Meters.	Feet.	Meters.	Feet.	Meters.	Feet.
250	820.20833	300	984.25000	350	1,148.29167	400	1,312.33333	450	1,476.37500
1	823.48917	1	987.53083	1	1,151.57250	1	1,315.61417	1	1,479.65583
2	826.77000	2	990.81167	2	1,154.85333	2	1,318.89500	2	1,482.93667
3	830.05083	3	994.09250	3	1,158.13417	3	1,322.17583	3	1,486.21750
4	833.33167	4	997.37333	4	1,161.41500	4	1,325.45667	4	1,489.49833
5	836.61250	5	1,000.65417	5	1,164.69583	5	1,328.73750	5	1,492.77917
6	839.89333	6	1,003.93500	6	1,167.97667	6	1,332.01833	6	1,496.06000
7	843.17417	7	1,007.21583	7	1,171.25750	7	1,335.29917	7	1,499.34083
8	846.45500	8	1,010.49667	8	1,174.53833	8	1,338.58000	8	1,502.62167
9	849.73583	9	1,013.77750	9	1,177.81917	9	1,341.86083	9	1,505.90250
260	853.01667	310	1,017.05833	360	1,181.10000	410	1,345.14167	460	1,509.18333
1	856.29750	1	1,020.33917	1	1,184.38083	1	1,348.42250	1	1,512.46417
2	859.57833	2	1,023.62000	2	1,187.66167	2	1,351.70333	2	1,515.74500
3	862.85917	3	1,026.90083	3	1,190.94250	3	1,354.98417	3	1,519.02583
4	866.14000	4	1,030.18167	4	1,194.22333	4	1,358.26500	4	1,522.30667
5	869.42083	5	1,033.46250	5	1,197.50417	5	1,361.54583	5	1,525.58750
6	872.70167	6	1,036.74333	6	1,200.78500	6	1,364.82667	6	1,528.86833
7	875.98250	7	1,040.02417	7	1,204.06583	7	1,368.10750	7	1,532.14917
8	879.26333	8	1,043.30500	8	1,207.34667	8	1,371.38833	8	1,535.43000
9	882.54417	9	1,046.58583	9	1,210.62750	9	1,374.66917	9	1,538.71083
270	885.82500	320	1,049.86667	370	1,213.90833	420	1,377.95000	470	1,541.99167
1	889.10583	1	1,053.14750	1	1,217.18917	1	1,381.23083	1	1,545.27250
2	892.38667	2	1,056.42833	2	1,220.47000	2	1,384.51167	2	1,548.55333
3	895.66750	3	1,059.70917	3	1,223.75083	3	1,387.79250	3	1,551.83417
4	898.94833	4	1,062.99000	4	1,227.03167	4	1,391.07333	4	1,555.11500
5	902.22917	5	1,066.27083	5	1,230.31250	5	1,394.35417	5	1,558.39583
6	905.51000	6	1,069.55167	6	1,233.59333	6	1,397.63500	6	1,561.67667
7	908.79083	7	1,072.83250	7	1,236.87417	7	1,400.91583	7	1,564.95750
8	912.07167	8	1,076.11333	8	1,240.15500	8	1,404.19667	8	1,568.23833
9	915.35250	9	1,079.39417	9	1,243.43583	9	1,407.47750	9	1,571.51917
280	918.66333	330	1,082.67500	380	1,246.71667	430	1,410.75833	480	1,574.80000
1	921.91417	1	1,085.95583	1	1,249.99750	1	1,414.03917	1	1,578.08083
2	925.19500	2	1,089.23667	2	1,253.27833	2	1,417.32000	2	1,581.36167
3	928.47583	3	1,092.51750	3	1,256.55917	3	1,420.60083	3	1,584.64250
4	931.75667	4	1,095.79833	4	1,259.84000	4	1,423.88167	4	1,587.92333
5	935.03750	5	1,099.07917	5	1,263.12083	5	1,427.16250	5	1,591.20417
6	938.31833	6	1,102.36000	6	1,266.40167	6	1,430.44333	6	1,594.48500
7	941.59917	7	1,105.64083	7	1,269.68250	7	1,433.72417	7	1,597.76583
8	944.88000	8	1,108.92167	8	1,272.96333	8	1,437.00500	8	1,601.04667
9	948.16083	9	1,112.20250	9	1,276.24417	9	1,440.28583	9	1,604.32750
290	951.44167	340	1,115.48333	390	1,279.52500	440	1,443.56667	490	1,607.60833
1	954.72250	1	1,118.76417	1	1,282.80583	1	1,446.84750	1	1,610.88917
2	958.00333	2	1,122.04500	2	1,286.08667	2	1,450.12833	2	1,614.17000
3	961.28417	3	1,125.32583	3	1,289.36750	3	1,453.40917	3	1,617.45083
4	964.56500	4	1,128.60667	4	1,292.64833	4	1,456.69000	4	1,620.73167
5	967.84583	5	1,131.88750	5	1,295.92917	5	1,459.97083	5	1,624.01250
6	971.12667	6	1,135.16833	6	1,299.21000	6	1,463.25167	6	1,627.29333
7	974.40750	7	1,138.44917	7	1,302.49083	7	1,466.53250	7	1,630.57417
8	977.68833	8	1,141.73000	8	1,305.77167	8	1,469.81333	8	1,633.85500
9	980.96917	9	1,145.01083	9	1,309.05250	9	1,473.09417	9	1,637.13583
								500	1,640.41667

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11.—Lengths—Meters to Feet (Continued).

500–750.

Meters.	Feet.	Meters.	Feet.	Meters.	Feet.	Meters.	Feet.	Meters.	Feet.
500	1,640.41667	**550**	1,804.45833	**600**	1,968.50000	**650**	2,132.54167	**700**	2,296.58333
1	1,643.69750	1	1,807.73917	1	1,971.78083	1	2,135.82250	1	2,299.86417
2	1,646.97833	2	1,811.02000	2	1,975.06167	2	2,139.10333	2	2,303.14500
3	1,650.25917	3	1,814.30083	3	1,978.34250	3	2,142.38417	3	2,306.42583
4	1,653.54000	4	1,817.58167	4	1,981.62333	4	2,145.66500	4	2,309.70667
5	1,656.82083	5	1,820.86250	5	1,984.90417	5	2,148.94583	5	2,312.98750
6	1,660.10167	6	1,824.14333	6	1,988.18500	6	2,152.22667	6	2,316.26833
7	1,663.38250	7	1,827.42417	7	1,991.46583	7	2,155.50750	7	2,319.54917
8	1,666.66333	8	1,830.70500	8	1,994.74667	8	2,158.78833	8	2,322.83000
9	1,669.94417	9	1,833.98583	9	1,998.02750	9	2,162.06917	9	2,326.11083
510	1,673.22500	**560**	1,837.26667	**610**	2,001.30833	**660**	2,165.35100	**710**	2,329.39167
1	1,676.50583	1	1,840.54750	1	2,004.58917	1	2,168.63083	1	2,332.67250
2	1,679.78667	2	1,843.82833	2	2,007.87000	2	2,171.91167	2	2,335.95333
3	1,683.06750	3	1,847.10917	3	2,011.15083	3	2,175.19250	3	2,339.23417
4	1,686.34833	4	1,850.39000	4	2,014.43167	4	2,178.47333	4	2,342.51500
5	1,689.62917	5	1,853.67083	5	2,017.71250	5	2,181.75417	5	2,345.79583
6	1,692.91000	6	1,856.95167	6	2,020.99333	6	2,185.03500	6	2,349.07667
7	1,696.19083	7	1,860.23250	7	2,024.27417	7	2,188.31583	7	2,352.35750
8	1,699.47167	8	1,863.51333	8	2,027.55500	8	2,191.59667	8	2,355.63833
9	1,702.75250	9	1,866.79417	9	2,030.83583	9	2,194.87750	9	2,358.91917
520	1,706.03333	**570**	1,870.07500	**620**	2,034.11667	**670**	2,198.15833	**720**	2,362.20000
1	1,709.31417	1	1,873.35583	1	2,037.39750	1	2,201.43917	1	2,365.48093
2	1,712.59500	2	1,876.63667	2	2,040.67833	2	2,204.72000	2	2,368.76167
3	1,715.87583	3	1,879.91750	3	2,043.95917	3	2,208.00083	3	2,372.04250
4	1,719.15667	4	1,883.19833	4	2,047.24000	4	2,211.28167	4	2,375.32333
5	1,722.43750	5	1,886.47917	5	2,050.52083	5	2,214.56250	5	2,378.60417
6	1,725.71833	6	1,889.76000	6	2,053.80167	6	2,217.84333	6	2,381.88500
7	1,728.99917	7	1,893.04083	7	2,057.08250	7	2,221.12417	7	2,385.16583
8	1,732.28000	8	1,896.32167	8	2,060.36333	8	2,224.40500	8	2,388.44667
9	1,735.56183	9	1,899.60250	9	2,063.64417	9	2,227.68583	9	2,391.72750
530	1,738.84167	**580**	1,902.88333	**630**	2,066.92500	**680**	2,230.96667	**730**	2,395.00833
1	1,742.12250	1	1,906.16417	1	2,070.20583	1	2,234.24750	1	2,398.28917
2	1,745.40333	2	1,909.44500	2	2,073.48667	2	2,237.52833	2	2,401.57000
3	1,748.68417	3	1,912.72583	3	2,076.76750	3	2,240.80917	3	2,404.85083
4	1,751.96500	4	1,916.00667	4	2,080.04833	4	2,244.09000	4	2,408.13167
5	1,755.24583	5	1,919.28750	5	2,083.32917	5	2,247.37083	5	2,411.41250
6	1,758.52667	6	1,922.56833	6	2,086.61000	6	2,250.65167	6	2,414.69333
7	1,761.80750	7	1,925.84917	7	2,089.89083	7	2,253.93250	7	2,417.97417
8	1,765.08833	8	1,929.13000	8	2,093.17167	8	2,257.21333	8	2,421.25500
9	1,768.36917	9	1,932.41083	9	2,096.45250	9	2,260.49417	9	2,424.53583
540	1,771.65000	**590**	1,935.69167	**640**	2,099.73333	**690**	2,263.77500	**740**	2,427.81667
1	1,774.93083	1	1,938.97250	1	2,103.01417	1	2,267.05583	1	2,431.09750
2	1,778.21167	2	1,942.25333	2	2,016.29500	2	2,270.33667	2	2,434.37833
3	1,781.49250	3	1,945.53417	3	2,109.57583	3	2,273.61750	3	2,437.65917
4	1,784.77333	4	1,948.81500	4	2,112.85667	4	2,276.89833	4	2,440.94000
5	1,788.05417	5	1,952.09583	5	2,116.13750	5	2,280.17917	5	2,444.22083
6	1,791.33500	6	1,955.37667	6	2,119.41833	6	2,283.46000	6	2,447.50167
7	1,794.61583	7	1,958.65750	7	2,122.69917	7	2,286.74083	7	2,450.78250
8	1,797.89667	8	1,961.93833	8	2,125.98000	8	2,290.02167	8	2,454.06333
9	1,801.17750	9	1,965.21917	9	2,129.26083	9	2,293.30250	9	2,457.34417
								750	2,460.62500

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11.—Lengths—Meters to Feet (Concluded).

750-1000.

Meters.	Feet.	Meters.	Feet.	Meters.	Feet.	Meters.	Feet.	Meters.	Feet.
750	2,460.62500	800	2,624.66667	850	2,788.70833	900	2,952.75000	950	3,116.79167
1	2,463.90583	1	2,627.94750	1	2,791.98917	1	2,956.03083	1	3,120.07250
2	2,467.18667	2	2,631.22833	2	2,795.27000	2	2,959.31167	2	3,123.35333
3	2,470.46750	3	2,634.50917	3	2,798.55083	3	2,962.59250	3	3,126.63417
4	2,473.74833	4	2,637.79000	4	2,801.83167	4	2,965.87333	4	3,129.91500
5	2,477.02917	5	2,641.07083	5	2,805.11250	5	2,969.15417	5	3,133.19583
6	2,480.31000	6	2,644.35167	6	2,808.39333	6	2,972.43500	6	3,136.47667
7	2,483.59083	7	2,647.63250	7	2,811.67417	7	2,975.71583	7	3,139.75750
8	2,486.87167	8	2,650.91333	8	2,814.95500	8	2,978.99667	8	3,143.03833
9	2,490.15250	9	2,654.19417	9	2,818.23583	9	2,982.27750	9	3,146.31917
760	2,493.43333	810	2,657.47500	860	2,821.51667	910	2,985.55833	960	3,149.60000
1	2,496.71417	1	2,660.75583	1	2,824.79750	1	2,988.83917	1	3,152.88083
2	2,499.99500	2	2,664.03667	2	2,828.07833	2	2,992.12000	2	3,156.16167
3	2,503.27583	3	2,667.31750	3	2,831.35917	3	2,995.40083	3	3,159.44250
4	2,506.55667	4	2,670.59833	4	2,834.64000	4	2,998.68167	4	3,162.72333
5	2,509.83750	5	2,673.87917	5	2,837.92083	5	3,001.96250	5	3,166.00417
6	2,513.11833	6	2,677.16000	6	2,841.20167	6	3,005.24333	6	3,169.28500
7	2,516.39917	7	2,680.44083	7	2,844.48250	7	3,008.52417	7	3,172.56583
8	2,519.68000	8	2,683.72167	8	2,847.76333	8	3,011.80500	8	3,175.84667
9	2,522.96083	9	2,687.00250	9	2,851.04417	9	3,015.08583	9	3,179.12750
770	2,526.24167	820	2,690.28333	870	2,854.32500	920	3,018.36667	970	3,182.40833
1	2,529.52250	1	2,693.56417	1	2,857.60583	1	3,021.64750	1	3,185.68917
2	2,532.80333	2	2,696.84500	2	2,860.88667	2	3,024.92833	2	3,188.97000
3	2,536.08417	3	2,700.12583	3	2,864.16750	3	3,028.20917	3	3,192.25083
4	2,539.36500	4	2,703.40667	4	2,867.44833	4	3,031.49000	4	3,195.53167
5	2,542.64583	5	2,706.68750	5	2,870.72917	5	3,034.77083	5	3,198.81250
6	2,545.92667	6	2,709.96833	6	2,874.01000	6	3,038.05167	6	3,202.09333
7	2,549.20750	7	2,713.24917	7	2,877.29083	7	3,041.33250	7	3,205.37417
8	2,552.48833	8	2,716.53000	8	2,880.57167	8	3,044.61333	8	3,208.65500
9	2,555.76917	9	2,719.81083	9	2,883.85250	9	3,047.89417	9	3,211.93583
780	2,559.05000	830	2,723.09167	880	2,887.13333	930	3,051.17500	980	3,215.21667
1	2,562.33083	1	2,726.37250	1	2,890.41417	1	3,054.45583	1	3,218.49750
2	2,565.61167	2	2,729.65333	2	2,893.69500	2	3,057.73667	2	3,221.77833
3	2,568.89250	3	2,732.93417	3	2,896.97583	3	3,061.01750	3	3,225.05917
4	2,572.17333	4	2,736.21500	4	2,900.25667	4	3,064.29833	4	3,228.34000
5	2,575.45417	5	2,739.49583	5	2,903.53750	5	3,067.57917	5	3,231.62083
6	2,578.73500	6	2,742.77667	6	2,906.81833	6	3,070.86000	6	3,234.90167
7	2,582.01583	7	2,746.05750	7	2,910.09917	7	3,074.14083	7	3,238.18250
8	2,585.29667	8	2,749.33833	8	2,913.38000	8	3,077.42167	8	3,241.46333
9	2,588.57750	9	2,752.61917	9	2,916.66083	9	3,080.70250	9	3,244.74417
790	2,591.85833	840	2,755.90000	890	2,919.94167	940	3,083.98333	990	3,248.03500
1	2,595.13917	1	2,759.18083	1	2,923.22250	1	3,087.26417	1	3,251.30583
2	2,598.42000	2	2,762.46167	2	2,926.50333	2	3,090.54500	2	3,254.58667
3	2,601.70083	3	2,765.74250	3	2,929.78417	3	3,093.82583	3	3,257.86750
4	2,604.98167	4	2,769.02333	4	2,933.06500	4	3,097.10667	4	3,261.14833
5	2,608.26250	5	2,772.30417	5	2,936.34583	5	3,100.38750	5	3,264.42917
6	2,611.54333	6	2,775.58500	6	2,939.62667	6	3,103.66833	6	3,267.71000
7	2,614.82417	7	2,778.86583	7	2,942.90750	7	3,106.94917	7	3,270.99083
8	2,618.10500	8	2,782.14667	8	2,946.18833	8	3,110.23000	8	3,274.27167
9	2,621.38583	9	2,785.42750	9	2,949.46917	9	3,113.51083	9	3,277.55250
								1000	3,280.83333

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12.—AREAS.

	Sq. Ins.	Sq. Feet.	Sq. Yds.	Acres.
1 *sq. millimeter* (sq. m m) =	.00155000	.00001076	.....	
100 sq. millimeters =				
1 *sq. centimeter* (sq. c m) =	.15499969	.00107639	.0001196	
100 sq. centimeters =				
1 *sq. decimeter* (sq. d m) =	15.499969	.10763867	.0119599	
100 sq. decimeters (= 1 centare) =				
1 *sq. meter*. =	1549.9969	10.763867	1.1959853	.0002471
100 sq. meters (= 1 are) =				
1 *sq. dekameter* (sq. D m) =	154999.69	1076.3867	119.59853	.0247104
100 sq. dekameters (= 1 hectare)				
= 1 *sq. hectometer*. =	Sq. Miles	107638.67	11959.853	2.4710439
100 sq. hectometers =				
1 *sq. kilometer* (sq. K m) =	.3861	10763867	1195985.3	247.10439
100 sq. kilometers =				
1 *sq. myriameter* (sq. M m) =	38.61			24710.439

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13.—Areas, Equivalents. 1-10.

Square Inches.		Square Millimeters.	Square Inches.		Square Centimeters.	Square Feet.		Square Meters.
0.00155	=	1	0.1550	=	1	1	=	0.09290
0.00310	=	2	0.3100	=	2	2	=	0.18581
0.00465	=	3	0.4650	=	3	3	=	0.27871
0.00620	=	4	0.6200	=	4	4	=	0.37161
0.00775	=	5	0.7750	=	5	5	=	0.46452
0.00930	=	6	0.9300	=	6	6	=	0.55742
0.01085	=	7	1	=	6.452	7	=	0.65032
0.01240	=	8	1.0850	=	7	8	=	0.74323
0.01395	=	9	1.2400	=	8	9	=	0.83613
1	=	645.16	1.3950	=	9	10.764	=	1
2	=	1,290.33	2	=	12.903	21.528	=	2
3	=	1,935.49	3	=	19.355	32.292	=	3
4	=	2,580.65	4	=	25.807	43.055	=	4
5	=	3,225.81	5	=	32.258	53.819	=	5
6	=	3,870.98	6	=	38.710	64.583	=	6
7	=	4,516.14	7	=	45.161	75.347	=	7
8	=	5,161.30	8	=	51.613	86.111	=	8
9	=	5,806.46	9	=	58.065	96.875	=	9

Square Yards.		Square Meters.	Square Miles.		Square Kilometers.	Acres.		Hectars.
1	=	0.8361	0.3861	=	1	1	=	0.4047
1.1960	=	1	0.7722	=	2	2	=	0.8094
2	=	0.6723	1	=	2.5900	2.471	=	1
2.3920	=	2	1.1583	=	3	3	=	1.2141
3	=	2.5084	1.5444	=	4	4	=	1.6187
3.5880	=	3	1.9305	=	5	4.942	=	2
4	=	3.3445	2	=	5.1800	5	=	2.0234
4.7839	=	4	2.3166	=	6	6	=	2.4281
5	=	4.1807	2.7027	=	7	7	=	2.8328
5.9799	=	5	3	=	7.7700	7.413	=	3
6	=	5.0168	3.0888	=	8	8	=	3.2375
7	=	5.8529	3.4749	=	9	9	=	3.6422
7.1759	=	6	4	=	10.3600	9.884	=	4
8	=	6.6890	5	=	12.9500	12.355	=	5
8.3719	=	7	6	=	15.5400	14.826	=	6
9	=	7.5252	7	=	18.1300	17.297	=	7
9.5679	=	8	8	=	20.7200	19.768	=	8
10.7639	=	9	9	=	23.3100	22.239	=	9

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14.—Land Measure (Square).

	1 *square inch*	= 6.4516 sq. centimeters
144 square inches	= 1 *square foot*	= 0.09290341 sq. meter.
9 square feet	= 1 *square yard*	= 0.836131 sq. meter.
30¼ square yards (= 272¼ sq. ft.)	= 1 *square rod*	= 25.292954 sq. meters.
40 square rods (= 1210 sq. yds. = 10890 sq. ft.)	= 1 *rood*	= 0.1011718 hectars.
4 roods (= 160 sq. rods = 4840 sq. yds. = 43560 sq. ft.)	= 1 *acre*	= 0.4046873 hectars.
640 acres (= 3,097,600 sq. yds. = 27,878,400 sq. ft.)	= 1 *square mile*	= 258.99985 hectars.
36 square miles (= 23040 acres = 1,003,622,400 sq. ft.)	= 1 *township*	= 9323.9945 hectars.

15.—Texas Land Measure.

(Also used in Mexico, New Mexico, Arizona and California.)

26,000,000	sq. varas (sq. of 5,099	varas) = 1 league and 1 labor	=	4,605.5	acres.
1,000,000	sq. varas (sq. of 1,000	varas) = 1 labor	=	177.136	acres.
25,000,000	sq. varas (sq. of 5,000	varas) = 1 league	=	4,428.4	acres.
12,500,000	sq. varas (sq. of 3,535.5	varas) = ½ league	=	2,214.2	acres.
8,333,333	sq. varas (sq. of 2,886.7	varas) = ⅓ league	=	1,476.13	acres.
6,250,000	sq. varas (sq. of 2,500	varas) = ¼ league	=	1,107.1	acres.
7,225,600	sq. varas (sq. of 2,688	varas)	=	1,280	acres.
3,612,800	sq. varas (sq. of 1,900.8	varas) = 1 section	=	640	acres.
1,806,400	sq. varas (sq. of 1,344	varas) = ½ section	=	320	acres.
902,200	sq. varas (sq. of 950.44	varas) = ¼ section	=	160	acres.
451,600	sq. varas (sq. of 672	varas) = ⅛ section	=	80	acres.
225,800	sq. varas (sq. of 475	varas) = 1.16 section	=	40	acres.
5,645.376	sq. varas (sq. of 75.137	varas) = 4,840 sq. yd.	=	1	acre.

To find the number of acres in any number of square varas, multiply the latter by 177 (or to be more exact, by 177½).

1 vara = 33⅓ inches 1,900.8 varas = 1 mile

16.—Weights and Measures of the Philippines.

1 pulgada (12 lineas)	= .927 inches.	1 libra (16 onso)	=	1.0144	lb. av.
1 pie	= 11.125 inches.	1 arroba	=	25.360	lb. av.
1 vara	= 33.375 inches.	1 catty (16 tael)	=	1.394	lb. av.
1 gantah	= .8796 gallon.	1 pecul (100 catty)	=	139.482	lb. av.
1 caban	= 21.991 gallons.				

17.—Volumes.

	Cu. Ins.	Cu. Feet.	Cu. Yds.
(1 *cubic c m*) = 1 *milliliter* (m l) =	.06102338	.00003531	.....
10 milliliters (= 10 cubic c m) = 1 *centiliter* (c l) =	.61023378	.00035314	.....
10 centiliters (= $\frac{1}{10}$ cubic d m) = 1 *deciliter* d l) =	6.1023378	.00353145	.....
10 deciliters (= 1 *cubic d m*) = 1 *liter* =	61.023378	.03531445	.00130794
10 liters (= 10 cubic d m) 1 *dekaliter* (D l) =	610.23378	.35314455	.01307943
10 dekaliters (= $\frac{1}{10}$ cubic meter) = 1 *hectoliter* (H l) =	6102.3378	3.5314455	.13079428
10 hectoliters (= 1 *cubic meter*) = 1 *kiloliter* (K l) =	61023.378	35.314455	1.3079428
10 kiloliters = 1 *myrialiter* (M l) =	610233.78	353.14455	13.079428

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18.—Volumes. Equivalents. 1-10.

Cubic Inches.	Cubic Millimeters.	Cubic Inches.	Cubic Centimet's	Cubic Feet.	Cubic Meters.	Cubic Yards.	Cubic Meters.
0.000061 =	1	0.0610 =	1	1	= 0.02832	1	= 0.7645
0.000122 =	2	0.1220 =	2	2	= 0.05663	1.3079	= 1
0.000183 =	3	0.1831 =	3	3	= 0.08495	2	= 1.5291
0.000244 =	4	0.2441 =	4	4	= 0.11327	2.6159	= 2
0.000305 =	5	0.3051 =	5	5	= 0.14159	3	= 2.2937
0.000366 =	6	0.3661 =	6	6	= 0.16990	3.9238	= 3
0.000427 =	7	0.4272 =	7	7	= 0.19822	4	= 3.0582
0.000488 =	8	0.4882 =	8	8	= 0.22654	5	= 3.8228
0.000549 =	9	0.5492 =	9	9	= 0.25485	5.2318	= 4
1	= 16,387.2	1	= 16.3872	35.314	= 1	6	= 4.5874
2	= 32,774.3	2	= 32.7743	70.629	= 2	6.5397	= 5
3	= 49,161.5	3	= 49.1615	105.943	= 3	7	= 5.3519
4	= 65,548.6	4	= 65.5486	141.258	= 4	7.8477	= 6
5	= 81,935.8	5	= 81.9358	176.572	= 5	8	= 6.1165
6	= 98,323.0	6	= 98.3230	211.887	= 6	9	= 6.8810
7	= 114,710.1	7	= 114.7101	247.201	= 7	9.1556	= 7
8	= 131,097.3	8	= 131.0973	282.516	= 8	10.4635	= 8
9	= 147,484.5	9	= 147.4845	317.830	= 9	11.7715	= 9

19.—Cubic Measure.

1 *cubic inch* = 16.3872 cubic centimeters.
1728 cubic inches . . . . . = 1 *cubic foot* = .02832 cubic meter.
27 cubic feet (= 46656 cu. ins.) = 1 *cubic yard* = 0.7646 cubic meter.
16 cubic feet (= 27648 cu. ins.) = 1 *cord foot* = 0.45307 cubic meter.
8 cord feet (= $4\frac{2}{27}$ cu. yds. =
128 cu. ft. = 221184 cu. ins.) = 1 *cord* (*wood*) = 3.6246 cubic meter.

20.—Capacities (Liquid).

	U. S. Apoth. Scruples	U. S. Apoth. Drams.	U. S. Liquid Ounces.	U. S. Liquid Quarts.
(1 cubic c m) = 1 *milliliter* (m l)... =	.81153168	.27051056	.03381382	.00105666
10 milliliters (= 10 cubic c m) = 1 *centiliter* (c l).... =	8.1153168	2.7051056	.33813820	.01056682
10 centiliters (= $\frac{1}{10}$ cubic d m) = 1 *deciliter* (d l).... =	U. S. Liquid	27.051056	3.3813820	.10566819
10 deciliters (= 1 cubic d m) = 1 *liter*..... =	Gallons. .26417047	270.51056	33.813820	1.0566819
10 liters (= 10 cubic d m) = 1 *dekaliter* (D l)... =	2.6417047	2705.1056	338.13820	10.566819
10 dekaliters ($\frac{1}{10}$ cubic meter) = 1 *hectoliter* (H l)... =	26.417047	27051.056	3381.3820	105.66819
10 hectoliters (= 1 *cubic meter*) = 1 *kiloliter* (K l).... =	264.17047	.....	33813.820	1056.6819
10 kiloliters..... = 1 *myrialiters* (M l). =	2641.7047	.....	338138.20	10566.819

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21.—Capacities, Equivalents. 1-10.

Millilitters. (cc.)	U. S. Liquid Oz.	Millilitters. (cc.)	U. S. Apothecaries' Drams.	U. S. Apothecaries' Scruples.	Millilitters. (cc.)	U. S. Liquid Quarts.	Liters.	U. S. Liquid Gallons.	Liters.
1	= 0.03381	1	= 0.2705	0.8115 =	1	1	= 0.94636	0.26417 =	1
2	= 0.06763	2	= 0.5410	1 =	1.2322	1.05668 =	1	0.52834 =	2
3	= 0.10144	3	= 0.8115	1.6231 =	2	2	= 1.89272	0.79251 =	3
4	= 0.13526	3.6967 =	1	2 =	2.4645	2.11336 =	2	1	= 3.78543
5	= 0.16907	4	= 1.0820	2.4346 =	3	3	= 2.83908	1.05668 =	4
6	= 0.20288	5	= 1.3525	3 =	3.6967	3.17005 =	3	1.32085 =	5
7	= 0.23670	6	= 1.6231	3.2461 =	4	4	= 3.78543	1.58502 =	6
8	= 0.27051	7	= 1.8936	4 =	4.9290	4.22673 =	4	1.84919 =	7
9	= 0.30432	7.3934 =	2	4.0577 =	5	5	= 4.73179	2	= 7.57087
29.574 =	1	8	= 2.1641	4.8692 =	6	5.28341 =	5	2.11336 =	8
59.147 =	2	9	= 2.4346	5 =	6.1612	6	= 5.67815	2.37753 =	9
88.721 =	3	11.0901 =	3	5.6807 =	7	6.34009 =	6	3	= 11.35630
118.295 =	4	14.7869 =	4	6 =	7.3934	7	= 6.62451	4	= 15.14174
147.869 =	5	18.4836 =	5	6.4923 =	8	7.39677 =	7	5	= 18.92717
177.442 =	6	22.1803 =	6	7 =	8.6257	8	= 7.57088	6	= 22.71261
207.016 =	7	25.8770 =	7	7.3038 =	9	8.45345 =	8	7	= 26.49804
236.590 =	8	29.5737 =	8	8 =	9.8579	9	= 8.51723	8	= 30.28348
266.163 =	9	33.2704 =	9	9	= 11.0901	9.51014 =	9	9	= 34.06891

22.—Liquid Measure.

		1 *gill*	= 0.1183	liters
4 gills	=	1 *pint*	= 0.47318	"
2 pints (= 8 gills)	=	1 *quart*	= 0.94636	"
4 quarts (= 8 pints = 32 gills)	=	1 *gallon*	= 3.78543	"
31½ gallons (= 126 quarts = 252 pints)	=	1 *barrel*	= 119.2412	"
2 barrels (= 63 gals. = 252 qts. = 504 pts.)	=	1 *hogshead*	= 238.4824	"
2 hogsheads (= 4 bbls. = 126 gals = 504 qts.)	=	1 *pipe, or butt*	= 476.9647	"
2 pipes (= 8 bbls. = 252 gals. = 1008 qts.)	=	1 *tun*	= 953.9295	"

1 tiera = 42 gals. 1 puncheon = 84 gals.

23.—Apothecaries' Measure (Fluid).

		1 *minim (drop)*	= 0.00006161	liters
60 minims	=	1 *fluid drachm*	= 0.0036967	"
8 fluid drachms (= 480 minims)	=	1 *fluid ounce*	= 0.029574	"
16 fluid ounces (= 128 drachms = 7680 minims)	=	1 *pint*	= 0.473179	"
8 pints (= 128 fluid ounces = 1024 drachms = 61440 minims)	=	1 *gallon*	= 3.785434	"

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24.—Capacities (Dry).

	U. S. Dry Pints.	U. S. Dry Quarts.	U. S. Pecks.	U. S. Bushels.
(1 *cubic c m*) = 1 *milliliter* (m l)... =	.00181616	.00090808	.....	.....
10 milliliters (= 10 cubic c m) = 1 *centiliter* (c l).... =	.01816155	.00908078	.00113510	.....
10 centiliters (= $\frac{1}{10}$ cubic d m) = 1 deciliter (d l).... =	.18161551	.09080775	.01135097	.00283774
10 deciliters (= 1 *cubic d m*) = 1 *liter*..... =	1.8161551	.90807754	.11350969	.02837742
10 liters (= 10 cubic d m) = 1 *dekaliter* (D l)... =	18.161551	9.0807754	1.1350969	.28377423
10 dekaliters (= $\frac{1}{10}$ cubic meter) = 1 *hectoliter* H l).... =	181.61551	90.807754	11.350969	2.8377423
10 hectoliters (= 1 *cubic meter*) = 1 *kilolter* (K l).... =	1816.1551	908.07754	113.50969	28.377423
10 kiloliters..... = 1 *myrialiter* (M l).. =	18161.551	9080.7754	1135.0969	283.77423

25.—Capacities, Equivalents. 1-10.

U. S. Dry Quarts.	Liters.	U. S. Pecks.	Liters.	Deka-liters.	U. S. Pecks.	U. S. Bushels.	Hecto-liters.	U. S. Bushels per Acre.	Hecto-liters per Hectar.
0.9081 =	1	0.11351 =	1	0.8810 =	1	1 =	0.35239	1 =	0.87078
1 =	1.1012	0.22702 =	2	1 =	1.1351	2 =	0.70479	1.14840 =	1
1.8162 =	2	0.34053 =	3	1.7620 =	2	2.83774 =	1	2 =	1.74156
2 =	2.2025	0.45404 =	4	2 =	2.2702	3 =	1.05718	2.29680 =	2
2.7242 =	3	0.56755 =	5	2.6429 =	3	4 =	1.40957	3 =	2.61233
3 =	3.3037	0.68106 =	6	3 =	3.4053	5 =	1.76196	3.44519 =	3
3.6323 =	4	0.79457 =	7	3.5239 =	4	5.67548 =	2	4 =	3.48311
4 =	4.4049	0.90808 =	8	4 =	4.5404	6 =	2.11436	4.59359 =	4
4.5404 =	5	1 =	8.80982	4.4049 =	5	7 =	2.46675	5 =	4.35389
5 =	5.5061	1.02157 =	9	5 =	5.6755	8 =	2.61914	5.74199 =	5
5.4485 =	6	2 =	17.61964	5.2859 =	6	8.51323 =	3	6 =	5.22467
6 =	6.6074	3 =	26.42946	6 =	6.8106	9 =	3.17154	6.89039 =	6
6.3565 =	7	4 =	35.23928	6.1669 =	7	11.35097 =	4	7 =	6.09545
7 =	7.7086	5 =	44.04910	7 =	7.9457	14.18871 =	5	8 =	6.96622
7.2646 =	8	6 =	52.85892	7.0479 =	8	17.02645 =	6	8.03879 =	7
8 =	8.8098	7 =	61.66874	7.9288 =	9	19.86420 =	7	9 =	7.83700
8.1727 =	9	8 =	70.47856	8 =	9.0808	22.70194 =	8	9.18719 =	8
9 =	9.9110	9 =	79.28838	9 =	10.2159	25.53968 =	9	10.33558 =	9

26.—Dry Measure.

1 *pint*..... = 0.55061 liter.
2 pints..... = 1 *quart*..... = 1.10123 liters.
8 quarts (= 16 pints)..... = 1 *peck*..... = 8.80982 liters.
4 pecks (= 32 quarts = 64 pints)..... = 1 *struck bushel* = 0.35239 hectoliter.

Note.—1 heaped bushel = 1¼ struck bushels The cone of the heaped bushel must not be less than 6 inches high.

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27.—Masses (Weights).

	Grains.	Avoir. Ounces.	Troy Ounces.	Troy Pounds.
(*1 *cubic m m*) 1 *milligram* (m g) . =	.01543236	.....	.....	.....
10 milligrams (*10 cubic m m) = 1 *centigram* (c g) . . =	.15432356	.00035274	.00032151	.....
10 centigrams (*$\frac{1}{10}$ cubic c m) = 1 *decigram* (d g) . . . =	1.5432356	.00352740	.00321507	.00026792
10 decigrams (*1 *cubic c m*) = 1 *gram*. . . . . =	15.432356	.03527396	.03215074	.00267923
	Avoir. Pounds.			
10 grams (*10 cubic c m) = 1 *dekagram* (Dg) . . =	.02204622	.35273957	.32150742	.02679229
10 dekagrams (*1 deciliter) = 1 *hectogram* (H g) . =	.22046223	3.5273957	3.2150742	.26792285
10 hectograms (*1 *liter*) = 1 *kilogram* (Kilo). =	2.2046223	35.273957	32.150742	2.6792285
10 kilograms (*10 liters) = 1 *myriagram* (Mg) =	22.046223	352.73957	321.50742	26.792285
10 myriagram (*1 hectoliter) = 1 *quintal*. . . . . =	220.46223	3527.3957	3215.0742	267.92285
10 quintals (*1 *cubic meter*) = 1 *millor or tonneau* =	2204.6223	35273.957	32150.742	2679.2285

*Equivalent quantity of water at max. density.

28.—Weights, Equivalents. 1-10.

Grams.	Grams.	Avoirdupois Ounces.	Grams.	Troy Ounces.	Grams.	Avoirdupois Pounds.	Kilograms.	Troy Pounds.	Kilograms.
1	= 0.06480	0.03527	= 1	0.03215	= 1	1	= 0.45359	1	= 0.37324
2	= 0.12960	0.07055	= 2	0.06430	= 2	2	= 0.90718	2	= 0.74648
3	= 0.19440	0.10582	= 3	0.09645	= 3	2.20462	= 1	2.67923	= 1
4	= 0.25920	0.14110	= 4	0.12860	= 4	3	= 1.36078	3	= 1.11973
5	= 0.32399	0.17637	= 5	0.16075	= 5	4	= 1.81437	4	= 1.49297
6	= 0.38879	0.21164	= 6	0.19290	= 6	4.40924	= 2	5	= 1.86621
7	= 0.45359	0.24692	= 7	0.22506	= 7	5	= 2.26796	5.35846	= 2
8	= 0.51839	0.28219	= 8	0.25721	= 8	6	= 2.72155	6	= 2.23945
9	= 0.58319	0.31747	= 9	0.28936	= 9	6.61387	= 3	7	= 2.61269
15.4324	= 1	1	= 28.3495	1	= 31.10348	7	= 3.17515	8	= 2.98593
30.8647	= 2	2	= 56.6991	2	= 62.20696	8	= 3.62874	8.03769	= 3
46.2971	= 3	3	= 85.0486	3	= 93.31044	8.81849	= 4	9	= 3.35918
61.7294	= 4	4	= 113.3981	4	= 124.41392	9	= 4.08233	10.71691	= 4
77.1618	= 5	5	= 141.7476	5	= 155.51740	11.02311	= 5	13.39614	= 5
92.5941	= 6	6	= 170.0972	6	= 186.62088	13.22773	= 6	16.07537	= 6
108.0265	= 7	7	= 198.4467	7	= 217.72437	15.43236	= 7	18.75460	= 7
123.4589	= 8	8	= 226.7962	8	= 248.82785	17.63698	= 8	21.43383	= 8
138.8912	= 9	9	= 255.1457	9	= 279.93133	19.84160	= 9	24.11306	= 9

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29.—*Apothecaries Weight.

	= 1 *grain*	= 0.06480 gram.
20 grains	= 1 *scruple*	= 1.29598 grams.
3 scruples (= 60 grains)	= 1 *dram*	= 3.88794 grams.
8 drams (= 24 scruple = 480 grains)	= 1 *ounce*	= 31.10348 grams.
12 ounces (= 96 drams = 228 scruples = 5760 grains)	= 1 *pound*	= 0.37324 kilogram.

30.—*Troy Weight.

	1 *grain*	= 0.06480 gram.
24 grains	= 1 *pennyweight*	= 1.55517 grams.
20 pennyweights (= 480 grains)	= 1 *ounce*	= 31.10348 grams.
12 ounces (= 240 pennyweights = 5760 grains)	= 1 *pound*	= 0.37324 kilogram.

31.—Avoirdupois Weight (Short Tons).

	1 *grain*	= 0.06480	gram.
27 11/32 grains (= 27.34375 grains)	= 1 *dram*	= 1.771845	grams.
16 drams (= 437½ grains)	= 1 *ounce*	= 28.3495	grams.
16 ounces (= 256 drams = 7000 grains)	= 1 *pound*	= 0.4535924	kilogram.
25 pounds (= 400 ounces)	= 1 *quarter*	= 11.339811	kilograms.
4 quarters	= 1 *hundred weight*	= 45.35924	kilograms.
20 hundred weight (2000 lbs.)	= 1 *ton*	= 907.18486	kilograms.

32.—Avoirdupois Weight (Long Ton).

	1 *grain*	= 0.06480	gram.
27 11/32 grains (= 27.34375 grains)	= 1 *dram*	= 1.771845	grams.
16 drams	= 1 *ounce*	= 28.3495	grams.
16 ounces	= 1 *pound*	= 0.4535924	kilogram.
112 pounds	= 1 *hundred weight*	= 50.80235	kilograms.
20 hundred weight (2240 lbs.)	= 1 *ton*	= 1016.047	kilograms.

* The grain, ounce and pound Apothecary and Troy weight are respectively equivalent.

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33.—Comparison of the Various Tons and Pounds in use in the United States.

From 1 to 10 Units.

Long Tons.	Short Tons.	Metric Tons.	Kilograms.	Avoirdupois Pounds.	Troy Pounds.
.00036735	.00041143	.00037324	.37324	.822857	1
.00044643	.00050000	.00045359	.45359	1	1.21528
.00073469	.00082286	.00074648	.74648	1.64571	2
.00098286	.00100000	.00090718	.90718	2	2.43056
.00098421	.00110231	.00100000	1	2.20462	2.67923
.00110204	.00123429	.00111973	1.11973	2.46857	3
.00133929	.00150000	.00136078	1.36078	3	3.64583
.00146939	.00164571	.00149297	1.49297	3.29143	4
.00178571	.00200000	.00181437	1.81437	4	4.86111
.00183673	.00205714	.00186621	1.86621	4.11429	5
.00196841	.00220462	.00200000	2	4.40924	5.35846
.00220408	.00246857	.00223945	2.23945	4.93714	6
.00223214	.00250000	.00226796	2.26796	5	6.07639
.00257143	.00288000	.00261269	2.61269	5.76000	7
.00267857	.00300000	.00272155	2.72155	6	7.29167
.00293878	.00329143	.00298593	2.98593	6.58286	8
.00295262	.00330693	.00300000	3	6.61387	8.03769
.00312500	.00350000	.00317515	3.17515	7	8.50694
.00330612	.00370286	.00335918	3.35918	7.40571	9
.00357143	.00400000	.00362874	3.62874	8	9.72222
.00393683	.00440924	.00400000	4	8.81849	10.71691
.00401786	.00450000	.00408233	4.08233	9	10.93750
.00492103	.00551156	.00500000	5	11.0231	13.39614
.00590524	.00661387	.00600000	6	13.2277	16.07537
.00688944	.00771618	.00700000	7	15.4324	18.75460
.00787365	.00881849	.00800000	8	17.6370	21.43383
.00885786	.00992080	.0090000	9	19.8416	24.11306
.99287	1	.90718	907.18	2,000.00	2,430.56
.98421	1.10231	1	1,000.00	2,204.62	2,679.23
1	1.12000	1.01605	1,016.05	2,240.00	2,722.22
1.78571	2	1.81437	1,814.37	4,000.00	4,861.11
1.96841	2.20464	2	2,000.00	4,409.24	5,358.46
2	2.24000	2.03209	2,032.09	4,480.00	5,444.44
2.67857	3	2.72155	2,721.55	6,000.00	7,291.67
2.95262	3.30693	3	3,000.00	6,613.87	8,037.69
3	3.36000	3.04814	3,048.14	6,720.00	8,166.67
3.57143	4	3.62874	3,628.74	8,000.00	9,722.22
3.93683	4.40924	4	4,000.00	8,818.49	10,716.91
4	4.48000	4.06419	4,064.19	8,960.00	10,888.89
4.46429	5	4.53592	4,535.92	10,000.00	12,152.78
4.92103	5.51156	5	5,000.00	11,023.11	13,396.14
5	5.60000	5.08024	5,080.24	11,200.00	13,611.11
5.35714	6	5.44311	5,443.11	12,000.00	14,583.33
5.90524	6.61387	6	6,000.00	13,227.73	16,075.37
6	6.72000	6.09628	6,096.28	13,440.00	16,333.33
6.25000	7	6.35029	6,350.29	14,000.00	17,013.89
6.88944	7.71618	7	7,000.00	15,432.36	18,754.60
7	7.84000	7.11232	7,112.32	15,680.00	19,055.56
7.14286	8	7.25748	7,257.48	16,000.00	19,444.44
7.87365	8.81849	8	8,000.00	17,636.98	21,433.83
8	8.96000	8.12838	8,128.38	17,920.00	21,777.78
8.03571	9	8.16466	8,164.66	18,000.00	21,875.00
8.85786	9.92080	9	9,000.00	19,841.60	24,113.06
9	10.08000	9.14442	9,144.42	20,160.00	24,500.00

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34.—Simple and Compound Units in Common Use, Equivalents.

Base: 1 Meter = 39.37 Inches, as per U. S. Law.

A	To reduce A to M.		To reduce M to A.		M
	Mult. by	Log.	Log.	Mult. by	
LENGTH.					
Mils (1000ths of an in.)	.0254	8.4048346	1.5951654	39.37	Millimeters
100ths of an inch	.254	9.4048346	0.5951654	3.937	Millimeters
64ths of an inch	.3968758	9.5986546	0.4013454	2.51968	Millimeters
Inches	2.540005	0.4048346	9.5951654	.3937	Centimeters
Feet	.3048006	9.4840158	0.5159842	3.28083^3	Meters
Yards	.9144018	9.9611371	0.0388629	1.09361^1	Meters
Rods	5.029210	0.7014998	9.2985002	.1988384	Meters
Chains (66 ft.)	20.11684	1.3035597	8.6964403	.0497096	Meters
Stations (100 ft.)	30.48006	1.4840158	8.5159842	.0328083	Meters
Miles	1.609347	0.2066497	9.7933503	.62137	Kilometers
AREA (Square or circular).					
Square mils	.0006452	6.8096692	3.1903308	1549.997	Sq. millimeters
Square inches	6.451626	0.8096692	9.1903308	.1549997	Sq. centimeters
Square feet	.0929034	8.9680316	1.0319684	10.76387	Sq. meters
Square yards	.8361307	9.9222742	0.0777258	1.195985	Sq. meters
Square rods	25.29295	1.4029996	8.5970004	.0395367	Sq. meters
Acres	0.4046873	9.6071196	0.3928804	2.4710439	Hectars
Quarter sections (160 a.)	64.75	1.8112402	8.1887598	.015444	Hectars
Sections (sq. miles)	258.99985	2.4132995	7.5867005	.003861	Hectars
Townships (36 sec.)	9323.9945	3.9696020	6.0303980	.00010725	Hectars
VOLUME (Cubic or globular).					
Cubic inches	16.38716	1.2145038	8.7854962	.0610234	Cubic cent'mtrs
Cubic feet	.02831702	8.4520475	1.5479525	35.31445	Cubic meters
Cubic yards	.7645594	9.8834113	0.1165887	1.3079428	Cubic meters
Acre feet	43560	4.6390879	5.3609121	.00002296	Cubic feet
CAPACITY (Liquid).					
Quarts (U. S.)	.9463586	9.9760558	0.0239442	1.056682	Liters
Gallons (U. S.)	3.785434	0.5781158	9.4218842	.264170	Liters
Barrels (31½ galls.)	119.2412	2.0764264	7.9235736	.0083864	Liters
CAPACITY (Dry).					
Quarts (U. S.)	1.101228	0.0418771	9.9581229	.9080775	Liters
Bushels (U. S. struck)	35.23928	1.5470270	8.4529730	.0283774	Liters

Examples to illustrate the use of above table:

45 mils = 45 × .0254 millimeters. 16 centimeters = 16 × .3937 inches.

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M.—SIMPLE AND COMPOUND UNITS IN COMMON USE, EQUIVALENTS. (Cont'd.)

A	To reduce A to M.		To reduce M to A.		M
	Mult. by	Log.	Log.	Mult. by	
WEIGHT.					
Ounces (Avoir.)	28.3496	1.4525461	8.5474539	.035274	Grams
Pounds (Avoir.)	.453593	9.6566658	0.3433342	2.20462	Kilograms
Tons (2000 lbs.)	907.186	2.9576964	7.0423036	.00110231	Kilograms
Tons (2000 lbs.)	.907186	9.9576964	0.0423036	1.10231	Metric tons
Tons (2240 lbs.)	1016.05	3.0069141	6.9930859	.00098421	Kilograms
Tons (2240 lbs.)	1.01605	0.0069141	9.9930859	.984206	Metric tons
MOMENTS.					
Inch-Pounds	1152.13	3.0615006	6.9384994	.000868	Centimeter-grms
Foot-Pounds	.138255	9.1406818	0.8593182	7.23300	Meter-kilograms.
STRESS PER AREA.					
Pounds per sq. in	.0703067	8.8469968	1.1530032	14.2234	Kilog.p.sq.cent'r.
Pounds per sq. ft.	4.88243	0.6886361	9.3113639	.204816	Kilog. p.sq.meter
Tons (2000 lbs.) p. sq. ft.	9.76486	0.9896661	9.0103339	.102408	Met.tonsp.sq.mtr
WEIGHT PER VOLUME.					
Pounds per cu. in	27.6797	1.4421621	8.5578379	.0361275	Grms.p.cu.cent'r.
Pounds per cu. ft.	16.0184	1.2046184	8.7953816	.0624283	Kilog. p. cu.m'tr
Tons (2000 lbs.)p.cu.yd.	1.18655	0.0742851	9.9257149	.8427813	Met.tonsp.cu.mtr
VELOCITY.					
Feet per second	.304801	9.4840158	0.5159842	3.28083$^{\vee}$3	Meters p. second
Feet per second	.01136364	8.0555173	1.9444827	88.	Miles p. minute
Feet per second	.681$^{\vee}$81	9.8336686	0.1663314	1.46$^{\vee}$6	Miles per hour
Feet per minute	.01136364	8.0555173	1.9444827	88.	Miles per hour
Miles per minute	26.82245	1.4284984	8.5715016	.0372822	Meters p. second
Miles per minute	1.609347	0.2066497	9.7933503	.62137	Kilomtrs. p. min.
Miles per hour	1.609347	0.2066497	9.7933503	.62137	Kilomtrs. p. hour
ACCELERATION.					
Feet per sec. per sec. (g = 32.2—, av.)	.30480	9.4840158	0.5159842	3.28083$^{\vee}$3	Meters p.sc.p.sc. (g = 9.81—, av.)

Examples to illustrate the use of above table:

100 foot-pounds = 100 × .138255 kg.-meters. 10 miles per min. = 10 × 88 ft. per sec.

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34.—SIMPLE AND COMPOUND UNITS IN COMMON USE, EQUIVALENTS. (Concl'd.)

A	To reduce A to M.		To reduce M to A.		M
	Mult. by	Log.	Log.	Mult. by	
DISCHARGE (Cu. ft., Gallons or Liters.)					
Cu. ft. per second.....	60.	1.7781513	8.2218487	.016$^{\vee}$6	Cu. ft. p. minute
Cu. ft. per second.....	3600.	3.5563025	6.4436975	.00027$^{\vee}$7	Cu. ft. per hour
Cu. ft. per second.....	86.4	1.9365137	8.0634863	.01157407	Ths'd.cu.ft.p.day
Cu. ft. per second.....	2.592	0.4136350	9.5863650	.3858025	Mill.cu.ft.p. mth.
Cu. ft. per second.....	31.536	1.4988066	8.5011934	.0317098	Mill.cu.ft. p. year
Cu. ft. per minute.....	60.	1.7781513	8.2218487	.016$^{\vee}$6	Cu. ft. per hour
Cu. ft. per minute.....	1440.	3.1583625	6.8416375	.000694$^{\vee}$4	Cu. ft. per day
Cu. ft. per minute.....	43.2	1.6354837	8.3645163	.02314815	Thsnd.cu.ft.p.mo
Cu. ft. per minute.....	525.6	2.7206554	7.2793446	.00190259	Thsnd.cu.ft.p.yr.
Cu. ft. per minute.....	7.480521	0.8739318	9.1260682	.1336805$^{\vee}$5	Galls.(U.S.)p. m.
Cu. ft. per minute.....	28.31702	1.4520475	8.5479525	.03531445	Liters p. minute
Gallons per minute.....	3.785434	0.5781158	9.4218842	.26417047	Liters p. minute
Cu. ft. per second.....	1.983471	0.2974258	9.7025742	.5041667	Acre-ft. per day
WORK AND POWER (Power = Rate of Work.)					
Foot-pounds.....	.138255	9.1406818	0.8593182	7.23300	Meter-Kilogms.
Foot-pounds.....	1.356284	0.1323508	9.8676492	.737308	Joules
Meter-Kilograms.....	9.81	0.9916690	9.0083310	.101937	Joules
Foot-pounds.....	.0012853	7.1090204	2.8909796	778.	Pnd.-deg. (Fahr.)
Meter-kilograms.....	.0092969	7.9683386	2.0316614	107.5626	Pnd.-deg. (Fahr.)
Foot-pounds.....	.0012853	7.1090204	2.8909796	778.	Brit. therm.units
Foot-pounds.....	.0003239	6.5104139	3.4895861	3087.35	Kilog'm-deg. (C.)
Meter-kilograms.....	.0023428	7.3697321	2.6302679	426.843	Kilog'm-deg. (C.)
Foot-pounds per sec.....	.0018$^{\vee}$18	7.2596373	2.7403627	550.	Mechanical H. P.
Foot-pounds per sec.....	.0013563	7.1323508	2.8676492	737.308	Electric H. P.
Foot-pounds per sec.....	.0018434	7.2656205	2.7343795	542.475	Metric H. P.
Foot-pounds per min.....	.00003$^{\vee}$03	5.4814861	4.5185139	33000.	Mechanical H. P.
Foot-pounds per min.....	.0000226	5.3541995	4.6458005	44238.51	Electric H. P.
Foot-pounds per min.....	.00003072	5.4874692	4.5125308	32548.49	Metric H. P.
Foot-pounds per hour.....	.0000008$^{\vee}$05	3.7033348	6.2966652	1980000.	Mechanical H. P.
Foot-pounds per hour.....	$.0_6$37675*	3.5760482	6.4239518	2654311.	Electric H. P.
Foot-pounds per hour.....	$0_6$51206*	3.7093179	6.2906821	1952909.	Metric H. P.
Meter-kilog. per min.....	$.0_3$21918*	6.3408043	3.6591957	4562.424	Mechanical H. P.
Meter-kilog per min.....	$.0_3$16350*	6.2135178	3.7864822	6116.207	Electric H. P.
Meter-kilog. per min.....	.0002$^{\vee}$2	6.3467875	3.6532125	4500.	Metric H. P.
Meter-kilog. per hour.....	$.0_5$36530*	4.5626530	5.4373470	273745.5	Mechanical H. P.
Meter-kilog. per hour.....	$.0_5$27250*	4.4353665	5.5646335	366972.5	Electric H. P.
Meter-kilog. per hour.....	$.0_5$37$^{\vee}$037*	4.5686362	5.4313638	270000.	Metric H. P.
Mechanical H. P.....	.7459666	9.8727135	0.1272865	1.34056	Electric H. P.
Mechanical H. P.....	1.01387	0.0059832	9.9940168	.9863177	Metric H. P.
Electric H. P.....	1.35916	0.1332697	9.8667303	.73575	Metric H. P.

Note.—1 electric horse-power = 1 kilowatt = 1000 watts = 1000 joules per second.

1 metric horse-power = 75 meter-kilograms per second.

Examples to illustrate the use of above table:

8 meter-kilograms = 8 × 9.81 joules. 10 British thermal units = 10 × 778 ft.-lbs.

*$.0_6$37675 = .000 000 37675; $0_6$37$^{\vee}$037 = 0.000 003 703 7037——; etc.

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35.—Electrical, Mechanical and Heat Units—Equivalent Values.

(See also preceding table.)

1 *Foot-pound*
= 1.356284 joules.
= 0.138255 kilogram-meters.
= 0.00000037675 kilowatt hours.
= 0.0012853 heat units.
= 0.0000005˘05 horse-power hour.

1 *Kilogram-meter*
= 9.81 joules.
= 7.23300 ft.-lbs.
= 0.0000036530 horse-power hour.
= 0.0000027250 kilowatt hour.
= 0.0092969 heat units.

1 *Joule*
= 0.737308 ft.-lbs.
= 0.101937 kilogram-meters.
= 0.000947697 heat units.
= 1 watt second.
= 0.00000027˘7 kilowatt hour.
= 0.000000372378 horse-power hour.

1 *Heat Unit (B.T.U.)*
= 1055.18932 joules.
= 1055.18932 watt-seconds.
= 778 ft.-lbs.
= 107.5626 kilogram-meters.
= 0.00029211 kilowatt hour.
= 0.00039293 horse-power hour.
= 0.0000688 lb. carbon oxidized.*
= 0.001036 lb. water evaporated from and at 212° F.*

1 *Watt*
= 1 joule per second.
= 0.00134056 horse power.
= 3.411711 heat units per hour.
= 0.737308 ft.-lb. per second.
= 0.0035 lb. water evaporated per hour.*
= 44.23851 ft.-lbs. per minute.
= 0.00135916 metric horse power.

1 *Kilowatt*
= 1000 watts.
= 1.34056 horse power.
= 2654311 ft.-lbs. per hour.
= 44.23851 ft.-lbs. per minute.
= 737.308 ft.-lbs. per second.
= 3411.711 heat units per hour.
= 56.86185 heat units per minute.
= 0.9476975 heat units per second.
= 0.2275 lb. carbon oxidized per hour.*
= 3.53 lbs. water evaporated per hour from and at 212° F.*

1 *Watt Per Square Inch*
= 8.19 heat units per sq. ft. per min.*
= 6371 ft.-lbs. per sq. ft. per min.*
= 0.193 horse power per sq. ft.*

1 *Kilowatt-hour*
= 1000 watt-hours.
= 1.34056 horse-power hours.
= 2654311 ft.-lbs.
= 3600000 joules.
= 3411.711 heat units.
= 366972.5 kilogram meters.
= 0.235 lb. carbon oxidized with perfect efficiency.
= 3.53 lbs. water evaporated from and at 212° F.*
= 22.75 lbs. of water raised from 62° to 212° F.*

1 *Horse Power*
= 745.9566 watts.
= 0.7459566 kilowatts.
= 550 ft-lbs. per second.
= 33000 ft.-lbs. per minute.
= 2544.987 heat units per hour.
= 42.41645 heat units per minute.
= 0.706941 heat units per second.
= 0.175 lbs. carbon oxidized per hour.*
= 2.64 lbs. water evaporated per hour from and at 212° F.*

1 *Horse Power-hour*
= 0.7459566 kilowatt-hours.
= 1980000 ft.-lbs.
= 2544.987 heat units.
= 273745.5 kilogram-meters.
= 0.175 lb. carbon oxidized with perfect efficiency.*
= 2.64 lbs. water evaporated from and at 212° F.*
= 17.0 lbs. water raised from 62° to 212° F.*

1 *Lb. Carbon Oxidized with Perfect Efficiency**
= 1.11 lb. Anthracite coal oxidized.
= 2.5 lbs. dry wood oxidized.
= 21 cubic feet illuminating gas.
= 4.26 kilowatt-hours.
= 5.71 horse-power hours.
= 11315000 ft.-lbs.
= 15 lbs. water evaporated from and at 212° F.*
= 14544 heat units.

1 *Lb. Water Evaporated from and at 212° F.**
= 0.283 kilowatt-hour.
= 0.379 horse-power hour.
= 965.7 heat units.
= 103900 kilogram-meters.
= 1019000 joules.
= 751300 ft.-lbs.
= 0.0664 lb. of carbon oxidized.

1 *Heat Unit per square ft. per min.**
= 0.122 watt per square inch.
= 0.0176 kilowatt per sq. ft.
= 0.0236 horse power per sq. ft.

* Values by H. W. Leonard.— See The Electrical Engineer, Feb. 25, 1895.

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36.—Foreign Weights and Measures.

(Compiled by the Department of State.)

Denominations.	Where Used.	American Equivalents.	Denominations.	Where Used.	American Equivalents.
Almude	Portugal	4.422 gallons.	Catty	Japan	1.31 pounds.
Ardeb	Egypt	7.6907 bushels.	do	Java, Siam and Malacca	1.35 pounds.
Are	Metric	0.02471 acre.	do	Sumatra	2.12 pounds.
Arobe	Paraguay	25 pounds.	Centaro	Central America	4.2631 gallons.
Arratel or libra	Portugal	1.011 pounds.	Centner	Bremen and Brunswick	117.5 pounds.
Arroba (dry)	Argentine Republic	25.3175 pounds.	do	Darmstadt	110.24 pounds.
do	Brazil	32.38 pounds.	do	Denmark and Norway	110.11 pounds.
do	Cuba	25.3664 pounds.	do	Nuremberg	112.43 pounds.
do	Portugal	32.38 pounds.	do	Prussia	113.44 pounds.
do	Spain	25.36 pounds.	do	Sweden	93.7 pounds.
do	Venezuela	25.4024 pounds.	do	Vienna	123.5 pounds.
Arroba (liquid)	Cuba, Spain and Venezuela	4.263 gallons.	do	Zollverein	110.24 pounds.
Arshine	Russia	28 inches.	do	Double or metric	220.46 pounds.
Arshine (square)	do	5.44 square feet.	Chetvert	Russia	5.7748 bushels.
Artel	Morocco	1.12 pounds.	Chih	China	14 inches.
Baril	Argentine Republic and Mexico	20.0787 gallons.	Coyan	Sarawak	3,098 pounds.
Barrel	Malta (customs)	11.4 gallons.	do	Siam (Koyan)	2,667 pounds.
do	Spain (raisins)	100 pounds.	Cuadra	Argentine Republic	4.2 acres.
Berkovets	Russia	361.12 pounds.	do	Paraguay	78.9 yards.
Bongkal	India	832 grains.	do	Paraguay (square)	8.077 square feet.
Bouw	Sumatra	7,096.5 square meters.	do	Uruguay	Nearly 2 acres.
Bu	Japan	0.1 inch.	Cubic meter	Metric	35.3 cubic feet.
Butt (wine)	Spain	140 gallons.	Cwt. (hundred wt.)	British	112 pounds.
Caffiso	Malta	5.4 gallons.	Dessiatine	Russia	2.6997 acres.
Candy	India (Bombay)	529 pounds.	do	Spain	1,599 bushels.
do	India (Madras)	500 pounds.	Drachme	Greece	Half ounce.
Cantar	Morocco	113 pounds.	Fanega (dry)	Central America	1.5745 bushels.
do	Syria (Damascus)	575 pounds.	do	Chili	2.575 bushels.
do	Turkey	124.7036 pounds.	do	Cuba	1.599 bushels.
Cantaro (cantar)	Malta	175 pounds.	do	Mexico	1.54728 bushels.
Carga	Mexico and Salvador	300 pounds.	do	Morocco	Strike fanega, 70 lbs.; full fanega, 118 lbs.
Catty	China	1.333 1-3 (1 1-3) pounds.	do	Uruguay (double)	7.776 bushels.

36.—FOREIGN WEIGHTS AND MEASURES.—Continued.

Denominations.	Where Used.	American Equivalents.	Denominations.	Where Used.	American Equivalents.
Fanega (dry)	Uruguay (single)	3.888 bushels.	Libra (pound)	Cuba	1.0161 pounds.
do	Venezuela	1.599 bushels.	do	Mexico	1.01465 pounds.
Fanega (liquid)	Spain	16 gallons.	do	Peru	1.0143 pounds.
Feddan	Egypt	1.03 acres.	do	Portugal	1.011 pounds.
Frail (raisins)	Spain	50 pounds.	do	Spain	1.0144 pounds.
Frasco	Argentine Republic	2.5096 quarts.	do	Uruguay	1.0143 pounds.
do	Mexico	2.5 quarts.	do	Venezuela	1.0161 pounds.
Frasla	Zanzibar	35 pounds.	Liter	Metric	1.0567 quarts.
Fuder	Luxemburg	264.17 gallons.	Livre (pound)	Greece	1.1 pounds.
Funt	Russia	0.9028 pound.	do	Guiana	1.0791 pounds.
Garnice	Russian Poland	0.88 gallon.	Load	England (timber)	Square, 50 cubic feet; unhewn, 40 cubic feet; inch planks, 600 superficial feet.
Gram	Metric	15.432 grains.			
Hectare	do	2.471 acres.			
Hectoliter (dry)	do	2.838 bushels.			
do (liquid)	do	26.417 gallons.	Manzana	Costa Rica	1 5-6 acres.
Joch	Austria-Hungary	1.422 acres.	do	Nicaragua and Salvador	1.727 acres.
Ken	Japan	6 feet.	Marc	Bolivia	0.507 pound.
Kilogram (kilo)	Metric	2.2046 pounds.	Maund	India	82 2-7 pounds.
Kilometer	do	0.621376 mile.	Meter	Metric	39.37 inches.
Klafter	Russia	216 cubic feet.	Mil	Denmark	4.68 miles.
Koku	Japan	4.9629 bushels.	do	Denmark (geographical)	4.61 miles.
Korree	Russia	3.5 bushels.	Milla	Nicaragua and Honduras	1.1493 miles.
Kwan	Japan	8.28 pounds.	Morgen	Prussia	0.63 acre.
Last	Belgium and Holland	85.134 bushels.	Oke	Egypt	2.7225 pounds.
do	England (dry malt)	82.52 bushels.	do	Greece	2.84 pounds.
do	Germany	2 metric tons (4,480 pounds.)	do	Hungary	3.0817 pounds.
			do	Turkey	2.82838 pounds.
do	Prussia	112.29 bushels.	do	Hungary and Wallachia	2.5 pints.
do	Russian Poland	11⅔ bushels.	Pic	Egypt	21½ inches.
do	Spain (salt)	4,760 pounds.	Picul	Borneo and Celebes	135.64 pounds.
League (land)	Paraguay	4,633 acres.	do	China, Japan and Sumatra	133 1-3 pounds.
Li	China	2,115 feet.	do	Java	135.1 pounds.
Libra (pound)	Argentine Republic	1.0127 pounds.	do	Philippine Islands	137.9 pounds.
do	Central America	1.043 pounds.	Pie	Argentine Republic	0.9478 foot.
do	Chili	1.014 pounds.	do	Spain	0.91407 foot.

36.—FOREIGN WEIGHTS AND MEASURES.—Concluded.

Denominations.	Where Used.	American Equivalents.	Denominations.	Where Used.	American Equivalents.
Pik	Turkey	27.9 inches.	Sun	Japan	1.193 inches.
Pood	Russia	36.112 pounds.	Tael	Cochin China	590.75 grains (troy).
Pund (pound)	Denmark and Sweden	1.102 pounds	Tan	Japan	0.25 acre.
Quarter	Great Britain	8.252 bushels.	To	do	2 pecks.
do	London (coal)	36 bushels.	Ton	Space measure	40 cubic feet.
Quintal	Argentine Republic	101.42 pounds.	Tonde (cereals)	Denmark	3.94783 bushels.
do	Brazil	130.06 pounds.	Tondeland	do.	1.36 acres.
do	Castile, Chili, Mexico and Peru	101.41 pounds.	Tsubo	Japan	6 feet square.
do	Greece	123.2 pounds.	Tsun	China	1.41 inches.
do	Newfoundland (fish)	112 pounds.	Tunna	Sweden	4.5 bushels.
do	Paraguay	100 pounds.	Tunnland	Sweden	1.22 acres.
do	Syria	125 pounds.	Vara	Argentine Republic	34.1208 inches.
do	Metric	220.46 pounds.	do	Central America	32.87 inches.
Rottle	Palestine	6 pounds.	do	Chili and Peru	33.367 inches.
do	Syria	5¼ pounds.	do	Cuba	33.384 inches.
Sagene	Russia	7 feet.	do	Curacao	33.375 inches.
Salm	Malta	490 pounds.	do	Mexico	33 inches.
Se	Japan	0.02451 acre.	do	Paraguay	34 inches.
Seer	India	1 pound 13 ounces.	do	Spain	0.914117 yard.
Shaku	Japan	11.9305 inches.	do	Venezuela	33.384 inches.
Sho	do	1.6 quarts.	Vedro	Russia	2.707 gallons.
Standard (St. Petersburg)	Lumber measure	165 cubic feet.	Vergees	Isle of Jersey	71.1 square rods.
Stone	British	14 pounds.	Verst	Russia	0.663 mile.
Suerte	Uruguay	2,700 cuadras (see cuadra).	Vlocka	Russian Poland	41.98 acres.

Numbers.

37.—Abstract Numbers.

10 units	= 1 ten	=	10
10 tens	= 1 hundred	=	100
10 hundreds	= 1 thousand	=	1 000
10 thousands	= 1 ten thousand	=	10 000
10 ten thousands	= 1 hundred thousand	=	100 000
10 hundred thousand	= 1 million	=	1 000 000

A pure decimal, where the first significant figure is far removed from the decimal point, may be abbreviated by a subscript to the first cipher to indicate the number of ciphers at the right of the decimal point and to the left of the first significant figure. Thus,

2 millionths,	or .000002,	may be written,	$.0_5 2$.
65 hundred millionths,	or .00000065,	may be written,	$.0_6 65$.
4 billionths,	or .000000004,	may be written,	$.0_8 4$.

38.—Duodecimo Numbers.

12 units = 1 dozen.
12 dozen = 1 gross = 144. (20 units = 1 score.)
12 gross = 1 great gross = 1728.

39.—Paper.

24 sheets = 1 quire. 2 reams = 1 bundle = 960.
20 quires = 1 ream = 480. 5 bundles = 1 bale = 4800.

Money.

40.—United States Money.

10 mills (m) = 1 cent (ct.) (Unit is $1.)
10 cents = 1 dime (d.) 10 dollars = 1 eagle (E.)
10 dimes = 1 dollar ($.) 2 eagles = 1 double eagle (EE.)

41.—Foreign Money.

English Money: 4 farthings (far.) = 1 penny (d.); 12 pence = 1 shilling (s.); 20 shillings = 1 pound (£) = 1 sovereign (= $4.8665, U. S. money). 1 guinea = 21 shillings; 1 crown = 5 shillings; 1 florin = 2 shillings.
French Money: 10 centimes = 1 decime; 10 decimes = 1 franc (fr.) (= $0.193, U. S. money).
German Money: 100 pfennig = 1 mark (= $0.238, U. S. money).
Italian Money: 100 centesimi = 1 lira (= $0.193, U. S. money).
Russian Money: 100 copecks = 1 ruble (= $0.515, U. S. money).
Austro-Hungarian Money: 100 kreutzers = 1 florin.

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42.—VALUE OF FOREIGN COINS IN UNITED STATES MONEY. (Proclaimed by the Secretary of the Treasury, October 1, 1905.*)

Country.	Standard.	Monetary Unit.	Value in U. S. Gold Dollar.	Coins.
Argent. R.	Gold.	Peso	$0.96,5	Gold: argentine ($4.82,4) and ½ argentine. Silver, peso and divisions.
Austria-H.	Gold.	Crown	.20,3	Gold: 10 and 20 crowns. Silver: 1 and 5 crowns.
Belgium	Gold.	Franc	.19,3	Gold: 10 and 20 francs. Silver: 5 francs.
Bolivia	Silver.	Boliviano	.44,1	Silver: boliviano and divisions.
Brazil	Gold.	Milreis	.54,6	Gold: 5, 10, and 20 milreis. Silver: ½, 1, and 2 milreis.
Canada	Gold.	Dollar	1.00	
Cent. Am.	Silver.	Peso†	.44,1	Silver: peso and divisions.
Chile	Gold.	Peso	.36,5	Gold: escudo ($1.82,5), doubloon ($3.65), and condor ($7.30). Silver: peso and divisions.
China	Silver.	Tael: Shanghai	.66.1	
		Tael: Haikwan	‖ .73.6	
		Tael: Canton	.72.3	
Colombia	Gold.	Dollar	1.00	Gold: condor ($9.64,7) and double-condor. Silver: peso.
Costa Rica	Gold.	Colon	.46,5	Gold: 2, 5, 10, and 20 colons ($9.30,7). Silver: 5, 10, 25, and 50 centimos.
Denmark	Gold.	Crown	.26,8	Gold: 10 and 20 crowns.
Ecuador	Gold.	Sucre	.48,7	Gold: 10 sucres ($4.86,65). Silver: sucre and divisions.
Egypt	Gold.	Pound (100 piasters)	4.94,3	Gold: pound (100 piasters), 5, 10, 20, and 50 piasters. Silver: 1, 2, 5, 10, and 20 piasters.
France	Gold.	Franc	.19,3	Gold: 5, 10, 20, 50, and 100 frs. Silver: 5 frs.
Germany	Gold.	Mark	.23,8	Gold: 5, 10, and 20 marks.
Gt. Britain	Gold.	Pound sterling	4.86,6½	Gold: sovereign (pound sterling) and ½ sovereign.
Greece	Gold.	Drachma	.19,3	Gold: 5, 10, 20, 50, and 100 drachmas. Silver: 5 drachmas.
Hayti	Gold.	Gourde	.96,5	Gold: 1, 2, 5, and 10 gourdes. Silver: gourde and divisions.
India	Gold.	Pound sterling‡	4.86,6½	Gold: sov. ($4.86,65). Silver: rupee and divisions.
Italy	Gold.	Lira	.19,3	Gold: 5, 10, 20, 50, and 100 lire. Silver, 5 lire.
Japan	Gold.	Yen	.49,8	Gold: 1, 2, 5, 10, and 20 yen. Silver: 10, 20, and 50 sen.
Mexico	Gold.	Peso ¶	.49,8	Gold: 5 and 10 pesos. Silver: dollar (or peso)** and divisions.
Neth'lands	Gold.	Florin	.40,2	Gold: 10 florins. Silver: ½, 1, and 2½ florins.
N'foundl'd	Gold.	Dollar	1.01,4	Gold: 2 dollars ($2.02,7).
Norway	Gold.	Crown	.26,8	Gold: 10 and 20 crowns.
Panama	Gold.	Balboa	1.00,0	Gold: 1, 2½, 5, 10, and 20 balboas. Silver: peso and divisions.
Peru	Gold.	Sol	.48,7	Gold: libra ($4.86,65). Silver: sol and divisions.
Portugal	Gold.	Milreis	1.08	Gold: 1, 2, 5, and 10 milreis.
Russia	Gold.	Ruble	.51,5	Gold: 5, 7½, 10, and 15 rubles. Silver: 5, 10, 15, 20, 25, 50, and 100 copeks.
Spain	Gold.	Peseta	.19,3	Gold: 25 pesetas. Silver: 5 pesetas.
Sweden	Gold.	Crown	.26,8	Gold: 10 and 20 crowns.
Switzerland	Gold.	Franc	.19,3	Gold: 5, 10, 20, 50, and 100 francs. Silver: 5 francs.
Turkey	Gold.	Piaster	.04,4	Gold: 25, 50, 100, 250, and 500 piasters.
Uruguay	Gold.	Peso	1.03,4	Gold: peso. Silver: peso and divisions.
Venezuela	Gold.	Bolivar	.19,3	Gold: 5, 10, 20, 50, and 100 bolivars. Silver: 5 bolivars.

* The coins of silver-standard countries are valued by their pure silver contents, at the average market price of silver for the three months preceding the date of this circular. † Not including Costa Rica. ‡ The sovereign is the standard coin of India, but the rupee ($0.44,8) is the

42a.—VALUE OF FOREIGN COINS AND PAPER NOTES IN AMERICAN MONEY BASED UPON THE VALUES EXPRESSED IN TABLE 42.

Number.	British £ Sterling.	German Mark	French Franc, Italian Lira.	Chinese Tael (Haikwan.)	Dutch Florin.	Japanese Yen, Mexican Peso.	Russian Gold Ruble.	Austrian Crown
1	4.86,6½	$0.23,8	$0.19,3	$0.73,6	$0.40,2	$0.49,8	$0.51,5	$0.20,3
2	9.73,3	0.47,6	0.38,6	1.47,2	0.80,4	0.99,6	1.03	0.40,6
3	14.59,9½	0.71,4	0.57,9	2.20,8	1.20,6	1.49,4	1.54,5	0.60,9
4	19.46,6	0.95,2	0.77,2	2.94,4	1.60,8	1.99,2	2.06	0.81,2
5	24.33,2½	1.19	0.96,5	3.68,0	2.01	2.49,0	2.57,5	1.01,5
6	29.19,9	1.42,8	1.15,8	4.41,6	2.41,2	2.98,8	3.09	1.21,8
7	34.06,5½	1.66,6	1.35,1	5.15,2	2.81,4	3.48,6	3.60,5	1.42 1
8	38.93,2	1.90,4	1.54,4	5.88,8	3.21,6	3.98,4	4.12	1.62,4
9	43.79,8½	2.14,2	1.73,7	6.62,4	3.61,8	4.48,2	4.63,5	1.82,7
10	48.66,5	2.38	1.93	7.36,0	4.02	4.98,0	5.15	2.03
20	97.33	4.76	3.86	14.72,0	8.04	9.96,0	10.30	4.06
30	145.99,5	7.14	5.79	22.08,0	12.06	14.94,0	15.45	6.09
40	194.66	9.52	7.72	29.44,0	16.08	19.92,0	20.60	8.12
50	243.32,5	11.90	9.65	36.80,0	20.10	24.90,0	25.75	10.15
100	486.65	23.80	19.30	73.60,0	40.20	49.80,0	51.50	20.30

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43.—Comparison of Prices.

French and German prices for metric units, British prices for Imperial units, and United States prices for United States standard weights and measures.

[Based upon the circular of the Secretary of the Treasury dated October 1, 1902, fixing the legal equivalent of the (German) mark at 23.8 cents, of the (French) franc at 19.3 cents, and the British pound sterling at $4.8665.]

Francs per Kilogram.		Dollars per Avoir. Lb.	Francs per Meter.		Dollars per Yard.	Francs per Liter.		Dollars per U. S. Liquid Gal.	Francs per Hectolitter.		Dollars per Bushel.	Shillings per British Imp. Gal.		Dollars per U. S. Liquid Gal.
1	=	.088	1	=	.176	1	=	.731	1	=	.068	1	=	.203
2	=	.175	2	=	.353	2	=	.461	2	=	.136	2	=	.405
3	=	.263	3	=	.529	3	=	2.192	3	=	.204	3	=	.608
4	=	.350	4	=	.705	4	=	2.922	4	=	.272	4	=	.810
5	=	.438	5	=	.882	5	=	3.653	5	=	.340	5	=	1.013
6	=	.525	6	=	1.058	6	=	4.384	6	=	.408	6	=	1.216
7	=	.613	7	=	1.234	7	=	5.114	7	=	.476	7	=	1.418
8	=	.700	8	=	1.411	8	=	5.844	8	=	.544	8	=	1.621
9	=	.788	9	=	1.587	9	=	6.575	9	=	.612	9	=	1.824
11.423	=	1	5.667	=	1	1.369	=	1	14.703	=	1	4.935	=	1
22.846	=	2	11.334	=	2	2.738	=	2	29.407	=	2	9.871	=	2
34.269	=	3	17.000	=	3	4.106	=	3	44.110	=	3	14.806	=	3
45.691	=	4	22.667	=	4	5.475	=	4	58.813	=	4	19.742	=	4
57.115	=	5	28.334	=	5	6.844	=	5	73.517	=	5	24.677	=	5
68.537	=	6	34.001	=	6	8.213	=	6	88.220	=	6	29.612	=	6
79.960	=	7	39.668	=	7	9.581	=	7	102.923	=	7	34.548	=	7
91.383	=	8	45.334	=	8	10.950	=	8	117.627	=	8	39.483	=	8
102.806	=	9	51.001	=	9	12.319	=	9	132.330	=	9	44.419	=	9

Marks per Kilogram.		Dollars per Avoir. Lb.	Marks per Meter.		Dollars per Yard.	Marks per Liter.		Dollars per U. S. Liquid Gal.	Marks per Hectolitter.		Dollars per Bushel.	Shillings per British Bushel.		Dollars per U. S. Bushels.
1	=	.108	1	=	.218	1	=	.901	1	=	.084	1	=	.236
2	=	.216	2	=	.435	2	=	1.802	2	=	.168	2	=	.472
3	=	.324	3	=	.653	3	=	2.703	3	=	.252	3	=	.707
4	=	.432	4	=	.871	4	=	3.604	4	=	.335	4	=	.943
5	=	.540	5	=	1.088	5	=	4.505	5	=	.419	5	=	1.179
6	=	.648	6	=	1.306	6	=	5.406	6	=	.503	6	=	1.415
7	=	.756	7	=	1.523	7	=	6.307	7	=	.587	7	=	1.650
8	=	.864	8	=	1.741	8	=	7.207	8	=	.671	8	=	1.886
9	=	.972	9	=	1.959	9	=	8.108	9	=	.755	9	=	2.122
9.263	=	1	4.595	=	1	1.110	=	1	11.923	=	1	4.241	=	1
18.526	=	2	9.190	=	2	2.220	=	2	23.847	=	2	8.483	=	2
27.789	=	3	13.785	=	3	3.330	=	3	35.770	=	3	12.724	=	3
37.052	=	4	18.380	=	4	4.440	=	4	47.693	=	4	16.965	=	4
46.316	=	5	22.975	=	5	5.550	=	5	59.616	=	5	21.207	=	5
55.579	=	6	27.570	=	6	6.660	=	6	71.540	=	6	25.448	=	6
64.842	=	7	32.165	=	7	7.770	=	7	83.463	=	7	29.689	=	7
74.105	=	8	36.760	=	8	8.880	=	8	95.386	=	8	33.931	=	8
83.368	=	9	41.355	=	9	9.990	=	9	107.310	=	9	38.172	=	9

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Miscellaneous.

44.—Time Measure.

1 *second* (s) = 15 seconds ($0^\circ - 00' - 15''$) of longitude.
60 seconds . . . . . 1 *minute* (m) = 15 minutes ($0^\circ - 15'$) of longitude.
60 minutes (= 3600 seconds) =
1 *hour* (h) = 15 degrees (15°) of longitude.
24 hours (= 1440 m = 86400 s) =
1 *solar day* (d) = 360 degrees (360°) of longitude.
7 days (= 168 h = 10080 m = 604800 s) =
1 *week*.
52 weeks (= 365 d = 8760 h = 525600 m) =
1 *common year*.
52 weeks (= 366 d = 8784 h = 527040 m) =
1 *leap year*.
100 years (= 75 common + 25 leap) =
1 *century*.

45.—Circular Measure.

= 1 *second* ($''$) = $\frac{1}{15}$ (.06$^{\vee}$6) second of time.
60 seconds . . . . . = 1 *minute* ($'$) = 4 seconds of time.
60 minutes (= 3600 seconds) = 1 *degree* ($^\circ$) = 4 minutes of time.
30 degrees (= $1800' = 108000''$) = 1 *sign* = the angle at the base of a right triangle whose altitude is 1, hypothenuse 2, and base $\sqrt{3}$.
90 degrees (= $5400' = 324000''$)
= 1 *right angle* ($\llcorner$) = 6 hours of time.
180 degrees (= $10800' = 648000''$)
= 1 *semi-circumference* = π = 3.14159265. . . .
360 degrees (= $21600' = 1296000''$)
= 1 *circumference* = 2π . radius = π . diameter.

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5.—ALGEBRA.

An algebraic equation is a shorthand mathematical expression, and every such expression may be transformed by observing certain algebraic rules. The first letters of the alphabet, $a, b, c, \ldots$, represent the known quantities of the equation, and the last letters, $\ldots x, y, z$, the unknown. *Members* of an equation are separated by the sign of equality (=); *terms* of a member are separated by plus (+) and minus (−) signs; and *factors* of a term are separated by the signs of multiplication (×) or (.), either expressed or understood.

Any *factor* or *set of factors* of a term may be considered the *denomination* of that term. Thus, in $a\,x\sqrt{y}$, either a, x, $\sqrt{y}$, ax, $a\sqrt{y}$, $x\sqrt{y}$ or $ax\sqrt{y}$ may be chosen as the denomination, as may be expedient.

Addition and Subtraction.—Place *like terms* (terms having the same denomination) in the same column, and reduce fractions to a common denominator, before adding or subtracting:

$$4x + 5x^2y + \sqrt{d} + \frac{1}{\sqrt{x}} \qquad \frac{x^2}{a+b} + \frac{y^2}{a+b} = \frac{x^2+y^2}{a+b}$$

$$-2x + 3x^2y - 2\sqrt{d} + \frac{2}{\sqrt{x}} \qquad \frac{(a+b)^2}{x} - \frac{(a+b)^2}{y} = \frac{(y-x)\,(a+b)^2}{x\,y}$$

$$\text{Sum} = 2x + 8x^2y - \sqrt{d} + \frac{3}{\sqrt{x}} \qquad 3\sqrt{3x} \pm \sqrt{12x} = (3 \pm 2)\sqrt{3x}$$

$$\text{Diff.} = 6x + 2x^2y - 3\sqrt{d} - \frac{1}{\sqrt{x}}$$

Note.—In subtraction, reverse the + and − signs of the subtrahend and proceed as in addition.

Exponents.—The following hints are given without discussion:

$$n \times n \times n = nnn = nn^2 = n^2n = n^{1+2} = n^{2+1} = n^3 = \frac{1}{n^{-3}} = n^{\frac{9}{3}} = \sqrt{n^6}$$

$$n\sqrt{xy} = n\sqrt{x}\sqrt{y} = n\,x^{\frac{1}{2}}\,y^{\frac{1}{2}} = \frac{x^{\frac{1}{2}}\,y^{\frac{1}{2}}}{n^{-1}} = \frac{n}{x^{-\frac{1}{2}}\,y^{-\frac{1}{2}}} = \sqrt{n^2xy} = (n^2\,x\,y)^{\frac{1}{2}}$$

$$\sqrt[3]{\frac{-8x^2}{27y^6}} = \frac{\sqrt[3]{-8x^2}}{\sqrt[3]{27y^6}} = \frac{(-8)^{\frac{1}{3}}\,(x^2)^{\frac{1}{3}}}{(27)^{\frac{1}{3}}\,(y^6)^{\frac{1}{3}}} = \frac{-2x}{3y^2} = -\frac{2x}{3y^2}$$

$$x^{\frac{3}{4}} = \sqrt[4]{x^3}. \quad a^{\frac{4}{9}} = \sqrt[9]{a^4}. \quad a^{\frac{x}{n}} = \sqrt[n]{a^x}. \quad n^{-\frac{x}{n}} = \frac{1}{n^{\frac{x}{n}}} = \frac{1}{\sqrt[n]{n^x}}. \quad \left(\frac{a}{x}\right)^n = \frac{a^n}{x^n}$$

$$x^{n-n} = x^n\,x^{-n} = x^0 = \frac{x^n}{x^n} = 1. \quad a^0 = 1. \quad n^0 = 1^*$$

$$x^n\,x^n = x^{n+n} = x^{2n}. \quad a^n\,a^x = a^{n+x}. \quad a^{n-x} = a^n a^{-x} = \frac{a^n}{a^x}.$$

$$(x^m)^n = x^{mn}. \quad (x^4)^5 = x^{20}. \quad (m^x)^{-n} = m^{-nx} = \frac{1}{m^{nx}}. \quad (x^2y^3)^2 = x^4y^6.$$

*Any number whose exponent is 0, is equal to 1.

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Multiplication and Powers.

Monomials: $n^2 \times n^{\frac{1}{2}} = n^2 n^{\frac{1}{2}} = n^{2\frac{1}{2}} = n^{\frac{5}{2}} = \sqrt{n^5} = \frac{n^5}{n^2}$.

$a^3 \times a\, n^2 = a^4 n^2$. $2\sqrt{3} \times 3\sqrt{5} = 6\sqrt{15}$. $3\sqrt{a}.\, x \times \sqrt{a}.\, x = 3ax^2$.

Like signs give *plus*: $a \times b = ab$. $\quad -a \times (-b) = ab$.

Unlike signs give *minus*: $-a \times b = -ab$. $\quad a \times (-b.) = -ab$.

Polynomials: Multiply each term of the mutliplicand by each term of the multiplier, placing like terms of the products in "column," and add.

$$\begin{array}{r} 3\,a\,x^2 + 4\sqrt{5} \\ \sqrt{5} \\ \hline \end{array} \qquad \begin{array}{r} a\,b^3 - a^2\,b \\ a\,b^3 - a^2\,b^2 \\ \hline -a^2b^4 + a^4\,b^3 \\ a^2\,b^5 - a^3\,b^4 \\ \hline a^2\,b^5 - 2a^3\,b^4 + a^4b^3 \end{array}$$

Product $= 3\,a\,x^2\sqrt{5} + 20$ $\qquad$ Product $= a^2\,b^5 - 2a^3\,b^4 + a^4b^3$
$= a^2\,(b^5 - 2\,ab^4 + a^2\,b^2)$.
$= a^2\,b^2\,(b - a)^2$.

$(x \pm y)^2 = x^2 \pm 2\,x\,y + y^2$
$(x + y)\,(x - y) = x^2 - y^2$. } These two formulas are the foundation of many short methods in arithmetic. (See page 11).

$(x \pm y)^3 = x^3 \pm 3\,x^2\,y + 3\,x\,y^2 \pm y^3$.
$(x + y)\,(x^2 - x\,y + y^2) = x^3 + y^3$.
$(x - y)\,(x^2 + x\,y + y^2) = x^3 - y^3$.
$(x + y + z)^2 = x^2 + y^2 + z^2 + 2\,x\,y + 2\,x\,z + 2\,y\,z$.
$(x + y + z)^3 = x^3 + y^3 + z^3 + 3\,x^2\,(y + z) + 3\,y^2\,(x + z) + 3\,z^2\,(x + y) + 6\,x\,y\,z$.

Binomial formula for expanding the sum or difference of two numbers, to any power:

$$(a \pm x)^n = a^n \pm na^{n-1}\,x + \frac{n(n-1)}{1 \times 2}\,a^{n-2}\,x^2 \pm \frac{n(n-1)(n-2)}{1 \times 2 \times 3}\,a^{n-3}\,x^3 + \ldots$$

Thus: $$(a+x)^5 = a^5 + 5a^4x + \frac{5 \times 4}{1 \times 2}\,a^3x^2 + \frac{5 \times 4 \times 3}{1 \times 2 \times 3}\,a^2x^3 + \frac{5 \times 4 \times 3 \times 2}{1 \times 2 \times 3 \times 4}\,ax^4 + \frac{5 \times 4 \times 3 \times 2 \times 1}{1 \times 2 \times 3 \times 4 \times 5}\,x^5$$

$$= a^5 + 5\,a^4\,x + 10\,a^3\,x^2 + 10\,a^2\,x^3 + 5\,a\,x^4 + x^5.$$

$$(a + x)^{\frac{1}{2}} = a^{\frac{1}{2}} + \tfrac{1}{2}a^{-\frac{1}{2}}\,x + \frac{\frac{1}{2}(-\frac{1}{2})}{1 \times 2}\,a^{-1\frac{1}{2}}\,x^2 + \frac{\frac{1}{2}(-\frac{1}{2})\,(-1\frac{1}{2})}{1 \times 2 \times 3}\,a^{-2\frac{1}{2}}x^3 + \frac{\frac{1}{2}\,(-\frac{1}{2})\,(-1\frac{1}{2})\,(-2\frac{1}{2})}{1 \times 2 \times 3 \times 4}\,a^{-3\frac{1}{2}}\,x^4 \ldots\ldots$$

$$= a^{\frac{1}{2}} + \frac{x}{2\sqrt{a}} - \frac{x^2}{2 \times 2^2\sqrt{a^3}} + \frac{3\,x^3}{2 \times 3 \times 2^3\sqrt{a^5}} - \frac{3 \times 5\,x^4}{2 \times 3 \times 4 \times 2^4\sqrt{a^7}} + \frac{3 \times 5 \times 7\,x^5}{2 \times 3 \times 4 \times 5 \times 2^5\sqrt{a^9}} \ldots\ldots$$

$$(a + x)^{\frac{1}{3}} = a^{\frac{1}{3}} + \tfrac{1}{3}\,a^{-\frac{2}{3}}\,x + \frac{\frac{1}{3}\,(-\frac{2}{3})}{1 \times 2}\,a^{-1\frac{2}{3}}\,x^2 + \frac{\frac{1}{3}(-\frac{2}{3})(-1\frac{2}{3})}{1 \times 2 \times 3}\,a^{-2\frac{2}{3}}\,x^3 + \frac{\frac{1}{3}\,(-\frac{2}{3})\,(-1\frac{2}{3})\,(-2\frac{2}{3})}{1 \times 2 \times 3 \times 4}\,a^{-3\frac{2}{3}}\,x^4 \ldots\ldots$$

$$= a^{\frac{1}{3}} + \frac{x}{3\sqrt[3]{a^2}} - \frac{2x^2}{2 \times 3^2\sqrt[3]{a^5}} + \frac{2 \times 5x^3}{2 \times 3 \times 3^3\sqrt[3]{a^8}} - \frac{2 \times 5 \times 8\,x^4}{2 \times 3 \times 4 \times 3^4\sqrt[3]{a^{11}}} + \frac{2 \times 5 \times 8 \times 11\,x^5}{2 \times 3 \times 4 \times 5 \times 3^5\sqrt[3]{a^{14}}} \ldots\ldots$$

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Division and Roots.

$x^2 + x^2 = \frac{x^2}{x^2} = x^2x^{-2} = x^0 = 1.$ $\quad x^3 + x^3 = x^{3-3} = x^3.$

$a^2b^2 + ab = (a^{2-1})\,(b^{2-1}) = ab^2.$ $\quad \frac{a}{x} + \frac{c}{n} = \frac{a}{x} \times \frac{n}{c} = \frac{an}{cx}.$

Division:

$$\begin{array}{l} x+3)\,x^2 + 5x + 6\,(x+2 \\ \underline{\quad x^2 + 3x} \\ \quad\;\; 2x + 6 \\ \quad\;\; \underline{2x + 6} \end{array}$$

$$\frac{\sqrt{3}}{\sqrt{5}} = \sqrt{\frac{3}{5}} = \sqrt{\frac{15}{25}} = \frac{\sqrt{15}}{\sqrt{25}} = \frac{\sqrt{15}}{\pm 5} = \pm\tfrac{1}{5}\sqrt{15}$$

$$\frac{ax^2\sqrt[n]{cy}}{ax\sqrt[n]{c}} = x\sqrt[n]{y}. \quad \frac{10\sqrt{.8}}{3\sqrt{5}} = \frac{\sqrt{80}}{3\sqrt{5}} = \tfrac{1}{3}\sqrt{16} = \frac{\pm 4}{3} = \pm 1\tfrac{1}{3}$$

Square Root by Binomial formula:

Example: Find the square root of 5?

Solution: In $(a+x)^{\frac{1}{2}}$ let $a = 4$ and $x = 1$; thence by expansion (see page 101)

$$(a+x)^{\frac{1}{2}} = a^{\frac{1}{2}} + \frac{x}{2\sqrt{a}} - \frac{x^2}{8\sqrt{a^3}} + \frac{x^3}{16\sqrt{a^5}} - \frac{5x^4}{128\sqrt{a^7}} + \frac{7x^5}{256\sqrt{a^9}} \cdots\cdots$$

$$= 2 + \frac{1}{2\times 2} - \frac{1}{8\times 8} + \frac{1}{16\times 32} - \frac{5}{128\times 128} \cdots\cdots$$

$$= 2 + .25 - .015625 + .001953 - .000305 = 2.236\pm.$$

Cube Root by Binomial formula:

Example: Find the cube root of 9?

Solution: 8 is the cube of 2 and 1 less than 9; hence from expansion formula, page 101, there is obtained,

$(a+x)^{\frac{1}{3}} = (8+1)^{\frac{1}{3}} = 8^{\frac{1}{3}}(1+\frac{1}{8})^{\frac{1}{3}} = 2\,(1+\frac{1}{8})^{\frac{1}{3}}.$ But

$(a+x)^{\frac{1}{3}} = a^{\frac{1}{3}} + \frac{x}{3\sqrt[3]{a^2}} - \frac{x^2}{3^2\sqrt[3]{a^5}} + \frac{5x^3}{3^4\sqrt[3]{a^8}} - \frac{10x^4}{3^5\sqrt[3]{a^{11}}} + \ldots,$ Therefore

$$\sqrt[3]{9} = 2(1+\tfrac{1}{8})^{\frac{1}{3}} = 2\left[1 + \frac{1}{3\times 8} - \frac{1}{3^2\times 8^2} + \frac{5}{3^4\times 8^3} - \frac{10}{3^5\times 8^4} \cdots\cdots\right]$$

$$= 2[1 + .0416\overset{\vee}{6} - .0017361\overset{\vee}{1} + .00012054 - .00001005 \ldots\ldots]$$

$$= 2 \times 1.04004 = 2.08008. \text{ Ans.}$$

Completing the Square.—This is performed by adding a certain amount c^2 (a third term), to the first member of the (affected quadratic) equation to make it a perfect square, and the same amount to the second member so as to preserve the equality.

Example 1: Find the value of x in the equation $x^2 + 4x = 21$?

Solution: By adding c^2 to both members we have $x^2 + 4\,x + c^2 = c^2 + 21$, in which the first member is a perfect square, the middle term, $4x$, being equal* to $2\sqrt{x^2}\sqrt{c^2}$, whence $c^2 = 2^2 = 4$,

and $\quad x^2 + 4\,x + 2^2 = 4 + 21 = 25.$

Extracting the sq. rt., $\quad x + 2 = \pm 5$

and $\quad x = \pm 5 - 2 = 3$ or -7. Ans.

Note.—Make the *first term* a perfect square before completing the square of the first member of the equation. This may be done by multiplying or dividing the whole equation by a constant. The first term must be *positive*.

* The square of a member of two terms = *the square of the first* + *twice the product of the two* + *the square of the second*.

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Example 2: Solve $2ax^2 + 5bx = 30$? Here,

Mult. by 2, div. by a and add c^2: $4x^2 + \frac{10b}{a}x + c^2 = c^2 + \frac{60}{a}$

Extracting the sq. rt., $2x + c = \sqrt{c^2 + \frac{60}{a}}$, in which

$$\frac{10bx}{a} = 2(2x \times c), \text{ or } c = \frac{5b}{2a}; \quad \therefore\ 2x = \pm\sqrt{\frac{25b^2}{4a^2} + \frac{60}{a}} - \frac{5b}{2a}$$

$$\therefore\ x = \pm\tfrac{1}{2}\sqrt{\frac{25b^2}{4a^2} + \frac{60}{a}} - \frac{5b}{4a}. \text{ Ans.}$$

Remarks on apparent fallacy of the above. To prove that $4 = 5$?

Let $16 - 36 = 25 - 45$

and to complete the square, let $c^2 = \frac{81}{4}$; then

$$16 - 36 + \frac{81}{4} = 25 - 45 + \frac{81}{4}.$$

Extracting the sq. rt., $4 - \frac{9}{2} = 5 - \frac{9}{2}$, hence, apparently, $4 = 5$ (or $-\frac{1}{2} = +\frac{1}{2}$). | But, $\pm(4 - \frac{9}{2}) = \mp(5 - \frac{9}{2})$, whence, really, $4 - 4\frac{1}{2} = -5 + 4\frac{1}{2}$.

In a somewhat similar way it may also be "shown" that $2 = 1$, $1 = 0$, etc.; but the discrepancies are always apparent upon inspection.

Simultaneous Equations.—To solve any problem, *the number of equations must be equal to the number of unknown quantities.* This can be understood quite readily by considering that each equation is really the equation of a curve of some kind. Thus, in Fig. 1, let curve A be represented by the equation $y = mx + c$, and curve B by the equation $y^2 = 4ax$. The equations of these curves are simultaneous equations when the curves intersect at any point, as p, with common coordinates x_0 and y_0; and the problem is solved by determining the values of these coordinates. Thus, by substitution, $y_0^2 = (mx + c)^2 = 4ax$;

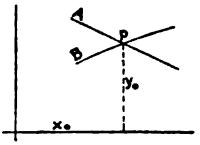

Fig. 1.

and $x_0 = \frac{y - c}{m} = \frac{y^2}{4a}$; from which we have only to solve the equation $(mx + c)^2 = 4ax$, or the equation $\frac{y - c}{m} = \frac{y^2}{4a}$, each one containing only one unknown quantity, x or y.

Simultaneous equations may be solved by three methods of eliminating all but one unknown quantity:

(1) Elimination by substitution, as above.

(2) Elimination by addition or subtraction, and by substitution.

Example: $3x + 4y = 18$. Mult. by 2: $6x + 8y = 36$
$2x + 3y = 13$. Mult. by 3: $6x + 9y = 39$

Whence by subtraction, $y = 3$

And, by substituting this value in the first equation,
$3x + 4 \times 3 = 18$; $x = 2$.

(3) Elimination by comparison.

Example: $3x + 4y = 18$.
$2x + 3y = 13$. $x = \frac{18 - 4y}{3} = \frac{13 - 3y}{2}$; $\therefore\ y = 3$, $x = 2$.

If more than two equations are given, eliminate one unknown quantity by combining *two* of the equations, and proceed until one equation, with one unknown quantity, remains. Then solve for that unknown quantity and substitute its value in one of the equations to obtain the value of another unknown quantity. Proceed in this manner, substituting the values thus obtained in another equation; and so on.

For Cubic Equations, see Plane Trigonometry.

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6.—LOGARITHMS OF NUMBERS.

Logarithms are useful in finding the product, quotient, powers and roots of numbers. A *system* of logarithms may be founded on any *base*, as a. If the base a raised to the mth power $= M$, then m is the logarithm of M to the base a; and conversely, M is the *anti-logarithm* (that is, the number corresponding to the logarithm) of m, to the same base. Thus,

Let $M =$ a number; m its log to base a. Then $\log_a M = m$; or $a^m = M$.
Let $N =$ a number; n its log to base a. Then $\log_a N = n$; or $a^n = N$.

The following formulas and methods illustrate the use of logarithms in the process above mentioned:—

To:	Formula:	Process: find anti-logarithm of
Multiply M by N.	$MN = a^m a^n = a^{m+n}$	(Log of M) + (log of N)
Divide M by N.	$\frac{M}{N} = \frac{a^m}{a^n} = a^{m-n}$	(Log of M) − (log of N)
Raise M to the Nth power.	$M^N = (a^m)^N = a^{mN}$	(Log of M) × N.
Extract the Nth root of M.	$\sqrt[N]{M} = \sqrt[N]{a^m} = a^{\frac{m}{N}}$	(Log of M) ÷ N.

Two systems of logarithms are in use, namely:

Common or Briggs System.
(Founded by Briggs.)
In general use for all practical purposes.

Base $a = 10$.

$a^n = N \therefore \log N = n.$

$10^0 = 1 \therefore \log 1 = 0.$

$10^1 = 10 \therefore \log 10 = 1.$

$10^2 = 100 \therefore \log 100 = 2.$

$10^{2.48} = 302 \therefore \log 302 = 2.48+$

Hyperbolic, Natural or Naperian System.
(Founded by Napier.)
Used in pure mathematical discussion and in steam engineering.
Base $= e = 2.7182818\ldots$
Derivation of e:

$e = \left(1+\frac{1}{x}\right)^x$ as x approaches infinity.

$$e = 1+1+\frac{1}{1.2}+\frac{1}{1.2.3}+\frac{1}{1.2.3.4}+\ldots$$
$$= 1+1+\tfrac{1}{2}+\tfrac{1}{6}+\tfrac{1}{24}+\ldots\ldots$$
$$= 2.7182818\ldots\ldots$$

The naperian log. of a number = its common log. × 2.3025851....
The common log. of a number = its hyperbolic log. × 0.4342945....
Common log. of 2.3025851 = 0.3622157; of 0.4342945 = 9.6377843 − 10.

Note from the above that—

The naperian log. of the naperian base ($e = 2.7182818\ldots$) = unity.
The common log. of the naperian base, 2.7182818.... = 0.4342945.
The common log. of the common base ($a = 10$) = unity.
The naperian log. of the common base, 10, = 2.3025851.

Common or Briggs System.—The logarithm of a number is composed of the *characteristic*, or integral portion to the left of the decimal point, and the *mantissa* or decimal fraction. The mantissa is all that appears in any table of logarithms and the degree of accuracy is dependent on the number of decimal places used in the mantissa. Vega's tables, to seven decimal places, are recommended for general office use in city surveying, and where the results, in general, are required accurately to the sixth or seventh significant figure.

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Table 1, following, to five decimal places, will be found compact and convenient where the result to five significant figures is sufficiently accurate. Tables to six decimal places, unless arranged on the "Vega system" (in which case they would comprise 180 pages—too bulky for any hand book) are not recommended.

In the logarithm of any number the mantissa is independent of the position of the decimal point, while on the contrary the characteristic is dependent only on the position of the first significant figure of the number with relation to the decimal point. Thus in the following examples:

(*a*)	log. 4021.7	= 3.60441		The use of the positive characteristic is generally preferred, omitting the (−10) in ordinary cases.
(*b*)	log. 402.17	= 2.60441		
(*c*)	log. 40.217	= 1.60441		
(*d*)	log. 4.0217	= 0.60441		
(*e*)	log. .40217	= $\bar{1}$.60441	= 9.60441 − 10	
(*f*)	log. .040217	= $\bar{2}$.60441	= 8.60441 − 10	

it will be seen that the characteristic is equal, *algebraically*, to the number of places minus one, which the first significant figure of the number occupies to the *left* of the decimal point. In (*a*) the characteristic is 3; in (*b*), 2; in (*d*), 0; in (*e*), − 1; and in (*f*), − 2. Some mathematicians prefer the use of the negative characteristic, but most of them employ the "positive," by algebraically adding 10 to the integer and placing −10 to the right of the mantissa or omitting the latter (−10) altogether. For example, log .040217 = 8.60441, the −10 being understood and the value of the characteristic being, of course, − 2. In the case of finding the root of (or dividing) a pure decimal, however, the −10 must be employed.

Example: Find the fourth root of .0081?

Solution:

log .0081	= 7.90849	−10
Quotient obtained by dividing by 4	= 1.97712	− 2.5
	2.5	
∴ Antilog or fourth root = 0.3. Ans.	9.47712	−10

To find the logarithm of a number.

Example: Find the log of 678.57?

Solution: The characteristic is 3 − 1 = 2. The mantissa for the first four figures, 67.85, is read directly from Table 1, and = .83155. To this, however, must be added $^7/_{10}$ (the next figure of the number is 7) of the difference between .83155 and the log of 67.86 = .83161. This difference is 6 and in the proportional parts (P. P.) column under 6 and opposite 7 will be found the value 4.2, call 4, which added to .83155 = 83159. Hence the log of 678.57 = 2.83159. Ans.

To find the anti-logarithm (number corresponding to a logarithm).

Example: 1.92513 is the logarithm of what number?

Solution: This is the reverse of finding the logarithm of a number. Neglecting, for the present, the characteristic, the next lower mantissa to .92513 is .92511 and the number corresponding is 8416. The difference between .92511 and the next higher mantissa in Table 1, .92516 is 5, and the proportional difference $\frac{.92513 - .92511}{.92516 - .92511}$ or $\frac{2}{5}$ calls for .4 to be added to the fourth figure, i. e., 4 to the fifth place of the number, as shown in the P. P. column. Therefore the number, disregarding the decimal point, is 84164. The characteristic, 1, calls for two places to the left of the decimal point, hence the antilog of 1.92513 is 84.164. Ans.

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To multiply one number by another. (Add the logs.)

Example.—Multiply 23.142 by 85.7?

Solution.—

log 23.142	=	1.36436
		4 = P. P. for 2.
log 85.7	=	1.93298
∴ Antilog = product = 1983.3 Ans.		3.29738
		32
		6 = P. P. = 3 –

To divide one number by another. (Subtract the logs.)

Example.— Divide 846.94 by 36.42?

Solution.—

log 846.94	=	2.92785
log 36.42	=	1.56134
∴ Antilog = quotient = 23.255 Ans.		1.36651
		42
		9 = P. P. = 5 –

To raise a number to any power. (Multiply the log by the index of the power.)

Example.—25.3^3 = ?

Solution.—

log 25.3	=	1.40312
Multiply by 3 :		3
∴ Antilog = 3rd power = 1619.4 Ans.		4.20936
		25
		11 P. P. = 4

To extract the root of a number. (Divide the log by the index of the root.)

Example 1. $\sqrt[5]{286.12}$ = ?

Solution.—

log 286.12	=	2.45655
Divide by 5 :		
∴ Antilog = 5th root = 3.1096 Ans.		0.49131
		122
		9 P. P. = 6

Example 2.—Find the square root of the fifth power of 23.2?

Solution.—The square root of the fifth power of $N = N^{\frac{5}{2}}$, therefore multiply the log of 23.2 by $2\frac{1}{2}$ or, better, divide by .4.

log 23.2	=	1.36549
Divide by .4 :		
∴ Antilog = $\sqrt{23.2^5}$ = 2592.5 Ans.		3.41372
		63
		9 P. P. = 5 +

Example 3.—$\sqrt{.09463}$ = ?

Solution—

log .09463	=	8.97603 – 10.
Dividing by 2	=	4.48801.5 – 5.
	–	5
∴ Antilog = square root = .30762 Ans.		9.48801.5 – 10
		799
		2.5

Naperian, Natural or Hyperbolic System.

A table of naperian logarithms of numbers from 1 to 10, advancing by hundredths, is given as a part of Table 1 of the common logarithms of numbers. The range of this table may be extended greatly, to include the logarithms of other numbers, as follows:

For logarithms of numbers from 1 to 10 advancing by hundredths and ten thousandths; use the "difference" column in the table. Thus, to find the log of 4.6953, add $^{53}/_{100}$ of the difference, 213, to the log of 4.69, equals 1.54543 + 113 = 1.54656. Ans.

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For logarithms of numbers from 10 to 100: Divide the number by 10, find the log of the quotient, from the table, and add 2.302585 (the log of 10). Thus, to find the log of 46.72. Log 4.672 = 1.54116 + 43, which, added to 2.302585 = 3.84418. Ans.

Rule.—Add 2.302585 to the log of $^1/_{10}$ of the number.

For logarithms of numbers from 100 to 1000: Divided the number by 100, find the log of the quotient, from the table, and add 2 × 2.302585 = 4.60517 (the log of 100).

Rule.—Add 4.60517 to the log of $^1/_{100}$ of the number.

In general, to find the naperian logarithm of any number, add $n \times$ 2.302585 to the naperian log of $\frac{1}{n} \times$ the number; the factor n to be selected and some power of 10, so as to bring the new number within the range of the table, i. e., from 1 to 10.

Log of 10.		Multiple:	Log of 10.		Multiple:
2.3025851	× 2 =	4.6051702	2.3025851	× 5 =	11.5129255
	× 3 =	6.9077553		× 6 =	13.8155106
	× 4 =	9.2103404		× 7 =	16.1180957

To find the antilog, proceed in a manner inverse to the above method of finding the log.

Example.—Of what number is 4.85203 the log?

Solution.—The highest multiple of 2.3025851 below the given log is 4.60517, obtained by the factor 2 corresponding with the factor 100 of the number. From the table of naperian logarithms, Table 1, we find that the number corresponding to the log of the difference, 0.24686, is 1.28. But the given log, 4.85203, is of a number 100 times as large, therefore its antilog is 1.28 × 100 = 128. Ans.

Given by log	= 4.85203
Multiple	= 4.60517
Diff.	= 0.24686

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I.—Logarithms.

No.	Common Logarithms of Numbers.														Naperian.		
	L.	O	1	2	3	4	5	6	7	8	9		P.	P.	No.	Log.	Dif.
100	00	000	043	087	130	173	217	260	303	346	389		44	43	42	1.00	.00000	995
1		432	475	518	561	604	647	689	732	775	817	1	4	4	4	1.01	.00995	985
2		860	903	945	988	030	072	115	157	199	242	2	9	9	8	1.02	.01980	976
3	01	284	326	368	410	452	494	536	578	620	662	3	13	13	13	1.03	.02956	966
4		703	745	787	828	870	912	953	995	036	078	4	18	17	17	1.04	.03922	957
105	02	119	160	202	243	284	325	366	407	449	490	5	22	22	21	1.05	.04879	948
6		531	572	612	653	694	735	776	816	857	898	6	26	26	25	1.06	.05827	939
7		938	979	019	060	100	141	181	222	262	302	7	31	30	29	1.07	.06766	930
8	03	342	383	423	463	503	543	583	623	663	703	8	35	34	34	1.08	.07696	922
9		743	782	822	862	902	941	981	021	060	100	9	40	39	38	1.09	.08618	913
110	04	139	179	218	258	297	336	376	415	454	493		41	40	39	1.10	.09531	905
1		532	571	610	650	689	727	766	805	844	883	1	4	4	4	1.11	.10436	897
2		922	961	999	038	077	115	154	192	231	269	2	8	8	8	1.12	.11333	889
3	05	308	346	385	423	461	500	538	576	614	652	3	12	12	12	1.13	.12222	881
4		690	729	767	805	843	881	918	956	994	032	4	16	16	16	1.14	.13103	873
115	06	070	108	145	183	221	258	296	333	371	408	5	21	20	20	1.15	.13976	866
6		446	483	521	558	595	633	670	707	744	781	6	25	24	23	1.16	.14842	858
7		819	856	893	930	967	004	041	078	115	151	7	29	28	27	1.17	.15700	851
8	07	188	225	262	298	335	372	408	445	482	518	8	33	32	31	1.18	.16551	844
9		555	591	628	664	700	737	773	809	846	882	9	37	36	35	1.19	.17395	837
120		918	954	990	027	063	099	135	171	207	243		38	37	36	1.20	.18232	830
1	08	279	314	350	386	422	458	493	529	565	600	1	4	4	4	1.21	.19062	823
2		636	672	707	743	778	814	849	884	920	955	2	8	7	7	1.22	.19885	816
3		991	026	061	096	132	167	202	237	272	307	3	11	11	11	1.23	.20701	810
4	09	342	377	412	447	482	517	552	587	621	656	4	15	15	14	1.24	.21511	803
125		691	726	760	795	830	864	899	934	968	003	5	19	19	18	1.25	.22314	797
6	10	037	072	106	140	175	209	243	278	312	346	6	23	22	22	1.26	.23111	791
7		380	415	449	483	517	551	585	619	653	687	7	27	26	25	1.27	.23902	784
8		721	755	789	823	857	890	924	958	992	025	8	30	30	29	1.28	.24686	778
9	11	059	093	126	160	193	227	261	294	327	361	9	34	33	32	1.29	.25464	772
130		394	428	461	494	528	561	594	628	661	694		35	34	33	1.30	.26236	767
1		727	760	793	826	860	893	926	959	992	024	1	4	3	3	1.31	.27003	760
2	12	057	090	123	156	189	222	254	287	320	352	2	7	7	7	1.32	.27763	755
3		385	418	450	483	516	548	581	613	646	678	3	11	10	10	1.33	.28518	749
4		710	743	775	808	840	872	905	937	969	001	4	14	14	13	1.34	.29267	743
135	13	033	066	098	130	162	194	226	258	290	322	5	18	17	17	1.35	.30010	738
6		354	386	418	450	481	513	545	577	609	640	6	21	20	20	1.36	.30748	733
7		672	704	735	767	799	830	862	893	925	956	7	25	24	23	1.37	.31481	727
8		988	019	051	082	114	145	176	208	239	270	8	28	27	26	1.38	.32208	722
9	14	301	333	364	395	426	457	489	520	551	582	9	32	31	30	1.39	.32930	717
140		613	644	675	706	737	768	799	829	860	891		32	31	30	1.40	.33647	712
1		922	953	983	014	045	076	106	137	168	198	1	3	3	3	1.41	.34359	707
2	15	229	259	290	320	351	381	412	442	473	503	2	6	6	6	1.42	.35066	701
3		534	564	594	625	655	685	715	746	776	806	3	10	9	9	1.43	.35767	697
4		836	866	897	927	957	987	017	047	077	107	4	13	12	12	1.44	.36464	692
145	16	137	167	197	227	256	286	316	346	376	406	5	16	16	15	1.45	.37156	688
6		435	465	495	524	554	584	613	643	673	702	6	19	19	18	1.46	.37844	682
7		732	761	791	820	850	879	909	938	967	997	7	22	22	21	1.47	.38526	678
8	17	026	056	085	114	143	173	202	231	260	289	8	26	25	24	1.48	.39204	674
9		319	348	377	406	435	464	493	522	551	580	9	29	28	27	1.49	.39878	669
150		609	638	667	696	725	754	782	811	840	869					1.50	.40547	

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1.—Logarithms.—Continued.

No.	Common Logarithms of Numbers.														Naperian.		
	L.	0	1	2	3	4	5	6	7	8	9		P.	P.	No.	Log.	Dif.
150	17	609	638	667	696	725	754	782	811	840	869		29	28	1.50	.40547	664
1		898	926	955	984	013	041	070	099	127	156	1	3	3	1.51	.41211	660
2	18	184	213	241	270	298	327	355	384	412	441	2	6	6	1.52	.41871	656
3		469	498	526	554	583	611	639	667	696	724	3	9	8	1.53	.42527	651
4		752	780	808	837	865	893	921	949	977	005	4	12	11	1.54	.43178	647
155	19	033	061	089	117	145	173	201	229	257	285	5	15	14	1.55	.43825	644
6		312	340	368	396	424	451	479	507	535	562	6	17	17	1.56	.44469	639
7		590	618	645	673	700	728	756	783	811	838	7	20	20	1.57	.45108	634
8		866	893	921	948	976	003	030	058	085	112	8	23	22	1.58	.45742	631
9	20	140	167	194	222	249	276	303	330	358	385	9	26	25	1.59	.46373	627
160		412	439	466	493	520	548	575	602	629	656		27	26	1.60	.47000	623
1		683	710	737	763	790	817	844	871	898	925	1	3	3	1.61	.47623	620
2		952	978	005	032	059	085	112	139	165	192	2	5	5	1.62	.48243	615
3	21	219	245	272	299	325	352	378	405	431	458	3	8	8	1.63	.48858	612
4		484	511	537	564	590	617	643	669	696	722	4	11	10	1.64	.49470	608
165		748	775	801	827	854	880	906	932	958	985	5	14	13	1.65	.50078	604
6	22	011	037	063	089	115	141	167	194	220	246	6	16	16	1.66	.50682	600
7		272	298	324	350	376	401	427	453	479	505	7	19	18	1.67	.51282	597
8		531	557	583	608	634	660	686	712	737	763	8	22	21	1.68	.51879	594
9		789	814	840	866	891	917	943	968	994	019	9	24	23	1.69	.52473	590
170	23	045	070	096	121	147	172	198	223	249	274		25	24	1.70	.53063	586
1		300	325	350	376	401	426	452	477	502	528	1	3	2	1.71	.53649	583
2		553	578	603	629	654	679	704	729	754	779	2	5	5	1.72	.54232	580
3		805	830	855	880	905	930	955	980	005	030	3	8	7	1.73	.54812	577
4	24	055	080	105	130	155	180	204	229	254	279	4	10	10	1.74	.55389	573
175		304	329	353	378	403	428	452	477	502	527	5	13	12	1.75	.55962	569
6		551	576	601	625	650	674	699	724	748	773	6	15	14	1.76	.56531	567
7		797	822	846	871	895	920	944	969	993	018	7	18	17	1.77	.57098	563
8	25	042	066	091	115	139	164	188	212	237	261	8	20	19	1.78	.57661	561
9		285	310	334	358	382	406	431	455	479	503	9	23	22	1.79	.58222	557
180		527	551	575	600	624	648	672	696	720	744		24	23	1.80	.58779	554
1		768	792	816	840	864	888	912	935	959	983	1	2	2	1.81	.59333	551
2	26	007	031	055	079	102	126	150	174	198	221	2	5	5	1.82	.59884	548
3		245	269	293	316	340	364	387	411	435	458	3	7	7	1.83	.60432	545
4		482	505	529	553	576	600	623	647	670	694	4	10	9	1.84	.60977	542
185		717	741	764	788	811	834	858	881	905	928	5	12	12	1.85	.61519	539
6		951	975	998	021	045	068	091	114	138	161	6	14	14	1.86	.62058	536
7	27	184	207	231	254	277	300	323	346	370	393	7	17	16	1.87	.62594	533
8		416	439	462	485	508	531	554	577	600	623	8	19	18	1.88	.63127	531
9		646	669	692	715	738	761	784	807	830	852	9	22	21	1.89	.63658	527
190		875	898	921	944	967	989	012	035	058	081		22	21	1.90	.64185	525
1	28	103	126	149	171	194	217	240	262	285	307	1	2	2	1.91	.64710	523
2		330	353	375	398	421	443	466	488	511	533	2	4	4	1.92	.65233	519
3		556	578	601	623	646	668	691	713	735	758	3	7	6	1.93	.65752	517
4		780	803	825	847	870	892	914	937	959	981	4	9	8	1.94	.66269	514
195	29	003	026	048	070	092	115	137	159	181	203	5	11	11	1.95	.66783	511
6		226	248	270	292	314	336	358	380	403	425	6	13	13	1.96	.67294	509
7		447	469	491	513	535	557	579	601	623	645	7	15	15	1.97	.67803	507
8		667	688	710	732	754	776	798	820	842	863	8	18	17	1.98	.68310	503
9		885	907	929	951	973	994	016	038	060	081	9	20	19	1.99	.68813	502
200	30	103	125	146	168	190	211	233	255	276	298				2.00	.69315	

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1.—Logarithms.—Continued.

No.	Common Logarithms of Numbers.													Naperian.		
	L.	0	1	2	3	4	5	6	7	8	9	P.	P.	No.	Log.	Dif.
200	30	103	125	146	168	190	211	233	255	276	298		22	2.00	.69315	498
1		320	341	363	384	406	428	449	471	492	514	1	2	2.01	.69813	497
2		535	557	578	600	621	643	664	685	707	728	2	4	2.02	.70310	494
3		750	771	792	814	835	856	878	899	920	942	3	7	2.03	.70804	491
4		963	984	006	027	048	069	091	112	133	154	4	9	2.04	.71295	489
205	31	175	197	218	239	260	281	302	323	345	366	5	11	2.05	.71784	487
6		387	408	429	450	471	492	513	534	555	576	6	13	2.06	.72271	484
7		597	618	639	660	681	702	723	744	765	785	7	15	2.07	.72755	482
8		806	827	848	869	890	911	931	952	973	994	8	18	2.08	.73237	479
9	32	015	035	056	077	098	118	139	160	181	201	9	20	2.09	.73716	478
210		222	243	263	284	305	325	346	366	387	408		21	2.10	.74194	475
1		428	449	469	490	510	531	552	572	593	613	1	2	2.11	.74669	473
2		634	654	675	695	715	736	756	777	797	818	2	4	2.12	.75142	470
3		838	858	879	899	919	940	960	980	001	021	3	6	2.13	.75612	469
4	33	041	062	082	102	122	143	163	183	203	224	4	8	2.14	.76081	466
215		244	264	284	304	325	345	365	385	405	425	5	11	2.15	.76547	464
6		445	465	486	506	526	546	566	586	606	626	6	13	2.16	.77011	463
7		646	666	686	706	726	746	766	786	806	826	7	15	2.17	.77473	459
8		846	866	885	905	925	945	965	985	005	025	8	17	2.18	.77932	458
9	34	044	064	084	104	124	143	163	183	203	223	9	19	2.19	.78390	456
220		242	262	282	301	321	341	361	380	400	420		20	2.20	.78846	453
1		439	459	479	498	518	537	557	577	596	616	1	2	2.21	.79299	452
2		635	655	674	694	713	733	753	772	792	811	2	4	2.22	.79751	449
3		830	850	869	889	908	928	947	967	986	005	3	6	2.23	.80200	448
4	35	025	044	064	083	102	122	141	160	180	199	4	8	2.24	.80648	445
225		218	238	257	276	295	315	334	353	372	392	5	10	2.25	.81093	443
6		411	430	449	468	488	507	526	545	564	583	6	12	2.26	.81536	442
7		603	622	641	660	679	698	717	736	755	774	7	14	2.27	.81978	440
8		793	813	832	851	870	889	908	927	946	965	8	16	2.28	.82418	437
9		984	003	021	040	059	078	097	116	135	154	9	18	2.29	.82855	436
230	36	173	192	211	229	248	267	286	305	324	342		19	2.30	.83291	434
1		361	380	399	418	436	455	474	493	511	530	1	2	2.31	.83725	432
2		549	568	586	605	624	642	661	680	698	717	2	4	2.32	.84157	430
3		736	754	773	791	810	829	847	866	884	903	3	6	2.33	.84587	428
4		922	940	959	977	996	014	033	051	070	088	4	8	2.34	.85015	427
235	37	107	125	144	162	181	199	218	236	254	273	5	10	2.35	.85442	424
6		291	310	328	346	365	383	401	420	438	457	6	11	2.36	.85866	423
7		475	493	511	530	548	566	585	603	621	639	7	13	2.37	.86289	421
8		658	676	694	712	731	749	767	785	803	822	8	15	2.38	.86710	419
9		840	858	876	894	912	931	949	967	985	003	9	17	2.39	.87129	418
240	38	021	039	057	075	093	112	130	148	166	184		18	2.40	.87547	416
1		202	220	238	256	274	292	310	328	346	364	1	2	2.41	.87963	414
2		382	399	417	435	453	471	489	507	525	543	2	4	2.42	.88377	412
3		561	578	596	614	632	650	668	686	703	721	3	5	2.43	.88789	411
4		739	757	775	792	810	828	846	863	881	899	4	7	2.44	.89200	409
245		917	934	952	970	987	005	023	041	058	076	5	9	2.45	.89609	407
6	39	094	111	129	146	164	182	199	217	235	252	6	11	2.46	.90016	406
7		270	287	305	322	340	358	375	393	410	428	7	13	2.47	.90422	404
8		445	463	480	498	515	533	550	568	585	602	8	14	2.48	.90826	402
9		620	637	655	672	690	707	724	742	759	777	9	16	2.49	.91228	401
250		794	811	829	846	863	881	898	915	933	950			2.50	.91629	

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1.—Logarithms.—Continued.

No.	Common Logarithms of Numbers.													Naperian.		
	L.	0	1	2	3	4	5	6	7	8	9	P.	P.	No.	Log.	Dif.
250	39	794	811	829	846	863	881	898	915	933	950		18	2.50	.91629	399
1		967	985	002	019	037	054	071	088	106	123	1	2	2.51	.92028	398
2	40	140	157	175	192	209	226	243	261	278	295	2	4	2.52	.92426	396
3		312	329	346	364	381	398	415	432	449	466	3	5	2.53	.92822	394
4		483	500	518	535	552	569	586	603	620	637	4	7	2.54	.93216	393
255		654	671	688	705	722	739	756	773	790	807	5	9	2.55	.93609	392
6		824	841	858	875	892	909	926	943	960	976	6	11	2.56	.94001	390
7		993	010	027	044	061	078	095	111	128	145	7	13	2.57	.94391	388
8	41	162	179	196	212	229	246	263	280	296	313	8	14	2.58	.94779	387
9		330	347	363	380	397	414	430	447	464	481	9	16	2.59	.95166	385
260		497	514	531	547	564	581	597	614	631	647		17	2.60	.95551	384
1		664	681	697	714	731	747	764	780	797	814	1	2	2.61	.95935	382
2		830	847	863	880	896	913	929	946	963	979	2	3	2.62	.96317	381
3		996	012	029	045	062	078	095	111	127	144	3	5	2.63	.96698	380
4	42	160	177	193	210	226	243	259	275	292	308	4	7	2.64	.97078	378
265		325	341	357	374	390	406	423	439	455	472	5	9	2.65	.97456	377
6		488	504	521	537	553	570	586	602	619	635	6	10	2.66	.97833	375
7		651	667	684	700	716	732	749	765	781	797	7	12	2.67	.98208	374
8		813	830	846	862	878	894	911	927	943	959	8	14	2.68	.98582	372
9		975	991	008	024	040	056	072	088	104	120	9	15	2.69	.98954	371
270	43	136	152	169	185	201	217	233	249	265	281		16	2.70	.99325	370
1		297	313	329	345	361	377	393	409	425	441	1	2	2.71	.99695	368
2		457	473	489	505	521	537	553	569	584	600	2	3	2.72	1.00063	367
3		616	632	648	664	680	696	712	727	743	759	3	5	2.73	1.00430	366
4		775	791	807	823	838	854	870	886	902	917	4	6	2.74	1.00796	364
275		933	949	965	981	996	012	028	044	059	075	5	8	2.75	1.01160	363
6	44	091	107	122	138	154	170	185	201	217	232	6	10	2.76	1.01523	362
7		248	264	279	295	311	326	342	358	373	389	7	11	2.77	1.01885	360
8		404	420	436	451	467	483	498	514	529	545	8	13	2.78	1.02245	359
9		560	576	592	607	623	638	654	669	685	700	9	14	2.79	1.02604	358
280		716	731	747	762	778	793	809	824	840	855		15	2.80	1.02962	356
1		871	886	902	917	932	948	963	979	994	010	1	2	2.81	1.03318	356
2	45	025	040	056	071	086	102	117	133	148	163	2	3	2.82	1.03674	354
3		179	194	209	225	240	255	271	286	301	317	3	5	2.83	1.04028	352
4		332	347	362	378	393	408	423	439	454	469	4	6	2.84	1.04380	352
285		484	500	515	530	545	561	576	591	606	621	5	8	2.85	1.04732	350
6		637	652	667	682	697	712	728	743	758	773	6	9	2.86	1.05082	349
7		788	803	818	834	849	864	879	894	909	924	7	11	2.87	1.05431	348
8		939	954	969	984	000	015	030	045	060	075	8	12	2.88	1.05779	347
9	46	090	105	120	135	150	165	180	195	210	225	9	14	2.89	1.06126	345
290		240	255	270	285	300	315	330	345	359	374		14	2.90	1.06471	344
1		389	404	419	434	449	464	479	494	509	523	1	1	2.91	1.06815	343
2		538	553	568	583	598	613	627	642	657	672	2	3	2.92	1.07158	342
3		687	702	716	731	746	761	776	790	805	820	3	4	2.93	1.07500	341
4		835	850	864	879	894	909	923	938	953	967	4	6	2.94	1.07841	340
295		982	997	012	026	041	056	070	085	100	114	5	7	2.95	1.08181	338
6	47	129	144	159	173	188	202	217	232	246	261	6	8	2.96	1.08519	337
7		276	290	305	319	334	349	363	378	392	407	7	10	2.97	1.08856	336
8		422	436	451	465	480	494	509	524	538	553	8	11	2.98	1.09192	335
9		567	582	596	611	625	640	654	669	683	698	9	13	2.99	1.09527	334
300		712	727	741	756	770	784	799	813	828	842			3.00	1.09861	

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1.—Logarithms.—Continued.

No.	Common Logarithms of Numbers.													Naperian.		
	L.	0	1	2	3	4	5	6	7	8	9	P.	P.	No.	Log.	Dif.
300	47	712	727	741	756	770	784	799	813	828	842		15	3.00	1.09861	333
1		857	871	885	900	914	929	943	958	972	986	1	2	3.01	1.10194	332
2	48	001	015	029	044	058	073	087	101	116	130	2	3	3.02	1.10526	330
3		144	159	173	187	202	216	230	244	259	273	3	5	3.03	1.10856	330
4		287	302	316	330	344	359	373	387	401	416	4	6	3.04	1.11186	328
305		430	444	458	473	487	501	515	530	544	558	5	8	3.05	1.11514	327
6		572	586	601	615	629	643	657	671	686	700	6	9	3.06	1.11841	327
7		714	728	742	756	770	785	799	813	827	841	7	11	3.07	1.12168	325
8		855	869	883	897	911	926	940	954	968	982	8	12	3.08	1.12493	324
9		996	010	024	038	052	066	080	094	108	122	9	14	3.09	1.12817	323
310	49	136	150	164	178	192	206	220	234	248	262		14	3.10	1.13140	322
1		276	290	304	318	332	346	360	374	388	402	1	1	3.11	1.13462	321
2		415	429	443	457	471	485	499	513	527	541	2	3	3.12	1.13783	320
3		554	568	582	596	610	624	638	651	665	679	3	4	3.13	1.14103	319
4		693	707	721	734	748	762	776	790	803	817	4	6	3.14	1.14422	318
315		831	845	859	872	886	900	914	927	941	955	5	7	3.15	1.14740	317
6		969	982	996	010	024	037	051	065	079	092	6	8	3.16	1.15057	316
7	50	106	120	133	147	161	174	188	202	215	229	7	10	3.17	1.15373	315
8		243	256	270	284	297	311	325	338	352	365	8	11	3.18	1.15688	314
9		379	393	406	420	433	447	461	474	488	501	9	13	3.19	1.16002	313
320		515	529	542	556	569	583	596	610	623	637		13	3.20	1.16315	312
1		651	664	678	691	705	718	732	745	759	772	1	1	3.21	1.16627	311
2		786	799	813	826	840	853	866	880	893	907	2	3	3.22	1.16938	310
3		920	934	947	961	974	987	001	014	028	041	3	4	3.23	1.17248	309
4	51	055	068	081	095	108	121	135	148	162	175	4	5	3.24	1.17557	308
325		188	202	215	228	242	255	268	282	295	308	5	7	3.25	1.17865	308
6		322	335	348	362	375	388	402	415	428	441	6	8	3.26	1.18173	306
7		455	468	481	495	508	521	534	548	561	574	7	9	3.27	1.18479	305
8		587	601	614	627	640	654	667	680	693	706	8	10	3.28	1.18784	305
9		720	733	746	759	772	786	799	812	825	838	9	12	3.29	1.19089	303
330		851	865	878	891	904	917	930	943	957	970		13	3.30	1.19392	303
1		983	996	009	022	035	048	061	075	088	101	1	1	3.31	1.19695	301
2	52	114	127	140	153	166	179	192	205	218	231	2	3	3.32	1.19996	301
3		244	257	270	284	297	310	323	336	349	362	3	4	3.33	1.20297	300
4		375	388	401	414	427	440	453	466	479	492	4	5	3.34	1.20597	299
335		504	517	530	543	556	569	582	595	608	621	5	7	3.35	1.20896	298
6		634	647	660	673	686	699	711	724	737	750	6	8	3.36	1.21194	297
7		763	776	789	802	815	827	840	853	866	879	7	9	3.37	1.21491	297
8		892	905	917	930	943	956	969	982	994	007	8	10	3.38	1.21788	295
9	53	020	033	046	058	071	084	097	110	122	135	9	12	3.39	1.22083	295
340		148	161	173	186	199	212	224	237	250	263		12	3.40	1.22378	293
1		275	288	301	314	326	339	352	364	377	390	1	1	3.41	1.22671	293
2		403	415	428	441	453	466	479	491	504	517	2	2	3.42	1.22964	292
3		529	542	555	567	580	593	605	618	631	643	3	4	3.43	1.23256	291
4		656	668	681	694	706	719	732	744	757	769	4	5	3.44	1.23547	290
345		782	794	807	820	832	845	857	870	882	895	5	6	3.45	1.23837	290
6		908	920	933	945	958	970	983	995	008	020	6	7	3.46	1.24127	288
7	54	033	045	058	070	083	095	108	120	133	145	7	8	3.47	1.24415	288
8		158	170	183	195	208	220	233	245	258	270	8	10	3.48	1.24703	287
9		283	295	307	320	332	345	357	370	382	394	9	11	3.49	1.24990	286
350		407	419	432	444	456	469	481	494	506	518			3.50	1.25276	

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1.—Logarithms.—Continued.

No.	Common Logarithms of Numbers.													Naperian.		
	L.	0	1	2	3	4	5	6	7	8	9	P.	P.	No.	Log.	Dif.
350	54	407	419	432	444	456	469	481	494	506	518		13	3.50	1.25276	286
1		531	543	555	568	580	593	605	617	630	642	1	1	3.51	1.25562	284
2		654	667	679	691	704	716	728	741	753	765	2	3	3.52	1.25846	284
3		777	790	802	814	827	839	851	864	876	888	3	4	3.53	1.26130	283
4		900	913	925	937	949	962	974	986	998	011	4	5	3.54	1.26413	282
355	55	023	035	047	060	072	084	096	108	121	133	5	7	3.55	1.26695	281
6		145	157	169	182	194	206	218	230	242	255	6	8	3.56	1.26976	281
7		267	279	291	303	315	328	340	352	364	376	7	9	3.57	1.27257	279
8		388	400	413	425	437	449	461	473	485	497	8	10	3.58	1.27536	279
9		509	522	534	546	558	570	582	594	606	618	9	12	3.59	1.27815	278
360		630	642	654	666	678	691	703	715	727	739			3.60	1.28093	278
1		751	763	775	787	799	811	823	835	847	859	1		3.61	1.28371	276
2		871	883	895	907	919	931	943	955	967	979	2		3.62	1.28647	276
3		991	003	015	027	038	050	062	074	086	098	3		3.63	1.28923	275
4	56	110	122	134	146	158	170	182	194	205	217	4		3.64	1.29198	275
365		229	241	253	265	277	289	301	312	324	336	5		3.65	1.29473	273
6		348	360	372	384	396	407	419	431	443	455	6		3.66	1.29746	273
7		467	478	490	502	514	526	538	549	561	573	7		3.67	1.30019	272
8		585	597	608	620	632	644	656	667	679	691	8		3.68	1.30291	272
9		703	714	726	738	750	761	773	785	797	808	9		3.69	1.30563	270
370		820	832	844	855	867	879	891	902	914	926		12	3.70	1.30833	270
1		937	949	961	972	984	996	008	019	031	043	1	1	3.71	1.31103	269
2	57	054	066	078	089	101	113	124	136	148	159	2	2	3.72	1.31372	269
3		171	183	194	206	217	229	241	252	264	276	3	4	3.73	1.31641	268
4		287	299	310	322	334	345	357	368	380	392	4	5	3.74	1.31909	267
375		403	415	426	438	449	461	473	484	496	507	5	6	3.75	1.32176	266
6		519	530	542	553	565	576	588	600	611	623	6	7	3.76	1.32442	265
7		634	646	657	669	680	692	703	715	726	738	7	8	3.77	1.32707	265
8		749	761	772	784	795	807	818	830	841	852	8	10	3.78	1.32972	265
9		864	875	887	898	910	921	933	944	955	967	9	11	3.79	1.33237	263
380		978	990	001	013	024	035	047	058	070	081			3.80	1.33500	263
1	58	092	104	115	127	138	149	161	172	184	195	1		3.81	1.33763	262
2		206	218	229	240	252	263	274	286	297	309	2		3.82	1.34025	261
3		320	331	343	354	365	377	388	399	410	422	3		3.83	1.34286	261
4		433	444	456	467	478	490	501	512	524	535	4		3.84	1.34547	260
385		546	557	569	580	591	602	614	625	636	647	5		3.85	1.34807	260
6		659	670	681	692	704	715	726	737	749	760	6		3.86	1.35067	258
7		771	782	794	805	816	827	838	850	861	872	7		3.87	1.35325	259
8		883	894	906	917	928	939	950	961	973	984	8		3.88	1.35584	257
9		995	006	017	028	040	051	062	073	084	095	9		3.89	1.35841	257
390	59	106	118	129	140	151	162	173	184	195	207		11	3.90	1.36098	256
1		218	229	240	251	262	273	284	295	306	318	1	1	3.91	1.36354	255
2		329	340	351	362	373	384	395	406	417	428	2	2	3.92	1.36609	255
3		439	450	461	472	483	494	506	517	528	539	3	3	3.93	1.36864	254
4		550	561	572	583	594	605	616	627	638	649	4	4	3.94	1.37118	254
395		660	671	682	693	704	715	726	737	748	759	5	6	3.95	1.37372	252
6		770	780	791	802	813	824	835	846	857	868	6	7	3.96	1.37624	253
7		879	890	901	912	923	934	945	956	966	977	7	8	3.97	1.37877	251
8		988	999	010	021	032	043	054	065	076	086	8	9	3.98	1.38128	251
9	60	097	108	119	130	141	152	163	173	184	195	9	10	3.99	1.38379	250
400		206	217	228	239	249	260	271	282	293	304			4.00	1.38629	

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1.—Logarithms.—Continued.

No.	Common Logarithms of Numbers.													Napierian.		
	L.	0	1	2	3	4	5	6	7	8	9	P.	P.	No.	Log.	Dif.
400	60	206	217	228	239	249	260	271	282	293	304		11	4.00	1.38629	250
1		314	325	336	347	358	369	379	390	401	412	1	1	4.01	1.38879	249
2		423	433	444	455	466	477	487	498	509	520	2	2	4.02	1.39128	249
3		531	541	552	563	574	584	595	606	617	627	3	3	4.03	1.39377	247
4		638	649	660	670	681	692	703	713	724	735	4	4	4.04	1.39624	248
405		746	756	767	778	788	799	810	821	831	842	5	6	4.05	1.39872	246
6		853	863	874	885	895	906	917	927	938	949	6	7	4.06	1.40118	246
7		959	970	981	991	002	013	023	034	045	055	7	8	4.07	1.40364	246
8	61	066	077	087	098	109	119	130	140	151	162	8	9	4.08	1.40610	244
9		172	183	194	204	215	225	236	247	257	268	9	10	4.09	1.40854	245
410		278	289	300	310	321	331	342	352	363	374			4.10	1.41099	243
1		384	395	405	416	426	437	448	458	469	479	1		4.11	1.41342	243
2		490	500	511	521	532	542	553	563	574	584	2		4.12	1.41585	243
3		595	606	616	627	637	648	658	669	679	690	3		4.13	1.41828	242
4		700	711	721	731	742	752	763	773	784	794	4		4.14	1.42070	241
415		805	815	826	836	847	857	868	878	888	899	5		4.15	1.42311	241
6		909	920	930	941	951	962	972	982	993	003	6		4.16	1.42552	240
7	62	014	024	034	045	055	066	076	086	097	107	7		4.17	1.42792	239
8		118	128	138	149	159	170	180	190	201	211	8		4.18	1.43031	239
9		221	232	242	252	263	273	284	294	304	315	9		4.19	1.43270	238
420		325	335	346	356	366	377	387	397	408	418		10	4.20	1.43508	238
1		428	439	449	459	469	480	490	500	511	521	1	1	4.21	1.43746	238
2		531	542	552	562	572	583	593	603	613	624	2	2	4.22	1.43984	236
3		634	644	655	665	675	685	696	706	716	726	3	3	4.23	1.44220	236
4		737	747	757	767	778	788	798	808	818	829	4	4	4.24	1.44456	236
425		839	849	859	870	880	890	900	910	921	931	5	5	4.25	1.44692	235
6		941	951	961	972	982	992	002	012	022	033	6	6	4.26	1.44927	234
7	63	043	053	063	073	083	094	104	114	124	134	7	7	4.27	1.45161	234
8		144	155	165	175	185	195	205	215	225	236	8	8	4.28	1.45395	234
9		246	256	266	276	286	296	306	317	327	337	9	9	4.29	1.45629	232
430		347	357	367	377	387	397	407	417	428	438			4.30	1.45861	233
1		448	458	468	478	488	498	508	518	528	538	1		4.31	1.46094	232
2		548	558	568	579	589	599	609	619	629	639	2		4.32	1.46326	231
3		649	659	669	679	689	699	709	719	729	739	3		4.33	1.46557	230
4		749	759	769	779	789	799	809	819	829	839	4		4.34	1.46787	231
435		849	859	869	879	889	899	909	919	929	939	5		4.35	1.47018	229
6		949	959	969	979	988	998	008	018	028	038	6		4.36	1.47247	229
7	64	048	058	068	078	088	098	108	118	128	137	7		4.37	1.47476	229
8		147	157	167	177	187	197	207	217	227	237	8		4.38	1.47705	228
9		246	256	266	276	286	296	306	316	326	335	9		4.39	1.47933	227
440		345	355	365	375	385	395	404	414	424	434		9	4.40	1.48160	227
1		444	454	464	473	483	493	503	513	523	532	1	1	4.41	1.48387	227
2		542	552	562	572	582	591	601	611	621	631	2	2	4.42	1.48614	226
3		640	650	660	670	680	689	699	709	719	729	3	3	4.43	1.48840	225
4		738	748	758	768	777	787	797	807	816	826	4	4	4.44	1.49065	225
445		836	846	856	865	875	885	895	904	914	924	5	4.5	4.45	1.49290	225
6		933	943	953	963	972	982	992	002	011	021	6	5.4	4.46	1.49515	224
7	65	031	040	050	060	070	079	089	099	108	118	7	6	4.47	1.49739	223
8		128	137	147	157	167	176	186	196	205	215	8	7	4.48	1.49962	223
9		225	234	244	254	263	273	283	292	302	312	9	8	4.49	1.50185	223
450		321	331	341	350	360	369	379	389	398	408			4.50	1.50408	

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1.—Logarithms.—Continued.

No.	Common Logarithms of Numbers.													Naperian.		
	L.	0	1	2	3	4	5	6	7	8	9	P.	P.	No.	Log.	Dif.
450	65	321	331	341	350	360	369	379	389	398	408		10	4.50	1.50408	222
1		418	427	437	447	456	466	475	485	495	504	1	1	4.51	1.50630	221
2		514	523	533	543	552	562	571	581	591	600	2	2	4.52	1.50851	221
3		610	619	629	639	648	658	667	677	686	696	3	3	4.53	1.51072	221
4		706	715	725	734	744	753	763	772	782	792	4	4	4.54	1.51293	221
																220
455		801	811	820	830	839	849	858	868	877	887	5	5	4.55	1.51513	219
6		896	906	916	925	935	944	954	963	973	982	6	6	4.56	1.51732	219
7		992	001	011	020	030	039	049	058	068	077	7	7	4.57	1.51951	219
8	66	087	096	106	115	124	134	143	153	162	172	8	8	4.58	1.52170	218
9		181	191	200	210	219	229	238	247	257	266	9	9	4.59	1.52388	218
460		276	285	295	304	314	323	332	342	351	361			4.60	1.52606	217
1		370	380	389	398	408	417	427	436	445	455	1		4.61	1.52823	216
2		464	474	483	492	502	511	521	530	539	549	2		4.62	1.53039	217
3		558	567	577	586	596	605	614	624	633	642	3		4.63	1.53256	215
4		652	661	671	680	689	699	708	717	727	736	4		4.64	1.53471	216
465		745	755	764	773	783	792	801	811	820	829	5		4.65	1.53687	215
6		839	848	857	867	876	885	894	904	913	922	6		4.66	1.53902	214
7		932	941	950	960	969	978	987	997	006	015	7		4.67	1.54116	214
8	67	025	034	043	052	062	071	080	089	099	108	8		4.68	1.54330	213
9		117	127	136	145	154	164	173	182	191	201	9		4.69	1.54543	213
470		210	219	228	237	247	256	265	274	284	293		9	4.70	1.54756	213
1		302	311	321	330	339	348	357	367	376	385	1	1	4.71	1.54969	212
2		394	403	413	422	431	440	449	459	468	477	2	2	4.72	1.55181	212
3		486	495	504	514	523	532	541	550	560	569	3	3	4.73	1.55393	211
4		578	587	596	605	614	624	633	642	651	660	4	4	4.74	1.55604	210
475		669	679	688	697	706	715	724	733	742	752	5	4.5	4.75	1.55814	211
6		761	770	779	788	797	806	815	825	834	843	6	5.4	4.76	1.56025	210
7		852	861	870	879	888	897	906	916	925	934	7	6	4.77	1.56235	209
8		943	952	961	970	979	988	997	006	015	024	8	7	4.78	1.56444	209
9	68	034	043	052	061	070	079	088	097	106	115	9	8	4.79	1.56653	209
480		124	133	142	151	160	169	178	187	196	205			4.80	1.56862	208
1		215	224	233	242	251	260	269	278	287	296	1		4.81	1.57070	207
2		305	314	323	332	341	350	359	368	377	386	2		4.82	1.57277	208
3		395	404	413	422	431	440	449	458	467	476	3		4.83	1.57485	206
4		485	494	502	511	520	529	538	547	556	565	4		4.84	1.57691	207
485		574	583	592	601	610	619	628	637	646	655	5		4.85	1.57898	206
6		664	673	681	690	699	708	717	726	735	744	6		4.86	1.58104	205
7		753	762	771	780	789	797	806	815	824	833	7		4.87	1.58309	206
8		842	851	860	869	878	886	895	904	913	922	8		4.88	1.58515	204
9		931	940	949	958	966	975	984	993	002	011	9		4.89	1.58719	205
490	69	020	028	037	046	055	064	073	082	090	099		8	4.90	1.58924	203
1		108	117	126	135	144	152	161	170	179	188	1	1	4.91	1.59127	204
2		197	205	214	223	232	241	249	258	267	276	2	1.6	4.92	1.59331	203
3		285	294	302	311	320	329	338	346	355	364	3	2.4	4.93	1.59534	203
4		373	381	390	399	408	417	425	434	443	452	4	3	4.94	1.59737	202
495		461	469	478	487	496	504	513	522	531	539	5	4	4.95	1.59939	202
6		548	557	566	574	583	592	601	609	618	627	6	5	4.96	1.60141	201
7		636	644	653	662	671	679	688	697	705	714	7	5.6	4.97	1.60342	201
8		723	732	740	749	758	767	775	784	793	801	8	6.4	4.98	1.60543	201
9		810	819	827	836	845	854	862	871	880	888	9	7	4.99	1.60744	200
500		897	906	914	923	932	940	949	958	966	975			5.00	1.60944	

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1.—Logarithms.—Continued.

No.	Common Logarithms of Numbers.											P.	P.	Naperian.		
	L.	0	1	2	3	4	5	6	7	8	9			No.	Log.	Dif.
500	69	897	906	914	923	932	940	949	958	966	975		9	5.00	1.60944	200
1		984	992	001	010	018	027	036	044	053	062	1	1	5.01	1.61144	199
2	70	070	079	088	096	105	114	122	131	140	148	2	2	5.02	1.61343	199
3		157	165	174	183	191	200	209	217	226	234	3	3	5.03	1.61542	199
4		243	252	260	269	278	286	295	303	312	321	4	4	5.04	1.61741	199
505		329	338	346	355	364	372	381	389	398	406	5	4.5	5.05	1.61939	198
6		415	424	432	441	449	458	467	475	484	492	6	5.4	5.06	1.62137	198
7		501	509	518	526	535	544	552	561	569	578	7	6	5.07	1.62334	197
8		586	595	603	612	621	629	638	646	655	663	8	7	5.08	1.62531	197
9		672	680	689	697	706	714	723	731	740	749	9	8	5.09	1.62728	197
510		757	766	774	783	791	800	808	817	825	834			5.10	1.62924	196
1		842	851	859	868	876	885	893	902	910	919	1		5.11	1.63120	196
2		927	935	944	952	961	969	978	986	995	003	2		5.12	1.63315	195
3	71	012	020	029	037	046	054	063	071	079	088	3		5.13	1.63511	196
4		096	105	113	122	130	139	147	155	164	172	4		5.14	1.63706	194
515		181	189	198	206	214	223	231	240	248	257	5		5.15	1.63900	195
6		265	273	282	290	299	307	315	324	332	341	6		5.16	1.64094	194
7		349	357	366	374	383	391	399	408	416	425	7		5.17	1.64287	193
8		433	441	450	458	466	475	483	492	500	508	8		5.18	1.64481	194
9		517	525	533	542	550	559	567	575	584	592	9		5.19	1.64673	192
520		600	609	617	625	634	642	650	659	667	675		8	5.20	1.64866	193
1		684	692	700	709	717	725	734	742	750	759	1	1	5.21	1.65058	192
2		767	775	784	792	800	809	817	825	834	842	2	1.6	5.22	1.65250	192
3		850	858	867	875	883	892	900	908	917	925	3	2.4	5.23	1.65441	191
4		933	941	950	958	966	975	983	991	999	008	4	3	5.24	1.65632	191
525	72	016	024	032	041	049	057	066	074	082	090	5	4	5.25	1.65823	191
6		099	107	115	123	132	140	148	156	165	173	6	5	5.26	1.66013	190
7		181	189	198	206	214	222	230	239	247	255	7	5.6	5.27	1.66203	190
8		263	272	280	288	296	304	313	321	329	337	8	6.4	5.28	1.66393	190
9		346	354	362	370	378	387	395	403	411	419	9	7	5.29	1.66582	189
530		428	436	444	452	460	469	477	485	493	501			5.30	1.66771	189
1		509	518	526	534	542	550	558	567	575	583	1		5.31	1.66959	188
2		591	599	607	616	624	632	640	648	656	665	2		5.32	1.67147	188
3		673	681	689	697	705	713	722	730	738	746	3		5.33	1.67335	188
4		754	762	770	779	787	795	803	811	819	827	4		5.34	1.67523	188
535		835	843	852	860	868	876	884	892	900	908	5		5.35	1.67710	187
6		916	925	933	941	949	957	965	973	981	989	6		5.36	1.67896	186
7		997	006	014	022	030	038	046	054	062	070	7		5.37	1.68083	187
8	73	078	086	094	102	111	119	127	135	143	151	8		5.38	1.68269	186
9		159	167	175	183	191	199	207	215	223	231	9		5.39	1.68455	186
540		239	247	255	263	272	280	288	296	304	312		7	5.40	1.68640	185
1		320	328	336	344	352	360	368	376	384	392	1	0.7	5.41	1.68825	185
2		400	408	416	424	432	440	448	456	464	472	2	1.4	5.42	1.69010	185
3		480	488	496	504	512	520	528	536	544	552	3	2	5.43	1.69194	184
4		560	568	576	584	592	600	608	616	624	632	4	3	5.44	1.69378	184
545		640	648	656	664	672	679	687	695	703	711	5	3.5	5.45	1.69562	184
6		719	727	735	743	751	759	767	775	783	791	6	4.2	5.46	1.69745	183
7		799	807	815	823	830	838	846	854	862	870	7	5	5.47	1.69928	183
8		878	886	894	902	910	918	926	933	911	949	8	5.6	5.48	1.70111	183
9		957	965	973	981	989	997	005	013	020	028	9	6.3	5.49	1.70293	182
550	74	036	044	052	060	068	076	084	092	099	107			5.50	1.70475	182

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1.—Logarithms.—Continued.

No.	Common Logarithms of Numbers.													Naperian.		
	L. 0	1	2	3	4	5	6	7	8	9	P.	P.		No.	Log.	Dif.
550	74 036	044	052	060	068	076	084	092	099	107				5.50	1.70475	181
1	115	123	131	139	147	155	162	170	178	186	1			5.51	1.70656	182
2	194	202	210	218	225	233	241	249	257	265	2			5.52	1.70838	181
3	273	280	288	296	304	312	320	327	335	343	3			5.53	1.71019	180
4	351	359	367	374	382	390	398	406	414	421	4			5.54	1.71199	181
555	429	437	445	453	461	468	476	484	492	500	5			5.55	1.71380	180
6	507	515	523	531	539	547	554	562	570	578	6			5.56	1.71560	180
7	586	593	601	609	617	624	632	640	648	656	7			5.57	1.71740	179
8	663	671	679	687	695	702	710	718	726	733	8			5.58	1.71919	179
9	741	749	757	764	772	780	788	796	803	811	9			5.59	1.72098	179
560	819	827	834	842	850	858	865	873	881	889		8		5.60	1.72277	178
1	896	904	912	920	927	935	943	950	958	966	1	1		5.61	1.72455	178
2	974	981	989	997	005	012	020	028	035	043	2	1.6		5.62	1.72633	178
3	75 051	059	066	074	082	089	097	105	113	120	3	2.4		5.63	1.72811	177
4	128	136	143	151	159	166	174	182	189	197	4	3		5.64	1.72988	178
565	205	213	220	228	236	243	251	259	266	274	5	4		5.65	1.73166	176
6	282	289	297	305	312	320	328	335	343	351	6	5		5.66	1.73342	177
7	358	366	374	381	389	397	404	412	420	427	7	5.6		5.67	1.73519	176
8	435	443	450	458	465	473	481	488	496	504	8	6.4		5.68	1.73695	176
9	511	519	526	534	542	549	557	565	572	580	9	7		5.69	1.73871	176
570	587	595	603	610	618	626	633	641	648	656				5.70	1.74047	175
1	664	671	679	686	694	702	709	717	724	732	1			5.71	1.74222	175
2	740	747	755	762	770	778	785	793	800	808	2			5.72	1.74397	175
3	815	823	831	838	846	853	861	868	876	884	3			5.73	1.74572	174
4	891	899	906	914	921	929	937	944	952	959	4			5.74	1.74746	174
575	967	974	982	989	997	005	012	020	027	035	5			5.75	1.74920	174
6	76 042	050	057	065	072	080	087	095	103	110	6			5.76	1.75094	173
7	118	125	133	140	148	155	163	170	178	185	7			5.77	1.75267	173
8	193	200	208	215	223	230	238	245	253	260	8			5.78	1.75440	173
9	268	275	283	290	298	305	313	320	328	335	9			5.79	1.75613	173
580	343	350	358	365	373	380	388	395	403	410		7		5.80	1.75786	172
1	418	425	433	440	448	455	462	470	477	485	1	0.7		5.81	1.75958	172
2	492	500	507	515	522	530	537	545	552	559	2	1.4		5.82	1.76130	172
3	567	574	582	589	597	604	612	619	626	634	3	2		5.83	1.76302	171
4	641	649	656	664	671	678	686	693	701	708	4	3		5.84	1.76473	171
585	716	723	730	738	745	753	760	768	775	782	5	3.5		5.85	1.76644	171
6	790	797	805	812	819	827	834	842	849	856	6	4.2		5.86	1.76815	170
7	864	871	879	886	893	901	908	916	923	930	7	5		5.87	1.76985	171
8	938	945	953	960	967	975	982	989	997	004	8	5.6		5.88	1.77156	170
9	77 012	019	026	034	041	048	056	063	070	078	9	6.3		5.89	1.77326	169
590	085	093	100	107	115	122	129	137	144	151				5.90	1.77495	170
1	159	166	173	181	188	195	203	210	217	225	1			5.91	1.77665	169
2	232	240	247	254	262	269	276	283	291	298	2			5.92	1.77834	168
3	305	313	320	327	335	342	349	357	364	371	3			5.93	1.78002	169
4	379	386	393	401	408	415	422	430	437	444	4			5.94	1.78171	168
595	452	459	466	474	481	488	495	503	510	517	5			5.95	1.78339	168
6	525	532	539	546	554	561	568	576	583	590	6			5.96	1.78507	168
7	597	605	612	619	627	634	641	648	656	663	7			5.97	1.78675	167
8	670	677	685	692	699	706	714	721	728	735	8			5.98	1.78842	167
9	743	750	757	764	772	779	786	793	801	808	9			5.99	1.79009	167
600	815	822	830	837	844	851	859	866	873	880				6.00	1.79176	

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1.—Logarithms.—Continued.

No.	Common Logarithms of Numbers.											P. P.	Naperian.		
	L. 0	1	2	3	4	5	6	7	8	9			No.	Log.	Dif.
600	77 815	822	830	837	844	851	859	866	873	880		8	6.00	1.79176	166
1	887	895	902	909	916	924	931	938	945	952	1	1	6.01	1.79342	167
2	960	967	974	981	988	996	003	010	017	025	2	1.6	6.02	1.79509	166
3	78 032	039	046	053	061	068	075	082	089	097	3	2.4	6.03	1.79675	165
4	104	111	118	125	132	140	147	154	161	168	4	3	6.04	1.79840	166
605	176	183	190	197	204	211	219	226	233	240	5	4	6.05	1.80006	165
6	247	254	262	269	276	283	290	297	305	312	6	5	6.06	1.80171	165
7	319	326	333	340	347	355	362	369	376	383	7	5.6	6.07	1.80336	164
8	390	398	405	412	419	426	433	440	447	455	8	6.4	6.08	1.80500	165
9	462	469	476	483	490	497	504	512	519	526	9	7	6.09	1.80665	164
610	533	540	547	554	561	569	576	583	590	597			6.10	1.80829	164
1	604	611	618	625	633	640	647	654	661	668	1		6.11	1.80993	163
2	675	682	689	696	704	711	718	725	732	739	2		6.12	1.81156	163
3	746	753	760	767	774	781	789	796	802	810	3		6.13	1.81319	163
4	817	824	831	838	845	852	859	866	873	880	4		6.14	1.81482	163
615	888	895	902	909	916	923	930	937	944	951	5		6.15	1.81645	163
6	958	965	972	979	986	993	000	007	014	021	6		6.16	1.81808	162
7	79 029	036	043	050	057	064	071	078	085	092	7		6.17	1.81970	162
8	099	106	113	120	127	134	141	148	155	162	8		6.18	1.82132	162
9	169	176	183	190	197	204	211	218	225	232	9		6.19	1.82294	161
620	239	246	253	260	267	274	281	288	295	302		7	6.20	1.82455	161
1	309	316	323	330	337	344	351	358	365	372	1	0.7	6.21	1.82616	161
2	379	386	393	400	407	414	421	428	435	442	2	1.4	6.22	1.82777	161
3	449	456	463	470	477	484	491	498	505	511	3	2	6.23	1.82938	160
4	518	525	532	539	546	553	560	567	574	581	4	3	6.24	1.83098	160
625	588	595	602	609	616	623	630	637	644	650	5	3.5	6.25	1.83258	160
6	657	664	671	678	685	692	699	706	713	720	6	4.2	6.26	1.83418	160
7	727	734	741	748	754	761	768	775	782	789	7	5	6.27	1.83578	159
8	796	803	810	817	824	831	837	844	851	858	8	5.6	6.28	1.83737	159
9	865	872	879	886	893	900	906	913	920	927	9	6.3	6.29	1.83896	159
630	934	941	948	955	962	969	975	982	989	996			6.30	1.84055	159
1	80 003	010	017	024	030	037	044	051	058	065	1		6.31	1.84214	158
2	072	079	085	092	099	106	113	120	127	134	2		6.32	1.84272	158
3	140	147	154	161	168	175	182	188	195	202	3		6.33	1.84530	158
4	209	216	223	229	236	243	250	257	264	271	4		6.34	1.84688	157
635	277	284	291	298	305	312	318	325	332	339	5		6.35	1.84845	158
6	346	353	359	366	373	380	387	393	400	407	6		6.36	1.85003	157
7	414	421	428	434	441	448	455	462	468	475	7		6.37	1.85160	157
8	482	489	496	502	509	516	523	530	536	543	8		6.38	1.85317	156
9	500	557	564	570	577	584	591	598	604	611	9		6.39	1.85473	157
640	618	625	632	638	645	652	659	665	672	679		6	6.40	1.85630	156
1	686	693	699	706	713	720	726	733	740	747	1	0.6	6.41	1.85786	156
2	754	760	767	774	781	787	794	801	808	814	2	1.2	6.42	1.85942	155
3	821	828	835	841	848	855	862	868	875	882	3	1.8	6.43	1.86097	156
4	889	895	902	909	916	922	929	936	943	949	4	2.4	6.44	1.86253	155
645	956	963	969	976	983	990	996	003	010	017	5	3	6.45	1.86408	155
6	81 023	030	037	043	050	057	064	070	077	084	6	3.6	6.46	1.86563	155
7	090	097	104	111	117	124	131	137	144	151	7	4.2	6.47	1.86718	154
8	158	164	171	178	184	191	198	024	211	218	8	4.8	6.48	1.86872	154
9	224	231	238	245	251	258	265	271	278	285	9	5.4	6.49	1.87026	154
650	291	298	305	311	318	325	331	338	345	351			6.50	1.87180	

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1.—Logarithms.—Continued.

No.	Common Logarithms of Numbers.													Naperian.		
	L.	0	1	2	3	4	5	6	7	8	9	P.	P.	No.	Log.	Dif.
650	81	291	298	305	311	318	325	331	338	345	351			6.50	1.87180	154
1		358	365	371	378	385	391	398	405	411	418	1		6.51	1.87334	153
2		425	431	438	445	451	458	465	471	478	485	2		6.52	1.87487	154
3		491	498	505	511	518	525	531	538	544	551	3		6.53	1.87641	153
4		558	564	571	578	584	591	598	604	611	617	4		6.54	1.87794	153
655		624	631	637	644	651	657	664	671	677	684	5		6.55	1.87947	152
6		690	697	704	710	717	723	730	737	743	750	6		6.56	1.88099	152
7		757	763	770	776	783	790	796	803	809	816	7		6.57	1.88251	152
8		823	829	836	842	849	856	862	869	875	882	8		6.58	1.88403	152
9		889	895	902	908	915	921	928	935	941	948	9		6.59	1.88555	152
660		954	961	968	974	981	987	994	000	007	014		7	6.60	1.88707	151
1	82	020	027	033	040	046	053	060	066	073	079	1	0.7	6.61	1.88858	152
2		086	092	099	105	112	119	125	132	138	145	2	1.4	6.62	1.89010	150
3		151	158	164	171	178	184	191	197	204	210	3	2	6.63	1.89160	151
4		217	223	230	236	243	249	256	263	269	276	4	3	6.64	1.89311	151
665		282	289	295	302	308	315	321	328	334	341	5	3.5	6.65	1.89462	150
6		347	354	360	367	373	380	387	393	400	406	6	4.2	6.66	1.89612	150
7		413	419	426	432	439	445	452	458	465	471	7	5	6.67	1.89762	150
8		478	484	491	497	504	510	517	523	530	536	8	5.6	6.68	1.89912	149
9		543	549	556	562	569	575	582	588	595	601	9	6.3	6.69	1.90061	150
670		607	614	620	627	633	640	646	653	659	666			6.70	1.90211	149
1		672	679	685	692	698	705	711	718	724	730	1		6.71	1.90360	149
2		737	743	750	756	763	769	776	782	789	795	2		6.72	1.90509	149
3		802	808	814	821	827	834	840	847	853	860	3		6.73	1.90658	148
4		866	872	879	885	892	898	905	911	918	924	4		6.74	1.90806	148
675		930	937	943	950	956	963	969	975	982	988	5		6.75	1.90954	148
6		995	001	008	014	020	027	033	040	046	052	6		6.76	1.91102	148
7	83	059	065	072	078	085	091	097	104	110	117	7		6.77	1.91250	148
8		123	129	136	142	149	155	161	168	174	181	8		6.78	1.91398	147
9		187	193	200	206	213	219	225	232	238	245	9		6.79	1.91545	147
680		251	257	264	270	276	283	289	296	302	308		6	6.80	1.91692	147
1		315	321	327	334	340	347	353	359	366	372	1	0.6	6.81	1.91839	147
2		378	385	391	398	404	410	417	423	429	436	2	1.2	6.82	1.91986	146
3		442	448	455	461	467	474	480	487	493	499	3	1.8	6.83	1.92132	147
4		506	512	518	525	531	537	544	550	556	563	4	2.4	6.84	1.92279	146
685		569	575	582	588	594	601	607	613	620	626	5	3	6.85	1.92425	146
6		632	639	645	651	658	664	670	677	683	689	6	3.6	6.86	1.92571	145
7		696	702	708	715	721	727	734	740	746	753	7	4.2	6.87	1.92716	146
8		759	765	771	778	784	790	797	803	809	816	8	4.8	6.88	1.92862	145
9		822	828	835	841	847	853	860	866	872	879	9	5.4	6.89	1.93007	145
690		885	891	897	904	910	916	923	929	935	942			6.90	1.93152	145
1		948	954	960	967	973	979	985	992	998	004	1		6.91	1.93297	145
2	84	011	017	023	029	036	042	048	055	061	067	2		6.92	1.93442	144
3		073	080	086	092	098	105	111	117	123	130	3		6.93	1.93586	144
4		136	142	148	155	161	167	173	180	186	192	4		6.94	1.93730	144
695		198	205	211	217	223	230	236	242	248	255	5		6.95	1.93874	144
6		261	267	273	280	286	292	298	305	311	317	6		6.96	1.94018	144
7		323	330	336	342	348	354	361	367	373	379	7		6.97	1.94162	143
8		386	392	398	404	410	417	423	429	435	442	8		6.98	1.94305	143
9		448	454	460	466	473	479	485	491	497	504	9		6.99	1.94448	143
700		510	516	522	528	535	541	547	553	559	566			7.00	1.94591	

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1.—Logarithms.—Continued.

No.	Common Logarithms of Numbers.													Naperian.		
	L.	0	1	2	3	4	5	6	7	8	9	P.	P.	No.	Log.	Dif.
700	84	510	516	522	528	535	541	547	553	559	566		7	7.00	1.94591	143
1		572	578	584	590	597	603	609	615	621	628	1	0.7	7.01	1.94734	143
2		634	640	646	653	658	665	671	677	683	689	2	1.4	7.02	1.94876	143
3		696	702	708	714	720	726	733	739	745	751	3	2	7.03	1.95019	142
4		757	763	770	776	782	788	794	800	807	813	4	3	7.04	1.95161	142
705		819	825	831	837	844	850	856	862	868	874	5	3.5	7.05	1.95303	141
6		880	887	893	899	905	911	917	924	930	936	6	4.2	7.06	1.95444	142
7		942	948	954	960	967	973	979	985	991	997	7	5	7.07	1.95586	141
8	85	003	009	016	022	028	034	040	046	052	058	8	5.6	7.08	1.95727	142
9		065	071	077	083	089	095	101	107	114	120	9	6.3	7.09	1.95869	140
710		126	132	138	144	150	156	163	169	175	181			7.10	1.96009	141
1		187	193	199	205	211	217	224	230	236	242	1		7.11	1.96150	141
2		248	254	260	266	272	278	285	291	297	303	2		7.12	1.96291	140
3		309	315	321	327	333	339	345	352	358	364	3		7.13	1.96431	140
4		370	376	382	388	394	400	406	412	418	425	4		7.14	1.96571	140
715		431	437	443	449	455	461	467	473	479	485	5		7.15	1.96711	140
6		491	497	503	509	516	522	528	534	540	546	6		7.16	1.96851	140
7		552	558	564	570	576	582	588	594	600	606	7		7.17	1.96991	139
8		612	618	625	631	637	643	649	655	661	667	8		7.18	1.97130	139
9		673	679	685	691	697	703	709	715	721	727	9		7.19	1.97269	139
720		733	739	745	751	757	763	769	775	781	788		6	7.20	1.97408	139
1		794	800	806	812	818	824	830	836	842	848	1	0.6	7.21	1.97547	138
2		854	860	866	872	878	884	890	896	902	908	2	1.2	7.22	1.97685	138
3		914	920	926	932	938	944	950	956	962	968	3	1.8	7.23	1.97824	138
4		974	980	986	992	998	004	010	016	022	028	4	2.4	7.24	1.97962	138
725	86	034	040	046	052	058	064	070	076	082	088	5	3	7.25	1.98100	138
6		094	100	106	112	118	124	130	136	141	147	6	3.6	7.26	1.98238	138
7		153	159	165	171	177	183	189	195	201	207	7	4.2	7.27	1.98376	137
8		213	219	225	231	237	243	249	255	261	267	8	4.8	7.28	1.98513	137
9		273	279	285	291	297	303	308	314	320	326	9	5.4	7.29	1.98650	137
730		332	338	344	350	356	362	368	374	380	386			7.30	1.98787	137
1		392	398	404	410	415	421	427	433	439	445	1		7.31	1.98924	137
2		451	457	463	469	475	481	487	493	499	504	2		7.32	1.99061	137
3		510	516	522	528	534	540	546	552	558	564	3		7.33	1.99198	136
4		570	576	581	587	593	599	605	611	617	623	4		7.34	1.99334	136
735		629	635	641	646	652	658	664	670	676	682	5		7.35	1.99470	136
6		688	694	700	705	711	717	723	729	735	741	6		7.36	1.99606	136
7		747	753	759	764	770	776	782	788	794	800	7		7.37	1.99742	135
8		806	812	817	823	829	835	841	847	853	859	8		7.38	1.99877	136
9		864	870	876	882	888	894	900	906	911	917	9		7.39	2.00013	135
740		923	929	935	941	947	953	958	964	970	976		5	7.40	2.00148	135
1		982	988	994	999	005	011	017	023	029	035	1	0.5	7.41	2.00283	135
2	87	040	046	052	058	064	070	075	081	087	093	2	1	7.42	2.00418	135
3		099	105	111	116	122	128	134	140	146	151	3	1.5	7.43	2.00553	134
4		157	163	169	175	181	186	192	198	204	210	4	2	7.44	2.00687	134
745		216	221	227	233	239	245	251	256	262	268	5	2.5	7.45	2.00821	135
6		274	280	286	291	297	303	309	315	320	326	6	3	7.46	2.00956	133
7		332	338	344	349	355	361	367	373	379	384	7	3.5	7.47	2.01089	134
8		390	396	402	408	413	419	425	431	437	442	8	4	7.48	2.01223	134
9		448	454	460	466	471	477	483	489	495	500	9	4.5	7.49	2.01357	133
750		506	512	518	523	529	535	541	547	552	558			7.50	2.01490	

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1.—Logarithms.—Continued.

No.	Common Logarithms of Numbers.													Naperian.		
	L.	O	1	2	3	4	5	6	7	8	9	P.	P.	No.	Log.	Dif.
750	87	506	512	518	523	529	535	541	547	552	558			7.50	2.01490	134
1		564	570	576	581	587	593	599	604	610	616	1		7.51	2.01624	133
2		622	628	633	639	645	651	656	662	668	674	2		7.52	2.01757	133
3		679	685	691	697	703	708	714	720	726	731	3		7.53	2.01890	132
4		737	743	749	754	760	766	772	777	783	789	4		7.54	2.02022	133
755		795	800	806	812	818	823	829	835	841	846	5		7.55	2.02155	132
6		852	858	864	869	875	881	887	892	898	904	6		7.56	2.02287	132
7		910	915	921	927	933	938	944	950	955	961	7		7.57	2.02419	132
8		967	973	978	984	990	996	001	007	013	018	8		7.58	2.02551	132
9	88	024	030	036	041	047	053	058	064	070	076	9		7.59	2.02683	132
760		081	087	093	098	104	110	116	121	127	133		6	7.60	2.02815	131
1		138	144	150	156	161	167	173	178	184	190	1	0.6	7.61	2.02946	132
2		196	201	207	213	218	224	230	235	241	247	2	1.2	7.62	2.03078	131
3		252	258	264	270	275	281	287	292	298	304	3	1.8	7.63	2.03209	131
4		309	315	321	326	332	338	343	349	355	360	4	2.4	7.64	2.03340	131
765		366	372	377	383	389	395	400	406	412	417	5	3	7.65	2.03471	130
6		423	429	434	440	446	451	457	463	468	474	6	3.6	7.66	2.03601	131
7		480	485	491	497	502	508	513	519	525	530	7	4.2	7.67	2.03732	130
8		536	542	547	553	559	564	570	576	581	587	8	4.8	7.68	2.03862	130
9		593	598	604	610	615	621	627	632	638	643	9	5.4	7.69	2.03992	130
770		649	655	660	666	672	677	683	689	694	700			7.70	2.04122	130
1		705	711	717	722	728	734	739	745	750	756	1		7.71	2.04252	129
2		762	767	773	779	784	790	795	801	807	812	2		7.72	2.04381	130
3		818	824	829	835	840	846	852	857	863	868	3		7.73	2.04511	129
4		874	880	885	891	897	902	908	913	919	925	4		7.74	2.04640	129
775		930	936	941	947	953	958	964	969	975	981	5		7.75	2.04769	129
6		986	992	997	003	009	014	020	025	031	037	6		7.76	2.04898	129
7	89	042	048	053	059	064	070	076	081	087	092	7		7.77	2.05027	129
8		098	104	109	115	120	126	131	137	143	148	8		7.78	2.05156	128
9		154	159	165	170	176	182	187	193	198	204	9		7.79	2.05284	128
780		209	215	221	226	232	237	243	248	254	260		5	7.80	2.05412	128
1		265	271	276	282	287	293	298	304	310	315	1	0.5	7.81	2.05540	128
2		321	326	332	337	343	348	354	360	365	371	2	1	7.82	2.05668	128
3		376	382	387	393	398	404	409	415	421	426	3	1.5	7.83	2.05796	128
4		432	437	443	448	454	459	465	470	476	481	4	2	7.84	2.05924	127
785		487	492	498	504	509	515	520	526	531	537	5	2.5	7.85	2.06051	128
6		542	548	553	559	564	570	575	581	586	592	6	3	7.86	2.06179	127
7		597	603	609	614	620	625	631	636	642	647	7	3.5	7.87	2.06306	127
8		653	658	664	669	675	680	686	691	697	702	6	4	7.88	2.06433	127
9		708	713	719	724	730	735	741	746	752	757	9	4.5	7.89	2.06560	126
790		763	768	774	779	785	790	796	801	807	812			7.90	2.06686	127
1		818	823	829	834	840	845	851	856	862	867	1		7.91	2.06813	126
2		873	878	883	889	894	900	905	911	916	922	2		7.92	2.06939	126
3		927	933	938	944	949	955	960	966	971	977	3		7.93	2.07065	126
4		982	988	993	998	004	009	015	020	026	031	4		7.94	2.07191	126
795	90	037	042	048	053	059	064	069	075	080	086	5		7.95	2.07317	126
6		091	097	102	108	113	119	124	129	135	140	6		7.96	2.07443	125
7		146	151	157	162	168	173	179	184	189	195	7		7.97	2.07568	126
8		200	206	211	217	222	227	233	238	244	249	8		7.98	2.07694	125
9		255	260	266	271	276	282	287	293	298	304	9		7.99	2.07819	125
800		309	314	320	325	331	336	342	347	352	358			8.00	2.07944	

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1.—Logarithms.—Continued.

No.	Common Logarithms of Numbers.													Naperian.		
	L.	0	1	2	3	4	5	6	7	8	9	P.	P.	No.	Log.	Diff.
800	90	309	314	320	325	331	336	342	347	352	358			8.00	2.07944	125
1		363	369	374	380	385	390	396	401	407	412	1		8.01	2.08069	125
2		417	423	428	434	439	445	450	455	461	466	2		8.02	2.08194	124
3		472	477	482	488	493	499	504	509	515	520	3		8.03	2.08318	125
4		526	531	536	542	547	553	.558	563	569	574	4		8.04	2.08443	124
805		580	585	590	596	601	607	612	617	623	628	5		8.05	2.08567	124
6		634	639	644	650	655	660	666	671	677	682	6		8.06	2.08691	124
7		687	693	698	703	709	714	720	725	730	736	7		8.07	2.08815	124
8		741	747	752	757	763	768	773	779	784	789	8		8.08	2.08939	124
9		795	800	806	811	816	822	827	832	838	843	9		8.09	2.09063	123
810		849	854	859	865	870	875	881	886	891	897		6	8.10	2.09186	124
1		902	907	913	918	924	929	934	940	945	950	1	0.6	8.11	2.09310	123
2		956	961	966	972	977	982	988	993	998	004	2	1.2	8.12	2.09433	123
3	91	009	014	020	025	030	036	041	046	052	057	3	1.8	8.13	2.09556	123
4		062	068	073	078	084	089	094	100	105	110	4	2.4	8.14	2.09679	122
815		116	121	126	132	137	142	148	153	158	164	5	3	8.15	2.09802	122
6		169	174	180	185	190	196	201	206	212	217	6	3.6	8.16	2.09924	122
7		222	228	233	238	243	249	254	259	265	270	7	4.2	8.17	2.10047	122
8		275	281	286	291	297	302	307	312	318	323	8	4.8	8.18	2.10169	122
9		328	334	339	344	350	355	360	365	371	376	9	5.4	8.19	2.10291	122
820		381	387	392	397	403	408	413	418	424	429			8.20	2.10413	122
1		434	440	445	450	455	461	466	471	477	482	1		8.21	2.10535	122
2		487	492	498	503	508	514	519	524	529	535	2		8.22	2.10657	122
3		540	545	551	556	561	566	572	577	582	587	3		8.23	2.10779	121
4		593	598	603	609	614	619	624	630	635	640	4		8.24	2.10900	121
825		645	651	656	661	666	672	677	682	687	693	5		8.25	2.11021	121
6		698	703	709	714	719	724	730	735	740	745	6		8.26	2.11142	121
7		751	756	761	766	772	777	782	787	793	798	7		8.27	2.11263	121
8		803	808	814	819	824	829	834	840	845	850	8		8.28	2.11384	121
9		855	861	866	871	876	882	887	892	897	903	9		8.29	2.11505	121
830		908	913	918	924	929	934	939	944	950	955		5	8.30	2.11626	120
1		960	965	971	976	981	986	991	997	002	007	1	0.5	8.31	2.11746	120
2	92	012	018	023	028	033	038	044	049	054	059	2	1	8.32	2.11866	120
3		065	070	075	080	085	091	096	101	106	111	3	1.5	8.33	2.11986	120
4		117	122	127	132	137	143	148	153	158	163	4	2	8.34	2.12106	120
835		169	174	179	184	189	195	200	205	210	215	5	2.5	8.35	2.12226	120
6		221	226	231	236	241	247	252	257	262	267	6	3	8.36	2.12346	119
7		273	278	283	288	293	298	304	309	314	319	7	3.5	8.37	2.12465	120
8		324	330	335	340	345	350	355	361	366	371	8	4	8.38	2.12585	119
9		376	381	387	392	397	402	407	412	418	423	9	4.5	8.39	2.12704	119
840		428	433	438	443	449	454	459	464	469	474			8.40	2.12823	119
1		480	485	490	495	500	505	511	516	521	526	1		8.41	2.12942	119
2		531	536	542	547	552	557	562	567	572	578	2		8.42	2.13061	119
3		583	588	593	598	603	609	614	619	624	629	3		8.43	2.13180	118
4		634	639	645	650	655	660	665	670	675	681	4		8.44	2.13298	119
845		686	691	696	701	706	711	716	722	727	732	5		8.45	2.13417	118
6		737	742	747	752	758	763	768	773	778	783	6		8.46	2.13535	118
7		788	793	799	804	809	814	819	824	829	834	7		8.47	2.13653	118
8		840	845	850	855	860	865	870	875	881	886	8		8.48	2.13771	118
9		891	896	901	906	911	916	921	927	932	937	9		8.49	2.13889	118
850		942	947	952	957	962	967	973	978	983	988			8.50	2.14007	

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1.—Logarithms.—Continued.

No.	Common Logarithms of Numbers. L.	0	1	2	3	4	5	6	7	8	9	P. P.		Naperian. No.	Log.	Dif.
850	92	942	947	952	957	962	967	973	978	983	988		6	8.50	2.14007	117
1		993	998	003	008	013	018	024	029	034	039	1	0.6	8.51	2.14124	118
2	93	044	049	054	059	064	069	075	080	085	090	2	1.2	8.52	2.14242	117
3		095	100	105	110	115	120	125	131	136	141	3	1.8	8.53	2.14359	117
4		146	151	156	161	166	171	176	181	186	192	4	2.4	8.54	2.14476	117
855		197	202	207	212	217	222	227	232	237	242	5	3	8.55	2.14593	117
6		247	252	258	263	268	273	278	283	288	293	6	3.6	8.56	2.14710	117
7		298	303	308	313	318	323	328	334	339	344	7	4.2	8.57	2.14827	116
8		349	354	359	364	369	374	379	384	389	394	8	4.8	8.58	2.14943	117
9		399	404	409	414	420	425	430	435	440	445	9	5.4	8.59	2.15060	116
860		450	455	460	465	470	475	480	485	490	495			8.60	2.15176	116
1		500	505	510	515	520	526	531	536	541	546	1		8.61	2.15292	117
2		551	556	561	566	571	576	581	586	591	596	2		8.62	2.15409	115
3		601	606	611	616	621	626	631	636	641	646	3		8.63	2.15524	116
4		651	656	661	666	671	676	682	687	692	697	4		8.64	2.15640	116
865		702	707	712	717	722	727	732	737	742	747	5		8.65	2.15756	115
6		752	757	762	767	772	777	782	787	792	797	6		8.66	2.15871	116
7		802	807	812	817	822	827	832	837	842	847	7		8.67	2.15987	115
8		852	857	862	867	872	877	882	887	892	897	8		8.68	2.16102	115
9		902	907	912	917	922	927	932	937	942	947	9		8.69	2.16217	115
870		952	957	962	967	972	977	982	987	992	997		5	8.70	2.16332	115
1	94	002	007	012	017	022	027	032	037	042	047	1	0.5	8.71	2.16447	115
2		052	057	062	067	072	077	082	086	091	096	2	1	8.72	2.16562	115
3		101	106	111	116	121	126	131	136	141	146	3	1.5	8.73	2.16677	114
4		151	156	161	166	171	176	181	186	191	196	4	2	8.74	2.16791	114
875		201	206	211	216	221	226	231	236	240	245	5	2.5	8.75	2.16905	115
6		250	255	260	265	270	275	280	285	290	295	6	3	8.76	2.17020	114
7		300	305	310	315	320	325	330	335	340	345	7	3.5	8.77	2.17134	114
8		349	354	359	364	369	374	379	384	389	394	8	4	8.78	2.17248	113
9		399	404	409	414	419	424	429	433	438	443	9	4.5	8.79	2.17361	114
880		448	453	458	463	468	473	478	483	488	493			8.80	2.17475	114
1		498	503	507	512	517	522	527	532	537	542	1		8.81	2.17589	113
2		547	552	557	562	567	571	576	581	586	591	2		8.82	2.17702	113
3		596	601	606	611	616	621	626	630	635	640	3		8.83	2.17816	113
4		645	650	655	660	665	670	675	680	685	689	4		8.84	2.17929	113
885		694	699	704	709	714	719	724	729	734	738	5		8.85	2.18042	113
6		743	748	753	758	763	768	773	778	783	787	6		8.86	2.18155	112
7		792	797	802	807	812	817	822	827	832	836	7		8.87	2.18267	113
8		841	846	851	856	861	866	871	876	880	885	8		8.88	2.18380	113
9		890	895	900	905	910	915	919	924	929	934	9		8.89	2.18493	112
890		939	944	949	954	959	963	968	973	978	983		4	8.90	2.18605	112
1		988	993	998	002	007	012	017	022	027	032	1	0.4	8.91	2.18717	113
2	95	036	041	046	051	056	061	066	071	075	080	2	0.8	8.92	2.18830	112
3		085	090	095	100	105	109	114	119	124	129	3	1.2	8.93	2.18942	112
4		134	139	143	148	153	158	163	168	173	177	4	1.6	8.94	2.19054	111
895		182	187	192	197	202	207	211	216	221	226	5	2	8.95	2.19165	112
6		231	236	240	245	250	255	260	265	270	274	6	2.4	8.96	2.19277	112
7		279	284	289	294	299	303	308	313	318	323	7	2.8	8.97	2.19389	111
8		328	332	337	342	347	352	357	361	366	371	8	3.2	8.98	2.19500	111
9		376	381	386	390	395	400	405	410	415	419	9	3.6	8.99	2.19611	111
900		424	429	434	439	444	448	453	458	463	468			9.00	2.19722	

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1.—Logarithms.—Continued.

No.	Common Logarithms of Numbers.													Naperian.		
	L.	0	1	2	3	4	5	6	7	8	9	P.	P.	No.	Log.	Dif.
900	95	424	429	434	439	444	448	453	458	463	468			9.00	2.19722	112
1		473	477	482	487	492	497	501	506	511	516	1		9.01	2.19834	110
2		521	525	530	535	540	545	550	554	559	564	2		9.02	2.19944	111
3		569	574	578	583	588	593	598	602	607	612	3		9.03	2.20055	111
4		617	622	626	631	636	641	646	650	655	660	4		9.04	2.20166	110
905		665	670	674	679	684	689	694	698	703	708	5		9.05	2.20276	111
6		713	718	722	727	732	737	742	746	751	756	6		9.06	2.20387	110
7		761	766	770	775	780	785	789	794	799	804	7		9.07	2.20497	110
8		809	813	818	823	828	832	837	842	847	852	8		9.08	2.20607	110
9		856	861	866	871	875	880	885	890	895	899	9		9.09	2.20717	110
910		904	909	914	918	923	928	933	938	942	947		5	9.10	2.20827	110
1		952	957	961	966	971	976	980	985	990	995	1	0.5	9.11	2.20937	110
2		999	004	009	014	019	023	028	033	038	042	2	1	9.12	2.21047	110
3	96	047	052	057	061	066	071	076	080	085	090	3	1.5	9.13	2.21157	109
4		095	099	104	109	114	118	123	128	133	137	4	2	9.14	2.21266	109
915		142	147	152	156	161	166	171	175	180	185	5	2.5	9.15	2.21375	110
6		190	194	199	204	209	213	218	223	227	232	6	3	9.16	2.21485	109
7		237	242	246	251	256	261	265	270	275	280	7	3.5	9.17	2.21594	109
8		284	289	294	298	303	308	313	317	322	327	8	4	9.18	2.21703	109
9		332	336	341	346	350	355	360	365	369	374	9	4.5	9.19	2.21812	108
920		379	384	388	393	398	402	407	412	417	421			9.20	2.21920	109
1		426	431	435	440	445	450	454	459	464	468	1		9.21	2.22029	109
2		473	478	483	487	492	497	501	506	511	515	2		9.22	2.22138	108
3		520	525	530	534	539	544	548	553	558	562	3		9.23	2.22246	108
4		567	572	577	581	586	591	595	600	605	609	4		9.24	2.22354	108
925		614	619	624	628	633	638	642	647	652	656	5		9.25	2.22462	108
6		661	666	670	675	680	685	689	694	699	703	6		9.26	2.22570	108
7		708	713	717	722	727	731	736	741	745	750	7		9.27	2.22678	108
8		755	759	764	769	774	778	783	788	792	797	8		9.28	2.22786	108
9		802	806	811	816	820	825	830	834	839	844	9		9.29	2.22894	107
930		848	853	858	862	867	872	876	881	886	890		4	9.30	2.23001	108
1		895	900	904	909	914	918	923	928	932	937	1	0.4	9.31	2.23109	107
2		942	946	951	956	960	965	970	974	979	984	2	0.8	9.32	2.23216	108
3		988	993	997	002	007	011	016	021	025	030	3	1.2	9.33	2.23324	107
4	97	035	039	044	049	053	058	063	067	072	077	4	1.6	9.34	2.23431	107
935		081	086	090	095	100	104	109	114	118	123	5	2	9.35	2.23538	107
6		128	132	137	142	146	151	155	160	165	169	6	2.4	9.36	2.23645	106
7		174	179	183	188	192	197	202	206	211	216	7	2.8	9.37	2.23751	107
8		220	225	230	234	239	243	248	253	257	262	8	3.2	9.38	2.23858	107
9		267	271	276	280	285	290	294	299	304	308	9	3.6	9.39	2.23965	106
940		313	317	322	327	331	336	340	345	350	354			9.40	2.24071	106
1		359	364	368	373	377	382	387	391	396	400	1		9.41	2.24177	107
2		405	410	414	419	424	428	433	437	442	447	2		9.42	2.24284	106
3		451	456	460	465	470	474	479	483	488	493	3		9.43	2.24390	106
4		497	502	506	511	516	520	525	529	534	539	4		9.44	2.24496	105
945		543	548	552	557	562	566	571	575	580	585	5		9.45	2.24601	106
6		589	594	598	603	607	612	617	621	626	630	6		9.46	2.24707	106
7		635	640	644	649	653	658	663	667	672	676	7		9.47	2.24813	105
8		681	685	690	695	699	704	708	713	717	722	8		9.48	2.24918	106
9		727	731	736	740	745	749	754	759	763	768	9		9.49	2.25024	105
950		772	777	782	786	791	795	800	804	809	813			9.50	2.25129	

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1.—Logarithms.—Continued.

No.	Common Logarithms of Numbers.												Naperian.		
	L. 0	1	2	3	4	5	6	7	8	9	P.	P.	No.	Log.	Dif.
950	97 772	777	782	786	791	795	800	804	809	813			9.50	2.25129	105
1	818	823	827	832	836	841	845	850	855	859	1		9.51	2.25234	105
2	864	868	873	877	882	886	891	896	900	905	2		9.52	2.25339	105
3	909	914	918	923	928	932	937	941	946	950	3		9.53	2.25444	105
4	955	959	964	968	973	978	982	987	991	996	4		9.54	2.25549	105
955	98 000	005	009	014	019	023	028	032	037	041	5		9.55	2.25654	105
6	046	050	055	059	064	068	073	078	082	087	6		9.56	2.25759	104
7	091	096	100	105	109	114	118	123	127	132	7		9.57	2.25863	105
8	137	141	146	150	155	159	164	168	173	177	8		9.58	2.25968	104
9	182	186	191	195	200	204	209	214	218	223	9		9.59	2.26072	104
960	227	232	236	241	245	250	254	259	263	268		5	9.60	2.26176	104
1	272	277	281	286	290	295	299	304	308	313	1	0.5	9.61	2.26280	104
2	318	322	327	331	336	340	345	349	354	358	2	1	9.62	2.26384	104
3	363	367	372	376	381	385	390	394	399	403	3	1.5	9.63	2.26488	104
4	408	412	417	421	426	430	435	439	444	448	4	2	9.64	2.26592	104
965	453	457	462	466	471	475	480	484	489	493	5	2.5	9.65	2.26696	103
6	498	502	507	511	516	520	525	529	534	538	6	3	9.66	2.26799	104
7	543	547	552	556	561	565	570	574	579	583	7	3.5	9.67	2.26903	103
8	588	592	597	601	605	610	614	619	623	628	8	4	9.68	2.27006	103
9	632	637	641	646	650	655	659	664	668	673	9	4.5	9.69	2.27109	104
970	677	682	686	691	695	700	704	709	713	717			9.70	2.27213	103
1	722	726	731	735	740	744	749	753	758	762	1		9.71	2.27316	103
2	767	771	776	780	784	789	793	798	802	807	2		9.72	2.27419	102
3	811	816	820	825	829	834	838	843	847	851	3		9.73	2.27521	103
4	856	860	865	869	874	878	883	887	892	896	4		9.74	2.27624	103
975	900	905	909	914	918	923	927	932	936	941	5		9.75	2.27727	102
6	945	949	954	958	963	967	972	976	981	985	6		9.76	2.27829	103
7	989	994	998	003	007	012	016	021	025	029	7		9.77	2.27932	102
8	99 034	038	043	047	052	056	061	065	069	074	8		9.78	2.28034	102
9	078	083	087	092	096	100	105	109	114	118	9		9.79	2.28136	102
980	123	127	131	136	140	145	149	154	158	162		4	9.80	2.28238	102
1	167	171	176	180	185	189	193	198	202	207	1	0.4	9.81	2.28340	102
2	211	216	220	224	229	233	238	242	247	251	2	0.8	9.82	2.28442	102
3	255	260	264	269	273	277	282	286	291	295	3	1.2	9.83	2.28544	102
4	300	304	308	313	317	322	326	330	335	339	4	1.6	9.84	2.28646	101
985	344	348	352	357	361	366	370	374	379	383	5	2	9.85	2.28747	102
6	388	392	396	401	405	410	414	419	423	427	6	2.4	9.86	2.28849	101
7	432	436	441	445	449	454	458	463	467	471	7	2.8	9.87	2.28950	101
8	476	480	484	489	493	498	502	506	511	515	8	3.2	9.88	2.29051	101
9	520	524	528	533	537	542	546	550	555	559	9	3.6	9.89	2.29152	101
990	564	568	572	577	581	585	590	594	599	603			9.90	2.29253	101
1	607	612	616	621	625	629	634	638	642	647	1		9.91	2.29354	101
2	651	656	660	664	669	673	677	682	686	691	2		9.92	2.29455	101
3	695	699	704	708	712	717	721	726	730	734	3		9.93	2.29556	101
4	739	743	747	752	756	760	765	769	774	778	4		9.94	2.29657	100
995	782	787	791	795	800	804	808	813	817	822	5		9.95	2.29757	101
6	826	830	835	839	843	848	852	856	861	865	6		9.96	2.29858	100
7	870	874	878	883	887	891	896	900	904	909	7		9.97	2.29958	100
8	913	917	922	926	930	935	939	944	948	952	8		9.98	2.30058	100
9	957	961	965	970	974	978	983	987	991	996	9		9.99	2.30158	
1000	000 000	043	087	130	174	217	260	304	347	391			10.00	2.302585	

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Slide Rules—Logarithmic slide rules are instruments graduated on a logarithmic basis for performing calculations involving multiplication (including powers) and division (including roots and reciprocals) of numbers.

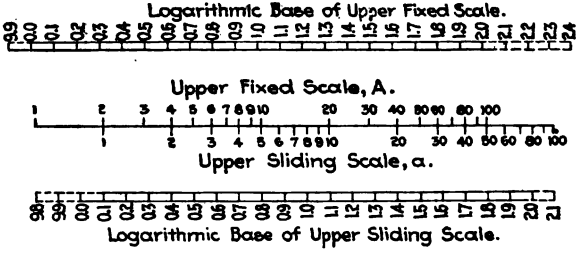

Fig. 1.

The principle of the slide rule is very simple, although some of the instruments themselves are complex and expensive.

The plain slide rule, Fig. 1, is usually from ten to eighteen inches long. (The longer the scale, the more accurate.) It consists of the upper and lower "fixed" scales A and B, graduated on one piece and grooved to receive the sliding piece, on which are graduated the scales a and b. Note that scales A and a are similar, as are also B and b; but that the former are in double series (from 1 to 10) while the latter are in single series only. The advantage of this system will be explained below.

Fig. 1 shows the upper fixed and sliding scales, A and a, together with their logarithmic bases. These two scales (or the two lower ones, either) may be used in performing any simple operation in multiplication or division. For instance, as the scales A and a are now set we can find the product of any number multiplied by 2; or, inversely, the quotient of any number divided by 2. Thus, $2 \times 1 = 2$; $2 \times 2 = 4$; $2 \times 3 = 6$, etc. Likewise, $10 \div 2 = 5$; $20 \div 2 = 10$; etc. Note that the logarithm of the product = the *sum* of the logarithms of the factors; thus, log of 40 (= 1.6) is equal to log of 20 + log of 2 = 1.3 + 0.3. Similarly, the log of the quotient is the log dividend *minus* the log divisior. Of course the logarithms themselves do not appear on the slide rule, but the numbers are arranged so their logarithms form series equally spaced, and the principle remains. To multiply any number q by n: Set 1 of scale a opposite q of scale A, and opposite n of scale a read the product p on scale A. To divide any number p by n: Set n of scale a opposite p of scale A, and opposite 1 of scale a read the quotient q on scale A. To find the reciprocal of a number: Divide 1 by that number; *or*, invert the sliding scale, end for end, with ends of both scales, A and a, opposite, and the reciprocal of any number on one scale, as A, will be found directly opposite on the other scale, as a.

To facilitate operations, and for accuracy, each slide rule is provided with a *movable index* with a vertical hair line.

Fig. 2 shows the ordinary slide rule with double scale. With the movable index, the square root of any number on scale A will be found

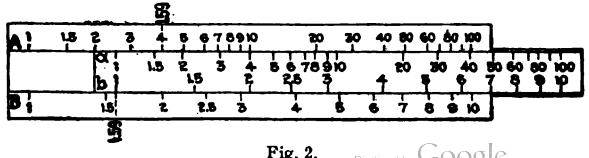

Fig. 2.

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directly below on scale B. In like manner, the square of any number on scale B is found opposite, on scale A. Furthermore, the cube of any number on scale B may be found by multiplying its corresponding square on scale A by the number itself on scale a, reading the cube on scale A. Thus, Fig. 2, the cube of 1.59 (scale B) is found on scale A opposite 1.59 of scale a, and is equal to 4. Clearly, then, the cube root of any number n (scale A) is found by making the reading on scale a, opposite n, equal to the reading on scale B opposite 1 of scale b, and these equal readings are the cube root of the number. Thus, the cube root of 4 = 1.59.

Thatcher's calculating instrument is a cylinder four inches in diameter and about eighteen inches long, which acts as a sliding scale inside a framework of twenty parallel bars forming the fixed scales. This instrument is by far the best on the market. It was sold formerly at $25.00, a price barely exceeding the cost of manufacture, and is now listed at $35.00; and with reading glass, $45.00. Results may be obtained to four or five decimal places. It is nearly as accurate as a five-place logarithmic table.

Slide rules are used in all logarithmic and trigonometric operations, and no engineer should be without one. Books giving full directions in the use of the slide rule can be obtained for from 25 to 75 cents. They explain the method of solving such equations as:

$$y = \frac{ax}{b};\ y = \frac{ax}{b^2};\ y = \frac{ax^2}{b};\ y = a\frac{x^2}{b^2};\ y = \sqrt{\frac{ax}{b}};\ y = a\sqrt{\frac{x}{b}};\ \text{etc.}$$

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7.—PLANE GEOMETRY

(See also Mensuration.)

Angles and Lines.—

Straight line: $a\,b$. Oblique lines: ab, cd.
Broken line: $a\,o\,d$. Oblique angles: A, B.
Acute angle: A. Adjacent angles: A, B; A, C.
Obtuse angle: B. Reflex angle: Greater than 180° and less than 360°.

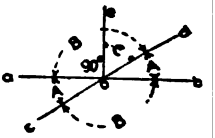


Fig. 1.

Complementary angles: A and C $(A+C=90°)$; each is the complement of the other.
Supplementary angles: A and B $(A+B=180°)$; each is the supplement of the other.
Right angle: $a\,o\,e$ $(=90°)$; $e\,o$ is perpendicular to $a\,b$.
Straight angle: $a\,o\,b$ $(=180°)$; sides form two right angles.
Conjugate angles: Two angles, about a point, whose sum equals 360°.
Vertical angles: A and A; B and B.

Triangles.—

Triangle: Plane bounded by three straight sides, Fig. 2.
Perimeter: Sum of the sides.
Angles: The interior angles (sum = 180°).

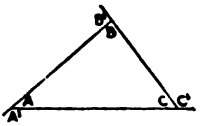


Fig. 2.

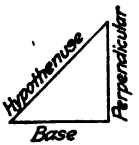


Fig. 3.

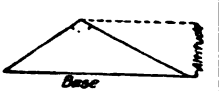
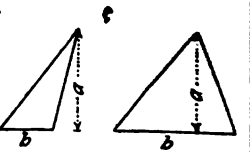


Fig. 4.

Successive partial exterior angles: A' B' and C' (sum = 360°).
Right triangle: One of its angles a right angle, Fig. 3.
Scalene " No two of its sides equal, Fig. 4.
Obtuse " One of its angles obtuse, Fig. 4.

Fig. 5.

Fig. 6.



Figs. 7.

Isosceles triangle: Two of its sides equal, Fig. 5.
Acute " All of its angles acute, Fig. 5.
Equilateral " All of its sides equal, Fig. 6.
Equiangular " All of its angles equal, Fig. 6.
Area of any triangle = ½ base × altitude.

Quadrilaterals.—

Quadrilateral: Plane bounded by four straight sides, Fig. 8.
Sum of interior angles $(A, B, C, D) = 360°$;
Sum of partial successive exterior angles $(A', B', C', D') = 360°$;
Sum of exterior angles (complete) = 1080°.
Square: All sides equal, and each angle 90°, Fig. 9.
Rectangle: Opposite sides parallel, and each angle 90°, Fig. 10.
Parallelogram: Opposite sides parallel, Fig. 11.

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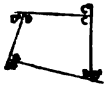
Fig. 8.

Fig. 9.

Fig. 10.

Fig. 11.

Fig 12.

Fig. 13.

Fig. 14.

Fig. 15.

Rhomboid: Opposite sides parallel, and all angles oblique, Fig. 12.
Rhombus: All sides equal, all angles oblique, Fig. 13.
Trapezoid: Two sides, only, parallel, Fig. 14.
Trapezium: No two sides parallel, Fig. 15.

Polygons (General).—

Polygon: Plane bounded by straight sides, Fig. 16.
3 = triangle, 4 = quadrilateral, 5 = pentagon, 6 = hexagon, 7 = heptagon, 8 = octagon, 9 = nonagon, 10 = decagon, 11 = undecagon, 12 = dodecagon.
Sum of interior angles (A, B, C, etc.) = 180° × (number of sides − 2).
Sum of partial successive exterior angles (A', B', C', etc.) = 360°.
Sum of exterior angles (complete) = 360° + number of sides × 180°.

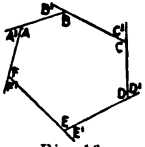

Fig. 16.

Regular Polygons.—

Regular polygon: All sides equal and all angles equal, Fig. 17.
Circle: A regular polygon with an infinite number of sides.
Radius of polygon: Radius (r) of circumscribed circle.
Apothem of polygon: Radius (a) of inscribed circle.
Perimeter: Sum of the sides.
Area: ½ apothem × perimeter.

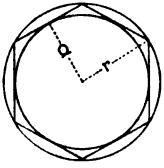

Fig. 17.

Circle.—

Tangent: Touches circumference at one point.
Secant: Intersects circumference at two points.
Diameter: Intersects center and is limited by circumference.
Radius: Distance from center to circumference.
Chord: A secant limited by the circumference.
Arc: A portion of the circumference.
Segment: Area bounded by arc and chord.
Sector: Area bounded by two radii and the intercepted arc.
Quadrant: Sector equal to quarter of a circle.
Semicircle: Sector equal to half a circle.

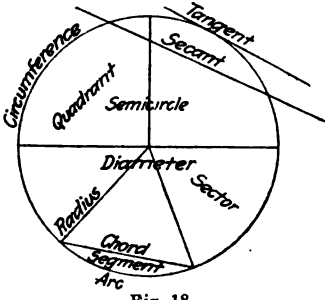

Fig. 18.

Circumference = diameter × π. $\pi = 3.1415927$.

Diameter = circumference × $\frac{1}{\pi}$. $\frac{1}{\pi} = 0.3183099$.

Area of circle = diameter² × $\frac{\pi}{4}$. $\frac{\pi}{4} = 0.7853982$.

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Problems in Construction of Figures.—

Intercepting circles: Common chord is at right angle to line joining centers of circles, Fig. 19. Used in laying off 90°.

To lay off 90°, 45°, 60° and 30°:

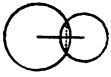

Fig. 19.

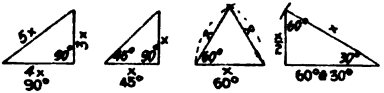

Fig 20.

Center of circle: Intersection of perpendiculars bisecting any two chords, Fig. 21.

An inscribed angle in a semicircle = 90°, Fig. 22.

An angle from a point on the circumference is measured by half the intercepted arc (angle = ½ arc), Fig. 23.

An angle included by a tangent and adjacent chord is measured by half the intercepted arc; (angle = ½ arc), Fig. 24.

Fig. 21.

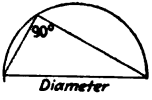

Fig. 22.

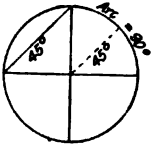

Fig. 23.

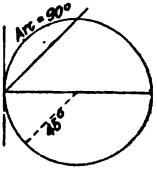

Fig. 24.

Equal angles from a point, o, on the circumference subtend equal arcs and equal chords, Fig. 25.

Railway curve: The two preceding propositions are fundamental in the laying out of railway curves. The chord is usually 100 ft. with a

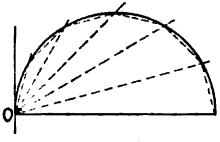

Fig. 25.

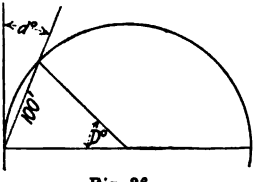

Fig. 26.

Fig. 27.

central angle $D°$ which equals the degree of curvature. The deflection angle $d°$ is always one-half the central angle $D°$; hence for one chord, or station, the deflection angle is one-half the degree of curvature.

Circle inscribed in a triangle: Center of circle is intersection of lines bisecting the angles, Fig. 27. Radius is shortest distance to either side.

Circle circumscribing a triangle: Center of circle is intersection of perpendiculars bisecting the sides, Fig. 28.

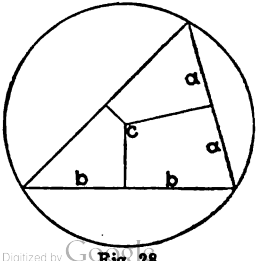

Fig. 28.

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Proportion by segments of chords: $ab = cd$; $\frac{a}{c} = \frac{d}{b}$; $\frac{a}{d} = \frac{c}{b}$. ade and cbf are similar triangles, Fig. 29.

Mean proportional: By similar triangles, $\frac{a}{c} = \frac{c}{b}$, whence c is a mean proportional between a and b ($a\,b = c^2$), Fig. 30.

Inscribed square and octagon; Circumscribed circle, Fig. 31.

Circumscribed square and octagon; Inscribed circle, Fig. 32.

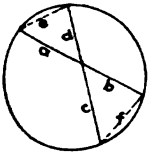

Fig. 29.

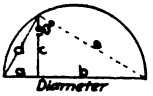

Fig. 30.

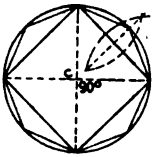

Fig. 31.

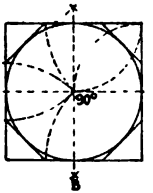

Fig. 32.

Pentagon inscribed in a circle: bd is one side of the pentagon (5 sides). Process: Bisect radius at a; $ac = ab$; $bd = bc$. Step off de, ef, etc., $= bd$, and connect, Fig. 33.

Decagon inscribed in a circle: Bisect the circular arcs of an inscribed pentagon, making twice the number of sides, Fig. 34.

Pentagon of given side ab: Erect the perpendicular $bc = \frac{1}{2}$ side ab; produce ac to d so that $cd = cb$; then $bd = be = ae =$ radius of pentagon $=$ radius of circumscribed circle, with center at e. Step off $a\,f$, $f\,g$, etc., $= ab$, and connect, Fig. 35.

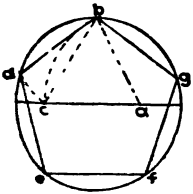

Fig. 33.

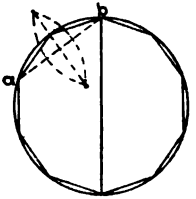

Fig. 34.

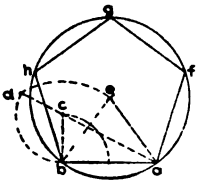

Fig. 35.

Hexagon inscribed in a circle: Each side is equal to the radius, Fig. 36.

Equilateral Triangle: Connect alternate points of hexagon, making half the number of sides, Fig. 37.

Dodecagon: Bisect the circular arcs of an inscribed hexagon, making twice the number of sides, Fig. 38.

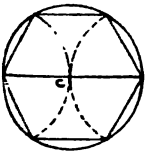

Fig. 36.

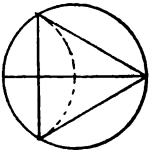

Fig. 37.

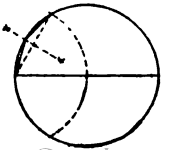

Fig. 38.

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8.—SOLID GEOMETRY.

Planes, Angles and Lines.—

Plane: Determined by (1) two parallel lines; (2) two intersecting lines; (3) three points not in the same straight line; (4) a straight line and a point outside of it.

Straight line: Intersection of two planes not parallel, as *a b*, Fig. 1.

Dihedral angle: The angle between two planes, measured at right angle to each plane and to the line of their intersection, or edge, as *A*, Fig. 1.

Right dihedral angle: A dihedral angle that is 90°, Fig. 2.

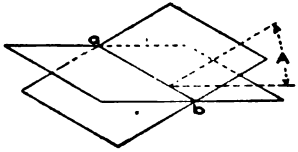

Fig. 1.

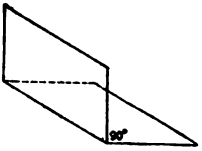

Fig. 2.

Other angles: Acute, obtuse, complementary, supplementary, adjacent, etc., as in Plane Geometry.

Coordinate planes: *V* and *H*, Fig. 3, are planes perpendicular to each other; hence, any line *v* in one plane, if perpendicular to *a b*, is perpendicular also to the other plane.

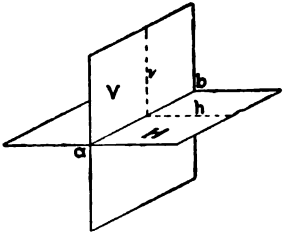

Fig. 3.

Polyhedrons.—

Polyhedron: Solid bounded by planes.

Tetrahedron: 4 triangular faces; 6 edges, Fig. 4.

Hexahedron: 6 square faces; 12 edges, Fig. 5.

Fig. 4.

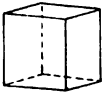

Fig. 5.

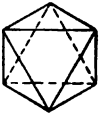

Fig. 6.

Fig. 7.

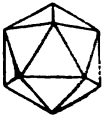

Fig. 8.

Octahedron: 8 triangular faces; 12 edges, Fig. 6.

Dodecahedron: 12 pentagonal faces; 30 edges, Fig. 7.

Icosahedron: 20 triangular faces; 30 edges, Fig. 8.

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Prisms.—

Prism: A polyhedron with two opposite faces parallel and equal polygons the other faces parallelograms, Fig. 9.
Right prism: Lateral edges perpendicular to bases, Fig. 10.
Regular prism: A right prism whose bases are regular polygons.
Oblique prism: Lateral edges are oblique to bases, Fig. 11.
Triangular prism: Bases are triangles, Fig. 11.

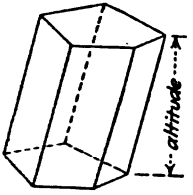

Fig. 9.

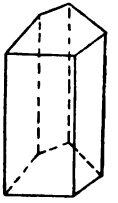

Fig. 10.

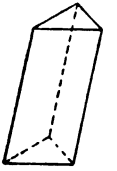

Fig. 11.

Quadrangular prism: Bases are quadrilaterals, Fig. 12.
Parallelopiped: Prism whose bases are parallelograms.
Right parallelopiped: Lateral edges are perpendicular to the bases, Fig. 12.
Rectangular parallelopiped: Six faces are rectangles, Fig. 12.
Cube: Parallelopiped whose six faces are squares, Fig. 13.
Volume of any prism = area of base × altitude.
Truncated prism: Portion included between base and plane section oblique to base, Fig. 14.

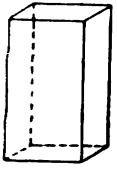

Fig. 12.

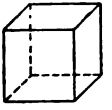

Fig. 13.

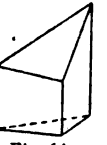

Fig. 14.

Pyramids.—

Pyramid: Polyhedron whose base is a polygon and whose sides are triangles joining a common apex or top, Fig. 15.
Altitude: Perpendicular distance from apex to plane of base.
Triangular pyramid: Base is a triangle (∴ solid is a tetrahedron).
Quadrangular pyramid: Base is a quadrilateral.
Regular pyramid: Base is a polygon, and apex is directly over its center, Fig. 16.

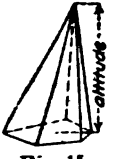

Fig. 15.

Fig. 16.

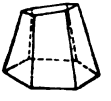

Fig. 17.

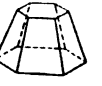

Fig. 18.

Irregular pyramid: One that is not regular.
Volume of any pyramid: ⅓ area of base × altitude.
Slant height: Distance along any lateral face from its apex to the middle of its base.

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Truncated pyramid: That portion between the base and a plane section cutting all the sides, Fig. 17.
Frustum of a pyramid: That portion between the base and a plane section parallel with the base, Fig. 18.

Cylinders.—

Cylinder = circular cylinder: Two bases are circular and parallel; all sections parallel with the bases are circular, Fig. 19.
Elliptic cylinder: Two bases are elliptical and parallel; all sections parallel with the bases are elliptical.

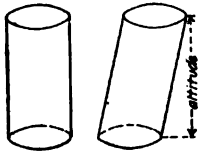

Fig. 19.

Right cylinder: Elements are perpendicular to the bases.
Oblique cylinder: Elements are oblique to the bases.
Volume of any cylinder = area of base × altitude.

Cones.—

Cone = circular cone: Base and all sections parallel with it are circular; elements composing the sides meet at a common apex or top, Fig. 20.
Elliptic cone: Base and all sections parallel with it are elliptical.
Altitude: Perpendicular distance from apex to plane of base.
Right cone: Cone with axis perpendicular to base.
Oblique cone: Cone with axis oblique to base.

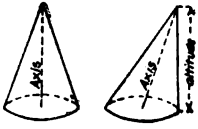

Fig. 20.

Fig. 21.

Fig. 22.

Volume of any cone: ⅓ area of base × altitude.
Truncated cone: That portion between the base and a plane cutting all the elements, Fig. 21.
Frustum of a cone: That portion between the base and a plane section parallel with the base, Fig. 22.

Spheres.—

Sphere: Solid whose every section is circular, Fig. 23.
Radius: Distance from center to surface.
Diameter: Two radii forming one straight line.
Great circle: The largest plane section (cuts center of sphere).
Pole of a circle: End of diameter or axis perpendicular to the plane of the circle.

Fig. 23.

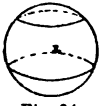

Fig. 24.

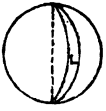

Fig. 25.

Fig. 26.

Fig. 27.

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Arc of a great circle: Shortest surface distance between two points on surface of sphere.
Polar distance of a circle: Distance from nearest pole to circumference of circle.
Quadrant: Polar distance of a great circle.
Surface of sphere $= 4\pi$ radius^2.
Volume of sphere $= \frac{4}{3}\pi$ radius^3.
Zone: Portion of surface between two parallel planes, Fig. 24.
Lune: Portion of surface between two semi-circumferences of great circles, Fig. 25.
Spherical segment: Portion of sphere between two parallel planes, Fig. 26.
Spherical pyramid: Apex corresponds to center of sphere, sides are radial planes, and base is a spherical polygon Fig. 27.
Spherical cone: Cone with spherical base, Fig. 27.

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9.—PLANE TRIGONOMETRY.

Plane Trigonometry deals with the functions of plane triangles, and shows how to compute the unknown elements when certain of the known elements are given. The six elements of a triangle are its three sides and three angles; and three of these, one of which must be a side, must be known in order to solve the triangle.

Trigonometric functions are the ratios of the sides of a right angle triangle. There are six primary ratios, namely, *sine*, *cosine*, *tangent*, *cotangent*, *secant* and *cosecant*. In addition to these, however, there are two secondary functions sometimes employed, namely, *versed sine* and *coversed sine*; and two tertiary functions seldom used, namely, *exsecant* and *coexsecant*.

The following trigonometric functions or ratios refer to Fig. 1, in which —

h = hypothenuse,
p = perpendicular,
b = base,
A = angle at base,
B = angle opposite base.
sine A = cosine B, etc.

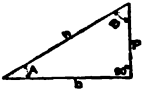

Fig. 1.

PRIMARY FUNCTIONS:

$\sin A\ (=\cos B) = \frac{p}{h}$.

$\cos A\ (=\sin B) = \frac{b}{h}$.

$\tan A\ (=\cot B) = \frac{p}{b}$.

$\cot A\ (=\tan B) = \frac{b}{p}$.

$\sec A\ (=\csc B) = \frac{h}{b}$.

$\csc A\ (=\sec B) = \frac{h}{p}$.

SECONDARY FUNCTIONS:

$\text{vrs } A\ (=\text{cvs } B) = 1-\cos A = 1-\frac{b}{h} = \frac{h-b}{h}$.

$\text{cvs } A\ (=\text{vrs } B) = 1-\sin A = 1-\frac{p}{h} = \frac{h-p}{h}$.

TERTIARY FUNCTIONS:

$\text{xsc } A\ (=\text{cxc } B) = \sec A - 1 = \frac{h}{b} - 1 = \frac{h-b}{b}$.

$\text{cxc } A\ (=\text{xsc } B) = \csc A - 1 = \frac{h}{p} - 1 = \frac{h-p}{p}$.

1.—EQUIVALENT VALUES OF PRIMARY FUNCTIONS OF ANY ANGLE x IN PURE TERMS OF EACH OF THE OTHER FUNCTIONS.

sin	*cos*	*tan*	*cot*	*sec*	*csc*
$\sin x$	$=\sqrt{1-\cos^2 x}$	$=\frac{\tan x}{\sqrt{1+\tan^2 x}}$	$=\frac{1}{\sqrt{1+\cot^2 x}}$	$=\frac{\sqrt{\sec^2 x-1}}{\sec x}$	$=\frac{1}{\csc x}$
$\sqrt{1-\sin^2 x}$	$=\cos x$	$=\frac{1}{\sqrt{1+\tan^2 x}}$	$=\frac{\cot x}{\sqrt{1+\cot^2 x}}$	$=\frac{1}{\sec x}$	$=\frac{\sqrt{\csc^2 x-1}}{\csc x}$
$\frac{\sin x}{\sqrt{1-\sin^2 x}}$	$=\frac{\sqrt{1-\cos^2 x}}{\cos x}$	$=\tan x$	$=\frac{1}{\cot x}$	$=\sqrt{\sec^2 x-1}$	$=\frac{1}{\sqrt{\csc^2 x-1}}$
$\frac{\sqrt{1-\sin^2 x}}{\sin x}$	$=\frac{\cos x}{\sqrt{1-\cos^2 x}}$	$=\frac{1}{\tan x}$	$=\cot x$	$=\frac{1}{\sqrt{\sec^2 x-1}}$	$=\sqrt{\csc^2 x-1}$
$\frac{1}{\sqrt{1-\sin^2 x}}$	$=\frac{1}{\cos x}$	$=\sqrt{1+\tan^2 x}$	$=\frac{\sqrt{1+\cot^2 x}}{\cot x}$	$=\sec x$	$=\frac{\csc x}{\sqrt{\csc^2 x-1}}$
$\frac{1}{\sin x}$	$=\frac{1}{\sqrt{1-\cos^2 x}}$	$=\frac{\sqrt{1+\tan^2 x}}{\tan x}$	$=\sqrt{1+\cot^2 x}$	$=\frac{\sec x}{\sqrt{\sec^2 x-1}}$	$=\csc x$

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Other Equivalent Values of Primary Functions of Any Angle x and (Angle x)^2.

$\sin x = \dfrac{\cos x}{\cot x} = \cos x \tan x.$	$\sin^2 x = 1 - \cos^2 x.$	$\dfrac{1}{\sin^2 x} = 1 + \dfrac{\cos^2 x}{\sin^2 x}$
$\cos x = \dfrac{\sin x}{\tan x} = \sin x \cot x.$	$\cos^2 x = 1 - \sin^2 x.$	$\dfrac{1}{\cos^2 x} = 1 + \dfrac{\sin^2 x}{\cos^2 x}$
$\tan x = \dfrac{\sin x}{\cos x} = \sin x \sec x.$	$\tan^2 x = \sec^2 x - 1.$	$\dfrac{1}{\tan^2 x} = \dfrac{\cos^2 x}{\sin^2 x}$
$\cot x = \dfrac{\cos x}{\sin x} = \cos x \csc x.$	$\cot^2 x = \csc^2 x - 1.$	$\dfrac{1}{\cot^2 x} = \dfrac{\sin^2 x}{\cos^2 x}$
$\sec x = \dfrac{\tan x}{\sin x} = \tan x \csc x.$	$\sec^2 x = 1 + \tan^2 x.$	$\dfrac{1}{\sec^2 x} = \dfrac{\sin^2 x}{\tan^2 x}$
$\csc x = \dfrac{\cot x}{\cos x} = \cot x \sec x.$	$\csc^2 x = 1 + \cot^2 x.$	$\dfrac{1}{\csc^2 x} = \dfrac{\cos^2 x}{\cot^2 x}$

Values of Trigonometric functions in the four quadrants.—In Fig. 2 the angle x_1 lies wholly within the first quadrant, 0° to 90°; x_2 extends into the second quadrant, comprising angles from 90° to 180°; x_3 extends into the third quadrant, comprising angles from 180° to 270°; and x_4 extends into the fourth quadrant, comprising angles from 270° to 360°.

It will be noticed that in all cases the sine falls perpendicularly upon the axis $X-X$ and that the cosine falls perpendicularly upon the coordinate axis $Y-Y$. Assuming arbitrarily that, when the sine is *above* the axis of $X-X$, it is *plus* and when *below*, it is *minus*; also, that when the cosine is to the *right* of the axis $Y-Y$ it is *plus*, and when to the *left* it is *minus*; we obtain the following sign values of the fundamental sign functions:

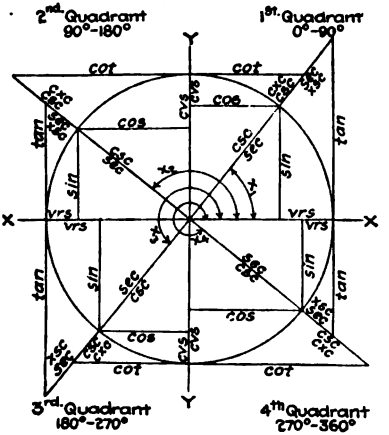

Fig. 2.

	1st quad.	2nd quad.	3rd quad.	4th quad.
Sine.....	+	+	−	−
Cosine.....	+	−	−	+

The sign values may further be extended to the other trigonometric functions by remembering that

$$\tan x = \frac{\sin x}{\cos x},\ \cot x = \frac{\cos x}{\sin x},\ \sec x = \frac{1}{\cos x},\ \csc x = \frac{1}{\sin x},$$

$$\text{vrs}\ x = 1 - \cos x,\ \text{cvs}\ x = 1 - \sin x,\ \text{xsc}\ x = \sec x - 1,\ \text{cxc}\ x = \csc x - 1;$$

and that in either multiplication or division, like signs give plus and unlike, minus. Hence the following table:

	1st quad.	2nd quad.	3rd quad.	4th quad.
Sine, cosecant, coexsecant..	+	+	−	−
Cosine, secant, exsecant....	+	−	−	+
Tangent, cotangent.....	+	−	+	−
Versed sine, coversed sine .	+	+	+	+

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2.—Natural Functions of Angles from 0° to 360°.

	Angle	Sin	Cos	Tan	Cot	Sec	Csc	Vrs	Cvs	Xsc	Cxc
1st Quadrant	0°	0	1	0	∞	1	∞	0	1	0	∞
	15°										
	30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{2-\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{2\sqrt{3}-3}{3}$	1
	45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$	$\frac{2-\sqrt{2}}{2}$	$\frac{2-\sqrt{2}}{2}$	$\sqrt{2}-1$	$\sqrt{2}-1$
	60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\frac{1}{2}$	$\frac{2-\sqrt{3}}{2}$	1	$\frac{2\sqrt{3}-3}{3}$
	75°										
	90°	1	0	∞	0	∞	1	1	0	∞	0
2nd Quadrant	105°										
	120°	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	-2	$\frac{2\sqrt{3}}{3}$	$\frac{3}{2}$	$\frac{2-\sqrt{3}}{2}$	-3	$\frac{2\sqrt{3}-3}{3}$
	135°	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	-1	$-\sqrt{2}$	$\sqrt{2}$	$\frac{2+\sqrt{2}}{2}$	$\frac{2-\sqrt{2}}{2}$	$-(\sqrt{2}+1)$	$\sqrt{2}-1$
	150°	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$	$-\frac{2\sqrt{3}}{3}$	2	$\frac{2+\sqrt{3}}{2}$	$\frac{1}{2}$	$-\frac{2\sqrt{3}+3}{3}$	1
	165°										
	180°	0	-1	0	∞	-1	∞	2	1	-2	∞
3rd Quadrant	195°										
	210°	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$-\sqrt{3}$	$-\frac{2\sqrt{3}}{3}$	-2	$\frac{2+\sqrt{3}}{2}$	$\frac{3}{2}$	$-\frac{2\sqrt{3}+3}{3}$	-3
	225°	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1	1	$-\sqrt{2}$	$-\sqrt{2}$	$\frac{2+\sqrt{2}}{2}$	$\frac{2+\sqrt{2}}{2}$	$-(\sqrt{2}+1)$	$-(\sqrt{2}+1)$
	240°	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	-2	$-\frac{2\sqrt{3}}{3}$	$\frac{3}{2}$	$\frac{2+\sqrt{3}}{2}$	-3	$-\frac{2\sqrt{3}+3}{3}$
	255°										
	270°	-1	0	∞	0	∞	-1	1	2	∞	-2
4th Quadrant	285°										
	300°	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	2	$-\frac{2\sqrt{3}}{3}$	$\frac{1}{2}$	$\frac{2+\sqrt{3}}{3}$	1	$-\frac{2\sqrt{3}+3}{3}$
	315°	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1	-1	$\sqrt{2}$	$-\sqrt{2}$	$\frac{2-\sqrt{2}}{2}$	$\frac{2+\sqrt{2}}{2}$	$\sqrt{2}-1$	$-(\sqrt{2}+1)$
	330°	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	-2	$\frac{2-\sqrt{3}}{2}$	$\frac{3}{2}$	$\frac{2\sqrt{3}-3}{3}$	-3
	345°										
	360°	0	1	0	∞	1	∞	0	1	0	∞

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Functions of complement, supplement, etc., of any angle x.

The complement of any angle x is $90°-x$. Thus, in Fig. 1, page 136, B is the complement of A.

The supplement of any angle x is $180°-x$, etc.

In the following Table, the first four primary functions of angles extending into either of the four quadrants (see Fig. 2, page 137) are reduced to the functions of angles not greater than 90°.

	1st Quadrant.	2nd Quadrant.
$\sin x$	$\sin x_1 = \cos(90° - x_1)$	$\sin x_2 = \sin(180° - x_2)$ $\sin x_2 = \cos(x_2 - 90°)$
$\cos x$	$\cos x_1 = \sin(90° - x_1)$	$\cos x_2 = -\cos(180° - x_2)$ $\cos x_2 = -\sin(x_2 - 90°)$
$\tan x$	$\tan x_1 = \cot(90° - x_1)$	$\tan x_2 = -\tan(180° - x_2)$ $\tan x_2 = -\cot(x_2 - 90°)$
$\cot x$	$\cot x_1 = \tan(90° - x_1)$	$\cot x_2 = -\cot(180° - x_2)$ $\cot x_2 = -\tan(x_2 - 90°)$

	3rd Quadrant.	4th Quadrant.
$\sin x$	$\sin x_3 = -\sin(x_3 - 180°)$ $\sin x_3 = -\cos(270° - x_3)$	$\sin x_4 = -\sin(360° - x_4)$ $\sin x_4 = -\cos(x_4 - 270°)$
$\cos x$	$\cos x_3 = -\cos(x_3 - 180°)$ $\cos x_3 = -\sin(270° - x_3)$	$\cos x_4 = \cos(360° - x_4)$ $\cos x_4 = \sin(x_4 - 270°)$
$\tan x$	$\tan x_3 = \tan(x_3 - 180°)$ $\tan x_3 = \cot(270° - x_3)$	$\tan x_4 = -\tan(360° - x_4)$ $\tan x_4 = -\cot(x_4 - 270°)$
$\cot x$	$\cot x_3 = \cot(x_3 - 180°)$ $\cot x_3 = \tan(270° - x_3)$	$\cot x_4 = -\cot(360° - x_4)$ $\cot x_4 = -\tan(x_4 - 270°)$

Functions of the sum of two angles $(x+y)$.

$$\sin(x+y) = \sin x \cos y + \cos x \sin y.$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y.$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}.$$

$$\cot(x+y) = \frac{\cot x \cot y - 1}{\cot y + \cot x}.$$

$$\sin x + \sin y = 2 \sin \tfrac{1}{2}(x+y) \cos \tfrac{1}{2}(x-y).$$

$$\cos x + \cos y = 2 \cos \tfrac{1}{2}(x+y) \cos \tfrac{1}{2}(x-y).$$

Functions of the difference of two angles $(x-y)$.

$$\sin(x-y) = \sin x \cos y - \cos x \sin y.$$

$$\cos(x-y) = \cos x \cos y + \sin x \sin y.$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}.$$

$$\cot(x-y) = \frac{\cot x \cot y + 1}{\cot y - \cot x}.$$

$$\sin x - \sin y = 2 \cos \tfrac{1}{2}(x+y) \sin \tfrac{1}{2}(x-y).$$

$$\cos x - \cos y = -2 \sin \tfrac{1}{2}(x+y) \sin \tfrac{1}{2}(x-y).$$

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Functions of half an angle($\frac{1}{2}x$).

$$\sin \tfrac{1}{2}x = \frac{\sin x}{2\cos \frac{1}{2}x} = \sqrt{\frac{\text{vers}\, x}{2}} = \sqrt{\frac{1-\cos x}{2}}.$$

$$\cos \tfrac{1}{2}x = \frac{\sin x}{2\sin \frac{1}{2}x} = \sqrt{\frac{1+\cos x}{2}}.$$

$$\tan \tfrac{1}{2}x = \frac{1-\cos x}{\sin x} = \frac{\sin x}{1+\cos x} = \operatorname{cosec} x - \cot x = \sqrt{\frac{1-\cos x}{1+\cos x}}.$$

$$\cot \tfrac{1}{2}x = \frac{1+\cos x}{\sin x} = \frac{\sin x}{1-\cos x} = \frac{\sin x}{\text{vers}\, x} = \frac{1}{\operatorname{cosec} x - \cot x}.$$

Functions of twice an angle ($2x$).

$$\sin 2x = 2\sin x\cos x = \frac{2\tan x}{1+\tan^2 x}.$$

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1 = \frac{1-\tan^2 x}{1+\tan^2 x}.$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x} = \frac{\sin 3x - \sin x}{\cos 3x + \cos x}.$$

$$\cot 2x = \frac{\cot^2 x - 1}{2\cot x}.$$

Functions of three times an angle ($3x$).

$$\sin 3x = 3\sin x - 4\sin^3 x.$$
$$\cos 3x = 4\cos^3 x - 3\cos x.$$

$$\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}.$$

Functions of four times an angle ($4x$).

$$\sin 4x = 4\sin x\cos x - 8\sin^3 x\cos x.$$
$$\cos 4x = 1 - 8\cos^2 x + 8\cos^4 x.$$

$$\tan 4x = \frac{4\tan x - 4\tan^3 x}{1 - 6\tan^2 x + \tan^4 x}.$$

Inverse Trigonometric Functions.—In logarithms we have seen that the anti-logarithm N is a number whose logarithm is n; that is, it is the number corresponding to the logarithm n. Similarly, in Trigonometry, we have—

The anti-sine of s = angle whose sine is s = angle corresponding to sine s. Thus, $\sin A = s$, or $A = \sin^{-1} s$ (Reads A = anti-sine or inverse sine of s.)

The anti-cosine of c = angle whose cosine is c = angle corresponding to cosine c. Thus, $\cos A = c$, or $A = \cos^{-1} c$ (Reads A = anti-cosine or inverse cosine of c.).

And so on with the remaining functions.

This must not be confused with the negative exponent, -1, as $n^{-1} = \frac{1}{n}$; $x^{-1} = \frac{1}{x}$; $(\sin x)^{-1} = \frac{1}{\sin x}$; $(\cos y)^{-1} = \frac{1}{\cos y}$; etc. But $\sin A = \sin(\sin^{-1} s) = s$; $\cos A = \cos(\cos^{-1} c) = c$; etc.

Examples: $\frac{1}{2} = \sin 30°$ ∴ the anti-sine of $\frac{1}{2}$ is $30°$.

$\frac{\sqrt{2}}{2} = \cos 45°$ ∴ the anti-cosine of $\frac{\sqrt{2}}{2}$ is $45°$.

$\sqrt{3} = \tan 60°$ ∴ the anti-tangent of $\sqrt{3}$ is $60°$.

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Natural and logarithmic trigonometric functions.—On pages 144 to 175 will be found tables (3 and 4) of natural functions, and on pages 176 to 198 a table (5) of logarithmic functions, of angles up to 90° or in the first quadrant. For angles greater than 90° see Table 1 of functions of any angle reduced to function of an angle not greater than 90°. In general, the following rules are convenient to memorize:

Sine	of an	angle	= the cosine	of its	complement.
"	"	"	= " sine	"	supplement.
Tangent	"	"	= " cotangent	"	complement.
"	"	"	= " −tangent	"	supplement.

In using the natural functions the processes of multiplication and division have to be performed, while the logarithmic functions are designed to reduce these to the simple processes of addition and subtraction. Logarithmic functions are simply logarithms of the natural functions.

The tables are as follows:

Table 3, page 144, Natural Sines, Tangents, Cotangents, Cosines, (Versed Sines, Coversed Sines).

Table 4, page 167, Natural Secants, Cosecants, (Exsecants, Coexsecants).

Table 5, page 176, Logarithmic Sines, Tangents, Cotangents, Cosines (Secants, Cosecants).

Solution of right angle triangles.—The six primary functions are all that need be employed in the solution of any triangle:

(1) $\sin A\ (=\cos B) = \frac{p}{h}$, Whence, $p = h \sin A = h \cos B$.

(2) $\cos A\ (=\sin B) = \frac{b}{h}$, " $b = h \cos A = h \sin B$.

(3) $\tan A\ (=\cot B) = \frac{p}{b}$, " $p = b \tan A = b \cot B$.

(4) $\cot A\ (=\tan B) = \frac{b}{p}$ " $b = p \cot A = p \tan B$.

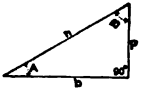

Fig. 3.

(5) $\sec A\ (=\csc B) = \frac{h}{b}$ " $h = b \sec A = b \csc B$; or $h = \frac{b}{\cos A} = \frac{b}{\sin B}$

(6) $\csc A\ (=\sec B) = \frac{h}{p}$, " $h = p \csc A = p \sec B$; or $h = \frac{p}{\sin A} = \frac{p}{\cos B}$

[Also $h^2 = b^2 + p^2$. Whence $h = \sqrt{b^2+p^2}$; $b = \sqrt{h^2-p^2}$; $p = \sqrt{h^2-b^2}$.]

Example 1.

Given: $A = 32°$, $h = 20.3$.

Required: p.

Solution: (1) $p = h \sin A$.

By natural functions.

$\sin A = \sin 32° = .52992$

$p = .52992 \times 20.3 = 10.757$. Ans.

By logarithmic functions.

$\log 20.3 = 1.30750$

$\log \sin 32° = 9.72421$

Ans. 10.757. 1.03171

Example 2.

Given: $p = 25$, $b = 20$.

Required: Angle B.

Solution: (4) $\tan B = \frac{b}{p}$.

By natural functions.

$\tan B = \frac{b}{p} = \frac{20}{25} = .8$.

Angle $B = \tan^{-1} .8 = 38°-40'$. Ans.

By logarithmic functions.

$\log 20 = 1.30103$

$\log 25 = 1.39794$

Ans. $38° - 40'$ 9.90309

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Example 3.

Given: $A = 16° - 10'$, $p = 40$.
Required: h.
Solution: (6) $h = p + \sin A$.

By natural functions.

$\sin A = \sin 16° - 10' = 0.27843$.
$h = 40 + 0.27843 = 143.66$. Ans.

By logarithmic functions.

$\log 40$ =	1.60206
$\log \sin 16° - 10'$ =	9.44472
Ans. 143.66	2.15734

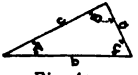

Fig. 4.

Solution of any triangle.—The following principles, easily memorized, lead to the solution of any triangle, right or oblique.

Sides are proportional to sines of opposite angles .....(1)

Thus, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

Sin ang opp given side : sin ang opp req side :: given side : req side....(2)

Thus, $\frac{\sin B}{\sin A} = \frac{\text{given side } b}{\text{required side } a}$, in which A, B and b are given.

Sum of sides : diff :: tang half sum of other two angles : tang half diff....(3)

Thus, $\frac{b+c}{b-c} = \frac{\tan \frac{1}{2}(B+C)}{\tan \frac{1}{2}(B-C)}$, in which A, b and c are given.

The square of any side as $a = a^2 = b^2 + c^2 - 2\,bc \cos A$ .....(4)

Thus, $\cos A = \frac{b^2 + c^2 - a^2}{2\,bc}$, in which a, b and c are given.

$$\sin \tfrac{1}{2}A = \sqrt{\frac{(s-b)\ (s-c)}{b\,c}}, \text{ in which } s = \frac{a+b+c}{2}, \text{ given.}$$

$$\tan \tfrac{1}{2}A = \sqrt{\frac{(s-b)\ (s-c)}{s\,(s-a)}}, \text{ in which } s = \frac{a+b+c}{2}, \text{ given.}$$

Rules in conjunction with the above:

Given: One side and two angles. Solve for "Req side" in (2).
Given: Two sides and angle opposite one of them. Solve for "Req side" and for one of the angles, in (2).
Given: Two sides and included angle. Solve for "Tang half diff" in (3).
Given: Three sides.
(*a*). Solve for $\cos A$, in (4), unless A is very small.
(*b*). Solve for $\sin \frac{1}{2} A$, in (4), unless A is very large.
(*c*). Solve for $\tan \frac{1}{2} A$, in (4), in general preferred.

If all the angles are required we may use the formulas:

$$\tan \tfrac{1}{2} A = \frac{r}{s-a}, \text{ in which } r = \sqrt{\frac{(s-a)\ (s-b)\ (s-c)}{s}};$$

$$\tan \tfrac{1}{2} B = \frac{r}{s-b};\ \tan \tfrac{1}{2} C = \frac{r}{s-c}.$$

Circular Measure.—It is sometimes convenient in mathematical calculations to express angles in circular measure. The unit of circular measure is an angle subtended by an arc whose length is equal to the radius of the circle. The value of such an angle in common measure $= 57° - 17' - 45'' = 57.2958°$, called a radian. The number of radians in 180° is 3.141592, and as this value is equal to π we call $180° = \pi$ in circular measure. Hence,

$$2\pi = 360°$$
$$\pi = 180°$$
$$\frac{\pi}{2} = 90°.$$

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Cubic Equations.—The general form of a cubic equation is

$$ax^3+bx^2+cx+d=0 \quad \ldots\ldots (1)$$

Dividing by a, we have

$$x^3+\frac{b}{a}x^2+\frac{c}{a}x+\frac{d}{a}=0 \quad \ldots\ldots (2)$$

By substitution, this can be reduced to the second general form

$$x^3+Bx^2+Cx+D=0 \quad \ldots\ldots (3)$$

To eliminate x^2, let $x=y-\frac{B}{3}$, then equation (3) reduces to

$$y^3+y\left(C-\frac{B^2}{3}\right)+\left(\frac{2B^3}{27}-\frac{BC}{3}+D\right)=0 \quad \ldots\ldots (4)$$

By substitution, this can be reduced to the third general form

$$y^3+px+q=0 \quad \ldots\ldots (5)$$

Now if we let $y=u+v$, equation (5) reduces to

$$u^3+v^3+(u+v)(3uv+p)+q=0 \quad \ldots\ldots (6)$$

Whence, $3uv+p=0$; $u^3+v^3+q=0$; $u^3v^3=-\frac{p^3}{27}$; and $u^3+v^3=-q$

And we have for the final equations

$$u^3=-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}} \quad \ldots\ldots (7)$$

$$v^3=-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}} \quad \ldots\ldots (8)$$

Therefore $y=u+v=\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$ ..... (9)

And $\quad x=y-\frac{B}{3}$ (see above). ..... (10)

Note that the above is a purely algebraic solution, which can obtain only when $\frac{q}{4}+\frac{p^3}{27}$ is equal to or greater than 0. When $\frac{q}{4}+\frac{p^3}{27}<0$ we have to resort to the trigometric solution, following.

Trigonometric Solution.—In the equation, (9),

$$y=\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$$

if $\frac{q}{4}+\frac{p^3}{27}<0$, then the roots are imaginary and y is the sum of two imaginary quantities. To solve the equation for y, under these conditions, proceed as follows: Let $-\frac{q}{2}=n\cos\theta$, and $\frac{q^2}{4}+\frac{p^3}{27}=-n^2\sin^2\theta$; whence, $n=\sqrt{-\frac{p^3}{27}}$, and $\cos\theta=-\frac{q}{2n}$. From this, the values of the three roots, y, are

$$2\sqrt[3]{n}\cos\frac{\theta}{3},\quad -2\sqrt[3]{n}\cos\left(60^\circ-\frac{\theta}{3}\right),\quad 2\sqrt[3]{n}\cos\left(120^\circ-\frac{\theta}{3}\right).$$

And $$x=y-\frac{B}{3}.$$

These calculations can be made readily by the use of logarithms. It is well to insert the value of x, thus found, in the original equation, (1), and solve as a check.

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3.—Natural Sines, Tangents, Cotangents, Cosines.

(Versed sine = 1 − cosine; coversed sine = 1 − sine.)

0° ′	Sine.	Tang.	Cotang.	Cosine.		1° ′	Sine.	Tang.	Cotang.	Cosine.	
0	.0000000	.000000	Infinite	1.000000	60	0	.0174524	.017455	57.28996	.9998477	60
1	.0002909	.000291	3437.746	1.000000	59	1	.0177432	.017746	56.35059	.9998426	59
2	.0005818	.000582	1718.873	.9999998	58	2	.0180341	.018037	55.44151	.9998374	58
3	.0008727	.000872	1145.915	.9999996	57	3	.0183249	.018328	54.56130	.9998321	57
4	.0011636	.001163	859.4363	.9999993	56	4	.0186158	.018619	53.70858	.9998267	56
5	.0014544	.001454	687.5488	.9999989	55	5	.0189066	.018910	52.88211	.9998213	55
6	.0017453	.001745	572.9572	.9999985	54	6	.0191974	.019201	52.08067	.9998157	54
7	.0020362	.002036	491.1060	.9999979	53	7	.0194883	.019492	51.30315	.9998101	53
8	.0023271	.002327	429.7175	.9999973	52	8	.0197791	.019783	50.54850	.9998044	52
9	.0026180	.002618	381.9709	.9999966	51	9	.0200699	.020074	49.81572	.9997986	51
10	.0029089	.002908	343.7737	.9999958	50	10	.0203608	.020365	49.10388	.9997927	50
11	.0031998	.003199	312.5213	.9999949	49	11	.0206516	.020656	48.41208	.9997867	49
12	.0034907	.003490	286.4777	.9999939	48	12	.0209424	.020947	47.73950	.9997807	48
13	.0037815	.003781	264.4408	.9999928	47	13	.0212332	.021238	47.08534	.9997745	47
14	.0040724	.004072	245.5519	.9999917	46	14	.0215241	.021529	46.44886	.9997683	46
15	.0043633	.004363	229.1816	.9999905	45	15	.0218149	.021820	45.82935	.9997620	45
16	.0046542	.004654	214.8576	.9999892	44	16	.0221057	.022111	45.22614	.9997556	44
17	.0049451	.004945	202.2187	.9999878	43	17	.0223965	.022402	44.63859	.9997492	43
18	.0052360	.005236	190.9841	.9999863	42	18	.0226873	.022693	44.06611	.9997426	43
19	.0055268	.005526	180.9322	.9999847	41	19	.0229781	.022984	43.50812	.9997360	41
20	.0058177	.005817	171.8854	.9999831	40	20	.0232690	.023275	42.96407	.9997292	40
21	.0061086	.006108	163.7001	.9999813	39	21	.0235598	.023566	42.43346	.9997224	39
22	.0063995	.006399	156.2590	.9999795	38	22	.0238506	.023857	41.91579	.9997156	38
23	.0066904	.006690	149.4650	.9999776	37	23	.0241414	.024148	41.41058	.9997086	37
24	.0069813	.006981	143.2371	.9999756	36	24	.0244322	.024439	40.91741	.9997015	36
25	.0072721	.007272	137.5075	.9999736	35	25	.0247230	.024730	40.43583	.9996943	35
26	.0075630	.007563	132.2185	.9999714	34	26	.0250138	.025021	39.96546	.9996871	34
27	.0078539	.007854	127.3213	.9999692	33	27	.0253046	.025312	39.50589	.9996798	33
28	.0081448	.008145	122.7739	.9999668	32	28	.0255954	.025603	39.05677	.9996724	32
29	.0084357	.008436	118.5401	.9999644	31	29	.0258862	.025894	38.61773	.9996649	31
30	.0087265	.008726	114.5886	.9999619	30	30	.0261769	.026185	38.18845	.9996573	30
31	.0090174	.009017	110.8920	.9999593	29	31	.0264677	.026477	37.76861	.9996497	29
32	.0093083	.009308	107.4264	.9999567	28	32	.0267585	.026768	37.35789	.9996419	28
33	.0095992	.009599	104.1709	.9999539	27	33	.0270493	.027059	36.95600	.9996341	27
34	.0098900	.009890	101.1069	.9999511	26	34	.0273401	.027350	36.56265	.9996262	26
35	.0101809	.010181	98.21794	.9999482	25	35	.0276309	.027641	36.17759	.9996182	25
36	.0104718	.010472	95.48947	.9999452	24	36	.0279216	.027932	35.80055	.9996101	24
37	.0107627	.010763	92.90848	.9999421	23	37	.0282124	.028223	35.43128	.9996020	23
38	.0110535	.011054	90.46333	.9999389	22	38	.0285032	.028514	35.06954	.9995937	23
39	.0113444	.011345	88.14357	.9999357	21	39	.0287940	.028805	34.71511	.9995854	21
40	.0116353	.011636	85.93979	.9999323	20	40	.0290847	.029097	34.36777	.9995770	20
41	.0119261	.011927	83.84350	.9999289	19	41	.0293755	.029388	34.02730	.9995684	19
42	.0122170	.012217	81.84704	.9999254	18	42	.0296662	.029679	33.69350	.9995599	18
43	.0125079	.012508	79.94343	.9999218	17	43	.0299570	.029970	33.36619	.9995512	17
44	.0127987	.012799	78.12634	.9999181	16	44	.0302478	.030261	33.04517	.9995424	16
45	.0130896	.013090	76.39000	.9999143	15	45	.0305385	.030552	32.73026	.9995336	15
46	.0133805	.013381	74.72916	.9999105	14	46	.0308293	.030843	32.42129	.9995247	14
47	.0136713	.013672	73.13899	.9999065	13	47	.0311200	.031135	32.11809	.9995157	13
48	.0139622	.013963	71.61507	.9999025	12	48	.0314108	.031426	31.82051	.9995066	12
49	.0142530	.014254	70.15334	.9998984	11	49	.0317015	.031717	31.52839	.9994974	11
50	.0145439	.014545	68.75008	.9998942	10	50	.0319922	.032008	31.24157	.9994881	10
51	.0148348	.014836	67.40185	.9998900	9	51	.0322830	.032299	30.95992	.9994788	9
52	.0151256	.015127	66.10547	.9998856	8	52	.0325737	.032591	30.68330	.9994693	8
53	.0154165	.015418	64.85800	.9998812	7	53	.0328644	.032882	30.41158	.9994598	7
54	.0157073	.015709	63.65674	.9998766	6	54	.0331552	.033173	30.14461	.9994502	6
55	.0159982	.016000	62.49915	.9998720	5	55	.0334459	.033464	29.88229	.9994405	5
56	.0162890	.016291	61.38290	.9998673	4	56	.0337366	.033755	29.62449	.9994308	4
57	.0165799	.016582	60.30582	.9998625	3	57	.0340274	.034047	29.37110	.9994209	3
58	.0168707	.016873	59.26587	.9998577	2	58	.0343181	.034338	29.12200	.9994110	2
59	.0171616	.017164	58.26117	.9998527	1	59	.0346088	.034629	28.87708	.9994009	1
60	.0174524	.017455	57.28996	.9998477	0	60	.0348995	.034920	28.63625	.9993908	0
	Cosine.	Cotang	Tang.	Sine.	′		Cosine.	Cotang	Tang.	Sine.	′
					89°						88°

Note.—Secant = 1 + cosine; cosecant = 1 + sine.

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2—Natural Sines, Tangents, Cotangents, Cosines.—(Continued).

(Versed sine = 1 − cosine; coversed sine = 1 − sine.)

2° '	Sine.	Tang.	Cotang.	Cosine.		3° '	Sine.	Tang.	Cotang.	Cosine.	
0	.0348995	.034920	28.63625	.9993908	60	0	.0523360	.052407	19.08113	.9986295	60
1	.0351902	.035212	28.39939	.9993806	59	1	.0526264	.052699	18.97552	.9986143	59
2	.0354809	.035503	28.16642	.9993704	58	2	.0529169	.052991	18.87106	.9985989	58
3	.0357716	.035794	27.93723	.9993600	57	3	.0532074	.053282	18.76775	.9985835	57
4	.0360623	.036085	27.71174	.9993495	56	4	.0534979	.053574	18.66556	.9985680	56
5	.0363530	.036377	27.48985	.9993390	55	5	.0537883	.053866	18.56447	.9985524	55
6	.0366437	.036668	27.27148	.9993284	54	6	.0540788	.054158	18.46447	.9985367	54
7	.0369344	.036959	27.05655	.9993177	53	7	.0543693	.054449	18.36553	.9985209	53
8	.0372251	.037250	26.84498	.9993069	52	8	.0546597	.054741	18.26765	.9985050	52
9	.0375158	.037542	26.63669	.9992960	51	9	.0549502	.055033	18.17080	.9984891	51
10	.0378065	.037833	26.43160	.9992851	50	10	.0552406	.055325	18.07497	.9984731	50
11	.0380971	.038124	26.22963	.9992740	49	11	.0555311	.055616	17.98015	.9984570	49
12	.0383878	.038416	26.03073	.9992629	48	12	.0558215	.055908	17.88631	.9984408	48
13	.0386785	.038707	25.83482	.9992517	47	13	.0561119	.056200	17.79344	.9984245	47
14	.0389692	.038998	25.64183	.9992404	46	14	.0564024	.056492	17.70152	.9984081	46
15	.0392598	.039290	25.45170	.9992290	45	15	.0566928	.056784	17.61055	.9983917	45
16	.0395505	.039581	25.26436	.9992176	44	16	.0569832	.057075	17.52051	.9983751	44
17	.0398411	.039872	25.07975	.9992060	43	17	.0572736	.057367	17.43138	.9983585	43
18	.0401318	.040164	24.89782	.9991944	42	18	.0575640	.057659	17.34315	.9983418	42
19	.0404224	.040455	24.71851	.9991827	41	19	.0578544	.057951	17.25580	.9983250	41
20	.0407131	.040746	24.54175	.9991709	40	20	.0581448	.058243	17.16933	.9983082	40
21	.0410037	.041038	24.36750	.9991590	39	21	.0584352	.058535	17.08372	.9982912	39
22	.0412944	.041329	24.19571	.9991470	38	22	.0587256	.058827	16.99895	.9982742	38
23	.0415850	.041621	24.02632	.9991350	37	23	.0590160	.059119	16.91502	.9982570	37
24	.0418757	.041912	23.85927	.9991228	36	24	.0593064	.059410	16.83191	.9982398	36
25	.0421663	.042203	23.69453	.9991106	35	25	.0595967	.059702	16.74961	.9982225	35
26	.0424569	.042495	23.53205	.9990983	34	26	.0598871	.059994	16.66811	.9982052	34
27	.0427475	.042786	23.37177	.9990859	33	27	.0601775	.060286	16.58739	.9981877	33
28	.0430382	.043078	23.21366	.9990734	32	28	.0604678	.060578	16.50745	.9981701	32
29	.0433288	.043369	23.05767	.9990609	31	29	.0607582	.060870	16.42827	.9981525	31
30	.0436194	.043660	22.90376	.9990482	30	30	.0610485	.061162	16.34985	.9981348	30
31	.0439100	.043952	22.75189	.9990355	29	31	.0613389	.061454	16.27217	.9981170	29
32	.0442006	.044243	22.60201	.9990227	28	32	.0616292	.061746	16.19522	.9980991	28
33	.0444912	.044535	22.45409	.9990098	27	33	.0619196	.062038	16.11899	.9980811	27
34	.0447818	.044826	22.30809	.9989968	26	34	.0622099	.062330	16.04348	.9980631	26
35	.0450724	.045118	22.16398	.9989837	25	35	.0625002	.062622	15.96866	.9980450	25
36	.0453630	.045409	22.02171	.9989706	24	36	.0627905	.062914	15.89454	.9980267	24
37	.0456536	.045701	21.88125	.9989573	23	37	.0630808	.063206	15.82110	.9980084	23
38	.0459442	.045992	21.74256	.9989440	22	38	.0633711	.063498	15.74833	.9979900	22
39	.0462347	.046284	21.60563	.9989306	21	39	.0636614	.063790	15.67623	.9979716	21
40	.0465253	.046575	21.47040	.9989171	20	40	.0639517	.064082	15.60478	.9979530	20
41	.0468159	.046867	21.33685	.9989035	19	41	.0642420	.064375	15.53398	.9979343	19
42	.0471065	.047158	21.20494	.9988899	18	42	.0645323	.064667	15.46381	.9979156	18
43	.0473970	.047450	21.07466	.9988761	17	43	.0648226	.064959	15.39427	.9978968	17
44	.0476876	.047741	20.94596	.9988623	16	44	.0651129	.065251	15.32535	.9978779	16
45	.0479781	.048033	20.81882	.9988484	15	45	.0654031	.065543	15.25705	.9978589	15
46	.0482687	.048325	20.69322	.9988344	14	46	.0656934	.065835	15.18934	.9978399	14
47	.0485592	.048616	20.56911	.9988203	13	47	.0659836	.066127	15.12224	.9978207	13
48	.0488498	.048908	20.44648	.9988061	12	48	.0662739	.066419	15.05572	.9978015	12
49	.0491403	.049199	20.32530	.9987919	11	49	.0665641	.066712	14.98978	.9977821	11
50	.0494308	.049491	20.20555	.9987775	10	50	.0668544	.067004	14.92441	.9977627	10
51	.0497214	.049782	20.08719	.9987631	9	51	.0671446	.067296	14.85961	.9977433	9
52	.0500119	.050074	19.97021	.9987486	8	52	.0674349	.067588	14.79537	.9977237	8
53	.0503024	.050366	19.85459	.9987340	7	53	.0677251	.067880	14.73167	.9977040	7
54	.0505929	.050657	19.74029	.9987194	6	54	.0680153	.068173	14.66852	.9976843	6
55	.0508835	.050949	19.62729	.9987046	5	55	.0683055	.068465	14.60591	.9976645	5
56	.0511740	.051241	19.51558	.9986898	4	56	.0685957	.068757	14.54383	.9976445	4
57	.0514645	.051532	19.40513	.9986748	3	57	.0688859	.069049	14.48227	.9976245	3
58	.0517550	.051824	19.29592	.9986598	2	58	.0691761	.069342	14.42123	.9976045	2
59	.0520455	.052116	19.18793	.9986447	1	59	.0694663	.069634	14.36069	.9975843	1
60	.0523360	.052407	19.08113	.9986295	0	60	.0697565	.069926	14.30066	.9975641	0
	Cosine.	Cotang.	Tang.	Sine.	'		Cosine.	Cotang.	Tang.	Sine.	'
					87°						86°

Note.—Secant = 1 + cosine. Cosecant = 1 + sine.

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3.—Natural Sines, Tangents, Cotangents, Cosines.—(Continued).

(Versed sine = 1 − cosine; coversed sine = 1 − sine.)

4° ′	Sine.	Tang.	Cotang.	Cosine.		5° ′	Sine.	Tang.	Cotang.	Cosine.	
0	.0697565	.069926	14.30066	.9975641	60	0	.0871557	.087488	11.43005	.9961947	60
1	.0700467	.070219	14.24113	.9975437	59	1	.0874455	.087781	11.39188	.9961693	59
2	.0703368	.070511	14.18209	.9975233	58	2	.0877353	.088074	11.35397	.9961438	58
3	.0706270	.070803	14.12353	.9975028	57	3	.0880251	.088368	11.31630	.9961183	57
4	.0709171	.071096	14.06545	.9974822	56	4	.0883148	.088661	11.27888	.9960926	56
5	.0712073	.071388	14.00785	.9974615	55	5	.0886046	.088954	11.24171	.9960669	55
6	.0714974	.071680	13.95071	.9974408	54	6	.0888943	.089247	11.20478	.9960411	54
7	.0717876	.071973	13.89404	.9974199	53	7	.0891840	.089540	11.16808	.9960152	53
8	.0720777	.072265	13.83782	.9973990	52	8	.0894738	.089834	11.13163	.9959892	52
9	.0723678	.072558	13.78206	.9973780	51	9	.0897635	.090127	11.09541	.9959631	51
10	.0726580	.072850	13.72673	.9973569	50	10	.0900532	.090420	11.05943	.9959370	50
11	.0729481	.073143	13.67185	.9973357	49	11	.0903429	.090713	11.02367	.9959107	49
12	.0732382	.073435	13.61740	.9973145	48	12	.0906326	.091007	10.98815	.9958844	48
13	.0735283	.073727	13.56339	.9972931	47	13	.0909223	.091300	10.95285	.9958580	47
14	.0738184	.074020	13.50979	.9972717	46	14	.0912119	.091593	10.91777	.9958315	46
15	.0741085	.074312	13.45662	.9972502	45	15	.0915016	.091887	10.88292	.9958049	45
16	.0743986	.074605	13.40386	.9972286	44	16	.0917913	.092180	10.84828	.9957783	44
17	.0746887	.074897	13.35151	.9972069	43	17	.0920809	.092473	10.81387	.9957515	43
18	.0749787	.075190	13.29957	.9971851	44	18	.0923706	.092767	10.77967	.9957247	44
19	.0752688	.075482	13.24803	.9971633	41	19	.0926602	.093060	10.74568	.9956978	41
20	.0755589	.075775	13.19688	.9971413	40	20	.0929499	.093354	10.71191	.9956708	40
21	.0758489	.076068	13.14612	.9971193	39	21	.0932395	.093647	10.67834	.9956437	39
22	.0761390	.076360	13.09575	.9970972	38	22	.0935291	.093940	10.64499	.9956165	38
23	.0764290	.076653	13.04576	.9970750	37	23	.0938187	.094234	10.61184	.9955892	37
24	.0767190	.076945	12.99616	.9970528	36	24	.0941083	.094527	10.57889	.9955620	36
25	.0770091	.077238	12.94692	.9970304	35	25	.0943979	.094821	10.54615	.9955345	35
26	.0772991	.077531	12.89805	.9970080	34	26	.0946875	.095114	10.51360	.9955070	34
27	.0775891	.077823	12.84955	.9969854	33	27	.0949771	.095408	10.48126	.9954794	33
28	.0778791	.078116	12.80141	.9969628	32	28	.0952666	.095701	10.44911	.9954517	32
29	.0781691	.078409	12.75363	.9969401	31	29	.0955562	.095995	10.41715	.9954240	31
30	.0784591	.078701	12.70620	.9969173	30	30	.0958458	.096289	10.38539	.9953962	30
31	.0787491	.078994	12.65912	.9968945	29	31	.0961353	.096582	10.35382	.9953683	29
32	.0790391	.079287	12.61239	.9968715	28	32	.0964248	.096876	10.32244	.9953403	28
33	.0793290	.079579	12.56599	.9968485	27	33	.0967144	.097169	10.29125	.9953122	27
34	.0796190	.079872	12.51994	.9968254	26	34	.0970039	.097463	10.26024	.9952840	26
35	.0799090	.080165	12.47422	.9968022	25	35	.0972934	.097757	10.22942	.9952557	25
36	.0801989	.080458	12.42883	.9967789	24	36	.0975829	.098050	10.19878	.9952274	24
37	.0804889	.080750	12.38376	.9967555	23	37	.0978724	.098344	10.16833	.9951990	23
38	.0807788	.081043	12.33902	.9967321	22	38	.0981619	.098638	10.13805	.9951705	22
39	.0810687	.081336	12.29460	.9967085	21	39	.0984514	.098932	10.10795	.9951419	21
40	.0813587	.081629	12.25050	.9966849	20	40	.0987408	.099225	10.07803	.9951132	20
41	.0816486	.081922	12.20671	.9966612	19	41	.0990303	.099519	10.04828	.9950844	19
42	.0819385	.082215	12.16323	.9966374	18	42	.0993197	.099813	10.01871	.9950556	18
43	.0822284	.082507	12.12006	.9966135	17	43	.0996092	.100107	9.989305	.9950266	17
44	.0825183	.082800	12.07719	.9965895	16	44	.0998986	.100400	9.960072	.9949976	16
45	.0828082	.083093	12.03462	.9965655	15	45	.1001881	.100694	9.931008	.9949685	15
46	.0830981	.083386	11.99234	.9965414	14	46	.1004775	.100988	9.902112	.9949393	14
47	.0833880	.083679	11.95037	.9965172	13	47	.1007669	.101282	9.873382	.9949101	13
48	.0836778	.083972	11.90868	.9964929	12	48	.1010563	.101576	9.844816	.9948807	12
49	.0839677	.084265	11.86728	.9964685	11	49	.1013457	.101870	9.816414	.9948513	11
50	.0842576	.084558	11.82616	.9964440	10	50	.1016351	.102164	9.788173	.9948217	10
51	.0845474	.084851	11.78533	.9964195	9	51	.1019245	.102458	9.760092	.9947921	9
52	.0848373	.085144	11.74477	.9963948	8	52	.1022138	.102752	9.732171	.9947625	8
53	.0851271	.085437	11.70450	.9963701	7	53	.1025032	.103046	9.704407	.9947327	7
54	.0854169	.085730	11.66449	.9963453	6	54	.1027925	.103339	9.676800	.9947028	6
55	.0857067	.086023	11.62476	.9963204	5	55	.1030819	.103634	9.649347	.9946729	5
56	.0859966	.086316	11.58529	.9962954	4	56	.1033712	.103928	9.622048	.9946428	4
57	.0862864	.086609	11.54609	.9962704	3	57	.1036605	.104222	9.594902	.9946127	3
58	.0865762	.086902	11.50715	.9962452	2	58	.1039499	.104516	9.567906	.9945825	2
59	.0868660	.087195	11.46847	.9962200	1	59	.1042392	.104810	9.541061	.9945523	1
60	.0871557	.087488	11.43005	.9961947	0	60	.1045285	.105104	9.514364	.9945219	0
	Cosine.	Cotang	Tang.	Sine.	′		Cosine.	Cotang	Tang.	Sine.	
					85°						8

Note.—Secant = 1 + cosine. Cosecant = 1 + sine.

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3.—Natural Sines, Tangents, Cotangents, Cosines.—(Continued).

(Versed sine = 1 − cosine; coversed sine = 1 − sine.)

6°						7°					
′	Sine.	Tang.	Cotang.	Cosine.	′	′	Sine.	Tang.	Cotang.	Cosine.	′
0	.1045285	.105104	9.514364	.9945219	60	0	.1218693	.122784	8.144346	.9925462	60
1	.1048178	.105398	9.487814	.9944914	59	1	.1221581	.123079	8.124807	.9925107	59
2	.1051070	.105692	9.461411	.9944609	58	2	.1224468	.123375	8.105359	.9924751	58
3	.1053963	.105986	9.435153	.9944303	57	3	.1227355	.123670	8.086004	.9924394	57
4	.1056856	.106280	9.409038	.9943996	56	4	.1230241	.123965	8.066739	.9924037	56
5	.1059748	.106575	9.383066	.9943688	55	5	.1233128	.124261	8.047564	.9923679	55
6	.1062641	.106869	9.357235	.9943379	54	6	.1236015	.124556	8.028479	.9923319	54
7	.1065533	.107163	9.331545	.9943070	53	7	.1238901	.124852	8.009483	.9922959	53
8	.1068425	.107457	9.305993	.9942760	52	8	.1241788	.125147	7.990575	.9922599	52
9	.1071318	.107751	9.280580	.9942448	51	9	.1244674	.125442	7.971755	.9922237	51
10	.1074210	.108046	9.255303	.9942136	50	10	.1247560	.125738	7.953022	.9921874	50
11	.1077102	.108340	9.230162	.9941823	49	11	.1250446	.126033	7.934375	.9921511	49
12	.1079994	.108634	9.205156	.9941510	48	12	.1253332	.126329	7.915815	.9921147	48
13	.1082885	.108929	9.180283	.9941195	47	13	.1256218	.126624	7.897339	.9920782	47
14	.1085777	.109223	9.155543	.9940880	46	14	.1259104	.126920	7.878948	.9920416	46
15	.1088669	.109517	9.130934	.9940563	45	15	.1261990	.127216	7.860642	.9920049	45
16	.1091560	.109812	9.106456	.9940246	44	16	.1264875	.127511	7.842419	.9919682	44
17	.1094452	.110106	9.082107	.9939928	43	17	.1267761	.127807	7.824279	.9919314	43
18	.1097343	.110401	9.057886	.9939610	42	18	.1270646	.128103	7.806221	.9918944	42
19	.1100234	.110695	9.033793	.9939290	41	19	.1273531	.128398	7.788245	.9918574	41
20	.1103125	.110989	9.009826	.9938969	40	20	.1276416	.128694	7.770350	.9918204	40
21	.1106017	.111284	8.985984	.9938648	39	21	.1279302	.128990	7.752536	.9917832	39
22	.1108908	.111578	8.962266	.9938326	38	22	.1282186	.129285	7.734802	.9917459	38
23	.1111799	.111873	8.938672	.9938003	37	23	.1285071	.129581	7.717148	.9917086	37
24	.1114689	.112168	8.915200	.9937679	36	24	.1287956	.129877	7.699573	.9916712	36
25	.1117580	.112462	8.891850	.9937355	35	25	.1290841	.130173	7.682076	.9916337	35
26	.1120471	.112757	8.868620	.9937029	34	26	.1293725	.130469	7.664658	.9915961	34
27	.1123361	.113051	8.845510	.9936703	33	27	.1296609	.130764	7.647317	.9915584	33
28	.1126252	.113346	8.822518	.9936375	32	28	.1299494	.131060	7.630053	.9915206	32
29	.1129142	.113641	8.799644	.9936047	31	29	.1302378	.131356	7.612865	.9914828	31
30	.1132032	.113935	8.776887	.9935719	30	30	.1305262	.131652	7.595754	.9914449	30
31	.1134922	.114230	8.754246	.9935389	29	31	.1308146	.131948	7.578717	.9914069	29
32	.1137812	.114525	8.731719	.9935058	28	32	.1311030	.132244	7.561756	.9913688	28
33	.1140702	.114819	8.709307	.9934727	27	33	.1313913	.132540	7.544869	.9913306	27
34	.1143592	.115114	8.687008	.9934395	26	34	.1316797	.132836	7.528057	.9912923	26
35	.1146482	.115409	8.664822	.9934062	25	35	.1319681	.133132	7.511317	.9912540	25
36	.1149372	.115703	8.642747	.9933728	24	36	.1322564	.133428	7.494651	.9912155	24
37	.1152261	.115998	8.620783	.9933393	23	37	.1325447	.133724	7.478057	.9911770	23
38	.1155151	.116293	8.598929	.9933057	22	38	.1328330	.134020	7.461535	.9911384	22
39	.1158040	.116588	8.577183	.9932721	21	39	.1331213	.134316	7.445085	.9910997	21
40	.1160929	.116883	8.555546	.9932384	20	40	.1334096	.134612	7.428706	.9910610	20
41	.1163818	.117178	8.534017	.9932045	19	41	.1336979	.134909	7.412397	.9910221	19
42	.1166707	.117473	8.512594	.9931706	18	42	.1339862	.135205	7.396159	.9909832	18
43	.1169596	.117767	8.491277	.9931367	17	43	.1342744	.135501	7.379990	.9909442	17
44	.1172485	.118062	8.470065	.9931026	16	44	.1345627	.135797	7.363891	.9909051	16
45	.1175374	.118357	8.448957	.9930685	15	45	.1348509	.136094	7.347861	.9908659	15
46	.1178263	.118652	8.427953	.9930342	14	46	.1351392	.136390	7.331898	.9908266	14
47	.1181151	.118947	8.407051	.9929999	13	47	.1354274	.136686	7.316004	.9907873	13
48	.1184040	.119242	8.386251	.9929655	12	48	.1357156	.136983	7.300178	.9907478	12
49	.1186928	.119537	8.365553	.9929310	11	49	.1360038	.137279	7.284418	.9907083	11
50	.1189816	.119832	8.344955	.9928965	10	50	.1362919	.137575	7.268725	.9906687	10
51	.1192704	.120127	8.324457	.9928618	9	51	.1365801	.137872	7.253098	.9906290	9
52	.1195593	.120423	8.304058	.9928271	8	52	.1368683	.138168	7.237537	.9905893	8
53	.1198481	.120718	8.283757	.9927922	7	53	.1371564	.138465	7.222042	.9905494	7
54	.1201368	.121013	8.263554	.9927573	6	54	.1374445	.138761	7.206611	.9905095	6
55	.1204256	.121308	8.243448	.9927224	5	55	.1377327	.139058	7.191245	.9904694	5
56	.1207144	.121603	8.223438	.9926873	4	56	.1380208	.139354	7.175943	.9904293	4
57	.1210031	.121898	8.203523	.9926521	3	57	.1383089	.139651	7.160705	.9903891	3
58	.1212919	.122194	8.183704	.9926169	2	58	.1385970	.139947	7.145530	.9903489	2
59	.1215806	.122489	8.163978	.9925816	1	59	.1388850	.140244	7.130419	.9903085	1
60	.1218693	.122784	8.144346	.9925462	0	60	.1391731	.140540	7.115369	.9902681	0
	Cosine.	Cotang.	Tang.	Sine.	′		Cosine.	Cotang.	Tang.	Sine.	′
					83°						82°

Note.—Secant = 1 + cosine. Cosecant = 1 + sine.

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8.—Natural Sines, Tangents, Cotangents, Cosines.—(Continued.)

(Versed sine = 1 − cosine; coversed sine = 1 − sine.)

8° ′	Sine.	Tang.	Cotang.	Cosine.		9° ′	Sine.	Tang.	Cotang.	Cosine.
0	.1391731	.140540	7.115369	.9902681	60	0	.1564345	.158384	6.313751	.9876883
1	.1394612	.140837	7.100382	.9902275	59	1	.1567218	.158682	6.301886	.9876428
2	.1397492	.141134	7.085457	.9901869	58	2	.1570091	.158980	6.290065	.9875972
3	.1400372	.141430	7.070593	.9901462	57	3	.1572963	.159279	6.278286	.9875514
4	.1403252	.141727	7.055790	.9901055	56	4	.1575836	.159577	6.266551	.9875057
5	.1406132	.142024	7.041048	.9900646	55	5	.1578708	.159875	6.254858	.9874598
6	.1409012	.142321	7.026366	.9900237	54	6	.1581581	.160174	6.243208	.9874138
7	.1411892	.142617	7.011744	.9899826	53	7	.1584453	.160472	6.231600	.9873678
8	.1414772	.142914	6.997180	.9899415	52	8	.1587325	.160770	6.220034	.9873216
9	.1417651	.143211	6.982678	.9899003	51	9	.1590197	.161069	6.208510	.9872754
10	.1420531	.143508	6.968233	.9898590	50	10	.1593069	.161367	6.197027	.9872291
11	.1423410	.143805	6.953847	.9898177	49	11	.1595940	.161666	6.185586	.9871827
12	.1426289	.144102	6.939519	.9897762	48	12	.1598812	.161964	6.174186	.9871363
13	.1429168	.144399	6.925248	.9897347	47	13	.1601683	.162263	6.162827	.9870897
14	.1432047	.144696	6.911035	.9896931	46	14	.1604555	.162561	6.151508	.9870431
15	.1434926	.144993	6.896879	.9896514	45	15	.1607426	.162860	6.140230	.9869964
16	.1437805	.145290	6.882780	.9896096	44	16	.1610297	.163159	6.128992	.9869496
17	.1440684	.145587	6.868737	.9895677	43	17	.1613167	.163457	6.117794	.9869027
18	.1443562	.145884	6.854750	.9895258	42	18	.1616038	.163756	6.106636	.9868557
19	.1446440	.146181	6.840819	.9894838	41	19	.1618909	.164055	6.095517	.9868087
20	.1449319	.146478	6.826943	.9894416	40	20	.1621779	.164353	6.084438	.9867615
21	.1452197	.146775	6.813122	.9893994	39	21	.1624650	.164652	6.073397	.9867143
22	.1455075	.147072	6.799356	.9893572	38	22	.1627520	.164951	6.062396	.9866670
23	.1457953	.147369	6.785644	.9893148	37	23	.1630390	.165250	6.051434	.9866196
24	.1460830	.147667	6.771986	.9892723	36	24	.1633260	.165548	6.040510	.9865722
25	.1463708	.147964	6.758382	.9892298	35	25	.1636129	.165847	6.029624	.9865246
26	.1466585	.148261	6.744831	.9891872	34	26	.1638999	.166146	6.018777	.9864770
27	.1469463	.149559	6.731334	.9891445	33	27	.1641868	.166445	6.007967	.9864293
28	.1472340	.148856	6.717889	.9891017	32	28	.1644738	.166744	5.997195	.9863815
29	.1475217	.149153	6.704496	.9890588	31	29	.1647607	.167043	5.986461	.9863336
30	.1478094	.149451	6.691156	.9890159	30	30	.1650476	.167342	5.975764	.9862856
31	.1480971	.149748	6.677867	.9889728	29	31	.1653345	.167641	5.965104	.9862375
32	.1483848	.150045	6.664630	.9889297	28	32	.1656214	.167940	5.954481	.9861894
33	.1486724	.150343	6.651444	.9888865	27	33	.1659082	.168239	5.943895	.9861412
34	.1489601	.150640	6.638310	.9888432	26	34	.1661951	.168539	5.933345	.9860929
35	.1492477	.150938	6.625225	.9887998	25	35	.1664819	.168838	5.922832	.9860445
36	.1495353	.151235	6.612191	.9887564	24	36	.1667687	.169137	5.912355	.9859960
37	.1498230	.151533	6.599208	.9887128	23	37	.1670556	.169436	5.901913	.9859475
38	.1501106	.151830	6.586273	.9886692	22	38	.1673423	.169735	5.891508	.9858988
39	.1503981	.152128	6.573389	.9886255	21	39	.1676291	.170035	5.881138	.9858501
40	.1506857	.152426	6.560553	.9885817	20	40	.1679159	.170334	5.870804	.9858013
41	.1509733	.152723	6.547767	.9885378	19	41	.1682026	.170633	5.860505	.9857524
42	.1512608	.153021	6.535029	.9884939	18	42	.1684894	.170933	5.850241	.9857035
43	.1515484	.153319	6.522339	.9884498	17	43	.1687761	.171232	5.840011	.9856544
44	.1518359	.153617	6.509698	.9884057	16	44	.0690628	.171532	5.829817	.9856053
45	.1521234	.153914	6.497104	.9883615	15	45	.1693495	.171831	5.819657	.9855561
46	.1524109	.154212	6.484558	.9883172	14	46	.1696362	.172130	5.809531	.9855068
47	.1526984	.154510	6.472059	.9882728	13	47	.1699228	.172430	5.799440	.9854574
48	.1529858	.154808	6.459607	.9882284	12	48	.1702095	.172730	5.789382	.9854079
49	.1532733	.155106	6.447201	.9881838	11	49	.1704961	.173029	5.779358	.9853583
50	.1535607	.155404	6.434842	.9881392	10	50	.1707828	.173329	5.769368	.9853087
51	.1538482	.155701	6.422530	.9880945	9	51	.1710694	.173628	5.759412	.9852590
52	.1541356	.155999	6.410263	.9880497	8	52	.1713560	.173928	5.749488	.9852092
53	.1544230	.156297	6.398042	.9880048	7	53	.1716425	.174228	5.739598	.9851593
54	.1547104	.156595	6.385866	.9879599	6	54	.1719291	.174527	5.729741	.9851093
55	.1549978	.156893	6.373735	.9879148	5	55	.1722156	.174827	5.719917	.9850593
56	.1552851	.157191	6.361650	.9878697	4	56	.1725022	.175127	5.710125	.9850091
57	.1555725	.157490	6.349609	.9878245	3	57	.1727887	.175427	5.700366	.9849589
58	.1558598	.157788	6.337612	.9877792	2	58	.1730752	.175727	5.690639	.9849086
59	.1561472	.158086	6.325660	.9877338	1	59	.1733617	.176027	5.680944	.9848582
60	.1564345	.158384	6.313751	.9876883	0	60	.1736482	.176327	5.671281	.9848078
	Cosine.	Cotang.	Tang.	Sine.	′		Cosine.	Cotang.	Tang.	Sine.

81°

Note.—Secant = 1 + cosine. Cosecant = 1 + sine.

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3.—Natural Sines, Tangents, Cotangents, Cosines.—(Continued.)

(Versed sine = 1 − cosine; coversed sine = 1 − sine).

10° ′	Sine.	Tang.	Cotang.	Cosine.		11° ′	Sine.	Tang.	Cotang.	Cosine.
0	.1736482	.176327	5.671281	.9848078	60	0	.1908090	.194380	5.144554	.9816272
1	.1739346	.176626	5.661650	.9847572	59	1	.1910945	.194682	5.136576	.9815716
2	.1742211	.176926	5.652051	.9847066	58	2	.1913801	.194984	5.128622	.9815160
3	.1745075	.177226	5.642483	.9846558	57	3	.1916656	.195286	5.120692	.9814603
4	.1747939	.177527	5.632947	.9846050	56	4	.1919510	.195588	5.112785	.9814045
5	.1750803	.177827	5.623442	.9845542	55	5	.1922365	.195890	5.104902	.9813486
6	.1753667	.178127	5.613968	.9845032	54	6	.1925220	.196192	5.097042	.9812927
7	.1756531	.178427	5.604524	.9844521	53	7	.1928074	.196494	5.089206	.9812366
8	.1759395	.178727	5.595112	.9844010	52	8	.1930928	.196796	5.081392	.9811805
9	.1762258	.179027	5.585730	.9843498	51	9	.1933782	.197098	5.073602	.9811243
10	.1765121	.179327	5.576378	.9842985	50	10	.1936636	.197400	5.065835	.9810680
11	.1767984	.179628	5.567057	.9842471	49	11	.1939490	.197703	5.058090	.9810116
12	.1770847	.179928	5.557766	.9841956	48	12	.1942344	.198005	5.050369	.9809552
13	.1773710	.180228	5.548505	.9841441	47	13	.1945197	.198307	5.042670	.9008986
14	.1776573	.180529	5.539274	.9840924	46	14	.1948050	.198610	5.034993	.9808420
15	.1779435	.180829	5.530072	.9840407	45	15	.1950903	.198912	5.027339	.9807853
16	.1782298	.181129	5.520900	.9839889	44	16	.1953756	.199214	5.019707	.9807285
17	.1785160	.181430	5.511757	.9839370	43	17	.1956609	.199517	5.012098	.9806716
18	.1788022	.181730	5.502644	.9838850	42	18	.1959461	.199819	5.004511	.9806147
19	.1790884	.182031	5.493560	.9838330	41	19	.1962314	.200122	4.996945	.9805576
20	.1793746	.182331	5.484505	.9837808	40	20	.1965166	.200424	4.989402	.9805005
21	.1796607	.182632	5.475478	.9837286	39	21	.1968018	.200727	4.981881	.9804433
22	.1799469	.182933	5.466481	.9836763	38	22	.1970870	.201030	4.974381	.9803860
23	.1802330	.183233	5.457512	.9836239	37	23	.1973722	.201332	4.966903	.9803286
24	.1805191	.183534	5.448571	.9835715	36	24	.1976573	.201635	4.959447	.9802712
25	.1808052	.183835	5.439659	.9835189	35	25	.1979425	.201938	4.952012	.9802136
26	.1810913	.184135	5.430775	.9834663	34	26	.1982276	.202240	4.944599	.9801560
27	.1813774	.184436	5.421918	.9834136	33	27	.1985127	.202543	4.937206	.9800983
28	.1816635	.184737	5.413090	.9833608	32	28	.1987978	.202846	4.929835	.9800405
29	.1819495	.185038	5.404290	.9833079	31	29	.1990829	.203149	4.922485	.9799827
30	.1822355	.185339	5.395517	.9832549	30	30	.1993679	.203452	4.915157	.9799247
31	.1825215	.185639	5.386771	.9832019	29	31	.1996530	.203755	4.907849	.9798667
32	.1828075	.185940	5.378053	.9831487	28	32	.1999380	.204058	4.900562	.9798086
33	.1830935	.186241	5.369363	.9830955	27	33	.2002230	.204361	4.893295	.9797504
34	.1833795	.186542	5.360699	.9830422	26	34	.2005080	.204664	4.886049	.9796921
35	.1836654	.186843	5.352062	.9829888	25	35	.2007930	.204967	4.878824	.9796337
36	.1839514	.187144	5.343452	.9829353	24	36	.2010779	.205270	4.871620	.9795752
37	.1842373	.187446	5.334869	.9828818	23	37	.2013629	.205573	4.864435	.9795167
38	.1845232	.187747	5.326313	.9828282	22	38	.2016478	.205876	4.857271	.9794581
39	.1848091	.188048	5.317783	.9827744	21	39	.2019327	.206180	4.850128	.9793994
40	.1850949	.188349	5.309279	.9827206	20	40	.2022176	.206483	4.843004	.9793406
41	.1853808	.188650	5.300801	.9826668	19	41	.2025024	.206786	4.835901	.9792818
42	.1856666	.188952	5.292350	.9826128	18	42	.2027873	.207090	4.828817	.9792228
43	.1859524	.189253	5.283925	.9825587	17	43	.2030721	.207393	4.821753	.9791638
44	.1862382	.189554	5.275525	.9825046	16	44	.2033569	.207696	4.814709	.9791047
45	.1865240	.189855	5.267151	.9824504	15	45	.2036418	.208000	4.807685	.9790455
46	.1868098	.190157	5.258803	.9823961	14	46	.2039265	.208303	4.800680	.9789862
47	.1870956	.190458	5.250480	.9823417	13	47	.2042113	.208607	4.793695	.9789268
48	.1873813	.190760	5.242183	.9822873	12	48	.2044961	.208910	4.786730	.9788674
49	.1876670	.191061	5.233911	.9822327	11	49	.2047808	.209214	4.779783	.9788079
50	.1879528	.191363	5.225664	.9821781	10	50	.2050655	.209518	4.772856	.9787483
51	.1882385	.191664	5.217442	.9821234	9	51	.2053502	.209821	4.765949	.9786886
52	.1885241	.191966	5.209245	.9820686	8	52	.2056349	.210125	4.759060	.9786288
53	.1888098	.192268	5.201073	.9820137	7	53	.2059195	.210429	4.752190	.9785689
54	.1890954	.192569	5.192926	.9819587	6	54	.2062042	.210733	4.745340	.9785090
55	.1893811	.192871	5.184803	.9819037	5	55	.2064888	.211036	4.738508	.9784490
56	.1896667	.193173	5.176705	.9818485	4	56	.2067734	.211340	4.731695	.9783889
57	.1899523	.193474	5.168631	.9817933	3	57	.2070580	.211644	4.724901	.9783287
58	.1902379	.193776	5.160581	.9817380	2	58	.2073426	.211948	4.718125	.9782684
59	.1905234	.194078	5.152555	.9816826	1	59	.2076272	.212252	4.711368	.9782080
60	.1908090	.194380	5.144554	.9816272	0	60	.2079117	.212556	4.704630	.9781476
	Cosine.	Cotang.	Tang.	Sine.	′		Cosine.	Cotang.	Tang.	Sine.

79°

Note.—Secant = 1 + cosine. Cosecant = 1 + sine.

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3.—Natural Sines, Tangents, Cotangents, Cosines.—(Continued.)

(Versed sine = 1 − cosine; coversed sine = 1 − sine.)

12°

′	Sine.	Tang.	Cotang.	Cosine.	′
0	.2079117	.212556	4.704630	.9781476	60
1	.2081962	.212860	4.697910	.9780871	59
2	.2084807	.213164	4.691208	.9780265	58
3	.2087652	.213468	4.684524	.9779658	57
4	.2090497	.213773	4.677859	.9779050	56
5	.2093341	.214077	4.671212	.9778441	55
6	.2096186	.214381	4.664583	.9777832	54
7	.2099030	.214685	4.657972	.9777222	53
8	.2101874	.214990	4.651378	.9776611	52
9	.2104718	.215294	4.644803	.9775999	51
10	.2107561	.215598	4.638245	.9775386	50
11	.2110405	.215903	4.631705	.9774773	49
12	.2113248	.216207	4.625183	.9774159	48
13	.2116091	.216512	4.618678	.9773544	47
14	.2118934	.216816	4.612190	.9772928	46
15	.2121777	.217121	4.605720	.9772311	45
16	.2124619	.217425	4.599268	.9771693	44
17	.2127462	.217730	4.592832	.9771075	43
18	.2130304	.218035	4.586414	.9770456	42
19	.2133146	.218340	4.580012	.9769836	41
20	.2135988	.218644	4.573628	.9769215	40
21	.2138829	.218949	4.567261	.9768593	39
22	.2141671	.219254	4.560911	.9767970	38
23	.2144512	.219559	4.554577	.9767347	37
24	.2147353	.219864	4.548260	.9766723	36
25	.2150194	.220169	4.541960	.9766098	35
26	.2153035	.220474	4.535677	.9765472	34
27	.2155876	.220779	4.529410	.9764845	33
28	.2158716	.221084	4.523160	.9764217	32
29	.2161556	.221389	4.516926	.9763589	31
30	.2164396	.221694	4.510708	.9762960	30
31	.2167236	.221999	4.504507	.9762330	29
32	.2170076	.222305	4.498322	.9761699	28
33	.2172915	.222610	4.492153	.9761067	27
34	.2175754	.222915	4.486000	.9760435	26
35	.2178593	.223221	4.479863	.9759802	25
36	.2181432	.223526	4.473742	.9759168	24
37	.2184271	.223831	4.467637	.9758533	23
38	.2187110	.224137	4.461548	.9757897	22
39	.2189948	.224442	4.455475	.9757260	21
40	.2192786	.224748	4.449418	.9756623	20
41	.2195624	.225054	4.443376	.9755985	19
42	.2198462	.225359	4.437350	.9755345	18
43	.2201300	.225665	4.431339	.9754706	17
44	.2204137	.225971	4.425343	.9754065	16
45	.2206974	.226276	4.419364	.9753423	15
46	.2209811	.226582	4.413399	.9752781	14
47	.2212648	.226888	4.407450	.9752138	13
48	.2215485	.227194	4.401516	.9751494	12
49	.2218321	.227500	4.395597	.9750849	11
50	.2221158	.227806	4.389694	.9750203	10
51	.2223994	.228112	4.383805	.9749556	9
52	.2226830	.228418	4.377931	.9748909	8
53	.2229666	.228724	4.372073	.9748261	7
54	.2232501	.229030	4.366229	.9747612	6
55	.2235337	.229336	4.360400	.9746962	5
56	.2238172	.229642	4.354586	.9746311	4
57	.2241007	.229949	4.348786	.9745660	3
58	.2243842	.230255	4.343001	.9745008	2
59	.2246676	.230561	4.337231	.9744355	1
60	.2249511	.230868	4.331475	.9743701	0
	Cosine.	Cotang.	Tang.	Sine.	′

13°

′	Sine.	Tang.	Cotang.	Cosine.	
0	.2249511	.230868	4.331475	.9743701	6
1	.2252345	.231174	4.325734	.9743046	
2	.2255179	.231481	4.320007	.9742390	
3	.2258013	.231787	4.314295	.9741734	
4	.2260846	.232094	4.308597	.9741077	
5	.2263680	.232400	4.302913	.9740419	5
6	.2266513	.232707	4.297244	.9739760	
7	.2269346	.233014	4.291588	.9739100	
8	.2272179	.233320	4.285947	.9738439	
9	.2275012	.233627	4.280319	.9737778	
10	.2277844	.233934	4.274706	.9737116	5
11	.2280677	.234241	4.269107	.9736453	
12	.2283509	.234547	4.263521	.9735789	
13	.2286341	.234854	4.257950	.9735124	
14	.2289172	.235161	4.252392	.9734458	
15	.2292004	.235468	4.246848	.9733792	4
16	.2294835	.235775	4.241317	.9733125	
17	.2297666	.236082	4.235800	.9732457	
18	.2300497	.236390	4.230297	.9731789	
19	.2303328	.236697	4.224808	.9731119	
20	.2306159	.237004	4.219331	.9730449	4
21	.2308989	.237311	4.213869	.9729777	
22	.2311819	.237618	4.208419	.9729105	
23	.2314649	.237926	4.202983	.9728432	
24	.2317479	.238233	4.197560	.9727759	
25	.2320309	.238541	4.192151	.9727084	3
26	.2323138	.238848	4.186754	.9726409	
27	.2325967	.239156	4.181371	.9725733	
28	.2328796	.239463	4.176001	.9725056	
29	.2331625	.239771	4.170644	.9724378	
30	.2334454	.240078	4.165299	.9723699	3
31	.2337282	.240386	4.159968	.9723020	
32	.2340110	.240694	4.154650	.9722339	
33	.2342938	.241001	4.149344	.9721658	
34	.2345766	.241309	4.144051	.9720976	
35	.2348594	.241617	4.138771	.9720294	2
36	.2351421	.241925	4.133504	.9719610	
37	.2354248	.242233	4.128249	.9718926	
38	.2357075	.242541	4.123007	.9718240	
39	.2359902	.242849	4.117778	.9717554	
40	.2362729	.243157	4.112561	.9716867	
41	.2365555	.243465	4.107356	.9716180	
42	.2368381	.243773	4.102164	.9715491	
43	.2371207	.244081	4.096985	.9714802	
44	.2374033	.244390	4.091817	.9714112	
45	.2376859	.244698	4.086662	.9713421	
46	.2379684	.245006	4.081519	.9712729	
47	.2382510	.245315	4.076389	.9712036	
48	.2385335	.245623	4.071270	.9711343	
49	.2388159	.245932	4.066164	.9710649	
50	.2390984	.246240	4.061070	.9709953	
51	.2393808	.246549	4.055987	.9709258	
52	.2396633	.246857	4.050917	.9708561	
53	.2399457	.247166	4.045859	.9707862	
54	.2402280	.247475	4.040812	.9707165	
55	.2405104	.247783	4.035777	.9706466	
56	.2407927	.248092	4.030755	.9705766	
57	.2410751	.248401	4.025744	.9705065	
58	.2413574	.248710	4.020744	.9704363	
59	.2416396	.249019	4.015757	.9703660	
60	.2419219	.249328	4.010780	.9702957	
	Cosine.	Cotang.	Tang.	Sine.	′

77°

Note.—Secant = 1 + cosine. Cosecant = 1 + sine.

Google

3.—Natural Sines, Tangents, Cotangents, Cosines.—(Continued.)

(Versed sine = 1 − cosine; coversed sine = 1 − sine.)

14° 15°

′	Sine.	Tang.	Cotang.	Cosine.		′	Sine.	Tang.	Cotang.	Cosine.	
0	.2419219	.249328	4.010780	.9702957	60	0	.2588190	.267949	3.732050	.9659258	60
1	.2422041	.249637	4.005816	.9702253	59	1	.2591000	.268261	3.727713	.9658505	59
2	.2424863	.249946	4.000863	.9701548	58	2	.2593810	.268572	3.723384	.9657751	58
3	.2427685	.250255	3.995922	.9700842	57	3	.2596619	.268884	3.719065	.9656996	57
4	.2430507	.250564	3.990992	.9700135	56	4	.2599428	.269196	3.714756	.9656240	56
5	.2433329	.250873	3.986073	.9699428	55	5	.2602237	.269508	3.710455	.9655484	55
6	.2436150	.251182	3.981166	.9698720	54	6	.2605045	.269820	3.706164	.9654726	54
7	.2438971	.251491	3.976271	.9698011	53	7	.2607853	.270132	3.701883	.9653968	53
8	.2441792	.251801	3.971386	.9697301	52	8	.2610662	.270444	3.697610	.9653209	52
9	.2444613	.252110	3.966513	.9696591	51	9	.2613469	.270757	3.693346	.9652449	51
10	.2447433	.252420	3.961651	.9695879	50	10	.2616277	.271069	3.689092	.9651689	50
11	.2450254	.252729	3.956801	.9695167	49	11	.2619085	.271381	3.684847	.9650927	49
12	.2453074	.253038	3.951961	.9694453	48	12	.2621892	.271694	3.680611	.9650165	48
13	.2455894	.253348	3.947133	.9693740	47	13	.2624699	.272006	3.676384	.9649402	47
14	.2458713	.253658	3.942315	.9693025	46	14	.2627506	.272318	3.672166	.9648638	46
15	.2461533	.253967	3.937509	.9692309	45	15	.2630312	.272631	3.667957	.9647873	45
16	.2464352	.254277	3.932714	.9691593	44	16	.2633118	.272943	3.663757	.9647108	44
17	.2467171	.254587	3.927929	.9690875	43	17	.2635925	.273256	3.659566	.9646341	43
18	.2469990	.254896	3.923156	.9690157	42	18	.2638730	.273569	3.655384	.9645574	42
19	.2472809	.255206	3.918393	.9689438	41	19	.2641536	.273881	3.651211	.9644806	41
20	.2475627	.255516	3.913642	.9688719	40	20	.2644342	.274194	3.647046	.9644037	40
21	.2478445	.255826	3.908901	.9687998	39	21	.2647147	.274507	3.642891	.9643268	39
22	.2481263	.256136	3.904171	.9687277	38	22	.2649952	.274820	3.638744	.9642497	38
23	.2484081	.256446	3.899451	.9686555	37	23	.2652757	.275133	3.634606	.9641726	37
24	.2486899	.256756	3.894742	.9685832	36	24	.2655561	.275445	3.630477	.9640954	36
25	.2489716	.257066	3.890044	.9685108	35	25	.2658366	.275758	3.626356	.9640181	35
26	.2492533	.257376	3.885357	.9684383	34	26	.2661170	.276071	3.622244	.9639407	34
27	.2495350	.257686	3.880680	.9683658	33	27	.2663973	.276385	3.618141	.9638633	33
28	.2498167	.257997	3.876014	.9682931	32	28	.2666777	.276698	3.614046	.9637858	32
29	.2500984	.258307	3.871358	.9682204	31	29	.2669581	.277011	3.609960	.9637081	31
30	.2503800	.258617	3.866713	.9681476	30	30	.2672384	.277324	3.605883	.9636305	30
31	.2506616	.258928	3.862078	.9680748	29	31	.2677187	.277637	3.601814	.9635527	29
32	.2509432	.259238	3.857453	.9680018	28	32	.2677989	.277951	3.597754	.9634748	28
33	.2512248	.259548	3.852839	.9679288	27	33	.2680792	.278264	3.593702	.9633969	27
34	.2515063	.259859	3.848235	.9678557	26	34	.2683594	.278578	3.589659	.9633189	26
35	.2517879	.260169	3.843642	.9677825	25	35	.2686396	.278891	3.585624	.9632408	25
36	.2520694	.260480	3.839059	.9677092	24	36	.2689198	.279205	3.581597	.9631626	24
37	.2523508	.260791	3.834486	.9676358	23	37	.2692000	.279518	3.577579	.9630843	23
38	.2526323	.261101	3.829923	.9675624	22	38	.2694801	.279832	3.573569	.9630060	22
39	.2529137	.261412	3.825370	.9674888	21	39	.2697602	.280145	3.569568	.9629275	21
40	.2531952	.261723	3.820828	.9674152	20	40	.2700403	.280459	3.565574	.9628490	20
41	.2534766	.262034	3.816295	.9673415	19	41	.2703204	.280773	3.561590	.9627704	19
42	.2537579	.262345	3.811773	.9672678	18	42	.2706004	.281087	3.557613	.9626917	18
43	.2540393	.262656	3.807260	.9671939	17	43	.2708805	.281401	3.553644	.9626130	17
44	.2543206	.262967	3.802758	.9671200	16	44	.2711605	.281715	3.549684	.9625342	16
45	.2546019	.263278	3.798266	.9670459	15	45	.2714404	.282029	3.545732	.9624552	15
46	.2548832	.263589	3.793783	.9669718	14	46	.2717204	.282343	3.541788	.9623762	14
47	.2551645	.263900	3.789310	.9668977	13	47	.2720003	.282657	3.537852	.9622972	13
48	.2554458	.264211	3.784848	.9668234	12	48	.2722802	.282971	3.533925	.9622180	12
49	.2557270	.264522	3.780395	.9667490	11	49	.2725601	.283285	3.530005	.9621387	11
50	.2560082	.264833	3.775951	.9666746	10	50	.2728400	.283599	3.526093	.9620594	10
51	.2562894	.265145	3.771518	.9666001	9	51	.2731198	.283914	3.522190	.9619800	9
52	.2565705	.265456	3.767094	.9665255	8	52	.2733997	.284228	3.518294	.9619005	8
53	.2568517	.265768	3.762680	.9664508	7	53	.2736794	.284543	3.514407	.9618210	7
54	.2571328	.266079	3.758276	.9663761	6	54	.2739592	.284857	3.510527	.9617413	6
55	.2574139	.266390	3.753881	.9663012	5	55	.2742390	.285172	3.506655	.9616616	5
56	.2576950	.266702	3.749496	.9662263	4	56	.2745187	.285486	3.502791	.9615818	4
57	.2579760	.267014	3.745120	.9661513	3	57	.2747984	.285801	3.498935	.9615019	3
58	.2582570	.267325	3.740754	.9660762	2	58	.2750781	.286115	3.495087	.9614219	2
59	.2585381	.267637	3.736398	.9660011	1	59	.2753577	.286430	3.491247	.9613418	1
60	.2588190	.267949	3.732050	.9659258	0	60	.2756374	.286745	3.487414	.9612617	0
	Cosine.	Cotang.	Tang.	Sine.	′		Cosine.	Cotang.	Tang.	Sine.	′

75° 74°

Note.—Secant = 1 + cosine. Cosecant = 1 + sine.

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3.—Natural Sines, Tangents, Cotangents, Cosines.—(Continued.)

(Versed sine = 1 − cosine; coversed sine = 1 − sine.)

16° ′	Sine.	Tang.	Cotang.	Cosine.		17° ′	Sine.	Tang.	Cotang.	Cosine.	
0	.2756374	.286745	3.487414	.9612617	60	0	.2923717	.305730	3.270852	.9563048	60
1	.2759170	.287060	3.483589	.9611815	59	1	.2926499	.306048	3.267452	.9562197	59
2	.2761965	.287375	3.479772	.9611012	58	2	.2929280	.306367	3.264059	.9561345	58
3	.2764761	.287690	3.475963	.9610208	57	3	.2932061	.306685	3.260672	.9560492	57
4	.2767556	.288005	3.472161	.9609403	56	4	.2934842	.307003	3.257292	.9559639	56
5	.2770352	.288320	3.468367	.9608598	55	5	.2937623	.307321	3.253918	.9558785	55
6	.2773147	.288635	3.464581	.9607792	54	6	.2940403	.307640	3.250550	.9557930	54
7	.2775941	.288950	3.460802	.9606984	53	7	.2943183	.307958	3.247189	.9557074	53
8	.2778736	.289265	3.457031	.9606177	52	8	.2945963	.308277	3.243834	.9556218	52
9	.2781530	.289580	3.453267	.9605368	51	9	.2948743	.308595	3.240486	.9555361	51
10	.2784324	.289896	3.449512	.9604558	50	10	.2951522	.308914	3.237143	.9554502	50
11	.2787118	.290211	3.445763	.9603748	49	11	.2954302	.309233	3.233807	.9553643	49
12	.2789911	.290526	3.442022	.9602937	48	12	.2957081	.309551	3.230478	.9552784	48
13	.2792704	.290842	3.438289	.9602125	47	13	.2959859	.309870	3.227154	.9551923	47
14	.2795497	.291157	3.434563	.9601312	46	14	.2962638	.310189	3.223837	.9551062	46
15	.2798290	.291473	3.430844	.9600499	45	15	.2965416	.310508	3.220526	.9550199	45
16	.2801083	.291789	3.427133	.9599684	44	16	.2968194	.310827	3.217221	.9549336	44
17	.2803875	.292104	3.423429	.9598869	43	17	.2970971	.311146	3.213922	.9548473	43
18	.2806667	.292420	3.419733	.9598053	42	18	.2973749	.311465	3.210630	.9547608	42
19	.2809459	.292736	3.416044	.9597236	41	19	.2976526	.311784	3.207344	.9546743	41
20	.2812251	.293052	3.412362	.9596418	40	20	.2979303	.312103	3.204063	.9545876	40
21	.2815042	.293368	3.408688	.9595600	39	21	.2982079	.312422	3.200789	.9545009	39
22	.2817833	.293683	3.405021	.9594781	38	22	.2984856	.312742	3.197521	.9544141	38
23	.2820624	.293999	3.401361	.9593961	37	23	.2987632	.313061	3.194259	.9543273	37
24	.2823415	.294316	3.397708	.9593140	36	24	.2990408	.313381	3.191003	.9542403	36
25	.2826205	.294632	3.394063	.9592318	35	25	.2993184	.313700	3.187754	.9541533	35
26	.2828995	.294948	3.390424	.9591496	34	26	.2995959	.314020	3.184510	.9540662	34
27	.2831785	.295264	3.386793	.9590672	33	27	.2998734	.314339	3.181272	.9539790	33
28	.2834575	.295580	3.383169	.9589848	32	28	.3001509	.314659	3.178040	.9538917	32
29	.2837364	.295897	3.379553	.9589023	31	29	.3004284	.314979	3.174814	.9538044	31
30	.2840153	.296213	3.375943	.9588197	30	30	.3007058	.315298	3.171594	.9537170	30
31	.2842942	.296529	3.372340	.9587371	29	31	.3009832	.315618	3.168380	.9536294	29
32	.2845731	.296846	3.368745	.9586543	28	32	.3012606	.315938	3.165172	.9535418	28
33	.2848520	.297163	3.365156	.9585715	27	33	.3015380	.316258	3.161970	.9534542	27
34	.2851308	.297479	3.361575	.9584886	26	34	.3018153	.316578	3.158774	.9533664	26
35	.2854096	.297796	3.358000	.9584056	25	35	.3020926	.316898	3.155584	.9532786	25
36	.2856884	.298112	3.354433	.9583226	24	36	.3023699	.317218	3.152399	.9531907	24
37	.2859671	.298429	3.350872	.9582394	23	37	.3026471	.317538	3.149220	.9531027	23
38	.2862458	.298746	3.347319	.9581562	22	38	.3029244	.317859	3.146047	.9530146	22
39	.2865246	.299063	3.343772	.9580729	21	39	.3032016	.318179	3.142880	.9529264	21
40	.2868032	.299380	3.340232	.9579895	20	40	.3034788	.318499	3.139719	.9528382	20
41	.2870819	.299697	3.336699	.9579060	19	41	.3037559	.318820	3.136563	.9527499	19
42	.2873605	.300014	3.333173	.9578225	18	42	.3040331	.319140	3.133414	.9526615	18
43	.2876391	.300331	3.329654	.9577389	17	43	.3043102	.319461	3.130270	.9525730	17
44	.2879177	.300648	3.326141	.9576552	16	44	.3045872	.319781	3.127131	.9524844	16
45	.2881963	.300965	3.322636	.9575714	15	45	.3048643	.320102	3.123999	.9523958	15
46	.2884748	.301283	3.319137	.9574875	14	46	.3051413	.320423	3.120872	.9523071	14
47	.2887533	.301600	3.315645	.9574035	13	47	.3054183	.320744	3.117750	.9522183	13
48	.2890318	.301917	3.312159	.9573195	12	48	.3056953	.321064	3.114635	.9521294	12
49	.2893103	.302235	3.308681	.9572354	11	49	.3059723	.321385	3.111525	.9520404	11
50	.2895887	.302552	3.305209	.9571512	10	50	.3062492	.321706	3.108421	.9519514	10
51	.2898671	.302870	3.310743	.9570669	9	51	.3065261	.322027	3.105322	.9518623	9
52	.2901455	.303187	3.298285	.9569825	8	52	.3068030	.322348	3.102229	.9517731	8
53	.2904239	.303505	3.294833	.9568981	7	53	.3070798	.322670	3.099141	.9516838	7
54	.2907022	.303823	3.291387	.9568136	6	54	.3073566	.322991	3.096059	.9515944	6
55	.2909805	.304141	3.287948	.9567290	5	55	.3076334	.323312	3.092983	.9515050	5
56	.2912588	.304458	3.284516	.9566443	4	56	.3079102	.323633	3.089912	.9514154	4
57	.2915371	.304776	3.281090	.9565595	3	57	.3081869	.323955	3.086846	.9513258	3
58	.2918153	.305094	3.277671	.9564747	2	58	.3084636	.324276	3.083786	.9512361	2
59	.2920935	.305412	3.274258	.9563898	1	59	.3087403	.324598	3.080732	.9511464	1
60	.2923717	.305730	3.270852	.9563048	0	60	.3090170	.324919	3.077683	.9510565	0
	Cosine.	Cotang.	Tang.	Sine.	′		Cosine.	Cotang.	Tang.	Sine.	′
					73°						72°

Note.—Secant = 1 + cosine. Cosecant = 1 + sine.

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3.—Natural Sines, Tangents, Cotangents, Cosines.—(Continued.)

(Versed sine = 1 − cosine; coversed sine = 1 − sine.)

18° 19°

′	Sine.	Tang.	Cotang.	Cosine.		′	Sine.	Tang.	Cotang.	Cosine.	
0	.3090170	.324919	3.077683	.9510565	60	0	.3255682	.344327	2.904210	.9455186	60
1	.3092936	.325241	3.074640	.9509666	59	1	.3258432	.344653	2.901468	.9454238	59
2	.3095702	.325563	3.071602	.9508766	58	2	.3261182	.344978	2.898731	.9453290	58
3	.3098468	.325884	3.068569	.9507865	57	3	.3263932	.345304	2.895998	.9452341	57
4	.3101234	.326206	3.065542	.9506963	56	4	.3266681	.345629	2.893270	.9451391	56
5	.3103999	.326528	3.062520	.9506061	55	5	.3269430	.345955	2.890546	.9450441	55
6	.3106764	.326850	3.059503	.9505157	54	6	.3272179	.346281	2.887827	.9449489	54
7	.3109529	.327172	3.056492	.9504253	53	7	.3274928	.346606	2.885113	.9448537	53
8	.3112294	.327494	3.053487	.9503348	52	8	.3277676	.346932	2.882403	.9447584	52
9	.3115058	.327816	3.050486	.9502443	51	9	.3280424	.347258	2.879697	.9446630	51
10	.3117822	.328138	3.047491	.9501536	50	10	.3283172	.347584	2.876997	.9445675	50
11	.3120586	.328461	3.044501	.9500629	49	11	.3285919	.347910	2.874300	.9444720	49
12	.3123349	.328783	3.041517	.9499721	48	12	.3288666	.348236	2.871608	.9443764	48
13	.3126112	.329105	3.038538	.9498812	47	13	.3291413	.348563	2.868921	.9442807	47
14	.3128875	.329428	3.035564	.9497902	46	14	.3294160	.348889	2.866238	.9441849	46
15	.3131638	.329750	3.032595	.9496991	45	15	.3296906	.349215	2.863560	.9440890	45
16	.3134400	.330073	3.029632	.9496080	44	16	.3299653	.349542	2.860886	.9439931	44
17	.3137163	.330395	3.026673	.9495168	43	17	.3302398	.349868	2.858216	.9438971	43
18	.3139925	.330718	3.023720	.9494255	42	18	.3305144	.350195	2.855551	.9438010	42
19	.3142686	.331041	3.020772	.9493341	41	19	.3307889	.350521	2.852891	.9437048	41
20	.3145448	.331363	3.017830	.9492426	40	20	.3310634	.350848	2.850234	.9436085	40
21	.3148209	.331686	3.014892	.9491511	39	21	.3313379	.351175	2.847583	.9435122	39
22	.3150969	.332009	3.011960	.9490595	38	22	.3316123	.351501	2.844935	.9434157	38
23	.3153730	.332332	3.009033	.9489678	37	23	.3318867	.351828	2.842292	.9433192	37
24	.3156490	.332655	3.006110	.9488760	36	24	.3321611	.352155	2.839653	.9432227	36
25	.3159250	.332978	3.003193	.9487842	35	25	.3324355	.352482	2.837019	.9431260	35
26	.3162010	.333302	3.000282	.9486922	34	26	.3327098	.352809	2.834389	.9430293	34
27	.3164770	.333625	2.997375	.9486002	33	27	.3329841	.353136	2.831763	.9429324	33
28	.3167529	.333948	2.994473	.9485081	32	28	.3332584	.353464	2.829142	.9428355	32
29	.3170288	.334271	2.991576	.9484159	31	29	.3335326	.353791	2.826525	.9427386	31
30	.3173047	.334595	2.988685	.9483237	30	30	.3338069	.354118	2.823912	.9426415	30
31	.3175805	.334918	2.985798	.9482313	29	31	.3340810	.354446	2.821304	.9425444	29
32	.3178563	.335242	2.982916	.9481389	28	32	.3343552	.354773	2.818700	.9424471	28
33	.3181321	.335566	2.980040	.9480464	27	33	.3346293	.355101	2.816100	.9423498	27
34	.3184079	.335889	2.977168	.9479538	26	34	.3349034	.355428	2.813504	.9422525	26
35	.3186836	.336213	2.974301	.9478612	25	35	.3351775	.355756	2.810913	.9421550	25
36	.3189593	.336537	2.971439	.9477684	24	36	.3354516	.356084	2.808326	.9420575	24
37	.3192350	.336861	2.968583	.9476756	23	37	.3357256	.356411	2.805743	.9419598	23
38	.3195106	.337185	2.965731	.9475827	22	38	.3359996	.356739	2.803164	.9418621	22
39	.3197863	.337509	2.962884	.9474897	21	39	.3362735	.357067	2.800590	.9417644	21
40	.3200619	.337833	2.960042	.9473966	20	40	.3365475	.357395	2.798019	.9416665	20
41	.3203374	.338157	2.957205	.9473035	19	41	.3368214	.357723	2.795453	.9415686	19
42	.3206130	.338481	2.954372	.9472103	18	42	.3370953	.358051	2.792891	.9414705	18
43	.3208885	.338805	2.951545	.9471170	17	43	.3373691	.358380	2.790333	.9413724	17
44	.3211640	.339129	2.948722	.9470236	16	44	.3376429	.358708	2.787780	.9412743	16
45	.3214395	.339454	2.945905	.9469301	15	45	.3379167	.359036	2.785230	.9411760	15
46	.3217149	.339778	2.943092	.9468366	14	46	.3381905	.359365	2.782685	.9410777	14
47	.3219903	.340103	2.940284	.9467430	13	47	.3384642	.359693	2.780144	.9409793	13
48	.3222657	.340427	2.937480	.9466493	12	48	.3387379	.360022	2.777606	.9408808	12
49	.3225411	.340752	2.934682	.9465555	11	49	.3390116	.360350	2.775073	.9407822	11
50	.3228164	.341077	2.931888	.9464616	10	50	.3392852	.360679	2.772544	.9406835	10
51	.3230917	.341401	2.929099	.9463677	9	51	.3395589	.361008	2.770019	.9405848	9
52	.3233670	.341726	2.926315	.9462736	8	52	.3398325	.361337	2.767499	.9404860	8
53	.3236422	.342051	2.923535	.9461795	7	53	.3401060	.361666	2.764982	.9403871	7
54	.3239174	.342376	2.920761	.9460854	6	54	.3403796	.361994	2.762469	.9402881	6
55	.3241926	.342701	2.917990	.9459911	5	55	.3406531	.362324	2.759960	.9401891	5
56	.3244678	.343026	2.915225	.9458968	4	56	.3409265	.362653	2.757456	.9400899	4
57	.3247429	.343351	2.912464	.9458023	3	57	.3412000	.362982	2.754955	.9399907	3
58	.3250180	.343677	2.909708	.9457078	2	58	.3414734	.363311	2.752458	.9398914	2
59	.3252931	.344002	2.906957	.9456132	1	59	.3417468	.363640	2.749966	.9397921	1
60	.3255682	.344327	2.904210	.9455186	0	60	.3420201	.363970	2.747477	.9396926	0
	Cosine.	Cotang.	Tang.	Sine.	′		Cosine.	Cotang.	Tang.	Sine.	′

71° 70°

Note.—Secant = 1 ÷ cosine. Cosecant = 1 ÷ sine.

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3.—Natural Sines, Tangents, Cotangents, Cosines.—(Continued.)

(Versed sine = 1 − cosine; coversed sine = 1 − sine.)

20° 21°

′	Sine.	Tang.	Cotang.	Cosine.		′	Sine.	Tang.	Cotang.	Cosine.	
0	.3420201	.363970	2.747477	.9396926	60	0	.3583679	.383864	2.605089	.9335804	60
1	.3422935	.364299	2.744992	.9395931	59	1	.3586395	.384197	2.602825	.9334761	59
2	.3425668	.364629	2.742512	.9394935	58	2	.3589110	.384531	2.600565	.9333718	58
3	.3428400	.364958	2.740035	.9393938	57	3	.3591825	.384865	2.598309	.9332673	57
4	.3431133	.365288	2.737562	.9392940	56	4	.3594540	.385199	2.596056	.9331628	56
5	.3433865	.365618	2.735093	.9391942	55	5	.3597254	.385533	2.593806	.9330582	55
6	.3436597	.365948	2.732628	.9390943	54	6	.3599968	.385867	2.591560	.9329535	54
7	.3439329	.366277	2.730167	.9389943	53	7	.3602682	.386202	2.589317	.9328488	53
8	.3442060	.366607	2.727710	.9388942	52	8	.3605395	.386536	2.587078	.9327439	52
9	.3444791	.366937	2.725256	.9387940	51	9	.3608108	.386870	2.584842	.9326390	51
10	.3447521	.367268	2.722807	.9386938	50	10	.3610821	.387205	2.582609	.9325340	50
11	.3450252	.367598	2.720362	.9385934	49	11	.3613534	.387539	2.580380	.9324290	49
12	.3452982	.367928	2.717920	.9384930	48	12	.3616246	.387874	2.578153	.9323238	48
13	.3455712	.368258	2.715482	.9383925	47	13	.3618958	.388209	2.575931	.9322186	47
14	.3458441	.368589	2.713048	.9382920	46	14	.3621669	.388543	2.573711	.9321133	46
15	.3461171	.368919	2.710618	.9381913	45	15	.3624380	.388878	2.571495	.9320079	45
16	.3463900	.369250	2.708192	.9380906	44	16	.3627091	.389213	2.569283	.9319024	44
17	.3466628	.369580	2.705769	.9379898	43	17	.3629802	.389548	2.567073	.9317969	43
18	.3469357	.369911	2.703351	.9378889	42	18	.3632512	.389883	2.564867	.9316912	42
19	.3472085	.370242	2.700936	.9377880	41	19	.3635222	.390218	2.562664	.9315855	41
20	.3474812	.370572	2.698525	.9376869	40	20	.3637932	.390554	2.560464	.9314797	40
21	.3477540	.370903	2.696118	.9375858	39	21	.3640641	.390889	2.558268	.9313739	39
22	.3480267	.371234	2.693714	.9374846	38	22	.3643351	.391224	2.556075	.9312679	38
23	.3482994	.371565	2.691314	.9373833	37	23	.3646059	.391560	2.553885	.9311619	37
24	.3485720	.371896	2.688919	.9372820	36	24	.3648768	.391895	2.551699	.9310558	36
25	.3488447	.372227	2.686526	.9371806	35	25	.3651476	.392231	2.549516	.9309496	35
26	.3491173	.372559	2.684138	.9370790	34	26	.3654184	.392567	2.547335	.9308434	34
27	.3493898	.372890	2.681753	.9369774	33	27	.3656891	.392902	2.545159	.9307370	33
28	.3496624	.373221	2.679372	.9368758	32	28	.3659599	.393238	2.542985	.9306306	32
29	.3499349	.373553	2.676995	.9367740	31	29	.3662306	.393574	2.540815	.9305241	31
30	.3502074	.373884	2.674621	.9366722	30	30	.3665012	.393910	2.538647	.9304176	30
31	.3504798	.374216	2.672251	.9365703	29	31	.3667719	.394246	2.536483	.9303109	29
32	.3507523	.374547	2.669885	.9364683	28	32	.3670425	.394582	2.534323	.9302042	28
33	.3510246	.374879	2.667522	.9363662	27	33	.3673130	.394918	2.532165	.9300974	27
34	.3512970	.375211	2.665163	.9362641	26	34	.3675836	.395255	2.530011	.9299905	26
35	.3515693	.375543	2.662808	.9361618	25	35	.3678541	.395591	2.527859	.9298835	25
36	.3518416	.375875	2.660456	.9360595	24	36	.3681246	.395928	2.525711	.9297765	24
37	.3521139	.376207	2.658108	.9359571	23	37	.3683950	.396264	2.523566	.9296694	23
38	.3523862	.376539	2.655764	.9358547	22	38	.3686654	.396601	2.521424	.9295622	22
39	.3526584	.376871	2.653423	.9357521	21	39	.3689358	.396937	2.519286	.9294549	21
40	.3529306	.377203	2.651086	.9356495	20	40	.3692061	.397274	2.517150	.9293475	20
41	.3532027	.377536	2.648753	.9355468	19	41	.3694765	.397611	2.515018	.9292401	19
42	.3534748	.377868	2.646423	.9354440	18	42	.3697468	.397948	2.512889	.9291326	18
43	.3537469	.378201	2.644096	.9353412	17	43	.3700170	.398285	2.510762	.9290250	17
44	.3540190	.378533	2.641774	.9352382	16	44	.3702872	.398622	2.508639	.9289173	16
45	.3542910	.378866	2.639454	.9351353	15	45	.3705574	.398959	2.506519	.9288096	15
46	.3545630	.379198	2.637139	.9350321	14	46	.3708276	.399296	2.504403	.9287017	14
47	.3548350	.379531	2.634827	.9349289	13	47	.3710977	.399634	2.502289	.9285938	13
48	.3551070	.379864	2.632518	.9348257	12	48	.3713678	.399971	2.500178	.9284858	12
49	.3553789	.380197	2.630213	.9347223	11	49	.3716379	.400308	2.498070	.9283778	11
50	.3556508	.380530	2.627912	.9346189	10	50	.3719079	.400646	2.495966	.9282696	10
51	.3559226	.380863	2.625614	.9345154	9	51	.3721780	.400984	2.493864	.9281614	9
52	.3561944	.381196	2.623319	.9344119	8	52	.3724479	.401321	2.491766	.9280531	8
53	.3564662	.381529	2.621028	.9343082	7	53	.3727179	.401659	2.489670	.9279447	7
54	.3567380	.381862	2.618741	.9342045	6	54	.3729878	.401997	2.487578	.9278363	6
55	.3570097	.382196	2.616457	.9341007	5	55	.3732577	.402335	2.485488	.9277277	5
56	.3572814	.382529	2.614176	.9339968	4	56	.3735275	.402673	2.483402	.9276191	4
57	.3575531	.382863	2.611899	.9338928	3	57	.3737973	.403011	2.481319	.9275104	3
58	.3578248	.383196	2.609625	.9337888	2	58	.3740671	.403349	2.479238	.9274016	2
59	.3580964	.383530	2.607355	.9336846	1	59	.3743369	.403687	2.477161	.9272928	1
60	.3583679	.383864	2.605089	.9335804	0	60	.3746066	.404026	2.475086	.9271839	0
	Cosine.	Cotang.	Tang.	Sine.	′		Cosine.	Cotang.	Tang.	Sine.	

69°

Note.—Secant = 1 ÷ cosine. Cosecant = 1 ÷ sine.

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3.—Natural Sines, Tangents, Cotangents, Cosines.—(Continued.)

(Versed sine = 1 − cosine; coversed sine = 1 − sine.)

22° 23°

′	Sine.	Tang.	Cotang.	Cosine.		′	Sine.	Tang.	Cotang.	Cosine.	
0	.3746066	.404026	2.475086	.9271839	60	0	.3907311	.424474	2.355852	.9205049	60
1	.3748763	.404364	2.473015	.9270748	59	1	.3909989	.424818	2.353948	.9203912	59
2	.3751459	.404703	2.470947	.9269658	58	2	.3912666	.425161	2.352046	.9202774	58
3	.3754156	.405041	2.468881	.9268566	57	3	.3915343	.425505	2.350148	.9201635	57
4	.3756852	.405380	2.466819	.9267474	56	4	.3918019	.425848	2.348251	.9200496	56
5	.3759547	.405719	2.464759	.9266380	55	5	.3920695	.426192	2.346358	.9199356	55
6	.3762243	.406057	2.462703	.9265286	54	6	.3923371	.426536	2.344467	.9198215	54
7	.3764938	.406396	2.460649	.9264192	53	7	.3926047	.426880	2.342578	.9197073	53
8	.3767632	.406735	2.458598	.9263096	52	8	.3928722	.427223	2.340692	.9195931	52
9	.3770327	.407074	2.456551	.9262000	51	9	.3931397	.427568	2.338809	.9194788	51
10	.3773021	.407413	2.454506	.9260902	50	10	.3934071	.427912	2.336928	.9193644	50
11	.3775714	.407753	2.452464	.9259805	49	11	.3936745	.428256	2.335050	.9192499	49
12	.3778408	.408092	2.450425	.9258706	48	12	.3939419	.428600	2.333174	.9191353	48
13	.3781101	.408431	2.448389	.9257606	47	13	.3942093	.428944	2.331301	.9190207	47
14	.3783794	.408771	2.446355	.9256506	46	14	.3944766	.429289	2.329431	.9189060	46
15	.3786486	.409110	2.444325	.9255405	45	15	.3947439	.429633	2.327563	.9187912	45
16	.3789178	.409450	2.442298	.9254303	44	16	.3950111	.429978	2.325697	.9186763	44
17	.3791870	.409790	2.440273	.9253201	43	17	.3952783	.430323	2.323834	.9185614	43
18	.3794562	.410129	2.438251	.9252097	42	18	.3955455	.430668	2.321974	.9184464	42
19	.3797253	.410469	2.436233	.9250993	41	19	.3958127	.431012	2.320116	.9183313	41
20	.3799944	.410809	2.434217	.9249888	40	20	.3960798	.431357	2.318260	.9182161	40
21	.3802634	.411149	2.432204	.9248782	39	21	.3963468	.431703	2.316407	.9181009	39
22	.3805324	.411489	2.430193	.9247676	38	22	.3966139	.432048	2.314557	.9179855	38
23	.3808014	.411830	2.428186	.9246568	37	23	.3968809	.432393	2.312709	.9178701	37
24	.3810704	.412170	2.426181	.9245460	36	24	.3971479	.432738	2.310863	.9177546	36
25	.3813393	.412510	2.424180	.9244351	35	25	.3974148	.433084	2.309020	.9176391	35
26	.3816082	.412851	2.422181	.9243242	34	26	.3976818	.433429	2.307180	.9175234	34
27	.3818770	.413191	2.420185	.9242131	33	27	.3979486	.433775	2.305342	.9174077	33
28	.3821459	.413532	2.418191	.9241020	32	28	.3982155	.434120	2.303506	.9172919	32
29	.3824147	.413872	2.416201	.9239908	31	29	.3984823	.434466	2.301673	.9171760	31
30	.3826834	.414213	2.414213	.9238795	30	30	.3987491	.434812	2.299842	.9170601	30
31	.3829522	.414554	2.412228	.9237682	29	31	.3990158	.435158	2.298014	.9169440	29
32	.3832209	.414895	2.410246	.9236567	28	32	.3992825	.435504	2.296188	.9168279	28
33	.3834895	.415236	2.408267	.9235452	27	33	.3995492	.435850	2.294365	.9167118	27
34	.3837582	.415577	2.406290	.9234336	26	34	.3998158	.436196	2.292544	.9165955	26
35	.3840268	.415918	2.404316	.9233220	25	35	.4000825	.436542	2.290725	.9164791	25
36	.3842953	.416259	2.402345	.9232102	24	36	.4003490	.436889	2.288909	.9163627	24
37	.3845639	.416601	2.400377	.9230984	23	37	.4006156	.437235	2.287095	.9162462	23
38	.3848324	.416942	2.398411	.9229865	22	38	.4008821	.437582	2.285284	.9161297	22
39	.3851008	.417284	2.396449	.9228745	21	39	.4011486	.437928	2.283475	.9160130	21
40	.3853693	.417625	2.394488	.9227624	20	40	.4014150	.438275	2.281669	.9158963	20
41	.3856377	.417967	2.392531	.9226503	19	41	.4016814	.438622	2.279865	.9157795	19
42	.3859060	.418309	2.390576	.9225381	18	42	.4019478	.438969	2.278063	.9156626	18
43	.3861744	.418650	2.388625	.9224258	17	43	.4022141	.439316	2.276264	.9155456	17
44	.3864427	.418992	2.386675	.9223134	16	44	.4024804	.439663	2.274467	.9154286	16
45	.3867110	.419334	2.384729	.9222010	15	45	.4027467	.440010	2.272672	.9153115	15
46	.3869792	.419676	2.382785	.9220884	14	46	.4030129	.440357	2.270880	.9151943	14
47	.3872474	.420019	2.380844	.9219758	13	47	.4032791	.440705	2.269090	.9150770	13
48	.3875156	.420361	2.378906	.9218632	12	48	.4035453	.441052	2.267303	.9149597	12
49	.3877837	.420703	2.376970	.9217504	11	49	.4038114	.441400	2.265518	.9148422	11
50	.3880518	.421046	2.375037	.9216375	10	50	.4040775	.441747	2.263735	.9147247	10
51	.3883199	.421388	2.373106	.9215246	9	51	.4043436	.442095	2.261955	.9146072	9
52	.3885880	.421731	2.371179	.9214116	8	52	.4046096	.442443	2.260177	.9144895	8
53	.3888560	.422073	2.369254	.9212986	7	53	.4048756	.442791	2.258401	.9143718	7
54	.3891240	.422416	2.367331	.9211854	6	54	.4051416	.443139	2.256628	.9142540	6
55	.3893919	.422759	2.365411	.9210722	5	55	.4054075	.443487	2.254857	.9141361	5
56	.3896598	.423102	2.363494	.9209589	4	56	.4056734	.443835	2.253088	.9140181	4
57	.3899277	.423445	2.361580	.9208455	3	57	.4059393	.444183	2.251322	.9139001	3
58	.3901955	.423788	2.359668	.9207320	2	58	.4062051	.444531	2.249558	.9137819	2
59	.3904633	.424131	2.357759	.9206185	1	59	.4064709	.444880	2.247796	.9136637	1
60	.3907311	.424474	2.355852	.9205049	0	60	.4067366	.445228	2.246036	.9135455	0
	Cosine.	Cotang.	Tang.	Sine.	′		Cosine.	Cotang.	Tang.	Sine.	′

67° 66°

Note.—Secant = 1 + cosine. Cosecant = 1 + sine.

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3.—Natural Sines, Tangents, Cotangents, Cosines.—(Continued.)

(Versed sine = 1 − cosine; coversed sine = 1 − sine.)

24° ′	Sine.	Tang.	Cotang.	Cosine.		25° ′	Sine.	Tang.	Cotang.	Cosine.	
0	.4067366	.445228	2.246036	.9135455	60	0	.4226183	.466307	2.144506	.9063078	60
1	.4070024	.445577	2.244279	.9134271	59	1	.4228819	.466661	2.142879	.9061848	59
2	.4072681	.445926	2.242524	.9133087	58	2	.4231455	.467016	2.141253	.9060618	58
3	.4075337	.446274	2.240772	.9131902	57	3	.4234090	.467370	2.139630	.9059386	57
4	.4077993	.446623	2.239021	.9130716	56	4	.4236725	.467725	2.138008	.9058154	56
5	.4080649	.446972	2.237273	.9129529	55	5	.4239360	.468079	2.136389	.9056922	55
6	.4083305	.447321	2.235528	.9128342	54	6	.4241994	.468434	2.134771	.9055688	54
7	.4085960	.447670	2.233784	.9127154	53	7	.4244628	.468789	2.133155	.9054454	53
8	.4088615	.448020	2.232043	.9125965	52	8	.4247262	.469143	2.131542	.9053219	52
9	.4091269	.448369	2.230304	.9124775	51	9	.4249895	.469498	2.129930	.9051983	51
10	.4093923	.448718	2.228567	.9123584	50	10	.4252528	.469853	2.128321	.9050746	50
11	.4096577	.449068	2.226833	.9122393	49	11	.4255161	.470209	2.126713	.9049509	49
12	.4099230	.449417	2.225100	.9121201	48	12	.4257793	.470564	2.125108	.9048271	48
13	.4101883	.449767	2.223370	.9120008	47	13	.4260425	.470919	2.123504	.9047032	47
14	.4104536	.450117	2.221643	.9118815	46	14	.4263056	.471275	2.121903	.9045792	46
15	.4107189	.450467	2.219917	.9117620	45	15	.4265687	.471630	2.120303	.9044551	45
16	.4109841	.450817	2.218194	.9116425	44	16	.4268318	.471986	2.118705	.9043310	44
17	.4112492	.451167	2.216473	.9115229	43	17	.4270949	.472342	2.117110	.9042068	43
18	.4115144	.451517	2.214754	.9114033	42	18	.4273579	.472697	2.115516	.9040825	42
19	.4117795	.451867	2.213037	.9112835	41	19	.4276208	.473053	2.113924	.9039582	41
20	.4120445	.452217	2.211323	.9111637	40	20	.4278838	.473409	2.112334	.9038338	40
21	.4123096	.452568	2.209611	.9110438	39	21	.4281467	.473765	2.110747	.9037093	39
22	.4125745	.452918	2.207901	.9109238	38	22	.4284095	.474122	2.109161	.9035847	38
23	.4128395	.453269	2.206193	.9108038	37	23	.4286723	.474478	2.107577	.9034600	37
24	.4131044	.453620	2.204487	.9106837	36	24	.4289351	.474834	2.105995	.9033353	36
25	.4133693	.453970	2.202784	.9105635	35	25	.4291979	.475191	2.104415	.9032105	35
26	.4136342	.454321	2.201083	.9104432	34	26	.4294606	.475548	2.102836	.9030856	34
27	.4138990	.454672	2.199384	.9103228	33	27	.4297233	.475904	2.101260	.9029606	33
28	.4141638	.455023	2.197687	.9102024	32	28	.4299859	.476261	2.099686	.9028356	32
29	.4144285	.455375	2.195992	.9100819	31	29	.4302485	.476618	2.098114	.9027105	31
30	.4146932	.455726	2.194299	.9099613	30	30	.4305111	.476975	2.096543	.9025853	30
31	.4149579	.456077	2.192609	.9098406	29	31	.4307736	.477332	2.094975	.9024600	29
32	.4152226	.456429	2.190921	.9097199	28	32	.4310361	.477689	2.093408	.9023347	28
33	.4154872	.456780	2.189234	.9095990	27	33	.4312986	.478047	2.091843	.9022092	27
34	.4157517	.457132	2.187551	.9094781	26	34	.4315610	.478404	2.090280	.9020838	26
35	.4160163	.457483	2.185869	.9093572	25	35	.4318234	.478762	2.088720	.9019582	25
36	.4162808	.457835	2.184189	.9092361	24	36	.4320857	.479119	2.087161	.9018325	24
37	.4165453	.458187	2.182511	.9091150	23	37	.4323481	.479477	2.085603	.9017068	23
38	.4168097	.458539	2.180836	.9089938	22	38	.4326103	.479835	2.084048	.9015810	22
39	.4170741	.458891	2.179163	.9088725	21	39	.4328726	.480193	2.082495	.9014551	21
40	.4173385	.459243	2.177492	.9087511	20	40	.4331348	.480551	2.080943	.9013292	20
41	.4176028	.459596	2.175822	.9086297	19	41	.4333970	.480909	2.079394	.9012031	19
42	.4178671	.459948	2.174155	.9085082	18	42	.4336591	.481267	2.077846	.9010770	18
43	.4181313	.460301	2.172491	.9083866	17	43	.4339212	.481625	2.076300	.9009508	17
44	.4183956	.460653	2.170828	.9082649	16	44	.4341832	.481984	2.074756	.9008246	16
45	.4186597	.461006	2.169167	.9081432	15	45	.4344453	.482342	2.073214	.9006982	15
46	.4189239	.461359	2.167509	.9080214	14	46	.4347072	.482701	2.071674	.9005718	14
47	.4191880	.461711	2.165852	.9078995	13	47	.4349692	.483060	2.070135	.9004453	13
48	.4194521	.462064	2.164198	.9077775	12	48	.4352311	.483418	2.068599	.9003188	12
49	.4197161	.462417	2.162546	.9076554	11	49	.4354930	.483777	2.067064	.9001921	11
50	.4199801	.462771	2.160895	.9075333	10	50	.4357548	.484136	2.065531	.9000654	10
51	.4202441	.463124	2.159247	.9074111	9	51	.4360166	.484495	2.064000	.8999386	9
52	.4205080	.463477	2.157601	.9072888	8	52	.4362784	.484855	2.062471	.8998117	8
53	.4207719	.463831	2.155957	.9071665	7	53	.4365401	.485214	2.060944	.8996848	7
54	.4210358	.464184	2.154315	.9070440	6	54	.4368018	.485573	2.059418	.8995578	6
55	.4212996	.464538	2.152675	.9069215	5	55	.4370634	.485933	2.057895	.8994307	5
56	.4215634	.464891	2.151037	.9067989	4	56	.4373251	.486293	2.056373	.8993035	4
57	.4218272	.465245	2.149402	.9066762	3	57	.4375866	.486652	2.054853	.8991763	3
58	.4220909	.465599	2.147768	.9065535	2	58	.4378482	.487012	2.053334	.8990489	2
59	.4223546	.465953	2.146136	.9064307	1	59	.4381097	.487372	2.051818	.8989215	1
60	.4226183	.466307	2.144506	.9063078	0	60	.4383711	.487732	2.050303	.8987940	0
	Cosine.	Cotang.	Tang.	Sine.	′		Cosine.	Cotang.	Tang.	Sine.	

65°

Note.—Secant = 1 + cosine. Cosecant = 1 + sine.

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3.—Natural Sines, Tangents, Cotangents, Cosines.—(Continued.)

(Versed sine = 1 − cosine; coversed sine = 1 − sine.)

′	Sine.	Tang.	Cotang.	Cosine.		′	Sine.	Tang.	Cotang.	Cosine.	
	26°					**27°**					
0	.4383711	.487732	2.050303	.8987940	60	0	.4539905	.509525	1.962610	.8910065	60
1	.4386326	.488092	2.048791	.8986665	59	1	.4542497	.509891	1.961200	.8908744	59
2	.4388940	.488453	2.047280	.8985389	58	2	.4545088	.510258	1.959791	.8907423	58
3	.4391553	.488813	2.045770	.8984112	57	3	.4547679	.510625	1.958383	.8906100	57
4	.4394166	.489173	2.044263	.8982834	56	4	.4550269	.510991	1.956978	.8904777	56
5	.4396779	.489534	2.042757	.8981555	55	5	.4552859	.511358	1.955573	.8903453	55
6	.4399392	.489894	2.041254	.8980276	54	6	.4555449	.511725	1.954171	.8902128	54
7	.4402004	.490255	2.039751	.8978996	53	7	.4558038	.512093	1.952770	.8900803	53
8	.4404615	.490616	2.038251	.8977715	52	8	.4560627	.512460	1.951371	.8899476	52
9	.4407227	.490977	2.036753	.8976433	51	9	.4563216	.512827	1.949973	.8898149	51
10	.4409838	.491338	2.035256	.8975151	50	10	.4565804	.513195	1.948577	.8896822	50
11	.4412448	.491699	2.033761	.8973868	49	11	.4568392	.513562	1.947112	.8895493	49
12	.4415059	.492061	2.032268	.8972584	48	12	.4570979	.513930	1.945789	.8894164	48
13	.4417668	.492422	2.030776	.8971299	47	13	.4573566	.514298	1.944398	.8892834	47
14	.4420278	.492783	2.029287	.8970014	46	14	.4576153	.514665	1.943008	.8891503	46
15	.4422887	.493145	2.027799	.8968727	45	15	.4578739	.515033	1.941620	.8890171	45
16	.4425496	.493507	2.026313	.8967440	44	16	.4581325	.515401	1.940233	.8888839	44
17	.4428104	.493868	2.024828	.8966153	43	17	.4583910	.515770	1.938848	.8887506	43
18	.4430712	.494230	2.023346	.8964864	42	18	.4586496	.516138	1.937464	.8886172	42
19	.4433319	.494592	2.021865	.8963575	41	19	.4589080	.516506	1.936082	.8884838	41
20	.4435927	.494954	2.020386	.8962285	40	20	.4591665	.516875	1.934702	.8883503	40
21	.4438534	.495317	2.018908	.8960994	39	21	.4594248	.517244	1.933323	.8882166	39
22	.4441140	.495679	2.017433	.8959703	38	22	.4596832	.517612	1.931945	.8880830	38
23	.4443746	.496041	2.015959	.8958411	37	23	.4599415	.517981	1.930569	.8879492	37
24	.4446352	.496404	2.014486	.8957118	36	24	.4601998	.518350	1.929195	.8878154	36
25	.4448957	.496766	2.013016	.8955824	35	25	.4604580	.518719	1.927822	.8876815	35
26	.4451562	.497129	2.011547	.8954529	34	26	.4607162	.519089	1.926451	.8875475	34
27	.4454167	.497492	2.010080	.8953234	33	27	.4609744	.519458	1.925081	.8874134	33
28	.4456771	.497855	2.008615	.8951938	32	28	.4612325	.519827	1.923713	.8872793	32
29	.4459375	.498218	2.007151	.8950641	31	29	.4614906	.520197	1.922347	.8871451	31
30	.4461978	.498581	2.005689	.8949344	30	30	.4617486	.520567	1.920982	.8870108	30
31	.4464581	.498944	2.004229	.8948045	29	31	.4620066	.520936	1.919618	.8868765	29
32	.4467184	.499308	2.002771	.8946746	28	32	.4622646	.521306	1.918256	.8867420	28
33	.4469786	.499671	2.001314	.8945446	27	33	.4625225	.521676	1.916896	.8866075	27
34	.4472388	.500035	1.999859	.8944146	26	34	.4627804	.522046	1.915537	.8864730	26
35	.4474990	.500398	1.998405	.8942844	25	35	.4630382	.522417	1.914179	.8863383	25
36	.4477591	.500762	1.996953	.8941542	24	36	.4632960	.522787	1.912823	.8862036	24
37	.4480192	.501126	1.995503	.8940240	23	37	.4635538	.523157	1.911469	.8860688	23
38	.4482792	.501490	1.994055	.8938936	22	38	.4638115	.523528	1.910116	.8859339	22
39	.4485392	.501854	1.992608	.8937632	21	39	.4640692	.523899	1.908764	.8857989	21
40	.4487992	.502218	1.991163	.8936326	20	40	.4643269	.524269	1.907414	.8856639	20
41	.4490591	.502583	1.989720	.8935021	19	41	.4645845	.524640	1.906066	.8855288	19
42	.4493190	.502947	1.988278	.8933714	18	42	.4648420	.525011	1.904719	.8853936	18
43	.4495789	.503312	1.986838	.8932406	17	43	.4650996	.525382	1.903373	.8852584	17
44	.4498387	.503676	1.985400	.8931098	16	44	.4653571	.525754	1.902029	.8851230	16
45	.4500984	.504041	1.983963	.8929789	15	45	.4656145	.526125	1.900687	.8849876	15
46	.4503582	.504406	1.982528	.8928480	14	46	.4658719	.526496	1.899346	.8848522	14
47	.4506179	.504771	1.981095	.8927169	13	47	.4661293	.526868	1.898006	.8847166	13
48	.4508775	.505136	1.979663	.8925858	12	48	.4663866	.527240	1.896668	.8845810	12
49	.4511372	.505501	1.978233	.8924546	11	49	.4666439	.527612	1.895332	.8844453	11
50	.4513967	.505866	1.976805	.8923234	10	50	.4669012	.527983	1.893997	.8843095	10
51	.4516563	.506232	1.975378	.8921920	9	51	.4671584	.528356	1.892663	.8841736	9
52	.4519158	.506597	1.973953	.8920606	8	52	.4674156	.528728	1.891331	.8840377	8
53	.4521753	.506963	1.972529	.8919291	7	53	.4676727	.529100	1.890000	.8839017	7
54	.4524347	.507329	1.971107	.8917975	6	54	.4679298	.529472	1.888671	.8837656	6
55	.4526941	.507694	1.969687	.8916659	5	55	.4681869	.529845	1.887343	.8836295	5
56	.4529535	.508060	1.968268	.8915342	4	56	.4684839	.530217	1.886017	.8834933	4
57	.4532128	.508426	1.966851	.8914024	3	57	.4687009	.530590	1.884692	.8833569	3
58	.4534721	.508792	1.965436	.8912705	2	58	.4689578	.530963	1.883369	.8832206	2
59	.4537313	.509159	1.964022	.8911385	1	59	.4692147	.531336	1.882047	.8830841	1
60	.4539905	.509525	1.962610	.8910065	0	60	.4694716	.531709	1.880726	.8829476	0
	Cosine.	Cotang.	Tang.	Sine.	′		Cosine.	Cotang.	Tang.	Sine.	′
					63°						**62°**

Note.—Secant = 1 + cosine. Cosecant = 1 + sine.

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3.—Natural Sines, Tangents, Cotangents, Cosines.—(Continued.)

(Versed sine = 1 − cosine; coversed sine = 1 − sine.)

28°

′	Sine.	Tang.	Cotang.	Cosine.	
0	.4694716	.531709	1.880726	.8829476	60
1	.4697284	.532082	1.879407	.8828110	59
2	.4699852	.532455	1.878089	.8826743	58
3	.4702419	.532829	1.876773	.8825376	57
4	.4704986	.533202	1.875458	.8824007	56
5	.4707553	.533576	1.874145	.8822638	55
6	.4710119	.533950	1.872833	.8821269	54
7	.4712685	.534324	1.871523	.8819898	53
8	.4715250	.534698	1.870214	.8818527	52
9	.4717815	.535072	1.868906	.8817155	51
10	.4720380	.535446	1.867600	.8815782	50
11	.4722944	.535820	1.866295	.8814409	49
12	.4725508	.536195	1.864992	.8813035	48
13	.4728071	.536569	1.863690	.8811660	47
14	.4730634	.536944	1.862389	.8810284	46
15	.4733197	.537319	1.861090	.8808907	45
16	.4735759	.537694	1.859792	.8807530	44
17	.4738321	.538069	1.858496	.8806152	43
18	.4740882	.538444	1.857201	.8804774	42
19	.4743443	.538819	1.855908	.8803394	41
20	.4746004	.539195	1.854615	.8802014	40
21	.4748564	.539570	1.853325	.8800633	39
22	.4751124	.539946	1.852035	.8799251	38
23	.4753683	.540322	1.850747	.8797869	37
24	.4756242	.540698	1.849461	.8796486	36
25	.4758801	.541074	1.848176	.8795102	35
26	.4761359	.541450	1.846892	.8793717	34
27	.4763917	.541826	1.845609	.8792332	33
28	.4766474	.542202	1.844328	.8790946	32
29	.4769031	.542579	1.843049	.8789559	31
30	.4771588	.542955	1.841770	.8788171	30
31	.4774144	.543332	1.840494	.8786783	29
32	.4776700	.543709	1.839218	.8785394	28
33	.4779255	.544086	1.837944	.8784004	27
34	.4781810	.544463	1.836671	.8782613	26
35	.4784364	.544840	1.835399	.8781222	25
36	.4786919	.545217	1.834129	.8779830	24
37	.4789472	.545595	1.832861	.8778437	23
38	.4792026	.545972	1.831593	.8777043	22
39	.4794579	.546350	1.830327	.8775649	21
40	.4797131	.546728	1.829062	.8774254	20
41	.4799683	.547106	1.827799	.8772858	19
42	.4802235	.547484	1.826537	.8771462	18
43	.4804786	.547862	1.825276	.8770064	17
44	.4807337	.548240	1.824017	.8768666	16
45	.4809888	.548618	1.822759	.8767268	15
46	.4812438	.548997	1.821502	.8765868	14
47	.4814987	.549375	1.820247	.8764468	13
48	.4817537	.549754	1.818993	.8763067	12
49	.4820086	.550133	1.817740	.8761665	11
50	.4822634	.550512	1.816489	.8760263	10
51	.4825182	.550891	1.815239	.8758859	9
52	.4827730	.551270	1.813990	.8757455	8
53	.4830277	.551650	1.812743	.8756051	7
54	.4832824	.552029	1.811496	.8754645	6
55	.4835370	.552409	1.810252	.8753239	5
56	.4837916	.552789	1.809008	.8751832	4
57	.4840462	.553168	1.807766	.8750425	3
58	.4843007	.553548	1.806525	.8749016	2
59	.4845552	.553928	1.805286	.8747607	1
60	.4848096	.554309	1.804047	.8746197	0
	Cosine.	Cotang.	Tang.	Sine.	′

29°

′	Sine.	Tang.	Cotang.	Cosine.	
0	.4848096	.554309	1.804047	.8746197	60
1	.4850640	.554689	1.802810	.8744786	59
2	.4853184	.555069	1.801575	.8743375	58
3	.4855727	.555450	1.800340	.8741963	57
4	.4858270	.555831	1.799107	.8740550	56
5	.4860812	.556211	1.797875	.8739137	55
6	.4863354	.556592	1.796645	.8737722	54
7	.4865895	.556973	1.795416	.8736307	53
8	.4868436	.557355	1.794188	.8734891	52
9	.4870977	.557736	1.792961	.8733475	51
10	.4873517	.558117	1.791736	.8732058	50
11	.4876057	.558499	1.790512	.8730640	49
12	.4878597	.558881	1.789289	.8729221	48
13	.4881136	.559262	1.788067	.8727801	47
14	.4883674	.559644	1.786847	.8726381	46
15	.4886212	.560026	1.785628	.8724960	45
16	.4888750	.560409	1.784410	.8723538	44
17	.4891288	.560791	1.783194	.8722116	43
18	.4893825	.561173	1.781979	.8720693	42
19	.4896361	.561556	1.780765	.8719269	41
20	.4898897	.561939	1.779552	.8717844	40
21	.4901433	.562321	1.778340	.8716419	39
22	.4903968	.562704	1.777130	.8714993	38
23	.4906503	.563087	1.775921	.8713566	37
24	.4909038	.563471	1.774714	.8712138	36
25	.4911572	.563854	1.773507	.8710710	35
26	.4914105	.564237	1.772302	.8709281	34
27	.4916638	.564621	1.771098	.8707851	33
28	.4919171	.565005	1.769895	.8706420	32
29	.4921704	.565388	1.768694	.8704989	31
30	.4924236	.565772	1.767494	.8703557	30
31	.4926767	.566156	1.766295	.8702124	29
32	.4929298	.566541	1.765097	.8700691	28
33	.4931829	.566925	1.763900	.8699256	27
34	.4934359	.567309	1.762705	.8697821	26
35	.4936889	.567694	1.761511	.8696386	25
36	.4939419	.568079	1.760318	.8694949	24
37	.4941948	.568463	1.759126	.8693512	23
38	.4944476	.568848	1.757936	.8692074	22
39	.4947005	.569233	1.756747	.8690636	21
40	.4949532	.569619	1.755559	.8689196	20
41	.4952060	.570004	1.754372	.8687756	19
42	.4954587	.570389	1.753186	.8686315	18
43	.4957113	.570775	1.752002	.8684874	17
44	.4959639	.571161	1.750819	.8683431	16
45	.4962165	.571547	1.749637	.8681988	15
46	.4964690	.571933	1.748456	.8680544	14
47	.4967215	.572319	1.747276	.8679100	13
48	.4969740	.572705	1.746098	.8677655	12
49	.4972264	.573091	1.744921	.8676209	11
50	.4974787	.573478	1.743745	.8674762	10
51	.4977310	.573864	1.742570	.8673314	9
52	.4979833	.574251	1.741396	.8671866	8
53	.4982355	.574638	1.740224	.8670417	7
54	.4984877	.575025	1.739053	.8668967	6
55	.4987399	.575412	1.737883	.8667517	5
56	.4989920	.575799	1.736714	.8666066	4
57	.4992441	.576187	1.735546	.8664614	3
58	.4994961	.576574	1.734380	.8663161	2
59	.4997481	.576962	1.733214	.8661708	1
60	.5000000	.577350	1.732050	.8660254	0
	Cosine.	Cotang.	Tang.	Sine.	′

61° 60°

Note.—Secant = 1 + cosine. Cosecant = 1 + sine.

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3—Natural Sines, Tangents, Cotangents, Cosines.—(Continued.)

(Versed sine = 1 − cosine; coversed sine = 1 − sine.)

30° ′	Sine.	Tang.	Cotang.	Cosine.		31° ′	Sine.	Tang.	Cotang.	Cosine.	
0	.5000000	.577350	1.732050	.8660254	60	0	.5150381	.600860	1.664279	.8571673	60
1	.5002519	.577738	1.730887	.8658799	59	1	.5152874	.601256	1.663183	.8570174	59
2	.5005037	.578126	1.729726	.8657344	58	2	.5155367	.601652	1.662088	.8568675	58
3	.5007556	.578514	1.728565	.8655887	57	3	.5157859	.602049	1.660994	.8567175	57
4	.5010073	.578902	1.727406	.8654430	56	4	.5160351	.602445	1.659901	.8565674	56
5	.5012591	.579291	1.726247	.8652973	55	5	.5162842	.602841	1.658809	.8564173	55
6	.5015107	.579679	1.725090	.8651514	54	6	.5165333	.603238	1.657718	.8562671	54
7	.5017624	.580068	1.723934	.8650055	53	7	.5167824	.603635	1.656629	.8561168	53
8	.5020140	.580457	1.722779	.8648595	52	8	.5170314	.604032	1.655540	.8559664	52
9	.5022655	.580846	1.721626	.8647134	51	9	.5172804	.604429	1.654452	.8558160	51
10	.5025170	.581235	1.720473	.8645673	50	10	.5175293	.604826	1.653366	.8556655	50
11	.5027685	.581624	1.719322	.8644211	49	11	.5177782	.605224	1.652280	.8555149	49
12	.5030199	.582013	1.718172	.8642748	48	12	.5180270	.605621	1.651196	.8553643	48
13	.5032713	.582403	1.717023	.8641284	47	13	.5182758	.606019	1.650112	.8552135	47
14	.5035227	.582793	1.715875	.8639820	46	14	.5185246	.606417	1.649030	.8550627	46
15	.5037740	.583182	1.714728	.8638355	45	15	.5187733	.606814	1.647949	.8549119	45
16	.5040252	.583572	1.713582	.8636889	44	16	.5190219	.607213	1.646868	.8547609	44
17	.5042765	.583962	1.712438	.8635423	43	17	.5192705	.607611	1.645789	.8546099	43
18	.5045276	.584352	1.711294	.8633956	42	18	.5195191	.608009	1.644711	.8544588	42
19	.5047788	.584743	1.710152	.8632488	41	19	.5197676	.608408	1.643633	.8543077	41
20	.5050298	.585133	1.709011	.8631019	40	20	.5200161	.608806	1.642557	.8541564	40
21	.5052809	.585524	1.707871	.8629549	39	21	.5202646	.609205	1.641482	.8540051	39
22	.5055319	.585914	1.706732	.8628079	38	22	.5205130	.609604	1.640408	.8538538	38
23	.5057828	.586305	1.705595	.8626608	37	23	.5207613	.610003	1.639335	.8537023	37
24	.5060338	.586696	1.704458	.8625137	36	24	.5210096	.610402	1.638263	.8535508	36
25	.5062846	.587087	1.703323	.8623664	35	25	.5212579	.610801	1.637191	.8533992	35
26	.5065355	.587478	1.702189	.8622191	34	26	.5215061	.611201	1.636121	.8532475	34
27	.5067863	.587870	1.701055	.8620717	33	27	.5217543	.611601	1.635052	.8530958	33
28	.5070370	.588261	1.699923	.8619243	32	28	.5220024	.612000	1.633984	.8529440	32
29	.5072877	.588653	1.698792	.8617768	31	29	.5222505	.612400	1.632917	.8527921	31
30	.5075384	.589045	1.697663	.8616292	30	30	.5224986	.612800	1.631851	.8526402	30
31	.5077890	.589436	1.696534	.8614815	29	31	.5227466	.613201	1.630786	.8524881	29
32	.5080396	.589828	1.695406	.8613337	28	32	.5229945	.613601	1.629722	.8523360	28
33	.5082901	.590221	1.694280	.8611859	27	33	.5232424	.614001	1.628659	.8521839	27
34	.5085406	.590613	1.693155	.8610380	26	34	.5234903	.614402	1.627597	.8520316	26
35	.5087910	.591005	1.692030	.8608901	25	35	.5237381	.614803	1.626536	.8518793	25
36	.5090414	.591398	1.690907	.8607420	24	36	.5239859	.615204	1.625476	.8517269	24
37	.5092918	.591791	1.689785	.8605939	23	37	.5242336	.615605	1.624417	.8515745	23
38	.5095421	.592183	1.688664	.8604457	22	38	.5244813	.616006	1.623359	.8514219	22
39	.5097924	.592576	1.687544	.8602975	21	39	.5247290	.616407	1.622302	.8512693	21
40	.5100426	.592969	1.686426	.8601491	20	40	.5249766	.616809	1.621246	.8511167	20
41	.5102928	.593363	1.685308	.8600007	19	41	.5252241	.617210	1.620192	.8509639	19
42	.5105429	.593756	1.684191	.8593523	18	42	.5254717	.617612	1.619138	.8508111	18
43	.5107930	.594150	1.683076	.8597037	17	43	.5257191	.618014	1.618085	.8506582	17
44	.5110431	.594543	1.681962	.8595551	16	44	.5259665	.618416	1.617033	.8505053	16
45	.5112931	.594937	1.680848	.8594064	15	45	.5262139	.618818	1.615982	.8503522	15
46	.5115431	.595331	1.679736	.8592576	14	46	.5264613	.619221	1.614932	.8501991	14
47	.5117930	.595725	1.678625	.8591088	13	47	.5267085	.619623	1.613882	.8500459	13
48	.5120429	.596119	1.677515	.8589599	12	48	.5269558	.620026	1.612834	.8498927	12
49	.5122927	.596514	1.676406	.8588109	11	49	.5272030	.620429	1.611787	.8497394	11
50	.5125425	.596908	1.675298	.8586619	10	50	.5274502	.620832	1.610741	.8495860	10
51	.5127923	.597303	1.674192	.8585127	9	51	.5276973	.621235	1.609696	.8494325	9
52	.5130420	.597697	1.673086	.8583635	8	52	.5279443	.621638	1.608652	.8492790	8
53	.5132916	.598092	1.671981	.8582143	7	53	.5281914	.622041	1.607609	.8491254	7
54	.5135413	.598487	1.670878	.8580649	6	54	.5284383	.622445	1.606567	.8489717	6
55	.5137908	.598882	1.669775	.8579155	5	55	.5286853	.622848	1.605526	.8488179	5
56	.5140404	.599278	1.668674	.8577660	4	56	.5289322	.623252	1.604485	.8486641	4
57	.5142899	.599673	1.667574	.8576164	3	57	.5291790	.623656	1.603446	.8485102	3
58	.5145393	.600069	1.666474	.8574668	2	58	.5294258	.624060	1.602408	.8483562	2
59	.5147887	.600464	1.665376	.8573171	1	59	.5296726	.624465	1.601370	.8482022	1
60	.5150381	.600860	1.664279	.8571673	0	60	.5299193	.624869	1.600334	.8480481	0
	Cosine.	Cotang	Tang.	Sine.	′		Cosine.	Cotang	Tang.	Sine.	′
					59°						58°

Note.—Secant = 1 ÷ cosine. Cosecant = 1 ÷ sine.

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3.—Natural Sines, Tangents, Cotangents, Cosines.—(Continued.)

(Versed sine = 1 − cosine; coversed sine = 1 − sine.)

32°						33°				
′	Sine.	Tang.	Cotang.	Cosine.		′	Sine.	Tang.	Cotang.	Cosine.
0	.5299193	.624869	1.600334	.8480481	60	0	.5446390	.649407	1.539865	.8386706
1	.5301659	.625273	1.599299	.8478939	59	1	.5448830	.649821	1.538884	.8385121
2	.5304125	.625678	1.598264	.8477397	58	2	.5451269	.650235	1.537905	.8383536
3	.5306591	.626083	1.597231	.8475853	57	3	.5453707	.650649	1.536927	.8381950
4	.5309057	.626488	1.596198	.8474309	56	4	.5456145	.651063	1.535949	.8380363
5	.5311521	.626893	1.595167	.8472765	55	5	.5458583	.651477	1.534972	.8378775
6	.5313986	.627298	1.594136	.8471219	54	6	.5461020	.651891	1.533996	.8377187
7	.5316450	.627704	1.593107	.8469673	53	7	.5463456	.652306	1.533021	.8375598
8	.5318913	.628109	1.592078	.8468126	52	8	.5465892	.652721	1.532047	.8374009
9	.5321376	.628515	1.591050	.8466579	51	9	.5468328	.653136	1.531074	.8372418
10	.5323839	.628921	1.590023	.8465030	50	10	.5470763	.653551	1.530102	.8370827
11	.5326301	.629327	1.588997	.8463481	49	11	.5473198	.653996	1.529130	.8369236
12	.5328763	.629733	1.587973	.8461932	48	12	.5475632	.654381	1.528160	.8367643
13	.5331224	.630139	1.586949	.8460381	47	13	.5478066	.654797	1.527190	.8366050
14	.5333685	.630546	1.585926	.8458830	46	14	.5480499	.655212	1.526221	.8364456
15	.5336145	.630953	1.584904	.8457278	45	15	.5482932	.655628	1.525253	.8362862
16	.5338605	.631359	1.583883	.8455726	44	16	.5485365	.656044	1.524286	.8361266
17	.5341065	.631766	1.582862	.8454172	43	17	.5487797	.656460	1.523320	.8359670
18	.5343523	.632173	1.581843	.8452618	42	18	.5490228	.656877	1.522354	.8358074
19	.5345982	.632581	1.580825	.8451064	41	19	.5492659	.657293	1.521389	.8356476
20	.5348440	.632988	1.579807	.8449508	40	20	.5495090	.657710	1.520426	.8354878
21	.5350898	.633395	1.578791	.8447952	39	21	.5497520	.658127	1.519463	.8353279
22	.5353355	.633803	1.577776	.8446395	38	22	.5499950	.658544	1.518501	.8351680
23	.5355812	.634211	1.576761	.8444838	37	23	.5502379	.658961	1.517540	.8350080
24	.5358268	.634619	1.575747	.8443279	36	24	.5504807	.659378	1.516579	.8348479
25	.5360724	.635027	1.574735	.8441720	35	25	.5507236	.659796	1.515620	.8346877
26	.5363179	.635435	1.573723	.8440161	34	26	.5509663	.660213	1.514661	.8345275
27	.5365634	.635844	1.572712	.8438600	33	27	.5512091	.660631	1.513703	.8343672
28	.5368089	.636252	1.571702	.8437039	32	28	.5514518	.661049	1.512746	.8342068
29	.5370543	.636661	1.570693	.8435477	31	29	.5516944	.661467	1.511790	.8340463
30	.5372996	.637070	1.569685	.8433914	30	30	.5519370	.661885	1.510835	.8338858
31	.5375449	.637479	1.568678	.8432351	29	31	.5521795	.662304	1.509880	.8337252
32	.5377902	.637888	1.567672	.8430787	28	32	.5524220	.662722	1.508927	.8335646
33	.5380354	.638297	1.566666	.8429222	27	33	.5526645	.663141	1.507974	.8334038
34	.5382806	.638707	1.565662	.8427657	26	34	.5529069	.663560	1.507022	.8332430
35	.5385257	.639116	1.564659	.8426091	25	35	.5531492	.663979	1.506071	.8330822
36	.5387708	.639526	1.563656	.8424524	24	36	.5533915	.664398	1.505121	.8329212
37	.5390158	.639936	1.562654	.8422956	23	37	.5536338	.664817	1.504171	.8327602
38	.5392608	.640346	1.561654	.8421388	22	38	.5538760	.665237	1.503222	.8325991
39	.5395058	.640756	1.560654	.8419819	21	39	.5541182	.665657	1.502275	.8324380
40	.5397507	.641167	1.559655	.8418249	20	40	.5543603	.666076	1.501328	.8322768
41	.5399955	.641577	1.558657	.8416679	19	41	.5546024	.666496	1.500382	.8321155
42	.5402403	.641988	1.557660	.8415108	18	42	.5548444	.666917	1.499436	.8319541
43	.5404851	.642399	1.556663	.8413536	17	43	.5550864	.667337	1.498492	.8317927
44	.5407298	.642810	1.555668	.8411963	16	44	.5553283	.667758	1.497548	.8316312
45	.5409745	.643221	1.554674	.8410390	15	45	.5555702	.668178	1.496605	.8314696
46	.5412191	.643632	1.553680	.8408816	14	46	.5558121	.668599	1.495663	.8313080
47	.5414637	.644044	1.552688	.8407241	13	47	.5560539	.669020	1.494722	.8311463
48	.5417082	.644456	1.551696	.8405666	12	48	.5562956	.669441	1.493782	.8309845
49	.5419527	.644867	1.550705	.8404090	11	49	.5565373	.669863	1.492842	.8308226
50	.5421971	.645279	1.549715	.8402513	10	50	.5567790	.670284	1.491903	.8306607
51	.5424415	.645691	1.548726	.8400936	9	51	.5570206	.670706	1.490965	.8304987
52	.5426859	.646104	1.547738	.8399357	8	52	.5572621	.671128	1.490028	.8303366
53	.5429302	.646516	1.546751	.8397778	7	53	.5575036	.671550	1.489092	.8301745
54	.5431744	.646929	1.545764	.8396199	6	54	.5577451	.671972	1.488157	.8300123
55	.5434187	.647341	1.544779	.8394618	5	55	.5579865	.672394	1.487222	.8298500
56	.5436628	.647754	1.543794	.8393037	4	56	.5582279	.672816	1.486288	.8296877
57	.5439069	.648167	1.542810	.8391455	3	57	.5584692	.673239	1.485355	.8295252
58	.5441510	.648580	1.541828	.8389873	2	58	.5587105	.673662	1.484423	.8293628
59	.5443951	.648994	1.540846	.8388290	1	59	.5589517	.674085	1.483491	.8292002
60	.5446390	.649407	1.539865	.8386706	0	60	.5591929	.674508	1.482561	.8290376
	Cosine.	Cotang.	Tang.	Sine.	′		Cosine.	Cotang.	Tang.	Sine.

57°

Note.—Secant = 1 + cosine. Cosecant = 1 + sine.

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3—Natural Sines, Tangents, Cotangents, Cosines.—(Continued.)

(Versed sine = 1 − cosine; coversed sine = 1 − sine.)

34° ′	Sine.	Tang.	Cotang.	Cosine.		35° ′	Sine.	Tang.	Cotang.	Cosine.	
0	.5591929	.674508	1.482561	.8290376	60	0	.5735764	.700207	1.428148	.8191520	60
1	.5594340	.674931	1.481631	.8288749	59	1	.5738147	.700641	1.427264	.8189852	59
2	.5596751	.675355	1.480702	.8287121	58	2	.5740529	.701074	1.426381	.8188182	58
3	.5599162	.675779	1.479773	.8285493	57	3	.5742911	.701508	1.425498	.8186512	57
4	.5601572	.676202	1.478846	.8283864	56	4	.5745292	.701943	1.424617	.8184841	56
5	.5603981	.676626	1.477919	.8282234	55	5	.5747672	.702377	1.423736	.8183169	55
6	.5606390	.677050	1.476993	.8280603	54	6	.5750053	.702811	1.422856	.8181497	54
7	.5608798	.677475	1.476068	.8278972	53	7	.5752432	.703246	1.421976	.8179824	53
8	.5611206	.677899	1.475144	.8277340	52	8	.5754811	.703681	1.421097	.8178151	52
9	.5613614	.678324	1.474221	.8275708	51	9	.5757190	.704116	1.420220	.8176476	51
10	.5616021	.678749	1.473298	.8274074	50	10	.5759568	.704551	1.419342	.8174801	50
11	.5618428	.679174	1.472376	.8272440	49	11	.5761946	.704986	1.418466	.8173125	49
12	.5620834	.679599	1.471455	.8270806	48	12	.5764323	.705422	1.417590	.8171449	48
13	.5623239	.680024	1.470535	.8269170	47	13	.5766700	.705858	1.416715	.8169772	47
14	.5625645	.680450	1.469615	.8267534	46	14	.5769076	.706294	1.415840	.8168094	46
15	.5628049	.680875	1.468696	.8265897	45	15	.5771452	.706730	1.414967	.8166416	45
16	.5630453	.681301	1.467778	.8264260	44	16	.7773827	.707166	1.414094	.8164736	44
17	.5632857	.681727	1.466861	.8262622	43	17	.5776202	.707602	1.413222	.8163056	43
18	.5635260	.682153	1.465945	.8260983	42	18	.5778576	.708039	1.412350	.8161376	42
19	.5637663	.682580	1.465029	.8259343	41	19	.5780950	.708476	1.411479	.8159695	41
20	.5640066	.683006	1.464114	.8257703	40	20	.5783323	.708913	1.410609	.8158013	40
21	.5642467	.683433	1.463200	.8256062	39	21	.5785696	.709350	1.409740	.8156330	39
22	.5644869	.683860	1.462287	.8254420	38	22	.5788069	.709787	1.408871	.8154647	38
23	.5647270	.684287	1.461374	.8252778	37	23	.5790440	.710225	1.408003	.8152963	37
24	.5649670	.684714	1.460463	.8251135	36	24	.5792812	.710663	1.407136	.8151278	36
25	.5652070	.685141	1.459552	.8249491	35	25	.5795183	.711100	1.406270	.8149593	35
26	.5654469	.685569	1.458642	.8247847	34	26	.5797553	.711539	1.405404	.8147906	34
27	.5656868	.685996	1.457732	.8246202	33	27	.5799923	.711977	1.404539	.8146220	33
28	.5659267	.686424	1.456824	.8244556	32	28	.5802292	.712415	1.403674	.8144532	32
29	.5661665	.686852	1.455916	.8242909	31	29	.5804661	.712854	1.402811	.8142844	31
30	.5664062	.687281	1.455009	.8241262	30	30	.5807030	.713293	1.401948	.8141155	30
31	.5666459	.687709	1.454102	.8239614	29	31	.5809397	.713732	1.401086	.8139466	29
32	.5668856	.688137	1.453197	.8237965	28	32	.5811765	.714171	1.400224	.8137775	28
33	.5671252	.688566	1.452292	.8236316	27	33	.5814132	.714610	1.399363	.8136084	27
34	.5673648	.688995	1.451388	.8234666	26	34	.5816498	.715050	1.398503	.8134393	26
35	.5676043	.689424	1.450485	.8233015	25	35	.5818864	.715490	1.397644	.8132701	25
36	.5678437	.689853	1.449582	.8231364	24	36	.5821230	.715929	1.396785	.8131008	24
37	.5680832	.690283	1.448680	.8229712	23	37	.5823595	.716369	1.395927	.8129314	23
38	.5683225	.690712	1.447779	.8228059	22	38	.5825959	.716810	1.395069	.8127620	22
39	.5685619	.691142	1.446879	.8226405	21	39	.5828323	.717250	1.394213	.8125925	21
40	.5688011	.691572	1.445980	.8224751	20	40	.5830687	.717691	1.393357	.8124229	20
41	.5690403	.692002	1.445081	.8223096	19	41	.5833050	.718131	1.392501	.8122532	19
42	.5692795	.692432	1.444183	.8221440	18	42	.5835412	.718572	1.391647	.8120835	18
43	.5695187	.692863	1.443286	.8219784	17	43	.5837774	.719014	1.390793	.8119137	17
44	.5697577	.693293	1.442389	.8218127	16	44	.5840136	.719455	1.389940	.8117439	16
45	.5699968	.693724	1.441494	.8216469	15	45	.5842497	.719897	1.389087	.8115740	15
46	.5702357	.694155	1.440599	.8214811	14	46	.5844857	.720338	1.388235	.8114040	14
47	.5704747	.694586	1.439704	.8213152	13	47	.5847217	.720780	1.387384	.8112339	13
48	.5707136	.695018	1.438811	.8211492	12	48	.5849577	.721222	1.386534	.8110638	12
49	.5709524	.695449	1.437918	.8209832	11	49	.5851936	.721665	1.385684	.8108936	11
50	.5711912	.695881	1.437026	.8208170	10	50	.5854294	.722107	1.384835	.8107234	10
51	.5714299	.696313	1.436135	.8206509	9	51	.5856652	.722550	1.383986	.8105530	9
52	.5716686	.696745	1.435245	.8204846	8	52	.5859010	.722993	1.383139	.8103826	8
53	.5719073	.697177	1.434355	.8203183	7	53	.5861367	.723436	1.382292	.8102122	7
54	.5721459	.697609	1.433466	.8201519	6	54	.5863724	.723879	1.381445	.8100416	6
55	.5723844	.698042	1.432578	.8199854	5	55	.5866080	.724322	1.380600	.8098710	5
56	.5726229	.698474	1.431690	.8198189	4	56	.5868435	.724766	1.379755	.8097004	4
57	.5728614	.698907	1.430803	.8196523	3	57	.5870790	.725210	1.378910	.8095296	3
58	.5730998	.699340	1.429917	.8194856	2	58	.5873145	.725654	1.378067	.8093588	2
59	.5733381	.699774	1.429032	.8193189	1	59	.5875499	.726098	1.377224	.8091879	1
60	.5735764	.700207	1.428148	.8191520	0	60	.5877853	.726542	1.376381	.8090170	0
	Cosine.	Cotang.	Tang.	Sine.	′		Cosine.	Cotang.	Tang.	Sine.	′
					55°						54°

Note.—Secant = 1 + cosine. Cosecant = 1 + sine.

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3.—Natural Sines, Tangents, Cotangents, Cosines.—(Continued.)

(Versed sine = 1 − cosine; coversed sine = 1 − sine.)

36°						37°					
′	Sine.	Tang.	Cotang.	Cosine.		′	Sine.	Tang.	Cotang.	Cosine.	
0	.5877853	.726542	1.376381	.8090170	60	0	.6018150	.753554	1.327044	.7986355	60
1	.5880206	.726987	1.375540	.8088460	59	1	.6020473	.754010	1.326242	.7984604	59
2	.5882558	.727431	1.374699	.8086749	58	2	.6022795	.754466	1.325439	.7982853	58
3	.5884910	.727876	1.373859	.8085037	57	3	.6025117	.754923	1.324636	.7981100	57
4	.5887262	.728321	1.373019	.8083325	56	4	.6027439	.755379	1.323837	.7979347	56
5	.5889613	.728767	1.372180	.8081612	55	5	.6029760	.755836	1.323036	.7977594	55
6	.5891964	.729212	1.371342	.8079899	54	6	.6032080	.756294	1.322237	.7975839	54
7	.5894314	.729658	1.370504	.8078185	53	7	.6034400	.756751	1.321437	.7974084	53
8	.5896663	.730104	1.369667	.8076470	52	8	.6036719	.757209	1.320639	.7972329	52
9	.5899012	.730550	1.368831	.8074754	51	9	.6039038	.757666	1.319841	.7970572	51
10	.5901361	.730996	1.367995	.8073038	50	10	.6041356	.758124	1.319044	.7968815	50
11	.5903709	.731442	1.367161	.8071321	49	11	.6043674	.758582	1.318247	.7967058	49
12	.5906057	.731889	1.366326	.8069603	48	12	.6045991	.759041	1.317451	.7965299	48
13	.5908404	.732336	1.365493	.8067885	47	13	.6048308	.759499	1.316655	.7963540	47
14	.5910750	.732783	1.364660	.8066166	46	14	.6050624	.759958	1.315861	.7961780	46
15	.5913096	.733230	1.363827	.8064446	45	15	.6052940	.760417	1.315066	.7960020	45
16	.5915442	.733677	1.362996	.8062726	44	16	.6055255	.760876	1.314273	.7958259	44
17	.5917787	.734125	1.362165	.8061005	43	17	.6057570	.761336	1.313480	.7956497	43
18	.5920132	.734573	1.361335	.8059283	42	18	.6059884	.761795	1.312687	.7954735	42
19	.5922476	.735021	1.360505	.8057560	41	19	.6062198	.762255	1.311895	.7952972	41
20	.5924819	.735469	1.359676	.8055837	40	20	.6064511	.762715	1.311104	.7951208	40
21	.5927163	.735917	1.358848	.8054113	39	21	.6066824	.763175	1.310314	.7949444	39
22	.5929505	.736366	1.358020	.8052389	38	22	.6069136	.763636	1.309523	.7947678	38
23	.5931847	.736814	1.357193	.8050664	37	23	.6071447	.764096	1.308734	.7945913	37
24	.5934189	.737263	1.356367	.8048938	36	24	.6073758	.764557	1.307945	.7944146	36
25	.5936530	.737712	1.355541	.8047211	35	25	.6076069	.765018	1.307157	.7942379	35
26	.5938871	.738162	1.354716	.8045484	34	26	.6078379	.765480	1.306369	.7940611	34
27	.5941211	.738611	1.353891	.8043756	33	27	.6080689	.765941	1.305582	.7938843	33
28	.5943550	.739061	1.353068	.8042028	32	28	.6082998	.766403	1.304796	.7937074	32
29	.5945889	.739511	1.352244	.8040299	31	29	.6085306	.766864	1.304010	.7935304	31
30	.5948228	.739961	1.351422	.8038569	30	30	.6087614	.767327	1.303225	.7933533	30
31	.5950566	.740411	1.350600	.8036838	29	31	.6089922	.767789	1.302440	.7931762	29
32	.5952904	.740861	1.349779	.8035107	28	32	.6092229	.768251	1.301656	.7929990	28
33	.5955241	.741312	1.348958	.8033375	27	33	.6094535	.768714	1.300873	.7928218	27
34	.5957577	.741763	1.348139	.8031642	26	34	.6096841	.769177	1.300090	.7926445	26
35	.5959913	.742214	1.347319	.8029909	25	35	.6099147	.769640	1.299308	.7924671	25
36	.5962249	.742665	1.346501	.8028175	24	36	.6101452	.770103	1.298526	.7922896	24
37	.5964584	.743117	1.345683	.8026440	23	37	.6103756	.770567	1.297745	.7921121	23
38	.5966918	.743568	1.344865	.8024705	22	38	.6106060	.771030	1.296964	.7919345	22
39	.5969252	.744020	1.344049	.8022969	21	39	.6108363	.771494	1.296185	.7917569	21
40	.5971586	.744472	1.343233	.8021232	20	40	.6110666	.771958	1.295405	.7915792	20
41	.5973919	.744924	1.342417	.8019495	19	41	.6112969	.772423	1.294627	.7914014	19
42	.5976251	.745377	1.341602	.8017756	18	42	.6115270	.772887	1.293848	.7912235	18
43	.5978583	.745829	1.340788	.8016018	17	43	.6117572	.773352	1.293071	.7910456	17
44	.5980915	.746282	1.339975	.8014278	16	44	.6119873	.773817	1.292294	.7908676	16
45	.5983246	.746735	1.339162	.8012538	15	45	.6122173	.774282	1.291517	.7906896	15
46	.5985577	.747188	1.338350	.8010797	14	46	.6124473	.774748	1.290742	.7905115	14
47	.5987906	.747642	1.337538	.8009056	13	47	.6126772	.775213	1.289966	.7903333	13
48	.5990236	.748095	1.336727	.8007314	12	48	.6129071	.775679	1.289192	.7901550	12
49	.5992565	.748549	1.335917	.8005571	11	49	.6131369	.776145	1.288418	.7899767	11
50	.5994893	.749003	1.335107	.8003827	10	50	.6133666	.776611	1.287644	.7897983	10
51	.5997221	.749457	1.334298	.8002083	9	51	.6135964	.777078	1.286871	.7896198	9
52	.5999549	.749911	1.333490	.8000338	8	52	.6138260	.777544	1.286099	.7894413	8
53	.6001876	.750366	1.332682	.7998593	7	53	.6140556	.778011	1.285327	.7892627	7
54	.6004202	.750821	1.331875	.7996847	6	54	.6142852	.778478	1.284556	.7890841	6
55	.6006528	.751276	1.331068	.7995100	5	55	.6145147	.778946	1.283786	.7889054	5
56	.6008854	.751731	1.330262	.7993352	4	56	.6147442	.779413	1.283016	.7887266	4
57	.6011179	.752186	1.329457	.7991604	3	57	.6149736	.779881	1.282246	.7885477	3
58	.6013503	.752642	1.328652	.7989855	2	58	.6152029	.780349	1.281477	.7883688	2
59	.6015827	.753098	1.327848	.7988105	1	59	.6154322	.780817	1.280709	.7881898	1
60	.6018150	.753554	1.327044	.7986355	0	60	.6156615	.781285	1.279941	.7880108	0
	Cosine.	Cotang.	Tang.	Sine.	′		Cosine.	Cotang.	Tang.	Sine.	′
					53°						52°

Note.—Secant = 1 + cosine. Cosecant = 1 + sine.

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3.—Natural Sines, Tangents, Cotangents, Cosines.—(Continued.)

(Versed sine = 1 − cosine; coversed sine = 1 − sine.)

38° 39°

′	Sine.	Tang.	Cotang.	Cosine.		′	Sine.	Tang.	Cotang.	Cosine.	
0	.6156615	.781285	1.279941	.7880108	60	0	.6293204	.809784	1.234897	.7771460	60
1	.6158907	.781754	1.279174	.7878316	59	1	.6295464	.810265	1.234162	.7769629	59
2	.6161198	.782222	1.278407	.7876524	58	2	.6297724	.810747	1.233429	.7767797	58
3	.6163489	.782691	1.277641	.7874732	57	3	.6299983	.811230	1.232696	.7765965	57
4	.6165780	.783161	1.276876	.7872939	56	4	.6302242	.811712	1.231963	.7764132	56
5	.6168069	.783630	1.276111	.7871145	55	5	.6304500	.812195	1.231231	.7762298	55
6	.6170359	.784100	1.275347	.7869350	54	6	.6306758	.812678	1.230499	.7760464	54
7	.6172648	.784570	1.274583	.7867555	53	7	.6309015	.813161	1.229768	.7758629	53
8	.6174936	.785040	1.273820	.7865759	52	8	.6311272	.813644	1.229038	.7756794	52
9	.6177224	.785510	1.273057	.7863963	51	9	.6313528	.814128	1.228308	.7754957	51
10	.6179511	.785980	1.272295	.7862165	50	10	.6315784	.814611	1.227578	.7753121	50
11	.6181798	.786451	1.271534	.7860367	49	11	.6318039	.815095	1.226849	.7751283	49
12	.6184084	.786922	1.270773	.7858569	48	12	.6320293	.815580	1.226121	.7749445	48
13	.6186370	.787393	1.270013	.7856770	47	13	.6322547	.816064	1.225393	.7747606	47
14	.6188655	.787864	1.269253	.7854970	46	14	.6324800	.816549	1.224665	.7745767	46
15	.6190939	.788336	1.268494	.7853169	45	15	.6327053	.817034	1.223938	.7743926	45
16	.6193224	.788808	1.267735	.7851368	44	16	.6329306	.817519	1.223212	.7742086	44
17	.6195507	.789280	1.266977	.7849566	43	17	.6331557	.818004	1.222486	.7740244	42
18	.6197790	.789752	1.266219	.7847764	42	18	.6333809	.818490	1.221761	.7738402	43
19	.6200073	.790224	1.265462	.7845961	41	19	.6336059	.818976	1.221036	.7736559	41
20	.6202355	.790697	1.264706	.7844157	40	20	.6338310	.819462	1.220312	.7734716	40
21	.6204636	.791170	1.263950	.7842352	39	21	.6340559	.819948	1.219588	.7732872	39
22	.6206917	.791643	1.263195	.7840547	38	22	.6342808	.820435	1.218865	.7731027	38
23	.6209198	.792116	1.262440	.7838741	37	23	.6345057	.820922	1.218142	.7729182	37
24	.6211478	.792590	1.261686	.7836935	36	24	.6347305	.821409	1.217419	.7727336	36
25	.6213757	.793064	1.260932	.7835127	35	25	.6349553	.821896	1.216698	.7725489	35
26	.6216036	.793537	1.260179	.7833320	34	26	.6351800	.822384	1.215976	.7723642	34
27	.6218314	.794012	1.259426	.7831511	33	27	.6354046	.822871	1.215256	.7721794	33
28	.6220592	.794486	1.258674	.7829702	32	28	.6356292	.823359	1.214535	.7719945	32
29	.6222870	.794961	1.257923	.7827892	31	29	.6358537	.823847	1.213816	.7718096	31
30	.6225146	.795435	1.257172	.7826082	30	30	.6360782	.824336	1.213097	.7716246	30
31	.6227423	.795911	1.256421	.7824270	29	31	.6363026	.824825	1.212378	.7714395	29
32	.6229698	.796386	1.255672	.7822459	28	32	.6365270	.825314	1.211660	.7712544	28
33	.6231974	.796861	1.254922	.7820646	27	33	.6367513	.825803	1.210942	.7710692	27
34	.6234248	.797337	1.254174	.7818833	26	34	.6369756	.826292	1.210225	.7708840	26
35	.6236522	.797813	1.253426	.7817019	25	35	.6371998	.826782	1.209508	.7706986	25
36	.6238796	.798289	1.252678	.7815205	24	36	.6374240	.827271	1.208792	.7705132	24
37	.6241069	.798765	1.251931	.7813390	23	37	.6376481	.827762	1.208076	.7703278	23
38	.6243342	.799242	1.251184	.7811574	22	38	.6378721	.828252	1.207361	.7701423	22
39	.6245614	.799719	1.250438	.7809757	21	39	.6380961	.828742	1.206646	.7699567	21
40	.6247885	.800196	1.249693	.7807940	20	40	.6383201	.829233	1.205932	.7697710	20
41	.6250156	.800673	1.248948	.7806123	19	41	.6385440	.829724	1.205219	.7695853	19
42	.6252427	.801151	1.248204	.7804304	18	42	.6387678	.830216	1.204505	.7693996	18
43	.6254696	.801628	1.247460	.7802485	17	43	.6389916	.830707	1.203793	.7692137	17
44	.6256965	.802106	1.246716	.7800665	16	44	.6392153	.831199	1.203081	.7690278	16
45	.6259235	.802584	1.245974	.7798845	15	45	.6394390	.831691	1.202369	.7688418	15
46	.6261503	.803063	1.245232	.7797024	14	46	.6396626	.832183	1.201658	.7686558	14
47	.6263771	.803541	1.244490	.7795202	13	47	.6398862	.832675	1.200947	.7684697	13
48	.6266038	.804020	1.243749	.7793380	12	48	.6401097	.833168	1.200237	.7682835	12
49	.6268305	.804499	1.243008	.7791557	11	49	.6403332	.833661	1.199527	.7680973	11
50	.6270571	.804979	1.242268	.7789733	10	50	.6405566	.834154	1.198818	.7679110	10
51	.6272837	.805458	1.241529	.7787909	9	51	.6407799	.834648	1.198109	.7677246	9
52	.6275102	.805938	1.240790	.7786084	8	52	.6410032	.835141	1.197401	.7675382	8
53	.6277366	.806418	1.240051	.7784258	7	53	.6412264	.835635	1.196693	.7673517	7
54	.6279631	.806898	1.239313	.7782431	6	54	.6414496	.836129	1.195986	.7671652	6
55	.6281894	.807378	1.238576	.7780604	5	55	.6416728	.836624	1.195279	.7669785	5
56	.6284157	.807859	1.237839	.7778777	4	56	.6418958	.837118	1.194573	.7667918	4
57	.6286420	.808340	1.237103	.7776949	3	57	.6421189	.837613	1.193867	.7666051	3
58	.6288682	.808821	1.236367	.7775120	2	58	.6423418	.838108	1.193162	.7664183	2
59	.6290943	.809302	1.235631	.7773290	1	59	.6425647	.838604	1.192457	.7662314	1
60	.6293204	.809784	1.234897	.7771460	0	60	.6427876	.839099	1.191753	.7660444	0
	Cosine.	Cotang.	Tang.	Sine.	′		Cosine.	Cotang.	Tang.	Sine.	′

51° 50°

Note.—Secant = 1 + cosine. Cosecant = 1 + sine.

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3.—Natural Sines, Tangents, Cotangents, Cosines.—(Continued.)

(Versed sine = 1 − cosine; coversed sine = 1 − sine.)

40°						41°					
′	Sine.	Tang.	Cotang.	Cosine.	‖	′	Sine.	Tang.	Cotang.	Cosine.	
0	.6427876	.839099	1.191753	.7660444	60	0	.6560590	.869286	1.150368	.7547096	60
1	.6430104	.839595	1.191049	.7658574	59	1	.6562785	.869797	1.149692	.7545187	59
2	.6433332	.840091	1.190346	.7656704	58	2	.6564980	.870308	1.149017	.7543278	58
3	.6434559	.840587	1.189643	.7654832	57	3	.6567174	.870820	1.148342	.7541368	57
4	.6436785	.841084	1.188941	.7652960	56	4	.6569367	.871331	1.147668	.7539457	56
5	.6439011	.841581	1.188239	.7651087	55	5	.6571560	.871843	1.146994	.7537546	55
6	.6441236	.842078	1.187538	.7649214	54	6	.6573752	.872355	1.146321	.7535634	54
7	.6443461	.842575	1.186837	.7647340	53	7	.6575944	.872868	1.145648	.7533721	53
8	.6445685	.843073	1.186136	.7645465	52	8	.6578135	.873380	1.144976	.7531808	52
9	.6447909	.843570	1.185437	.7643590	51	9	.6580326	.873893	1.144304	.7529894	51
10	.6450132	.844068	1.184737	.7641714	50	10	.6582516	.874406	1.143632	.7527980	50
11	.6452355	.844567	1.184038	.7639838	49	11	.6584706	.874920	1.142961	.7526065	49
12	.6454577	.845065	1.183340	.7637960	48	12	.6586895	.875433	1.142290	.7524149	48
13	.6456798	.845564	1.182642	.7636082	47	13	.6589083	.875947	1.141620	.7522233	47
14	.6459019	.846063	1.181944	.7634204	46	14	.6591271	.876462	1.140950	.7520316	46
15	.6461240	.846562	1.181247	.7632325	45	15	.6593458	.876976	1.140281	.7518398	45
16	.6463460	.847062	1.180551	.7630445	44	16	.6595645	.877491	1.139612	.7516480	44
17	.6465679	.847561	1.179855	.7628564	43	17	.6597831	.878006	1.138944	.7514561	43
18	.6467898	.848061	1.179159	.7626683	42	18	.6600017	.878521	1.138276	.7512641	42
19	.6470116	.848561	1.178464	.7624802	41	19	.6602202	.879037	1.137608	.7510721	41
20	.6472334	.849062	1.177769	.7622919	40	20	.6604386	.879552	1.136941	.7508800	40
21	.6474551	.849563	1.177075	.7621036	39	21	.6606570	.880068	1.136274	.7506879	39
22	.6476767	.850064	1.176382	.7619152	38	22	.6608754	.880585	1.135608	.7504957	38
23	.6478984	.850565	1.175688	.7617268	37	23	.6610936	.881101	1.134942	.7503034	37
24	.6481199	.851066	1.174996	.7615383	36	24	.6613119	.881618	1.134277	.7501111	36
25	.6483414	.851568	1.174303	.7613497	35	25	.6615300	.882135	1.133612	.7499187	35
26	.6485628	.852070	1.173612	.7611611	34	26	.6617482	.882653	1.132947	.7497262	34
27	.6487842	.852572	1.172920	.7609724	33	27	.6619662	.883170	1.132283	.7495337	33
28	.6490056	.853075	1.172229	.7607837	32	28	.6621842	.883688	1.131620	.7493411	32
29	.6492268	.853577	1.171539	.7605949	31	29	.6624022	.884206	1.130957	.7491484	31
30	.6494480	.854080	1.170849	.7604060	30	30	.6626200	.884725	1.130294	.7489557	30
31	.6496692	.854583	1.170160	.7602170	29	31	.6628379	.885244	1.129632	.7487629	29
32	.6498903	.855087	1.169471	.7600280	28	32	.6630557	.885763	1.128970	.7485701	28
33	.6501114	.855591	1.168782	.7598389	27	33	.6632734	.886282	1.128308	.7483772	27
34	.6503324	.856095	1.168094	.7596498	26	34	.6634910	.886801	1.127647	.7481842	26
35	.6505533	.856599	1.167407	.7594606	25	35	.6637087	.887321	1.126987	.7479912	25
36	.6507742	.857103	1.166720	.7592713	24	36	.6639262	.887841	1.126327	.7477981	24
37	.6509951	.857608	1.166033	.7590820	23	37	.6641437	.888361	1.125667	.7476049	23
38	.6512158	.858113	1.165347	.7588926	22	38	.6643612	.888882	1.125008	.7474117	22
39	.6514366	.858618	1.164661	.7587031	21	39	.6645785	.889403	1.124349	.7472184	21
40	.6516572	.859124	1.163976	.7585136	20	40	.6647959	.889924	1.123690	.7470251	20
41	.6518778	.859629	1.163291	.7583240	19	41	.6650131	.890445	1.123032	.7468317	19
42	.6520984	.860135	1.162607	.7581343	18	42	.6652304	.890967	1.122375	.7466382	18
43	.6523189	.860641	1.161923	.7579446	17	43	.6654475	.891489	1.121718	.7464446	17
44	.6525394	.861148	1.161240	.7577548	16	44	.6656646	.892011	1.121061	.7462510	16
45	.6527598	.861655	1.160557	.7575650	15	45	.6658817	.892534	1.120405	.7460574	15
46	.6529801	.862162	1.159874	.7573751	14	46	.6660987	.893056	1.119749	.7458636	14
47	.6532004	.862669	1.159192	.7571851	13	47	.6663156	.893579	1.119094	.7456699	13
48	.6534206	.863176	1.158511	.7569951	12	48	.6665325	.894103	1.118439	.7454760	12
49	.6536408	.863684	1.157830	.7568050	11	49	.6667493	.894626	1.117784	.7452821	11
50	.6538609	.864192	1.157149	.7566148	10	50	.6669661	.895150	1.117130	.7450881	10
51	.6540810	.864700	1.156469	.7564246	9	51	.6671828	.895674	1.116476	.7448941	9
52	.6543010	.865209	1.155789	.7562343	8	52	.6673994	.896199	1.115823	.7446999	8
53	.6545209	.865718	1.155110	.7560439	7	53	.6676160	.896723	1.115170	.7445058	7
54	.6547408	.866227	1.154431	.7558535	6	54	.6678326	.897248	1.114518	.7443115	6
55	.6549607	.866736	1.153753	.7556630	5	55	.6680490	.897773	1.113866	.7441173	5
56	.6551804	.867246	1.153075	.7554724	4	56	.6682655	.898299	1.113214	.7439229	4
57	.6554002	.867755	1.152397	.7552818	3	57	.6684818	.898825	1.112563	.7437285	3
58	.6556198	.868265	1.151721	.7550911	2	58	.6686981	.899351	1.111912	.7435340	2
59	.6558395	.868776	1.151044	.7549004	1	59	.6689144	.899877	1.111262	.7433394	1
60	.6560590	.869286	1.150368	.7547096	0	60	.6691306	.900404	1.110612	.7431448	0
	Cosine.	Cotang.	Tang.	Sine.	′		Cosine.	Cotang.	Tang.	Sine.	
					49°						

Note.—Secant = 1 + cosine. Cosecant = 1 + sine.

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3.—Natural Sines, Tangents, Cotangents, Cosines.—(Continued.)

(Versed sine = 1 − cosine; coversed sine = 1 − sine.)

′	Sine.	Tang.	Cotang.	Cosine.		′	Sine.	Tang.	Cotang.	Cosine.	
	42°						43°				
0	.6691206	.900404	1.110612	.7431448	60	0	.6819984	.932515	1.072368	.7313537	60
1	.6698468	.900930	1.109963	.7429502	59	1	.6832111	.933059	1.071743	.7311553	59
2	.6705728	.901456	1.109314	.7427554	58	2	.6824237	.933603	1.071118	.7309568	58
3	.6712989	.901985	1.108665	.7425606	57	3	.6826363	.934147	1.070494	.7307583	57
4	.6719948	.902513	1.108017	.7423658	56	4	.6828489	.934692	1.069870	.7305597	56
5	.6726208	.903041	1.107369	.7421708	55	5	.6830613	.935238	1.069246	.7303610	55
6	.6734266	.903569	1.106721	.7419758	54	6	.6832738	.935783	1.068623	.7301623	54
7	.6736424	.904097	1.106075	.7417808	53	7	.6834861	.936329	1.068000	.7299635	53
8	.6738582	.904626	1.105428	.7415857	52	8	.6836984	.936875	1.067377	.7297646	52
9	.6740739	.905155	1.104782	.7413905	51	9	.6839107	.937421	1.066755	.7295657	51
10	.6742895	.905685	1.104136	.7411953	50	10	.6841229	.937968	1.066134	.7293668	50
11	.6745051	.906214	1.103491	.7410000	49	11	.6843350	.938515	1.065512	.7291677	49
12	.6747206	.906744	1.102846	.7408046	48	12	.6845471	.939062	1.064891	.7289686	48
13	.6749361	.907274	1.102201	.7406092	47	13	.6847591	.939610	1.064271	.7287695	47
14	.6751515	.907805	1.101557	.7404137	46	14	.6849711	.940157	1.063651	.7285703	46
15	.6752668	.908336	1.100914	.7402181	45	15	.6851830	.940706	1.063031	.7283710	45
16	.6755821	.908867	1.100270	.7400225	44	16	.6853948	.941254	1.062411	.7281716	44
17	.6757973	.909398	1.099628	.7398268	43	17	.6856066	.941803	1.061792	.7279722	43
18	.6760125	.909930	1.098985	.7396311	42	18	.6858184	.942352	1.061174	.7277728	42
19	.6762276	.910461	1.098343	.7394353	41	19	.6860300	.942901	1.060556	.7275732	41
20	.6764427	.910994	1.097702	.7392394	40	20	.6862416	.943451	1.059938	.7273736	40
21	.6766577	.911526	1.097060	.7390435	39	21	.6864532	.944001	1.059320	.7271740	39
22	.6768727	.912059	1.096420	.7388475	38	22	.6866647	.944551	1.058703	.7269743	38
23	.6770876	.912592	1.095779	.7386515	37	23	.6868761	.945102	1.058086	.7267745	37
24	.6773024	.913125	1.095139	.7384553	36	24	.6870875	.945653	1.057470	.7265747	36
25	.6775173	.913659	1.094500	.7382592	35	25	.6872988	.946204	1.056854	.7263748	35
26	.6777319	.914192	1.093861	.7380629	34	26	.6875101	.946755	1.056238	.7261748	34
27	.6779466	.914727	1.093222	.7378666	33	27	.6877213	.947307	1.055623	.7259748	33
28	.6781612	.915261	1.092584	.7376703	32	28	.6879325	.947859	1.055008	.7257747	32
29	.6783757	.915796	1.091946	.7374738	31	29	.6881435	.948411	1.054394	.7255746	31
30	.6785903	.916331	1.091308	.7372773	30	30	.6883546	.948964	1.053780	.7253744	30
31	.6788046	.916866	1.090671	.7370808	29	31	.6885655	.949517	1.053166	.7251741	29
32	.6790190	.917402	1.090034	.7368842	28	32	.6887765	.950070	1.052553	.7249738	28
33	.6792333	.917937	1.089398	.7366875	27	33	.6889873	.950624	1.051940	.7247734	27
34	.6794476	.918474	1.088762	.7364908	26	34	.6891981	.951178	1.051327	.7245729	26
35	.6796618	.919010	1.088126	.7362940	25	35	.6894089	.951732	1.050715	.7243724	25
36	.6798760	.919547	1.087491	.7360971	24	36	.6896195	.952287	1.050103	.7241719	24
37	.6770901	.920084	1.086857	.7359002	23	37	.6898302	.952842	1.049492	.7239712	23
38	.6773041	.920621	1.086223	.7357033	22	38	.6900407	.953397	1.048880	.7237705	22
39	.6775181	.921159	1.085588	.7355061	21	39	.6902512	.953952	1.048270	.7235698	21
40	.6777320	.921696	1.084955	.7353090	20	40	.6904617	.954508	1.047659	.7233690	20
41	.6779459	.922235	1.084322	.7351118	19	41	.6906721	.955064	1.047049	.7231681	19
42	.6781597	.922773	1.083689	.7349146	18	42	.6908824	.955620	1.046440	.7229671	18
43	.6783734	.923312	1.083057	.7347173	17	43	.6910927	.956177	1.045831	.7227661	17
44	.6785871	.923851	1.082425	.7345199	16	44	.6913029	.956734	1.045222	.7225651	16
45	.6788007	.924391	1.081793	.7343225	15	45	.6915131	.957291	1.044613	.7223640	15
46	.6790143	.924930	1.081162	.7341250	14	46	.6917232	.957849	1.044005	.7221628	14
47	.6792278	.925470	1.080532	.7339275	13	47	.6919332	.958407	1.043397	.7219615	13
48	.6794413	.926010	1.079901	.7337299	12	48	.6921432	.958965	1.042790	.7217602	12
49	.6796547	.926550	1.079271	.7335322	11	49	.6923531	.959524	1.042183	.7215589	11
50	.6798681	.927091	1.078642	.7333345	10	50	.6925630	.960082	1.041576	.7213574	10
51	.6800813	.927632	1.078013	.7331367	9	51	.6927728	.960642	1.040970	.7211559	9
52	.6802946	.928173	1.077384	.7329388	8	52	.6929825	.961201	1.040364	.7209544	8
53	.6805078	.928715	1.076756	.7327409	7	53	.6931922	.961761	1.039758	.7207528	7
54	.6807209	.929257	1.076128	.7325429	6	54	.6934018	.962321	1.039153	.7205511	6
55	.6809339	.929799	1.075500	.7323449	5	55	.6936114	.962881	1.038548	.7203494	5
56	.6811469	.930342	1.074873	.7321467	4	56	.6938209	.963442	1.037944	.7201476	4
57	.6813599	.930884	1.074246	.7319486	3	57	.6940304	.964003	1.037340	.7199457	3
58	.6815728	.931428	1.073620	.7317503	2	58	.6942398	.964565	1.036736	.7197438	2
59	.6817856	.931971	1.072994	.7315521	1	59	.6944491	.965126	1.036133	.7195418	1
60	.6819984	.932515	1.072368	.7313537	0	60	.6946584	.965688	1.035530	.7193398	0
	Cosine.	Cotang	Tang.	Sine.	′		Cosine.	Cotang	Tang.	Sine.	′
					47°						46°

Note.—Secant = 1 + cosine. Cosecant = 1 + sine.

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3.—Natural Sines, Tangents, Cotangents, Cosines.—(Concluded.)

(Versed sine = 1 − cosine; coversed sine = 1 − sine.)

44° 44°

′	Sine.	Tang.	Cotang.	Cosine.		′	Sine.	Tang.	Cotang.	Cosine.	
0	.6946584	.965688	1.035530	.7193398	60	30	.7009093	.982697	1.017607	.7132504	30
1	.6948676	.966251	1.034927	.7191377	59	31	.7011167	.983269	1.017015	.7130465	29
2	.6950767	.966813	1.034325	.7189355	58	32	.7013241	.983841	1.016423	.7128426	28
3	.6952858	.967376	1.033723	.7187333	57	33	.7015314	.984414	1.015832	.7126385	27
4	.6954949	.967939	1.033122	.7185310	56	34	.7017387	.984987	1.015241	.7124344	26
5	.6957039	.968503	1.032520	.7183287	55	35	.7019459	.985560	1.014651	.7122303	25
6	.6959128	.969067	1.031919	.7181263	54	36	.7021531	.986133	1.014061	.7120260	24
7	.6961217	.969631	1.031319	.7179238	53	37	.7023601	.986707	1.013471	.7118218	23
8	.6963305	.970196	1.030719	.7177213	52	38	.7025672	.987282	1.012881	.7116174	22
9	.6965392	.970761	1.030119	.7175187	51	39	.7027741	.987856	1.012292	.7114130	21
10	.6967479	.971326	1.029520	.7173161	50	40	.7029811	.988431	1.011703	.7112086	20
11	.6969565	.971891	1.028921	.7171134	49	41	.7031879	.989006	1.011115	.7110041	19
12	.6971651	.972457	1.028322	.7169106	48	42	.7033947	.989582	1.010527	.7107995	18
13	.6973736	.973023	1.027724	.7167078	47	43	.7036014	.990158	1.009939	.7105948	17
14	.6975821	.973590	1.027126	.7165049	46	44	.7038081	.990734	1.009352	.7103901	16
15	.6977905	.974156	1.026528	.7163019	45	45	.7040147	.991311	1.008764	.7101854	15
16	.6979988	.974724	1.025931	.7160989	44	46	.7042213	.991888	1.008178	.7099806	14
17	.6982071	.975291	1.025334	.7158959	43	47	.7044278	.992465	1.007591	.7097757	13
18	.6984153	.975859	1.024738	.7156927	42	48	.7046342	.993042	1.007005	.7095707	12
19	.6986234	.976427	1.024141	.7154895	41	49	.7048406	.993620	1.006420	.7093657	11
20	.6988315	.976995	1.023546	.7152863	40	50	.7050469	.994199	1.005834	.7091607	10
21	.6990396	.977564	1.022950	.7150830	39	51	.7052532	.994777	1.005249	.7089556	9
22	.6992476	.978133	1.022355	.7148796	38	52	.7054594	.995356	1.004665	.7087504	8
23	.6994555	.978702	1.021760	.7146762	37	53	.7056655	.995935	1.004080	.7085451	7
24	.6996633	.979272	1.021166	.7144727	36	54	.7058716	.996515	1.003496	.7083398	6
25	.6998711	.979842	1.020572	.7142691	35	55	.7060776	.997095	1.002913	.7081345	5
26	.7000789	.980412	1.019978	.7140655	34	56	.7062835	.997675	1.002329	.7079291	4
27	.7002866	.980983	1.019385	.7138618	33	57	.7064894	.998256	1.001746	.7077236	3
28	.7004942	.981554	1.018792	.7136581	32	58	.7066953	.998837	1.001164	.7075180	2
29	.7007018	.982125	1.018199	.7134543	31	59	.7069011	.999418	1.000581	.7073124	1
30	.7009093	.982697	1.017607	.7132504	30	60	.7071068	1.00000	1.000000	.7071068	0
	Cosine.	Cotang	Tang.	Sine.	′		Cosine.	Cotang	Tang.	Sine.	′

45° 45°

Note.—Secant = 1 + cosine. Cosecant = 1 + sine.

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4.—Natural Secants, Cosecants, (Exsecants, Coexsecants).*

Secants.

′	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	
0	1.00000	1.00015	1.00061	1.00137	1.00244	1.00382	1.00551	1.00751	1.00983	1.01247	60
1	.00000	.00016	.00062	.00139	.00246	.00385	.00554	.00755	.00987	.01251	59
2	.00000	.00016	.00063	.00140	.00248	.00387	.00557	.00758	.00991	.01256	58
3	.00000	.00017	.00064	.00142	.00250	.00390	.00560	.00762	.00995	.01261	57
4	.00000	.00017	.00065	.00143	.00252	.00392	.00563	.00765	.00999	.01265	56
5	1.00000	1.00018	1.00066	1.00145	1.00254	1.00395	1.00566	1.00769	1.01004	1.01270	55
6	.00000	.00018	.00067	.00147	.00257	.00397	.00569	.00773	.01008	.01275	54
7	.00000	.00019	.00068	.00148	.00259	.00400	.00573	.00776	.01012	.01279	53
8	.00000	.00020	.00069	.00150	.00261	.00403	.00576	.00780	.01016	.01284	52
9	.00000	.00020	.00070	.00151	.00263	.00405	.00579	.00784	.01020	.01289	51
10	1.00000	1.00021	1.00072	1.00153	1.00265	1.00408	1.00582	1.00787	1.01024	1.01294	50
11	.00001	.00021	.00073	.00155	.00267	.00411	.00585	.00791	.01029	.01298	49
12	.00001	.00022	.00074	.00156	.00269	.00413	.00588	.00795	.01033	.01303	48
13	.00001	.00023	.00075	.00158	.00271	.00416	.00592	.00799	.01037	.01308	47
14	.00001	.00023	.00076	.00159	.00274	.00419	.00595	.00802	.01041	.01313	46
15	1.00001	1.00024	1.00077	1.00161	1.00276	1.00421	1.00598	1.00806	1.01046	1.01318	45
16	.00001	.00024	.00078	.00163	.00278	.00424	.00601	.00810	.01050	.01322	44
17	.00001	.00025	.00079	.00164	.00280	.00427	.00604	.00813	.01054	.01327	43
18	.00001	.00026	.00081	.00166	.00282	.00429	.00608	.00817	.01059	.01332	42
19	.00002	.00026	.00082	.00168	.00284	.00432	.00611	.00821	.01063	.01337	41
20	1.00002	1.00027	1.00083	1.00169	1.00287	1.00435	1.00614	1.00825	1.01067	1.01342	40
21	.00002	.00028	.00084	.00171	.00289	.00438	.00617	.00828	.01071	.01346	39
22	.00002	.00028	.00085	.00173	.00291	.00440	.00621	.00832	.01076	.01351	38
23	.00002	.00029	.00087	.00175	.00293	.00443	.00624	.00836	.01080	.01356	37
24	.00002	.00030	.00088	.00176	.00296	.00446	.00627	.00840	.01084	.01361	36
25	1.00003	1.00031	1.00089	1.00178	1.00298	1.00449	1.00630	1.00844	1.01089	1.01366	35
26	.00003	.00031	.00090	.00180	.00300	.00451	.00634	.00848	.01093	.01371	34
27	.00003	.00032	.00091	.00182	.00302	.00454	.00637	.00851	.01097	.01376	33
28	.00003	.00033	.00093	.00183	.00305	.00457	.00640	.00855	.01102	.01381	32
29	.00004	.00034	.00094	.00185	.00307	.00460	.00644	.00859	.01106	.01386	31
30	1.00004	1.00034	1.00095	1.00187	1.00309	1.00463	1.00647	1.00863	1.01111	1.01391	30
31	.00004	.00035	.00097	.00189	.00312	.00465	.00650	.00867	.01115	.01395	29
32	.00004	.00036	.00098	.00190	.00314	.00468	.00654	.00871	.01119	.01400	28
33	.00005	.00037	.00099	.00192	.00316	.00471	.00657	.00875	.01124	.01405	27
34	.00005	.00037	.00100	.00194	.00318	.00474	.00660	.00878	.01128	.01410	26
35	1.00005	1.00038	1.00102	1.00196	1.00321	1.00477	1.00664	1.00882	1.01133	1.01415	25
36	.00005	.00039	.00103	.00198	.00323	.00480	.00667	.00886	.01137	.01420	24
37	.00006	.00040	.00104	.00200	.00326	.00482	.00671	.00890	.01142	.01425	23
38	.00006	.00041	.00106	.00201	.00328	.00485	.00674	.00894	.01146	.01430	22
39	.00006	.00041	.00107	.00203	.00330	.00488	.00677	.00898	.01151	.01435	21
40	1.00007	1.00042	1.00108	1.00205	1.00333	1.00491	1.00681	1.00902	1.01155	1.01440	20
41	.00007	.00043	.00110	.00207	.00335	.00494	.00684	.00906	.01160	.01445	19
42	.00007	.00044	.00111	.00209	.00337	.00497	.00688	.00910	.01164	.01450	18
43	.00008	.00045	.00113	.00211	.00340	.00500	.00691	.00914	.01169	.01455	17
44	.00008	.00046	.00114	.00213	.00342	.00503	.00695	.00918	.01173	.01461	16
45	1.00009	1.00047	1.00115	1.00215	1.00345	1.00506	1.00698	1.00922	1.01178	1.01466	15
46	.00009	.00048	.00117	.00216	.00347	.00509	.00701	.00926	.01182	.01471	14
47	.00009	.00048	.00118	.00218	.00350	.00512	.00705	.00930	.01187	.01476	13
48	.00010	.00049	.00120	.00220	.00352	.00515	.00708	.00934	.01191	.01481	12
49	.00010	.00050	.00121	.00222	.00354	.00518	.00712	.00938	.01196	.01486	11
50	1.00011	1.00051	1.00122	1.00224	1.00357	1.00521	1.00715	1.00942	1.01200	1.01491	10
51	.00011	.00052	.00124	.00226	.00359	.00524	.00719	.00946	.01205	.01496	9
52	.00011	.00053	.00125	.00228	.00362	.00527	.00722	.00950	.01209	.01501	8
53	.00012	.00054	.00127	.00230	.00364	.00530	.00726	.00954	.01214	.01506	7
54	.00012	.00055	.00128	.00232	.00367	.00533	.00730	.00958	.01219	.01512	6
55	1.00013	1.00056	1.00130	1.00234	1.00369	1.00536	1.00733	1.00962	1.01223	1.01517	5
56	.00013	.00057	.00131	.00236	.00372	.00539	.00737	.00966	.01228	.01522	4
57	.00014	.00058	.00133	.00238	.00374	.00542	.00740	.00970	.01233	.01527	3
58	.00014	.00059	.00134	.00240	.00377	.00545	.00744	.00975	.01237	.01532	2
59	.00015	.00060	.00136	.00242	.00379	.00548	.00747	.00979	.01242	.01537	1
60	1.00015	1.00061	1.00137	1.00244	1.00382	1.00551	1.00751	1.00983	1.01247	1.01543	0
	89°	88°	87°	86°	85°	84°	83°	82°	81°	80°	′

Cosecants.

* Exsecant = secant—1; coexsecant = cosecant − 1.

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4.—Natural Secants, Cosecants (Exsecants, Coexsecants).*—(Cont'd.)

Secants.

′	10°	11°	12°	13°	14°	15°	16°	17°	18°	19°	
0	1.01543	1.01872	1.02234	1.02630	1.03061	1.03528	1.04030	1.04569	1.05146	1.05762	6
1	.01548	.01877	.02240	.02637	.03069	.03536	.04039	.04578	.05156	.05773	5
2	.01553	.01883	.02247	.02644	.03076	.03544	.04047	.04588	.05166	.05783	5
3	.01558	.01889	.02253	.02651	.03084	.03552	.04056	.04597	.05176	.05794	5
4	.01564	.01895	.02259	.02658	.03091	.03560	.04065	.04606	.05186	.05805	5
5	1.01569	1.01901	1.02266	1.02665	1.03099	1.03568	1.04073	1.04616	1.05196	1.05815	5
6	.01574	.01906	.02272	.02672	.03106	.03576	.04082	.04625	.05206	.05826	5
7	.01579	.01912	.02279	.02679	.03114	.03584	.04091	.04635	.05216	.05836	5
8	.01585	.01918	.02285	.02686	.03121	.03592	.04100	.04644	.05226	.05847	5
9	.01590	.01924	.02291	.02693	.03129	.03601	.04108	.04653	.05236	.05858	5
10	1.01595	1.01930	1.02298	1.02700	1.03137	1.03609	1.04117	1.04663	1.05246	1.05869	5
11	.01601	.01936	.02304	.02707	.03144	.03617	.04126	.04672	.05256	.05879	4
12	.01606	.01941	.02311	.02714	.01352	.03625	.04135	.04682	.05266	.05890	4
13	.01611	.01947	.02317	.02721	.03159	.03633	.04144	.04691	.05276	.05901	4
14	.01616	.01953	.02323	.02728	.03167	.03642	.04152	.04700	.05286	.05911	4
15	1.01622	1.01959	1.02330	1.02735	1.03175	1.03650	1.04161	1.04710	1.05297	1.05922	4
16	.01627	.01965	.02336	.02742	.03182	.03658	.04170	.04719	.05307	.05933	4
17	.01633	.01971	.02343	.02749	.03190	.03666	.04179	.04729	.05317	.05944	4
18	.01638	.01977	.02349	.02756	.03198	.03674	.04188	.04738	.05327	.05955	4
19	.01643	.01983	.02356	.02763	.03205	.03683	.04197	.04748	.05337	.05965	4
20	1.01649	1.01989	1.02362	1.02770	1.03213	1.03691	1.04206	1.04757	1.05347	1.05976	4
21	.01654	.01995	.02369	.02777	.03221	.03699	.04214	.04767	.05357	.05987	3
22	.01659	.02001	.02375	.02784	.03228	.03708	.04223	.04776	.05367	.05998	3
23	.01665	.02007	.02382	.02791	.03236	.03716	.04232	.04786	.05378	.06009	3
24	.01670	.02013	.02388	.02799	.03244	.03724	.04241	.04795	.05388	.06020	3
25	1.01676	1.02019	1.02395	1.02806	1.03251	1.03732	1.04250	1.04805	1.05398	1.06030	3
26	.01681	.02025	.02402	.02813	.03259	.03741	.04259	.04815	.05408	.06041	3
27	.01687	.02031	.02408	.02830	.03267	.03749	.04268	.04824	.05418	.06052	3
28	.01692	.02037	.02415	.02827	.03275	.03758	.04277	.04834	.05429	.06063	3
29	.01698	.02043	.02421	.02834	.03282	.03766	.04286	.04843	.05439	.06074	3
30	1.01703	1.02049	1.02428	1.02842	1.03290	1.03774	1.04295	1.04853	1.05449	1.06085	3
31	.01709	.02055	.02435	.02849	.03298	.03783	.04304	.04863	.05460	.06096	2
32	.01714	.02061	.02441	.02856	.03306	.03791	.04313	.04872	.05470	.06107	2
33	.01720	.02067	.02448	.02863	.03313	.03799	.04322	.04882	.05480	.06118	2
34	.01725	.02073	.02454	.02870	.03321	.03808	.04331	.04891	.05490	.06129	2
35	1.01731	1.02079	1.02461	1.02878	1.03329	1.03816	1.04340	1.04901	1.05501	1.06140	2
36	.01736	.02085	.02468	.02885	.03337	.03825	.04349	.04911	.05511	.06151	2
37	.01742	.02091	.02474	.02892	.03345	.03833	.04358	.04920	.05521	.06162	2
38	.01747	.02097	.02481	.02899	.03353	.03842	.04367	.04930	.05532	.06173	2
39	.01753	.02103	.02488	.02907	.03360	.03850	.04376	.04940	.05542	.06184	2
40	1.01758	1.02110	1.02494	1.02914	1.03368	1.03858	1.04385	1.04950	1.05552	1.06195	2
41	.01764	.02116	.02501	.02921	.03376	.03867	.04394	.04959	.05563	.06206	1
42	.01769	.02122	.02508	.02928	.03384	.02875	.04403	.04969	.05573	.06217	1
43	.01775	.02128	.02515	.02936	.03392	.03884	.04413	.04979	.05584	.06228	1
44	.01781	.02134	.02521	.02943	.03400	.03892	.04422	.04989	.05594	.06239	1
45	1.01786	1.02140	1.02528	1.02950	1.03408	1.03901	1.04431	1.04998	1.05604	1.06250	1
46	.01792	.02146	.02535	.02958	.03416	.03909	.04440	.05008	.05615	.06261	1
47	.01798	.02153	.02542	.02965	.03424	.03918	.04449	.05018	.05625	.06272	1
48	.01803	.02159	.02548	.02972	.03432	.03927	.04458	.05028	.05636	.06283	1
49	.01809	.02165	.02555	.02980	.03439	.03935	.04468	.05038	.05646	.06295	1
50	1.01815	1.02171	1.02562	1.02987	1.03447	1.03944	1.04477	1.05047	1.05657	1.06306	1
51	.01820	.02178	.02569	.02994	.03455	.03952	.04486	.05057	.05667	.06317	
52	.01826	.02184	.02576	.03002	.03463	.03961	.04495	.05067	.05678	.06328	
53	.01832	.02190	.02582	.03009	.03471	.03969	.04504	.05077	.05688	.06339	
54	.01837	.02196	.02589	.03017	.03479	.03978	.04514	.05087	.05699	.06350	
55	1.01843	.02203	1.02596	1.03024	1.03487	1.03987	1.04523	1.05097	1.05709	1.06362	
56	.01849	.02209	.02603	.03032	.03495	.03995	.04532	.05107	.05720	.06373	
57	.01854	.02215	.02610	.03039	.03503	.04004	.04541	.05116	.05730	.06384	
58	.01860	.02221	.02617	.03046	.03512	.04013	.04551	.05126	.05741	.06395	
59	.01866	.02228	.02624	.03054	.03520	.04021	.04560	.05136	.05751	.06407	
60	1.01872	1.02234	1.02630	1.03061	1.03528	1.04030	1.04569	1.05146	1.05762	1.06418	
	79°	78°	77°	76°	75°	74°	73°	72°	71°	70°	

Cosecants.

* Exsecant = secant − 1; coexsecant = cosecant − 1.

Google

4.—Natural Secants, Cosecants (Exsecants, Coexsecants).*—(Cont'd.)

Secants.

′	20°	21°	22°	23°	24°	25°	26°	27°	28°	29°	
0	1.06418	1.07115	1.07853	1.08636	1.09464	1.10338	1.11260	1.12233	1.13257	1.14335	60
1	.06429	.07126	.07866	.08649	.09478	.10353	.11276	.12249	.13275	.14354	59
2	.06440	.07138	.07879	.08663	.09492	.10368	.11292	.12266	.13292	.14372	58
3	.06452	.07150	.07892	.08676	.09506	.10383	.11308	.12283	.13310	.14391	57
4	.06463	.07162	.07904	.08690	.09520	.10398	.11323	.12299	.13327	.14409	56
5	1.06474	1.07174	.07917	1.08703	1.09535	1.10413	1.11339	1.12316	1.13345	1.14428	55
6	.06486	.07186	.07930	.08717	.09549	.10428	.11355	.12333	.13362	.14446	54
7	.06497	.07199	.07943	.08730	.09563	.10443	.11371	.12349	.13380	.14465	53
8	.06508	.07211	.07955	.08744	.09577	.10458	.11387	.12366	.13398	.14483	52
9	.06520	.07223	.07968	.08757	.09592	.10473	.11403	.12383	.13415	.14502	51
10	1.06531	1.07235	1.07981	1.08771	1.09606	1.10488	1.11419	1.12400	1.13433	1.14521	50
11	.06542	.07247	.07994	.08784	.09620	.10503	.11435	.12416	.13451	.14539	49
12	.06554	.07259	.08006	.08798	.09635	.10518	.11451	.12433	.13468	.14558	48
13	.06565	.07271	.08019	.08811	.09649	.10533	.11467	.12450	.13486	.14576	47
14	.06577	.07283	.08032	.08825	.09663	.10549	.11483	.12467	13504	.14595	46
15	1.06588	1.07295	1.08045	1.08839	1.09678	1.10564	1.11499	1.12484	1.13521	1.14614	45
16	.06600	.07307	.08058	.08852	.09692	.10579	.11515	.12501	.13539	.14632	44
17	.06611	.07320	.08071	.08866	.09707	.10594	.11531	.12518	.13557	.14651	43
18	.06622	.07332	.08084	.08880	.09721	.10609	.11547	.12534	.13575	.14670	42
19	.06634	.07344	.08097	.08893	.09735	.10625	.11563	.12551	.13593	.14689	41
20	1.06645	1.07356	1.08109	1.08907	1.09750	1.10640	1.11579	1.12568	1.13610	1.14707	40
21	.06657	.07368	.08122	.08921	.09764	.10655	.11595	.12585	.13628	.14726	39
22	.06668	.07380	.08135	.08934	.09779	.10670	.11611	.12602	.13646	.14745	38
23	.06680	.07393	.08148	.08948	.09793	.10686	.11627	.12619	.13664	.14764	37
24	.06691	.07405	.08161	.08962	.09808	.10701	.11643	.12636	.13682	.14782	36
25	1.06703	1.07417	1.08174	1.08975	1.09822	1.10716	1.11659	1.12653	1.13700	1.14801	35
26	.06715	.07429	.08187	.08989	.09837	.10731	.11675	.12670	.13718	.14820	34
27	.06726	.07442	.08200	.09003	.09851	.10747	.11691	.12687	.13735	.14839	33
28	.06738	.07454	.08213	.09017	.09866	.10762	.11708	.12704	.13753	.14858	32
29	.06749	.07466	.08226	.09030	.09880	.10777	.11724	.12721	.13771	.14877	31
30	1.06761	1.17479	1.08239	1.09044	1.09895	1.10793	1.11740	1.12738	1.13789	1.14896	30
31	.06773	.07491	.08252	.09058	.09909	.10808	.11756	.12755	.13807	.14914	29
32	.06784	.07503	.08265	.09072	.09924	.10824	.11772	.12772	.13825	.14933	28
33	.06796	.07516	.08278	.09086	.09939	.10839	.11789	.12789	.13843	.14952	27
34	.06807	.07528	.08291	.09099	.09953	.10854	.11805	.12807	.13861	.14971	26
35	1.06819	1.07540	1.08305	1.09113	1.09968	1.10870	1.11821	1.12824	1.13879	1.14990	25
36	.06831	.07553	.08318	.09127	.09982	.10885	.11838	.12841	.13897	.15009	24
37	.06843	.07565	.08331	.09141	.09997	.10901	.11854	.12858	.13916	.15028	23
38	.06854	.07578	.08344	.09155	.10012	.10916	.11870	.12875	.13934	.15047	22
39	.06866	.07590	.08357	.09169	.10026	.10932	.11886	.12892	.13952	.15066	21
40	1.06878	1.07602	1.08370	1.09183	1.10041	1.10947	1.11903	1.12910	1.13970	1.15085	20
41	.06889	.07615	.08383	.01197	.10055	.10963	.11919	.12927	.13988	.15105	19
42	.06901	.07627	.08397	.09211	.10071	.10978	.11936	.12944	.14006	.15124	18
43	.06913	.07640	.08410	.09224	.10085	.10994	.11952	.12961	.14024	.15143	17
44	.06925	.07652	.08423	.09238	.10100	.11009	.11968	.12979	.14042	.15162	16
45	1.06936	1.07665	1.08436	1.09252	1.10115	1.11025	1.11985	1.12996	1.14061	1.15181	15
46	.06948	.07677	.08449	.09266	.10130	.11041	.12001	.13013	.14079	.15200	14
47	.06960	.07690	.08463	.09280	.10144	.11056	.12018	.13031	.14097	.15219	13
48	.06972	.07702	.08476	.09294	.10159	.11072	.12034	.13048	.14115	.15239	12
49	.06984	.07715	.08489	.09308	.10174	.11087	.12051	.13065	.14134	.15258	11
50	1.06995	1.07727	1.08503	1.09323	1.10189	1.11103	1.12067	1.13083	1.14152	1.15277	10
51	.07007	.07740	.08516	.09337	.10204	.11119	.12084	.13100	.14170	.15296	9
52	.07019	.07752	.08529	.09351	.10218	.11134	.12100	.13117	.14188	.15315	8
53	.07031	.07765	.08542	.09365	.10233	.11150	.12117	.13135	.14207	.15335	7
54	.07043	.07778	.08556	.09379	.10248	.11166	.12133	.13152	.14225	.15354	6
55	1.07055	1.07790	1.08569	1.09393	1.10263	1.11181	1.12150	1.13170	1.14243	1.15373	5
56	.07067	.07803	.08582	.09407	.10278	.11197	.12166	.13187	.14262	.15393	4
57	.07079	.07816	.08596	.09421	.10293	.11213	.12183	.13205	.14280	.15412	3
58	.07091	.07828	.08609	.09435	.10308	.11229	.12199	.13222	.14299	.15431	2
59	.07103	.07841	.08623	.09449	.10323	.11244	.12216	.13240	.14317	.15451	1
60	1.07115	1.07853	1.08636	1.09464	1.00338	1.11260	1.12233	1.13257	1.14335	1.15470	0
	69°	68°	67°	66°	65°	64°	63°	62°	61°	60°	′

Cosecants.

* Exsecant = secant − 1; coexsecant = cosecant − 1.

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4.—Natural Secants, Cosecants (Exsecants, Coexsecants).*—(Cont'd.)

Secants.

′	30°	31°	32°	33°	34°	35°	36°	37°	38°	39°	
0	1.15470	1.16663	1.17918	1.19236	1.20622	1.22077	1.23607	1.25214	1.26902	1.28676	64
1	.15489	.16684	.17939	.19259	.20645	.22102	.23633	.25241	.26931	.28706	5
2	.15509	.16704	.17961	.19281	.20669	.22127	.23659	.25269	.26960	.28737	5
3	.15528	.16725	.17982	.19304	.20693	.22152	.23685	.25296	.26988	.28767	5
4	.15548	.16745	.18004	.19327	.20717	.22177	.23711	.25324	.27017	.28797	5
5	1.15567	1.16766	1.18025	1.19349	1.20740	1.22202	1.23738	1.25351	1.27046	1.28828	5
6	.15587	.16786	.18047	.19372	.20764	.22227	.23764	.25379	.27075	.28858	5
7	.15606	.16806	.18068	.19394	.20788	.22252	.23790	.25406	.27104	.28889	5
8	.15626	.16827	.18090	.19417	.20812	.22277	.23816	.25434	.27133	.28919	5
9	.15645	.16848	.18111	.19440	.20836	.22302	.23843	.25462	.27162	.28950	5
10	1.15665	1.16868	1.18133	1.19463	1.20859	1.22327	1.23869	1.25489	1.27191	1.28980	5
11	.15684	.16889	.18155	.19485	.20883	.22352	.23895	.25517	.27221	.29011	4
12	.15704	.16909	.18176	.19508	.20907	.22377	.23922	.25545	.27250	.29042	4
13	.15724	.16930	.18198	.19531	.22931	.23402	.25948	.25572	.27279	.29072	4
14	.15743	.16950	.18220	.19554	.20955	.22428	.23975	.25600	.27308	.29103	4
15	1.15763	1.16971	1.18241	1.19576	1.20979	1.22453	1.24001	1.25628	1.27337	1.29133	4
16	.15782	.16992	.18263	.19599	.21003	.22478	.24028	.25656	.27366	.29164	4
17	.15802	.17012	.18285	.19622	.21027	.22503	.24054	.25683	.27396	.29195	4
18	.15822	.17033	.18307	.19645	.21051	.22528	.24081	.25711	.27425	.29226	4
19	.15841	.17054	.18328	.19668	.21075	.22554	.24107	.25739	.27454	.29256	4
20	1.15861	1.17075	1.18350	1.19691	1.21099	1.22579	1.24134	1.25767	1.27483	1.29287	4
21	.15881	.17095	.18372	.19713	.21123	.22604	.24160	.25795	.27513	.29318	3
22	.15901	.17116	.18394	.19736	.21147	.22629	.24187	.25823	.27542	.29349	3
23	.15920	.17137	.18416	.19759	.21171	.22655	.24213	.25851	.27572	.29380	3
24	.15940	.17158	.18437	.19782	.21195	.22680	.24240	.25879	.27601	.29411	3
25	1.15960	1.17178	1.18459	1.19805	1.21220	1.22706	1.24267	1.25907	1.27630	1.29442	3
26	.15980	.17199	.18481	.19828	.21244	.22731	.24293	.25935	.27660	.29473	3
27	.16000	.17220	.18503	.19851	.21268	.22756	.24320	.25963	.27689	.29504	3
28	.16019	.17241	.18525	.19874	.21292	.22782	.24347	.25991	.27719	.29535	3
29	.16039	.17262	.18547	.19897	.21316	.22807	.24373	.26019	.27748	.29566	3
30	1.16059	1.17283	1.18569	1.19920	1.21341	1.22833	1.24400	1.26047	1.27778	1.29597	3
31	.16079	.17304	.18591	.19944	.21365	.22858	.24427	.26075	.27807	.29628	2
32	.16099	.17325	.18613	.19967	.21389	.22884	.24454	.26104	.27837	.29659	2
33	.16119	.17346	.18635	.19990	.21414	.22909	.24481	.26132	.27867	.29690	2
34	.16139	.17367	.18657	.20013	.21438	.22935	.24508	.26160	.27896	.29721	2
35	1.16159	1.17388	1.18679	1.20036	1.21462	1.22960	1.24534	1.26188	1.27926	1.29752	2
36	.16179	.17409	.18701	.20059	.21487	.22986	.24561	.26216	.27956	.29784	2
37	.16199	.17430	.18723	.20083	.21511	.23012	.24588	.26245	.27985	.29815	2
38	.16219	.17451	.18745	.20106	.21535	.23037	.24615	.26273	.28015	.29846	2
39	.16239	.17472	.18767	.20129	.21560	.23063	.24642	.26301	.28045	.29877	2
40	1.16259	1.17493	1.18790	1.20152	1.21584	1.23089	1.24669	1.26330	1.28075	1.29909	2
41	.16279	.17514	.18812	.20176	.21609	.23114	.24696	.26358	.28105	.29940	
42	.06299	.17535	.18834	.20199	.21633	.23140	.24723	.26387	.28134	.29971	
43	.16319	.17556	.18856	.20222	.21658	.23166	.24750	.26415	.28164	.30003	
44	.16339	.17577	.18878	.20246	.21682	.23192	.24777	.26443	.28194	.30034	
45	1.16359	1.17598	1.18901	1.20269	1.21707	1.23217	1.24804	1.26472	1.28224	1.30066	
46	.16380	.17620	.18923	.20292	.21731	.23243	.24832	.26500	.28254	.30097	
47	.16400	.17641	.18945	.20316	.21756	.23269	.24859	.26529	.28284	.30129	
48	.16420	.17662	.18967	.20339	.21781	.23295	.24886	.26557	.28314	.30160	
49	.16440	.17683	.18990	.20363	.21805	.23321	.24913	.26586	.28344	.30192	
50	1.16460	1.17704	1.19012	1.20386	1.21830	1.23347	1.24940	1.26615	1.28374	1.30223	
51	.16481	.17726	.19034	.20410	.21855	.23373	.24967	.26643	.28404	.30255	
52	.16501	.17747	.19057	.20433	.21879	.23399	.24995	.26672	.28434	.30287	
53	.16521	.17768	.19079	.20457	.21904	.23424	.25022	.26701	.28464	.30318	
54	.16541	.17790	.19102	.20480	.21929	.23450	.25049	.26729	.28495	.30350	
55	1.16562	1.17811	1.19124	1.20504	1.21953	1.23476	1.25077	1.26758	1.28525	1.30382	
56	.16582	.17832	.19146	.20527	.21978	.23502	.25104	.26787	.28555	.30413	
57	.16602	.17854	.19169	.20551	.22003	.23529	.25131	.26815	.28585	.30445	
58	.16623	.17875	.19191	.20575	.22028	.23555	.25159	.26844	.28615	.30477	
59	.16643	.17896	.19214	.20598	.22053	.23581	.25186	.26873	.28646	.30509	
60	1.16663	1.17918	1.19236	1.20622	1.22077	1.23607	1.25214	1.26902	1.28676	1.30541	
	59°	58°	57°	56°	55°	54°	53°	52°	51°	50°	

Cosecants.

* Exsecant = secant − 1; coexsecant = cosecant − 1.

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4.—Natural Secants, Cosecants (Exsecants, Coexsecants).*—(Cont'd.)

Secants.

′	40°	41°	42°	43°	44°	45°	46°	47°	48°	49°	
0	1.30541	1.32501	1.34563	1.36733	1.39016	1.41421	1.43956	1.46628	1.49448	1.52425	60
1	.30573	.32535	.34599	.36770	.39055	.41463	.43999	.46674	.49496	.52476	59
2	.30605	.32568	.34634	.36807	.39095	.41504	.44042	.46719	.49544	.52527	58
3	.30636	.32602	.34669	.36844	.39134	.41545	.44086	.46765	.49593	.52579	57
4	.30668	.32636	.34704	.36881	.39173	.41586	.44129	.46811	.49641	.52630	56
5	1.30700	1.32669	1.34740	1.36919	1.39212	1.41627	1.44173	1.46857	1.49690	1.52681	55
6	.30732	.32703	.34775	.36956	.39251	.41669	.44217	.46903	.49738	.52732	54
7	.30764	.32737	.34811	.36993	.39291	.41710	.44260	.46949	.49787	.52784	53
8	.30796	.32770	.34846	.37030	.39330	.41752	.44304	.46995	.49835	.52835	52
9	.30829	.32804	.34882	.37068	.39369	.41793	.44347	.47041	.49884	.52886	51
10	1.30861	1.32838	1.34917	1.37105	1.39409	1.41835	1.44391	1.47087	1.49933	1.52938	50
11	.30893	.32872	.34953	.37143	.39448	.41876	.44435	.47134	.49981	.52989	49
12	.30925	.32905	.34988	.37180	.39487	.41918	.44479	.47180	.50030	.53041	48
13	.30957	.32939	.35024	.37218	.39527	.41959	.44523	.47226	.50079	.53092	47
14	.30989	.32973	.35060	.37255	.39566	.42001	.44567	.47272	.50128	.53144	46
15	1.31022	1.33007	1.35095	1.37293	1.39606	1.42042	1.44610	1.47319	1.50177	1.53196	45
16	.31054	.33041	.35131	.37330	.39646	.42084	.44654	.47365	.50226	.53247	44
17	.31086	.33075	.35167	.37368	.39685	.42126	.44698	.47411	.50275	.53299	43
18	.31119	.33109	.35203	.37406	.39725	.42168	.44742	.47458	.50324	.53351	42
19	.31151	.33143	.35238	.37443	.39764	.42210	.44787	.47504	.50373	.53403	41
20	1.31183	1.33177	1.35274	1.37481	1.39804	1.42251	1.44831	1.47551	1.50422	1.53455	40
21	.31216	.33211	.35310	.37519	.39844	.42293	.44875	.47598	.50471	.53507	39
22	.31248	.33245	.35346	.37556	.39884	.42335	.44919	.47644	.50521	.53559	38
23	.31281	.33279	.35382	.37594	.39924	.42377	.44963	.47691	.50570	.53611	37
24	.31313	.33314	.35418	.37632	.39963	.42419	.45007	.47738	.50619	.53663	36
25	1.31346	1.33348	1.35454	1.37670	1.40003	1.42461	1.45052	1.47784	1.50669	1.53715	35
26	.31378	.33382	.35490	.37708	.40043	.42503	.45096	.47831	.50718	.53768	34
27	.31411	.33416	.35526	.37746	.40083	.42545	.45141	.47878	.50767	.53820	33
28	.31443	.33451	.35562	.37784	.40123	.42587	.45185	.47925	.50817	.53872	32
29	.31476	.33485	.35598	.37822	.40163	.42630	.45229	.47972	.50866	.53924	31
30	1.31509	1.33519	1.35634	1.37860	1.40203	1.42672	1.45274	1.48019	1.50916	1.53977	30
31	.31541	.33554	.35670	.37898	.40243	.42714	.45319	.48066	.50966	.54029	29
32	.31574	.33588	.57307	.37936	.40283	.42756	.45363	.48113	.51015	.54082	28
33	.31607	.33622	.35743	.37974	.40324	.42799	.45408	.48160	.51065	.54134	27
34	.31640	.33657	.35779	.38012	.40364	.42841	.45452	.48207	.51115	.54187	26
35	1.31672	1.33691	1.35815	1.38051	1.40404	1.42883	1.45497	1.48254	1.51165	1.54240	25
36	.31705	.33726	.35852	.38089	.40444	.42926	.45542	.48301	.51215	.54292	24
37	.31738	.33760	.35888	.38127	.40485	.42968	.45587	.48349	.51265	.54345	23
38	.31771	.33795	.35924	.38165	.40525	.43011	.45631	.48396	.51314	.54398	22
39	.31804	.33830	.35961	.38204	.40565	.43053	.45676	.48443	.51364	.54451	21
40	1.31837	1.33864	1.35997	1.38242	1.40606	1.43096	1.45721	1.48491	1.51415	1.54504	20
41	.31870	.33899	.36034	.38280	.40646	.43139	.45766	.48538	.51465	.54557	19
42	.31903	.33934	.36070	.38319	.40687	.43181	.45811	.48586	.51515	.54610	18
43	.31936	.33968	.36107	.38357	.40727	.43224	.45856	.48633	.51565	.54663	17
44	.31969	.34003	.36143	.38396	.40768	.43267	.45901	.48681	.51615	.54716	16
45	1.32002	1.34038	1.36180	1.38434	1.40808	1.43310	1.45946	1.48728	1.51665	1.54769	15
46	.32035	.34073	.36217	.38473	.40849	.43352	.45992	.48776	.51716	.54822	14
47	.32068	.34108	.36253	.38512	.40890	.43395	.46037	.48824	.51766	.54876	13
48	.32101	.34142	.36290	.38550	.40930	.43438	.46082	.48871	.51817	.54929	12
49	.32134	.34177	.36327	.38589	.40971	.43481	.46127	.48919	.51867	.54982	11
50	1.32168	1.34212	1.36363	1.38628	1.41012	1.43524	1.46173	1.48967	1.51918	1.55036	10
51	.32201	.34247	.36400	.38666	.41053	.43567	.46218	.49015	.51968	.55089	9
52	.32234	.34282	.36437	.38705	.41093	.43610	.46263	.49063	.52019	.55143	8
53	.32267	.34317	.36474	.38744	.41134	.43653	.46309	.49111	.52069	.55196	7
54	.32301	.34352	.36511	.38783	.41175	.43696	.46354	.49159	.52120	.55250	6
55	1.32334	1.34387	1.36548	1.38822	1.41216	1.43739	1.46400	1.49207	1.52171	1.55303	5
56	.32368	.34423	.36585	.38860	.41257	.43783	.46445	.49255	.52222	.55357	4
57	.32401	.34458	.36622	.38899	.41298	.43826	.46491	.49303	.52273	.55411	3
58	.32434	.34493	.36659	.38938	.41339	.43869	.46537	.49351	.52323	.55465	2
59	.32468	.34528	.36696	.38977	.41380	.43912	.46582	.49399	.52374	.55518	1
60	1.32501	1.34563	1.36733	1.39016	1.41421	1.43956	1.46628	1.49448	1.52425	1.55572	0
	49°	48°	47°	46°	45°	44°	43°	42°	41°	40°	′

Cosecants.

* Exsecant = secant − 1; coexsecant = cosecant − 1.

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4.—Natural Secants, COSECANTS (EXSECANTS, COEXSECANTS).*—(Cont'd.)

Secants.

′	50°	51°	52°	53°	54°	55°	56°	57°	58°	59°
0	1.55572	1.58902	1.62427	1.66164	1.70130	1.74345	1.78829	1.83608	1.88708	1.94160
1	.55626	.58959	.62487	.66228	.70198	.74417	.78906	.83690	.88796	.94254
2	.55680	.59016	.62548	.66292	.70267	.74490	.78984	.83773	.88884	.94349
3	.55734	.59073	.62609	.66357	.70335	.74562	.79061	.83855	.88972	.94443
4	.55789	.59130	.62669	.66421	.70403	.74635	.79138	.83938	.89060	.94537
5	1.55843	1.59188	1.62730	1.66486	1.70472	1.74708	1.79216	1.84020	1.89148	1.94632
6	.55897	.59245	.62791	.66550	.70540	.74781	.79293	.84103	.89237	.94726
7	.55951	.59303	.62852	.66615	.70609	.74854	.79371	.84186	.89325	.94821
8	.56005	.59360	.62913	.66679	.70677	.74927	.79449	.84269	.89414	.94916
9	.56060	.59418	.62974	.66744	.70746	.75000	.79527	.84352	.89503	.95011
10	1.56114	1.59475	1.63035	1.66809	1.70815	1.75073	1.79604	1.84435	1.89591	1.95106
11	.56169	.59533	.63096	.66873	.70884	.75146	.79682	.84518	.89680	.95201
12	.56223	.59590	.63157	.66938	.70953	.75219	.79761	.84601	.89769	.95296
13	.56278	.59648	.63218	.67003	.71022	.75293	.79839	.84685	.89858	.95392
14	.56332	.59706	.63279	.67068	.71091	.75366	.79917	.84768	.89948	.95487
15	1.56387	1.59764	1.63341	1.67133	1.71160	1.75440	1.79995	1.84852	1.90037	1.95583
16	.56442	.59822	.63402	.67199	.71229	.75513	.80074	.84935	.90126	.95678
17	.56497	.59880	.63464	.67264	.71298	.75587	.80152	.85019	.90216	.95774
18	.56551	.59938	.63525	.67329	.71368	.75661	.80231	.85103	.90305	.95870
19	.56606	.59996	.63587	.67394	.71437	.75734	.80309	.85187	.90395	.95966
20	1.56661	1.60054	1.63648	1.67460	1.71506	1.75808	1.80388	1.85271	1.90485	1.96062
21	.56716	.60112	.63710	.67525	.71576	.75882	.80467	.85355	.90575	.96158
22	.56771	.60171	.63772	.67591	.71646	.75956	.80546	.85439	.90665	.96255
23	.56826	.60229	.63834	.67656	.71715	.76031	.80625	.85523	.90755	.96351
24	.56881	.60287	.63895	.67722	.71785	.76105	.80704	.85608	.90845	.96448
25	1.56937	1.60346	1.63957	1.67788	1.71855	1.76179	1.80783	1.85692	1.90935	1.96544
26	.56992	.60404	.64019	.67853	.71925	.76253	.80862	.85777	.91026	.96641
27	.57047	.60463	.64081	.67919	.71995	.76328	.80942	.85861	.91116	.96738
28	.57103	.60521	.64144	.67985	.72065	.76402	.81021	.85946	.91207	.96835
29	.57158	.60580	.64206	.68051	.72135	.76477	.81101	.86031	.91297	.96932
30	1.57213	1.60639	1.64268	1.68117	1.72205	1.76552	1.81180	1.86116	1.91388	1.97029
31	.57269	.60698	.64330	.68183	.72275	.76626	.81260	.86201	.91479	.97127
32	.57324	.60756	.64393	.68250	.72346	.76701	.81340	.86286	.91570	.97224
33	.57380	.60815	.64455	.68316	.72416	.76776	.81419	.86371	.91661	.97322
34	.57436	.60874	.64518	.68382	.72487	.76851	.81499	.86457	.91752	.97420
35	1.57491	1.60933	1.64580	1.68449	1.72557	1.76926	1.81579	1.86542	1.91844	1.97517
36	.57547	.60992	.64643	.68515	.72628	.77001	.81659	.86627	.91935	.97615
37	.57603	.61051	.64705	.68582	.72698	.77077	.81740	.86713	.92027	.97713
38	.57659	.61111	.64768	.68648	.72769	.77152	.81820	.86799	.92118	.97811
39	.57715	.61170	.64831	.68715	.72840	.77227	.81900	.86885	.92210	.97910
40	1.57771	1.61229	1.64894	1.68782	1.72911	1.77303	1.81981	1.86970	1.92302	1.98008
41	.57827	.61288	.64957	.68848	.72982	.77378	.82061	.87056	.92394	.98107
42	.57883	.61348	.65020	.68915	.73053	.77454	.82142	.87142	.92486	.98205
43	.57939	.61407	.65083	.68982	.73124	.77530	.82222	.87229	.92578	.98304
44	.57995	.61467	.65146	.69049	.73195	.77606	.82303	.87315	.92670	.98403
45	1.58051	1.61526	1.65209	1.69116	1.73267	1.77681	1.82384	1.87401	1.92762	1.98502
46	.58108	.61586	.65272	.69183	.73338	.77757	.82465	.87488	.92855	.98601
47	.58164	.61646	.65336	.69250	.73409	.77833	.82546	.87574	.92947	.98700
48	.58221	.61705	.65399	.69318	.73481	.77910	.82627	.87661	.93040	.98799
49	.58277	.61765	.65462	.69385	.73552	.77986	.82709	.87748	.93133	.98899
50	1.58333	1.61825	1.65526	1.69452	1.73624	1.78062	1.82790	1.87834	1.93226	1.98998
51	.58390	.61885	.65589	.69520	.73696	.78138	.82871	.87921	.93319	.99098
52	.58447	.61945	.65653	.69587	.73768	.78215	.82953	.88008	.93412	.99198
53	.58503	.62005	.65717	.69655	.73840	.78291	.83034	.88095	.93505	.99298
54	.58560	.62065	.65780	.69723	.73911	.78368	.83116	.88183	.93598	.99398
55	1.58617	1.62125	1.65844	1.69790	1.73983	1.78445	1.83198	1.88270	1.93692	1.99498
56	.58674	.62185	.65908	.69858	.74056	.78521	.83280	.88357	.93785	.99598
57	.58731	.62246	.65972	.69926	.74128	.78598	.83362	.88445	.93879	.99698
58	.58788	.62306	.66036	.69994	.74200	.78675	.83444	.88532	.93973	.99799
59	.58845	.62366	.66100	.70062	.74272	.78752	.83526	.88620	.94066	.99899
60	1.58902	1.62427	1.66164	1.70130	1.74345	1.78829	1.83608	1.88708	1.94160	2.00000
	39°	38°	37°	36°	35°	34°	33°	32°	31°	30°

Cosecants.

* Exsecant = secant − 1; coexsecant = cosecant − 1.

Google

4.—Natural Secants, COSECANTS (EXSECANTS, COEXSECANTS).*—(Cont'd.)

Secants.

′	60°	61°	62°	63°	64°	65°	66°	67°	68°	69°	
0	2.00000	2.06267	2.13005	2.20269	2.28117	2.36620	2.45859	2.55930	2.66947	2.79043	60
1	.00101	.06375	.13122	.20395	.28253	.36768	.46020	.56106	.67139	.79254	59
2	.00202	.06483	.13239	.20521	.28390	.36916	.46181	.56282	.67332	.79466	58
3	.00303	.06592	.13356	.20647	.28526	.37064	.46342	.56458	.67525	.79679	57
4	.00404	.06701	.13473	.20773	.28663	.37212	.46504	.56634	.67718	.79891	56
5	2.00505	2.06809	2.13590	2.20900	2.28800	2.37361	2.46665	2.56811	2.67911	2.80104	55
6	.00607	.06918	.13707	.21026	.28937	.37509	.46827	.56988	.68105	.80318	54
7	.00708	.07027	.13825	.21153	.29074	.37658	.46989	.57165	.68299	.80531	53
8	.00810	.07137	.13942	.21280	.29211	.37808	.47152	.57342	.68494	.80746	52
9	.00912	.07246	.14060	.21407	.29349	.37957	.47314	.57520	.68689	.80960	51
10	2.01014	2.07356	2.14178	2.21535	2.29487	2.38107	2.47477	2.57698	2.68884	2.81175	50
11	.01116	.07465	.14296	.21662	.29625	.38256	.47640	.57876	.69079	.81390	49
12	.01218	.07575	.14414	.21790	.29763	.38406	.47804	.58054	.69275	.81605	48
13	.01320	.07685	.14533	.21918	.29901	.38556	.47967	.58233	.69471	.81821	47
14	.01422	.07795	.14651	.22045	.30040	.38707	.48131	.58412	.69667	.82037	46
15	2.01525	2.07905	2.14770	2.22174	2.30179	2.38857	2.48295	2.58591	2.69864	2.82254	45
16	.01628	.08015	.14889	.22302	.30318	.39008	.48459	.58771	.70061	.82471	44
17	.01730	.08126	.15008	.22430	.30457	.39159	.48624	.58950	.70258	.82688	43
18	.01833	.08236	.15127	.22559	.30596	.39311	.48789	.59130	.70455	.82906	42
19	.01936	.08347	.15246	.22688	.30735	.39462	.48954	.59311	.70653	.83124	41
20	2.02039	2.08458	2.15366	2.22817	2.30875	2.39614	2.49119	2.59491	2.70851	2.83342	40
21	.02143	.08569	.15485	.22946	.31015	.39766	.49284	.59672	.71050	.83561	39
22	.02246	.08680	.15605	.23075	.31155	.39918	.49450	.59853	.71249	.83780	38
23	.02349	.08791	.15725	.23205	.31295	.40070	.49616	.60035	.71448	.83999	37
24	.02453	.08903	.15845	.23334	.31436	.40222	.49782	.60217	.71647	.84219	36
25	2.02557	2.09014	2.15965	2.23464	2.31576	2.40375	2.49948	2.60399	2.71847	2.84439	35
26	.02661	.09126	.16085	.23594	.31717	.40528	.50115	.60581	.72047	.84659	34
27	.02765	.09238	.16206	.23724	.31858	.40681	.50282	.60763	.72247	.84880	33
28	.02869	.09350	.16326	.23855	.31999	.40835	.50449	.60946	.72448	.85102	32
29	.02973	.09462	.16447	.23985	.32140	.40988	.50617	.61129	.72649	.85323	31
30	2.03077	2.09574	2.16568	2.24116	2.32282	2.41142	2.50784	2.61313	2.72850	2.85545	30
31	.03182	.09686	.16689	.24247	.32424	.41296	.50952	.61496	.73052	.85767	29
32	.03286	.09799	.16810	.24378	.32566	.41450	.51120	.61680	.73254	.85990	28
33	.03391	.09911	.16932	.24509	.32708	.41605	.51289	.61864	.73456	.86213	27
34	.03496	.10024	.17053	.24640	.32850	.41760	.51457	.62049	.73659	.86437	26
35	2.03601	2.10137	2.17175	2.24772	2.32993	2.41914	2.51626	2.62234	2.73862	2.86661	25
36	.03706	.10250	.17297	.24903	.33135	.42070	.51795	.62419	.74065	.86885	24
37	.03811	.10363	.17419	.25035	.33278	.42225	.51965	.62604	.74269	.87109	23
38	.03916	.10477	.17541	.25167	.33422	.42380	.52134	.62790	.74473	.87334	22
39	.04022	.10590	.17663	.25300	.33565	.42536	.52304	.62976	.74677	.87560	21
40	2.04128	2.10704	2.17786	2.25432	2.33708	2.42692	2.52474	2.63162	2.74881	2.87785	20
41	.04233	.10817	.17909	.25565	.33852	.42848	.52645	.63348	.75086	.88011	19
42	.04339	.10931	.18031	.25697	.33996	.43005	.52815	.63535	.75292	.88238	18
43	.04445	.11045	.18154	.25830	.34140	.43162	.52986	.63722	.75497	.88465	17
44	.04551	.11159	.18277	.25963	.34284	.43318	.53157	.63909	.75703	.88692	16
45	2.04658	2.11274	2.18401	2.26097	2.34429	2.43476	2.53329	2.64097	2.75909	2.88920	15
46	.04764	.11388	.18524	.26230	.34573	.43633	.53500	.64285	.76116	.89148	14
47	.04870	.11503	.18648	.26364	.34718	.43790	.53672	.64473	.76323	.89376	13
48	.04977	.11617	.18772	.26498	.34863	.43948	.53845	.64662	.76530	.89605	12
49	.05084	.11732	.18895	.26632	.35009	.44106	.54017	.64851	.76737	.89834	11
50	2.05191	2.11847	2.19019	2.26766	2.35154	2.44264	2.54190	2.65040	2.76945	2.90063	10
51	.05298	.11963	.19144	.26900	.35300	.44423	.54363	.65229	.77154	.90293	9
52	.05405	.12078	.19268	.27035	.35446	.44582	.54536	.65419	.77362	.90524	8
53	.05512	.12193	.19393	.27169	.35592	.44741	.54709	.65609	.77571	.90754	7
54	.05619	.12309	.19517	.27304	.35738	.44900	.54883	.65799	.77780	.90986	6
55	2.05727	2.12425	2.19642	2.27439	2.35885	2.45059	2.55057	2.65989	2.77990	2.91217	5
56	.05835	.12540	.19767	.27574	.36031	.45219	.55231	.66180	.78200	.91449	4
57	.05942	.12657	.19892	.27710	.36178	.45378	.55405	.66371	.78410	.91681	3
58	.06050	.12773	.20018	.27845	.36325	.45539	.55580	.66563	.78621	.91914	2
59	.06158	.12889	.20143	.27981	.36473	.45699	.55755	.66755	.78832	.92147	1
60	2.06267	2.13005	2.20269	2.28117	2.36620	2.45859	2.55930	2.66947	2.79043	2.92380	0
	29°	28°	27°	26°	25°	24°	23°	22°	21°	20°	′

Cosecants.

* Exsecant = secant − 1; coexsecant = cosecant − 1.

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4.—Natural Secants, Cosecants (Exsecants, Coexsecants).*—(Cont'd

Secants.

′	70°	71°	72°	73°	74°	75°	76°	77°	78°	79°
0	2.92380	3.07155	3.23607	3.42030	3.62796	3.86370	4.13357	4.44541	4.80973	5.24064
1	.92614	.07415	.23897	.42356	.63164	.86790	.13839	.45102	.81633	.24870
2	.92849	.07675	.24187	.42683	.63533	.87211	.14323	.45664	.82294	.25558
3	.93083	.07936	.24478	.43010	.63903	.87633	.14809	.46228	.82956	.26248
4	.93318	.08197	.24770	.43337	.64274	.88056	.15295	.46793	.83621	.27241
5	2.93554	3.08459	3.25062	3.43666	3.64645	3.88479	4.15782	4.47360	4.84288	5.28036
6	.93790	.08721	.25355	.43995	.65018	.88904	.16271	.47928	.84956	.28833
7	.94026	.08983	.25648	.44324	.65391	.89330	.16761	.48498	.85627	.29634
8	.94263	.09246	.25942	.44655	.65765	.89756	.17252	.49069	.86299	.30436
9	.94500	.09510	.26237	.44986	.66140	.90184	.17744	.49642	.86973	.31241
10	2.94737	3.09774	3.26531	3.45317	3.66515	3.90613	4.18238	4.50216	4.87649	5.32049
11	.94975	.10038	.26827	.45650	.66892	.91042	.18733	.50791	.88327	.32859
12	.95213	.10303	.27123	.45983	.67269	.91473	.19228	.51368	.89007	.33671
13	.95452	.10568	.27420	.46316	.67647	.91904	.19725	.51947	.89689	.34486
14	.95691	.10834	.27717	.46651	.68025	.92337	.20224	.52527	.90373	.35304
15	2.95931	3.11101	3.28015	3.46986	3.68405	3.92770	4.20723	4.53109	4.91058	5.36124
16	.96171	.11367	.28313	.47321	.68785	.93204	.21224	.53692	.91746	.36947
17	.96411	.11635	.28612	.47658	.69167	.93640	.21726	.54277	.92436	.37772
18	.96652	.11903	.28912	.47995	.69549	.94076	.22229	.54863	.93128	.38600
19	.96893	.12171	.29212	.48333	.69931	.94514	.22734	.55451	.93821	.39430
20	2.97135	3.12440	3.29512	3.48671	3.70315	3.94952	4.23239	4.56041	4.94517	5.40263
21	.97377	.12709	.29814	.49010	.70700	.95392	.23746	.56632	.95215	.41099
22	.97619	.12979	.30115	.49350	.71085	.95832	.24255	.57224	.95914	.41937
23	.97862	.13249	.30418	.49691	.71471	.96274	.24764	.57819	.96616	.42778
24	.98106	.13520	.30721	.50032	.71858	.96716	.25275	.58414	.97320	.43622
25	2.98349	3.13791	3.31024	3.50374	3.72246	3.97160	4.25787	4.59012	4.98025	5.44468
26	.98594	.14063	.31328	.50716	.72635	.97604	.26300	.59611	.98733	.45317
27	.98838	.14335	.31633	.51060	.73024	.98050	.26814	.60211	.99443	.46169
28	.99083	.14608	.31939	.51404	.73414	.98497	.27330	.60813	5.00155	.47023
29	.99329	.14881	.32244	.51748	.73806	.98944	.27847	.61417	.00869	.47881
30	2.99574	3.15155	3.32551	3.52094	3.74198	3.99393	4.28366	4.62023	5.01585	5.48740
31	.99821	.15429	.32858	.52440	.74591	.99843	.28885	.62630	.02303	.49603
32	3.00067	.15704	.33166	.52787	.74984	4.00293	.29406	.63238	.03024	.50468
33	.00315	.15979	.33474	.53134	.75379	.00745	.29929	.63849	.03746	.51337
34	.00562	.16255	.33783	.53482	.75775	.01198	.30452	.64461	.04471	.52208
35	3.00810	3.16531	3.34092	3.53831	3.76171	4.01652	4.30977	4.65074	5.05197	5.53081
36	.01059	.16808	.34403	.54181	.76568	.02107	.31503	.65690	.05926	.53958
37	.01308	.17085	.34713	.54531	.76966	.02563	.32031	.66307	.06657	.54837
38	.01557	.17363	.35025	.54883	.77365	.03020	.32560	.66925	.07390	.55720
39	.01807	.17641	.35336	.55235	.77765	.03479	.33090	.67545	.08125	.56605
40	3.02057	3.17920	3.35649	3.55587	3.78166	4.03938	4.33622	4.68167	5.08863	5.57493
41	.02308	.18199	.35962	.55940	.78568	.04398	.34154	.68791	.09602	.58383
42	.02559	.18479	.36276	.56294	.78970	.04860	.34689	.69417	.10344	.59277
43	.02810	.18759	.36590	.56649	.79374	.05322	.35224	.70044	.11088	.60174
44	.03062	.19040	.36905	.57005	.79778	.05786	.35761	.70673	.11835	.61073
45	3.03315	3.19322	3.37221	3.57361	3.80183	4.06251	4.36299	4.71303	5.12583	5.61976
46	.03568	.19604	.37537	.57718	.80589	.06717	.36839	.71935	.13334	.62881
47	.03821	.19886	.37854	.58076	.80996	.07184	.37380	.72569	.14087	.63790
48	.04075	.20169	.38171	.58434	.81404	.07652	.37923	.73205	.14843	.64701
49	.04329	.20453	.38489	.58794	.81813	.08121	.38466	.73843	.15599	.65616
50	3.04584	3.20737	3.38808	3.59154	3.82223	4.08591	4.39012	4.74482	5.16359	5.66533
51	.04839	.21021	.39128	.59514	.82633	.09063	.39558	.75123	.17121	.67454
52	.05094	.21306	.39448	.59876	.83045	.09535	.40106	.75766	.17886	.68377
53	.05350	.21592	.39768	.60238	.83457	.10009	.40656	.76411	.18652	.69304
54	.05607	.21878	.40089	.60601	.83871	.10484	.41206	.77057	.19421	.70234
55	3.05864	3.22165	3.40411	3.60965	3.84285	4.10960	4.41759	4.77705	5.20193	5.71166
56	.06121	.22452	.40734	.61330	.84700	.11437	.42312	.78355	.20966	.72102
57	.06379	.22740	.41057	.61695	.85116	.11915	.42867	.79007	.21742	.73041
58	.06637	.23028	.41381	.62061	.85533	.12394	.43424	.79661	.22521	.73983
59	.06896	.23317	.41705	.62428	.85951	.12875	.43982	.80316	.23301	.74929
60	3.07155	3.23607	3.42030	3.62796	3.86370	4.13357	4.44541	4.80973	5.24084	5.75877
	19°	18°	17°	16°	15°	14°	13°	12°	11°	10°

Cosecants.

* Exsecant = secant − 1; coexsecant = cosecant − 1.

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4.—Natural Secants, Cosecants (Exsecants, Coexsecants).*—(Cond'd.)

Secants.

′	80°	81°	82°	83°	84°	85°	86°	87°	88°	89°	
0	5.75877	6.39245	7.18530	8.20551	9.56677	11.47371	14.33559	19.10732	28.65371	57.29869	60
1	.76829	.40422	.20020	.22500	.59332	.51199	.39547	.21397	.89440	58.26976	59
2	.77784	.41602	.21517	.24457	.62002	.55052	.45586	.32182	29.13917	59.27431	58
3	.78742	.42787	.23019	.26425	.64687	.58932	.51676	.43088	.38812	60.31411	57
4	.79703	.43977	.24529	.28402	.67387	.62837	.57817	.54119	.64137	61.39105	56
5	5.80667	6.45171	7.26044	8.30388	9.70103	11.66769	14.64011	19.65275	29.89903	62.50715	55
6	.81635	.46369	.27566	.32384	.72833	.70728	.70258	.76560	30.16120	63.66460	54
7	.82606	.47572	.29095	.34390	.75579	.74714	.76558	.87976	.42802	64.86572	53
8	.83581	.48779	.30630	.36405	.78341	.78727	.82913	.99524	.69960	66.11304	52
9	.84558	.49991	.32171	.38431	.81119	.82768	.89323	20.11208	.97607	67.40927	51
10	5.85539	6.51208	7.33719	8.40466	9.83912	11.86837	14.95788	20.23028	31.25758	68.75736	50
11	.86524	.52429	.35274	.42511	.86722	.90934	15.02310	.34989	.54425	70.16047	49
12	.87511	.53655	.36835	.44566	.89547	.95060	.08890	.47093	.83623	71.62285	48
13	.88502	.54886	.38403	.46632	.92389	.99214	.15527	.59341	32.13366	73.14583	47
14	.89497	.56121	.39978	.48707	.95248	12.03397	.22223	.71737	.43671	74.73586	46
15	5.90495	6.57361	7.41560	8.50793	9.98123	12.07610	15.28979	20.84283	32.74554	76.39655	45
16	.91496	.58606	.43148	.52889	10.01015	.11852	.35795	.96982	33.06030	78.13274	44
17	.92501	.59855	.44743	.54996	.03923	.16125	42672	21.09838	.38118	79.94968	43
18	.93509	.61110	.46346	.57113	.06849	.20427	.49611	.22852	.70835	81.85315	42
19	.94521	.62369	.47955	.59241	.09792	.24761	.56614	.36027	34.04199	83.84947	41
20	5.95536	6.63633	7.49571	8.61379	10.12752	12.29125	15.63679	21.49368	34.38232	85.94561	40
21	.96555	.64902	.51194	.63528	.15730	.33521	.70810	.62876	.72952	88.14924	39
22	.97577	.66176	.52825	.65688	.18725	.37948	.78005	.76555	35.08380	90.46886	38
23	.98603	.67454	.54462	.67859	.21739	.42408	.85268	.90409	.44539	92.91387	37
24	.99633	.68738	.56107	.70041	.24770	.46900	.92597	22.04440	.81452	95.49471	36
25	6.00666	6.70027	7.57759	8.72234	10.27819	12.51424	15.99995	22.18653	36.19141	98.22303	35
26	.01703	.71321	.59418	.74438	.30887	.55982	16.07462	.33050	.57633	101.1119	34
27	.02743	.72620	.61085	.76653	.33973	.60572	.14999	.47635	.96953	104.1757	33
28	.03787	.73924	.62759	.78880	.37077	.65197	.22607	.62413	37.37127	107.4311	32
29	.04834	.75233	.64441	.81118	.40201	.69856	.30287	.77386	.78185	110.8966	31
30	6.05886	6.76547	7.66130	8.83367	10.43343	12.74550	16.38041	22.92559	38.20155	114.5930	30
31	.06941	.77866	.67826	.85628	.46505	.79278	.45869	23.07935	.63068	118.5444	29
32	.08000	.79191	.69530	.87901	.49685	.84042	.53772	.23520	39.06957	122.7780	28
33	.09062	.80521	.71242	.90186	.52886	.88841	.61751	.39316	.51855	127.3253	27
34	.10129	.81856	.72962	.92482	.56106	.93677	.69808	.55329	.97797	132.2223	26
35	6.11199	6.83196	7.74689	8.94791	10.59346	12.98549	16.77944	23.71563	40.44820	137.5111	25
36	.12273	.84542	.76424	.97111	.62605	13.03458	.86159	.88022	.92963	143.2406	24
37	.13350	.85893	.78167	.99444	.65885	.08400	.94456	24.04712	41.42266	149.4684	23
38	.14432	.87250	.79918	9.01788	.69186	.13388	17.02835	.21637	.92772	156.2625	22
39	.15517	.88612	.81677	.04146	.72507	.18411	.11297	.38802	42.44525	163.7033	21
40	6.16607	6.89979	7.83443	9.06515	10.75849	13.23472	17.19843	24.56212	42.97571	171.8883	20
41	.17700	.91352	.85218	.08897	.79212	.28572	.28476	.73873	43.51961	180.9350	19
42	.18797	.92731	.87001	.11292	.82596	.33712	.37196	.91790	44.07746	190.9868	18
43	.19898	.94115	.88792	.13699	.86001	.38891	.46005	25.09969	.64980	202.2212	17
44	.21004	.95505	.90592	.16120	.89428	.44112	.54903	.28414	45.23720	214.8600	16
45	6.22113	6.96900	7.92400	9.18553	10.92877	13.49373	17.63893	25.47134	45.84026	229.1839	15
46	.23226	.98301	.94216	.20999	.96348	.54676	.72975	.66132	46.45963	245.5540	14
47	.24343	.99708	.96040	.23459	.99841	.60021	.82152	.85417	47.09596	264.4427	13
48	.25464	7.01120	.97873	.25931	11.03356	.65408	.91424	26.04994	.74997	286.4795	12
49	.26590	.02538	.99714	.28417	.06894	.70838	18.00794	.24869	48.42241	312.5230	11
50	6.27719	7.03962	8.01565	9.30917	11.10455	13.76312	18.10262	26.45051	49.11406	343.7752	10
51	.28853	.05392	.03423	.33430	.14039	.81829	.19830	.65546	.82576	381.9723	9
52	.29991	.06828	.05291	.35957	.17646	.87391	.29501	.86360	50.55840	429.7187	8
53	.31133	.08269	.07167	.38497	.21277	.92999	.39274	27.07503	51.31290	491.1070	7
54	.32279	.09717	.09052	.41052	.24932	.98651	.49153	.28981	52.09027	572.9581	6
55	6.33429	7.11171	8.10946	9.43620	11.28610	14.04350	18.59139	27.50804	52.89156	687.5496	5
56	.34584	.12630	.12849	.46203	.32313	.10096	.69233	.72978	53.71790	859.4369	4
57	.35743	.14096	.14760	.48800	.36040	.15889	.79438	.95513	54.57046	1145.916	3
58	.36906	.15568	.16681	.51411	.39792	.21730	.89755	28.18417	55.45053	1718.874	2
59	.38073	.17046	.18612	.54037	.43569	.27620	19.00185	.41700	56.35946	3437.747	1
60	6.39245	7.18530	8.20551	9.56677	11.47371	14.33559	19.10732	28.65371	57.29869	Infinite	0
	9°	8°	7°	6°	5°	4°	3°	2°	1°	0°	′

Cosecants.

* Exsecant = secant − 1; coexsecant = cosecant − 1.

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5.—Logarithmic Sines, Tangents, Cotangents, Cosines.

(Secants, Cosecants.)*

0° ′	Sine.	Tang.	Cotang.	Cosine.		′	Sine.	Tang.	Cotang.	Cosine.	1°
0	Inf. Neg.	Inf. Neg.	Infinite.	10.00000	60	0	8.24186	8.24192	11.75808	9.99993	6
1	6.46373	6.46373	13.53627	.00000	59	1	.24903	.24910	.75090	.99993	5
2	.76476	.76476	.23524	.00000	59	2	.25609	.25616	.74384	.99993	5
3	6.94085	6.94085	13.05915	.00000	57	3	.26304	.26312	.73688	.99993	5
4	7.06579	7.06579	12.93421	.00000	56	4	.26988	.26996	.73004	.99992	5
5	7.16270	7.16270	12.83730	10.00000	55	5	8.27661	8.27669	11.72331	9.99992	5
6	.24188	.24188	.75812	.00000	54	6	.28324	.28332	.71668	.99992	5
7	.30882	.30882	.69118	.00000	53	7	.28977	.28986	.71014	.99992	5
8	.36682	.36682	.63318	.00000	52	8	.29621	.29629	.70371	.99992	5
9	.41797	.41797	.58203	.00000	51	9	.30255	.30263	.69737	.99991	5
10	7.46373	7.46373	12.53627	10.00000	50	10	8.30879	8.30888	11.69112	9.99991	5
11	.50512	.50512	.49488	.00000	49	11	.31495	.31505	.68495	.99991	4
12	.54291	.54291	.45709	.00000	48	12	.32103	.32112	.67888	.99990	4
13	.57767	.57767	.42233	.00000	47	13	.32702	.32711	.67289	.99990	4
14	.60985	.60986	.39014	.00000	46	14	.33292	.33302	.66698	.99990	4
15	7.63982	7.63982	12.36018	10.00000	45	15	8.33875	8.33886	11.66114	9.99990	4
16	.66784	.66785	.33215	10.00000	44	16	.34450	.34461	.65539	.99989	4
17	.69417	.69418	.30582	9.99999	43	17	.35018	.35029	.64971	.99989	4
18	.71900	.71900	.28100	.99999	42	18	.35578	.35590	.64410	.99989	4
19	.74248	.74248	.25752	.99999	41	19	.36131	.36143	.63857	.99989	4
20	7.76475	7.76476	12.23524	9.99999	40	20	8.36678	8.36689	11.63311	9.99988	4
21	.78594	.78595	.21405	.99999	39	21	.37217	.37229	.62771	.99988	3
22	.80615	.80615	.19385	.99999	38	22	.37750	.37762	.62238	.99988	3
23	.82545	.82546	.17454	.99999	37	23	.38276	.38289	.61711	.99987	3
24	.84393	.84394	.15606	.99999	36	24	.38796	.38809	.61191	.99987	3
25	7.86166	7.86167	12.13833	9.99999	35	25	8.39310	8.39323	11.60677	9.99987	3
26	.87870	.87871	.12129	.99999	34	26	.39818	.39832	.60168	.99986	3
27	.89509	.89510	.10490	.99999	33	27	.40320	.40334	.59666	.99986	3
28	.91088	.91089	.08911	.99999	32	28	.40816	.40830	.59170	.99986	3
29	.92612	.92613	.07387	.99998	31	29	.41307	.41321	.58679	.99985	3
30	7.94084	7.94086	12.05914	9.99998	30	30	8.41792	8.41807	11.58193	9.99985	3
31	.95508	.95510	.04490	.99998	29	31	.42272	.42287	.57713	.99985	2
32	.96887	.96889	.03111	.99998	28	32	.42746	.42762	.57238	.99984	2
33	.98223	.98225	.01775	.99998	27	33	.43216	.43232	.56768	.99984	2
34	7.99520	7.99522	12.00478	.99998	26	34	.43680	.43696	.56304	.99984	2
35	8.00779	8.00781	11.99219	9.99998	25	35	8.44139	8.44156	11.55844	9.99983	2
36	.02002	.02004	.97996	.99998	24	36	.44594	.44611	.55389	.99983	2
37	.03192	.03194	.96806	.99997	23	37	.45044	.45061	.54939	.99983	2
38	.04350	.04353	.95647	.99997	22	38	.45489	.45507	.54493	.99982	2
39	.05478	.05481	.94519	.99997	21	39	.45930	.45948	.54052	.99982	2
40	8.06578	8.06581	11.93419	9.99997	20	40	8.46366	8.46385	11.53615	9.99982	2
41	.07650	.07653	.92347	.99997	19	41	.46799	.46817	.53183	.99981	
42	.08696	.08700	.91300	.99997	18	42	.47226	.47245	.52755	.99981	
43	.09718	.09722	.90278	.99997	17	43	.47650	.47669	.52331	.99981	
44	.10717	.10720	.89280	.99996	16	44	.48069	.48089	.51911	.99980	
45	8.11693	8.11696	11.88304	9.99996	15	45	8.48485	8.48505	11.51495	9.99980	
46	.12647	.12651	.87349	.99996	14	46	.48896	.48917	.51083	.99979	
47	.13581	.13585	.86415	.99996	13	46	.49304	.49325	.50675	.99979	
48	.14495	.14500	.85500	.99996	12	48	.49708	.49729	.50271	.99979	
49	.15391	.15395	.84605	.99996	11	49	.50108	.50130	.49870	.99978	
50	8.16268	8.16273	11.83727	9.99995	10	50	8.50504	8.50527	11.49473	9.99978	
51	.17128	.17133	.82867	.99995	9	51	.50897	.50920	.49080	.99977	
52	.17971	.17976	.82024	.99995	8	52	.51287	.51310	.48690	.99977	
53	.18798	.18804	.81196	.99995	7	53	.51673	.51696	.48304	.99977	
54	.19610	.19616	.80384	.99995	6	54	.52055	.52079	.47921	.99976	
55	8.20407	8.20413	11.79587	9.99994	5	55	8.52434	8.52459	11.47541	9.99976	
56	.21189	.21195	.78805	.99994	4	56	.52810	.52835	.47165	.99975	
57	.21958	.21964	.78036	.99994	3	57	.53183	.53208	.46792	.99975	
58	.22713	.22720	.77280	.99994	2	58	.53552	.53578	.46422	.99974	
59	.23456	.23462	.76538	.99994	1	59	.53919	.53945	.46055	.99974	
60	8.24186	8.24192	11.75808	9.99993	0	60	8.54282	8.54308	11.45692	9.99974	
	Cosine.	Cotang.	Tang.	Sine.	′		Cosine.	Cotang.	Tang.	Sine.	
					89°						

*Log secant = colog cosine = 1 − log cosine; log cosecant = colog sine = 1 − log sine.

Ex.—Log sec 0° 30′ = 10.00002. *Ex.*—Log cosec 0° 30′ = 12.0591

5.—Logarithmic Sines, Tangents, Cotangents, Cosines. (Secants, Cosecants.)*—(Cont'd.)

2° ′	Sine.	Tang.	Cotang.	Cosine.		3° ′	Sine.	Tang.	Cotang.	Cosine.	
0	8.54282	8.54308	11.45692	9.99974	60	0	8.71880	8.71940	11.28060	9.99940	60
1	.54642	.54669	.45331	.99973	59	1	.72120	.72181	.27819	.99940	59
2	.54999	.55027	.44973	.99973	58	2	.72359	.72420	.27580	.99939	58
3	.55354	.55382	.44618	.99972	57	3	.72597	.72659	.27341	.99938	57
4	.55706	.55734	.44266	.99972	56	4	.72834	.72896	.27104	.99938	56
5	8.56054	8.56083	11.43917	9.99971	55	5	8.73069	8.73132	11.26868	9.99937	55
6	.56400	.56429	.43571	.99971	54	6	.73303	.73366	.26634	.99936	54
7	.56743	.56773	.43227	.99970	53	7	.73535	.73600	.26400	.99936	53
8	.57084	.57114	.42886	.99970	52	8	.73767	.73832	.26168	.99935	52
9	.57421	.57452	.42548	.99969	51	9	.73997	.74063	.25937	.99934	51
10	8.57757	8.57788	11.42212	9.99969	50	10	8.74226	8.74292	11.25708	9.99934	50
11	.58089	.58121	.41879	.99968	49	11	.74454	.74521	.25479	.99933	49
12	.58419	.58451	.41549	.99968	48	12	.74680	.74748	.25252	.99932	48
13	.58747	.58779	.41221	.99967	47	13	.74906	.74974	.25026	.99932	47
14	.59072	.59105	.40895	.99967	46	14	.75130	.75199	.24801	.99931	46
15	8.59395	8.59428	11.40572	9.99967	45	15	8.75353	8.75423	11.24577	9.99930	45
16	.59715	.59749	.40251	.99966	44	16	.75575	.75645	.24355	.99929	44
17	.60033	.60068	.39932	.99966	43	17	.75795	.75867	.24133	.99929	43
18	.60349	.60384	.39616	.99965	42	18	.76015	.76087	.23913	.99928	42
19	.60662	.60698	.39302	.99964	41	19	.76234	.76306	.23694	.99927	41
20	8.60973	8.61009	11.38991	9.99964	40	20	8.76451	8.76525	11.23475	9.99926	40
21	.61282	.61319	.38681	.99963	39	21	.76667	.76742	.23258	.99926	39
22	.61589	.61626	.38374	.99963	38	22	.76883	.76958	.23042	.99925	38
23	.61894	.61931	.38069	.99962	37	23	.77097	.77173	.22827	.99924	37
24	.62196	.62234	.37766	.99962	36	24	.77310	.77387	.22613	.99923	36
25	8.62497	8.62535	11.37465	9.99961	35	25	8.77522	8.77600	11.22400	9.99923	35
26	.62795	.62834	.37166	.99961	34	26	.77733	.77811	.22189	.99922	34
27	.63091	.63131	.36869	.99960	33	27	.77943	.78023	.21978	.99921	33
28	.63385	.63426	.36574	.99960	32	28	.78152	.78232	.21768	.99920	32
29	.63678	.63718	.36282	.99959	31	29	.78360	.78441	.21559	.99920	31
30	8.63968	8.64009	11.35991	9.99959	30	30	8.78568	8.78649	11.21351	9.99919	30
31	.64256	.64298	.35702	.99958	29	31	.78774	.78855	.21145	.99919	29
32	.64543	.64585	.35415	.99958	28	32	.78979	.79061	.20939	.99917	28
33	.64827	.64870	.35130	.99957	27	33	.79183	.79266	.20734	.99917	27
34	.65110	.65154	.34846	.99956	26	34	.79386	.79470	.20530	.99916	26
35	8.65391	8.65435	11.34565	9.99956	25	35	8.79588	8.79673	11.20327	9.99915	25
36	.65670	.65715	.34285	.99955	24	36	.79789	.79875	.20125	.99914	24
37	.65947	.65993	.34007	.99955	23	37	.79990	.80076	.19924	.99913	23
38	.66223	.66269	.33731	.99954	22	38	.80189	.80277	.19723	.99913	22
39	.66497	.66543	.33457	.99954	21	39	.80388	.80476	.19524	.99912	21
40	8.66769	8.66816	11.33184	9.99953	20	40	8.80585	8.80674	11.19326	9.99911	20
41	.67039	.67087	.32913	.99952	19	41	.80782	.80872	.19128	.99910	19
42	.67308	.67356	.32644	.99952	18	42	.80978	.81068	.18932	.99909	18
43	.67575	.67624	.32376	.99951	17	43	.81173	.81264	.18736	.99909	17
44	.67841	.67890	.32110	.99951	16	44	.81367	.81459	.18541	.99908	16
45	8.68104	8.68154	11.31846	9.99950	15	45	8.81560	8.81653	11.18347	9.99907	15
46	.68367	.68417	.31583	.99949	14	46	.81752	.81846	.18154	.99906	14
47	.68627	.68678	.31322	.99949	13	47	.81944	.82038	.17962	.99905	13
48	.68886	.68938	.31062	.99948	12	48	.82134	.82230	.17770	.99904	12
49	.69144	.69196	.30804	.99948	11	49	.82324	.82420	.17580	.99904	11
50	8.69400	8.69453	11.30547	9.99947	10	50	8.82513	8.82610	11.17390	9.99903	10
51	.69654	.69708	.30292	.99946	9	51	.82701	.82799	.17201	.99902	9
52	.69907	.69962	.30038	.99946	8	52	.82888	.82987	.17013	.99901	8
53	.70159	.70214	.29786	.99945	7	53	.83075	.83175	.16825	.99900	7
54	.70409	.70465	.29535	.99944	6	54	.83261	.83361	.16639	.99899	6
55	8.70658	8.70714	11.29286	9.99944	5	55	8.83446	8.83547	11.16453	9.99898	5
56	.70905	.70962	.29038	.99943	4	56	.83630	.83732	.16268	.99898	4
57	.71151	.71208	.28792	.99942	3	57	.83813	.83916	.16084	.99897	3
58	.71395	.71453	.28547	.99942	2	58	.83996	.84100	.15900	.99896	2
59	.71638	.71697	.28303	.99941	1	59	.84177	.84282	.15718	.99895	1
60	8.71880	8.71940	11.28060	9.99940	0	60	8.84358	8.84464	11.15536	9.99894	0
	Cosine.	Cotang.	Tang.	Sine.	′		Cosine.	Cotang.	Tang.	Sine.	′
					87°						86°

*Log secant = colog cosine = 1 − log cosine; log cosecant = colog sine = 1 − log sine.

Ex.—Log sec 2°- 30′ = 10.00041. *Ex.*—Log cosec 2°- 30′ = 11.36032.

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5.—Logarithmic Sines, Tangents, Cotangents, Cosines.—(Cont'd.)

(Secants, Cosecants.)*

4° ′	Sine.	Tang.	Cotang.	Cosine.		5° ′	Sine.	Tang.	Cotang.	Cosine.
0	8.84358	8.84464	11.15536	9.99894	60	0	8.94030	8.94195	11.05805	9.99834
1	.84539	.84646	.15354	.99893	59	1	.94174	.94340	.05660	.99833
2	.84718	.84826	.15174	.99892	58	2	.94317	.94485	.05515	.99832
3	.84897	.85006	.14994	.99891	57	3	.94461	.94630	.05370	.99831
4	.85075	.85185	.14815	.99891	56	4	.94603	.94773	.05227	.99830
5	8.85252	8.85363	11.14637	9.99890	55	5	8.94746	8.94917	11.05083	9.99829
6	.85429	.85540	.14460	.99889	54	6	.94887	.95060	.04940	.99828
7	.85605	.85717	.14283	.99888	53	7	.95029	.95202	.04798	.99827
8	.85780	.85893	.14107	.99887	52	8	.95170	.95344	.04656	.99825
9	.85955	.86069	.13931	.99886	51	9	.95310	.95486	.04514	.99824
10	8.86128	8.86243	11.13757	9.99885	50	10	8.95450	8.95627	11.04373	9.99823
11	.86301	.86417	.13583	.99884	49	11	.95589	.95767	.04233	.99822
12	.86474	.86591	.13409	.99883	48	12	.95728	.95908	.04092	.99821
13	.86645	.86763	.13237	.99882	47	13	.95867	.96047	.03953	.99820
14	.86816	.86935	.13065	.99881	46	14	.96005	.96187	.03813	.99819
15	8.86987	8.87106	11.12894	9.99880	45	15	8.96143	8.96325	11.03675	9.99817
16	.87156	.87277	.12723	.99879	44	16	.96280	.96464	.03536	.99816
17	.87325	.87447	.12553	.99879	43	17	.96417	.96602	.03398	.99815
18	.87494	.87616	.12384	.99878	42	18	.96553	.96739	.03261	.99814
19	.87661	.87785	.12215	.99877	41	19	.96689	.96877	.03123	.99813
20	8.87829	8.87953	11.12047	9.99876	40	20	8.96825	8.97013	11.02987	9.99812
21	.87995	.88120	.11880	.99875	39	21	.96960	.97150	.02850	.99810
22	.88161	.88287	.11713	.99874	38	22	.97095	.97285	.02715	.99809
23	.88326	.88453	.11547	.99873	37	23	.97229	.97421	.02579	.99808
24	.88490	.88618	.11382	.99872	36	24	.97363	.97556	.02444	.99807
25	8.88654	8.88783	11.11217	9.99871	35	25	8.97496	8.97691	11.02309	9.99806
26	.88817	.88948	.11052	.99870	34	26	.97629	.97825	.02175	.99804
27	.88980	.89111	.10889	.99869	33	27	.97762	.97959	.02041	.99803
28	.89142	.89274	.10726	.99868	32	28	.97894	.98092	.01908	.99802
29	.89304	.89437	.10563	.99867	31	29	.98026	.98225	.01775	.99801
30	8.89464	8.89598	11.10402	9.99866	30	30	8.98157	8.98358	11.01642	9.99800
31	.89625	.89760	.10240	.99865	29	31	.98288	.98490	.01510	.99798
32	.89784	.89920	.10080	.99864	28	32	.98419	.98622	.01378	.99797
33	.89943	.90080	.09920	.99863	27	33	.98549	.98753	.01247	.99796
34	.90102	.90240	.09760	.99862	26	34	.98679	.98884	.01116	.99795
35	8.90260	8.90399	11.09601	9.99861	25	35	8.98808	8.99015	11.00985	9.99793
36	.90417	.90557	.09443	.99860	24	36	.98937	.99145	.00855	.99792
37	.90574	.90715	.09285	.99859	23	37	.99066	.99275	.00725	.99791
38	.90730	.90872	.09128	.99858	22	38	.99194	.99405	.00595	.99790
39	.90885	.91029	.08971	.99857	21	39	.99322	.99534	.00466	.99788
40	8.91040	8.91185	11.08815	9.99856	20	40	8.99450	8.99662	11.00338	9.99787
41	.91195	.91340	.08660	.99855	19	41	.99577	.99791	.00209	.99786
42	.91349	.91495	.08505	.99854	18	42	.99704	8.99919	11.00081	.99785
43	.91502	.91650	.08350	.99853	17	43	.99830	9.00046	10.99954	.99783
44	.91655	.91803	.08197	.99852	16	44	8.99956	.00174	.99826	.99782
45	8.91807	8.91957	11.08043	9.99851	15	45	9.00082	9.00301	10.99699	9.99781
46	.91959	.92110	.07890	.99850	14	46	.00207	.00427	.99573	.99780
47	.92110	.92262	.07738	.99848	13	47	.00332	.00553	.99447	.99778
48	.92261	.92414	.07586	.99847	12	48	.00456	.00679	.99321	.99777
49	.92411	.92565	.07435	.99846	11	49	.00581	.00805	.99195	.99776
50	8.92561	8.92716	11.07284	9.99845	10	50	9.00704	9.00930	10.99070	9.99775
51	.92710	.92866	.07134	.99844	9	51	.00828	.01055	.98945	.99773
52	.92859	.93016	.06984	.99843	8	52	.00951	.01179	.98821	.99772
53	.93007	.93165	.06835	.99842	7	53	.01074	.01303	.98697	.99771
54	.93154	.93313	.06687	.99841	6	54	.01196	.01427	.98573	.99769
55	8.93301	8.93462	11.06538	9.99840	5	55	9.01318	9.01550	10.98450	9.99768
56	.93448	.93609	.06391	.99839	4	56	.01440	.01673	.98327	.99767
57	.93594	.93756	.06244	.99838	3	57	.01561	.01796	.98204	.99765
58	.93740	.93903	.06097	.99837	2	58	.01682	.01918	.98082	.99764
59	.93885	.94049	.05951	.99836	1	59	.01803	.02040	.97960	.99763
60	8.94030	8.94195	11.05805	9.99834	0	60	9.01923	9.02162	10.97838	9.99761
	Cosine.	Cotang.	Tang.	Sine.	′		Cosine.	Cotang.	Tang.	Sine.
					85°					84

*Log secant = colog cosine = 1 − log cosine; log cosecant = colog sine = 1 − log sine.

Ex.—Log sec 4° − 30′ = 10.00134. *Ex.*—Log cosec 4° − 30′ = 11.10536.

5.—Logarithmic Sines, Tangents, Cotangents, Cosines.—(Cont'd.)
(Secants, Cosecants.)*

6°

′	Sine.	Tang.	Cotang.	Cosine.		′	Sine.	Tang.	Cotang.	Cosine.	
0	9.01923	9.02162	10.97838	9.99761	60	0	9.08589	9.08914	10.91086	9.99675	60
1	.02043	.02283	.97717	.99760	59	1	.08692	.09019	.90981	.99674	59
2	.02163	.02404	.97596	.99759	58	2	.08795	.09123	.90877	.99672	58
3	.02283	.02525	.97475	.99757	57	3	.08897	.09227	.90773	.99670	57
4	.02402	.02645	.97355	.99756	56	4	.08999	.09330	.90670	.99669	56
5	9.02520	9.02766	10.97234	9.99755	55	5	9.09101	9.09434	10.90566	9.99667	55
6	.02639	.02885	.97115	.99753	54	6	.09202	.09537	.90463	.99666	54
7	.02757	.03005	.96995	.99752	53	7	.09304	.09640	.90360	.99664	53
8	.02874	.03124	.96876	.99751	52	8	.09405	.09742	.90258	.99663	52
9	.02992	.03242	.96758	.99749	51	9	.09506	.09845	.90155	.99661	51
10	9.03109	9.03361	10.96639	9.99748	50	10	9.09606	9.09947	10.90053	9.99659	50
11	.03226	.03479	.96521	.99747	49	11	.09707	.10049	.89951	.99658	49
12	.03342	.03597	.96403	.99745	48	12	.09807	.10150	.89850	.99656	48
13	.03458	.03714	.96286	.99744	47	13	.09907	.10252	.89748	.99655	47
14	.03574	.03832	.96168	.99742	46	14	.10006	.10353	.89647	.99653	46
15	9.03690	9.03948	10.96052	9.99741	45	15	9.10106	9.10454	10.89546	9.99651	45
16	.03805	.04065	.95935	.99740	44	16	.10205	.10555	.89445	.99650	44
17	.03920	.04181	.95819	.99738	43	17	.10304	.10656	.89344	.99648	43
18	.04034	.04297	.95703	.99737	42	18	.10402	.10756	.89244	.99647	42
19	.04149	.04413	.95587	.99736	41	19	.10501	.10856	.89144	.99845	41
20	9.04262	9.04528	10.95472	9.99734	40	20	9.10599	9.10956	10.89044	9.99643	40
21	.04376	.04643	.95357	.99733	39	21	.10697	.11056	.88944	.99642	39
22	.04490	.04758	.95242	.99731	38	22	.10795	.11155	.88845	.99640	38
23	.04603	.04873	.95127	.99730	37	23	.10893	.11254	.88746	.99638	37
24	.04715	.04987	.95013	.99728	36	24	.10990	.11353	.88647	.99637	36
25	9.04828	9.05101	10.94899	9.99727	35	25	9.11087	9.11452	10.88548	9.99635	35
26	.04940	.05214	.94786	.99726	34	26	.11184	.11551	.88449	.99633	34
27	.05052	.05328	.94672	.99724	33	27	.11281	.11649	.88351	.99632	33
28	.05164	.05441	.94559	.99723	32	28	.11377	.11747	.88253	.99630	32
29	.05275	.05553	.94447	.99721	31	29	.11474	.11845	.88155	.99629	31
30	9.05386	9.05666	10.94334	9.99720	30	30	9.11570	9.11943	10.88057	9.99627	30
31	.05497	.05778	.94222	.99718	29	31	.11666	.12040	.87960	.99625	29
32	.05607	.05890	.94110	.99717	28	32	.11761	.12138	.87862	.99624	28
33	.05717	.06002	.93998	.99716	27	33	.11857	.12235	.87765	.99622	27
34	.05827	.06113	.93887	.99714	26	34	.11952	.12332	.87668	.99620	26
35	9.05937	9.06224	10.93776	9.99713	25	35	9.12047	9.12428	10.87572	9.99618	25
36	.06046	.06335	.93665	.99711	24	36	.12142	.12525	.87475	.99617	24
37	.06155	.06445	.93555	.99710	23	37	.12236	.12621	.87379	.99615	23
38	.06264	.06556	.93444	.99708	22	38	.12331	.12717	.87283	.99613	22
39	.06372	.06666	.93334	.99707	21	39	.12425	.12813	.87187	.99612	21
40	9.06481	9.06775	10.93225	9.99705	20	40	9.12519	9.12909	10.87091	9.99610	20
41	.06589	.06885	.93115	.99704	19	41	.12612	.13004	.86996	.99608	19
42	.06696	.06994	.93006	.99702	18	42	.12706	.13099	.86901	.99607	18
43	.06804	.07103	.92897	.99701	17	43	.12799	.13194	.86806	.99605	17
44	.06911	.07211	.92789	.99699	16	44	.12892	.13289	.86711	.99603	16
45	9.07018	9.07320	10.92680	9.99698	15	45	9.12985	9.13384	10.86616	9.99601	15
46	.07124	.07428	.92572	.99696	14	46	.13078	.13478	.86522	.99600	14
47	.07231	.07536	.92464	.99695	13	47	.13171	.13573	.86427	.99598	13
48	.07337	.07643	.92357	.99693	12	48	.13263	.13667	.86333	.99596	12
49	.07442	.07751	.92249	.99692	11	49	.13355	.13761	.86239	.99595	11
50	9.07548	9.07858	10.92142	9.99690	10	50	9.13447	9.13854	10.86146	9.99593	10
51	.07653	.07964	.92036	.99689	9	51	.13539	.13948	.86052	.99591	9
52	.07758	.08071	.91929	.99687	8	52	.13630	.14041	.85959	.99589	8
53	.07863	.08177	.91823	.99686	7	53	.13722	.14134	.85866	.99588	7
54	.07968	.08283	.91717	.99684	6	54	.13813	.14227	.85773	.99586	6
55	9.08072	9.08389	10.91611	9.99683	5	55	9.13904	9.14320	10.85680	9.99584	5
56	.08176	.08495	.91505	.99681	4	56	.13994	.14412	.85588	.99582	4
57	.08280	.08600	.91400	.99680	3	57	.14085	.14504	.85496	.99581	3
58	.08383	.08705	.91295	.99678	2	58	.14175	.14597	.85403	.99579	2
59	.08486	.08810	.91190	.99677	1	59	.14266	.14688	.85312	.99577	1
60	9.08589	9.08914	10.91086	9.99675	0	60	9.14356	9.14780	10.85220	9.99575	0
	Cosine.	Cotang.	Tang.	Sine.	′		Cosine.	Cotang.	Tang.	Sine.	′

83° 7° 82°

*Log secant = colog cosine = 1 − log cosine; log cosecant = colog sine = 1 − log sine.

Ex.—Log sec 6°−30′ = 10.00280. *Ex.*—Log cosec 6°−30′ = 10.94614.

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5.—Logarithmic Sines, Tangents, Cotangents, Cosines.—(Cont'd.) (Secants, Cosecants.)*

8°

′	Sine.	Tang.	Cotang.	Cosine.	
0	9.14356	9.14780	10.85220	9.99575	60
1	.14445	.14872	.85128	.99574	59
2	.14535	.14963	.85037	.99572	58
3	.14624	.15054	.84946	.99570	57
4	.14714	.15145	.84855	.99568	56
5	9.14803	9.15236	10.84764	9.99566	55
6	.14891	.15327	.84673	.99565	54
7	.14980	.15417	.84583	.99563	53
8	.15069	.15506	.84492	.99561	52
9	.15157	.15598	.84402	.99559	51
10	9.15245	9.15688	10.84312	9.99557	50
11	.15333	.15777	.84223	.99556	49
12	.15421	.15867	.84133	.99554	48
13	.15508	.15956	.84044	.99552	47
14	.15596	.16046	.83954	.99550	46
15	9.15683	9.16135	10.83865	9.99548	45
16	.15770	.16224	.83776	.99546	44
17	.15857	.16312	.83688	.99545	43
18	.15944	.16401	.83599	.99543	42
19	.16030	.16489	.83511	.99541	41
20	9.16116	9.16577	10.83423	9.99539	40
21	.16203	.16665	.83335	.99537	39
22	.16289	.16753	.83247	.99535	38
23	.16374	.16841	.83159	.99533	37
24	.16460	.16928	.83072	.99532	36
25	9.16545	9.17016	10.82984	9.99530	35
26	.16631	.17103	.82897	.99528	34
27	.16716	.17190	.82810	.99526	33
28	.16801	.17277	.82723	.99524	32
29	.16886	.17363	.82637	.99522	31
30	9.16970	9.17450	10.82550	9.99520	30
31	.17055	.17536	.82464	.99518	29
32	.17139	.17622	.82378	.99517	28
33	.17223	.17708	.82292	.99515	27
34	.17307	.17794	.82206	.99513	26
35	9.17391	9.17880	10.82120	9.99511	25
36	.17474	.17965	.82035	.99509	24
37	.17558	.18051	.81949	.99507	23
38	.17641	.18136	.81864	.99505	22
39	.17724	.18221	.81779	.99503	21
40	9.17807	9.18306	10.81694	9.99501	20
41	.17890	.18391	.81609	.99499	19
42	.17973	.18475	.81525	.99497	18
43	.18055	.18560	.81440	.99495	17
44	.18137	.18644	.81356	.99494	16
45	9.18220	9.18728	10.81272	9.99492	15
46	.18302	.18812	.81188	.99490	14
47	.18383	.18896	.81104	.99488	13
48	.18465	.18979	.81021	.99486	12
49	.18547	.19063	.80937	.99484	11
50	9.18628	9.19146	10.80854	9.99482	10
51	.18709	.19229	.80771	.99480	9
52	.18790	.19312	.80688	.99478	8
53	.18871	.19395	.80605	.99476	7
54	.18952	.19478	.80522	.99474	6
55	9.19033	9.19561	10.80439	9.99472	5
56	.19113	.19643	.80357	.99470	4
57	.19193	.19725	.80275	.99468	3
58	.19273	.19807	.80193	.99466	2
59	.19353	.19889	.80111	.99464	1
60	9.19433	9.19971	10.80029	9.99462	0
	Cosine.	Cotang.	Tang.	Sine.	′

9°

′	Sine.	Tang.	Cotang.	Cosine.	
0	9.19433	9.19971	10.80029	9.99462	
1	.19513	.20053	.79947	.99460	
2	.19592	.20134	.79866	.99458	
3	.19672	.20216	.79784	.99456	
4	.19751	.20297	.79703	.99454	
5	9.19830	9.20378	10.79622	9.99453	
6	.19909	.20459	.79541	.99450	
7	.19988	.20540	.79460	.99448	
8	.20067	.20621	.79379	.99446	
9	.20145	.20701	.79299	.99444	
10	9.20223	9.20782	10.79218	9.99442	
11	.20302	.20862	.79138	.99440	
12	.20380	.20942	.79058	.99438	
13	.20458	.21022	.78978	.99436	
14	.20535	.21102	.78898	.99434	
15	9.20613	9.21183	10.78818	9.99432	
16	.20691	.21261	.78739	.99429	
17	.20768	.21341	.78659	.99427	
18	.20845	.21420	.78580	.99425	
19	.20922	.21499	.78501	.99423	
20	9.20999	9.21578	10.78422	9.99421	
21	.21076	.21657	.78343	.99419	
22	.21153	.21736	.78264	.99417	
23	.21229	.21814	.78186	.99415	
24	.21306	.21893	.78107	.99413	
25	9.21382	9.21971	10.78029	9.99411	
26	.21458	.22049	.77951	.99409	
27	.21534	.22127	.77873	.99407	
28	.21610	.22205	.77795	.99404	
29	.21685	.22283	.77717	.99402	
30	9.21761	9.22361	10.77639	9.99400	
31	.21836	.22438	.77562	.99398	
32	.21912	.22516	.77484	.99396	
33	.21987	.22593	.77407	.99394	
34	.22062	.22670	.77330	.99392	
35	9.22137	9.22747	10.77253	9.99390	
36	.22211	.22824	.77176	.99388	
37	.22286	.22901	.77099	.99385	
38	.22361	.22977	.77023	.99383	
39	.22435	.23054	.76946	.99381	
40	9.22509	9.23130	10.76870	9.99379	
41	.22583	.23206	.76794	.99377	
42	.22657	.23283	.76717	.99375	
43	.22731	.23359	.76641	.99372	
44	.22805	.23435	.76565	.99370	
45	9.22878	9.23510	10.76490	9.99368	
46	.22952	.23586	.76414	.99366	
47	.23025	.23661	.76339	.99364	
48	.23098	.23737	.76263	.99362	
49	.23171	.23812	.76188	.99359	
50	9.23244	9.23887	10.76113	9.99357	
51	.23317	.23962	.76038	.99355	
52	.23390	.24037	.75963	.99353	
53	.23462	.24112	.75888	.99351	
54	.23535	.24186	.75814	.99348	
55	9.23607	9.24261	10.75739	9.99346	
56	.23679	.24335	.75665	.99344	
57	.23752	.24410	.75590	.99342	
58	.23823	.24484	.75516	.99340	
59	.23895	.24558	.75442	.99337	
60	9.23967	9.24632	10.75368	9.99335	
	Cosine.	Cotang.	Tang.	Sine.	′

81°

*Log secant = colog cosine = 1 − log cosine; log cosecant = colog sine = 1 − log sine.

Ex.—Log sec 8° − 30′ = 10.00480. *Ex.*—Log cosec 8° − 30′ = 10.83030

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c Sines, TANGENTS, COTANGENTS, COSINES.—(Cont'd.)
(SECANTS, COSECANTS.)*

11°

Cotang.	Cosine.		′	Sine.	Tang.	Cotang.	Cosine.	
10.75368	9.99335	60	0	9.28060	9.28865	10.71135	9.99195	60
.75294	.99333	59	1	.28125	.28933	.71067	.99192	59
.75221	.99331	58	2	.28190	.29000	.71000	.99190	58
.75147	.99328	57	3	.28254	.29067	.70933	.99187	57
.75074	.99326	56	4	.28319	.29134	.70866	.99185	56
10.75000	9.99324	55	5	9.28384	9.29201	10.70799	9.99182	55
.74927	.99322	54	6	.28448	.29268	.70732	.99180	54
.74854	.99319	53	7	.28512	.29335	.70665	.99177	53
.74781	.99317	52	8	.28577	.29402	.70598	.99175	52
.74708	.99315	51	9	.28641	.29468	.70532	.99172	51
10.74635	9.99313	50	10	9.28705	9.29535	10.70465	9.99170	50
.74563	.99310	49	11	.28769	.29601	.70399	.99167	49
.74490	.99308	48	12	.28833	.29668	.70332	.99165	48
.74418	.99306	47	13	.28896	.29734	.70266	.99162	47
.74345	.99304	46	14	.28960	.29800	.70200	.99160	46
10.74273	9.99301	45	15	9.29024	9.29866	10.70134	9.99157	45
.74201	.99299	44	16	.29087	.29932	.70068	.99155	44
.74129	.99297	43	17	.29150	.29998	.70002	.99152	43
.74057	.99294	42	18	.29214	.30064	.69936	.99150	42
.73985	.99292	41	19	.29277	.30130	.69870	.99147	41
10.73914	9.99290	40	20	9.29340	9.30195	10.69805	9.99145	40
.73842	.99288	39	21	.29403	.30261	.69739	.99142	39
.73771	.99285	38	22	.29466	.30326	.69674	.99140	38
.73699	.99283	37	23	.29529	.30391	.69609	.99137	37
.73628	.99281	36	24	.29591	.30457	.69543	.99135	36
10.73557	9.99278	35	25	9.29654	9.30522	10.69478	9.99132	35
.73486	.99276	34	26	.29716	.30587	.69413	.99130	34
.73415	.99274	33	27	.29779	.30652	.69348	.99127	33
.73345	.99271	32	28	.29841	.30717	.69283	.99124	32
.73274	.99269	31	29	.29903	.30782	.69218	.99122	31
10.73203	9.99267	30	30	9.29966	9.30846	10.69154	9.99119	30
.73133	.99264	29	31	.30028	.30911	.69089	.99117	29
.73063	.99262	28	32	.30090	.30975	.69025	.99114	28
.72992	.99260	27	33	.30151	.31040	.68960	.99112	27
.72922	.99257	26	34	.30213	.31104	.68896	.99109	26
10.72852	9.99255	25	35	9.30275	9.31168	10.68832	9.99106	25
.72782	.99252	24	36	.30336	.31233	.68767	.99104	24
.72712	.99250	23	37	.30398	.31297	.68703	.99101	23
.72643	.99248	22	38	.30459	.31361	.68639	99099	22
.72573	.99245	21	39	.30521	.31425	.68575	99096	21
10.72504	9.99243	20	40	9.30582	9.31489	10.68511	9.99093	20
.72434	.99241	19	41	.30643	.31552	.68448	.99091	19
.72365	.99238	18	42	.30704	.31616	.68384	.99088	18
.72296	.99236	17	43	.30765	.31679	.68321	.99086	17
.72227	.99233	16	44	.30826	.31743	.68257	.99083	16
10.72158	9.99231	15	45	9.30887	9.31806	10.68194	9.99080	15
.72089	.99229	14	46	.30947	.31870	.68130	99078	14
.72020	.99226	13	47	.31008	.31933	.68067	.99075	13
.71951	.99224	12	48	.31068	.31996	.68004	.99072	12
.71883	.99221	11	49	.31129	.32059	.67941	.99070	11
10.71814	9.99219	10	50	9.31189	9.32122	10.67878	9.99067	10
.71746	.99217	9	51	.31250	.32185	.67815	.99064	9
.71677	.99214	8	52	.31310	.32248	.67752	.99062	8
.71609	.99212	7	53	.31370	.32311	.67689	.99059	7
.71541	.99209	6	54	.31430	.32373	.67627	.99056	6
10.71473	9.99207	5	55	9.31490	9.32436	10.67564	9.99054	5
.71405	.99204	4	56	.31549	.32498	.67502	.99051	4
.71338	.99202	3	57	.31609	.32561	.67439	.99048	3
.71270	.99200	2	58	.31669	.32623	.67377	.99046	2
.71202	.99197	1	59	.31728	.32685	.67315	.99043	1
10.71135	9.99195	0	60	9.31788	9.32747	10.67253	9.99040	0
Tang.	Sine.	′		Cosine.	Cotang.	Tang.	Sine.	′
		79°						78°

colog cosine = 1 − log cosine; log cosecant = colog sine =

0° − 30′ = 10.00733. *Ex.*—Log cosec 10° − 30′ = 10.73937.

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5.—Logarithmic Sines, Tangents, Cotangents, Cosines.—(Cont'd.)

(Secants, Cosecants.)*

12° ′	Sine.	Tang.	Cotang.	Cosine.		13° ′	Sine.	Tang.	Cotang.	Cosine.	
0	9.31788	9.32747	10.67253	9.99040	60	0	9.35209	9.36336	10.63664	9.98872	60
1	.31847	.32810	.67190	.99038	59	1	.35263	.36394	.63606	.98869	59
2	.31907	.32872	.67128	.99035	58	2	.35318	.36452	.63548	.98867	58
3	.31966	.32933	.67067	.99032	57	3	.35373	.36509	.63491	.98864	57
4	.32025	.32995	.67005	.99030	56	4	.35427	.36566	.63434	.98861	56
5	9.32084	9.33057	10.66943	9.99027	55	5	9.35481	9.36624	10.63376	9.98858	55
6	.32143	.33119	.66881	.99024	54	6	.35536	.36681	.63319	.98855	54
7	.32202	.33180	.66820	.99022	53	7	.35590	.36738	.63262	.98852	53
8	.32261	.33242	.66758	.99019	52	8	.35644	.36759	.63205	.98849	52
9	.32319	.33303	.66697	.99016	51	9	.35698	.36852	.63148	.98846	51
10	9.32378	9.33365	10.66635	9.99013	50	10	9.35752	9.36909	10.63091	9.98843	50
11	.32437	.33426	.66574	.99011	49	11	.35806	.36966	.63034	.98840	49
12	.32495	.33487	.66513	.99008	48	12	.35860	.37023	.62977	.98837	48
13	.32553	.33548	.66452	.99005	47	13	.35914	.37080	.62920	.98834	47
14	.32612	.33609	.66391	.99002	46	14	.35968	.37137	.62863	.98831	46
15	9.32670	9.33670	10.66330	9.99000	45	15	9.36022	9.37193	10.62807	9.98828	45
16	.32728	.33731	.66269	.98997	44	16	.36075	.37250	.62750	.98825	44
17	.32786	.33792	.66208	.98994	43	17	.36129	.37306	.62694	.98822	43
18	.32844	.33853	.66147	.98991	42	18	.36182	.37363	.62637	.98819	42
19	.32902	.33913	.66087	.98989	41	19	.36236	.37419	.62581	.98816	41
20	9.32960	9.33974	10.66026	9.98986	40	20	9.36289	9.37476	10.62524	9.98813	40
21	.33018	.34034	.65966	.98983	39	21	.36342	.37532	.62468	.98810	39
22	.33075	.34095	.65905	.98980	38	22	.36395	.37588	.62412	.98807	38
23	.33133	.34155	.65845	.98978	37	23	.36449	.37644	.62356	.98804	37
24	.33190	.34215	.65785	.98975	36	24	.36502	.37700	.62300	.98801	36
25	9.33248	9.34276	10.65724	9.98972	35	25	9.36555	9.37756	10.62244	9.98798	35
26	.33305	.34336	.65664	.98969	34	26	.36608	.37812	.62188	.98795	34
27	.33362	.34396	.65604	.98967	33	27	.36660	.37868	.62132	.98792	33
28	.33420	.34456	.65544	.98964	32	28	.36713	.37924	.62076	.98789	32
29	.33477	.34516	.65484	.98961	31	29	.36766	.37980	.62020	.98786	31
30	9.33534	9.34576	10.65424	9.98958	30	30	9.36819	9.38035	10.61965	9.98783	30
31	.33591	.34635	.65365	.98955	29	31	.36871	.38091	.61909	.98780	29
32	.33647	.34695	.65305	.98953	28	32	.36924	.38147	.61853	.98777	28
33	.33704	.34755	.65245	.98950	27	33	.36976	.38202	.61798	.98774	27
34	.33761	.34814	.65186	.98947	26	34	.37028	.38257	.61743	.98771	26
35	9.33818	9.34874	10.65126	9.98944	25	35	9.37081	9.38313	10.61687	9.98768	25
36	.33874	.34933	.65067	.98941	24	36	.37133	.38368	.61632	.98765	24
37	.33931	.34992	.65008	.98938	23	37	.37185	.38423	.61577	.98762	23
38	.33987	.35051	.64949	.98936	22	38	.37237	.38479	.61521	.98759	22
39	.34043	.35111	.64889	.98933	21	39	.37289	.38534	.61466	.98756	21
40	9.34100	9.35170	10.64830	9.98930	20	40	9.37341	9.38589	10.61411	9.98753	20
41	.34156	.35229	.64771	.98927	19	41	.37393	.38644	.61356	.98750	19
42	.34212	.35288	.64712	.98924	18	42	.37445	.38699	.61301	.98746	18
43	.34268	.35347	.64653	.98921	17	43	.37497	.38754	.61246	.98743	17
44	.34324	.35405	.64595	.98919	16	44	.37549	.38808	.61192	.98740	16
45	9.34380	9.35464	10.64536	9.98916	15	45	9.37600	9.38863	10.61137	9.98737	15
46	.34436	.35523	.64477	.98913	14	46	.37652	.38918	.61082	.98734	14
47	.34491	.35581	.64419	.98910	13	47	.37703	.38972	.61028	.98731	13
48	.34547	.35640	.64360	.98907	12	48	.37755	.39027	.60973	.98728	12
49	.34602	.35698	.64302	.98904	11	49	.37806	.39082	.60918	.98725	11
50	9.34658	9.35757	10.64243	9.98901	10	50	9.37858	9.39136	10.60864	9.98722	10
51	.34713	.35815	.64185	.98898	9	51	.37909	.39190	.60810	.98719	9
52	.34769	.35873	.64127	.98896	8	52	.37960	.39245	.60755	.98517	8
53	.34824	.35931	.64069	.98893	7	53	.38011	.39299	.60701	.98712	7
54	.34879	.35989	.64011	.98890	6	54	.38062	.39353	.60647	.98709	6
55	9.34934	9.36047	10.63953	9.98887	5	55	9.38113	9.39407	10.60593	9.98706	5
56	.34989	.36105	.63895	.98884	4	56	.38164	.39461	.60539	.98703	4
57	.35044	.36163	.63837	.98881	3	57	.38215	.39515	.60485	.98700	3
58	.35099	.36221	.63779	.98878	2	58	.38266	.39569	.60431	.98697	2
59	.35154	.36279	.63721	.98875	1	59	.38317	.39623	.60377	.98694	1
60	9.35209	9.36336	10.63664	9.98872	0	60	9.38368	9.39677	10.60323	9.98690	0
	Cosine.	Cotang.	Tang.	Sine.	′		Cosine.	Cotang.	Tang.	Sine.	′
					77°						76°

*Log secant = colog cosine = 1 − log cosine; log cosecant = colog sine = 1 − log sine.

Ex.—Log sec 12° − 30′ = 10.01042. *Ex.*—Log cosec 12° − 30′ = 10.66466.

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5.—Logarithmic Sines, Tangents, Cotangents, Cosines.—(Cont'd.)

(Secants, Cosecants.)*

14° '	Sine.	Tang.	Cotang.	Cosine.		15° '	Sine.	Tang.	Cotang.	Cosine.	
0	9.38368	9.39677	10.60323	9.98690	60	0	9.41200	9.42805	10.57195	9.98494	60
1	.38418	.39731	.60269	.98687	59	1	.41347	.42856	.57144	.98491	59
2	.38469	.39785	.60215	.98684	58	2	.41394	.42906	.57094	.98488	58
3	.38519	.39838	.60162	.98681	57	3	.41441	.42957	.57043	.98484	57
4	.38570	.39892	.60108	.98678	56	4	.41488	.43007	.56993	.98481	56
5	9.38620	9.39945	10.60055	9.98675	55	5	9.41535	9.43057	10.56943	9.98477	55
6	.38670	.39999	.60001	.98671	54	6	.41582	.43108	.56892	.98474	54
7	.38721	.40052	.59948	.98668	53	7	.41628	.43158	.56842	.98471	53
8	.38771	.40106	.59894	.98665	52	8	.41675	.43208	.56792	.98467	52
9	.38821	.40159	.59841	.98662	51	9	.41722	.43258	.56742	.98464	51
10	9.38871	9.40212	10.59788	9.98659	50	10	9.41768	9.43308	10.56692	9.98460	50
11	.38921	.40266	.59734	.98656	49	11	.41815	.43358	.56642	.98457	49
12	.38971	.40319	.59681	.98652	48	12	.41861	.43408	.56592	.98453	48
13	.39021	.40372	.59628	.98649	47	13	.41908	.43458	.56542	.98450	47
14	.39071	.40425	.59575	.98646	46	14	.41954	.43508	.56492	.98447	46
15	9.39121	9.40478	10.59522	9.98643	45	15	9.42001	9.43558	10.56442	9.98443	45
16	.39170	.40531	.59469	.98640	44	16	.42047	.43607	.56393	.98440	44
17	.39220	.40584	.59416	.98636	43	17	.42093	.43657	.56343	.98436	43
18	.39270	.40636	.59364	.98633	42	18	.42140	.43707	.56293	.98433	42
19	.39319	.40689	.59311	.98630	41	19	.42186	.43756	.56244	.98429	41
20	9.39369	9.40742	10.59258	9.98627	40	20	9.42232	9.43806	10.56194	9.98426	40
21	.39418	.40795	.59205	.98623	39	21	.42278	.43855	.56145	.98422	39
22	.39467	.40847	.59153	.98620	38	22	.42324	.43905	.56095	.98419	38
23	.39517	.40900	.59100	.98617	37	23	.42370	.43954	.56046	.98415	37
24	.39566	.40952	.59048	.98614	36	24	.42416	.44004	.55996	.98412	36
25	9.39615	9.41005	10.58995	9.98610	35	25	9.42461	9.44053	10.55947	9.98409	35
26	.39664	.41057	.58943	.98607	34	26	.42507	.44102	.55898	.98405	34
27	.39713	.41109	.58891	.98604	33	27	.42553	.44151	.55849	.98402	33
28	.39762	.41161	.58839	.98601	32	28	.42599	.44201	.55799	.98398	32
29	.39811	.41214	.58786	.98597	31	29	.42644	.44250	.55750	.98395	31
30	9.39860	9.41266	10.58734	9.98594	30	30	9.42690	9.44299	10.55701	9.98391	30
31	.39909	.41318	.58682	.98591	29	31	.42735	.44348	.55652	.98388	29
32	.39958	.41370	.58630	.98588	28	32	.42781	.44397	.55603	.98384	28
33	.40006	.41422	.58578	.98584	27	33	.42826	.44446	.55554	.98381	27
34	.40055	.41474	.58526	.98581	26	34	.42872	.44495	.55505	.98377	26
35	9.40103	9.41526	10.58474	9.98578	25	35	9.42917	9.44544	10.55456	9.98373	25
36	.40152	.41578	.58422	.98574	24	36	.42962	.44592	.55408	.98370	24
37	.40200	.41629	.58371	.98571	23	37	.43008	.44641	.55359	.98366	23
38	.40249	.41681	.58319	.98568	22	38	.43053	.44690	.55310	.98363	22
39	.40297	.41733	.58267	.98565	21	39	.43098	.44738	.55262	.98359	21
40	9.40346	9.41784	10.58216	9.98561	20	40	9.43143	9.44787	10.55213	9.98356	20
41	.40394	.41836	.58164	.98558	19	41	.43188	.44836	.55164	.98352	19
42	.40442	.41887	.58113	.98555	18	42	.43233	.44884	.55116	.98349	18
43	.40490	.41939	.58061	.98551	17	43	.43278	.44933	.55067	.98345	17
44	.40538	.41990	.58010	.98548	16	44	.43323	.44981	.55019	.98342	16
45	9.40586	9.42041	10.57959	9.98545	15	45	9.43367	9.45029	10.54971	9.98338	15
46	.40634	.42093	.57907	.98541	14	46	.43412	.45078	.54922	.98334	14
47	.40682	.42144	.57856	.98538	13	47	.43457	.45126	.54874	.98331	13
48	.40730	.42195	.57805	.98535	12	48	.43502	.45174	.54826	.98327	12
49	.40778	.42246	.57754	.98531	11	49	.43546	.45222	.54778	.98324	11
50	9.40825	9.42297	10.57703	9.98528	10	50	9.43591	9.45271	10.54729	9.98320	10
51	.40873	.42348	.57652	.98525	9	51	.43635	.45319	.54681	.98317	9
52	.40921	.42399	.57601	.98521	8	52	.43680	.45367	.54633	.98313	8
53	.40968	.42450	.57550	.98518	7	53	.43724	.45415	.54585	.98309	7
54	.41016	.42501	.57499	.98515	6	54	.43769	.45463	.54537	.98306	6
55	9.41063	9.42552	10.57448	9.98511	5	55	9.43813	9.45511	10.54489	9.98302	5
56	.41111	.42603	.57397	.98508	4	56	.43857	.45559	.54441	.98299	4
57	.41158	.42653	.57347	.98505	3	57	.43901	.45606	.54394	.98295	3
58	.41205	.42704	.57296	.98501	2	58	.43946	.45654	.54346	.98291	2
59	.41252	.42755	.57245	.98498	1	59	.43990	.45702	.54298	.98288	1
60	9.41300	9.42805	10.57195	9.98494	0	60	9.44034	9.45750	10.54250	9.98284	0
	Cosine.	Cotang.	Tang.	Sine.	'		Cosine.	Cotang.	Tang.	Sine.	'
					75°						74°

*Log secant = colog cosine = 1 − log cosine; log cosecant = colog sine = 1 − log sine.

Ex.—Log sec 14° − 30′ = 10.01406. *Ex.*—Log cosec 14° − 30′ = 10.60140.

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5.—Logarithmic Sines, Tangents, Cotangents, Cosines.—(Cont'd.)
(Secants, Cosecants.)*

16° ′	Sine.	Tang.	Cotang.	Cosine.		17° ′	Sine.	Tang.	Cotang.	Cosine.	
0	9.44034	9.45750	10.54250	9.98284	60	0	9.46594	9.48534	10.51466	9.98060	6
1	.44078	.45797	.54203	.98281	59	1	.46635	.48579	.51421	.98056	5
2	.44122	.45845	.54155	.98277	58	2	.46676	.48624	.51376	.98052	5
3	.44166	.45892	.54108	.98273	57	3	.46717	.48669	.51331	.98048	5
4	.44210	.45940	54060	.98270	56	4	.46758	.48714	.51286	.98044	5
5	9.44253	9.45987	10.54013	9.98266	55	5	9.46800	9.48759	10.51241	9.98040	5
6	.44297	.46035	.53965	.98262	54	6	.46841	.48804	.51196	.98036	5
7	.44341	.46082	.53918	.98259	53	7	.46882	.48849	.51151	.98032	5
8	.44385	.46130	.53870	.98255	52	8	.46923	.48894	.51106	.98029	5
9	.44428	.46177	.53823	.98251	51	9	.46964	.48939	.51061	.98025	5
10	9.44472	9.46224	10.53776	9.98248	50	10	9.47005	9.48984	10.51016	9.98021	5
11	.44516	.46271	.53729	.98244	49	11	.47045	.49029	.50971	.98017	4
12	.44559	.46319	.53681	.98240	48	12	.47086	.49073	.50927	.98013	4
13	.44602	.46366	.53634	.98237	47	13	.47127	.49118	.50882	.98009	4
14	.44646	.46413	.53587	.98233	46	14	.47168	.49163	.50837	.98005	4
15	9.44689	9.46460	10.53540	9.98229	45	15	9.47209	9.49207	10.50793	9.98001	4
16	.44733	.46507	.53493	.98226	44	16	.47249	.49252	.50748	.97997	4
17	.44776	.46554	.53446	.98222	43	17	.47290	.49296	.50704	.97993	4
18	.44819	.46601	.53399	.98218	42	18	.47330	.49341	.50659	.97989	4
19	.44862	.46648	.53352	.98215	41	19	.47371	.49385	.50615	.97986	4
20	9.44905	9.46694	10.53306	9.98211	40	20	9.47411	9.49430	10.50570	9.97982	4
21	.44948	.46741	.53259	.98207	39	21	.47452	.49474	.50526	.97978	3
22	.44992	.46788	.53212	.98204	38	22	.47492	.49519	.50481	.97974	3
23	.45035	.46835	.53165	.98200	37	23	.47533	.49563	.50437	.97970	3
24	.45077	.46881	.53119	.98196	36	24	.47573	.49607	.50393	.97966	3
25	9.45120	9.46928	10.53072	9.98192	35	25	9.47613	9.49652	10.50348	9.97962	3
26	.45163	.46975	.53025	.98189	34	26	.47654	.49696	.50304	.97958	3
27	.45206	.47021	.52979	.98185	33	27	.47694	.49740	.50260	.97954	3
28	.45249	.47068	.52932	.98181	32	28	.47734	.49784	.50216	.97950	3
29	.45292	.47114	.52886	.98177	31	29	.47774	.49828	.50172	.97946	3
30	9.45334	9.47160	10.52840	9.98174	30	30	9.47814	9.49872	10.50128	9.97942	3
31	.45377	.47207	.52793	.98170	29	31	.47854	.49916	.50084	.97938	2
32	.45419	.47253	.52747	.98166	28	32	.47894	.49960	.50040	.97934	2
33	.45462	.47299	.52701	.98162	27	33	.47934	.50004	.49996	.97930	2
34	.45504	.47346	.52654	.98159	26	34	.47974	.50048	.49952	.97926	2
35	9.45547	9.47392	10.52608	9.98155	25	35	9.48014	9.50092	10.49908	9.97922	2
36	.45589	.47438	.52562	.98151	24	36	.48054	.50136	.49864	.97918	2
37	.45632	.47484	.52516	.98147	23	37	.48094	.50180	.49820	.97914	2
38	.45674	.47530	.52470	.98144	22	38	.48133	.50223	.49777	.97910	2
39	.45716	.47576	.52424	.98140	21	39	.48173	.50267	.49733	.97906	2
40	9.45758	9.47622	10.52378	9.98136	20	40	9.48213	9.50311	10.49689	9.97902	2
41	.45801	.47668	.52332	.98132	19	41	.48252	.50355	.49645	.97898	1
42	.45843	.47714	.52286	.98129	18	42	.48292	.50398	.49602	.97894	1
43	.45885	.47760	.52240	.98125	17	43	.48332	.50442	.49558	.97890	1
44	.45927	.47806	.52194	.98121	16	44	.48371	.50485	.49515	.97886	1
45	9.45969	9.47852	10.52148	9.98117	15	45	9.48411	9.50529	10.49471	9.97882	1
46	.46011	.47897	.52103	.98113	14	46	.48450	.50572	.49428	.97878	1
47	.46053	.47943	.52057	.98110	13	47	.48490	.50616	.49384	.97874	1
48	.46095	.47989	.52011	.98106	12	48	.48529	.50659	.49341	.97870	1
49	.46136	.48035	.51965	.98102	11	49	.48568	.50703	.49297	.97866	1
50	9.46178	9.48080	10.51920	9.98098	10	50	9.48607	9.50746	10.49254	9.97861	1
51	.46220	.48126	.51874	.98094	9	51	.48647	.50789	.49211	.97857	
52	.46262	.48171	.51829	.98090	8	52	.48686	.50833	.49167	.97853	
53	.46303	.48217	.51783	.98087	7	53	.48725	.50876	.49124	.97849	
54	.46345	.48262	.51738	.98083	6	54	.48764	.50919	.49081	.97845	
55	9.46386	9.48307	10.51693	9.98079	5	55	9.48803	9.50962	10.49038	9.97841	
56	.46428	.48353	.51647	.98075	4	56	.48842	.51005	.48995	.97837	
57	.46469	.48398	.51602	.98071	3	57	.48881	.51048	.48952	.97833	
58	.46511	.48443	.51557	.98067	2	58	.48920	.51092	.48908	.97829	
59	.46552	.48489	.51511	.98063	1	59	.48959	.51135	.48865	.97825	
60	9.46594	9.48534	10.51466	9.98060	0	60	9.48998	9.51178	10.48822	9.97821	
	Cosine.	Cotang.	Tang.	Sine.	′		Cosine.	Cotang.	Tang.	Sine.	′
					73°						72

*Log secant = colog cosine = 1 − log cosine; log cosecant = colog sine = 1 − log sine.

Ex.—Log sec 16° − 30′ = 10.01826. *Ex.*—Log cosec 16° − 30′ = 10.54666

5.—Logarithmic Sines, Tangents, Cotangents, Cosines.—(Cont'd.)
(Secants, Cosecants.)*

18° '	Sine.	Tang.	Cotang.	Cosine.		19° '	Sine.	Tang.	Cotang.	Cosine.	
0	9.49998	9.51178	10.48822	9.97821	60	0	9.51264	9.53697	10.46303	9.97567	60
1	.49037	.51221	.48779	.97817	59	1	.51301	.53738	.46262	.97563	59
2	.49076	.51264	.48736	.97812	58	2	.51338	.53779	.46221	.97558	58
3	.49115	.51306	.48694	.97808	57	3	.51374	.53820	.46180	.97554	57
4	.49153	.51349	.48651	.97804	56	4	.51411	.53861	.46139	.97550	56
5	9.49192	9.51392	10.48608	9.97800	55	5	9.51447	9.53902	10.46098	9.97545	55
6	.49231	.51435	.48565	.97796	54	6	.51484	.53943	.46057	.97541	54
7	.49269	.51478	.48522	.97792	53	7	.51520	.53984	.46016	.97536	53
8	.49308	.51520	.48480	.97788	52	8	.51557	.54025	.45975	.97532	52
9	.49347	.51563	.48437	.97784	51	9	.51593	.54065	.45935	.97528	51
10	9.49385	9.51606	10.48394	9.97779	50	10	9.51629	9.54106	10.45894	9.97523	50
11	.49424	.51648	.48352	.97775	49	11	.51666	.54147	.45853	.97519	49
12	.49462	.51691	.48309	.97771	48	12	.51702	.54187	.45813	.97515	48
13	.49500	.51734	.48266	.97767	47	13	.51738	.54228	.45772	.97510	47
14	.49539	.51776	.48224	.97763	46	14	.51774	.54269	.45731	.97506	46
15	9.49577	9.51819	10.48181	9.97759	45	15	9.51811	9.54309	10.45691	9.97501	45
16	.49615	.51861	.48139	.97754	44	16	.51847	.54350	.45650	.97497	44
17	.49654	.51903	.48097	.97750	43	17	.51883	.54390	.45610	.97492	43
18	.49692	.51946	.48054	.97746	42	18	.51919	.54431	.45569	.97488	42
19	.49730	.51988	.48012	.97742	41	19	.51955	.54471	.45529	.97484	41
20	9.49768	9.52031	10.47969	9.97738	40	20	9.51991	9.54512	10.45488	9.97479	40
21	.49806	.52073	.47927	.97734	39	21	.52027	.54552	.45448	.97475	39
22	.49844	.52115	.47885	.97729	38	22	.52063	.54593	.45407	.97470	38
23	.49882	.52157	.47843	.97725	37	23	.52099	.54633	.45367	.97466	37
24	.49920	.52200	.47800	.97721	36	24	.52135	.54673	.45327	.97461	36
25	9.49958	9.52242	10.47758	9.97717	35	25	9.52171	9.54714	10.45286	9.97457	35
26	.49996	.52284	.47716	.97713	34	26	.52207	.54754	.45246	.97453	34
27	.50034	.52326	.47674	.97708	33	27	.52242	.54794	.45206	.97448	33
28	.50072	.52368	.47632	.97704	32	28	.52278	.54835	.45165	.97444	32
29	.50110	.52410	.47590	.97700	31	29	.52314	.54875	.45125	.97439	31
30	9.50148	9.52452	10.47548	9.97696	30	30	9.52350	9.54915	10.45085	9.97435	30
31	.50185	.52494	.47506	.97691	29	31	.52385	.54955	.45045	.97430	29
32	.50223	.52536	.47464	.97687	28	32	.52421	.54995	.45005	.97426	28
33	.50261	.52578	.47422	.97683	27	33	.52456	.55035	.44965	.97421	27
34	.50298	.52620	.47380	.97679	26	34	.52492	.55075	.44925	.97417	26
35	9.50336	9.52661	10.47339	9.97674	25	35	9.52527	9.55115	10.44885	9.97412	25
36	.50374	.52703	.47297	.97670	24	36	.52563	.55155	.44845	.97408	24
37	.50411	.52745	.47255	.97666	23	37	.52598	.55195	.44805	.97403	23
38	.50449	.52787	.47213	.97662	22	38	.52634	.55235	.44765	.97399	22
39	.50486	.52829	.47171	.97657	21	39	.52669	.55275	.44725	.97394	21
40	9.50523	9.52870	10.47130	9.97653	20	40	9.52705	9.55315	10.44685	9.97390	20
41	.50561	.52912	.47088	.97649	19	41	.52740	.55355	.44645	.97385	19
42	.50598	.52953	.47047	.97645	18	42	.52775	.55395	.44605	.97381	18
43	.50635	.52995	.47005	.97640	17	43	.52811	.55434	.44566	.97376	17
44	.50673	.53037	.46963	.97636	16	44	.52846	.55474	.44526	.97372	16
45	9.50710	9.53078	10.46922	9.97632	15	45	9.52881	9.55514	10.44486	9.97367	15
46	.50747	.53120	.46880	.97628	14	46	.52916	.55554	.44446	.97363	14
47	.50784	.53161	.46839	.97623	13	47	.52951	.55593	.44407	.97358	13
48	.50821	.53202	.46798	.97619	12	48	.52986	.55633	.44367	.97353	12
49	.50858	.53244	.46756	.97615	11	49	.53021	.55673	.44327	.97349	11
50	9.50896	9.53285	10.46715	9.97610	10	50	9.53056	9.55712	10.44288	9.97344	10
51	.50933	.53327	.46673	.97606	9	51	.53092	.55752	.44248	.97340	9
52	.50970	.53368	.46632	.97602	8	52	.53126	.55791	.44209	.97335	8
53	.51007	.53409	.46591	.97597	7	53	.53161	.55831	.44169	.97331	7
54	.51043	.53450	.46550	.97593	6	54	.53196	.55870	.44130	.97326	6
55	9.51080	9.53492	10.46508	9.97589	5	55	9.53231	9.55910	10.44090	9.97322	5
56	.51117	.53533	.46467	.97584	4	56	.53266	.55949	.44051	.97317	4
57	.51154	.53574	.46426	.97580	3	57	.53301	.55989	.44011	.97312	3
58	.51191	.53615	.46385	.97576	2	58	.53336	.56028	.43972	.97308	2
59	.51227	.53656	.46344	.97571	1	59	.53370	.56067	.43933	.97303	1
60	9.51264	9.53697	10.46303	9.97567	0	60	9.53405	9.56107	10.43893	9.97299	0
	Cosine.	Cotang.	Tang.	Sine.	'		Cosine.	Cotang.	Tang.	Sine.	'
					71°						70°

*Log secant = colog cosine = 1 − log cosine; log cosecant = colog sine = 1 − log sine.

Ex.—Log sec 18° − 30′ = 10.02304. *Ex.*—Log cosec 18° − 30′ = 10.49852.

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5.—Logarithmic Sines, Tangents, Cotangents, Cosines.—(Cont'd.)

(Secants, Cosecants.)*

20° ′	Sine.	Tang.	Cotang.	Cosine.		21° ′	Sine.	Tang.	Cotang.	Cosine.	
0	9.53405	9.56107	10.43893	9.97299	60	0	9.55433	9.58418	10.41582	9.97015	60
1	.53440	.56146	.43854	.97294	59	1	.55466	.58455	.41545	.97010	59
2	.53475	.56185	.43815	.97289	58	2	.55499	.58493	.41507	.97005	58
3	.53509	.56224	.43776	.97285	57	3	.55532	.58531	.41469	.97001	57
4	.53544	.56264	.43736	.97280	56	4	.55564	.58569	.41431	.96996	56
5	9.53578	9.56303	10.43697	9.97276	55	5	9.55597	9.58606	10.41394	9.96991	55
6	.53613	.56342	.43658	.97271	54	6	.55630	.58644	.41356	.96986	54
7	.53647	.56381	.43619	.97266	53	7	.55663	.58681	.41319	.96981	53
8	.53682	.56420	.43580	.97262	52	8	.55695	.58719	.41281	.96976	52
9	.53716	.56459	.43541	.97257	51	9	.55728	.58757	.41243	.96971	51
10	9.53751	9.56498	10.43502	9.97252	50	10	9.55761	9.58794	10.41206	9.96966	50
11	.53785	.56537	.43463	.97248	49	11	.55793	.58832	.41168	.96962	49
12	.53819	.56576	.43424	.97243	48	12	.55826	.58869	.41131	.96957	48
13	.53854	.56615	.43385	.97238	47	13	.55858	.58907	.41093	.96952	47
14	.53888	.56654	.43346	.97234	46	14	.55891	.58944	.41056	.96947	46
15	9.53922	9.56693	10.43307	9.97229	45	15	9.55923	9.58981	10.41019	9.96942	45
16	.53957	.56732	.43268	.97224	44	16	.55956	.59019	.40981	.96937	44
17	.53991	.56771	.43229	.97220	43	17	.55988	.59056	.40944	.96932	43
18	.54025	.56810	.43190	.97215	42	18	.56021	.59094	.40906	.96927	42
19	.54059	.56849	.43151	.97210	41	19	.56053	.59131	.40869	.96922	41
20	9.54093	9.56887	10.43113	9.97206	40	20	9.56085	9.59168	10.40832	9.96917	40
21	.54127	.56926	.43074	.97201	39	21	.56118	.59205	.40795	.96912	39
22	.54161	.56965	.43035	.97196	38	22	.56150	.59243	.40757	.96907	38
23	.54195	.57004	.42996	.97192	37	23	.56182	.59280	.40720	.96903	37
24	.54229	.57042	.42958	.97187	36	24	.56215	.59317	.40683	.96898	36
25	9.54263	9.57081	10.42919	9.97182	35	25	9.56247	9.59354	10.40646	9.96893	35
26	.54297	.57120	.42880	.97178	34	26	.56279	.59391	.40609	.96888	34
27	.54331	.57158	.42842	.97173	33	27	.56311	.59429	.40571	.96883	33
28	.54365	.57197	.42803	.97168	32	28	.56343	.59466	.40534	.96878	32
29	.54399	.57235	.42765	.97163	31	29	.56375	.59503	.40497	.96873	31
30	9.54433	9.57274	10.42726	9.97159	30	30	9.56408	9.59540	10.40460	9.96868	30
31	.54466	.57312	.42688	.97154	29	31	.56440	.59577	.40423	.96863	29
32	.54500	.57351	.42649	.97149	28	32	.56472	.59614	.40386	.96858	28
33	.54534	.57389	.42611	.97145	27	33	.56504	.59651	.40349	.96853	27
34	.54567	.57428	.42572	.97140	26	34	.56536	.59688	.40312	.96848	26
35	9.54601	9.57466	10.42534	9.97135	25	35	9.56568	9.59725	10.40275	9.96843	25
36	.54635	.57504	.42496	.97130	24	36	.56599	.59762	.40238	.96838	24
37	.54668	.57543	.42457	.97126	23	37	.56631	.59799	.40201	.96833	23
38	.54702	.57581	.42419	.97121	22	38	.56663	.59835	.40165	.96828	22
39	.54735	.57619	.42381	.97116	21	39	.56695	.59872	.40128	.96823	21
40	9.54769	9.57658	10.42342	9.97111	20	40	9.56727	9.59909	10.40091	9.96818	20
41	.54802	.57696	.42304	.97107	19	41	.56759	.59946	.40054	.96813	19
42	.54836	.57734	.42266	.97102	18	42	.56790	.59983	.40017	.96808	18
43	.54869	.57772	.42228	.97097	17	43	.56822	.60019	.39981	.96803	17
44	.54903	.57810	.42190	.97092	16	44	.56854	.60056	.39944	.96798	16
45	9.54936	9.57849	10.42151	9.97087	15	45	9.56886	9.60093	10.39907	9.96793	15
46	.54969	.57887	.42113	.97083	14	46	.56917	.60130	.39870	.96788	14
47	.55003	.57925	.42075	.97078	13	47	.56949	.60166	.39834	.96783	13
48	.55036	.57963	.42037	.97073	12	48	.56980	.60203	.39797	.96778	12
49	.55069	.58001	.41999	.97068	11	49	.57012	.60240	.39760	.96772	11
50	9.55102	9.58039	10.41961	9.97063	10	50	9.57044	9.60276	10.39724	9.96767	10
51	.55136	.58077	.41923	.97059	9	51	.57075	.60313	.39687	.96762	9
52	.55169	.58115	.41885	.97054	8	52	.57107	.60349	.39651	.96757	8
53	.55202	.58153	.41847	.97049	7	53	.57138	.60386	.39614	.96752	7
54	.55235	.58191	.41809	.97044	6	54	.57169	.60422	.39578	.96747	6
55	9.55268	9.58229	10.41771	9.97039	5	55	9.57201	9.60459	10.39541	9.96742	5
56	.55301	.58267	.41733	.97035	4	56	.57232	.60495	.39505	.96737	4
57	.55334	.58304	.41696	.97030	3	57	.57264	.60532	.39468	.96732	3
58	.55367	.58342	.41658	.97025	2	58	.57295	.60568	.39432	.96727	2
59	.55400	.58380	.41620	.97020	1	59	.57326	.60605	.39395	.96722	1
60	9.55433	9.58418	10.41582	9.97015	0	60	9.57358	9.60641	10.39359	9.96717	0
	Cosine.	Cotang.	Tang.	Sine.	′		Cosine.	Cotang.	Tang.	Sine.	′
					69°						68°

*Log secant = colog cosine = 1 − log cosine; log cosecant = colog sine = 1 − log sine.

Ex.—Log sec 20° − 30′ = 10.02841. *Ex.*—Log cosec 20° − 30′ = 10.45567.

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5.—Logarithmic Sines, Tangents, Cotangents, Cosines.—(Cont'd.)
(Secants, Cosecants.)*

22° 23°

′	Sine.	Tang.	Cotang.	Cosine.		′	Sine.	Tang.	Cotang.	Cosine.	
0	9.57356	9.60641	10.39359	9.96717	60	0	9.59188	9.62785	10.37215	9.96408	60
1	.57389	.60677	.39323	.96711	59	1	.59218	.62820	.37180	.96397	59
2	.57420	.60714	.39286	.96706	58	2	.59247	.62855	.37145	.96392	58
3	.57451	.60750	.39250	.96701	57	3	.59277	.62890	.37110	.96387	57
4	.57482	.60786	.39214	.96696	56	4	.59307	.62926	.37074	.96381	56
5	9.57514	9.60823	10.39177	9.96691	55	5	9.59336	9.62961	10.37039	9.96376	55
6	.57545	.60859	.39141	.96686	54	6	.59366	.62996	.37004	.96370	54
7	.57576	.60896	.39105	.96681	53	7	.59396	.63031	.36969	.96365	53
8	.57607	.60931	.39069	.96676	52	8	.59425	.63066	.36934	.96360	52
9	.57638	.60967	.39033	.96670	51	9	.59455	.63101	.36899	.96354	51
10	9.57669	9.61004	10.38996	9.96665	50	10	9.59484	9.63135	10.36865	9.96349	50
11	.57700	.61040	.38960	.96660	49	11	.59514	.63170	.36830	.96343	49
12	.57731	.61076	.38924	.96655	48	12	.59543	.63205	.36795	.96338	48
13	.57762	.61112	.38888	.96650	47	13	.59573	.63240	.36760	.96333	47
14	.57793	.61148	.38852	.96645	46	14	.59602	.63275	.36725	.96327	46
15	9.57824	9.61184	10.38816	9.96640	45	15	9.59632	9.63310	10.36690	9.96322	45
16	.57855	.61220	.38780	.96634	44	16	.59661	.63345	.36655	.96316	44
17	.57885	.61256	.38744	.96629	43	17	.59690	.63379	.36621	.96311	43
18	.57916	.61292	.38708	.96624	42	18	.59720	.63414	.36586	.96305	42
19	.57947	.61328	.38672	.96619	41	19	.59749	.63449	.36551	.96300	41
20	9.57978	9.61364	10.38636	9.96614	40	20	9.59778	9.63484	10.36516	9.96294	40
21	.58008	.61400	.38600	.96608	39	21	.59808	.63519	.36481	.96289	39
22	.58039	.61436	.38564	.96603	38	22	.59837	.63553	.36447	.96284	38
23	.58070	.61472	.38528	.96598	37	23	.59866	.63588	.36412	.96278	37
24	.58101	.61508	.38492	.96593	36	24	.59895	.63623	.36377	.96273	36
25	9.58131	9.61544	10.38456	9.96588	35	25	9.59924	9.63657	10.36343	9.96267	35
26	.58162	.61579	.38421	.96582	34	26	.59954	.63692	.36308	.96262	34
27	.58192	.61615	.38385	.96577	33	27	.59983	.63726	.36274	.96256	33
28	.58223	.61651	.38349	.96572	32	28	.60012	.63761	.36239	.96251	32
29	.58253	.61687	.38313	.96567	31	29	.60041	.63796	.36204	.96245	31
30	9.58284	9.61722	10.38278	9.96562	30	30	9.60070	9.63830	10.36170	9.96240	30
31	.58314	.61758	.38242	.96556	29	31	.60099	.63865	.36135	.96234	29
32	.58345	.61794	.38206	.96551	28	32	.60128	.63899	.36101	.96229	28
33	.58375	.61830	.38170	.96546	27	33	.60157	.63934	.36066	.96223	27
34	.58406	.61865	.38135	.96541	26	34	.60186	.63968	.36032	.96218	26
35	9.58436	9.61901	10.38099	9.96535	25	35	9.60215	9.64003	10.35997	9.96212	25
36	.58467	.61936	.38064	.96530	24	36	.60244	.64037	.35963	.96207	24
37	.58497	.61972	.38028	.96525	23	37	.60273	.64072	.35928	.96201	23
38	.58527	.62008	.37992	.96520	22	38	.60302	.64106	.35894	.96196	22
39	.58557	.62043	.37957	.96514	21	39	.60331	.64140	.35860	.96190	21
40	9.58588	9.62079	10.37921	9.96509	20	40	9.60359	9.64175	10.35825	9.96185	20
41	.58618	.62114	.37886	.96504	19	41	.60388	.64209	.35791	.96179	19
42	.58648	.62150	.37850	.96498	18	42	.60417	.64243	.35757	.96174	18
43	.58678	.62185	.37815	.96493	17	43	.60446	.64278	.35722	.96168	17
44	.58709	.62221	.37779	.96488	16	44	.60474	.64312	.35688	.96162	16
45	9.58739	9.62256	10.37744	9.96483	15	45	9.60503	9.64346	10.35654	9.96157	15
46	.58769	.62292	.37708	.96477	14	46	.60532	.64381	.35619	.96151	14
47	.58799	.62327	.37673	.96472	13	47	.60561	.64415	.35585	.96146	13
48	.58829	.62362	.37638	.96467	12	48	.60589	.64449	.35551	.96140	12
49	.58859	.62398	.37602	.96461	11	49	.60618	.64483	.35517	.96135	11
50	9.58889	9.62433	10.37567	9.96456	10	50	9.60646	9.64517	10.35483	9.96129	10
51	.58919	.62468	.37532	.96451	9	51	.60675	.64552	.35448	.96123	9
52	.58949	.62504	.37496	.96445	8	52	.60704	.64586	.35414	.96118	8
53	.58979	.62539	.37461	.96440	7	53	.60732	.64620	.35380	.96112	7
54	.59009	.62574	.37426	.96435	6	54	.60761	.64654	.35346	.96107	6
55	9.59039	9.62609	10.37391	9.96429	5	55	9.60789	9.64688	10.35312	9.96101	5
56	.59069	.62645	.37355	.96424	4	56	.60818	.64722	.35278	.96095	4
57	.59098	.62680	.37320	.96419	3	57	.60846	.64756	.35244	.96090	3
58	.59128	.62715	.37285	.96413	2	58	.60875	.64790	.35210	.96084	2
59	.59158	.62750	.37250	.96408	1	59	.60903	.64824	.35176	.96079	1
60	9.59188	9.62785	10.37215	9.96403	0	60	9.60931	9.64858	10.35142	9.96073	0
	Cosine.	Cotang.	Tang.	Sine.	′		Cosine.	Cotang.	Tang.	Sine.	′

67° 66°

*Log secant = colog cosine = 1 − log cosine; log cosecant = colog sine = 1 − log sine.

Ex.—Log sec 22° - 30′ = 10.03438. *Ex.*—Log cosec 22° - 30′ = 10.41716.

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5.—Logarithmic Sines, Tangents, Cotangents, Cosines.—(Cont'd.)

(Secants, Cosecants.)*

24° ′	Sine.	Tang.	Cotang.	Cosine.	‖ ′	25° Sine.	Tang.	Cotang.	Cosine.	
0	9.60931	9.64858	10.35142	9.96073	60	0	9.62595	9.66867	10.33133	9.95728	60
1	.60960	.64892	.35108	.96067	59	1	.62622	.66900	.33100	.95722	59
2	.60988	.64926	.35074	.96062	58	2	.62649	.66933	.33067	.95716	58
3	.61016	.64960	.35040	.96056	57	3	.62676	.66966	.33034	.95710	57
4	.61045	.64994	.35006	.96050	56	4	.62703	.66999	.33001	.95704	56
5	9.61073	9.65028	10.34972	9.96045	55	5	9.62730	9.67032	10.32968	9.95698	55
6	.61101	.65062	.34938	.96039	54	6	.62757	.67065	.32935	.95692	54
7	.61129	.65096	.34904	.96034	53	7	.62784	.67098	.32902	.95686	53
8	.61158	.65130	.34870	.96028	52	8	.62811	.67131	.32869	.95680	52
9	.61186	.65164	.34836	.96022	51	9	.62838	.67163	.32837	.95674	51
10	9.61214	9.65197	10.34803	9.96017	50	10	9.62865	9.67196	10.32804	9.95668	50
11	.61242	.65231	.34769	.96011	49	11	.62892	.67229	.32771	.95663	49
12	.61270	.65265	.34735	.96005	48	12	.62918	.67262	.32738	.95657	48
13	.61298	.65299	.34701	.96000	47	13	.62945	.67295	.32705	.95651	47
14	.61326	.65333	.34667	.95994	46	14	.62972	.67327	.32673	.95645	46
15	9.61354	9.65366	10.34634	9.95988	45	15	9.62999	9.67360	10.32640	9.95639	45
16	.61382	.65400	.34600	.95982	44	16	.63026	.67393	.32607	.95633	44
17	.61411	.65434	.34566	.95977	43	17	.63052	.67426	.32574	.95627	43
18	.61438	.65467	.34533	.95971	42	18	.63079	.67458	.32542	.95621	42
19	.61466	.65501	.34499	.95965	41	19	.63106	.67491	.32509	.95615	41
20	9.61494	9.65535	10.34465	9.95960	40	20	9.63133	9.67524	10.32476	9.95609	40
21	.61522	.65568	.34432	.95954	39	21	.63159	.67556	.32444	.95603	39
22	.61550	.65602	.34398	.95948	38	22	.63186	.67589	.32411	.95597	38
23	.61578	.65636	.34364	.95942	37	23	.63213	.67622	.32378	.95591	37
24	.61606	.65669	.34331	.95937	36	24	.63239	.67654	.32346	.95585	36
25	9.61634	9.65703	10.34297	9.95931	35	25	9.63266	9.67687	10.32313	9.95579	35
26	.61662	.65736	.34264	.95925	34	26	.63292	.67719	.32281	.95573	34
27	.61689	.65770	.34230	.95920	33	27	.63319	.67752	.32248	.95567	33
28	.61717	.65803	.34197	.95914	32	28	.63345	.67785	.32215	.95561	32
29	.61745	.65837	.34163	.95908	31	29	.63372	.67817	.32183	.95555	31
30	9.61773	9.65870	10.34130	9.95902	30	30	9.63398	9.67850	10.32150	9.95549	30
31	.61800	.65904	.34096	.95897	29	31	.63425	.67882	.32118	.95543	29
32	.61828	.65937	.34063	.95891	28	32	.63451	.67915	.32085	.95537	28
33	.61856	.65971	.34029	.93885	27	33	.63478	.67947	.32053	.95531	27
34	.61883	.66004	.33996	.95879	26	34	.63504	.67980	.32020	.95525	26
35	9.61911	9.66038	10.33962	9.95873	25	35	9.63531	9.68012	10.31988	9.95519	25
36	.61939	.66071	.33929	.95868	24	36	.63557	.68044	.31956	.95513	24
37	.61966	.66104	.33896	.95862	23	37	.63583	.68077	.31923	.95507	23
38	.61994	.66138	.33862	.95856	22	38	.63610	.68109	.31891	.95500	22
39	.62021	.66171	.33829	.95850	21	39	.63636	.68142	.31858	.95494	21
40	9.62049	9.66204	10.33796	9.95844	20	40	9.63662	9.68174	10.31826	9.95488	20
41	.62076	.66238	.33762	.95839	19	41	.63689	.68206	.31794	.95482	19
42	.62104	.66271	.33729	.95833	18	42	.63715	.68239	.31761	.95476	18
43	.62131	.66304	.33696	.95827	17	43	.63741	.68271	.31729	.95470	17
44	.62159	.66337	.33663	.95821	16	44	.63767	.68303	.31697	.95464	16
45	9.62186	9.66371	10.33629	9.95815	15	45	9.63794	9.68336	10.31664	9.95458	15
46	.62214	.66404	.33596	.95810	14	46	.63820	.68368	.31632	.95452	14
47	.62241	.66437	.33563	.95804	13	47	.63846	.68400	.31600	.95446	13
48	.62268	.66470	.33530	.95798	12	48	.63872	.68432	.31568	.95440	12
49	.62296	.66503	.33497	.95792	11	49	.63898	.68465	.31535	.95434	11
50	9.62323	9.66537	10.33463	9.95786	10	50	9.63924	9.68497	10.31503	9.95427	10
51	.62350	.66570	.33430	.95780	9	51	.63950	.68529	.31471	.95421	9
52	.62377	.66603	.33397	.95775	8	52	.63976	.68561	.31439	.95415	8
53	.62405	.66636	.33364	.95769	7	53	.64002	.68593	.31407	.95409	7
54	.62432	.66669	.33331	.95763	6	54	.64028	.68626	.31374	.95403	6
55	9.62459	9.66702	10.33298	9.95757	5	55	9.64054	9.68658	10.31342	9.95397	5
56	.62486	.66735	.33265	.95751	4	56	.64080	.68690	.31310	.95391	4
57	.62513	.66768	.33232	.95745	3	57	.64106	.68722	.31278	.95384	3
58	.62541	.66801	.33199	.95739	2	58	.64132	.68754	.31246	.95378	2
59	.62568	.66834	.33166	.95733	1	59	.64158	.68786	.31214	.95372	1
60	9.62595	9.66867	10.33133	9.95728	0	60	9.64184	9.68818	10.31182	9.95366	0
	Cosine.	Cotang.	Tang.	Sine.	′		Cosine.	Cotang.	Tang.	Sine.	′
					65°						64°

*Log secant = colog cosine = 1 − log cosine; log cosecant = colog sine = 1 − log sine.

Ex.—Log sec 24°− 30′ = 10.04098. *Ex.*—Log cosec 24°− 30′ = 10.38227.

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5.—Logarithmic Sines, Tangents, Cotangents, Cosines.—(Cont'd.)
(Secants, Cosecants.)*

26° ′	Sine.	Tang.	Cotang.	Cosine.		27° ′	Sine.	Tang.	Cotang.	Cosine.	
0	9.64184	9.68818	10.31182	9.95366	60	0	9.65705	9.70717	10.29283	9.94988	60
1	.64210	.68850	.31150	.95360	59	1	.65729	.70748	.29252	.94982	59
2	.64236	.68882	.31118	.95354	58	2	.65754	.70779	.29221	.94975	58
3	.64262	.68914	.31086	.95348	57	3	.65779	.70810	.29190	.94969	57
4	.64288	.68946	.31054	.95341	56	4	.65804	.70841	.29159	.94962	56
5	9.64313	9.68978	10.31022	9.95335	55	5	9.65828	9.70873	10.29127	9.94956	55
6	.64339	.69010	.30990	.95329	54	6	.65853	.70904	.29096	.94949	54
7	.64365	.69042	.30958	.95323	53	7	.65878	.70935	.29065	.94943	53
8	.64391	.69074	.30926	.95317	52	8	.65902	.70966	.29034	.94936	52
9	.64417	.69106	.30894	.95310	51	9	.65927	.70997	.29003	.94930	51
10	9.64442	9.69138	10.30862	9.95304	50	10	9.65952	9.71028	10.28972	9.94923	50
11	.64468	.69170	.30830	.95298	49	11	.65976	.71059	.28941	.94917	49
12	.64494	.69202	.30798	.95292	48	12	.66001	.71090	.28910	.94911	48
13	.64519	.69234	.30766	.95286	47	13	.66025	.71121	.28879	.94904	47
14	.64545	.69266	.30734	.95279	46	14	.66050	.71153	.28847	.94898	46
15	9.64571	9.69298	10.30702	9.95273	45	15	9.66075	9.71184	10.28816	9.94891	45
16	.64596	.69329	.30671	.95267	44	16	.66099	.71215	.28785	.94885	44
17	.64622	.69361	.30639	.95261	43	17	.66124	.71246	.28754	.94878	43
18	.64647	.69393	.30607	.95254	42	18	.66148	.71277	.28723	.94871	42
19	.64673	.69425	.30575	.95248	41	19	.66173	.71308	.28692	.94865	41
20	9.64698	9.69457	10.30543	9.95242	40	20	9.66197	9.71339	10.28661	9.94858	40
21	.64724	.69488	.30512	.95236	39	21	.66221	.71370	.28630	.94852	39
22	.64749	.69520	.30480	.95229	38	22	.66246	.71401	.28599	.94845	38
23	.64775	.69552	.30448	.95223	37	23	.66270	.71431	.28569	.94839	37
24	.64800	.69584	.30416	.95217	36	24	.66295	.71462	.28538	.94832	36
25	9.64826	9.69615	10.30385	9.95211	35	25	9.66319	9.71493	10.28507	9.94826	35
26	.64851	.69647	.30353	.95204	34	26	.66343	.71524	.28476	.94819	34
27	.64877	.69679	.30321	.95198	33	27	.66368	.71555	.28445	.94813	33
28	.64902	.69710	.30290	.95192	32	28	.66392	.71586	.28414	.94806	32
29	.64927	.69742	.30258	.95185	31	29	.66416	.71617	.28383	.94799	31
30	9.64953	9.69774	10.30226	9.95179	30	30	9.66441	9.71648	10.28352	9.94793	30
31	.64978	.69805	.30195	.95173	29	31	.66465	.71679	.28321	.94786	29
32	.65003	.69837	.30163	.95167	28	32	.66489	.71709	.28291	.94780	28
33	.65029	.69868	.30132	.95160	27	33	.66513	.71740	.28260	.94773	27
34	.65054	.69900	.30100	.95154	26	34	.66537	.71771	.28229	.94767	26
35	9.65079	9.69932	10.30068	9.95148	25	35	9.66562	9.71802	10.28198	9.94760	25
36	.65104	.69963	.30037	.95141	24	36	.66586	.71833	.28167	.94753	24
37	.65130	.69995	.30005	.95135	23	37	.66610	.71863	.28137	.94747	23
38	.65155	.70026	.29974	.95129	22	38	.66634	.71894	.28106	.94740	22
39	.65180	.70058	.29942	.95122	21	39	.66658	.71925	.28075	.94734	21
40	9.65205	9.70089	10.29911	9.95116	20	40	9.66682	9.71955	10.28045	9.94727	20
41	.65230	.70121	.29879	.95110	19	41	.66706	.71986	.28014	.94720	19
42	.65255	.70152	.29848	.95103	18	42	.66731	.72017	.27983	.94714	18
43	.65281	.70184	.29816	.95097	17	43	.66755	.72048	.27952	.94707	17
44	.65306	.70215	.29785	.95090	16	44	.66779	.72078	.27922	.94700	16
45	9.65331	9.70247	10.29753	9.95084	15	45	9.66803	9.72109	10.27891	9.94694	15
46	.65356	.70278	.29722	.95078	14	46	.66827	.72140	.27860	.94687	14
47	.65381	.70309	.29691	.95071	13	47	.66851	.72170	.27830	.94680	13
48	.65406	.70341	.29659	.95065	12	48	.66875	.72201	.27799	.94674	12
49	.65431	.70372	.29628	.95059	11	49	.66899	.72231	.27769	.94667	11
50	9.65456	9.70404	10.29596	9.95052	10	50	9.66922	9.72262	10.27738	9.94660	10
51	.65481	.70435	.29565	.95046	9	51	.66946	.72293	.27707	.94654	9
52	.65506	.70466	.29534	.95039	8	52	.66970	.72323	.27677	.94647	8
53	.65531	.70498	.29502	.95033	7	53	.66994	.72354	.27646	.94640	7
54	.65556	.70529	.29471	.95027	6	54	.67018	.72384	.27616	.94634	6
55	9.65580	9.70560	10.29440	9.95020	5	55	9.67042	9.72415	10.27585	9.94627	5
56	.65605	.70592	.29408	.95014	4	56	.67066	.72445	.27555	.94620	4
57	.65630	.70623	.29377	.95007	3	57	.67090	.72476	.27524	.94614	3
58	.65655	.70654	.29346	.95001	2	58	.67113	.72506	.27494	.94607	2
59	.65680	.70685	.29315	.94995	1	59	.67137	.72537	.27463	.94600	1
60	9.65705	9.70717	10.29283	9.94988	0	60	9.67161	9.72567	10.27433	9.94593	0
	Cosine.	Cotang.	Tang.	Sine.	′		Cosine.	Cotang.	Tang.	Sine.	′
					63°						62°

*Log secant = colog cosine = 1 − log cosine; log cosecant = colog sine = 1 − log sine.

Ex.—Log sec 26° − 30′ = 10.04821. *Ex.*—Log cosec 26° − 30′ = 10.35047.

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5.—Logarithmic Sines, Tangents, Cotangents, Cosines.—(Cont'd.)
(Secants, Cosecants.)*

28° ′	Sine.	Tang.	Cotang.	Cosine.		29° ′	Sine.	Tang.	Cotang.	Cosine.
0	9.67161	9.72567	10.27433	9.94593	60	0	9.68557	9.74375	10.25625	9.94182
1	.67185	.72598	.27402	.94587	59	1	.68580	.74405	.25595	.94175
2	.67208	.72628	.27372	.94580	58	2	.68603	.74435	.25565	.94168
3	.67232	.72659	.27341	.94573	57	3	.68625	.74465	.25535	.94161
4	.67256	.72689	.27311	.94567	56	4	.68648	.74494	.25506	.94154
5	9.67280	9.72720	10.27280	9.94560	55	5	9.68671	9.74524	10.25476	9.94147
6	.67303	.72750	.27250	.94553	54	6	.68694	.74554	.25446	.94140
7	.67327	.72780	.27220	.94546	53	7	.68716	.74583	.25417	.94132
8	.67350	.72811	.27189	.94540	52	8	.68739	.74613	.25387	.94126
9	.67374	.72841	.27159	.94533	51	9	.68762	.74643	.25357	.94119
10	9.67398	9.72872	10.27128	9.94526	50	10	9.68784	9.74673	10.25327	9.94112
11	.67421	.72902	.27098	.94519	49	11	.68807	.74702	.25298	.94105
12	.67445	.72932	.27068	.94513	48	12	.68829	.74732	.25268	.94098
13	.67468	.72963	.27037	.94506	47	13	.68852	.74762	.25238	.94090
14	.67492	.72993	.27007	.94499	46	14	.68875	.74791	.25209	.94083
15	9.67515	9.73023	10.26977	9.94492	45	15	9.68897	9.74821	10.25179	9.94076
16	.67539	.73054	.26946	.94485	44	16	.68920	.74851	.25149	.94069
17	.67562	.73084	.26916	.94479	43	17	.68942	.74880	.25120	.94062
18	.67586	.73114	.26886	.94472	42	18	.68965	.74910	.25090	.94055
19	.67609	.73144	.26856	.94465	41	19	.68987	.74939	.25061	.94048
20	9.67633	9.73175	10.26825	9.94458	40	20	9.69010	9.74969	10.25031	9.94041
21	.67656	.73205	.26795	.94451	39	21	.69032	.74998	.25002	.94034
22	.67680	.73235	.26765	.94445	38	22	.69055	.75028	.24972	.94027
23	.67703	.73265	.26735	.94438	37	23	.69077	.75058	.24942	.94020
24	.67726	.73295	.26705	.94431	36	24	.69100	.75087	.24913	.94012
25	9.67750	9.73326	10.26674	9.94424	35	25	9.69122	9.75117	10.24883	9.94005
26	.67773	.73356	.26644	.94417	34	26	.69144	.75146	.24854	.93998
27	.67796	.73386	.26614	.94410	33	27	.69167	.75176	.24824	.93991
28	.67820	.73416	.26584	.94404	32	28	.69189	.75205	.24795	.93984
29	.67843	.73446	.26554	.94397	31	29	.69212	.75235	.24765	.93977
30	9.67866	9.73476	10.26524	9.94390	30	30	9.69234	9.75264	10.24736	9.93970
31	.67890	.73507	.26493	.94383	29	31	.69256	.75294	.24706	.93963
32	.67913	.73537	.26463	.94376	28	32	.69279	.75323	.24677	.93955
33	.67936	.73567	.26433	.94369	27	33	.69301	.75353	.24647	.93948
34	.67959	.73597	.26403	.94362	26	34	.69323	.75382	.24618	.93941
35	9.67982	9.73627	10.26373	9.94355	25	35	9.69345	9.75411	10.24589	9.93934
36	.68006	.73657	.26343	.94349	24	36	.69368	.75441	.24559	.93927
37	.68029	.73687	.26313	.94342	23	37	.69390	.75470	.24530	.93920
38	.68052	.73717	.26283	.94335	22	38	.69412	.75500	.24500	.93912
39	.68075	.73747	.26253	.94328	21	39	.69434	.75529	.24471	.93905
40	9.68098	9.73777	10.26223	9.94321	20	40	9.69456	9.75558	10.24442	9.93898
41	.68121	.73807	.26193	.94314	19	41	.69479	.75588	.24412	.93891
42	.68144	.73837	.26163	.94307	18	42	.69501	.75617	.24383	.93884
43	.68167	.73867	.26133	.94300	17	43	.69523	.75647	.24353	.93876
44	.68190	.73897	.26103	.94293	16	44	.69545	.75676	.24324	.93869
45	9.68213	9.73927	10.26073	9.94286	15	45	9.69567	9.75705	10.24295	9.93862
46	.68237	.73957	.26043	.94279	14	46	.69589	.75735	.24265	.93855
47	.68260	.73987	.26013	.94273	13	47	.69611	.75764	.24236	.93847
48	.68283	.74017	.25983	.94266	12	48	.69633	.75793	.24207	.93840
49	.68305	.74047	.25953	.94259	11	49	.69655	.75822	.24178	.93833
50	9.68328	9.74077	10.25923	9.94252	10	50	9.69677	9.75852	10.24148	9.93826
51	.68351	.74107	.25893	.94245	9	51	.69699	.75881	.24119	.93819
52	.68374	.74137	.25863	.94238	8	52	.69721	.75910	.24090	.93811
53	.68397	.74166	.25834	.94231	7	53	.69743	.75939	.24061	.93804
54	.68420	.74196	.25804	.94224	6	54	.69765	.75969	.24031	.93797
55	9.68443	9.74226	10.25774	9.94217	5	55	9.69787	9.75998	10.24002	9.93789
56	.68466	.74256	.25744	.94210	4	56	.69809	.76027	.23973	.93782
57	.68489	.74286	.25714	.94203	3	57	.69831	.76056	.23944	.93775
58	.68512	.74316	.25684	.94196	2	58	.69853	.76086	.23914	.93768
59	.68534	.74345	.25655	.94189	1	59	.69875	.76115	.23885	.93760
60	9.68557	9.74375	10.25625	9.94182	0	60	9.69897	9.76144	10.23856	9.93753
	Cosine.	Cotang.	Tang.	Sine.	′		Cosine.	Cotang.	Tang.	Sine.

61°

*Log secant = colog cosine = 1 − log cosine; log cosecant = colog sine = 1 − log sine.

Ex.—Log sec 28°- 30′ = 10.05610. *Ex.*—Log cosec 28° - 30′ = 10.321

, TANGENTS, COTANGENTS, COSINES.—(Cont'd.)
(SECANTS, COSECANTS.)*

31°

.	Cosine.		′	Sine.	Tang.	Cotang.	Cosine.	
	9.98753	60	0	9.71184	9.77877	10.22123	9.93307	60
	.93746	59	1	.71205	.77906	.22094	.93299	59
	.93738	58	2	.71226	.77935	.22065	.93291	58
	.93731	57	3	.71247	.77963	.22037	.93284	57
	.93724	56	4	.71268	.77992	.22008	.93276	56
	9.93717	55	5	9.71289	9.78020	10.21980	9.93269	55
	.93709	54	6	.71310	.78049	.21951	.93261	54
	.93702	53	7	.71331	.78077	.21923	.93253	53
	.93695	52	8	.71352	.78106	.21894	.93246	52
	.93687	51	9	.71373	.78135	.21865	.93238	51
	9.93680	50	10	9.71393	9.78163	10.21837	9.93230	50
	.93673	49	11	.71414	.78192	.21808	.93223	49
	.93665	48	12	.71435	.78220	.21780	.93215	48
	.93658	47	13	.71456	.78249	.21751	.93207	47
	.93650	46	14	.71477	.78277	.21723	.93200	46
	9.93643	45	15	9.71498	9.78306	10.21694	9.93192	45
	.93636	44	16	.71519	.78334	.21666	.93184	44
	.93628	43	17	.71539	.78363	.21637	.93177	43
	.93621	42	18	.71560	.78391	.21609	.93169	42
	.93614	41	19	.71581	.78419	.21581	.93161	41
	9.93606	40	20	9.71602	9.78448	10.21552	9.93154	40
	.93599	39	21	.71622	.78476	.21524	.93146	39
	.93591	38	22	.71643	.78505	.21495	.93138	38
	.93584	37	23	.71664	.78533	.21467	.93131	37
	.93577	36	24	.71685	.78562	.21438	.93123	36
	9.93569	35	25	9.71705	9.78590	10.21410	9.93115	35
	.93562	34	26	.71726	.78618	.21382	.93107	34
	.93554	33	27	.71747	.78647	.21353	.93100	33
	.93547	32	28	.71767	.78675	.21325	.93092	32
	.93539	31	29	.71788	.78704	.21296	.93084	31
	9.93532	30	30	9.71809	9.78732	10.21268	9.93077	30
	.93525	29	31	.71829	.78760	.21240	.93069	29
	.93517	28	32	.71850	.78789	.21211	.93061	28
	.93510	27	33	.71870	.78817	.21183	.93053	27
	.93502	26	34	.71891	.78845	.21155	.93046	26
	9.93495	25	35	9.71911	9.78874	10.21126	9.93038	25
	.93487	24	36	.71932	.78902	.21098	.93030	24
	.93480	23	37	.71952	.78930	.21070	.93022	23
	.93472	22	38	.71973	.78959	.21041	.93014	22
	.93465	21	39	.71994	.78987	.21013	.93007	21
	9.93457	20	40	9.72014	9.79015	10.20985	9.92999	20
	.93450	19	41	.72034	.79043	.20957	.92991	19
	.93442	18	42	.72055	.79072	.20928	.92983	18
	.93435	17	43	.72075	.79100	.20900	.92976	17
	.93427	16	44	.72096	.79128	.20872	.92968	16
	9.93420	15	45	9.72116	9.79156	10.20844	9.92960	15
	.93412	14	46	.72137	.79185	.20815	.92952	14
	.93405	13	47	.72157	.79213	.20787	.92944	13
	.93397	12	48	.72177	.79241	.20759	.92936	12
	.93390	11	49	.72198	.79269	.20731	.92929	11
	9.93382	10	50	9.72218	9.79297	10.20703	9.92921	10
	.93375	9	51	.72238	.79326	.20674	.92913	9
	.93367	8	52	.72259	.79354	.20646	.92905	8
	.93360	7	53	.72279	.79382	.20618	.92897	7
	.93352	6	54	.72299	.79410	.20590	.92889	6
	9.93344	5	55	9.72320	9.79438	10.20562	9.92881	5
	.93337	4	56	.72340	.79466	.20534	.92874	4
	.93329	3	57	.72360	.79495	.20505	.92866	3
	.93322	2	58	.72381	.79523	.20477	.92858	2
	.93314	1	59	.72401	.79551	.20449	.92850	1
	9.93307	0	60	9.72421	9.79579	10.20421	9.92842	0
	Sine.	′		Cosine.	Cotang.	Tang.	Sine.	′

59° 58°

osine = 1 − log cosine; log cosecant = colog sine =
= 10.06468. *Ex.*—Log cosec 30° − 30′ = 10.29453.

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5.—Logarithmic Sines, Tangents, Cotangents, Cosines.—(Cont'd.)

(Secants, Cosecants.)*

32°

′	Sine.	Tang.	Cotang.	Cosine.	
0	9.72421	9.79579	10.20421	9.92842	60
1	.72441	.79607	.20393	.92834	59
2	.72461	.79635	.20365	.92826	58
3	.72482	.79663	.20337	.92818	57
4	.72502	.79691	.20309	.92810	56
5	9.72522	9.79719	10.20281	9.92803	55
6	.72542	.79747	.20253	.92795	54
7	.72562	.79776	.20224	.92787	53
8	.72582	.79804	.20196	.92779	52
9	.72602	.79832	.20168	.92771	51
10	9.72622	9.79860	10.20140	9.92763	50
11	.72643	.79888	.20112	.92755	49
12	.72663	.79916	.20084	.92747	48
13	.72683	.79944	.20056	.92739	47
14	.72703	.79972	.20028	.92731	46
15	9.72723	9.80000	10.20000	9.92723	45
16	.72743	.80028	.19972	.92715	44
17	.72763	.80056	.19944	.92707	43
18	.72783	.80084	.19916	.92699	42
19	.72803	.80112	.19888	.92691	41
20	9.72823	9.80140	10.19860	9.92683	40
21	.72843	.80168	.19832	.92675	39
22	.72863	.80195	.19805	.92667	38
23	.72883	.80223	.19777	.92659	37
24	.72902	.80251	.19749	.92651	36
25	9.72922	9.80279	10.19721	9.92643	35
26	.72942	.80307	.19693	.92635	34
27	.72962	.80335	.19665	.92627	33
28	.72982	.80363	.19637	.92619	32
29	.73002	.80391	.19609	.92611	31
30	9.73022	9.80419	10.19581	9.92603	30
31	.73041	.80447	.19553	.92595	29
32	.73061	.80474	.19526	.92587	28
33	.73081	.80502	.19498	.92579	27
34	.73101	.80530	.19470	.92571	26
35	9.73121	9.80558	10.19442	9.92563	25
36	.73140	.80586	.19414	.92555	24
37	.73160	.80614	.19386	.92546	23
38	.73180	.80642	.19358	.92538	22
39	.73200	.80669	.19331	.92530	21
40	9.73219	9.80697	10.19303	9.92522	20
41	.73239	.80725	.19275	.92514	19
42	.73259	.80753	.19247	.92506	18
43	.73278	.80781	.19219	.92498	17
44	.73298	.80808	.19192	.92490	16
45	9.73318	9.80836	10.19164	9.92482	15
46	.73337	.80864	.19136	.92473	14
47	.73357	.80892	.19108	.92465	13
48	.73377	.80919	.19081	.92457	12
49	.73396	.80947	.19053	.92449	11
50	9.73416	9.80975	10.19025	9.92441	10
51	.73435	.81003	.18997	.92433	9
52	.73455	.81030	.18970	.92425	8
53	.73474	.81058	.18942	.92416	7
54	.73494	.81086	.18914	.92408	6
55	9.73513	9.81113	10.18887	9.92400	5
56	.73533	.81141	.18859	.92392	4
57	.73552	.81169	.18831	.92384	3
58	.73572	.81196	.18804	.92376	2
59	.73591	.81224	.18776	.92367	1
60	9.73611	9.81252	10.18748	9.92359	0
	Cosine.	Cotang.	Tang.	Sine.	′

33°

′	Sine.	Tang.	Cotang.	Cosine.	
0	9.73611	9.81252	10.18748	9.92359	6
1	.73630	.81279	.18721	.92351	5
2	.73650	.81307	.18693	.92343	5
3	.73669	.81335	.18665	.92335	5
4	.73689	.81362	.18638	.92326	5
5	9.73708	9.81390	10.18610	9.92318	5
6	.73727	.81418	.18582	.92310	4
7	.73747	.81445	.18555	.92302	4
8	.73766	.81473	.18527	.92293	4
9	.73785	.81500	.18500	.92285	4
10	9.73805	9.81528	10.18472	9.92277	5
11	.73824	.81556	.18444	.92269	4
12	.73843	.81583	.18417	.92260	4
13	.73863	.81611	.18389	.92252	4
14	.73882	.81638	.18362	.92244	4
15	9.73901	9.81666	10.18334	9.92235	4
16	.73921	.81693	.18307	.92227	4
17	.73940	.81721	.18279	.92219	4
18	.73959	.81748	.18252	.92211	4
19	.73978	.81776	.18224	.92202	4
20	9.73997	9.81803	10.18197	9.92194	4
21	.74017	.81831	.18169	.92186	3
22	.74036	.81858	.18142	.92177	3
23	.74055	.81886	.18114	.92169	3
24	.74074	.81913	.18087	.92161	3
25	9.74093	9.81941	10.18059	9.92152	3
26	.74113	.81968	.18032	.92144	3
27	.74132	.81996	.18004	.92136	3
28	.74151	.82023	.17977	.92127	3
29	.74170	.82051	.17949	.92119	3
30	9.74189	9.82078	10.17922	9.92111	3
31	.74208	.82106	.17894	.92102	2
32	.74227	.82133	.17867	.92094	2
33	.74246	.82161	.17839	.92086	2
34	.74265	.82188	.17812	.92077	2
35	9.74284	9.82215	10.17785	9.92069	2
36	.74303	.82243	.17757	.92060	2
37	.74322	.82270	.17730	.92052	2
38	.74341	.82298	.17702	.92044	2
39	.74360	.92325	.17675	.92035	2
40	9.74379	9.82352	10.17648	9.92027	2
41	.74398	.82380	.17620	.92018	1
42	.74417	.82407	.17593	.92010	1
43	.74436	.82435	.17565	.92002	1
44	.74455	.82462	.17538	.91993	1
45	9.74474	9.82489	10.17511	9.91985	1
46	.74493	.82517	.17483	.91976	1
47	.74512	.82544	.17456	.91968	1
48	.74531	.82571	.17429	.91959	1
49	.74549	.82599	.17401	.91951	1
50	9.74568	9.82626	10.17374	9.91942	1
51	.74587	.82653	.17347	.91934	1
52	.74606	.82681	.17319	.91925	1
53	.74625	.82708	.17292	.91917	1
54	.74644	.82735	.17265	.91908	1
55	9.74662	9.82762	10.17238	9.91900	1
56	.74681	.82790	.17210	.91891	1
57	.74700	.82817	.17183	.91883	1
58	.74719	.82844	.17156	.91874	1
59	.74737	.82871	.17129	.91866	1
60	9.74756	9.82899	10.17101	9.91857	1
	Cosine.	Cotang.	Tang.	Sine.	5

57°

*Log secant = colog cosine = 1 − log cosine; log cosecant = colog sine = 1 − log sine.

Ex.—Log sec 32° − 30′ = 10.07397. *Ex.*—Log cosec 32° − 30′ = 10.2697[illegible]

5.—Logarithmic Sines, Tangents, Cotangents, Cosines.—(Cont'd.)

(Secants, Cosecants.)*

34° '	Sine.	Tang.	Cotang.	Cosine.		35° '	Sine.	Tang.	Cotang.	Cosine.	
0	9.74756	9.82899	10.17101	9.91857	60	0	9.75859	9.84523	10.15477	9.91336	60
1	.74775	.82926	.17074	.91849	59	1	.75877	.84550	.15450	.91328	59
2	.74794	.82953	.17047	.91840	58	2	.75895	.84576	.15424	.91319	58
3	.74812	.82980	.17020	.91832	57	3	.75913	.84603	.15397	.91310	57
4	.74831	.83008	.16992	.91823	56	4	.75931	.84630	.15370	.91301	56
5	9.74850	9.83035	10.16965	9.91815	55	5	9.75949	9.84657	10.15343	9.91292	55
6	.74868	.83062	.16938	.91806	54	6	.75967	.84684	.15316	.91283	54
7	.74887	.83089	.16911	.91798	53	7	.75985	.84711	.15289	.91274	53
8	.74906	.83117	.16883	.91789	52	8	.76003	.84738	.15262	.91266	52
9	.74924	.83144	.16856	.91781	51	9	.76021	.84764	.15236	.91257	51
10	9.74943	9.83171	10.16829	9.91772	50	10	9.76039	9.84791	10.15209	9.91248	50
11	.74961	.83198	.16802	.91763	49	11	.76057	.84818	.15182	.91239	49
12	.74980	.83225	.16775	.91755	48	12	.76075	.84845	.15155	.91230	48
13	.74999	.83252	.16748	.91746	47	13	.76093	.84872	.15128	.91221	47
14	.75017	.83280	.16720	.91738	46	14	.76111	.84899	.15101	.91212	46
15	9.75036	9.83307	10.16693	9.91729	45	15	9.76129	9.84925	10.15075	9.91203	45
16	.75054	.83334	.16666	.91720	44	16	.76146	.84952	.15048	.91194	44
17	.75073	.83361	.16639	.91712	43	17	.76164	.84979	.15021	.91185	43
18	.75091	.83388	.16612	.91703	42	18	.76182	.85006	.14994	.91176	42
19	.75110	.83415	.16585	.91695	41	19	.76200	.85033	.14967	.91167	41
20	9.75128	9.83442	10.16558	9.91686	40	20	9.76218	9.85059	10.14941	9.91158	40
21	.75147	.83470	.16530	.91677	39	21	.76236	.85086	.14914	.91149	39
22	.75165	.83497	.16503	.91669	38	22	.76253	.85113	.14887	.91141	38
23	.75184	.83524	.16476	.91660	37	23	.76271	.85140	.14860	.91132	37
24	.75202	.83551	.16449	.91651	36	24	.76289	.85166	.14834	.91123	36
25	9.75221	9.83578	10.16422	9.91643	35	25	9.76307	9.85193	10.14807	9.91114	35
26	.75239	.83605	.16395	.91634	34	26	.76324	.85220	.14780	.91105	34
27	.75258	.83632	.16368	.91625	33	27	.76342	.85247	.14753	.91096	33
28	.75276	.83659	.16341	.91617	32	28	.76360	.85273	.14727	.91087	32
29	.75294	.83686	.16314	.91608	31	29	.76378	.85300	.14700	.91078	31
30	9.75313	9.83713	10.16287	9.91599	30	30	9.76395	9.85327	10.14673	9.91069	30
31	.75331	.83740	.16260	.91591	29	31	.76413	.85354	.14646	.91060	29
32	.75350	.83768	.16232	.91582	28	32	.76431	.85380	.14620	.91051	28
33	.75368	.83795	.16205	.91573	27	33	.76448	.85407	.14593	.91042	27
34	.75386	.83822	.16178	.91565	26	34	.76466	.85434	.14566	.91033	26
35	9.75405	9.83849	10.16151	9.91556	25	35	9.76484	9.85460	10.14540	9.91023	25
36	.75423	.83876	.16124	.91547	24	36	.76501	.85487	.14513	.91014	24
37	.75441	.83903	.16097	.91538	23	37	.76519	.85514	.14486	.91005	23
38	.75459	.83930	.16070	.91530	22	38	.76537	.85540	.14460	.90996	22
39	.75478	.83957	.16043	.91521	21	39	.76554	.85567	.14433	.90987	21
40	9.75496	9.83984	10.16016	9.91512	20	40	9.76572	9.85594	10.14406	9.90978	20
41	.75514	.84011	.15989	.91504	19	41	.76590	.85620	.14380	.90969	19
42	.75533	.84038	.15962	.91495	18	42	.76607	.85647	.14353	.90960	18
43	.75551	.84065	.15935	.91486	17	43	.76625	.85674	.14326	.90951	17
44	.75569	.84092	.15908	.91477	16	44	.76642	.85700	.14300	.90942	16
45	9.75587	9.84119	10.15881	9.91469	15	45	9.76660	9.85727	10.14273	9.90933	15
46	.75605	.84146	.15854	.91460	14	46	.76677	.85754	.14246	.90924	14
47	.75624	.84173	.15827	.91451	13	47	.76695	.85780	.14220	.90915	13
48	.75642	.84200	.15800	.91442	12	48	.76712	.85807	.14193	.90906	12
49	.75660	.84227	.15773	.91433	11	49	.76730	.85834	.14166	.90896	11
50	9.75678	9.84254	10.15746	9.91425	10	50	9.76747	9.85860	10.14140	9.90887	10
51	.75696	.84280	.15720	.91416	9	51	.76765	.85887	.14113	.90878	9
52	.75714	.84307	.15693	.91407	8	52	.76782	.85913	.14087	.90869	8
53	.75732	.84334	.15666	.91398	7	53	.76800	.85940	.14060	.90860	7
54	.75751	.84361	.15639	.91389	6	54	.76817	.85967	.14033	.90851	6
55	9.75769	9.84388	10.15612	9.91381	5	55	9.76835	9.85993	10.14007	9.90842	5
56	.75787	.84415	.15585	.91372	4	56	.76852	.86020	.13980	.90832	4
57	.75805	.84442	.15558	.91363	3	57	.76870	.86046	.13954	.90823	3
58	.75823	.84469	.15531	.91354	2	58	.76887	.86073	.13927	.90814	2
59	.75841	.84496	.15504	.91345	1	59	.76904	.86100	.13900	.90805	1
60	9.75859	9.84523	10.15477	9.91336	0	60	9.76922	9.86126	10.13874	9.90796	0
	Cosine.	Cotang.	Tang.	Sine.	'		Cosine.	Cotang.	Tang.	Sine.	'
					55°						54°

*Log secant = colog cosine = 1 − log cosine; log cosecant = colog sine = 1 − log sine.

Ex.—Log sec 34° − 30′ = 10.08401. *Ex.*—Log cosec 34° − 30′ = 10.24687.

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5.—Logarithmic Sines, Tangents, Cotangents, Cosines—(Cont'd.) (Secants, Cosecants.)*

36° ′	Sine.	Tang.	Cotang.	Cosine.		37° ′	Sine.	Tang.	Cotang.	Cosine.	
0	9.76922	9.86126	10.13874	9.90796	60	0	9.77946	9.87711	10.12289	9.90235	60
1	.76939	.86153	.13847	.90787	59	1	.77963	.87738	.12262	.90225	59
2	.76957	.86179	.13821	.90777	58	2	.77980	.87764	.12236	.90216	58
3	.76974	.86206	.13794	.90768	57	3	.77997	.87790	.12210	.90206	57
4	.76991	.86232	.13768	.90759	56	4	.78013	.87817	.12183	.90197	56
5	9.77009	9.86259	10.13741	9.90750	55	5	9.78030	9.87843	10.12157	9.90187	55
6	.77026	.86285	.13715	.90741	54	6	.78047	.87869	.12131	.90178	54
7	.77043	.86312	.13688	.90731	53	7	.78063	.87895	.12105	.90168	53
8	.77061	.86338	.13662	.90722	52	8	.78080	.87922	.12078	.90159	52
9	.77078	.86365	.13635	.90713	51	9	.78097	.87948	.12052	.90149	51
10	9.77095	9.86392	10.13608	9.90704	50	10	9.78113	9.87974	10.12026	9.90139	50
11	.77112	.86418	.13582	.90694	49	11	.78130	.88000	.12000	.90130	49
12	.77130	.86445	.13555	.90685	48	12	.78147	.88027	.11973	.90120	48
13	.77147	.86471	.13529	.90676	47	13	.78163	.88053	.11947	.90111	47
14	.77164	.86498	.13502	.90667	46	14	.78180	.88079	.11921	.90101	46
15	9.77181	9.86524	10.13476	9.90657	45	15	9.78197	9.88105	10.11895	9.90091	45
16	.77199	.86551	.13449	.90648	44	16	.78213	.88131	.11869	.90082	44
17	.77216	.86577	.13423	.90639	43	17	.78230	.88158	.11842	.90072	43
18	.77233	.86603	.13397	.90630	42	18	.78246	.88184	.11816	.90063	42
19	.77250	.86630	.13370	.90620	41	19	.78263	.88210	.11790	.90053	41
20	9.77268	9.86656	10.13344	9.90611	40	20	9.78280	9.88236	10.11764	9.90043	40
21	.77285	.86683	.13317	.90602	39	21	.78296	.88262	.11738	.90034	39
22	.77302	.86709	.13291	.90592	38	22	.78313	.88289	.11711	.90024	38
23	.77319	.86736	.13264	.90583	37	23	.78329	.88315	.11685	.90014	37
24	.77336	.86762	.13238	.90574	36	24	.78346	.88341	.11659	.90005	36
25	9.77353	9.86789	10.13211	9.90565	35	25	9.78362	9.88367	10.11633	9.89995	35
26	.77370	.86815	.13185	.90555	34	26	.78379	.88393	.11607	.89985	34
27	.77387	.86842	.13158	.90546	33	27	.78395	.88420	.11580	.89976	33
28	.77405	.86868	.13132	.90537	32	28	.78412	.88446	.11554	.89966	32
29	.77422	.86894	.13106	.90527	31	29	.78428	.88472	.11528	.89956	31
30	9.77439	9.86921	10.13079	9.90518	30	30	9.78445	9.88498	10.11502	9.89947	30
31	.77456	.86947	.13053	.90509	29	31	.78461	.88524	.11476	.89937	29
32	.77473	.86974	.13026	.90499	28	32	.78478	.88550	.11450	.89927	28
33	.77490	.87000	.13000	.90490	27	33	.78494	.88577	.11423	.89918	27
34	.77507	.87027	.12973	.90480	26	34	.78510	.88603	.11397	.89908	26
35	9.77524	9.87053	10.12947	9.90471	25	35	9.78527	9.88629	10.11371	9.89898	25
36	.77541	.87079	.12921	.90462	24	36	.78543	.88655	.11345	.89888	24
37	.77558	.87106	.12894	.90452	23	37	.78560	.88681	.11319	.89879	23
38	.77575	.87132	.12868	.90443	22	38	.78576	.88707	.11293	.89869	22
39	.77592	.87158	.12842	.90434	21	39	.78592	.88733	.11267	.89859	21
40	9.77609	9.87185	10.12815	9.90424	20	40	9.78609	9.88759	10.11241	9.89849	20
41	.77626	.87211	.12789	.90415	19	41	.78625	.88786	.11214	.89840	19
42	.77643	.87238	.12762	.90405	18	42	.78642	.88812	.11188	.89830	18
43	.77660	.87264	.12736	.90396	17	43	.78658	.88838	.11162	.89820	17
44	.77677	.87290	.12710	.90386	16	44	.78674	.88864	.11136	.89810	16
45	9.77694	9.87317	10.12683	9.90377	15	45	9.78691	9.88890	10.11110	9.89801	15
46	.77711	.87343	.12657	.90368	14	46	.78707	.88916	.11084	.89791	14
47	.77728	.87369	.12631	.90358	13	47	.78723	.88942	.11058	.89781	13
48	.77744	.87396	.12604	.90349	12	48	.78739	.88968	.11032	.89771	12
49	.77761	.87422	.12578	.90339	11	49	.78756	.88994	.11006	.89761	11
50	9.77778	9.87448	10.12552	9.90330	10	50	9.78772	9.89020	10.10980	9.89752	10
51	.77795	.87475	.12525	.90320	9	51	.78788	.89046	.10954	.89742	9
52	.77812	.87501	.12499	.90311	8	52	.78805	.89073	.10927	.89732	8
53	.77829	.87527	.12473	.90301	7	53	.78821	.89099	.10901	.89722	7
54	.77846	.87554	.12446	.90292	6	54	.78837	.89125	.10875	.89712	6
55	9.77862	9.87580	10.12420	9.90282	5	55	9.78853	9.89151	10.10849	9.89702	5
56	.77879	.87606	.12394	.90273	4	56	.78869	.89177	.10823	.89693	4
57	.77896	.87633	.12367	.90263	3	57	.78886	.89203	.10797	.89683	3
58	.77913	.87659	.12341	.90254	2	58	.78902	.89229	.10771	.89673	2
59	.77930	.87685	.12315	.90244	1	59	.78918	.89255	.10745	.89663	1
60	9.77946	9.87711	10.12289	9.90235	0	60	9.78934	9.89281	10.10719	9.89653	0
	Cosine.	Cotang.	Tang.	Sine.	′ 53°		Cosine.	Cotang.	Tang.	Sine.	′ 52°

*Log secant = colog cosine = 1 − log cosine; log cosecant = colog sine = 1 − log sine.

Ex.—Log sec 36° − 30′ = 10.09482. *Ex.*—Log cosec 36° − 30′ = 10.22561.

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5.—Logarithmic Sines, Tangents, Cotangents, Cosines.—(Cont'd.)
(Secants, Cosecants.)*

38° ′	Sine.	Tang.	Cotang.	Cosine.		39° ′	Sine.	Tang.	Cotang.	Cosine.	
0	9.78934	9.89281	10.10719	9.89653	60	0	9.79887	9.90837	10.09163	9.89050	60
1	.78950	.89307	.10693	.89643	59	1	.79903	.90863	.09137	.89040	59
2	.78967	.89333	.10667	.89633	58	2	.79918	.90889	.09111	.89030	58
3	.78983	.89359	.10641	.89624	57	3	.79934	.90914	.09086	.89020	57
4	.78999	.89385	.10615	.89614	56	4	.79950	.90940	.09060	.89009	56
5	9.79015	9.89411	10.10589	9.89604	55	5	9.79965	9.90966	10.09034	9.88999	55
6	.79031	.89437	.10563	.89594	54	6	.79981	.90992	.09008	.88989	54
7	.79047	.89463	.10537	.89584	53	7	.79996	.91018	.08982	.88978	53
8	.79063	.89489	.10511	.89574	52	8	.80012	.91043	.08957	.88968	52
9	.79079	.89515	.10485	.89564	51	9	.80027	.91069	.08931	.88958	51
10	9.79095	9.89541	10.10459	9.89554	50	10	9.80043	9.91095	10.08905	9.88948	50
11	.79111	.89567	.10433	.89544	49	11	.80058	.91121	.08879	.88937	49
12	.79128	.89593	.10407	.89534	48	12	.80074	.91147	.08853	.88927	48
13	.79144	.89619	.10381	.89524	47	13	.80089	.91172	.08828	.88917	47
14	.79160	.89645	.10355	.89514	46	14	.80105	.91198	.08802	.88906	46
15	9.79176	9.89671	10.10329	9.89504	45	15	9.80120	9.91224	10.08776	9.88896	45
16	.79192	.89697	.10303	.89495	44	16	.80136	.91250	.08750	.88886	44
17	.79208	.89723	.10277	.89485	43	17	.80151	.91276	.08724	.88875	43
18	.79224	.89749	.10251	.89475	42	18	.80166	.91301	.08699	.88865	42
19	.79240	.89775	.10225	.89465	41	19	.80182	.91327	.08673	.88855	41
20	9.79256	9.89801	10.10199	9.89455	40	20	9.80197	9.91353	10.08647	9.88844	40
21	.79272	.89827	.10173	.89445	39	21	.80213	.91379	.08621	.88834	39
22	.79288	.89853	.10147	.89435	38	22	.80228	.91404	.08596	.88824	38
23	.79304	.89879	.10121	.89425	37	23	.80244	.91430	.08570	.88813	37
24	.79319	.89905	.10095	.89415	36	24	.80259	.91456	.08544	.88803	36
25	9.79335	9.89931	10.10069	9.89405	35	25	9.80274	9.91482	10.08518	9.88793	35
26	.79351	.89957	.10043	.89395	34	26	.80290	.91507	.08493	.88782	34
27	.79367	.89983	.10017	.89385	33	27	.80305	.91533	.08467	.88772	33
28	.79383	.90009	.09991	.89375	32	28	.80320	.91559	.08441	.88761	32
29	.79399	.90035	.09965	.89364	31	29	.80336	.91585	.08415	.88751	31
30	9.79415	9.90061	10.09939	9.89354	30	30	9.80351	9.91610	10.08390	9.88741	30
31	.79431	.90086	.09914	.89344	29	31	.80366	.91636	.08364	.88730	29
32	.79447	.90112	.09888	.89334	28	32	.80382	.91662	.08338	.88720	28
33	.79463	.90138	.09862	.89324	27	33	.80397	.91688	.08312	.88709	27
34	.79478	.90164	.09836	.89314	26	34	.80412	.91713	.08287	.88699	26
35	9.79494	9.90190	10.09810	9.89304	25	35	9.80428	9.91739	10.08261	9.88688	25
36	.79510	.90216	.09784	.89294	24	36	.80443	.91765	.08235	.88678	24
37	.79526	.90242	.09758	.89284	23	37	.80458	.91791	.08209	.88668	23
38	.79542	.90268	.09732	.89274	22	38	.80473	.91816	.08184	.88657	22
39	.79558	.90294	.09706	.89264	21	39	.80489	.91842	.08158	.88647	21
40	9.79573	9.90320	10.09680	9.89254	20	40	9.80504	9.91868	10.08132	9.88636	20
41	.79589	.90346	.09654	.89244	19	41	.80519	.91893	.08107	.88626	19
42	.79605	.90371	.09629	.89233	18	42	.80534	.91919	.08081	.88615	18
43	.79621	.90397	.09603	.89223	17	43	.80550	.91945	.08055	.88605	17
44	.79636	.90423	.09577	.89213	16	44	.80565	.91971	.08029	.88594	16
45	9.79652	9.90449	10.09551	9.89203	15	45	9.80580	9.91996	10.08004	9.88584	15
46	.79668	.90475	.09525	.89193	14	46	.80595	.92022	.07978	.88573	14
47	.79684	.90501	.09499	.89183	13	47	.80610	.92048	.07952	.88563	13
48	.79699	.90527	.09473	.89173	12	48	.80625	.92073	.07927	.88552	12
49	.79715	.90553	.09447	.89162	11	49	.80641	.92099	.07901	.88542	11
50	9.79731	9.90578	10.09422	9.89152	10	50	9.80656	9.92125	10.07875	9.88531	10
51	.79746	.90604	.09396	.89142	9	51	.80671	.92150	.07850	.88521	9
52	.79762	.90630	.09370	.89132	8	52	.80686	.92176	.07824	.88510	8
53	.79778	.90656	.09344	.89122	7	53	.80701	.92202	.07798	.88499	7
54	.79793	.90682	.09318	.89112	6	54	.80716	.92227	.07773	.88489	6
55	9.79809	9.90708	10.09292	9.89101	5	55	9.80731	9.92253	10.07747	9.88478	5
56	.79825	.90734	.09266	.89091	4	56	.80746	.92279	.07721	.88468	4
57	.79840	.90759	.09241	.89081	3	57	.80762	.92304	.07696	.88457	3
58	.79856	.90785	.09215	.89071	2	58	.80777	.92330	.07670	.88447	2
59	.79872	.90811	.09189	.89060	1	59	.80792	.92356	.07644	.88436	1
60	9.79887	9.90837	10.09163	9.89050	0	60	9.80807	9.92381	10.07619	9.88425	0
	Cosine.	Cotang.	Tang.	Sine.	′		Cosine.	Cotang.	Tang.	Sine.	′
					51°						50°

*Log secant = colog cosine = 1 − log cosine; log cosecant = colog sine = − log sine.

Ex.—Log sec 38° 30′ = 10 10646. *Ex.*—Log cosec 38° 30′ = 10.20585.

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5.—Logarithmic Sines, Tangents, Cotangents, Cosines.—(Cont'd.) (Secants, Cosecants.)*

40° ′	Sine.	Tang.	Cotang.	Cosine.		41° ′	Sine.	Tang.	Cotang.	Cosine.	
0	9.80807	9.92381	10.07619	9.88425	60	0	9.81694	9.93916	10.06084	9.87778	60
1	.80822	.92407	.07593	.88415	59	1	.81709	.93942	.06058	.87767	59
2	.80837	.92433	.07567	.88404	58	2	.81723	.93967	.06033	.87756	58
3	.80852	.92458	.07542	.88394	57	3	.81738	.93993	.06007	.87745	57
4	.80867	.92484	.07516	.88383	56	4	.81753	.94018	.05982	.87734	56
5	9.80882	9.92510	10.07490	9.88372	55	5	9.81767	9.94044	10.05956	9.87723	55
6	.80897	.92535	.07465	.88362	54	6	.81781	.94069	.05931	.87712	54
7	.80912	.92561	.07439	.88351	53	7	.81796	.94095	.05905	.87701	53
8	.80927	.92587	.07413	.88340	52	8	.81810	.94120	.05880	.87690	52
9	.80942	.92612	.07388	.88330	51	9	.81825	.94146	.05854	.87679	51
10	9.80957	9.92638	10.07362	9.88319	50	10	9.81839	9.94171	10.05829	9.87668	50
11	.80972	.92663	.07337	.88308	49	11	.81854	.94197	.05803	.87657	49
12	.80987	.92689	.07311	.88298	48	12	.81868	.94222	.05778	.87646	48
13	.81002	.92715	.07285	.88287	47	13	.81882	.94248	.05752	.87635	47
14	.81017	.92740	.07260	.88276	46	14	.81897	.94273	.05727	.87624	46
15	9.81032	9.92766	10.07234	9.88266	45	15	9.81911	9.94299	10.05701	9.87613	45
16	.81047	.92792	.07208	.88255	44	16	.81926	.94324	.05676	.87601	44
17	.81061	.92817	.07183	.88244	43	17	.81940	.94350	.05650	.87590	43
18	.81076	.92843	.07157	.88234	42	18	.81955	.94375	.05625	.87579	42
19	.81091	.92868	.07132	.88223	41	19	.81969	.94401	.05599	.87568	41
20	9.81106	9.92894	10.07106	9.88212	40	20	9.81983	9.94426	10.05574	9.87557	40
21	.81121	.92920	.07080	.88201	39	21	.81998	.94452	.05548	.87546	39
22	.81136	.92945	.07055	.88191	38	22	.82012	.94477	.05523	.87535	38
23	.81151	.92971	.07029	.88180	37	23	.82026	.94503	.05497	.87524	37
24	.81166	.92996	.07004	.88169	36	24	.82041	.94528	.05472	.87513	36
25	9.81180	9.93022	10.06978	9.88158	35	25	9.82055	9.94554	10.05446	9.87501	35
26	.81195	.93048	.06952	.88148	34	26	.82069	.94579	.05421	.87490	34
27	.81210	.93073	.06927	.88137	33	27	.82084	.94604	.05396	.87479	33
28	.81225	.93099	.06901	.88126	32	28	.82098	.94630	.05370	.87468	32
29	.81240	.93124	.06876	.88115	31	29	.82112	.94655	.05345	.87457	31
30	9.81254	9.93150	10.06850	9.88105	30	30	9.82126	9.94681	10.05319	9.87446	30
31	.81269	.93175	.06825	.88094	29	31	.82141	.94706	.05294	.87434	29
32	.81284	.93201	.06799	.88083	28	32	.82155	.94732	.05268	.87423	28
33	.81299	.93227	.06773	.88072	27	33	.82169	.94757	.05243	.87412	27
34	.81314	.93252	.06748	.88061	26	34	.82184	.94783	.05217	.87401	26
35	9.81328	9.93278	10.06722	9.88051	25	35	9.82198	9.94808	10.05192	9.87390	25
36	.81343	.93303	.06697	.88040	24	36	.82212	.94834	.05166	.87378	24
37	.81358	.93329	.06671	.88029	23	37	.82226	.94859	.05141	.87367	23
38	.81373	.93354	.06646	.88018	22	38	.82240	.94884	.05116	.87356	22
39	.81387	.93380	.06620	.88007	21	39	.82255	.94910	.05090	.87345	21
40	9.81402	9.93406	10.06594	9.87996	20	40	9.82269	9.94935	10.05065	9.87334	20
41	.81417	.93431	.06569	.87985	19	41	.82283	.94961	.05039	.87322	19
42	.81431	.93457	.06543	.87975	18	42	.82297	.94986	.05014	.87311	18
43	.81446	.93482	.06518	.87964	17	43	.82311	.95012	.04988	.87300	17
44	.81461	.93508	.06492	.87953	16	44	.82326	.95037	.04963	.87289	16
45	9.81475	9.93533	10.06467	9.87942	15	45	9.82340	9.95062	10.04938	9.87277	15
46	.81490	.93559	.06441	.87931	14	46	.82354	.95088	.04912	.87266	14
47	.81505	.93584	.06416	.87920	13	47	.82368	.95113	.04887	.87255	13
48	.81519	.93610	.06390	.87909	12	48	.82382	.95139	.04861	.87243	12
49	.81534	.93636	.06364	.87898	11	49	.82396	.95164	.04836	.87232	11
50	9.81549	9.93661	10.06339	9.87887	10	50	9.82410	9.95190	10.04810	9.87221	10
51	.81563	.93687	.06313	.87877	9	51	.82424	.95215	.04785	.87209	9
52	.81578	.93712	.06288	.87866	8	52	.82439	.95240	.04760	.87198	8
53	.81592	.93738	.06262	.87855	7	53	.82453	.95266	.04734	.87187	7
54	.81607	.93763	.06237	.87844	6	54	.82467	.95291	.04709	.87176	6
55	9.81622	9.93789	10.06211	9.87833	5	55	9.82481	9.95317	10.04683	9.87164	5
56	.81636	.93814	.06186	.87822	4	56	.82495	.95342	.04658	.87153	4
57	.81651	.93840	.06160	.87811	3	57	.82509	.95368	.04632	.87141	3
58	.81665	.93865	.06135	.87800	2	58	.82523	.95393	.04607	.87130	2
59	.81680	.93891	.06109	.87789	1	59	.82537	.95418	.04582	.87119	1
60	9.81694	9.93916	10.06084	9.87778	0	60	9.82551	9.95444	10.04556	9.87107	0
	Cosine.	Cotang.	Tang.	Sine.	′		Cosine.	Cotang.	Tang.	Sine.	

49°

*Log secant = colog cosine = 1 − log cosine; log cosecant = colog sine = 1 − log sine.

Ex.—Log sec 40°- 30′ = 10.11895. *Ex.*—Log cosec 40°- 30′ = 10.187

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5.—Logarithmic Sines, Tangents, Cotangents, Cosines.—(Cont'd.)
(Secants, Cosecants.)*

′	Sine.	Tang.	Cotang.	Cosine.		′	Sine.	Tang.	Cotang.	Cosine.	
	42°						43°				
0	9.82551	9.95444	10.04556	9.87107	60	0	9.83378	9.96966	10.03034	9.86413	60
1	.82565	.95469	.04531	.87096	59	1	.83392	.96991	.03009	.86401	59
2	.82579	.95495	.04505	.87085	58	2	.83405	.97016	.02984	.86389	58
3	.82593	.95520	.04480	.87073	57	3	.83419	.97042	.02958	.86377	57
4	.82607	.95545	.04455	.87062	56	4	.83432	.97067	.02933	.86366	56
5	9.82621	9.95571	10.04429	9.87050	55	5	9.83446	9.97092	10.02908	9.86354	55
6	.82635	.95596	.04404	.87039	54	6	.83459	.97118	.02882	.86342	54
7	.82649	.95622	.04378	.87028	53	7	.83473	.97143	.02857	.86330	53
8	.82663	.95647	.04353	.87016	52	8	.83486	.97168	.02832	.86318	52
9	.82677	.95672	.04328	.87005	51	9	.83500	.97193	.02807	.86306	51
10	9.82691	9.95698	10.04302	9.86993	50	10	9.83513	9.97219	10.02781	9.86295	50
11	.82705	.95723	.04277	.86982	49	11	.83527	.97244	.02756	.86283	49
12	.82719	.95748	.04252	.86970	48	12	.83540	.97269	.02731	.86271	48
13	.82733	.95774	.04226	.86959	47	13	.83554	.97295	.02705	.86259	47
14	.82747	.95799	.04201	.86947	46	14	.83567	.97320	.02680	.86247	46
15	9.82761	9.95825	10.04175	9.86936	45	15	9.83581	9.97345	10.02655	9.86235	45
16	.82775	.95850	.04150	.86924	44	16	.83594	.97371	.02629	.86223	44
17	.82788	.95875	.04125	.86913	43	17	.83608	.97396	.02604	.86211	43
18	.82802	.95901	.04099	.86902	42	18	.83621	.97421	.02579	.86200	42
19	.82816	.95926	.04074	.86890	41	19	.83634	.97447	.02553	.86188	41
20	9.82830	9.95952	10.04048	9.86879	40	20	9.83648	9.97472	10.02528	9.86176	40
21	.82844	.95977	.04023	.86867	39	21	.83661	.97497	.02503	.86164	39
22	.82858	.96002	.03998	.86855	38	22	.83674	.97523	.02477	.86152	38
23	.82872	.96028	.03972	.86844	37	23	.83688	.97548	.02452	.86140	37
24	.82885	.96053	.03947	.86832	36	24	.83701	.97573	.02427	.86128	36
25	9.82899	9.96078	10.03922	9.86821	35	25	9.83715	9.97598	10.02402	9.86116	35
26	.82913	.96104	.03896	.86809	34	26	.83728	.97624	.02376	.86104	34
27	.82927	.96129	.03871	.86798	33	27	.83741	.97649	.02351	.86092	33
28	.82941	.96155	.03845	.86786	32	28	.83755	.97674	.02326	.86080	32
29	.82955	.96180	.03820	.86775	31	29	.83768	.97700	.02300	.86068	31
30	9.82968	9.96205	10.03795	9.86763	30	30	9.83781	9.97725	10.02275	9.86056	30
31	.82982	.96231	.03769	.86752	29	31	.83795	.97750	.02250	.86044	29
32	.82996	.96256	.03744	.86740	28	32	.83808	.97776	.02224	.86032	28
33	.83010	.96281	.03719	.86728	27	33	.83821	.97801	.02199	.86020	27
34	.83022	.96307	.03693	.86717	26	34	.83834	.97826	.02174	.86008	26
35	9.83037	9.96332	10.03668	9.86705	25	35	9.83848	9.97851	10.02149	9.85996	25
36	.83051	.96357	.03643	.86694	24	36	.83861	.97877	.02123	.85984	24
37	.83065	.96383	.03617	.86682	23	37	.83874	.97902	.02098	.85972	23
38	.83078	.96408	.03592	.86670	22	38	.83887	.97927	.02073	.85960	22
39	.83092	.96433	.03567	.86659	21	39	.83901	.97953	.02047	.85948	21
40	9.83106	9.96459	10.03541	9.86647	20	40	9.83914	9.97978	10.02022	9.85936	20
41	.83120	.96484	.03516	.86635	19	41	.83927	.98003	.01997	.85924	19
42	.83133	.96510	.03490	.86624	18	42	.83940	.98029	.01971	.85912	18
43	.83147	.96535	.03465	.86612	17	43	.83954	.98054	.01946	.85900	17
44	.83161	.96560	.03440	.86600	16	44	.83967	.98079	.01921	.85888	16
45	9.83174	9.96586	10.03414	9.86589	15	45	9.83980	9.98104	10.01896	9.85876	15
46	.83188	.96611	.03389	.86577	14	46	.83993	.98130	.01870	.85864	14
47	.83202	.96636	.03364	.86565	13	47	.84006	.98155	.01845	.85851	13
48	.83215	.96662	.03338	.86554	12	48	.84020	.98180	.01820	.85839	12
49	.83229	.96687	.03313	.86542	11	49	.84033	.98206	.01794	.85827	11
50	9.83242	9.96712	10.03288	9.86530	10	50	9.84046	9.98231	10.01769	9.85815	10
51	.83256	.96738	.03262	.86518	9	51	.84059	.98256	.01744	.85803	9
52	.83270	.96763	.03237	.86507	8	52	.84072	.98281	.01719	.85791	8
53	.83283	.96788	.03212	.86495	7	53	.84085	.98307	.01693	.85779	7
54	.83297	.96814	.03186	.86483	6	54	.84098	.98332	.01668	.85766	6
55	9.83310	9.96839	10.03161	9.86472	5	55	9.84112	9.98357	10.01643	9.85754	5
56	.83324	.96864	.03136	.86460	4	56	.84125	.98383	.01617	.85742	4
57	.83338	.96890	.03110	.86448	3	57	.84138	.98408	.01592	.85730	3
58	.83351	.96915	.03085	.86436	2	58	.84151	.98433	.01567	.85718	2
59	.83365	.96940	.03060	.86425	1	59	.84164	.98458	.01542	.85706	1
60	9.83378	9.96966	10.03034	9.86413	0	60	9.84177	9.98484	10.01516	9.85693	0
	Cosine.	Cotang.	Tang.	Sine.	′		Cosine.	Cotang.	Tang.	Sine.	′
					47°						46°

*Log secant = colog cosine = 1 − log cosine; log cosecant = colog sine = 1 − log sine.

Ex.—Log sec 42° − 30′ = 10.13237. *Ex.*—Log cosec 42° − 30′ = 10.17032.

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5.—Logarithmic Sines, Tangents, Cotangents, Cosines.—(Concl'd.)
(Secants, Cosecants.)

44° 44°

′	Sine.	Tang.	Cotang.	Cosine.		′	Sine.	Tang.	Cotang.	Cosine.	
0	9.84177	9.98484	10.01516	9.85693	60	30	9.84566	9.99242	10.00758	9.85324	3
1	.84190	.98509	.01491	.85681	59	31	.84579	.99267	.00733	.85312	2
2	.84203	.98534	.01466	.85669	58	32	.84592	.99293	.00707	.85299	2
3	.84216	.98560	.01440	.85657	57	33	.84605	.99318	.00682	.85287	2
4	.84229	.98585	.01415	.85645	56	34	.84618	.99343	.00657	.85274	2
5	9.84242	9.98610	10.01390	9.85632	55	35	9.84630	9.99368	10.00632	9.85262	2
6	.84255	.98635	.01365	.85620	54	36	.84643	.99394	.00606	.85250	2
7	.84269	.98661	.01339	.85608	53	37	.84656	.99419	.00581	.85237	2
8	.84282	.98686	.01314	.85596	52	38	.84669	.99444	.00556	.85225	2
9	.84295	.98711	.01289	.85583	51	39	.84682	.99469	.00531	.85212	2
10	9.84308	9.98737	10.01263	9.85571	50	40	9.84694	9.99495	10.00505	9.85200	2
11	.84321	.98762	.01238	.85559	49	41	.84707	.99520	.00480	.85187	1
12	.84334	.98787	.01213	.85547	48	42	.84720	.99545	.00455	.85175	1
13	.84347	.98812	.01188	.85534	47	43	.84733	.99570	.00430	.85162	1
14	.84360	.98838	.01162	.85522	46	44	.84745	.99596	.00404	.85150	1
15	9.84373	9.98863	10.01137	9.85510	45	45	9.84758	9.99621	10.00379	9.85137	1
16	.84385	.98888	.01112	.85497	44	46	.84771	.99646	.00354	.85125	1
17	.84398	.98913	.01087	.85485	43	47	.84784	.99672	.00328	.85112	1
18	.84411	.98939	.01061	.85473	42	48	.84796	.99697	.00303	.85100	1
19	.84424	.98964	.01036	.85460	41	49	.84809	.99722	.00278	.85087	1
20	9.84437	9.98989	10.01011	9.85448	40	50	9.84822	9.99747	10.00253	9.85074	1
21	.84450	.99015	.00985	.85436	39	51	.84835	.99773	.00227	.85062	
22	.84463	.99040	.00960	.85423	38	52	.84847	.99798	.00202	.85049	
23	.84476	.99065	.00935	.85411	37	53	.84860	.99823	.00177	.85037	
24	.84489	.99090	.00910	.85399	36	54	.84873	.99848	.00152	.85024	
25	9.84502	9.99116	10.00884	9.85386	35	55	9.84885	9.99874	10.00126	9.85012	
26	.84515	.99141	.00859	.85374	34	56	.84898	.99899	.00101	.84999	
27	.84528	.99166	.00834	.85361	33	57	.84911	.99924	.00076	.84986	
28	.84540	.99191	.00809	.85349	32	58	.84923	.99949	.00051	.84974	
29	.84553	.99217	.00783	.85337	31	59	.84936	.99975	.00025	.84961	
30	9.84566	9.99242	10.00758	9.85324	30	60	9.84949	10.00000	10.00000	9.84949	
	Cosine.	Cotang.	Tang.	Sine.	′		Cosine.	Cotang.	Tang.	Sine.	′

45° 45

5a.—Table for Finding the Logarithmic Sines and Tangents of Small Angles.

[Values of *S* and *T* in Formulas Below.*]

A.	A(sec.).	S.	A.	A(sec.).	T.	A.	A(sec.).	T.
0°00′00″	0000″	4.68557	0°00′00″	000″	4.68557	1°25′40″	5140″	4.6856
0°40′10″	2410″	57	0°03′00″	180″	57	1°25′50″	5150″	6
0°40′20″	2420″	56	0°03′20″	200″	58	1°30′20″	5420″	6
0°57′00″	3420″	56	0°28′40″	1720″	58	1°30′30″	5430″	6
0°57′10″	3430″	55	0°28′50″	1730″	59	1°34′40″	5680″	6
1°09′50″	4190″	55	0°40′20″	2420″	59	1°34′50″	5690″	6
1°10′00″	4200″	54	0°40′30″	2430″	60	1°39′00″	5940″	6
1°20′40″	4840″	54	0°49′30″	2970″	60	1°39′10″	5950″	7
1°20′50″	4850″	53	0°49′40″	2980″	61	1°43′00″	6180″	7
1°30′10″	5410″	53	0°57′10″	3430″	61	1°43′10″	6190″	7
1°30′20″	5420″	52	0°57′20″	3440″	62	1°46′50″	6410″	7
1°38′50″	5930″	52	1°03′50″	3830″	62	1°47′00″	6420″	7
1°39′00″	5940″	51	1°04′00″	3840″	63	1°50′40″	6640″	7
1°46′50″	6410″	51	1°10′00″	4200″	63	1°50′50″	6650″	7
1°47′00″	6420″	50	1°10′10″	4210″	64	1°54′10″	6850″	7
1°54′10″	6850″	50	1°15′30″	4530″	64	1°54′20″	6860″	7
1°54′20″	6860″	49	1°15′40″	4540″	65	1°57′40″	7060″	7
2°01′00″	7260″	49	1°20′50″	4850″	65	1°57′50″	7070″	7
2°01′10″	7270″	48	1°21′00″	4860″	66	2°01′10″	7270″	7

* Log sin A = log A (seconds) + S. Log tan A = log A (seconds) + T.

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ERICAL TRIGONOMETRY.

—Spherical Trigonometry, in its broadest sense, of spherical pyramids; and, more directly, of the pherical pyramids.

cal pyramid (Fig. 1) is a triangular pyramid cut nter of the sphere being the apex of the pyramid, ce forming the base. The base is therefore bounded *cles*.

e is a term applied to the outline of the base of a amid; thus, Fig. 1, $A\ B\ C\ A$ is a spherical triangle. Trigonometry are confined usually to the solution of he practical application leads us into the fields of nd Astronomy.

ctions or quantities in any ollows (Fig. 1): Radius, r;

....	A	B	C ;
....	a	b	c ;
....	a_1	b_1	c_1 ;

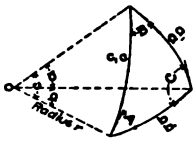

Fig. 1.

ius as unity, and remember- cles are proportional to the angles, it is seen that the to six *primary* functions, l angles A, B. C, and the Three of these must be ve the other three. The c, are always shown on the in Figs. 2 and 3.

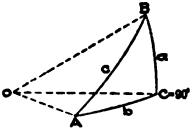

Fig. 2.

l Triangles.—The spherical is a right spherical triangle, e and o the center of the formulas are given for triangles.

Formulas:

$$\frac{\text{s } B}{\text{s } b}. \qquad \sin B = \frac{\sin b}{\sin c} = \frac{\cos A}{\cos a}.$$

$$\text{an } A = \frac{\tan a}{\sin b}. \qquad \cos B = \frac{\tan a}{\tan c}. \qquad \tan B = \frac{\tan b}{\sin a}$$

$$\text{).} \qquad \cos c = \cot A \cot B.$$

ormulas are all that are necessary, as the value of may be determined by transposition. Thus, from

e, $\sin a = \dfrac{\tan b}{\tan B}$, and so on.

the side b if the angle $A = 18° - 20'$ and the hypoth-

$A = \dfrac{\tan b}{\tan c}$, we have, $\tan b = \tan c \cos A$;

$$\begin{aligned} \log \tan 48° - 30' &= 0.05319 \\ \log \cos 18° - 20' &= 9.97738 \\ \hline \text{gle } b = 47° - 01', \text{ as } 0.03057 &= \log \tan b. \end{aligned}$$

e (arc) b may be obtained from the table of Circular ollowing).

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Oblique Spherical Triangles.—

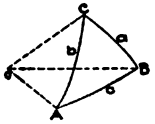

Fig. 3.

Formulas.

$$\frac{\sin A}{\sin B} = \frac{\sin a}{\sin b}. \qquad \frac{\sin B}{\sin C} = \frac{\sin b}{\sin c}. \qquad \frac{\sin A}{\sin C} = \frac{\sin a}{\sin c}.$$

$$\begin{aligned}
\cos a &= \cos b \cos c + \sin b \sin c \cos A.\\
\cos b &= \cos c \cos a + \sin c \sin a \cos B.\\
\cos c &= \cos a \cos b + \sin a \sin b \cos C.\\
\cos A &= -\cos B \cos C + \sin B \sin C \cos a.\\
\cos B &= -\cos C \cos A + \sin C \sin A \cos b.\\
\cos C &= -\cos A \cos B + \sin A \sin B \cos c.
\end{aligned}$$

$$\sin \tfrac{1}{2} A = \sqrt{\frac{\sin (s-b) \sin (s-c)}{\sin b \sin c}} \qquad \sin \tfrac{1}{2} a = \sqrt{-\frac{\cos S \cos (S-A)}{\sin B \sin C}}$$

$$\sin \tfrac{1}{2} B = \sqrt{\frac{\sin (s-c) \sin (s-a)}{\sin c \sin a}} \qquad \sin \tfrac{1}{2} b = \sqrt{-\frac{\cos S \cos (S-B)}{\sin C \sin A}}$$

$$\sin \tfrac{1}{2} C = \sqrt{\frac{\sin (s-a) \sin (s-b)}{\sin a \sin b}} \qquad \sin \tfrac{1}{2} c = \sqrt{-\frac{\cos S \cos (S-C)}{\sin A \sin B}}$$

$$\cos \tfrac{1}{2} A = \sqrt{\frac{\sin s \sin (s-a)}{\sin b \sin c}} \qquad \cos \tfrac{1}{2} a = \sqrt{\frac{\cos (S-B) \cos (S-C)}{\sin B \sin C}}$$

$$\cos \tfrac{1}{2} B = \sqrt{\frac{\sin s \sin (s-b)}{\sin c \sin a}} \qquad \cos \tfrac{1}{2} b = \sqrt{\frac{\cos (S-C) \cos (S-A)}{\sin C \sin A}}$$

$$\cos \tfrac{1}{2} C = \sqrt{\frac{\sin s \sin (s-c)}{\sin a \sin b}} \qquad \cos \tfrac{1}{2} c = \sqrt{\frac{\cos (S-A) \cos (S-B)}{\sin A \sin B}}$$

$$\tan \tfrac{1}{2} A = \sqrt{\frac{\sin (s-b) \sin (s-c)}{\sin s \sin (s-a)}} \qquad \tan \tfrac{1}{2} a = \sqrt{-\frac{\cos S \cos (S-A)}{\cos (S-B) \cos (S-C)}}$$

$$\tan \tfrac{1}{2} B = \sqrt{\frac{\sin (s-c) \sin (s-a)}{\sin s \sin (s-b)}} \qquad \tan \tfrac{1}{2} b = \sqrt{-\frac{\cos S \cos (S-B)}{\cos (S-C) \cos (S-A)}}$$

$$\tan \tfrac{1}{2} C = \sqrt{\frac{\sin (s-a) \sin (s-b)}{\sin s \sin (s-c)}} \qquad \tan \tfrac{1}{2} c = \sqrt{-\frac{\cos S \cos (S-C)}{\cos (S-A) \cos (S-B)}}$$

Note.—In the above formulas, $s = \frac{1}{2}(a+b+c)$; $S = \frac{1}{2}(A+B+C)$.

$$\frac{\sin \frac{1}{2}(A+B)}{\sin \frac{1}{2}(A-B)} = \frac{\tan \frac{1}{2} c}{\tan \frac{1}{2}(a-b)} \qquad \frac{\cos \frac{1}{2}(A+B)}{\cos \frac{1}{2}(A-B)} = \frac{\tan \frac{1}{2} c}{\tan \frac{1}{2}(a+b)}$$

$$\frac{\sin \frac{1}{2}(a+b)}{\sin \frac{1}{2}(a-b)} = \frac{\cot \frac{1}{2} C}{\tan \frac{1}{2}(A-B)} \qquad \frac{\cos \frac{1}{2}(a+b)}{\cos \frac{1}{2}(a-b)} = \frac{\cot \frac{1}{2} C}{\tan \frac{1}{2}(A+B)}$$

General Rules.

1. Sines of the sides are proportional to the sines of their opposite angles. Example.—In Fig. 2, $\sin A : \sin a :: \sin C : \sin c$.
2. Cosine of any side equals the product of the cosines of the other two sides, plus the product of their sines and the cosine of their included angle. Example.—In Fig. 2, $\cos a = \cos b \cos c + \sin b \sin c \cos A$.

Special Cases.

Case I. Given one side (c) and two adjacent angles (A and B).

1st. Solve for b and c in the following:

$$\tan \tfrac{1}{2}(b-a) = \sin \tfrac{1}{2}(B-A) \operatorname{cosec} \tfrac{1}{2}(B+A) \tan \tfrac{1}{2} c.$$
$$\tan \tfrac{1}{2}(b+a) = \cos \tfrac{1}{2}(B-A) \sec \tfrac{1}{2}(B+A) \tan \tfrac{1}{2} c.$$

2nd. Solve for C in the following:

$$\cot \tfrac{1}{2} C = \sin \tfrac{1}{2}(b+a) \operatorname{cosec} \tfrac{1}{2}(b-a) \tan \tfrac{1}{2}(B-A).$$

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Case II. Given two sides (b and c) and their included angle (A).

1st. Solve for B and C in the following:

$$\tan \tfrac{1}{2}(B-C) = \sin \tfrac{1}{2}(b-c) \operatorname{cosec} \tfrac{1}{2}(b+c) \cot \tfrac{1}{2} A;$$
$$\tan \tfrac{1}{2}(B+C) = \cos \tfrac{1}{2}(b-c) \sec \tfrac{1}{2}(b+c) \cot \tfrac{1}{2} A.$$

2nd. Solve for a in the following:

$$\tan \tfrac{1}{2} a = \sin \tfrac{1}{2}(B+C) \operatorname{cosec} \tfrac{1}{2}(B-C) \tan \tfrac{1}{2}(b-c).$$

Case III. Given the three sides (a, b and c). Solve for A, B and C.

$$\tan \tfrac{1}{2} A = \frac{k}{\sin(s-a)}; \quad \tan \tfrac{1}{2} B = \frac{k}{\sin(s-b)}; \quad \tan \tfrac{1}{2} C = \frac{k}{\sin(s-c)};$$

in which $k = \sqrt{\dfrac{\sin(s-a)\sin(s-b)\sin(s-c)}{\sin s}}$,

and $s = \tfrac{1}{2}(a+b+c)$.

Case IV. Given the three angles (A, B and C). Solve for a, b and c.

$$\tan \tfrac{1}{2} a = K \cos(S-A); \quad \tan \tfrac{1}{2} b = K \cos(S-B); \quad \tan \tfrac{1}{2} c = K \cos(S-C);$$

in which $K = \sqrt{-\dfrac{\cos S}{\cos(S-A)\cos(S-B)\cos(S-C)}}$,

and $S = \tfrac{1}{2}(A+B+C)$.

Case V. Given two sides (a and b) and the angle (B) opposite one of them.

1st. Solve for A in the following:

$$\sin A = \sin a \operatorname{cosec} b \sin B.$$

2nd. Solve for C and c in the following:

$$\cot \tfrac{1}{2} C = \sin \tfrac{1}{2}(b+a) \operatorname{cosec} \tfrac{1}{2}(b-a) \tan \tfrac{1}{2}(B-A);$$
$$\tan \tfrac{1}{2} c = \sin \tfrac{1}{2}(B+A) \operatorname{cosec} \tfrac{1}{2}(B-A) \tan \tfrac{1}{2}(b-a).$$

Case VI. Given two angles (A and B) and the side (b) opposite one of them.

1st. Solve for a in the following:

$$\sin a = \sin A \operatorname{cosec} B \sin b.$$

2nd. Solve for c and C in the following:

$$\tan \tfrac{1}{2} c = \sin \tfrac{1}{2}(B+A) \operatorname{cosec} \tfrac{1}{2}(B-A) \tan \tfrac{1}{2}(b-a);$$
$$\cot \tfrac{1}{2} C = \sin \tfrac{1}{2}(b+a) \operatorname{cosec} \tfrac{1}{2}(b-a) \tan \tfrac{1}{2}(B-A).$$

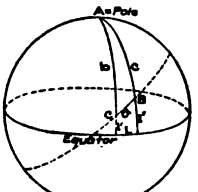

Fig. 4.

Distance between two points on the Earth's surface.—Given the latitudes and longitudes of two points B and C, Fig. 4, let it be required to find the shortest distance, a, between them. Select a third point, A, at the nearest pole; and connect the points ABC with arcs of great circles. Produce the meridian lines AB and AC to the equator. Then will l' be the latitude of C; l', the latitude of B; and L, the difference of longitude; and, in the spherical triangle ABC, $b = 90° - l'$, $c = 90° - l'$, and the angle $A = L$. Solve for Case II, preceding.

The Celestial Sphere.—In solving astronomical problems, the center of the earth is assumed to be the *center* of the celestial sphere; and its axis produced, the *axis* of said sphere. The extremeties of the axis produced are the *poles*; and the great circle whose plane is perpendicular to the axis (at the center), is the *equator*. A plumb line at any given point is called the *vertical line*, and if produced it intersects the celestial sphere in the *zenith* (above) and in the *nadir* (below). The zenith and nadir are the poles of any vertical line; and a great circle whose plane is perpendicular to said vertical line, forms the *horizon*. The *meridian* is the great circle whose plane passes through the zenith and the poles, and the points where it intersects the horizon are called the *north* and *south* points. The *prime vertical* is the great circle whose plane passes through the zenith perpendicular to the meridian, and the points where it intersects the horizon are called the *east* and *west* points.

A *vertical circle* is a great circle whose plane passes through the poles of a vertical line, i. e., through the zenith and nadir. It is therefore perpendicular to the horizon. The *azimuth* of a star is the arc *measured on the horizon* between the north point (or the south point) and the vertical circle passing through the star; hence it is the tangential angle at the zenith, between the meridian and said vertical circle. The *altitude*, h of a star is

its distance from the horizon measured on a vertical circle. The *zenith distance* $= z = 90° - h$.

An *hour circle* is any great circle whose plane passes through the poles (and perpendicular to the equator). The *hour angle*, P, of any star is the tangential angle which its hour circle measures (*westward* from and) with the meridian, at the pole; the arc of the angle is measured at the equator. The *right ascension*, $R.\ A.$, of a star is the distance on the equator from the *vernal equinox* (the point where the sun crosses the equator from south to north*), *eastward* to the hour circle of the star. The *declination*, δ, of a star is its distance from the equator, measured on an hour circle. The *polar distance*, p, $= 90° - \delta$.

Astronomical Time.—A *solar day* is our common day of twenty-four hours. Any particular solar day is an *apparent solar day*, and is the interval between two successive passages of the sun across the same meridian. A *mean solar day* is the average length of the solar days in a *tropical* year. A tropical year contains, according to Hansen and Olufsen, 365.2422008 mean solar days; according to Bessel, 365.24222 mean solar days. On account of the earth's revolution around the sun, there is *one more sidereal day* in a year than solar days. Hence, according to Bessel there are 366.-24222 sidereal days in a year, and we have the ratios—

1 solar day = 1.0027379 sidereal day = 1 sid. day + 3*m* 56.555*s* sid. time.
1 sid. day = 0.9972696 solar day = 1 solar day − 3*m* 55.909*s* solar time.

The *equation of time* is the quantity (time) to be "added," algebraically, to the *apparent* solar time to give the *mean* solar time. Its value for any day of the year may be found in the *solar ephemeris* tables of the Nautical Almanac.

The following are conversion tables for *mean solar* time and *sidereal* time:

1.—MEAN SOLAR TIME REDUCED TO SIDEREAL TIME.

Mean Hours	Sidereal Time	Mean Minutes	Sidereal Time	Mean Seconds	Sidereal Time
1	1h 0m 9.856s	1	1m 0.164s	1	1.003s
2	2 0 19.713	2	2 0.329	2	2.005
3	3 0 29.569	3	3 0.493	3	3.008
4	4 0 39.426	4	4 0.657	4	4.011
5	5 0 49.282	5	5 0.821	5	5.014
6	6 0 59.139	6	6 0.986	6	6.016
7	7 1 8.995	7	7 1.150	7	7.019
8	8 1 18.852	8	8 1.314	8	8.022
9	9 1 28.708	9	9 1.478	9	9.025
10	10 1 38.565	10	10 1.643	10	10.027
24	24 3 56.555	30	30 4.928	30	30.082

2.—SIDEREAL TIME REDUCED TO MEAN SOLAR TIME.

Sidereal Hours	Mean Solar Time.	Sidereal Minutes	Mean Solar Time	Sidereal Seconds	Mean Solar Time
1	0h 59m 50.170s	1	0m 59.836s	1	0.997s
2	1 59 40.341	2	1 59.672	2	1.995
3	2 59 30.511	3	2 59.509	3	2.992
4	3 59 20.682	4	3 59.345	4	3.989
5	4 59 10.852	5	4 59.181	5	4.986
6	5 59 1.023	6	5 59.017	6	5.984
7	6 58 51.193	7	6 58.853	7	6.981
8	7 58 41.364	8	7 58.689	8	7.978
9	8 58 31.534	9	8 58.526	9	8.975
10	9 58 21.704	10	9 58.362	10	9.973
24	23 56 4.091	30	29 55.085	30	29.918

* The *autumnal equinox* is the point where the sun crosses the equator from *north to south*.

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11.—MENSURATION.

A.—PLANE SURFACES, LINES, ETC.

Triangle.—Three sides. *Area* $= \frac{bh}{2}$; $= \frac{1}{2}\, ab \sin C$; $= \sqrt{s\,(s-a)\,(s-b)\,(s-c)}$, in which $s = \frac{a+b+c}{2}$. Length of $c = \sqrt{a^2+b^2+2\,ab\cos C}$. *Center of gravity* is $\frac{h}{3}$ above b. Angles $A+B+C=180°$.

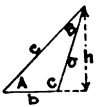

Fig. 1.

Right-angled triangle.—Angle $C=90°$. *Area* $= \frac{ab}{2}$. Hypothenuse $c = \sqrt{a^2+b^2}$; hence $a = \sqrt{c^2-b^2} = \sqrt{(c+b)(c-b)}$; $b = \sqrt{c^2-a^2}$. If $a=b$, then $c = 1.4142\,a = 1.4142\,b$, or $a=b=0.7071\,c$.

Acute-angled triangle.—Each angle less than 90°.

Obtuse-angled triangle.—One angle greater than 90° (as Fig. 1).

Isosceles triangle.—Two sides (therefore two angles) equal.

Equilateral triangle.—Three sides (therefore three angles) equal. See Table 1, Regular Polygons.

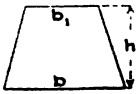

Fig. 2.

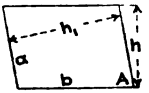

Fig. 3.

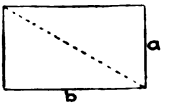

Fig. 4.

Quadrilateral.—Four Sides.

Trapezium.—An irregular quadrilateral; no two sides parallel. *Area* may be obtained by cutting it into two triangles, and solving.

Trapezoid.—Two sides only, parallel, Fig. 2. *Area* $= \frac{h}{2}\,(b+b_1)$. See Section 29 for *center of gravity*.

Parallelogram.—Includes the rhomboid, rhombus, rectangle and square. *Area* = length of one side multiplied by perpendicular distance to parallel side opposite.

Rhomboid.—A "skewed" rectangle, Fig. 3; opposite sides (therefore opposite angles) equal. *Area* $= bh = ba \sin A = ab \sin A = ah_1$ (Fig. 3). *Cen. of grav* is at intersection of two diagonals.

Rectangle.—Opposite sides equal and parallel, as with the rhomboid; but angles 90°, Fig. 4. Side $b>a$. *Area* $= ab$. Diagonal $= \sqrt{a^2+b^2}$. *Cen. of grav.* equi-distant from parallel sides, and at intersection of diagonals.

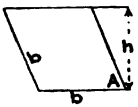

Fig. 5.

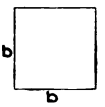

Fig. 6.

Rhombus.—Same as rhomboid (Fig. 3) but with all sides equal, Fig. 5. *Area* $= bh = b^2 \sin A$.

Square.—All sides equal; angles 90°, Fig. 6. *Area* $= b^2$. Diagonal $= \sqrt{2\,b^2} = 1.4142\,b$. Side b = diagonal $\times$ 0.7071.

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Regular Polygon.—Any number of sides, from the triangle, with three sides, to the circle, with an ∞ number of sides. In Table 1, following,

s = length of each side $= 2\,R \sin \frac{1}{2}\,\alpha = 2\,r \tan \frac{1}{2}\,\alpha$;
n = number of sides in polygon ($n\,s$ = perimeter);
R = radius of circumscribing circle $= \frac{1}{2}\,s \operatorname{cosec} \frac{1}{2}\,\alpha = r \sec \frac{1}{2}\,\alpha$;
r = apothem = radius of inscribed circle $= R \cos \frac{1}{2}\,\alpha = \frac{1}{2}\,s \cot \frac{1}{2}\,\alpha$;
α = angle subtended from center by each side $= 360 \div n$;
β = internal angle $= 180° - \alpha$;

Fig. 7.

$$Area = \frac{s\,r\,n}{2} = \frac{\text{perimeter} \times \text{apothem}}{2} = \tfrac{1}{2}\,n\,s\,R \cos \tfrac{1}{2}\,\alpha = n\,r^2 \tan \tfrac{1}{2}\,\alpha.$$

1.—Table of Regular Polygons (Fig. 7).

Number of Sides n	Name of Polygon	Central Angle α	Internal Angle β	Side s: Equals R times	Side s: Equals r times	Outer radius R: Equals s times	Outer radius R: Equals r times	Inner radius r: Equals s times	Inner radius r: Equals R times
3	Triangle	120°-00′-00″	60°-00′-00″	1.73205	3.46410	.577350	2.00000	.28868	.50000
4	Square	90 -00 -00	90 -00 -00	1.41421	2.00000	.707107	1.41421	.50000	.70711
5	Pentagon	72 -00 -00	108 -00 -00	1.17557	1.45308	.850651	1.23607	.68819	.80903
6	Hexagon	60 -00 -00	120 -00 -00	1.00000	1.15470	1.000000	1.15470	.86603	.86603
7	Heptagon	51 -25 -43	128 -34 -17	.86777	.96315	1.152382	1.10992	1.03827	.90097
8	Octagon	45 -00 -00	135 -00 -00	.76537	.82843	1.306563	1.08239	1.20711	.92388
9	Nonagon	40 -00 -00	140 -00 -00	.68404	.72794	1.461902	1.06418	1.37374	.93969
10	Decagon	36 -00 -00	144 -00 -00	.61803	.64984	1.618034	1.05146	1.53884	.95106
11	Undecagon	32 -43 -38	147 -16 -22	.56346	.58725	1.774733	1.04222	1.70287	.95942
12	Dodecagon	30 -00 -00	150 -00 -00	.51764	.53590	1.931854	1.03528	1.86603	.96593
∞	Circle	0	180°	0	0	∞	Unity	∞	Unity

Number of Sides n	Name of Polygon	Perimeter p: Equals s times	Perimeter p: Equals R times	Perimeter p: Equals r times	Area A: Equals s^2 times	Area A: Equals R^2 times	Area A: Equals r^2 times	Area A: Equals p times
3	Triangle	3	5.19615	10.39230	4330127	1.29904	5.19615	$r \div 2$
4	Square	4	5.65685	8.00000	1.0000000	2.00000	4.00000	••
5	Pentagon	5	5.87785	7.26542	1.720474	2.37764	3.63271	••
6	Hexagon	6	6.00000	6.92820	2.598076	2.59808	3.46410	••
7	Heptagon	7	6.07437	6.74204	3.633924	2.73641	3.36502	••
8	Octagon	8	6.12293	6.62741	4.828427	2.82843	3.31370	••
9	Nonagon	9	6.15636	6.55146	6.181824	2.89254	3.27573	••
10	Decagon	10	6.18034	6.49838	7.694088	2.93893	3.24920	••
11	Undecagon	11	6.19812	6.45977	9.365639	2.97352	3.22989	••
12	Dodecagon	12	6.21166	6.43078	11.196152	3.00000	3.21539	••
∞	Circle	∞	6.28319	6.28319	∞	3.14159	3.14159	••

Circle.—Infinite number of sides.

Circumference, $p = \pi d = 3.1416\,d, = 2\,\pi r = 6.2832\,r$;

Area, $a = \frac{\pi\,d^2}{4} = 0.7854\,d^2, = \pi\,r^2 = 3.1416\,r^2$;

Diameter, $d = \frac{p}{\pi} = 0.318310\,p, = 2\sqrt{\frac{a}{\pi}} = 1.12838\sqrt{a}$;

Radius, $r = \frac{p}{2\,\pi} = 0.159155\,p, = \sqrt{\frac{a}{\pi}} = 0.56419\sqrt{a}$.

Fig. 8.

$\pi = 3.141592\,6536$ Log $= 0.497\,149\,8727$ $\Big\|$ $\frac{1}{\pi} = 0.318\,309\,8862$ Log $= 9.502\,850\,127$

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*TABULAR VALUES OF COMBINATIONS OF π, WITH LOGS.

Number.	Logarithm.	Number.	Logarithm.	Number.	Logarithm.
$2\pi = 6.283185$	0.7981799	$\frac{2}{\pi} = 0.636620$	9.8038801	$\frac{\pi}{2} = 1.570796$	0.1961199
$3\pi = 9.424778$	0.9742712	$\frac{3}{\pi} = 0.954930$	9.9799714	$\frac{\pi}{3} = 1.047198$	0.0200286
$4\pi = 12.566371$	1.0992099	$\frac{4}{\pi} = 1.273240$	0.1049101	$\frac{\pi}{4} = 0.785398$	9.8950899
$5\pi = 15.707963$	1.1961199	$\frac{5}{\pi} = 1.591549$	0.2018201	$\frac{\pi}{5} = 0.628319$	9.7981799
$6\pi = 18.849556$	1.2753012	$\frac{6}{\pi} = 1.909859$	0.2810014	$\frac{\pi}{6} = 0.523599$	9.7189986
$7\pi = 21.991149$	1.3422479	$\frac{7}{\pi} = 2.228169$	0.3479481	$\frac{\pi}{7} = 0.448799$	9.6520519
$8\pi = 25.132741$	1.4002399	$\frac{8}{\pi} = 2.546479$	0.4059401	$\frac{\pi}{8} = 0.392699$	9.5940599
$9\pi = 28.274334$	1.4513924	$\frac{9}{\pi} = 2.864789$	0.4570926	$\frac{\pi}{9} = 0.349066$	9.5429074
Note.—In circular		$\frac{180^\circ}{\pi} = 57.295780$	1.7581226	$\frac{\pi}{180^\circ} = .01745329$	8.2418774
measure, $\pi = 180^\circ$		$\frac{360^\circ}{\pi} = 114.59156$	2.0591526	$\frac{\pi}{360^\circ} = .00087266$	7.9408474
$= 10800$ min.		$\frac{10800'}{\pi}$ 3437.7468	3.5362739	$\frac{\pi}{10800'} = .00029089$	6.4637261
$= 648000$ sec.		$\frac{648000''}{\pi} = 206264.81$	5.3144251	$\frac{\pi}{648000''} = .000004848$	4.6855749
$\pi^2 = 9.86960$	0.9942997	$\pi^3 = 31.00628$	1.4914496	$\pi^4 = 97.4091$	1.9885995
$\frac{1}{\pi^2} = 0.10132$	9.0057003	$\frac{1}{\pi^3} = 0.03225$	8.5085504	$\frac{1}{\pi^4} = 0.01027$	8.0114005
$\sqrt{\pi} = 1.77245$	0.2485749	$\sqrt[4]{\pi} = 1.46459$	0.1657166	$\sqrt[4]{\pi} = 1.33134$	0.1242875
$\sqrt{2\pi} = 2.50663$	0.3990899	$\sqrt[4]{2\pi} = 1.84527$	0.2660600	$\sqrt[4]{2\pi} = 1.58323$	0.1995450
$\sqrt{\frac{1}{\pi}} = 0.56419$	9.7514251	$\sqrt[3]{\frac{1}{\pi}} = 0.68278$	9.8342834	$\sqrt[4]{\frac{1}{\pi}} = 0.75113$	9.8757125
$\sqrt{\frac{2}{\pi}} = 0.79788$	9.9019401	$\sqrt[3]{\frac{2}{\pi}} = 0.86025$	9.9346267	$\sqrt[4]{\frac{2}{\pi}} = 0.89324$	9.9509700
$\sqrt{\frac{\pi}{2}} = 1.25331$	0.0980599	$\sqrt[3]{\frac{\pi}{2}} = 1.16245$	0.0653733	$\sqrt[4]{\frac{\pi}{2}} = 1.11952$	0.0490300

* For multiples 1 to 9, see next page.

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TABULAR VALUES OF COMBINATIONS OF π, WITH LOGS.—Concluded.
(Multiples 1 to 9.)

	π*	Log.		$\frac{1}{\pi}$	Log.		$\sqrt{\frac{1}{\pi}}$	Log.
1	3.1415927	0.4971499	1	0.3183099	9.5028501	1	0.5641896	9.7514251
2	6.2831853	0.7981799	2	0.6366198	9.8038801	2	1.1283792	0.0524551
3	9.4247780	0.9742712	3	0.9549297	9.9799714	3	1.6925688	0.2285464
4	12.5663706	1.0992099	4	1.2732395	0.1049101	4	2.2567583	0.3534851
5	15.7079633	1.1961199	5	1.5915494	0.2018201	5	2.8209479	0.4503951
6	18.8495559	1.2753012	6	1.9098593	0.2810014	6	3.3851375	0.5295764
7	21.9911486	1.3422479	7	2.2281692	0.3479481	7	3.9493271	0.5965231
8	25.1327412	1.4002399	8	2.5464791	0.4059401	8	4.5135167	0.6545151
9	28.2743339	1 4513924	9	2.8647890	0.4570926	9	5.0777063	0 7056676

	π^2	Log.		$\frac{1}{\pi^2}$	Log.		$\sqrt{\pi}$	Log.
1	9.8696044	0.9942997	1	0.1013210	9.0057003	1	1.7724539	0.2485749
2	19.7392088	1.2953297	2	0.2026420	9.3067303	2	3.5449077	0.5496049
3	29.6088132	1.4714210	3	0 3039631	9.4828216	3	5.3173616	0.7256963
4	39.4784176	1.5963597	4	0.4052841	9.6077603	4	7.0898154	0.8506349
5	49.3480220	1.6932697	5	0.5066051	9.7046703	5	8.8622693	0.9475449
6	59.2176264	1.7724510	6	0.6079261	9.7838516	6	10.6347231	1.0267262
7	69.0872308	1.8393977	7	0.7092471	9.8507983	7	12.4071770	1.0936729
8	79.9568352	1.8973897	8	0.8105682	9.9087903	8	14.1796308	1.1516649
9	88.8264396	1.9485422	9	0.9118892	9.9599428	9	15.9520847	1.2028174

	π^3	Log.		$\frac{1}{\pi^3}$	Log.		$\sqrt[3]{\pi}$	Log.
1	31.0062767	1.4914496	1	0.0322515	8.5085504	1	1.4645919	0.1657166
2	62.0125534	1.7924796	2	0.0645030	8.8095804	2	2.9291838	0.4667466
3	93.0188300	1.9685709	3	0.0967545	8.9856717	3	4.3937756	0.6428379
4	124.0251067	2.0935096	4	0.1290060	9.1106104	4	5.8583675	0.7677766
5	155.0313834	2.1904196	5	0.1612575	9.2075204	5	7.3229594	0.8646866
6	186.0376601	2.2696009	6	0.1935090	9.2867017	6	8.7875513	0.9438679
7	217.0439368	2.3365476	7	0.2257605	9.3536484	7	10.2521432	1.0108146
8	248.0502134	2.3945396	8	0.2580120	9.4116404	8	11.7167351	1.0688066
9	279.0564901	2.4456921	9	0.2902635	9.4627929	9	13.1813269	1.1199591

	π^4	Log.		$\frac{1}{\pi^4}$	Log.		$\sqrt[3]{\frac{1}{\pi}}$	Log.
1	97.4090909	1.9885995	1	0.0102660	8.0114005	1	0.6827841	9.8342834
2	194.8181818	2.2896295	2	0.0205320	8.3124305	2	1.3655681	0.1353134
3	292.2272727	2.4657208	3	0.0307979	8.4885218	3	2.0483522	0.3114047
4	389.6363636	2.5906595	4	0.0410639	8.6134605	4	2.7311363	0.4363434
5	487.0454545	2.6875695	5	0.0513299	8.7103705	5	3.4139203	0.5332534
6	584.4545453	2.7667508	6	0 0615959	8.7895518	6	4.0967044	0.6124347
7	681.8636362	2.8336975	7	0.0718619	8.8564985	7	4.7794885	0 6793814
8	779.2727271	2.8916895	8	0.0821278	8.9144905	8	5.4622725	0.7373734
9	876.6818180	2.9428420	9	0.0923938	8.9656430	9	6.1450566	0.7885259

Logarithms of Numbers 1 to 9, for Reference.

Log 1 = 0.0000000	Log 4 = 0.6020600	Log 7 = 0.8450980
" 2 = 0.3010300	" 5 = 0.6989700	" 8 = 0.9030900
" 3 = 0.4771213	" 6 = 0.7781513	" 9 = 0.9542425

*See Table 11, pages 224 and 225, for values of π when multiplied by any whole number or decimal. (See Foot-note to Table 11.) Table 12 can be used in a similar manner.

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and **Chord.**—

unctions ($\alpha + \beta = 360°$).
angle in degrees, $< 180°$.
angle in degrees, $> 180°$.

ons,
th of arc a to radius 1.
th of arc b to radius 1.

ance to *cen. of grav.* g of arc a.
ance to *cen. of grav.* G of arc b.

Formulas:

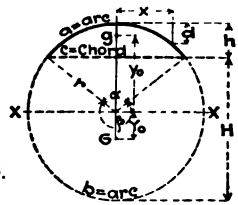

Fig. 9.

$= \frac{c}{2r}$. $\text{Cos}\,\tfrac{1}{2}\alpha = \frac{r-h}{r}$; $\cos\tfrac{1}{2}\beta = \frac{H-r}{r}$.

$\frac{}{h)}$; $\tan\tfrac{1}{2}\beta = \frac{c}{2(H-r)}$. $\text{Vers}\,\tfrac{1}{2}\alpha = \frac{h}{r}$; $\text{vers}\,\tfrac{1}{2}\beta = \frac{H}{r}$.

$\tan\tfrac{1}{2}\beta = \frac{2H}{c}$.

745329 α; $\beta_1 = \frac{\pi\beta}{180°} = 0.01745329\,\beta$

01745 $r\,\alpha$; Arc $b = r\,\beta_1 = 0.01745\,r\,\beta$.

$\frac{\alpha}{c} = \frac{.01745\,h\,\alpha}{\text{vers}\,\tfrac{1}{2}\alpha}$. $= \frac{.01745\,c\,\beta}{2\sin\tfrac{1}{2}\beta} = \frac{.01745\,H\,\beta}{\text{vers}\,\tfrac{1}{2}\beta}$.

$\alpha = 2\,r\sin\tfrac{1}{2}\beta = 2\sqrt{h(2r-h)} = 2\sqrt{H(2r-H)}$
$\tan\tfrac{1}{2}\alpha = 2(r-h)\tan\tfrac{1}{2}\beta = 2(H-r)\tan\tfrac{1}{2}\alpha = 2(H-r)$
$\tan\tfrac{1}{2}\beta$
$\alpha = 2h\cot\tfrac{1}{2}\beta = 2(2r-H)\cot\tfrac{1}{2}\alpha = 2(2r-H)\cot\tfrac{1}{2}\beta$.

ord $\left(\frac{c}{2}\right)$ is a mean proportional between h and H; thus

$= 2\sqrt{hH}$.

$$\left.\begin{aligned} \frac{\sin\tfrac{1}{2}\alpha}{\alpha_1} &= r\frac{\text{length of chord } c}{\text{length of arc } a} = \frac{rc}{a}. \\ \frac{\sin\tfrac{1}{2}\beta}{\beta_1} &= r\frac{\text{length of chord } c}{\text{length of arc } b} = \frac{rc}{b}. \end{aligned}\right\} \frac{y_0}{Y_0} = \frac{b}{a} = \frac{\beta}{\alpha} = \frac{\beta_1}{\alpha_1}.$$

57.29578; log = 1.7581226 (see under Circle).

olution s generated by arc a revolving about axis $X-X$
plied by length of path traveled by point g;

in which θ is the angle of revolution, in degrees)

$\frac{\alpha}{}$, the general equation. Thus, to get the surface of a

$180°$, $\theta = 360°$; then $s = 4\pi r^2 = 12.566371\,r^2$.

te of arc a H = middle ordinate of arc A
$\alpha) = r\,\text{vers}\,\tfrac{1}{2}\alpha$ $= r(1+\cos\tfrac{1}{2}\beta)$

$\left(\frac{c}{2}\right)^2 = \frac{c}{2}\tan\tfrac{1}{2}\alpha$. $= r + \sqrt{r^2 - \left(\frac{c}{2}\right)^2} = \frac{c}{2}\tan\tfrac{1}{2}\beta$.

te at any point distant x from center $= h - d$, in which
ence, ordinate $= h - r + \sqrt{r^2 - x^2}$.

$$\frac{180a}{\alpha} = \frac{57.29578a}{\alpha} = \frac{h}{1-\cos\tfrac{1}{2}\alpha} = \frac{c}{2\sin\tfrac{1}{2}\alpha} = \frac{h}{\text{vers}\,\tfrac{1}{2}\alpha};$$

$$\frac{180b}{\beta} = \frac{57.29578b}{\beta} = \frac{H}{1+\cos\tfrac{1}{2}\beta} = \frac{c}{2\sin\tfrac{1}{2}\beta} = \frac{H}{\text{vers}\,\tfrac{1}{2}\beta}.$$

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VALUES OF α_1 AND β_1 (Fig. 9)

2.—LENGTHS OF CIRCULAR ARCS FOR RADIUS 1.

(Note that the arc is directly proportional to central angle and to radius.)

Deg.	Length.	Deg.	Length.	Min.	Length.	Sec.	Length.
15°	0.261799 3878	1°	.017453 2925	1′	.000290 8882	1″	.000004 8481
30°	0.523598 7756	2°	.034906 5850	2′	.000581 7764	2″	.000009 6963
45°	0.785398 1634	3°	.052359 8776	3′	.000872 6646	3″	.000014 5444
60°	1.047197 5512	4°	.069813 1701	4′	.001163 5528	4″	.000019 3925
75°	1.308996 9390	5°	.087266 4626	5′	.001454 4410	5″	.000024 2407
90°	1.570796 3268	6°	.104719 7551	6′	.001745 3293	6″	.000029 0888
135°	2.356194 4902	7°	.122173 0476	7′	.002036 2175	7″	.000033 9370
180°	3.141592 6536	8°	.139626 3402	8′	.002327 1057	8″	.000038 7851
270°	4.712388 9804	9°	.157079 6327	9′	.002617 9939	9″	.000043 6332
360°	6.283185 3072	10°	.174532 9252	10′	.002908 8821	10″	.000048 4814

Problem.—Find the length of circular arc corresponding to a central angle of 237°—46′—58.3″ and to a radius of 6387.42.

Solution.—From Table 2.

200°	= 3.490659	
30°	= .523599	
7°	= .122173	
40′	= .011636	
6′	= .001745	
50″	= .000242	
8″	= .000039	
.3″	= .000001	
log	4.150094	= 0.6180580
log	6387.42	= 3.8053255
Ans.	26508.4	4.4233835

Table 3, following, has been prepared from Table 2.

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VALUES OF α_1 AND β_1 (Fig. 9)

3.—LENGTHS OF CIRCULAR ARCS FOR RADIUS 1.

(Deg., Min., Sec., are central angles; Length is length of subtended arc.)

Deg.	Length.	Deg	Length.	Deg.	Length.	Min.	Length.	Sec.	Length.
1	.0174533	61	1.0646508	121	2.1118484	1	.0002909	1	.0000048
2	.0349066	62	1.0821041	122	2.1293017	2	.0005818	2	.0000097
3	.0523599	63	1.0995574	123	2.1467550	3	.0008727	3	.0000145
4	.0698132	64	1.1170107	124	2.1642083	4	.0011636	4	.0000194
5	.0872665	65	1.1344640	125	2.1816616	5	.0014544	5	.0000242
6	.1047198	66	1.1519173	126	2.1991149	6	.0017453	6	.0000291
7	.1221730	67	1.1693706	127	2.2165682	7	.0020362	7	.0000339
8	.1396263	68	1.1868239	128	2.2340214	8	.0023271	8	.0000388
9	.1570796	69	1.2042772	129	2.2514747	9	.0026180	9	.0000436
10	.1745329	70	1.2217305	130	2.2689280	10	.0029089	10	.0000485
11	.1919862	71	1.2391838	131	2.2863813	11	.0031998	11	.0000533
12	.2094395	72	1.2566371	132	2.3038346	12	.0034907	12	.0000582
13	.2268928	73	1.2740904	133	2.3212879	13	.0037815	13	.0000630
14	.2443461	74	1.2915436	134	2.3387412	14	.0040724	14	.0000679
15	.2617994	75	1.3089969	135	2.3561945	15	.0043633	15	.0000727
16	.2792527	76	1.3264502	136	2.3736478	16	.0046542	16	.0000776
17	.2967060	77	1.3439035	137	2.3911011	17	.0049451	17	.0000824
18	.3141593	78	1.3613568	138	2.4085544	18	.0052360	18	.0000873
19	.3316126	79	1.3788101	139	2.4260077	19	.0055269	19	.0000921
20	.3490659	80	1.3962634	140	2.4434610	20	.0058178	20	.0000970
21	.3665191	81	1.4137167	141	2.4609142	21	.0061087	21	.0001018
22	.3839724	82	1.4311700	142	2.4783675	22	.0063995	22	.0001067
23	.4014257	83	1.4486233	143	2.4958208	23	.0066904	23	.0001115
24	.4188790	84	1.4660766	144	2.5132741	24	.0069813	24	.0001164
25	.4363323	85	1.4835299	145	2.5307274	25	.0072722	25	.0001212
26	.4537856	86	1.5009832	146	2.5481807	26	.0075631	26	.0001261
27	.4712389	87	1.5184364	147	2.5656340	27	.0078540	27	0001309
28	.4886922	88	1.5358897	148	2.5830873	28	.0081449	28	.0001357
29	.5061455	89	1.5533430	149	2.6005406	29	.0084358	29	.0001406
30	.5235988	90	1.5707963	150	2.6179939	30	.0087266	30	.0001454
31	.5410521	91	1.5882496	151	2.6354472	31	.0090175	31	.0001503
32	.5585054	92	1.6057029	152	2.6529005	32	.0093084	32	0001551
33	.5759587	93	1.6231562	153	2.6703538	33	.0095993	33	0001600
34	.5934119	94	1.6406095	154	2.6878070	34	.0098902	34	.0001648
35	.6108652	95	1.6580628	155	2.7052603	35	.0101811	35	.0001697
36	.6283185	96	1.6755161	156	2.7227136	36	.0104720	36	.0001745
37	.6457718	97	1.6929694	157	2.7401669	37	0107629	37	.0001794
38	.6632251	98	1.7104227	158	2.7576202	38	.0110538	38	.0001842
39	.6806784	99	1.7278760	159	2.7750735	39	.0113446	39	.0001891
40	.6981317	100	1.7453293	160	2.7925268	40	.0116355	40	.0001939
41	.7155850	101	1.7627825	161	2.8099801	41	.0119264	41	.0001988
42	.7330383	102	1.7802358	162	2.8274334	42	.0122173	42	.0002036
43	.7504916	103	1.7976891	163	2.8448867	43	.0125082	43	.0002085
44	.7679449	104	1.8151424	164	2.8623400	44	.0127991	44	.0002133
45	.7853982	105	1.8325957	165	2.8797933	45	.0130900	45	.0002182
46	.8028515	106	1.8500490	166	2.8972466	46	0133809	46	.0002230
47	.8203047	107	1.8675023	167	2.9146999	47	.0136717	47	.0002279
48	.8377580	108	1.8849556	168	2.9321531	48	.0139626	48	.0002327
49	.8552113	109	1.9024089	169	2.9496064	49	.0142535	49	.0002376
50	.8726646	110	1.9198622	170	2.9670597	50	.0145444	50	.0002424
51	.8901179	111	1.9373155	171	2.9845130	51	.0148353	51	.0002473
52	.9075712	112	1.9547688	172	3.0019663	52	.0151262	52	.0002521
53	.9250245	113	1.9722221	173	3.0194196	53	.0154171	53	.0002570
54	.9424778	114	1.9896753	174	3.0368729	54	.0157080	54	.0002618
55	.9599311	115	2.0071286	175	3.0543262	55	.0159989	55	.0002666
56	.9773844	116	2.0245819	176	3.0717795	56	.0162897	56	.0002715
57	.9948377	117	2.0420352	177	3.0892328	57	.0165806	57	.0002763
58	1.0122910	118	2.0594885	178	3.1066861	58	.0168715	58	.0002812
59	1.0297443	119	2.0769418	179	3.1241394	59	.0171624	59	.0002860
60	1.0471976	120	2.0943951	180	3.1415927	60	.0174533	60	.0002909

Note.—Length of arc is directly proportional to central angle and to radius.

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4.—Lengths of Circular Arcs for Chord 1.

(Note.—Multiply the tabular number by the length of chord.)

Rise ÷ Chord.	Thousandths									
	.000	.001	.002	.003	.004	.005	.006	.007	.008	.009
.00	1.00000	1.00002	1.00002	1.00003	1.00004	1.00007	1.00010	1.00013	1.00017	1.00022
.01	.00027	.00032	.00038	.00045	.00053	.00061	.00069	.00078	.00087	.00097
.02	00107	.00117	.00128	.00140	.00153	.00167	.00182	.00196	.00210	.00225
.03	.00240	.00256	.00272	.00289	.00307	.00327	.00345	.00364	.00384	.00405
.04	.00426	.00447	.00469	.00492	.00515	.00539	.00563	.00587	.00612	.00638
.05	1.00665	1.00692	1.00720	1.00748	1.00776	1.00805	1.00834	1.00864	1.00895	1.00926
.06	.00957	.00989	.01021	.01054	.01088	.01123	.01158	.01193	.01228	.01264
.07	.01302	.01338	.01376	.01414	.01453	.01493	.01533	.01573	.01614	.01656
.08	.01698	.01741	.01784	.01828	.01872	.01916	.01961	.02006	.02052	.02098
.09	.02146	.02192	.02240	.02289	.02339	.02389	.02440	.02491	.02542	.02594
.10	1.02646	1.02698	1.02752	1.02806	1.02860	1.02914	1.02970	1.03026	1.03082	1.03138
.11	.03196	.03254	.03312	.03371	.03430	.03490	.03551	.03611	.03672	.03734
.12	.03797	.03860	.03923	.03987	.04051	.04116	.04181	.04247	.04313	.04380
.13	.04447	.04515	.04584	.04652	.04722	.04792	.04862	.04932	.05003	.05075
.14	.05147	.05220	.05293	.05367	.05441	.05516	.05591	.05667	.05743	.05819
.15	1.05896	1.05973	1.06051	1.06130	1.06209	1.06288	1.06368	1.06449	1.06530	1.06611
.16	.06693	.06775	.06858	.06941	.07025	.07109	.07194	.07279	.07365	.07451
.17	.07537	.07624	.07711	.07799	.07888	.07977	.08066	.08156	.08246	.08337
.18	.08428	.08519	.08611	.08704	.08797	.08890	.08984	.09079	.09174	.09269
.19	.09365	.09461	.09557	.09654	.09752	.09850	.09949	.10048	.10147	.10247
.20	1.10347	1.10447	1.10548	1.10650	1.10752	1.10855	1.10958	1.11062	1.11165	1.11269
.21	.11374	.11479	.11584	.11690	.11796	.11904	.12011	.12118	.12225	.12333
.22	.12444	.12554	.12664	.12774	.12885	.12997	.13108	.13219	.13331	.13443
.23	.13557	.13671	.13785	.13900	.14015	.14131	.14247	.14363	.14480	.14597
.24	.14714	.14832	.14951	.15070	.15189	.15308	.15428	.15549	.15670	.15791
.25	1.15912	1.16034	1.16156	.16279	1.16402	1.16526	1.16650	1.16774	1.16899	1.17024
.26	.17150	.17276	.17403	.17530	.17657	.17784	.17912	.18040	.18169	.18298
.27	.18429	.18559	18689	.18820	.18951	.19082	.19214	.19346	.19479	.19612
.28	.19746	.19880	.20014	.20149	.20284	.20419	.20555	.20691	.20827	.20964
.29	.21102	.21239	.21377	.21515	.21654	.21794	.21933	.22073	.22213	.22354
.30	1.22495	1.22636	1.22778	1.22920	1.23063	1.23206	1.23349	1.23492	1.23636	1.23781
.31	.23926	.24070	.24216	.24361	.24507	.24654	.24801	.24948	.25095	.25243
.52	.25391	.25540	.25689	.25838	.25988	.26138	.26288	.26437	.26588	.26740
.33	.26892	.27044	.27196	.27349	.27502	.27656	.27810	.27964	.28118	.28273
.34	.28428	.28583	.28739	.28895	.29052	.29209	.29366	.29523	.29681	.29839
.35	1.29997	1.30156	1.30315	1.30474	1.30634	1.30794	1.30954	1.31115	1.31276	1.31437
.36	.31599	.31761	.31923	.32086	.32249	.32413	.32577	.32741	.32905	.33068
.37	.33234	.33399	.33564	.33730	.33896	.34063	.34229	.34396	.34563	.34731
.38	.34899	.35068	.35237	.35406	.35575	.35744	.35914	.36084	.36254	.36425
.39	.36596	.36767	.36939	.37111	.37283	.37455	.37628	.37801	.37974	.38148
.40	1.38322	1.38496	1.38671	1.38846	1.39021	1.39196	1.39372	1.39548	1.39724	1.39900
.41	.40077	.40254	.40432	.40610	.40788	.40966	.41145	.41324	.41503	.41683
.42	.41861	.42041	.42221	.42402	.42583	.42764	.42945	.43127	.43309	.43491
.43	.43673	.43856	.44039	.44222	.44405	.44589	.44773	.44957	.45142	.45327
.44	.45512	.45697	.45883	.46069	.46255	.46441	.46628	.46815	.47002	.47189
.45	1.47377	1.47565	1.47753	1.47942	1.48131	1.48320	1.48509	1.48699	1.48889	1.49079
.46	.49269	.49460	.49651	.49842	.50033	.50224	.50416	.50608	.50800	.50993
.47	.51185	.51378	.51571	.51764	.51958	.52152	.52346	.52541	.52736	.52931
.48	.53126	.53322	.53518	.53714	.53910	.54106	.54302	.54499	.54696	.54893
.49	.55091	.55289	.55487	.55685	.55884	.56083	.56282	.56481	.56681	.56880
.50	1.57080									

Ex.—For a chord of 215 ft. and rise of 18 ft.: The rise ÷ chord = 0.083721; and the corresponding arc = 215 × (1.01828 + .721 × .00044) = 215 × 1.0186 = 219.00 ft. *Ans.*

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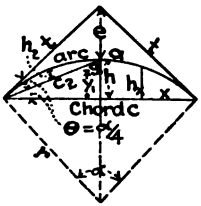

Fig. 10.

Flat Circular Arc.—

Exact formulas:

c_2 = chord of half arc $= \sqrt{h^2 + \left(\frac{c}{2}\right)^2} = \frac{c}{2} \sec \frac{1}{2} \alpha$;

e = external secant $= r\,(\sec \frac{1}{2} \alpha - 1) = r\,(\text{exsec}\, \frac{1}{2} \alpha)$

t = tangent distance $= r \tan \frac{1}{2} \alpha = \frac{c}{2} \sec \frac{1}{2} \alpha$.

Approximate formulas:

a = arc $= \frac{8c_2 - c}{3}$; $\qquad h_x = \frac{x\,(c-x)}{2r} = \frac{4\,h\,x}{c^2}(c-x)$;

c = chord $= 8\,c_2 - 8\,a$; $\qquad h_1$ = quarter ordinate $= \frac{3}{4}\,h$ $\left(\text{when } x = \frac{c}{4}\right)$;

$e = h$; $\qquad t = \frac{c}{2}\sqrt{\frac{c^2}{4\,r^2} + 1}$;

h = rise $= \frac{c^2}{8r}\left(\therefore r = \frac{c^2}{8h}\right)$; $\qquad y_1 = \frac{3}{4}\,h$. ($g$ = cen. of grav. of arc.)

The coefficients in Table 5 (a and b), following, give an idea of the approximation of the above formulas; and they will also be found convenient to use in some cases where fairly correct values, simply, are sought.

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5.—COEFFICIENTS TO BE USED WITH APPROXIMATE FORMULAS (Page, preceding) TO GIVE EXACT VALUES. (Fig. 10.)

(*a*)—BASED ON CENTRAL ANGLE α = 10°, 20°, 30°, etc.

(Note.—Multiply result from approximate formula, by Coefficient.)

Central Angle α =	10°	20°	30°	40°	50°	60°
Values of $\frac{h}{c}$ =	.02183	.04374	.06583	.08816	.11085	.13397
Values of $\frac{a}{c}$ =	1.00127	1.00510	1.01152	1.02060	1.03245	1.04720
Values of $\frac{c}{a}$ =	.99873	.99493	.98862	.97982	.96857	.95493
Values of $\frac{c}{r}$ =	.17431	.34730	.51764	.68404	.84524	1.00000
Formulas.	**Coefficients.***					
$a = \frac{8c_2 - c}{3}$ ×	1.00000	1.00000	.99999	.99997	.99992	.99984
$c = (8c_2 - 3a)$ ×	1.00000	.99999	.99997	.99991	.99977	.99951
†$e = h$ ×	1.00382	1.01543	1.03528	1.06418	1.10338	1.15470
‡$h = \frac{c^2}{8r}$ ×	.99810	.99240	.98296	.96985	.95315	.93301
†$h_1 = \frac{3}{4}h$ ×	1.00017	1.00064	1.00143	1.00256	1.00401	1.00581
†$h_2 = \frac{1}{4}h$ ×	1.00050	1.00191	1.00429	1.00765	1.01199	1.01738
†$y_1 = \frac{3}{8}h$ ×	1.00014	1.00052	1.00116	1.00204	1.00321	1.00464

* Coefficient for $a = \frac{3}{8} \cdot \frac{8 \sin \frac{1}{2}\alpha - \sin \frac{1}{2}\alpha}{\text{arc}}$; for $c = \frac{8 \sin \frac{1}{2}\alpha - 1\frac{1}{2}\,\text{arc}}{\sin \frac{1}{2}\alpha}$; for $e = \sec \frac{1}{2}\alpha$; for $h = \cos^2 \frac{1}{2}\alpha$; for $h_1 = \frac{3}{4} \cdot \frac{1 - \cos \frac{1}{2}\alpha}{\cos \frac{1}{2}\alpha - \cos \frac{1}{2}\alpha}$; for $h_2 = \frac{4(1 - \cos \frac{1}{2}\alpha)}{1 - \cos \frac{1}{2}\alpha}$; for $y_1 = \frac{3}{8} \cdot \frac{1 - \cos \frac{1}{2}\alpha}{\frac{c}{a} - \cos \frac{1}{2}\alpha}$.

† Increase in coefficient is nearly parabolic; that is, nearly proportional to α^2; thus, the coefficient of h_1 for α = 18°, is $1 + .00064 \times \frac{(18)^2}{(20)^2} = 1.00052$, using the coefficient of the nearest angle.

‡ Decrease in coefficient is nearly parabolic.

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5.—Coefficients to be used with Approximate Formulas.—Concluded.

(b) Based on Rise to Chord, $\frac{h}{c}$, of Arc.

(Note.—Multiply result from approximate formula, by Coefficient.)

Values of $\frac{h}{c}$ =		.02	.04	.06	.08	.10	.12
Values of α =		9° 09′ 45″	18° 17′ 44″	27° 22′ 16″	36° 21′ 40″	45° 14′ 23″	53° 58′ 59″
Values of $\frac{r}{c}$ =		6.3	3.1	2.1	1.6	1.3	1.1
Values of $\frac{c}{r}$ =		.16	.32	.47	.62	.77	.91
Formulas.		Coefficients.					
†$e = k$	×	1.003	1.013	1.029	1.052	1.083	1.122
‡$k = \frac{c^2}{8r}$	×	.998	.994	.986	.975	.962	.946
†$k_1 = \frac{2}{3}k$	×	1.000	1.001	1.001	1.002	1.003	1.005
†$k_2 = \frac{1}{3}k$	×	1.0004	1.0016	1.0036	1.0063	1.0098	1.0140

† Increase in coefficient is nearly parabolic.

‡ Decrease in coefficient is nearly parabolic.

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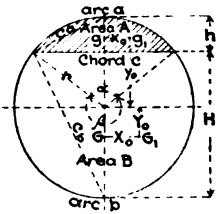

Fig. 11.

Circular Segment; and Half Segment.—

Diameter $d = 2r$.

Point g =	position	segment A	with	$x = 0$; $y = y_0$.
g_1 =	of center	$\frac{1}{2}$ (segment A)	coordi-	$x = x_0$; $y = y_0$.
G =	of grav-	segment B	nates	$x = 0$; $y = Y_0$.
G_1 =	ity of	$\frac{1}{2}$ (segment B)		$x = X_0$; $y = Y_0$.

Formulas for Center of Gravity (Fig. 11): (Geometrical)

For smaller segment: $\alpha = 0°$ to $180°$

$$x_0 = \tfrac{1}{3}r\frac{4\sin^2\frac{1}{2}\alpha - \sin^2\frac{1}{2}\alpha\cos\frac{1}{2}\alpha}{\frac{1}{2}\alpha_1 - \sin\frac{1}{2}\alpha\cos\frac{1}{2}\alpha} = \tfrac{1}{3}r\frac{c_a^2 - \frac{c^2}{4}(r-h)}{\text{area } A \text{ of segment}}$$

$$y_0 = \tfrac{2}{3}r\frac{\sin^3\frac{1}{2}\alpha}{\frac{1}{2}\alpha_1 - \sin\frac{1}{2}\alpha\cos\frac{1}{2}\alpha} = \tfrac{1}{12}\cdot\frac{c^3}{\text{area } A \text{ of segment}}$$

For larger segment: $\beta = 180°$ to $360°$

$$X_0 = \tfrac{1}{3}r\frac{4\sin^2\frac{1}{2}\beta + \sin^2\frac{1}{2}\beta\cos\frac{1}{2}\beta}{\frac{1}{2}\beta_1 + \sin\frac{1}{2}\beta\cos\frac{1}{2}\beta} = \tfrac{1}{3}r\frac{c_b^2 + \frac{c^2}{4}(H-r)}{\text{area B of segment}}$$

$$Y_0 = \tfrac{2}{3}r\frac{\sin^3\frac{1}{2}\beta}{\frac{1}{2}\beta_1 + \sin\frac{1}{2}\beta\cos\frac{1}{2}\beta} = \tfrac{1}{12}\cdot\frac{c^3}{\text{area B of segment}}$$

Or, $y_0 = \dfrac{c^3}{12A}$; $Y_0 = \dfrac{c^3}{12B}$; $\dfrac{y_0}{Y_0} = \dfrac{B}{A}$ when $\alpha + \beta = 360°$.

Note that $\sin\frac{1}{2}\alpha = \sin\frac{1}{2}\beta$; $\cos\frac{1}{2}\alpha = \cos\frac{1}{2}\beta$; but $\sin\frac{1}{2}\alpha = \cos\frac{1}{2}\beta$, etc. Table 6, below, gives tabular values.

6.—Values (Coefficients) of x_0, y_0, X_0 and Y_0 for Various Values of α and β. (Fig. 11.)

(Multiply the tabular Coefficient by the radius.)

α	$x_0 =$ r times	$y_0 =$ r times	β	$X_0 =$ r times	$Y_0 =$ r times	α	$x_0 =$ r times	$y_0 =$ r times	β	$X_0 =$ r times	$Y_0 =$ r times
0°	.00000	Unity	360°	.42441	.00000						
30°	.09728	.97957	330°	.42565	.00369	120°	.33920	.70502	240°	.44162	.17133
60°	.18930	.91994	300°	.42211	.03301	150°	.39058	.56734	210°	.44162	.28849
90°	.27124	.82587	270°	.43972	.08252	180°	.42441	.42441	180°	.42441	.42441

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Circular Segment (Fig 11; Tables 7, 8).—
greater than a semi-circle, i. e., α not $> 180°$.
less than a semi-circle, i. e., β not $< 180°$.

$$-h) = r\left(\frac{a}{2} - r \sin \tfrac{1}{2}\alpha \cos \tfrac{1}{2}\alpha\right) = \frac{r}{2}(a - r \sin \alpha). \quad (1)$$

$$H - r) = r\left(\frac{b}{2} + r \sin \tfrac{1}{2}\beta \cos \tfrac{1}{2}\beta\right) = \frac{r}{2}(b + r \sin \beta). \quad (2)$$

(2), above, are General Formulas.]

and b are given in terms of r, c and h; whence—

$$\left.\begin{array}{l} \ldots) - \sin \alpha \\ \hline \end{array}\right] = \text{area of sector} - \text{area of triangle between chord and radii.} \quad (3)$$

$$\left.\begin{array}{l} \ldots + \sin \beta \\ \hline \end{array}\right] = \text{area of sector} + \text{area of triangle between chord and radii.} \quad (4)$$

$$\left.\begin{array}{l} \ldots) - \sin \alpha \\ \hline \end{array}\right] \text{in which } \sin \tfrac{1}{2}\alpha = \frac{c}{2r};\ \tan \tfrac{1}{4}\alpha = \frac{2h}{c}. \quad (5)$$

$$\left.\begin{array}{l} \ldots + \sin \beta \\ \hline \end{array}\right] \text{in which } \sin \tfrac{1}{2}\beta = \frac{c}{2r};\ \tan \tfrac{1}{4}\beta = \frac{c}{2h}. \quad (6)$$

$$\left.\begin{array}{l} \ldots) - \sin \alpha \\ \hline \ldots \alpha \end{array}\right] \text{in which vers } \tfrac{1}{2}\alpha = \frac{h}{r} = 2 \sin^2 \tfrac{1}{4}\alpha. \quad (7)$$

$$\left.\begin{array}{l} \ldots) - \sin \alpha \\ \hline \ldots \text{rs } \tfrac{1}{2}\alpha \end{array}\right] \text{in which } \tan \tfrac{1}{4}\alpha = \frac{2h}{c}. \quad (8)$$

and central angle are given use equation (1), (2),
ntral angle, use (5) or (6); rise and chord, use (5),
d rise, use (7). See also foot-note to Table 8.

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7.—Table of Circular Segments—Areas, Etc.

To find the area: Multiply the tabular coefficient in column 4 by r^2; or the coefficient in column 5 by $h \times c$.

1	2	3	4	5	1	2	3	4	5
Central Angle α Deg.	Rise h / Rad r	Rise h / Chord c	Area = (Rad)² times	Area = $h\,c$ times	Central Angle α Deg.	Rise h / Rad r	Rise h / Chord c	Area = (Rad)² times	Area = $h\,c$ times
1°	.00004	.00218	.0000004	.6667	51°	.0974	.1131	.0564860	.67344
2	.00015	.00436	.0000035	.6667	52	.1012	.1154	.0597802	.67373
3	.00034	.00655	.0000119	.6667	53	.1051	.1177	.0631945	.67400
4	.00061	.00873	.0000284	.6667	54	.1090	.1200	.0667304	.67429
5	.00095	.01091	.0000554	.6667	55	.1130	.1223	.0703895	.67458
6	.00137	.01309	.0000956	.6667	56	.1171	.1247	.0741734	.67488
7	.00187	.01528	.0001518	.6667	57	.1212	.1270	.0780835	.67519
8	.00244	.01746	.0002266	.6668	58	.1254	.1293	.0821214	.67550
9	.00308	.01965	.0003226	.6669	59	.1296	.1316	.0862885	.67582
10	.00381	.02183	.0004424	.66690	60	.1340	.1340	.0905861	.67614
11	.00460	.02402	.0005886	.66697	61	.1384	.1363	.0950156	.67647
12	.00548	.02620	.0007639	.66702	62	.1428	.1387	.0995783	.67681
13	.00643	.02839	.0009709	.66708	63	.1474	.1410	.1042755	.67715
14	.00745	.03058	.0012121	.66716	64	.1520	.1434	.1091084	.67750
15	.00856	.03277	.0014902	.66725	65	.1566	.1457	.1140781	.67786
16	.00973	.03496	.0018076	.66731	66	.1613	.1481	.1191859	.67822
17	.01098	.03716	.0021671	.66740	67	.1661	.1505	.1244328	.67859
18	.01231	.03935	.0025711	.66749	68	.1710	.1529	.1298200	.67897
19	.01371	.04155	.0030222	.66758	69	.1759	.1553	.1353484	.67935
20	.01519	.04374	.0035229	.66769	70	.1808	.1576	.1410189	.67974
21	.01675	.04594	.0040756	.66779	71	.1859	.1601	.1468326	.68014
22	.01837	.04814	.0046829	.66790	72	.1910	.1625	.1527903	.68054
23	.02008	.05035	.0053473	.66802	73	.1961	.1649	.1588928	.68095
24	.02185	.05255	.0060712	.66814	74	.2014	.1673	.1651410	.68136
25	.02370	.05476	.0068570	.66826	75	.2066	.1697	.1715356	.68179
26	.02563	.05697	.0077073	.66840	76	.2120	.1722	1780773	.68222
27	.02763	.05918	.0086242	.66853	77	.2174	.1746	.1847667	.68265
28	.02970	.06139	.0096103	.66868	78	.2229	.1771	.1916046	.68310
29	.03185	.06361	.0106679	.66882	79	.2284	.1795	.1985915	.68355
30	.03407	.06583	.0117994	.66897	80	.2340	.1820	.2057278	.68401
31	.03637	.06805	.0130070	.66913	81	.2396	.1845	.2130142	.68448
32	.03874	.07027	.0142930	.66929	82	.2453	.1869	.2204509	.68495
33	.04118	.07250	.0156598	.66946	83	.2510	.1894	.2280385	.68543
34	.04370	.07473	.0171095	.66964	84	.2569	.1919	.2357773	.68592
35	.04628	.07696	.0186444	.66982	85	.2627	.1944	.2436676	.68641
36	.04894	.07919	.0202666	.67000	86	.2686	.1970	.2517095	.68692
37	.05168	.08143	.0219784	.67019	87	.2746	.1995	.2599035	.68743
38	.05448	.08367	.0237818	.67039	88	.2807	.2020	.2682495	.68795
39	.05736	.08592	.0256790	.67059	89	.2868	.2046	.2767477	.68848
40	.06031	.08816	.0276721	.67079	90	.2929	.2071	.2853982	.68902
41	.06333	.09041	.0297630	.67101	91	.2991	.2097	.2942010	.68956
42	.06642	.09267	.0319538	.67122	92	.3053	.2122	.3031561	.69011
43	.06958	.09493	.0342466	.67145	93	.3116	.2148	.3122634	.69067
44	.07282	.09719	.0366432	.67168	94	.3180	.2174	.3215227	.69123
45	.07612	.09946	.0391457	.67193	95	.3244	.2200	.3309340	.69181
46	.07950	.10173	.0417558	.67215	96	.3309	.2226	.3404971	.69239
47	.08294	.10400	.0444755	.67240	97	.3374	.2252	.3502116	.69299
48	.08645	.10628	.0473066	.67265	98	.3439	.2279	.3600773	.69359
49	.09004	.10856	.0502504	.67292	99	.3506	.2305	.3700938	.69420
50	.09369	.11085	.0533101	.67318	100	.3572	.2332	.3802607	.69482

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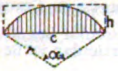

CIRCULAR SEGMENTS—AREAS, ETC.—Concluded.

ultiply the tabular coefficient in column 4 by r^2; or the coefficient in column 5 by $h \times c$.

4	5	1	2	3	4	5
Area =		Central Angle α Deg.	Rise h / Rad r	Rise h / Chord c	Area =	
(Rad)² times	$h\,c$ times				(Rad)² times	$h\,c$ times
.3905777	.69545	141°	.6662	.3534	.9157969	.72916
.4010441	.69608	142	.6744	.3566	.9313530	.73026
.4116595	.69673	143	.6827	.3599	.9470029	.73137
.4224234	.69738	144	.6910	.3633	.9627444	.73250
.4333350	.69805	145	.6993	.3666	.9785755	.73364
.4443937	.69872	146	.7076	.3700	.9944939	.73480
.4555988	.69941	147	.7160	.3734	1.0104975	.73598
.4669495	.70010	148	.7244	.3768	1.0265840	.73717
.4784451	.70080	149	.7328	.3802	1.0427512	.73838
.4900848	.70151	150	.7412	.3837	1.0589969	.73960
.5018675	.70223	151	.7496	.3871	1.0753188	.74084
.5137924	.70295	152	.7581	.3906	1.0917144	.74210
.5258586	.70371	153	.7666	.3942	1.1081816	.74337
.5380649	.70446	154	.7751	.3977	1.1247180	.74466
.5504104	.70522	155	.7836	.4013	1.1413210	.74597
.5628940	.70600	156	.7921	.4049	1.1579885	.74730
.5755144	.70678	157	.8006	.4085	1.1747179	.74865
.5882705	.70758	158	.8092	.4122	1.1915068	.75001
.6011611	.70838	159	.8178	.4158	1.2083528	.75140
.6141849	.70920	160	.8264	.4195	1.2252534	.75280
.6273405	.71003	161	.8350	.4233	1.2422059	.75422
.6406268	.71087	162	.8436	.4270	1.2592082	.75566
.6540422	.71172	163	.8522	.4308	1.2762575	.75713
.6675853	.71258	164	.8608	.4346	1.2933513	.75861
.6812548	.71345	165	.8695	.4385	1.3104871	.76011
.6950489	.71434	166	.8781	.4424	1.3276623	.76164
.7089613	.71523	167	.8868	.4463	1.3448744	.76318
.7230053	.71615	168	.8955	.4502	1.3621207	.76475
.7371644	.71707	169	.9042	.4542	1.3793987	.76634
.7514418	.71800	170	.9128	.4582	1.3967058	.76795
.7658359	.71895	171	.9215	.4622	1.4140393	.76959
.7803449	.71991	172	.9302	.4663	1.4313966	.77124
.7499671	.72088	173	.9390	.4704	1.4487752	.77293
.8097007	.72187	174	.9477	.4745	1.4661722	.77463
.8245438	.72287	175	.9564	.4786	1.4835852	.77636
.8394947	.72388	176	.9651	.4828	1.5010115	.77812
.8545517	.72491	177	.9738	.4871	1.5184484	.77990
.8697119	.72595	178	.9826	.4913	1.5358933	.78171
.8849743	.72701	179	.9913	.4957	1.5533435	.78354
.9003667	.72808	180	unity	.5000	1.5707963	.78540

$\frac{c}{h}+h\Big)$; $h=r-\sqrt{r^2-\frac{c^2}{4}}$; $c=2\sqrt{h\,(2\,r-h)}$. If the seg-

a semi-circle and we denote the rise by H $(=2\,r-h)$,

$H=r+\sqrt{r^2-\frac{c^2}{4}}$; $c=2\sqrt{H\,(2r-H)}$. The area of a seg-

emi-circle, i. e., with rise H, = area of circle $(3.1416\,r^2)$
rise h $(=2\,r-H)$.

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8.—*Areas of Segments of Circles for Diameter 1.

To find the area of any circular segment: Multiply the tabular are below, corresponding to the particular value of rise ÷ *diam.*, by the squar of the *diam.*

Note that the *diam.* = rise + (half chord)² ÷ rise; hence, (*diam.*)² = $\left[\frac{1+(\text{half chord})^2}{\text{rise}}\right]^2$; and, rise ÷ *diam.* = rise² ÷ [rise² + (half chord)²].

Rise ÷ Diam.	Thousandths.									
	.000	.001	.002	.003	.004	.005	.006	.007	.008	.00
.00	.000000	.000042	.000119	.000219	.000337	.000471	.000619	.000779	.000952	.0011
.01	.001329	.001533	.001746	.001969	.002199	.002438	.002685	.002940	.003202	.0034
.02	.003749	.004032	.004322	.004619	.004922	.005231	.005546	.005867	.006194	.0065
.03	.006866	.007209	.007559	.007913	.008273	.008638	.009008	.009383	.009763	.0101
.04	.010538	.010932	.011331	.011734	.012142	.012555	.012971	.013393	.013818	.0142
.05	.014681	.015119	.015561	.016008	.016458	.016912	.017369	.017831	.018297	.0187
.06	.019239	.019716	.020197	.020681	.021168	.021660	.022155	.022653	.023155	.0236
.07	.024168	.024680	.025196	.025714	.026236	.026761	.027290	.027821	.028356	.0289
.08	.029435	.029979	.030526	.031077	.031630	.032186	.032746	.033308	.033873	.0344
.09	.035012	.035586	.036162	.036742	.037324	.037909	.038497	.039087	.039681	.0402
.10	.040875	.041477	.042081	.042687	.043296	.043908	.044523	.045140	.045759	.0463
.11	.047006	.047633	.048262	.048894	.049529	.050165	.050805	.051446	.052090	.0527
.12	.053385	.054037	.054690	.055346	.056004	.056664	.057327	.057991	.058658	.0593
.13	.059999	.060673	.061349	.062027	.062707	.063389	.064074	.064761	.065449	.0661
.14	.066833	.067528	.068225	.068924	.069626	.070329	.071034	.071741	.072450	.0731
.15	.073875	.074590	.075307	.076026	.076747	.077470	.078194	.078921	.079650	.0803
.16	.081112	.081847	.082582	.083320	.084060	.084801	.085545	.086290	.087037	.0878
.17	.088536	.089288	.090042	.090797	.091555	.092314	.093074	.093837	.094601	.0953
.18	.096135	.096904	.097675	.098447	.099221	.099997	.100774	.101553	.102334	.1031
.19	.103900	.104686	.105472	.106261	.107051	.107843	.108636	.109431	.110227	.1110
.20	.111824	.112625	.113427	.114231	.115036	.115842	.116651	.117460	.118271	.1190
.21	.119898	.120713	.121530	.122348	.123167	.123988	.124811	.125634	.126459	.1272
.22	.128114	.128943	.129773	.130605	.131438	.132273	.133109	.133946	.134784	.1356
.23	.136465	.137307	.138151	.138996	.139842	.140689	.141538	.142388	.143239	.1440
.24	.144945	.145800	.146656	.147513	.148371	.149231	.150091	.150953	.151816	.1526
.25	.153546	.154413	.155281	.156149	.157019	.157891	.158763	.159636	.160511	.1613
.26	.162263	.163141	.164020	.164900	.165781	.166663	.167546	.168431	.169316	.1702
.27	.171090	.171978	.172868	.173758	.174650	.175542	.176436	.177330	.178226	.1791
.28	.180020	.180918	.181818	.182718	.183619	.184522	.185425	.186329	.187235	.1881
.29	.189048	.189956	.190865	.191774	.192685	.193597	.194509	.195423	.196337	.1972
.30	.198168	.199085	.200003	.200922	.201841	.202762	.203683	.204605	.205528	.2064
.31	.207376	.208302	.209228	.210155	.211083	.212011	.212941	.213871	.214802	.2157
.32	.216666	.217600	.218534	.219469	.220404	.221341	.222278	.223216	.224154	.2250
.33	.226034	.226974	.227916	.228858	.229801	.230745	.231689	.232634	.233580	.2345
.34	.235473	.236421	.237369	.238319	.239268	.240219	.241170	.242122	.243074	.2440
.35	.244980	.245935	.246890	.247845	.248801	.249758	.250715	.251673	.252632	.2535
.36	.254551	.255511	.256472	.257433	.258395	.259358	.260321	.261285	.262249	.2632
.37	.264179	.265145	.266111	.267078	.268046	.269014	.269982	.270951	.271921	.2728
.38	.273861	.274832	.275804	.276776	.277748	.278721	.279695	.280669	.281643	.2826
.39	.283593	.284569	.285545	.286521	.287499	.288476	.289454	.290432	.291411	.2923
.40	.293370	.294350	.295330	.296311	.297292	.298274	.299256	.300238	.301221	.3022
.41	.303187	.304171	.305156	.306140	.307125	.308110	.309096	.310082	.311068	.3120
.42	.313042	.314029	.315017	.316005	.316993	.317981	.318970	.319959	.320949	.3219
.43	.322928	.323919	.324909	.325900	.326891	.327883	.328874	.329866	.330858	.3318
.44	.332843	.333836	.334829	.335823	.336816	.337810	.338804	.339799	.340793	.3417
.45	.342783	.343778	.344773	.345768	.346764	.347760	.348756	.349752	.350749	.3517
.46	.352742	.353739	.354736	.355733	.356730	.357728	.358725	.359723	.360721	.3617
.47	.362717	.363715	.364714	.365712	.366711	.367710	.368708	.369707	.370706	.3717
.48	.372704	.373704	.374703	.375702	.376702	.377701	.378701	.379701	.380700	.3817
.49	.382700	.383700	.384699	.385699	.386699	.387699	.388699	.389699	.390699	.3916
.50	.392699									

*Calculated from formula: Area $A = 0.3926990817 - \left[x(r^2 - x^2)^{\frac{1}{2}} + r^2 \sin^{-1}\frac{x}{r}\right]$

in which r = radius of circle, and x = radius − rise = 0.5 − rise.

Note that $\sin^{-1}\frac{x}{r}$ = the angle (in *circular measure*) whose natural sine $\frac{x}{r}$. For angles reduced to circular measure, see Tables 2 and 3.

Circular Ring; and Half Circular Ring.—

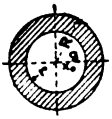

Fig. 12.

Let R = radius of outer circle $= \frac{D}{2}$;

r = radius of inner circle $= \frac{d}{2}$;

R_e = radius of circle with area equivalent to that circular ring;

A = area of circular ring; $\pi = 3.1416$.

Then $R_e = \sqrt{R^2 - r^2} = \sqrt{(R+r)(R-r)} = \frac{1}{2}\sqrt{(D+d)(D-d)} = \sqrt{\frac{A}{\pi}}$;

$$A = \pi R_e^2 = \pi(R^2 - r^2) = \pi(R+r)(R-r) = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(D+d)(D-d);$$

$$R = \sqrt{r^2 + \frac{A}{\pi}} = \sqrt{r^2 + R_e^2}; \qquad D = \sqrt{d^2 + \frac{4A}{\pi}};$$

$$r = \sqrt{R^2 - \frac{A}{\pi}} = \sqrt{R^2 - R_e^2}; \qquad d = \sqrt{D^2 - \frac{4A}{\pi}}.$$

Formula for Center of Gravity (Fig. 12):

For half circular ring, $x_0 = \frac{0.42441\,(R^3 - r^3)}{(R^2 - r^2)} = \frac{0.21221\,(D^3 - d^3)}{(D^2 - d^2)}$ = distance to cen. of grav. g.

The above value of x_0 may be obtained from Pappus's Theorem, page 243, by using it inversely. Thus, we know that the volume generated by the revolution of a plane area (lying wholly on one side of an axis) about its axis = the area × the path described by its center of gravity. Hence, volume $V = 2\pi x_0 A$; or

$$x_0 = \frac{V}{2\pi A} = \frac{\text{volume of spherical shell}}{2\pi \,.\, \text{Area of half circular ring}} = \frac{1}{2\pi} \cdot \frac{\frac{4}{3}\pi(R^3 - r^3)}{\frac{\pi}{2}(R^2 - r^2)}$$

$$= \frac{4}{3\pi} \cdot \frac{R^3 - r^3}{R^2 - r^2}; \text{ in which } \frac{4}{3\pi} = 0.42441.$$

Pappus's Theorem is useful in finding the centers of gravity of any figures (lines or areas) of revolution.

Zone, and Half Zone, of Circle.—

Area of zone $= Z$ = area of circle $(3.1416\, r^2) - (A + B)$; in which r = radius of circle, A = area of upper segment, and B = area of lower segment. (See preceding tables and formulas for areas of segments.)

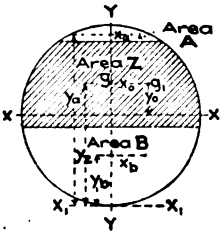

Fig. 13.

Formulas for Center of Gravity (Fig. 13):

Let g = cen. of grav. of zone, with coordinates $x = 0$, $y = y_0$;

g_1 = cen. of grav. of half zone, with coordinates $x = x_0$, $y = y_0$;

A = area of upper segment, with cen. of grav. dist. y_a from axis $X_1 - X_1$;

Z = area of zone, with cen. of grav. distant y_z from $X_1 - X_1$

B = area of lower segment, with cen. of grav. dist. y_b from $X_1 - X_1$;

C = area of circle $(= 3.1416 r^2)$, with cen. of grav. distant r from $X_1 - X_1$;

Then, $x_0 = \frac{0.42441\, r\, C - A x_a - B x_b}{Z}$;

$y_0 = y_z - r = \frac{C r - A y_a - B y_b}{Z} - r.$

Ordinates x_a, x_b, y_a and y_b may be solved by use of formulas in connection with Fig. 11.

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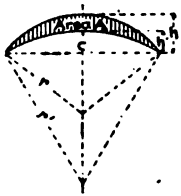

Fig. 14.

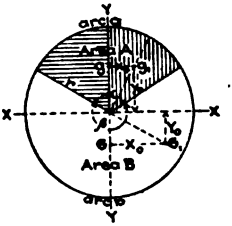

Fig. 15.

Circular Lune (Fig. 14).—

Area A = area of segment with rise h *minus* area of segment with rise h'.
(Common chord c.)

See preceding Tables, 7 and 8, of Circular Segments.

Circular Sector; and Half Sector (Fig. 15).—

$$\text{Point } \left.\begin{array}{l} g = \\ g_1 = \\ G = \\ G_1 = \end{array}\right\} \begin{array}{l}\text{position}\\ \text{of center}\\ \text{of grav-}\\ \text{ity of}\end{array} \left\{\begin{array}{c}\text{sector } A\\ \tfrac{1}{2}(\text{sector } A)\\ \text{sector } B\\ \tfrac{1}{2}\ (\text{sector } B)\end{array}\right\} \begin{array}{l}\text{with}\\ \text{coordi-}\\ \text{nates.}\end{array} \left\{\begin{array}{l} x=0;\ y=y_0\\ x=x_0;\ y=y_0\\ x=0;\ y=Y_0\\ x=X_0;\ y=Y_0\end{array}\right.$$

Formulas for Center of Gravity and Area (Fig. 15):

$$y_0 = \tfrac{2}{3}\, r \frac{\sin \frac{1}{2}\alpha}{\frac{1}{2}\alpha_1} = \tfrac{4}{3}\, r \frac{\sin \frac{1}{2}\alpha}{\alpha_1} = \tfrac{4}{3}\, r^2 \frac{\sin \frac{1}{2}\alpha}{a} = \tfrac{2}{3}\, r^3 \frac{\sin \frac{1}{2}\alpha}{A},$$

$$x_0 = y_0 \tan \tfrac{1}{4}\alpha = \tfrac{4}{3}\, r \frac{\text{vers } \frac{1}{2}\alpha}{\alpha_1} = \tfrac{4}{3}\, r^2 \frac{\text{vers } \frac{1}{2}\alpha}{a} = \tfrac{2}{3}\, r^3 \frac{\text{vers } \frac{1}{2}\alpha}{A},$$

$$Y_0 = \tfrac{2}{3}\, r \frac{\sin \frac{1}{2}\beta}{\frac{1}{2}\beta_1} = \tfrac{4}{3}\, r \frac{\sin \frac{1}{2}\beta}{\beta_1} = \tfrac{4}{3}\, r^2 \frac{\sin \frac{1}{2}\beta}{b} = \tfrac{2}{3}\, r^3 \frac{\sin \frac{1}{2}\beta}{B},$$

$$X_0 = Y_0 \tan \tfrac{1}{4}\beta = \tfrac{4}{3}\, r \frac{\text{vers } \frac{1}{2}\beta}{\beta_1} = \tfrac{4}{3}\, r^2 \frac{\text{vers } \frac{1}{2}\beta}{b} = \tfrac{2}{3}\, r^3 \frac{\text{vers } \frac{1}{2}\beta}{B},$$

$$\textit{Area } A = \tfrac{1}{2}\, r^2 \alpha_1 = \tfrac{1}{2}\, a\, r = \tfrac{2}{3}\, r^3 \frac{\sin \frac{1}{2}\alpha}{y_0} = \tfrac{2}{3}\, r^3 \frac{\text{vers } \frac{1}{2}\alpha}{x_0},$$

$$\textit{Area } B = \tfrac{1}{2}\, r^2 \beta_1 = \tfrac{1}{2}\, b\, r = \tfrac{2}{3}\, r^3 \frac{\sin \frac{1}{2}\beta}{Y_0} = \tfrac{2}{3}\, r^3 \frac{\text{vers } \frac{1}{2}\beta}{X_0};$$

In which a = length of arc of sector of area A and central angle α,
b = length of arc of sector of area B and central angle β,
α_1 = .0174533 α (degrees), (See Tables 2 and 3 of length of
β_1 = .0174533 β (degrees). circular arcs to radius 1.)

Relations of Circle and Square.—

Let D = *diam.* of circle; C = *circum.* of circle; A = area of circle.
d = *diag.* of square; s = side of square; p = perimeter of square;
a = area of square.

$$\text{Then, } D = \frac{C}{\pi} = \frac{4A}{C};\ C = \pi D = \frac{4A}{D};\ A = \frac{\pi}{4} D^2 = \frac{CD}{4} = \frac{C^2}{4\pi};$$

$$d = s\sqrt{2} = \frac{p}{4}\sqrt{2} = \sqrt{2a};\ s = \frac{d}{\sqrt{2}} = \frac{p}{4} = \sqrt{a};\ p = 2d\sqrt{2} = 4s = 4\sqrt{a};$$

$$a = \frac{d^2}{2} = s^2 = \frac{p^2}{16}.$$

(For convenience of calculation: $\pi = 3.141592+$, log $= 0.4971499$;
$\frac{\pi}{4} = 0.785398+$, log $= 9.8950899$; $\frac{1}{\pi} = 0.318310-$, log $= 9.5028501$;
log $4 = 0.6020600$; log $\frac{1}{4} = 9.3979400$; log $2 = 0.3010300$;
$\sqrt{2} = 1.414214-$, log $= 0.1505150$; $\frac{1}{\sqrt{2}} = 0.707107$, log $= 9.8494850$.)

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9.—Table of Relations of Circle and Square.

Equality.	Circle. C = Circumference. A = Area.	Square. p = Perimeter. a = Area.
$D = d$ (Square inscribed in circle.)	$D = \sqrt{2} \cdot s = 1.41421\ s$	$s = \frac{\sqrt{2}}{2} \cdot D = 0.70711\ D$
	$= \frac{\sqrt{2}}{4} \cdot p = 0.35355\ p$	$p = 2\sqrt{2} \cdot D = 2.82843\ D$
	$= \sqrt{2} \cdot \sqrt{a} = 1.41421\sqrt{a}$	$a = \frac{1}{2} \cdot D^2 = 0.50000\ D^2$
	$C = \pi \cdot d = 3.14159\ d$	$d = \frac{1}{\pi} \cdot C = 0.31831\ C$
	$= \pi\sqrt{2} \cdot s = 4.44288\ s$	$s = \frac{\sqrt{2}}{2\pi} \cdot C = 0.22508\ C$
	$= \frac{\pi\sqrt{2}}{4} \cdot p = 1.11072\ p$	$p = \frac{2\sqrt{2}}{\pi} \cdot C = 0.90032\ C$
	$= \pi\sqrt{2} \cdot \sqrt{a} = 4.44288\sqrt{a}$	$a = \frac{1}{2\pi^2} \cdot C^2 = 0.05066\ C^2$
	$A = \frac{\pi}{4} \cdot d^2 = 0.78540\ d^2$	$d = \frac{2}{\sqrt{\pi}} \cdot \sqrt{A} = 1.12838\sqrt{A}$
	$= \frac{\pi}{2} \cdot s^2 = 1.57080\ s^2$	$s = \sqrt{\frac{2}{\pi}} \cdot \sqrt{A} = 0.79788\sqrt{A}$
	$= \frac{\pi}{32} \cdot p^2 = 0.09817\ p^2$	$p = \frac{4\sqrt{2}}{\sqrt{\pi}} \cdot \sqrt{A} = 3.1915\sqrt{A}$
	$= \frac{\pi}{\ } \cdot a = 1.57080\ a$	$a = \frac{2}{\ } \cdot A = 0.63662\ A$

Equality.	Circle. C = Circumference. A = Area.	Square. p = Perimeter. a = Area.
$D = s$ (Circle inscribed in square.)	$D = \frac{\sqrt{2}}{2} \cdot d = 0.70711\ d$	$d = \sqrt{2} \cdot D = 1.41421\ D$
	$= \frac{1}{4} \cdot p = 0.25000\ p$	$p = 4 \cdot D = 4.00000\ D$
	$= \sqrt{a} = 1.00000\sqrt{a}$	$a = D^2 = 1.00000\ D^2$
	$C = \pi \cdot s = 3.14159\ s$	$s = \frac{1}{\pi} \cdot C = 0.31831\ C$
	$= \frac{\pi\sqrt{2}}{2} \cdot d = 2.22144\ d$	$d = \frac{\sqrt{2}}{\pi} \cdot C = 0.45016\ C$
	$= \frac{\pi}{4} \cdot p = 0.78540\ p$	$p = \frac{4}{\pi} \cdot C = 1.27324\ C$
	$= \pi \cdot \sqrt{a} = 3.14159\sqrt{a}$	$a = \frac{1}{\pi^2} \cdot C^2 = 0.10132\ C^2$
	$A = \frac{\pi}{4} \cdot s^2 = 0.78540\ s^2$	$s = \frac{2}{\sqrt{\pi}} \cdot \sqrt{A} = 1.12838\sqrt{A}$
	$= \frac{\pi}{8} \cdot d^2 = 0.39270\ d^2$	$d = \frac{2\sqrt{2}}{\sqrt{\pi}} \cdot \sqrt{A} = 1.59577\sqrt{A}$
	$= \frac{\pi}{64} \cdot p^2 = 0.04909\ p^2$	$p = \frac{8}{\sqrt{\pi}} \cdot \sqrt{A} = 4.51352\sqrt{A}$
	$= \frac{\pi}{\ } \cdot a = 0.78540\ a$	$a = \frac{4}{\ } \cdot A = 1.27324\ A$

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9.—TABLE OF RELATIONS OF CIRCLE AND SQUARE.—Concluded.

Equality	Circle. C = Circumference. A = Area.	Square. p = Perimeter. a = Area.
$C = p$ (Equal perimeters.)	$C = 2\sqrt{2} \cdot d = 2.82843\ d$	$d = \frac{\sqrt{2}}{4} \cdot C = 0.35355\ C$
	$= 4 \cdot s = 4.00000\ s$	$s = \frac{1}{4} \cdot C = 0.25000\ C$
	$= 4 \cdot \sqrt{a} = 4.00000\sqrt{a}$	$a = \frac{1}{16} \cdot C^2 = 0.06250\ C^2$
	$D = \frac{1}{\pi} \cdot p = 0.31831\ p$	$p = \pi \cdot D = 3.14159\ D$
	$= \frac{2\sqrt{2}}{\pi} \cdot d = 0.90032\ d$	$d = \frac{\pi\sqrt{2}}{4} \cdot D = 1.11072\ D$
	$= \frac{4}{\pi} \cdot s = 1.27324\ s$	$s = \frac{\pi}{4} \cdot D = 0.78540\ D$
	$= \frac{4}{\pi} \cdot \sqrt{a} = 1.27324\ \sqrt{a}$	$a = \frac{\pi^2}{16} \cdot D^2 = 0.61685\ D^2$
	$A = \frac{1}{4\pi} \cdot p^2 = 0.07958\ p^2$	$p = 2\sqrt{\pi} \cdot \sqrt{A} = 3.54491\sqrt{A}$
	$= \frac{2}{\pi} \cdot d^2 = 0.63662\ d^2$	$d = \frac{\sqrt{2\pi}}{2} \cdot \sqrt{A} = 1.25331\sqrt{A}$
	$= \frac{4}{\pi} \cdot s^2 = 1.27324\ s^2$	$s = \frac{\sqrt{\pi}}{2} \cdot \sqrt{A} = 0.88623\sqrt{A}$
	$= \frac{4}{\pi} \cdot a = 1.27324\ a$	$a = \frac{\pi}{4} \cdot A = 0.78540\ A$

Equality	Circle. C = Circumference. A = Area.	Square p = Perimeter. a = Area.
$A = a$ (Equal areas.)	$A = \frac{1}{2} \cdot d^2 = 0.50000\ d^2$	$d = \sqrt{2} \cdot \sqrt{A} = 1.41421\sqrt{A}$
	$= s^2 = 1.00000\ s^2$	$s = \sqrt{A} = 1.00000\sqrt{A}$
	$= \frac{1}{16} \cdot p^2 = 0.06250\ p^2$	$p = 4 \cdot \sqrt{A} = 4.00000\sqrt{A}$
	$D = \frac{2}{\sqrt{\pi}} \cdot \sqrt{a} = 1.12838\sqrt{a}$	$a = \frac{\pi}{4} \cdot D^2 = 0.78540\ D^2$
	$= \sqrt{\frac{2}{\pi}} \cdot d = 0.79788\ d$	$d = \frac{\sqrt{2\pi}}{2} \cdot D = 1.25331\ D$
	$= \frac{2}{\sqrt{\pi}} \cdot s = 1.12838\ s$	$s = \frac{\sqrt{\pi}}{2} \cdot D = 0.88623\ D$
	$= \frac{1}{2\sqrt{\pi}} \cdot p = 0.28209\ p$	$p = 2\sqrt{\pi} \cdot D = 3.54491\ D$
	$C = 2\sqrt{\pi} \cdot \sqrt{a} = 3.54491\sqrt{a}$	$a = \frac{1}{4\pi} \cdot C = 0.07958\ C$
	$= \sqrt{2\pi} \cdot d = 2.50663\ d$	$d = \frac{\sqrt{2}}{2\pi} \cdot C = 0.22508\ C$
	$= 2\sqrt{\pi} \cdot s = 3.54491\ s$	$s = \frac{1}{2\sqrt{\pi}} \cdot C = 0.28209\ C$
	$= \frac{\sqrt{\pi}}{2} \cdot p = 0.88623\ p$	$p = \frac{2}{\sqrt{\pi}} \cdot C = 1.12838\ C$

To reduce circular inches to square inches, multiply by $\frac{\pi}{4} = 0.78540$. To reduce square inches to circular inches, multiply by $\frac{4}{\pi} = 1.27324$.

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F A FOOT FOR EACH 1/64 OF AN INCH.

4"	5"	6"	7"	8"	9"	10"	11"	Inch
333	.4167	.5000	.5833	.6667	.7500	.8333	.9167	0
346	.4180	.5013	.5846	.6680	.7518	.8346	.9180	1-64
359	.4193	.5026	.5859	.6693	.7526	.8359	.9193	1-32
372	.4206	.5039	.5872	.6706	.7539	.8372	.9206	3-64
385	.4219	.5052	.5885	.6719	.7552	.8385	.9219	1-16
398	.4232	.5065	.5898	.6732	.7565	.8398	.9232	5-64
411	.4245	.5078	.5911	.6745	.7578	.8411	.9245	3-32
424	.4258	.5091	.5924	.6758	.7591	.8424	.9258	7-64
437	.4271	.5104	.5937	.6771	.7604	.8437	.9271	1-8
451	.4284	.5117	.5951	.6784	.7617	.8451	.9284	9-64
464	.4297	.5130	.5964	.6797	.7630	.8464	.9297	5-32
477	.4310	.5143	.5977	.6810	.7643	.8477	.9310	11-64
490	.4323	.5156	.5990	.6823	.7656	.8490	.9323	3-16
503	.4336	.5169	.6003	.6836	.7669	.8503	.9336	13-64
516	.4349	.5182	.6016	.6849	.7682	.8516	.9349	7-32
529	.4362	.5195	.6029	.6862	.7695	.8529	.9362	15-64
542	.4375	.5208	.6042	.6875	.7708	.8542	.9375	1-4
555	.4388	.5221	.6055	.6888	.7721	.8555	.9388	17-64
568	.4401	.5234	.6068	.6901	.7734	.8568	.9401	9-32
581	.4414	.5247	.6081	.6914	.7747	.8581	.9414	19-64
594	.4427	.5260	.6094	.6927	.7760	.8594	.9427	5-16
607	.4440	.5273	.6107	.6940	.7773	.8607	.9440	21-64
620	.4453	.5286	.6120	.6953	.7786	.8620	.9453	11-32
633	.4466	.5299	.6133	.6966	.7799	.8633	.9466	23-64
646	.4479	.5312	.6146	.6979	.7812	.8646	.9479	3/8
659	.4492	.5326	.6159	.6992	.7826	.8559	.9492	25-64
672	.4505	.5339	.6172	.7005	.7839	.8672	.9505	13-32
685	.4518	.5352	.6185	.7018	.7852	.8685	.9518	27-64
698	.4531	.5365	.6198	.7031	.7865	.8698	.9531	7-16
711	.4544	.5378	.6211	.7044	.7878	.8711	.9544	29-64
724	.4557	.5391	.6224	.7057	.7891	.8724	.9557	15-32
737	.4570	.5404	.6237	.7070	.7904	.8737	.9570	31-64
750	.4583	.5417	.6250	.7083	.7917	.8750	.9583	1/2
763	.4596	.5430	.6263	.7096	.7930	.8763	.9596	33-64
776	.4609	.5443	.6276	.7109	.7943	.8776	.9609	17-32
789	.4622	.5456	.6289	.7122	.7956	.8789	.9622	35-64
802	.4635	.5469	.6302	.7135	.7969	.8802	.9635	9-16
815	.4648	.5482	.6315	.7148	.7982	.8815	.9648	37-64
828	.4661	.5495	.6328	.7161	.7995	.8828	.9661	19-32
841	.4674	.5508	.6341	.7174	.8008	.8841	.9674	39-64
854	.4688	.5521	.6354	.7188	.8021	.8854	.9688	5/8
867	.4701	.5534	.6367	.7201	.8034	.8867	.9701	41-64
880	.4714	.5547	.6380	.7214	.8047	.8880	.9714	21-32
893	.4727	.5560	.6393	.7227	.8060	.8893	.9727	43-64
906	.4740	.5573	.6406	.7240	.8073	.8906	.9740	11-16
919	.4753	.5586	.6419	.7253	.8086	.8919	.9753	45-64
932	.4766	.5599	.6432	.7266	.8099	.8932	.9766	23-32
945	.4779	.5612	.6445	.7279	.8112	.8945	.9779	47-64
958	.4792	.5625	.6458	.7292	.8125	.8958	.9792	3/4
971	.4805	.5638	.6471	.7305	.8138	.8971	.9805	49-64
984	.4818	.5651	.6484	.7318	.8151	.8984	.9818	25-32
997	.4831	.5664	.6497	.7331	.8164	.8997	.9831	51-64
010	.4844	.5677	.6510	.7344	.8177	.9010	.9844	13-16
023	.4857	.5690	.6523	.7357	.8190	.9023	.9857	53-64
036	.4870	.5703	.6536	.7370	.8203	.9036	.9870	27-32
049	.4883	.5716	.6549	.7383	.8216	.9049	.9883	55-64
062	.4896	.5729	.6562	.7396	.8229	.9062	.9896	7/8
076	.4909	.5742	.6576	.7409	.8242	.9076	.9909	57-64
089	.4922	.5755	.6589	.7422	.8255	.9089	.9922	29-32
102	.4935	.5768	.6602	.7435	.8268	.9102	.9935	59-64
115	.4948	.5781	.6615	.7448	.8281	.9115	.9948	15-16
128	.4961	.5794	.6628	.7461	.8294	.9128	.9961	61-64
141	.4974	.5807	.6641	.7474	.8307	.9141	.9974	31-32
154	.4987	.5820	.6654	.7487	.8320	.9154	.9987	63-64

of an inch reduced to decimals, see Table 8, Ele-

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11.—Circumferences *C* of Circles for given—

Circumferences are directly proportional to the diameters.

D.	C. 0	1	2	3	4	5	6	7	8	
0.0	.000000	.031416	.062832	.094248	.125664	.157080	.188496	.219911	.251327	.2
.1	.314159	.345575	.376991	.408407	.439823	.471239	.502655	.534071	.565487	.5
.2	.628319	.659734	.691150	.722566	.753982	.785398	.816814	.848230	.879646	.9
.3	.942478	.973894	1.00531	1.03673	1.06814	1.09956	1.13097	1.16239	1.19381	1.7
.4	1.25664	1.28805	1.31947	1.35088	1.38230	1.41372	1.44513	1.47655	1.50796	1.3
0.5	1.57080	1.60221	1.63363	1.66504	1.69646	1.72788	1.75929	1.79071	1.82212	1.8
.6	1.88496	1.91637	1.94779	1.97920	2.01062	2.04204	2.07345	2.10487	2.13628	2.1
.7	2.19911	2.23053	2.26195	2.29336	2.32478	2.35619	2.38761	2.41903	2.45044	2.4
.8	2.51327	2.54469	2.57611	2.60752	2.63894	2.67035	2.70177	2.73319	2.76460	2.7
.9	2.82743	2.85885	2.89027	2.92168	2.95310	2.98451	3.01593	3.04734	3.07876	3.0
1.0	3.14159	3.17301	3.20442	3.23584	3.26726	3.29867	3.33009	3.36150	3.39292	3.4
.1	3.45575	3.48717	3.51858	3.55000	3.58142	3.61283	3.64425	3.67566	3.70708	3.7
.2	3.76991	3.80133	3.83274	3.86416	3.89557	3.92699	3.95841	3.98982	4.02124	4.0
.3	4.08407	4.11549	4.14690	4.17832	4.20973	4.24115	4.27257	4.30398	4.33540	4.3
.4	4.39823	4.42965	4.46106	4.49248	4.52389	4.55531	4.58673	4.61814	4.64956	4.6
1.5	4.71239	4.74381	4.77522	4.80664	4.83805	4.86947	4.90088	4.93230	4.93672	4.9
.6	5.02655	5.05796	5.08938	5.12080	5.15221	5.18363	5.21504	5.24646	5.27788	5.3
.7	5.34071	5.37212	5.40354	5.43496	5.46637	5.49779	5.52920	5.56062	5.59203	5.6
.8	5.65487	5.68628	5.71770	5.74911	5.78053	5.81195	5.84336	5.87478	5.90619	5.9
.9	5.96903	6.00044	6.03186	6.06327	6.09469	6.12611	6.15752	6.18894	6.22035	6.2
2.0	6.28319	6.31460	6.34602	6.37743	6.40885	6.44026	6.47168	6.50310	6.53451	6.5
.1	6.59734	6.62876	6.66018	6.69159	6.72301	6.75442	6.78584	6.81726	6.84867	6.8
.2	6.91150	6.94292	6.97434	7.00575	7.03717	7.06858	7.10000	7.13142	7.16283	7.
.3	7.22566	7.25708	7.28849	7.31991	7.35133	7.38274	7.41416	7.44557	7.47699	7.7
.4	7.53982	7.57124	7.60265	7.63407	7.66549	7.69690	7.72832	7.75973	7.79115	7.
2.5	7.85398	7.88540	7.91681	7.94823	7.97965	8.01106	8.04248	8.07389	8.10531	8.
.6	8.16814	8.19956	8.23097	8.26239	8.29380	8.32522	8.35664	8.38805	8.41947	8.
.7	8.48230	8.51372	8.54513	8.57655	8.60796	8.63938	8.67080	8.70221	8.73363	8.
.8	8.79646	8.82788	8.85929	8.89071	8.92212	8.95354	8.98495	9.01637	9.04779	9.
.9	9.11062	9.14203	9.17345	9.20487	9.23628	9.26770	9.29911	9.33053	9.36195	9.
3.0	9.42478	9.45619	9.48761	9.51903	9.55044	9.58186	9.61327	9.64469	9.67611	9.
.1	9.73894	9.77035	9.80177	9.83319	9.86460	9.89602	9.92743	9.95885	9.99026	10
.2	10.0531	10.0845	10.1159	10.1473	10.1788	10.2102	10.2416	10.2730	10.3044	10
.3	10.3673	10.3987	10.4301	10.4615	10.4929	10.5243	10.5558	10.5872	10.6186	10
.4	10.6814	10.7128	10.7442	10.7757	10.8071	10.8385	10.8699	10.9013	10.9327	10
3.5	10.9956	11.0270	11.0584	11.0898	11.1212	11.1527	11.1841	11.2155	11.2469	11
.6	11.3097	11.3411	11.3726	11.4040	11.4354	11.4668	11.4982	11.5296	11.5611	11
.7	11.6239	11.6553	11.6867	11.7181	11.7496	11.7810	11.8124	11.8438	11.8752	11
.8	11.9381	11.9695	12.0009	12.0323	12.0637	12.0951	12.1265	12.1580	12.1894	12
.9	12.2522	12.2836	12.3150	12.3465	12.3779	12.4093	12.4407	12.4721	12.5035	12
4.0	12.5664	12.5978	12.6292	12.6606	12.6920	12.7235	12.7549	12.7863	12.8177	12
.1	12.8805	12.9119	12.9434	12.9748	13.0062	13.0376	13.0690	13.1004	13.1319	13
.2	13.1947	13.2261	13.2575	13.2889	13.3204	13.3518	13.3832	13.4146	13.4460	13
.3	13.5088	13.5403	13.5717	13.6031	13.6345	13.6659	13.6973	13.7288	13.7602	13
.4	13.8230	13.8544	13.8858	13.9173	13.9487	13.9801	14.0115	14.0429	14.0743	14
4.5	14.1372	14.1686	14.2000	14.2314	14.2628	14.2942	14.3257	14.3571	14.3885	14
.6	14.4513	14.4827	14.5142	14.5456	14.5770	14.6084	14.6398	14.6712	14.7027	14
.7	14.7655	14.7969	14.8283	14.8597	14.8911	14.9226	14.9540	14.9854	15.0168	15
.8	15.0796	15.1111	15.1425	15.1739	15.2053	15.2367	15.2681	15.2996	15.3310	15
.9	15.3938	15.4252	15.4566	15.4881	15.5195	15.5509	15.5823	15.6137	15.6451	15
5.0	15.7080	15.7394	15.7708	15.8022	15.8336	15.8650	15.8965	15.9279	15.9593	15

Diameter may be obtained from circumferences by inverse interpol

Note.—Area of surface of *Sphere* = diameter × circumference = *D*

Values in this table are also multiples of π.

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—DIAMETERS D, IN DECIMALS.
This table may be used like logarithmic tables.

D.	C. 0	1	2	3	4	5	6	7	8	9
5.0	15.7080	15.7394	15.7708	15.8022	15.8336	15.8650	15.8965	15.9279	15.9593	15.9907
.1	16.0221	16.0535	16.0850	16.1164	16.1478	16.1792	16.2106	16.2420	16.2734	16.3049
.2	16.3363	16.3677	16.3991	16.4305	16.4619	16.4934	16.5248	16.5562	16.5876	16.6190
.3	16.6504	16.6819	16.7133	16.7447	16.7761	16.8075	16.8389	16.8704	16.9018	16.9332
.4	16.9646	16.9960	17.0274	17.0588	17.0903	17.1217	17.1531	17.1845	17.2159	17.2473
5.5	17.2788	17.3102	17.3416	17.3730	17.4044	17.4358	17.4673	17.4987	17.5301	17.5615
.6	17.5929	17.6243	17.6558	17.6872	17.7186	17.7500	17.7814	17.8128	17.8442	17.8757
.7	17.9071	17.9385	17.9699	18.0013	18.0327	18.0642	18.0956	18.1270	18.1584	18.1898
.8	18.2212	18.2527	18.2841	18.3155	18.3469	18.3783	18.4097	18.4411	18.4726	18.5040
.9	18.5354	18.5668	18.5982	18.6296	18.6611	18.6925	18.7239	18.7553	18.7867	18.8181
6.0	18.8496	18.8810	18.9124	18.9438	18.9752	19.0064	19.0381	19.0695	19.1009	19.1323
.1	19.1637	19.1951	19.2265	19.2580	19.2894	19.3208	19.3522	19.3836	19.4150	19.4465
.2	19.4779	19.5093	19.5407	19.5721	19.6035	19.6350	19.6664	19.6978	19.7292	19.7606
.3	19.7920	19.8235	19.8549	19.8863	19.9177	19.9491	19.9805	20.0119	20.0434	20.0748
.4	20.1062	20.1376	20.1690	20.2004	20.2319	20.2633	20.2947	20.3261	20.3575	20.3889
6.5	20.4204	20.4518	20.4832	20.5146	20.5460	20.5374	20.6088	20.6403	20.6717	20.7031
.6	20.7345	20.7659	20.7973	20.8288	20.8602	20.8916	20.9230	20.9544	20.9858	21.0173
.7	21.0487	21.0801	21.1115	21.1429	21.1743	21.2058	21.2372	21.2686	21.3000	21.3314
.8	21.3628	21.3942	21.4257	21.4571	21.4885	21.5199	21.5513	21.5827	21.6142	21.6456
.9	21.6770	21.7084	21.7398	21.7712	21.8027	21.8341	21.8655	21.8969	21.9283	21.9597
7.0	21.9911	22.0226	22.0540	22.0854	22.1168	22.1482	22.1796	22.2111	22.2425	22.2739
.1	22.3053	22.3367	22.3681	22.3996	22.4310	22.4624	22.4938	22.5252	22.5566	22.5881
.2	22.6195	22.6509	22.6823	22.7137	22.7451	22.7765	22.8080	22.8394	22.8708	22.9022
.3	22.9336	22.9650	22.9965	23.0279	23.0593	23.0907	23.1221	23.1535	23.1850	23.2164
.4	23.2478	23.2792	23.3106	23.3420	23.3734	23.4049	23.4363	23.4677	23.4991	23.5305
7.5	23.5619	23.5934	23.6248	23.6562	23.6876	23.7190	23.7504	23.7819	23.8133	23.8447
.6	23.8761	22.9075	23.9389	23.9704	24.0018	24.0332	24.0646	24.0960	24.1274	24.1588
.7	24.1903	24.2217	24.2531	24.2845	24.3159	24.3473	24.3788	24.4102	24.4416	24.4730
.8	24.5044	24.5358	24.5673	24.5987	24.6301	24.6615	24.6929	24.7243	24.7558	24.7872
.9	24.8186	24.8500	24.8814	24.9128	24.9442	24.9757	25.0071	25.0385	25.0699	25.1013
8.0	25.1327	25.1642	25.1956	25.2270	25.2584	25.2898	25.3212	25.3527	25.3841	25.4155
.1	25.4469	25.4783	25.5097	25.5411	25.5726	25.6040	25.6354	25.6668	25.6982	25.7296
.2	25.7611	25.7925	25.8239	25.8553	25.8867	25.9181	25.9496	25.9810	26.0124	26.0438
.3	26.0752	26.1066	26.1381	26.1695	26.2009	26.2323	26.2637	26.2951	26.3265	26.3580
.4	26.3894	26.4208	26.4522	24.4836	26.5150	26.5465	26.5779	26.6093	26.6407	26.6721
8.5	26.7035	26.7350	26.7664	26.7978	26.8292	26.8606	26.8920	26.9234	26.9549	26.9863
.6	27.0177	27.0491	27.0805	27.1119	27.1434	27.1748	27.2062	27.2376	27.2690	27.3004
.7	27.3319	27.3633	27.3947	27.4261	27.4575	27.4889	27.5204	27.5518	27.5832	27.6146
.8	27.6460	27.6774	27.7088	27.7403	27.7717	27.8031	27.8345	27.8659	27.8973	27.9288
.9	27.9602	27.9916	28.0230	28.0544	28.0858	28.1173	28.1487	28.1801	28.2115	28.2429
9.0	28.2743	28.3058	28.3372	28.3686	28.4000	28.4314	28.4628	28.4942	28.5257	28.5571
.1	28.5885	28.6199	28.6513	28.6827	28.7142	28.7456	28.7770	28.8084	28.8398	28.8712
.2	28.9027	28.9341	28.9655	28.9969	29.0283	29.0597	29.0911	29.1226	29.1540	29.1854
.3	29.2168	29.2482	29.2796	29.3111	29.3425	29.3739	29.4053	29.4367	29.4681	29.4996
.4	29.5310	29.5624	29.5938	29.6252	29.6566	29.6881	29.7195	29.7509	29.7823	29.8137
9.5	29.8451	29.8765	29.9080	29.9394	29.9708	30.0022	30.0336	30.0650	30.0965	30.1279
.6	30.1593	30.1907	30.2221	30.2535	30.2850	30.3164	30.3478	30.3792	30.4106	30.4420
.7	30.4734	30.5049	30.5363	30.5677	30.5991	30.6305	30.6619	30.6934	30.7248	30.7562
.8	30.7876	30.8190	30.8504	30.8819	30.9133	30.9447	30.9761	31.0075	31.0389	31.0704
.9	31.1018	31.1332	31.1646	31.1960	31.2274	31.2588	31.2903	31.3217	31.3531	31.3845
10.0	31.4159	31.4473	31.4788	31.5102	31.5416	31.5730	31.6044	31.6358	31.6673	31.6987

Diff. between any two successive circumferences = .0314+; hence, if the diameter is extended to the third decimal place (thousandths), add .00314+ multiplied by the thousandth figure). *Ex.—Diam.* = 7.523; then *circum.* = 23.6248 + .0094 = 23.6342.

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12.—Circumferences of Circles for Given—

Circumferences are directly proportional to the diameters.

Decimal		.0000	.083⌄3	.1250	.166⌄6	.2500	.333⌄3	.3750	.416⌄6
		Fractions—as 12ths of a Foot (Inches), and 8ths of an Inch.							
Ft	12ths	$\frac{0}{12}$	$\frac{1}{12}$	$\frac{1.5}{12}$	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{4.5}{12}$	$\frac{5}{12}$
Ins.	8ths	$\frac{0}{8}$		$\frac{1}{8}$		$\frac{2}{8}$		$\frac{3}{8}$	
First column = Diameter in Feet or Inches for Decimals; Feet for 12ths; Inches for 8ths	0	.000000	.261799	.392699	.523599	.785398	1.04720	1.17810	1.30900
	1	3.14159	3.40339	3.53429	3.66519	3.92699	4.18879	4.31969	4.45059
	2	6.28319	6.54498	6.67588	6.80678	7.06858	7.33038	7.46128	7.59218
	3	9.42478	9.68658	9.81748	9.94838	10.2102	10.4720	10.6029	10.7338
	4	12.5664	12.8282	12.9591	13.0900	13.3518	13.6136	13.7445	13.8754
	5	15.7080	15.9698	16.1007	16.2316	16.4934	16.7552	16.8861	17.0170
	6	18.8496	19.1114	19.2423	19.3732	19.6350	19.8968	20.0277	20.1586
	7	21.9911	22.2529	22.3838	22.5147	22.7765	23.0383	23.1692	23.3001
	8	25.1327	25.3945	25.5254	25.6563	25.9181	26.1799	26.3108	26.4417
	9	28.2743	28.5361	28.6670	28.7979	29.0597	29.3215	29.4524	29.5833
	10	31.4159	31.6773	31.8086	31.9395	32.2013	32.4631	32.5940	32.7249
	11	34.5575	34.8193	34.9502	35.0811	35.3429	35.6047	35.7356	35.8665
	12	37.6991	37.9609	38.0918	38.2227	38.4845	38.7463	38.8772	39.0081
	13	40.8407	41.1025	41.2334	41.3643	41.6261	41.8879	42.0188	42.1497
	14	43.9823	44.2441	44.3750	44.5059	44.7677	45.0295	45.1604	45.2913
	15	47.1239	47.3857	47.5166	47.6475	47.9093	48.1711	48.3020	48.4329
	16	50.2655	50.5273	50.6582	50.7891	51.0509	51.3127	51.4436	51.5745
	17	53.4071	53.6689	53.7998	53.9307	54.1925	54.4543	54.5852	54.7161
	18	56.5487	56.8105	56.9414	57.0723	57.3341	57.5959	57.7268	57.8577
	19	59.6903	59.9521	60.0830	60.2139	60.4757	60.7375	60.8684	60.9993
	20	62.8319	63.0937	63.2246	63.3555	63.6173	63.8791	64.0100	64.1409
	21	65.9734	66.2352	66.3661	66.4970	66.7588	67.0206	67.1515	67.2824
	22	69.1150	69.3768	69.5077	69.6386	69.9004	70.1622	70.2931	70.4240
	23	72.2566	72.5184	72.6493	72.7802	73.0420	73.3038	73.4347	73.5656
	24	75.3982	75.6600	75.7909	75.9218	76.1836	76.4454	76.5763	76.7072
	25	78.5398	78.8016	78.9325	79.0634	79.3252	79.5870	79.7179	79.8488
	26	81.6814	81.9432	82.0741	82.2050	82.4668	82.7286	82.8595	82.9904
	27	84.8230	85.0848	85.2157	85.3466	85.6084	85.8702	86.0011	86.1320
	28	87.9646	88.2264	88.3573	88.4882	88.7500	89.0118	89.1427	89.2736
	29	91.1062	91.3680	91.4989	91.6298	91.8916	92.1534	92.2843	92.4152
	30	94.2478	94.5096	94.6405	94.7714	94.0332	95.2950	95.4259	95.5568
	31	97.3894	97.6512	97.7821	97.9130	98.1748	98.4366	98.5675	98.6984
	32	100.531	100.793	100.924	101.055	101.316	101.578	101.709	101.840
	33	103.673	103.934	104.065	104.196	104.458	104.720	104.851	104.982
	34	106.814	107.076	107.207	107.338	107.600	107.861	107.992	108.123
	35	109.956	110.218	110.348	110.479	110.741	111.003	111.134	111.265
	36	113.097	113.359	113.490	113.621	113.883	114.145	114.145	114.406
	37	116.239	116.501	116.632	116.763	117.024	117.286	117.417	117.548
	38	119.381	119.642	119.773	119.904	120.166	120.428	120.559	120.690
	39	122.522	122.784	122.915	123.046	123.308	123.569	123.700	123.831
	40	125.664	125.926	126.056	126.187	126.449	126.711	126.842	126.973
	41	128.805	129.067	129.198	129.329	129.591	129.852	129.983	130.114
	42	131.947	132.209	132.340	132.470	132.732	132.994	133.125	133.256
	43	135.088	135.350	135.481	135.612	135.874	136.136	136.267	136.397
	44	138.230	138.492	138.623	138.754	139.015	139.277	139.408	139.539
	45	141.372	141.633	141.764	141.895	142.157	142.419	142.550	142.681
	46	144.513	144.775	144.906	145.037	145.299	145.560	145.691	145.822
	47	147.655	147.917	148.048	148.178	148.440	148.702	148.833	148.964
	48	150.796	151.058	151.189	151.320	151.582	151.844	151.975	152.106
	49	153.938	154.200	154.331	154.462	154.723	154.985	155.116	155.247
	50	157.080	157.341	157.472	157.603	157.865	158.127	158.258	158.389

Diameters may be obtained from circumferences by inverse interpolation.

Note.—Area of surface of *Sphere* = diameter × circumference.

The Circumferences are in the same Denomination as that for which the First Column is used. Ex:—Dia. = 14.125 ft. = 14 ft. 1½ ins.: then, Circumference = 44.375 ft.

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—DIAMETERS, IN FEET AND INCHES; AND IN INCHES.

This table may be used like logarithmic tables.

Decimal	.5000	.583̆3	.6250	.666̆6	.7500	.833̆3	.8750	.916̆6
	Fractions—as 12ths of a Foot (Inches), and 8ths of an Inch.							
Ft. 12ths	6/12	7/12	7.5/12	8/12	9/12	10/12	10.5/12	11/12
Ins. 8ths	4/8		5/8		6/8		7/8	
0	1.57080	1.83260	1.96350	2.09440	2.35619	2.61799	2.74889	2.87979
1	4.71239	4.97419	5.10509	5.23599	5.49779	5.75959	5.89049	6.02139
2	7.85398	8.11578	8.24668	8.37758	8.63938	8.90118	9.03208	9.16298
3	10.9956	11.2574	11.3883	11.5192	11.7810	12.0428	12.1737	12.3046
4	14.1372	14.3990	14.5299	14.6608	14.9226	15.1844	15.3153	15.4462
5	17.2788	17.5406	17.6715	17.8024	18.0642	18.3260	18.4569	18.5878
6	20.4204	20.6822	20.8131	20.9440	20.2058	21.4675	21.5984	21.7293
7	23.5619	23.8237	23.9546	24.0855	24.3473	24.6091	24.7400	24.8709
8	26.7035	26.9653	27.0962	27.2271	27.4889	27.7507	27.8816	28.0125
9	29.8451	30.1069	30.2378	30.3687	30.6305	30.8923	31.0232	31.1541
10	32.9867	33.2485	33.3794	33.5103	33.7721	34.0339	34.1648	34.2957
11	36.1283	36.3901	36.5210	36.6519	36.9137	37.1755	37.3064	37.4373
12	39.2699	39.5317	39.6626	39.7935	40.0553	40.3171	40.4480	40.5789
13	42.4115	42.6733	42.8042	42.9351	43.1969	43.4587	43.5896	43.7205
14	45.5531	45.8149	45.9458	46.0767	46.3385	46.6003	46.7312	46.8621
15	48.6947	48.9565	49.0874	49.2183	49.4801	49.7419	49.8728	50.0037
16	51.8363	52.0981	52.2290	52.3599	52.6217	72.8835	53.0144	53.1453
17	54.9779	55.2397	55.3706	55.5015	55.7633	56.0251	56.1560	56.2869
18	58.1195	58.3813	58.5122	58.6431	58.9049	59.1667	59.2976	59.4285
19	61.2611	61.5229	61.6538	61.7847	62.0465	62.3083	62.4392	62.5701
20	64.4026	64.6644	64.7953	64.9262	65.1880	65.4498	65.5807	65.7116
21	67.5442	67.8060	67.9369	68.0678	68.3296	68.5914	68.7223	68.8532
22	70.6858	70.9476	71.0785	71.2094	71.4712	71.7330	71.8639	71.9948
23	73.8274	74.0892	74.2201	74.3510	74.6128	74.8746	75.0055	75.1364
24	76.9690	77.2308	77.3617	77.4926	77.7544	78.0162	78.1471	78.2780
25	80.1106	80.3724	80.5033	80.6342	80.8960	81.1578	81.2887	81.4196
26	83.2522	83.5140	83.6449	83.7758	83.0376	84 2994	84.4303	84.5612
27	86.3938	86.6556	86.7865	86.9174	87.1792	87.4410	87.5719	87.7028
28	89.5354	89.7972	89.9281	90.0590	90.3208	90.5826	90.7135	90.8444
29	92.6770	92.9388	93.0697	93.2006	93.4624	93.7242	93.8551	93.9860
30	95.8186	96.0804	96.2113	96.3422	96.6040	96.8658	96.9967	97.1276
31	98.9602	99.2220	99.3529	99.4838	99.7456	100.007	100.138	100.269
32	102.102	102.364	102.494	102.625	102.887	103.149	103.280	103.411
33	105.243	105.505	105.636	105.767	106.029	106.291	106.421	106.552
34	108.385	108.647	108.778	108.909	109.170	109.432	109.563	109.694
35	111.527	111.788	111.919	112.050	112.312	112.574	112.705	112.836
36	114.668	114.930	115.061	115.192	115.454	115.715	115.846	115.977
37	117.810	118.072	118.202	118.333	118.596	118.857	118.988	119.119
38	120.951	121.213	121.344	121.475	121.737	121.999	122.129	122.260
39	124.093	124.355	124.486	124.617	124.878	125.140	125.271	125.402
40	127.235	127.496	127.627	127.758	128.020	128.282	128.413	128.543
41	130.376	130.638	130.769	130.900	131.161	131.423	131.554	131.685
42	133.518	133.779	133.910	134.041	134.303	134.565	134.696	134.827
43	136.659	136.921	137.052	137.183	137.445	137.706	137.837	137.968
44	139.801	140.063	140.194	140.324	140.586	140.848	140.979	141.110
45	142.942	143.204	143.335	143.466	143.728	143.990	144.121	144.251
46	146.084	146.346	146.477	146.608	146.869	147.131	147.262	147.393
47	149.226	149.487	149.618	149.749	150.011	150.273	150.404	150.535
48	152.367	152.629	152.760	152.891	153.153	153.414	153.545	153.676
49	155.509	155.771	155.902	156.032	156.294	156.556	156.687	156.818
50	158.650	158.912	159.043	159.174	159.436	159.698	159.829	159.959

First Column = Diameter in Feet or Inches for Decimals: Feet for 12ths; Inches for 8ths.

Diff. between any two successive circumferences in 12ths = .2618; hence if the diameter is extended beyond 12ths, this *diff.* must be used proportionately. *Diff.* between any two successive circumferences in 8ths = .3927; hence, if the diameter is extended beyond 8ths, this *diff.* must be used proportionately.

The Circumferences are in the same Denomination as that for which the First Column is used.

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12.—Circumferences of Circles for Given—

Circumferences are directly proportional to the diameters.

Decimal		.0000	.083⅓	.1250	.166⅔	.2500	.333⅓	.3750	.416⅔
		Fractions—as 12ths of a Foot (Inches), and 8ths of an Inch.							
Ft.	12ths	0/12	1/12	1.5/12	2/12	3/12	4/12	4.5/12	5/12
Ins.	8ths	0/8		1/8		2/8		3/8	
First Column = Diameter in Feet or Inches for Decimals; Feet for 12ths; Inches for 8ths.									
	50	157.080	157.341	157.472	157.603	157.865	158.127	158.258	158.389
	51	160.221	160.483	160.614	160.745	161.007	161.268	161.399	161.530
	52	163.363	163.625	163.756	163.886	164.148	164.410	164.541	164.672
	53	166.504	166.766	166.897	167.028	167.290	167.552	167.683	167.813
	54	169.646	169.908	170.039	170.170	170.431	170.693	170.824	170.955
	55	172.788	173.049	173.180	173.311	173.573	173.835	173.966	174.097
	56	175.929	176.191	176.322	176.453	176.715	176.976	177.107	177.238
	57	179.071	179.333	179.463	179.594	179.856	180.118	180.249	180.380
	58	182.212	182.474	182.605	182.736	182.998	183.260	183.390	183.521
	59	185.354	185.616	185.747	185.878	186.139	186.401	186.532	186.663
	60	188.496	188.757	188.888	189.019	189.281	189.543	189.674	189.805
	61	191.637	191.899	192.030	192.161	192.423	192.684	192.815	192.946
	62	194.779	195.041	195.171	195.302	195.564	195.826	195.957	196.088
	63	197.920	198.182	198.313	198.444	198.706	198.968	199.098	199.229
	64	201.062	201.324	201.455	201.586	201.847	202.109	202.240	202.371
	65	204.204	204.465	204.596	204.727	204.989	205.251	205.382	205.513
	66	207.345	207.607	207.738	207.869	208.131	208.392	208.523	208.654
	67	210.487	210.749	210.879	211.010	211.272	211.534	211.665	211.796
	68	213.628	213.890	214.021	214.152	214.414	214.675	214.806	214.937
	69	216.770	217.032	217.163	217.293	217.555	217.817	217.948	218.079
	70	219.911	220.173	220.304	220.435	220.697	220.959	221.090	221.220
	71	223.053	223.315	223.446	223.577	223.838	224.100	224.231	224.362
	72	226.195	226.456	226.587	226.718	226.980	227.242	227.373	227.504
	73	229.336	229.598	229.729	229.860	230.122	230.383	230.514	230.645
	74	232.478	232.740	232.871	233.001	233.263	233.525	233.656	233.787
	75	235.619	235.881	236.012	236.143	236.405	236.667	236.798	236.928
	76	238.761	239.023	239.154	239.285	239.546	239.808	239.939	240.070
	77	241.903	242.164	242.295	242.426	242.688	242.950	243.081	243.212
	78	245.044	245.306	245.437	245.568	245.830	246.091	246.222	246.353
	79	248.186	248.448	248.579	248.709	248.971	249.233	249.364	249.495
	80	251.327	251.589	251.720	251.851	252.113	252.375	252.506	252.636
	81	254.469	254.731	254.862	254.993	255.254	255.516	255.647	255.778
	82	257.611	257.872	258.003	258.134	258.396	258.658	258.789	258.920
	83	260.752	261.014	261.145	261.276	261.538	261.799	261.930	262.061
	84	263.894	264.156	264.286	264.417	264.679	264.941	265.072	265.203
	85	267.035	267.297	267.428	267.559	267.821	268.083	268.213	268.344
	86	270.177	270.439	270.570	270.701	270.962	271.224	271.355	271.486
	87	273.319	273.580	273.711	273.842	274.104	274.366	274.497	274.628
	88	276.460	276.722	276.853	276.984	277.246	277.507	277.638	277.769
	89	279.602	279.864	279.994	280.125	280.387	280.649	280.780	280.911
	90	282.743	283.005	283.136	283.267	283.529	283.791	283.921	284.052
	91	285.885	286.147	286.278	286.409	286.670	286.932	287.063	287.194
	92	289.027	289.288	289.419	289.550	289.812	290.074	290.205	290.336
	93	292.168	292.430	292.561	292.692	292.954	293.215	293.346	293.477
	94	295.310	295.572	295.702	295.833	296.095	296.357	296.488	296.619
	95	298.451	298.713	298.844	298.975	299.237	299.498	299.629	299.760
	96	301.593	301.855	301.986	302.116	302.378	302.640	302.771	302.902
	97	304.734	304.996	305.127	305.258	305.520	305.782	305.913	306.043
	98	307.876	308.138	308.269	308.400	308.661	308.923	309.054	309.185
	99	311.018	311.279	311.410	311.541	311.803	312.065	312.196	312.327
	100	314.159	314.421	314.552	314.683	314.945	315.206	315.337	315.468

Diameters may be obtained from circumferences by inverse interpolation.

Note.—Area of surface of *Sphere* = diameter × circumference.

The Circumferences are in the same Denomination as that for which the First Column is used. Ex.—Dia. = 62.25 ins. = 62⅜ = 62½ ins.: then, Circumference = 195.564 ins.

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—DIAMETERS IN FEET AND INCHES; AND IN INCHES.—Concluded.

This table may be used like logarithmic tables.

Decimal		.5000	.583⅓	.6250	.666⅔	.7500	.833⅓	.8750	.916⅔
		Fractions—as 12ths of a Foot (Inches), and 8ths of an Inch.							
Ft.	12ths	$\frac{6}{12}$	$\frac{7}{12}$	$\frac{7.5}{12}$	$\frac{8}{12}$	$\frac{9}{12}$	$\frac{10}{12}$	$\frac{10.5}{12}$	$\frac{11}{12}$
Ins.	8ths	$\frac{4}{8}$		$\frac{5}{8}$		$\frac{6}{8}$		$\frac{7}{8}$	
First Column = Diameter in Feet or Inches for Decimals; Feet for 12ths; Inches for 8ths.									
50		158.650	158.912	159.043	159.174	159.436	159.698	159.829	159.959
51		161.792	162.054	162.185	162.316	162.577	162.839	162.970	163.101
52		164.934	165.195	165.326	165.457	165.719	165.981	166.112	166.243
53		168.075	168.337	168.468	168.599	168.861	169.122	169.253	169.384
54		171.217	171.479	171.609	171.740	172.002	172.264	172.395	172.526
55		174.358	174.620	174.751	174.882	175.144	175.406	175.536	175.667
56		177.500	177.762	177.893	178.024	178.285	178.547	178.678	178.809
57		180.642	180.903	181.034	181.165	181.427	181.689	181.820	181.951
58		183.783	184.045	184.176	184.307	184.569	184.830	184.961	185.092
59		186.925	187.187	187.317	187.448	187.710	187.972	188.103	188.234
60		190.066	190.328	190.459	190.590	190.852	191.114	191.244	191.375
61		193.208	193.470	193.601	191.732	193.993	194.255	194.386	194.517
62		196.350	196.611	196.742	196.873	197.135	197.397	197.528	197.659
63		199.491	199.753	199.884	200.015	200.277	200.538	200.669	200.800
64		202.633	202.895	203.025	203.156	203.418	203.680	203.811	203.942
65		205.774	206.036	206.167	206.298	206.560	206.822	206.952	207.083
66		208.916	209.178	209.309	209.440	209.701	209.963	210.094	210.225
67		212.058	212.319	212.450	212.581	212.843	213.105	213.236	213.367
68		215.199	215.461	215.592	215.723	215.984	216.246	216.377	216.508
69		218.341	218.602	218.733	218.864	219.126	219.388	219.519	219.650
70		221.482	221.744	221.875	222.006	222.268	222.529	222.660	222.791
71		224.624	224.886	225.017	225.147	225.409	225.671	225.802	225.933
72		227.765	228.027	228.158	228.289	228.551	228.813	228.944	229.074
73		230.907	231.169	231.300	231.431	231.692	231.954	232.085	232.216
74		234.049	234.310	234.441	234.572	234.834	235.096	235.227	235.358
75		237.190	237.452	237.583	237.714	237.976	238.237	238.368	238.499
76		240.332	240.594	240.725	240.855	241.117	241.379	241.510	241.641
77		243.473	243.735	243.866	243.997	244.259	244.521	244.652	244.782
78		246.615	246.877	247.008	247.139	247.400	247.662	247.793	247.924
79		249.757	250.018	250.149	250.280	250.542	250.804	250.935	251.066
80		252.898	253.160	253.291	253.422	253.684	253.945	254.076	254.207
81		256.040	256.302	256.433	256.563	256.825	257.087	257.218	257.349
82		259.181	259.443	259.574	259.705	259.967	260.229	260.359	260.490
83		262.323	262.585	262.716	262.847	263.108	263.370	263.501	263.632
84		265.465	265.726	265.857	265.988	266.250	266.512	266.643	266.774
85		268.606	268.868	268 999	269.130	269.392	269.653	269.784	269.915
86		271.748	272.010	272.140	272.271	272.533	272.795	272.926	273.057
87		274.889	275.151	275.282	275.413	275.675	275.937	276.067	276.198
88		278.031	278.293	278.424	278.555	278.816	279.078	279.209	279.340
89		281.173	281.434	281.565	281.696	281.958	282.220	282.351	282.482
90		284.314	284.576	284.707	284.838	285.100	285.361	285.492	285.623
91		287.456	287.718	287.848	287.979	288.241	288.503	288.634	288.765
92		290.597	290.859	290.990	291.121	291.383	291.645	291.775	291.906
93		293.739	294.001	294.132	294.263	294.524	294 786	294.917	295.048
94		296.881	297.142	297.273	297.404	297.666	297.928	298.059	298.190
95		300.022	300.284	300.415	300.546	300.807	301.069	301.200	301.331
96		303.164	303.425	303.556	303.687	303.949	304.211	304.342	304.473
97		306.305	306.567	306.698	306.829	307.091	307.352	307.483	307.614
98		309.447	309.709	309.840	309.970	310.232	310.494	310.625	310.756
99		312.588	312.850	312.981	313.112	313.374	313.636	313.767	313.897
100		315.730	315.992	316.123	316.254	316.515	316.777	316.908	317.039

Diff. between any two successive circumferences in 12ths = .2618; hence, if the diameter is extended beyond 12ths, this *diff.* must be used proportionately. *Diff.* between any two successive circumferences in 8ths = .3927; hence, if the diameter is extended beyond 8ths, this *diff.* must be used proportionately.

The Circumferences are in the same Denomination as that for which the First Column is used.

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13.—Areas of Circles in Square Inches for Given—

Note.—Diameters in inches are in heavy type, with fractions of an inch in column "opposite the areas. The range of this table may be enlarged by noting that areas of circles are proportional to the *squares* of their diameters. *Spheres*: Surface = 4 × given areas. Volume = $\frac{4}{3}$ *diam.* × given areas. (Assuming *diam.* of sphere = *diam.* of circle.)

	0		3		8	16	24	32	40
1-64	.00019	.....	7.0686	...	50.265	201.06	452.39	804.25	1256.64
1-32	.00077	1-16	7.3662	¼	51.849	204.23	457.11	810.54	1264.50
1-16	.00307	¼	7.6699	½	53.456	207.39	461.86	816.86	1272.39
3-32	.00690	3-16	7.9798	¾	55.088	210.60	466.64	823.21	1280.31
⅛	.01227	½	8.2958	1	56.745	213.82	471.44	829.58	1288.25
5-32	.01917	5-16	8.6179	¼	58.426	217.08	476.26	835.97	1296.21
3-16	.02761	¾	8.9462	½	60.132	220.35	481.11	842.39	1304.20
7-32	.03758	7-16	9.2806	¾	61.862	223.65	485.98	848.83	1312.21
					9	17	25	33	41
¼	.04909	⅛	9.6211	.	63.617	226.98	490.87	855.30	1320.25
9-32	.06213	9-16	9.9678	¼	65.397	230.33	495.79	861.79	1328.32
5-16	.07670	¼	10.321	½	67.201	233.71	500.74	868.21	1336.40
11-32	.09281	11-16	10.680	¾	69.029	237.10	505.71	874.85	1344.51
⅜	.11045	¾	11.045	1	70.882	240.53	510.71	881.41	1352.65
13-32	.12962	13-16	11.416	¼	72.760	243.98	515.72	888.00	1360.81
7-16	.15033	⅞	11.793	½	74.662	247.45	520.77	894.62	1369.00
15-32	.17257	15-16	12.177	¾	76.589	250.95	525.84	901.26	1377.21
			4		10	18	26	34	42
	.19635	.....	12.566	...	78.540	254.47	530.93	907.92	1385.44
17-32	.22166	1-16	12.962	¼	80.516	258.02	536.05	914.61	1393.70
9-16	.24850	⅛	13.364	½	82.516	261.59	541.19	921.32	1401.98
19-32	.27688	3-16	13.772	¾	84.541	265.18	546.35	928.06	1410.29
⅝	.30680	¼	14.186	1	86.590	268.80	551.55	934.82	1418.62
21-32	.33824	5-16	14.607	¼	88.664	272.45	556.76	941.61	1426.98
11-16	.37122	⅜	15.033	½	90.763	276.12	562.00	948.42	1435.36
23-32	.40574	7-16	15.466	¾	92.886	279.81	567.27	955.25	1443.77
					11	19	27	35	43
¾	.44179	½	15.904	...	95.033	283.53	572.56	962.11	1452.20
25-32	.47937	9-16	16.349	¼	97.205	287.27	577.87	969.00	1460.65
13-16	.51849	⅝	16.800	½	99.402	291.04	583.21	975.91	1469.13
27-32	.55914	11-16	17.257	¾	101.62	294.83	588.57	982.84	1477.63
⅞	.60132	¾	17.721	1	103.87	298.65	593.96	989.80	1486.17
29-32	.64504	13-16	18.190	¼	106.14	302.49	599.37	996.78	1494.72
15-16	.69029	⅞	18.665	½	108.43	306.35	604.81	1003.78	1503.30
31-32	.73708	15-16	19.147	¾	110.75	310.24	610.27	1010.82	1511.90
	1		5		12	20	28	36	44
.....	.78540	.....	19.635	...	113.10	314.16	615.75	1017.87	1520.53
1-16	.88664	1-16	20.129	¼	115.47	318.10	621.26	1024.95	1529.18
⅛	.99402	⅛	20.629	½	117.86	322.06	626.80	1032.06	1537.86
3-16	1.1075	3-16	21.135	¾	120.28	326.05	632.36	1039.19	1546.55
¼	1.2272	¼	21.648	1	122.72	330.06	637.94	1046.35	1555.28
5-16	1.3530	5-16	22.166	¼	125.19	334.10	643.55	1053.52	1564.03
⅜	1.4849	⅜	22.691	½	127.68	338.16	649.18	1060.73	1572.81
7-16	1.6230	7-16	23.221	¾	130.19	342.25	654.84	1067.95	1581.61
					13	21	29	37	45
½	1.7671	½	23.758	...	132.73	346.36	660.52	1075.21	1590.43
9-16	1.9175	9-16	24.301	¼	135.30	350.50	666.23	1082.48	1599.28
⅝	2.0739	⅝	24.850	½	137.89	354.66	671.96	1089.79	1608.15
11-16	2.2365	11-16	25.406	¾	140.50	358.84	677.71	1097.11	1617.04
¾	2.4053	¾	25.967	1	143.14	363.05	683.49	1104.46	1625.97
13-16	2.5802	13-16	26.535	¼	145.80	367.28	689.30	1111.84	1634.92
⅞	2.7612	⅞	27.109	½	148.49	371.54	695.13	1119.24	1643.89
15-16	2.9483	15-16	27.688	¾	151.20	375.83	700.98	1126.66	1652.88
	2		6		14	22	30	38	46
.....	3.1416	.....	28.274	...	153.94	380.13	706.86	1134.11	1661.90
1-16	3.3410	⅛	29.465	¼	156.70	384.46	712.76	1141.59	1670.95
⅛	3.5466	¼	30.680	½	159.48	388.82	718.69	1149.08	1680.01
3-16	3.7583	⅜	31.919	¾	162.30	393.20	724.64	1156.61	1689.10
¼	3.9761	½	33.183	1	165.13	397.61	730.62	1164.15	1698.23
5-16	4.2000	⅝	34.472	¼	167.99	402.04	736.62	1171.73	1707.37
⅜	4.4301	¾	35.785	½	170.87	406.49	742.64	1179.32	1716.54
7-16	4.6664	⅞	37.122	¾	173.78	410.97	748.69	1186.94	1725.73
			7		15	23	31	39	47
½	4.9087	.....	38.485	...	176.71	415.48	754.77	1194.59	1734.94
9-16	5.1572	⅛	39.871	¼	179.67	420.00	760.87	1202.26	1744.18
⅝	5.4119	¼	41.282	½	182.65	424.56	766.99	1209.95	1753.45
11-16	5.6727	⅜	42.718	¾	185.66	429.13	773.14	1217.67	1762.73
¾	5.9396	½	44.179	1	188.69	433.74	779.31	1225.42	1772.05
13-16	6.2126	⅝	45.664	¼	191.75	438.36	785.51	1233.18	1781.39
⅞	6.4918	¾	47.173	½	194.83	443.01	791.73	1240.98	1790.76
15-16	6.7771	⅞	48.707	¾	197.93	447.69	797.98	1248.79	1800.14

AMETERS IN INCHES AND FRACTIONS.

	72		80	88	96	104		
0	4071.5	...	5026.5	6082.1	7238.2	8494.9	..	
5	4085.7	1/4	5042.3	6099.4	7257.1	8515.3	1/4	
2	4099.8	1/2	5058.0	6116.7	7276.0	8535.8	1/2	
8	4114.0	3/4	5073.8	6134.1	7294.9	8556.2	3/4	
5	4128.2	1/8	5089.6	6151.4	7313.8	8576.7	1/8	
1	4142.5	3/8	5105.4	6168.8	7332.8	8597.3	3/8	
8	4156.8	5/8	5121.2	6186.2	7351.8	8617.8	5/8	
8	4171.1	7/8	5137.1	6203.7	7370.8	8638.4	7/8	
	73		81	89	97	105		
3	4185.4	...	5153.0	6221.1	7389.8	8659.0	..	
1	4199.7	1/4	5168.9	6238.6	7408.9	8679.6	1/4	
9	4214.1	1/2	5184.9	6256.1	7428.0	8700.3	1/2	
7	4228.5	3/4	5200.8	6273.7	7447.1	8721.0	3/4	
5	4242.9	1/8	5216.8	6291.2	7466.2	8741.7	1/8	
4	4257.4	3/8	5232.8	6308.8	7485.3	8762.4	3/8	
3	4271.8	5/8	5248.9	6326.4	7504.5	8783.2	5/8	
2	4286.3	7/8	5264.9	6344.1	7523.7	8803.9	7/8	
	74		82	90	98	106		
2	4300.8	..	5281.0	6361.7	7543.0	8824.7	..	
2	4315.4	1/4	5297.1	6379.4	7562.2	8845.6	1/4	
2	4329.9	1/2	5313.3	6397.1	7581.5	8866.4	1/2	
2	4344.5	3/4	5329.4	6414.9	7600.8	8887.3	3/4	
3	4359.2	1/8	5345.6	6432.6	7620.1	8908.2	1/8	
3	4373.8	3/8	5361.8	6450.4	7639.5	8929.1	3/8	
4	4388.5	5/8	5378.1	6468.2	7658.9	8950.1	5/8	
5	4403.1	7/8	5394.3	6486.0	7678.3	8971.0	7/8	
	75		83	91	99	107		
7	4417.9	..	5410.6	6503.9	7697.7	8992.0	..	
8	4432.6	1/4	5426.9	6521.8	7717.1	9013.0	1/4	
0	4447.4	1/2	5443.3	6539.7	7736.6	9034.1	1/2	
2	4462.2	3/4	5459.6	6557.6	7756.1	9055.2	3/4	
5	4477.0	1/8	5476.0	6575.5	7775.6	9076.3	1/8	
7	4491.8	3/8	5492.4	6593.5	7795.2	9097.4	3/8	
0	4506.7	5/8	5508.8	6611.5	7814.8	9119.4	5/8	
3	4521.5	7/8	5525.3	6629.6	7834.4	9139.7	7/8	
	76		84	92	100	108		
7	4536.5	...	5541.8	6647.6	7854.0	9160.9	..	
0	4551.4	1/4	5558.3	6665.7	7873.6	9182.1	1/4	
4	4566.4	1/2	5574.8	6683.8	7893.3	9203.3	1/2	
8	4581.3	3/4	5591.4	6701.9	7913.0	9224.6	3/4	
3	4596.3	1/8	5607.9	6720.1	7932.7	9245.9	1/8	
7	4611.4	3/8	5624.5	6738.2	7952.5	9267.2	3/8	
2	4626.4	5/8	5641.2	6756.4	7972.2	9288.6	5/8	
7	4641.5	7/8	5657.8	6774.7	7992.0	9309.9	7/8	
	77		85	93	101	109		
3	4656.6	...	5674.5	6792.9	8011.8	9331.3	.	
8	4671.8	1/4	5691.2	6811.2	8031.7	9352.7	1/4	
4	4686.9	1/2	5707.9	6829.5	8051.6	9374.2	1/2	
0	4702.1	3/4	5724.7	6847.8	8071.4	9395.6	3/4	
7	4717.3	1/8	5741.5	6866.1	8091.4	9417.1	1/8	
3	4732.5	3/8	5758.3	6884.5	8111.3	9438.6	3/8	
0	4747.8	5/8	5775.1	6902.9	8131.3	9460.2	5/8	
7	4763.1	7/8	5791.9	6921.3	8151.3	9481.7	7/8	
	78		86	94	102	110		
5	4778.4	...	5808.8	6939.8	8171.3	9503.3	..	
2	4793.7	1/4	5825.7	6958.2	8191.3	9524.9	1/4	
0	4809.0	1/2	5842.6	6976.7	8211.4	9546.6	1/2	
8	4824.4	3/4	5859.6	6995.3	8231.5	9568.2	3/4	
6	4839.8	1/8	5876.5	7013.8	8251.6	9589.9	1/8	
5	4855.2	3/8	5893.5	7032.4	8271.7	9611.6	3/8	
4	4870.7	5/8	5910.6	7051.0	8291.9	9633.3	5/8	
3	4886.2	7/8	5927.6	7069.6	8312.1	9655.1	7/8	
	79		87	95	103	111		
2	4901.7	..	5944.7	7088.2	8332.3	9676.9	..	
1	4917.2	1/4	5961.8	7106.9	8352.5	9698.7	1/4	
1	4932.7	1/2	5978.9	7125.6	8372.8	9720.5	1/2	
1	4948.3	3/4	5996.0	7144.3	8393.1	9742.4	3/4	
2	4963.8	1/8	6013.2	7163.0	8413.4	9764.3	1/8	
2	4979.5	3/8	6030.4	7181.8	8433.7	9786.2	3/8	
3	4995.2	5/8	6047.6	7200.6	8454.1	9808.1	5/8	
4	5010.9	7/8	6064.9	7219.4	8474.5	9830.1	7/8	

Ex.— For *dia.* of 64 ins., area equals 3217.0 sq. ins. For *dia.* of 66⅝ ins., area equals 3460.2 sq. ins.
Ex.— For *dia.* of $\frac{3}{8}$ in., area is ¼ of that for $\frac{3}{16}$ in. For *dia.* of $50\frac{3}{16}$ in., area is ¼ of that for 100¼ ins. For *dia.* of 200¼ ins., area is 4 × that for 100¼ ins.

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14.—Areas of Circles in Square Feet* for Given—

Note.—Diameters in feet are in heavy type, with decimals of a foot "in column" opposite the areas.

	0	6	12	18	24	30	36	42	48	
.0	.000000	28.27433	113.0973	254.4690	452.3893	706.8583	1017.8760	1385.4424	1809.5574	.0
.1	.007854	29.22467	114.9901	257.3043	456.1671	711.5786	1023.5387	1392.0476	1817.1050	.1
.2	.031416	30.19071	116.8987	260.1553	459.9606	716.3145	1029.2172	1398.6685	1824.6684	.2
.3	.070686	31.17245	118.8229	263.0220	463.7698	721.0662	1034.9113	1405.3051	1832.2475	.3
.4	.125664	32.16991	120.7628	265.9044	467.5947	725.8336	1040.6212	1411.9574	1839.8423	.4
.5	.196350	33.18307	122.7185	268.8025	471.4352	730.6166	1046.3467	1418.6254	1847.4528	.5
.6	.282743	34.21194	124.6898	271.7163	475.2916	735.4154	1052.0880	1425.3092	1855.0790	.6
.7	.384845	35.25652	126.6769	274.6459	479.1636	740.2299	1057.8449	1432.0086	1862.7210	.7
.8	.502655	36.31681	128.6796	277.5911	483.0513	745.0601	1063.6176	1438.7238	1870.3786	.8
.9	.636173	37.39281	130.6981	280.5521	486.9547	749.9060	1069.4060	1445.4546	1878.0519	.9
	1	**7**	**13**	**19**	**25**	**31**	**37**	**43**	**49**	
.0	.785398	38.48451	132.7323	283.5287	490.8739	754.7676	1075.2101	1452.2012	1885.7410	.0
.1	.950332	39.59192	134.7822	286.5211	494.8087	759.6450	1081.0299	1458.9635	1893.4457	.1
.2	1.13097	40.71504	136.8478	289.5292	498.7592	764.5380	1086.8654	1465.7415	1901.1662	.2
.3	1.32732	41.85387	138.9291	292.5530	502.7255	769.4467	1092.7166	1472.5352	1908.9024	.3
.4	1.53938	43.00840	141.0261	295.5925	506.7075	774.3712	1098.5835	1479.3446	1916.6543	.4
.5	1.76715	44.17865	143.1388	298.6477	510.7052	779.3113	1104.4662	1486.1697	1924.4218	.5
.6	2.01062	45.36460	145.2672	301.7186	514.7185	784.2672	1110.3645	1493.0105	1932.2051	.6
.7	2.26980	46.56626	147.4114	304.8052	518.7476	789.2388	1116.2786	1499.8670	1940.0041	.7
.8	2.54469	47.78362	149.5712	307.9075	522.7924	794.2260	1122.2083	1506.7393	1947.8189	.8
.9	2.83529	49.01670	151.7468	311.0255	526.8529	799.2290	1128.1538	1513.6272	1955.6493	.9
	2	**8**	**14**	**20**	**26**	**32**	**38**	**44**	**50**	
.0	3.14159	50.26548	153.9380	314.1593	530.9292	804.2477	1134.1149	1520.5308	1963.4954	.0
.1	3.46361	51.52997	156.1450	317.3087	535.0211	809.2821	1140.0918	1527.4503	1971.3572	.1
.2	3.80133	52.81017	158.3677	320.4739	539.1287	814.3322	1146.0844	1534.3853	1979.2348	.2
.3	4.15476	54.10608	160.6061	323.6547	543.2521	819.3980	1152.0927	1541.3360	1987.1280	.3
.4	4.52389	55.41769	162.8602	326.8513	547.3911	824.4795	1158.1167	1548.3025	1995.0370	.4
.5	4.90874	56.74502	165.1300	330.0636	551.5459	829.5768	1164.1564	1555.2847	2002.9617	.5
.6	5.30929	58.08805	167.4155	333.2916	555.7163	834.6898	1170.2118	1562.2826	2010.9020	.6
.7	5.72555	59.44679	169.7167	336.5353	559.9025	839.8184	1176.2830	1569.2962	2018.8581	.7
.8	6.15752	60.82123	172.0336	339.7947	564.1044	844.9628	1182.3698	1576.3255	2026.8299	.8
.9	6.60520	62.31139	174.3662	343.0698	568.3220	850.1228	1188.4724	1583.3706	2034.8174	.9
	3	**9**	**15**	**21**	**27**	**33**	**39**	**45**	**51**	
.0	7.06858	63.61725	176.7146	346.3606	572.5553	855.2986	1194.5906	1590.4313	2042.8206	.0
.1	7.54768	65.03882	179.0786	349.6671	576.8043	860.4901	1200.7246	1597.5077	2050.8395	.1
.2	8.04248	66.47610	181.4584	352.9894	581.0690	865.6973	1206.8742	1604.5999	2058.8742	.2
.3	8.55299	67.92909	183.8539	356.3273	585.3494	870.9202	1213.0396	1611.7077	2066.9245	.3
.4	9.07920	69.39778	186.2650	359.6809	589.6455	876.1588	1219.2207	1618.8313	2074.9905	.4
.5	9.62113	70.88218	188.6919	363.0503	593.9574	881.4131	1225.4175	1625.9705	2083.0723	.5
.6	10.17876	72.38229	191.1345	366.4354	598.2849	886.6831	1231.6300	1633.1255	2091.1697	.6
.7	10.75210	73.89811	193.5928	369.8361	602.6282	891.9688	1237.8582	1640.2962	2099.2829	.7
.8	11.34115	75.42964	196.0668	373.2526	606.9871	897.2703	1244.1021	1647.4826	2107.4118	.8
.9	11.94591	76.97687	198.5565	376.6848	611.3618	902.5874	1250.3617	1654.6847	2115.5563	.9
	4	**10**	**16**	**22**	**28**	**34**	**40**	**46**	**52**	
.0	12.56637	78.53982	201.0619	380.1327	615.7522	907.9203	1256.6371	1661.9025	2123.7166	.0
.1	13.20254	80.11847	203.5831	383.5963	620.1582	913.2688	1262.9281	1669.1360	2131.8926	.1
.2	13.85442	81.71282	206.1199	387.0756	624.5800	918.6331	1269.2348	1676.3852	2140.0843	.2
.3	14.52201	83.32289	208.6724	390.5707	629.0175	924.0131	1275.5573	1683.6502	2148.2917	.3
.4	15.20531	84.94867	211.2407	394.0814	633.4707	929.4088	1281.8955	1690.9308	2156.5149	.4
.5	15.90431	86.59015	213.8246	397.6078	637.9397	934.8202	1288.2493	1698.2272	2164.7537	.5
.6	16.61903	88.24734	216.4243	401.1500	642.4243	940.2473	1294.6189	1705.5392	2173.0082	.6
.7	17.34945	89.92024	219.0397	404.7078	646.9246	945.6901	1301.0042	1712.8670	2181.2785	.7
.8	18.09557	91.60884	221.6708	408.2814	651.4407	951.1486	1307.4052	1720.2105	2189.5644	.8
.9	18.85741	93.31316	224.3176	411.8707	655.9724	956.6228	1313.8219	1727.5697	2197.8661	.9
	5	**11**	**17**	**23**	**29**	**35**	**41**	**47**	**53**	
.0	19.63495	95.03318	226.9801	415.4756	660.5199	962.1128	1320.2543	1734.9445	2206.1834	.0
.1	20.42821	96.76891	229.6583	419.0963	665.0800	967.6184	1326.7024	1742.3351	2214.5165	.1
.2	21.23717	98.52035	232.3522	422.7327	669.6619	973.1397	1333.1663	1749.7414	2222.8653	.2
.3	22.06183	100.2875	235.0618	426.3848	674.2565	978.6768	1339.6458	1757.1635	2231.2298	.3
.4	22.90221	102.0703	237.7871	430.0526	678.8668	984.2296	1346.1410	1764.6012	2239.6100	.4
.5	23.75829	103.8689	240.5282	433.7361	683.4928	989.7980	1352.6520	1772.0546	2248.0059	.5
.6	24.63009	105.6832	243.2849	437.4354	688.1345	995.3822	1359.1786	1779.5237	2256.4175	.6
.7	25.51759	107.5132	246.0574	441.1503	692.7919	1000.9821	1365.7210	1787.0086	2264.8448	.7
.8	26.42079	109.3588	248.8456	444.8809	697.4650	1006.5977	1372.2791	1794.5091	2273.2879	.8
.9	27.33971	111.2202	251.6494	448.6273	702.1538	1012.2290	1378.8529	1802.0254	2281.7466	.9

* Or any other denomination. Note that areas of circles are proportional to the *squares* of their diameters, and that the range of the table may therefore be extended greatly.

Spheres: Surface = 4 × above areas. Volume = ⅔ *diam.* × above areas. (Assuming *diam.* of sphere = *diam.* of circle.)

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TERS IN FEET* AND TENTHS.

decimal point in the diameter = 2 decimal points
a. of 5.41, area equals 22.987.

	72	78	84	90	96	
1944	4071.5041	4778.3624	5541.7694	6361.7251	7238.2295	.0
5695	4082.8217	4790.6225	5554.9720	6375.8701	7253.3170	.1
9603	4094.1550	4802.8983	5568.1902	6390.0308	7268.4202	.2
3669	4105.5040	4815.1897	5581.4242	6404.2073	7283.5391	.3
7891	4116.8687	4827.4969	5594.6739	6418.3994	7298.6737	.4
2270	4128.2491	4839.8198	5607.9392	6432.6073	7313.8240	.5
6807	4139.6452	4852.1584	5621.2203	6446.8308	7328.9901	.6
1500	4151.0571	4864.5128	5634.5171	6461.0701	7344.1718	.7
6351	4162.4846	4876.8828	5647.8296	6475.3251	7359.3693	.8
1359	4173.9279	4889.2685	5661.1578	6489.5958	7374.5824	.9
7	73	79	85	91	97	
6524	4185.3868	4901.6699	5674.5017	6503.8822	7389.8113	.0
1845	4196.8615	4914.0871	5687.8614	6518.1843	7405.0559	.1
7324	4208.3519	4926.5199	5701.2367	6532.5021	7420.3162	.2
2960	4219.8579	4938.9685	5714.6277	6546.8356	7435.5922	.3
8754	4231.3797	4951.4328	5728.0345	6561.1848	7450.8839	.4
4704	4242.9172	4963.9127	5741.4569	6575.5497	7466.1913	.5
0811	4254.4704	4976.4084	5754.8951	6589.9304	7481.5144	.6
7075	4266.0394	4988.9198	5768.3490	6604.3267	7496.8532	.7
3497	4277.6240	5001.4469	5781.8185	6618.7388	7512.2078	.8
0075	4289.2243	5013.9897	5795.3038	6633.1666	7527.5780	.9
68	74	80	86	92	98	
6811	4300.8403	5026.5482	5808.8048	6647.6100	7542.9640	.0
2704	4312.4721	5039.1225	5822.3215	6662.0692	7558.3656	.1
0754	4324.1195	5051.7124	5835.8539	6676.5441	7573.7830	.2
7960	4335.7827	5064.3180	5849.4020	6691.0347	7589.2161	.3
5324	4347.4616	5076.9394	5862.9659	6705.5410	7604.6648	.4
2845	4359.1562	5089.5764	5876.5454	6720.0630	7620.1293	.5
0523	4370.8664	5102.2292	5890.1407	6734.6007	7635.6095	.6
8359	4382.5924	5114.8977	5903.7516	6749.1542	7651.1054	.7
6351	4394.3341	5127.5819	5917.3783	6763.7233	7666.6170	.8
4500	4406.0916	5140.2818	5931.0206	6778.3081	7682.1444	.9
69	75	81	87	93	99	
2807	4417.8647	5152.9974	5944.6787	6792.9087	7697.6874	.0
1270	4429.6535	5165.7287	5958.3525	6807.5250	7713.2461	.1
9891	4441.4580	5178.4757	5972.0420	6822.1569	7728.8206	.2
8668	4453.2783	5191.2384	5985.7472	6836.8046	7744.4107	.3
7603	4465.1142	5204.0168	5999.4681	6851.4680	7760.0166	.4
6695	4476.9659	5216.8110	6013.2047	6866.1471	7775.6382	.5
5944	4488.8332	5229.6208	6026.9570	6880.8419	7791.2754	.6
5350	4500.7163	5242.4463	6040.7250	6985.5524	7806.9284	.7
4913	4512.6151	5255.2876	6054.5088	6910.2786	7822.5971	.8
4633	4524.5296	5268.1446	6068.3082	6925.0205	7838.2815	.9
70	76	82	88	94	100	
4510	4536.4598	5281.0173	6082.1234	6939.7782	7853.9816	.0
4544	4548.4057	5293.9056	6095.9542	6954.5515	7869.6975	.1
4736	4560.3673	5306.8097	6109.8008	6969.3406	7885.4290	.2
5084	4572.3446	5319.7295	6123.6631	6984.1453	7901.1762	.3
5590	4584.3377	5332.6650	6137.5411	6998.9658	7916.9392	.4
6252	4596.3464	5345.6162	6151.4348	7013.8019	7932.7178	.5
7072	4608.3708	5358.5832	6165.3442	7028.6538	7948.5122	.6
8049	4620.4110	5371.5658	6179.2693	7043.5214	7964.3222	.7
9182	4632.4669	5384.5641	6193.2101	7058.4047	7980.1480	.8
0473	4644.5384	5397.5782	6207.1666	7073.3037	7995.9895	.9
71	77	83	89	95	101	
1921	4656.6257	5410.6079	6221.1388	7088.2184	8011.8467	.0
3526	4668.7287	5423.6534	6235.1268	7103.1488	8027.7196	.1
5289	4680.8474	5436.7146	6249.1304	7118.0950	8043.6082	.2
7208	4692.9818	5449.7915	6263.1498	7133.0568	8059.5125	.3
9284	4705.1319	5462.8840	6277.1848	7148.0343	8075.4325	.4
1518	4717.2977	5475.9923	6291.2356	7163.0276	8091.3682	.5
3908	4729.4792	5489.1163	6305.3021	7178.0366	8107.3197	.6
6456	4741.6765	5502.2561	6319.3843	7193.0612	8123.2868	.7
9160	4753.8894	5515.4115	6333.4822	7208.1016	8139.2697	.8
2022	4766.1181	5528.5826	6347.5958	7223.1577	8155.2682	.9

denomination. Diameters and areas must be in the *same*
meters, feet or inches, etc.
s of circles are proportional to the *squares* of their dia-
he range of the table may therefore be extended greatly.

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15.—Areas of Circles in Square Feet for Given—

Note.—Diameters in feet are in heavy type, with inches "in column opposite the areas.

	0	**5**	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**
0	.000000	19.6350	78.5398	176.715	314.159	490.874	706.858	962.113	1256.64	1590.
1	.005454	20.2949	79.8543	178.684	316.783	494.152	710.791	966.700	1261.88	1596.
2	.021817	20.9658	81.1796	180.663	319.417	497.441	714.734	971.298	1267.13	1602.
3	.049087	21.6475	82.5159	182.654	322.062	500.740	718.688	975.906	1272.39	1608.
4	.087266	22.3402	83.8631	184.656	324.719	504.051	722.654	980.526	1277.67	1614.
5	.136354	23.0438	85.2212	186.668	327.386	507.373	726.630	985.157	1282.95	1620.
6	.196350	23.7583	86.5901	188.692	330.064	510.705	730.617	989.798	1288.25	1625.
7	.267254	24.4837	87.9700	190.726	332.752	514.049	734.615	994.450	1293.56	1631.
8	.349066	25.2200	89.3609	192.772	335.452	517.403	738.623	999.114	1298.87	1637.
9	.441786	25.9672	90.7626	194.828	338.163	520.768	742.643	1003.788	1304.20	1643.
10	.545415	26.7254	92.1752	196.895	340.885	524.144	746.674	1008.47	1309.54	1649.
11	.659953	27.4944	93.5987	198.973	343.617	527.531	750.715	1013.17	1314.89	1655.
	1	**6**	**11**	**16**	**21**	**26**	**31**	**36**	**41**	**46**
0	.785398	28.2743	95.0332	201.062	346.361	530.929	754.768	1017.88	1320.25	1661.
1	.921752	29.0652	96.4785	203.162	349.115	534.338	758.831	1022.59	1325.63	1667.
2	1.069014	29.8669	97.9348	205.273	351.880	537.758	762.905	1027.32	1331.01	1673.
3	1.22718	30.6796	99.4020	207.394	354.656	541.188	766.990	1032.06	1336.40	1680.
4	1.39626	31.5032	100.8800	209.527	357.443	544.630	771.086	1036.81	1341.81	1686.
5	1.57625	32.3377	102.369	211.670	360.241	548.082	775.193	1041.57	1347.23	1692.
6	1.76715	33.1831	103.869	213.825	363.050	551.546	779.311	1046.35	1352.65	1698.
7	1.96895	34.0394	105.380	215.990	365.870	555.020	783.440	1051.13	1358.09	1704.
8	2.18166	34.9066	106.901	218.166	368.701	558.505	787.580	1055.92	1363.54	1710.
9	2.40528	35.7847	108.434	220.353	371.542	562.001	791.730	1060.73	1369.00	1716.
10	2.63981	36.6737	109.978	222.551	374.395	565.508	795.892	1065.55	1374.47	1722.
11	2.88525	37.5737	111.532	224.760	377.258	569.026	800.064	1070.37	1379.95	1728.
	2	**7**	**12**	**17**	**22**	**27**	**32**	**37**	**42**	**47**
0	3.14159	38.4845	113.097	226.980	380.133	572.555	804.248	1075.21	1385.44	1734.
1	3.40885	39.4063	114.674	229.211	383.018	576.095	808.442	1080.06	1390.95	1741.
2	3.68701	40.3389	116.261	231.452	385.914	579.646	812.647	1084.92	1396.46	1747.
3	3.97608	41.2825	117.859	233.705	388.821	583.207	816.863	1089.79	1401.98	1753.
4	4.27606	42.2370	119.468	235.969	391.739	586.780	821.090	1094.67	1407.52	1759.
5	4.58694	43.2024	121.088	238.243	394.668	590.363	825.328	1099.56	1413.07	1765.
6	4.90874	44.1786	122.718	240.528	397.608	593.597	829.577	1104.47	1418.63	1772.
7	5.24144	45.1658	124.360	242.824	400.559	597.563	833.837	1109.38	1424.19	1778.
8	5.58505	46.1640	126.013	245.131	403.520	601.179	838.107	1114.31	1429.77	1784.
9	5.93957	47.1730	127.676	247.450	406.493	604.806	842.389	1119.24	1435.36	1790.
10	6.30500	48.1929	129.351	249.778	409.476	608.444	846.681	1124.19	1440.97	1797.
11	6.68134	49.2237	131.036	252.118	412.470	612.092	850.984	1129.15	1446.58	1803.
	3	**8**	**13**	**18**	**23**	**28**	**33**	**38**	**43**	**48**
0	7.06858	50.2655	132.732	254.469	415.476	615.752	855.299	1134.11	1452.20	1809.
1	7.46674	51.3181	134.439	256.831	418.492	619.423	859.624	1139.09	1457.84	1815.
2	7.87580	52.3817	136.157	259.203	421.519	623.104	863.960	1144.09	1463.48	1822.
3	8 2957	53.4562	137.886	261.587	424.557	626.797	868.307	1149.09	1469.14	1828.
4	8.72665	54.5415	139.626	263.981	427.606	630.500	872.665	1154.10	1474.80	1834.
5	9.16843	55.6378	141.377	266.386	430.665	634.214	877.033	1159.12	1480.48	1841.
6	9.62113	56.7450	143.139	268.803	433.736	637.940	881.413	1164.16	1486.17	1847.
7	10.08473	57.8631	144.911	271.230	436.818	641.676	885.804	1169.20	1491.87	1853.
8	10.5592	58.9921	146.695	273.668	439.910	645.423	890.205	1174.26	1497.58	1860.
9	11.0447	60.1320	148.489	276.117	443.014	649.181	894.618	1179.32	1503.30	1866.
10	11.5410	61.2829	150.295	278.576	446.128	652.949	899.041	1184.40	1509.03	1872.
11	12.0482	62.4446	152.111	281.047	449.253	656.729	903.475	1189.49	1514.78	1879.
	4	**9**	**14**	**19**	**24**	**29**	**34**	**39**	**44**	**49**
0	12.5664	63.6173	153.938	283.529	452.389	660.520	907.920	1194.59	1520.53	1885.
1	13.0954	64.8008	155.776	286.021	455.536	664.321	912.376	1199.70	1526.30	1892.
2	13.6354	65.9953	157.625	288.525	458.694	668.134	916.843	1204.82	1532.07	1898.
3	14.1863	67.2006	159.485	291.039	461.863	671.957	921.321	1209.95	1537.86	1905.
4	14.7480	68.4169	161.356	293.564	465.043	675.791	925.810	1215.10	1543.66	1911.
5	15.3207	69.6441	163.237	296.101	468.234	679.637	930.310	1220.25	1549.47	1917.
6	15.9043	70.8822	165.130	298.648	471.435	683.495	934.820	1225.42	1555.28	1924.
7	16.4988	72.1312	167.033	301.206	474.648	687.360	939.342	1230.59	1561.12	1930.
8	17.1042	73.3911	167.948	303.775	477.871	691.238	943.874	1235.78	1566.96	1937.
9	17.7205	74.6619	170.873	306 354	481.105	695.126	948.417	1240.98	1572.81	1943.
10	18.3478	75.9436	172.809	308.945	484.351	699.026	952.972	1246.19	1578.67	1950.
11	18.9859	77.2363	174.757	311.547	487.607	702.937	957.537	1251.41	1584.55	1956.

Areas of circles are proportional to the *squares* of their diameters.

Spheres: Surface = 4 × above areas. Volume = ⅔ *diam.* × above area (Assuming *diam.* of sphere = *diam.* of circle.)

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ETERS IN FEET AND INCHES.

of 65 ft., area equals 3318.31 sq. ft.
of 90′ – 6″, area equals 6432.61 sq. ft.

65	70	75	80	85	90	95
3318.31	3848.45	4417.86	5026.55	5674.50	6361.73	7088.22
3326.82	3857.62	4427.69	5037.03	5685.63	6373.51	7100.66
3335.35	3866.80	4437.52	5047.51	5696.78	6385.31	7113.11
3343.88	3875.99	4447.37	5058.01	5707.93	6397.12	7125.57
3352.43	3885.19	4457.22	5068.52	5719.09	6408.94	7138.05
3360.99	3894.40	4467.09	5079.04	5730.27	6420.77	7150.53
3369.55	3903.63	4476.97	5089.58	5741.46	6432.61	7163.03
3378.13	3912.86	4486.85	5100.12	5752.65	6444.46	7175.53
3386.72	3922.10	4496.75	5110.67	5763.86	6456.32	7188.05
3395.33	3931.36	4506.66	5121.24	5775.08	6468.20	7200.58
3403.94	3940.63	4516.58	5131.81	5786.31	6480.08	7213.12
3412.56	3949.90	4526.52	5142.40	5797.55	6491.98	7225.67
66	**71**	**76**	**81**	**86**	**91**	**96**
3421.19	3959.19	4536.46	5153.00	5808.80	6503.88	7238.23
3429.84	3968.49	4546.41	5163.61	5820.07	6515.80	7250.80
3438.49	3977.80	4556.38	5174.22	5831.34	6527.73	7263.38
3447.16	3987.12	4566.35	5184.86	5842.63	6539.67	7275.98
3455.84	3996.45	4576.34	5195.50	5853.92	6551.62	7288.58
3464.53	4005.80	4586.34	5206.15	5865.23	6563.58	7301.20
3473.23	4015.15	4596.35	5216.81	5876.55	6575.55	7313.82
3481.94	4024.52	4606.37	5227.48	5887.87	6587.53	7326.46
3490.66	4033.89	4616.40	5238.17	5899.21	6599.53	7339.11
3499.39	4043.28	4626.44	5248.87	5910.56	6611.53	7351.77
3508.13	4052.68	4636.49	5259.57	5921.92	6623.55	7364.44
3516.89	4062.08	4646.55	5270.29	5933.30	6635.57	7377.12
67	**72**	**77**	**82**	**87**	**92**	**97**
2525.65	4071.50	4656.63	5281.02	5944.68	6647.61	7389.81
3534.43	4080.93	4666.71	5291.76	5956.07	6659.66	7402.51
3543.21	4090.38	4676.81	5302.51	5967.48	6671.72	7415.23
3552.01	4099.83	4686.91	5313.27	5978.89	6683.79	7427.95
3560.82	4109.29	4697.03	5324.04	5990.32	6695.87	7440.69
3569.64	4118.76	4707.16	5334.82	6001.76	6707.96	7453.43
3578.47	4128.25	4717.30	5345.62	6013.20	6720.06	7466.19
3587.31	4137.74	4727.45	5356.42	6024.66	6732.18	7478.96
3596.16	4147.25	4737.61	5367.24	6036.13	6744.30	7491.74
3605.03	4156.77	4747.78	5378.06	6047.61	6756.44	7504.53
3613.90	4166.30	4757.96	5388.90	6059.11	6768.58	7517.33
3622.79	4175.84	4768.16	5399.75	6070.61	6780.74	7530.14
68	**73**	**78**	**83**	**88**	**93**	**98**
3631.68	4185.39	4778.36	5410.61	6082.12	6792.91	7542.96
3640.59	4194.95	4788.58	5421.48	6093.65	6805.09	7555.80
3649.51	4204.52	4798.80	5432.36	6105.18	6817.28	7568.64
3658.43	4214.10	4809.04	5443.25	6016.73	6829.48	7581.50
3667.37	4223.70	4819.29	5454.15	6128.29	6841.69	7594.36
3676.32	4233.30	4829.55	5465.07	6139.86	6853.91	7607.24
3685.29	4292.92	4839.82	5475.99	6151.43	6866.15	7620.13
3694.26	4252.54	4850.10	5486.93	6163.02	6878.39	7633.03
3703.24	4262.18	4860.39	5497.87	6174.63	6890.65	7645.94
3712.23	4271.83	4870.70	5508.83	6186.24	6902.91	7658.86
3721.24	4281.49	4881.01	5519.80	6197.86	6915.19	7671.79
3730.25	4291.16	4891.33	5530.78	6209.49	6927.48	7684.73
69	**74**	**79**	**84**	**89**	**94**	**99**
3739.28	4300.84	4901.67	5541.77	6221.14	6939.78	7697.69
3748.32	4310.53	4912.02	5552.77	6232.79	6952.09	7710.65
3757.37	4320.24	4922.37	5563.78	6244.46	6964.41	7723.63
3766.43	4329.95	4932.74	5574.81	6256.14	6976.74	7736.61
3775.50	4339.67	4943.12	5585.84	6267.83	6989.08	7749.61
3784.58	4349.41	4953.51	5596.88	6279.53	7001.44	7762.62
3793.67	4359.16	4963.91	5607.94	6291.24	7013.80	7775.64
3802.77	4368.91	4974.32	5619.01	6302.96	7026.18	7788.68
3811.89	4378.68	4984.75	5630.08	6314.69	7038.56	7801.71
3821.01	4388.46	4995.18	5641.17	6326.43	7050.96	7814.76
3830.15	4398.25	5005.63	5652.27	6338.18	7063.37	7827.82
3839.29	4408.05	5016.08	5663.38	6349.95	7075.79	7840.90

in *dia.*, area = 0.7853981634 sq. ft.
in *dia.*, area = 0.0054541539 sq. ft.

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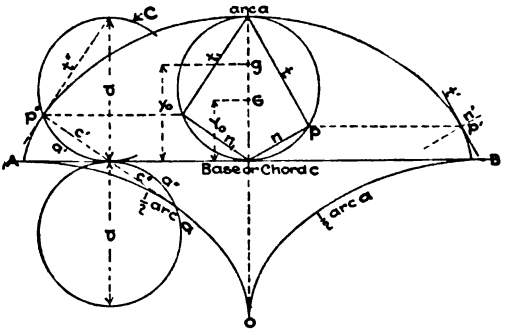

Fig. 16.

Cycloid.—If a "generating" circle C of diameter d is rolled along a straight base or chord c, any point as p'', starting from a point A, will have traced a cycloidal arc a, from A to B, when the generating circle has performed a complete revolution. Moreover, the *evolute* of the cycloid is composed of two half-arcs shown below the base and meeting at the point O. Thus, $A\,O = \frac{1}{2}$ arc a.

Properties of the Cycloid (Fig. 16):

Length of arc $a = 4\,d = 4$ times *diam.* of generating circle $= \frac{4\,c}{\pi} = 1.27324\ c$.

Length of chord $c = \pi\,d = 3.1416\,d = \frac{\pi\,a}{4} = 0.7854\ a$.

Area of cycloidal segment (*bet.* a and c) $= 3 \times$ area of generating circle $= \frac{3}{4}\pi d^2 = \frac{3}{4}\,d\,c$.

Distance y_0 from base to cen of grav g of cycloidal arc (line) $= \frac{2}{3}\,d$.

Distance Y_0 from base to cen of grav G of cycloidal segment (surface above c) $= \frac{8}{15}\,d$.

Tangent t' at point p' is parallel with t; t'' at point p'' is par with t_1.

Normal n' at point p' is parallel with n; $c'\,c''$ at point p'' is par with n_1.

Note that $a' = a'' =$ length along base from A to intersection of d with c.

The extremities of $c'\,c''$ touch the cycloidal arc above, and the evolute below.

Area contained between the base and the evolute (below c) $= \frac{1}{4}\,\pi\,d^2 = \frac{1}{4}\,d\,c$.

If the half-arc $A\,O$ is inverted it forms a curve along which a body will descend, by gravity, from O to A in the least space of time.

For Motion of Falling Bodies on the Cycloidel Curve, see page 286.
For Equation of Cycloid, see page 260.

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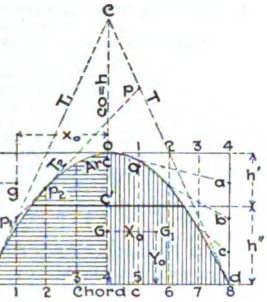

Fig. 17.

Segment; Parabolic Half-Segment; and Parabolic
n of Parabola, see page 257.)
bola (Fig. 17):

ord; $\frac{h}{c}$ may have any value desired.

$$=2h\sqrt{\frac{c^2}{16\,h^2}+1}+\frac{c^2}{8\,h}\log^*\frac{4h}{c}\left(1+\sqrt{\frac{c^2}{16\,h^2}+1}\right).\ \text{(Exact.)}$$

$$=2\sqrt{\frac{c^2}{4}+\tfrac{4}{3}h^2}.\quad\text{(Approximate†.)}$$

$h=\frac{2}{3}$ base $\times$ altitude $=\frac{2}{3}\,c\,h$.

ent of height $h'=c\sqrt{\frac{h'}{h}}$.

$$h'=\tfrac{2}{3}\,c'h'=\tfrac{2}{3}\,c\,h'\sqrt{\frac{h'}{h}}.$$

$$=\tfrac{2}{3}\,c\left(h-h'\sqrt{\frac{h'}{h}}\right).$$

$l, A\,O\,B, =\frac{1}{2}\,h\times\frac{c}{2}=\frac{1}{4}\,c\,h.$

of half segment of height $h=\frac{2}{3}\times\frac{c}{2}=\frac{2}{12}c.$

and G_1 of segment and half segment $(h)=\frac{3}{8}\,h.$

f spandril $=\frac{2}{3}\times\frac{c}{2}=\frac{2}{3}\,c.$

f spandril $=\frac{7}{10}\,h.$

at d; T_1 is tangent at A; T_2 at p_2.

$$\frac{A\,p_1}{T_1}=\frac{Cp}{T}=\frac{02}{c}.$$

ural) log = common (Briggs) log *mult* by 2.3025851,
is 0.3622157.

lues, *mult* result from *approx* formula by 0.99975
0.9972 when $\frac{h}{c}=0.2$; by 0.99 when $\frac{h}{c}=0.3$; by
0.923 when $h=c$.

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Parabola may be drawn (1) by drawing T_2 in various positions between T and T_1, varying the distance $A\,p_1 = C\,p$ and using the preceding equations for position of p_2, although the latter is not absolutely necessary.

Parabola may be drawn (2) by the method adopted for the right-hand half of arc a (Fig. 17): dividing the half-chord and the height into equal spaces (any number) and joining points of intersection of verticals with corresponding inclined lines, as $1-5$ with $O-a$, $2-6$ with $O-b$, etc.

Parabola may be drawn (3) by laying off ordinates from the base. Thus, if base is divided into 8 parts the middle ordinate $= (4)^2\,k = k$, in which k = a constant $= \frac{1}{16}\,k$. Then the ordinates at 0 and 8 are $0 \times 8k$; at 1 and 7 are $1 \times 7\,k$; at 2 and 6 are $12\,k$; at 3 and 5 are $15\,k$; at 4 is $16\,k$. It matters not how many divisions of the base are used, whether odd or even. If odd, say 11, the middle ordinate $= (5.5)^2\,k$. Also, the ordinate midway between 1 and 2 (Fig. 17) $= 1.5 \times 6.5\,k$.

16.—Lengths of Parabolic Arcs for Chord (Base) 1.

(The final figure may not be exact in some cases.)

Height div. by Chord.	Length of Arc = Chord mult. by	Height div. by Chord.	Length of Arc = Chord mult. by	Height div. by Chord.	Length of Arc = Chord mult. by
.01	1.000 267	.11	1.031 389	.21	1.107 516
.02	1.001 066	.12	1.037 171	.22	1.117 128
.03	1.002 396	.13	1.043 395	.23	1.127 053
.04	1.004 251	.14	1.050 048	.24	1.137 278
.05	1.006 627	.15	1.057 116	.25	1.147 794
.06	1.009 519	.16	1.064 587	.26	1.158 588
.07	1.012 918	.17	1.072 447	.27	1.169 651
.08	1.016 814	.18	1.080 684	.28	1.180 971
.09	1.021 199	.19	1.089 281	.29	1.192 540
.10	1.026 061	.20	1.098 230	.30	1.204 347

Calculated from the exact equation, preceding.

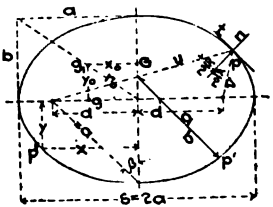

Fig. 18.

Ellipse.—The Ellipse is a flattened circle. (See Analytic Geometry, Fig. 9, page 258.)

Notation and Methods of Drawing the Ellipse (Fig. 18):

$$a = \text{semi-major axis} = \frac{d}{e} = \sqrt{b^2+d^2} = \tfrac{1}{2}\,s = \tfrac{1}{2}(u+v) = \frac{x}{\sqrt{1-\frac{y^2}{b^2}}} = \frac{bx}{\sqrt{b^2-y^2}}.$$

$$b = \text{semi-minor axis} = d\sqrt{\frac{1}{e^2}-1} = \sqrt{a^2-d^2} = \sqrt{(a+d)(a-d)} = \frac{y}{\sqrt{1-\frac{x^2}{a^2}}} = \frac{ay}{\sqrt{a^2-y^2}}$$

$$e = \text{eccentricity of ellipse} = \frac{d}{a} = \sin\beta = \frac{\sqrt{a^2-b^2}}{a} = \sqrt{1-\frac{b^2}{a^2}}$$

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$d = \frac{1}{2}$ (focal distance) $= \sqrt{a^2 - b^2} = \sqrt{(a+b)(a-b)}$.

s (= span) = major axis $= 2a = u + v = \dfrac{2d}{e} = 2\sqrt{b^2 + d^2}$.

$$\left.\begin{array}{l} x = \text{variable abscissa} = a\sqrt{1 - \dfrac{y^2}{b^2}}. \\ y = \text{variable ordinate} = b\sqrt{1 - \dfrac{x^2}{a^2}}. \end{array}\right\} \text{Sometimes used in platting } (p'') \text{ or laying out the ellipse.}$$

$u + v$ = a constant $= 2a$ (see point p); common method of drawing the ellipse.

$a - b$ = constant distance *bet* axis on line of "elliptic compass" (see pt. p').

Other Properties of the Ellipse:

Dist x_0 to cen of grav g_1 of quadrant or g of half-ellipse $= 0.42441\,a$.

Dist y_0 to cen of grav g_1 of quadrant or G of half-ellipse $= 0.42441\,b$.

Dist r_0 to cen of grav g_1 of quadrant $= \sqrt{x_0^2 + y_0^2} = 0.42441\sqrt{a^2 + b^2}$.

Line n is normal to the ellipse at point p, and bisects angle α.

Line t is tangent to the ellipse at point p, and is at $\llcorner$ with n.

Area of ellipse $= \pi a b = \dfrac{\pi b d}{e} = \dfrac{\pi}{2} s b = \dfrac{\pi}{2} b (u + v)$. ($\pi = 3.1416$.)

$= 0.7854 \times$ major axis $\times$ minor axis.

FORMULAS FOR CIRCUMFERENCE OR PERIMETER OF ELLIPSE:

Method 1.—Let l = perimeter; s = major axis $= 2a$; $e^2 = (\text{eccentricity})^2 = \dfrac{d^2}{a^2} = \dfrac{(a+b)(a-b)}{a^2}$.

Terms 1 2 3 4 5 etc.

$$\text{Then } l = \pi\left[s\left(1 - \frac{1}{2^2}e^2 - \frac{3}{2^2.4^2}e^4 - \frac{3^2.5}{2^2.4^2.6^2}e^6 - \frac{3^2.5^2.7}{2^2.4^2.6^2.8^2}e^8 - \frac{3^2.5^2.7^2.9}{2^2.4^2.6^2.8^2.10^2}e^{10} - \frac{3^2.5^2.7^2.9^2.11}{2^2.4^2.6^2.8^2.10^2.12^2}e^{12} - \ldots\right)\right] \quad (1)$$

1 2 3 4 5 6 7 etc.

$$\text{Then } l = \pi\left[s\left(1 - \frac{1}{4}e^2 - \frac{3}{64}e^4 - \frac{5}{256}e^6 - \frac{175}{16384}e^8 - \frac{441}{65536}e^{10} - \frac{4851}{1048576}e^{12} - \ldots\right)\right] \quad (2)$$

Formulas (1) and (2) may be expressed: $l = \pi[s(k)]$ . . . . . (3) in which the continued series k is a coefficient of s, making sk the diameter of a circle whose circumference = perimeter of the ellipse.

To facilitate the use of equation (2): Log $\pi = 0.4971499$; log $\frac{1}{4} = 9.3979400$; log $\frac{3}{64} = 8.6709413$; log $\frac{5}{256} = 8.2907300$; log $\frac{175}{16384} = 8.0286181$; log $\frac{441}{65536} = 7.8279587$; log $\frac{4851}{1048576} = 7.6652314$. Note that logarithms of e^4, e^6, e^8, e^{10}, e^{12} are 2, 3, 5, 6 times log e^2.

Problem 1.—The major and minor axes of an ellipse are 36 and 24 ft., respectively. Find the perimeter?

Solution.—Major axis $s = 36$; $a = 18$; $b = 12$; $e^2 = \dfrac{180}{18^2}$; log $e^2 = 9.7447275$.

Using 5-place logarithms, we have for the value of k, by terms:

1	2	3	4	5	6	7
Log $\frac{1}{4}$ =	9.39794	$\frac{3}{64}$ = 8.67094	8.29073	8.02862	7.82796	7.66523
Log e^2 =	9.74473	e^4 = 9.48946	e^6 = 9.23418	e^8 = 8.97891	e^{10} = 8.72364	e^{12} = 8.46837
Sum =	9.14267	8.16040	7.52491	7.00753	6.55160	6.13360

and numbers corresponding to above logarithms are below:

$\therefore k = 1.00000 - 0.13889 - 0.01447 - 0.00335 - 0.00102 - 0.00036 - 0.00014$ minus the sum of the value of terms above the 7th, which, by inspection, we will assume to equal 0.00007. Hence, $k = 0.8417$; and the perimeter $l = \pi s k = 3.1416 \times 36 \times 0.8417 = 95.194$ ft. *Ans.*

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Method 2.—Let l = perimeter; a = semi-major axis; b = semi-minor axis; $E=\frac{a-b}{a+b}$.

Terms: 1 2 3 4 5

$$\text{Then } l=\pi\,(a+b)\left[1+\frac{1}{2^2}E^2+\frac{1}{2^2.4^2}E^4+\frac{3^2}{2^2.4^2.6^2}E^6+\frac{3^2.5^2}{2^2.4^2.6^2.8^2}E^8\right.$$

6 7

$$\left.+\frac{3^2.5^2.7^2}{2^2.4^2.6^2.8^2.10^2}E^{10}+\frac{3^2.5^2.7^2.9^2}{2^2.4^2.6^2.8^2.10^2.12^2}E^{12}+\right] \quad (4)$$

Terms: 1 2 3 4 5

$$\text{Then } l=\pi\,(a+b)\left[1+\frac{E^2}{4}+\frac{E^4}{64}+\frac{E^6}{256}+\frac{25\,E^8}{16384}\right.$$

6 7

$$\left.+\frac{49\,E^{10}}{65536}+\frac{441\,E^{12}}{1048576}+\right] \quad (5)$$

Formulas (4) and (5) may be expressed: $l=\pi(a+b)K$ . . . . . (6) in which $(a+b)K$ is the diameter of a circle whose circumference = perimeter of the ellipse.

To facilitate the use of equation (5): Log $\pi=0.4971499$; log $\frac{1}{4}=9.397400$; log $\frac{1}{64}=8.1938200$; log $\frac{1}{256}=7.5917600$; log $\frac{25}{16384}=7.1835201$; log $\frac{49}{65536}=6.8737162$; log $\frac{441}{1048576}=6.6238387$. Note that logarithms of E^2, E^4, E^6, E^8, E^{10}, E^{12} are 2, 4, 6, 8, 10, 12 times log E.

Problem 2.—Solve problem 1 by formula (5)?

Solution.—$a=18$; $b=12$; $a+b=30$; $a-b=6$; $E=0.2$; log $E=9.3010300$. Using 5-place logarithms, we have for the value of K, by terms:

1	2	3	4	5	6	7
Log $\frac{1}{4}=9.39794$	$\frac{1}{64}=8.19382$		7.59176	7.18352	6.87372	6.62384
Log $E^2=8.60206$	$E^4=7.20412$	$E^6=5.80618$	$E^8=4.40824$	$E^{10}=3.01030$	$E^{12}=1.61236$	
Sum $=\overline{2}.00000$	$\overline{5}.39794$	$\overline{7}.39794$	$\overline{9}.59176$	$\overline{11}.88402$	$\overline{12}.23620$	

and numbers corresponding to above logarithms are below:

$\therefore K=1+0.010000+0.000025+0.00000025+-----+-----+-----$ $=1.0100253$; and the perimeter $l=\pi(a+b)K=95.193$. *Ans.*

Comparison of Methods 1 *and* 2.—A mere glance at the solution of Problems 1 and 2, illustrating the two preceding methods of calculating the perimeter of the ellipse, clearly shows the *superiority of Method* 2: The 5th, 6th, 7th, etc., terms giving values so small as to be negligible in the present instance. Moreover, equation (5), with the accompanying logarithmic values given just below it, will be found quite as rapid to use, in many cases, as many of our so-called approximate formulas, with, in addition, the advantage of accuracy.

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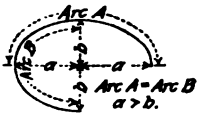
17.—Lengths of Semi-Elliptic Arcs, A or B For a = Unity, and for Successive Values of $\frac{b}{a}$.



Note.—To find A or B: Multiply values of coefficient C, in the table, by length of semi-major axis, or a. Thus, $A = B = Ca$.

[Calculated from Formula (4–5).*]

$\frac{b}{a}$	C	Diff.	$\frac{b}{a}$	C	Diff.	$\frac{b}{a}$	C	Diff.
.00	2.00000	.00061	.33	2.22395	.01064	.66	2.63494	.01396
.01	2.00061	.00132	.34	2.23459	.01078	.67	2.64890	.01403
.02	2.00193	.00201	.35	2.24537	.01092	.68	2.66293	.01409
.03	2.00394	.00263	.36	2.25629	.01106	.69	2.67702	.01416
.04	2.00657	.00314	.37	2.26735	.01119	.70	2.69118	.01423
.05	2.00971	.00363	.38	2.27854	.01132	.71	2.70541	.01429
.06	2.01334	.00406	.39	2.28986	.01145	.72	2.71970	.01435
.07	2.01740	.00448	.40	2.30131	.01157	.73	2.73405	.01441
.08	2.02188	.00487	.41	2.31288	.01169	.74	2.74846	.01447
.09	2.02675	.00523	.42	2.32457	.01181	.75	2.76293	.01454
.10	2.03198	.00559	.43	2.33638	.01193	.76	2.77747	.01459
.11	2.03757	.00592	.44	2.34831	.01204	.77	2.79206	.01465
.12	2.04349	.00622	.45	2.36035	.01214	.78	2.80671	.01470
.13	2.04971	.00653	.46	2.37249	.01226	.79	2.82141	.01476
.14	2.05624	.00681	.47	2.38475	.01235	.80	2.83617	.01481
.15	2.06305	.00709	.48	2.39710	.01246	.81	2.85098	.01486
.16	2.07014	.00735	.49	2.40956	.01255	.82	2.86584	.01492
.17	2.07749	.00760	.50	2.42211	.01266	.83	2.88076	.01497
.18	2.08509	.00784	.51	2.43477	.01275	.84	2.89573	.01502
.19	2.09293	.00807	.52	2.44752	.01284	.85	2.91075	.01507
.20	2.10100	.00831	.53	2.46036	.01293	.86	2.92582	.01512
.21	2.10931	.00851	.54	2.47329	.01303	.87	2.94094	.01517
.22	2.11782	.00873	.55	2.48632	.01311	.88	2.95611	.01521
.23	2.12655	.00893	.56	2.49943	.01319	.89	2.97132	.01526
.24	2.13548	.00913	.57	2.51262	.01328	.90	2.98658	.01531
.25	2.14461	.00931	.58	2.52590	.01336	.91	3.00189	.01535
.26	2.15392	.00950	.59	2.53926	.01344	.92	3.01724	.01539
.27	2.16342	.00967	.60	2.55270	.01352	.93	3.03263	.01544
.28	2.17309	.00985	.61	2.56622	.01360	.94	3.04807	.01549
.29	2.18294	.01002	.62	2.57982	.01367	.95	3.06356	.01552
.30	2.19296	.01017	.63	2.59349	.01374	.96	3.07908	.01557
.31	2.20313	.01034	.64	2.60723	.01382	.97	3.09465	.01561
.32	2.21347	.01048	.65	2.62105	.01389	.98	3.11026	.01564
.33	2.22395		.66	2.63494		.99	3.12590	.01569
						1.00	3.14159	

* Number of terms used in Formula (4) in calculation of this table: 50 terms for $\frac{b}{a} = 0.01$; 35, for $\frac{b}{a} = 0.06$; 20, for $\frac{b}{a} = 0.15$; 13, for $\frac{b}{a} = 0.25$; 7, for $\frac{b}{a} = 0.50$; 5, for $\frac{b}{a} = 0.75$; 4, for $\frac{b}{a} = 0.90$; 3, for $\frac{b}{a} = 0.98$; and 2 terms for $\frac{b}{a} = 0.99$.

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Elliptic Segment; and Chord.—

Let A = areas of elliptic segment with chord C_a;

B = area of elliptic segment with chord C_b;

a = semi-major axis = rad of large circle;

b = semi-minor axis = rad of small circle;

$b-b'$ = rise of segment A;

$a-a'$ = rise of segment B.

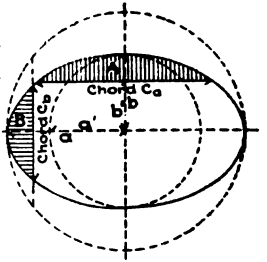

Fig. 19.

Then, length of chord $C_a = 2a\sqrt{1-\left(\frac{b'}{b}\right)^2}$;

length of chord $C_b = 2b\sqrt{1-\left(\frac{a'}{a}\right)^2}$.

Area segment A : area whole ellipse :: area *seg* small circle : area small circle.

$$\therefore A = (\text{area } seg \text{ small circle with same chord } C_a) \times \frac{a}{b} \ldots\ldots (1)$$

Area segment B : area whole ellipse :: area *seg* large circle : area large circle,

$$\therefore B = (\text{area } seg \text{ large circle with same chord } C_b) \times \frac{b}{a} \ldots\ldots (2)$$

(See Tables 7 and 8 of Circular Segments, preceding.)

Problem 1.—Find the area of segment A of the ellipse $a=10$, $b=8$, whose chord is distant $b'=5$ from and parallel with the major axis?

Solution.—Diam of small circle = 16, and middle rise k $(=b-b')$ of arc from chord $=8-5=3$. Now from Table 8, of Circular Segments, the area corresponding to $\frac{\text{rise } 3}{\text{diam } 16}$, or $.1875, = .101943$ $diam^2 = .101943 \times 4b^2$; and multiplying this value by $\frac{a}{b}$ (see Equation 1) we have,

$$\text{Area } A = .101943 \times 4\,a\,b = .101943 \times 320 = 32.622. \quad \textit{Ans.}$$

It is to be noted that area A = area B when $\frac{b'}{b} = \frac{a'}{a}$.

Problem 2.—What is the length of chord C_a of the ellipse given in Problem 1?

Solution.—From the above formula, $C_a = 2a\sqrt{1-\left(\frac{b'}{b}\right)^2} = 20\sqrt{1-\frac{25}{64}} =$ 15.612. *Ans.*

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B.—SOLIDS.

Pappus's Theorem.—If a plane curve l or area a lies wholly on one side of a straight line as axis in its own plane, the surface S or volume V generated by its whole or partial revolution about that axis is:

$S = l \times$ length of path p traversed by cen of grav g of line; or $S = lp$;
$V = a \times$ length of path P traversed by cen of grav G of area; or $V = aP$.

If x_0 = distance from axis to g, X_0 = distance from axis to G } then for one complete revolution,

$p = {}^*2\pi x_0$; $\therefore$ $S = 2\pi l x_0$, and $x_0 = S \div 2\pi l$ ..... (1)

$P = {}^*2\pi X_0$; $\therefore$ $V = 2\pi a X_0$, and $X_0 = V \div 2\pi a$ ..... (2)

Thus, equations (1) and (2) are used for finding the surfaces and volumes of the sphere, cone, cylinder, torus (cylindrical ring), paraboloid, ellipsoid, etc.; also of their sectors, segments, zones and frustums.

It is to be noted also that these equations enable us to find the centers of gravity of their lines and areas when their lines, surfaces and volumes are known.

Prismoidal Formula.—The volume V of a prismoid is equal to the length l multiplied by the mean area A; and A is equal to $\frac{1}{6}$ (sum of end areas, a_1 and a_2, + 4 times the middle area a_m); thus

$$V = lA = \frac{l}{6}(a_1 + 4a_m + a_2) \quad \ldots\ldots\ldots\ldots (3)$$

A prismoid is a solid having *parallel* end faces or areas, joined together by *regular* surfaces or sides, as the sides of prisms, cylinders, cones, pyramids, wedges, or their frustums, or any lateral combination of same. The prismoidal formula will apply also to the sphere, hemisphere and spherical segment; to warped-surface solids where the warp is continuous between ends of solid; to railroad cuttings that can be decomposed into prisms, wedges, etc.; to two equal cones arranged like an hour glass with bases as end areas; to the conical wedge bounded on one side by a plane radiating from the apex of cone; to the frustums of same; and to many other solids.

18.—The Five Regular Polyhedrons.

(All dihedral or solid angles are equal, and all faces regular polygons. Five only.)

Name.	Bounded by	Total Surface S = (1 edge)^2 times	Total Volume V = (1 edge)^3 times	Apothem a, or radius of inscribed sphere, = 1 edge times	Radius r, or radius of circumscribed sphere, = 1 edge times
Tetrahedron.....	4 △'s	1.7320508	0.1178513	0.2041	0.6124
Cube (hexahedron)	6 □'s	6.0000000	1.0000000	0.5000	0.8660
Octahedron.....	8 △'s	3.4641016	0.4714045	0.4082	0.7071
Dodecahedron....	12 ⬠'s	20.6457788	7.6631189	1.1136	1.4013
Icosahedron.....	20 △'s	8.6602540	2.1816950	0.7558	0.9511

The volume V of any regular polyhedron is equal to its surface S times one-third its apothem a; or, $V = \frac{1}{3}Sa$; $\therefore a = 3V \div S$.

* $2\pi = 6.283185$.

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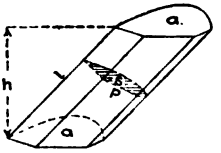

Fig. 20.

Prisms and Cylinders.—A *prism* is a solid with *parallel ends* and *parallel side edges*. Hence the ends will be equal and similar polygons (regular or irregular), and the sides will be parallelograms. A *cylinder* is a prism with an infinite number of sides. The ends of the cylinder may be circular, elliptic, or of any curvature.

Area.—The surface of any prism or cylinder, whether right or oblique, is equal to the two end areas + the perimeter p of any right section s multiplied by the length l of any lateral element: or $S = 2a + pl$ (Fig. 20).

Volume.—The volume of any prism or cylinder, whether right or oblique, is equal to the area of any right section s multiplied by the length l of any lateral element; or $V = s\,l$ (Fig. 20).

Also, volume equals area of either end multiplied by the vertical distance between the end faces; or, $V = a\,h$ (Fig. 20).

Frustum of Prism or Cylinder.—Prism or cylinder with end faces not parallel (Fig. 21).

Volume.—Let g_1 = *cen of grav* of end area a_1; g_2 of any sectional area a_2; g_3 of end area a_3. Then

$V = a_1h_1$; (h_1 = *vert dist* from g_3 to plane a_1).
$V = a_3h_3$; (h_3 = *vert dist* from g_1 to plane a_3).
$V = a_2h_2$; (h_2 is *vert* to plane a_2, *bet* g_1 and g_3.)

In general, V = area a of any plane section multiplied by the perpendicular distance h between planes passing through centers of gravity of end areas and parallel with the said plane section. If a is a right section a_0, $V = a_0l$. These formulas also enable us to find the relation between certain elements, as $a_0l = a_1h_1 = a_2h_2 = a_3h_3$.

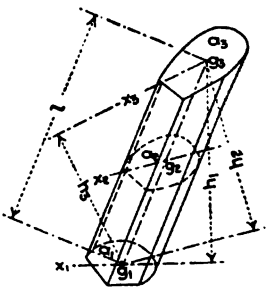

Fig. 21.

Note that Fig. 21 becomes a *circular cylindric ungula* when the right section a_0 is a circle, and hence $l = \frac{1}{2}$ (longest side + shortest side).

Circular Cylindric Frustum.—This is a special case of the preceding in which a_0 is a right circular section whose perimeter is p. Then,

Volume $V = a_0l = \frac{1}{2}\,a_0(l_1 + l_2)$;
$V = a_1h_1$; (h_1 is *perp* to plane a_1.)
$V = a_3h_3$; (h_3 is *perp* to plane a_3.)
Area $A = a_1 + a_3 + pl = a_1 + a_3 + p\,(l_1 + l_2)$.

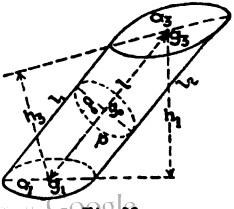

Fig. 22.

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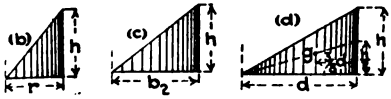

Fig. 23b. Fig. 23c. Fig. 23d.

Half-Wedges. — The following formulas give the lf-wedges cut from circular cylinders; h being the easured along the element of the cylinder at a_1. , (b), (c) and (d)—as follows:—

ess than radius r; lower edge c_1.

rea of base at b_1) $(r-b_1)$]. (Fig. 23a)

$= \frac{h}{b_1}$ [$c_1 r$ − (length of arc a_1) $(r-b_1)$]. (Fig. 23a)

radius of cylinder; lower edge $= d$.

(Fig. 23b)

$= 2rh = dh$. (Fig. 23b)

$> r$ and $<$ diameter d; lower edge $= c_2$.

rea of base at b_2) (b_2-r)]. (Fig. 23c)

$= \frac{h}{b_2}$ [$c_2 r$ + ($2\pi r$ = arc a_2) (b_2-r)]. (Fig. 23c)

diameter of cylinder; lower edge at a_2.

(area of circular base). (Fig. 23d)

$= \pi rh$. (Fig. 23d)

whether figure is right or oblique. (Fig. 23d)

r *right or oblique* figure, h being *perp* height.
face, for *right* figure only. For total surface, add d base (circular).

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19.—Properties of Hollow Cylinders (Pipes, Tanks or Wells), One Foot in Length.

Note that Areas, Volumes, Capacities and Weights are proportional to the *squares* of the diameters. 1728 cu. ins. = 7.4805 gallons = 1 cu. ft. = 62.5 lbs. (nearly) of water; 231 cu. ins. = 1 gallon; 201.974 gallons = 1 cu. yd.

Diam. Ins.	Diam. Ft.	Hydraulic Mean Radius $= \frac{d}{4}$.	Circum. Ins.	Circum. = Surface. Ft.	Volume. Cu. Ins.	Area = Volume. Ft.	Volume. Cu. Yds.	Capacity Gallons.	Weight of Water, Pounds.
⅛	.0104	.0026	.392699	.032725	.147262	.000085	.000003	.00064	.00533
3-16	.0156	.0039	.589049	.049087	.331340	.000192	.000007	.00143	.01198
¼	.0208	.0052	.785398	.065450	.589049	.000341	.000013	.00255	.02131
⅜	.0312	.0078	1.17810	.098175	1.32536	.000767	.000028	.00574	.04794
½	.0417	.0104	1.57080	.130900	2.35619	.001364	.000051	.01020	.08522
⅝	.0521	.0130	1.96350	.163625	3.68155	.002131	.000079	.01594	.13316
¾	.0625	.0156	2.35619	.196350	5.30144	.003068	.000114	.02295	.19175
⅞	.0729	.0182	2.74889	.229074	7.21585	.004176	.000155	.03124	.26099
1	.0833	.0208	3.14159	.261799	9.42478	.005454	.000202	.04080	.34088
1¼	.1042	.0261	3.92699	.327249	14.7262	.008523	.000316	.06375	.53263
1½	.1250	.0312	4.71239	.392699	21.2058	.012272	.000455	.09180	.76699
1¾	.1458	.0365	5.49779	.458149	28.8634	.016703	.000619	.12495	1.04396
2	.1667	.0417	6.28319	.523599	37.6991	.021817	.000808	.16320	1.3635
2¼	.1875	.0469	7.06858	.589049	47.7129	.027612	.001023	.20655	1.7257
2½	.2083	.0521	7.85398	.654498	58.9049	.034088	.001263	.25500	2.1305
2¾	.2292	.0573	8.63938	.719948	71.2749	.041247	.001528	.30855	2.5779
3	.2500	.0625	9.42478	.785398	84.8230	.049087	.001818	.36720	3.0680
3½	.2917	.0729	10.9956	.916298	115.454	.066813	.002475	.49980	4.1759
4	.3333	.0833	12.5664	1.04720	150.796	.087266	.003232	.65280	5.4541
4½	.3750	.0937	14.1372	1.17810	190.852	.110447	.004091	.82620	6.9029
5	.4167	.1042	15.7080	1.30900	235.619	.136354	.005050	1.02000	8.5221
5½	.4583	.1146	17.2788	1.43990	285.100	.164988	.006111	1.2342	10.312
6	.5	.1250	18.8496	1.57080	339.292	.196350	.007272	1.4688	12.272
6½	.5417	.1354	20.4204	1.70170	398.197	.230438	.008535	1.7238	14.402
7	.5833	.1458	21.9911	1.83260	461.814	.267254	.009898	1.9992	16.703
7½	.6250	.1562	23.5619	1.96350	530.144	.306796	.011363	2.2950	19.175
8	.6667	.1667	25.1327	2.09440	603.186	.349066	.012928	2.6112	21.817
8½	.7083	.1771	26.7035	2.22529	680.940	.394063	.014595	2.9478	24.629
9	.7500	.1875	28.2743	2.35619	763.407	.441786	.016362	3.3048	27.612
9½	.7917	.1979	29.8451	2.48709	850.586	.492237	.018331	3.6822	30.765
10	.8333	.2083	31.4159	2.61799	942.478	.545415	.020201	4.0800	34.088
10½	.8750	.2187	32.9867	2.74889	1039.08	.601320	.022271	4.4982	37.582
11	.9167	.2292	34.5575	2.87979	1140.40	.659953	.024443	4.9368	41.247
11½	.9583	.2396	36.1283	3.01069	1246.43	.721312	.026715	5.3958	45.082
12	1.	.25	37.6991	3.14159	1357.17	.785398	.029089	5.8752	49.087
13	1.0833	.2708	40.8407	3.40339	1592.79	.921752	.034139	6.8952	57.609
14	1.1667	.2917	43.9823	3.66519	1847.26	1.06901	.03959	7.9968	66.813
15	1.2500	.3125	47.1239	3.92699	2120.58	1.22718	.04545	9.1800	76.699
16	1.3333	.3333	50.2655	4.18879	2412.74	1.29626	.05171	10.445	87.266
17	1.4167	.3542	53.4071	4.45059	2723.76	1.57625	.05838	11.791	98.516
18	1.5	.375	56.5487	4.71239	3053.63	1.76715	.06545	13.219	110.45
19	1.5833	.3958	59.6903	4.97419	3402.34	1.96895	.07292	14.729	123.06
20	1.6667	.4167	62.8319	5.23599	3769.91	2.18166	.08080	16.320	136.35
22	1.8333	.4583	69.1150	5.75959	4561.59	2.63981	.09777	19.747	164.99
24	2.	.5	75.3982	6.28319	5428.67	3.14159	.11636	23.501	196.35

The Circumference is proportional to the Diameter.

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s OF HOLLOW CYLINDERS.—Concluded.

Circum. = Surface. Ft.	Volume. Cu. Ins.	Area = Volume. Ft.	Volume. Cu. Yds.	Capacity Gallons.	Weight of Water. Pounds.
6.80678	6371.15	3.68701	.13656	27.581	230.44
7.33038	7389.03	4.27606	.15837	31.987	267.25
7.85398	8482.30	4.90874	.18181	36.720	306.80
8.37758	9650.97	5.58505	.20685	41.779	349.07
8.90118	10895.0	6.30500	.23352	47.165	394.06
9.42478	12214.5	7.06858	.26180	52.877	441.79
9.94838	14609.4	7.87580	.29170	63.244	492.24
10.4720	15079.6	8.72665	.32321	65.280	545.42
10.9956	16625.3	9.62113	.35634	71.971	601.32
11.5192	18246.4	10.5592	.39108	78.989	659.95
12.0428	19942.8	11.5410	.42744	86.333	721.31
12.5664	21714.7	12.5664	.46542	94.003	785.40
13.0900	23561.9	13.6354	.50501	102.000	852.21
13.6136	25484.6	14.7480	.54622	110.32	921.75
14.1372	27482.7	15.9043	.58905	118.97	994.02
14.6608	29556.1	17.1042	.63349	127.95	1069.0
15.7080	33929.2	19.6350	.72722	146.88	1227.2
17.2788	41054.3	23.7583	.87994	177.72	1484.9
18.8496	48858.0	28.2743	1.0472	211.51	1767.1
20.4204	57340.3	33.1831	1.2290	248.23	2073.9
21.9911	66501.2	38.4845	1.4254	287.88	2405.3
23.5619	76340.7	44.1786	1.6362	330.48	2761.2
25.1327	86858.8	50.2655	1.8617	376.01	3141.6
26.1799	94247.8	54.5415	2.0201	408.00	3408.8
28.2743	109931.	63.6173	2.3562	475.89	3976.1
31.4159	135717.	78.5398	2.9089	587.52	4908.7
34.5575	164217.	95.0332	3.5197	710.90	5939.6
37.6991	195432.	113.097	4.1888	846.03	7068.6
40.8407	229361.	132.732	4.9160	992.91	8295.8
43.9823	266005.	153.938	5.7014	1151.5	9621.1
47.1239	305363.	176.715	6.5450	1028.2	11045.
50.2655	357435.	201.062	7.4467	1547.3	12566.
53.4071	392222.	226.980	8.4067	1697.9	14186.
56.5487	439722.	254.469	9.4248	1903.6	15904.
59.6903	525938.	283.529	10.501	2276.8	17721.
62.8319	542867.	314.159	11.636	2350.1	19635.
69.1150	656869.	380.133	14.079	2843.6	23758.
75.3982	781729.	452.389	16.755	3384.1	28274.
78.5398	848230.	490.874	18.181	3672.0	30680.
81.6814	917446.	530.929	19.664	3971.6	33183.
87.9646	.....	615.752	22.806	4606.1	38484.
94.2478	.....	706.858	26.180	5287.7	44179.
100.531	.....	804.248	29.787	6116.2	50265.
106.814	.....	907.920	33.627	6791.7	56745.
113.097	.....	1017.88	37.699	7614.3	63617.
119.381	.....	1134.11	42.004	8483.7	70882.
125.664	.....	1256.64	46.542	9400.3	78540.
131.947	.....	1385.44	51.313	10364.	86590.
138.230	.....	1520.53	56.316	11374.	95033.
144.513	.....	1661.90	61.552	12432.	103869.
150.796	.....	1809.56	67.021	13536.	113097.
157.080	.....	1963.50	72.722	14688.	122719.

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Pyramid and Pyramidic Frustum.—A "regular" pyramid is one in which the base is a regular polygon; if not, it is "irregular." If the *axis*, from the *apex* to the cen of grav *g* of the base, is perp to the base it is a "right" pyramid; if not, it is "oblique." Fig. 24 shows an oblique pyramid and frustum together forming a right pyramid.

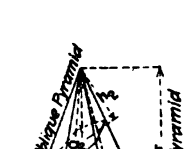

Fig. 24.

Volume V_r of right pyramid $= \frac{1}{3}$ (area of base $\times$ perp height) $= \frac{1}{3} a_1 h_1$.

Volume V_o of oblique pyramid $= \frac{1}{3}$ (area of base $\times$ perp height) $= \frac{1}{3} a_2 h_2$.

Volume V_f of pyramidic frustum $= V_r - V_o = \frac{1}{3}(a_1 h_1 - a_2 h_2)$.

If a_2 is parallel with a_1, applying the prismoidal formula, volume V_f of frustum $= \frac{h_1 - h_2}{3}\left(a_1 + a_2 + \sqrt{a_1 a_2}\right)$, whether pyramid is right or oblique, regular or irregular.

The *area* of the *sides* of a right regular pyramid $= \frac{1}{2}$ (perimeter of base $\times$ least slant height); of a right regular frustum with parallel faces $= \frac{1}{2}$ (perimeter of top + perimeter of base) $\times$ least slant height.

For area of an oblique or irregular pyramid or frustum, the sides must be calculated—as triangles, or as trapezoids or trapeziums, respectively. No simple general formula will apply.

Center of gravity of pyramid, whether right of oblique, lies in the axis, and one-fourth its length from the base.

Cone and Conic Frustum.— A cone may be considered as a pyramid with an infinite number of sides. If the axis from the *apex* to the cen of grav *g* of the base, is perp to the base it is a "right" cone; if not, it is "oblique." Generally speaking, a right cone is understood to have a circular base; and an oblique cone, to have an elliptic base. Such cones, however, are sometimes termed right- and oblique circular cones, to distinguish them from right- and oblique elliptic cones whose base of right cone is an ellipse. Note that an oblique circular cone may be cut from a right elliptic cone; or that an oblique elliptic cone may be cut from either a circular or an elliptic cone. Fig. 25 shows an oblique cone and frustum together forming a right cone.

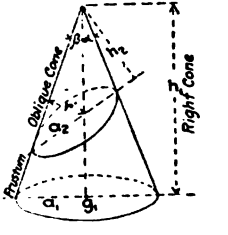

Fig. 25.

Volume V_r of *right* cone $= \frac{1}{3}$ (area of base $\times$ perp height) $= \frac{1}{3} a_1 h_1$.

Volume V_o of *oblique* cone $= \frac{1}{3}$ (area of base $\times$ perp height) $= \frac{1}{3} a_2 h_2$.

Volume V_f of conic *frustum* $= V_r - V_o = \frac{1}{3}(a_1 h_1 - a_2 h_2)$.

If a_2 is parallel with a_1, applying the prismoidal formula, volume V_f of frustum $= \frac{h_1 - h_2}{3}\left(a_1 + a_2 + \sqrt{a_1 a_2}\right)$.*

The *area* of the curved surface (side) of a *right cone* $= \frac{1}{2}$ (perim of base $\times$ slant height); of a *right frustum* with parallel faces $= \frac{1}{2}$(perim of top + perim of base) $\times$ slant height.

---

* If the bottom and top faces of the frustum are circles with radii r_1 and r_2, respectively, then $a_1 = \pi r_1^2$ and $a_2 = \pi r_2^2$. If they are elliptical, use the formula: area of ellipse $= \pi ab$, in which a = semi-major axis and b = semi-minor axis. $\pi = 3.1416$.

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The *area* of the curved surface (side) of an *oblique elliptic cone* of height h_2 (Fig. 25), and which has been *cut from a right circular cone*, $= \frac{a_2 h_2}{r'}$, in which h_2 = perp height, a_2 = area of elliptic base, and $r' = h_2 \frac{\sin \beta}{\cos \alpha}$ = perp dist from side of right circular cone to point where axis of same pierces base a_2 of oblique cone. Hence, area $= \frac{a_2 h_2}{r'} = \frac{a_2 \cos \alpha}{\sin \beta}$. Also, area $= \frac{3}{r'}$ (volume of oblique cone) $= \frac{3V_o}{r'}$.

Center of gravity of cone, whether right or oblique, lies in the axis and one-fourth its length from the base.

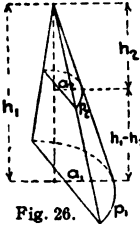

Fig. 26.

Conic Wedge and Frustum from right cone.—If the wedge is cut from the cone by a plane passing through the apex (Fig. 26), with a_2 parallel to a_1, we have,

$V_1 = \frac{1}{3} a_1 h_1$; $A_1 = \frac{1}{2} h_1 p_1$. ($p_1$ = perim of a_1.)
$V_2 = \frac{1}{3} a_2 h_2$; $A_2 = \frac{1}{2} h_2 p_2$. ($p_2$ = perim of a_2.)
Volume of frustum $= V_1 - V_2 = \frac{1}{3}(a_1 h_1 - a_2 h_2)$.
Area of frustum $= A_1 - A_2 = \frac{1}{2}(h_1 p_1 - h_2 p_2)$; (ends not included).

Volume of frustum may also be derived from the prismoidal formula;

thus, $V_f = \frac{h_1 - h_2}{6}(a_1 + a_2 + 4a_m)$ in which a_m, the area of the middle section, $= \frac{1}{2}\sqrt{a_1 a_2} + \frac{1}{2}(a_1 + a_2)$; or, $V_f = \frac{h_1 - h_2}{3}\left(a_1 + a_2 + \sqrt{a_1 a_2}\right)$.

Note.—The above formulas for *volume*, V_1, V_2 and V_f, will apply also if the cone is oblique. For an oblique circular cone the above formulas for *area* will not apply; but see formulas for area of oblique elliptic cone above.

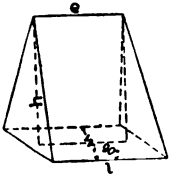

Fig. 27.

Wedge.—In Fig. 27 let w = width, and l = length of base, which must be a parallelogram but not necessarily a rectangle; let h be the perp height of cutting edge above the base; and e the length of cutting edge, which must be parallel with l. Then volume $V = \frac{wh}{6}(2l + e)$. The two ends and two faces of height h may slope in any direction; the two ends need not be parallel.

Sphere.—A sphere is generated by the complete revolution of a semi circle (Fig. 28) about its diameter. The *volume* of the sphere is equal to the area of the semicircle X the path described by its center of gravity G; the *area of the surface* of the sphere is equal to the length of semi-circular arc X the path described by its center of gravity g. Any section of a sphere cut by a plane is a circle; and if the plane cuts the center, it is a great circle. The axis of a sphere always lies in the plane of a great circle. The ratio of volume to surface of a sphere is greater than that of any other solid.

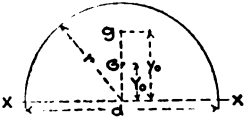

Fig. 28.

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Distance Y_0 *to* G *of semi-circle* (*area*) $= \frac{4}{3} \cdot \frac{r}{\pi} = 0.42441\ 31816\, r.$

Distance y_0 *to* g *of semi-circular arc* $= \frac{2r}{\pi} = 0.63661\ 97724\, r.$

Area of semicircle $= \frac{\pi r^2}{2} = 1.57079\ 63268\, r^2.$

Length of semi-circular arc $= \pi r = 3.14159\ 26536\, r.$

The following are some of the relations of Volume, Surface, Area of Great Cicrle, Circumference, Diameter and Radius, of the sphere:

Volume of Sphere $= \frac{4}{3}\pi$ radius$^3 = 4.18879$ radius$^3 = \frac{\pi}{6}$ diam^3

$= 0.52360$ diam$^3 = \frac{1}{6\pi^2}$ circum$^3 = 0.016887$ circum^3

$= \frac{4}{3}$ radius $\times$ area great circle $= \frac{2}{3}$ diam $\times$ area great circle
$= \frac{1}{3}$ radius $\times$ surface of sphere $= \frac{1}{6}$ diam $\times$ surface of sphere

$= \frac{\pi\sqrt{3}}{2}$ ($= 2.161128$) volume of inscribed cube*

$= \frac{\pi}{6}$ ($= 0.52360$) volume of circumscribing cube

$= \sqrt{3}$ ($= 1.732051$) volume of maximum inscribed cylinder†
$= \frac{3}{4}$ volume of circumscribing cylinder.

Volumes of spheres are as the *cubes* of their radii or their diameters.

Surface of Sphere $= 4\pi$ radius$^2 = 12.56637$ radius$^2 = \pi$ diam^2

$= 3.14159$ diam$^2 = \frac{1}{\pi}$ circum$^2 = 0.31831$ circum$^2 =$ diam $\times$ circum

$= 4 \times$ area great circle $=$ area of circle whose radius is equal to diam of sphere $= 3 \times$ volume $\div$ radius of sphere.

$= \frac{\pi}{2}$ ($= 1.57080$) area of surface of inscribed cube

$= \frac{\pi}{6}$ ($= 0.52360$) area of surface of circumscribing cube

$= 4 \times$ area of convex surface of inscribed cylinder of *max* convex surface‡
$=$ convex surface of circumscribing cylinder

$= 3\sqrt{3}$ ($= 5.19615$) convex surface of inscribed cone of *max* convex surface¶.

Surfaces of spheres are as the *squares* of their radii or their diameters.

Area of Great Circle of Sphere $= \pi$ radius$^2 = 3.141593$ radius$^2 = \frac{\pi}{4}$ diam$^2 =$ 0.785398 diam$^2 = \frac{r}{2}$ circum of sphere $= \frac{1}{4}$ area of surface of sphere $= \frac{3}{2}$ volume of sphere $\div$ diam.

---

* Edge of inscribed cube $= \frac{1}{\sqrt{3}}$ ($= 0.57735$) diam of sphere.

† Altitude of inscribed cylinder $= \frac{2}{3}\sqrt{3}$ ($= 1.15470$) rad of sphere;
Diam of base of inscribed cylinder $= 2\sqrt{\frac{2}{3}}$ ($= 1.63299$) rad of sphere.

‡ Altitude of inscribed cylinder $=$ diam of base $= \sqrt{2}$ radius of sphere.

¶ Height of inscribed cone $= \frac{2}{3}$ diam of sphere; diam of base of inscribed cone $= \frac{1}{3}\sqrt{2}$ ($= 0.4714$) diam of sphere.

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Circumference of Sphere $= 2\pi$ radius $= 6.283185$ radius $= \pi$ diam $= 3.141593$ diam = (area of surface of sphere) ÷ diam $= \sqrt{\pi}$ surface of sphere $=$ 1.77245 $\sqrt{\text{surface of sphere}} = \sqrt[3]{6\pi^2}$ (= 3.89778) $\sqrt[3]{\text{volume of sphere}} =$ $\sqrt[3]{59.21763}$ volume of sphere.

Diameter of Sphere $= 2$ radii $= \frac{1}{\pi}$ circum $= 0.318310$ circum

$= \frac{2}{\sqrt{\pi}}$ (= 1.12838) $\sqrt{\text{area great circle}} = \sqrt{1.273240 \text{ area great circle}}$

$= \sqrt{\frac{1}{\pi}}$ (= 0.56419) $\sqrt{\text{surface of sphere}} = \sqrt{0.318310 \text{ surface of sphere}}$

$= \sqrt[3]{\frac{6}{\pi}}$ (= 1.240701) $\sqrt[3]{\text{volume of sphere}} = \sqrt[3]{1.909859 \text{ volume of sphere}}$.

Radius of Sphere $= \frac{1}{2}$ diam $= \frac{1}{2\pi}$ circum $= 0.159155$ circum

$= \sqrt{\frac{1}{\pi}}$ (= 0.56419) $\sqrt{\text{area great circle}} = \sqrt{0.318310 \text{ area great circle}}$

$= \frac{1}{2}\sqrt{\frac{1}{\pi}}$ (= 0.282095) $\sqrt{\text{surface of sphere}} = \sqrt{0.079577 \text{ surface of sphere}}$

$= \sqrt[3]{\frac{3}{4\pi}}$ (= 0.620350) $\sqrt[3]{\text{volume of sphere}} = \sqrt[3]{0.238732 \text{ volume of sphere}}$.

20.—Areas of the Surfaces of Spheres.

(Surfaces of spheres are proportional to the *squares* of their diameters.)

Diam.	Area Surface	Logarithm	Diam.	Area Surface	Logarithm
1/64	0.000 766 99	6.884 7899	1/64 · 1/12	0.000 005 326 322	4.726 4274
1/32	0.003 067 96	7.486 8499	1/32 · 1/12	0.000 021 305 29	5.328 4874
1/16	0.012 271 85	8.088 9099	1/16 · 1/12	0.000 085 221 16	5.930 5474
1/8	0.049 0874	8.690 9699	1/8 · 1/12	0.000 340 8846	6.532 6074
1/4	0.196 350	9.293 0299	1/4 · 1/12	0.001 363 538	7.134 6674
1/2	0.785 398	9.895 0899	1/2 · 1/12	0.005 454 154	9.736 7274
1	3.141 593	0.497 1499	1/12	0.021 816 616	8.338 7874
0.1	0.031 415 93	8.497 1499	2/12	0.087 266 46	8.940 8474
0.2	0.125 6637	9.099 2099	3/12	0.196 349 55	9.293 0299
0.3	0.282 7433	9.451 3924	4/12	0.349 0659	9.542 9074
0.4	0.502 6548	9.701 2699	5/12	0.545 4154	9.736 7274
0.5	0.785 398	9.895 0899	6/12	0.785 398	9.895 0899
0.6	1.130 973	0.053 4524	7/12	1.069 014	0.028 9835
0.7	1.539 380	0.187 3460	8/12	0.396 264	0.144 9674
0.8	2.010 619	0.303 3299	9/12	1.767 146	0.247 2724
0.9	2.544 690	0.405 6349	10/12	2.181 662	0.338 7874
1.0	3.141 593	0.497 1499	11/12	2.639 811	0.421 5728

Ex.—Surface of sphere $\frac{1}{64}$ in. in diam $= 0.0_2 76699$ sq. in.; therefore, surface of sphere $\frac{5}{64}$ in. in diam $=$ $0.0_2 76699 \times 5^2 = 0.019175$ sq. in.

Ex.—Surface of sphere 7 units in diameter $= 1.539380 \times 10^2 = 153.9380$.

Ex.—Surface of sphere $\frac{1}{2}$ in. in diam $= 0.0_3 3408846$ sq. ft.; therefore, surface of sphere $\frac{5}{8}$ ins. in diam $=$ $0.0_3 3408846 \times 50^2 = 0.8522115$ sq. ft.

Ex.—Surface of sphere 11 ins. in diameter $= 2.639811$ sq. ft.

Remarks.—For Surfaces of Spheres of various diameters, see—

(a) Foot note to Table 11, of Circles, with diameters in decimals.
(b) Foot-note to Table 12, of Circles, with diameters in 8ths and 12ths.
(c) Foot-note to Table 13, of Circles, with diameters in inches and fractions.
(d) Foot-note to Table 14, of Circles, with diameters in decimals.
(e) Foot-note to Table 15, of Circles, with diameters in feet and inches.

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21.—Volumes of Spheres

(Volumes of spheres are proportional to the *cubes* of their diameters.)

Diam.	Volume	Logarithm	Diam.	Volume	Logarithm
$\frac{1}{64}$	0.000 001 997 37	4.300 4586	$\frac{1}{64}\cdot\frac{1}{16}$	0.000 000 001 155 885	1.062 9149
$\frac{1}{32}$	0.000 015 978 96	5.203 5486	$\frac{1}{32}\cdot\frac{1}{16}$	0.000 000 009 247 085	1.966 0049
$\frac{1}{16}$	0.000 127 8317	6.106 6386	$\frac{1}{16}\cdot\frac{1}{16}$	0.000 000 073 9767	2.869 0049
$\frac{1}{8}$	0.001 022 654	7.009 7286	$\frac{1}{8}\cdot\frac{1}{16}$	0.000 000 591 8136	3.772 1849
$\frac{1}{4}$	0.008 181 23	7.912 8186	$\frac{1}{4}\cdot\frac{1}{16}$	0.000 004 734 509	4.675 2749
$\frac{1}{2}$	0.065 449 84	8.815 9086	$\frac{1}{2}\cdot\frac{1}{16}$	0.000 037 876 07	5.578 3649
1	0.523 5988	9.718 9986	$\frac{1}{16}$	0.000 303 008 54	6.481 4549
0.1	0.000 523 5988	6.718 9986	$\frac{2}{16}$	0.002 424 069	7.384 5449
0.2	0.004 188 790	7.622 0886	$\frac{3}{16}$	0.008 181 232	7.912 8187
0.3	0.014 137 17	8.150 3624	$\frac{4}{16}$	0.019 392 55	8.287 6349
0.4	0.033 510 32	8.525 1786	$\frac{5}{16}$	0.037 876 07	8.578 3649
0.5	0.065 449 84	8.815 9086	$\frac{6}{16}$	0.065 449 84	8.815 9086
0.6	0.113 0973	9.053 4524	$\frac{7}{16}$	0.103 931 94	9.016 7490
0.7	0.179 5944	9.254 2927	$\frac{8}{16}$	0.155 1404	9.190 7249
0.8	0.268 0826	9.428 2686	$\frac{9}{16}$	0.220 8932	9.344 1824
0.9	0.381 7017	9.581 7241	$\frac{10}{16}$	0.303 0085	9.481 4549
1.0	0.523 5988	9.718 9986	$\frac{11}{16}$	0.403 3044	9.605 6330

Ex.—Volume of sphere $\frac{5}{16}$ in. in diam $= 0.0_5199737$ cu. in.; therefore, volume of sphere $\frac{5}{64}$ in. in diam $= 0.0_5199737 \times 5^3 = 0.00024967$ cu. in.

Ex.—Volume of sphere 7 units in diameter $= 0.1795944 \times 10^3 = 179.5944$.

Ex.—Volume of sphere $\frac{1}{8}$ in. in diam $= 0.0_65918136$ cu. ft.; therefore, volume of sphere $\frac{40}{8}$ ins. in diam $= 0.0_65918136 \times 50^3 = 0.0739767$ cu. ft.

Ex.—Volume of sphere 11 ins. in diameter $= 0.4033044$ cu. ft.

Remarks.—For Volumes of spheres of various diameters, see—

(a) Foot note to Table 13, of Circles, with diameters in inches and fractions
(b) Foot-note to Table 14, of Circles, with diameters in decimals.
(c) Foot-note to Table 15, of Circles, with diameters in feet and inches.

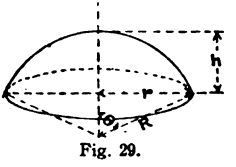

Fig. 29.

Spherical Segment.—

Let a = area of base of segment $= \pi r^2$;
A = area of great circle $= \pi R^2$;
h = height of segment.

Then $r = R \sin \theta$; $h = R \text{ vers } \theta = r \tan \frac{1}{2} \theta$.

Volume of segment $= \frac{h}{2}\left(a + \frac{\pi h^2}{3}\right) = \frac{\pi h}{6}(3r^2 + h^2) = 0.52360\, h\,(3r^2 + h^2)$

$= \pi h^2 \left(R - \frac{h}{3}\right) = 3.14159\, h^2 \left(R - \frac{h}{3}\right)$.

Convex Surface of segment $= 2\pi Rh = \frac{3h}{2R^2}$ volume of sphere $= \frac{h}{2R}$ surface of sphere $= \frac{2h}{R}$ area great circle $= \frac{2hA}{R} = h \times$ circumference of great circle. It is thus seen that area of convex surface is directly proportional to height h of segment. Hence, from Table of Spheres find the total surface of the sphere for *diam* $= 2R$, and multiply this result by $\frac{h}{2R}$; or, better, from Table 11 or 12, of Circles, multiply the circumference of its great circle by h.

Area of Base of segment $= \pi r^2$.

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Spherical Zone.—*Volume* of zone = volume of sphere *minus* volume of segments (see Spherical Segment, page 252) $= \frac{\pi}{2} H \left(\frac{H^2}{3} + r_1^2 + r_2^2\right) =$ $1.570796\ H \left(\frac{H^2}{3} + r_1^2 + r_2^2\right)$.

Convex Surface of zone = surface of sphere *minus* convex surface of segments (see Spherical Segment, page 252) $= 2\pi RH = H \times$ circum of great circle.

Areas of Bases $= \pi r_1^2$ and πr_2^2.

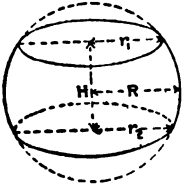

Fig. 30.

Hollow Sphere.—Let R = radius to outside surface, and r = radius to inside surface. Then,

Volume $= \frac{4}{3}\pi (R^3 - r^3) = 4.18879\,(R^3 - r^3)$. May also be taken from Table of Spheres, by deducting volume of the lesser from volume of greater sphere.

Surface = convex surface + concave surface $= 4\pi (R^2 + r^2) = 12.56637$ *times* $(R^2 + r^2)$. May also be taken from Table of Spheres, by adding together the surfaces of the larger and the smaller spheres.

Circular Segmental Ring.—Let g be the cen of grav of area A of segmental section, with o' as a center, forming a ring about the axis $X-X$. Also let $y_0 + d$ = radial distance from said axis to center of grav g of segment. Then,

Volume of ring $= 2\pi A\,(y_0 + d)$. For values of y_0 and A see Fig. 11, also Tables 1 and 8 of Circular Segments.

Note that when $d = 0$, Fig. 31 will represent a sphere with a hollow cylindrical core, points o' and o'' being identical with o. The above formula holds true for *any* section A, provided that $y_0 + d$ = *dist* to cen of grav g.

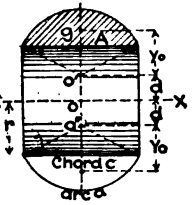

Fig. 31.

Convex Surface of ring $= 2\pi a\,(Y_0 + d)$, in which a = length of circular arc, and $Y_0 + d$ = *dist* to *cen of grav* of arc. (See Fig. 9; also Tables 2, 3 and 4, of Circular Arcs.)

Concave Surface of ring $= 2\pi rc$, in which c = chord of arc = width of ring. (See Fig. 9; also Tables 2 and 3 of Circular Arcs.)

Regular Circular Ring.—For any circular ring where section $s-s$ is a regular polygon, circle, ellipse, etc., so that center of grav of area A and of perimeter P of section lies midway between outer and inner circumferences Fig. 32, we have,

Volume of ring $= \frac{\pi}{2}\,(D + d) \times$ area A of section $s-s$.

Surface of ring $= \frac{\pi}{2}\,(D + d) \times$ perimeter P of section $s-s$.

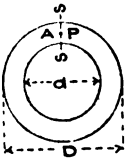

Fig. 32.

Circular Spindle—Generated by revolving circular arc a or segment A about an axis, as $X-X$.

Volume $= 3.14159 \left[\frac{c^3}{6} - A\left(\frac{c^2}{4h} - h\right)\right]$

$= 3.14159 \left(\frac{c^3}{6} - 2Ad\right)$.

Surface $= 3.14159 \left[h\,(a + c) - \frac{c^2}{4h}\,(a - c)\right]$

$= 6.283185\,(cr - ad) = 2\pi(cr - ad)$.

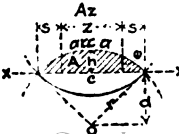

Fig. 33.

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Middle Zone (and *Frustum*) of circular spindle.—Let s (Fig. 33) be height of middle zone; then, if A_z = area of zone of segment,

$$Volume = 3.14159 \left[\frac{s}{4}\left(c^2 - \frac{s^2}{3}\right) - 2A_z d\right].$$

Convex Surface = 6.283185 × length of "zone-arc" a_z × *dist* from axis $X-X$ to cen of grav of a_z.

Segment of circular spindle.—Let s (Fig. 33) be height of segment; then, *Volume* = ½ (vol of spindle *minus* volume of middle frustum).

Convex Surface = ½ (surf of spindle *minus* convex surface of middle zone).

Parabolic Spindle.—Generated by revolving parabolic arc a (assume same Fig. 33) or segment A, about axis $X-X$.

Volume = $\frac{8}{15}\pi ch^2 = 1.675516\, ch^2 = \frac{8}{15}$ volume of circumscribing cylinder.

Convex Surface = 6.283185 × length of arc a × dist from axis $X-X$ to cen of grav of arc.

Middle Zone (and *Frustum*) of parabolic spindle.—Let s (Fig. 33) be height of middle zone, assuming arc a to be parabolic. Then, as $\frac{\pi}{15} = 0.209440$,

Volume = $0.209440\, s[8h^2 + 3s^2 + 4hs]$. Note that $s = h\left(1 - \frac{s}{c}\right)$.

(The above formula may be used in determining the contents of casks whose staves are parabolic.)

Convex Surface = 6.283185 × length of "zone-arc" a_z × dist from axis $X-X$ to cen of grav of a_z.

Segment of parabolic spindle.—Let s (Fig. 33) be height of segment; then, *Volume* = ½ (volume of spindle *minus* volume of middle frustum).

Convex Surface = ½ (surface of spindle *minus* convex surface of middle zone).

Cycloidal Spindle.—Generated by revolving cycloidal arc a (assume same Fig. 33) or segment A about axis $X-X$.

Volume = $\frac{5}{8}\pi^2 h^2 = 6\ 16850h^2 = \frac{5}{16\pi}c^2 = 0.198944c^2 = \frac{5\pi^2}{512}a^2 = 0.09638\, a^2 = \frac{5}{8}\pi\, ch^2 = 1.96350\, ch^2 = \frac{5}{8}$ volume of circumscribing cylinder. Note that $h = \frac{c}{\pi} = \frac{a}{4}$.

Convex Surface = $\frac{16\pi}{3}h^2 = 16.75516h^2 = \frac{16}{3\pi}c^2 = 1.69765c^2 = \frac{\pi}{3}a^2 = 1.04720\, a^2 = \frac{16}{3}ch$.

Paraboloid or Parabolic Conoid.—Generated by revolving one-half parabola of arc a, height h, base c, and area A (Fig. 34), about its *vertical* axis $Y-Y$.

Volume = $\frac{\pi}{8}c^2h = 0.39270\, c^2h$ = area of base × half the height.

Volume of parabolic segment of height $h' = \frac{\pi}{8}c'^2h'$.

Volume of parabolic frustum of height $h'' = \frac{\pi}{8}(c^2h - c'^2h')$. Note that $c' = c\sqrt{\frac{h'}{h}}$.

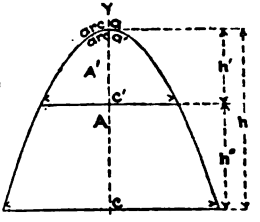

Fig. 34.

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l by—

ound its *short* diam d as the fixed axis; whence,

$0.52360\, dD^2$.

ound its *long* diam D as the fixed axis; whence,

$0.52360\, Dd^2$;

f circumscribing cylinder, (1) of length d and
ngth D and diam d.

cannot be determined by any practical working
's Theorem, preceding, of course applies.

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12.—ANALYTIC GEOMETRY.

(See also Mensuration.)

Analytic Geometry treats of the algebraic analysis of geometrica figures.

Plane Analytic Geometry, including what is commonly termed Coni Sections or sections cut from cones, deals with the analysis of plane curves referred to two coordinate axes, X and Y. The coordinate distances to an point p (Fig. 1) of the figure are x and y; x being measured from parallel to X, and y being measured from X parallel to Y. One, usually x is called the *abscissa*, and the other, usually y, is called the *ordinate*, to th point p. They are dependent variables for any particular figure. Th coordinate axes, X and Y, lie in the plane of the figure, and their point o intersection is called the *origin*. If the axes are at right angle with eac other they are called "rectangular coordinate axes," and the variables and y are called 'rectangular coordinates." This is the usual method.

Solid Analytic Geometry deals with the analysis of solid figures and i therefore sometimes termed Analytic Geometry of Space. Three coordinat axes are used, namely, X Y and Z, intersecting at the origin; and an point in space may be located by the three coordinates x, y and z, measure parallel to the respective axes.

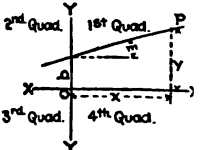

Fig. 1.

Straight Line.—Equation: $y=mx+b$.

X and Y are coordinate axes.

x = *abscissa*, measured from axis Y to any point p.

y = *ordinate*, measured from axis X to any point p.

m = tangent of angle of inclination $= \dfrac{y-b}{x}$.

b = a constant = distance on axis Y from the origin O to the straight line.

Note.—For any point p in 1st qudrant, x and y are plus; 2nd quadrant x is $-$, y is $+$; 3rd quadrant, x and y are minus; 4th quadrant, x is $+$ y is $-$. For angle upward to the right, m is $+$; downward to the right m is $-$. Straight line cuts Y, *above* origin O when b is $+$; *below*, when b is $-$.

Problem. 1—A straight line cuts the axis of Y 10 feet above the origin and makes an angle of $(+)$ $6°-10'$ with the axis of X. Solve?

Solution.—Natural tang of $6°-10'=.108$; $b=10$; hence the equation

$$y=.108\,x+10$$

from which the value of y may be obtained for any value of x, or vice versa

Thus, if $x=0$, $y=10$; $x=1$, $y=10.108$; $y=0$, $x=-\dfrac{10}{.108}=-92.6$; etc.

Problem 2.—Find the point of intersection of the two following lines and plat them: $y=-\frac{1}{2}x+2$, and $y=x-8$.

Solution.—

Equations:	Platting:
(1) $y=-\frac{1}{2}x+2$	When $y=0$, $x=4$; $x=0$, $y=2$.
(2) $y=x-8$	$y=0$, $x=8$; $x=0$, $y=-8$.

diff. $0=-1\frac{1}{2}x+10$

$\therefore \frac{3}{2}x=10$

$\therefore x=\frac{20}{3}=6\frac{2}{3}$.

Substituting value of $x=6\frac{2}{3}$ in either of the above equations, we obtain $y=-1\frac{1}{3}$.

$\therefore$ the coordinates of the point p at intersection of the two lines are

$$x=+6\tfrac{2}{3}, \text{ and } y=-1\tfrac{1}{3}.$$

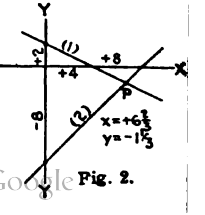

Fig. 2.

This is also a graphical illustration of Simultaneous Equations, page 103.

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Circle.—*Origin at center of circle.* Equation of circle: $r^2 = x^2 + y^2$,

whence $r = \pm\sqrt{x^2+y^2}$; $x = \pm\sqrt{r^2-y^2}$; $y = \pm\sqrt{r^2-x^2}$.

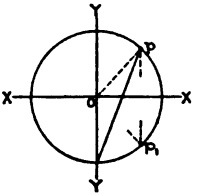

Fig. 3.

Problem.—A circle of 10 ft. radius is cut by a vertical line 8 ft. to the right of its center at the points p and p_1. What is the length of the chord pp_1?

Solution.—$y = \pm\sqrt{r^2-x^2} = \pm\sqrt{100-64} = \pm\sqrt{36} = \pm 6$. Hence, for p, $y = +6$, and for p_1, $y = -6$. ∴ the length of chord = 12 ft.

Intersection of circle with an oblique straight line.

Problem.—At what point in the above circle will the straight line $y = 2x - 10$ intersect?

Solution.—Equating the value of y^2 for the circle and for the straight line, $y^2 = r^2 - x^2 = (2x-10)^2$, we have, eliminating y and making $r^2 = 100$,

$$100 - x^2 = 4x^2 - 40x + 100$$

$$\text{whence} \quad 5x^2 = 40x$$

$$\therefore \quad x = 8, \text{ or } 0;$$

and substituting these values of x in the above equation of the straightline, we have, for $x = 8$, $y = 6$; for $x = 0$, $y = -10$. Hence the line intersects the circle at a point $x = 8$, $y = 6$; which is the same point as p in the above figure. It also intercepts the axis of X at a point $x = 5$; and the axis of Y at a point $y = -10$, at which point it also cuts the lowest point of the circle.

Equation of Tangent to the circle: $x_1x + y_1y = r^2$.

Problem.—The equation of a circle (Fig. 4) is $x^2 + y^2 = 100$. Find the equation of the tangent to the circle at point p, whose abscissa x is 8?

Solution.—Substituting the value of $x(=8)$ in the equation of the circle, we have

$$y = +\sqrt{100-64} = +6, \textit{ plus}, \text{ since } p \text{ is } \textit{above} \text{ the axis of } X;$$

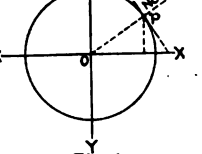

Fig. 4.

and substituting these values of $x = 8$ and $y = 6$ in the above equation of the tangent, and remembering that they correspond with x_1 and y_1 of that equation, we have, for equation of tangent, $8x + 6y = 100$. This tangent cuts the axis of X at $x = \frac{100}{8}$, and the axis of Y at $y = \frac{100}{6}$ (by making $y = 0$ in the first case and $x = 0$ in the second).

Equation of Normal to the circle: $y_1x - x_1y = 0$. Let it be required to find the normal to the circle (Fig. 4) whose equation is $x^2 + y^2 = 100$. Proceeding as in the above solution for the tangent, we obtain for the normal, $6x - 8y = 0$; or, $y = \frac{3}{4}x$. Clearly, this passes through the origin, at O, for when $x = 0$, $y = 0$.

Origin not at center of circle. Equation: $r^2 = (x-a)^2 + (y-b)^2$, whence $r = \pm\sqrt{(x-a)^2 + (y-b)^2}$; $x = a \pm \sqrt{r^2 - (y-b)^2}$; $y = b \pm \sqrt{r^2 - (x-a)^2}$.

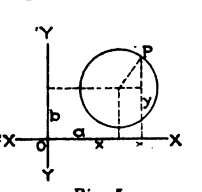

Fig. 5.

Equation of above circle may be reduced to $x^2 + y^2 - 2ax - 2by + a^2 + b^2 - r^2 = 0$; or letting $-2a = H$, $-2b = J$, and $a^2 + b^2 - r^2 = K$, we have, $x^2 + y^2 + Hx + Jy + K = 0$.

Equation of tangent to circle:

$$x_1x + y_1y + \frac{H}{2}(x + x_1) + \frac{J}{2}(y + y_1) + K = 0.$$

Parabola.—Next to the circle this is the most useful curve. (See, also, Mensuration, page 237.)

Equation: $y^2 = nx$ (n may be any quantity), whence $y = \pm\sqrt{nx}$; $n = \frac{y^2}{x}$; $x = \frac{y^2}{n}$.

The *latus rectum* (L L) $= 2y$ at a point where $y = 2x$. To find this point—

$$y = 2x = \sqrt{nx}$$

$$\therefore \quad 4x^2 = nx$$

$$\therefore \quad x = \frac{n}{4}; \text{ and } n = 4x = 2y = \text{length of latus rectum.}$$

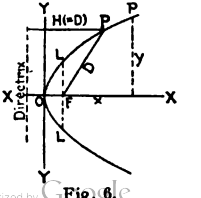

Fig. 6.

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The *focus* (F) is at the intersection of the latus rectum with the axis X, a distance of $x = \frac{n}{4}$ from the origin, O.

The *directrix* is a line parallel with the axis of Y and distant $\frac{n}{4}$ (the focal distance) to the *left* of it, so that the axis of Y lies midway between the directrix and the latus rectum. The horizontal distance H from the directrix to any point, as P, on the parabola is equal to the direct distance D from the focus to that point (Fig. 6).

Problem 1.—At what points does the circle, whose radius is 3, intersect the parabola whose value of $n = 8$?

Solution.— circle: $y^2 = r^2 - x^2$

parabola: $y^2 = 8x$

(diff.) $x^2 + 8x = r^2 = 9$

$\therefore\ x = 1;\ y = 2\sqrt{2}$ or $-2\sqrt{2}$.

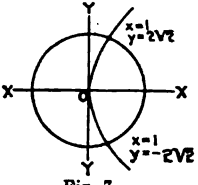

Fig. 7.

Problem 2.—What is the equation of a parabola (that is, the value of n) whose base is 16, and altitude 12?

Solution.—When $x =$ the altitude $= 12$; $y =$ half the base $= 8$. Then $n = \frac{y^2}{x} = \frac{64}{12} = \frac{16}{3}$, and the equation of the above parabola is $y^2 = \frac{16}{3}x$, so y can be determined for any value of x (measured from the top downward). Thus, for $x = 3$ $y = \sqrt{16} = 4$; $x = 6$, $y = 4\sqrt{2}$; $x = 9$, $y = 4\sqrt{3}$; $x = 12$, $y = 4\sqrt{4} = 8$.

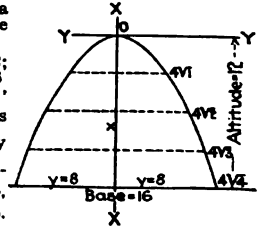

Fig. 8.

Equation of Tangent to the parabola: $y_1 y = \frac{n}{2}(x + x_1)$.

Equation of Normal to the parabola: $y_1 x + \frac{n}{2} y = x_1 y_1 + \frac{n}{2} y_1$.

Radius of Curvature r of the parabola $y^2 = nx$ is,

At any point, $r = \frac{4}{n^2}\left(y^2 + \frac{n^2}{4}\right)^{\frac{3}{2}}$.

At the vertex, O, $r = \frac{n}{2}$.

Note.—The equation of the parabola is variously given as $y^2 = nx$, $y^2 = 2px$, $y^2 = 4ax$, etc. It is plain that in these equations $n = 2p = 4a$, etc.

Ellipse.—The ellipse is a flattened circle.

Equation: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, whence $b^2x^2 + a^2y^2 = a^2b^2$;

$x = \pm \frac{a}{b}\sqrt{b^2 - y^2}$; $y = \pm \frac{b}{a}\sqrt{a^2 - x^2}$;

$a = \frac{bx}{\pm\sqrt{b^2 - y^2}}$ = semi-major axis;

$b = \frac{ay}{\pm\sqrt{a^2 - x^2}}$ = semi-minor axis.

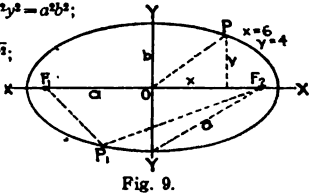

Fig. 9.

Foci (singular, *focus*). F_1 and F_2 are foci, so situated that the length of any broken line $F_1 P_1 F_2$, joining any point P_1 of the curve, is a constant. Therefore, the distance from either focus to either extremity of the minor axis is equal to one-half the major axis, or a.

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Problem.—The major axis of an ellipse is 20 ($a=10$), and the minor axis 10 ($b=5$). For a point P whose abscissa $x=6$, what is the value of the ordinate y? (See Fig. 9.)

Solution.— $y=\frac{b}{a}\sqrt{a^2-x^2}=\frac{5}{10}\sqrt{100-36}=\frac{1}{2}\sqrt{64}=4.$ *Ans.*

Tangent to the ellipse. Let T (Fig. 10) be the tangent point of the tangent PT to the *auxiliary circle*; then will the point t, lying vertically under T, be the tangent point of the tangent Pt to the ellipse, drawn from the same point P, lying on the major axis.

Equation of Tangent to the ellipse: $b^2x_1x+a^2y_1y=a^2b^2$.

Equation of Normal to the ellipse: $a^2y_1x-b^2x_1y=(a^2-b^2)\,x_1y_1$.

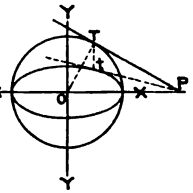

Fig. 10.

Radius of Curvature r of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is,

At any point, $r=\frac{(a^4y^2+b^4x^2)^{\frac{3}{2}}}{a^4b^4}$.

At extremity of major axis, $r=\frac{b^2}{a}$.

At extremity of minor axis, $r=\frac{a^2}{b}$.

The *false ellipse* or *oval* is simply an approximation to the true ellipse. For instance, the true ellipse consists of an *infinite* number of infinitesimal arcs with radii gradually decreasing in length from $r=\frac{a^2}{b}$ at extremity of minor axis, to $r=\frac{b^2}{a}$ at extremity of major axis; whereas the false ellipse or oval consists of a *definite* number of arcs with radii decreasing as above. The semi-false ellipse is sometimes called a *many-centered arch*; the most common forms for bridges are the 5-centered and 3-centered. These terms may also be applied to the false elliptic arc even if it comprises only part of the semi-false ellipse.

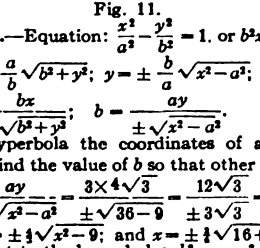

Fig. 11.

Hyperbola (Fig. 11).—Equation: $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, or $b^2x^2-a^2y^2=a^2b^2$.....(1)

whence, $x=\pm\frac{a}{b}\sqrt{b^2+y^2}$; $y=\pm\frac{b}{a}\sqrt{x^2-a^2}$;

$$a=\frac{bx}{\pm\sqrt{b^2+y^2}};\quad b=\frac{ay}{\pm\sqrt{x^2-a^2}}.$$

Problem.—In a hyperbola the coordinates of a point P are, $x=6$, $y=4\sqrt{3}$; and $a=3$. Find the value of b so that other points can be plotted?

Solution.— $b=\frac{ay}{\pm\sqrt{x^2-a^2}}=\frac{3\times4\sqrt{3}}{\pm\sqrt{36-9}}=\frac{12\sqrt{3}}{\pm3\sqrt{3}}=\pm4.$

Then for any point, $y=\pm\frac{4}{3}\sqrt{x^2-9}$; and $x=\pm\frac{3}{4}\sqrt{16+y^2}$.

Equation of Tangent to the hyperbola: $b^2x_1x-a^2y_1y=a^2b^2$.

Equation of Normal to the hyperbola: $a^2y_1x+b^2x_1y=(a^2+b^2)x_1y_1$.

Equilateral Hyperbola.—The hyperbola becomes equilateral when the asymptotes are perpendicular to each other: hence the two axes, $2a$ and $2b$ (Fig. 11) are equal: hence $a=b$ in Equation (1), page 259.

The equilateral hyperbola is a very useful curve. It enters into the solution of the Howe-truss brace problem (see Section 33).

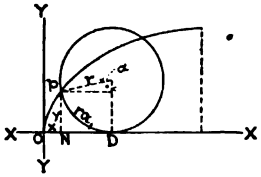

Fig. 12.

Cycloid (Fig. 12).—Equation: $x=r\,\text{vers}^{-1}\frac{y}{r}-\sqrt{2ry-y^2}$, in which $\text{vers}^{-1}\frac{y}{r}$ = the angle, in circular measure, whose vers $=\frac{y}{r}$.

Hence, $x=r\alpha_1-ND=OD-ND=ON$ (Fig. 12). Also,

$x=r(\alpha_1-\sin\alpha)$; ($\alpha_1=0.0174533\,\alpha$ in degrees.)

$y=r(1-\cos\alpha)=r\,\text{vers}\,\alpha$;

$r=x+(\alpha_1-\sin\alpha)=y+\text{vers}\,\alpha$.

Radius of Curvature $r=2\sqrt{2ry}$ = twice the length of the normal. (The normal to the curve at p is a straight line pD.)

For other Properties of the Cycloid, see page 236.

Catenary and Modified Catenary.—(See Suspension Bridges, and Arches.)

Spiral of Archimedes.—Equation: $r=a\theta$.

Logarithmic Spiral.—Equation: $r=a^\theta$.

Hyperbolic Spiral.—Equation: $r\theta=a$.

Lemniscate of Bernouilli.—Equation: $r^2=a^2\cos 2\theta$.

Helix or Screw.—A line traced from a fixed point bearing on a cylinder which is made to revolve, and at the same time to advance, uniformly. The *pitch* = the spacing of the lines or threads of the screw. Note that if the cylinder is developed, i. e., represented on a flat surface, all the lines or threads will be straight.

Common Spiral.—A line traced from a point bearing on a revolving plate and advancing uniformly outward from its center.

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13.—DESCRIPTIVE GEOMETRY.

Descriptive Geometry deals with graphical methods of representing magnitudes and of solving problems in Geometry.

Perspective pictures an object as if seen from a finite distance. Fig. 1 shows the perspective of a rectangular box. Points *VP* are *vanishing points* on the *horizon line HL*, which is supposed to be on a level with the eye. *PS* is the *point of sight*, directly opposite the eye and at the intersection of *HL* with the *vertical line VL*. All lines which are parallel in the object must meet at a common vanishing point. If the lines are horizontal in the object the *VP* lies on *HL*. If the lines are vertical in the object the *VP* lies on the line *VL*, at an infinite distance from *PS*; therefore, vertical lines in the object are shown vertical in the perspective. Note that the ground line, *GL*, is horizontal. The perspective of the object may occupy any position with reference to *PS*, *HL*, and *VL*; i.e., *HL* may lie above or below the object, or cut it, and *VL* may lie to the right or left of the object, or cut it.

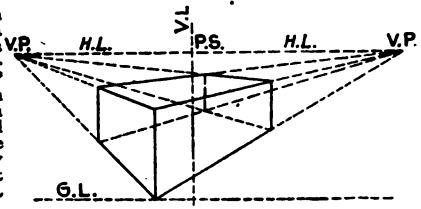

Fig. 1.

Cabinet projection pictures an object as if seen from an infinite distance. The ground line *GL* is at an angle of 45° with the horizontal, and lines parallel with its direction are drawn to ½ scale. (Fig. 2.)

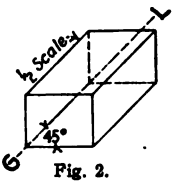

Fig. 2.

Isometric projection pictures an object as if seen from an infinite distance, with the ground line *GL* 30° with the horizontal; and all lines drawn to full scale. (Fig. 3.)

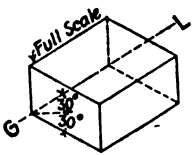

Fig. 3.

ORTHOGRAPHIC PROJECTION.

Orthographic projection, or Descriptive Geometry proper, deals graphically with the problems of position and dimension of the (point,) line, surface and solid in space. The ground line *GL* is the intersection of two coordinate planes, *V* (vertical) and *H* (horizontal), forming four dihedral angles — 1st, 2nd, 3rd and 4th (Fig. 4).

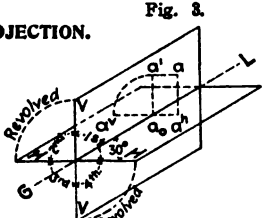

Fig. 4.

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Revolved planes. — In the isometric figure, Fig. 4, a is a point in space, a^h is its vertical projection on the horizontal plane H, and a' is its horizontal projection on the vertical plane V; further, a_0 is the horizontal projection of a^h and the vertical projection of a', on the ground line. If now the vertical plane V is revolved on the ground line GL, so as to coincide with the horizontal plane H, the projection a' of a will fall on a^v, so that the distance $a^v a_0 = a' a_0 = a a^h$, and $a^h a_0 = a a'$.

By revolving the ground line to a horizontal position as in Fig. 5, with the vertical plane above and the horizontal plane below, it will be seen that the point a in space may be represented by its projections, a^v and a^h.

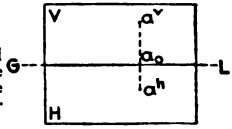

Fig. 5.

Projection of the point.—A point may be situated in the 1st, 2nd, 3rd or 4th dihedral angle, or it may be situated in one of the composite planes. Fig. 6 illustrates the system of lettering adopted to show the position of any point. A *point* in space is designated by a *small* letter, and its projection by the same letter with v or h written above, as an index.

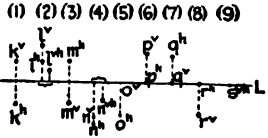

Fig. 6.

Dihedral angle in which point is situated:	1°	2°	3°	4°			
Plane " " " " " :	H	V	V	H	front	upper	back	lower	On ground line

Note.—In cases (2) and (4), l^{vh} and n^{vh} represent points l and n respectively equidistant from the coordinate planes.

Projection of the right line.—The position of a right line in space may be determined by the projection of two of its points. Fig. 7 illustrates the system of lettering adopted to show the position of any line. A *line* in space is designated by a *capital* letter, and its projection by the same letter with v or h written above, as an index.

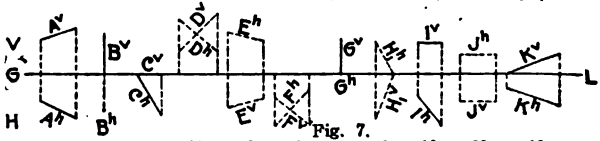

Fig. 7.

Dihed ang:	1°	1°		2°	3°	4°		3°	1°	3°	1°
Plane:	H						V				
	front						upper				

Remarks.—B and K pierce H; C, G and H_1 pierce the ground line; I pierces V, and is parallel to H; B and G are perpendicular to H; and J is parallel to H and V.

Projection of two lines.

Remarks.—

(1). M and L intersect at a, therefore the projections of their intersection are a^v and a^h.

(2). N and O are not in the same plane, as the intersections of their projections are not the projections of a common point.

(3). P and Q are parallel, as their common projections are parallel.

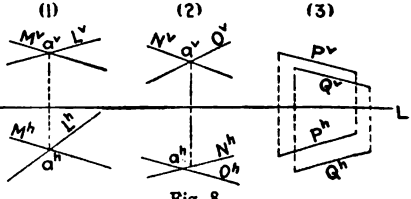

Fig. 8.

Note.—Two lines which are parallel or intersect, represent a plane, and the horizontal and vertical projections of their point of intersection must lie in the same perpendicular line.

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Projection of the plane.—A plane is determined by three points, or a point and a right line, or two right lines parallel or intersecting. The lines of intersection of a plane in space with the coordinate planes are called *traces*. Fig. 9 shows the vertical and horizontal traces of various planes in the 1st dihedral angle.

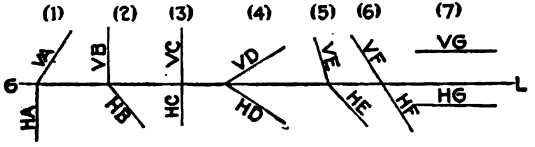

Fig. 9.

Plane *A*, perpendicular to vertical plane.	Plane *B*, perpendicular to horizontal plane.	Plane *C*, perpendicular to both coordinate planes.	Traces of *D* make acute angles with ground line.	Traces of *E*.	Traces of *F* make supplementary angles with ground line.	Plane *G*, parallel to ground line.

Problems of Construction.—In problems of construction the following conventional lines are employed:

Principal lines (data and results), *full* when visible and *dots* when invisible.
Auxiliary lines (minor lines), small *dashes* and *dots*.
Construction lines, (joining projections of the same points in space), fine *dashes*.

Problem 1.—*To find the projections of a line in space and also its length.* (Fig. 10.)

A right line *A* in the 1st dihedral angle pierces the *H* plane at a point *n*, 3 ft. from the ground line, and the *V* plane at a point *m*, 5 ft. from the ground line; the distance between the projections of the points, measured parallel with the ground line, being 15 ft. What is the length of the line *A* in space?

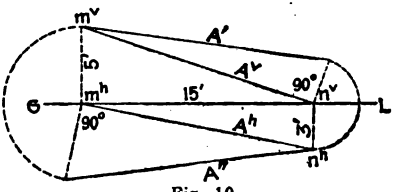

Fig. 10.

Solution.—Let the points *n* and *m* be represented respectively by their vertical and horizontal projections n^v, n^h and m^v, m^h; then will A^v, joining n^v and m^v, and A^h, joining n^h and m^h, be the respective vertical and horizontal projections of the line *A*; and the length of the line will equal A' or A'', A' being the hypothenuse of a right triangle whose base is A^v and altitude 3 ft.; and A'' the hypothenuse of a right triangle with base A^h and altitude 5 ft. Hence, by scale, $A = 16.1$ ft. (Analytically, $A = \sqrt{15^2 + 5^2 + 3^2} = \sqrt{259} = 16.09+$ ft.) Ans.

Problem 2.—*To find where a given right line A*, shown *in the 1st dihedral angle, pierces the coordinate planes V* and *H*? (Fig. 11.)

Solution.—Let A^v and A^h be the vertical and horizontal projections of the line *A*; then will m^h, the point where the horizontal projection of *A* intercepts the ground line, be the horizontal projection of the point where the line pierces the *V* plane (at m^v); and n^v, the point where the vertical projection of *A* intercepts the ground line, be the vertical projection of the point where the line pierces the *H* plane (at n^h). Therefore, the line *A* pierces the (upper)

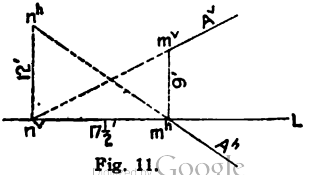

Fig. 11.

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V plane at m^v, by scale 9 ft. from the ground line; and it pierces the (back) H plane at n^h, by scale 12 ft. from the ground line; the points being horizontally (parallel with the ground line) 17½ ft. apart. It will be observed that the portion of the line between m^v and n^h is in the 2nd dihedral angle.

Problem 3.—*To pass a plane P through three given points not in the same straight line.* (Fig. 12.)

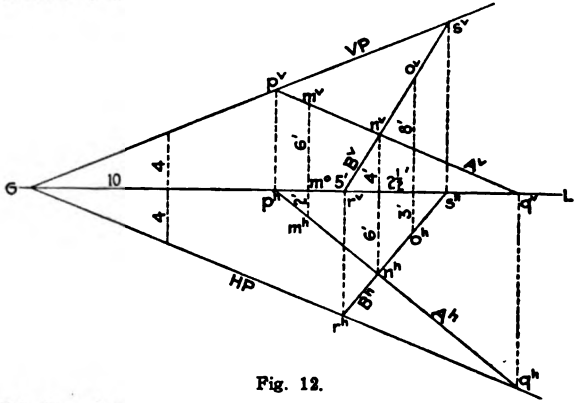

Fig. 12.

Solution.—Let m, n and o be three points in space, in the 1st dihedral angle, so that the horizontal distance (parallel with ground line) between the projections of m and n is 5 ft., and between those of n and o, is 2½ ft. Also let the perpendicular distances from the points in space, to the coordinate planes be as follows: m to H, 6ft.; m to V, 2 ft.; n to H, 4 ft.; n to V, 6 ft.; o to H, 8 ft., and o to V, 3 ft. That is, m is 6 ft. from the H plane and 2 ft. from the V plane, etc.

Let A^h be the horizontal projection of a line A passing through the points m and n, then A^h will pass through m^h and n^h; also where A^h intercepts the ground line GL (Problem 2), p^h is the horizontal projection of the point where the line A pierces the V plane, at p^v. In a similar way, A pierces the H plane at q^h. By like analysis, B^v and B^h are the vertical and horizontal projections of the line B, passing through the points n and o in space; and s^v and r^h are the points where the line pierces the V and H planes, respectively. It is evident that p^v and s^v are points on the vertical trace VP of the plane P, and that r^h and q^h lie in its horizontal trace, HP.

By scale, the tangent of the angle which the trace of each plane makes with the ground line is .400, and the point of intersection, at G, of the two traces at the ground line, is distant 20 ft. from m^o of point m.

This problem is very useful in structural shop details, such as finding the end bevel of hip and valley rafters, in framing, etc.

Problem 4.—*To find the point b where a perpendicular line L from a given point a pierces a given plane P; also the length of L.* (Fig. 13.)

Data.—Let a^v and a^h be the vertical and horizontal projections of the point a, and let VP and HP be the traces of the plane P.

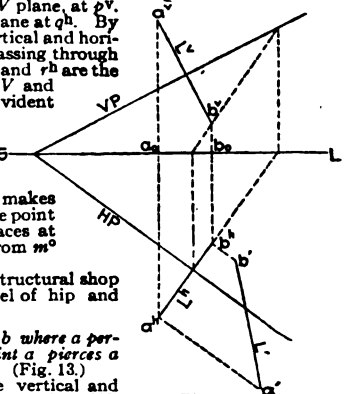

Fig. 13.

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a line in space is perpendicular to a plane, the projec- are respectively perpendicular to the traces of the plane; Therefore, draw L^v from a^v perpendicular to VP, and L^h ular to HP, as the vertical and horizontal projections of the ular to the plane P. Find b^v and b^h, the vertical and tions of b. From the extremities of L^h lay off, at right a^v, and $b^h\ b' = b_0\ b^v$. Then L' is the length of the line L

To find the angle between two given planes, P and Q.

–Let the two planes 14, intersect at the ss a plane R through the line L, perpen- Q (it will also be their common line cutting these planes nd R_2, and cutting its horizontal trace the lines R_1 and R_2, measure the required e planes. he planes P and Q nd line at points 3 he vertical trace VP ake an angle of 65° ine; HP, an angle of HQ, 60°. The re- een the planes is α, as follows:

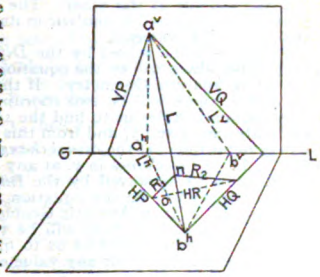

Fig. 14.

Fig. 15, lay 3 ft. apart les of the es P and Q. VP and ertical and es of the d HQ, the ontal traces and L^h will projection intersection R perpen- ne L, at n, s P and Q; zontal trace R be per- horizontal the line L, ne R will be both planes ve the tri- e line L is , around its is $a^v a^h$ so the ground will L' rep- of the line endicular to

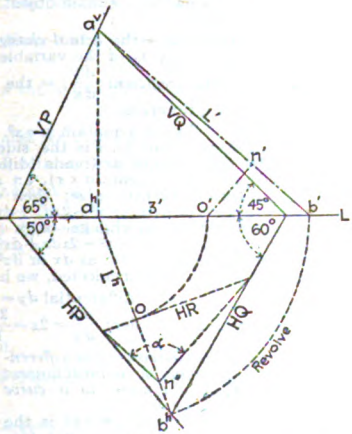

Fig. 15.

itude of the triangular portion of the plane R between the H and the planes P and Q. Revolve this triangle on n will n, the apex of the triangle, fall at n'', and on'' will required angle between the planes is α.

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14.—THE CALCULUS.

The Calculus furnishes us with direct and exact methods for solving many ploblems which could be solved otherwise only by indirect approximations; and the application of its principles provides us with useful working formulas in Mensuration, Geometry, Mechanics and other subjects.

The two processes in calculus are *differentiation* and *integration*, each being the inverse of the other. The former comprises the subject of Differential Calculus, and is *analytic* in its nature; the latter comprises Integral Calculus, and is *synthetic*.

Problems to be solved by the *Differential Calculus* may be reduced to an equation which will be the equation of some curve referred to coordinate axes, as in Analytic Geometry. If there are two *variables* in the equation, as x and y, there will be two coordinate axes, X and Y. The process of *differentiation* enables us to find the *slope of the curve* at any point p whose coordinates are x and y; and from this angle of slope, say with the axis of X, we may determine (1) the *actual change* in y for a corresponding change in x, and (2) the *rate of change* in y, at any point on the curve.

Problems to be solved by the *Integral Calculus* may be reduced to an equation which will be the equation of the *slope of some curve* referred to coordinate axes, as in Analytic Geometry. If there are *two variables* in the equation, as x and y, there will be two coordinate axes, X and Y. The process of *integration* enables us to find the *equation of the curve*, i. e., the value of the ordinate y for any value of the abscissa x.

A. DIFFERENTIAL CALCULUS.

Differentiation has for its main object the determination, from the equation of the problem, of—

1st, the *differential* dy, = the *actual change* in a function, due to a corresponding change dx of the variable x upon which it depends.

2nd, the *differential coefficient* $\frac{dy}{dx}$, = the *rate of change* of a function.

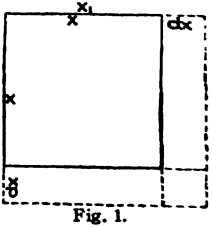

Fig. 1.

To illustrate: In the equation $y=x^2$ let $y=$ the area of a square of which x is the side. Now increase x by the distance dx (reads "differential of x," and does not mean $d\times x$), an amount *smaller than any numerical value*; then will the new area $y_1=x_1^2=(x+dx)^2=x^2+2xdx+dx^2$; and the increment or actual change in y will be $dy=y_1-y=x^2+2xdx+dx^2-x^2=2xdx+dx^2$. But as any infinitesimal quantity, as dx or dx^2, which is purely additive may be neglected, we have,

$$\text{differential } dy=2xdx, \tag{1}$$

and the differential coefficient $\frac{dy}{dx}=2x=\frac{2x}{1}$. (2)

Differential Coefficient.—*The differential coefficient is simply the natural tangent of the angle which a tangent to a curve makes with the axis of X.*

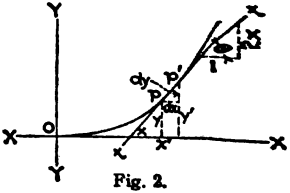

Fig. 2.

The preceding equation, $y=x^2$, is the equation of the parabola, $y=nx^2$ (see Analytic Geometry), in which $n=1$. Let Fig. 2 represent such a curve and let x and y be the coordinates of any point p. Also let p' be another point so that $x+dx=x'$ and $y+dy=y'$; then will $^*\frac{dy}{dx}=$

* Note that $dy=$ increment of $y=y'-y$, and $dx=$ increment of $x=x'-x$.

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the angle θ of a straight line passing through the
ints pp' are practically identical, as dx is smaller
ıe; hence the line tt, passing through the point
ırve at the point p whose coordinates are x and y.

he length of the side of a square whose area would
s the side, for a small increment?

differential coefficient $\frac{dy}{dx}$ equal to 8 in equation (2),

$=4$. Ans.

l. — We have seen, Fig. 2, that $\frac{dy}{dx}\left(=\frac{2x}{1}\right)$ repre-
gent to the parabola passing through the point p
and y. If, now, we assign a definite value to either
other may be determined from the equation $y=x^2$,

Thus, let $x=\frac{1}{2}$; then $y=\frac{1}{4}$, and $\frac{dy}{dx}=2x=2\times\frac{1}{2}=1$.
the *tangent* to the curve at the point p whose co-
$\frac{1}{4}$, is $\frac{dy}{dx}=1$; and hence the angle which it makes

This angle, 45°, means that for any increment in
ose length is $\frac{1}{2}$, the area will receive an equal incre-
nple, let $x=1\frac{1}{2}$; then, from $y=x^2$, $y=2\frac{1}{4}$; and from
angle which the *tangent* to the curve at the point
h the axis of X, $=$ the anti-tangent of $3=71°-34'$.

differential coefficient of y with respect to x, so is

ential coefficient of x with respect to y; that is,

represents the slope of a *normal* to the parabola

se coordinates are x and y.

$\frac{dx}{dy}$ are, respectively, the slopes of the *tangent* and
erred to the axis of X.

tion.—*The differential coefficient of—*
ct to itself is 1.

$=8$; $\frac{dy}{dx}=0$.

ny number of quantities is the *algebraic* sum of their
ts.

$+\frac{dv}{dx}$. $y=4x^3-5x^2$; $\frac{dy}{dx}=12x^2-10x$.

iables, is the sum of the products of each variable by
oefficient of the other.

$+u\frac{dv}{dx}$. $y=3x^2(4x+1)$; $\frac{dy}{dx}=(4x+1)\ 6x+(3x^2)4$.

tant and a variable, is the product of the constant
ial coefficient of the variable.

. $y=5x^3$; $\frac{dy}{dx}=5(3x^2)$.

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(f) *a fraction*, is the differential coefficient of the numerator multiplied b the denominator, minus the differential coefficient of the denominato multiplied by the numerator, this difference being divided by the squa of the denominator.

$$y=\frac{u}{v};\ \frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}.$$

$$y=\frac{3x^2+2}{2x};\ \frac{dy}{dx}=\frac{2x(6x+0)-(3x^2+2)2}{4x^2}=\frac{3x^2-2}{2x^2}.$$

(g) *any power of a variable*, is the product of the exponent, the power wit exponent diminished by 1, and the differential coefficient of the variabl

$$y=u^n;\ \frac{dy}{dx}=nu^{n-1}\frac{du}{dx}.$$

$$y=2(x^2+3)^3;\ \frac{dy}{dx}=(3\times 2)(x^2+3)^2\times 2x=12x(x^2+3)^2.$$

Maxima and Minima.—One of the principal uses of the differential calculus is to find the maximum or minimum value of a function. In any curve, as for instance the ellipse, $a^2y^2+b^2x^2=a^2b^2$ (see Analytic Geometry), let it be required to find the maximum value of the ordinate y. From the above equation, placing y in the first member, we have $a^2y^2=a^2b^2-b^2x^2$. Differentiating, $2a^2\,y\,dy=0-2b^2\,xdx$; whence the differential coefficient $\frac{dy}{dx}=-\frac{2b^2x}{2a^2y}=-\frac{b^2x}{a^2y}=$ tangent of

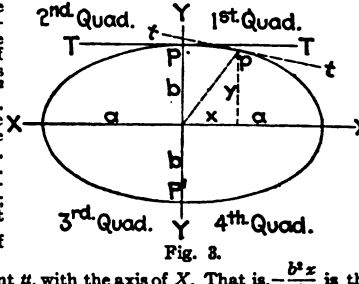

Fig. 3.

angle of inclination of the tangent tt, with the axis of X. That is, $-\frac{b^2x}{a^2y}$ is th value of m in the equation of the straight line tt. (See Analytic Geometry

For any tangent to the ellipse, if m is a *minus* quantity, the point contact p is in the 1st or 3rd quadrants, and if it is a *plus* quantity the poi of contact is in the 2nd or 4th quadrants. This is made evident by notin whether the tangent line tt slopes upward to the right or downward to th right. If the point of contact p is in the first quadrant, x and y are *pl* and $\frac{dy}{dx}=-\frac{b^2}{a^2}\left(\frac{x}{y}\right)=-\frac{b^2x}{a^2y}$; if in the second quadrant, x is *minus* and y is *plu* hence $\frac{dy}{dx}=-\frac{b^2}{a^2}\left(\frac{-x}{y}\right)=\frac{b^2x}{a^2y}$; if in the 3rd quadrant, $\frac{dy}{dx}=-\frac{b^2}{a^2}\left(\frac{-x}{-y}\right)=-\frac{b^2x}{a^2y}$; and if in the 4th quadrant, $\frac{dy}{dx}=-\frac{b^2}{a^2}\left(\frac{x}{-y}\right)=\frac{b^2x}{a^2y}$. *Thus, we are ab t.) study the slope of a curve* at any point.

By inverse analysis we may find the values of x and y (including the maximum and minimum values) by assigning definite values to the slo $\frac{dy}{dx}$. Thus, when y is a maximum we know that the tangent line tt mu move so as to be parallel with the axis of X; hence, it will coincide with th tangent line TT, touching the point P at the upper extremity of the min axis; or with a parallel tangent through the point P' at its lower extremit And, when y is a minimum, tt will be perpendicular to the axis of X, touc ing points on the curve at either extremity of the major axis. In the form case, $\frac{dy}{dx}\left(=\pm\frac{b^2x}{a^2y}\right)=0$; whence $x=0$. which value substituted in t general equation makes $y=\pm b$, a maximum. And in the latter ca $\frac{dy}{dx}\left(=\pm\frac{b^2x}{a^2y}\right)=\infty$; whence $y=0$, a minimum.

Maxima and Minima obtain at points where $\frac{dy}{dx}$ *changes from* + *to or from* − *to* +.

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Problem 1.—It is desired to suspend a weight W vertically beneath a point P situated midway between two level supports A and B. For this purpose two diagonal rods, AW and BW, of equal length, are used. At what angle α shall the rods be inclined so that a minimum amount of metal will be required?

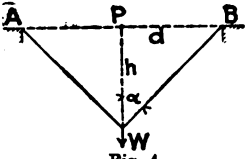

Fig. 4.

Solution.—The stress in each rod $= \frac{W}{2}$ secant α, and the length of each rod $= d$ cosecant $\alpha = h$ secant α; hence, the amount of metal required in the rods would be, making n a constant, and h variable,

$$y = n\left(\frac{W}{2}\sec\alpha\right)(h\sec\alpha)$$
$$= \frac{nhW}{2}\sec^2\alpha = \frac{nhW}{2}(1+\tan^2\alpha)$$
$$= \frac{nhW}{2} + \frac{nhW}{2}\cdot\frac{d^2}{h^2}$$
$$= \frac{nhW}{2} + \frac{nd^2W}{2h},$$

whence $\frac{dy}{dh} = \frac{nW}{2} - \frac{2nd^2W}{4h^2} = 0$, for minimum;

$$\therefore\ 4nWh^2 = 4nWd^2$$
$$\therefore\ h = d, \text{ or } \alpha = 45°.$$

This proves that the most economical angle for a truss diagonal is 45°.

Problem 2.—It is proposed to build a railroad from A, a point opposite B and distant 30 miles, to D a point 50 miles below B. The cost of grading on the line BC is \$8,000 per mile, while the cost of grading from A diagonally to any point C is \$12,000 per mile. Find the position of the point C so that the entire cost of grading will be a minimum.

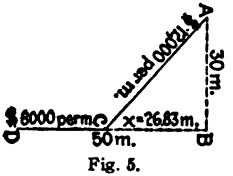

Fig. 5.

Solution.—Let y = the cost of grading the line, and from the data,

$$y = \$12{,}000\,(\overline{AB}^2 + \overline{BC}^2)^{\frac{1}{2}} + \$8{,}000\,(BD - BC).$$

Let $BC = x$, $AB = 30$, and $BD = 50$; then

$$y = 12000\,(900 + x^2)^{\frac{1}{2}} + 8000\,(50 - x).$$
$$\frac{dy}{dx} = 6000\,(900 + x^2)^{-\frac{1}{2}}(2x) - 8000 = 0 \text{ for minimum;}$$
$$\therefore\ \frac{12000x}{\sqrt{900 + x^2}} = 8000$$
$$\therefore\ x = 26.83 \text{ miles. Ans.}$$

Note.—The *entire* cost of construction and the cost of operation would of course enter as factors to reduce the distance between C and D, for economy.

Problem 3.—What is the maximum cylinder, in volume, that can be inscribed in a sphere?

Solution.—Let R = radius of sphere; r = radius, and h = altitude, of cylinder. Let V = volume of cylinder; then,

$$V = \pi r^2 h = \pi h\left(R^2 - \frac{h^2}{4}\right),$$

in which V = a function of h, both being variable, with R constant. Differentiating and making $\frac{dV}{dh} = 0$, we have,

$$\frac{dV}{dh} = \pi R^2 - \tfrac{3}{4}\pi h^2 = 0;$$
$$\therefore h = \frac{2}{\sqrt{3}}R = \tfrac{2}{3}\sqrt{3}\ R. \text{ Ans.}$$

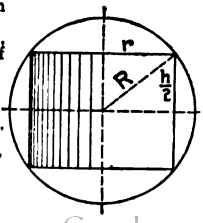

Fig. 6.

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Other Forms of Differentiation.—There are two forms in which the result of differentiation may be expressed, namely, *differential coefficient* or simply *differential:*

$$y = x^2; \quad \frac{dy}{dx} = 2x \quad = \text{differential coefficient.}$$

$$y = x^2; \quad dy = 2x\,dx \quad = \text{differential.}$$

Hence the differential coefficient $\frac{dy}{dx}$ may be considered as a fraction, the numerator dy being the perpendicular, and the denominator dx the base, of an infinitesimal triangle of which the hypothenuse is equal to $\sqrt{dx^2+dy^2}$. Multiplying the differential coefficient by dx we obtain the differential $dy = 2x\,dx$. It is well to keep this graphical analysis in mind in dealing with problems involving both Differential and Integral Calculus, as it then loses much of its haze and mystery.

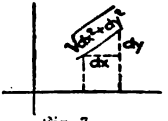

Fig. 7.

Besides the Algebraic forms (*a*) to (*g*), which are *universal in their application*, the following formulas for differentiation of special functions will be found useful for reference. (For Algebraic Functions, see pages 267, 268.)

Logarithmic and Exponential Functions:

(h) $y = \log_a x; \quad dy = \frac{\log_a e\,dx}{x}.$

$y = \log_e x; \quad dy = \frac{dx}{x}.$

(i) $y = \log_a u; \quad \frac{dy}{dx} = \log_a e\,\frac{\frac{du}{dx}}{u}; \quad dy = \log_a e\,\frac{du}{u}.$

$y = \log_e u; \quad \frac{dy}{dx} = \frac{\frac{du}{dx}}{u}; \quad dy = \frac{du}{u}.$

(j) $y = a^u; \quad \frac{dy}{dx} = \log_e a \cdot a^u\,\frac{du}{dx}; \quad dy = \log_e a \cdot a^u\,du.$

(k) $y = e^u; \quad \frac{dy}{dx} = e^u\,\frac{du}{dx}; \quad dy = e^u\,du.$

(l) $y = u^v; \quad \frac{dy}{dx} = vu^{v-1}\frac{du}{dx} + \log_e u \cdot u^v\frac{dv}{dx}; \quad dy = vu^{v-1}du + \log_e u \cdot u^v dv.$

Trigonometric Functions:

(m) $y = \sin u; \quad \frac{dy}{dx} = \cos u\,\frac{du}{dx}; \quad dy = \cos u\,du.$

(n) $y = \cos u; \quad \frac{dy}{dx} = -\sin u\,\frac{du}{dx}; \quad dy = -\sin u\,du.$

(o) $y = \tan u; \quad \frac{dy}{dx} = \sec^2 u\,\frac{du}{dx}; \quad dy = \sec^2 u\,du.$

(p) $y = \cot u; \quad \frac{dy}{dx} = -\operatorname{cosec}^2 u\,\frac{du}{dx}; \quad dy = -\operatorname{cosec}^2 u\,du.$

(q) $y = \sec u; \quad \frac{dy}{dx} = \sec u \tan u\,\frac{du}{dx}; \quad dy = \sec u \tan u\,du.$

(r) $y = \operatorname{cosec} u; \quad \frac{dy}{dx} = -\operatorname{cosec} u \cot u\,\frac{du}{dx}; \quad dy = -\operatorname{cosec} u \cot u\,du.$

(s) $y = \operatorname{vers} u; \quad \frac{dy}{dx} = \sin u\,\frac{du}{dx}; \quad dy = \sin u\,du.$

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RSE TRIGONOMETRIC FUNCTIONS:

$$\frac{dy}{dx}=\frac{\frac{du}{dx}}{\sqrt{1-u^2}}; \qquad dy=\frac{du}{\sqrt{1-u^2}}.$$

$$\frac{dy}{dx}=-\frac{\frac{du}{dx}}{\sqrt{1-u^2}}; \qquad dy=-\frac{du}{\sqrt{1-u^2}}.$$

$$\frac{dy}{dx}=\frac{\frac{du}{dx}}{1+u^2}; \qquad dy=\frac{du}{1+u^2}.$$

$$\frac{dy}{dx}=-\frac{\frac{du}{dx}}{1+u^2}; \qquad dy=-\frac{du}{1+u^2}.$$

$$\frac{dy}{dx}=\frac{\frac{du}{dx}}{u\sqrt{u^2-1}}; \qquad dy=\frac{du}{u\sqrt{u^2-1}}.$$

$$\frac{dy}{dx}=-\frac{\frac{du}{dx}}{u\sqrt{u^2-1}}; \qquad dy=-\frac{du}{u\sqrt{u^2-1}}$$

$$\frac{dy}{dx}=\frac{\frac{du}{dx}}{\sqrt{2u-u^2}}; \qquad dy=\frac{du}{\sqrt{2u-u^2}}.$$

XPANSION OF FUNCTIONS.

$\frac{1}{1+x}=1-x+x^2-x^3+x^4$------.

ation is employed in the expansion of functions and . Let $y=u$ be any equation in which u is a function fferential coefficient $\left(=\frac{dy}{dx}\right)$ is the diff coef of y; $\left(\frac{d^2y}{dx^2}\right)$ is the diff coef of the first diff coef; the 3rd diff ef of the 2nd diff coef, etc. For example, let $y=3x^4$; $3x^3$, etc.

m enables us to expand into a series any function u y the use of successive differential coefficients. Let expanded, and assume

x^2+Ex^4——, using Indeterminate Coefficients. (1)

value of u successively and making $x=0$ in all iation, we have,

Making $x=0$:

)	$y=A,$	or $A=y.$
Ex^3——.	$\frac{dy}{dx}=B,$	or $B=\frac{dy}{dx}.$
Ex^4——.	$\frac{d^2y}{dx^2}=2C,$	or $C=\frac{1}{2}\left(\frac{d^2y}{dx^2}\right).$
——.	$\frac{d^3y}{dx^3}=2\cdot3\,D,$	or $D=\frac{1}{2\cdot3}\left(\frac{d^3y}{dx^3}\right).$
	$\frac{d^4y}{dx^4}=2\cdot3\cdot4\,E,$	or $E=\frac{1}{2\cdot3\cdot4}\left(\frac{d^4y}{dx^4}\right).$
- - - - - -	etc.	- - - - - - - - - - - - - -

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Substituting the above values of the indeterminate coefficients in equation (1) we have the following working equation:

$$u=y+\frac{dy}{dx}\cdot x+\frac{d^2y}{dx^2}\cdot\frac{x^2}{2}+\frac{d^3y}{dx^3}\cdot\frac{x^3}{2\cdot 3}+\frac{d^4y}{dx^4}\cdot\frac{x^4}{2\cdot 3\cdot 4}+ - - - - - \ldots\ldots\ldots\ldots (2)$$

remembering that $y=u$ when, in all the successive differential coefficients, $x=0$.

Examples in the use of equation (2).—It is to be noted that Maclauren's Theorem may be considered a *special case* of Taylor's Theorem, following. Also, the Binomial Formula (see Algebra) is a special case of Maclauren's Theorem.

Expansion of a few common functions:

$$\log_e(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\frac{x^5}{5}- - - - -. \quad \text{Naperian system.}$$

$$\log_a(1+x)=M\left(x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\frac{x^5}{5}- - - - - -\right) \quad \text{Common system.}$$

$$\therefore M=\frac{\text{Common log }(1+x)}{\text{Naperian log }(1+x)}=\frac{1}{2.30258509}=.4342944819=\text{the modulus of the common system}=\text{common log } e=\log_a e.$$

$$e=2+\tfrac{1}{2}+\frac{1}{2\cdot 3}+\frac{1}{2\cdot 3\cdot 4}+\frac{1}{2\cdot 3\cdot 4\cdot 5}+ - - - - - -=2.718281828+.$$

$$e^x=1+\frac{x}{1}+\frac{x^2}{1\cdot 2}+\frac{x^3}{1\cdot 2\cdot 3}+\frac{x^4}{1\cdot 2\cdot 3\cdot 4}- - - - - - -.$$

$$\sin\alpha=\alpha_1-\frac{\alpha_1^3}{1\cdot 2\cdot 3}+\frac{\alpha_1^5}{1\cdot 2\cdot 3\cdot 4\cdot 5}-\frac{\alpha_1^7}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7}+ - - - - - -;$$

$$\text{thus, nat sine of } 30^\circ=\frac{\pi}{6}-\frac{1}{\lfloor 3}\left(\frac{\pi}{6}\right)^3+\frac{1}{\lfloor 5}\left(\frac{\pi}{6}\right)^5-\frac{1}{\lfloor 7}\left(\frac{\pi}{6}\right)^7+ - - -.$$

$$\cos\alpha=1-\frac{\alpha_1^2}{1\cdot 2}+\frac{\alpha_1^4}{1\cdot 2\cdot 3\cdot 4}-\frac{\alpha_1^6}{\lfloor 6}+\frac{\alpha_1^8}{\lfloor 8}- - - - -;$$

$$\text{thus, nat cos of } 20^\circ=1-\tfrac{1}{2}\left(\frac{\pi}{9}\right)^2+\frac{1}{\lfloor 4}\left(\frac{\pi}{9}\right)^4-\frac{1}{\lfloor 6}\left(\frac{\pi}{9}\right)^6+ - - - - -.$$

$$\frac{\pi}{4}=1-\tfrac{1}{3}+\tfrac{1}{5}-\tfrac{1}{7}+\tfrac{1}{9}- - - - - -. \qquad \therefore \pi=3.1415926536.$$

Taylor's Theorem enables us to expand a function of the sum of two variables arranged according to the ascending powers of one of the variables. Let u, = a function of $(x+h)$, be the function to be developed. Then, for a working formula,

$$u=y+\frac{dy}{dx}\cdot h+\frac{d^2y}{dx^2}\cdot\frac{h^2}{2}+\frac{d^3y}{dx^3}\cdot\frac{h^3}{1\cdot 2\cdot 3}+ - - - - - \ldots\ldots\ldots\ldots (3)$$

remembering that $y=u$ when, in all the successive differential coefficients of u, $h=0$. (See Maclauren's Theorem, preceding, for illustration.)

Expansion of a few common functions:

$$\log(x+h)=\log x+\frac{h}{x}-\frac{h^2}{2x^2}+\frac{h^3}{3x^3}-\frac{h^4}{4x^4}+ - - - - -.$$

$$\tan(x+h)=\tan x+h\sec^2 x+h^2\sec^2 x\tan x+\frac{h^3}{3}\sec^2 x(1+3\tan^2 x)+ - - - - -$$

$$\log(1+\sin x)=x-\frac{x^2}{2}+\frac{x^3}{6}-\frac{x^4}{12}+ - - - - -.$$

B. INTEGRAL CALCULUS.

Integral Calculus is the process of *summation* or the adding up of an infinite number of infinitesimal quantities. Its operation is the inverse of differentiation. In the equation $y=x^2$, by differentiation we have $dy=2xdx$, hence the summation or integration of $2xdx=\int 2xdx=x^2\left(=2\frac{x^2}{2}\right)$. Suppose, for example, we wish to find the *area* of a triangle whose altitude

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is h and base b, Fig. 8. If we let dA represent, in general, the area of any vertical strip of length y and width dx, distant x from the axis of Y, then the total area of the triangle will equal the summation of all these strips between the limits $x=b$ and $x=0$; or, area $=A=\int_{x=0}^{x=b} dA = \int_{x=0}^{x=b} ydx$.

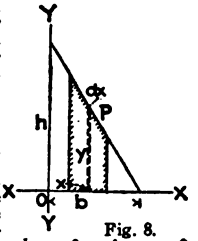

Fig. 8.

The main difficulty in Integral Calculus is in forming the equations to be integrated and recognizing them as related to certain fundamental forms whose integrals are well known, many of which are given in tabluar form in the following pages. In the above case, x and y are dependent variables, x increasing as y decreases, and vice versa. When $x=b$, $y=0$; when $x=0$, $y=h$; and the point P, whose coordinates are x and y, moves along the hypothenuse in a straight line between the axes of X and Y, as x and y vary. The equation of this straight line, of which $y=mx+b$ is the general equation, is

$$y=-\frac{h}{b}x+h$$

because the tangent of inclination m of the hypothenuse with the axis of X is $-\frac{h}{b}$ (minus, because downward to the right), and the value of b where the hypothenuse intercepts the axis of Y is h. But $dA=ydx=$ the area of an infinitesimal strip of length y, and from the above equation, $ydx=-\frac{h}{b}xdx+hdx$; hence,

$$\text{Area}=A=\int_{x=0}^{x=b}-\frac{h}{b}x\,dx+hdx$$

which, by certain fundamental forms given in the table, reduces to

$$A=-\frac{h}{b}\cdot\frac{x^2}{2}+hx.$$

Substituting the value of $x=b$ in the equation, we have

$$\text{Area } A=-\frac{h}{b}\cdot\frac{b^2}{2}+hb=\tfrac{1}{2}bh. \quad \text{Ans.}$$

Definite Integration a Method of Limits.—In the preceding illustration of the area of the triangle we referred to the upper limit $x=b$, and the lower limit $x=0$. We will now find the area of a *part* of the triangle or that etched portion, Fig. 8, between the upper limit $x=\frac{3}{4}b$, and the lower limit $x=\frac{1}{4}b$.

Thus, $$A_1=\int_{x=\frac{1}{4}b}^{x=\frac{3}{4}b}-\frac{h}{b}x\,dx+hdx$$

After integrating, $$A_1=\left[-\frac{h}{b}\frac{x^2}{2}+hx\right]_{x=\frac{1}{4}b}^{x=\frac{3}{4}b}$$

Substituting $\frac{3}{4}b$ and $\frac{1}{4}b$ for x, $$A_1=\underbrace{\left(-\frac{h}{2b}\cdot\frac{9b^2}{16}+\tfrac{3}{4}bh\right)}_{\text{upper}}-\underbrace{\left(-\frac{h}{2b}\cdot\frac{b^2}{16}+\frac{bh}{4}\right)}_{\text{lower}}$$

$\therefore$ partial area $A_1=\frac{bh}{4}$. Ans.

Note.—Subtract the value obtained by using the lower limit, from the value obtained by using the upper limit.

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Formulas for Integration.—

(A) $\int ax^n dx = \frac{ax^{n+1}}{n+1}$. $\int x^4 dx = \frac{x^5}{5}$; $\int 2x^3 dx = 2 \cdot \frac{x^4}{4} = \frac{x^4}{2}$.

(B) $\int \frac{dx}{x} = \log x$. $\int \frac{adx}{x} = a \log x$.

(C) $\int a^x dx = \frac{a^x}{\log a}$. $\int a^x \log a \, dx = a^x$.

(D) $\int e^x dx = e^x$. $\int a^x e^x dx = \frac{a^x e^x}{1 + \log a}$.

(E) $\int \cos x \, dx = \sin x$. $\int \cos 2x \, dx = \sin (x+a) \cos (x-a)$.

(F) $\int \sin x \, dx = -\cos x$. $\int \frac{2dx}{\sin 2x} = \log \tan x$.

(G) $\int \sec^2 x \, dx = \tan x$.

(H) $\int \operatorname{cosec}^2 x \, dx = -\cot x$. $\int \operatorname{cosec} x \, dx = \log \sqrt{\frac{1 - \cos x}{1 + \cos x}}$.

(I) $\int \sec x \tan x \, dx = \sec x$.

(J) $\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x$.

(K) $\int \tan x dx = \log \sec x$.

(L) $\int \cot x \, dx = \log \sin x$.

(M) $\int \sec x \, dx = \log (\sec x + \tan x) = \log \tan \left(\frac{x}{2} + \frac{\pi}{4}\right)$.

(N) $\int \operatorname{cosec} x \, dx = \log (\operatorname{cosec} x - \cot x) = \log \tan \frac{x}{2} = \log \sqrt{\frac{1 - \cos x}{1 + \cos x}}$.

(O) $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} = -\frac{1}{a} \cot^{-1} \frac{x}{a}$.

(P) $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \frac{x-a}{x+a} = \frac{1}{2a} \log \frac{a-x}{a+x}$.

(Q) $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} = -\cos^{-1} \frac{x}{a}$.

(R) $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log \left(x + \sqrt{x^2 \pm a^2}\right)$.

(S) $\int \frac{dx}{a\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{x}{a} = -\frac{1}{a} \operatorname{cosec}^{-1} \frac{x}{a}$.

(T) $\int \frac{dx}{\sqrt{2ax - x^2}} = \operatorname{vers}^{-1} \frac{x}{a}$.

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(U) $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}.$

(V) $\int x^2\sqrt{a^2-x^2}\,dx = \frac{x}{8}(2x^2-a^2)\sqrt{a^2-x^2} + \frac{a^4}{8}\sin^{-1}\frac{x}{a}.$

Areas of Plane Curves.—

Formula (1). $A = \int_{a_1}^{a_2} y\,dx$ (Fig. 9.)

Formula (2). $A = \int_{b_1}^{b_2} (a-x)\,dy$ (Fig. 10.)

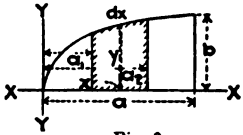

Fig. 9.

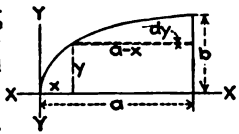

Fig. 10.

Let Figs. 9 and 10 each represent one-half a parabola whose base is $2b$ and altitude a. Required the area of the figure? From the general equation of the parabola, $y^2=nx$, $n=\frac{y^2}{x}$. When $x=a$, $y=b$; therefore $n=\frac{y^2}{x}=\frac{b^2}{a}$, and the equation of the parabola in the figure is $y^2=\frac{b^2}{a}x$. From this equation, $y=b\frac{x^{\frac{1}{2}}}{a^{\frac{1}{2}}}$ and $x=\frac{ay^2}{b^2}$.

Method (1), *vertical strips.*

$$A = \int_0^a y\,dx = \int_0^a \frac{b}{a^{\frac{1}{2}}}x^{\frac{1}{2}}\,dx.$$

$$= \left[\frac{b}{a^{\frac{1}{2}}}\cdot\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^a$$

$= \frac{2}{3}ab.$ Ans.

Method (2), *horizontal strips.*

$$A = \int_0^b (a-x)\,dy = \int_0^b \left(a-\frac{ay^2}{b^2}\right)dy.$$

$$= \left[ay - \frac{a}{b^2}\frac{y^3}{3}\right]_0^b$$

Equivalent values. $= ab - \frac{1}{3}ab = \frac{2}{3}ab.$ Ans.

It is thus seen that the same result, $\frac{2}{3}ab$, is obtained whether we assume the strips to be vertical and summate horizontally or whether we assume the strips to be horizontal and summate vertically.

Again, required the area of the *shaded portion* of the parabola in Fig. 9, Formula (1), between the limits $a_2=4$ and $a_1=2$:

$$A_1 = \int_2^4 y\,dx = \int_2^4 \frac{b}{a^{\frac{1}{2}}}x^{\frac{1}{2}}\,dx = \left[\frac{b}{a^{\frac{1}{2}}}\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_2^4 = \left[\frac{2}{3}\frac{b}{a^{\frac{1}{2}}}x^{\frac{3}{2}}\right]_2^4$$

$$= \frac{16}{3}\cdot\frac{b}{\sqrt{a}} - \frac{4\sqrt{2}}{3}\cdot\frac{b}{\sqrt{a}} = \frac{16-4\sqrt{2}}{3}\cdot\frac{b}{\sqrt{a}}.$$ Ans.

Lengths of Plane Curves.—

Formula: Length $L = \int dL = \int \sqrt{dx^2+dy^2}.$ (See Fig. 7.)

Use $L = \int \left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}}dx$, when x is the independent variable.

Use $L = \int \left[1+\left(\frac{dx}{dy}\right)^2\right]^{\frac{1}{2}}dy$, when y is the independent variable.

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Problem.—The catenary, $y=\frac{a}{2}\left(e^{\frac{x}{a}}+e^{-\frac{x}{a}}\right)$, is the curve which a cable assumes when suspended at both ends, as at A and B, Fig. 11. Required the length of the catenary from the vertex V, on the axis of Y, to any point p whose coordinates are x and y?

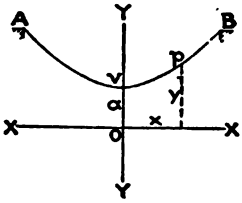

Fig. 11.

Solution.— $$y=\frac{a}{2}\left(e^{\frac{x}{a}}+e^{-\frac{x}{a}}\right)$$

hence, $$\frac{dy}{dx}=\tfrac{1}{2}\left(e^{\frac{x}{a}}-e^{-\frac{x}{a}}\right)$$

and $$dL=\tfrac{1}{2}\left(e^{\frac{x}{a}}+e^{-\frac{x}{a}}\right)dx\ \left(=\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx\right)$$

with limits $x=x$, and $x=0$, $L=\frac{1}{2}\int_0^x\left(e^{\frac{x}{a}}+e^{-\frac{x}{a}}\right)dx$

$\therefore$ length from V to $P=L=\frac{a}{2}\left(e^{\frac{x}{a}}-e^{-\frac{x}{a}}\right)$. Ans.

Areas of Curved Surfaces, or surfaces of revolution.—

Formula (1). $S=2\pi\int y\,ds=2\pi\int y\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}}dx$. (About axis of X.)

Formula (2). $S=2\pi\int x\,ds=2\pi\int x\left[1+\left(\frac{dx}{dy}\right)^2\right]^{\frac{1}{2}}dy$ (About axis of Y.)

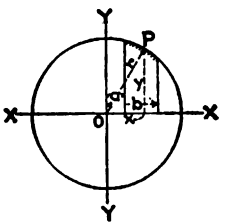

Fig. 12.

Problem.—Let it be required to find the area of the *shaded zone* of a sphere of radius r, Fig. 12.

Solution.—Use Formula (1):

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Equation of circle is $x^2+y^2=r^2$,
whence, $y^2=r^2-x^2$ or $y=\sqrt{r^2-x^2}$.

$$\frac{dy}{dx}=-\frac{x}{y} \quad \text{and} \quad \left(\frac{dy}{dx}\right)^2=\frac{x^2}{y^2}.$$

Substituting value of $\left(\frac{dy}{dx}\right)^2$ in Formula (1) there is obtained,

$$S=2\pi\int_{x=a}^{x=b} y\left(1+\frac{x^2}{y^2}\right)^{\frac{1}{2}}dx$$

$$=2\pi\int_{x=a}^{x=b}(y^2+x^2)^{\frac{1}{2}}\,dx=2\pi\left[rx\right]_{x=a}^{x=b}=2\pi r\,(b-a). \quad \text{Ans.}$$

Volumes, or planes of revolution.—

Formula (1). $V=\pi\int_{x=b}^{x=a} y^2dx$ (Plane revolved about axis of X.)

Formula (2). $V=\pi\int_{z=d}^{z=c} x^2dy$ (Plane revolved about axis of Y.)

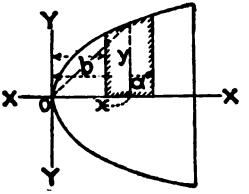

Fig. 13.

Problem.—Find the volume generated by the shaded portion of the parabola $y^2=4x$, Fig. 13, about the axis of X.

Solution.—From the equation of the parabola and Formula (1), we have,

$$V=\pi\int_b^a y^2dx=\int_b^a 4x\,dx$$

$$=\pi\left[2x^2\right]_b^a=2\pi(a^2-b^2). \quad \text{Ans.}$$

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15.—MECHANICS.

General Discussion.

The subject of Mechanics deals fundamentally with *matter* as *ma* at *rest* and in *motion*; and with its accompanying phenomena.

All matter has *weight* (W) and its *mass* (M), for any particular locality, is directly proportional to its weight. The weight of a body is the attracitve *force* (F) of gravity existing between that body and the earth.

Unit of weigh $W = 1$ pound.
Unit of mass $M = ?$
Unit of force $F = 1$ pound.

Motion is a purely relative term: used to express the relation existi between two or more bodies when one or more of them is in a state "unrest."

If a body starting from rest is allowed to fall unrestrained toward the earth, its *rate of motion*, or *velocity* (v), through *space* (h or s) will increase constantly with the *time* (t) occupied in its descent, and this *increase in velocity* per second of time is called *acceleration* (g) or (a). Such a motion as described is called *uniformly accelerated motion*, the reverse of *uniformly retarded motion*; each of which is a form of *uniformly varying motion*.

Unit of veloci $v = 1$ ft. per se
Unit of space h or $s = 1$ foot
Unit of time $t = 1$ second.
Unit of accel. $g = 32.18 \pm$ ft.
Unit of accel. $a =$ any dist. when due to other forces than gravity.

Moreover, if, during its descent and after the body has acquired certain velocity, the action of gravity should suddenly cease, then accele tion would also cease and the body would continue with a constant *uniform motion*.

Fundamental Equations of Motion and Force.—In their relations one another, any force F acting on a mass M so as to produce accelerati a, may be considered analogous to the force of gravity W acting on t mass M and producing gravity acceleration g. Thus,

Weight = mass × g acceleration. Or, $W = Mg = \frac{Fg}{a}$; $g = \frac{W}{M} = \frac{Wa}{F}$; $M = \frac{W}{g}$.

Force = mass × a acceleration. $F = Ma = \frac{Wa}{g}$; $a = \frac{F}{M} = \frac{Fg}{W}$; $M =$

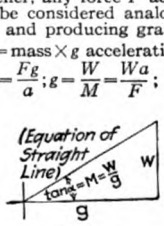

Fig. 1.

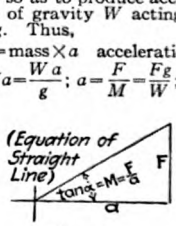

Fig. 2.

Ex. 1.—A car weighing 60 tons is impelled on a level track by a ho zontal force of 1 ton. Assuming the rolling friction at 8 lbs. per ton, fi the acceleration?

Ans.— $a = \frac{Fg}{W} = \frac{(2000 - 480)\ 32.2}{60 \times 2000} = 0.408$ ft. per sec.

Note.—The acceleration × the length of time of the acting force = t velocity in ft. per second (see Ex. 3).

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MOTION.

I. Uniform motion: v_0 constant, no acceleration.—With uniform motion a body moves at a constant velocity v_0, imparted to it by a force which has been removed; hence there is no acceleration. Note that v_0 becomes the initial velocity in uniformly varying (accelerated or retarded) motions, following.

$$v_0=\frac{h}{t}=\frac{h_1}{t_1}=\frac{h-h_1}{t-t_1};\ h=v_0t;\ t=\frac{h}{v_0}.$$

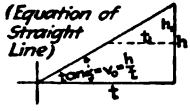

Fig. 3.

$$v_0=\frac{s}{t}=\frac{s_1}{t_1}=\frac{s-s_1}{t-t_1};\ s=v_0t;\ t=\frac{s}{v_0}.$$

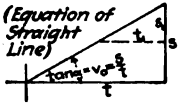

Fig. 4.

Ex. 2.—What velocity will be required to travel 16 miles in three-quarters of an hour?

Ans.— $v_0=\frac{s}{t}=\frac{16\times5280}{45\times60}=31.29$ ft. per sec.

Relative uniform motion.—A man walking *forward* with a velocity v_0', in a train moving with a velocity v_0'', acquires an actual velocity of $V_0=v_0'+v_0''$. Similarly, if walking *backward* in the train he would acquire a velocity of $v_0=v_0'-v_0''$, in the direction in which he is walking; or, $v_0=v_0''-v_0'$, in the direction of the moving train.

II. Uniformly accelerated motion; no initial velocity, v_0.—Problems that come under this head are those in which the body starts from rest and is acted upon by a constant force in the direction of its motion. A train starting from a station, or a body falling from a height, are examples. In the following, friction is neglected:

(*a*).—ACCELERATION, TIME AND VELOCITY.

$$v=gt;\ t=\frac{v}{g};\ g=\frac{v}{t}=\frac{v_1}{t_1}=\frac{v-v_1}{t-t_1}.$$

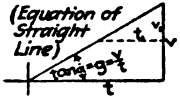

Fig. 5.

$$v=at;\ t=\frac{v}{a};\ a=\frac{v}{t}=\frac{v_1}{t_1}=\frac{v-v_1}{t-t_1}.$$

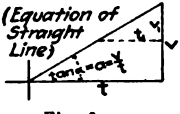

Fig. 6.

Ex. 3.—What velocity will a body acquire at the end of 10 seconds, falling in a vacuum?

Ans.— $v=gt=32.2\times10=322$ ft. per sec.

(*b*).—TIME, VELOCITY AND DISTANCE.

$$h=\frac{vt}{2};\ t=\frac{2h}{v};\ v=\frac{2h}{t}=\frac{2\,(h-h_1)}{t-t_1}.$$

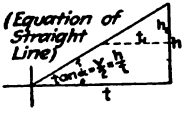

Fig. 7.

$$s=\frac{vt}{2};\ t=\frac{2s}{v};\ v=\frac{2s}{t}=\frac{2\,(s-s_1)}{t-t_1}.$$

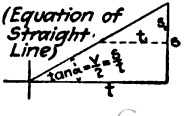

Fig. 8.

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Ex. 4.—A body falling in a vacuum attains a velocity of 322 ft. p sec. at the end of 10 seconds. Through what height has it fallen?

Ans.— $h = \frac{vt}{2} = \frac{1}{2}(322 \times 10) = 1610$ ft.

Note that $\frac{v}{2}$ is the *average* velocity for the time t.

(c).—VELOCITY, DISTANCE AND ACCELERATION.

$g = \frac{v^2}{2h}$; $h = \frac{v^2}{2g}$; $v = \sqrt{2gh} = 8.02\sqrt{h}$.

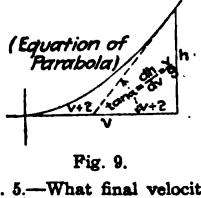

Fig. 9.

$a = \frac{v^2}{2s}$; $s = \frac{v^2}{2a}$; $v = \sqrt{2as}$.

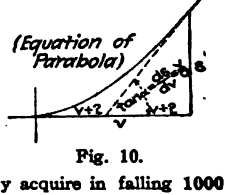

Fig. 10.

Ex. 5.—What final velocity will a body acquire in falling 1000 ft. the earth?

Ans.—Final velocity $= v = 8.02\sqrt{h} = 8.02 \times 31.62 = 253.6$ ft. per sec.

(d).—DISTANCE, ACCELERATION AND TIME.

$t = \sqrt{\frac{2h}{g}} = .249\sqrt{h}$; $g = \frac{2h}{t^2}$; $h = \frac{gt^2}{2} = 16.1t^2$.

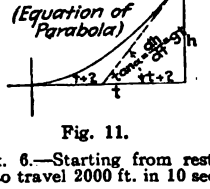

Fig. 11.

$t = \sqrt{\frac{2s}{a}}$; $a = \frac{2s}{t^2}$; $s = \frac{at^2}{2}$.

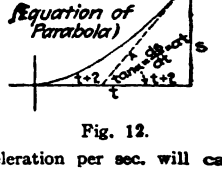

Fig. 12.

Ex. 6.—Starting from rest, what acceleration per sec. will caus body to travel 2000 ft. in 10 seconds?

Ans.— $a = \frac{2s}{t^2} = \frac{4000}{100} = 40$ ft. every second.

Note that the term "acceleration" means "rate or acceleration second" = increase in velocity per second.

III. Uniformly accelerated motion with positive initial velocity This is a case of uniformly varying motion in which the constant in velocity v_0 is in the *same direction* as the accelerated velocity v produce some constant, acting force. Let $V = v + v_0$ = the resultant velocity at time t measured from the instant that v begins to act; h or s = the dist traveled in that time; g or a = the rate of acceleration per second. the following relations exist, neglecting friction:

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(A).—Acceleration, Time and Velocity.

$V=v+v_0=gt+v_0;\ v=V-v_0;\ v_0=V-gt.$

$t=\frac{V-v_0}{g}=\frac{v}{g}.\quad g=\frac{V-v_0}{t}=\frac{v}{t}.$

$V=v+v_0=at+v_0;\ v=V-v_0;\ v_0=V-at.$

$t=\frac{V-v_0}{a}=\frac{v}{a}.\quad a=\frac{V-v_0}{t}=\frac{v}{t}.$

Ex. 7.—A stone is thrown from a balloon vertically downward toward the earth with a velocity of 100 ft. per sec., striking the earth with a velocity of 1000 ft. per sec. What is the time of its descent?

Ans.— $t=\frac{V-v_0}{g}=\frac{1000-100}{32.2}=28.0$ seconds.

(B).—Time, Velocity and Distance.

$h=\frac{t}{2}(V+v_0)=t\left(\frac{v}{2}+v_0\right);$

$t=\frac{2h}{V+v_0}=\frac{h}{\frac{v}{2}+v_0}.$

$V=\frac{2h}{t}-v_0;\ v=2\left(\frac{h}{t}-v_0\right);\ v_0=\frac{h}{t}-\frac{v}{2}.$

$s=\frac{t}{2}(V+v_0)=t\left(\frac{v}{2}+v_0\right);$

$t=\frac{2s}{V+v_0}=\frac{s}{\frac{v}{2}+v_0}.$

$V=\frac{2s}{t}-v_0;\ v=2\left(\frac{s}{t}-v_0\right);\ v_0=\frac{s}{t}-\frac{v}{2}.$

Ex. 8.—At what height above the ground is the balloon in Ex. 7, preceding?

Ans.— $h=\frac{t}{2}(V+v_0)=14\times1100=15400$ ft.

(C).—Velocity, Distance and Acceleration.

$g=\frac{V^2-v_0^2}{2h}=\frac{v(v+2v_0)}{2h}=\frac{v(2V-v)}{2h}.$

$h=\frac{V^2-v_0^2}{2g}=\frac{v(v+2v_0)}{2g}=\frac{v(2V-v)}{2g}.$

$V=\sqrt{2gh+v_0^2};\ v_0=\sqrt{V^2-2gh};\ v=\sqrt{v_0^2+2gh}-v_0.$

$a=\frac{V^2-v_0^2}{2s}=\frac{v(v+2v_0)}{2s}=\frac{v(2V-v)}{2s}.$

$s=\frac{V^2-v_0^2}{2a}=\frac{v(v+2v_0)}{2a}=\frac{v(2V-v)}{2a}.$

$V=\sqrt{2as+v_0^2};\ v_0=\sqrt{V^2-2as};\ v=\sqrt{v_0^2+2as}-v_0.$

Ex. 9.—At a point one mile above the earth's surface a rifle ball is fired downward to the earth with an initial velocity of 2200 ft. per sec. With what velocity does it strike the earth?

Ans.— $V=\sqrt{2gh+v_0^2}=\sqrt{2\times32.2\times5280+(2200)^2}=2276$ ft. per sec.

(D).—Distance, Acceleration and Time.

$t=\sqrt{\left(\frac{v_0}{g}\right)^2+\frac{2h}{g}}-\frac{v_0}{g}.$

$g=\frac{2}{t}\left(\frac{h}{t}-v_0\right).\quad h=t\left(\frac{gt}{2}+v_0\right).$

$t=\sqrt{\left(\frac{v_0}{a}\right)^2+\frac{2s}{a}}-\frac{v_0}{a}.$

$a=\frac{2}{t}\left(\frac{s}{t}-v_0\right).\quad s=t\left(\frac{at}{2}+v_0\right).$

Ex. 10.—In what length of time will a body travel 2000 ft., if the initial velocity is 20 ft. per sec., and the acceleration 10 ft. per sec.?

Ans.— $t=\sqrt{\left(\frac{v_0}{a}\right)^2+\frac{2s}{a}}-\frac{v_0}{a}=\sqrt{\left(\frac{20}{10}\right)^2+\frac{4000}{10}}-\frac{20}{10}=18.1$ seconds.

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IV. Uniformly accelerated motion with negative initial velocity v_0.—This is a case of uniformly varying motion in which the constant, initial velocity v_0 is *opposite in direction* to the accelerated velocity v produced by some constant, acting force. Let $v' = v - v_0$ = the resultant velocity, in the direction of v at any time t measured from the instant v begins to act; h or s = the distance (algebraic) traveled in that time; g or a = the rate of acceleration per second. Then the following relations exist, neglecting friction:

(*A'*).—ACCELERATION, TIME AND VELOCITY.

$$v' = v - v_0 = gt - v_0;\ v = gt;\ v_0 = gt - v'. \qquad t = \frac{v' + v_0}{g} = \frac{v}{g}. \qquad g = \frac{v' + v_0}{t} = \frac{v}{t}.$$

$$v' = v - v_0 = at - v_0;\ v = at;\ v_0 = at - v'. \qquad t = \frac{v' + v_0}{a} = \frac{v}{a}. \qquad a = \frac{v' + v_0}{t} = \frac{v}{t}.$$

Ex. 11.—From a point distant h (unknown) above the earth, a rock is thrown vertically upward with a velocity of 200 ft. per sec., and in falling strikes the earth with a velocity of 400 ft. per sec. What length of time is the stone in the air?

Ans.— $t = \frac{v' + v_0}{g} = \frac{400 + 200}{32.2} = 18.63$ seconds.

(*B'*).—TIME, VELOCITY AND DISTANCE.

$$h = \frac{t}{2}(v' - v_0). \qquad t = \frac{2h}{v' - v_0}.$$

$$v' = \frac{2h}{t} + v_0;\ v_0 = v' - \frac{2h}{t};\ v = 2\left(\frac{h}{t} + v_0\right).$$

$$s = \frac{t}{2}(v' - v_0). \qquad t = \frac{2s}{v' - v_0}.$$

$$v' = \frac{2s}{t} + v_0;\ v_0 = v' - \frac{2s}{t};\ v = 2\left(\frac{s}{t} + v_0\right).$$

Ex. 12.—In example 11, preceding, find the distance h of the starting point above the ground?

Ans.— $h = \frac{t}{2}(v' - v_0) = \frac{18.63}{2}(400 - 200) = 1863$ ft.

Note.—From formulas (c) we find that the rock ascended $h = \frac{v_0^2}{2g} = \frac{(200)^2}{2g} = 621$ ft.; and then descended $h = \frac{v'^2}{2g} = \frac{(400)^2}{2g} = 2484$ ft. See Ex. 13, illustrating this.

(*C'*).—VELOCITY, DISTANCE AND ACCELERATION.

$$g = \frac{v'^2 - v_0^2}{2h}. \qquad h = \frac{v'^2 - v_0^2}{2g}.$$

$$v' = \sqrt{2gh + v_0^2};\ v_0 = \sqrt{v'^2 - 2gh}.$$

$$a = \frac{v'^2 - v_0^2}{2s}. \qquad s = \frac{v'^2 - v_0^2}{2a}.$$

$$v' = \sqrt{2as + v_0^2};\ v_0 = \sqrt{v'^2 - 2as}.$$

Ex. 13.—In Example 11, find the distance h of the starting point above the ground, using the acceleration instead of the time (Ex. 12)?

Ans.— $h = \frac{v'^2 - v_0^2}{2g} = \frac{(400)^2 - (200)^2}{64.4} = 1863$ ft.

(*D'*).—DISTANCE, ACCELERATION AND TIME.

$$t = \sqrt{\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}} + \frac{v_0}{g}.$$

$$g = \frac{2}{t}\left(\frac{h}{t} + v_0\right). \qquad h = t\left(\frac{gt}{2} - v_0\right).$$

$$t = \sqrt{\left(\frac{v_0}{a}\right)^2 + \frac{2s}{a}} + \frac{v_0}{a}.$$

$$a = \frac{2}{t}\left(\frac{s}{t} + v_0\right). \qquad s = t\left(\frac{at}{2} - v_0\right).$$

Ex. 14.—In Example 11, find the distance h of the starting point above the ground, using the time, acceleration and initial velocity?

Ans.— $h = t\left(\frac{gt}{2} - v_0\right) = 1863$ ft.

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1.—Falling Bodies.*

(k = height of fall; t = time in seconds; v = final veloc. in ft. per sec.)

$v = gt = 32.16t$, $\quad = \sqrt{2gh} = 8.02\sqrt{h}$.

$t = \frac{v}{g} = .031095\ v$, $\quad = \sqrt{\frac{2h}{g}} = .2494\sqrt{h}$.

$h = \frac{v^2}{2g} = .015547\ v^2$, $\quad = \frac{gt^2}{2} = 16.08\ t^2$.

Veloc. v	Time t	Height h	Veloc. v	Time t	Height h	Veloc. v	Time t	Height h	Veloc. v	Time t	Height h
.1	.00311	.00016	42.	1.3060	27.425	490.	15.237	3732.8	1040.	32.339	16816.
.2	.00622	.00062	44.	1.3682	30.099	500.	15.547	3886.7	1060.	32.961	17469.
.3	.00933	.00140	46.	1.4304	32.897	510.	15.858	4043.8	1080.	33.583	18134.
.4	.01244	.00249	48.	1.4926	35.820	520.	16.169	4203.9	1100.	34.204	18812.
.5	.01555	.00389	50.	1.5547	38.867	530.	16.480	4367.2	1120.	34.826	19502.
.6	.01866	.00560	55.	1.7102	47.030	540.	16.791	4533.5	1140.	35.448	20205.
.7	.02177	.00762	60.	1.8657	55.969	550.	17.102	4703.0	1160.	36.070	20920.
.8	.02488	.00995	65.	2.0212	65.686	560.	17.413	4875.5	1180.	36.692	21648.
.9	.02799	.01259	70.	2.1766	76.180	570.	17.724	5051.2	1200.	37.314	22388.
1.0	.03109	.01555	75.	2.3321	87.452	580.	18.035	5230.1	1250.	38.869	24292.
1.2	.03731	.02239	80.	2.4876	99.501	590.	18.346	5411.9	1300.	40.423	26274.
1.4	.04353	.03047	85.	2.6431	112.33	600.	18.657	5596.9	1350.	41.978	28334.
1.6	.04975	.03980	90.	2.7985	125.93	610.	18.968	5785.0	1400.	43.532	30472.
1.8	.05597	.05037	95.	2.9540	140.31	620.	19.279	5976.3	1450.	45.088	32688.
2.0	.06219	.06219	100.	3.1095	155.47	630.	19.590	6170.6	1500.	46.642	34981.
2.25	.06996	.07871	110.	3.4204	188.12	640.	19.901	6368.0	1550.	48.197	37352.
2.50	.07774	.09717	120.	3.7314	223.88	650.	20.212	6568.6	1600.	49.752	39800.
2.75	.08551	.11757	130.	4.0423	262.74	660.	20.523	6772.3	1650	51.307	42327.
3.0	.09328	.13992	140.	4.3533	304.72	670.	20.834	6979.0	1700.	52.865	44931.
3.5	.10883	.19045	150.	4.6642	349.81	680.	21.145	7188.9	1750.	54.416	47613.
4.0	.12438	.24875	160.	4.9752	398.00	690.	21.455	7401.9	1800.	55.971	50372.
4.5	.13993	.31483	170.	5.2865	449.31	700.	21.766	7618.0	1850.	57.526	53210.
5.0	.15547	.38867	180.	5.5971	503.72	710.	22.077	7837.2	1900.	59.081	56125.
5.5	.17102	.47030	190.	5.9081	561.25	720.	22.388	8059.6	1950.	60.635	59117.
6.0	.18657	.55969	200.	6.2190	621.88	730.	22.699	8285.0	2000.	62.190	62188.
6.5	.20212	.65686	210.	6.5299	685.62	740.	23.010	8513.5	2100.	65.299	68562.
7.0	.21766	.76180	220.	6.8409	752.47	750.	23.321	8745.2	2200.	68.409	75247.
7.5	.23321	.87452	230.	7.1518	822.44	760.	23.632	8979.9	2300.	71.518	82245.
8.0	.24876	.99501	240.	7.4628	895.51	770.	23.943	9217.8	2400.	74.628	89551.
8.5	.26431	1.1233	250.	7.7737	971.69	780.	24.254	9458.8	2500.	77.737	97169.
9.0	.27985	1.2593	260.	8.0847	1051.0	790.	24.565	9702.9	2600.	80.847	105100.
9.5	.29540	1.4031	270.	8.3956	1133.4	800.	24.876	9950.1	2700.	83.956	113340.
10.0	.31095	1.5547	280.	8.7066	1218.9	810.	25.187	10200.	2800.	87.066	121890.
11.	.34204	1.8812	290.	9.0175	1307.5	820.	25.498	10454.	2900.	90.175	130750.
12.	.37314	2.2388	300.	9.3285	1399.2	830.	25.809	10710.	3000.	93.285	139920.
13.	.40423	2.6274	310.	9.6394	1494.1	840	26.120	10970.	3200.	99.504	159200.
14.	.43533	3.0472	320.	9.9504	1592.0	850.	26.431	11233.	3400.	105.73	179720.
15.	.46642	3.4981	330.	10.2613	1693.1	860.	26.742	11499.	3600.	111.94	201490.
16.	.49752	3.9800	340.	10.572	1797.2	870.	27.053	11768.	3800.	118.16	224500.
17.	.52865	4.4931	350.	10.883	1904.5	880.	27.364	12040.	4000.	124.38	248750.
18.	.55971	5.0372	360.	11.194	2014.9	890.	27.675	12315.	4200.	130.60	274250.
19.	.59081	5.6125	370.	11.505	2128.4	900.	27.985	12593.	4400.	136.82	300990.
20.	.62190	6.2188	380.	11.816	2245.0	910.	28.296	12874.	4600.	143.04	328970.
22.	.68409	7.5247	390.	12.127	2364.7	920.	28.607	13159.	4800.	149.26	358200.
24.	.74628	8.9551	400.	12.438	2487.5	930.	28.918	13447.	5000.	155.47	388670.
26.	.80847	10.510	410.	12.749	2613.5	940.	29.229	13737.	5500.	171.02	470300.
28.	.87066	12.189	420.	13.060	2742.5	950.	29.540	14031.	6000.	186.57	559690.
30.	.93285	13.992	430.	13.371	2874.6	960.	29.851	14328.	6500.	202.12	656860.
32.	.99504	15.920	440.	13.682	3009.9	970.	30.162	14628.	7000.	217.66	761800.
34.	1.0572	17.972	450.	13.993	3148.3	980.	30.473	14931.	7500.	233.21	874520.
36.	1.1194	20.149	460.	14.304	3289.7	990.	30.784	15238.	8000.	248.76	995010.
38.	1.1816	22.450	470.	14.615	3434.3	1000.	31.095	15547.	9000.	279.85	1259300.
40.	1.2438	24.875	480.	14.926	3582.0	1020.	31.717	16175.	10000.	310.95	1554700.

* Ex.—A body falling from a height h of 398 ft. reaches the earth in 5 seconds, attaining a *final* velocity v of 160 ft. per sec. If the body were shot vertically upward with an *initial* velocity of 160 ft. per sec. it would reach the starting point, 398 ft. above the earth, in 5 sec. Each motion would be just the inverse of the other. See, also, table on page 1155.

Summary of Preceding Motion Formulas.

Notation:

a = rate of acceleration in feet per second.
g = gravity acceleration in feet per second.
v = velocity at time t, in ft. per sec., due to acceleration only.
v_0 = constant or uniform or initial velocity in ft. per sec.
V = $v+v_0$ = resultant velocity in ft. per sec. with initial velocity *positive*.
v' = $v-v_0$ = resultant velocity in ft. per sec. with initial velocity *negative*.
s = direct distance in ft. from point of starting. (Used with a.)
h = direct distance in ft. from point of starting. (Used with g.)
t = time in seconds after starting.

Formulas:

I. Uniform motion; no acceleration.

$$v_0=\frac{h}{t}=\frac{s}{t}.\quad h=v_0t;\ s=v_0t.\quad t=\frac{h}{v_0}=\frac{s}{v_0}.$$

II. Uniformly accelerated motion; no initial velocity.

$$v=gt=at=\frac{2h}{t}=\frac{2s}{t}=\sqrt{2gh}=\sqrt{2as}.\quad h=\frac{vt}{2}=\frac{v^2}{2g}=\frac{gt^2}{2};\ s=\frac{vt}{2}=\frac{v^2}{2a}=\frac{at^2}{2}.$$

$$t=\frac{v}{g}=\frac{v}{a}=\frac{2h}{v}=\frac{2s}{v}=\sqrt{\frac{2h}{g}}=\sqrt{\frac{2s}{a}}.\quad g=\frac{v}{t}=\frac{v^2}{2h}=\frac{2h}{t^2};\ a=\frac{v}{t}=\frac{v^2}{2s}=\frac{2s}{t^2}.$$

III. Uniformly accelerated motion; initial velocity positive.

$$V=gt+v_0=at+v_0=\frac{2h}{t}-v_0=\frac{2s}{t}-v_0=\sqrt{2gh+v_0^2}=\sqrt{2as+v_0^2}.$$

$$v_0=V-gt=V-at=\frac{h}{t}-\frac{v}{2}=\frac{s}{t}-\frac{v}{2}=\sqrt{V^2-2gh}=\sqrt{V^2-2as}.$$

$$v=gt=at=2\left(\frac{h}{t}-v_0\right)=2\left(\frac{s}{t}-v_0\right)=\sqrt{v_0^2+2gh}-v_0=\sqrt{v_0^2+2as}-v_0.$$

$$h=\frac{t}{2}(V+v_0)=\frac{V^2-v_0^2}{2g}=t\left(\frac{gt}{2}+v_0\right);\ s=\frac{t}{2}(V+v_0)=\frac{V^2-v_0^2}{2a}=t\left(\frac{at}{2}+v_0\right).$$

$$g=\frac{v}{t}=\frac{V-v_0}{t}=\frac{V^2-v_0^2}{2h}=\frac{2}{t}\left(\frac{h}{t}-v_0\right);\ a=\frac{v}{t}=\frac{V-v_0}{t}=\frac{V^2-v_0^2}{2s}=\frac{2}{t}\left(\frac{s}{t}-v_0\right).$$

IV. Uniformly accelerated motion; initial velocity negative.

$$v'=gt-v_0=at-v_0=\frac{2h}{t}+v_0=\frac{2s}{t}+v_0=\sqrt{2gh+v_0^2}=\sqrt{2as+v_0^2}.$$

$$v_0=gt-v'=at-v'=v'-\frac{2h}{t}=v'-\frac{2s}{t}=\sqrt{v'^2-2gh}=\sqrt{v'^2-2as}.$$

$$v=gt=at=2\left(\frac{h}{t}+v_0\right)=2\left(\frac{s}{t}+v_0\right)=\sqrt{2gh+v_0^2}+v_0=\sqrt{2as+v_0^2}+v_0.$$

$$g=\frac{v}{t}=\frac{v'+v_0}{t}=\frac{v'^2-v_0^2}{2h}=\frac{2}{t}\left(\frac{h}{t}+v_0\right);\ a=\frac{v}{t}=\frac{v'+v_0}{t}=\frac{v'^2-v_0^2}{2s}=\frac{2}{t}\left(\frac{s}{t}+v_0\right).$$

The resultant of two constant velocities in different directions is a motion in a straight line, and may be termed the parallelogram of motions or the triangle of motions. Let v_0' and v_0'' represent two velocities of magnitude and direction shown in Figs. 13 and 14, and making any angle θ with each other. Then will V_0 be the resultant velocity both in magnitude and direction:

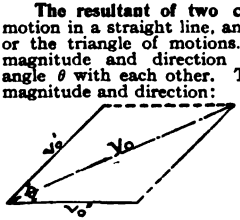

Fig. 13.

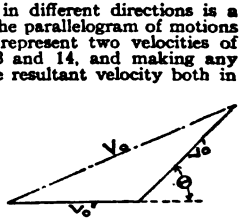

Fig. 14.

From Trigonometry, $V_0^2=v_0'^2+v_0''^2+2v_0'v_0''\cos\theta$;

$$\therefore\ V_0=\sqrt{v_0'^2+v_0''^2+2v_0'v_0''\cos\theta}.$$

Note.—The polygon of motions is analogous to the polygon of forces (see Figs. 31 to 35).

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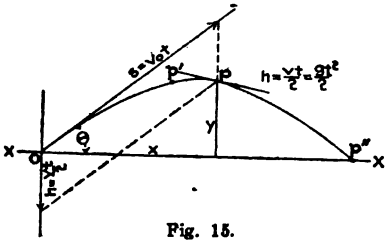

Fig. 15.

Parabolic Motion.—The path of a projectile, or of a jet from a nozzle, illustrates the resultant of a constant velocity v_0 in one direction, and a variable velocity v in another direction. Let x and y (Fig. 15) be the coordinates of any point p in the path of the resultant curve; then

$x = v_0 t \cos\theta$ = horizontal distance traveled;

$y = t\left(v_0 \sin\theta - \frac{gt}{2}\right)$ = vertical distance traveled

$$= x \tan\theta - \frac{gx^2}{2v_0^2 \cos^2\theta};$$

$\frac{dy}{dx} = \tan\theta - \frac{gx}{v_0^2 \cos^2\theta}$ = nat tan of angle which the tangent to the curve makes with the axis of X. Making $\frac{dy}{dx} = 0$, we obtain the coordinates x' and y' of the point p' at the vertex of the curve; thus,

$x = x' = \frac{v_0^2 \sin\theta \cos\theta}{g}$; and $y = y' = \frac{v_0^2 \sin^2\theta}{2g}$.

When p falls to the point p'' on the axis of X, $y = y'' = 0$; and $x = x'' = \frac{2v_0^2 \sin\theta \cos\theta}{g}$. $\therefore x'' = 2x'$, or the horizontal distance from o to p'' is double that from o to the vertex p'.

The general equation of the time t occupied in moving from o to any point p is

$$t = \sqrt{\left(\frac{v_0 \sin\theta}{g}\right)^2 - \frac{2y}{g}} + \frac{v_0 \sin\theta}{g}.$$

In moving from o to the vertex p', $\left(\text{making } y = \frac{v_0^2 \sin^2\theta}{2g}\right)$,

$$t = \frac{v_0 \sin\theta}{g}.$$

In moving from o to p'' on the axis of X, (making $y = 0$),

$$t = \frac{2v_0 \sin\theta}{g}.$$

The above discussion is a general case of which the following are special cases:

Case 1. *Initial velocity horizontal* ($\theta = 0$).—

$x = v_0 t.$ $\qquad y = -\frac{gt^2}{2}.$

Case 2. *Initial velocity vertical* ($\theta = 90°$).—

$x = 0.$ $\qquad y = t\left(v_0 - \frac{gt}{2}\right).$

(The last is a case of Uniformly Retarded Motion, being the reverse of "Uniformly Accelerated Motion with negative initial velocity," preceding. Note that y corresponds with h of the preceding Motion formulas; and that when y becomes a minus quantity, p is below the starting point.)

When the value of y is a minus quantity, in any of the above cases, it shows that the projectile p has fallen *below* the axis of X.

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Circular Motion.—Let p be a point on the rim of a fly-wheel of radius r, and revolving at n revolutions per unit of time. Then the velocity v of the point p is

$$v = 2\pi r n.$$

(Note that v takes the compound denomination of r and n; i.e., if $r =$ rad in *ft.*, and $n =$ rev per *sec.*, then $v =$ veloc in *ft.* per *sec.*)

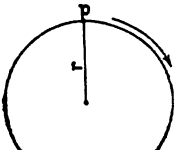

Fig. 16.

Ex. 15.—A driving rope traveling at the rate of 300 ft. per min runs over a pulley 4 ft. in *diam.* How many revolutions does the pulley make per min?

Ans.— $n = \frac{v}{2\pi r} = \frac{300}{4\pi} = 23.87$ rev per min. Ans.

Motion on Inclined Plane.—Neglecting friction, bodies falling from A would reach a (on inclined plane Aa), b (on inclined plane Ab), and H (distant $2r$ vertically below) in the same length of time t. Thus,

from general formula, $t = \sqrt{\frac{2h}{g}}$ for AH;

as $h = \frac{l}{\sin \alpha}$, $t = \sqrt{\frac{2l}{g \sin \alpha}}$ for Aa;

as $h = \frac{l'}{\sin \beta}$, $t = \sqrt{\frac{2l'}{g \sin \beta}}$ for Ab.

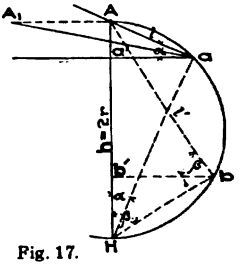

Fig. 17.

Moreover, the velocities *at the same elevation* would be equal; thus, the velocity of the body at a, descending on the inclined plane Aa, would be equal to the velocity of the body at a', falling freely through space. Similarly, the velocities at b and b' would be equal.

The following formulas relate to the inclined plane Aa, making an angle α with the horizontal:

$$v = gt \sin \alpha = \sqrt{2gl \sin \alpha}. \quad l = \frac{gt^2 \sin \alpha}{2} = \frac{v^2}{2g \sin \alpha}.$$

$$t = \frac{v}{g \sin \alpha} = \sqrt{\frac{2l}{g \sin \alpha}}.$$

The time occupied in the descent of bodies down inclined planes of the *same height* varies as the length of the planes. Thus, let $A_1a = 2 \times Aa = 2l$; then the time occupied in falling from A_1 to a would be double that occupied in falling from A to a.

Motion on Cycloidal Curve. — The cycloid (Fig. 18) is often called the curve of quickest descent. Let ACB be the cycloidal curve (of length $4d$) generated by a circle of diam d rolled along the plane AB. Then will a body starting at A fall to C and to B quicker than by any other curve. (For Properties of the Cycloid, see page 236.)

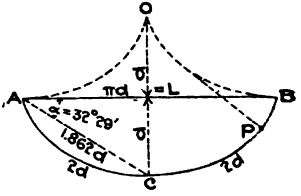

Fig. 18.

Time occupied in reaching C: $t = \frac{\pi}{2}\sqrt{\frac{2d}{g}} = \frac{1}{2}\sqrt{\frac{2\pi L}{g}}$.

Time occupied in reaching B: $t = \pi\sqrt{\frac{2d}{g}} = \sqrt{\frac{2\pi L}{g}}$.

By cycloidal curve.

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Time occupied in descent from A to C on the *inclined plane AC*, is $t=\sqrt{\frac{2l}{g \sin \alpha}}=1.862\sqrt{\frac{2d}{g}}$, as against $t=1.5708\sqrt{\frac{2d}{g}}$ for the cycloid. (See also Simple Cycloidal Pendulum, below.)

Simple Circular Pendulum.—This consists of a mass suspended from a fixed point O by a thread, of length l, considered as having no weight. The length of time t for a single vibration from A to B is

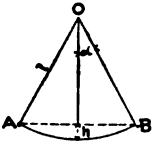

Fig. 19.

$t=\pi\sqrt{\frac{l}{g}}\left(1+\frac{h}{8l}\right)$ (Nearly exact.)

$t=\pi\sqrt{\frac{l}{g}}$ (Approximate but most frequently used; practically exact when α does not exceed 2°30′).

From the second equation we get, $l=\frac{gt^2}{\pi^2}$; $g=\frac{\pi^2 l}{t^2}$. The length of a pendulum which will give a single vibration in 1 second of time is $l=\frac{g}{\pi^2}$ ft. $=0.101321\,g$ ft. The value of g for any locality may be obtained from the formula*—

$$g=32.16954\,(1-0.00284\cos 2\lambda)\left(1-\frac{2h}{R}\right) \text{ ft. per sec.}$$

in which λ = latitude of the place,
h = its elevation in feet above sea level,
R = radius in feet of the earth at the latitude λ,
= 20 887 510 $(1+0.00164\cos 2\lambda)$,
= 20 900 000 ft., approximately.

The value of g in England is 32.2 ft.; in New York City, 32.17. The value of g usually assumed in the U. S. is 32.16; and of $\sqrt{2g}$, is 8.02, in hydraulics. The length of a pendulum which will make a single vibration in one second is therefore $l=0.101321\,g=0.101321\times 32.16=3.258$ ft. $=39.1$ ins., or a little less than 1 meter ($=3.28$ ft. $=39.37$ ins.).

The compound circular pendulum consists of a pendulum in which the mass is more or less distributed, instead of being concentrated at a single point as above (Fig. 19). It may be reduced to the simple circular pendulum by finding the distance l from O to the *center of oscillation*, otherwise termed the *center of percussion*. Assuming the whole pendulum to be rigid, the center of percussion is such a point that when struck sharply at right angle to the direction of the pendulum the latter will begin to oscillate or vibrate as a simple pendulum without producing any shock at its upper end or axis O. If the pendulum is a rod of uniform cross-section, and homogeneous, the center of percussion† will be a point distant $l=\frac{2}{3}$ of the length of the rod from O.

Simple Cycloidal Pendulum.—Let Op ($=2d$), Fig. 18, be the length of the pendulum vibrating between the cycloidal arcs OA and OB. Then will Op be the evolute of the cycloid ACB, and the mass p will trace the cycloidal curve. Hence the motion of the mass p will be the same whether it is swinging from O or simply rolling freely on ACB, friction neglected. Moreover, for any given cycloid the time of descent from *any point* on the curve will be $t=\pi\sqrt{\frac{d}{2g}}$; and the time of one oscillation, $t=\pi\sqrt{\frac{2d}{g}}$. In order that the time of one oscillation may be one second, make $2d=\frac{g}{\pi^2}=0.101321\,g$; ∴ the length of the cycloidal pendulum, $=2d$, will be the same length as the Simple Pendulum, preceding. The cycloidal pendulum is exact for any angle of vibration, while the simple pendulum is practically correct only for small arcs if the formula is adhered to strictly.

* See also formula for g under Gravity Acceleration, Weights and Specific Gravities of Materials (Section 27).

† See formula for Center of Percussion, page 303.

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DYNAMICS.

Work, Power, Energy, Etc.

Force.— In the preceding equations of motion, the mass of the movin body is not considered—simply abstract motions. When, however, th moving body is considered as having *mass*, it will take on the conceptio of *force* (=mass×acceleration), *work* (=force×distance), *power* (=rat of work=work÷time), and *energy* (=capacity for work); also "*impulse* (=force×time), and "*momentum*" (=mass×velocity),—remembering th the momentum imparted to a body is equal to the impulse which produc it. ("Impact" or "collision" is a blow, or pressure of such short duratio that it cannot be measured, between two bodies. See page 303.)

Considering "Force" F to be a *constant, unbalanced, resultant for* acting on a *total mass* M of *weight* W during the *time* t, the above relatio may be summarized as follows:

(a). Fundamental Relations; Resultant Force F Constant.
(Distance not included.)

Impulse = force × time = mass × velocity = momentum.....(

$$\frac{\text{weight} \times \text{velocity}}{g} = \text{"} \quad \text{.....}($$

Therefore,

	Mass	$M=\frac{Ft}{v}=\frac{F}{a}=\frac{W}{g}$	..... (
(Force-)	Accel	$a=\frac{F}{M}=\frac{Fg}{W}=\frac{v}{t}$	in ft. per sec..... (
	Weight	$W=Mg=\frac{Fg}{a}=\frac{Fgt}{v}$	in pounds. ..... (
(Gravity-)	Accel	$g=\frac{W}{M}=\frac{Wa}{F}=\frac{Wv}{Ft}$	in ft. per sec..... (
	Force	$F=\frac{Mv}{t}=\frac{Wa}{g}=\frac{Wv}{gt}$	in pounds..... (
	Time	$t=\frac{Mv}{F}=\frac{Wv}{Fg}=\frac{v}{a}$	in seconds..... (
	Velocity	$v=\frac{Ft}{M}=\frac{Fgt}{W}=at$	in ft. per sec..... (

Remarks.—The above formulas are for problems in which the for and therefore acceleration, is constant. Equated values may be subs tuted from one formula in another.

Problem 1.—A railroad train weighing 600 tons, 5 minutes after starti attains a velocity of 30 miles per hour. Find the tractive force of t engine drawing the train, the resistance being 8 lbs. per ton.?

Solution.—Tractive force T= unbalanced force F+resistance

From formula (7), $F=\frac{Wv}{gt}=\frac{600\times2000\times30\times5280}{32.16\times5\times60\times3600}=5473$ lbs.; and R $600\times8=4800$ lbs.; $\therefore T=F+R=5473+4800=10273$ lbs. Ans.

ATWOOD'S MACHINE.

Problem 2.—Atwood's machine consists of a flexible cord passing over a (frictionless) pulley and supporting equal weights at each end, with provision for an "unbalanced" weight at either end. In Fig. 20, the balanced weights are 5 lbs., and the unbalanced weight is 2 lbs. Find the acceleration of the weights, and the tension on the cord?

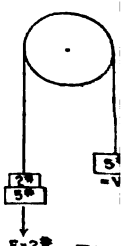

Fig.

Solution.—From formula (4), acceleration $a=\frac{Fg}{W}=\frac{2\times32.2}{12}=5.37$ ft. per sec.; and $\frac{a}{g}=\frac{F}{W}=\frac{2}{12}=\frac{1}{6}$. In considering the tension T on the cord, we have to consider the weight W' whose mass is M' and acceleration a

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d; thus, $T = F' + W' =$ (formula 7) $\frac{W'a}{g} + W' = \frac{5}{6} + 5 =$

Problem 2 corresponds with the resistance R in Prob-

d the acceleration, and tension
1, assuming no friction? (Note
wood's Machine, Fig. 20.)

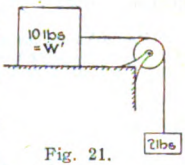

Fig. 21.

eration $a =$ (formula 7) $\frac{Fg}{W} =$

and $\frac{a}{g} = \frac{2}{12} = \frac{1}{6}$. As there is no
resistance, hence the tension

$+ R\ (R = 0) = F' =$ (formula 7) $\frac{W'a}{g} = \frac{10}{6} = 1\frac{2}{3}$ lbs. Ans.

e total force F ($= 2$ lbs.), $\frac{2}{12}$ or $\frac{1}{6}$ of it is employed in

, and the balance or $\frac{5}{6}$ ($= 1\frac{2}{3}$ lbs.) is employed in moving
ence the tension in the rope, as above.)
bove examples (2 and 3) the velocity at the end of say
9) $= v = at = 5.37 \times 10 = 53.7$ ft. per sec.

eral Relations; Resultant Force F Constant.
(Distance included.)

$$= Ft = Mat = \frac{Wat}{g} = \frac{2E}{at} = \frac{2Pt}{v} = \frac{2K}{at}\ (= N) \quad \left\{ \begin{array}{l} = \text{the result if total force} \\ \text{were expended} \\ \text{in one second.} \end{array} \right\} \quad (10)$$

$$= Mv = \frac{Wv}{g} = \frac{Fv}{a} = \frac{2E}{v} = \frac{2P}{a} = \frac{2K}{v}\ (= Z) \quad (11)$$

$$= \frac{W}{g} = \frac{F}{a} = \frac{K}{as} = \frac{Pt}{as} = \frac{E}{as} = \frac{Z}{at} = \frac{N}{v} \quad \ldots\ldots\ldots\ldots (12)$$

$$= \frac{F}{M} = \frac{Fg}{W} = \frac{v}{t} = \frac{2s}{t^2} = \frac{v^2}{2s} = \frac{Kg}{Ws} = \frac{2P}{Ft} = \frac{2P}{Z} = \frac{2P}{N} \quad \ldots (13)$$

$$= Mg = \frac{Fg}{a} = \frac{Kg}{as} = \frac{Pgt}{as} = \frac{Zg}{at} = \frac{Ng}{v} \quad \ldots\ldots\ldots\ldots (14)$$

$$= \frac{W}{M} = \frac{Wa}{F} \quad \ldots\ldots\ldots\ldots (15)$$

$$= Ma = \frac{Wa}{g} = \frac{Wv}{gt} = \frac{2Ws}{gt^2} = \frac{K}{s} = \frac{Pt}{s} = \frac{E}{s} = \frac{Z}{t} = \frac{Na}{v} \quad \ldots (16)$$

$$= \frac{v}{a} = \frac{2s}{v} = \sqrt{\frac{2s}{a}} = \frac{Wvs}{Kg} = \frac{2K}{Fv} = \frac{K}{P} = \frac{Fs}{P} = \frac{2P}{Fa} = \frac{Z}{F} \quad \ldots (17)$$

$$= at = \frac{2s}{t} = \sqrt{2as} = \frac{2K}{Ft} = \frac{2P}{F} = \frac{2E}{Z} = \frac{Na}{F} = \frac{2K}{N} \quad \ldots (18)$$

$$= \frac{vt}{2} = \frac{v^2}{2a} = \frac{at^2}{2} = \frac{K}{F} = \frac{Pt}{F} = \frac{E}{F} \quad \ldots\ldots\ldots\ldots (19)$$

$$= Fs = \frac{Was}{g} = \frac{Wvs}{gt} = \frac{2Ws^2}{gt^2} = \frac{Fvt}{2} = \frac{Fv^2}{2a} = \frac{Fat^2}{2} = Pt =$$

$$\frac{Zat}{2} = \frac{Nv}{2}\ (= E) \quad \ldots\ldots\ldots\ldots (20)$$

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Power $\left(\frac{\text{ft.-lbs.}}{\text{p. sec.}}\right)$ $P = \frac{K}{t} = \frac{Fs}{t} = \frac{Was}{gt} = \frac{Wvs}{gt^2} = \frac{2Ws^2}{gt^2} = \frac{Fv}{2} = \frac{Fv^2}{2at} = \frac{Fat}{2} = \frac{Za}{2} = \frac{Nv}{2t}$ ..... (21)

Energy (ft.-lbs.) $E = \frac{Mv^2}{2} = \frac{Wv^2}{2g} = \frac{Fv^2}{2a}$ = same values as for K, above $(=K)$ (22)

(Remarks.—The above formulas are for problems in which the force, and therefore acceleration, is constant. Equated values may be substituted from one formula in another.)

Problem 4.—How much energy is expended in raising a weight of 800 lbs., 10 ft.?

Solution.—From formula (20), $E = K = Fs = 800 \times 10 = 8000$ ft.-lbs. of energy or work. Ans.

Problem 5.—A weight of 4000 lbs. is raised 100 ft. in 5 seconds. Find the tension in the hoisting rope, the acceleration being uniform? (See Fig. 22.)

Solution.—Tension T = unbalanced force F + the resistance R $(=W)$ = (formula 16) $\frac{2Ws}{gt^2} + W = \frac{4000 \times 200}{32.2 \times 25} + 4000 = 994 + 4000 = 4994$ lbs. Ans.

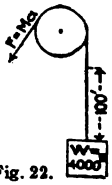

Fig. 22.

Work.—Work is the transformation of energy, and is simply force $(F) \times$ distance (s). The foot-pound (one pound raised one foot high, or its equivalent) is generally considered the unit of work unless other units are specified. The element of *time* is not a factor.

The Fundamental Formulas for Work are—

When the force F is constant, Work $(K) = Fs$ ..... (23)

When the force F is uniformly variable, Work $(K) = \int_{s_0}^{s_1} F ds$ ..... (24)

In which s_1 and s_0 are the limiting values of s, in feet.

Problem 6.—A rope weighing 6 lbs. per lin. ft. and 400-ft. long, is suspended at one end from a drum. Find the least number of ft.-lbs. of work required to wind up 100-ft. of it?

Solution.—Let s (Fig. 23) be the variable length of the hanging rope; then $6s$ will be its weight, and $6s\,ds$ the work done in raising it through the distance ds. Using formula (24), the total work done in raising the rope through 100-ft. of distance is

$$K = \int_{s_0}^{s_1} F ds = \int_{300}^{400} 6s ds = \left[\frac{6s^2}{2}\right]_{300}^{400} = 3\,(400^2 - 300^2)$$

$$= 210\,000 \text{ ft.-lbs. Ans.}$$

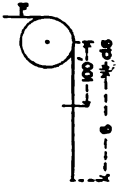

Fig. 23.

This is represented graphically in Fig. 24, the vertical ordinate F being the force applied for any length s of the hanging rope, and the shaded area representing the total least work performed. Thus, to wind up the whole rope the least amount of work required would be (making $s_0 = 0$) $\frac{6}{2} \cdot s_1^2 = 3 \times 400^2 = 480\,000$ ft.-lbs. = area whole triangle.

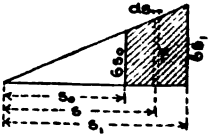

Fig. 24.

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: is the *rate* of work, that is, it is the amount of work
:ime. It bears the same relation to work that velocity

: as velocity $= \frac{\text{space traveled}}{\text{time}}$.

ver may be:

nd-second unit $= 1$ lb. raised 1 ft. in 1 sec.

nd-minute unit $= 1$ lb. raised 1 ft. in 1 min.

wer unit $\begin{cases} = 550 \text{ foot-pounds per sec.} \\ = 33000 \text{ foot-pounds per min.} \end{cases}$

ire of work per unit of time.

r power, with the force F constant, is—

$$\frac{\text{space}}{\text{me}} = \frac{Fs}{t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

locomotive draws a load of 600 tons at the rate of 20
resistance being 8 lbs. per ton. Find the horse-power?

$$\frac{Fs}{t} = \frac{8 \times 600 \times 20 \times 5280}{60} \text{ ft.-lbs. per min.}$$

$$= \frac{8 \times 600 \times 20 \times 5280}{60 \times 33000} \text{ horse power.}$$

$= 256$ h. p. Ans.

power $\left(\frac{Fs}{t}\right)$ is the rate of work Fs, it is possible through
pliances to accomplish a great amount of work with
le power, by increasing the time t of performing the
rsa. That the ancients understood this is evidenced by
ver, inclined plane, wedge, screw and pulley, in raising
V, greater than any available direct force F. All prob-
cter really come under the principle of *leverage*, simple

Notation.

ce in lbs. applied to first lever,

in ft. that F_1, or the initial lever end, moves in time t,
conds,

intermediate forces in lbs. at successive joints of com-
pound lever,

distances in ft. the intermediate joints move in time t,

force in lbs. at end of last lever = weight of mass raised
n ft. that F, or the final lever end, moves in time t.

General Formulas.

las: Power $P = \frac{F_1 s_1}{t} = \frac{Fs}{t}$; Work $K = F_1 s_1 = Fs \ldots$ (26)

mulas: Power $P = \frac{F_1 s_1}{t} = \frac{F's'}{t} = \frac{F''s''}{t} = \frac{F'''s'''}{t} = \text{etc.} = \frac{Fs}{t}$ (27)

Work $K = F_1 s_1 = F's' = F''s'' = F'''s''' = \text{etc.} = Fs$ (28)

nulas neglect friction or loss of energy in the mechanism;
e that the total power and work *applied* to the mechan-
into useful work.

—The force F_1 re-
e weight W (Fig.
mula (26), $F_1 =$
nce $\frac{l}{l_1} = \frac{s}{s_1}$).

Fig. 25.

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Compound Lever.—Let F_1 be the force required to raise W. F_1 and W may be equated with F' acting at the joint of the two levers. Thus,

$F_1 s_1 = F's' = Fs$; but $s' = \frac{s_1 l_1}{l_1'} = \frac{sl}{l'}$;

$\therefore \frac{s}{s_1} = \frac{l_1 l'}{l l_1'}$; $\therefore F_1 = F\frac{s}{s_1} = W\frac{l_1 l'}{l l_1'}$.

Fig. 26.

Note that the acting force multiplied by the continued product of the *left-hand* lever arms = the weight raised multiplied by the continued product of the *right-hand* lever arms. This principle holds true in belting and gearing also.

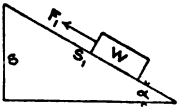

Fig. 27.

Inclined Plane.—Let W be a weight to be moved up an inclined plane of length s_1 and height s, by the force F_1. Then $F_1 s_1 = Ws$, or $F_1 = W\frac{s}{s_1}$, in which $\frac{s}{s_1} = \sin \alpha$.

Wedge.—The wedge is a double inclined plane.

Screw.—The screw is an inclined plane wound around a cylinder. The formula $\frac{F_1 s_1}{t} = \frac{Fs}{t}$ applies; in which F_1 is the acting force at the end of a lever attached to the screw; s_1 = distance traversed by the lever end in the time t; F = the force acted against by the screw; and s the progressive distance traversed by the screw in the time t. Or, we may use the formulas

$$F_1\sqrt{(2\pi l)^2 + \left(\frac{2pl}{d}\right)^2} = Fp \quad \text{(Exact)} \ldots\ldots\ldots\ldots (20)$$

$$F_1(2\pi l) = Fp \quad \text{(Approx.)} \ldots\ldots\ldots\ldots (30)$$

in which l = length of lever from *cen* of screw to applied force F_1, and p = pitch of screw, of *diam* d.

Pulley.—From formula (26), $F_1 s_1 = Ws$, in which W moves the distance s when F_1 moves the distance s_1. In Fig. 28 it is evident that $s_1 = 2s$; hence,

$$F_1 = W\frac{s}{s_1} = \frac{W}{2}.$$

Another method of determining the forces in pulley ropes is by "cutting sections." Thus, imagine all the forces to be in equilibrium, and draw a cutting plane *ab*. Then it follows that each of the two ropes cut, supports half the weight W, and the cutting plane *cb* shows that F_1 has the same stress $\frac{W}{2}$.

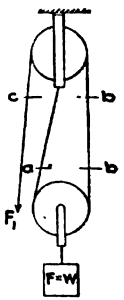

Fig. 28.

Simple and compound pulleys are thus often more easily analyzed by the "method of sections," although the principle of "work" applies universally.

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Toggle.—If the weight W is raised by the force F_1, use formula (26); thus, $F_1 s_1 = Ws$, or $F_1 = W\frac{s}{s_1} = W\frac{2b}{h}$. (It is to be noted that the required force F_1 gradually decreases as b decreases, and that an almost infinite force W may be raised by a finite force F_1 when b approaches zero.)

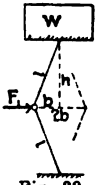

Fig. 29.

Proof: Let F_1 force the toggle joint to the right the infinitesimal distance s_1; then W will be raised the infinitesimal distance s, that is, $\frac{s}{2}$ for each lever arm l. Now as l remains constant, we have, before the movement, $l^2 = b^2 + h^2$; and after the movement, $l^2 = (b - s_1)^2 + \left(h + \frac{s}{2}\right)^2$. Equating, $b^2 + h^2 = b^2 - 2bs_1 + s_1^2 + h^2 + hs + \frac{s^2}{4}$. Remembering that the infinitesimal quantities s_1^2 and $\frac{s^2}{4} = 0$, this reduces to $hs = 2bs_1$, or $\frac{s}{s_1} = \frac{2b}{h}$. The same result may be obtained by the parallelogram of forces indicated in Fig. 29.

Impulse and Momentum.—If a constant force F acts for a length of time t on a Mass M, the latter will acquire a velocity v at the end of that time. Furthermore, the product Ft, called *impulse*, is equal to the product Mv, called *momentum*; or

$$(\text{Impulse}, Z =)\quad Ft = Mv \quad (= \text{Momentum}, N).$$

The acting force F (lbs.) may be any amount acting through any length of time t (sec.), or it may be a force t-times as large as F acting for one second on the mass; in either case the velocity v of the mass at the end of that time (t seconds in the first case and *one* second in the last case) will be the same, and therefore the momentum will be the same. Hence the momentum of a mass M $\left(= \frac{W}{g}\right)$ moving with a velocity v is equal to that force which in one second will give it that velocity; or, inversely, the amount of force which in one second will bring it to rest.

If the acting force is uniformly variable instead of constant, we have the equation

$$\int M dv = \int F dt \quad \text{or} \quad M\,(v_1 - v_0) = \int_{t_0}^{t_1} F dt$$

in which v_0 = the velocity when t equals t_0,
and v_1 = the velocity when t equals t_1.

Energy.—Energy is capacity for work. The energy of a body is measured by the amount of work which it is capable of performing, called *potential energy*; or by the amount of work it does perform, called *kinetic energy*.

For illustration, a cannon ball weighing 60 lbs. is raised from A to B, a distance of 20 feet. The work performed in raising it is, therefore, $\int F ds = Fs = 60 \times 20 = 1200$ ft.-lbs. The ball at B may now be considered to possess, by virtue of its position with reference to A, a potential (or statical) energy equivalent to 1200 ft.-lbs.; and which, if the ball is allowed to fall back to A, will be entirely expended in the form of kinetic (or actual) energy when it reaches A and its velocity is destroyed.

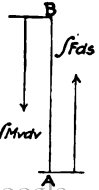

Fig. 30.

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From the preceding equation, $\int F dt = \int M dv$, multiplying both

of the equation by v, and remembering that $v dt = ds$, there is obtained,

Work in ascent stored up at B as potential energy = $\left\} \int F ds = \int_{0 \text{ at } B}^{v_1 \text{ at } A} M v dv \right\}$ = Kinetic energy in descent expended at A

Or, the general formula, $\int F ds = \frac{1}{2} M (v_1^2 - v_0^2)$ .....

If the initial velocity $v_0 = 0$, then $v_1 = v$ and we have,

$$\text{Energy } E = \tfrac{1}{2} M v^2 = \frac{W v^2}{2g} \text{ .....}$$

Or, as $h = \frac{v^2}{2g}$ (IIc), $E = Wh = Fs =$ work K.....

That is, the energy expended = the work performed.

Problem 8.—How much energy is expended in shooting a cannon weighing 50 lbs. with an initial velocity of 2300 ft. per sec?

Solution.—From formula (32) energy in ft.-lbs. $= \frac{W v^2}{2g} = \frac{50 \times 52900}{2 \times 32.1}$

4,112,250 ft.-lbs. Ans.

If fired vertically it would ascend, from formula (19),

$$h = \frac{E}{F} = \frac{E}{W} = \frac{4\ 112\ 250}{50} = 82\ 245 \text{ ft.}$$

RESULTANT FORCES.

Composition and Resolution of Forces.—On page 284 are explained parallelogram and triangle of uniform motions composed of two cons initial velocities v_0' and v_0'', producing the resultant initial velocity From Mechanics we learn that: *Forces are proportional to the velocities w they will impart to a given body in a unit of time.* Hence, if F_1 impar velocity v_0', and F_2 a velocity v_0'', then will R, the resultant of F_1 an (Figs. 31 and 32) of the parallelogram or triangle of forces, corres with V_0, the resultant of v_0' and v_0'' (Figs. 13 and 14) of the parallelog or triangle of motions. Thus, Fig. 31,

$$R^2 = F_1^2 + F_2^2 + 2 F_1 F_2 \cos \theta,$$

$$\text{or } R = \sqrt{F_1^2 + F_2^2 + 2 F_1 F_2 \cos \theta}.$$

When $\theta = 90°$, $\cos \theta = 0$, and

$$R = \sqrt{F_1^2 + F_2^2}.$$

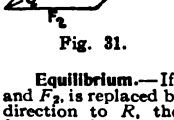

Fig. 31.

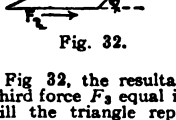

Fig. 32.

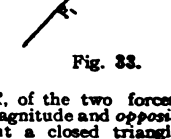

Fig. 33.

Equilibrium.—If, in Fig 32, the resultant R, of the two force and F_2, is replaced by a third force F_3 equal in magnitude and *opposi* direction to R, then will the triangle represent a closed triangl forces, or forces which if allowed to act at any common point P (Fig will be in equilibrium. Clearly, any one of the forces of a closed angle of forces is equal and opposite to the resultant of the other tw

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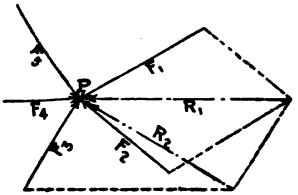

Fig. 34.

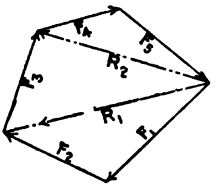

Fig. 35.

Polygon of Forces.—It is evident, from the preceding demonstration, that any number of forces as F_1, F_2, F_3, F_4 and F_5, acting in equilibrium at a common point P, Fig. 34, will, if drawn consecutively in rotation in the direction of the forces, form a closed polygon of forces, Fig. 35. For any polygon may be cut into triangles by the diagonal lines R_1, R_2, etc., each diagonal being the resultant of two other forces, and considered as replacing them. Thus R_1 replaces F_1 and F_2 so that we may consider only four forces acting, namely, R_1, F_3, F_4 and F_5; again, R_2 replaces R_1 ($=F_1$ and F_2) and F_3 so only three forces remain, namely, R_2, F_4 and F_5—a triangle of forces. Hence, as R_2, F_4 and F_5 are in equilibrium, so are all the forces composing the sides of the polygon in equilibrium. The principle of the force polygon, representing static equilibrium of forces, is fundamental to *Graphical Statics*, in the determination of stresses in structures.

Moments and Reactions.—Let AB be a horizontal lever 50-ft. long, fixed at, and free to rotate around, its left hand end A (Fig. 36). If now a weight $W = 60\#$ is applied vertically downward at a point $33\frac{1}{3}$-ft. distant from A, tending to cause the rod to rotate around A in a right hand motion, it is evident that some force R_2 applied at the other end B of the lever will preserve equilibrium or prevent rotation. The value of R_2 is obtained by taking moments M about A, as origin; thus,

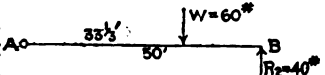

Fig. 36.

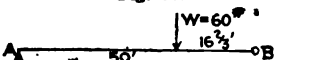

Fig. 37.

$\Sigma M = 0$: $33\frac{1}{3}\,W = 50\,R_2$; $\therefore R_2 = \frac{2}{3}\,W = 40\#$.

In a similar manner, if the lever is considered as fixed at, and free to rotate around, B (Fig. 37), we have $\Sigma M = 0$: $16\frac{2}{3}W = 50R_1$; $\therefore R_1 = \frac{1}{3}W = 20\#$.

Fig. 38 is the combined result of the two operations above, considering the lever AB as a beam 50-ft. long supporting the weight $W = 60\#$, producing a reaction of $20\#$ at A and $40\#$ at B. The weight of the beam itself is not considered.

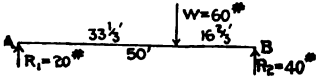

Fig. 38.

Center of Gravity and Resultant of a System of Parallel Forces.—The line of the resultant of a system of parallel forces passes through the *center of gravity* of the system. In Fig. 39, let P_1 ($=10\#$), P_2 ($=20\#$) and P_3 ($=30\#$) be a system of parallel forces or loads acting on a rigid body AB, at points 25 ft. apart. Then will R ($=60\#$) acting at the point O, be the resultant of the above forces and capable of replacing them in certain problems where the reactions are to be determined. In Alligation, page 57, we have a similar problem, that is, finding the average cost of a

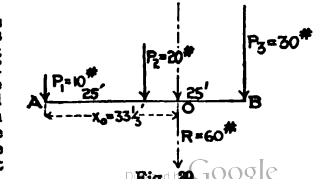

Fig. 39.

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mixture where barrels of cement replace pounds, and cents replace of the present problem; and in which the *origin of moments* is 200 ft. left of A. As a matter of fact, the origin of moments may be at any p but for convenience it will be assumed at A in the present case. taking moments about A, of the loads P_1, P_2 and P_3, we have—

Distance to center or gravity of resultant $= x_0 = \frac{\Sigma Px}{\Sigma P}$;

ΣPx = sum of moments of all the forces about center of moments A;
ΣP = sum of all the forces acting $= R$; whence,

$$x_0 = \frac{\Sigma Px}{\Sigma P} = \frac{\Sigma M}{R};\ \Sigma M = x_0 R;\ R = \frac{\Sigma M}{x_0}.$$

Taking moments about A as origin:

		ΣM
P_1	10×0	$= 0$
P_2	20×25	$= 500$
P_3	30×50	$= 1500$
$P_1 + P_2 + P_3$	$= 60$	2000
R	$\Sigma P \times x_0$	$= \Sigma M$

Therefore, $R = 60$ lbs.; and $x_0 = \frac{\Sigma M}{\Sigma P} = \frac{2000}{60} = 33\frac{1}{3}$ ft.

Note the similarity of analysis of this problem of concentrated force the following problem of a distributed force.

Resultant of a Distributed Force.—The resultant of a distrib force is equal to the total force, acting in a line passing through its center of gravity. We have seen (Fig. 39), that the horizontal distance x_0 to the line of the resultant of any system of forces, from any point taken as the origin of moments, is equal to the sum of the moments of the forces about that point, divided by the sum of the forces. The same principle holds true with a distributed force as with a system of forces or concentrated loads.

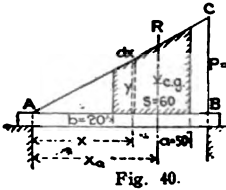

Fig. 40.

Problem 9.—Let AB, Fig. 40, be a girder of 60 ft. span, s, loaded w distributed force whose intensity, at any point distant x from the left ment, is the ordinate y from the line AB to the line AC; that is, the inte of the force increases uniformly from 0 lbs. at A, to 100 lbs. per lin. B, so that $y = \frac{100\,x}{s}$ lbs. 1*st*, find the general equation of the resulta and its distance x_0 from the left abutment A; 2*nd*, find R and x_0 fo span completely loaded from A to B; 3*rd*, find R and x_0 for the span k from $b = 20'$ to $a = 50'$.

Solution.—1st. $x_0 = \frac{\Sigma Px}{\Sigma P} = \frac{\text{summation of } x\,.\,ydx \text{ forces}}{\text{summation of } ydx \text{ forces}} = \frac{\Sigma M}{R}$.

$$x_0 = \frac{\int_b^a x\left(\frac{100\,x}{s}\right)dx}{\int_b^a \frac{100\,x}{s}\,dx} = \frac{\frac{100}{s}\int_b^a x^2 dx}{\frac{100}{s}\int_b^a x\,dx} = \frac{\frac{100}{s}\left[\frac{x^3}{3}\right]_b^a}{\frac{100}{s}\left[\frac{x^2}{2}\right]_b^a} = \frac{a^3-b^3}{3} \Big/ \frac{a^2-b^2}{2} = \tfrac{2}{3} \times \frac{a^3-}{a^2-}$$

In which $\frac{a^3-b^3}{3} \times \frac{100}{s}$ = sum of the moments of all the infinitesimal

$xydx = \Sigma M$;

$\frac{a^2-b^2}{2} \times \frac{100}{s}$ = sum of all the infinitesimal forces $ydx = R$;

$\tfrac{2}{3} \times \frac{a^3-b^3}{a^2-b^2}$ = distance from A to center of gravity of the tesimal forces $= x_0$;

a = upper limit of any assigned value to x;
b = lower limit of any assigned value to x.

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2nd. For the span completely loaded from A to B, the upper limit of $x=a=60'$, and the lower limit of $x=b=0$. Substituting these values in the general equation, we have,

$$R=\frac{a^2-b^2}{2}\times\frac{100}{s}=\frac{3600-0}{2}\times\frac{100}{60}=3000 \text{ lbs.};\ x_0=\tfrac{2}{3}\cdot\frac{a^3-b^3}{a^2-b^2}=\tfrac{2}{3}a=40 \text{ ft.}$$

3rd. For a span loaded between $x=a=50'$, and $x=b=20'$, we have,

$$R=\frac{a^2-b^2}{2}\times\frac{100}{s}=\frac{2100}{2}\times\frac{100}{60}=1750 \text{ lbs.};\ x_0=\tfrac{2}{3}\cdot\frac{a^3-b^3}{a^2-b^2}=\tfrac{2}{3}\cdot\frac{117000}{2100}=37\tfrac{1}{7} \text{ ft.}$$

In the same manner we may find the resultant and center of gravity of any distributed force, as parabolic, circular, elliptical, etc., by substituting the value of y in the equation of the curve before summing up all the *moments* of the ydx forces for $\Sigma Px=\Sigma M$, and all the ydx forces for $\Sigma P=R$. Note that the resultant R = the area of the curve, and the distance x_0 = the distance to its center of gravity.

Centrifugal Force.—Centrifugal force is that component of a force acting radially outward when a body moves along a curve with a certain velocity. In the general equation (16) of force, $F=Ma$, the acceleration a in the present instance is $\frac{v^2}{r}$ in which v = velocity of the moving body in feet per second, and r = radius of curve in feet. Hence,

$$\text{Centrifugal force}=F_c=M\frac{v^2}{r}=\frac{W}{g}\cdot\frac{v^2}{r}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (34)$$

Problem 10.—Find the tension in a string 10 ft. long, fixed at one end, with a weight of 40 lbs. fastened at the other and revolving around a circle, with the string as a radius, at a velocity of 30 revolutions per minute?

Solution.—In formula (34) F_c is the tension in lbs. in the string; and v, in feet per second, is $\frac{30}{60}.2\pi r$, or $v^2=\pi^2 r^2$.

$$\text{Therefore, tension, } F_c=\frac{40}{32.16}.\pi^2 r$$

$$=\frac{40\times 9.87\times 10}{32.16}=122.8 \text{ lbs.}$$

Note that in the above case, F_c is a concentrated resultant force acting normal to the curve at *one* point, and r (= radius of curve) is the distance from center of curve to cen of grav of moving body. Compare above problem with the two following.

Problem 11.—Find the radial pressure which a train weighing 1000 tons and moving 20 miles per hour, will produce on a track laid on a 7 degree curve.

Solution.—The radius of a 7° curve is 819 ft. Use formula (34):

$$F_c=\frac{W}{g}\cdot\frac{v^2}{r}=\frac{1000\times\left(\frac{20\times 5280}{60\times 60}\right)^2}{32.2\times 819} \text{ tons}$$

$$=32.63 \text{ tons}=65300 \text{ lbs. Ans.}$$

Note that in the above case the force F_c is a distributed force acting normal to the curve, throughout the length of the train.

Problem 12.—Given a ring (Fig. 41) 4 ft. in diameter, weighing 100 lbs., and revolving about its axis at a speed of 1000 revolutions per minute. Find the tension stress in the ring?

Solution.—[Note.—Distance of center of gravity of the semi-circular ring from center of circle $=r_0=\frac{2r}{\pi}=\frac{Diam}{\pi}=\frac{4}{\pi}$ (see page 207).]

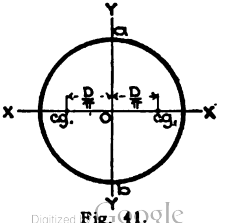

Fig. 41.

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Consider half the weight of the ring acting at each center of gravity of the semi-circle on the axis of X, causing tension at points a and b on the axis of Y. Then the total stress at a and b, from formula (34) is

$$F_c = \frac{W_1}{g} \cdot \frac{v_0^2}{r_0} \quad \text{in which} \begin{cases} W_1 = 50 \\ v_0 = 2\pi\, r_0 n = \frac{400}{3} \\ (n = \text{revolutions per second} = \frac{100}{3}) \end{cases}$$

$$= \frac{50}{32.2} \cdot \frac{400 \times 400 \times \pi}{3 \times 3 \times 4} = 21690 \text{ lbs.}$$

$\therefore \frac{1}{2} F_c = 10845$ lbs. = tension at a or b, or at any point. Ans.

Problem 13.—Prove that in order for a train to press normally upon the track in going around a curve of radius r (feet), the outer rail must be elevated an amount $e = \frac{G v^2}{g r} = \frac{G v^2}{32.2\, r}$ ..... (35)

in which e = elevation of outer rail in feet,
v = velocity of train in feet per second,
G = gauge of track in feet,
r = radius of curve in feet.

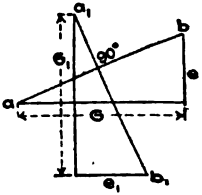

Fig. 42.

Solution.—The train is exerting two component forces, one horizontally due to centrifugal force and the other vertically due to gravitation, which can be reduced to one resultant force acting normal to the calculated inclined surface of elevated track. In Fig. 42 let e_1 represent intensity of centrifugal force, and $G_1 = W$ = weight of train; then $a_1 b_1$ is the resultant pressure which, in order to be normal to the track, must be at right angle to ab, G being the gauge (practically) and e the required elevation.

Through similar triangles, $\frac{e}{e_1} = \frac{G}{G_1}$; but from formula (34), $e_1 = F_c = \frac{W}{g}\frac{v^2}{r}$ and $G_1 = W$,

$$\therefore e = \frac{G e_1}{G_1} = \frac{G \frac{W}{g}\frac{v^2}{r}}{W}$$

$$\therefore e = \frac{G v^2}{g r} = \frac{G v^2}{32.2 r}. \qquad \textit{Required proof.}$$

Whence, elevation outer rail $= \frac{\text{gauge of track} \times (\text{velocity of train})^2}{32.2 \times \text{radius of curve in feet}}$.

MOMENTS OF INERTIA, ETC., OF PLANE SURFACES.

Bending Moment—Resisting Moment.—*We* will now consider the relation between the *moments of the outer forces* of a girder and the *moments of the inner forces*. For example, let Fig. 43 represent a wooden beam 4" wide, 6" deep and 10' long between supports, and loaded uniformly with 120 lbs per lin. ft. Consider the bending moment (of outer forces) and resisting moment (of inner forces) about the center of moments o, situated at the center of the span on the neutral axis $X-X$ of the beam. Then from the nature of the loading,

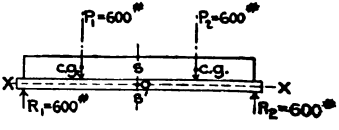

Fig. 43.

$R_1 = P_1 = P_2 = R_2 = \frac{1}{2}$ total load $= \frac{10 \times 120}{2} = 600$ lbs.; P_1 and P_2 each being the resultant of half the total load; R_1 and R_2 the reactions at the points of support. Taking moments of the outer forces to the left of the section at o, we have—

Bending moment $= M_B = R_1 \times 5 - P_1 \times 2\frac{1}{2} = 1500$ ft.-lbs. $= 18000$ inch-lbs.

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8000 inch-lbs. must of course be resisted by the moment acting on the right of the vertical section $s-s'$ passing her words, the resist- equal the bending mo- 44 represent an en- he beam at $s-s'$; F_c he compressive forces of the section *above* $o-o$; and F_t the re- ile forces acting to the on *below* the neutral as the neutral axis is the section, will $F_c=$ moments about o, we

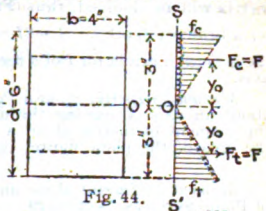

Fig. 44.

$M_R = 2\,Fy_0 = M_R = 18000$ inch-lbs., whence $F=\frac{9000}{y_0}$ lbs.

s the resultant of a distributed force varying in intensity center of the section to f_c at the top and f_t at the bottom so that the stress in any fiber is directly proportional to the neutral axis; hence, $f_c=f_t=f=$ the intensity of *outer* are inch; and the resultant forces $F\,F$ act in a line pas- enter of gravity of each distributed force (see Fig. 40), ance from the neutral axis of the beam to the outer fiber;

As the beam is 4 inches wide, we have $F=\frac{f}{2}\cdot\frac{bd}{2}=4\times 3$

ee above), or $f=750$ lbs per sq. in. That is, the maxi-

er stress on the beam is 750 lbs. per sq. in. (See also (36a) for the *ordinary methods* of finding the maximum

ent = Moment of Inertia $\times\frac{f}{y}=\frac{fI}{y}$.—The general formula resistance of a beam of any section, is

$$\frac{My}{I} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (36)$$

ter fiber stress per square inch,
ce in inches from neutral axis to outer fiber,
nt of inertia of section, about the neutral axis.

r beam of breadth b and depth d, $I=\frac{bd^3}{12}$ and $y=\frac{d}{2}$; hence,

ces to $M_R=\frac{1}{6}fbd^2$; $f=\frac{6\,M_R}{bd^2}$; $b=\frac{6\,M_R}{fd^2}$; $d=\sqrt{\frac{6\,M_R}{fb}}$ (36a)

d to agree with the preceding analysis, Fig. 44.

ertia* = Area $\times$ **(radius of gyra-** moment of inertia, I, about an y plane surface as bh, *resting on* um of the products of the areas of rips $x.dy$ by the squares of their he axis. Thus, for the rectangle xis of X, where $x=b$, and y varies $y=0$, we have, for beam of height h,

$$=I_h=\int_0^h b\,y^2\,dy=\left[\frac{by^3}{3}\right]_0^h=\frac{bh^3}{3}\,.. (37)$$

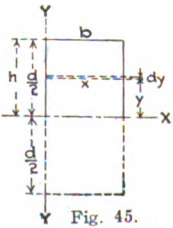

Fig. 45.

s of inertia of plane figures, see Properties and Tables, Section 29; also, see Properties and Tables of Steel Shapes,

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If now another similar and equal rectangle is added *below* the axis of X, making the total height $2h=d$, it is evident that the above moment of inertia will be doubled; thus (Fig. 45),

for a beam of depth d, $I_d=2I_h=\frac{2}{3}bh^3=\frac{bd^3}{12}$ ..... (38)

the general formula for I of a rectangular beam, of depth d, about its neutral axis.

Moment of Inertia about a Parallel Axis.—Let I be the moment of inertia about an axis X passing through the *cen of grav* of the plane figure; I', the moment of inertia about a *parallel axis* X', distant a from X; and A, the area of the plane figure; then will

$$I'=I+Aa^2.$$

Moment of Inertia about an Inclined Axis.—See Properties and Tables of Plane Surfaces, Section 29.

Radius of Gyration $=\sqrt{\frac{\text{Moment of Inertia}}{\text{Area}}}=\sqrt{\frac{I}{A}}$.—The radius of gyration r of a plane surface as bh, where moment of inertia is I_h, and area A_h, is the distance $r=y_1$, from the axis of X to a point (or line) at which if the whole *area* were concentrated the moment of inertia would remain the same.

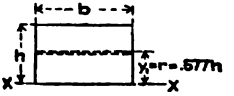

Fig. 46.

From the above and equation (37) we have, since $I_h=\frac{bh^3}{3}$ and $A_h=bh$,

$$r=\sqrt{\frac{I_h}{A_h}}=\sqrt{\frac{bh^3}{3}\div bh}=\frac{h}{\sqrt{3}}=\frac{h\sqrt{3}}{3}=.577\,h \text{ ..... (39)}$$

And for a beam of depth $d=2h$, with axis through *cen* of *grav*, equation (38),

$$r=\sqrt{\frac{I_d}{A_d}}=\sqrt{\frac{bd^3}{12}\div bd}=\frac{d}{\sqrt{12}}=\frac{d\sqrt{3}}{2\times 3}=\frac{.577d}{2}=.577\,h \text{ ..... (39a)}$$

In other words, the radius of gyration r is the same for a rectangle bh resting on the axis of X, as it is for a rectangle bd (in which $d=2h$) about its neutral axis passing through the center of the section.

Caution.—The radius of gyration r $(=y_1)$, Fig. 46, must not be confused with the distance y_0, Fig. 44, for—

r = distance from axis of X to cen of grav of moment-*areas*, while
y_0 = " " " " " " " " moment-*forces*.

In the rectangle bd which, for simplicity, has been referred to throughout the discussion,

$$y_0=\frac{M}{2F}=\frac{\frac{fI}{y}}{\frac{bdf}{2}}=\frac{2I}{bdy}=\frac{\frac{2\,bd^3}{12}}{\frac{bd^2}{2}}=\frac{d}{3}=\tfrac{2}{3}h;\quad \therefore y_0=\frac{2\sqrt{3}}{3}r=1.155r \text{ (40)}$$

$$r=\sqrt{\frac{My}{fA}}=\sqrt{\frac{I}{A}}=\sqrt{\frac{I}{bd}}=\sqrt{\frac{bd^3}{12bd}}=\frac{d}{\sqrt{12}}=\frac{\sqrt{3}\,h}{3};\quad \therefore r=\frac{\sqrt{3}}{2}y_0=.866\,y_0 \text{ (41)}$$

whence, $y_0^2=\frac{4}{3}r^2=\frac{d^2}{9}=\frac{4}{9}h^2$; and $r^2=\frac{3}{4}y_0^2=\frac{d^2}{12}=\frac{h^2}{3}$ ..... (42)

For other sections than rectangular, these ratios will not hold, necessarily.

Problem 14.—Find the moment of inertia I and radius of gyration r of a circular section (complete circle) of diameter D, about its diameter.

Solution.—The moment of inertia of a complete circle is 4 times that of the quadrant, Fig. 47; and I of the quadrant equals $I_q=\int_0^R y^2\,x\,dy$, because $x\,dy$ is the area of each infinitesimal strip, and y^2 is the

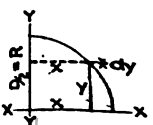

Fig. 47.

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tance from the axis of X. From the equation of the , $x = \sqrt{R^2 - y^2}$. Substituting this value of x in the above tegrating by formula V of Integral Calculus, page 275,

$$\overline{y^2}\, dy = \left[\frac{y}{8}(2y^2 - R^2)\sqrt{R^2 - y^2} + \frac{R^4}{8}\sin^{-1}\frac{y}{R} \right]_0^R$$

$$= \frac{R^4}{8}\sin^{-1} 1; \text{ but anti-sine of } 1 = 90° = \frac{\pi}{2},$$

ore $I_q = \dfrac{\pi R^4}{16}$;

$$\left.\begin{aligned} 4\, I_q &= \frac{\pi R^4}{4}; \\ \therefore I &= \frac{\pi D^4}{64}. \end{aligned}\right\} = \text{moment of inertia of the circle} \ldots\ldots (43)$$

tion $r = \sqrt{\dfrac{I}{A}} = \sqrt{\dfrac{\pi D^4}{64} \div \dfrac{\pi D^2}{4}} = \dfrac{D}{4} = \dfrac{R}{2}$ ..... (44)

Rectangular and Circular Beams Compared.—The general nce of a beam is $M = \dfrac{f\,I}{y}$; or $\dfrac{M}{f} = \dfrac{I}{y}$.* Now if two beams nd equal loading are to have the same *safe resistance* it eir outer fiber stresses f, as well as their resisting moments ctively equal. It remains then simply to equate the section

two beams. Thus—

beam of breadth b and depth d, $\dfrac{I}{y} = \dfrac{bd^2}{6}$.

m of diameter D, $\dfrac{I}{y} = \dfrac{\pi D^4}{64} \div \dfrac{D}{2} = \dfrac{\pi D^3}{32}$.

ve values of $\dfrac{I}{y}$, we have $\dfrac{\pi D^3}{32} = \dfrac{bd^2}{6}$.

diameter of the circular beam, $D = 2\sqrt[3]{\dfrac{2\,bd^2}{3\pi}}$,

epth of the rectangular beam, $d = \frac{1}{2}\sqrt{\dfrac{3\,\pi\,D^3}{b}}$,

idth of the rectangular beam, $b = \dfrac{3\,\pi\,D^3}{16\,d^2}$.

beam of $6\frac{1}{2}$ ins. *diam* will give the same safe resistance as m 4″ wide and 6″ deep, Fig. 44.

vity.—The distance to the center of gravity of any plane d axis, is equal to the sum of the moments of each infini- e figure about that axis, divided by the total area of the

is X_1, distance $y_0 = \dfrac{\int x\,y\,dy}{\int x\,dy}$ ..... (45)

its value from the equation of the figure;

is Y_1, distance $x_0 = \dfrac{\int y\,x\,dx}{\int y\,dx}$ ..... (46)

its value from the equation of the figure.

the *section modulus*, and is equal to the moment of inertia istance of the most extreme fiber from the neutral axis. s. 39 and 40.

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It is not necessary, however, to find the moments of *infinitesimal* areas; in fact, the figure may be divided into any number of areas of such shape that their centers of gravity are easily determined. Then the distance from the fixed axis to the center of gravity of the plane figure, is equal to the sum of the products of each area into the distance of its center of gravity from that axis, and this divided by the total area of the figure. Furthermore, the areas whose moments are thus obtained may form one plane figure or any number of plane figures. In the latter case, the center of gravity obtained would be the center of gravity of all the figures. The *exact point* of the center of gravity is determined from *two* coordinate axes.

The center of gravity of a plane figure or system of plane figures or areas, connected or isolated, is the position of the resultant of a uniformly distributed force over the figure or areas; hence a thin sheet, of any outline, may be balanced on a point of support applied at its center of gravity.

MOMENTS OF INERTIA, ETC., OF SOLIDS.

Moment of Inertia.—The moment of inertia of a solid body, about a fixed axis, is the sum of the products of the weight of each infinitesimal particle of matter (composing the body) into the square of its distance from said axis. Hence, the moment of inertia is obtained by the use of the Integral Calculus. An approximation to exact values may be obtained by assuming a *definite* number of *very small* particles, multiplying the weight of each by the square of the distance from its center of gravity to said axis, and finding the sum of these products.

For an axis X passing through the *cen* of *grav* of the body, the moment of inertia $I_x = \Sigma wr^2$; where w = the weight of each element, and r = its *dist* from the axis X.

For an axis X' parallel with and distant d from X, the moment of inertia $I'_x = I_x + Wd^2$; where W = the total weight of the body. Hence, knowing the value of I_x about an axis passing through the *cen* of *gravity*, the value of I'_x about any axis parallel with it can easily be obtained.

2.—MOMENT OF INERTIA OF REGULAR SOLIDS.

Description.	About axis X through $c.$ of $g.$	About ‖ axis X' dist d from X
Sphere of total weight W, and radius r.....	$\frac{2}{5}Wr^2$	$W\left(\frac{2}{5}r^2+d^2\right)$
Circular plate of rad r; axis perp to plate ..	$\frac{Wr^2}{2}$	$W\left(\frac{r^2}{2}+d^2\right)$
Circular cylinder of length $2l$, and radius r — Axis longitudinal...	$\frac{Wr^2}{2}$	$W\left(\frac{r^2}{2}+d^2\right)$
Circular cylinder of length $2l$, and radius r — Axis perp to axis of cylind.....	$W\left(\frac{l^2}{3}+\frac{r^2}{4}\right)$	$W\left(\frac{l^2}{3}+\frac{r^2}{4}+d^2\right)$
Circular ring, outer rad r, inner rad r_1; axis perp	$W\left(\frac{r^2-r_1^2}{2}\right)$	$W\left(\frac{r^2-r_1^2}{2}+d^2\right)$
Rod or bar of uniform cross-section and length $2l$; axis perp to length of rod.....	$\frac{Wl^2}{3}$	$W\left(\frac{l^2}{3}+d^2\right)$

Radius of Gyration.—The radius of gyration of a solid about a fixed axis, is the square root of the quotient obtained by dividing the moment of inertia about that axis, by the total weight of the body. Thus, from Table 2, above, the radius of gyration of a circular plate about a *perp* axis through its center $= \sqrt{\frac{Wr^2}{2} \div W} = \frac{r}{\sqrt{2}} = \frac{r\sqrt{2}}{2}$.

The radius of gyration of a body is the distance from the axis to the *center of gyration* or point at which if the whole mass were concentrated the moment of inertia would remain the same.

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Center of Gravity.—The distance to the center of gravity of any body from a fixed plane, is equal to the sum of the moments of the weight of each infinitesimal particle of matter of the body into its *perp* distance from that plane, divided by the weight of the body.

It is not necessary, however, to find the moments of *infinitesimal* particles; in fact, the body may be divided into any number of parts of such shape that their centers of gravity are easily determined. Then the distance from the fixed plane to the center of gravity of the body, is equal to the sum of the products of the weight of each mass into the distance of its center of gravity from that plane, and this divided by the total weight of the body. Furthermore, the masses whose moments are thus obtained may form one body or any number of bodies. In the latter case, the center of gravity obtained would be the center of gravity of all the bodies. The *exact point* of the center of gravity is determined from *three* coordinate planes.

If the center of gravity of a solid is supported on a point, the whole body will be in unstable equilibrium.

Center of Oscillation = Center of Percussion, *about Axis O.*—The *center of oscillation P* of a mass swinging or oscillating about a fixed axis passing through *O*, is a point at which if the whole mass were concentrated the oscillation would remain the same. The *center of percussion P* of the mass suspended from the axis at *O*, is that point at which if a horizontal force be applied there will be no shock at *O*. The two points *P* are identical. Let any mass *M*, Fig. 48, be suspended from an axis at *O*. Let *G* be the *cen* of *grav* of the mass, *C* the center of gyration, and *P* the center of oscillation or percussion; and let y_0, r and l be the respective distances to these points from the plane *X*; r being the radius of gyration of the mass about *O*, and l the radius of oscillation. Then will r be a mean proportional between y_0 and l, so that

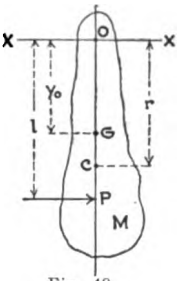

Fig. 48.

$$\text{radius of oscillation } l = \frac{r^2}{y_0} = \frac{(\text{radius of gyration})^2}{\text{dist to cen of grav}} \quad \ldots\ldots\ldots\ldots\ldots (47)$$

To illustrate: For a rod of length L and of uniform section, suspended freely from its upper end: $y_0 = \frac{L}{2}$, $r^2 = \frac{L^2}{3}$, and $l = \frac{2}{3}L$.

Note that *O* and *P* are interchangeable; that is, if *P* becomes the *axis*, then *O* will be the center of oscillation or percussion.

The theory of the compound pendulum is based on the above principles.

Impact or Collision.—The "line of impact" of two bodies in collision is a line *normal to the surfaces* at the "point of contact," irrespective of the relative motion of the two bodies. Impact is—

Central, when the line of impact coincides with the line joining the centers of gravity of the two bodies; *eccentric*, when it does not.

Direct, when the line of impact coincides with the line of relative motion of the two bodies; *oblique*, when it does not.

Central impact may be either direct or oblique.

Direct impact may be either central or eccentric.

In all cases of impact, action = reaction.

Notation.

m_1, m_2, = the respective masses,

c_1, c_2, = the respective velocities before impact,

v', v'', = the respective velocities at any given instant during impact,

v_1, v_2, = the respective velocities after impact,

v, = common velocity at time of greatest compression,

e, = coefficient of restitution in imperfectly elastic contact.

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Central Impact Formulas.

Velocity at time of greatest compression, $v = \frac{m_1 c_1 + m_2 c_2}{m_1 + m_2}$ ..... (48)

Coefficient of restitution, $e = \frac{\text{veloc regained after compression}}{\text{velocity lost during compression}} = \frac{v - v_1}{c_1 - v} = \frac{v_2 - v}{v - c_2}$ ..... (49)

For perfectly elastic bodies, e would $= 1$; but as all bodies are imperfectly elastic, e is always less than unity, showing that there is a loss of energy during impact.

Loss of energy due to inelastic impact $= \frac{m_1 m_2}{2(m_1 + m_2)}(c_1 - c_2)^2$ ..... (50)

Loss of energy due to imperfectly elastic impact $= (1 - e^2)\frac{m_1 m_2}{2(m_1 + m_2)}(c_1 - c_2)^2$ ..... (51)

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16.—THEORY OF STRESSES IN STRUCTURES.

OUTER AND INNER FORCES.

A structure is designed to resist safely the loads which may come upon it. These loads (which include the weight of the structure itself) together with their attendant reactions, are called the external or *outer forces*; while the stresses which these outer forces produce in the members of the structure themselves, are called the internal or *inner forces*; thus—

Outer forces = { (a.) Loads (= live, dead, wind, snow, etc.).
(b.) Reactions.

Inner forces = (c.) Stresses in the members.

Assuming that the *loads* acting on the structure are known, the first step is to find one or more of the *reactions*; and from these outer forces the *stresses* in the members may be determined by the use of the analytical or the graphical methods, or by the two methods combined. In the operation of finding the reactions and stresses it is well to remember the following principles:

PRINCIPLES OF STATIC EQUILIBRIUM.

First.—The algebraic sum of the moments* of the outer forces about any point is equal to zero; from which, considering right-hand (clockwise) moments *plus* and left-hand (un-clockwise) moments *minus*, or vice versa, we have—

Summation of moments = 0: $\Sigma M = 0$ ..... (1)

Second.—The algebraic sum of the components, in any given direction, of the outer forces is equal to zero; from which, considering those downward or to the right, *plus*, and those upward or to the left, *minus*, or vice versa, we have—

Summation of vertical components = 0: $\Sigma V = 0$ ..... (2)
Summation of horizontal components = 0: $\Sigma H = 0$ ..... (3)

Third.—For clearness and simplicity of calculation, in the application of equations (1), (2) and (3), the stresses in certain members of the truss or frame-work may temporarily be considered as outer forces, by cutting the structure in two by certain planes (straight or curved) properly intersecting the members in question—considering that portion of the structure on one side of the cutting plane (usually the left side) as a complete structure; and the stresses in the members so cut, as outer forces acting on the section to produce equilibrium. That is, these *imaginary* outer forces are the stresses to be calculated. (See Figs. 1 and 2.) Generally, only three† *active* members may be cut; thus—

Cut three active members; intersection of either two is origin of moments for calculating the stress in the third member. .... (4)

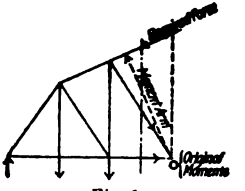

Fig. 1.

* The moment of a force about a given point, as origin, is the product of the force and the shortest distance from the origin to the line of the force.
† See "Notes," page 726, for cutting of *four* active members.

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PRINCIPAL METHODS OF CALCULATION.

Method of Moments.—This method of calculation, using equation (is usually employed in determining the reactions at the points of suppo the stresses in the chord members of bridges and roofs and in the web me bers where the chord members are not parallel; the wind stresses in t main vertical members of towers, buildings and similar structures; the co pressive stresses in masonry dams; etc.

Method of Shears.—This method, using equations (2) and (3), is usua employed in calculating the stresses in the web members of any frame-wo connecting parallel chords, as the lateral systems of bridges; the web me bers of simple bridge and roof trusses; the bracing of towers; the sh in dams; etc.

The stress in any *vertical* web member which "takes up" *all* the she is equal to the algebraic sum of the vertical components of the outer for to the left of the section cutting the member; and the stress in any *diago* member is equal to the vertical shear multiplied by the secant of angle inclination of that member with the vertical.

The term "vertical" may be used in any uniformly specific directio usually at right angle with the axis of the structure and parallel with t direction of the prevailing outer forces. To epitomize:

Stress in any member carrying all the shear = vertical shear × secant of angle of inclination with the vertical. } ....

The above five laws are fundamental in the determination of the stres in any structure which is statically determinate.

Graphical Methods.—The most common methods of determin stresses is by graphics, which may be performed by "moment diagran or by diagrams involving the principle of the "triangle of forces" (Mechanics, Figs. 31 to 35). The latter is called Maxwell's method s is the most used. (See Figs. 4, etc., following.)

PRACTICAL APPLICATION OF PRECEDING PRINCIPLES.

CASE A. LOADS AND REACTIONS VERTICAL.

Problem.—Find the dead-load stresses in a 6-panel Pratt truss 120-ft. span, 23 ft. in height center to center of chords; assuming t dead load per truss at 500 lbs. per lin. ft. and acting at the lower pa points, Fig. 2.

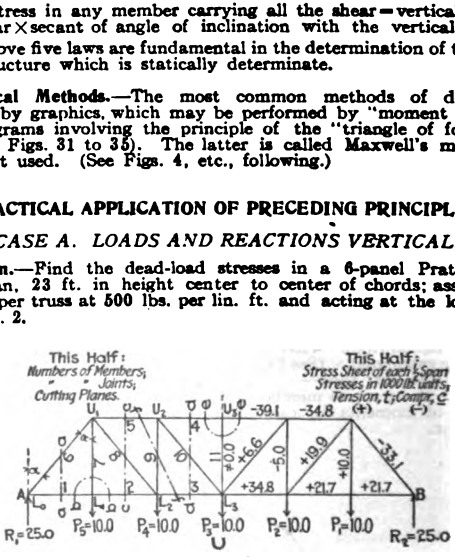

Fig. 2.

Calculation.—Stresses are usually in thousand-pound units and cal lated to the nearest hundred pounds; thus, 64.7=64700 lbs.; 64.0=64000 l 64=64 lbs., etc. That is, where the stress is abbreviated to the thousand-po unit there should be one figure to the right of the decimal point, whet it is a significant figure or merely zero, to show that the stress is in abbreviated form. The sign (−) before, or (*c*) after, the stress indic that the member is in compression, as −64.7=64.7*c*; while the signs (and (*t*) indicate tension, as +64.7=64.7*t*.

Load per panel per truss = 500 × 20 = 10000 lbs. = 10.0.

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Reaction at Left Support.—Assuming the panel lengths as unity and taking moments about the right hand abutment B, using equation (1), $\Sigma M=0$, we have,

$$R_1 \times 6 - 10.0\ (5+4+3+2+1) = 0.$$

Hence, reaction $R_1 = \frac{150.0}{6} = 25.0$, acting upward.

Lengths of Members.—All horizontal or chord members are 20-ft.; all verticals (posts and end suspenders) are 23-ft.; hence, all diagonals (bars and end posts) are

$$\sqrt{20^2+23^2} = 30.48 - \text{ft.} \quad \text{Thus, hor} = 20;\ \text{ver} = 23;\ \text{diag} = 30.48.$$

Trigonometric Ratios.—The angle α, Fig. 2, is the angle of inclination of the diagonal members with the vertical; and its two functions, $\tan\alpha$ and $\sec\alpha$, are respectively employed in calculating the stresses in the chords and diagonal members of bridges with parallel chords.

Bending Moments and Chord Stresses:

Let O, Fig. 3, be the origin of moments at any panel point of either chord, obtained by cutting three active members, see equation (4), and let it be required to find the stress in the opposite chord member so cut. Let any number of panel loads P_1 act to the left of O, and any number of panel loads P_2 act to the right of O, on the span Nl; in which

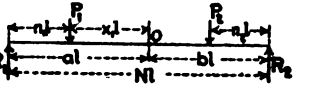

Fig. 3.

l = length in feet of one panel,
N = number of panels in the span,
n_1, a = number of panels from R_1 to P_1 and O, respectively,
n_2, b = number of panels from R_2 to P_2 and O, respectively.

$$\text{Then, Bending moment at } O = M_0 = \frac{\Sigma P_1 n_1 . bl + \Sigma P_2 n_2 . a l}{N} \quad \ldots\ldots\ldots (6)$$

And if d represents the effective depth of truss, we have,

$$\text{Stress in chord member} = \frac{M_0}{d} = \frac{M_0 \tan\alpha}{l} = \frac{\Sigma P_1 n_1 b + \Sigma P_2 n_2 a}{N} \tan\alpha \ldots (7)$$

which are the general equations for finding the bending moments and chord stresses without first finding the reactions.

Equations (6) and (7) are, however, very little used, and it is probably best for the beginner to employ equations (8) and (9), following, in which the reaction R_1 must first be determined.

$$\text{Then, Bending moment at } O = M_0 = R_1 . al - \Sigma P_1 . x_1 l \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

$$\text{And, Stress in chord member} = \frac{M_0}{d} = (R_1 a - \Sigma P_1 x_1) \tan\alpha \ldots\ldots\ldots\ldots (9)$$

in which x_1 = number of panels from P_1 to O.

Shears and Web Stresses:

Let O, Fig. 3, be the *line* of shear cutting usually two or three active members, including one web member which takes up all the shear. Then—

Vertical shear at $O = S_0 = R_1 - \Sigma P_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$

Stress in web member $= (R_1 - \Sigma P_1) \sec\alpha \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$

in which α = angle of inclination of web member with the vertical,
and ΣP_1 = sum of all loads to left of cutting *line*.

Stresses in Chord Members by Method of Moments: $\Sigma M = 0$.—In addition to determining the *amount* of stress in the various members of a structure, it is of course necessary that the *kind* of stress, whether tensile or compressive, be known in each case. Certain rules may be followed in determining the amount and kind of stresses in statically determined structures, as included in the following steps:

First.—Draw a plane *through* the structure cutting *three active* members, including the one to be calculated. The point of intersection of any two of the members will be the *center of moments* for determining the stress in the third.

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Second.—For the member in question, find the intersection of the other two members or "center of moments" about which the moments of the outer forces, acting on the section to the left of the cutting plane, are to be taken. These will include the reaction at the left abutment; and the moment equation, $\Sigma M = 0$, will include also the required stress assumed temporarily as an outer force and acting at the right of the cutting plane.

Third.—Take moments about the center of moments, *assuming* that the above last mentioned unknown force (the required stress) is *plus* (+) and acts away *from* the section. All other forces tending to produce moments in the same direction as the above may be considered as (+) forces and their moments as (+) moments; while all other outer forces, that is, forces tending to produce moments in the *opposite* direction may be considered as (−) forces and their moments as (−) moments.

Fourth.—If, in the equation $\Sigma M = 0$, the result of the required stress is a (+) quantity, the member is in tension; while if it is a (−) quantity the member is in compression. Thus,

Assume required stress as $+S$ acting away *from* the section.....(12)
If the result of S is +, the member is in tension.....(13)
" " " " S is −, " " " " compression.....(14)

Table 1 is based on the above methods and rules, which will be found useful to the young engineer. The experienced engineer can usually tell the *kind* of stress by inspection, but there are many cases in complex structures where the rules have to be followed.

1.—SHOWING CALCULATION OF CHORD STRESSES, FIG. 2.

Member.	Cutting Plane	Method	Center of Moments.	Equation.	Calculation.	Stress.	Remarks.
2	$C-C$	$\Sigma M=0$	U_1	$S_2 = R_1 \tan \alpha$	$25.0 \times .87 = +21.7$	$21.7t$	$R_1 = 25.0$; $\tan \alpha = .87$
1	U_1-L_1	$\Sigma M=0$	U_1	$S_1 = R_1 \tan \alpha$	.....	$21.7t$	Also, from $\Sigma H = 0$, $S_1 = S_2$
3	$d-d$	$\Sigma M=0$	U_2	$S_3 = (R_1 \times 2 - P_5 \times 1) \tan \alpha$	$40.0 \times .87 = +34.8$	$34.8t$	
4	$d-d$	$\Sigma M=0$	L_3	$S_4 = -(3R - 3P) \tan \alpha$	$-45.0 \times .87 = -39.1$	$39.1c$	$3P = P_5 \times 2 + P_4 \times 1$.
5	$C-C$	$\Sigma M=0$	L_2	$S_5 = -(2R - P) \tan \alpha$	$-40.0 \times .87 = -34.8$	$34.8c$	$2R = R_1 \times 2$

Stresses in Web Members by Method of Shears: $\Sigma V = 0$.—The shear method is usually adopted where a single web member in a panel takes up all the shear, as in Fig. 2, in which top and bottom chords are parallel and at right angle with the prevailing outer forces. In such cases the following steps may be employed:

First.—Draw a plane, cutting the web member in question and also the upper and lower chords. It is clearly evident that the stresses in the chords can have no vertical component to assist in "taking up" the shear, since they are at right angle with the shear, hence it must be taken up by the web or shear member.

Second.—Find the algebraic sum of the vertical outer forces to the *left* of the cutting plane, and this will be equal in magnitude and *opposite* in direction to the *vertical* component of the required stress in the member, acting at the right of the cutting plane to produce equilibrium. If the required stress acts away *from* the cutting plane the member is in tension (+), while if it acts *toward* the cutting plane the member is in compression (−).

Table 2 is based on the above rules, although the algebraic signs giving direction can usually be dispensed with.

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ING CALCULATION OF WEB STRESSES—Fig. 2.

Equation.	Calculation.	Stress.	Remarks.
$S_6=-R_1 \sec \alpha$	$\left\{\begin{matrix}-25.0\times1.325= \\ -33.1\end{matrix}\right\}$	33.1c	$\left\{\begin{matrix}R_1=25.0; \\ \sec\alpha=1.325.\end{matrix}\right.$
$S_1=S_2-0$	21.7−0=21.7	21.7t	See Table 1.
$S_7=P_5-0$	10.0−0=10.0	10.0t	Cutting plane, curved.
$S_8=(R_1-P_5)\sec \alpha$	15.0×1.325=19.9	19.9t	
$S_9=-(R-2P)$	−25.0+20.0=−5.0	5.0c	
$S_{10}=(R-2P)\sec \alpha$	5.0×1.325=6.6	6.6t	
$S_{11}=0$		0.0	Cutting plane, curved.

e span and loads being symmetrical it is necessary to
s to the center of the span only, the other half span being
n.

um.—In addition to the outer forces being in equilibrium
tions (1), (2) and (3), it is also true that all forces, inner
t any panel point or joint of the truss are in equilibrium
e equations. Fig. 4 (using above calculations of 120-
l explain:

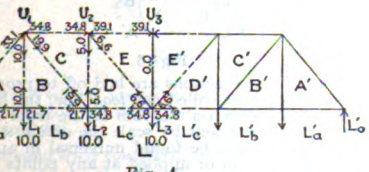

Fig. 4.

acting *toward* the joints denote *compression*; *from* the

.—In the above figure the inner forces or stresses in the
ented as outer forces acting at the several panel points,
l that the forces at each joint will form a closed polygon
equilibrium (see Mechanics, Figs. 34, 35). Thus, the
triangle; at L_1, a rectangle; at U_1, a trapezium; etc.,
).

sed, to distinguish the stress in the stress diagram corres-
nber in the skeleton diagram of the truss, is called Bow's
mple, the line BC in the stress diagram, Fig. 6, repre-
he diagonal member lying between the triangular spaces
leton diagram, Fig. 4. The space U covers the whole
truss from L_0 to L'_0, while the space L is made to cover
e truss between the same points; so that AU corresponds
U_2; AL to L_0L_1; BL to L_1L_2; etc. Some engineers prefer
ters around the outside of the truss *between the lines of*
r instance, instead of L being used universally for the
e space, to use L_a, L_b, L_c, etc., between L_0L_1, L_1L_2,

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L_2L_3, etc., or to use entirely different letters, as F, G, H, etc.; while some prefer the use of the letters X and Y to represent, respectively, the upper and lower outside spaces of the truss. There is, however, no real principle involved in these various customs.

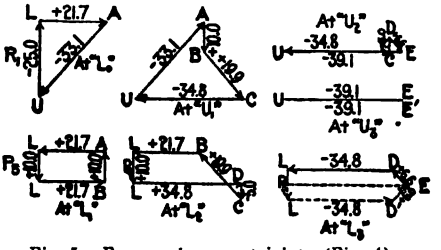

Fig. 5. Force polygons at joints (Fig. 4).

Graphical Method.—The principle of force polygons at the joints of a structure has led to the rapid solution of stresses by the graphical method. It will be noticed that Figs. 5, if "fitted together," will form the complete graphical stress diagram of half the truss—Fig. 6, showing the complete stress diagram of the whole truss, the two halves being symmetrical.

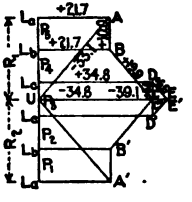

Fig. 6.

Hence if the loads and reactions are laid off to proper scale, forming a closed polygon of outer forces called the *load line*, the lines drawn parallel with the members of the skeleton diagram of the truss and properly intersecting one another, will represent by scale the actual stresses in the members. The following rules will be found universal in application for loads and reactions in any direction or applied at any points of the structure.

GENERAL RULES FOR STRESS DIAGRAMS.

Order of Considering Forces Around Joints.—

1°. *Draw skeleton of truss accurately to a good scale.*

2°. *Show T loads at top joints and B loads at bottom joints of truss.*

Note.—In ordinary cases where T and B loads *at the abutments* are vertically downward and opposite in direction to the reactions at those points, these loads may be omitted—so far as the stresses in the trusses are concerned—and the reactions diminished accordingly: but the total reactions must be considered in designing the supports themselves. Where the loads and reactions are not in the same straight line, both must be included.

3°. *Find the reactions—R_1 = left-hand; R_2 = right-hand.*

4°. *Load-line = reactions and loads forming closed polygon of outer forces.*

Note.—Begin with one of the reactions and draw, in the directions of the forces, the closed polygon of outer forces—reactions and loads—considering them in right-handed (clockwise) or left-handed (unclockwise) order around the truss, as per the following:

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5°. *Outer forces around truss considered in clockwise order*:

Beginning with R_1—(mainly for T loads, at top of truss)—

$$\left\{\begin{array}{l}\text{Draw stress diagram to left of load line;}\\ \text{Consider forces around joints in clockwise order}\end{array}\right\}.(15)$$

Beginning with R_2—(mainly for B loads, at bottom of truss)—

$$\left\{\begin{array}{l}\text{Draw stress diagram to left of load line;}\\ \text{Consider forces around joints in clockwise order}\end{array}\right\}.(16)$$

For (15) For (17) For (19)

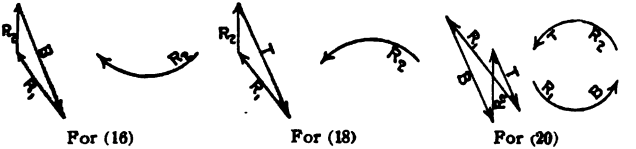
6°. *Outer forces around truss considered in un-clockwise order*:

Beginning with R_1—(mainly for B loads, at bottom of truss)—

$$\left\{\begin{array}{l}\text{Draw stress diagram to right of load line;}\\ \text{Consider forces around joints in un-clockwise order}\end{array}\right\}.(17)$$

Beginning with R_2—(mainly for T loads, at top of truss)—

$$\left\{\begin{array}{l}\text{Draw stress diagram to right of load line;}\\ \text{Consider forces around joints in un-clockwise order}\end{array}\right\}.(18)$$



For (16) For (18) For (20)

For trusses loaded with T loads, at top, and B loads, at bottom chords, (15) and (16) may be combined in (19); while (17) and (18) may be combined in (20); as follows:

7°. *Outer forces around truss considered in clockwise order*:

$$\left\{\begin{array}{l}\text{Beginning with } R_1 \text{ or } R_2\text{—(for both } T \text{ and } B \text{ loads)—}\\ \text{Draw stress diagram to left of load line;}\\ \text{Consider forces around joints in clockwise order}\end{array}\right\} \quad (19)$$

8°. *Outer forces around truss considered in un-clockwise order*:

$$\left\{\begin{array}{l}\text{Beginning with } R_1 \text{ or } R_2\text{—(for both } B \text{ and } T \text{ loads)—}\\ \text{Draw stress diagram to right of load line;}\\ \text{Consider forces around joints in un-clockwise order}\end{array}\right\} \quad (20)$$

Remarks.—The load-line diagrams above, illustrating (15) to (20) inclusive, simply show the *order* of considering the forces at the joints, but not their direction. Note that if the outer forces are all vertical, the load line will be a vertical line.

Order of Considering Joints.—So far, we have considered the order of drawing the outer forces to form the load-line polygon; and the order of considering the forces around the joints for a stress diagram either to *right* or *left* of the load line. We will now consider the order of proceeding from one joint to another of the truss in drawing the stress diagram; bearing in mind that it is sometimes essential to begin at the end of the truss which

has the larger reaction, as a stress diagram is simply a system of triangulation and the larger the initial base the better. There are other reasons, however, which must be considered; namely, the nature of the loading and the general convenience of always starting from the one end. In trusses which support moving loads the loads are made to come on from the right, and the left reaction is taken as the initial base of the stress diagram. In other words, we usually start with joint L_0 of the truss.

In following from joint to joint we must select the one which has, for the given loading, *only two unknown stresses* whose directions are known, because—with one force given, in magnitude and direction—

Two unknown forces only, whose directions are known, can be solved by a "triangle of forces;" and this fact determines the order of considering joints. } (21)

PRACTICAL APPLICATION OF PRECEDING GRAPHICAL RULES.

Graphical Solution of Case *A*; Loads at Bottom Joints.—Data: 6 panels at 20-ft. = 120-ft. span; height of truss = 23 ft.; load per joint = 10,000 lbs.; reactions, each = 25,000 lbs.

Draw the truss very carefully to scale, large enough so lines in the stress diagram can be drawn parallel with the members of the truss, with sufficient accuracy.

In accordance with (17), beginning at the left-hand end of the truss, lay off consecutively in direction and intensity, by scale, the outer forces $R_1 = 25.0$ (upward); P_5, P_4, P_3, P_2, $P_1 = 10.0 + 10.0 + 10.0 + 10.0 + 10.0$ (downward); and $R_2 = 25.0$ (upward).

Draw the stress diagram as per Fig. 6, considering the order of the forces in un-clockwise order around each joint and noting that when a stress is drawn in direction away *from* the joint, the member is in tension; while if drawn in direction *toward* the joint, the member is in compression. The arrows in Figs. 5 show the direction of the stresses of each polygon of forces, indicating the nature of the stress in each case.

Table 3 shows the order of considering the joints, the given and required forces at each joint, and the nature of the stresses in the members.

3.—GRAPHICAL METHOD OF SOLVING CASE *A*, FIGS. 4, 5 AND 6.

Order of Considering Joints.	Name of Joint.	Forces (Fig. 4).		New Intersection (Fig. 6).	Direction of forces in force polygon.		Remarks.
		Given.	To find.		*From* the joint, denoting *tension* (+).	*Toward* the joint, denoting *compression* (−)	
1	L_0	R_1	LA, UA	A	$LA = +21.7$	$UA = -33.1$	
2	L_1	AL, L_1L_2	LB, AB	B	$LB = +21.7$ $AB = +10.0$		
3	U_1	UA, AB	UC, BC	C	$BC = +19.9$	$UC = -34.8$	
4	L_2	CB, BL_1, L_1L_3	LD, CD	D	$LD = +34.8$	$CD = -5.0$	
5	U_2	UC, CD	UE, DE	E	$DE = +6.6$	$UE = -39.1$	
6	U_3	UE	EE', UE'	E'	$EE' = \pm 0.0$	$UE' = -39.1$	Point $E = E'$.
7	L_3	EE', DE, DL, L_3L'.	LD', $E'D'$	D'	$D'E = +6.6$	$LD' = +34.8$	

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ads at Top Joints.—The following is a graphical solution of —the loads at the joints acting at the top instead of at e truss. Two methods are shown, namely, (15) and (18). ress diagram is a supplement of the other, that is, revolved

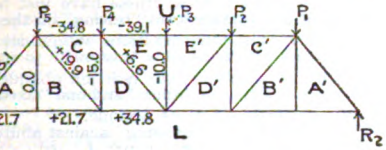
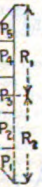


Fig. 7.



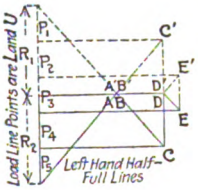


Fig. 9.

eactions, clockwise. (18). Loads and reactions, unclockwise. nd joints, clockwise. Forces around joints, unclockwise. ompare the above with the diagrams obtained for bottom gs. 5 and 6, and note that the difference in the stresses is ertical members.

ads at Top and Bottom Joints.—Before leaving the analysis n let us consider another case—top cord joints loaded with

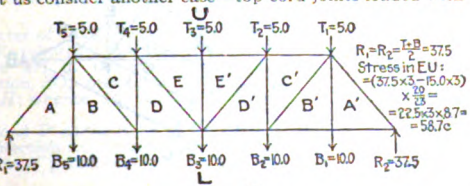


Fig. 10.

his case either (19) or (20) may be used. We will use (19)— ram to left of load line (R_1, T, R_2, B).

reactions clock-

nd joints clock-

he stress dia- t-hand half of omplete in full the right-hand n dotted lines. mmetrical and aded, hence one diagram is all in practice.

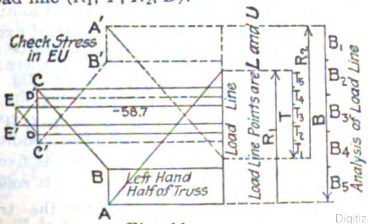


Fig. 11.

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Reactions in Any Direction.—The foregoing elementary principles will apply to any case of loading and reactions, whether vertical or oblique; remembering that the load line is a polygon of forces in which the resultant loads and resultant reactions are respectively equal in magnitude and opposite in direction. There are four cases, as follows:

Case A.—Loads and reactions vertical (these have just been considered).
Example.—Truss of an ordinary span "resting" on the abutments.

Case B.—Resultant of loads oblique; reactions oblique and parallel.
Example. Roof truss with both ends "fastened" to supports.

Case C.—Resultant of loads oblique; one reaction oblique, one vertical.
Example.—Roof truss with one "roller" and one "fixed" end.

Case D.—The loads vertical; reactions oblique.
Example.—Three-hinged arch, "pressing" against abutments.

Case B; Roof Truss; Both Ends Fixed; Wind on One Side.—A roof truss with both ends *fastened* to the supports, and wind pressure on one side, Fig. 12. The assumption made here is that the horizontal components of R_1 and R_2 are directly proportional to the reactions themselves; in other words, the end that has the greater reaction resists the greater horizontal thrust.

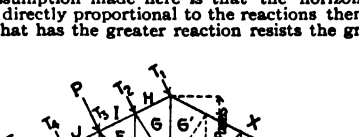


Fig. 12.

Note.—P is the resultant of all the T loads: T_1 and T_5 are each one-half of T_2, T_3 or T_4, the last named three being full panel loads. For height of truss = 2 panels, the resultant P cuts the lower chord 2½ panels from the left-hand end, so that $R_1 = 5\frac{1}{2}P \div 8$, and $R_2 = 2\frac{1}{2}P \div 8$.

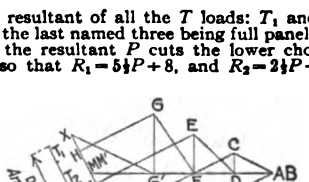


Fig. 13.

(18). Loads and reactions, un-clockwise.
Forces around joints, un-clockwise.

Remarks.—This case is seldom used excepting perhaps for short spans. *Case C*, which will next be treated, can be applied universally. Note that stress diagram, Fig. 13, indicates that there is no stress in any of the *dotted* web members of the truss shown in Fig. 12, G' being made to include the whole triangular space comprising the right-hand half of the truss.

Case C; Roof Truss; One Roller End; Wind on Either Side.—Roof truss with wind pressure on left side; two conditions as follows:

Ca.—Left end is roller end; right end is fixed end. See Figs. 14 and 15.

Cb.—Left end is fixed end; right end is roller end. See Figs. 14 and 16.

Fig. 15 in the stress diagram for the truss (Fig. 14) treated as per Case *Ca*. The load line is composed of P, R_1, and R_2. P, the resultant of

all the T loads, is known in intensity, direction and position, passing through T_3 normal to the roof; R_1 is known in direction and position, passing vertically through L. Let O be the point of intersection of P and R_1 and considering it the center of moments of all the outer forces, we have, from $\Sigma M = 0$, that R_2 must be in the direction RO, passing through the center of moments O.

The stress diagram, Fig. 15, is drawn by rule (18), page 311:

Fig. 14— (18).—Loads and reactions, un-clockwise.
Forces around joints, un-clockwise.

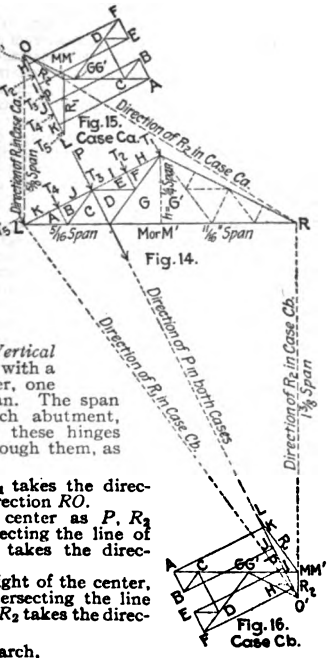
Note.—As the left end is the roller end the reaction R_1 must be vertical.

Fig. 16 is the stress diagram for the same truss, treated as per Case *Cb* by similar analysis.

(15). Loads and reactions, clockwise.
Forces around joints, clockwise.

Note.—As the right-hand end is the roller end the reaction R_2 must be vertical.

Remarks.—The two conditions (*a*) and (*b*) of Case *C* are applied in practice to the roof truss for *maximum* wind stress in each of the members. In addition to these the members of course are designed for snow-, dead-, and live-loads, if any. For these latter cases, however, the reactions are vertical and the graphical methods are simple. (See Roofs, Section 46).



Case D. Three-hinged Arch; Vertical Loads.—Three-hinged arch (Fig. 17) with a load P acting on the left-hand girder, one panel from the center of the span. The span is hinged in the center and at each abutment, and as there can be no moments at these hinges the lines of reactions must pass through them, as follows:

(*a*.) For a load at center of span, R_1 takes the direction LO, and R_2 takes the direction RO.

(*b*). For a load at the left of the center as P, R_2 takes the direction RO, intersecting the line of the force P at H; whence R_1 takes the direction LH.

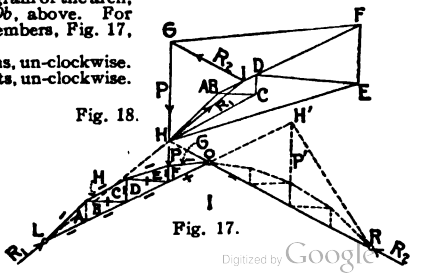
(*c*.) Similarly, for a load P' at the right of the center, R_1 takes the direction LO, intersecting the line of the force P' at H'; whence R_2 takes the direction RH'.

Fig. 18 is a stress diagram of the arch, for the load P, Case *Db*, above. For this load the dotted members, Fig. 17, have no stress.

(18). Loads and reactions, un-clockwise.
Forces around joints, un-clockwise.

Note.—R_2 and P determine R_1.

Remarks.— The + and − signs for the members indicate respectively whether they are in tension or compression.



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17.—NATURAL HISTORY OF MATERIALS.

(For Weights and Specific Gravities, See Section 27.)

A.—CHEMICAL.

Composition of Matter.—If a drop of water could be magnified to size of the earth it is estimated that the countless *atoms* of which the d is composed would each appear to be about the size of a bird's egg. would also be seen that all the atoms would be in groups of three, e group or *molecule* being composed of two atoms of hydrogen and on oxygen. Subject the drop of water to a galvanic current and it will ce to exist as water, being decomposed into its two *elements*—hydrogen oxygen. In other words, the molecules forming the *compound*, water, become disintegrated, all the atoms of hydrogen passing off as hydro gas, and all the atoms of oxygen passing off as oxygen gas. Conversely now the two evolved gases are mixed and sufficiently heated they will unite with explosive force into the same molecular composition as bef re-forming the drop of water. In the above processes, the decomposit and recomposition of the substance water, none of the atoms have b divided or destroyed.

Old Atomic Theory.—Up to about the year 1898 the last sentence of preceding paragraph might have been concluded about as follows, generally accepted: "Because the atom is indivisible and indestructi representing the smallest particle of an element of *matter*; hence when say that matter is indestructible we mean simply that the atoms of wh all matter is composed are indestructible."

Recent Discoveries.—The recent discoveries of the Hertzian ray and practical use in wireless telegraphy; of the Röntgen or X-ray, by mean the Crooke's or vacuum tube, 1895; of the Becquerel ray, exhibiting radio-activity properties of the element uranium, in 1896; and later, Madame Curie and others, of similar properties in the elements thori polonium, radium and actinium, have resulted in shaking the old ato theory of Dalton, which had lasted for a century.

The Corpuscular Theory.—It is now believed that each atom, previo supposed to represent the smallest particle of matter, is really composed a great number of smaller particles called *corpuscles*. For instance, smallest and lightest atom known, that of hydrogen, contains about such corpuscles; while one of the heaviest atoms, that of radium, cont about 200,000. It is further submitted that these corpuscles are so infini small that the unoccupied space in the atom is almost infinitely great comparison, and that they are vibrating or traveling through this sp with velocities of many thousands of feet per second. This becomes all more amazing when it is stated that the diameter of a molecule, which atoms compose, is in the neighborhood of $\frac{1}{2500000000}$ of an inch.

The corpuscular theory has been advanced in order to explain som the recent discoveries cited above. Radium, for instance, is consta giving off heat to surrounding objects without any visible source of sup The heat is evidently furnished by the breaking up or disintegration of atoms; i. e., by the transformation of part of the corpuscular energy of atoms, into heat. The balance of the energy is dissipated in the form various kinds of rays. On this theory the laws of the conservation of en still hold, for it has been observed that all of the radio-active substa gradually lose in weight. It is estimated that radium will disperse al $\frac{1}{2}$ its mass in 1500 years, and $\frac{9}{10}$ of it in 10000 years. Thorium and uran act much more slowly, requiring in the first instance about 1000 mi years, and in the second instance 10000 million years. Moreover, all stances are supposed to be radio-active to some extent, i. e., their atoms gradually breaking up into corpuscles and the corpuscular energy is b transformed into other kinds of energy. In line with this argument, wit the scent of flowers and of many substances.

Looking at the subject, then, from this point of view we may cons *matter* as the great storehouse of *energy;* and some idea of the efficienc

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e had from the following: The hydrogen-oxygen flame is known, yet the amount of heat energy given off spon-efinite quantity of radium is 30,000 times as great as that ne combustion of an equal weight of hydrogen. In the m the *atoms* burst, releasing the stored-up corpuscular ne case of the hydrogen the *molecules* burst and release the mic energies only. The corpuscular energy is vastly the imated that the total work of disintegration of a small produces heat equivalent to the generation of one horse-r! How to utilize, economically, the tremendous amount n matter is a problem. We have seen that the ordinary tion releases but an infinitesimal part of this stored energy, t only a small percentage of even this is made available Scientists are even now further speculating on the possi-scles themselves being divisible into more minute particles greater sources of energy, some of them advancing the corpuscles are made up of the invisible and universal Others speculate on the possibility of corpuscles being

Theory.—Long before the recent discoveries, which led nt of part of the atomic theory of Dalton, chemists pre-any future time it were found possible to transmute one another, there was little doubt that any one element of transmuted into any other element of a different kind. d Soddy are credited with the discovery of the transmu-emanations into an entirely different element, helium; l discovery has brought forth new theories regarding the atter:

conception of the ultimate unit is the electron, and this, n an electrical conception, is in reality a material concep-n the atom of matter. The electron could be defined as ence known capable of isolation and of free movement t is a definite amount or 'charge' of negative electricity; allest possible amount known to exist; for electricity, no has been shown to consist of discrete particles or units, y space continuously. Unlike the atoms of matter, only on is known, consisting of the same amount or charge of y with identical properties in all its various manifestations. at each atom of matter contains in the normal condition on, which it is capable of losing, and conversely that it may one electron more than it normally possesses without deep-nange. An atom with one or more electrons less than it rmal state is positively charged and is often called a *positive* atom with one or more electron in excess is a *negative ion.*"

—*Scientific American.*

—At present there are about 90 known elements, that is, atoms are all alike in the particular element but different other element. Table 1 gives a fairly complete list of eir symbols and some of their properties. The 2nd column eneral way only, the division of the elements into three nce, A, B, c. In the third column the names of those ommon and widely distributed are in SMALL CAPITALS; are rare are printed in *italics*. The atomic weights are tems, both of which are frequently used:

ary weight of an atom of hydrogen = 1 ($\therefore O = 15.87$).
ary weight of an atom of oxygen = 16 ($\therefore H = 1.008$).

(*a*), is the most used. The 7th column shows which are ents; they are marked Met.; the other 13 are metalloids. mns give the order in which many of the elements exist e earth, air and sea combined; and opposite the nine iven the percentage by quantity of the total mass. Thus, d to comprise one-half, silicon one-quarter, aluminum one-e total. The 10th column shows the state of the element mperatures, as solid, liquid and gaseous. The third from ives the molecular volumes of some of the elements; that oms are single, double, triple, etc., in the molecule. The s are self-explanatory.

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1.—The Chemical Elements.

No.	Importance A, B, c.	*Name.* (Date of Identification.)	Symbol.	Atomic Weights. $H=1$.	Atomic Weights. $O=16$.	Metallic Elements = Met.	Order of quantity and total per cent in Earth, Water and Air on the Globe.		State* at ordinary temperatures.	Arrangement of atoms† 1, 2, 3, 4, 5, 6.	Melting point, av. C°	Boiling point, av. C°	Chemical Equivalent (=atomic weight + valency).
1	A	Aluminum.....(1828)	Al	26.9	27.1	Met.	3	7.3%	S	.....	725	.....	9
2	B	Antimony.....(a. 1450)	Sb	119.3	120.2	Met.	.....	.....	S	.....	437	1400	39.8 in *ous*, 24 in *ic*...
3	c	*Argon* [1% of atmosphere] (1895)	A	39.6	39.9	Met.	.....	.....	S	.....	−190	−187	.....
4	B	Arsenic.....(a. 1694	As	74.4	75.0	Met.	.....	.....	S	4	.....	.....	24.8 in *ous*, 14.9 in *ic*.
5	B	Barium.....(1808)	Ba	136.4	137.4	Met.	17	.....	S	.....	.....	.....	68.2
6	c	*Beryllium* (*Glucinum*) (1827)	Be	9.0	9.1	Met.	.....	.....	S	.....	.....	.....	4.5
7	B	Bismuth.....(a. 1450)	Bi	206.9	208.5	Met.	.....	.....	S	.....	265	1270	69
8	B	Boron.....(1808)	B	10.9	11.0	.....	.....	.....	S	.....	.....	.....	3.6
9	A	Bromine.....(1826)	Br	79.4	80.0	.....	13	.....	L	2	−7	61	79.4
10	B	Cadmium.....(1817)	Cd	111.6	112.4	Met.	.....	.....	S	1	315	810	55.8
11	c	*Caesium*.....(1860)	Cs	132.0	133.0	Met.	.....	.....	S	.....	27	.....	66.0
12	A	Calcium.....(1808)	Ca	39.8	40.1	Met.	5	3.5%	S	.....	.....	.....	19.9
13	A	Carbon.....	C	11.9	12.0	.....	11	.....	S	.....	.....	.....	6.
14	c	*Cerium*.....(1803)	Ce	139.0	140.0	Met.	.....	.....	S	.....	.....	.....	.....
15	A	Chlorine.....(1774)	Cl	35.2	35.5	.....	12	.....	G	2	.....	−34	35.2
16	B	Chromium.....(1797)	Cr	51.7	52.1	Met.	19	.....	S	.....	.....	.....	25.9 in *ous*, 17.2 in *ic*.
17	B	Cobalt.....(1735)	Co	58.6	59.0	Met.	.....	.....	S	.....	1650	.....	29.3
18	A	*Columbium* (*Niobium*) (1846)	Cb	93.3	94.0	Met.	.....	.....	S	.....	.....	.....	.....
19	A	Copper.....(B. C.)	Cu	63.1	63.6	Met.	.....	.....	S	.....	1165	.....	31.6
20	c	*Erbium*.....(1843)	E	164.8	166.0	Met.	.....	.....	S	.....	.....	.....	.....
21	B	Fluorine.....(1887)	F	18.9	19.0	.....	.....	.....	G	2	.....	.....	18.9
22	c	*Gadolinium*.....(1899)	Gd	155.0	156.0	Met.	.....	.....	S	.....	.....	.....	.....
23	B	*Gallium*.....(1875)	Ga	69.5	70.0	Met.	.....	.....	L	.....	30	.....	.....
24	c	*Germanium*.....(1886)	Ge	71.9	72.5	Met.	.....	.....	S	.....	900	.....	.....
		Glucinum (see *Beryllium*)											

33	c	*Lanthanum*.....(1841)	La	137.9	138.9	Met.	.....	.....	S	.....	.....	.....	.....	.....
34	A	Lead.....(B. C.)	Pb	205.4	206.9	Met.	.....	.....	S	.....	328	1525	102.7	
35	c	Lithium.....(1817)	Li	7.0	7.0	Met.	.....	.....	S	.....	.....	.....	7.	
36	B	MAGNESIUM.....(1808)	Mg	24.2	24.4	Met.	6	2.5%	S	.....	650	1100	12.1	
37	B	Manganese.....(1774)	Mn	54.6	55.0	Met.	15	.....	S	.....	1600	.....	27.3	
38	A	Mercury.....(a. 300 B.C.)	Hg	198.5	200.0	Met.	.....	.....	L	1	−40	357	198.5 in *ous*, 99.3 in *ic*.	
39	c	Molybdenum.....(1872)	Mo	95.3	96.0	Met.	.....	.....	S	.....	.....	.....	.....	
40	c	*Neodymium*.....	Ne	142.5	143.6	Met.	.....	.....	S	.....	.....	.....	.....	
41	c	*Neon*.....(1898)	....	19.9	20.0	Met.	.....	.....	S	.....	.....	.....	.....	
42	B	Nickel.....(1751)	Ni	58.3	58.7	Met.	.....	.....	S	.....	1525	.....	29.1	
		Niobium (see *Columbium*)												
43	A	NITROGEN.....(1772)	N	13.0	14.0	.....	18	.....	G	2	−203	−194	13.	
44	c	*Osmium*.....(1803)	Os	189.6	191.0	Met.	.....	.....	S	.....	2500	.....	189.6	
45	A	OXYGEN [Ozone $=O_3$] (1774)	O	15.9	16.0	.....	1	50.0%	G	2, 3‡	−210	−181	8.	
46	c	Palladium.....(1803)	Pd	105.7	106.5	Met.	.....	.....	S	.....	1655	.....	52.8 in *ous*, 26.4 in *ic*.	
47	A	Phosphorus.....(1669)	P	30.8	31.0	.....	14	.....	S	4	44	290	6.2 in phosphates.	
48	B	Platinum.....(1736)	Pt	193.3	194.8	Met.	.....	.....	S	.....	1830	.....	96.6 in *ous*, 48.3 in *ic*.	
49	A	POTASSIUM.....(1807)	K	38.9	39.2	Met.	8	2.2%	S	.....	60	700	38.9	
50	c	*Praseodymium*.....	Pr	139.4	140.5	Met.	.....	.....	S	.....	.....	.....	.....	
51	c	Radium.....(1888)	Ra	223.3	225.0	Met.	.....	.....	S	.....	.....	.....	.....	
52	c	*Rhodium*.....(1804)	Rh	102.2	103.0	Met.	.....	.....	S	.....	2000	.....	51.1 in *ous*, 34.1 in *ic*.	
53	c	*Rubidium*.....(1860)	Rb	84.8	85.4	Met.	.....	.....	S	.....	38	.....	84.8	
54	c	*Ruthenium*.....(1843)	Ru	100.9	101.7	Met.	.....	.....	S	.....	.....	.....	50.5 in *ous*, 33.6 in *ic*.	
55	c	*Samarium*.....	Sm	148.9	150.0	Met.	.....	.....	S	.....	.....	.....	.....	
56	c	*Scandium*.....(1879)	Sc	43.8	44.1	Met.	.....	.....	S	.....	.....	.....	.....	
57	c	*Selenium*.....(1817)	Se	78.6	79.2	.....	.....	.....	S	2	217	680		
58	A	SILICON.....(1823)	Si	28.2	28.4	.....	2	25.3%	S	.....	1400	.....	7.0	

* Solid (S), liquid (L) or gaseous (G) state. a. antedates. ‡ as ozone.
† 1, monatomic; 2, diatomic; 3, triatomic; 4, tetratomic; 5, pentatomic; 6, hexatomic.

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1.—THE CHEMICAL ELEMENTS.—Concluded.

No.	Importance A, B, c.	*Name.* (Date of Identification.)	Symbol.	Atomic Weight. $H=1$.	Atomic Weight. $O=16$	Metallic Elements = Met.	Order of quantity and total per cent in Earth, Water and Air on the Globe.		State* at ordinary temperatures.	Arrangement of atoms† 1, 2, 3, 4, 5, 6.	Melting point, av. C°	Boiling point, av. C°	Chemical Equivalent (=atomic weight ÷ valency).
59	A	Silver.....(B. C.)	Ag	107.1	107.9	Met.	.....	.....	S	.....	1000	.....	107.1
60	A	SODIUM.....(1807)	Na	22.9	23.1	Met.	7	2.3%	S	.....	94	800	22.9
61	B	Strontium.....(1808)	Sr	86.9	87.6	Met.	.....	.....	S	.....	.....	.....	43.5
62	A	Sulphur.....(B.C.)	S	31.8	32.1	.....	16	.....	S	2‡	118	448	31.8
63	c	*Tantalum*.....(1802)	Ta	181.6	183.0	Met.	.....	.....	S	.....	.....	.....	.....
64	c	*Tellurium*.....(1772)	Te	126.6	127.6	Met.	.....	.....	S	.....	475	.....	.....
65	c	*Terbium*.....(1843)	Tb	158.8	160.0	Met.	.....	.....	S	.....	.....	.....	.....
66	c	*Thallium*.....(1861)	Tl	202.6	204.1	Met.	.....	.....	S	.....	.....	.....	202.6 in *ous*, 67.5 in *ic*.
67	c	*Thorium*.....(1829)	Th	230.8	232.5	Met.	.....	.....	S	.....	.....	.....	.....
68	c	*Thulium*.....	Tm	169.7	171.0	Met.	.....	.....	S	.....	.....	.....	.....
69	A	Tin.....(B. C.)	Sn	118.1	119.0	Met.	.....	.....	S	.....	231	1525	59.0 in *ous*, 29.5 in *ic*.
70	c	Titanium.....(1791)	Ti	47.7	48.1	Met.	10	.....	S	.....	.....	.....	23.9 in *ous*, 11.9 in *ic*.
71	c	Tungsten.....(1781)	W	182.6	184.0	Met.	.....	.....	S	.....	.....	.....	91.3 in *ous*.
72	c	Uranium.....(1841)	U	236.7	238.5	Met.	.....	.....	S	.....	.....	.....	118.4 in *ous*.
73	c	Vanadium.....(1830)	V	50.8	51.2	Met.	.....	.....	S	.....	.....	.....	16.9 in *ous*.
74	c	*Xenon*.....(1898)	X	127.0	128.0	Met.	.....	.....	S	.....	.....	.....	.....
75	c	*Ytterbium*.....	Yb	171.7	173.0	Met.	.....	.....	S	.....	.....	.....	.....
76	.....	*Yttrium*.....(1794)	Yt	88.3	89.0	Met.	.....	.....	S	.....	.....	.....	.....
77	A	Zinc.....(a. 1700)	Zn	64.9	65.4	Met.	.....	.....	S	1	416	915	32.5
78	c	Zirconium.....(1789)	Zr	89.9	90.6	Met.	.....	.....	S	.....	.....	.....	.....
79	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
80	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....

* Solid (S), liquid (L) or gaseous (G) state. ‡ Sulphur is sometimes hexatomic. a. antedates.
† 1, monatomic; 2, diatomic; 3, triatomic; 4, tetratomic; 5, pentatomic; 6, hexatomic.

Compounds.—A compound is a chemical union of two or more elements, each molecule of the compound being a perfect likeness in miniature of the compound itself. It differs from a *mixture* in that no chemical change takes place in the latter, the original molecules of each of the substances mixed remaining intact.

Compound substances are generally named so as

(a) To denote the constituent elements.
(b) To denote the kind of molecular grouping of the elements.
(c) To denote the nature of a resulting compound by the addition or subtraction of some particular kind of element.

Simple Combinations,——ides.—Many of the non-metallic elements when combined with a simple basic one take on the suffix *ide* or *et*, as, for instance,

Iron	and oxygen	form	iron oxide (oxide of iron).
Lead	" chlorine	"	lead chloride (chloride of lead).
Potassium	" bromine	"	potassium bromide (etc.).
Hydrogen	" sulphur	"	hydrogen sulphide (sulphuret of hydrogen).
Copper	" sulphur	"	copper sulphide (etc.).

But by reason of the fact that two substances often unite in different proportions the names of the resulting compounds are so contrived by Latin prefixes and suffixes as to indicate quite directly its molecular composition. Thus,

*Sub*oxide of copper	Cu_2O	Ratio of O is <	1 : 1
*Proto*xide of manganese (=*mono*xide)	MnO	" "	1 : 1
*Deuto*xide of lead	PbO_2	" " >	1 : 1
*Sesqui*oxide of manganese	Mn_2O_3	" "	3 : 2
*Bi*noxide of silica (=*di*oxide)	SiO_2	" "	2 : 1
*Ter*oxide of chromium (=*tri*oxide)	CrO_3	" "	3 : 1
*Per*oxide of Nickel	Ni_2O_3	" "	max.

The same prefixes may be applied to other compounds, as chlorine, sulphur, etc.

Acids, Bases and Salts.—When an *acid* and a *base* or *alkali* are brought together chemically in the proper proportions, they neutralize each other, and the resulting chemical products are, 1st, *water* and 2nd, *salt*.

An *acid* may be defined as a substance containing hydrogen, which it readily exchanges for a metal when treated with a metal or metal compound called a *base*.

A *base* is a substance containing a metal combined with hydrogen and oxygen, which metal it readily exchanges for hydrogen when treated with an acid.

A *salt* is a neutral substance, one of the products of the action of an *acid* on a *base*, the other product being *water*.

Oxides and Hydroxides.—An *oxide* is a compound of oxygen, more especially with a *metal* or *metalloid*. The principal classes of oxides are (1) basic or metallic oxides, and (2) acid oxides or acid anhydrides. Examples: (1), calcium oxide (CaO); (2), sulphuric anhydride (SO_3). A *hydroxide* may be formed (1) by treating oxides with water, and (2) by decomposing salts by the addition of soluble hydroxides to their solutions. Examples: (1), $CaO+H_2O=Ca(OH)_2$; (2), $MgSO_4+2\,NaOH=Na_2SO_4+Mg(OH)_2$.

Acid Combinations.—An acid is a compound containing, in each molecule, one or more atoms of hydrogen which *may be* displaced by a metal or by a compound possessing metallic functions. If the acid molecule contains one hydrogen atom it is *monobasic*; two hydrogen atoms, bibasic; three hydrogen atoms, tribasic; more than one hydrogen atom, polybasic, etc.

When acid contains oxygen the suffix —*ic* is usually given to the characteristic element of the compound, as sulphur*ic* acid for H_2SO_4. But if two acids can be formed with oxygen from the same characteristic element, that one which is the more highly oxidized has the suffix —*ic* while the other, the less oxidized, takes on the suffix —*ous*. In cases where more than two acids are formed by different proportions of oxygen the above are

further modified by the preffixes *hyper*=, meaning over, and *hypo*=, meaning under. Thus we may have according to the relative proportions of oxygen present, say in the sulphur acids:

(a)	*hypo*sulphurous acid,	$H_2 OS_2$.	(Least proportion of oxygen.)
(b)	sulphur*ous* "	$H_2 S O_4$.	(Less proportion of oxygen.)
(c)	*hyper*sulphur*ous* "		(*Per* often used instead of *hyper*.)
(d)	*hypo*sulphur*ic* "		(This is preferable to c.)
(e)	sulphur*ic* "	$H_2 SO_4$	(Greater proportion of oxygen.)
(f)	*hyper*sulphur*ic* "	$H_2 S_2 O_8$	(Greatest proportion of oxygen.)

Salts.—We have seen that an acid contains one or more atoms of hydrogen which *may be* replaced by metallic atoms or basic radicals. When such a change takes place in an acid the resulting compound is a salt. The names of the salts so formed are related to the acids which produce them, as

——*ic* acids	form	——*ate* salts.	
——*ous* "	"	——*ite* "	
hypo——*ous* "	"	*hypo*——*ite* "	

Thus, sulphur*ic* acid ($H_2 SO_4$) and soda (Na) form sulph*ate* of soda ($Na_2 SO_4$); *hypo*sulphur*ous* acid ($H_2 SO_3$) and soda form *hypo*sulph*ite* of soda, etc.

In line with the above definition we may say further that any base in which one or more of the hydrogen atoms have been replaced by non-metallic atoms or acid radicals, is a salt.

A salt may be formed by compounding a metallic oxide with an anhydride.

Periodic Law.—"The properties of the elements, as well as the forms and properties of the compounds, are in periodic dependence on, or form periodic functions of, the atomic weights of the elements." This law, conceived by Newlands and later perfected by Mendeléeff, Meyer and others, is a fundamental law of Chemistry If the elements are arranged in series on an ascending scale, according to their atomic weights, this series can be made to exhibit Family Groups showing common characteristics, in much the same manner as the natural divisions or classifications of the mineral, vegetable and animal kingdoms.

Table 2 is such a grouping, by Meyer. The inclined lines indicate a spiral or continuous series if the table were wrapped around a cylinder. The vertical columns show the Groups, I, II, III, etc., with sub-divisions A and B. Each group has common characteristics, while those of the sub-divisions are still more in common. The vacant spaces are for other elements, most of which have not yet been discovered or their atomic weights determined. It is a significant fact that by means of this law Mendeléeff foretold the existence of some of the elements long before their discovery, and calculated their atomic weights correctly.

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II.—NATURAL SYSTEM OF THE ELEMENTS (MEYER.)

H, 1.00							
A I. B	A II. B	A III. B	A IV. B	A V. B	A VI. B	A VII. B	VIII.
Li, 7.0	*Be*, 9.0	*B*, 10.9	*C*, 11.9	*N*, 13.0	*O*, 15.9	*F*, 18 9	
Na, 22.9	*Mg*, 24.2	*Al*, 26.9	*Si*, 28.2	*P*, 30.8	*S*, 31.8	*Cl*, 35 2	
K, 38.9	*Ca*, 39.8	*Sc*, 43 8	*Ti*, 47.7	*V*, 50 8	*Cr*, 51.7	*Mn*, 54 6	*Fe*, 55.5 *Co*, 58.6 *Ni*, 58.3
Cu, 63.1	*Zn*, 64.9	*Ga*, 69.5	*Ge*, 71.9	*As*, 74 4	*Se*, 78.6	*Br*, 79.4.	
Rb, 84.8	*Sr*, 86.9	*Yt*, 88 3	*Zr*, 89.9	*Cb*, 93 3	*Mo*, 95 3		*Ru*, 100.9 *Rh*, 102.2 *Pd*, 105,7
Ag, 107.1	*Cd*, 111.6	*In*, 113.1	*Sn*, 118.1	*Sb*, 119 3	*Te*, 126 6	*I*, 125.9	
Cs, 132.0	*Ba*, 136.4	*La*, 137.9	*Ce*, 139.0	*Di*.			
	E, 164.8	*Yb*, 171.7		*Ta*, 181.6	*W*, 182.6		*Os*, 189.6 *Ir*, 191.5 *Pt*, 193.3
Au, 195.7	*Hg*, 198.5	*Tl*, 202.6	*Pb*, 205.4	*Bi*, 206 9			
			Th, 230.8		*U*, 236.7		
R_2O.	RO.	R_2O_3.	RO_2.	RH_3. R_2O_5.	RH_2. RO_3.	RH. R_2O_7.	RO_4.

Chemical Substances and Their Common Names.—

(Knowledge Year Book.)

Alum = Sulphate of aluminum and potassium.
Aqua fortis = Nitric acid.
Aqua regia = Nitro-hydrochloric acid.
Calomel = Mercurous chloride.
Carbolic acid = Phenol.
Caustic potash = Potassium hydrate.
Caustic soda = Sodium hydrate.
Chalk = Calcium carbonate.
Copperas = Sulphate of iron.
Corrosive sublimate = Mercurous chloride.
Cream of tartar = Bitartrate of potassium.
Epsom salts = Magnesium sulphate.
Fire damp = Light carburetted hydrogen, methane.
Glaubers' salt = Sodium sulphate.
Grape sugar = Glucose.
Goulard water = Basic acetate of lead.
Iron pyrites = Sulphide of iron.
Jewelers' putty = Oxide of tin.
Laughing gas = Nitrous oxide.
Lime = Calcium oxide.
Lunar caustic = Silver nitrate.
Mosaic gold = Bisulphide of tin.
Muriatic acid = Hydrochloric acid.
Plaster of paris = Calcium sulphate.
Realgar = Sulphide of arsenic.
Red lead = Oxide of lead.
Rochelle salt = Sodium potassium tartrate.
Sal ammoniac = Ammonium chloride.
Salt, common = Sodium chloride.
Salt of tartar (potash) = Potassium carbonate.
Saltpetre = Potassium nitrate.
Salt of lemon = Oxalic acid.
Slacked lime = Calcium hydrate.
Soda, washing = Calcium carbonate.
Soda, baking = Sodium bicarbonate.
Soda = Sodium carbonate.
Spirits of hartshorn = Solution of Ammonia.
Spirits of salt = Hydrochloric acid.
Sugar of lead = Lead acetate.
Tartar emetic = Potassium antimony tartrate.
Verdigris = Basic acetate of copper.
Vermillion = Sulphide of mercury.
Vinegar = Dilute Acetic acid.
Vitriol, blue = Copper sulphate.
green = Ferrous sulphate.
oil of = Sulphuric acid.
white = Zinc sulphate.
Volatile alkali = Ammonia.

B.—MINERALOGICAL.

Minerals Compose Rocks.—Mineralogical substances include all thos natural objects which belong to the Inorganic Kingdom, whether solid liquid or gaseous. A *mineral* may be defined as a natural, inorganic, homo geneous, chemical substance; while a *rock* is made up of one or more miner masses mixed in fairly constant proportions throughout.

(*a*) **Hardness.**—The hardness of a mineral is its resistance to abrasion There are ten recognized degrees of hardness, typified by the followin minerals:

1. *Talc.*—Soft and greasy; easily scratched by finger nail.
2. *Gypsum.*—Not easily scratched by the finger nail.
3. *Calcite.*—Scratches copper coin; is scratched by same.
4. *Fluorite.*—Not scratched by copper; and does not scratch glas
5. *Apatite.*—Barely scratches glass; easily scratched by a knife.
6. *Orthoclase.*—Easily scratches glass; barely scratched by a file.
7. *Quartz.*—Not scratched by a knife.
8. *Topaz.*—Not scratched by a file.
9. *Sapphire.*—About $^7/_{100}$ as hard as diamond.
10. *Diamond.*—Hardest substance known.

(*b* to *l*). **Other Physical Characteristics.**—Minerals are often identified the above characteristics of hardness and also by the kinds and degrees the following:

(b) tenacity, (c) cleavage, (d) fracture, (e) feel, (f) taste, (g) od (h) lustre, (i) color, (j) transparency, (k) translucency, (l) opaq ness, etc.

3.—Classification of Important Mineral Species.*

I. Native Elements.

A Series.—Non-basic or electro-positive elements.

1. Gold Group.—Gold, silver, hydrogen, potassium, sodium, etc.
2. Iron Group.—Platinum, palladium, mercury, copper, iron, zinc, lead, cobalt, nickel, chromium, manganese, calcium, magnesium, etc.
3. Tin Group.—Tin, titanium, zirconium, etc.

B Series.—Elements generally electro-negative.

1. Arsenic Group.—Arsenic, antimony, bismuth, phosphorus, vanadium, etc.
2. Sulphur Group.—Sulphur, tellurium, selenium.
3. Carbon-Silicon Group.—Carbon, silicon, diamond, graphite.

C Series.—Elements always negative.

1 Chlorine, bromine, iodine.
2. Fluorine.
3. Oxygen.

II. Sulphides.—Sulphides, tellurides, selenides, arsenides, antimonides, bismuthides.

A. Binary Compounds.—Sulphides and tellurides of metals of the sulphur and arsenic groups.

(a) Realgar Group. RS. Realgar, $As\,S$.
(b) Orpiment Group. $R_2\,S_3$. Orpiment, $As_2\,S_3$
Stibuite, $Sb_2\,S_3$
Bismuthinite, $Bi_2\,S_3$.
(c) Tetradymite, $Bi_2\,(Te, S)_3$.
(d) Molybdenite Group. $R\,S_2$. Molybdenite, $Mo\,S_2$.

B. Binary compounds.—Sulphides, tellurides, etc., of metals of the gold iron and tin groups.

1. Basic Division.—Dyscrasite, $Ag_4\,Sb$; $Ag_6\,Sb$. Domeykite, $Cu_3\,As$.
2. Proto Division. RS (or $R_2\,S$), $R\,Se$, $R\,Te$.
(a) Galenite Group. Argentite, $Ag_2\,S$. Crookesite, $(Cu_2, Tl, Ag)\,Se$.
Galenite, $Pb\,S$.
(b) Blend Group. Sphalerite, $Zn\,S$.
(c) Chalcocite Group.
(d) Pyrrhotite Group. Cinnabar, $Hg\,S$ (or $Hg_2\,S_2$). Millerite, $Ni\,S$.
Pyrrhotite, $Fe_7\,S_8$ mostly.
Greenockite, $Cd\,S$ (or $Cd_2\,S_2$).
3. Deuto or Pyrite Division.
(a) Pyrite Group. Pyrite, $Fe\,S_2$. Chalcopyrite, $Cu\,Fe\,S_2$.
(b) Marcasite Group. Marcasite, $Fe\,S_2$.
Arsenopyrite, $Fe\,As\,S = Fe\,S_2 + Fe\,As_2$.
Sylvanite.

C. Ternary Compounds.—Sulpharsenites, sulphantimonites, sulphobismuthites.

(a) Group I. $R\,(As, Sb)_2\,S_4 = RS + (As\,Sb)_2\,S_3$.
(b) Sub Group. $R_3\,(As, Sb, Bi)_4\,S_9 = 3\,RS + 2(As, Sb, Bi)_2\,S_3$.
(c) Group II. $R_2\,(Sb, As)_2\,S_3 = 2RS + (Sb, As)_2\,S_3$.
(d) Group III. $R_3(As, Sb)_2\,S_6 = 3RS + (As, Sb)_2\,S_3$.
(e) Group IV. $R_4(As, Sb, Bi)_2\,S_7 = 4RS + (As, Sb, Bi)_2\,S_3$.
(f) Group V. $R_5(As, Sb)_2\,S_8 = 5RS + (As, Sb)_2\,S_3$.

III. Chlorides, Bromides, Iodides.

A. Anhydrous Chlorides. $R(Cl, Bi, I)$; $R_2(Cl, Br, I)$; $R\,Cl_3$.

Halite, $NaCl$. Calomel, $HgCl$.
Sylvite, KCl. Sal Ammoniac, NH_4Cl.

B. Hydrous Chlorides.

C. Oxychlorides.

IV. Fluorides.

A. Anhydrous Fluorides. Fluorite, $Ca\,F_2$.
Cryolite, $Na_6\,Al\,F_{12}$ (or $6\,Na\,F + Al\,F_3$).

B. Hydrous Fluorides.

* Mostly in accordance with Dana's Classification.

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3.—Classification of Important Minerals.—Cont'd.

V. Oxygen Compounds.

α. Oxides.

A. Oxides of metals of the gold, iron and tin groups.

1. Anhydrous oxides.

(a) Protoxides. RO or (R_2O). Cuprite, Cu_2O. Zincite, ZnO. Tenorite, CuO.

(b) Sesquioxides, RO_3. Corundum, Al_2O_3. Hematite, Fe_2O_3.

(c) Compounds of protoxides and sesquioxides, RR_2O_4 (or $RO+R_2O_3$).

Spinel Group.—Spinel, $MgAl_2O_4(=MgO+Al_2O_3)$.
Magnetite, $FeFe_2O_4$(or Fe_3O_4) $=FeO+Fe_2O_3$.
Franklinite, $(Fe, Zn, Mn)(Fe, Mn)_2O_4$.
Chromite, $FeCr_2O_4$, or $(Fe, Mg, Cr)(Al, Fe, Cr)_2O_4$.
Uranite, $U_3O_8(UO_2+2UO_3)$.

(d) Deutoxides. RO_2.

2. Hydrous oxides. Targite, $H_2F_2O_7$ Diapose, H_2AlO_4.
Gothite, $H_2FeO_4=H_6Fe_2O_6+2Fe_2O_3$.
Manganite, $H_2MnO_4=H_8Mn_2O_6+2MnO_3$.
Limonite, $H_6Fe_2O_9=H_6Fe_2O_6+Fe_2O_3$.

B. Oxides of metals of the arsenic and sulphur groups.
Arsenolite, As_2O_3. Bismite, Bi_2O_3.
Molybdite, MoO_3. Tungstite, WO_3.

C. Oxides of the carbon-silicon group. Quartz, SiO_2.
Tridymite, SiO_2. Opal, SiO_2.

β. Ternary Oxide Compounds.

A. Silicates.

1. Anhydrous silicates.

1° Bisilicates. $RSiO_3$.

(a) Amphibole Group.—Wollastonite, $CaSiO_3$.
Pyroxene (=Augite), $RSiO_3$: R may be $Ca, Mg, Fe, Mn, Zn, Ka_2, Na_2$.

(b) Amphibole Section. Amphibole, $RSiO_3$ (var., Hornblende).
Beryl, $Be_3Al_2Si_6O_{18}$.

2° Unisilicates, R_2SiO_4.

(a) Chrysolite Group.—Chrysolite, $(Mg, Fe)_2SiO_4$. Olivene.
(b) Willemite Group.—Willemite, Zn_2SiO_4.
(d) Garnet Group.—$R_3RSi_3O_{12}$.
(e) Vesuvianite Group.
(f) Epidote Group.—Epidote, $H_2Ca_4R_3Si_6O_{26}$.
(k) Mica Group.—Biotite. Muscovite.
(l) Scapolite Group.
(m) Nephelite Group.
(n) Feldspar Group.—Anarthite. Labradorite. Andesite. Oligoclase. Albite. Orthoclase.

3° Subsilicates.

(a) Chondrodite. Tourmaline.
(b) Andalusite. Fibrolite, $AlSiO_5$. Cyanite, $AlSiO_5$
Topaz, $AlSiO_5$. Euclase, $H_2Be_2AlSi_2O_{10}$.
Datolite, $H_2Ca_2B_2Si_2O_{10}$. Titamite, $CaTiSiO_5$.
(c) Staurolite.

2. Hydrous Silicates.—I. General Section.
Bisilicates.—Pectotite; lanmontite; okenite; crysocolla; alipite.
Unisilicates.—Calamine; prenite.—Thorite, Pyrosmalite.
Subsilicates.—Allophane.

II. Zeolite Section.

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3.—CLASSIFICATION OF IMPORTANT MINERALS.—Cont'd.

Oxygen Compounds—Cont'd.

III. Margarophyllite Section.

Bisilicates.—Talc; pyrophyllite; sepiolite.

Unisilicates.

Serpentine Group.
Kaolinite Group.
Pinite Group.
Hydro-mica Group.

Subsilicates.

Chlorite Group.

B. Tantalates, Columbates.

C. Phosphates, Arsenates, Vanadates, etc.

Anhydrous.

Xenotime, $Y_2 P_2 O_8$.

Apatite Group.—Apatite. Vanadinite.
Antimonates.

Hydrous.

Antimonates.
Nitrates. Nitre, $K N O_3$. Soda nitre, $Na NO_3$.

D. Borates. Boracite, $Mg_7 B_{16} Cl_2 O_3 = 2Mg_2 B_8 O_{15} + Mg Cl_2$.
Borax, $Na_2 B_4 O_7 + 10\,aq = 2(Na BO_2 + HBO_2) + 9aq$.

E. Tungstates ($R W O_4$), Molybdates ($R Mo O_4$), Chromates ($R CiO_4$).

F. Sulphates.

Anhydrous $R SO_4$.

Barite Group. Barite, $Ba SO_4$. Anglesite, $Pb SO_4$.

Hydrous. Mirabilite, $Na_2 SO_4 + 10\,aq$.
Gypsum, $Ca SO_4 + 10\,aq$. var., selenite.
Epsomite, $Mg SO_4 + 7\,aq$.

Copperas Group.—Chalcanthite, $Ca SO_4 + 5\,aq$.

Alum and Halalrichite Groups.—Copiatite.
Aluminite $Al SO_6 + 9\,aq$.

Tellurates.—Montanite, $Bi_2 Fe O_6 + 2\,aq$.

G. Carbonates.

Anhydrous.

Calcite Group. $R CO_3$. Calcite, $Ca CO_3$.

1. Crystallized: Iceland spar, Fountainblue limestone, Satin spar.
2. Massive: Granular (Saccharroidal): Marble, Statuary marble, Shell marble, Lithographic stone, Breccia marble, Pudding stone marble, Hydraulic limestone.

   Soft Compact Limestone: Chalk, Calcareous marl. Concretionery, massive: Oolite.

   a. Stalactites.
   b. Stalagmites.
   c. Calc-sinter. Travertine, Calc-tufa.
   d. Agaric mineral.
   e. Rock-meal.

Dolomite, $(Ca, Mg) CO_3$.
Aukerite, $Ca CO_3 + Fe CO_3 + x\,(Ca Mg C_2 O_6)$.
Magnesite, $Mg CO_3$.
Siderite, $Fe CO_3$.
Rhodochrosite, $Mn CO_3$.
Smithsonite, $Zn CO_3$.
Aragonite Group.

Hydrous.

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3.—CLASSIFICATION OF IMPORTANT MINERALS.—Concl'd.

VI. Hydrocarbons.

1. Simple (no oxygen).
   - (a) Marsh Gas Series. $C_n H_{2n+2}$. Includes liquid naphthas and more volatile parts of petroleum; also scheercrite and chrismatite.
     Petroleum.—Mineral oil, Kerosene, Bergol, Erdol.
   - (b) Olefiant or Ethylene Series. $C_n H_{2n}$. Pittolium group of liquids or pettarsphalts (mineral tar) and the paraffines. Paraffin Group.
   - (c) Camphene Series. $C_n H_{2n-4}$.
   - (d) Benzole Series. $C_n H_{2n-6}$. Benzole liquids.
   - (e) Naphthalin Series. $C_n H_{2n-12}$.
     Naphthalin (found in Rangoon tar).

2. Oxygenated.
   Succinate (amber).

Appendix to Hydrocarbons.

Asphaltum.—Bitumen, Asphalt, Mineral pitch.—Mixture of different hydrocarbons, part of which are oxygenated.

Following substances are closely related to asphaltum:
Grahamite, Albertite, Pianzite, Wollongongite.

Mineral Coal.—Made up of different kinds of hydrocarbons with perhaps, in some cases, free carbon.

1. Anthracite.
   Bituminous.
2. Coking.
3. Non-coking.
4. Cannel coal.
5. Torbanite.
6. Brown coal.
7. Earth brown coal.

(*m*). **Blowpipe Characteristics.**—The following important flame colorations result from the blowpipes: *Carmine*, from lithium compounds; *scarlet*, from strontium compounds; *yellowish red*, from calcium compounds; *yellow*, from sodium and all its salts; *yellowish green*, from barium compounds, molybdenum sulphide and oxide; *pure green*, from compounds of tellurium or thallium; *emerald green*, from most copper compounds with hydrochloric acid; *bluish green*, from phosphoric acid and phosphates with sulphuric acid; *feeble green*, from antimony or ammonium compounds; *whitish green*, from zinc; *light blue*, from arsenic, selenium and lead; *azure blue*, from copper chloride; *violet*, from potassium compounds.

Some of the Important Minerals.

THE GOLD MINERALS.

Tellurides.—Sylvanite, calaverite, krennerite.
Ores.—Pyrite, arsenopyrite, pyrrhotite, and various sulphides, etc.

THE SILVER MINERALS.

Ordinary silver ores contain less than 1% of the silver compounds distributed through various minerals.

THE POTASSIUM MINERALS.

Chlorides.—Sylvite, carnallite, kainite. *Sulphates.*—Kalinite. *Nitrate.*—Nitre. The chlorides are the natural potash salts.

THE SODIUM MINERALS.

Chloride.—Halite. *Sulphate.*—Mirabilite. *Nitrate.*—Soda nitrate. *Carbonate.*—From Na_2CO_3, $HNaCO_3$. $2H_2O$.

THE LITHIUM MINERALS.

Used in medicine.

THE PLATINUM AND IRIDIUM MINERALS.

Metals.—Platinum, iridosmine. *Arsenide.*—Sperrylite. *Purified platinum.*—Used in incandescent lamps, for electrical contact points, and in the so-called "oxidizing of silver." *Iridosmine.*—Pointing gold pens; phosphide of iridium is used for pointing tools and stylographic pens and for knife edges in the most delicate balances.

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The Mercury Minerals.

Sulphide.—Cinnabar. *Chloride.*—Calomel.

Uses.—Mercury is used in extraction of gold and silver from their ores; manufacture of vermillion; barometers, thermometers; silvering mirrors; medicine.

The Copper Minerals.

Sulphides.—Chalcocite, bornite, chalcopyrite. *Sulpho-arsenite.*—Enargite. *Sulphoantimonite.*—Tetrohedrite. *Oxides.*—Cuprite, tenorite. *Basic chloride.*—Atocamite. *Sulphates.*—Chalcanthite. *Carbonates.*—Malachite, azurite. *Silicates.*—Chrysocolla, diastase.

Ores.—Chalcopyrite, bornite, native copper, cuprite, malachite, augite.

Uses of copper.—Electrical work; alloys with zinc and tin, such as brass, yellow metal, bronze, bell metal, German silver.

The Iron Minerals.

Metal.—Iron. *Sulphides and arsenides.*—Pyrrhotite, pyrite, marcasite, arsenopyrite, leucopyrite. *Oxides.*—Magnetite, franklinite, hematite, menaccanite, turgite, goethite, melanterite. *Sulphates.*—Copiatite, melanterite. *Phosphates.*— Vivianite, triphylite. *Arsenates.*—Scorodite, pharmacosiderite. *Carbonate.*—Siderite. *Chromate.*—Chromite.

Ground for paint.—Limonite, hematite. *Used as iron ores.*—Hematite, limonite, magnetite, siderite. *For extracting sulphur.*—Pyrite, marcasite, pyrrhotite. *For arsenic.*—Arsenopyrite. *For chromium.*—Potassium bichromate, potassium chromate, ferro-chromium. *For tungsten.*— Wolframite, scheelite. *For gold and silver.*—Pyrite, arsenopyrite. *For nickel.*—Pyrrhotite.

The Cadmium Minerals.

Sulphide.—Greenockite, is a yellow pigment of fixed color.

The Zinc Minerals.

Sulphide.—Sphalerate. *Oxide.*—Zincite. *Sulphate.*—Gostarite. *Carbonates.*—Smithsonite, hydrozincite. *Silicates.*—Willemite, Calamine.

Ores of zinc.—Spalerite, smithsonite, calamine, willemite, zincite. *Zinc oxide* also made from franklinite.

Uses of metallic zinc.—Galvanizing iron wire or sheets; manufacturing brass; sheet zinc, zinc dust. *Zinc white*, a paint, is zinc oxide ground in oil.

The Lead Minerals.

Ores.—Galenite, cerussite. *Uses of lead.*—Manufacture of white lead, preparation of red lead, litharge, shot, lead pipe, sheet lead. *Alloys.*—With antimony for type, and, friction-bearings.

The Cobalt Minerals.

Sulphides and arsenides.—Linnaeite, cobaltite, smaltite. *Arsenates.*—Erythrite.

Cobalt blue.—Cobalt and alumina compound. *Ainman's green.*—Compound of cobalt and zinc oxide.

The Nickel Minerals.

Sulphides.—Millerite, pentlandite, gersdorffite. *Arsenides.*—Nicolite, chloanthite. *Arsenate.*—Annabergite. *Carbonate.*—Zaratite. *Silicate.*—Garnierite.

Alloyed with copper, iron and arsenic in making German silver; with copper (25% *Ni* and 75% *Cu*) in the five-cent piece; with steel in making nickel steel.

The Manganese Minerals.

Sulphide.—Alabandite. *Oxides.*—Braunite, hausmaunite, pyrolusite, manganite, psilomelane. *Carbonate.*—Rhodochrosite, wad.

Alloys with iron, form: Spiegeleisen, ferro-manganese (for manufacture of steel). *Used as manganese ores.*—Pyrolusite, psilomelane.

The Calcium Minerals.

Limestone and marble are massive calcite and dolomite and are used largely in building construction. Limestone is used for hydraulic cements; and gypsum, for plaster of paris and wall plaster; also in paper making.

The Magnesium Minerals.

Calcined magnesite is used as a lining for converters in the basic process for the manufacture of steel.

The Tin Minerals.

Sulphide.—Stannite. *Oxide.*—Cassiterite (ore of tin).
Uses of tin.—Tin plate (sheet iron coated with tin) for making cans, kitchen utensils, etc. *Alloy.*—Bronze, bell metal, pewter, solder, tin amalgam.

The Titanium Minerals.

Oxide of titanium used for glazing porcelain.

The Thorium Minerals.

Oxide of thorium, thoria, constitutes about 99% (other 1% is cerium oxide) of the mantle of the Welsbach incandescent gas lamp.

The Arsenic Minerals.

Sulphides.—Orpiment, realgar. *Sources.*—From arsenides and arsenosulphides of iron, cobalt and copper. The white arsenic is a poisonous oxide, and is used in dyeing, medicine, sheep washing, calico printing, timber preservative, for fly paper, rat poisons and glass manufacture. Paris green is an arsenate of copper.

The Antimony Minerals.

Sulphides.—Stibnite, kermesite. *Oxide.*—Valentinite.
Uses.—The sulphide is used in vulcanizing rubber.

The Bismuth Minerals.

Uses.—Alloys with tin, lead and cadmium, *expand in cooling.*

The Sulphur and Tellurium Minerals.

Uses of Sulphur.—Manufacture of sulphuric acid, gunpowder, matches, rubber goods, bleaching, medicines, etc. Tellurium is closely related to sulphur in a chemical way.

The Uranium Minerals.

A small quantity of uranium in steel increases the elasticity and hardness. *Ore.*—Uraninite (principal source of radium).

The Molybdenum Minerals.

Sulphide.—Molybdenite. *Oxide.*—Molybdite.
Uses.—The metal is becoming important as an alloy with steel.

The Aluminum Minerals.

Ores.—Bauxite is the principal ore.
Uses of Aluminum.—Where lightness, strength and non-corrosiveness are desirable. It is replacing sheet copper and zinc and is used as bronze powder and alumina leaf for silvering letters and signs. It is replacing copper wire as electrical conductors. In metallurgy it is becoming important in welding pipes, rails in place, and steel castings. By adding less than 1% of aluminum to the melted metal it prevents blow-holes in castings of steel, copper and zinc.

The Boron Minerals.

Acid.—Sassolite. *Borates.*—Borax, ulexite, colemanite, boracite. Borax is used in welding; as a base of enamels on metal or porcelain; as a flux; as an antiseptic in packing meat; for powders, soaps, washing, dyeing, tanning.

The Hydrogen Minerals.

Water is the most important, one-eighth of its volume being hydrogen. Hydrogen is present in combination with carbon forming marsh gas, petroleum, ozocerite, etc.

The Carbon Minerals.

Diamond and graphite are pure carbon. It is present in a large number of solid, liquid and gaseous compounds, as natural gas, petroleum, asphalts, bitumens, fossil resins, mineral waxes, coal. Carbon is present in all organic matter, entering with plant life from the carbon dioxide of the air; and exists to a large extent in natural gas.

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THE SILICA MINERALS.

A.—Silica.

B.—Anhydrous Silicates.
- I. Disilicates and polysilicates.
- II. Metasilicates.
- III. Orthosilicates.
- IV. Subsilicates.

C.—Hydrous silicates.
- I. Zeolite Division.
- II. Mica Division.
- III. Serpentine and Talc Division.
- IV. Kaolin Division.

D.—Titano silicates.

Some of the Most Important Silicates:—

Granite.—Contains syenite, gneiss, basalt, diorite, andesite, which are compounds of silicates (usually three or more) and usually quartz, the feldspars and micas, pyroxene and amphibole.

Sandstone.—Made up of grains, mainly quartz, with perhaps feldspar, mica, or other mineral:

siliceous;
ferruginous;
calcareous;
argillaceous;
} According to the nature of the cement which binds the grains together.

Bluestone.—Hard, durable, fine-grained sandstone, cemented with siliceous material.

Slate.—Hardened clay. Used mostly for roofing, sinks, blackboards.

Fibrous talc and compact talc.—Used in paper manufacture. The latter, soapstone, is very valuable because refractory, for use in furnaces, crucibles, sinks, baths, hearths, electric switch-boards, cooking utensils; also used in paints, slate pencils, crayon, gas tips, as a lubricant, and in soap making.

Mica.—Muscovite, phlogopite, biotite, have become of great importance as non-conductors in electrical apparatus; also for stove and furnace doors.

Asbestos.—This is fibrous amphibole and serpentine. It is used in the manufacture of incombustible paper, cloth, cement, boiler and steam-pipe covering and packing rope or yarn for valves.

Serpentine.—A marble.

Feldspar.—Crushed and mixed with kaolin in the manufacture of porcelain.

Quartz.—Used in the manufacture of sandpaper, glass, porcelain, hone-stones, oilstones and as a flux. Ground flint is used as a scouring agent in soaps.

Infusorial earth.—Calcined and made into water filters, polishing powders, soap filling and boiler and steam-pipe covering.

Kaolinite and clay.—Clay is decomposed feldspar and other silicates. It lies in beds composed partly of some hydrous aluminum silicates, as kaolinite, but is usually mixed with quartz, mica, undecomposed feldspar, oxides and sulphides of iron. The industry includes the manufacture of common brick, paving brick, fire-brick, hydraulic cement, earthenware, stoneware, porcelain, terra cotta, sewer pipe, drain tile.

Fuller's earth.—A clay used in refining and clarifying mineral oils.

Rocks and Rock-Formations (Lithology).

COMPOSITION.—*The chemical* elements which enter chiefly into the composition of the rocks may be classed as metallic and non-metallic, as follows:

Metallic (basic) elements.—Aluminum, magnesium, calcium, iron, sodium potassium.

Non-metallic (acidic) elements.—Oxygen, silicon, carbon, sulphur, phosphorus, chlorine, fluorine, hydrogen.

These are combined in various ways, forming the numerous minerals which compose the rocks. Table 4, following, gives a list of the principal rock-forming minerals, arranged as per E. S. Dana's classification. It will be seen that silica and the silicates play a most important part. The calcareous group of minerals comprise mainly the lime and magnesium carbonates, and the lime phosphates and sulphates, so that lime is a very important constituent. Iron is found plentifully in most of our heavier rocks, being associated principally with oxygen, sulphur and carbon. The hydro-carbons furnish a most useful group of substances to the engineer.

4.—PRINCIPAL ROCK-FORMING MINERALS.

	Group.		Species.	Variety, etc.	Chemical. Composition.	Hardness	Sp. Gr.	Color.	Remarks.
	* Reference.	Name.							
1	*II, B, 3, a.*	Pyrite.	Pyrite.	(Firestone.)	$Fe\,S_2$	5	6.5	Yellow.	Most abundant and widely distributed sulphide.
2	*III, A.*	.....	Halite.	Rock salt.	$Na\,Cl$	2.5	2.0	Light.	Found with calcite, gypsum, clay, sandstone.
3	*V, ∝, A, 1. b.*	.....	Hematite.	(70% iron.)	$Fe\,O_3$	6	5	Reddish.	Source of nearly all red and brown colors in rocks and soils.
4	*V. ∝, A, 1, c.*	Spinel.	Magnetite.	(72.4% iron)	$Fe_3\,O_4$	6	5	Iron black.	Important iron ore; strongly magnetic.
5	*V, ∝, A, 2.*	.....	Limonite.	(60% iron.)	$H_6\,Fe_2\,O_9$	5	4	Dark brown.	Source of yellow and light-brown colors in rocks and soils.
6	*V, ∝ C.*	.....	Quartz.	Rock crystal	$Si\,O_2$	7	2.65	Light.	Prismatic and glassy.
				Chalcedony.	$Si\,O_2$	7	2.65	"	Translucent.
				Flint.	$Si\,O_2$	7	2.65	Dark.	Sub-translucent; contains carbon.
7	*V, β, A, 1, 1° a.*	Amphibole.	Pyroxene	(=Augite.)	$x\,si\,O_3$	5.5	3.4	Light.	Found in limestone, dolomite, basalt, etc.
8	*V, β, A, 1, 1° b*	"	.....	Hornblende.	$x\,Si\,O_3$	5.5	3.2	Black.	Black variety of amphibole.
9	*V, β, A, 1, 2°, d.*	Garnet.	.....	.....	*Si, Fe*, etc.	7	4.0	Brown.	Abundant in metamorphic rocks—schists and gneisses.
10	*V, β, A, 1, 2°, f.*	Epidote.	Epidote.	.....	*Si, Al, Co, Fe*	6.5	3.4	Greenish.	Aluminum-iron-calcium silicate. Secondary in igneous and metamorphic rocks.
11	*V, β, A, 1, 2°, k.*	Mica.	Biotile.	.....	*Si, Al, K, Mg, Fe*	2.5	2.9	Black.	*Mg K Fe* silicate; soft flexible layers.
12	"	"	Muscovite.	(Common.)	*Si, Al, K, H*	2	2.85	Light.	Essential in granite, mica-schist, gneiss.

13	*V, β, A*, 1, 2°, *n*	Feldspar.	Plagioclase.	Albite. Labradorite.	Lime-soda feldspars.	6 6	2.5 2.6	White. Dark gray.	Found in diorite. Plentiful in eruptive rocks.
14	"	"	Orthoclase.	.....	Potash feldspar.	6	2.5	Light.	Occurs in granite, gneiss, sienite, etc.
15	*V, β, A*, 1, 3°, *a*.	.....	Tourmaline.	.....	Aluminum boro-silicate.	7	3.0	Dark, etc.	Occurs in granite, gneiss, mica schists.
16	*V, β, A*, 1, 3°, *c*.	.....	Staurolite.	.....	*H, Fe, Al, Si*	7.5	3.5	Dark brown.	Accessory in metamorphic slates and schists.
17	*V, β, A*, 2, *III*.	Talc.	.....	.....	*H, Mg, Si*	1	2.6	Light.	Granular massive varieties are soapstones.
18	"	Serpentine.	Serpentine.	.....	*H, Mg, Si*	3.5	2.6	Greenish.	Tough, beautiful and durable stone.
19	"	Kaolinite.	Kaolinite.	China clay.	*H, Si, Al*	1	2.5	.....	Decomposed granite—quartz, mica, feldspar.
20	"	Hydro-mica	Margarodite.	.....	*H, Si, Al*, K	2.5	2.75	Greenish.	Similar in appearance to muscovite.
21	"	Chlorite.	.....	(Schist.)	*Si, Al, Mg, Fe*	1.5	....	Dark green.	Composed mostly of chlorite.
22	*V, β, F*.	.....	Gypsum.	Selenite.	*H, Ca, S, O*	2	2.3	Light.	The only rock-forming sulphate.
23	*V, β, G*.	Calsite.	Calcite.	(Anhydrous)	$Ca\ CO_3$	3	2.7	White.	Crystalline-massive—limestone, marble, chalk.
24	"	"	Dolomite.	(Common.)	*Ca, Mg*, CO_3	3.1	2.8	Light.	Similar to calcite.
25	"	"	"	(Brownspar)	.....	...	....	Brown.	Massive; contains iron.

* As per classification of Minerals, Table 3, preceding.

The following Table of the common rocks is compiled mainly from notes of Prof. W. O. Crosby of the Mass. Inst. Tech., and shows the various groups of rocks grading one into another. Thus,

I.—Gravel, conglomerate.
II.—Sand, sandstone, quartzite.
III.—Clay, slate, (porcelainite.)
IV. V.—Limestone, dolomite, magnesite, gypsum, rock-salt (phosphate rock).
VI. VII.—Gneiss, (strat. syenite,) (strat. diorite,) (norite,) mica schist, hornblende schist, hornblende rock, (schorl schist,) (graphite schist,) talc schist, chlorite schist, greensand, serpentine.
VIII, IX, X.—Granite, syenite, (diorite,) diabase, rhyolite, trachyte, obsidian, petrosilex, felsite, andesite, basalt, tachylite, porphyrite, melaphyr.

5.—The Common Rocks.

	No	Name.	Variety.	Composition.	Cementing Material.	Texture.	Remarks.
I.—Formation of conglomerate, sandstone (and brownstone)	1	Beach Gravel.	Pebbles.	Quartz, feldspar, granite, etc	.....	.....	Rounded by action of waves.
	2	Roxbury Puddingstone.	"	Consolidated pebbles, mud, sand.	Clayey paste.	.....	Originally beach gravel.
	3	Conglomerate.	"	Gravel and calcareous cement.	Carbonate of lime.	.....	" " "
	4	Limestone breccia.	"	Limestone pebbles and ferruginous cement.	Ferruginous.	.....	Not brecciated (crushed and recemented) L.
	5	Quartzose breccia.	Siliceous.	Quartz, red jasper, silica.	Silica.	.....	.....
	6	" puddingstone.	Vitreous.	do.	" (weak).	.....	Imperfectly cemented.
	7	" "	"	Small grains do.	" (strong).	.....	Looks like coarse granite.
	8	Conglomerate.	Quartz.	Quartz, feldspar, schist.	Ferruginous, arenaceous.	.....	Deposited by strong currents.
	9	"	Mica.	Micaceous rocks.	Argillaceous.	.....	From old rocks.
	10	Beach sand.	Quartz.	Quartz—vitreous.	Ferruginous.	.....	Grains sub-angular (not rounded.)
	11	Sandstone.	"	Sand as No. 10, but cemented.	Iron oxide.	Banded.	Bands parallel with stratification.
	12	"	Brownstone.	Quartz, feldspar, mica.	Ferruginous.	.....	Formed from old micaceous rocks.
II.—Composition of various kinds of sandstone.	13	"	Brownstone.	do. (micaceous).		Shaly.	Larger proportion of mica than No. 12.
	14	"	"	Quartz, feldspar.	.....	Fine grained.	Formed in New Jersey; little or no mica.
	15	"	Freestone.	Quartz. (quartzose)	.....	Friable.	Specks of iron oxide visible.
	16	"	"	Quartz, greenish clay "	.....	"	Iron oxide not visible.
	17	"	"	Nearly pure quartz "	Silica(weak).	"	Slightly stained with iron oxide.
	18	"	Arkose.	Quartz, feldspar, mica. (feldspathic)	.....	.....	Resembles granite.
	19	"	.....	Quartz, mica, clay.	.....	.....	Argillaceous, micaceous, carbonaceous (dark).
	20	"	Brownstone.	Quartz, ferric oxide.	Ferruginous.	Homogeneous.	No mica.
	21	"	.....	Quartz, ferrous compounds.	"	"	.....
	22	"	Flagstone.	Quartz, mica, clay.	Argillaceous.	Fine.	Splits easily.
	23	"	.....	Quartz.	Silica.	.....	.....
	24	"	Quartzite.	Quartz.	Silica.	.....	Metamorphosed sandstone; vitreous.

III.—Formation of slate and limestone.	25	Clay.	Common.	Kaolin and quartz flour.	.....	Soft earthy.	Forms the basis of nearly all kinds of slate.
	26	Slate.	Normal.	Consolidated clay.	.....	.....	Brown bands=surface oxidation of iron oxide.
	27	"	Sandy.	Consolidated clay, sand, mica	.....	Shaly.	A shaly, argillaceous sandstone.
	28	"	Stratified.	Quartz, mica (sand), clay.	Iron oxide.	Laminated.	Hardness due to excess of quartz.
	29	"	Massive.	Quartz.	Siliceous.	Compact, flinty.	From the celebrated trilobite quarry.
	30	"	.....	Brown clay and greenish gray sand.	.....	.....	Sedimentary under fluctuating currents.
	31	"	Roofing.	Fine and homogeneous.	.....	Laminated.	Perfect cleavage.
	32	"	Highly metamorphic.	Semi-micaceous.	Argillaceous.	.....	Imperfect cleavage.
	33	Tufa.	Basaltic.	Basalt lava.	"	Porous	Ashen color.
	34	Limestone.	Coquina.	Fragments of shells consolidated.	Carbonate of lime.	"	Imperfectly consolidated.
	35	"	.....	Coquina with clay filling.	"	.....	Interstices filled with clay.
	36	"	Tufaceous.	Deposition of $Ca\ Co_3$ on moss etc.	.....	Tufaceous.	Carbonate of lime and vegetable matter.
IV.—Composition of various kinds of limestone.	37	"	Calc tufa.	Travertine.	Calcareous.	Compact.	Deposited at mouth of spring or river.
	38	"	Coquina.	Finely ground corals, shells.	.....	Porous.	Finely fragmental.
	39	"	Earthy.	Calcite, fine clay (argillaceous).	.....	Oolitic.	Uncrystalline; large proportion of clay.
	40	"	Lithographic.	do.	.....	Compact.	do.
	41	"	"	do.	.....	.....	From an older formation than No. 40.
	42	"	Argillaceous.	A calcareous slate.	.....	Compact.	Carbonaceous (dark).
	43		Hydraulic.	Calcite, clay, magnesia, silica	.....	.....	When burned it makes a hydraulic cement.
	44	"	Oolitic.	Nearly pure limestone.	.....	Compact.	.....
	45	"	Fossil.	Coral formation.	.....	Crystalline.	Crystalline near fossil remains.
	46	"	Crinoidal.	"	.....	"	Can be polished and used as a marble.
	47	"	"	.	.....	"	Same as No. 47, but colored by iron oxide.
	48	"	Marble.	Calcite.	.....	Holocrystalline.	Pure, uncolored, non-fossiliferous

5.—THE COMMON ROCKS.—Continued.

	No.	Name.	Variety.	Composition.	Cementing Material.	Texture.	Remarks.
V.—Comparison of limestone, marble, dolomite, gypsum, rock salt.	49	Limestone.	Marble.	Pure calcite	.....	Crystalline.	Typical marble; pinkish tint.
	50	"	Crystalline.	Calcite; graphite, mica, iron.	.....	Friable.	Not suitable for building stone.
	51	"	"	Calcite; serpentine, graphite, etc.	.....	.....	Serpentine is secondary after pyroxene.
	52	"	Compact.	Calcite; chert (cryptocrystalline quartz).	.....	Compact.	Chert is much harder than calcite.
	53	Dolomite.	"	Dolomite.	.....	"	Very little calcite.
	54	"	Brecciated.	Dolomite; calcite, iron oxide.	.....	Crystalline.	Partly brecciated; not conglomerate.
	55		Crystalline.	Dolomite; tremolite (amphibole).	.....	"	Contains but little calcite.
	56	"	Coarsely "	Dolomite.	.....	"	Coarse, pure dolomite.
	57	"	.....	Dolomite, calcite.	.....	"	.....
	58	Limestone.	Onyx Marble.	Nearly pure calcite; some iron	.....	Banded.	Iron oxide and carbonate coloring.
	59	Gypsum.	.....	Gypsum.	.....	Crys. to compact.	Does not contain calcite nor dolomite.
	60	Halite.	Rock salt.	Sodium chloride; clay, iron oxide.	.....	Cubic cleavage.	Common rock salt.
VI.—Comparison of gneiss and mica schist.	61	Gneiss.	Typical.	Feldspar, quartz, mica, hornblende.	.....	Crys., banded.	The mica is mostly biotite.
	62	"	.....	Feldspar, quartz, biotite.	.....	"	Little or no hornblende.
	63	"	.....	Quartz, feldspar; garnet,	.....	Crys., laminated.	Almost binary; very little mica, no hornblende.
	64	"	Typical, massive.	Quartz, feldspar, hornblende.	.....	"	Abundance of feldspar makes it typical.
	65	"	Granitoid.	Quartz, feldspar.	.....	Crystalline.	Indistinctly laminated.
	66	"	Muscovite.	Quartz, feldspar, mica.	.....	Friable, granular.	Mica is muscovite.
	67	"	Granite (?).	Quartz, feldspar, mica (biotite).	.....	Crystalline.	Shows some indications of stratification.
	68	Mica schist.	Normal.	Mica, quartz; garnet.	.....	Crys.; schistose.	Mica (mostly muscovite); abund't.
	69	"	Metamorphic sandstones.	Mica, quartz.	.....	"	Granular aggregate.
	70	"		Mica, quartz.	.....	Granular, friable.	Used for making scythe stones.
	71	"	Typical.	Mica (muscovite) quartz; garnet.	.....	Crys.; schistose.	Laminated.
	72	"	.....	Muscovite, biotite, quartz; garnet.	.....	"	Contains prismatic staurolites.

VII.—Composition of various kinds of schists.	73	.....	Ottrelite.	Mica, clay; ottrelite, garnet.	.....	Crys.; schistose.	Illustrates change of argillaceous mica schist into slate.
	74	.....	.....	Nearly pure quartzite; little mica.	.....	"	Illustrates change of quartzose mica to quartzite and sandstone.
	75	Schist.	Hydromica.	Quartz, hydra-mica.	.....	"	.....
	76	"	Hornblende.	Quartz, hornblende.	.....	"	Micaceous, greenish and soft.
	77	"	"	Quartz, hornblende.	.....	"	Much more hornblende than in No. 76.
	78	"	Biotite.	Quartz, mica (biotite).	.....	Crys.; laminated.	Very little hornblende, if any.
	79	.....	Hornblende.	Pure hornblende rock.	.....	Crystalline.	Has high specific gravity.
	80	.....	Chlorite.	Nearly pure chlorite.	.....	Crys.; schistose.	Some hydromica, talc. Green color.
	81	Schist.	"	Similar to but more massive than No. 80.	.....	Crys.; schistose.	Not distinctly schistose.
	82	Talc schist.	Massive.	Soapstone.	.....	.....	Somewhat impure talc.
	83	Greensand.	Typical.	Glauconite; clay.	.....	Compact.	Earthy granular mineral.
	84	Serpentine.	Impure.	Massive impure serpentine.	.....	Compact.	Metamorphic eruptive rock.
VIII.—Comparison of granite, syenite and trap.	85	Granite.	Micaceous.	Quartz, feldspar, mica.	.....	Crystalline.	Mica is biotite and muscovite.
	86	"	.....	Quartz, feldspar, mica.	.....	"	" " chiefly biotite.
	87	"	Rockport.	Quartz, feldspar, hornblende mica.	.....	"	Feldspar is orthoclase; mica is biotite.
	88	"	Jonesboro.	do.	.....	"	Feldspar (pink) crystals are double (=Carlsbad twins.)
	89	"	.....	do.	.....	"	Feldspar (red) crystals are double (=Carlsbad twins.)
	90	"	.....	do.	.....	Finely "	A few grains of epidote secondary after hornblende.
	91	"	.....	do.	.....	" "	Some of the feldspar is pinkish.
	92	"	.....	do	.....	Crystalline.	Feldspar (orthoclase) red and green.
	93	Syenite.	Typical.	Feldspar (orthoclase), hornblende.	.....	"	No quartz. A little secondary biotite.
	94	Trap.	(=Diabase.)	Feldspar (plagioclase), biotite, hornlende.	.....	"	No quartz. Most of hornblende altered to biotite.
	95	"	Compact.	Feldspar, hornblende.	.....	"	Dark color; high specific gravity.
	96	"	.....	Feldspar (inc. plagioclase), hornblende.	.....	Crys.; porphyritic	Plagioclase crystals.

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5.—THE COMMON ROCKS.—Concluded.

	No.	Name.	Variety.	Composition.	Cementing. Material.	Texture.	Remarks.
IX.—Formation of rhyolite, trachyte and felsite from obsidian.	97	Rhyolite.	.....	Orthoclase, quartz, hornblende, biotite.	.....	Crystalline.	Not a typical volcanic rock.
	98	"	Typical.	Orthoclase, quartz, hornblende.	.....	Imp. crys.	Orthoclase is sanidine
	99	Trachyte.	"	Sanidine, hornblende, biotite.	.....	Crys.; porphyritic	Some of sanidine=Carlsbad twins. No quartz.
	100	"	.....	Feldspar (orthoclase ?)	.....	Crystalline.	No quartz nor accessories
	101	Obsidian.	Typical plain.	Orthoclase, etc.	.....	Highly vitreous.	Acid volcanic glass; conchoidal fracture.
	102	"	Banded.	do.	.....	Vitreous, banded.	Alternate glassy and stony layers.
	103	"	Spherulitic.	do.	.....	Vit.; spherulitic.	.....
	104	"	Pumice.	do.	.....	Vit.; vesicular.	.....
	105	Felsite.	=Petrosilex.	A completely devitrified obsidian. Acid.	.....	Compact.	Red color due to oxidation.
	106	"	.....	Formerly banded obsidian.	.....	Porphyritic.	Scattering crystals of feldspar.
	107	"	Typical.	A basic or true felsite.	.....	Finely banded.	Not porphyritic.
	108	"	Quartz porphyry.	Porphyritic with quartz and feldspar.	.....	Compact.	An acid felsite (petrosilex.)
X.—Comparison of basalt, tachylite and melaphyr.	109	"	Porphyritic.	Orthoclase, feldspar, hornblende.	.....	"	Porphyritic feldspar; hornblende crystal.
	110	"	"	do.	.....	"	Reddened by oxidation.
	111	Basalt.	Typical.	Augite, olivine, plagioclase, magnetite.	.....	Crys.; vitreous.	Dark color; high specific gravity.
	112	"	.....	Augite, olivine, plagioclase,	.....	Compact.	Augite and olivine, porphyritic.
	113	"	Recent lava.	Leucite, augite, olivine, plagioclase.	.....	Crys.; vesicular.	Leucite (crystals) replace plagioclase.
	114	"	.....	Vesicular basalt.	.....	Compact.	Normally blended.
	115	Tachylite.	Basaltic.	Olivine, plagioclase, etc.	.....	"	Heavier and not so glassy as obsidian.
	116	Melaphyr.	Greenstone.	Epidote, chlorite, quartz.	.....	Comp.; crystalline.	Once an amorphorus basalt like tachylite.
	117	"	Compact.	Plagioclase, epidote, chlorite.	.....	Amygdaloidal.	No. 113 with pores filled up.
	118	"	.....	do.	.....	Comp.; brecciated.	Fragments cemented by secondary epidote.

NOTES ON PRECEDING TABLE.

Class I.—This group of rocks illustrates the formation, by various cementing processes, of pudding stone and conglomerate from ordinary beach pebbles; limestone breccia from limestone pebbles; conglomerate from old broken rocks; and sandstone from sand. Gravel consists of pebbles worn by the action of water to the size not smaller than a pea, while sand is the result of greater wear, the grains being minute. Shingle is larger than gravel. Quartz is a large constituent of gravel owing to its resistence to abrasion, hence old gravel is a durable material for building operations. Clay is mainly decomposed feldspathic rock in place. It is made up principally of silica and alumina, and contains also lime, magneisa, iron oxides, etc. As a cement it is called argillaceous.

The rock cements are mostly argillaceous (clayey), carbonate of lime (calcite), ferruginous (iron compounds), silica, arenaceous (sandy-quartz sand), iron oxide, etc. Very often they are a combination of these or perhaps consist of certain of their elements.

Cement gravel is partially formed rock composed of cemented gravel and sand. It occurs in various degrees of hardness and is often difficult to classify by the engineer, either as earth or rock. A separate classification is often preferred, as it is usually more easily broken up by blasting than by plowing; that is, powder is cheaper in some cases than the specified "six-mule team."

Class II.—This is a distinct group—sandstones. It is composed of stratified grains of sand cemented together. The nature of the cementing material determines largely the hardness or crushing strength of the sandstone, silica being superior to carbonate of lime or iron oxide in both strength and durability. Quartz is the principal constituent of sandstone; but mica, feldspar, hornblende and augite are often present. The sandstones lie midway between the conglomerates (pebble base) and the shales (indurated clay base). They may also be said to occupy a structural position between almost loose sand, in which there is little or no cementing material, and the consolidated, metamorphosed varieties which resemble granite. If the cementing material is carbonite of lime and very abundant, it gradates into limestone.

Of the varieties, quartzite is the hardest; arkose contains much feldspar; freestone, as its name implies, is easily worked; brownstone is an impure sandstone, rather soft, and found in Connecticut and New Jersey; flagstone is shalely and used for pavements, and includes the bluestone variety.

Class III.—Slate is a metamorphic clay or shale, produced by great heat and pressure. The old cleavage planes of the shale are changed to new cleavage planes in the slate, and when these are perfect it forms the typical roofing slate. Slate is quarried in Maine, Vermont, New York, Pennsylvania, Maryland, Georgia, Minnesota, California.

Classes IV. and V.—These two groups comprise mainly the limestones, marbles and dolomites. Under Class III. it was seen that a sandstone containing an abundance of calcareous (carbonate of lime) cement grades into limestone when the cementing material increases to such an extent that it really forms the base of the rock, the sand diminishing in quantity and becoming merely accessory, forming the impurities. Pure limestone is carbonate of lime, the cementing material of many other rocks. Limestone may contain also a small amount of carbonate of magnesia and such impurities as silica, alumina, ferric oxide, etc. When, however, the proportion of carbonate of magnesia increases to a considerable extent the limestone grades into dolomite. Further, if it contains a large amount of the above so-called impurities—silica and alumina—it becomes hydraulic limestone. Coquina and tufacious limestones in Class III., and crinoidal limestone in Class IV, represent different sources of formation or origin.

Marble is a name formerly applied to all limestones of crystal and granular texture, capable of taking a polish; but it is now commercially used to apply to all limestones (even the non-crystalline) which can be polished.

Gypsum is composed of one-third lime ($Ca\ O$), nearly one-half sulphuric acid (H_2SO_4), and one-fifth water. Alabaster is a pure white variety, while gypsite contains more or less earthy impurities. Plaster of paris is obtained by heating gypsum to above 127° C. (261° F.), driving off about one-seventh of the water of crystallization, and then grinding to powder.

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Halite, No. 60, is sodium chloride (*Na Cl*) in the crystal form. Sodium chloride, common salt, plays a very important part in the manufacture of some high explosives.

Classes VI. and VII.—Gneiss is a name given to a class of metamorphic rocks composed essentially of the same elements as granite, namely, orthoclase, quartz and mica, with perhaps pyroxene, hornblende, etc. They may be either of igneous origin, like the granites and syenites, or of sedimentary origin, as the schists, presenting all graduations between these two classes. As they merge from the granitic to the schistose texture the feldspar disappears. Gneisses may be granitic (granite-gneiss), syenic (syenite-gneiss), hornblendic (hornblende-gneiss), etc., according to its composition.

Schist is a term applied to a metamorphic rock of slaty appearance. and of imperfect crystallization. Thus we have mica schist, hornblende schist, staurolitic schist, albite schist, etc. A slate is a schist of argillaceous character, forming a rather distinct class.

Soapstone, a magnesian silicate, is a talc schist and is very useful for hearthstones, sinks, etc.

Greensand or marl is composed mostly of glauconite (a silicate of iron and potash) and is used as a fertilizer.

Classes VIII., IX. and X.—These are distinctly igneous (eruptive) rocks, with more or less change in composition due to age of weathering.

Lava from an eruptive volcano is molten rock which subsequently hardens in cooling. Many attempts have been made to classify lava according to its composition or texture, but no satisfactory results have vet been attained. In density it varies from pumice, an extremely light, frothy, spongy form of obsidian (rhyolite) usually, to the heavy, compact basalt, which is usually the last of the lava flow.

Obsidian is a volcanic glass consisting approximately of silica (70 per cent), aluminum (12), calcium, potassium, sodium, iron, etc. The principal kinds of obsidian are rhyolite and trachyte.

Rhyolite has about the same chemical composition as granite, and from it, through weathering, etc., the latter is formed. Soda-rhyolites contain much sodium oxide; porphyritic-rhyolites are massive and compact. Felsite is a rhyolite which is not porphyritic.

Trachyte has about the same chemical composition as syenite, which, like granite, is formed through the action of weathering, changing slightly the chemical composition and, to a marked degree, the texture.

Basalt is a porphyritic rock of basaltic or columnar form. It is composed of hornblende, feldspar, augite, iron, and various other minerals. As it weathers very slowly it is synonymous with greenstone, trap, "basalt," etc., which comprise the so-called trap rocks. Melaphyr is a Tertiary basalt.

C.—BOTANICAL.

About three-eighths of the area of the United States, or 700,000,000 acres, is wooded; and of this, about 14 per cent comes under the United States Forest Service. The rapid depletion of our timber is a matter of grave concern, calling for abundant National and State reservations. The National forest reserves are situated usually at the heads of our tributary streams; serving, at the same time, to equalize the "run-off" of the precipitation, by retarding the melting of the snows in the spring of the year.

The following is a classification of the most important lumber trees of the United States and Canada:

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.—CLASSIFICATION OF IMPORTANT TREES.

	Height.	Dia.	Principal Localities.	Uses.
			The PINES. Genus, *Pinus*.	
			Soft Pines.	
s. ...	60′-120′	3′-4′	Atlantic States.	Construction; cabinet work; masts; boxes.
n P. ..	100′	4′-5′	Pacific Northwest.	Lumber in Montana and Idaho.
rtiana)	200′-220′ -300′	6′-10′	Oreg. to So. Calif., along W. Slopes Cascades, Sierra Nev.	Carpentry, doors, blinds, shingles. Sap yields sugar.
hite	40′-75′	2′-5′	E. slope Rocky Mts. from B.C. to Texas. Ariz and N. M.	Lumber in Eastern Montana, Ariz., and N. M.
			Pitch Pines.	
, Bull a.)...	150′-230′	5′-8′	Mountain slopes, Neb. to Pacific; B. C. to Lower Cal.	Lumber, ties, fencing, fuel.
ern P.	90′-120′	2′-2½′ -3′	Belt 125 m. wide, Va. to Fla., W. to Miss. R. Isolated, Ala., La., Tex.	Most widely used in the East for heavy construction; bridges, buildings, docks, masts, spars.
ield P.	80′-100′	abt. 2′ -3′	Coast: Cape May to Texas. Inland: Carolinas to Ark., La.	Lumber, h'vy cons.; docks, ships, cars, houses; ties.
.....	75′-120′	2′-3′	Nor. U. S. and Canada, Me. to Minn., south to Penn.	Cons.; buildings, bridges, docks; masts, spars.
.....	80′-100′ -120′	3′-4′	Conn. to Fla., W. Va., E. Tenn.; Gulf States to E. La.; So. Missouri to E. Texas. Largest size W. of Miss. R.	Inferior to Longleaf P. for heavy construction. Wood is softer and more easily worked for carpentry.
P..	100′	2½′-3′	So. Car. to So. Fla., and along Gulf to Pearl R., La.	Same uses as Longleaf pine.
.....	-120′		So. Carolina to Miss.	Ties and fuel.
			The LARCHES. Genus, *Larix*.	
ack ..	100′-250′	6′-8′	Columbia R. Basin; E'n Oreg., N'n Mont. and Idaho.	Ties, fence posts; durable; furniture and interior finish.
			The SPRUCES (1). Genus, *Picea*.	
.....	70′-80′ -100′	2′-3′	P. E. Island, N. Eng, N. Y., Alleg. Mts. to N. Carolina.	Lumber, paper pulp, flooring; sound'g bds. for musical instruments.
.....	60′-70′ -150′	-2′	N'n U. S., Canada and Alaska.	Inferior as lumber; superior for paper pulp.
.....)	-150′	4′-5′	Alberta, B. C., Mont., Idaho, Cascade Mts., Wash., Oreg.	Lumber, buildings, fuel, charcoal; bark tans leather.

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6.—CLASSIFICATION OF IMPORTANT TREES.—Cont'd.

Species.	Height.	Dia.	Principal Localities.	Uses.
The SPRUCES (1). Genus, *Picea*.—(Continued.)				
Sitka S., Tideland S. (*P. sitchensis.*)	100′-200′	3′-4′ -16′	Alaska, B. C., Wash., Oreg., to Mendocino Co., Cal.	Most imp. lum. Alaska; bldgs boats, fences, boxes, fuel.
The HEMLOCKS. Genus, *Tsuga*.				
Hemlock (*T. canadensis.*)	60′-70′ -100′	2′-4′	N. Scotia to E'n Minn.; N. Y., N. Eng., Pa. to Ala.; Mich., Wis.	Lumber, buildi ties; bark ta leather.
Western H. (*T. heterophylla.*)	100′-200′ -250′	6′-10′	S. E'n Alaska to C. Mendocino; east to Mont. and Idaho.	Lumber for bu ings; bark for ning.
Mountain H. (*T. mertensiana.*)	70′-100′ -150′	4′-5′	Alaska, B. C., Olympic Mts., Wash.; Cascades and Sierras.	Sometimes use lumber; bark tanning.
The SPRUCES (2). Genus, *Pseudotsuga*.				
Douglas S., Red Fir (*P. mucronata.*)	100′-150′ -300′	2′-3′ -10′	S'n B. C., W'n Wash. and Oreg., N'n Cal., N. M., Ariz., Texas.	Known as Ore Pine, Wash. I Superior for c struction; lum
The FIRS. Genus, *Abies*.				
White F. (*A. amabilis.*)	70′-100′ -200′	4′-6′	B. C., Olympic Mts.; Cascades, Oreg., Wash.	Largest and m com. Fir in O pic Mts. Int finish.
White F. (*A. grandis.*)	100′-250′ -300′	2′ -4′	Vancouver Id., Wash., N'n Oreg., Idaho, Mont.; N'n Cal.	Interior finish, packing boxe
Red F. "Larch." (*A. nobilis.*)	150′-200′ -250′	6′-8′	W'n Wash., Oreg.; N'n Cal. Cascades and Sierras.	Interior finish buildings, an packing case
The REDWOODS. Genus, *Sequoia*.				
Redwood (*S. sempervirens.*)	200′-300′ -325′	10′-15′ -28′	Coast, S'n Oreg. to Monterey Co., Cal.	Lumber, build shingles; pos ties, wine-bu
Big Tree (*S. wellingtonia.*)	275′-325′ -350′	20′-35′	Narrow area, W'n slope of Sierras. King's Riv.	Lumber, cons tion; shingles fencing.
The ARBOR VITAES. Genus, *Thuya*.				
White Cedar, Arbor Vitae (*T. occidentalis.*)	50′-60′	2′-3′ -6′	N. Scotia to Manitoba; central N. West; Allegheny Mts.	Fencing, ties, gles, tel. pol
Giant A.-V., Red Ced., Canoe Ced. (*T. plicata.*)	150′-200′	bottom -15′	Coast, Alaska to C. Mendocino.; Mts. E'ly to Idaho, Mont.	Finishing, hou doors, sashe cooperage.
The CYPRESSES. (1). Genus, *Cypressus*.				
Monterey C. (*C. marocarpa.*)	60′-70′	2′-3′ -6′	Coast of Cal., south of Monterey Bay..	Hedges, wind brakes; shin

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6.—CLASSIFICATION OF IMPORTANT TREES.—Cont'd.

Species.	Height.	Dia.	Principal Localities.	Uses.
The CYPRESSES (3). Genus, *Chamaecyparis*.				
"White Cedar." (*C. thyoides.*)	70′-80′	2′-2½′ -4′	Coast, Mass. to Fla., and west to Pearl Riv., Miss.	Finishing; shingles, cooperage, fencing, ties.
Sitka C., Yellow C. (*C. nootkatensis.*)	100′-120′	5′-6′	S. W'n Alaska, B. C., Cascade Mts., Wash., Oreg.	Shipbuilding; interior finish; furniture.
Lawson Cypress (*C. lawsoniana.*)	150′-200′	-12′	Coast Mts. of Oregon and Cal.	Int. finish and flooring; ties, fencing; shipbuilding.
The CYPRESSES (3). Genus, *Taxodium*.				
Bald Cypress (*T. distichum.*)	75′-150′	4′-5′ -12′	Del. to Fla.; Gulf Coast to Tex; Miss. Val. to Mo., Ind., Ill.	Lumber, flooring, shingles, cooperage, ties, fencing.
The WALNUTS. Genus, *Juglans*.				
Butternut, White W (*J. cinerea.*)	50′-100′	2′-3′	New Brunswick to Del. and to Dak.; Mid. West; Ga., Ala.	Cabinet work; interior finish.
Black W (*J. nigra.*)	75′-150′	4′-6′	Ontario to Fla.; west to Nebr. and Texas.	Cabinet work; interior finish; ship building.
The HICKORIES. Genus. *Hicoria*.				
Shellbark H., Shagbark H. (*H. ovata.*)	70′-90′ -120′	3′-4′	Me. to Del. to Fla.; N'n Ala., Miss.; west to Min., Neb.; south to Texas.	Agric. imp.; wagons and ax-handles.
Big Shellbark H. (*H. laciniosa.*)	120′	-3′	Ohio Basin, Middle West, and South East.	Agric. imp.; wagons; ax-handles.
Pignut, White H. (*H. glabra.*)	80′-90′	3′-4′	Me. to Fla., west thro Ont., Mich., Neb. South to Texas.	Agric. imp.; wagons; tools.
Mockernut H., Big Bud H. (*H. alba.*)	50′-80′ -100′	-3′	Ontario to Fla.; west to Kan. and Tex.	Agric. imp.; wagons; ax-handles.
Nutmeg H. (*H. myristicaeformis.*)	80′-100′	-2′	E'n S. Car.; Cen. Ala. Miss., to S'n Kan.	Lumber and fuel.
Pecan H. (*H. pecan.*)	100′-170′	4′-6′	S'n Ill., Ind., Ia.; Miss. Riv. States to Cen. Ala.	Fuel; valuable nut tree.
Bitternut H., Swamp H. (*H. minima.*)	60′-100′	2′-3′	Me. to Fla.; Ontario to Minn., Neb. and Tex.	Hoops and fuel.
Water H., Bitter Pecan. (*H. aquatica.*)	50′-80′ -100′	-2′	Swamp regions, Va. to Tex.; north along Miss. R. to Ill.	Fencing, fuel.
The POPLARS. Genus, *Populus*.				
Cottonwood (*P. deltoidea.*)	60′-100′ -150′	7′-8′	Quebec to N.W. Territory; south to Fla.; west to N.M.	Shelter and shade in prairie States.
Cottonwood (*P. fremontii.*)	100′	5′-6′	California. from Sacramento south; E'ly to Col., Tex.	Shade tree; fuel.

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6.—CLASSIFICATION OF IMPORTANT TREES.—Cont'd.

Species.	Height.	Dia.	Principal Localities.	Uses.
The POPLARS. Genus, *Populus*.—Continued.				
Aspen, Quaking Asp.... (*P. tremuloides.*)	40'-100'	-3'	Newfoundland to Alaska; south to N. J.: west to Penn., Ky., Neb., Rocky Mts., Coast Range.	Shelter and shade; now rapidly spread over Rocky Mt. areas swept by fires.
Balm of Gilead..... (*P. balsamifera.*)	60'-100'	6'-7'	Newf'dl'd to Alaska; S. to N. Y.; W. to Mich., Neb., Idaho.	Shade and shelter.
Black Cottonwood..... (*P. trichocarpa.*)	-200'	7'-8'	West coast, Alaska to S'n Cal.	Woodenware; staves of sugar barrels.
The WILLOWS. Genus, *Salix*.				
Black W..... (*S. nigra.*)	40'-60' -120'	-3'	Me. to Fla.; Rocky Mts.; Cal.	Borders of streams.
Golden Osier..... (*S. alba.*)	40'-60'	2'	Eastern N. America.	
The BIRCHES. Genus, *Betula*.				
Canoe B., Paper B..... (*B. papyrifera.*)	60'-70'	2'-3'	Labrador to Alaska; south to Long Id.; Penn., Cent. Mich., Minn., Neb., Black Hills, Mont., N.W. Wash.	Letter paper; bark for canoes, and various camp articles.
Yellow B., Grey B..... (*B. lutea.*)	-100'	3'-4'	Gulf of St. L. to Del., to N. Car., to Tenn. to Minn.	Furniture, boxes, wheel hubs, fuel.
The BEECHES. Genus, *Fagus*.				
Beech..... (*F. Americana.*)	70'-80' -120'	3'-4'	N. Scotia to L. Huron; N'n Wis.; south to Fla., Mo. and Tex.	Chairs, shoe-lasts, plane-stocks, tool handles; fuel.
The CHESTNUTS. Genus, *Castanea*.				
Chestnut..... (*C. dentata.*)	60'-100'	3'-4' -12'	S'n Me. to Mich.; south to Ind., Del.; to Ala., Miss.	Int. finish, furniture, ties, fencing, fuel.
Chinquapin..... (*C. pumila.*)	-50'	2'-3'	Penn. to Fla.; west to Ark. and Texas.	Large sizes for ties and fencing.
The OAKS. Genus, *Quercus*.				
White Oaks.				
Pacific Post O., Oregon White O..... (*Q. garryana.*)	60'-70' -100'	2'-3'	Vancouver Id.; W'n Wash., Oreg., and Cal.	Cabinet work, wagons, ship building, cooperage, fuel.
Live O..... (*Q. virginiana.*)	40'-75'	3'-4'	Coast and Islands, Va. to Fla.; Gulf and Lower Cal.	Excellent lumber; ship building.
White O..... (*Q. alba.*)	60'-100' -150'	3'-4' -8'	S'n Me. to Fla.; west to Minn., Kan. and Tex.	Construction; shipbuilding, ties, int. finish, cabinet work.
Burr O., Mossy-Cup O... (*Q. macrocarpa.*)	70'-180'	6'-7'	Nova Scotia to Montana; s'th to Penn., Tenn. and Tex.	Cons.; shipbuilding, ties, int. finish, cabinet work.
Swamp O., Overcup O... (*Q. lyrata.*)	70'-100'	2'-3'	Maryland to Fla; west to Missouri and Tex.	Confounded commercially with *Q. alba*, above.

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6.—Classification of Important Trees.—Cont'd.

Species.	Height.	Dia.	Principal Localities.	Uses.
White Oaks.—Continued.				
Post O., Iron O. (*Q. minor.*)	40′-50′ -100′	2′-3′	Mass. to Fla.; west to Missouri and Tex.	Ties, fencing, fuel; cooperage.
Chestnut O., Tan-bark O (*Q prinus.*)	60′-70′ -100′	3′-4′ -7′	Maine to Tenn.; Mts. in Ga. and Ala.	Cooperage, wheels, fencing, ties.
Yellow O. (*Q. scuminata.*)	80′-100′ -160′	3′-4′	Vt. to Minn.; south to Ala. and Tex.	Cooperape, wheels, fencing, ties.
Swamp White O. (*Q. plantanoides.*)	60′-70′ -100′	2′-3′ -9′	Me. to Great Lakes and Ia.; south to Md., Ga., Ky., Ark.	Cooperage, boat-building, fencing, ties, fuel.
Cow O., Basket O. (*Q. michauxii.*)	60′-100′	3′-7′	N'n Del. to Fla.; west to Ill., Mo. and Texas.	Fencing, ties, fuel; bark for tanning.
Black Oaks.				
Live O., Maul O., Gold-cup O. (*Q. chrysolepis.*)	40′-50′	3′-5′	W'n slopes Sierras and Coast Mts., Oreg., and Cal.; Ariz., N. M.	Wagons and agric. imp.; most val. Oak on Pac. Coast.
Pin O., Swamp Spanish O. (*Q. palustris.*)	70′-80′ -120′	2′-3′ -5′	Mass. to Del.; west to Wis. and Ark.	Cooperage; int. finish; shingles, clapboards.
Red O. (*Q. rubra.*)	70′-80′ -150′	3′-4′	N. Scotia to Minn.; south to Ga., Tenn. and Kan.	Interior finish; furniture; bark for tanning.
Scarlet O. (*Q. coccinea.*)	70′-80′	2′-3′	Me. to Fla.; west to Ohio Val., Minn., Neb., Mo.	Interior finish; furniture.
Texan (Red) O. (*Q. Texana.*)	50′-100′ -200′	7′-8′	Iowa to Ind.; south to Fla. and Texas.	Lumber, Miss. Val., better than *Q. rubra.*
Black O., Yellow O. (*Q. velutina.*)	70′-80′ -150′	3′-4′	Me. to Fla.; west to Minn., Kan., E'n Tex.	Fuel. Bark used in medicine, dyeing, tanning.
Spanish O. (*Q. digetata.*)	70′-80′	2′-3′	N. J. to Fla.; west to Mo. and Tex.	Light construction, and fuel; bark for tanning.
Water O. (*Q. nigra.*)	60′-80′	2′-3½′	Del. to Fla.; west thro Gulf States, Ky., Tenn., Ark. to Tex.	Fuel.
Willow O. (*Q. phellos.*)	70′-80′	2′-3′ -4′	N. Y. to Fla.; Gulf States to Tex.; N'ly in Mo., Ky., Tenn.	Clapboards; felles for wheels; some construction.
Laurel O., Shingle O. (*Q. imbricaria.*)	50′-60′ -100′	3′ -4′	Penn. to Ga.; west to Nebr. and Ark.	Clapboards and shingles; some construction.
The Elms. Genus, *Ulmus.*				
White E., American E (*U. americana.*)	75′-125′	6′-11′	Newfoundland to Fla.; west to Rocky Mts.	Wheel hubs, saddle trees, flooring, cooperage.
Slippery E., Red E. (*U. fulva.*)	60′-70′	2′	Ontario to Dak., Nebr.; south to Fla.; west to Tex.	Fencing, ties, wheel hubs, agric. implements.
Cedar E. (*U. crassifolia.*)	-80′	2′-3′	S'n Ark., Miss. and Tex.	Fencing and fuel.
The Sweet Gums. Genus, *Liquidambar.*				
Sweet G., Bilsted (*L. styraciflua.*)	80′-140′	4′-5′	Conn. to Mo.; south to Fla. and Tex.	Furniture, int. finish, shingles, fruit boxes, paving blocks, ties.

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6.—CLASSIFICATION OF IMPORTANT TREES.—Concl'd.

Specials.	Height.	Dia.	Principal Localities.	Uses.
The MAPLES. Genus, *Acer*.				
Red M., Scarlet M., Swamp M. (*A. rubrum.*)	80'-120'	3'-4½'	Eastern and Middle States, Lower Mississippi, Texas.	Tool handles, oars, furniture, wooden-ware; fuel.
Silver M., Soft M. (*A. saccharinum.*)	90'-120'	3'-4'	New Brunswick to Dak.; south to Fla. and Oklohoma.	Cheap furniture and flooring.
Oregon M., Broadleaf M. (*A. machrophyllum.*)	80'-100'	2'-3'	South Coast Alaska to San Diego, Cal.	Interior finish, furniture, ax- and broom-handles.
Sugar M., Rock M., Hard M. (*A. saccharum.*)	75'-120'	2'-3'	Great Lakes to Newfoundland. to Fla., to Neb., to Tex.	Buildings, flooring, furniture, boats, fuel.
Black M., Black SugarM. (*A. nigrum.*)	-80'	-3'	Dak. to Vt., to Va., Ky., Mo., Kan.	Buildings, flooring, furniture, boats, fuel.
The ASHES. Genus, *Fraxinus*.				
White A. (*F. americana.*)	-120'	5'-6'	N. Scotia to Fla.; west to Minn., Texas, Ohio R. Val.	Int. finish, stairs; tool handles, oars, furniture.
Black A. (*F. nigra.*)	80'-90'	1½'	Va. to Del. to Manitoba, Ill., Mo., Kan.	Furniture, fencing, barrel hoops, cabinet work.
Blue *A.* (*F. quadrangulata.*)	60'-70' 120'	2'-3'	Mich. to Mo., to Kan. to Ark.	Flooring, tool handles, vehicles.
Oregon A. (*F. oregona.*)	70'-80'	-4'	Coast, Puget Sound to San Francisco; Sierras.	Int. finish, furniture, wagons, cooperage, fuel.
Green A. (*F. lanceolata.*)	50'-60'	2'	L. Champl'n to Fla.; west to Ariz., Utah, Tex.	Int. finish, stairs; tool handles, oars, furniture.

INTERESTING FACTS ABOUT TREES.

Some of the Tallest Trees.—Big Tree (350 ft.), Redwood (325 ft.), Sugar Pine (300 ft.), Douglas Spruce (300 ft.), Western Hemlock (250 ft.), Western Larch (250 ft.).

Some of the Best Timber Trees.—Longleaf Pine, Douglas Spruce, White Pine, Norway Pine, Shortleaf Pine, Live Oak (*Q. virginiana*).

Some of the Best Lumber Trees.—All the timber trees make good lumber. In addition we may name: Most of the Pines, Spruces, Hemlocks, White Fir, Redwoods, Sitka Cypress, Lawson Cypress, some of the Walnuts and Hickories, many of the Oaks. Some of the Oaks, Ashes, Chestnuts, Maples, and Walnuts are used for interior finish.

Ages of Trees at Maturity.—White Pine, 250 yrs.; White Oak, 200 yrs.; Chestnut, 175 yrs.; Beech, 100 yrs.; Elms, 90 yrs.; White Ash, 80 yrs.; Birches, 50 yrs.

Relative Rapidity of Growth.—Silver Maple, White Elm, Red Maple, Sugar Maple, Chestnut, Red Oak, Pin Oak, Scarlet Oak, White Ash, White Oak.

Some Important Tree Products—Tar, Rosin and Turpentine are obtained from the *resin* or sap of the longleaf pine. Pine tar is also obtained by the burning, or rather smouldering, of the limbs and knots of the longleaf pine. The dry distillation of woods produces wood vinegar (used for dyeing), acetic acid (which is made into vinegar), and wood alcohol. The latter may also be obtained from sawdust, and is destined to become a most valuable product for heating and power. Wood charcoal is also a valuable product. All are familiar with the products of the Balsam and the Sugar Maple. Wood pulp is used in making paper.

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D.—ZOÖLOGICAL.

Forenote.—So far as known there are about 350,000 living species of animals, and 50,000 extinct specimens, making a grand total of about 400,000. The latest and most logical classification of the animal world, by Parker and Haswell, comprises 12 grand divisions or Phyllums, arranged according to structure—beginning with the lowest forms, the Protozoans or unicellular animals, and ending with the highest forms, the Mammals with Man at the head of the scale. In the evolution of the species the lower forms of life have played an important part in the development of the higher, just as today the whole animal kingdom forms a valued adjunct to the development and uses of man.

Among the insects we have the *honey*-bee; the *silk*-worm; the cochineal, found on certain species of cactus, for useful and harmless *dye*; the lac for *shellac*; the galls of the gall-fly for ink, etc.

Many fishes, including the sword-fish, codfish, sunfish, etc., yield oil. Oil is also yielded by many reptiles, by the whale, walrus, manatee, etc.

The whale, narwhal and elephant furnish us with ivory; the seal, beaver, and other fur-bearing animals, with furs; the alligator, goat, and many quadrupeds, with skins for leather; the latter, also, with horns and tallow; the sheep, with wool; sea-fowl, with guano; etc., etc.

A considerable percentage of the animal world supplies us with food; and a few animals produce useful work.

The following classification is now recognized by scientists as the most logical that has ever been submitted.

7.—*CLASSIFICATION OF ANIMALS.*

(After Parker and Haswell.)

Class. Subclass. Phyllum.

a. ***Protozoa.*** (Unicellular animals.)

I. Protozoa (Protozoans). One cell; or several cells of same kind.

1. *Rhizopoda* (Amoeba, etc.). With retractile pseudopodia.
2. *Mycetozoa* (Slimes). Terrestrial protozoa, plasmoidal.
3. *Mastigophora.* Without cilia, or sucking tentacles.
4. *Sporozoa.* Without appendages; internal parasites.
5. *Infusoria.* With sucking tentacles.

β. ***Metazoa.*** (Multicellular animals.)

II. Porifera (Sponges). Fixed; body-wall perforated, pores.

1. *Porifera.*
   (*a*). *Calcarea.* With skeleton or calcareous spicules.
   (*b*). *Non-Calcarea.* Where skeleton exists, composed of siliceous spicules.

III. Cœlenterata (Polyps, etc.). Radial structure; digestive cavity lined.

1. *Hydrozoa* (Hydroids, etc.). More than two rays; single cavity.
2. *Scyphozoa* (Jellyfishes). Many radii; cavity divided by radial partitions.
3. *Actinozoa.* Colonies or attached individuals.
   (*a*). *Zoantharia* (Sea-Anemones, Corals, etc.). Numerous tentacles, usually in 5's.
   (*b*). *Alcyonaria* (Sea-Fans, Red Corals, etc.). Eight tentacles.
4. *Ctenophora.* With two radii, and rows of sucking tentacles.

IV. Platyhelminthes (Flatworms). Body composed of loose cells.

1. *Turbellaria* (Planarians). Body covered with celia; one opening to alimentary tract.
2. *Trematoda* (Flukes). Parasitic, unsegmented; adult without celia.
3. *Cestoda* (Tapeworms). Without mouth or alimentary canal.
4. *Nemertinea.* Carnivorous and aquatic; with mouth, arms, and food canal.
   (*a*). *Palaeonemertinea.*
   (*b*). *Schizonemertea.*
   (*c*). *Hoplonemertea.*

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7.—CLASSIFICATION OF ANIMALS.—Cont'd.

Class. Subclass. Phyllum.

V. Nemathelminthes (Round worms). Usually—mouth, arms, alimentary tract.

1. *Nematoda* (Threadworms). With intestinal canal; parasitic.
2. *Acanthocephala*. Parasitic; no mouth or intestines.
3. *Chaetognatha* (Arrow worms). Developed nervous system; spiny.

VI. Trochelminthes (Wheel-animalcules). Larva in trochosphere form.

1. *Rotifera*. Microscopic.
2. *Dinophilea*. Minute; with 5 to 8 segments, usually ciliated.
3. *Gastrotricha*. Minute; ciliated on ventral surface.

VII. Molluscoida (Sea-Mats and Brachiopods). Aquatic.

1. *Polyzoa* (Bryozoans). Form colonies connected by one organic substance.
   (*a*). *Ectoprocta*. Anus, external.
   (*b*). *Endoprocta*. Anus, internal; bud, forming colonies.
2. *Phoronida*. Worm-like; born from ova, not by buds.
3. *Brachiopoda* (Lamp-shells). Body enc. in shell of two valves, usually on a stalk.

VIII. Echinodermata (Echinoderms). With intestinal walls.

1. *Asteroidea* (Starfish). Star-shaped; furrows under the arms.
2. *Ophiuroidea* (Brittle Stars). Arms not grooved.
3. *Echinoidea* (Sea-Urchins). Body, globular; armless.
4. *Holothuroidea* (Trepangs). Worm-like; with tentacles about mouth.
5. *Crinoidea* (Crinoides). Sessile; with cup-shaped body.
6. *Cystoidea* (Fossil). Globular and stalked (sessile).
7. *Blastoidea* (Fossil). Ovate; stalked.

IX. Annulata (Worms). Bilaterally segmented; without jointed legs.

1. *Chaetopoda* (Annelids). Composed of series of metameres, bearing cirri.
   (*a*) *Polychaeta* (Marine Annelids). Sexes distinct; ovaries and testes, many.
   (*b*) *Oligochaeta* (Fresh water and terrestrial). Sexes united; ovaries and testes, few.
2. *Myzostomida* (Crinoides). Unsegmented.
3. *Gephyrea* (Marine). Sessile; adult, without external segmentation.
4. *Archi-Annelida* (Often Parasitic). Minute; marine, segmented (faintly).
5. *Hirundinea* (Leeches). With ventral suckers.

X. Arthropoda (Insects, Crustaceans, etc.). Symmetrical, segmented.

1. *Crustacea* (Crustaceans). Aquatic, gill bearing; two pairs antennae, usually.
   (*a*) *Entomostraca* (Water-Fleas, etc.). Varied number of appendages.
   (*b*). *Malacostraca* (Crabs, Crayfish, etc.). With 19 pairs of appendages.
2. *Trilobita* (Extinct Trilobites). With head, thorax and abdomen.
3. *Onychophora* (Peripatus). With series of short walking appendages.
4. *Myriapoda* (Centipedes and Millipedes). Segments, each with 1 or 2 pairs of legs.
5. *Insecta* (Insects). With six thoracic legs, and usually with wings.
6. *Arachnida* (Spiders, Scorpions, etc.). Air-breathing; without antennae.

7.—CLASSIFICATION OF ANIMALS.—Concluded.

Class. Subclass. Subphylum. Phyllum.

XI. Mollusca (Mollusks). Unsegmented; muscular locomotion.

1. *Pelecypoda* (Bivalves). Gills leaf-like; two-valved shell.
2. *Amphineura* (Chitons). Symmetrical bi-laterally; anus at end of body.
3. *Gastropoda* (Gastropods). Unsym. body; shell (if any) univalve.
   (*a*). *Streptoneura* (Limpets, Whelks, etc.). Visceral commissures like a figure 8.
   (*b*). *Enthyneura* (Pulmonates Nudibranchs, etc.). Vis. com. not like a figure 8.
4. *Scaphopoda* (Marine). Mouth-lobes formed into a tube.
   (*a*). *Scaphopoda* (Tusk-shells.)
   (*b*). *Rhodope*. Minute; no shell; with sucking tentacles.
5. *Cephalopoda* (Cuttle-fish). Mouth surrounded by arms; foot funnel-shaped.
   (*a*). *Dibranchiata* (Squids, Octopods). Two sym. branchiae; tubular funnel.
   (*b*). *Tetrabranchiata* (Nautilus, Ammonites). Four branchiae; multilocular shell.

XII. Chordata (Chordates). Animals having a notochord.

1. (*A*). *Adelochorda* (Balanoglossus, etc.). Marine; with notochord as larvae.
2. (*B*). *Urochorda* (Ascidians). Marine; having a notochord when larvae.
   (*C*). *Vertebrata* (Vertebrates). Sym. bilaterally; having a backbone.
   (*I*). *Acrania* (Branchiostomidae, Amphioxus, etc.). Without a head.
   (*II*). *Craniata* (Fishes, Reptiles, Birds, Mammals).
1. *Cyclostomata* (Lampreys). Eel-like, without lower jaw; mouth, suctorial.
2. *Pisces* (Fishes). Aquatic; persistent gills, paired fins.
   (*a*). *Elasmobranchii* (Sharks and Rays). Cartilaginous skeleton; cranium not ossified.
   (*b*). *Holocephali* (Chimaeras). Cartilaginous; four pairs of gill slits.
   (*c*). *Teleostomi* (Sturgeons and ordinary Fish).
   (*d*). *Dipnoi* (Lungfishes). Fish-like animals with apparatus for breathing.
   (*e*). *Ostracodermi* (Cephalaspis, etc.).
3. *Amphibia* (Amphibians). With gills when leavae, lungs when adult. 5-toed feet.
4. *Reptilia* (Reptiles). Air-breathing; horny epidermal skeleton of scales.
5. *Aves* (Birds). Clothed with feathers.
   (*a*). *Archaeornithes* (Archaeopteryx). With prolonged tail of many vertebrae.
   (*b*). *Neornithes* (Modern Birds). Tail vertebrae compacted.
6. *Mammalia* (Mammals). Suckle their young; clothed with hair, more or less.
   (*a*). *Prototheria* (Didelphia). Mammals with oviducts separated.
   (*b*). *Theria* (Monodelphia). Mammals with oviducts more or less united.
      (b_1). *Metatheria* (Marsupials). Rudimentary birth; sheltered in pouch.
      (b_2). *Eutheria* (higher Mammals). Born in uterus; no pouch.

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18.—EXPLOSIVES.

An explosive is a mixture which, under certain disturbing influences, enters into rapid chemical reaction, forming expansive gases and evolving much heat. The substances mixed may be solid or liquid. Explosives may be classified according to the nature of the mixture, whether *mechanical* or *chemical*.

(a). MECHANICAL MIXTURES.

In mechanical mixtures the component elements remain intact until a high temperature is reached, when they chemically react and pass off as gases (mostly), causing the explosion.

Nitrate Mixtures.—This class comprises the mixture of a nitrate, embodying oxygen, with some base yielding carbon and usually containing sulphur. The explosion occurs when the oxygen leaves the nitrate and combines with the carbon, having a greater affinity for the latter. Gunpowder is typical of this class.

Gunpowder.—This is the oldest and one of the most common of explosives. It consists of a mixture of saltpetre (=nitre=potassium nitrate = KNO_3), charcoal (=carbon) and sulphur in various proportions according to the nature of the explosion desired. The black powder, formerly used almost exclusively in U. S. ordnance, was composed of 75 parts saltpetre, 15 parts charcoal and 10 parts sulphur; while the brown or sporting powder contains much more charcoal and considerably less sulphur, the proportions being, 79 parts saltpetre, 18 parts charcoal and 3 parts sulphur. These are now being superseded by the smokeless powder.

Blasting Powder.—This is made slower acting than the gunpowder by reducing the proportion of saltpetre to 66 parts, charcoal forming 15 parts, and sulphur 19 parts. Sodium nitrate or Chili saltpetre is now commonly used instead of the more expensive potassium nitrate.

Assuming the weight of powder at 62.5 lbs. per cu. ft. (spec. grav. = unity), the following table gives the weight of powder in a hole one ft. deep and of various diameters:

1.—WEIGHT OF POWDER IN A HOLE ONE FOOT DEEP.

Diam. Ins.	Weight. Lbs.	Weight (Avoir.) Lbs.	Ozs.	Diam. Ins.	Weight. Lbs.	Weight (Avoir.) Lbs.	Ozs.
¼	.0852	0 —	1.4	2½	2.131	2 —	2.1
⅜	.1332	0 —	2.1	3	3.068	3 —	1.1
½	.1917	0 —	3.1	3½	4.176	4 —	2.8
⅝	.2610	0 —	4.2	4	5.454	5 —	7.3
1	.3409	0 —	5.5	4½	6.903	6 —	14.5
1¼	.5326	0 —	8.5	5	8.522	8 —	8.4
1½	.7670	0	12.3	5½	10.312	10 —	5.0
1¾	1.044	1 —	0.7	6	12.27	12 —	4.4
2	1.364	1 —	5.8	6½	14.40	14 —	6.4

Note.—Weight in lbs. = 0.3409 × (diam. of hole in ins.)^2; therefore it is proportional to the square of the diameter of the hole.

Other Nitrate Explosive Mixtures.—The following mixtures are analogous to gunpowder in exploding at high temperatures:

Amide.—Ammonium nitrate, potassium nitrate, charcoal.

Azotine.—Sodium nitrate (69), carbon (15), sulphur (12), petroleum (4).

Carbo-azotine.—Potassium nitrate (61), soot (26), sulphur (14).

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Diorexine.—Potassium nitrate (50), sodium nitrate (25), sulphur (12), hard sawdust (13).

Johnite.—Potassium nitrate (75), sulphur (10), lignite (19), sodium picrate (3), potassium chlorate (2).

Petralite.—Potassium nitrate (64), charcoal (30), crude antimony (6).

Pyrolite.—Potassium nitrate (51), sodium nitrate (16), sulphur (20), sawdust (11), charcoal (2).

Chlorate Explosive Mixtures.—Potassium chlorate is a constituent in each of the following explosives, which are considered rather unsafe on account of spontaneous or premature reaction:

Asphaline.—Potassium chlorate (54), potassium nitrate and sulphate (4), bran (42).

Ehrhardt P.—Potassium chlorate, cream of tartar, powdered nutgalls, tannin.

Fontaine P.—Potassium chlorate, potassium picrate.

Horseley P.—Potassium chlorate (6), nutgalls (1), charcoal (1), nitroglycerin (72). (This is a dynamite.)

Michalowoski Blasting P.—Potassium chlorate (50), manganese dioxide (5), bran (45).

Oriental P.—Potassium nitrate and crude gamboze, potassium chlorate.

Pyronome.—Potassium nitrate (69), sulphur (9), charcoal (10), antimony (8), potassium chlorate (5), rye flour (4), potassium chromate.

Racharock.—Potassium chlorate (79), mono-nitrobenzine (21). Fresh mixed.

(*b*). CHEMICAL COMPOUNDS.

Nitro-substitution Explosive Mixtures.—The following are examples of nitric acid treatment of the hydrocarbons, producing compounds which at high temperatures become unstable:

Ammonites.—Ammonium nitrate (88), di-nitro napthalin (12).

Bellite.—Ammonium nitrate (5), meta-di-nitrobenzine (1), potassium nitrate.

Borlinetto P.—Picric acid (10), sodium (10), potassium chromate (8).

Extralite.—Ammonium nitrate, potassium chlorate, naphthalin.

Joveite.—Nitro-naphthalin, nitro-phenol, sodium nitrate.

Roburite.—Ammonium nitrate, chlorinated di-nitrobenzine.

Romite.—Ammonium nitrate, potassium chlorate, naphthalin.

Securite.—Ammonium nitrate, di-nitrobenzine.

Volney P.—Potassium nitrate, sulphur, nitro-naphthalin.

Nitric Acid Compounds.—When certain vegetable tissues, called cellulose, notably cotton, are treated with nitric acid the resulting cellulose nitrate is a very high explosive. The stronger treatments produce *guncotton* and the weaker, *pyroxylin*. Similarly, when the animal compounds gelatin, or glycerine, are treated with nitric acid there results *nitroglycerin*, a most powerful explosive. *Dynamite* consists of some absorbent soaked with nitroglycerin to protect the latter from decomposition and premature explosion.

Guncotton.—This is made by soaking cotton, or other form of cellulose, in nitric acid (one part) and sulphuric acid (three parts) for about a day, and then thoroughly washing. The resulting product is from 60 to 85 per cent heavier than the cellulose, depending upon the proportion and strengths of the acids and the general treatment. It is the safest explosive known in general handling and shipment. It is exploded by percussion when confined and highly compressed, but if ignited burns quietly without explosion.

Guncotton is used as an agent in various mixtures containing nitrates of which the following are examples:

Glyoxiline.—Guncotton, saltpetre, nitroglycerin,—in form of pellets.

Potentite.—Guncotton, saltpetre,—in form of cartridges.

Tonite.—Guncotton, barium nitrate,—in form of cartridges.

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Detonation.—In our classification of the various explosives under the two headings, Mechanical Mixtures and Chemical Compounds, we are confronted with the great natural law of gradation, which is universal. For instance, there are some animate objects which the naturalist is unable to classify distinctly as plants or animals, bordering on the division line and having some of the common attributes of both great orders in nature. So with explosives. Up to 1864, when Alfred Nobel, a Swedish engineer, began to put some of the higher explosives to practical use, the great principle of "detonation" or instantaneous explosion by "shock" was but vaguely known. His investigations with nitroglycerin led to the discovery that many explosives heretofore treated by ordinary combustion were far more powerful when subjected to "percussion," which converted the substance *immediately* into gas. In his experiments with nitroglycerin he used a percussion cap charged with fulminate of mercury as an initiatory explosive. Detonation is essential in the higher class of explosives as guncotton, nitroglycerin, dynamite, etc.

The following are some of the guncotton preparations resulting from Nobel's discovery:

Blasting gelatin.—Guncotton absorbed in nitroglycerin.

Gelatin dynamite.—Blasting gelatin and an absorbent.

Forcite.—Blasting gelatin (50), absorbent (50).

Gelignite.—Blasting gelatin (65), absorbent (35).

Smokeless Powders contain guncotton, nitrate, carbonate, nitroglycerin, and various other substances of like nature. The product is a horn like substance, which is cut into chords or grains.

Nitroglycerin.—To the engineer, this is one of the most useful and most powerful explosive agents. It is prepared by treating glycerin with strong nitric and sulphuric acids, producing the chemical compound, $C_3 H_5 N_3 O_9$. In its pure liquid form it burns quietly, producing carbon dioxide, hydrogen, nitrogen and water. But when gradually heated to 180° C. or when subjected to violent percussion it explodes, developing gases from 1200 to 1400 times the volume of the liquid, which in turn are further expanded by the great amount of heat evolved, to about 10,000 times the original bulk. Nitroglycerin when mixed with infusorial earth as an (inert) absorbent forms *dynamite*, which is sold under various trade names.

Dynamite.—As the liquid nitroglycerin is liable to explode by heat or decomposition, it is rendered safer by being combined with some protecting absorbent. The absorbent may be *inert* or *active*, naturally dividing the dynamite into two classes. The second class comprises by far the more important, as follows:

Atlas P.—Nitroglycerin (75), wood fibre (21), sodium nitrate (2), magnesium carbonate (2), Also lower grades.

Carbonite.—Nitroglycerin (25), wood dust (40), sodium nitrate (34), sodium carbonate (1).

Dualin.—Nitroglycerin (40), wood dust (30), potassium nitrate (20).

Giant P.—Nitroglycerin (40), resin (8), kieselguhr (8), sulphur (6), sodium nitrate (40).

Hercules P.—Nitroglycerin (40), wood fibre (11), sodium nitrate (45), sodium chloride (1), magnesium carbonate (1).

Judson P.—Nitroglycerin (5), cannel coal (15), sulphur (16), sodium nitrate (64).

Lithofracteur.—Nitroglycerin (54), kieselguhr (17), sulphur (7), barium nitrate (15), wood dust (2), manganese (2), soda (2), bran (1).

Meganite.—Nitroglycerin (60), nitrated wool and vegetable ivory (20), sodium nitrate (20).

Rhexite.—Nitroglycerin (64), decomposed wood (11), wood dust (7), sodium nitrate (18).

Safety nitro P.—Nitroglycerin (69), wood fibre (13), sodium nitrate (18).

Stonite.—Nitroglycerin (68), wood dust (4), kieselguhr (20), potassium nitrate (8).

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Vigorite.—Nitroglycerin (68), kieselguhr (20), potassium nitrate (7), carbonate, etc. (5).

Vulcan P.—Nitroglycerin (30), charcoal (11), sulphur (7), sodium nitrate (52).

Unmixed Explosives.—There is a certain class of explosives called Panclastites, consisting of two ingredients which separately are inexplosive but when mixed are considered fully as powerful as dynamite. They are shipped separatelv and mixed at the site when needed for use. The principal ingredients of the mixture are nitrogen tetroxide and carbon disulphide. They are not generally recommended for ordinary use by the class of men usually employed in blasting.

Percussion Caps.—Fulminate of mercury enters into the composition used for percussion caps and electric fuses employed in detonating charges of dynamite in blasting operations. It is prepared by adding alcohol to a nitric acid solution of mercury.

(c). The Handling and Use of Dynamite.

Dynamite as invented by Noebel in 1867 consisted of nitroglycerin absorbed by a porous, inert solid. The best absorbent was found to be a siliceous, infusorial earth known as kieselguhr, obtained in Hanover, Germany. When dried it is an impalpable, white powder of cellular structure, and is capable of absorbing three times its weight of nitroglycerin, giving the resultant dynamite the appearance and consistency of heavy brown sugar. Coupled with its absorbent, nitroglycerin is thus, in the form of dynamite, free from the danger of spontaneous explosion, and detonation from shocks of a moderate nature. In the loose powdered form, dynamite loses none of its explosive properties when exposed to natural temperatures; i.e., it explodes readily by the action of the fulminating mercury primer or cap. On the other hand, when the powder is compressed into cartridges and the temperature is reduced to between 50° and 42° F. it freezes and, like frozen nitroglycerin, is inexplosive. It loses only a small percentage of its explosive power on being saturated with water, hence its great value in submarine work. If unconfined and ignited by a flame (350° F) it burns freely and quietly without explosion.

Dynamite is usually compressed in sticks or cylinders called cartridges, varying from ¼ to 2 ins. in dia. and about 8 ins. long, more or less; or they are sold in any size and length ordered. The sticks are wrapped separately in paper, and packed in boxes, in layers cushioned with sawdust; usually 50 lbs., or 25 lbs., to the box.

In "freezing" weather these cartridges have to be "thawed" out. The term "freezing" applies to any temperature that chills the dynamite, usually in the neighborhood of 45° F, sometimes lower and sometimes higher, depending upon the grade, the quality of the absorbent, and the nature of the exposure. The "thawing" of dynamite consists in slightly warming it to take off the chill. It sometimes requires a temperature of 50° or 60° F to do this effectively, and it must not be allowed to chill again while being carried to the drill holes and loaded. The usual method of thawing dynamite is by a small out-door fire, but the disadvantages of this plan are: (1) waste of time in thawing, (2) greater or less danger attached to it, (3) inefficiency of thorough thawing so that the dynamite will explode with the greatest effect. In a pamphlet "Thawing Dynamite" published by the Aetna Powder Co., of Chicago, is an illustration of a Thaw House about 7x10 ft., fitted with steam coils and shelves for thawing. The shelves have a capacity of about 500 lbs. of dynamite, and about 1000 lbs. more can be stored in the house in boxes. The special caution for sweeping and cleaning merits careful attention.

In preparing the charge, the fulminating cap, to which the safety fuse has been attached, is inserted in the top of the cartridge, the neck of which is tied around the fuse with a string, and the charge placed in the hole and fired. If an electric blasting machine is used, a special cap is required.

Many of the dynamites are put up in two grades, No. 1, and No. 2, the former containing as high as 75% N.-G. Some of the powders are furnished in several grades, ranging from 75 down to 20% N.-G. In ordering dynamite it is necessary to state the percentage of nitroglycerin required, whether 30, 40, 50, 60, 75%, unless the name of the desired brand is known.

2.—Some of the Most Common Commercial Dynamites.

(Note.—The percentages of nitroglycerin are shown in parenthese

Aetna powder, No. 1 (65), No. 2 XX (50), No. 2 (40), No. 3 X (3
No. 4 X (25), No. 5 (15).

Atlas powder, A (75), B + (60), B (50), C + (45), C (40), D + (3
D (30), E + (25).
E (20), F + (15).

Colonia powder (40).

Carbonite (25).
Dualin (40 to 50).

Dynamite—Nobel's Kieselguhr—Old No. 1 (75), Old No. 2, (40), Old N
3, (25).

Forcite (49).

Gelignite (62½).

Giant powder, No. 1 (75), New No. 1 (50), No. 2 extra (45), No. 2 (4
No. 2 c (33), No. XXX (27), No. M (20).

Giant powder—Nobel's—No. 2 (20).

Hecla powder, No. 1 XX (75).

Hercules powder, No. 1 XX (75), No. 1 (65), No. 2 SSS (55), No. 2 SS (5
No 2 S (45), No. 2 (40), No. 3 S (35), No. 3 (30), No. 4 S (25), No.
(20).

Horsley powder (72).
Judson powder, FPF (20), FF (15).
Rendrock (40).
Vigorite (30 to 68).

Judson giant powder, No. 2 (4
Lithofracteur (54).
Stonite (68).
Vulcanite (30).

A List of Permissible Explosives for Use in Coal Mines.—Follow
brands, tested prior to Oct. 1, 1909: Aetna coal powder A, AA, B, C; Bi
minite No. 1; Black Diamond Nos. 3, 4; Carbonite Nos. 1, 2, 3, 1-L.
2-L. F.; Coalite Nos. 1, 2-D; Coal Special Nos. 1, 2; Collier dynamite N
2, 4, 5; Gyant A low-flame dynamite, C low-flame dynamite; Masur
M. L. F.; Meteor dynamite; Mine-ite A, B; Manobel; Tunelite Nos. 5, 6, 7

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19.—PRESERVATIVES.

PAINTS.

A paint consists of a pigment, vehicle, drier, and (usually) solvent. Sometimes the pigment is ground alone and then mixed with the vehicle and drier but usually they are ground together, and sealed in kegs or cans for shipment.

Pigments.—The following are the most important to the engineer:

White lead, consisting of a mixture of lead hydrate and lead carbonate, is universally employed for white paint. It is often adulterated, however, with heavyspar, gypsum, chalk, clay, sulphate of lead, etc., which renders it more or less inferior.

White zinc or zinc oxide is used for white paint; often adulterated with heavyspar.

Lampblack (mainly carbon) is one of the best pigments for black paint and for printer's ink. It is obtained by burning substances rich in carbon, with a low flame, that is, to incomplete combustion, so the carbon will not be burned. For this purpose resin, ozokerite, the by-product hydrocarbons from oil refineries, various kinds of oils gas, etc., are used.

Boneblack and *ivory black* are obtained by heating bones with exclusion of air.

Graphite is becoming one of our most useful paint pigments, especially as a protection to iron and steel from rust.

Iron oxides give brownish colors varying to red, yellow, etc., depending upon the natural ores, as ochres, bole, pozzuole earth, sienna earth, etc. The ground ores are often adulterated with heavyspar, gypsum, clay, etc., for cheapness and shading of colors.

Red lead (red oxide or minium) is more highly oxidized than white lead or litharge and is obtained by properly heating the latter in a furnace or oven, in contact with the air, to a certain temperature (about 400° to 500° C.). It is used as a red pigment and is also a very durable cement for glass and for water pipes. Commercially, as a pigment, it is often adulterated with brick dust, red bole, heavyspar, oxide of iron, etc.—the last named being particularly objectionable.

Verdigris is a pigment usually mixed with white lead to produce a green paint. It is made by exposing thin copper plates to the air in contact with acetic acid.

Ochre is a clay containing oxide of iron, and when dried and ground is used as a pigment. It is usually of a yellowish color but by burning it gradually changes to red by the oxidation of the iron present.

Umber differs from ochre in containing a quantity of oxide of manganese in the clay in addition to the oxide of iron. The unburned or "raw" umber is dried and ground as a pigment for brown paint; while the calcined or "burnt" umber yields a reddish brown color.

Sienna is a ferruginous earth like ochre, the unburned of "raw" sienna producing a yellowish-brown pigment; if "burnt" (before being ground) it yields a reddish-brown pigment.

Asbestos is a fibrous hornblende (amphibole) or crysotile (serpentine) of various colors as white, gray, red, green and black. Its value in paint lies in its fireproofing qualities when applied to wood, or other inflammable material.

Vehicles—A vehicle in paint corresponds to lime paste in common mortar, forming the *binder* when dry. In calcimine, the pigment is zinc-white and the vehicle glue-sizing diluted with water; tinting is produced by adding the various coloring matters. For common paints linseed oil is universally referred as a vehicle, although any kind of varnish may be used.

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Linseed oil is obtained from the seed of common flax, by (1) cold pressure, (2) hot pressure, (3) extraction. The first named process produces the food oil; the second and third, the article of commerce. The method of extraction is by the use of some solvent as benzine, naphtha, etc. Carbon disulphide yields fully 50% more oil than the cold-drawn process, but is objectionable when used with a lead pigment because the retained sulphur turns it dark. "Boiled" linseed oil is used as a vehicle for paints because drying is facilitated and it acts better with the pigment. Linseed oil is often adulterated with other oils as those of mustard seed, rape seed, hemp seed, cotton seed, etc. The engineer usually specifies "pure" raw or boiled linseed oil, the former being often preferred *alone* as a coating for steel when it leaves the shop.

Driers, such as compounds of lead, manganese and zinc, are simply oxidising agents for converting the oils from "vehicles" into "binders." Among the most common are litharge, red lead, lead acetate, oxides of manganese, borate of manganese, oxide of zinc, etc., or their various mixtures. These act as agents in conveying oxygen from the air to the oils, thus drying them.

Solvents.—A good solvent is slow to evaporate and flows sufficiently to efface the brush marks.

Turpentine is the best solvent. It is a repeated distillation of the resinous sap of certain coniferous trees, and dissolves readily the various substances used in paints. Only the pure turpentine should be accepted. It is composed entirely of carbon and hydrogen, but when exposed to the air it absorbs oxygen and turns yellow. It should be kept air-tight and away from the light. Turpentine has no substitute. Benzine, kerosene and some other oils are sometimes used either alone or as adulterants, but should be shunned.

House Paints.—White is obtained from white lead; black, from lampblack; red, from red lead (50) and red ochre (50); green, from verdigris (75) and white lead (25); chocolate, from spanish brown (96) and lampblack (4); stone color, from white lead (99) and burnt umber (1); lead color, from white lead (98) and lampblack (2).

The following table shows how various colors may be produced by a simple combination of two other colors. To produce "shades" of any color add black; to produce "tints" add white:

	Red.	Yellow.	Blue.	Orange.	Purple.	Green.
Red.....						
Yellow.....	Orange.					
Blue.....	Purple.	Green.				
Orange.....						
Purple...				Russet.		
Green.....				Olive.	Citrine	

Note.—Yellow and red produce orange; yellow and blue, green; purp and orange, russet; etc.

Special Paints.—These are made by mixing substances having speci properties, with the pigments:

Aluminum paint is made from powdered aluminum and contains abo 91% metallic aluminum, 6% aluminum oxide, 1½% silica, 1% wat Gas or air is forced under pressure into the molten metal which is vigorou stirred, and forms a powder in setting. This powder is crushed and r through sieves and polished. About 2 lbs. of the powder is mixed wi one gallon of varnish composed of 1.5 galls. turpentine, ½ gall. palest co varnish, 4 oz. palest terebine, 4 oz. carbonate of magnesia. The latter allowed to settle and the clear varnish is drawn off.

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Bronze paint is made by mixing filings of copper, brass, etc., with the pigment. It is used in painting iron and other materials.

Copper paint, mercury paint, arsenic paint and paris-green paint are poisonous to marine life and are used in painting ships bottoms. The copper paint is formed by mixing salts of copper with the pigment. The others are formed by mixing mercury, arsenic, and paris green, respectively, with the pigments used.

VARNISHES, LACQUERS, ETC.

Varnishes.—A varnish consists of a gum or resin dissolved in "spirit" or in oil, with perhaps some coloring matter added. The resinous substances are amber, animè, copal, mastic, resin, sandarac, shellac, for the lighter colors; and asphaltum and pitch for the dark colors. The solvents are "spirits," as ethyl alcohol, wood (methyl) alcohol, ether, benzol, chloroform, carbon disulphide, coal oils (light), turpentine; or "oils," as linseed, poppy, nut, hemp, castor, walnut, cotton-seed. The coloring matter is aloes, dragon's blood (a resin), tumeric, sanders-wood, saffron, anotto, indigo. The name of the varnish usually takes its name from the solvent, as *spirit varnishes* and *oil varnishes*.

Lacquering is the varnishing of polished metal surfaces, as brass. The lacquer is an alcoholic solution af shellac with some coloring matter added to give the desired tint. Being transparent or nearly so, the brass effect is visible while the metal is at the same time preserved from discoloring or oxidizing.

Japanning is varnishing with japan, a black varnish made with asphaltum (mostly) dissolved in turpentine.

GALVANIZING AND TINNING.

Galvanizing consists in dipping the iron or steel sheets, rods, bolts, wire, etc., in a bath of molten zinc. The coat is naturally very durable unless exposed to sulphurous smoke or other similarly destructive chemical agents.

Tinning consists in immersing the iron or steel sheets in molten tin and tallow. Tin plate, so called, is merely the thin metal sheet *coated* with tin. Terne plate is made by dipping the iron or steel sheets into a bath of tin and lead, being less expensive than tin plate and much inferior to it. It is used largely for roofing.

ELECTRO-PLATING.

This branch of Electro-Chemistry was evolved from discoveries made by Jacobi in 1838. By its use we are enabled to plate the cheaper and less durable metals with durable metal coatings. The agent employed is the electric current.

Electro-Chemistry is that branch of chemistry which treats of the chemical changes, produced by, or producing, electricity or electrical energy. It embraces the subjects of (1) Electrolysis, the decomposition of a chemical compound called an *electrolyte* into its constituent parts by an electric current; and (2) Electro-Metallurgy, the deposition of certain metals, as gold, silver, copper, etc., from their solutions by means of the slow action of an electric current. All electrolytes are either acids, bases or salts, and all electro-chemical reactions produce, or are produced by, these three classes of compounds. The transformation of chemical energy into electrical energy is through a system termed a voltaic or galvanic cell or battery, two or more *cells* in combination forming a *battery*.

Electrolysis—When an electrolytic substance is subjected to the action of an electric current it decomposes into *ions*, the *cations* (electropositive ions) seeking the "cathode" or negative pole, and the *anions* (electronegative ions) seeking the "anode" or positive pole. Thus, when water is decomposed the hydrogen atoms (cations) are attracted to the negative pole, while the oxygen atoms (anions) collect at the positive pole.

Electro-Metallurgy is an electrolytic process by which the metal in an ore is separated from its impurities. It is one of the common processes, the others being: Smelting or heating of the ore; Amalgamation by the use of mercury; and Extraction by chemical solutions.

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Electro-Plating embodies all the preceding principles. The article to be plated must first be cleaned thoroughly. It is dipped in a cleansing solution and then thoroughly rinsed with water so that none of the solution will remain. If necessary it is then placed in a scouring tray before going to the plating vat, where it is suspended in the plating solution from copper rods by means of short "slinging" wires of copper. The electric current used for depositing the metal from the plating solution may be obtained from any source which is convenient, either from a battery or dynamo. The following are some of the solutions used for electro-plating: For *gold* plating, a solution of gold cyanide and potassium cyanide; for *silver*, a solution of silver cyanide and potassium cyanide; for *copper*, an ammoniacal solution of copper and potassium cyanide; for *nickel*, the double sulphate of nickel and ammonia, or the double salt of nickel and ammonia; for *brass*, liquid ammonia and potassium cyanide added to a nitric acid solution of brass. Various other metals may be used for plating, as iron, tin, lead, platinum, bronze, etc.

Gold and silver may be deposited in excellent condition on a large number of the metals and alloys. Copper may be deposited on steel, iron, tinned iron, zinc; also on lead and tin, and their alloys; it is to be noted also that when any of these metals are to be gold-, silver-, or nickel-plated it is best to plate them with copper first, as copper plating adheres more firmly to them, and in turn is adhered to more firmly by the desired coatings. Nickel may be deposited on most of the common metals, including German silver, brass, and alloys of the soft metals.

PRESERVATION OF STEEL AND IRON.

Oiling, Painting, Asphalting, etc.—Many engineers specify that structural material on leaving the shop shall simply be coated with pure boiled linseed oil instead of being painted. This is doubtless good practice, as the oil penetrates the skin of the metal sufficiently to preserve it temporarily from rust and does not cover up any possible imperfections, which may be detected readily before erection. Among the best *paints* are the lead paints and the carbon paints. On ordinary structures they should last at least ten years if properly applied. Some of the best paint companies guarantee their paints for that period. Graphite paint and asphalt paint are also largely used. One gallon of paint will cover 600 sq. ft. and upward of metal, two coats; costing about $\frac{3}{10}$ to $\frac{5}{10}$ ct. per sq. ft.; or say, 2 to 3 cents per 100 lbs. of metal, for ordinary structures. For light bridges the cost per 100 lbs. would be greater; for heavy bridges and buildings, about 2 cents. This is exclusive of labor in painting.

Iron or steel thoroughly imbedded in cement or cement concrete is permanently protected from rust, but not from electrolysis.

For water pipes the following are used: Asphalt, coal tar (inferior), Smith's durable metal coating, Sabin process. The last named is a process of applying asphalt varnish, the pipe, after dipping, being baked for several hours at a temp. of 400° to 600°, thus producing an enamel coating.

Mill Scale on structural steel, can effectively be removed at the mill: (1) by "pickling" or cold rolling, at an expense of say 80 cents to $2.00 per ton; (2) by the sand blast at a cost of about ½ less; (3) by steel scrapers and wire brushes at considerably less expense if only ordinary cleaning is desired. Engineers differ as to the advisability of using the sand blast. The following opinions are expressed in Report of Committee E on Preservative Coatings for Iron and Steel, Proceedings A. S. T. M., Vol. VI: Mr. Phelps Johnson, Eng'r Dominion Br. Co., Limited, says: "In my experience I have found that an exposure to the weather for three or four months will ordinarily loosen nearly all the mill scale adhering to rolled material, and have considered that the subsequent removal by steel brushes of the slight coating of rust powder that is formed leaves the material in the best practicable shape for receiving the paint. I am of the opinion that the exposure to the weather may be prolonged to say 24 months without appreciable waste of metal or injurious pitting or roughening of the surfaces, and that there is but slight increase in the labor necessary to brush the surface clean for painting." Mr. Gustav Lindenthal, Consulting Engineer, says: "Mill scale is simply the magnetic iron oxide, which is insoluble in most acids, and an excellent preservative of the metal underneath, always provided that it adheres tightly to the surface of the metal. . . . I consider the use of the sand-blast for the cleaning of metal surfaces not only

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useless, but detrimental. . . . The problem of painting iron and steel is not yet solved, in spite of the volumes of papers, discussions and reports on investigations which encumber the bookshelves." Mr. A. H. Sabin, Chemist, says; "For *best work*, the cleaning of structural steel work by the use of the sand blast is probably the simplest and most satisfactory way to have it done. The great objection to this, as to all such work, is the cost. . . . . For *ordinary work*, the wire brush is an efficient means of getting rid of loose scale and dirt; but it is practically worthless for removing thick rust or anything which adheres closely. Much of such material may be removed by steel scrapers, but deeply corrugated spots should be cleaned out thoroughly with a chisel, and then well brushed." Mr. George E. Thackray, Structural Engineer, Cambria Steel Co., says: "Sand blasting is almost useless and impracticable, as I have seen sand blasting done, exposing the metal surface, which was soon covered with a thick coat of rust before the painters could reach it, although both the sand blasters and painters were working with expedition."

Mr. Leonard M. Cox, Civil Eng'r. U. S. Navy, in describing the protective coating on the floating dry dock "Dewey," says:* "The selection of a protective coating for the dock was made the subject of careful study. Samples of a number of the best known paints on the market, exclusive of the oxide paints, were applied to test-plates and subjected to different conditions. Three plates were coated with each sample, one was exposed to the weather at the company's works, a second was suspended half in air and half in water, and a third was submerged in the water of Chesapeake Bay. The tests extended over a period of 2 years, during the construction of the dock, and resulted in the choice of a mixture of red lead, white zinc and linseed oil in the following proportions: 100 lbs. of red lead, 15 lbs. of zinc ground in oil, and 5 galls. of linseed oil. It is only fair to state that in these tests a graphite paint manufactured in Detroit showed as good results as the red lead, but was not used because of the lack of available data bearing on its behavior in salt water.

"As the pontoons have more or less water in their bottoms at all times, except when self-docked, it was very necessary to provide adequate protection for their floors. Experiments with Bitumastic Enamel led to its application to the whole of the interior floors, and to all vertical bulkheads, braces, etc., to a height of 12 ins. The process of applying this mixture consists in a careful cleaning and drying of the metal, one coating of a solution, the function of which is to provide a surface to which the enamel will adhere, and a final heavy coating of enamel, from $\frac{1}{16}$ to $\frac{1}{8}$ in. in thickness, applied hot.

"Great care was taken to rid the hull plating of mill scale. The specification provided that all loose scale should be removed by hammering, scraping, and brushing with wire brushes, and to make its removal easier and surer, no paint was applied until a short time before launching. Nearly all the material, therefore, was exposed to the weather in the yard for periods ranging from 12 to 24 months. The existence of mill scale on the completed structure has such an important bearing on the subject of corrosion that the additional expense entailed by pickling would be a good investment in the way of insurance, and it is recommended that specifications for future docks include this requirement. It was also noticed that, in the process of weathering, the mill scale, where covered with paint, adhered closely and could not be removed without the use of the hammer and chisel. This scale, of course, will come off in time, and for this reason the requirement that all contact surfaces be given one coat of paint before assembling, should be limited, so as to apply to rolled shapes only."

PRESERVATION OF TIMBER.

Sources of Decay.—*Decay* may be due to (1) "wet rot," caused by the attack of certain large fungi which may enter the pores of standing timber or of timber used in damp places; (2) "fermentation," or decomposition of the cellulose tissues by the putrifying sap, which takes place in unseasoned timber; (3) "dry rot," which occurs in so-called seasoned timber, caused by the fungus *Merulius lachrymans* which can operate only in the presence of some moisture; (4) insect larvae and forest worms;

* Page 145, Vol. LVIII, Trans. A. S. C. E., in Paper No. 1042, entitled "The Naval Floating Dock—its advantages, design and construction."

(5) sea worms, as the teredo (*Teredo navalis*), limnoria and other marine wood borers which inhabit pure salt water and attack woodwork exposed to same. These sea worms were formerly confined to southern waters, but have gradually worked their way northward. (Not present in fresh water.)

Piling and timber grillage for submarine work need not be seasoned or treated with preservatives when below the action of the teredo, as timber will last thus for centuries. Piling in ordinary foundation work is practically preserved from contact with the atmosphere and the destructive elements, by being sealed in the surrounding moisture. It is customary to leave the bark on piles when driven in the water and to peel them when land driven. If exposed to teredo the piles are peeled and treated with some preservative. They may be copper sheeted, creosoted, tarred, kyanized, wrapped with tarred paper or other similar preparation secured by vertical strips of wood. Specially constructed piles are sometimes used, made up of strips of scantling, on the principle that teredo will not cross a seam between two strips; the composite pile thus made being sometimes treated also with a surface preservative. The process of encircling piles with tight casings, pumping out the water, and then applying steam, has proved to be expensive.

Creosoting consists in impregnating the wood with creosote oil (containing carbolic acid, cresylic acid, etc.) which is obtained from coal-tar distillations. The timber to be treated is placed in large iron cylinders, hermetically sealed and filled with steam at a low pressure, which softens and warms up the sap and woody fibres. The steam is then expelled and a partial vacuum produced (one-half atmosphere) with the air pumps, thus expunging the sap in the cells of the wood. The creosote oil is then turned into the cylinders and by means of hydraulic pressure is forced into the wood. There are many variations in the process. Creosoting is universally recognized as one of the best and most practical preservatives known. Timber should be framed before treatment as the surface saturation is strongest. Ordinary ties, 6x8—8, usually absorb about 25 pounds of creosote, costing from 40 to 60 cents apiece. Piling or timber exposed to the teredo require much more oil. The cost or creosoting piling may be assumed at 20 cents upward per lin. ft.; and timber $16. upward per M. B. M. Creosoting probably ranks first, and burnettizing second, as a timber preservative, on a commercial scale.

One valuable point to be noted in creosoting is that the hard and expensive woods like oak and longleaf pine do not absorb the creosote oil as rapidly nor as thoroughly as do the softer and less expensive woods like beech; so that the soft woods, thoroughly impregnated, really outlast the harder ones.

For a good description of a creosoting plant see the article entitled "New Tie and Timber Preserving Plant of the A. T. & S. F. Ry, at Somerville, Texas," *Eng. News*, May 3, 1906.

Burnettizing is similar in method to creosoting. The solution used is chloride of zinc, a compound of zinc and hydrochloric acid. The results are inferior to creosoting, especially for bridge timbers. For ties, the comparison is not so marked. The cost of burnettizing is about ⅓ to ½ that of creosoting, but much depends upon the uses for which the timber is required. As water leaches out the zinc chloride in a comparatively short time when the treated timber is constantly submerged, it is evident that burnettizing will not do for piling. It even loses its strength in positions exposed to heavy rains.

Miscellaneous Notes.—Where timber is used in construction, the first proper guard against the actions of decay is the selection of the right kind of wood. Cedar, redwood, juniper, cypress, etc., are best able to resist ordinary decay, due to alternate wetness and dryness, hence their special usefulness for fencing, telegraph poles, ties, shingles, etc. Decay is something that enters the wood at some time or other, either during the period of growth or subsequently. There is little doubt that if a timber is cut from a perfectly healthy tree and its surface hermetically sealed so as to keep the interior in a constant state of either extreme moisture or dryness, it will last for thousands of years. The earliest and most primitive method of preserving wood was that of charring the surface, practiced before the dawn of history. Subsequently, it was discovered that piling kept constantly moist would last for centuries: and that the life of timber would

be prolonged greatly if kept dry from the influences of the weather, etc. Witness some of our old covered wooden bridges and the wooden joists of our colonial houses.

In the West where wooden bridges are common there are four principal methods employed to prolong the life of the structure: (1) Wooden covering (not common); (2) seasoning the timber to 12 or 15 per cent of moisture, and painting; (3) covering the upper surfaces of chords and end posts with galvanized iron (# 22 to # 27), allowing the sheets to lap a few inches down the sides; (4) treating the timber before erection, or painting it after erection, with a liquid preparation known as carbolineum avenarius. This preparation is also much used in preserving flooring and wooden paving blocks.

A coating of coal tar forms a good preservative on any material, wood, iron or stone, but its appearance is often objectionable. Note the use of tarred paper for roofing.

Copper sheeting for piles and ships' bottoms is very effective against the action of sea worms.

It is claimed that the life of wooden posts can be lengthened by boring an auger hole in the top, filling same with salt, and sealing with a wooden plug. Probably the more effective method is to bevel the tops of the posts, and plane and paint them. Posts of all kinds which set in the ground may be painted with tar for a short distance above and below the ground level, and an additional covering of metal may also be used. Telegraph poles are frequently treated in this manner, and their life greatly lengthened.

Chloride of zinc is often used for the preservation of timber, and as a disinfectant.

Creosote oil tends, if anything, to *preserve* railway spikes in ties; it also has a *deterrent* effect on the ravages of the teredo on piling.

Creosote and zinc chloride together forms Rutgen's process.

Barshall or Hesselmann process—zinc chloride and glue.

Kyanizing—corrosive sublimate.

SEASONING OF TIMBER.

The following is a digest of Bulletin No. 41, Bureau of Forestry, U. S. Department of Agriculture, issued in 1903, comprising an article entitled "Seasoning of Timber," by Hermann von Schrenk, in charge of Mississippi Valley Laboratory, Bureau of Plant Industry:

Distribution of Water in Timber.—(*a*) *Local Distribution.*—Water may occur in wood in three conditions: (1) It forms the greater part (over 90 per cent) of the protoplasmic contents of the living cells; (2) it saturates the walls of all cells; and (3) it entirely or at least partly fills the cavities of the lifeless cells, fibers, and vessels; in the sapwood of pine it occurs in all three forms; in the heartwood only in the second form, it merely saturates the walls. Of 100 pounds of water associated with 100 pounds of dry-wood substance taken from 200 pounds of fresh sapwood of white pine, about 35 pounds are needed to saturate the cell walls, less than 5 pounds are contained in living cells, and the remaining 60 pounds partly fill the cavities of the wood fibers. This latter forms the sap as ordinarily understood. It is water brought from the soil, containing small quantities of mineral salts, and in certain species (maple, birch, etc.), it also contains at certain times a small percentage of sugar and other organic matter. These organic substances are the dissolved reserve food, stored during winter in the pith rays, etc., of the wood and bark; generally but a mere trace of them is to be found. From this it appears that the solids contained in the sap, such as albumen, gum, sugar, etc., can not exercise the influence on the strength of the wood which is so commonly claimed for them.

The wood next to the bark contains the most water. In the species which do not form heartwood the decrease toward the pith is gradual, but where this is formed the change from a more moist to a drier condition is usually quite abrupt at the sapwood limit. In longleaf pine, the wood of the outer 1 inch of a disk may contain 50 per cent of water, that of the next, or second inch, only 35 per cent, and that of the heartwood only 20 per cent. In such a tree the amount of water in any one section varies with the amount of sapwood, and is therefore greater for the upper than the lower cuts, greater for the limbs than stems, and greatest of all in the roots.

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Different trees, even of the same kind and from the same place, differ as to the amount of water they contain. A thrifty tree contains more water than a stunted one, and a young tree more than an old one, while the wood of all trees varies in its moisture relations with the season of the year.—Timber. By Filibert Roth (Bull. 10, Division of Forestry, U. S. Dept. of Agriculture, 1895.)

(*b*) *Seasonal Distribution.*—It is generally supposed that trees contain less water in winter than in summer. This is evidenced by the popular saying that "the sap is down in the winter." This is not always the case. Some trees contain as much water in winter as in summer, if not more. The average weight of lodgepole pine ties of the same size cut at Bozeman, Mont., in June, 1902, was 157 lbs.; in July, 144 lbs.; in Aug., 150 lbs.; in Sept., 157 lbs.; in Oct., 164 lbs. It is probable that this increase would keep up throughout the winter.

Relation of Water to Decay in Timber.—Low forms of plant life called fungi grow in wood, and by so doing disintegrate and dissolve portions of the wood fiber. As a result of this, the wood changes in its physical properties and is called decayed. When the fungus has extracted a sufficient amount of material, it forms, on the outside of the wood, fruiting bodies known as punks or toadstools, containing spores, which are blown about and infect sound wood. One of the most common of these wood-destroying fungi is called the *Lentinus lepidens*. The conditions necessary for the growth of these fungi are (1) water, (2) air, (3) organic food material, and (4) a certain amount of heat. The wood fiber and the organic substances found in the living cells of sapwood, such as albuminous substances, starch, sugar and oils, form the food supply necessary to start the growth of the fungus threads. A further requirement is oxygen; no growth will take place under water or in the ground at depths of 2 feet or more, the depth varying with the character of the soil. The best examples of this necessity for oxygen can be found in the way in which fence posts and telegraph and telephone poles decay at points just at or just below the surface of the ground, where there is a balance between the supply of air and of water.

For practical purposes water is the most important factor. Without water no fungus growth, and consequently no decay, is possible. "Dry rot," a form of decay in which the wood turns to a dry, brittle, charcoal-like substance, is commonly supposed to take place without any water. But such is not the case. The atmospheric moisture is sufficient to permit growth of the dry-rot fungus even if no moisture is contained in the wood. Too much water will prevent fungus growth, because it shuts off the air supply. The amount of water necessary for fungi growth is very small; and wood freshly cut contains more than enough, at all seasons of the year.

What Seasoning Is.—(*a*) *Difference Between Seasoned and Unseasoned Timber.*—Seasoning implies other changes besides the evaporation of water. Although we have as yet only a vague conception as to the exact nature of the difference between seasoned and unseasoned wood, it is very probable that one of these consists in changes in the albuminous substances in the wood fiber, and possibly also in the tannins, resins, and other incrusting substances. Whether the change in these substances is merely a drying out, or whether it consists in a partial decomposition, is as yet undetermined. Exposure to the wind and air, however, brings about changes in the wood which are of a nature that the wood becomes drier and more permeable. When seasoned by exposure to live steam, similar changes may take place. The water leaves the wood in the form of steam, while the organic compounds in the walls probably coagulate or disintegrate under the high temperature.

(*b*) *Manner of Evaporation of Water.*—The evaporation of water from timber takes place largely through the ends, i.e., in the direction of the longitudinal axis of the wood fibers. From the other surfaces it takes place very slowly out of doors, but with great rapidity in a kiln. The rate of evap. differs with the kind of timber and its shape. Thin boards and beams dry faster than thick ones; sapwood, faster than heartwood; and pine, faster than oak. Recent tests (not altogether conclusive) showed little difference in rate of evap. from sawed and hewn ties. Air-drying out of door takes from two months to a year, depending on kind of timber and climate. Wood which has been air-dried will absorb water in small quantities after a rain, or during damp weather, and lose most of it again

when a few warm, dry days follow. When soaked in water, seasoned timber absorbs it rapidly. It first enters the wood through the cell walls, and when these are soaked it will fill the cell lumen completely.

Seasoning and Preservative Treatment.—(*a*) *Seasoning and the Leaching of Salts.*—Where timber is chemically treated with salts dissolved in water, it is absolutely necessary to season it after the treating process, for two reasons: First, to prevent the rapid leaching out of the salts pressed into the wood; second, to prevent subsequent decay. In the case of ties, the leaching out takes place very rapidly when they are laid immediately after treatment.

(*b*) *Seasoning and the Processes of Preservation.*—The object of timber treatment is to get certain chemical compounds into the wood with as much thoroughness as possible. Because of its peculiar structure, wood will not allow of the penetration of liquids into its mass as does a sponge. The solution must work its way into the wood fibers through walls of wood substance. If a water solution is used for the impregnating material, it ought to fill every cell and permeate every wall, at least of the sapwood. The most successful method for timber treatment (excepting the boiling process) so far used consists in pressing the solution into the wood. If the wood cells and the walls are already full of water, it is evident that there will be great difficulty in making the water already in place give way to the solution. When the walls and cell cavities are free from water the process of absorption of a solution is facilitated. Besides this, prior seasoning not only brings about a reduction in the amount of water, but also results in a partial disintegration of the albuminous substances which offer more or less resistance to the entrance of solutions. The steaming of wood before treatment with solutions can never replace seasoning, as it can do no more than drive off part of the water, unless the temp. of the steam is sufficiently high to injure certain portions of the wood fiber itself.

Advantages of Seasoning.—The four principal advantages of seasoning timber may be enumerated as follows: (1) Seasoned timber lasts much longer than unseasoned; (2) Seasoning before chemical treatment greatly increases its effectiveness, and seasoning after treatment prevents the rapid leaching out of the salts introduced to preserve the timber; (3) The saving of freight due to a decrease in weight of 35 to 40% is sometimes a considerable item, even when compared with the total cost of the timber; (4) Seasoning allows the use of lower grade or softer timbers, so that red and swamp oak and beech are now being substituted for white oak, loblolly pine for longleaf pine, hemlock and tamarack for oak and pine, etc.; besides, the softer woods, being more porous, are more easily treated chemically.

To prevent checking and splitting of timbers while seasoning, many English roads use S irons, which are driven into the ends of timbers that begin to show such tendencies. Fig. 1 shows one of these irons of meduim size, and Fig. 2 shows the method of applying same. The effect is a great saving.

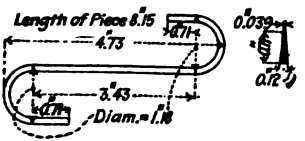

Fig. 1.

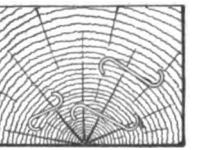

Fig. 2.

How Timber is Seasoned.—(*a*) *Kiln Drying.*—The French Eastern Ry. maintains at Amague a plant for completing the drying of its ties after they have been seasoned in the open air, which consists of four kilns. The structures are about 50 ft. long by 46 ft. wide, and contain two pairs of hot-air galleries, each pair of which is provided with an independent furnace and can be operated as a separate kiln. The air enters, at first cold, from the outside into a lower chamber of the furnace, and becomes gradually

heated in its upward progress. It is at last discharged into a hot-air chamber which occupies all the upper part of the kiln. From this it is carried down to the galleries in which the ties are dried by four vertical pipes, having a cross section of 18 by 18 ins. Two pipes open into each gallery, at the end of which the trams bearing the ties pass out after the drying has been completed. Each tram carries about 40 ties slightly separated from each other, so that all the faces may be in direct contact with the hot air in the galleries of the dry rooms and with the tar oil in the cylinders. The four kilns in all contain 16 galleries, with a capacity of 5 trams each, in all 80 small trams. It is thus possible to dry about 3200 ties at once. With an annual output of 400,000 ties, seventy-two hours would be allowed for the average drying period. The temperature of the galleries is at the maximum of 30° to 35° C. at the entrance, and 70° to 80° C. at the delivery. As the trams are taken from the cylinders one at a time, the drying is progressive, and the wood, for this reason, is less liable to split or warp. To turn out 400,000 ties the furnaces of the four dry rooms consumed about 200 tons of fine coal and 250 tons of the trimmings, from show machines, and wood trash and chips from the wood yard. This mixture develops a sufficient heat and offers the additional advantage of not wearing out the fire boxes by a two intense heat. The expense for fuel is about one-fifth of a cent for each tie.

(*b*) *Seasoning.*—Seasoning of timber has been carried on in a practical way for many years in Europe. Most of the European railroads season their ties for many months before they treat them. The Eastern French Railway piles its ties in open piles 11.4 ft. high, 8.8 ft. wide, and 8.8 to 65.6 ft. long. The ties are about 5 ft. apart. The ties are laid grillage fashion and spaced 4 ins. apart, except the two top tiers which are inclined (in the same direction) to shed water. At Amague, where kiln drying is subsequently used, oak ties are allowed to remain in piles for 15 to 20 months; and beech ties, 6 months. They are then kiln dried for from 60 to 80 hours at a temp. beginning at 35° and gradually brought to 75° C. Finally, they are treated with tar oil.

(*c*) *Seasoning by Steaming.*—Steaming is at best a makeshift and unless modified materially it can never replace open-air seasoning, supplemented possibly by kiln-drying. However, it ought to last longer than unsteamed, and where it is necessary to secure partially seasoned wood the steaming may do. The use of the vacuum pump does not materially improve matters, for it is not possible to maintain a sufficiently high temperature in a cylinder in which enough of a vacuum exists to insure the complete removal of all the water. [With the present state of our knowledge the injurious effect upon timber from high-temperature steam is not definitely known.]

(*d*) *Seasoning by Immersion in Water.*—It is very probable that immersion of timber for long periods in water materially hastens subsequent seasoning. The tannins, resins, albuminous materials, etc., which are deposited in the cells of the fibers of green wood, and which prevent rapid evaporation of the water, undergo changes when under water, probably due to the action of bacteria which can live without air, and in the course of time many of these substances are leached out of the wood. The cells thereby become more and more permeable to water, and when the wood is finally brought into the air the water escapes very rapidly and very evenly.

(*e*) *Seasoning by Boiling in Oil.*—It is sometimes claimed that all season ing preparatory to treatment with a substance like tar oil might be done away with by putting the green wood into a cylinder with the oil and heating to 225° F., thus driving off the water in the form of steam, after which the tar oil would penetrate readily into the wood. This is the basis of the so-called "Curtiss process" of timber treatment. But the same objection made for steaming holds here, i.e., in order to get a temperature of 212° F. in the center of the treated wood the outside temperature would have to be raised so high that the strength of the wood might be injured seriously. A company on the Pacific coast which treats red fir piling asserts that it avoids this danger by leaving the green timber in the tar oil at a temp. which never exceeds 225° F. for from 5 to 12 hours, until there is no further evidence of water vapor coming out of the wood. The tar oil is then run out, and a vacuum is created for about an hour, after which the oil is run in again and is kept in the cylinders under 100 lbs. pressure for from 10 to 12 hours, until the required amount of absorption has been reached (about 12 lbs. per cu. ft.).

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Conclusions and Recommendations.—Timber seasoning is a practical method for increasing the length of life of both untreated and treated timber. At the same time it forms the most important preliminary step to successful chemical treatment. The cost of seasoning is insignificant, while the returns amount to a considerable sum in the end. With the increased cost and scarcity of timber, every step leading toward a more economic use of our supply ought to receive attention. It is perhaps too soon to draw final conclusions, but the following general recommendations can confidently be made:

(1) Green timber should be piled in as open piles as possible as soon as it is cut, and so kept until it is dry. In the case of ties the 7 by 2 form of pile (tiers with 7 and 2 ties alternating) is the best. No timber should be treated chemically until it is dry.

(2) Timber treated with a preservative dissolved in water should be piled after treatment for several months at least to allow the water pressed into the wood with the salt to evaporate. Under no circumstances should timber freshly treated with a water solution be exposed to weathering influences.

CREOSOTE IN WELL-PRESERVED TIMBERS.

The following is a digest of Forest Service Circular 98, U. S. Dept. of Agriculture, issued May 9. 1907, comprising an article entitled "Quality and Character of Creosote in Well-Preserved Timbers," by Gellert Alleman, Prof. of Chem. in Swarthmore College:

"Of the various preservative processes for timber, those using coal-tar creosote are the most efficient, and, in the long run, are frequently the most economical as compared with the less expensive metallic-salts processes. Moreover, creosoted wood can be used for some purposes, as for salt-water piles, for which wood treated with metallic salts is but slightly more durable than untreated timber. Recent reports on the service of creosoted railroad ties, and of salt-water piles, have shown that, while proper treatment gives excellent results, much of this timber was not treated properly and has not lasted as it should. It is imperative that we should know, as completely as possible, just what constitutes efficient creosote treatment. This depends on three things—the amount of creosote, its character, and the thoroughness with which it penetrates the timber. The proper amount of creosote will depend upon the intended use of the timber. For instance, piles which must resist the attacks of marine borers need more creosote than telephone poles, and those in warm waters require more than those in cooler waters.

"The sort of creosote best suited to prevent decay and the inroads of marine borers can be ascertained only by many careful experiments. The best means for securing a maximum penetration of the oil is a problem complicated by many factors such as wood structure, moisture, etc. One way of approaching this problem involves a study of the nature of the creosotes present in timbers which have given long service. The results of a series of analyses of the oils present in such timbers forms the most important part of this paper. A brief account of the source and composition of coal-tar creosote precedes the description and discussion of the experiments.

Manufacture and Composition of Creosote.—(*a*) *Source and Composition of Coal Tar.*—When certain varieties of coal are heated in an oven or retort, in the absence of sufficient air for their combustion, the coal is decomposed and gas, tar, and coke are formed. The gas and tar rise from the heated mass and the coke remains in the retort. Coke and illuminating gas are manufactured in this way. Where coke is the main product desired the "beehive" oven is used and the gas and tar are not collected, but when the volatile materials are to be collected the "by-product" oven is used. In making illuminating gas the coke and tar are regarded as by-products, and one of the problems of managament is how to dispose of these by-products to advantage.

Coal tar is an extremely complex mixture of organic compounds, varying with different coals and with different treatments of the same coal, which will yield at the same plant various qualities of coke, gas and tar, depending on the amount of heat applied, the quantity of air admitted, and the season of the year. With a low heat a relatively small amount of gas and tar is evolved and the tar contains large quantities of compounds of the paraffin series; but with a high temp. much larger amounts of gas

and tar are obtained and the predominant compounds of the tar, in nearly all cases, are those of the aromatic series, as benzine, toluene, phenol, naphthalin, anthracene, etc.

(*b*) *Production of Creosote from Coal Tar.*—The first distillation of crude tar, in which several separate fractions are usually taken, is made in large iron retorts holding from 10 to 30 tons. The forms of the retorts and the manner of controlling the distillation vary more or less in different work. In some cases the still is provided with a thermometer enclosed in an iron tube screwed into the still head; in other cases the time for changing the receiver for various fractions is judged solely by the specific gravity and other properties of the distillates.

In Germany the fractions are frequently taken as follows: The temp. is that registered by the thermometer in the tar at the beginning of the distillation, but free from the oil and indicating the temp. of the vapor passing over when anthracene oil begins to distil: First light running up to 110° C.; light oils, 110° to 210° C.; carbolic oils, 210° to 240° C.; heavy or creosote oils, 240° to 270° C.; anthracene oils, 270° to 400° C.

At many English works the following fractions are taken with the thermometer placed as in the German procedure just cited: Light naphtha up to 110° C.; light oil, 110° to 170° C.; carbolic oils, 170° to 225° C.; creosote oils, 225° to 270° C.; anthracene oils, 270° to 360° C.

These temperatures are by no means universally accepted in the respective countries, and one or more fractions are often omitted; when, for example, it does not pay to extract carbolic acid, or when the demand for anthracene is limited.

Owing to the variable constitution of the tar and to the different temperatures between which fractions are taken, the products of this preliminary separation are ferquently widely different in physical character and chemical composition. In distilling according to the German method given above, the "first runnings" and "light oils" contain, among other things, benzene, toluene, and the xylenes; the "carbolic oils" contain phenol, the creosotes, and some naphthalin; the "creosote oils," small quantities of phenols, naphthalin, anthracene, and many other hydrocarbons; the "anthracene oils," anthracene, acridene, etc. The residue in the still is either soft or hard pitch, according to the point at which the distillation is stopped. When the anthracene oil is completely distilled, the residue is largely hard pitch or carbon, and this is used as a briquette binder and in the manufacture of electric light carbons. When the distillation is stopped at an earlier stage, soft pitch is obtained which contains a considerable quantity of the high-boiling tar constituents; and is used for roofing and for builders' paper. At present there is almost no market in America for hard pitch, whereas the demand for soft pitch for roofing is very great, which explains why the distillation of tars is not carried so far here as in some foreign works.

(*c*) *Statistics of Production and Importation of Creosote.*—The following table gives, approximately, the amount of Coal Tar produced in the U. S., also the amount of Creosote Oil produced and imported, in millions of gallons, for the years named:

Year.	Coal Tar.				Creosote Oil.				Remarks.
	From Gas Works.	From By-Product Ovens.	Total. Millions of Gals.	Price per Gallon.	Produced in U.S.	Imported.	Total. Millions of Gals.	Price. per Gallon.	
1898	24.38	4.02	28.40	3.7cts.	.....	.....	.....	.....	Present price of oil much higher.
1903	40.80	22.15	62.95	3.49 "	4.00	3.71	7.71	5.8cts.	
1904	41.73	27.77	69.50	3.04 "	4.85	3.78	8.63	6.3 "	
1905	43.64	36.38	80.02	2.73 "	5.80	7.75	13.55	5.4 "	

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The estimate for 1903 includes the tar produced at 1956 " by-product " ovens, assuming 8.5 gallons of tar per ton of coal coked. It is generally nated that 12.5 gallons per ton of coal may be obtained from gas works, that the general average from gas works and from ovens is a little over allons. Moreover, it is usually assumed that the average coal tar pro- d in this country contains at least 10% of oils which can be used as, dded to, creosote oil.

d) *Composition of Commercial Creosote.*—Technically speaking, the ion of oil passing over between 240° and 270° C. during the first dis- ion of the crude coal tar is known as " creosote oil," " heavy oil," or d oil of coal tar." In practice, however, the oily residues which in after extracting carbolic acid, naphthalin, and anthracene from various distillates in which they occur are added to the creosote oil, in consequence, many of the creosote oils of commerce contain con- able amounts of materials having boiling points higher than 270° C., lower than 240° C. As a matter of fact, it is the practice at nearly stilling plants to add to the " creosote well " or tank all those oils and ues which cannot profitably be worked over and used to greater com- ial advantage. The solvents which are used in the purification of thalin and of anthracene are sometimes added to the " creosote well," this accounts for the occasional presence of paraffin oil in creosote. The " creosote well " or tank is usually constructed of steel plates, s fitted with inclosed steam coils at the bottom, in order that the solid rials crystallizing out can be melted before the oil is delivered to tank tank steamers, or barrels. A stirring device is also frequently used cure uniformity in quality of supply.

The creosote oil of commerce contains phenol (carbolic acid), the ortho, , and para cresols, naphthalin, the α and β methyl-naphthalins (the er being a liquid, the latter a solid melting at 33° C.), anthracene, anthracene, arcidene, and small quantities of certain high-boiling and acids. When first distilled, creosote has a distinctly fluorescent rance, and is light green in cloor. There is strong evidence for the that some of the individual constituents in creosote oil combine with other and probably form new products.

t certain works, carbolic acid is extracted and the creosote oil coming such places is low in " tar acids;" at other places, naphthalin is of derable commercial importance and the creosote oils obtained from works contain little naphthalin. Usually anthracene separates napthalin, and in the event that the latter is frozen out, the former o lacking in the oil which is placed on the market. In America the ote oils usually contain large amounts of naphthalin, very small nts (about 5%) of phenols or cresols, and practically no anthracene he reason previously mentioned). It is evident that these variations nufacture result in creosotes differing greatly in physical and chemical erties; some are rather thin oils, some are almost entirely solid with thalin, and some are heavy oils with a large proportion of high-boiling ituents.

he different sorts of oils are believed to have different preservative s when injected into timber, but there is, unfortunately, a lack of rmity of opinion. Some investigators have advocated oils rich in ols, some those containing much naphthalin, some those containing ximum of the high-boiling compounds. But little has been published e subject.

nalyses of the Creosote Extracted from Timber Well Preserved after Service.—To determine what is, in fact, a good oil, a natural way is amine the composition of those oils which have protected treated r satisfactorily. This can be done by extracting and analyzing the rom timbers the exact history of whose service is known. The writer essor Alleman) secured from various sources a number of creosoted rs which had been in varied and extended use under markedly different tic conditions. The oils from these timbers were extracted and ana- with the following results:

esults of the Analyses.—The results of the analyses of the creosote cted from 37 different samples of wood are given in Table 1, following. majority of these samples were obtained from various English com- s using creosoted wood. Such details as could be learned concerning istory of the timbers are given in the footnotes.

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1.—RESULTS OF ANALYSES OF EXTRACTED OILS.

Sample.	Source.	Species.	Service.	Creosote per cubic foot.	Distillation of extracted oil. To 205° C.	205° to 245° C.	245° to 270° C.	270° to 320° C.	320° to 420° C.	Residue above 420° C.	Total.	Solid naphthalin from distillates.	Solid anthracene oil from distillates.	Tar acids.
			Years.	*Pounds.*	*Perct.*	*Perct.*	*Perct.*	*Perct.*	*Perct.*	*Perct.*	*Perct*	*Perct.*	*Perct.*	*Cm³.*
Tie No. 106	Glasgow and Southwestern Ry., Scotland.	Pine	16	4.06	.....	16.32	13.24	20.15	24.10	25.93	99.74	.....	.....	0.51
Tie No. 104	do	do	18	9.24	.....	9.37	18.17	27.54	21.38	23.01	99.47	.....	.....	.36
Tie No. 113	do	do	18	8.01	.....	9.75	18.54	24.96	22.42	24.07	99.74	.....	.....	1.13
Tie No. 109	do	do	16	8.29	.....	7.08	12.45	16.68	40.84	22.52	99.57	.....	.....	.74
Pile No. 118	Great Western Ry., Heath Div., England.	Yellow pine	47	7.68	.....	19.92	17.58	20.62	14.48	27.11	99.71	.....	32.15	.78
Tie No. 107	do	Baltic redwood	42	12.71	.....	9.45	12.30	27.56	33.48	17.87	100.66	.....	51.07	.96
Tie No. 105	do	do	*a* 33	5.08	.....	6.83	10.16	26.11	32.17	23.91	99.18	.....	.....	.....
Tie No. 103	London and Northwestern Ry. Co., England	do	19	14.07	.....	17.78	11.88	21.26	14.64	34.01	99.57	.....	35.14	.26
Tie No. 110	do	do	19	13.84	.....	18.23	16.61	23.01	12.78	29.11	99.74	.....	26.87	.68
Paving blocks Nos. 88 and 89.	Northeastern Ry. Co., England	Yellow pine	20	17.19	.....	20.13	10.27	12.18	27.46	29.78	99.82	21.46	57.03	1.19
Tie No. 114	do	Pine	30	7.41	.....	15.44	7.44	15.68	44.96	16.14	99.66	.....	.....	.37
Paving blocks Nos. 90 and 91.	City Engineer, Hull Corporation, Hull, England.	Yellow Pine	11	14.37	.....	21.03	24.45	7.68	25.06	21.78	100.00	24.93	.....	.87
Tie No. 108	Maryport and Carlisle Ry. Co., England.	.....	23	5.17	.....	10.59	12.61	28.56	20.32	27.87	99.95	.....	.....	.62
Tie No. 2	Highland Ry. Co	Scotch fir	20	5.93	.....	15.78	8.04	27.80	18.12	29.81	99.55	.....	48.14	.76
Tie No. 3	do	do	20	9.03	.....	10.15	16.32	20.54	12.63	40.02	99.66	.....	52.13	.54
Tie No. 41	do	do	22	12.76	.....	10.46	20.34	22.63	16.32	29.83	99.58	.....	.....	1.14
Tie No. 101	North British Ry. Co., Scotland	do	24	11.46	.....	8.32	26.36	23.42	29.58	11.86	99.54	.....	40.15	.38
Tie No. 102	do	do	21	8.19	.....	18.24	12.16	28.92	22.76	17.35	99.43	.....	.....	.21
Tie No. 112	do	do	14	7.21	0.47	7.65	8.03	17.58	38.88	27.97	100.58	.....	53.04	1.23
Pile No. 116	Clyde Navigation Trust, Glasgow.	Red pine	46	8.42	.....	9.44	16.92	29.68	32.08	11.03	99.15	.....	.....	.....
Pile No. 111	do	do	46	8.07	2.76	19.53	14.61	18.15	17.23	27.03	99.31			

Pile No. 4	do	do	46	12.61	.....	16.87	12.15	13.25	25.37	32.30	99.94	.....	54.42	1.07
Pile No. 5	do	Pitch pine	21	9.06	.....	13.56	10.52	20.34	31.24	23.92	99.58	.....	.....	1.78
Tie No. 8	do	do	22	12.36	.....	9.03	15.21	29.46	13.35	32.91	99.96	.....	41.16	1.37
Tie No. 9	do	Baltic redwood	16	9.42	.....	6.39	10.38	27.75	31.86	23.51	99.89	.....	52.14	1.15
Pile No. 81	Norfolk Creosoting Co	Loblolly pine	11	18.34	.....	38.88	13.76	13.12	10.08	24.02	99.86	34.47	37.24	.....
Pile No. 82	do	do	15	19.12	.....	51.50	12.35	3.74	11.61	21.03	100.23	48.62	34.15	.....
Pile No. 83	do	do	7	.....	Treated with creosote and resin.						Not analyzed.			.....
Pile No. 84	do	do	20	8.43	.....	13.56	15.78	14.49	19.77	36.13	99.83	.....	55.22	.....
Pile No. 85	do	do	17	13.21	.....	19.07	12.39	12.32	17.58	38.14	99.50	18.96	43.71	.....
Tie No. 86	E. B. Cushing, Southern Pacific Co., Houston, Tex.	do	22	b 19.36 c 16.14	...	22.53	13.47	22.63	18.58	22.37	99.58	22.53	46.18	.....
Pile No. 50	International Creosoting and Construction Co., Galveston, Tex.	do	29	17.63	1.26	27.60	22.43	31.22	12.02	5.13	99.66	25.41	48.15	.....
Pile No. 51	do	do	29	17.08	2.18	31.06	18.21	36.04	8.17	4.13	99.79	28.14	41.12	.....
Paving block No. 52.	do	do	34	18.81	.48	26.61	32.06	17.52	8.47	14.42	99.56	.....	50.67	.....
Paving block No. 53.	do	do	29	12.44	.68	17.57	18.78	37.52	16.62	8.56	99.73	3.68	53.89	.....
Paving blocks Nos. 54 and 55	do	do	9	d 6.07 e 5.46	9.62	14.41	19.27	41.74	11.23	3.40	99.67	.....	.....	.....
Cond't pipe No. 67	Bell Telephone Co	Pine	14	8.74	5.08	27.23	10.46	27.68	19.03	9.93	99.41	23.17	14.28	.....

a Twenty years as a tie and thirteen years as a fence post. *b* Center. *c* Under rail. *d* Paving block No. 54. *e* Paving block No. 55.

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GLASGOW AND SOUTHWESTERN RAILWAY, SCOTLAND:

Tie No. 106 taken out of the Main Line February 12, 1905, near Milliken Park; put in during 1889. Creosoted with 2½ gallons of gas-works creosote of 1.010 specific gravity at 60° F.

Tie No. 104 taken out of the Main Line February 6, 1905, near Elderslie Station; put in during 1887. Creosoted with about 2½ gallons of oil to the tie.

Tie No. 113 taken from the Main Line April 16, 1905, near Elderslie Station; put in during 1887. Creosoted that same year with about 2½ gallons of oil to the tie.

Tie No. 109, same history as No. 106.

GREAT WESTERN RAILWAY, HEATH DIVISION, ENGLAND:

Pile No. 118 was in salt water at New Milford 47 years; not decayed, but attacked by Limnoria.

Tie No. 107 was in sidings at Eastern Depot, Swansea, for 42 years.

Tie No. 105 served for 20 years in the Main Line at Hirwain (Vale of Neath), and afterwards was used as a fence post for 13 years.

Memel pile No. 1 was in a tidal river at Loughor for 53 years. Treated with crude coal tar and not analyzed.

Memel pile No. 2 was in salt water at Llanelly Docks for 58 years. Treated with crude coal tar and not analyzed.

LONDON AND NORTHWESTERN RAILWAY COMPANY, ENGLAND:

Ties No, 103 and No. 110 installed in the road for permanent way purposes in 1886; removed February, 11, 1905. They were creosoted with blast furnace Scotch oil in 1886.

NORTHEASTERN RAILWAY COMPANY, ENGLAND:

Paving blocks Nos. 88 and 89 removed in perfect condition after being in use at Hull for 20½ years.

Tie No. 114 was under water for 20 years and afterwards used as a tie under piles of timber in the dockyard for 10 years, a total service of 30 years.

CITY ENGINEER, HULL CORPORATION, HULL, ENGLAND:

Paving blocks Nos. 90 and 91 laid with close joints in 1894; removed in February, 1905, in perfect condition. Paving block No. 92 consisted of three broken blocks laid with open joints in 1892; not decayed when removed in 1905. Treated with creosote and pitch on the outside; not analysed.

MARYPORT AND CARLISLE RAILWAY COMPANY, ENGLAND:

Ties Nos. 108 and 137 creosoted in 1881, placed in track 1882, and removed in 1905. Tie No. 137 was not analyzed.

HIGHLAND RAILWAY COMPANY, SCOTLAND:

Ties Nos. 2, 3, and 41 were in service in a gravel ballast, damp bed, for 20 years, 20 years, and 22 years, respectively. These ties were seasoned two years before treatment and stacked six months after creosoting before being placed in the track.

NORTH BRITISH RAILWAY COMPANY, SCOTLAND:

Tie No. 101 was in the track 21 years; No. 102, 21 years; No. 112, 14 years; No. 100, 14 years (not analyzed). These ties were taken out at different parts of the system.

CLYDE NAVIGATION TRUST, GLASGOW:

Pile No. 116 is from the part of the pile which was above high-water level, and was therefore exposed to the air.

Pile No. 111 is from the part of the pile between high and low water. It was therefore exposed to the wash of the water.

Pile No. 117 is from the part of the pile which was always under water.

Pile No. 4 is from the part which was buried in the ground.

Pile No. 5 is from the part of a pile above high water; creosoted with 8 pounds per cubic foot.

Tie No. 8 was laid in slag in the year 1883.

Tie No. 9 is a part of a Baltic redwood tie laid in 1889. It was bedded in concrete and causewayed over with granite sets. Both of these ties were creosoted with 2½ gallons of gas creosote of not less than 1.010 specific gravity at 61° F.

Pile No. 115 is from a part of pile No. 5 between high and low water; not analyzed; creosoted with 8 pounds per cubic foot.

NORFOLK CREOSOTING COMPANY, NORFOLK, VA.:

Pile No. 81, section of a creosoted pile put in Santiago Harbor Cuba, April, 1887; taken out in perfect condition November, 1902.

Pile No. 82, section of a creosoted pile put in Tampico Bay in May, 1891; taken out in perfect condition October, 1902.

Pile No. 83, section of a pile treated with creosote and resin; removed after seven years badly attacked by Teredo.

Pile No. 84, section of a creosoted pile put in a drydock at Newport News, Va., May, 1881; removed in good condition October, 1901.

Pile No. 85, section of a creosoted pile in service for 17 years at Newport News, Va.

Tie No. 86 was in the track for 22 years at Houston, Texas.

INTERNATIONAL CREOSOTING AND CONSTRUCTION COMPANY, GALVESTON, TEX.:

Piles Nos. 50 and 51 were in Galveston Bay for 29 years.

Paving block No. 52 was in service in the street at New Orleans, La., for 34 years.

Paving block No. 53 was in service in Galveston for 29 years.

Paving blocks Nos. 54 and 55 were in use at Galveston for 9 years. They showed poor service.

BELL TELEPHONE COMPANY:

Conduit pipe No. 67 was in service as conduit at Philadelphia, Pa., for 14 years; removed in perfect condition to make extensions of service.

In the following Table (2) the 37 samples described above are grouped in classes and the average results of all the well preserved timbers are given for each class:

2.—ANALYSES OF EXTRACTED OILS.

Samples.	Average service.	Creosote to the cubic foot.	Distillation of extracted oil.								
			To 205° C.	205° to 245° C.	245° to 270° C.	270° to 320° C.	320° to 420° C.	Residue above 420° C.	Solid naphthalin from distillates.	Solid anthracene oil from distillates.	Tar acids.
	Yrs.	*Lbs.*	%	%	%	%	%	%	%	%	*Cm³.*
19 cross-ties	21.84	9.58	0.025	12.07	13.88	23.80	24.69	25.27	1.19	23.47	0.65
6 English piles	43.00	9.19	.46	16.92	15.31	21.06	22.77	23.04	.....	19.95	.61
6 American piles	20.20	15.64	.57	30.28	15.82	18.49	13.21	21.43	25.93	43.27	....
4 paving blocks	23.60	15.70	.29	21.34	21.39	18.73	19.40	18.64	12.52	40.40	.52
1 paving block poor service	9.00	5.77	9.62	14.41	19.27	41.74	11.23	3.40	.....	.....	....
1 conduit pipe	14.00	8.74	5.08	27.23	10.46	27.68	19.03	9.93	23.17	14.28	....
Average of 36 timbers giving good service	24.90	11.18	.36	17.37	15.18	22.00	21.71	23.09	6.98	27.81	.50

Discussion of the Analytical Results.—(*a*) *Quantity of Oil Found.*—The average figures show that the quantity of creosote in these long-service timbers was not excessive. Practically nothing is known of the amount of oil which was injected into the various samples. Six of the ties, Nos. 8, 9, 104, 106, 109, and 113, were said to have received 2½ (English) gallons of creosote each or 10.50 lbs. of oil per cu. ft., assuming the spec. grav. of the oil at 1.05 and the volume of each tie at 2½ cu. ft. The average amount present was 8.56 lbs. The American piles (all set in warm water, those farthest north being on the Virginia coast) show a much higher content of oil than the English samples. For the paving blocks, there is a considerable contrast between the quantity of oil present in those which gave good service, and the sample which was short lived. The difference in

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service was doubtless due to two factors, the quantity and the quality of the injected creosote; it is impossible to say which had the greater influence.

In general, the results tend to show that 10 lbs. of creosote per cubic foot is ample for railroad ties, and that piles require from 10 to 20 lbs., according to the location in which they are to be placed. If a creosote contains much light oil, a proportionately larger quantity must be used.

(*b*) *Character of the Extracted Creosote.*—A difficulty in the proper interpretation of the results of the analyses arises from our ignorance of the quality of the oil used in treating the various timbers. It is, therefore, possible to believe that the materials which have volatilized from the timbers have created an antiseptic environment which has been a most important factor in preserving the wood. Aside from this, however, the analyses furnish a strong argument in favor of the use of heavy oils. For example, tie No. 6 had seen but 14 years' service, two-thirds the average of the ties, and doubtless would not have suffered decay for many years to come, yet it does not contain more light oil than the average of the tie group, but, on the contrary, the creosote from this specimen was over half recovered as solid anthracene oil. If the constituents present in the timbers represented a non-efficient residue from which the effective light oils had evaporated, we should expect to find a relatively high proportion of light oils in this tie which had seen a shorter term of service. The natural interpretation of the results is that it is the heavy, high-boiling compounds which stay in timber and are an efficient barrier to the entrance of water and to the attacks of fungi and borers.

The creosotes recovered contained practically nothing which boiled below 205° C. The general average shows that 32.9% of the oils distilled below 270° C., and 66.95% above—that is, two-thirds above and one-third below this rather high temperature. Another noticeable fact is the large amount of solid anthracene oil recovered from the distillates of many samples, the highest being 57%.

A distinctive feature of the creosotes from American piles was the quantity of naphthalin which they contained. The average from this class of timbers was nearly 26%, and one sample showed over 48%. It appears probable that the creosotes used in treating these timbers contained much more naphthalin than the oils applied to the English piles. The results indicate that this substance possesses value for timber treatment, although it probably is inferior to anthracene oil. It is worth noting that these long-lived American piles contained more anthracene oil than naphthalin.

Perhaps the most striking thing is the disappearance of the tar acids. It is certainly conservative to place the original tar-acid content at 5%. Yet the extracted oils showed but one tenth of this amount. It is possible that the compounds, on account of their hydroxyl groups, have undergone chemical changes during the many years that they have been exposed to varying amounts of water and air, to the reactive lignin portion of the wood, and to the numerous compounds present in creosote. On the other hand, these phenol bodies have been volatilized or been washed from the timbers.

It appears, therefore, that light oils, boiling below 205° C., will not remain in timber, but that heavy oils, containing a high percentage of anthracene oil, will remain almost indefinitely and protect the wood from decay and boring animals. It is probable that naphthalin stays in wood for many years, but whether it is as valuable as anthracene oil is an open question. The value of the tar acids has apparently been overestimated by many persons, for although it has not been proved that they are valueless, they have been shown to possess poor staying qualities.

EXCERPTS AND REFERENCES.

The Protection of Ferric Structures from Corrosion (By M. P. Wood. Trans. A. S. M. E., 1901; Eng. News, Sept. 16, 1901.—(1) **Iron Oxide Pigments.**—To neutralize the sulphur element natural to the ore or developed in roasting, it is the common practice to add carbonate of lime (common chalk) to the amount of 5 to 10% by weight of the iron oxide. (2) **Boiled Oil vs. Pigment Coatings.**—Many engineers have abandoned the use of iron oxide and other uncertain patent paint compounds, but still adhere to the use of oil for the first coating. The writer believes that a good pigment paint is much better than oil coating. **Asphaltum Coatings.**—The so-called asphaltum paints in general have thus far proved to be quite as ineffective protective coatings as any of the iron oxide or miscellaneous

compound paints. **Linseed Oil.**—Discusses the processes of extracting the oil, etc. **Painting at the Mill.**—Does not advocate painting the iron or steel just after it has left the rolls or hammer, and while hot; but just before assembling.

The Painting and Sand-Blast Cleaning of Steel Bridges and Viaducts (By Geo. W. Lilly. Engrs. Club, Columbus, O., Feb. 1, 1902; Eng. News, April 24, 1902).—**Raw Linseed Oil** is said to make a better binder than boiled linseed oil, but it sets so slowly that in certain locations, such as viaducts subject to the blast, smoke and steam from locomotives, its use is inadvisable, because it will be filled with cinders and otherwise seriously injured before it is dry. **The Pigments** most commonly used for anti-rust paints on steel may be classed under the names red lead, iron oxide, carbon and graphite. Each of these has had its champion among men who have had considerable experience in the use of paints, while the experience of many experts has led to the conclusion that "red lead, oxide of iron, carbon and graphite all give results which average about the same." **Mixing and Applying the paint.**—All paints should be thoroughly mixed by machinery and subsequent grinding with a burr-stone mill if possible. But red lead, which is inclined to settle and harden somewhat in the mixture and make it difficult to spread and also diminish the coherence of the coating made by it, can be quite thoroughly mixed by hand when the facilities for machine mixing are not at hand; and this is advisable in most cases, for the reason that it is usually impossible to have it mixed at the time it is needed except it be done by hand. All painting should be done, if possible, when the temperature of the atmosphere is above 55° F., when little trouble will be experienced in spreading red lead or any other of the commonly used paints. If painting is done when the temperature is lower than this, the paint should be warmed up by placing it in a vessel, which is set in water heated to a temperature of 130° to 150° F., and each painter should be frequently supplied with warm paint when the paint in his bucket becomes cool. **Cleaning the Steel Before Painting.**—Before any paint or other coating of any kind is permitted to be applied to the iron or steel of a bridge or viaduct, all the scale, rust, dirt, grease and other foreign substances, as well as dead paint, should be removed from its surface, so that the coating may come into intimate contact with the clean surface of metal, and thus give the best condition for firm adhesion of the coating to the metal. The sand-blast has been given sufficient trial to make it reasonable to say that such cleaning as is necessary on new work at the shops—that is, removal of mill scale and some rust and grease—can be done at about ½ ct. per sq. ft. of steel surface cleaned, and possibly a little less. On this basis the cost per ton for cleaning steel plates would be: For plates 1" thick, 49 cts.; ¾" thick, 98 cts.; ½" thick, $1.96. For shapes: 7" I-beams, weighing 17.5 lbs. per ft., $1.35 per ton; 12" I-beams at 50 lbs. per ft., 80 cts.; the heavier sections costing less and the lighter sections costing more per ton. The average cost for cleaning most plate girder bridges would probably be about $1 per ton; and the cost for a truss bridge might vary from $1 per ton for heavy bridges to $1.75 per ton for light bridges. This article contains an illustrated description of the Newhouse sand-blast machine used in cleaning viaducts at Columbus, O.

Creosoting Wooden Poles for Electric Line Work (By W. E. Moore. Read before Nat'l Elec. Lt. Ass'n, at Cincinnati, O., May 20, 1902; Eng. News, May 29, 1902).—**Iron Poles** or creosote poles are as yet seldom used by lighting companies, though iron poles are extensively used for street railway purposes. **Wooden Poles.**—Wooden poles are usually of cedar, heart-sawed pine, cypress, juniper or redwood, and are generally used on account of low cost and the comparative safety with which workmen may handle the live wires when standing on the cross-arms, as wood is a fair insulator. Red-cedar and white-cedar poles, while they have a comparatively long life, have now become so scarce that it is extremely difficult to secure them in sufficient numbers of suitable sizes for electric light lines at any price in the Eastern or Southern market, though there is yet a considerable supply of white cedar in the Northwest. Heart-sawed pine poles have a somewhat longer life than cypress, ranging from 8 to 9 years; but sap pine, though readily secured in sticks of suitable size and necessary length, is never used, on account of its rapid decay. **Study of Preservation of Poles.**—The Augusta Ry. and Elec. Co. began about 9 years ago to study the problem of treating poles. The first experiment consisted in charring

the butts of the poles, up to about 1 ft. above the earth line, and then saturating them with a coal-tar paint; but this was found to be of little service. Painting the poles with various brands of preservative compounds sold under various trade names was then tried, but with little or no beneficial results. In the meantime the poles then in use, almost entirely of cypress, continued to rot out after an average life of about 5 or 6 years. **Creosoting Plant.**—Consists of a steel cylinder 6′ dia. and 102′ long, with heavy cast-iron heads, securely supported on hinges and arranged to be clamped against a fibrous gasket on the head of a cylinder so as to resist a hydrostatic pressure of 150 lbs. There is a narrow gage railway through it, with tracks continuing beyond ends of cylinder, which has a series of 1″ pipes laid from end to end and covering the bottom, and supplied with steam from an 80 H. P. return tubular boiler, the steam being superheated to a temperature of 400° to 500°. To the cylinder is also connected a direct-acting vacuum pump, 14″ x 24″, and, again, a direct-acting oil-pressure pump, 10″ x 18″. **Method of Treatment.**—Is fully described. **Cost of Treatment.**—Cost of average size (say 34 ft. long, 8″ dia. at top) cypress pole, from $1.75 to $2 each; and cost of creosoting, about $20 per M. B. M., or about twice the first cost of pole. Assumed that life of treated pole will be prolonged 4 to 6 times that of untreated. **Effect of Creosoting, on Linemen.**—Disagreeable in handling; dangerous to linemen when handling wires carrying 1000 to 3000 volts, as the creosoting lowers the electric resistivity of the timber.

Sand-Blast Cleaning of Structural Steel (By Geo. W. Lilly. Trans. A. S. C. E., Vol. L).

Creosoting Works of the Western Ry. of France (By J. M. Merklen. May number of "Revue Generale des Chemins de Fer;" Eng. News, July 27, 1905).—Full description of Plant, Method of Treatment, and illustrated Method of Piling Ties for Seasoning.

Tie and Timber Preserving Plant of the A. T. & S. F. Ry. at Somerville, Texas (Eng. News, May 3, 1906).—Description of Creosoting Process, Seasoning, Inspection and Marking, Creosoting Plant, Experimental Plant, Tanks, Gages and Pipe System, etc. Illustrated.

The Inspection of Treatment for the Protection of Timber by the Injection of Creosote Oil (By H. R. Stanford. Trans. A. S. C. E., Vol. LVI).

Coal Tar Paints (Eng. News, Aug. 16, 1906).—Discussions and references to other articles.

Corrosion of Steel in Reinforced Cinder Concrete (By W. H. Fox. Eng. News, May 23, 1907).—Following conclusions from results of experiments: In no case was any evidence found underneath the collars of neat cement which surrounded a portion of each steel specimen. To secure a dense homogeneous cinder concrete, a thorough tamping is necessary. A rich mixture, either a 1:1:3 or one in which the proportion of cement to aggregate is larger, should be used in all cases. The greatest care should be taken in mixing the materials, and it may be necessary to resort to the seemingly impractical method of coating the reinforcement with grout before placing in the concrete.

Cleaning Steelwork by Sand-Blast and Painting by Compressed Air (By De Witt C. Webb. Eng. News, Sept. 19, 1907).—**Plant.**—Following outfit, purchased at cost of $2090, delivered at U. S. Naval Station, Key West, Fla.: 1 hor. gasoline engine, 20 H. P.; 1 air compressor, capac. 90 ft. of free air per min. compressed to pres. of 30 lbs. per sq. in. in one stage, belt connected to engine; 1 rotary circulating pump, belt connec. to engine; 1 galv. steel water tank; 1 air receiver, 18″ x 54″; (above all mounted on steel framed wagon with wooden housing.) 2 sand-blast machines, at capac. of 2 cu. ft. of sand each; 2 paint spraying machines, one a hand machine of ¾-gal. capacity for one operator, the other of 10-gal. capac. for two operators; 100 lin. ft. of sand-blast hose; 200 lin. ft. of pneumatic hose for sand-blast machines; 400 lin. ft. of pneu. hose for painting machines; 4 khaki helmets, with mica-covered openings for the eyes; 200 lin. ft. of 2″ galv. pipe. **Cost of Cleaning.**—2000 sq. ft. of previously untouched surface was thoroughly cleaned and 7000 sq. ft. of hand cleaning was all gone over and much improved at a total cost for labor of $97.68, and for gasoline (at 19 cts. per gal.) of $16.15. **Cost of Painting.**—The coal-tar

paint originated by A C. Cunningham was used (see Eng. News, July 12, 1906), prepared with the following proportions (by volume): coal tar (4 parts), kerosene oil (1), Portland cement (1); cost, 15 cts. per gallon. Shed "A" required 64½ gals. for 9000 sq. ft. or 1 gal. to 140 sq. ft., at cost for labor of $28.16, and for gasoline of $3.80. Shed "B" required 86 gals. for 12500 sq. ft. or 1 gal. to 145 sq. ft., at cost for labor (including cleaning, painting, moving, setting up and removing) of $460, and for gasoline, $81.

Paints for Concrete (By G. D. White. Proc. A. S. T. M., Vol. IX, 1909).—With discussions.

Comparison of Various Processes of Preserving Timber (By G. B. Shipley, Eng. News, Oct. 14, 1909).—*Approximate cost of treating ties*, excluding royalty:—Burnetizing process: about ½-lb. dry zinc per cu. ft., $0.12 per tie. Wellhouse process: about ½-lb. zinc per cu. ft. plus glue and tannin, $0.16 per tie. Card process: about 1½-lbs. creosote and ½-lb. dry zinc per cu. ft., $0.18 per tie. Rueping process: about 6-lbs. creosote per cu. ft., $0.225 per tie. Lowry process: about 6-lbs. creosote per cu. ft., $0.225 per tie. Absorption process: about 6-lbs creosote per cu. ft., $0.23 per tie. Full cell process: about 10-lbs. creosote per cu. ft., $0.335 per tie. The above costs are based on creosote oil at $0.07 per gal. and dry zinc chloride at $0.04 per lb. It costs about ½-cent per tie to handle in the yard. *Timber*.—The cost of creosoting timber with 10 lbs. of creosote per cu. ft. will be about $8 per 1000 ft. B. M., and for each additional pound of creosote used add about $0.75 per 1000 ft. B. M. based on creosote oil at $0.07 per gal. Burnetizing, ½-lb. dry zinc chloride per cu. ft., $4 per M. B. M.

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20.—LUMBER AND LUMBERING.

Stumpage.—The following is a digest of Forest Service Circular 97, U. S. Dept. of Agriculture, issued April 24, 1907, and entitled "The Timber Supply of the United States," by R. S. Kellogg, Forest Inspector:

The percentage of the total lumber cut furnished by the principal regions since 1850, according to census figures, is as follows:

1.—GEOGRAPHICAL DISTRIBUTION OF TOTAL LUMBER PRODUCT.

Year.	North-eastern States.	Lake States.	Southern States.	Pacific States.
	Per cent.	*Per cent.*	*Per cent.*	*Per cent.*
1850	54.5	6.4	13.8	3.9
1860	36.2	13.6	16.5	6.2
1870	36.8	24.4	9.4	3.8
1880	24.8	33.4	11.9	3.5
1890	18.4	36.3	15.9	7.3
1900	16.0	27.4	25.2	9.6

The principal estimates of the stumpage of the U. S. made since 1880 are given in Table 2. The first, by Sargent (1880), in addition to being too low for almost every species considered, with the possible exception of the hardwoods, is notable for its omission of Douglas spruce—which exists today in greater quantity than any other of our valuable timbers—and yellow pine, another important species. The next estimate, that of Hotchkiss (1898), does not go into details. The next estimate, by Gannett (1900), was most carefully prepared. That by Fernow (1902) is a regional estimate. Long's estimate (1903) does not cover cypress, sugar pine, or hardwoods. Its principal point of interest is that it differs so radically—about 38%—from that of the census of 1903 upon the stumpage of yellow pine. The last estimate given in the table is that published in the "American Lumberman," Sept. 23, 1905. It is based primarily upon census data, with the addition of some species and with increased figures for others:

2.—ESTIMATES OF STUMPAGE OF THE UNITED STATES.

Kind of Timber.	Census, 1880.	Hotchkiss, 1898.	Census, 1900.	Fernow, 1902.	Long, 1903.	American Lumberman, 1905.
	M b'rd ft.	M board ft.	M board ft.	M board ft.	M b'rd ft.	M board ft.
White pine	87,755,000	.....	50,000,000	.....	60,000,000	.....
Eastern and northern pine	.....	.....	.....	.....	.....	55,000,000
Southern yellow pine	237,141,500	.....	300,000,000	.....	187,250,000	300,000,000
Eastern spruce	12,265,000	.....	50,000,000	.....	18,221,000	75,000,000
Eastern hemlock	20,165,000	.....	100,000,000	.....	56,571,000	100,000,000
Douglas fir	.....	.....	300,000,000	.....	260,000,000	350,000,000
Western yel. pine	.....	.....	125,000,000	.....	138,000,000	250,000,000
Cypress	*2,153,600	.....	65,000,000	.....	.....	65,000,000
Redwood	25,825,000	.....	75,000,000	.....	75,000,000	75,000,000
Cedar	22,800,000	.....	.....	.....	27,640,000	.....
Sugar pine	.....	.....	25,000,000	.....	.....	50,000,000
Other conifers	12,500,000	.....	.....	.....	.....	250,000,000
Total conifers	420,605,100	.....	1,090,000,000	.....	822,682,000	1,570,000,000
Total hardw'ds	435,685,000	.....	300,000,000	.....	.....	400,000,000
Region:						
Northern States	.....	100,000,000	.....	500,000,000	.....	.....
Southern States	.....	300,000,000	.....	700,000,000	.....	.....
Western States	.....	.....	.....	800,000,000	.....	.....
Pacific States	.....	1,000,000,000	.....	.....	.....	.....
Total	856,290,100	1,400,000,000	1,390,000,000	2,000,000,000	822,682,000	1,970,000,000

* Florida and Alabama only.

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The "Pacific Lumber Trade Journal," in the issue of January, 1907, gave the following estimate of the stumpage of the Pacific Coast, including Idaho, Montana, and British Columbia:

3.—ESTIMATED STUMPAGE OF CALIFORNIA, OREGON, WASHINGTON, IDAHO, MONTANA, AND BRITISH COLUMBIA.

Kind of timber.	M board feet.	Kind of timber.	M board feet.
Douglas fir	374,064,102	Spruce	25,419,215
Western and yellow pine.	175,586,520	Larch	5,078,601
Red cedar	78,961,383	Miscellaneous and hardwoods	5,700,000
Redwood	75,000,000		
Hemlock	60,848,259		
Sugar pine	50,000,000	Total	850,658,080

This total is credited by States as follows:

	M board feet.		M board feet.
Oregon	225,000,000	British Columbia	150,000,000
Washington	195,658,080	Idaho and Montana	100,000,000
California	180,000,000		

The present annual cut of some of the principal woods is as follows: *White Pine*—about 3 billion feet in the Lake States and 1 billion feet in other States; *Yellow Pine*—about 12 billion feet, or a little more than one-third the total cut of all species, and it is evident that within 10 to 15 years there will be a most serious shortage; *Spruce*—about 1½ billion feet, of which Maine furnishes about one-third; *Hemlock*—about 3 billion feet, of which Penn., Mich., and Wis. furnish about three-fourths (the cut of both eastern spruce and eastern himlock is decreasing, while that of western spruce and hemlock is increasing); *Douglas Spruce*—about 4½ billion feet (1¾ billion feet in 1900); *Western Yellow Pine*—about 1 billion feet, two-thirds of which is in the Pacific Coast States; *Redwood*—about 450 million feet, and increasing; *Cypress*—about 750 million feet, with Louisiana supplying about 65%; *Hardwoods*—about 5 billion feet, consisting of approximately 43% oak, 12% poplar, 9% maple, and lesser amounts of numerous other species.

To engineers who are basing cost of proposed work on former estimates which are incomplete in detail, the accompanying diagram, Fig. 1, may be

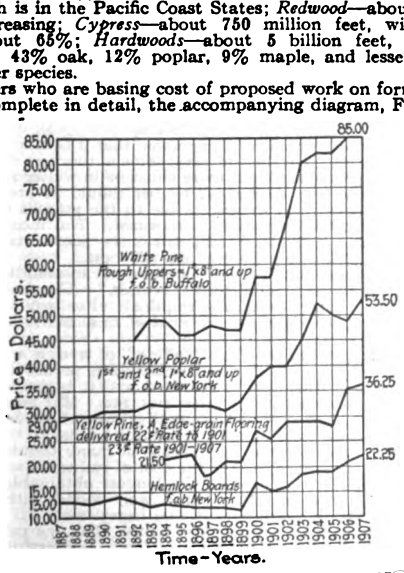

Fig. 1. Range of Lumber prices, 1887 to 1907.

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of interest as showing the marked advance in the prices of lumber during the past 20 years, and especially during the last decade.

Logging.—(*a*) *How Trees Grow.*—The section of an ordinary tree, through the trunk, discloses first the heartwood at the center, next the sapwood, and between the sapwood and the bark we find the *cambium.* If we call the bark the "coat" of the tree, we may call the cambium the (continuous) "undergarment," for it clothes every portion of the woody fiber from tip of root to end of stem, terminating in the leaves which are really an extension of the cambium itself. This slimy covering carries the life blood or sap of the tree. It has no definite thickness as it gradually merges into the bark on the one hand and into the woody fiber (sapwood) on the other. If the trunk or limb of a tree is completely girdled, exposing the cambium to the air, that portion of the tree above the girdle will die, as if amputated.

A tree breathes mainly through its leaves, and partly through the pores of the bark. The sap, containing various mineral substances as potassium, calcium, iron, sulphur, magnesium, phosphorous and nitrogen, ascends from the roots to the leaves, and is here met by the free oxygen and carbonic acid (CO_2) which the leaves breathe from the air. Various compounds, mainly *starch* ($C_6H_{10}O_5$), are formed in these chemical laboratories through the agency of the sun, and these new products, in solution, circulate back with the sap, build up the woody fiber of the tree and produce growth of trunk, roots, branches, leaves, and buds, generally. When the carbon dioxide is breathed into the leaves, much of the water of the sap is thrown off into the air, and this exhalation is called *transpiration*, a process similar to the perspiration of animals. Some trees will transpire considerably more than a hundred gallons of water in a day. When daylight ends, starch-making ceases, but the building up of the woody fibers goes on throughout the night.

Trees grow radially outward. Drive two spikes, spaced vertically, into the trunk of a tree and note that the space does not increase perceptibly as the tree grows taller. The trunk expands in diameter as the delicate cells near the cambium become thickened with starch from the down-flow sap into the woody fiber, and the up-flowing sap is forced outward through newer tubes. Finally the walls of the sapwood become hardened with mineral deposits and form heartwood. The alternate semi-annular rings are due to summer and winter growth. With the fall of the leaves the breathing takes place through the pores of the bark, and in winter the tree practically sleeps or hibernates.

(*b*) *Best Time for Cutting Timber.*—We usually speak of "winter cut" timber as the best, and some give as a reason, that it contains less moisture. This is hardly the case. On the contrary, some authorities claim that many woods contain quite as much if not more moisture in winter than in summer. The principal reasons why winter cut timber should be preferred are that it is harder and denser; not so susceptible to the attacks of forest fungi; and capable of being more perfectly seasoned. The most rapid growth of timber is in the early summer, and this is the poorest time for cutting. Winter and late fall cutting seasons are the best.

(*c*) *Volume of Standing Timber.*—The amount of standing timber on a certain tract is usually estimated by a "cruiser." This is done in various ways depending upon the accuracy required. For any individual tree, the contents is assumed equal to the area of base multiplied by one-half the height.

(*d*) *Transportation of Logs.*—The "felling" of trees is a very important operation, often producing splits and cracks which reduce the grade of the timber. After felling, the bark is removed and it is cut into the desired lengths for mill logs, spars, poles, piles, ties, etc. Mill logs are usually in even lengths, from 12 ft. upward, generally from 16 to 34 ft. They are dragged, rafted, flumed, or shipped (by logging trains) to the mill. Ocean rafts are sometimes made up at an expense of many thousands of dollars, and towed hundreds of miles to save the cost of rail transportation. These rafts are cigar shaped, composed of the longest obtainable logs, arranged scientifically with "broken joints," and well bound with heavy chains for strength in resisting the action of the heavy seas. If this method is contemplated, it is wise to *estimate*, ordinarily, that one raft in every two or three is liable to be "broken up" and lost. Successful rafting of this kind has been performed from Astoria, Oregon, to San Francisco, and from Nova Scotia to Jersey City, N. J.

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(*e*) *Scaling Logs.*—The determination of the number of thousand feet of lumber in logs, as a basis for selling, is usually made at the mill by an official scaler. The diam of the small end is measured in ins. by a scale rule, 4 ins. deducted, and the balance squared. The result is the number of ft. B. M. for a log 16 ft. in length—proportionate for logs of other lengths. Where logs are defective a reduction is made depending upon the judgment of the scaler.

Sawing the logs up into rough lumber, such as "sticks" (general term for large dimension timber), posts, beams, joists, ties, scantling (small dimension stuff), boards, shingles and laths, is done with the various saws, as band, circular (single or double), slab, gang, shingle, and lath saws. Rough lumber should be furnished "full-dimension" but not necessarily to exact dimension, as ordered. When exact dimensions are required the lumber should be ordered "sized."

"Sizing" consists in running the rough lumber through the planer or "sticker," so gaged that when planed it shall be exactly to ordered dimensions. In ordering say 12″ x 12″ to be planed on *all* sides, we generally say 12″ x 12″ *s* 4 *s*; if on *one* side, 12″ x 12″ *s* 1 *s*; etc.

Planing costs the purchaser, ordinarily, from $1.50 to $2.00 per *M*, but it is often desirable to order planed lumber where rough lumber might answer; thus, with so-called "permanent" wooden structures like Howe truss bridges, for instance, the cost of framing with planed lumber is less than with rough, and the life greater, to say nothing of the appearance. Rough dimension stuff which is to be planed is usually ordered about ¼″ large for each dimension, depending upon size of piece; less for smaller pieces.

Seasoning.—After being sawed, lumber should be seasoned more or less thoroughly before being used. If "open-stacked" under roof shelter for three months or longer it is in fairly good condition for ordinary outside construction, but for so-called thoroughly dried lumber a year or two is required. Heavy timbers of course require longer periods. Kiln-dried lumber is the common practice and is very satisfactory if properly done, and the temperature of the kiln is not too high. If steam is admitted the temperature may be from 160° to 170° F. for the harder woods and from 170° to 180° for the softer kinds, as pine. In dry kilns the temperature should be lower. The harder woods are preferably stacked in the open air for some months before being placed in the kilns, or they may be immediately kiln dried at low temperatures. (See also "Seasoning of Timber," under Preservatives, page 361).

Board Measure.—One ft. B. M. of lumber is equivalent to a board 1 in. thick, 12 ins. wide, and 1 ft. long, or $\frac{1}{12}$ cu. ft.; hence 1000 ft. B. M. (= 1 M. B. M.) contains 83⅓ cu. ft.

The following Table (4) of board measure embraces all the sizes ordinarily used in construction, and gives the ft. B. M. for lengths from 1 to 9, which may be used decimally for any lengths. Interpolation may be resorted to for dimensions not in the table, but this will rarely be necessary. Another method is to find the ft. B. M. for dimensions 2 or more times as large or small, and then factor the results accordingly. Note that for this purpose the list of 1-inch stuff is very comprehensive.

Where (exact) results are required to decimal places beyond those in the table—which will rarely be the case—a casual inspection will in most cases suffice. Thus, 1.083 = 1.083˘3, 3.167 = 3.16˘6, 1.042 = 1.0416˘6, etc. The "character" of the decimal for any particular length may generally be determined by examining other decimals in the same *line*, showing whether the decimal represents 6ths, 8ths, 12ths, 16ths, etc.

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4.—Feet Board Measure—Engineers' Table.*

Dimension in Inches.	Length in Feet.									Dimension in
	1	2	3	4	5	6	7	8	9	
½x1	.0417	.0833	.1250	.1667	.2083	.2500	.2917	.3333	.3750	½x1
⅝x1	.0521	.1042	.1563	.2083	.2604	.3125	.3646	.4167	.4688	⅝x1
¾x1½	.0651	.1302	.1953	.2604	.3255	.3906	.4557	.5208	.5859	¾x1½
⅞x1	.0625	.1250	.1875	.2500	.3125	.3750	.4375	.5000	.5625	⅞x1
⅞x1½	.0781	.1563	.2344	.3125	.3906	.4688	.5469	.6250	.7031	⅞x1½
⅞x1½	.0938	.1875	.2813	.3750	.4688	.5625	.6563	.7500	.8438	⅞x1½
¾x1	.0729	.1458	.2187	.2917	.3646	.4375	.5104	.5833	.6563	¾x1
¾x1½	.0911	.1823	.2734	.3646	.4557	.5469	.6380	.7292	.8203	¾x1½
¾x1¾	.1094	.2188	.3281	.4375	.5469	.6562	.7656	.8750	.9844	¾x1¾
¾x1¾	.1276	.2552	.3828	.5104	.6380	.7656	.8932	1.0208	1.1484	¾x1¾
1x1	.0833	.1667	.2500	.3333	.4167	.5000	.5833	.6667	.7500	1x1
1x1½	.1042	.2083	.3125	.4167	.5208	.6250	.7292	.8333	.9375	1x1½
1x1½	.1250	.2500	.3750	.5000	.6250	.7500	.8750	1.0000	1.1250	1x1½
1x1¾	.1458	.2917	.4375	.5833	.7292	.8750	1.0208	1.167	1.313	1x1¾
1x2	.1667	.3333	.5000	.6667	.8333	1.0000	1.167	1.333	1.500	1x2
1x2½	.1875	.3750	.5625	.7500	.9375	1.125	1.313	1.500	1.688	1x2½
1x2½	.2083	.4167	.6250	.8333	1.0417	1.250	1.458	1.667	1.875	1x2½
1x2¾	.2292	.4583	.6875	.9167	1.146	1.375	1.604	1.833	2.063	1x2¾
1x3	.2500	.5000	.7500	1.000	1.250	1.500	1.750	2.000	2.250	1x3
1x3¼	.2708	.5417	.8125	1.083	1.354	1.625	1.896	2.167	2.437	1x3¼
1x3½	.2917	.5833	.8750	1.167	1.458	1.750	2.042	2.333	2.625	1x3½
1x3¾	.3125	.6250	.9375	1.250	1.563	1.875	2.188	2.500	2.813	1x3¾
1x4	.3333	.6667	1.0000	1.333	1.667	2.000	2.333	2.667	3.000	1x4
1x4½	.3750	.7500	1.125	1.500	1.875	2.250	2.625	3.000	3.375	1x4½
1x5	.4167	.8333	1.250	1.667	2.083	2.500	2.917	3.333	3.750	1x5
1x5½	.4583	.9167	1.375	1.833	2.292	2.750	3.208	3.667	4.125	1x5½
1x6	.5000	1.0000	1.500	2.000	2.500	3.000	3.500	4.000	4.500	1x6
1x6½	.5417	1.083	1.625	2.167	2.708	3.250	3.792	4.333	4.875	1x6½
1x7	.5833	1.167	1.750	2.333	2.917	3.500	4.083	4.667	5.250	1x7
1x7½	.6250	1.250	1.875	2.500	3.125	3.750	4.375	5.000	5.625	1x7½
1x8	.6667	1.333	2.000	2.667	3.333	4.000	4.667	5.333	6.000	1x8
1x8½	.7083	1.417	2.125	2.833	3.542	4.250	4.958	5.667	6.375	1x8½
1x9	.7500	1.500	2.250	3.000	3.750	4.500	5.250	6.000	6.750	1x9
1x9½	.7917	1.583	2.375	3.167	3.908	4.750	5.542	6.333	7.125	1x9½
1x10	.8333	1.667	2.500	3.333	4.167	5.000	5.833	6.667	7.500	1x10
1x10½	.8750	1.750	2.625	3.500	4.375	5.250	6.125	7.000	7.875	1x10½
1x11	.9167	1.833	2.750	3.667	4.583	5.500	6.417	7.333	8.250	1x11
1x11½	.9583	1.917	2.875	3.833	4.792	5.750	6.708	7.667	8.625	1x11½
1x12	1.0000	2.000	3.000	4.000	5.000	6.000	7.000	8.000	9.000	1x12
1x12½	1.042	2.083	3.125	4.167	5.208	6.250	7.292	8.333	9.375	1x12½
1x13	1.083	2.167	3.250	4.333	5.417	6.500	7.583	8.667	9.750	1x13
1x13½	1.125	2.250	3.375	4.500	5.625	6.750	7.875	9.000	10.125	1x13½
1x14	1.167	2.333	3.500	4.667	5.833	7.000	8.167	9.333	10.50	1x14
1x14½	1.208	2.417	3.625	4.833	6.042	7.250	8.458	9.667	10.88	1x14½
1x15	1.250	2.500	3.750	5.000	6.250	7.500	8.750	10.000	11.25	1x15
1x15½	1.292	2.583	3.875	5.167	6.458	7.750	9.042	10.33	11.63	1x15½
1x16	1.333	2.667	4.000	5.333	6.667	8.000	9.333	10.67	12.00	1x16
1x17	1.417	2.833	4.250	5.667	7.083	8.500	9.917	11.33	12.75	1x17
1x18	1.500	3.000	4.500	6.000	7.500	9.000	10.500	12.00	13.50	1x18
1x19	1.583	3.167	4.750	6.333	7.917	9.500	11.083	12.67	14.25	1x19

*Ex.—Find the number of ft. B. M. in 2438 lin. ft. of 1"x6½"?

Solution.—2000 = 1083.3
400 = 216.7
30 = 16.25
8 = 4.333

Ans.—1321. ft. B. M

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4.—FEET BOARD MEASURE—ENGINEERS' TABLE.—Continued.

Dimension in Inches.	Length in Feet.									Dimension in Inches.
	1	2	3	4	5	6	7	8	9	
1 x20	1.667	3.333	5.000	6.667	8.333	10.00	11.67	13.33	15.00	1 x20
"x21	1.750	3.500	5.250	7.000	8.750	10.50	12.25	14.00	15.75	"x21
"x22	1.833	3.667	5.500	7.333	9.167	11.00	12.83	14.67	16.50	"x22
"x23	1.917	3.833	5.750	7.667	9.583	11.50	13.42	15.33	17.25	"x23
"x24	2.000	4.000	6.000	8.000	10.000	12.00	14.00	16.00	18.00	"x24
1¼x1¼	.1302	.2604	.3906	.5208	.6510	.7812	.9115	1.042	1.172	1¼x1¼
"x1½	.1563	.3125	.4688	.6250	.7813	.9375	1.0938	1.250	1.406	"x1½
"x1¾	.1823	.3646	.5469	.7292	.9115	1.0937	1.276	1.458	1.641	"x1¾
"x2	.2083	.4167	.6250	.8333	1.0417	1.250	1.458	1.667	1.875	"x2
"x2½	.2344	.4688	.7031	.9375	1.172	1.406	1.641	1.875	2.109	"x2½
1¼x2½	.2604	.5208	.7813	1.0417	1.302	1.563	1.823	2.083	2.344	1¼x2½
"x2¾	.2865	.5729	.8594	1.146	1.432	1.719	2.005	2.292	2.578	"x2¾
"x3	.3125	.6250	.9375	1.250	1.563	1.875	2.188	2.500	2.813	"x3
"x3½	.3646	.7292	1.0937	1.458	1.823	2.187	2.552	2.917	3.281	"x3½
"x4	.4167	.8333	1.250	1.667	2.083	2.500	2.917	3.333	3.750	"x4
1¼x4½	.4688	.9375	1.406	1.875	2.344	2.813	3.281	3.750	4.219	1¼x4½
"x5	.5208	1.0417	1.563	2.083	2.604	3.125	3.646	4.167	4.688	"x5
"x5½	.5729	1.146	1.719	2.292	2.865	3.438	4.010	4.583	5.156	"x5½
"x6	.6250	1.250	1.875	2.500	3.125	3.750	4.375	5.000	5.625	"x6
"x7	.7292	1.458	2.188	2.917	3.646	4 375	5.104	5.833	6.562	"x7
1¼x8	.8333	1.667	2.500	3.333	4.167	5.000	5.833	6.667	7.500	1¼x8
"x9	.9375	1.875	2.813	3.750	4.688	5.625	6.563	7.500	8.438	"x9
"x10	1.0417	2.083	3.125	4.167	5.208	6.250	7.292	8.333	9.375	"x10
"x11	1.146	2.292	3.438	4.583	5.729	6.875	8.021	9.167	10.313	"x11
"x12	1.250	2.500	3.750	5.000	6.250	7.500	8.750	10.00	11.25	"x12
1½x1½	.1875	.3750	.5625	.7500	.9375	1.125	1.313	1.500	1.688	1½x1½
"x1¾	.2188	.4375	.6563	.8750	1.0938	1.313	1.531	1.750	1.969	"x1¾
"x2	.2500	.5000	.7500	1.0000	1.250	1.500	1.750	2.000	2.250	"x2
"x2¼	.2813	.5625	.8438	1.1250	1.406	1.688	1.969	2.250	2.531	"x2¼
"x2½	.3125	.6250	9375	1.250	1.563	1.875	2.188	2.500	2.813	"x2½
1½x2¾	.3438	.6875	1.0313	1.375	1.719	2.063	2.406	2.750	3.094	1½x2¾
"x3	.3750	.7500	1.125	1.500	1.875	2.250	2.625	3.000	3.375	"x3
"x3½	.4375	.8750	1.313	1.750	2.188	2.625	3.063	3.500	3.938	"x3½
"x4	.5000	1.0000	1.500	2.000	2.500	3.000	3.500	4.000	4.500	"x4
"x4½	.5625	1.125	1.688	2.250	2.813	3.375	3.938	4.500	5.063	"x4½
1½x5	.6250	1.250	1.875	2.500	3.125	3.750	4.375	5.000	5.625	1½x5
"x5½	.6875	1.375	2.063	2.750	3.438	4.125	4.813	5.500	6.188	"x5½
"x6	.7500	1.500	2.250	3.000	3.750	4.500	5.250	6.000	6.750	"x6
"x7	.8750	1.750	2.625	3.500	4.375	5.250	6.125	7.000	7.875	"x7
"x8	1.0000	2.000	3.000	4.000	5.000	6.000	7.000	8.000	9.000	"x8
1½x9	1.125	2.250	3.375	4.500	5.625	6.750	7.875	9.000	10.125	1½x9
"x10	1.250	2.500	3.750	5.000	6.250	7.500	8.750	10.000	11.25	"x10
"x12	1.500	3.000	4.500	6.000	7.500	9.000	10.500	12.00	13.50	"x12
2 x2	.3333	.6667	1.000	1.333	1.667	2.000	2.333	2.667	3.000	2 x2
"x2¼	.3750	.7500	1.125	1.500	1.875	2.250	2.625	3.000	3.375	"x2¼
2 x2½	.4167	.8333	1.250	1.667	2.083	2.500	2.917	3.333	3.750	2 x2½
"x2¾	.4583	.9167	1.375	1.833	2.292	2.750	3.208	3.667	4.125	"x2¾
"x3	.5000	1.0000	1.500	2.000	2.500	3.000	3.500	4.000	4.500	"x3
"x3½	.5833	1.167	1.750	2.333	2.917	3.500	4.083	4.667	5.250	"x3½
"x4	.6667	1.333	2.000	2.667	3.333	4.000	4.667	5.333	6.000	"x4
2 x4½	.7500	1.500	2.250	3.000	3.750	4.500	5.250	6.000	6.750	2 x4½
"x5	.8333	1.667	2.500	3.333	4.167	5.000	5.833	6.667	7.500	"x5
"x5½	.9167	1.833	2.750	3.667	4.583	5.500	6.417	7.333	8.250	"x5½
"x6	1.0000	2.000	3.000	4.000	5.000	6.000	7.000	8.000	9.000	"x6
"x7	1.167	2.333	3.500	4.667	5.833	7.000	8.167	9.333	10.500	"x7

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4.—FEET BOARD MEASURE—ENGINEERS' TABLE.—Continued.

Dimension in Inches.	Length in Feet.									Dimension in Inches
	1	2	3	4	5	6	7	8	9	
2 x8	1.333	2.667	4.000	5.333	6.667	8.000	9.333	10.667	12.00	2 x8
"x9	1.500	3.000	4.500	6.000	7.500	9.000	10.500	12.00	13.50	"x9
"x10	1.667	3.333	5.000	6.667	8.333	10.00	11.67	13.33	15.00	"x10
"x11	1.833	3.667	5.500	7.333	9.167	11.00	12.83	14.67	16.50	"x11
"x12	2.000	4.000	6.000	8.000	10.00	12.00	14.00	16.00	18.00	"x12
2 x14	2.333	4.667	7.000	9.333	11.67	14.00	16.33	18.67	21.00	2 x14
"x15	2.500	5.000	7.500	10.000	12.50	15.00	17.50	20.00	22.50	"x15
"x16	2.667	5.333	8.000	10.67	13.33	16.00	18.67	21.33	24.00	"x16
2½x2½	.5208	1.0417	1.563	2.083	2.604	3.125	3.646	4.167	4.688	2½x2½
"x2¾	.5729	1.146	1.719	2.292	2.865	3.438	4.010	4.583	5.156	"x2¾
2½x3	.6250	1.250	1.875	2.500	3.125	3.750	4.375	5.000	5.625	2½x3
"x3½	.7292	1.458	2.188	2.917	3.646	4.375	5.104	5.833	6.562	"x3½
"x4	.8333	1.667	2.500	3.333	4.167	5.000	5.833	6.667	7.500	"x4
"x4½	.9375	1.875	2.813	3.750	4.688	5.625	6.563	7.500	8.438	"x4½
"x5	1.042	2.083	3.125	4.167	5.208	6.250	7.292	8.333	9.375	"x5
2½x5½	1.146	2.292	3.438	4.583	5.729	6.875	8.021	9.167	10.313	2½x5½
"x6	1.250	2.500	3.750	5.000	6.250	7.500	8.750	10.000	11.25	"x6
"x7	1.458	2.917	4.375	5.833	7.292	8.750	10.208	11.67	13.13	"x7
"x8	1.667	3.333	5.000	6.667	8.333	10.00	11.67	13.33	15.00	"x8
"x9	1.875	3.750	5.625	7.500	9.375	11.25	13.13	15.00	16.88	"x9
2½x10	2.083	4.167	6.250	8.333	10.417	12.50	14.58	16.67	18.75	2½x10
"x12	2.500	5.000	7.500	10.000	12.50	15.00	17.50	20.00	22.50	"x12
"x14	2.917	5.833	8.750	11.67	14.58	17.50	20.42	23.33	26.25	"x14
"x15	3.125	6.250	9.375	12.50	15.63	18.75	21.88	25.00	28.13	"x15
"x16	3.333	6.667	10.000	13.33	16.67	20.00	23.33	26.67	30.00	"x16
3 x3	.7500	1.500	2.250	3.000	3 750	4.500	5.250	6.000	6.750	3 x3
"x3½	.8750	1.750	2.625	3.500	4.375	5.250	6.125	7.000	7.875	"x3½
"x4	1.0000	2.000	3.000	4.000	5 000	6.000	7.000	8.000	9.000	"x4
"x4½	1.125	2.250	3.375	4.500	5.625	6.750	7.875	9.000	10.125	"x4½
"x5	1.250	2.500	3.750	5.000	6.250	7.500	8.750	10.000	11.25	"x5
3 x5½	1.375	2.750	4.125	5.500	6.875	8.250	9.625	11.00	12.38	3 x5½
"x6	1.500	3.000	4.500	6.000	7.500	9.000	10.500	12.00	13.50	"x6
"x7	1.750	3.500	5.250	7.000	8.750	10.500	12.25	14.00	15.75	"x7
"x8	2.000	4.000	6.000	8.000	10.000	12.00	14.00	16.00	18.00	"x8
"x9	2.250	4.500	6.750	9.000	11.25	13.50	15.75	18.00	20.25	"x9
3 x10	2.500	5.000	7.500	10.000	12.50	15.00	17.50	20.00	22.50	3 x10
"x11	2.750	5.500	8.250	11.00	13.75	16.50	19.25	22.00	24.75	"x11
"x12	3.000	6.000	9.000	12.00	15.00	18.00	21.00	24.00	27.00	"x12
"x13	3.250	6.500	9.750	13.00	16.25	19.50	22.75	26.00	29.25	"x13
"x14	3.500	7.000	10.500	14.00	17.50	21.00	24.50	28.00	31.50	"x14
3 x15	3.750	7.500	11.25	15.00	18.75	22.50	26.25	30.00	33.75	3 x15
"x16	4.000	8.000	12.00	16.00	20.00	24.00	28.00	32.00	36.00	"x16
"x18	4.500	9.000	13.50	18.00	22.50	27.00	31.50	36.00	40.50	"x18
3½x3½	1.021	2.042	3.063	4.083	5.104	6.125	7.146	8.167	9.188	3½x3½
"x4	1.167	2.333	3.500	4.667	5.833	7.000	8.167	9.333	10.667	"x4
3½x4½	1.313	2.625	3.938	5.250	6.563	7.875	9.188	10.500	11.813	3½x4½
"x5	1.458	2.917	4.375	5.833	7.292	8.750	10.208	11.67	13.13	"x5
"x5½	1.604	3.208	4.813	6.417	8.021	9.625	11.23	12.83	14.44	"x5½
"x6	1.750	3.500	5.250	7.000	8.750	10.500	12.25	14.00	15.75	"x6
"x7	2.042	4.083	6.125	8.167	10.208	12.25	14.29	16.33	18.38	"x7
3½x8	2.333	4.667	7.000	9.333	11.67	14.00	16.33	18.67	21.00	3½x8
"x9	2.625	5.250	7.875	10.500	13.13	15.75	18.38	21.00	23.63	"x9
"x10	2.917	5.833	8.750	11.67	14.58	17.50	20.42	23.33	26.25	"x10
"x11	3.208	6.417	9.625	12.83	16.04	19.25	22.46	25.67	28.88	"x11
"x12	3.500	7.000	10.500	14.00	17.50	21.00	24.50	28.00	31.50	"x12

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4.—FEET BOARD MEASURE—ENGINEERS' TABLE.—Continued.

Dimension in Inches.	Length in Feet.									Dimension in Inches.
	1	2	3	4	5	6	7	8	9	
3½x14	4.083	8.167	12.25	16.33	20.42	24.50	28.58	32.67	36.75	3½x14
"x15	4.375	8.750	13.13	17.50	21.88	26.25	30.63	35.00	39.38	"x15
"x16	4.667	9.333	14.00	18.67	23.33	28.00	32.67	37.33	42.00	"x16
"x17	4.958	9.917	14.88	19.83	24.79	29.75	34.71	39.67	44 63	"x17
"x18	5.250	10.500	15.75	21.00	26.25	31.50	36.75	42.00	47.25	"x18
4 x4	1.333	2.667	4.000	5.333	6.667	8.000	9.333	10.67	12.00	4 x4
"x4½	1.500	3.000	4.500	6.000	7.500	9.000	10.500	12.00	13.50	"x4½
"x5	1.667	3.333	5.000	6.667	8.333	10.00	11.67	13.33	15.00	"x5
"x5½	1.833	3.667	5.500	7.333	9.167	11.00	12.83	14.67	16.50	"x5½
"x6	2.000	4.000	6.000	8.000	10.000	12.00	14.00	16.00	18.00	"x6
4 x6½	2.167	4.333	6.500	8.667	10.83	13.00	15.17	17.33	19.50	4 x6½
"x7	2.333	4.667	7.000	9.333	11.67	14.00	16.33	18.67	21.00	"x7
"x8	2.667	5.333	8.000	10.667	13.33	16.00	18.67	21.33	24.00	"x8
"x9	3.000	6.000	9.000	12.00	15.00	18.00	21.00	24.00	27.00	"x9
"x10	3.333	6.667	10.000	13.33	16.67	20.00	23.33	26.67	30.00	"x10
4 x11	3.667	7.333	11.00	14.67	18.33	22.00	25.67	29.33	33.00	4 x11
"x12	4.000	8.000	12.00	16.00	20.00	24.00	28.00	32.00	36.00	"x12
"x14	4.667	9.333	14.00	18.67	23.33	28.00	32.67	37.33	42.00	"x14
"x15	5.000	10.000	15.00	20.00	25.00	30.00	35.00	40.00	45.00	"x15
"x16	5.333	10.67	16.00	21.33	26.67	32.00	37.33	42.67	48.00	"x16
4 x18	6.000	12.00	18.00	24.00	30.00	36.00	42.00	48.00	54.00	4 x18
4½x4½	1.688	3.375	5.063	6.750	8.438	10.13	11.81	13.50	15.19	4½x4½
"x5	1.875	3.750	5.625	7.500	9.375	11.25	13.13	15.00	16.88	"x5
"x5½	2.063	4.125	6.188	8.250	10.313	12.38	14.44	16.50	18.56	"x5½
"x6	2.250	4.500	6.750	9.000	11.25	13.50	15.75	18.00	20.25	"x6
4½x6½	2.438	4.875	7.313	9.750	12.19	14.63	17.06	19.50	21.94	4½x6½
"x7	2.625	5.250	7.875	10.500	13.13	15.75	18.38	21.00	23.63	"x7
"x8	3.000	6.000	9.000	12.00	15.00	18.00	21.00	24.00	27.00	"x8
"x9	3.375	6.750	10.125	13.50	16.88	20.25	23.63	27.00	30.38	"x9
"x10	3.750	7.500	11.25	15.00	18.75	22.50	26.25	30.00	33.75	"x10
4½x11	4.125	8.250	12.38	16.50	20.63	24.75	28.88	33.00	37.13	4½x11
"x12	4.500	9.000	13.50	18.00	22.50	27.00	31.50	36.00	40.50	"x12
"x14	5.250	10.500	15.75	21.00	26.25	31.50	36.75	42.00	47.25	"x14
"x15	5.625	11.25	16.88	22.50	28.13	33.75	39.38	45.00	50.63	"x15
"x16	6.000	12.00	18.00	24.00	30.00	36.00	42.00	48.00	54.00	"x16
4½x18	6.750	13.50	20.25	27.00	33.75	40.50	47.25	54.00	60.75	4½x18
5 x5	2.083	4.167	6.250	8.333	10.42	12.50	14.58	16.67	18.75	5 x5
"x5½	2.292	4.583	6.875	9.167	11.46	13.75	16.04	18.33	20.63	"x5½
"x6	2.500	5.000	7.500	10.000	12.50	15.00	17.50	20.00	22.50	"x6
"x6½	2.708	5.417	8.125	10.83	13.54	16.25	18.96	21.67	24.37	"x6½
5 x7	2.917	5.833	8.750	11.67	14.58	17.50	20.42	23.33	26.25	5 x7
"x7½	3.125	6.250	9.375	12.50	15.63	18.75	21.88	25.00	28.13	"x7½
"x8	3.333	6.667	10.000	13.33	16.67	20.00	23.33	26.67	30.00	"x8
"x9	3.750	7.500	11.25	15.00	18.75	22.50	26.25	30.00	33.75	"x9
"x10	4.167	8.333	12.50	16.67	20.83	25.00	29.17	33.33	37.50	"x10
5 x11	4.583	9.167	13.75	18.33	22.92	27.50	32.08	36.67	41.25	5 x11
"x12	5.000	10.000	15.00	20.00	25.00	30.00	35.00	40.00	45.00	"x12
"x14	5.833	11.67	17.50	23.33	29.17	35.00	40.83	46.67	52.50	"x14
"x15	6.250	12.50	18.75	25.00	31.25	37.50	43.75	50.00	56.25	"x15
"x16	6.667	13.33	20.00	26.67	33.33	40.00	46.67	53.33	60.00	"x16
5 x18	7.500	15.00	22.50	30.00	37.50	45.00	52.50	60.00	67.50	5 x18
5½x5½	2.521	5.042	7.563	10.08	12.60	15.13	17.65	20.17	22.69	5½x5½
"x6	2.750	5.500	8.250	11.00	13.75	16.50	19.25	22.00	24.75	"x6
"x6½	2.979	5.958	8.938	11.92	14.90	17.88	20.85	23.83	26.81	"x6½
"x7	3.208	6.417	9.625	12.83	16.04	19.25	22.46	25.67	28.88	"x7

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4.—Feet Board Measure—Engineers' Table.—Continued.

Dimension in Inches.	Length in Feet.								
	1	2	3	4	5	6	7	8	9
5½x7½	3.438	6.875	10.313	13.75	17.19	20.63	24.06	27.50	30 94
"x8	3.667	7.333	11.00	14.67	18.33	22.00	25.67	29.33	33.00
"x9	4.125	8.250	12.38	16.50	20.63	24.75	28.88	33.00	37.13
"x10	4.583	9.167	13.75	18.33	22.92	27.50	32.08	36.67	41.25
"x11	5.042	10.083	15.13	20.17	25.21	30.25	35.29	40.33	45.38
5½x12	5.500	11.00	16.50	22.00	27.50	33.00	38.50	44.00	49.50
"x14	6.417	12.83	19.25	25.67	32.08	38.50	44.92	51.33	57.75
"x15	6.875	13.75	20.63	27.50	34.38	41.25	48.13	55.00	61.88
"x16	7.333	14.67	22.00	29.33	36.67	44.00	51.33	58.67	66.00
"x18	8.250	16.50	24.75	33.00	41.25	49.50	57.75	66.00	74.25
6 x6	3.000	6.000	9.000	12.00	15.00	18.00	21.00	24 00	27.00
"x6½	3.250	6.500	9.750	13.00	16.25	19.50	22.75	26.00	29.25
"x7	3.500	7.000	10.500	14.00	17.50	21.00	24.50	28.00	31.50
"x7½	3.750	7.500	11.25	15.00	18.75	22.50	26.25	30.00	33.75
"x8	4.000	8.000	12.00	16.00	20.00	24.00	28.00	32.00	36.00
6 x9	4.500	9.000	13.50	18.00	22.50	27.00	31.50	36.00	40.50
"x10	5.000	10.000	15.00	20.00	25.00	30.00	35.00	40.00	45.00
"x11	5.500	11.00	16.50	22.00	27.50	33.00	38.50	44.00	49.50
"x12	6.000	12.00	18.00	24.00	30.00	36.00	42.00	48.00	54.00
"x14	7.000	14.00	21.00	28.00	35.00	42.00	49.00	56.00	63.00
6 x15	7.500	15.00	22.50	30.00	37.50	45.00	52.50	60.00	67.50
"x16	8.000	16.00	24.00	32.00	40.00	48.00	56.00	64.00	72.00
"x18	9.000	18.00	27.00	36.00	45.00	54.00	63.00	72.00	81.00
6½x6½	3.521	7.042	10.56	14.08	17.60	21.13	24.65	28.17	31.69
"x7	3.792	7.583	11.38	15.17	18.96	22.75	26.54	30.33	34.13
6½x7½	4.063	8.125	12.19	16.25	20.31	24.38	28.44	32.50	36.56
"x8	4.333	8.667	13.00	17.33	21.67	26.00	30.33	34.67	39.00
"x9	4.875	9.750	14.63	19.50	24.38	29.25	34.13	39.00	43.88
"x10	5.417	10.833	16.25	21.67	27.08	32.50	37.92	43.33	48.75
"x11	5.958	11.92	17.88	23.83	29.79	35.75	41.71	47.67	53.63
6½x12	6.500	13.00	19.50	26.00	32.50	39.00	45.50	52.00	58.50
"x14	7.583	15.17	22.75	30.33	37.92	45.50	53.08	60.67	68.25
"x15	8.125	16.25	24.38	32.50	40.63	48.75	56.88	65.00	73.13
"x16	8.667	17.33	26.00	34.67	43.33	52.00	60.67	69.33	78.00
"x18	9.750	19.50	29.25	39.00	48.75	58.50	68.25	78.00	87.75
7 x7	4.083	8.167	12.25	16.33	20.42	24.50	28.58	32.67	36.75
"x7½	4.375	8.750	13.13	17.50	21.88	26.25	30.63	35.00	39.38
"x8	4.667	9.333	14.00	18.67	23.33	28.00	32.67	37.33	42.00
"x9	5.250	10.500	15.75	21.00	26.25	31.50	36.75	42.00	47.25
"x10	5.833	11.67	17.50	23.33	29.17	35.00	40.83	46.67	52.50
7 x11	6.417	12.83	19.25	25.67	32.08	38.50	44.92	51.33	57.75
"x12	7.000	14.00	21.00	28.00	35.00	42.00	49.00	56.00	63.00
"x13	7.583	15.17	22.75	30.33	37.92	45.50	53.08	60.67	68.25
"x14	8.167	16.33	24.50	32.67	40.83	49.00	57.17	65.33	73.50
"x15	8.750	17.50	26.25	35.00	43.75	52.50	61.25	70.00	78.75
7 x16	9.333	18.67	28.00	37.33	46.67	56.00	65.33	74.67	84.00
"x18	10.500	21.00	31.50	42.00	52.50	63.00	73.50	84.00	94.50
7½x7½	4.688	9.375	14.06	18.75	23.44	28.13	32.81	37.50	42.19
"x8	5.000	10.000	15.00	20.00	25.00	30.00	35.00	40.00	45.00
"x8½	5.313	10.63	15.94	21.25	26.56	31.88	37.19	42.50	47.81
7½x9	5.625	11.25	16.88	22.50	28.13	33.75	39.38	45.00	50.63
"x10	6.250	12.50	18.75	25.00	31.25	37.50	43.75	50.00	56.25
"x11	6.875	13.75	20.63	27.50	34.38	41.25	48.13	55.00	61.88
"x12	7.500	15.00	22.50	30.00	37.50	45.00	52.50	60.00	67.50
"x13	8.125	16.25	24.38	32.50	40.63	48.75	56.88	65.00	73.13

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4.—FEET BOARD MEASURE—ENGINEERS' TABLE.—Continued.

Dimension in Inches.	Length in Feet.									Dimension in Inches.
	1	2	3	4	5	6	7	8	9	
7½x14	8.750	17.50	26.25	35.00	43.75	52.50	61.25	70.00	78.75	7½x14
"x15	9.375	18.75	28.13	37.50	46.88	56.25	65.63	75.00	84.38	"x15
"x16	10.000	20.00	30.00	40.00	50.00	60.00	70.00	80.00	90.00	"x16
"x18	11.25	22.50	33.75	45.00	56.25	67.50	78.75	90.00	101.25	"x18
8 x8	5.333	10.67	16.00	21.33	26.67	32.00	37.33	42.67	48.00	8 x8
8 x8½	5.667	11.33	17.00	22.67	28.33	34.00	39.67	45.33	51.00	8 x8½
"x9	6.000	12.00	18.00	24.00	30.00	36.00	42.00	48.00	54.00	"x9
"x10	6.667	13.33	20.00	26.67	33.33	40.00	46.67	53.33	60.00	"x10
"x11	7.333	14.67	22.00	29.33	36.67	44.00	51.33	58.67	66.00	"x11
"x12	8.000	16.00	24.00	32.00	40.00	48.00	56.00	64.00	72.00	"x12
8 x13	8.667	17.33	26.00	34.67	43.33	52.00	60.67	69.33	78.00	8 x13
"x14	9.333	18.67	28.00	37.33	46.67	56.00	65.33	74.67	84.00	"x14
"x15	10.000	20.00	30.00	40.00	50.00	60.00	70.00	80.00	90.00	"x15
"x16	10.67	21.33	32.00	42.67	53.33	64.00	74.67	85.33	96.00	"x16
"x18	12.00	24.00	36.00	48.00	60.00	72.00	84.00	96.00	108.00	"x18
8½x8½	6.021	12.04	18.06	24.08	30.10	36.13	42.15	48.17	54.19	8½x8½
"x9	6.375	12.75	19.13	25.50	31.88	38.25	44.63	51.00	57.38	"x9
"x9½	6.729	13.46	20.19	26.92	33.65	40.38	47.10	53.83	60.56	"x9½
"x10	7.083	14.17	21.25	28.33	35.42	42.50	49.58	56.67	63.75	"x10
"x11	7.792	15.58	23.38	31.17	38.96	46.75	54.54	62.34	70.13	"x11
8½x12	8.500	17.00	25.50	34.00	42.50	51.00	59.50	68.00	76.50	8½x12
"x13	9.208	18.42	27.63	36.83	46.04	55.25	64.46	73.67	82.88	"x13
"x14	9.917	19.83	29.75	39.67	49.58	59.50	69.42	79.33	89.25	"x14
"x15	10.63	21.25	31.88	42.50	53.13	63.75	74.38	85.00	95.63	"x15
"x16	11.33	22.67	34.00	45.33	56.67	68.00	79.33	90.67	102.00	"x16
8½x17	12.04	24.08	36.13	48.17	60.21	72.25	84.29	96.33	108.38	8½x17
"x18	12.75	25.50	38.25	51.00	63.75	76.50	89.25	102.00	114.75	"x18
9 x9	6.750	13.50	20.25	27.00	33.75	40.50	47.25	54.00	60.75	9 x9
"x9½	7.125	14.25	21.38	28.50	35.63	42.75	49.88	57.00	64.13	"x9½
"x10	7.500	15.00	22.50	30.00	37.50	45.00	52.50	60.00	67.50	"x10
9 x11	8.250	16.50	24.75	33.00	41.25	49.50	57.75	66.00	74.25	9 x11
"x12	9.000	18.00	27.00	36.00	45.00	54.00	63.00	72.00	81.00	"x12
"x13	9.750	19.50	29.25	39.00	48.75	58.50	68.25	78.00	87.75	"x13
"x14	10.50	21.00	31.50	42.00	52.50	63.00	73.50	84.00	94.50	"x14
"x15	11.25	22.50	33.75	45.00	56.50	67.50	78.75	90.00	101.25	"x15
9 x16	12.00	24.00	36.00	48.00	60.00	72.00	84.00	96.00	108.00	9 x16
"x17	12.75	25.50	38.25	51.00	63.75	76.50	89.25	102.00	114.75	"x17
"x18	13.50	27.00	40.50	54.00	67.50	81.00	94.50	108.00	121.50	"x18
"x20	15.00	30.00	45.00	60.00	75.00	90.00	105.00	120.00	135.00	"x20
9½x9½	7.521	15.04	22.56	30.08	37.60	45.13	52.65	60.17	67.69	9½x9½
9½x10	7.917	15.83	23.75	31.67	39.58	47.50	55.42	63.33	71.25	9½x10
"x11	8.708	17.42	26.13	34.83	43.54	52.25	60.96	69.67	78.38	"x11
"x12	9.500	19.00	28.50	38.00	47.50	57.00	66.50	76.00	85.50	"x12
"x13	10.29	20.58	30.88	41.17	51.46	61.75	72.04	82.34	92.63	"x13
"x14	11.08	22.17	33.25	44.33	55.42	66.50	77.58	88.67	99.75	"x14
9½x15	11.88	23.75	35.63	47.50	59.38	71.25	83.13	95.00	106.88	9½x15
"x16	12.67	25.33	38.00	50.67	63.33	76.00	88.67	101.33	114.00	"x16
"x17	13.46	26.92	40.38	53.83	67.29	80.75	94.21	107.67	121.13	"x17
"x18	14.25	28.50	42.75	57.00	71.25	85.50	99.75	114.00	128.25	"x18
"x20	15.83	31.67	47.50	63.33	79.17	95.00	110.83	126.67	142.50	"x20
10x10	8.333	16.67	25.00	33.33	41.67	50.00	58.33	66.67	75.00	10x10
"x11	9.167	18.33	27.50	36.67	45.83	55.00	64.17	73.33	82.50	"x11
"x12	10.00	20.00	30.00	40.00	50.00	60.00	70.00	80.00	90.00	"x12
"x13	10.83	21.67	32.50	43.33	54.17	65.00	75.83	86.67	97.50	"x13
"x14	11.67	23.33	35.00	46.67	58.33	70.00	81.67	93.33	105.00	"x14

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4.—Feet Board Measure—Engineers' Table.—Continued.

Dimension in Inches.	Length in Feet.									Dimension in Inches.
	1	2	3	4	5	6	7	8	9	
10x15	12.50	25.00	37.50	50.00	62.50	75.00	87.50	100.00	112.50	10x15
"x16	13.33	26.67	40.00	53.33	66.67	80.00	93.33	106.67	120.00	"x16
"x17	14.17	28.33	42.50	56.67	70.83	85.00	99.17	113.33	127.50	"x17
"x18	15.00	30.00	45.00	60.00	75.00	90.00	105.00	120.00	135.00	"x18
"x20	16.67	33.33	50.00	66.67	83.33	100.00	116.67	133.33	150.00	"x20
11x11	10.08	20.17	30.25	40.33	50.42	60.50	70.58	80.67	90.75	11x11
"x12	11.00	22.00	33.00	44.00	55.00	66.00	77.00	88.00	99.00	"x12
"x13	11.92	23.83	35.75	47.67	59.58	71.50	83.42	95.33	107.25	"x13
"x14	12.83	25.67	38.50	51.33	64.17	77.00	89.83	102.67	115.50	"x14
"x15	13.75	27.50	41.25	55.00	68.75	82.50	96.25	110.00	123.75	"x15
11x16	14.67	29.33	44.00	58.67	73.33	88.00	102.67	117.33	132.00	11x16
"x17	15.58	31.17	46.75	62.33	77.92	93.50	109.08	124.67	140.25	"x17
"x18	16.50	33.00	49.50	66.00	82.50	99.00	115.50	132.00	148.50	"x18
"x20	18.33	36.67	55.00	73.33	91.67	110.00	128.33	146.67	165.00	"x20
12x12	12.00	24.00	36.00	48.00	60.00	72.00	84.00	96.00	108.00	12x12
12x13	13.00	26.00	39.00	52.00	65.00	78.00	91.00	104.00	117.00	12x13
"x14	14.00	28.00	42.00	56.00	70.00	84.00	98.00	112.00	126.00	"x14
"x15	15.00	30.00	45.00	60.00	75.00	90.00	105.00	120.00	135.00	"x15
"x16	16.00	32.00	48.00	64.00	80.00	96.00	112.00	128.00	144.00	"x16
"x17	17.00	34.00	51.00	68.00	85.00	102.00	119.00	136.00	153.00	"x17
12x18	18.00	36.00	54.00	72.00	90.00	108.00	126.00	144.00	162.00	12x18
"x20	20.00	40.00	60.00	80.00	100.00	120.00	140.00	160.00	180.00	"x20
"x22	22.00	44.00	66.00	88.00	110.00	132.00	154.00	176.00	198.00	"x22
"x24	24.00	48.00	72.00	96.00	120.00	144.00	168.00	192.00	216.00	"x24
13x13	14.08	28.17	42.25	56.33	70.42	84.50	98.58	112.67	126.75	13x13
13x14	15.17	30.33	45.50	60.67	75.83	91.00	106.17	121.33	136.50	13x14
"x15	16.25	32.50	48.75	65.00	81.25	97.50	113.75	130.00	146.25	"x15
"x16	17.33	34.67	52.00	69.33	86.67	104.00	121.33	138.67	156.00	"x16
"x17	18.42	36.83	55.25	73.67	92.08	110.50	128.92	147.33	165.75	"x17
"x18	19.50	39.00	58.50	78.00	97.50	117.00	136.50	156.00	175.50	"x18
13x20	21.67	43.33	65.00	86.67	108.33	130.00	151.67	173.33	195.00	13x20
"x22	23.83	47.67	71.50	95.33	119.17	143.00	166.83	190.67	214.50	"x22
"x24	26.00	52.00	78.00	104.00	130.00	156.00	182.00	208.00	234.00	"x24
14x14	16.33	32.67	49.00	65.33	81.67	98.00	114.33	130.67	147.00	14x14
"x15	17.50	35.00	52.50	70.00	87.50	105.00	122.50	140.00	157.50	"x15
14x16	18.67	37.33	56.00	74.67	93.33	112.00	130.67	149.33	168.00	14x16
"x17	19.08	38.17	57.25	76.33	95.42	114.50	133.58	152.67	171.75	"x17
"x18	21.00	42.00	63.00	84.00	105.00	126.00	147.00	168.00	189.00	"x18
"x20	23.33	46.67	70.00	93.33	106.67	140.00	163.33	186.67	210.00	"x20
"x22	25.67	51.33	77.00	102.67	128.33	154.00	179.67	205.33	231.00	"x22
14x24	28.00	56.00	84.00	112.00	140.00	168.00	196.00	224.00	252.00	14x24
15x15	18.75	37.50	56.25	75.00	93.75	112.50	131.25	150.00	168.75	15x15
"x16	20.00	40.00	60.00	80.00	100.00	120.00	140.00	160.00	180.00	"x16
"x17	21.25	42.50	63.75	85.00	106.25	127.50	148.75	170.00	191.25	"x17
"x18	22.50	45.00	67.50	90.00	112.50	135.00	157.50	180.00	202.50	"x18
15x19	23.75	47.50	71.25	95.00	118.75	142.50	166.25	190.00	213.75	15x19
"x20	25.00	50.00	75.00	100.00	125.00	150.00	175.00	200.00	225.00	"x20
"x22	27.50	55.00	82.50	110.00	137.50	165.00	192.50	220.00	247.50	"x22
"x24	30.00	60.00	90.00	120.00	150.00	180.00	210.00	240.00	270.00	"x24
16x16	21.33	42.67	64.00	85.33	106.67	128.00	149.33	170.67	192.00	16x16
16x17	22.67	45.33	68.00	90.67	113.33	136.00	158.67	181.33	204.00	16x17
"x18	24.00	48.00	72.00	96.00	120.00	144.00	168.00	192.00	216.00	"x18
"x20	26.67	53.33	80.00	106.67	133.33	160.00	186.67	213.33	240.00	"x20
"x22	29.33	58.67	88.00	117.33	146.67	176.00	205.33	234.67	264.00	"x22
"x24	32.00	64.00	96.00	128.00	160 00	192.00	224.00	256.00	288.00	"x24

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4.—FEET BOARD MEASURE—ENGINEERS' TABLE.—Concluded.

Dimension in Inches.	Length in Feet.									Dimension in Inches.
	1	2	3	4	5	6	7	8	9	
17x17	24.08	48.17	72.25	96.33	120.42	144.50	168.58	192.67	216.75	17x17
"x18	25.50	51.00	76.50	102.00	127.50	153.00	178.50	204.00	229.50	"x18
"x19	26.92	53.83	80.75	107.67	134.58	161.50	188.42	215.33	242.25	"x19
"x20	28.33	56.67	85.00	113.33	141.67	170.00	198.33	226.67	255.00	"x20
"x22	31.17	62.33	93.50	124.67	155.83	187.00	218.17	249.33	280.50	"x22
17x24	34.00	68.00	102.00	136.00	170.00	204.00	238.00	272.00	306.00	17x24
18x18	27.00	54.00	81.00	108.00	135.00	162.00	189.00	216.00	243.00	18x18
"x19	28.50	57.00	85.50	114.00	142.50	171.00	199.50	228.00	256.50	"x19
"x20	30.00	60.00	90.00	120.00	150.00	180.00	210.00	240.00	270.00	"x20
"x22	33.00	66.00	99.00	132.00	165.00	198.00	231.00	264.00	297.00	"x22
18x24	36.00	72.00	108.00	144.00	180.00	216.00	252.00	288.00	324.00	18x24
20x20	33.33	66.67	100.00	133.33	166.67	200.00	233.33	266.67	300.00	20x20
"x22	36.67	73.33	110.00	146.67	183.33	220.00	256.67	293.33	330.00	"x22
"x24	40.00	80.00	120.00	160.00	200.00	240.00	280.00	320.00	360.00	"x24
22x22	40.33	80.67	121.00	161.33	201.67	242.00	282.33	322.67	363.00	22x22
22x24	44.00	88.00	132.00	176.00	220.00	264.00	308.00	352.00	396.00	22x24
24x24	48.00	96.00	144.00	192.00	240.00	288.00	336.00	384.00	432.00	24x24

Grading of Lumber.—Attention is here called to Forest Service Bulletin 71, U. S. Dept. of Agriculture, entitled "Rules and Specifications for the Grading of Lumber, adopted by the Various [18] Lumber Manufacturing Associations of the U. S." Compiled by E. R. Hodson, 1906.

The American Society for Testing Materials has undertaken the very difficult task of recommending Standard Specifications for the Grading of Structural Timber. The report of the Committee for 1906 will be found in Vol. VI of the proceedings, page 129, and is discussed under three headings: (1) Definitions of structural timber, (2) Standard defects, and (3) Standard names for structural timbers. Among the standard defects are the following: *Sound knot*—solid across its face, and as hard as the surrounding wood; *loose knot*—not firmly held in place by growth or position; *pith knot*—sound knot with pith hole not more than ¼ in. in dia in the center; *rotten knot*—not as hard as the surrounding wood; *pin knot*—sound knot not over ½ in. in dia.; *large knot*—sound knot more than 1½ ins. in dia.; *spike knot*—one sawn in a lengthwise direction; *pitch pockets*—openings in grain of wood containing more or less pitch or bark (*small* when not over ¼ in. wide, *standard* when not over ½ in. wide or 3 ins. long, *large* when over ½ in. wide or 3 ins. long); *wans*—bark or lack of wood on edge of timber; *shakes*—splits or checks between annular rings; *rot, dote, and red heart*—white or red rotten spots, dark discolorations not found in sound wood, etc.

Classification and Inspection of Yellow Pine Lumber.*—*General Rules.*—All lumber must be sound, commercial longleaf yellow pine (pine combining large coarse knots, with coarse grain, is excluded under these rules), well manufactured, full to size and saw butted, and shall be free from the following defects: Unsound, loose, and hollow knots, wormholes and knot holes, through shakes or round shakes that show on the surface; and shall be square edge unless otherwise specified. A through shake is hereby defined to be through or connected from side to side, or edge to edge, or edge to side. In the measurement of dressed lumber the width and thickness of the lumber before dressing must be taken—less than one inch thick shall be measured as one inch. The measurement of wane shall always apply to the lumber in the rough. Where terms one-half and two-thirds heart are used they shall be construed as referring to the area of the face on which measured.

* Interstate rules of 1905. Adopted by the Georgia-Florida Sawmill Association, Georgia Interstate Sawmill Association, South Carolina Lumber Association, New York Lumber Trade Association of New York City, Yellow Pine Exchange of New York City, The Lumbermen's Exchange of Philadelphia, The Lumbermen's Exchange of Baltimore.

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In the dressing of lumber, when not otherwise specified, one-eighth inch shall be taken off by each planer cut. All lumber grading higher than the grade for which it is sold shall be accepted as of the grade sold.

Classification.—Flooring shall embrace four, five and six quarter inches in thickness by three to six inches in width, excluding 1¼x6. For example: 1x3, 4, 5, and 6; 1¼x3, 4, 5, and 6; 1¼x3, 4, and 5. *Boards* shall embrace all thicknesses under 1½ ins. by over 6 ins. wide. For example: ¾, 1, 1¼, and 1½ ins. thick by over 6 ins. wide. *Plank* shall embrace all sizes from 1½ to under 6 ins. in thickness by over 6 ins. in width. For example: 1½, 2, 2½, 3, 3½, 4, 4½, 5, 5½, 5¾, by 6 and over in width. *Scantling* shall embrace all sizes exceeding 1½ ins. and under 6 ins in thickness, and from 2 to under 6 ins in width. For example: 2x2, 2x3, 2x4, 2x5, 3x3, 3x4, 3x5, 4x4, 4x5, and 5x5. *Dimension* sizes shall embrace all sizes 6 ins. and up in thickness by 6 ins. and up in width. For example: 6x6, 6x7, 7x7, 7x8, 8x9, and up. *Stepping* shall embrace 1 to 2½ ins. in thickness by 7 ins. and up in width. For example: 1, 1¼, 1½, 2, and 2½ x7, and up in width. *Rough Edge or Flitch* shall embrace all sizes 1 in. and up in thickness by 8 ins. and up in width, sawed on two sides only. For example: 1, 1¼, 2, 3, 4, and up thick by 8 ins. and up wide, sawed on two sides only.

Inspection.—Standard lumber shall be sound, sap no objection. Wane may be allowed ¼ of the width of the piece measured across face of wane, extending ¼ of the length on one corner, or its equivalent on two or more corners, provided that not over 10% of the pieces of any one size shall show such wane. *Merchantable* sizes under 9 ins. shall show some heart the entire length on one side; sizes 9 ins. and over shall show some heart the entire length on two opposite sides. Wane may be allowed ⅓ of the width of the piece measured across face or wane, and extending ⅓ of the length of the piece on one corner or its equivalent on two or more corners, provided that not over 10% of the pieces of any one size shall show such wane. *Prime* lumber.—Flooring shall show one heart face, free from through or round shakes or knots exceeding one inch in dia or more than four in a board on the face side. Boards 7 ins. and under wide shall show one heart face; over 7 ins. wide shall show ⅔ heart on both sides; all free from round or through shakes, large or unsound knots. Plank 7 ins. and under wide shall show one heart face; over 7 ins. wide shall show ⅔ heart on both sides; all free from round or through shakes, large or unsound knots. Scantling shall show three corners heart, free from through or round shakes or unsound knots. *Dimension sizes of Prime lumber.*—All square lumber shall show ⅔ heart on two sides and not less than ⅓ heart on two other sides. Other sizes shall show ⅔ heart on face and show heart ⅔ of length on edges, excepting when the width exceeds the thickness by 3 ins. or over; then it shall show heart on the edges for ⅓ the length. Stepping shall show 3 corners heart, free from shakes and all knots exceeding ½ in. in dia, and not more than 6 in a board. Rough Edge or Flitch shall be sawed from good heart timber, and shall be measured in the middle on the narrow face, free from injurious shakes or unsound knots. Wane on not over 5% of the pieces in any one size, shall be allowed as on merchantable quality.

Rules for Grading Fir, Spruce, Cedar, and Hemlock Lumber.*—*General Instructions.*—All lumber graded with special reference to its suitability for the use intended; therefore each piece is considered and its grade determined by its general character, including the sum of all its defects. "Yard Lumber" (dimension, common boards, finish, etc.) is graded from the face (best) side, except that when lumber which is dressed one side only is graded from the dressed side. Factory lumber (for doors, sashes, etc.) which must show on both sides, is always graded from the poorest side. Defects are taken in connection with the size of the piece, wider and longer pieces carrying more defects than smaller pieces in the same grade. Grade is determined at time of shipment and cannot be reconsidered after further working; a shipment of any grade to consist of a fair average of that grade. Material not conforming to standard sizes shall be governed by special contract. Standard lengths for all lumber are multiples of two feet, except that the standard lengths for flooring, ceiling, siding, rustic, and finish are

* Rail shipments. Digest of rules adopted March 30, 1906, by the Pacific Coast Lumber Manufacturers' Association, Southwestern Washington Lumber Manufacturers' Association, Oregon and Washington Lumber Manufacturers' Association.

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ot. Odd and fractional lengths shall be counted as next gth. All dressed lumber shall be measured and sold ough material used in its manufacture; and lumber one kness shall be counted as one inch thick.

gnized defects are knots (classed as sound or loose, ot holes, splits and checks (considered as to length and ark or lack of wood from any cause on edges of timber), rm holes, pitch seams (clearly defined openings between wood, are generally filled with granulated pitch, and a not admissible in any grade above No. 2 Common), nings between the grains of the wood containing more or rounded by sound grain wood), torn or chipped grain dressing against the grain and is more or less of a defect th or extent), discoloration (the result of various causes, onsidered a defect when the piece is damaged for the

-Defects in rough stock caused by improper manufac- l reduce the grade, unless they may be removed by dress- standard sizes. Imperfect manufacture in dressing n grain, broken knots, mismatching, insufficient tongue ng, ceiling, drop siding, etc., shall be considered defects grade accordingly as they are slight or serious in their the stock.

FIR.

d sorted with reference to the direction of the grain. ain) has the grain at or nearly at right angle with face dapted to flooring, stepping, etc., because it does not r. F. G. (flat grain) means that the grain is parallel or the face of the board; it is selected for finish because of orm.

1 Clear E. G., 3, 4 and 6 in.; free from all defects; angle n 45°. No. 2 Clear E. G., 3, 4 and 6 in.; angle of grain vill admit of slight roughness in dressing, and from 1 to pockets, or equivalent defects. No. 3 Clear E. G., 3, f grain not less than 45°; such defects admitted as will ty for cheap floors. No. 1 Clear F. G., 3, 4 and 6 in.; s, and all flat grain. No. 2 Clear F. G., 3, 4 and 6 in.; will admit of slight roughness in dressing, three close exceed 2 ins. in length, and 3 small tight smooth knots equivalent of combined defects. No. 3 Clear F. G., 3, lefects admitted as will not impair its utility for cheap g.

inch, Classed as No. 1 Clear, No. 2 Clear, No. 3 Clear; G. admissible.

6 inch. Graded same as Ceiling on the face side with t more than one grade lower.

Drop Siding, and Rustic.—Classed as No. 1 Clear (6 or 4, 6 or 8 in.), No. 3 Clear (4, 6 or 8 in.).

4 and 6 inch. Classed as No. 1 Clear, No. 2 Clear, No. 3

sed as No. 1 Clear (8 to 14 inch Clear), No. 2 Clear

ling.—Generally speaking, there should be no imper- ously impair the strength of the piece.

—4x4 and larger shall not be more than ¼ inch scant e evenly manufactured from sound stock, and must be at would materially weaken the piece. Timbers 10x10 2 inch wane on one corner, or its equivalent on two or fourth the length of the piece. Other sizes may have cts. Season checks and checks extending not over gth of the piece admissible.

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SPRUCE.

Names and Grades.—Flooring is classed as Clear, "A," and "B;" Finish, as First and Second Clear, Third Clear, Selects; Ceiling, as Clear, "A," and "B;" Partition, as Clear, "A," and "B;" Porch Decking, as Clear, "A," and "B;" Bevel Siding, as Clear, "A," "B," and "C;" Factory Stock, as Select and Better, No. 1 Shop, No. 2 Shop.

RED CEDAR.

Names and Grades.—Bevel Siding is classed as Clear, "A," and "B;" Ceiling, as No. 1, No. 2, No. 3; Finish, as No. 1, No. 2, No. 3; Corrugated Decking, as No. 2 and Better; Flooring, as No. 1, No. 2, No. 3.

HEMLOCK.

In a general way the rules for grading Fir and Spruce lumber are applied to Hemlock.

Shingles.†—*Perfections.*—18 inch, random widths, five butts must measure $2\frac{1}{16}$ inch plump in thickness when green, or $2\frac{1}{4}$ inches after drying. Must be well manufactured, strictly clear in every resect, and 90% vertical grain. Will not admit any shingle narrower than 3 inches.

Puget A.—Same thickness as Perfections. Must be well manufactured; will admit sound knots 8 inches from butt, 16-inch shims; also admits slash-grain shingles, otherwise must be clear. Will not admit shingles narrower than 2 inches.

Eureka.—18 inch, random widths, five butts must measure $2\frac{1}{16}$ inches in thickness when green, or 2 inches after drying. Must be well manufactured, strictly clear in every respect, and 90 per cent vertical grain. Will not admit any shingles narrower than 3 inches.

Skagit A.—Same thickness as Eureka. Must be well manufactured. Will admit sound knots 8 inches from butt; 16-inch shims. Also admit slash-grain shingles; otherwise must be clear. Will not admit shingles narrower than 2 inches.

Extra Clear.—16 inch, random widths, five butts must measure $2\frac{1}{16}$ inches in thickness when green, or 2 inches after drying. Must be well manufactured, strictly clear in every respect, and 90 per cent vertical grain. Will not admit any shingles narrower than 2 inches.

Choice A.—Same width and thickness as Extra Clear. Must be well manufactured. Will admit sound knots 6 inches from butt; also slash-grain shingles, wane edge, sap, 14-inch shims, $\frac{1}{2}$-inch knotholes or worm holes 6 inches from butt; otherwise must be clear.

*Extra *A**—16-inch ramdom widths, six butts must measure $2\frac{1}{16}$ inches green, or 2 inches after drying; must be well manufactured. Will admit sound knots 10 inches from butt; otherwise must be strictly clear, and 90 per cent. vertical grain. Will not admit any shingles narrower than 2 inches.

Standard A.—Same width and thickness as Extra *A*; must be well manufactured. Will admit sound knots 6 inches from butt, slash-grain shingles, wane edge, sap, 14-inch shims, $\frac{1}{2}$-inch knot holes or more holes 6 inches from butt; otherwise must be clear.

Shingles with the following defects are *culls*, and must not be put in any of the above grades: Rot, worm holes, except as above provided, check, shake, stub corners, tapering edges, rough, waney, or unevenly sawn.

Eighteen inch, 5 to $2\frac{1}{4}$ inch shingles, must be packed 20 courses per bunch, 5 bunches to the M. Eighteen inch, 5 to 2 inch, and 16-inch shingles, must be packed 25 courses to the bunch, 4 bunches to the M. All shingles must be packed in the regulation frame, full 20 inches in width, and no opening of more than $1\frac{1}{2}$ inches is admissible in any one course.

All shingles to be packed as closely as possible. Bands should not be shorter than $19\frac{1}{2}$ inches in length. Every bunch of shingles must be branded.

Dimension shingles are packed 24 courses in each bunch.

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† Oregon and Washington Lumber Manufacturers' Association.

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EXCERPTS AND REFERENCES.

Graphical Comparison of Various Log Rules (By A. H. Morse. Eng. News, April 9, 1908).—According to The Woodman's Handbook, by Prof. H. S. Graves, published as Bulletin 36 of the Bureau of Forestry, U. S. Dept. of Agric., Wash., D. C., more than 30 different "log rules" are in use in the U. S. All these rules profess to accomplish the same thing, viz., to provide means for ascertaining the number of board feet of 1-in. lumber which can be sawn from a log of given dia. and length; that is, to give the "scale" of a log in board feet. This article compares the principal log rules.

A Table of Lumber Weights (Eng. News, May 5, 1910).—The following table of weights of lumber represents the commercial classification, taken from the reports of the secretaries of the various lumber associations, and are accepted by the trade in place of actual weights.

Shipping Weights in lbs. per 1000 ft. B. M.

	Green from Saw.	Shipping Dry.	Well Seasoned.	Kiln Dried.		Green from Saw.	Shipping Dry.	Well Seasoned.	Kiln Dried.
Ash, black..	4600	....	3200	3000	Hickory....	6000	5000	4500	....
Ash, white..	4600	....	3800	3300	Mahogany..	4500	....	3500	....
Basswood ..	4200	2800	2500	2100	Maple, hard	5400	4150	3900	3400
Beech.....	5750	....	4000	....	Maple, soft.	5000	3650	3300	3000
Birch.....	5500	....	4000	....	Oak, red...	5500	4250	4000	3400
Buckeye....	....	2600	....	....	Oak, white.	5700	4500	4100	3600
Butternut..	4000	2800	2500	....	Pine, long lf.	4500	3500	....	....
Chestnut...	5000	3500	2800	2450	Pine, white.	3500	2500	2400	2200
Cherry.....	5000	4000	....	....	Poplar.....	3900	3000	2800	2400
Cottonwood.	4600	3100	2800	2400	Poplar Bay (Tap).....	4200	....	3000	....
Cypress	5000	3500	2900	....	Spruce, Adirondack..	3300	2700	2300	....
Elm, rock ..	5400	4300	4000	3500	Spruce, W. Va.....	3000	2700	2300	2200
Elm, soft...	4750	3300	3100	2900	Sycamore ..	4750	3200	3000	....
Gum	5400	3600	3300	3030	Walnut, Black	4900	4000	3800	....
Gum, sap...	5000	3300	3000	2750					
Hemlo'k, Pa.	4000	....	2500	....					
Hemlock, Nor.....	4500	3000	....	....					

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21.—METALLURGY.

Iron Ore.—Hematite (Fe_2O_3), an ore abundant around Lake Superior, in Alabama, etc., furnishes five-sixths of the iron manufactured in the United States; with limonite (one-ninth), and magnetite (one-sixteenth), next in order. Most of the ore has to be prepared for the blast-furnace. Thus, "sorting" the ore from foreign fragments; "washing" away the earthy matter; "concentrating" the crushed ore by magnetic "separators;" "calcinating" or driving off the volatile matter by heat; etc., are methods usually employed. Iron ore contains from 35 to 65% of iron, the balance being oxygen, phosphorus, sulphur, silica (sand), and other impurities.

Pig Iron is the cast iron product from a blast furnace. The furnace is shaped like a lamp chimney, and the charge is introduced at the top: layers of ore, limestone, fuel, ore, limestone, fuel*, etc., in regular sequence. By forcing a current of hot air at the bottom of the furnace into the heated mass a series of chemical reactions take place, producing molten iron, molten slag, and gases. The molten iron is drawn off after the slag and run into moulds or (cast) pigs.† From pig iron all kinds of iron and steel can be made. Among the uses to which pig-iron may be put are the following:

(*a*) Directly without refining, for foundry work.

(*b*) Refined by elimination of silica and phosphorus, then removal of carbon by dry-puddling, for wrought iron (not used in U. S.).

(*c*) Wet puddling, in removal of silica, phosphorus and carbon, for soft structural and blacksmith iron.

(*d*) Acid Bessemer, in removal of silica, manganese and carbon, for rails and structural steels (mild).

(*e*) Basic Bessemer, in removal of silica, manganese, carbon and phosphorus, for rails and structural steels (mild).

(*f*) Krupp washing, and Siemens-Martin process, for structural steels (mild).

(*g*) Charcoal hearths, for wrought iron; cementation furnace, for shear steel; crucible, for high grade tool steel.

Cast Iron.—In foundry work, for ordinary castings, the melt from the cupola is much more certain in composition than the direct casting from the unrefined pig. It gives a closer grain and somewhat increased strength. A good casting contains about 3.5 per cent. carbon, 1.5 per cent. silicon, not over 0.7 per cent. phosphorus, 0.6 per cent manganese, and a trace of sulphur. It shows a gray fracture, can be worked easily with chisel and file, completely fills the mold, chills with a smooth surface, has no blow holes, shrinks little in cooling. Large castings should contain less silicon than small; shrinkage may be decreased by lowering the manganese; to increase the strength the carbon, silicon and manganese are lowered and phosphorus eliminated as far as possible.

The above chemical properties may be varied to suit the special purposes for which the castings are intended. Mr. Henry Souther,‡ in Vol. V, Proc. A. S. T. M., page 218, says with reference to Hard Cast Iron which had given complaint in the machine shop: "Cast iron that chills may be called hard in the truest sense of the word. . . . In the last five or six years three separate complaints of hard iron have reached the writer and proved of so baffling a character that in each case visits were made to the machine shops. . . . It developed [citing one particular case] that small drills, ¼-in. or thereabout, were standing up with this iron just as well as any other, but the larger drills in the neighborhood of ½-in. and ¾-in. were dulling exactly as though the iron were charged with emery.

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* Charcoal, coke or anthracite coal.

† Pig iron contains about 93% of pure iron, 3 to 5% of carbon (pure coal), also some silicon, phosphorus, sulphur, etc.

‡ State Chemist, Hartford, Conn.

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The edges were being ground off and would last only a fraction of the time usual for the same drills in the same machine. Here was an unusual condition, thin iron working easily, thick iron on the same castings working with difficulty. The chemical results were normal, except manganese: Silicon 2.50, phosphorus .70, sulphur about .08, total carbon 3.50 and manganese .16. . . . Inasmuch as the only abnormal part in the analysis was shown in the manganese, that element was suspected, although there seemed to be no metallurgical reason for doing so. Means were taken to raise it to the neighborhood of .50, and as soon as this was done the difficulty disappeared in the machine shop and has not reappeared after some months. . . . This leads to the belief that there must be some carbide of iron or carbide of silicon that forms in the absence of a reasonable amount of manganese, and that does not form with manganese present."

Cast Steel.—*Steel for castings may be made by the open-hearth, crucible or Bessemer process. Castings to be annealed unless otherwise specified. Ordinary castings, those in which no physical requirements are specified, shall not contain over 0.40% carbon, nor over 0.08% phosphorus. Castings which are subjected to physical test shall not contain over 0.05% phosphorus, nor over 0.05% sulphur.

Tested castings shall be of three classes: Hard, Medium, and Soft, with minimum physical requirements as follows:

	Hard.	Medium.	Soft.
Tensile strength, lbs. per sq. in.....	85000.	70000.	60000.
Yield point, " " " "	38250.	31500.	27000.
Elongation, per cent in two ins.....	15.	18.	22.
Contraction of area, per cent.....	20.	25.	30.

For determining the above physical properties, standard turned specimens ½" in dia. and 2" gauged length shall be used.

For bending test, a specimen 1"x½" shall bend cold around a dia. of 1" without fracture on outside of bent portion, through an angle of 90° for "Medium" or 120° for "Soft" castings.

Malleable Castings.—An ordinary casting, unless it is too large, may be made malleable by decarburization. The original casting is preferably "white," indicating that is is low in carbon, and should practically be free from sulphur and phosphorus. The castings are packed in a pot (or retort) with oxide of iron, usually hematite, placed in a furnace, and kept at an orange or red heat for about a week, and then cooled very slowly. The carbon in the casting mostly unites with the oxygen of the ore, passing off as carbonic oxide. The casting is thus rendered malleable like crude wrought iron. Malleable castings are used for pipe specials which have to be threaded; for wood-stave (water) pipe shoes; and in general for delicate castings which are liable to be subjected to various kinds of stress.

Wrought Iron.—Wrought iron is purified pig iron. Although it is made "directly" from the ore the "indirect" process is generally used. The pig iron is oxidized in a puddling furnace whence the silicon, manganese, phosphorus and carbon impurities are removed. As with malleable cast iron the light grades, containing the least carbon and those practically free from sulphur and phosphorus, are selected. The wet-puddling process, in a reverboratory furnace, is the practice in the United States. A gray unrefined iron, containing more silica than the white, is used, mixed with a large quantity of hematite or magnetite of high grade used as reagents for oxidizing the impurities—the carbon passing off in the resulting carbon dioxide, and the other impurities, in the slag. The following shows the per cent. of composition of the pig and of the resulting muck-bar (puddle bar), in a typical case:

	Carbon.	Manganese.	Silicon.	Phosphorus.	Sulphur.
Pig (before melting).	2.55%	5.15%	3.20%	0.95%	0.05%
Muck-bar iron.....	0.65"	0.15"	0.10"	0.10"	0.01"

* Synopsis of Specifications adopted by letter-ballot of the A. S. T. M. on Sept. 1, 1905.

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The objection to this process is the enormous cost of fuel. The muck bars are re-heated and rolled into merchant bar—principally demanded by the blacksmith. (Soft or mild steels are generally replacing wrought iron.)

Mr. George Schuhmann, in the "Pilot" (official publication of the P. & R. Ry. Dept.), April, 1906, says:

"The purer the iron the higher is its melting point. Pig iron melts at about 2100 degrees F., steel at about 2500 degrees, and wrought iron at about 2800 degrees. The temperature in the puddling furnace is high enough to melt pig iron, but not high enough to keep wrought iron in a liquid state; therefore, as soon as the small particles of iron become purified they partly congeal (come to nature), forming a spongy mass in which small globules of iron are in a semi-plastic state, feebly cohering with fluid cinder filling the cavities between them. This sponge is divided by the puddler into lumps of about 200 lbs. each; these lumps or balls are taken to a steam hammer or a squeezer, where they are hammered or squeezed into elongated blocks (blooms), and while still hot, rolled out between the puddle rolls into bars 3 to 6 inches wide, about ¾-inch thick, 15 to 30 ft. long. These bars are called puddle bars or muck bars, and, owing to the large amount of cinder still contained therein, they have rather rough surfaces. The muck bars are cut up into pieces from 2 to 4 ft. long and piled on top of each other in so-called "piles" varying from 100 to 2000 lbs., according to the size product desired. These piles are heated in heating furnaces, and when white hot, are taken to the rolls to be welded together and rolled out into merchant iron in the shape of either sheets, plates, bars, or structural shapes as desired. When cold this material is sheared and straightened, and is then ready for the market.

After leaving the puddle furnace, wrought iron does not undergo any material change in its chemical composition, and the only physical change is an expulsion of a large portion of the cinder; the small cinder-coated globules of iron are welded together and the subsequent rolling back and forth will elongate these globules, giving the iron a fibrous structure, and reheating and rerolling will drive these fibers closer together, thus increasing the strength and ductility of the metal.

Steel.—For structural work, steel has almost entirely superseded wrought iron, being stronger and more cheaply manufactured. The latter, however, is still used largely for underground pipes, because better able to resist decomposition by the natural elements and also electrolysis. The following typical analyses of the products of the four principal processes of structural steel manufacture are here shown for approximate comparison, being in no way absolute:

	Acid Bessemer.		Basic Bessemer		Acid Open Hearth.		Basic Open Hearth.
	Soft S.	Rail S.	Soft S.	Med. H'd S.	Soft S.	Rail S.	
Combined carbon	.12%	.43%	.16%	.75%	.12%	.43%	.14%
Silicon	.04	.06	.06	.20	.....	.01	.....
Sulphur	.05	.06	.04	.04	.05	.06	.04
Phosphorus	.06	.07	.04	.04	.06	.07	.05
Manganese	.06	.50	.50	.70	.60	.80	1.20
Arsenic	.04	.05	.04	.04	.04	.04	.04

These processes consist in decarburizing and purifying the pig iron to a practically pure liquid wrought iron; recharging the liquid with carbon and manganese where necessary; and casting it into ingots. They will be described as briefly as possible.

Acid Bessemer Process.—The molten iron or pig from the blast furnace (direct process) or the cupola (indirect process) is poured into an "acid" lined "converter," in the bottom of which are a number of small holes

through which air is forced into the metal. The air oxidizes to impurities—carbon, silicon, manganese, etc.—the heat of combustion from the carbon and sulphur maintaining the metal in a fluid condition. The "acid" lining is a highly refractory, siliceous (about 93 per cent. silica) material, which removes the carbon, silicon and manganese, but is incapable of taking the phosphorus and sulphur away from the metal; hence the pig iron used must practically be free from these two latter elements, as they are not allowed to enter largely into steel. The impurities being thus removed it is now liquid wrought iron. Carbon, for strength and hardness, and manganese, for malleability in rolling, are added—the latter in the form of spiegeleisen or ferro-manganese. It is now steel and is poured into molds for cooling down into ingots. These ingots are charged into the soaking pit or vertical furnace to equalize before being rolled into blooms. Later, the blooms are (re-heated and) rolled into rails or into the various structural shapes desired.

Basic Bessemer Process.—Pig iron that is too high in phosphorus (2.5 to 3.0 per cent.) for the acid Bessemer process may be converted into steel by the basic Bessemer process provided it is low (below 0.50 per cent.) in silicon. The "basic" lining of the "converter" is made from a pure magnesian limestone (dolomite) containing about 20 per cent. of magnesia, but not over 1 or 2 per cent. of silica. The pig is usually remelted in the cupola and transferred to the converter—hard-burned limestone low in SiO_2 being added as a flux—where air is introduced as in the acid process. The impurities in the metal are burned out, the phosphorus furnishing the heat and uniting with the lime to form phosphate of lime. The liquid is now almost a chemically pure iron, and is recarburized as in the acid Bessemer process. The floating slag when cooled and ground is used as a fertilizer or for cement.

This process is not used much in the U. S.

Acid Open Hearth Process.—This process, like the acid Bessemer, calls for a pig low in sulphur and phosphorus because neither is eliminated. The Siemens furnace is a large hearth built of refractory siliceous material on which the metal is *melted*, by the combustion of gas admitted into the furnace alternately with air. No solid fuel is used. A typical pig iron would contain about the following: Carbon 3.5 per cent., silicon 2.2, sulphur 0.03, phosphorus 0.035, manganese 0.75.

This process is particularly adapted to producing steels high in carbon and to the exact percentages specified.

Basic Open Hearth Process.—In this process the lining of the hearth is made from the "basic" material—magnesian limestone—and like the basic Bessemer, a high phosphoric pig is used, say about as follows: Phosphorus, 1.80 per cent.; silicon, not over 1.50; sulphur, not over 0.50; manganese, 1.75; carbon, 3.50. Iron and steel scrap is also added to the melt. The method is about the same as for the acid O-H process, the impurities being separated from the bath partly by the air admitted and partly by the addition of iron oxides. The product is an excellent steel.

Cementation Process.—This process is still employed in making high grade carbon steels for tools, cutlery, etc. "Cement bars" are made from the purest and best grade iron (usually the Swedish) heated in contact with carbon (charcoal) to a red heat maintained for a week or more. The product is called "cement steel" or "blister steel." "Shear steel" is made from cement steel by cutting, piling, heating and rolling the cement bars—"double shear steel" being simply a duplication of the process. "Crucible steel" or "cast steel" is made from blister steel by cutting up and melting the bars in a black-lead pot, and cooling in the form of ingots.

Shear steel is used for plows and cheap edge tools, but not for high grade tools.

Crucible cast steel, made by melting the blister steel of the cementation process in crucibles, is a high grade steel, used for the best tools. In order to avoid the permanent set of internal stresses in castings, due to unequal shrinkage in cooling, they are **"annealed"** by being gradually

heated to temperatures between 800° (for high carbon steel) and 900° (for low carbon steels). Crucible steel is "tempered" for different uses, depending upon the percentage of carbon present, as follows:

Name of Temper.	Per cent. of Carbon.	Burning.	Welding.	Temper.	Remarks.
Razor ..	1.5	Easy.	Very difficult.	Extremely hard.	Razors, etc.
Saw-file	1.375	.....	.....	Hard.	Saws, etc.
Tool....	1.25	.....	Possible.	.....	Extreme care required in welding.
Spindle.	1.125	.....	.....	Medium.	Machine turning tools, cutters, etc.
Chisel ..	1.	.....	Quite easy.	.....	Cold chisels, hot sets, etc.
Set.....	0.875	.....	.....	Little.	Cold sets, etc.
Die.....	0.75	Not easy.	Easy.	Very little.	For hard surface tools as dies.

Steels very low in carbon can easily be welded but not tempered. Steel with carbon above ¾ per cent. can be tempered by heating to a high heat, and then quenching in water or other liquid.

Open Hearth Cast Steel.—The *acid* open hearth is generally preferred. The cast steel produced is adapted to machinery castings for steel vessels, cars, etc.; it contains from one-sixth to one-half of one per cent. carbon.

Harveyized steel plates for armor are mild steel plates, face-hardened on one side by carburization and sudden chilling; the other side may remain as mild steel or be decarburized into a softer metal.

Manganese steel is a steel containing about 14 per cent. manganese, and is made by melting ferro-manganese with carbon steel. It is very tough and hard, capable of being forged. It is used principally for car wheels, stamp heads and jaws for crushing machines; also for crossing frogs.

Vanadium steel is used for crusher jaws, gear wheels, etc. (See also page 399.)

Chrome steel is made by melting ferro-chrome with bar iron, in a crucible. It is very hard and can be forged readily. Contains about 1.8 per cent. chromium and 0.7 per cent. carbon.

Nickel steel is produced by adding nickel ore to the carbon steel in the open hearth bath, thus increasing the strength and elasticity. It is used in armor plate manufacture; and, with the probable future reduction in cost, will doubtless be used in steel bridges, especially of long spans, in the near future. (See, also, pages 398 and 399.)

Tungsten steel is very hard, difficult to forge, and is used for cutting-tools.

Alloys.—An alloy is a mixture of two or more metals by fusion. The number of possible mixtures, taking into consideration the varying proportions, is infinite, but the principal alloys useful to the engineer, are to a certain extent limited and well known. From a mechanical point of view the most useful metals for alloys are: copper (Cu), tin (Sn), zinc (Zn), lead (Pb), antimony (Sb), nickel (Ni), bismuth (Bi), aluminum (Al), iron (Fe), etc., about in the order mentioned. Cu is by far the most useful and may be classed as the primary metal; with Sn, Zn, Pb, and Sb, secondary; and Ni, Bi, Al, Fe, etc., tertiary, as follows:

Fe	*Sn*	Al
Pb	Cu	*Sb*
Ni	*Zn*	Bi

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Bronze (*Cu*, *Sn*, *X*) is the name generally given to those alloys consisting chiefly of copper and tin, with the former metal largely in excess. The following are some of the principal bronzes and their uses:

Name.	Composition (parts).			Remarks.
	Copper.	Tin.	Other metals.	
Medal B.....	95	5	.....	.....
Coin B.....	91	9	.....	.....
Gun B.....	90	10	.....	For navy ordnance.
Statuary B.....	89	11	.....	.....
Valve-metal.....	88	12	Zinc (2).	.....
Bell-metal.....	82	18	.....	.....
Speculum-metal...	68	32	.....	.....

Phosphor bronze is a bronze containing a small percentage (3±) of phosphorus properly introduced during fusion. It materially increases the strength and hardness of the metal and renders it less susceptible to oxidation and the action of the elements generally. Phosphor bronze is used for heavy journal bearings in the best machinery, and for wire.

Manganese bronze (*Cu*, *Sn*, *Mn*) is a bronze containing a small percentage (1+) of manganese, rendering the material tough and non-corrosive. It is largely used for propeller blades. A good mixture is *Cu* (88), *Sn* (10), *Mn* (2).

Aluminum bronze (*Cu*, *Al*), so-called, is usually composed of *Cu* (90) and *Al* (10) although these proportions may be varied greatly. It has a high tensile strength and is not easily corroded.

Silicon bronze (*Cu*, *Sn*, *Si*) is used for castings and for telegraph wires. It is an excellent conductor of electricity and has great tensile strength. The proportion of silicon is from 3 to 5 per cent. Tin may or may not be used with the copper.

Brass (*Cu*, *Zn*, *X*) includes the alloys which are composed mainly of copper and zinc. The principal ones are tabularized as follows:

Name.	Composition (parts).			Remarks.
	Copper.	Zinc.	Other metals.	
Valve-metal.....	83	15	Tin (2).	.....
Brazing-metal.....	75	25	.....	Flanges for copper pipe.
.....	70	30	.....	Works well under hammer.
Common B.....	67	33	.....	.....
Muntz-metal.....	60	40	.....	.....
Delta-metal.....	58	40	Iron.	.....
Tobin "Bronze" .	58	40	Tin, lead, iron.	.....
Thurston-metal...	56	42	Tin (2).	Thurston's maximum is 57, 42, 1.
Brazing-metal.....	50	50	.....	Solder for copper pipe flanges.
German silver.....	60	20	Nickel (20).	.....

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Tin-base Alloys.—

Name.	Composition (parts).				Remarks.
	Tin.	Zinc.	Lead.	Other Metals.	
Babbitt-metal..	89	.....	.....	{ *Sb*, 8 *Cu*, 3	Used for machinery bearings.
Pewter.....	85	.....	.....	15	Contains antimony, copper and bismuth.
Solder.....	67	33	.....	.....	.....
"	50	.....	50	.....	.....

Lead-base Alloys.—

Name.	Composition (parts).				Remarks.
	Lead.	Tin.	Antimony.	Bismuth.	
White-metal....	88	.....	12	.....	.....
Fusible plug....	86	10	.....	4	For steam boilers

Alzene, a new metal alloy composed of aluminum and zinc, is said to be as strong as cast iron, much more elastic, does not rust easily, and takes a very high polish. It is a German product. Future results of investigations are awaited with interest. The present proportions of the alloy are 2 parts aluminum and 1 part zinc. It is capable of filling out the most delicate lines and figures of forms in casting.

EXCERPTS AND REFERENCES.

Malleable Cast Iron (By H. E. Diller. Jl. of the Am. Foundrymen's Assn.; Eng. News, Dec. 11, 1902).—"Malleable" can be made up to 60 000 lbs. per sq. in., though this is not advisable as the shock-resisting qualities are sacrificed. Yet as specifications become more severe the general quality of this class of castings will be improved until we get a more reliable article, and which can better resist the encroachments of the steel casting. The article discusses the methods of manufacture.

Manufacture and Properties of Nickel-Steel (By A. L. Colby. Proc. A. S. T. M., 1903; Eng. News, Aug. 20, 1903.)—**Modulus of Elasticity.**—Young's modulus is practically the same for both tool steel containing 1.40% carbon and the mildest steel used in boilers. Their modulus of elasticity is in fact rarely found to be below 29,000,000 or above 31,000,000 and is generally taken at 29,500,000 or 30,000,000 in engineering calculations. The high nickel-steels, especially those containing 20% nickel or over, have a lower mod. of elas. than carbon-steel; but nickel-steels containing say 4% of nickel or less, such as are applicable for shafting, forgings, bridge construction, rails, etc., have the same mod. of elas. as carbon steels, viz., in the neighborhood of 29,000,000 lbs. per sq. in., many authorities claiming that this is true of even 5% nickel-steel. **Tensile Strength and Elastic Limit.**—Nickel-steel is chiefly distinguished from carbon-steel by its proportionately high elastic limit. If 3% nickel is alloyed with an open-hearth steel of 0.25% carbon, it produces a metal equal in tensile strength to a simple carbon-steel of 0.45% carbon, but having the advantageous ductility of the lower carbon-steel. On low carbon-steels not annealed, the addition of each 1% of nickel up to 5% causes approx., an increase of 5000 lbs. per sq. in. in the elastic limit, and 4000 lbs. in the ultimate or

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tensile strength. The influence of nickel on the elastic limit and ultimate strength increases with the percentage of carbon present; high carbon nickel-steels showing a greater gain than low carbon nickel steels. **Other Properties Discussed.**—Effect of compression; rigidity; cold and quench bending tests; hardness; resistance to torsion; resistance to wear or abrasion; expansion; effect of punching and shearing; segregation.

Notes on the Metallurgy of Steel (By Bradley Stoughton. Trans. A. S. C. E., Vol. LIV, Part E).—See pages 398 to 406 of Transactions for 112 references to this subject.

Vanadium Steel Alloys (By J. Kent Smith. Soc. of Chemical Industry, Liverpool; Eng. News, May 24, 1906).—The vanadium steel industry is altogether an English industry, 80% of the production being used for the construction of motor cars and omnibuses. The chrome-vanadium steels containing 10 to 20% vanadium show the most remarkable properties, the highest test yet obtained, after special treatment, being a maximum breaking strength of 103 tons per sq. in. The nickel-vanadium steels are also of great strength, but show a lower resistance to dynamic and torsional tests.

Manganese Bronze (By C, R. Spare. Proc. A. S. T. M., Vol. XIII, 1908).—Properties of, with discussions.

Vanadium Structural Steel (By G. L. Norris. Eng. Rec., June 4, 1910). —Table of results of tests of angles made from three different heats of chrome-vanadium steel. "The best chrome-vanadium steels for rolled shapes have from 0.18 to 0.25% carbon, 0.25 to 0.40% manganese, 0.4 to 0.6% chrome, and 0.15 to 0.20% vanadium."

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22.—BUILDING STONES AND CEMENTS.

(For Weights and Specific Gravities, See Section 27.)

This class of materials comprises all the non-metallic minerals of construction, including (I) Natural stones, (II) Cements, and (III) Artificial stones:

I. NATURAL BUILDING STONES.

These may be classified in three divisions, as follows:

A. Crystalline siliceous rocks, or those of the igneous types containing much silica, as granites and sienites.

B. Calcareous rocks, or those consisting mainly of lime, as limestones and marbles.

C. Fragmentary rocks, as sandstones and slates.

A. Crystalline Siliceous Rocks.

Granite.—Granite is an igneous rock of granular and acid composition and, contrary to former belief, is now supposed to be forming today, from fusion, deep in the earth's crust. It is composed mainly of orthoclase, quartz, mica and usually feldspar, the following chemical analysis being typical: Silica (70), alumina (15), iron oxides (4), sodium oxide (4), potassium oxide (4), lime (2), magnesia (1).

Depending upon the material presence in quantity, with quartz, of muscovite, hornblende or augite, we have respectively,

Muscovite granite. (Muscovite is a kind of mica.)

Hornblende granite,

Augite granite.

If the granite loses its quartz it grades into *Sienite.*

If potash feldspar is largely replaced by lime-soda feldspar it further grades into *Diorite.*

Granite is one of the most useful of building stones, but it is hard to work and cracks when subjected to great heat as evidenced by the Boston fire of 1871 on the Boston post office building, and by the recent Baltimore fire.

Basalt.—This term is applied to rocks of basaltic lava origin, comprising the so-called trap and the common greenstone. The basalts are very hard, heavy, and durable as building stone, the principal objection to their general use being the difficulty of working them,—quarrying and cutting. Basalt blocks are used principally as paving stone, resisting wear and tear to a remarkable degree.

Trap.—This includes a very wide range of extremely hard, crystalline rocks, and is sometimes applied outside the range of basalt. They are fine grained and usually dark-green in color. Broken trap rock makes an excellent base for concrete.

Greenstone.—When trap is altered by the presence of hornblende, chlorite, epidote, etc., it merges into greenstone, the green color being imparted by the hornblende.

B. Calcareous Rocks.

Limestone.—Pure limestone is carbonate of lime ($CaCO_3$), but there is sometimes present also carbonate of magnesia ($MgCo_3$) and certain so-called impurities, as alumina, silica, iron, etc. Carbonate of lime (CaO, CO_2) is a compound of lime (CaO) and carbonic acid (CO_2), the lime being an oxide of calcium,—an alkaline earth of specific gravity 3.18. Limestone and chalk are examples of carbonate of lime in the amorphous condition, while marble, aragonite and Iceland spar are varieties in the crystalline form.

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of lime is abundantly present in both the organic and , the great beds of limestone being thus provided with sources in their formation. The following are important:

Formed of the fossil remains of the crinoid species of as indicated by fragments of the coral stems. These Indiana, Iowa and Kansas.

posed of cemented fragments of shells and corals found he sea on the coast of Florida.

aceous rock composed mainly of sea shells of the order nd extensively on the coast of England and seems to l limestone; not used as a building stone.

mestone containing carbonate of lime ($CaCO_3$) and car- ($MgCO_3$) in large proportions. It may be either crys- ous (massive). The massive varieties containing iron *spar*. Dolomite is found in Vermont, Rhode Island, rsey, Missouri.

A limestone containing carbonate of lime ($CaCO_3$), sia ($MgCO_3$), silicon dioxide (SiO_2) and alumina, having dening under water after it has been burned.

ure of calcium carbonate (45%) and clay, with perhaps able as a building stone. Occurs in New Jersey, Ken- Virginia and the Carolinas.

limestone deposit formed at the mouths of springs and

e may be considered as a high grade limestone, crys- and capable of being polished. Those composed wholly e are white, while the various colorings in most of the the presence of foreign matter. Most of our marble is t, although it is obtained in many other states as Massa- , Tennessee, Maryland, Georgia, Pennsylvania, Arizona, .

ign marbles imported to this country are the following:

and gold, Carrara, Landscape, Nero antico.

ocatelle, Griotte.

antico, Parian, Pentellic, Rosso antico.

dian marble.

C. Fragmentary Rocks.

ndstone may be called a "sand conglomerate," formed As a building stone the cementing material is quite d, the whole mixture having been subjected to great Sandstones when thoroughly dried will absorb about weight of water; and in cold countries where buildings e action of the frost it is best to present the foliated the weather, in order to prevent flaking or frost chipping. quarried, usually lying *in situ* in layers of greater or uniform. A channelling machine is used in cutting out the thickness being determined by the strata.

e, obtained from Berea, Ohio, is a gritty stone well nd general masonry construction. It is a grayish stone.

one, quarried in New York (Medina) is largely used st. It is reddish and argillaceous.

York) sandstone is very hard and durable (sometimes a reddish yellow stone, found in New York, Virginia, igan.

lley sandstone is a Triassic brownstone, and a very im- one.

dstone is also Triassic.

ndstone composed mainly of quartz sand with siliceous ormation is the best. If metamorphosed into quartzite der and more durable.

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The best test of a sandstone is to expose it to the action of frost as in actual construction. This is the case with all the foregoing, which have been well tried and tested practically. An artificial freezing is sometimes applied to specimens from new quarries, which tests their power of resisting frost action. It consists in boiling the specimen 10 or 15 times in a strong solution of soda sulphate, exposing it to the action of the air for some hours after each boiling. The absorbed salt expands in crystallizing, similar to frost action, flaking the specimen to a greater or less extent.

Flagstones are thinly bedded sandstones, the cleavage being parallel with the beds (usually). Bluestone is a variety found in Pennsylvania, New York and New Jersey.

Slate.—Slate is formed from shale under great pressure and heat. The cleavage planes may or may not be identical with the original shale foliation, but usually crosses them at different angles, and even at right angle to the bedding as in that at Slatington, Penn.

Roofing slate is a true slate, being a hard, compact rock apparently not affected by the weather. It may be laid on boarding or on terra cotta roofing, the lower edge of each third layer overlapping the upper edge of the first by an inch or more. Japanned malleable iron nails are used to avoid rust. Vermont and Pennsylvania furnish most of the slate quarried in the United States. It is also distributed throughout the South, Northwest and California.

II. CEMENTS.

Materials with Cementing Properties may be mineral, vegetable or animal substances, either pure, mixed, or transmuted. The following substances have cementing qualities: *Mineral.*—Quartz, calcite and the iron ores; specifically, silica, lime, plaster of paris, sal ammoniac, sulphur, iron borings, brick dust, isinglass, clay, red lead, white lead, asphaltum, alum, copal, chalk, paraffin, gypsum, etc. *Vegetable.*—Gum, resin, wax and vegetable albumen; specifically, balsam, india rubber, rosin, starch, rice flour, wheat flour, mastic, etc. *Animal.*—Albumen, gelatin and glycerin; specifically, white of egg, lac, shellac, skim milk, cheese, beeswax, stearin, dextrin, etc. Gelatin is obtained by boiling animal substances as skins, hoofs, etc., in water.

The Solvents most common in the mixing of cements are water, alcohol, naphtha, vinegar, turpentine, linseed oil, benzine, petroleum, glycerin, ammonia, etc.

In the present discussion, cements are classed under Miscellaneous Cements, and (Builders) Cements.

MISCELLANEOUS CEMENTS.

Boiler cement.—To stop cracks and leaks in boilers and stoves. Dry powdered clay (6 parts), iron filings (1 part); mix to a paste with pure boiled linseed oil.

Coppersmith's cement.—To mend leaky joints in copper boilers. Bullocks blood thickened with quicklime; use immediately.

Fireproof cement.—To mend stone. Fine river sand (20), litharge (2), quicklime (1); mix to a thin paste with linseed oil.

Flour cement.—For general paste. To ½ pint water add 1 tablespoonful wheat flour slowly, stirring rapidly; heat until it boils, stirring. Adding a little powdered alum to the water strengthens the paste; a little brown sugar and corrosive sublimate will preserve it from turning mouldy.

Gas Fitters' cement.—Resin (4½), beeswax (1); melt, and stir in Venetian red (3); pour into iron- or oiled paper molds.

Iron cement.—For closing joints of iron pipes. Mix cast-iron borings or turnings (80) with sal-ammoniac (2), and flowers of sulphur (1). In using, add enough water to moisten; stir, and ram into joints. The sulphur is sometimes omitted.

Glue cement.—Glue (1), melted with water (least possible), and mixed with black resin (1) and red ochre (¼).

Steam-Pipe cement.—Grind good linseed oil varnish with equal weights of white lead, manganese oxide, and pipeclay.

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Keene's Marble cement.—For stucco work; will not stand weather. Baked gypsum or plaster of paris steeped in a saturated solution of alum and then recalcined and reduced to powder. To use, mix with water the same as plaster of paris.

For a complete list of cements and their preparation see The Scientific American Cyclopedia of Receipts, Notes and Queries; also Coolies' Cyclopedia of Practical Receipts.

(*BUILDERS*) *CEMENTS.*

Calcium (Ca) is a light-yellow metal whose specific gravity is 1.58, and which oxidizes at ordinary temperatures, hence it does not occur pure in nature, but is found abundantly as a component of calcite ($CaCO_3$), gypsum, dolomite, selenite, aragonite. Calcite or carbonate of lime is the principal constituent of limestone, marble and chalk. Calcium as a base plays a most important part in the limes, mortars and cements used in construction.

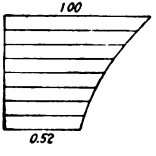
Lime (CaO), as its symbol implies, is an oxide of calcium, and may be obtained by placing calcium in contact with water, whence the latter is decomposed, forming lime and hydrogen gas (which latter escapes). Lime is a white, alkaline powder, of specific gravity 3.16. It may be obtained also by heating pure carbonate of lime or calcite ($CaCO_3 = CaO + CO_2$), whence the carbonic acid (Co_2) is driven off, leaving the lime. This is a pure lime and not the more or less impure, commercial article known to the engineer.

Common Lime is a more or less impure lime obtained by calcinating common limestone, composed mainly of $CaCO_3$, in kilns or furnaces, thereby driving off the carbonic acid and organic impurities. Its purity is dependent mainly upon the mineral purity of the carbonate which is burned. *Quicklime* or burned lime (CaO + impurities) is the name given to it as it comes from the kilns. In this state it is white and has a specific gravity of 3.16. *Slacked lime* is quicklime which has been slacked by exposure to the air, whence the term "air slacked" in contradistinction to "water slacked." In the former case it absorbs carbonic acid and moisture from the air. Water slacked or simply common slacked lime is a calcium hydroxide (CaH_2O_2). Its specific gravity is 2.1 or about two-thirds that of quicklime, showing that slacking increases its bulk about one-half. "Fat lime" is obtained from limestone containing 5 per cent or less of impurities, and when these latter amount to 6 per cent the lime is poor.

Common Lime Mortar.—This is composed of common lime mixed with the required amount of sand, and freshly slacked to a smooth paste. The proportions of the mixture of lime, water and sand vary according to the quality of the lime and the class of work for which it is intended. An average proportion for brickwork is, by bulk, 1 of lime, 2 to 3 of sand. It will not set under water and is used only where in contact with air it can set slowly by absorbing carbonic acid (and some moisture). Lime mortar may contain any proportion of sand or even no sand.

For a fat lime and a good quality of cement the adhesive properties of the resulting mortar, after being set, are about as follows for various mixtures, calling that of pure lime paste unity:

Lime.		Sand.	Cohesion.
1	:	0	1.00
1	:	0.5	.905
1	:	1	.82
1	:	1.5	.745
1	:	2	.68
1	:	2.5	.625
1	:	3	.58
1	:	3.5	.545
1	:	4	.52



Lime Plaster.—This consists approximately of 1 part quicklime, 2 parts sand, and to each 100 lbs. of the resulting mortar about ¾ bushel of cowhair or other fine fibre is sometimes added to give it coherent strength. It is applied on wooden or metal laths. Many patent plasters are manufactured in slabs at the factories and shipped ready to be put in place. They are often compounds of gypsum.

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Plaster of Paris.—When gypsum, a hydrated sulphate of lime ($CaSO_4 + 2H_2O$), is heated sufficiently, part of its water of crystallization is driven off, leaving the resulting composition ($CaSO_4)_2 + H_2O$, called plaster of paris. This is one of the simplest of the mineral cements, and in addition to its value in the arts it is used for cementing slabs of marble in building construction. This consists in simply mixing the plaster of paris to a creamy paste and applying it. Its setting consists in taking on water, again crystallizing into the hydrate, gypsum. Plaster of paris is a useful constituent in many of the delicate cements. Its strength may be increased by mixing with it a solution of thin glue, albumen (white of egg), or vegetable gum. Many architectural ornaments are made of plaster of paris mixed with about an equal amount of paper pulp and a solution of size.

Hydraulic Lime.—As common lime is made by burning common limestone, so is hydraulic lime obtained in a similar way from hydraulic limestone, which contains a large amount of silica and alumina. In burning, these latter combine with part of the lime, forming lime silicates and aluminates, the other portion of the lime remaining free. The silicates and aluminates possess the power of hardening under water. It is used in high-class masonry construction in Europe, being replaced by cements in the United States.

Hydraulic Cement.—These cements are capable of hardening under water—containing a large amount, 25 to 50 per cent., of silica and alumina. They may be classed as natural, Portland (semi-natural), and slag (artificial or puzzolanic) cements.

Natural Cement is obtained by simply burning the natural hydraulic limestone at a low temperature, and grinding the clinker very fine. It is manufactured in Ulster County, N. Y.; Louisville, Ky.; Cumberland, Md.; Utica, Ill., and Milwaukee, Wis. The limestone contains much clay, which supplies the required silica and alumina. The product as shipped is composed, approximately, of lime (42 parts), silica (28), alumina (10), iron oxide, magnesia and impurities (20). This cement is usually called Roman cement in Europe, and *Rosendale* cement in the U. S. It is cheaper than Portland cement, gains strength more slowly, and sets more quickly.

Portland Cement.—Good Portland cement contains the usual hydraulic cement ingredients about as follows, namely, lime (62), silica (23), alumina (8), and other impurities including iron oxide, magnesia, sulphuric acid (7). It is prepared by selecting such natural materials that when mixed, ground, and calcined, the product will be the required compound as above. Thus, the proper silicates and aluminates of lime are obtained from a mixture of argillaceous limestones of different chemical composition; from relatively pure limestone and clay or chalk and clay; and from marl and clay. The resulting clinker is then ground to a powder. Sand-ground cement, as its name indicates, is a mixture of sand and Portland cement ground together. It increases the bulk of the "cement" considerably without reducing its strength, provided the ratio of sand to cement is not greater than 1 to 1, and the grinding is done properly, and with a good quality of sharp, siliceous sand. Portland cement is stronger than Rosendale, sets more slowly, but acquires its strength more rapidly.

Slag Cement.—This is made by grinding furnace slag with lime, the slag containing the other necessary ingredients—silica and alumina (also a small percentage of impurities). It is not employed to any great extent in the United States.

Bitumen.—Bitumen is a mineral pitch comprising the various class of substances known as asphalt, maltha, petroleum, naphtha, natural gas, etc. Bitumen is decomposed vegetable matter comprising mainly carbon and hydrogen, but containing also oxygen, sulphur and nitrogen in small proportions. Bituminous substances are found associated with the carboniferous rocks in pre-volcanic regions. Thus we have bituminous coal, -limestone, -sandstone, -shale, etc.

Asphalt.—This is one of the principal bitumens and is supposed to have been formed by the hardening of its allied liquid substances, maltha (mineral tar) and pretroleum. Pitch Lake is a lake of asphalt on the island of Trinidad, and is controlled by the Trinidad Asphalt Co. of Philadelphia. Its product is of the finest quality, and supply apparently inexhaustible. Asphalt is a natural cement; its composition is practically unalterable

when exposed to the natural elements; and it is quite plastic and practically water-proof. Thus, it forms an excellent road, pavement and roofing material, and as a coating preservative for water pipes it is unexcelled. Asphalt *cement* is simply the refined asphalt tempered generally with the residue from petroleum oil. Asphalt *mastic* is prepared by mixing asphalt cement with sand and perhaps crushed limestone, or by mixing maltha, the poorer quality of bitumen, with natural rock asphalt. The mixture is heated, and cooled in molds ready for use. Asphalt *concrete* consists of broken stone or gravel with asphalt mastic used as a binder.

Manufacture of Portland Cement.

Raw Materials.—The materials used in the manufacture of Portland cement are carbonate of lime and clay: occurring either more or less naturally mixed, as in the argillaceous limestones; or separate in natural beds, as the chalk deposits of England and the various clay deposits common in many countries. In England, chalk is the principal form of carbonate of lime employed, and this is mechanically mixed with estuary mud. In Germany, the chief material is "mergel" (marl), a limestone rock of greater or less hardness, containing clay; also "weisenkalk," a pure soft marl composed mainly of carbonate of lime. In the United States the marls similar to those of Germany are found in Ohio, Indiana, Michigan and New York, and, mixed with clay, are largely used in the manufacture of Portland cement; but most of our cement is made from certain limestones containing sufficient clay and nearly free from magnesia, and found in certain localities, as Phillipsburg, N. J., and Lehigh Co., Penn.

The three distinct operations in the manufacture of Portland cement are (1) Pulverizing and mixing the raw materials, (2) Calcination or burning, (3) Grinding the clinker. These are discussed as follows:

Pulverizing and Mixing.—There are two processes in use for mixing the raw materials preparatory to calcination. These are known as the "wet" process and the "dry" process, each naturally adapted to the class of materials to be mixed.

Wet Process.—This process is used for such materials as chalk, soft marl and clay, which, by the admixture of a large quantity of water in a "wash mill," can be reduced to a homogeneous creamy condition of "slip" or "slurry." The most recent practice in the United States is to grind the mixture in a liquid state, run it into tanks where it is kept continually stirred, and thence to the rotary tubular furnaces where it is calcined.

Dry Process.—This process is particularly adapted to the treatment of argillaceous limestones or limestones containing sufficient clay so that very little mechanical mixing is necessary; notably the limestones of Phillipsburg, N. J., Lehigh Co., Penn., and of some localities in the West. Such material is run into a "grinding machine" where it is ground to a greater or less degree of fineness, depending upon the relative admixture of carbonate of lime and clay; i.e., if they occur intimately mixed in the same rock in the right proportion, very little grinding is required. The process becomes very expensive, however, when an almost pure limestone rock and a clayey shale form the ingredients to be mixed, in which case they must first be crushed to the size of small pebbles, then mixed and ground to an impalpable powder to produce an intimate blending.

Calcination.—After pulverizing and mixing, the material is conducted to the rotary tubular furnace, where it is burned at a temperature of about 1600° F. Such a furnace consists of a steel tube usually from 60 to 100 ft. long, and 6 to 10 ft. in diameter, and lined with fire brick. It is arranged to rotate on rollers and is placed on a grade of about 5%. The raw mixture to be calcined is introduced at the higher end of the tubular furnace and, as the latter revolves, the heated material forms into small balls which slowly gravitate to the lower end, fall into a conveyer, and are carried to the clinker storage-room. The degree of burning is one of the most important considerations in cement manufacture.

Grinding the Clinker.—After calcination, the resulting clinker passes to the "ball mills" where it is broken up into sand, with a small proportion of dust. It then goes to the "pulverizer," where it is reduced to the required fineness of powder—cement.

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Cement Testing.

Requisites of a Cement.—The principal requisites of a cement are that when combined with other materials of construction it shall possess the necessary resistance to disintegration, that is, (1) *strength* to resist safely the loads which may come upon it, and (2) *endurance* to withstand safely the continuous or repeated stresses to which it may be subjected, with the element "time" included. Certain factors affecting these qualities in a cement will now be considered.

Chemical Composition is one of the most important factors entering into both the strength and endurance of cement; hence, the raw materials should be selected in the proper proportions to give the required chemical mix in the finished product. The elements calcium, silicon, aluminum, carbon (and hydrogen) all unite with oxygen, forming oxides, the exact chemical composition being very complex, and varying greatly with different cements. Any foreign matter thus becomes an adulterant, tending to prevent "set" or hardening of the cement.

Set or Hardening of cement takes place when water is introduced into the anhydrous cement powder. It then becomes a hydrated compound in the form of an artificial stone. In Trans. A. S. C. E., Vol. LIV, Part F, pp. 48-52, Mr. R. L. Humphrey says: "This hydration or crystallization begins with the addition of water, and it is doubtful whether it ever ceases. . . . . Upon the addition of water, it begins immediately to hydrate in the from of fine needle-like crystals, and it is the intermeshing of these crystals that produces the hardening. They can be seen forming under a microscope immediately upon the addition of water to cement. The action is analogous to the formation of crystals from a saturated salt solution. If water is added to a cement that has commenced to set, and it is again mixed, the crystals already formed will be broken up, and the mass weakened by this retempering, Successive repetitions will gradually destroy all bond, and the cement having set or crystallized, the mass will become like so much inert sand." The phenomenon of hardening, then, "is purely a chemical one and is the result of the mechanical intermeshing of the crystals of silicate of lime, etc., which are formed by the hydration of the cement powder."

It will thus be seen, from the above, that cement, cement mortar, and concrete should be placed in the work immediately after mixing, and should not subsequently be disturbed.

Rate of Setting or time of setting in an important feature in construction work. It is necessary that the cement should not be so "quick-setting" as to prevent proper manipulation of the mix and placing it in the structure, nor so "slow-setting" as to require extra time to set, thereby retarding the work. The term "set" is here used in the sense of "initial set" or that point of crystallization where any disturbance or displacement of the material would effectually weaken it. Both "initial set" and "final set" or "hard set" are indefinite terms because crystallization or hardening is a continuous process which goes on indefinitely. Cements which do not set in two hours are considered slow-setting. On the other hand, 10 to 30 minutes is usually allowed any cement for manipulation and placing, before initial set should appear. The rate of setting of cement may be increased by the addition of lime, or plaster of paris or gypsum, but the resultant strength is thereby decreased.

Fineness is absolutely necessary to the thorough crystallization and consequent hardening, previously described. If the grains are coarse the crystals and their intermeshing will be imperfect, which will also be the case if the cement powder does not contain the proper mix of the raw materials, and the right degree or calcination. An ideal cement is one in which the clinker has been pulverized to a "flour" or impalpable powder, thoroughly anhydrous, and containing elements in such proportion that when water is added the hydrated compound will form into innumerable perfect crystals, with no free matter to obstruct their formation. Of course this condition can never be realized because (1) the grinding or pulverizing is never carried to such an extreme degree, and (2) all cements contain more or less free matter, as excess of lime, magnesia, iron, etc. These two classes of imperfections affect the strength of the cement, and the second class also affects its "soundness" and consequently its endurance.

Soundness is a negative term used to express the property which a cement has of *not* unduly expanding, contracting, checking or cracking during setting, hardening or crystallization. Such deformations are due to "active" impurities in the cement, such as free magnesia, lime, sulphur trioxide, etc., which produce internal stresses and thereby impair both its strength and durability. The class of impurities above mentioned should not be confused with certain "inactive" or inert impurities or adulterations which affect only the strength of the cement and not its endurance.

Inactive Adulterants are present to a greater or less extent in nearly all cement, the effect being simply to reduce its strength and commercial value. Other inert adulterants, as sand, in sand-cement mortar, and sand, broken stone, gravel, etc., in concrete, are added to or mixed with cement purely on the grounds of economy, in order to increase the bulk of the cementing material, allowing its strength to be impaired to a point consistent with necessary safety.

Method of Testing Cement.—The following is a digest of the standard method of testing cements, submitted as a progress report by a Committee of the Am. Soc. of Civil Engineers in 1903 and 1904, and adopted by the Am. Soc. for Testing Materials in 1904:

Selection of Sample.—Generally, one barrel in every ten to be sampled, the sample to be a fair average of contents of package; it shall be passed through a sieve having 20 meshes per lin. in. to remove lumps before testing. In obtaining sample from barrels or bags, an auger or a sampling iron should be used, reaching from side to center.

A chemical analysis, if required, may be made in accordance with the method outlined in the Journal of the Society of Chemical Industry, published Jan. 15, 1902. The determination of the principal constituents of cement—silica, alumina, iron oxide and lime—is not conclusive as an indication of quality.

Specific Gravity.—This is most conveniently made with Le Chatelier's apparatus, which consists of a flask (D), Fig. 1, of 120 cu. cm. (7.32 cu. ins.) capacity, the neck of which is about 20 cm. (7.87 ins.) long. In the middle of this neck is a bulb (C), above and below which are two marks (F) and (E). The volume between these marks is 20 cu. cm. (1.22 cu. ins.). The neck has a dia. of about 9 mm. (0.35 in.), and is graduated to tenths of cu. centimeters above the mark (F). Benzine (62° Baumé naphtha), or kerosene free from water, should be used in making the determination.

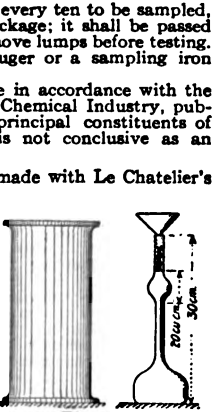

Fig. 1.

The specific gravity can be determined in two ways: *1st*, the flask is filled with either of these liquids to the lower mark (E), and 64 gr. (2.25 oz.) of powder, previously dried at 100° C. (212° F.) and cooled to the temp. of the liquid, is gradually introduced through the funnel (B) [the stem of which extends into the flask to the top of the bulb (C)], until the upper mark (F) is reached. The difference in weight between the cement remaining and the original quantity (64 gr.) is the weight which has displaced 20 cu. cm. *2nd*, the whole quantity of powder is introduced, and the level of the liquid rises to some division of the graduated neck. This reading plus 20 cu. cm. is the volume displaced by 64 gr. of powder.

The specific gravity is then obtained by the formula: Specific gravity = weight of cement ÷ displaced volume.

The flask, during the operation, is kept immersed in water in a jar (A), in order to avoid variations in the temperature of the liquid. Results from different trials should agree within 0.01. The apparatus is conveniently cleaned by inverting the flask over a glass jar, and shaking it vertically until the liquid starts to flow freely, repeating the operation several times.

More accurate determinations may be made with the picnometer.

Fineness.—The fineness is determined by measuring the residue retained on certain sieves, those known as No. 100 and No. 200 being recommended for this purpose. The sieves should be circular, about 20 cm. (7.87 ins.) in dia., 6 cm. (2.36 ins.) high, and provided with a pan 5 cm. (1.97 ins.) deep, and a cover.

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The wire cloth should be *woven* from brass wire having a dia. of 0.0045 in. for No. 100 sieve, and 0.0024 in. for No. 200 sieve. It should be mounted on the frames without distortion; the mesh should be regular in spacing and be within the limits: 96 to 100 meshes per lin. in. for No. 100, and 188 to 200 for No. 200. For the test, 50 to 100 grams (1.76 to 3.52 oz.) dried at a temperature of 212° F. prior to sieving, should be used.

The coarsely screened (and dried) sample is weighed and placed on the No. 200 sieve, which, with pan and cover attached is held in one hand in a slightly inclined position, and moved forward and backward, at the same time striking the side gently with the palm of the other hand, at the rate of about 200 strokes per min., and continued until not more than $\frac{1}{10}$ of 1% passes through after 1 minute of continuous sieving. The residue is weighed, then placed on the No. 100 sieve and the operation repeated. The results should be reported to the nearest $\frac{1}{10}$ of 1%. The work may be expedited by placing in the sieve a small quantity of large shot.

Normal Consistency.—The use of the proper percentage of water in making the pastes* from which pats, tests of setting and briquettes are made, is exceedingly important, affecting the results vitally. No method is entirely satisfactory, but the following is recommended:

Vicat Needle Test.

The apparatus used is known as the Vicat needle, which consists of a frame (K), Fig. 2, bearing a movable rod (L), with a cap (A) at one end, and at the other the cylinder (B), 1 cm. (0.39 in.) in dia.; the cap, rod and cylinder weighing 300 gr. (10.58 oz.). The rod, which can be held

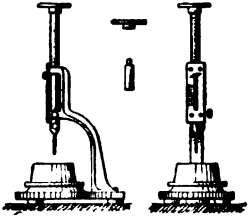

Fig. 2.

in any desired position by a screw (F), carries an indicator, which moves over a scale (graduated to centimeters) attached to the frame (K). The paste is held by a conical, hard-rubber ring (I), 7 cm. (2.76 ins.) in dia. at the base, 4 cm. (1.57 ins.) high, resting on a glass plate (J), about 10 cm. (3.94 ins.) square.

In making the determination, the same quantity of cement as will subsequently be used for each batch in making the briquettes (but not less than 500 grains) is kneaded into a paste, as described under "Mixing," and quickly formed into a ball with the hands, completing the operation by tossing it six times from one hand to the other, maintained 6 ins. apart; the ball is then pressed into the rubber ring through the larger opening, smoothed off, and placed (on its large end) on a glass plate and the smaller end smoothed off with a trowel; the paste, confined in the ring, resting on the plate, is placed under the rod bearing the cylinder, which is brought in contact with the surface and quickly released.

The paste is of normal consistency when the cylinder penetrates to a point in the mass 10 mm. (1.39 in.) below the top of the ring. Great care must be taken to fill the ring exactly to the top.

The trial pastes are made with varying percentages of water until the correct consistency is obtained.

The Committee has recommended, as normal, a paste the consistency of which is rather wet, because it believes that variations in the amount of compression to which the briquette is subjected in moulding are likely to be less with such a paste.

* The term "paste" is here used to designate a mixture of cement and water, and the word "mortar" a mixture of cement, sand and water.

Having determined in this manner the proper percentage of water required to produce a paste of normal consistency, the proper percentage for the mortars may be obtained from an empirical formula, which the Committee hopes to devise; but temporarily the following Table* may be used:

PERCENTAGE OF WATER FOR STANDARD SAND MORTARS.

Neat.	One Cement, Three Standard Ottawa Sand.	Neat.	One Cement, Three Standard Ottawa Sand.	Neat.	One Cement, Three Standard Ottawa Sand.
15	8.0	23	9.3	31	10.7
16	8.2	24	9.5	32	10.8
17	8.3	25	9.7	33	11.0
18	8.5	26	9.8	34	11.2
19	8.7	27	10.0	35	11.5
20	8.8	28	10.2	36	11.5
21	9.0	29	10.3	37	11.7
22	9.2	30	10.5	38	11.8

	1 to 1	1 to 2	1 to 3	1 to 4	1 to 5
Cement	500	333	250	200	167
Sand	500	666	750	800	833

Time of Setting.—The object of this test is to determine the time which has elapsed from the moment water is added until the paste ceases to be fluid and plastic (called the "initial set"), and also the time required for it to acquire a certain degree of hardness (called the "final" or "hard set"). The former of these is the most important, since, with the commencement of setting, the process of crystallization or hardening is said to begin. As a disturbance of this process may produce a loss of strength, it is desirable to complete the operation of mixing and moulding, or incorporating the mortar into the work, before the cement begins to set.

It is usual to measure arbitrarily the beginning and end of the setting by the penetration of weighted wires of given diameters. For this purpose the Vicat needle, already described, should be used. In making the test, a paste of normal consistency is moulded and placed under the rod (L), Fig. 2, as described under "Normal Consistency;" this rod, bearing the cap (D) at one end and the needle (H), 1 mm. (0.039 in.) in dia., at the other, weighing 300 gr. (10.58 oz.). The needle is then carefully brought in contact with the surface of the paste and quickly released. The setting is said to have commenced when the needle ceases to pass a point 5 mm. (0.20 in.) above the upper surface of the glass plate, and is said to have terminated the moment the needle does not sink visibly into the mass.

The test pieces should be stored in moist air during the test; this is accomplished by placing them on a rack over water contained in a pan and covered with a damp cloth, the cloth to be kept away from them by means of a wire screen; or they may be stored in a moist box or closet. Care should be taken to keep the needle clean, as the collection of cement on the sides of the needle retards the penetration, while cement on the point reduces the area and tends to increase the penetration.

The determination of the time of setting is only approximate, being materially affected by the temperature of the mixing water, the temperature and humidity of the air during the test, the percentage of water used, and the amount of moulding the paste receives.

Standard Sand.—The Committee recognizes the grave objections to the standard quartz now generally used, especially on account of its high percentage of voids, the difficulty of compacting in the moulds, and its lack of uniformity. It recommends, for the present, the natural sand from Ottawa, Ill., screened to pass a sieve having 20 meshes per lin. in., and re-

* Prepared by the Committee on Standard Specifications for Cements as a temporary expedient.

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tained on a sieve having 30 meshes per lin. in.; the wires to have diameters of 0.0165 and 0.0112 in., respectively, i.e., half the width of the opening in each case. Sand having passed the No. 20 sieve shall be considered standard when not more than 1% passes a No. 30 sieve after one minute continuous sifting of a 500-gram sample. The Sandusky Portland Cement Co., of Sandusky, O., has agreed to undertake the preparation of this sand, and to furnish it at a price only sufficient to cover the actual cost of preparation.

Form of Briquette.—While the form of the briquette recommended by a former Committee of the Society is not wholly satisfactory, this Committee is not prepared to suggest any change, other than rounding off the corners by curves of ½-in. radius, Fig. 3.

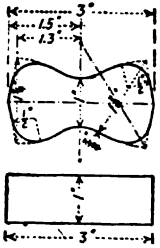

Fig. 3.

Moulds.—The moulds should be made of brass, bronze, or some equally non-corrodible material, having sufficient metal in the sides to prevent spreading during moulding. Gang moulds, as shown in Fig. 4, are preferred to single moulds, since the greater quantity of mortar that can be mixed for simultaneous moulding tends to produce greater uniformity in the results. They should be wiped out with an oily cloth before using.

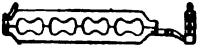

Fig. 4.

Mixing.—All proportions should be stated by weight; the quantity of water to be used should be stated as a percentage of the dry material. The metric system is recommended because of the convenient relation of the gram and the cubic centimeter. The temperature of the room and the mixing water should be as near 70° F. as it is practicable to maintain it.

The sand and cement should be thoroughly mixed dry, and on some non-absorbing surface, preferably plate glass. If an absorbing surface is used it should previously be dampened. The quantity of material to be mixed at one time depends on the number of test pieces to be made; about 1000 gr. (35.28 oz.) makes a convenient quantity to be mixed, especially by hand methods. The material is weighed and placed on the mixing table, and a crater formed in the center, into which the proper percentage of clean water is poured; the material on the outer edge is turned into the crater by the aid of a trowel. As soon as the water has been absorbed, which should not require more than one minute, the operation is completed by vigorously kneading with the hands for an additional 1½ minutes, the process being similar to that used in kneading dough. A sand-glass affords a convenient guide for the time of kneading. During the operation of mixing, the hands should be protected by gloves, preferably of rubber.

Moulding.—Having worked the paste or mortar to the proper consistency it is at once placed in the moulds by hand, being pressed in firmly with the fingers and smoothed off with a trowel without ramming. It should be heaped up on the upper surface of the mould, and, in smoothing off, the trowel should be drawn over the mould in such a manner as to exert a moderate pressure on the excess material. The mould should be turned over and the operation repeated. A check upon the uniformity of the mixing and moulding is afforded by weighing the briquettes just prior to immersion, or upon removal from the moist closet. Those varying in weight more than 3% from the average should be rejected.

Storage of the Test Pieces.—During the first 24 hours after moulding, the test pieces should be kept in moist air to prevent them from drying out. A moist closet or chamber is so easily devised that the use of the damp cloth should be abandoned if possible. Covering the test pieces with a damp cloth is objectionable, as commonly used, because the cloth may dry out unequally, and, in consequence, the test pieces are not all maintained under the same conditions. Where a moist closet is not available, a cloth may be used and kept uniformly wet by immersing the ends in water, and being kept from direct contact with the test pieces by means of a wire screen or some similar arrangement.

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A moist closet consists of a soapstone or slate box, or a metal lined wooden box—the metal lining being covered with felt and this felt kept wet. The bottom of the box is so constructed as to hold water, and the sides are provided with cleats for holding glass shelves on which to place the briquettes. Care should be taken to keep the air in the closet uniformly moist.

After 24 hours in moist air, the test pieces for longer periods of time should be immersed in water maintained as near 70° F. as practicable; they may be stored in tanks or pans, which should be of non-corrodible material.

Tensile Strength.—The tests may be made on any standard machine. A solid metal clip, as shown in Fig. 5, is recommended. This clip is to be used without cushioning at the points of contact with the test specimen. The bearing at each point of contact should be ¼ in. wide, and the distance between the centers of contact on the same clip should be 1¼ ins.

Fig. 5.

Test pieces should be broken as soon as they are removed from the water. Care should be observed in centering the briquettes in the testing machine, as cross-stains, produced by improper centering, tend to lower the breaking strength. The load should not be applied too suddenly, as it may produce vibration, the shock from which often breaks the briquette before the ultimate strength is reached. Care must be taken that the clips and the sides of the briquette be clean and free from grains of sand or dirt, which would prevent a good bearing. The load should be applied at the rate of 600 lbs. per minute. The average of the briquettes of each sample tested should be taken as the test, excluding any results which are manifestly faulty.

Constancy of Volume.—The object is to develop those qualities which tend to destroy the strength and durability of a cement. As it is highly essential to determine such qualities at once, tests of this character are for the most part made in a very short time, and are known, therefore, as *accelerated tests.* Failure is revealed by cracking, checking, swelling or disintegration, or all of these phenomena. A cement which remains perfectly sound is said to be of constant volume.

Tests for constancy of volume are divided into two classes: (1) normal tests, or those made in either air or water maintained at about 70° F., and (2) accelerated tests, or those made in air, steam or water at a temperature of 115° F. and upward. The test pieces should be allowed to remain 24 hours in moist air before immersion in water or steam, or preservation in air.

For these tests, pats, about 7½ cm. (2.95 ins.) in dia., 1½ cm. (0.49 in.) thick at the center, and tapering to a thin edge, should be made, upon a clean glass plate [about 10 cm. (3.94 ins.) square], from cement paste of normal consistency.

Normal Test.—A pat is immersed in water maintained as near 70° F. as possible for 28 days, and observed at intervals. A similar pat is maintained in air at ordinary temperature and observed at intervals.

Accelerated Test.—A pat is exposed in any convenient way in an atmosphere of steam, above boiling water, in a loosely closed vessel, for 3 hours.

To pass these tests satisfactorily, the pats should remain firm and hard, and show no signs of cracking, distortion or disintegration. Should the pat leave the plate, distortion may be detected best with a straight-edge applied to the surface which was in contact with the plate. In the present state of our knowledge it cannot be said that cement should necessarily be condemned simply for failure to pass the accelerated tests; nor can a cement be considered entirely satisfactory simply because it has passed these tests.

Specifications for Cement (A. S. T. M.).

The following specifications were adopted by the American Society for Testing Materials, Nov. 14, 1904:

General Conditions.—(1) All cement shall be inspected. (2) Cement may be inspected either at the place of manufacture or on the work. (3) In order to allow ample time for inspecting and testing, the cement should be stored in a suitable weather-tight building having the floor properly blocked or raised from the ground. (4) The cement shall be stored in such

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a manner as to permit easy access for proper inspection and identification of each shipment. (5) Every facility shall be provided by the contractor and a period of at least twelve days allowed for the inspection and necessary tests. (6) Cement shall be delivered in suitable packages with the brand and name of manufacture plainly marked thereon (7) A bag of cement shall contain 94 pounds of cement net. Each barrel of Portland cement shall contain 4 bags, and each barrel of natural cement shall contain 3 bags of the above net weight. (8) Cement failing to meet the 7-day requirements may be held awaiting the results of the 28-day tests before rejection. (9) All tests shall be made in accordance with the methods proposed by the Committee on Uniform Tests of Cement of the American Society of Civil Engineers, presented to the Society January 21, 1903, and amended January 20, 1904, with all subsequent amendments thereto. [See "Method of Testing Cement," page 407.]

(10) The acceptance or rejection shall be based on the following requirements:

(11) **Natural Cement.**—Definition.—This term shall be applied to the finely pulverized product resulting from the calcination of an argillaceous limestone at a temperature only sufficient to drive off the carbonic acid gas.

(12) *Specific Gravity.*—The specific gravity of the cement thoroughly dried at 100° C., shall not be less than 2.8.

(13) *Fineness.*—It shall leave by weight a residue of not more than 10% on the No. 100, and 30% on the No. 200 sieve.

(14) *Time of Setting.*—It shall develop initial set in not less than 10 minutes, and hard set in not less than 30 minutes nor more than 3 hours.

(15) *Tensile Strength.*—The minimum requirements for tensile strength for briquettes one inch in cross section shall be within the following limits, and shall show no retrogression in strength within the periods specified:*

Age.	Neat Cement.	Strength.
24 hours in moist air		50-100 lbs.
7 days (1 day in moist air, 6 days in water)		100-200 "
28 days (1 day in moist air, 27 days in water)		200-300 "
	One Part Cement, Three Parts Standard Sand.	
7 days (1 day in moist air, 6 days in water)		25- 75 "
28 days (1 day in moist air, 27 days in water)		75-150 "

(16) *Constancy of Volume.*—Pats of the neat cement about 3 ins. in diameter, ½-in. thick at center, tapering to a thin edge, shall be kept in moist air for a period of 24 hours. (*a*) A pat is then kept in air at normal temperature. (*b*) Another is kept in water maintained as near 70° F. as practicable.

(17) These pats are observed at intervals for at least 28 days, and, to satisfactorily pass the test, should remain firm and hard and show no signs of distortion, checking, cracking or disintegrating.

(18) **Portland Cement.**—Definition.—This term is applied to the finely pulverized product resulting from the calcination to incipient fusion of an intimate mixture of properly proportioned argillaceous and calcareous materials, and to which no addition greater than 3% has been made subsequent to calcination.

(19) *Specific Gravity.*—The specific gravity of the cement, thoroughly dried at 100° C., shall not be less than 3.10.

(20) *Fineness.*—It shall leave by weight a residue of not more than 8% on the No. 100, and not more than 25% on the No. 200 sieve.

(21) *Time of Setting.*—It shall develop initial set in not less than 30 minutes, but must develop hard set in not less than one hour nor more than ten hours.

* For example, the minimum requirement for the 24-hour neat cement test should be some specified value within the limits of 50 and 100 lbs., and so on for each period stated. [The consumer, when ordering, may fix the min. value.]

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(22) *Tensile Strength.*—The minimum requirements for tensile strength for briquettes one inch square in section shall be within the following limits, and shall show no retrogression in strength within the periods specified:*

Age.	Neat Cement.	Strength.
24 hours in moist air		150-200 lbs.
7 days (1 day in moist air, 6 days in water)		450-550 "
28 days (1 day in moist air, 27 days in water)		550-650 "
	One Part Cement, Three Parts Sand.	
7 days (1 day in moist air, 6 days in water)		150-200 "
28 days (1 day in moist air, 27 days in water)		200-300 "

(23) *Constancy of Volume.*—Pats of neat cement about 3 ins. in diameter, ½-in. thick at the center, and tapering to a thin edge, shall be kept in moist air for a period of 24 hours. (*a*) A pat is then kept in air at normal temperature and observed at intervals for at least 28 days. (*b*) Another pat is kept in water maintained as near 70° F. as practicable, and observed at intervals of at least 28 days. (*c*) A third pat is exposed in any convenient way in an atmosphere of steam, above boiling water, in a loosely closed vessel for five hours.

(24) These pats, to satisfactorily pass the requirements, shall remain firm and hard and show no signs of distortion, checking, cracking or disintegrating.

(25) *Sulphuric Acid and Magnesia.*—The cement shall not contain more than 1.75% of anhydrous sulphuric acid (SO_3), nor more than 4% of magnesia (MgO).

Specifications for Cement (Engrs. U. S. A.).

The following specifications† are from Professional Papers, No. 28, Corps of Engineers, U. S. A.

(*a*) **American Portland Cement.**—Shall be dry and free from lumps; the calcined product to contain at least 1.7 times as much lime, by weight, as of the materials which give the lime its hydraulic properties; and to be finely pulverized after said calcination, with subsequent additions or substitutions for regulating certain properties of technical importance not to exceed 2% of the calcined product.

The cement to be put up in strong, sound barrels, well lined with paper, or in stout cloth or canvas sacks; each package to be plainly labeled with name of brand and of manufacturer. Bidders will state the brand which they propose to furnish. The average weight per barrel shall not be less than 375 lbs. net. Four sacks shall contain one barrel of cement.

Tests may be made of the fineness, specific gravity, soundness, time of setting, and tensile strength of the cement.

(7) *Fineness.*—Ninety-two per cent of the cement must pass through a sieve made of No. 40 wire, Stubb's gauge, having 10,000 openings per square inch.

(8) *Specific Gravity.*—The specific gravity of the cement, as determined from a sample which has been carefully dried, shall be between 3.10 and 3.25.

(9) *Soundness.*—To test the soundness of the cement, at least two pats of neat cement, as taken from the package, mixed for five minutes with about 20 per cent of water by weight shall be made on glass, each pat about 3 inches in diameter and one-half inch thick at the center, tapering thence to a thin edge. The pats are to be kept under a wet cloth until finally set, when one is to be placed in fresh water for twenty-eight days. The second pat will be placed in water which will be raised to the boiling point for six

* For example, the minimum requirement for the 24-hour neat cement test should be some specified value within the limits of 150 and 200 lbs., and so on for each period stated. [The consumer, when ordering, may fix the minimum values.]

†A digest from the Report of Majors W. L. Marshall and S. S. Leach, and Capt. Spencer Crosby, Board of Engineer Officers, on testing Hydraulic Cements, with specifications for the several classes; Second Edition, with Corrections, 1902.

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hours, then allowed to cool. Neither should show distortion or cracks. The boiling test may or may not reject at the option of the engineer officer in charge.

(10) *Time of Setting.*—The cement shall not acquire its initial set in less than forty-five minutes and must have acquired its final set in ten hours.

(The following paragraph will be substituted for the above in case a quick-setting cement is desired:

The cement shall not acquire its initial set in less than twenty nor more than thirty minutes, and must have acquired its final set in not less than forty-five minutes nor in more than two and one-half hours.)

The pats made to test the soundness may be used in determining the time of setting. The cement is considered to have acquired its initial set when the pat will bear, without being appreciably indented, a wire one-twelfth inch in diameter loaded to weigh one-fourth pound. The final set has been acquired when the pat will bear, without being appreciably indented, a wire one twenty-fourth inch in diameter loaded to weigh 1 pound.

(11) *Tensile Strength.*—Briquettes made of neat cement, after being kept in air for twenty-four hours under a wet cloth and the balance of the time in water, shall develop tensile strength per sq. in. as follows: 7 days, 450 lbs; 28 days, 540 lbs. Briquettes made of 1 part cement and 3 parts standard sand, by weight, shall develop tensile strength per sq. in. as follows: 7 days, 140 lbs; 28 days, 220 lbs. (In case quick-setting cement is desired, the following tensile strengths shall be substituted for the above: Neat briquettes: 7 days, 400 lbs.: 28 days, 480 lbs. Briquettes of 1 part cement to 3 parts standard sand: 7 days, 120 lbs.; 28 days, 180 lbs.)

The highest result from each set of briquettes made at any one time, is to be considered the governing test. Any cement not showing an increase of strength in the 28-day tests over the 7-day tests will be rejected.

(*b*) **Natural Cement.**—The average net weight per barrel shall not be less than 300 lbs. (West of the Allegheny Mountains this may be 265 lbs.)

(7) *Fineness.*—At least 80 per cent of the cement must pass through a sieve made of No. 40 wire, Stubb's gauge, having 10,000 openings per square inch.

(8) *Time of Setting.*—The cement shall not acquire its initial set in less than twenty minutes and must have acquired its final set in four hours.

(9) The time of setting is to be determined from a pat of neat cement mixed for five minutes with 30 per cent of water by weight and kept under a wet cloth until finally set. The cement is considered to have acquired its initial set when the pat will bear, without being appreciably indented, a wire one-twelfth inch in diameter loaded to weigh one-fourth pound. The final set has been acquired when the pat will bear, without being appreciably indented, a wire one twenty-fourth inch in diameter loaded to weigh 1 pound.

(10) *Tensile Strength.*—Briquettes made of neat cement shall develop the following tensile strengths per square inch, after having been kept in air for twenty-four hours under a wet cloth and the balance of the time in water: at the end of 7 days, 90 lbs.; 28 days, 200 lbs. Briquettes made of 1 part cement and 1 part standard sand, by weight, shall develop the following tensile strengths pr sq. in.: 7 days, 60 lbs.; 28 days, 150 lbs.

(*c*) **Puzzolan Cement.**—The average weight per barrel shall not be less than 330 lbs. net. Four sacks shall contain one barrel of cement.

(7) *Fineness.*—Ninety-seven per cent of the cement must pass through a sieve made of No. 40 wire, Stubb's gauge, having 10,000 openings per square inch.

(8) *Specific Gravity.*—The specific gravity of the cement, as determined from a sample which has been carefully dried, shall be between 2.7 and 2.8.

(9) *Soundness.*—To test the soundness of cement, pats of neat cement mixed for five minutes with 18 per cent. of water by weight shall be made on glass, each pat about 3 inches in diameter and one-half inch thick at the center, tapering thence to a thin edge. The pats are to be kept under wet cloths until finally set, when they are to be placed in fresh water. They should not show distortion or cracks at the end of twenty-eight days.

(10) *Time of Setting.*—The cement shall not acquire its initial set in less than forty-five minutes and shall acquire its final set in ten hours. The pats made to test the soundness may be used in determining the time of setting. The cement is considered to have acquired its initial set when the pat will bear, without being appreciably indented, a wire one-twelfth inch in diameter loaded to one-fourth pound weight. The final set has been acquired when the pat will bear, without being appreciably indented, a wire one twenty-fourth inch in diameter loaded to 1 pound weight.

(11) *Tensile Strength.*—Briquettes made of neat cement, after being kept in air under a wet cloth for twenty-four hours and the balance of the time in water, shall develop tensile strengths per square inch as follows: After 7 days, 350 lbs.; 28 days, 500 lbs. Briquettes made of 1 part cement and 3 parts standard sand, by weight, shall develop tensile strength per sq. in. as follows: 7 days, 140 lbs., 28 days, 220 lbs.

III. ARTIFICIAL BUILDING STONES.

The following classification, which includes brick, is convenient in the present discussion:

(1) Brick, usually an argillaceous material, molded to the required form, and baked.
(2) Concrete, consisting of a matrix of cement and sand, properly mixed with a stone aggregate, *in situ.*
(3) Block stone, a hydraulic cement concrete specially treated and formed in moulds at the factory.

(1) Brick.

A good brick clay consists of a fairly pure hydrated silicate of alumina, but contains also more or less so-called impurities as iron, manganese, free silica (sand), potash, lime, etc. In making bricks the clay is well kneaded, molded, dried, and then burned in kilns.

Common brick is designated as "clinker," "hard" or "soft," denoting the degree of burning in the kiln. *Clinker* bricks have been overburned and usually have a blueish cast; they are partly vitrified, and brittle. *Hard* bricks are the best burned, red brick. *Soft* bricks are those which are under-burned, and are paler than the hard. In addition to the amount of burning, the color of brick is much affected by the quantity of iron or lime in the clay; the presence of iron peroxide produces a red brick while lime has a tendency to make it yellow. The common standard of brick in the United States is 8¼"x4"x2¼"; in England, 8¾"x4¼"x2¼".

Face brick is made from one of more specially selected clays, mixed if necessary to give the desired chemical properties and color. The bricks are pressed in the molding machine, hence the name "pressed brick." They are also sometimes re-pressed. The standard size in the United States is 8¼"x4"x2¼".

Glazed brick is a brick coated with enamel, a fusible salt. A common liquid glass for glazing is composed of silicic acid (23.2 per cent.), soda (5.9), potash (2.5), water (65.4). The glazing may be in various colors and consequently pretty patterns may be obtained. Glazed bricks are used in buildings and subways; they are hygienic as well as ornamental.

Vitrified brick.—Vitrification is obtained from the thorough burning of a highly siliceous clay or of a clay mixed with a large amount of siliceous sand.

Terra Cotta, meaning baked earth, is made of a specially graded clay or clays mixed with a certain amount of siliceous sand, if necessary, to secure the desired amount of vitrification. After molding and drying, the mixture is baked. The surface of terra cotta may also be glazed like that of the common glazed brick and tile.

Fire brick is used for the lining of furnaces, chimneys, etc. It is made from the natural fire-clay of which large beds are found in New Jersey and elsewhere. It is a high graded hydrated silicate of alumina, containing preferably not over 4 per cent. of impurities. The clay contains a large proportion of silica (sand), and sometimes more is added to insure its refractory quality. The manufacture is similar to that of common brick, but with the fire-clay the product is vitrified.

Paving brick is not so much vitrified as fire brick.

Sewer brick is ordinary, hard brick.

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(2) Concrete (*En-masse*).

(See, also, Concrete Masonry, page 439.)

*Concrete** is composed of a *matrix* possessing cementing prope thoroughly mixed and united with an *aggregate* or base composed of ments of hard, imperishable, mineral substances, the whole forming a compact, enduring mass. The matrix is the cement-sand-water mo while the aggregate is the crushed rock or gravel base.

Mixture.—The cement and sand are thoroughly mixed dry in the pr proportion, enough water being added subsequently to form a paste wet sand. After this matrix has been carefully kneaded, the aggre which has previously been washed of all dirt and other injurious ma is added, and the whole mass thoroughly mixed. It should be pu p'ce immediately and tamped, with the water just flushing the sur

An ideal concrete is one in which the entire surface of every grai sand is covered with the mortar paste, with the grains nearly touching other but with no voids; and in which also all interstices of the aggre are completely filled with the matrix, pressing firmly against every surf An unduly large proportion of cement may make the mass unnecessar strong and expensive, while too little cement will render it porous a possibly too weak. Where great strength is not essential a cheaper bra of cement may be used instead of the most expensive, perhaps used greater proportion, sufficient to fill all sand voids, thus rendering the m non-porous at no greater cost. This is an essential consideration in loc ities where frosts are liable to work injuriously upon the mass of concre if porous. As with cement, in regard to voids in the sand, so with the matr regarding voids in the aggregate. An excess of matrix means excessi cost, while a deficiency gives a porous concrete. The latter is avoided thorough tamping and proper flushing, and by varying the proportio if necessary.

Proportions.—The proportions, by bulk, in mixing concrete are usual based on 1 of cement, 2 to 3 of sand, and 4 to 7 of aggregate, dependi upon the nature and class of the work as well as the quality and kind materials available. Portland cement is the best, and costs more than t other brands. It is usually specified in first class work. In ordinary wo it often gives better results than the cheaper brands, at no greater co because it "goes further"; bearing in mind of course that if scantily us the matrix will be more or less porous. A greater proportion of sand c be used in the matrix if it is fine and siliceous, hence a little money spe on sand will sometimes save a much greater sum on cement. Lastly, mu less matrix and consequently less cement is required if the aggregate graded to different sizes, lessening the voids.

The voids in sand amount, generally, to about 33 or 34% of the gro bulk; in gravel, to about the same; in broken sandstone, to about 40 to 42 in broken trap, to about 45 to 50%. Generally, the percentage of voids greater in the freshly broken, hard, flint-like rocks, and less in the soft rock which leave the crusher with less-defined edges. With gravel, whe the edges have worn away, the percentage of voids is the least, hence grav is the most economical aggregate to use for concrete because less cemer or mortar, is required. For the same reason, basalt or trap are the mo expensive, although far superior for many purposes.

Economy in cement may be practiced by grading the size of materi as previously noted; thus, a coarse and a fine sand may be used togeth in forming the matrix, while the aggregate may consist of graded grav or gravel and broken stone.

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* Before hydraulic cement became recognized as an essential ing dient in concrete, lime of greater or less hydraulic properties was us sometimes mixed with hydraulic cement or asphalt, etc. Thus,

Kind of concrete.	*Matrix.*	*Aggregate.*
Hydraulic cement c.	Hydraulic cement mortar.	Broken stone, crushed brick, slag cinders, pebbles, gravel, etc.
" lime "	" lime "	
Asphalt cement "	Asphalt cement "	

"Concrete" is now universally recognized as hydraulic cement concre

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Cement-Sand Mix.—To get the proper proportion of cement to sand in the matrix: Determine the percentage of voids in the sand by noting the quantity *c* of water required to "flush" to the level in a water-tight box filled with an *s* quantity of the sand to be used; then the proportion of cement to sand should be about 1.1 *c* : *s*. It is to be noted that the amount of cement required is thus about 10% in excess of the voids in the sand, which is as it should be, as the sand grains will be coated with the cement and separated somewhat from each other. For example, using sand with 34% voids, the ratio of cement to sand would be about 1 : 2½. In practice this ratio is sometimes increased to 1 : 2, 1 : 1½, or 1 : 1, depending upon the character and importance of the work, and also upon the character and quality of the materials. A 1 : 2 mix is generally considered good practice for a cement mortar in first class masonry work; while a 1 : 1 mix makes an excellent wearing surface when used as a finishing coat for concrete walks.

Cement-Sand-Broken-stone Mix.—To get the proper proportion of *matrix* to *aggregate*: Determine the percentage of voids in the aggregate or broken stone by noting the quantity *m* of water required to "flush" to the level in a water-tight box filled with an *a* quantity of broken stone; then the proportion of matrix to aggregate should be about 1.1 *m* : a. For example, with 45% voids in the aggregate, the ratio of mortar to broken stone should be about 1 : 2.

Further, if we assume the sand voids at 34%, as above, the cement-sand-broken stone ratio would be about 1 : 2½ : 5. Note that the quantity of sand in the matrix, slightly increased, really determines the bulk of the matrix itself in estimating the above ratio. In practice, this ratio is *sometimes* increased to 1 : 2 : 4 mix for first class building walls, piers and buttresses; to 1 : 1½ : 3 mix for columns and heavy beams of concrete-steel; and to 1 : 1 : 2 mix for lighter reinforced members.

Size of Broken Stone.—The size to which the stone must be crushed or broken will be governed somewhat by the size of the concrete mass. For large bridge piers, stone which will pass through a 2½-in. ring in any way is often allowed. The most common specification, however, for such foundations stipulates the 2-in. ring. For certain classes of work both an upper and a lower limit are specified; as for instance, rock that will pass through a 2½-in. but not through a 1-in. ring, called the 2½"—1" grade; similarly, we may have a 2"—¾" grade, a 1½"—½" grade, etc. Much refinement in this respect is, however, unnecessarily expensive, excepting perhaps in building construction and notably in concrete-steel work. For the latter, sizes as small as the pea-grade size are used for reinforced floors.

(3) Block Stone.

By this is meant the commonly named "artificial stone" proper. It is manufactured usually at the factory and shipped to the site for erection. Among the most important kinds are the following:

Béton-Coignet.—This was invented by Coignet, a Frenchman, and consisted of Portland cement, siliceous hydraulic cement, and clean sharp sand, thoroughly mixed with a small quantity of water, and placed in molds to set, for use.

Sand bricks are manufactured from lime and sand mixed with water to the consistency of a mortar. This mortar is molded into bricks or blocks and then hardened by heating.

Portland stone is composed of a mixture of Portland cement with water and sand or gravel, placed in molds and thoroughly rammed for setting. It is made plain and also ornamental.

McMurtrie stone is made of Portland cement concrete to which is added a solution of castile soap and a solution of alum, forming compounds of alumina potash and certain acids, which tend to "set" the concrete quickly. It is not attacked by the weather.

Ransome stone.—The aggregate consists of broken granite, gravel or sand; the matrix, a silicate of soda and sand immersed in a hot solution of chloride of calcium. It makes a very good building stone.

Sorel stone.—The matrix consists of a solution of magnesium chloride added to the oxide of magnesium. The aggregate is a finely crushed or powdered stone of good quality. It is extremely hard and is used for imitation marble, emery wheels, etc.

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EXCERPTS AND REFERENCES.

Lutes and Cements Useful to Engineers (By S. S. Sadtler. Paper, Phila. Engrs. Club, June 4, 1904; Eng. News, Jan. 5, 1905).—**Water-Proof Compositions.**—Asphalt fluid coatings are useful for reservoir walls, concrete foundations, brick, wood, etc. Asphalt only partly dissolves in petroleum naphtha, but heated in a steam-jacketed kettle and not thinned out too much, a mixture of the two may be obtained in which the part of the asphalt not dissolved is held in suspension. Asphalt is entirely soluble in benzol or tuluol which are about the cheapest solvents for all the constituents of asphalt. Tar and pitch are sometimes used in this connection, but tar contains water, light oils, and free carbon, and does not wear as well as good refined asphalt; and pitch contains free carbon, which is sometimes objectionable when thinned out with a solvent. The asphalt alone is somewhat pervious to water, and this is improved by adding about one-fourth its weight of paraffin, and made better if in addition a little boiled linseed oil is added also. For thicker compositions, where body is required, asbestos stone powder, cement, etc., may be added as fillers. Lutes of linseed oil thickened with clay, asbestos, red or white lead, etc., are water-proof if made thick enough. These are much used for steam joints. Flaxseed meal made into a paste with water is often serviceable, the oil contained serving as a binder as the water evaporates. **Oil-Proof Compositions.**—The most useful lute for small leaks, etc., is the well-known "hektograph composition," as follows: Good glue or gelatin (2 parts), glycerine (1), water (7); this is applied warm and stiffens quickly on cooling. Another composition: a stiff paste of molasses and flour. Another preparation impervious to oil vapors is the "flaxseed poultice," mentioned above, which is proof to oil vapors. A stiff paste of glycerine and litharge makes one of the strongest cements (oil-proof, water-proof, acid-proof, etc.), forming a chemical combination and setting in a few minutes; if a little water is added it sets more slowly, often an advantage; it is mixed when required for use. Plaster of Paris wetted, by itself, or mixed with asbestos, straw, hair, etc., is useful. A solution of silicate of soda made into a stiff paste with carbonate of lime, hardens in 6 to 8 hours. **Acid-Proof Compositions.**—Several kinds given. **Other Proof Compositions.**—For hydrocarbon gases; chlorine; general purposes. Elastic cements; marine glue; gasket compositions; leather cements, stone, iron and crucible cements; etc.

Cost of Portland Cement.—Portland cement, in and around New York, costs from $1.10 to $1.25 per barrel, depending upon the quality and quantity required. The price varies greatly according to locality.

A Cement Which is Proof Against Sea-Water (By S. B. Newberry. Cement Age, Jan., 1907; Eng. News, Dec. 12, 1907).—This cement is an iron-ore cement and has been manufactured for several years in Germany. Its composition, as compared with an average commercial Portland is as follows:

	Portland Cement.	Ore-Port. Cement.		Port. Cem.	Ore-Port.
Silica	22.0	20.5	Magnesia..	1.5	1.5
Alumina	8.0	1.5	Sulph. Anhyd.	1.5	1.0
Iron Oxid	2.5	11.0	Alkalies, etc.	1.0	1.0
Lime	63.5	63.5		100.0	100.0

Important Illustrations—

Description.	Eng. Rec.
A briquette storage tank, St. Louis testing laboratory	Jul. 24, '09.

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23.—QUARRYING.

When a quarry, of whatever nature, is proposed to be opened, the site should be studied carefully with two main points in view, namely, (1) the most economic methods of mining the material, and (2) the best and cheapest means of transporting the product when mined. It is well to remember that of the two methods of transportation—water and rail—the former is the great "leveler" in the equalization or adjustment of freight rates. Consideration must also be had to water-supply and drainage, whether the machinery is operated by electric- or steam power. On opening the quarry, the surface stripping, consisting of organic and decomposed mineral matter, should be removed ahead of the quarrying. The cost of stripping and removing the weathered rock is usually a very considerable item, and in new quarries opened for local work this cost must be proportioned to the total yardage required.

Sand and Gravel.—An ideal sand and gravel plant is one in which the bank is sufficiently elevated to allow of hydraulic operations. The stream or streams of water may be furnished by gravity or by pumping. The gravel and sand are sluiced into revolving screens of two or more sizes which sort the material, and from which it passes into the bunkers, ready to be loaded on scows or cars. The revolving screens are not very durable and probably the best are common steel plate, perforated. The holes are punched the required size for sorting out the sand and gravel. The material as it passes into the bunkers is thoroughly washed and hence clean and ready for use.

The C. O. Bartlett and Snow Co., of Cleveland, Ohio, manufacture a movable combined gravel digging and screening plant, operated on a track. The particular machine illustrated in *Eng. News*, May 12, 1904, weighs 25 tons, is equipped with a 25-HP. engine and boiler, and has a capacity of 30 to 40 cu. yds. per hour. "The gravel and sand are discharged from the elevator buckets into a belt conveyor, which delivers to a rotary screen. The screen discharges into bins, from which the material is drawn off into gondola cars or wagons." The fine dirt, or dust, is rejected by a belt conveyor to a waste dump. The excavating buckets have a capacity of about three-fourths of a cubic yard.

Rip-rap.—This class includes all rock which may be irregular in size and shape, and especially used for "filling" in breakwater construction, rock-fill dams, slope walls, paving, and crushing into broken stone for concrete, etc. The idea in blasting this material is thoroughly to loosen as much of it as possible for the labor of drilling and cost of explosives, hence the blasts are usually heavy and deep seated. Dynamite high in nitroglycerin, say 75%, is best.

Block Stone.—Building stone is quarried in sizes considerably larger than the finished dimensions required, in order to allow for necessary waste in squaring and dressing. There are four methods employed, each one of which is economically suited to the special nature of the quarry, the material, and the product desired.

Hand Tools are employed where the stone is bedded in thin layers and can be worked with pick, sledge and bar, and by hand-drilling and wedging. Flagstones, for sidewalks, are easily hand-quarried, being a thin-bedded sandstone; also some limestones, shales, etc.

Channeling Machines are used in quarrying blocks of sandstone, limestone and marble. In many quarries the rock is bedded in various commercial thicknesses, the channels are cut to the required depth, and the blocks are wedged off in dimensions as ordered.

The first method used was that of "broach channeling" which consists of drilling rows of holes spaced about an inch apart, and then breaking down these spaces by means of a tool called a "broach."

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The "Steel Gang" channeler consists of a gang of tools or chisels arranged side by side and forming a narrow cutter several inches in length. Fig. 1 shows a "swivel head" channeler, size Z, with boiler, manufactured by the Sullivan Machinery Co., of Chicago. This company also manufactures the "rigid-head" channeler, used for vertical cutting, and the "undercutting" channeler, used for horizontal cutting; also various special types.

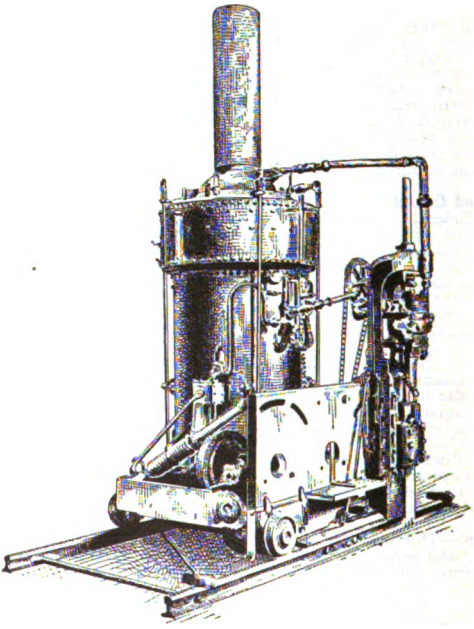

Fig. 1.

The many recent improvements in channeling machines have rendered them valuable not only in quarrying dimension stone but also in certain classes of rock excavation, notably in the excavation of channels or waterways through solid rock, sinking of large pits or shafts, etc., for water power, and in general where the sides of the finished work are required to be fairly smooth or regular.

The Z machine, together with all the Sullivan channelers of other types, may be operated by compressed air as well as steam. Air is preferable under conditions necessitating the use of a number of machines at points distant from the central power plant and from each other. In this case a suitable reheater is mounted on the machine to secure the utmost efficiency. The Z channeler used in constructing the wheel pits at Niagara Falls and the great canal of the Lake Superior Power Co., at Sault Ste. Marie, were driven by air as above described.

The approximate amount of air at 80 pounds pressure necessary to operate the Y and Z channelers is, without reheating, about 350 feet per minute. The size 6½ machine will use about 300 feet under the same conditions, while the VX channeler will use approximately 200 feet.

1.—Weights and Specifications of Sullivan Channelers.

Type.	Rigid Head with Boiler.	Rigid Head without Boiler.	Rigid Head with Boiler.	Rigid Head without Boiler.	Swivel Head with Boiler.	Swivel Head without Boiler.	Swivel Head with Boiler.	Swivel Head without Boiler.	Side Hill without Boiler.	Under Cutting without Boiler.
Size	Y	Y	6½	6½	Z	Z	6½	6½	VX	VA
Diameter of cylinder, in.	7	7	6½	6½	7	7	6½	6½	4½	3½
Gauge of track	4′ 11″	4′ 11″	4′ 11″	4′ 11″	4′ 11″	4′ 11″	4′ 11″	4′ 11″	3′ 3¼″	4′ 2″
Distance of cut from wall, in.	6	6	9	9	6	6	9	9	4	.....
Distance from center to center of cuts when machine is turned on track	6′ 5″	6′ 5″	6′ 5″	6′ 5″	6′ 5″	6′ 5″	6′ 5″	6′ 5″	4′ 4″	.....
*Height	10′ 1″	8′ 2″	10′ 1″	6′ 1″	10′ 1″	8′ 1″	10′ 1″	6′ 6″	†5′ 6″	3′ 1″
Length along track	6′ 6″	6′ 6″	6′ 6″	6′ 6″	6′ 6″	6′ 6″	6′ 6″	6′ 6″	4′ 3″	5′ 2″
Width	6′ 11″	6′ 11″	7′ 4″	7′ 4″	6′ 11″	6′ 11″	7′ 4″	7′ 4″	†4′ 4″	6′ 10″
Weight in Pounds.										
Machine separate	12,300	8,300	11,000	7,100	12,800	8,800	10,400	6,500	1,900	2,200
Equipment	4,500	4,500	4,400	4,400	4,500	4,500	4,400	4,400	2,300	3,000
Total	16,800	12,800	15,400	11,500	17,300	13,300	14,800	10,900	4,200	5,200

* Machines with boiler are measured to the top of the boiler without stack; machines without boiler to the top of the standard, when this is vertical.

† Height, standard horizontal, 1 foot 11 inches. Width, standard horizontal, 5 feet 6 inches.

Approximate boiler power required for channelers without boilers: Sizes Y and Z, 30 horse power; size 6½, 25 horse power; size VX, 20 horse power.

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Explosives are employed principally in quarrying the harder rocks, as granite, syenite, trap, etc.; but they are used extensively also in quarrying sandstone, limestone and marble, in the smaller quarries where canneling machines have not been introduced. In blasting, the explosive must not be so violent as to shatter the detached portions too much, hence a coarse gunpowder is mostly preferred instead of the instantaneous dynamite. Where dynamite is used it is generally of the lower grades, containing say 30 to 40% nitroglycerin for ordinary work, unless large masses of very heavy rock, like granite, are to be opened up, in which case 60% dynamite or even 75% dynamite is sometimes used. The liquid nitroglycerin so useful in submarine blasting is seldom employed in quarrying, being too violent, as well as dangerous to handle. Large blocks, detached from the quarry by blasting, are subsequently broken up into the required dimension, by drilling holes in line and wedging, that is, by plug and feather.

Rock Drills used in quarrying may be classed under three heads, as follows:

The Hammer- or "Jumper" Drill, which includes the smaller sizes such as may be operated by one man who also wields the hammer, weighing say 4½ lbs.; and also the larger sizes or jumpers proper, which are operated by one man turning and with two men striking (4½-lb. and 10-lb. hammers for two hand drilling, and two 10-lb. hammers for three-hand drilling).

The "Churn" Drill, a heavy drill 6 to 8 ft. in length, operated usually by two or three, sometimes six, men who raise the drill, let it fall into the hole, catch it on the rebound, etc.—the most economical hand drill for moderately deep holes, when nearly vertical.

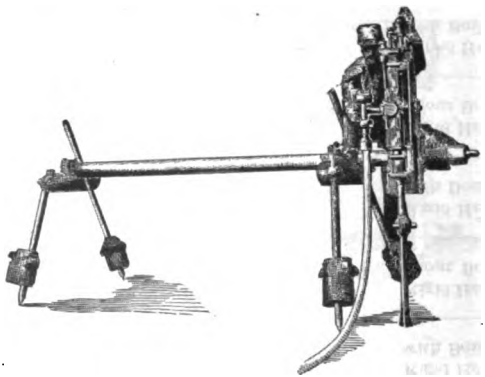

Fig. 2.

The "Percussion" Drill, which is simply a churn drill attached to the piston of a steam- or compressed air cylinder, and by far the most effective drill for quarrying, mining, tunneling and general rock excavation. Fig 2 illustrates a percussion drill of the Ingersoll-Rand type mounted on a "quarry bar," which arrangement is specially useful for drilling "plug and feather" holes, for "broaching" in granite and similar materials, for taking out "key blocks," for "lofting" work in quarrying material with a well defined cleavage, and for general contract work. The "Light 3-inch" bars have a length (over all) of 10 ft., a cutting length of 8 ft.-4 ins., are suitable for 2-2¼ in. cyl. dia. of drill, and weigh, with weights but without drill, 945 lbs. The "Standard 4½-inch" bars have a length (over all) of 12 ft., a cutting length of 10 ft., are suitable for 2½-3¼ in. cyl. dia. of drill, and weigh, with weights but without drill, 1625 lbs. The price of the Light bars is $175.00 and of the Standard bars $250.00.

Table 2, page 424, gives dimensions and weights of the Rand "Little Giant" percussion drill (Fig. 3).

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* Percussion drills may be mounted on the "tripod," which is a most common method; on the "column" or vertical bar; and on the "gadder" or adjustable carriage designed especially for quarry work where parallel holes are to be drilled in any plane. "Used in connection with the channeler it is applied in 'lofting,' or drilling the horizontal undercutting holes in material which has been channeled. Used in 'plug and feather work,' it breaks the large blocks cut free by the channelers." (See page 426.)

Compressed Air, used directly in splitting granite and creating working beds in the quarry, is a recent novelty practised successfully by the North Carolina Granite Corporation at Mt. Airy, N. C., and described fully in "Mine and Quarry" † of May, 1905. The quarry consists of a solid mass of granite without cleavage planes or beds in any direction and the compressed air is used to create artificial beds to which to work:

In the center of the sheet or area to be lifted, a drill hole two or three inches in diameter is sunk six or eight feet in depth, depending on the *greatest thickness* of stone required. The bottom of the hole is enlarged into a pocket by exploding half a stick of dynamite. A small charge of powder, about a handful, is then exploded in the pocket, thus starting a horizontal crack or cleavage across its greater diameter. Charges increasing in size are now exploded in the cavity, the drill hole being plugged at each blast, to confine the powder gases and thus exert a more or less constant force upon the stone. After the cleavage has extended to a radius of 75 or 100 feet in all directions, a pipe is cemented into the hole and connected by means of a globe valve, to the air pipe line from an air compressor Compressed air at 70 to 80 pounds pressure is gradually admitted and the cleavage rapidly extended until it comes out upon the hillside in a thin edge. A sheet of several acres in extent may be raised in this manner, affording a bed plane approximately horizontal, to which the quarrymen can work, thus securing stone of any required thickness. . . . The time [formerly] required for extending the cleavage by powder for 100 feet was between two and three weeks, while to split the larger area, between 100 and 225 feet radius, required only half an hour when compressed air was used. . . . Its equipment is modern in every respect, and includes 35 plug‡ drills, 3 Sullivan tripod drills, 4 Sullivan quarry bars, 15 surfacing machines, and 60 small hand tools. These are all operated by air power from a Sullivan Corliss air compressor of the two stage type, with a piston displacement of 2000 cubic feet of free air per minute at 78 r.p.m., against 80 to 100 pounds terminal pressure. The dimensions of the compressor are as follows: Steam cylinders, 16 and 28 in. by 42 in. stroke; air cylinders, 26 and 16 in. by 42-in. stroke.

Cost of Quarrying Rubble and Dimension Stone for the Buffalo, N. Y., Breakwater (By Emile Low. Eng. News, Oct. 20, 1904.—Tabulated costs, method of quarrying, and plant used are given. The cost for explosives, which includes powder, dynamite and fuses, per ton of stone, all kinds, quarried was: For May (1903), 1.3 cts.; July (1903), 2.0 cts.; August (1903), 2.1 cts. About 1 lb. of powder was used for every 7 tons of stone quarried, and 1 lb. of dynamite for every 67 tons of stone. One fuse was used for every 5½ tons of stone quarried. Total cost of quarrying 1 ton of stone, loading and placing same aboard of scows, as follows: Labor, 33 cts.; coal, 4 cts.; explosives, 2 cts.; miscellaneous, 5 cts.; total, 44 cts. This is exclusive of cost of plant and deterioration.

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* Pneumatic drills for hand service, like pneumatic riveters, are the latest development in rock drilling by machinery. They go by the various names of "Little Jap," "Little Imp," and "Plug" drills, and do remarkable work.

† Published by the Sullivan Machinery Co., Chicago.

‡ The plug drill is a pneumatic power drill for hand service similar to other pneumatic hand tools, as the pneumatic riveter, etc.

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2.—Rand "Little Giant" Percussion Rock Drills. (See page 422.)

Letter and number indicating type	O Model 5	O D Model 5	1D Light	I Model 5	2 D Model 5	3 D Model 5	3½ D Model 5	4 D Model 5	5 D Panama	7 Dunbar
Dimensions:										
Diameter of cylinder ... in.	2	2	2¼	2¼	2¾	3¼	3¼	3⅝	4½	5½
Length of stroke ... in.	4	5½	5½	5¾	6½	6¾	6¾	7½	8	8½
Length of drill from end of crank to end of chuck ... in.	34½	38	36½	39¾*	47½*	48½*	55*	55†	61‡	65
Depth of hole drilled without change of bit ... in.	12	15	18	18	18–24¶	24	24	30	36	30
Diameter of supply inlet (standard pipe) ... in.	¾	¾	¾	¾	¾	1	1	1	1¼	1½
Approximate strokes per minute with 60 pounds pressure at drill	600	550	500	500	450	400	400	325	.....	.....
Depth of vertical hole each machine will drill easily from 1 to ... ft.	4	5	6	6	10	14	16	20	30	.....
Diameter of holes drilled as desired from (at bottom) ... in.	1 1/16	1	1 1/16	1 1/16	1½	1¾	1¾	2	2¼	.....
Average work done per 10 hours in granite down holes, including time lost in setting drill and changing bits ... ft.	70	70	70	70	70	70	75	75	.....	.....
Diameter of octagon steel used ... in.	¾ or ⅞	¾ or ⅞	¾ or ⅞	¾ or ⅞	1 or 1¼	1¼ or 1½	1¼ or 1½	1⅜	1½	1¾
Size of shanks (diameter and lengths) ... in.	¾ x 5	¾ x 5	¾ x 5	¾ x 5	1 x 5½	1¼ x 6	1¼ x 6	1⅜ x 6½	1½ x 7	.....
Number of pieces in set of steels to drill holes in depths as stated	4	4	4	4	8 or 7	7	8	8	10	.....
Best size of boiler to give plenty of steam at high pressure ... h. p.	5	5	6	6	8	8	10	12	15	20 to 23
Best size of supply pipe to carry steam or air 100 to 200 feet ... in.	¾	¾	1	1	1¼	1½	1½	2	2	2½
Approximate Weights:										
Drill unmounted, with wrenches and fittings, not boxed ... lbs.	104	118	125	176	232	321	329	459	704	717
Drill unmounted, with wrenches and fittings, boxed ... lbs.	138	152	158	209	273	371	379	520	792	841
Tripod, without weights ... lbs.	80	80	90	90	150	181	181	333	437	No Tripod Used
Holding down weights, not boxed ... lbs.	102	102	210	210	309	345	345	399	573	
Holding down weights, boxed ... lbs.	116	116	228	228	331	368	368	426	600	
Drill, tripod, weights and wrenches, shipping weights ... lbs.	334	348	476	527	754	920	928	1279	1829	

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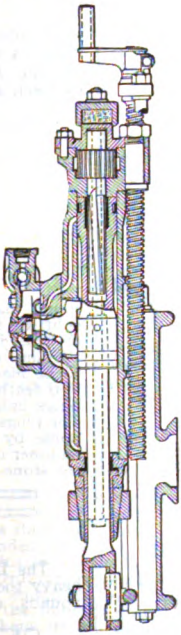

Fig. 3.—"Little Giant" (see page 422).

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24.—STONE CUTTING.

The following is from the report of the American Society's Committee* to secure uniformity of terms. The matter is re-arranged, for convenience, in parallel columns.

STONES CLASSED ACCORDING TO FINISH.

All stones used in building are divided into three classes, according to the finish of the surface, viz.:

I. Rough stones that are used as they come from the quarry.

II. Stones roughly squared and dressed.

III. Stones accurately squared and finely dressed.

In practice, the line of separation between them is not very distinctly marked, as one class gradually merges into the next.

I. **Unsquared Stones.** — This class covers all stones which are used as they come from the quarry, without other preparation than the removal of very acute angles and excessive projections from the general figure. The term "backing" which is frequently applied to this class of stone, is inappropriate, as it properly designates material used in a certain relative position in a wall, whereas stones of this kind may be used in any position.

TOOLS EMPLOYED.

The *Hand Hammer*, weighing from 2 to 5 pounds is used in drilling holes, and in pointing and chiseling the harder rocks.

Fig. 1.

The *Plug*, a truncated wedge of steel, and the *Feathers* of half-round malleable iron are used for splitting unstratified stone. A row of holes is made with the *Drill* on the line on which the fracture is to be made; in each of these holes two feathers are inserted, and the plugs lightly driven between them. The plugs are then gradually driven home by light blows of the hand hammer on each in succession until the stone splits.

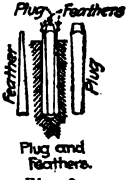

Fig. 2.

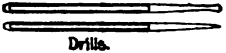

Fig. 3.

The Double Face Hammer is a heavy tool weighing from 20 to 30 pounds, used for roughly shaping stones as they come from the quarry and for knocking off projections. This is used only for the roughest work.

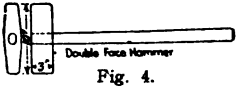

Fig. 4.

* Trans. Am. Soc. C. E., vol. vi., p. 297.

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II. **Squared Stones.**—This class covers all stones that are roughly squared and roughly dressed on beds and joints. The dressing is usually done with the face hammer or ax, or in soft stones with the tooth-ax. The distinction between this class and the third lies in the degree of closeness of the joints which is demanded. Where the dressing on the joints is such that the distance between the general planes of the surfaces of adjoining stones is one-half inch or more, the stones properly belong to this class.

(*a*) **Quarry-faced** stones are those whose faces are left untouched as they come from the quarry.

(*b*) **Pitch-faced** stones are those on which the arris is clearly defined by a line beyond which the rock is cut away by the pitching chisel, so as to give edges that are approximately true.

(*c*) **Drafted Stones** ase those on which the face is surrounded by a chisel draft, the space inside the draft being left rough. Ordinarily, however, this is done only on stones in which the cutting of the joints is such as to exclude them from this class.

In ordering stones of this class the specifications should always state the width of the bed and end joints which are expected, and also how far the surface of the face may project beyond the plane of the edge. In practice, the projection varies between 1 inch and 6 inches. It should also be specified whether or not the faces are to be drafted.

The *Face Hammer* has one blunt and one cutting end, and is used for the same purpose as the

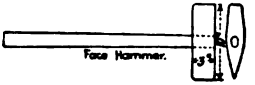

Fig. 5.

double face hammer where less weight is required. The cutting end is used for roughly squaring stones, preparatory to the use of finer tools.

The *Cavil* has one blunt and one pyramidal, or pointed, end, and weighs from 15 to 20 pounds.

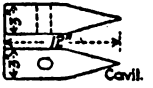

Fig. 6.

It is used in quarries for roughly shaping stone for transportation.

The *Pitching Chisel* is usually of 1½-inch octagonal steel, spread on the cutting edge to a rectangle of ⅜x2½ inches. It is used to make a

Fig. 7.

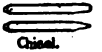

Fig. 8.

well-defined edge to the face of a stone, a line being marked on the joint surface to which the chisel is applied and the portion of the stone outside of the line broken off by a blow with the hand-hammer on the head of the chisel.

The *Chisel*, of round steel of ½ to ¾ inch in diameter and about 10 inches long, with one end brought to a cutting edge from ½ inch to 2 inches wide, is used for cutting drafts or margins on the face of stones.

The *Tooth Chisel* is the same as the chisel, except that the cutting edge is divided into teeth. It is used only on marbles and sandstones.

Fig. 9.

Fig. 10.

The *Splitting Chisel* is used chiefly on the softer, stratified stones, and sometimes on fine architectural carvings in granite.

III. **Cut Stones.**—This class covers all squared stones with smoothly dressed beds and joints. As a rule, all the edges of cut stones are drafted, and between the drafts the stone is smoothly dressed. The face, however, is often left rough when the constructions are massive.

Rough-pointed.—When it is necessary to remove an inch or more from the face of a stone, it is

Fig. 12.

done by the pick or heavy point until the projections vary from ½ inch to 1 inch. The stone is then said to be rough-pointed. In dressing limestone and granite, this operation precedes all others.

Fine-pointed.—If a smoother finish is desired, rough pointing is followed by fine pointing. It is done with a fine point. Fine

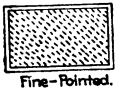

Fig. 14.

pointing is used only where the finish made by it is to be final, and never as a preparation for a final finish by another tool.

Crandalled.—This is only a speedy method of pointing, the effect being the same as fine pointing,

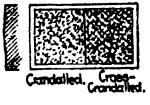

Fig. 16.

except that the dots on the stone are more regular. The variations of level are about ¼ inch, and the rows are made parallel. When other rows at right angles to the first are introduced, the stone is said to be *cross-crandalled.*

The *Mallet* is used [instead of the hand hammer, in pointing,

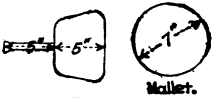

Fig. 11.

chiseling, etc.] where the softer limestones are to be cut.

The *Pick* somewhat resembles the pick used in digging, and is used for rough dressing, mostly on

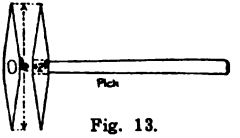

Fig. 13.

limestone and sandstone. Its length varies from 15 to 24 inches, the thickness of the eye being about 2 inches.

The *Point* is made of round or octagonal rods of steel, from ¼ inch to 1 inch in diameter. It is made

Fig. 15.

about 12 inches long with one end brought to a point. It is used until its length is reduced to about 5 inches. It is employed for dressing off the irregular surface of stones, either for a permanent finish or preparatory to the use of the ax. According to the hardness of the stone, either the hand-hammer or the mallet is used with it.

The *Crandall* is a malleable-iron bar about 2 feet long, flattened at one end. In this end is a slot,

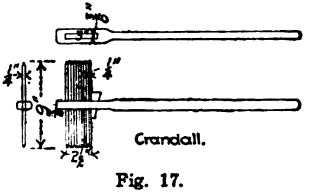

Fig. 17.

3 inches long and ⅜ inch wide. Through this slot are passed ten double-headed points of ¼-inch squared steel, 9 inches long, which are held in place by a key.

hammered, and These two vary r smoothness of produced. The

a patent ham- 12 to the inch; ecifications the e inch must be , 8-cut, 10-cut, of axing is to h chisel marks, rallel as far as is a final finish.

he tooth-ax is er of points, and ce of a stone in as fine pointing. ver, only a prep- hammering, and done without ng as the surface ficiently leveled.

The *Ax*, or *Pean Hammer*, has two opposite cutting edges. It is used for making drafts around the arris, or edge, of stones, and in

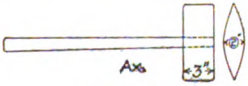

Fig. 19.

reducing faces, and sometimes joints, to a level. Its length is about 10 inches, and the cutting edge about 4 inches. It is used after the point and before the patent hammer.

The *Patent Hammer* is a double-headed tool so formed as to hold at each end a set of wide thin chisels. The tool is in two parts, which are held together by the bolts which hold the chisels. Lateral motion

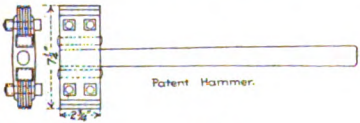

Fig. 20.

is prevented by four guards on one of the pieces. The tool without the teeth is $5\frac{1}{2}$x$2\frac{3}{4}$x$1\frac{1}{2}$ inches. The teeth are $2\frac{3}{4}$ inches wide. Their thickness varies from $\frac{1}{12}$ to $\frac{1}{8}$ of an inch. This tool is used for giving a finish to the surface of stones.

The *Tooth Ax* is like the ax, except that its cutting edges are divided into teeth, the number of

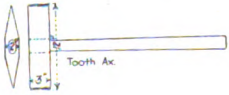

Fig. 21.

which varies with the kind of work required. This tool is not used in granite and gneiss cutting.

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Bush-hammered.—The roughnesses of a stone are pounded off by the bush hammer, and the stone is then said to be "bushed." This

Fig. 22.

kind of finish is dangerous on sandstone, as experience has shown that sandstone thus treated is very apt to scale. In dressing limestone which is to have a bush-hammered finish, the usual sequence of operation is (1) rough-pointing, (2) tooth-axing, and (3) bush-hammering.

Rubbed. — In dressing sandstone and marble, it is very common to give the stone a plane surface at once by the use of the stone-saw. Any roughnesses left by the saw are removed by rubbing with grit or sandstone. Such stones, therefore, have no margins. They are frequently used in architecture for string-courses, lintels, door-jambs, etc.; and they are also well adapted for use in facing the walls of lock-chambers and in other localities where a stone surface is liable to be rubbed by vessels or other moving bodies.

Diamond Panels. — Sometimes the space between the margins is sunk immediately adjoining them, and then rises gradually until the four planes form an apex at the middle of the panel. In general, such panels are callled diamond panels, and the one just described is called a sunk diamond panel. When the surface of the stone rises gradually from the inner lines of the margins to the middle of the panel, it is called a raised diamond panel. Both kinds of finish are common on bridge quoins and similar work. The details of this method should be given in the specifications.

The *Bush Hammer* is a square prism of steel whose ends are cut into a number of pyramidal points. The length of the hammer is from

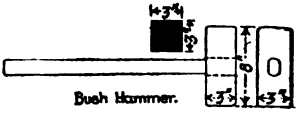

Fig. 23.

4 to 8 inches, and the cutting face from 2 to 4 inches square. The points vary in number and in size with the work to be done. One end is sometimes made with a cutting edge like that of the ax.

The *Machine Tools* used chiefly are the saws, planers, grinders, polishers, etc.

EXCERPTS AND REFERENCES.

Pneumatic Stone-Dressing Machines at the Wachusett Dam, Clinton, Mass. (Eng. News, June 30, 1904).—Illustrations of the Kotten and the Dallett pneumatic stone-dressing machines. No costs are given.

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25.—MASONRY.

Kinds of Masonry.—I. Stone masonry; II. Brick masonry; III. Concrete masonry; IV. Reinforced-concrete masonry. In addition to the above we may also have: V. Mixed masonry; VI. Concrete-block masonry; etc. (For Masonry Arches, see Sec. 44, Arches, page 763.)

CLASSIFICATION OF RAILROAD MASONRY.

(By Committee* of Am. Ry. Eng. & M. W. Assn.—See Proceedings, Vol. 7, 1906.)

Kind.	Material.	Description.	Manner of Work.	Dressing.	
				Joints or Beds.	Face of Surface.
Bridge, and Retaining-Wall	Stone	Dimension	Coursed	Smooth	Smooth Rock-f'ced
		Ashlar	Coursed Broken-coursed	Smooth Fine p'ted R'gh p'ted	Smooth Rock-f'ced
		Rubble	Uncoursed	R'gh-p'ted Scabbled	Rock-faced
	Concrete	Reinforced Plain Rubble			
Arch.....	Stone	Ashlar	Coursed	Smooth Fine-p'ted	Smooth Rock-f'ced
		Rubble	Uncoursed	Rough-p't Scabbled	Rock-faced
	Concrete	Reinforced Plain			
	Brick	No. 1	English Bond Flemish Bond		
Culvert...	Stone	Rubble Dry	Uncoursed	R'gh-p'ted Scabbled	Rock-faced
	Concrete	Reinforced Plain Rubble			
Dry.....	Stone	Rubble	Uncoursed		

I. STONE MASONRY.

Definitions of Parts of Wall.†—*Face*, the front surface of a wall; *Back*, the inside surface. *Facing*, the stones which form the face or outside of the wall; *Backing*, the stones which form the back of the wall; *Filling*, the interior of the wall. *Batter*, the slope of the surface of the wall; as, 1 on 12 = 1 inch horizontal to 1 foot vertical. *Course*, a horizontal layer of stone in

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† See Trans. Am. Soc. C. E., Vol. VI., as previously referred to under Sec. 24, Stone Cutting.

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the wall. If the stones of each layer are of equal thickness throughout it is termed "regular coursing;" if the thicknesses are unequal, the term "random" or "unequal coursing" is applied. *Joints*, the mortar layer between the stones. The horizontal joints are called "bed-joints," or simply "beds;" the vertical joints are sometimes called "builds." Usually the horizontal joints are called "beds" and the vertical ones "joints." *Coping*, a projecting course of heavy stones on the top of the wall to protect it. (A "weathered" coping is one whose top is beveled so as to act as a roof). *Pointing*, a better quality of mortar put in the face of the joints to help them resist weathering. *Bond*, the arrangement of stones in adjacent courses. *Stretcher*, a stone whose greatest dimension lies parallel to the face of the wall. *Header*, a stone whose greatest dimension lies perpendicular to the face of the wall. *Quoin*, a corner stone; a header for one face and a stretcher for the other. *Dowels*, straight bars or pins of iron which enter a hole in the upper side of one stone and also a hole in the lower side of the stone next above (for lateral stability). *Cramps*, bars of iron having the ends turned at right angles to the body of the bar, which ends enter holes in the upper side of adjacent stones.

Definitions of Kinds of Masonry.*—(For classification of Stones, see Sec. 24, Stone Cutting).

Rubble Masonry.—Masonry composed of unsquared stone, Fig. 1.

Uncoursed rubble.—Masonry composed of unsquared stones laid without any attempt at regular courses. Fig. 1.

Coursed rubble.—Unsquared-stone masonry which is leveled off at specified heights to an approximately horizontal surface (as at *a*, *b*, etc., Fig. 1).

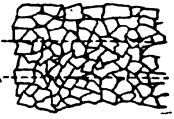

Fig. 1.

(*Squared rubble.*—Rubble masonry in which the stone may be required to be roughly shaped with the hammer, so as to fit approximately.)

Squared-Stone Masonry.—Masonry in which the stones are roughly squared and roughly dressed on beds and joints. (If the joints are as small as about one-half inch the classification might come under that of Ashlar).

There are five kinds.

Regarding character of face:

Quarry-faced masonry.—Face of stone is left as it comes from the quarry.

Pitch-faced masonry.—Face of stone is roughly dressed.

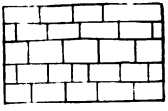
Fig. 2.

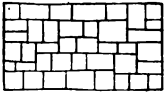
Fig. 3.

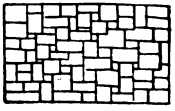
Fig. 4.

Regarding character of course:

Range-work (Fig. 2).—Masonry laid in continuous courses throughout.

Broken Range-work (Fig. 3).—Masonry laid in courses non-continuous or broken.

Random-work (Fig. 4).—Masonry not laid in courses at all.

Ashlar Masonry.—"Cut-stone masonry," or masonry composed of any of the various kinds of cut stone mentioned under Stone Cutting, Sec. 24. From its derivation ashlar apparently means large, square blocks; but practice seems to have made it synonymous with "cut-stone," and this secondary meaning has been retained for convenience.

Fig. 5 represents the face of an ashlar wall, coursed throughout.

Broken Ashlar (Fig. 6).—Cut-stone masonry in which the joints are not continuous.

* See Trans. Am. Soc. C. E., Vol. VI., as previously referred to under Sec. 24, Stone Cutting.

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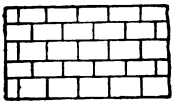

Fig. 5.

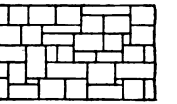

Fig. 6.

Small Ashlar.—Cut-stone masonry in which the stones are less than one foot in height; seldom used.

Rough Ashlar.—A term sometimes given to squared stone masonry, either "quarry-faced" or "pitch-faced" when laid as *range-work*, but it is more logical and more expressive to call such masonry *squared range-work*.

Dimension Stones.—Cut-stones, all of whose dimensions have been fixed in advance. If the specifications for ashlar masonry are so written as to prescribe the dimensions to be used, it will not be necessary to make a new class of such stones.

Range-work, whether of squared stones or of ashlar, is usually backed up with rubble masonry, which in such cases is specified as Coursed Rubble.

Whatever terms are employed in common use to various classes of masonry, it is not safe to trust to them alone in preparing specifications for construction, but every specification should contain an accurate description of the character and quality of the work desired. Whenever practicable, samples of such kind of cutting and masonry should be prepared beforehand, and exhibited to the persons who propose to undertake the work.

Specifications for Stone Masonry.—The following is a Committee Report* (amendments included) submitted to the Am. Ry. Eng. and M. W. Assn., March 1906, but has not formally been approved by the Association.—See Proceedings, Vol. 7.

GENERAL.

Stone.—1. Stone masonry shall be built of the kinds specially designated, with such arrangements of courses and bond as shown on the drawings or as directed. The stone shall be hard and durable, free from seams or other imperfections, of approved quality and shape, and in no case have less bed than rise, and shall be laid on their broadest beds, well bonded and solidly bedded. When liable to be affected by freezing, no unseasoned stone shall be laid.

Dressing.—2. Dressing shall be the best of the kind specified for each class of work. 3. Beds and joints or builds shall be square with each other, and dressed true and out of wind. Hollow beds will not be allowed. 4. All stones shall be dressed for laying on natural bed. 5. Marginal drafts should be neat and accurate. 6. Pitching shall be done to true lines and exact batter.

Mortar.—7. The sand and cement shall be mixed dry and in small batches in proportions as directed, on a suitable platform, which must be kept clean and free from all foreign matter; then water is to be added, and the whole remixed until the mass of mortar is thoroughly homogeneous, and leaves the hoe clean when drawn from it. It shall not be retempered after it has begun to set. Mechanical mixing to produce the same results may be permitted.

Laying.—8. All stones shall be laid on natural beds. Each stone shall be settled into place in full bed of mortar. 9. No stone shall be dropped or slid over the wall, but shall be placed without jarring the stones already laid. 10. No heavy hammering shall be allowed on the wall after a course is laid. 11. If a stone becomes loose after the mortar is set, it shall be relaid with fresh mortar. 12. Each stone shall be cleansed and dampened before laying. 13. Stones shall not be laid in freezing weather unless allowed by the engineer. If allowed, they shall be free from ice, snow or frost by warming, and laid in mortar made of heated sand and water, or, with proper precautions, mixed with brine in proportions of one lb. of salt to 18 galls. of water, when the temperature is 32° F. Add one ounce of salt for every degree of temperature below 32° F. 14. Stones shall be laid to exact lines and levels so as to give the required bond and thickness of mortar in beds and joints.

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Pointing.—15. Mortar in beds and joints of exposed faces shall be removed to a depth of not less than one in. before it has set. No pointing shall be done until the wall is complete and mortar set, nor when frost is in the stone. Wet the joints and fill again with mortar made of equal parts sand and Portland cement. It shall be pounded in with "set-in" or calking tool, and finished with a beading tool the width of the joint, used with a straight-edge.

CLASSIFICATION.

16. Stone masonry shall be classified under the following heads: Bridge and Retaining Wall Masonry, Arch Masonry, Culvert Masonry, and Dry Masonry.

Bridge and Retaining Wall Masonry.

17. Bridge and Retaining Wall masonry shall consist of two classes: (*a*) Ashlar (either coursed or broken-coursed), and (*b*) Rubble.

(*a*) Ashlar Masonry in Bridges and Retaining Walls.

18. In Ashlar masonry in bridges and retaining walls (either coursed or broken-coursed), the stone shall be large and well-proportioned. 19. No course shall be less than 14 ins. nor more than 30 ins. thick; the thickness of courses to diminish regularly from bottom to top.

Dressing.—20. The beds and joints or builds of face stones shall be fine-pointed, so that the mortar layer shall not exceed ¼ in. in thickness when the stones are laid. 21. Joints in face stones shall be full to the square for a depth equal to at least ½ the height of the course, but in no case less than 12 ins.

Facing or Surface Finish.—22. The exposed surface of each face stone will be rock-faced, and the edges pitched to true lines and exact batter; the face to have no projections over 3 ins. beyond the pitch lines. 23. A chisel draft 1½ ins. wide shall be cut at each exterior corner. 24. No holes for stone hooks shall be permitted to show in exposed surfaces. They must be handled with clamps, keys, lewis or dowels.

Stretchers.—25. Stretchers shall be not less than 4 ft. long, and to have at least 1¼ times as much bed as thickness of the course.

Headers.—26. Headers shall be not less than 4 ft. in length. They shall occupy one-fifth of the face of the wall, and no header shall have less than 18 ins. width of face, and when the course exceeds 18 ins. height, the width of face shall be not less than the height of course. Headers shall hold the size in the heart of the wall that they show on the face, and be so arranged that a header in a superior course shall be placed between two headers in a course below; but no header shall be laid over a joint, and no joint shall cover a header. They shall be similarly disposed in the back of the wall, interlocking with those in the face when the thickness of the wall will admit. When the wall is too thick to admit of such arrangement, stones of not less than 4 ft. in length shall be placed transversely in the heart of the wall to bond the two opposite sides of it.

Backing.—27. Backing shall be large, well-shaped stone, roughly bedded and jointed; the bed joints not to exceed 1 in., and vertical joints generally not to exceed 2 ins. No part or portion of vertical joints shall have a greater dimension than 6 ins., which void shall be thoroughly filled with spalls full bedded in cement mortar or filled with concrete. At least one-half of the backing stones shall be of the same size and character as the face stone and with parallel beds. 28. When face stone is backed with two courses, neither course shall be less than 8 ins. thick. 29. When the wall is 3 ft. thick or less, the face stone shall pass entirely through and no backing be allowed. 30. If the engineer so directs, the backing may be entirely of concrete, or back laid with headers and stretchers, as specified above, and heart of wall filled with concrete.

Bond.—31. The bond of stone on face, back and heart of wall shall not be less than 12 ins. Backing shall be laid to break joints with the face stone and with one another.

Coping.—32. Coping shall be dimension stone, holding full size throughout, proportioned for its loading, as marked on the drawings. 33. The beds, joints and top shall be fine-pointed. 34. The location of joints shall be determined by the position of the bed plates, and must be shown on the drawings.

Locks.—35. When required, in the judgment of the engineer, coping stones, stones in the wings and abutments, and stones on piers shall be secured together with iron clamps or dowels, their position being indicated by the engineer.

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ble Masonry in Bridges and Retaining Walls.

idge- and retaining-wall masonry shall consist of stones laid in irregular courses. The beds shall be parallel, nd lie horizontally in the wall. The bottom stones shall lat stones. The wall shall be compactly laid, having at surface of the back and face headers, so arranged as to ll the spaces in the heart of the wall filled with suitable , thoroughly bedded in cement mortar or filled with e joints must not be more than 1 in. in thickness.

Arch Masonry.

nry shall consist of the arch ring only, or that portion ios and extrados, and shall be of two classes: (*a*) Ashlar l (*b*) Rubble Arch Masonry.

(*a*) Ashlar Arch Masonry.

oirs shall be of full size throughout, and must have bond ness of the stone and dressed true to templet. 39. The and depth of voussoirs shall be shown on the drawings. the voussoirs and the intrados shall be fine-pointed. not exceed ⅜ in. 41. The exposed surface of the ring oth or rock-faced, with a marginal draft. 42. Voussoirs o simultaneously from both bench walls. 43. Backing ge stones shaped to fit the arch bonded to the spandrel ds of mortar. Concrete may also be used for backing. g is required, a thin coat of mortar or grout shall be ap- coat upon which shall be placed a covering of suitable erial. 45. Centers shall not be struck until directed. iers, spandrels, parapets, wing walls and copings shall specifications for Ashlar Masonry in Bridges and Re-

(*b*) Rubble Arch Masonry.

oirs shall be full size throughout, and must have bond ness of stone. 48. The depth of voussoirs shall be shown The beds need only be roughly dressed so as to bring nes. 50. Mortar joints shall not exceed 1 in. 51. The the ring stones shall be rock-faced, and the edges pitched Voussoirs shall be carried up simultaneously from both Backing shall consist of large stones shaped to fit the e spandrel and laid in full beds of mortar. Concrete for backing. 54. If waterproofing is required, a thin grout shall be applied for a finishing coat upon which overing of suitable waterproofing material. 56. Bench rels, parapet and wing walls shall be built under the Rubble Masonry in Bridges and Retaining Walls.

Culvert Masonry.

sonry shall be laid in cement mortar. The character uality of work shall be similar to that specified for Rub- dges and Retaining Walls. 58. One-half the top stones hall extend entirely across the wall. 59. The covering strong stone at least 12 ins. thick, or as shown on draw- e roughly dressed so as to make close joints with each their whole width at least 12 ins. over the side of walls. led under high embankments, as directed by the engineer gs. 60. End walls shall be covered with suitable coping.

Dry Masonry.

y shall include: (*a*) Dry Ret'g Walls, and (*b*) Slope Walls.

(*a*) Dry Retaining Walls.

ng walls shall include all dry rubble work for retaining imilar work. 63. Flat stone at least twice as wide as Beds and joints to be roughly dressed square to each stone. 64. Joints not to exceed ⅜ in. 65. The different be evenly distributed over the whole face of wall, gener- rgest stone in the lower part of wall. 66. The work ed and present a reasonably true and smooth surface, rojections. The wall is double-faced and self-sustaining

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(b) Slope Walls.

67. Slope walls shall be built of such thickness and slope as may be required by the engineer. No stones shall be used which do not reach through the wall. Stones shall be placed at right angles to the slopes. This wall is single-faced and built with steep slope simultaneously with the embankment which it is to protect.

Quantity of Masonry in Abutments.—Tables 1 and 2, following, were calculated from Fig. 7. Table 1 gives the quantities of masonry and steel in *one* abutment of various heights, and Table 2 is a supplementary table to facilitate the reduction of these quantities to weights.

1.—QUANTITIES IN ONE R. R. MASONRY *RIGHT ABUTMENTS OF VARIOUS HEIGHTS h. (See Fig. 7.)

Height h of Base of Rail above Top of Foundation. Ft.	A_1	A_2	A_3	A_4	F_1	F_2	F_3	F_4	R_1	R_2	R_3	R_4
	Masonry *Above* Foundation.				Masonry *In* Foundation.				Steel Rail In Foundation.			
	1-Track Road. Cu. Yds.	2-Track Road. Cu. Yds.	3-Track Road. Cu. Yds.	4-Track Road. Cu. Yds.	1-Track Road. Cu. Yds	2-Track Road. Cu. Yds.	3-Track Road. Cu. Yds.	4-Track Road. Cu. Yds.	1-Track Road. Lin. Ft.	2-Track Road. Lin. Ft.	3-Track Road. Lin Ft.	4-Track. Road. Lin. Ft.
8	21.2	48.4	64.8	78.8	29.6	44.2	57.8	68.4	285.	394.	509.	608.
9	38.8	58.8	78.6	94.5	33.2	48.5	62.8	74.1	308.	421.	541.	643.
10	47.6	70.8	94.0	112.	37.0	53.0	68.0	80.0	330.	450.	575.	680.
11	57.7	84.2	111.	131.	41.0	57.7	73.4	86.1	354.	481.	611.	719.
12	69.0	99.2	130.	152.	45.2	62.6	79.0	92.4	380.	514.	649.	760.
13	81.6	116.	150.	175.	49.6	67.7	84.8	98.9	408.	549.	689.	803.
14	95.4	134.	172.	200.	54.2	73.0	90.8	106.	438.	586.	731.	848.
15	111.	153.	195.	227.	59.0	78.5	97.0	113.	470.	625.	775.	895.
16	127.	174.	220.	255.	64.0	84.2	103.	120.	504.	666.	821.	944.
17	144.	196.	247.	285.	69.2	90.1	110.	127.	540.	709.	869.	995.
18	163.	220.	275.	317.	74.6	96.2	117.	134.	578.	754.	919.	1048.
19	183.	246.	305.	351.	80.2	103.	124.	142.	618.	801.	971.	1103.
20	205.	273.	336.	386.	86.0	109.	131.	150.	660.	850.	1025.	1160.
21	227.	301.	369.	423.	92.0	116.	138.	158.	704.	901.	1081.	1219.
22	251.	331.	404.	462.	98.2	123.	146.	166.	750.	954.	1139.	1280.
23	276.	363.	440.	503.	105.	130.	154.	175.	798.	1009.	1199.	1343.
24	302.	396.	478.	546.	111.	137.	162.	184.	848.	1066.	1261.	1408.
25	330.	430.	517.	591.	118.	145.	170.	193.	900.	1125.	1325.	1475.
26	359.	466.	558.	637.	125.	152.	178.	202.	954.	1186.	1391.	1544.
27	389.	503.	601.	685.	132.	160.	187.	211.	1010.	1249.	1459.	1615.
28	420.	542.	645.	735.	140.	168.	196.	220.	1068.	1314.	1529.	1688.
29	453.	583.	691.	787.	147.	177.	205.	230.	1128.	1381.	1601.	1763.
30	487.	625.	738.	840.	155.	185.	214.	240.	1190.	1450.	1675.	1840.
31	522.	668.	787.	895.	163.	194.	223.	250.	1254.	1521.	1751.	1919.
32	558.	713.	838.	952.	171.	203.	233.	260.	1320.	1594.	1829.	2000.
33	596.	760.	890.	1011.	180.	212.	243.	271.	1388.	1669.	1909.	2083.
34	634.	808.	944.	1072.	188.	221.	253.	282.	1458.	1746.	1991.	2168.
35	675.	857.	999.	1135.	197.	231.	263.	293.	1530.	1835.	2075.	2255.
36	716.	908.	1056.	1199.	206.	240.	273.	304.	1604.	1906.	2161.	2344.

Above by Formulas:

$A_1 = \frac{1}{2}(h-5)^2+3.2h.$ $F_1 = 0.1h^2+1.9h+8.$ $R_1 = h^2+3h+200.$

$A_2 = \frac{1}{2}(h-5)^2+5.2h.$ $F_2 = 0.1h^2+2.6h+17.$ $R_2 = h^2+10h+250.$

$A_3 = 0.8h^2+0.2h+12.$ $F_3 = 0.1h^2+3.3h+25.$ $R_3 = h^2+15h+325.$

$A_4 = 0.9h^2+0.4h+18.$ $F_4 = 0.1h^2+4.0h+30.$ $R_4 = h^2+18h+400.$

* For "skew" abutments, multiply quantities in table by secant of "skew-angle," i. e., angle which face of abut. makes with a *right angle* to center line.

2.—TABLE FOR FINDING WEIGHTS OF QUANTITIES IN TABLE 1.

C. Yds. of Masonry Or Lin. Ft. of Rail.	Weight of Masonry, in 1000 Lbs.			Weight of Steel, in Lbs.		
	Mas. at 145 Lbs. per cu. ft.	Mas. at 155 Lbs. per cu. ft.	Mas. at 160 Lbs. per cu. ft.	Rails at 60 Lbs. per yd.	Rails at 65 Lbs. per yd.	Rails at 70 Lbs. per yd.
1	2.915	4.185	4.32	20.	21.67	23.33
2	7.830	8.370	8.64	40.	43.33	46.67
3	11.745	12.555	12.96	60.	65.	70.
4	15.660	16.740	17.28	80.	86.67	93.33
5	19.575	20.925	21.60	100.	108.33	116.67
6	23 490	25.110	25.92	120.	130.	140.
7	27.405	29.295	30.24	140.	151.67	163.33
8	31.320	33.480	34.56	160.	173.33	186.67
9	35.235	37.665	38.88	180.	195.	210.
10	39.150	41.850	43.20	200.	216.67	233.33
100	391.5	418.5	432.	2000.	2167.	2333.
200	783.	837.	864.	4000.	4333.	4667.
300	1174.5	1255.5	1296.	6000.	6500.	7000.
400	1566.	1674.	1728.	8000.	8667.	9333.
500	1957.5	2092.5	2160.	10000.	10833.	11667.
1000	3915.	4185.	4320.	20000.	21667.	23333.

*Ex.—A masonry abutment contains 384 cu. yds. masonry at 155 lbs. per cu. ft., and 822 lin. ft. steel rails at 65 lbs per (lin) yd. Find the weight or load which the abutment alone produces on the pile foundation.

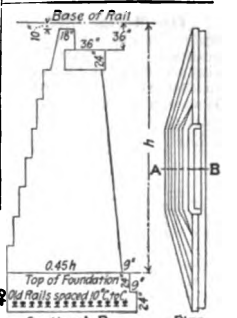

Fig. 7.

*Ans.—From columns 1, 3 and 6 in above table we have: Weight = (masonry) 1607.04 × 1000 + (steel) 18460 = 1,625,500 lbs. = 812.75 tons.

II. BRICK MASONRY.

Bond.—The terms "header" and "stretcher" are used in brickwork as in stonework. The 'bond" of a brick wall depends upon the arrangement of headers and stretchers, in the same or in adjacent courses.

English Bond (Fig. 8).—Alternate entire courses of headers and stretchers.

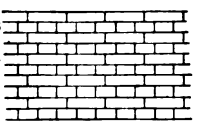
Fig. 8.

Modified English Bond.—Each course consisting entirely of headers or of stretchers, but not alternating as above; usually one course of headers to every four to six courses of stretchers. (Where wall is laid with one course of headers to every two or three courses of stretchers it is generally classed as English bond).

Flemish Bond (Fig. 9).—Each course made up of alternate headers and stretchers.

English bond is considered the strongest.

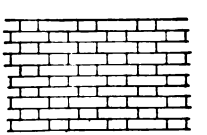
Fig. 9.

Brickwork is well adapted to all kinds of masonry construction where excessive strength and massive weight (as bridge abutments, piers, dams, etc., which call for concrete and stonework) are not particularly essential. Hence its use in building-walls (including the upper stories in tall buildings), tunnel linings, small arches, culverts, (street paving), sewers, etc. The convenience in handling and laying brick, in forming arches and "rounding" corners, makes it particularly useful in these classes of construction. In fire-resisting qualities it is superior to any natural building stone, with the exception possibly of sandstone, and is equal to the latter in this respect,

and infinitely superior to it (i.e., to most varieties) as regards frost action and absorption of moisture. Generally speaking, when compared with stonework, brick masonry lies between ashlar and common rubble masonry, both in cost and strength. Regarding cost, the writer has witnessed the price of common building brick fluctuate during the past year, in Brooklyn, between $16 and $7 per thousand. It is perhaps essential to note here that while the higher price was maintained the builders resorted largely to rubble and concrete construction, which finally brought the trust price to the lower figure. This may illustrate also that the "cost of work" obtained from any source must be used with judgment in connection with the details furnished therewith, and with the prevailing local conditions.

The Mortar used in brick masonry may vary considerably, depending upon the class of work. For instance, the best mortar usually specified is composed of 1 part Portland cement, and 2 parts clean, sharp sand, where the structure is exposed to considerable stress; while for the most ordinary brickwork, 1 part fresh, well-slaked lime and 2½ to 3 parts sand, will answer. Between these limits we may have various mixtures of Portland cement, natural cement, common lime, and sand. Thus:

Class *A*	1 Port. Cem.	Nat. Cem.	Com. Lime.	2	Clean, sharp Sand.
" A_1	1 " "	.....	.....	2½	" " "
" A_2	1 " "	.....	.....	3	" " "
" *B*	.....	1 " "	.....	2	" " "
" B_1	.....	1 " "	.....	2½	" " "
" B_2	.....	1 " "	.....	3	" " "
" *C*	.....	.....	1 " "	2	" " "
" C_1	.....	.....	1 " "	2½	" " "
" C_2	.....	.....	1 " "	3	" " "
" *D*	1 " "	1 " "	.....	4	" " "
" D_1	1 " "	1 " "	.....	5	" " "
" D_2	1 " "	1 " "	.....	6	" " "
" *E* etc.	1 " "	.....	1 Lime paste	2	" " "
" *F* etc.	.....	1 " "	1 " "	2	" " "

Class *A* is used in superior building construction, for railroad masonry in general, tunnel lining, and sewers; class *E*, for building-work of the very highest class; class *C*, for common brickwork as in buildings. Where cement and lime come in barrels, "barrel" measure may be used in determining the "parts" of cement, lime and sand. Fire brick should be laid in fire-clay mortar. Colored mortar is used for pressed brick facing.

Pressed-brick masonry, for facing, requires less mortar per cubic yard of masonry than the common-brick backing, because pressed bricks are a little larger than the common (which are 8¼x4x2¼ standard) to allow for thinner joints.

For any kind of masonry, the bricks should be wet before laying (except perhaps in freezing weather—unless special care is taken to warm the water and sand), and thoroughly bedded in mortar, and pressed or shoved into place.

3.—Quantities of Brick and Mortar in Brick Masonry.

(Brick, standard size—8¼x4x2¼.)

Thickness of Joint.	No. of Brick per Cu. Ft. of Masonry.	No. of Brick per Cu. Yd. of Masonry.	Cu. Ft. of Masonry. per 1000 Brick.	Cu. Yds. of Masonry per 1000 Brick.	Cu. Yds. of Mortar per Yd. of Masonry.	Cu. Yds. of Mortar per 1000 Brick.
zero.	23.3	628.4	43.0	1.59	.000	.000
⅛	21.1	568.6	47.5	1.76	.095	.167
¼	19.1	516.6	52.3	1.94	.180	.350
⅜	17.4	471.0	57.3	2.12	.250	.530
½	16.0	430.9	62.6	2.32	.314	.728
⅝	14.6	395.4	68.3	2.53	.371	.939

This table is calculated for massive masonry construction; allowance should be made for thin walls—bricks to increase and mortar to diminish slightly per volume of masonry.

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III. CONCRETE MASONRY.

No other class of masonry is so generally employed, especially for massive construction, as concrete. It is almost universally used for footings of heavy structures such as abutments, walls and piers for bridges, revetments (breakwaters), dams, and buildings; while it often enters largely, sometimes wholly, into these structures themselves.

Concrete footings are often reinforced with steel rails or I-beams, in one or more tiers, to distribute the loads (either above or below) more uniformly. Such construction may be said to form the connecting link between plain- and reinforced concrete. See Sec. 50, Foundations.

Rock Crushers are now almost wholly employed in breaking stone for concrete, in place of hand-breaking. By the last-named method a laborer is usually counted upon to break about one cu. yd. of trap or from 2 to 3 cu. yds. of limestone, in sizes to pass through a 2-in. ring, in a 10-hr. day. Much depends upon the size and shape of the rip-rap stone received for breaking. On the other hand, the capacities of machines run up to several hundred tons per day. A cubic yard of broken trap rock, assuming 45% voids, will weigh about 185x27x.55=about 2750 lbs. or 1⅜ tons; while a cubic yard of limestone, assuming 37½% voids, will weigh about the same. Unscreened broken stone, especially of the softer kinds, has less percentage of voids than the screened. The softer rocks in crushing assume a more rounded form, and break into more variable sizes, both of these conditions tending to reduce the percentage of voids. Spheres of uniform size, as cannon balls, may be piled in *large piles* in pyramidal form so that the percentage of voids will *approach* the lower limit of 25.963% voids.

Permanent crushers, often with extensive plants, are frequently installed at quarries. For this purpose, the gyratory type of crusher is preferred, a single machine being able to turn out as high as 2000 tons or more per 10-hr. day. Such a machine would weigh in the neighborhood of 50 tons, have a receiving opening of say 1½x5 ft., and require about 150 H. P. to operate. The smaller machines are less economical in the use of power. A machine of ½ the above capacity would require about ⅔ the power to operate, and would weigh about 30 tons; a machine of ¼ the above capacity would require about ½ the power, and weigh about 20 tons; ⅛ the capacity, ¼ the power, and weigh about 10 tons, etc. The above H. P.'s include power required to operate elevators and screens.

Portable crushers are furnished as low as about 4 tons in weight, with openings for 7x10 in. stone, capacity 50 to 60 tons per 10-hr. day, and requiring about 8 H. P. to operate. Small machines cost about $8.50 to $9 per capacity in tons per 10 hr. day. Large machines, from $7 to $8.

Hand crushers, costing about $30, will receive stone up to about 1½x3 ins.

The best crushers have jaws of manganese-steel, or other steel of equal hardness.

Concrete Mixers, or machines for mixing concrete, are indispensable at the present day on work of any magnitude. They may be divided into two classes, namely, "fixed" mixers, and "mechanical" mixers.

Fixed or Gravity Mixer consists of a steel trough fixed on an incline of say 4 to 4½ ins. horizontal to 12 ins. vertical, and some provided with internal projections in the form of steel pins or baffle plates, which deflect and "mix" the material, fed at the upper end, as it descends. These mixers are economically used in lining the bottoms and sides of reservoirs and in work of similar character where the mixing can be done at an elevation above the place for depositing the concrete.

Mechanical Mixers are of two types: "continuous" mixers, and "batch" mixers. The *continuous* mixers are provided with plows, shovels, or paddles which mix the material as fast as delivered, and discharge the mixture continuously. Portable machines of this type are used economically for street work.

Batch mixers of the rotary type are the most generally used. These machines with engine (gasoline), or engine and boiler (steam), are mounted on skids or on wheels, are very compact, and have a capacity of about 200 batches per 10-hr. day, each batch ordinarily containing ½ yd., ¾ yd., or 1 yd., depending on the "size" of the mixer. They are, however, made

in sizes ranging from 2 cu. ft. up to 2 cu. yds. The number of H. P. required is equal to size of batch in cu. yds. multiplied by 15, about. Many of the machines measure the proportions of the ingredients.

The Proportions of Portland Cement, Sand and Broken Stone are, 1 : 2 to 4 : 4 to 8, depending upon the quality of materials used and of the resulting concrete required. A good mix is 1 : 2½ : 5, while 1 : 3 : 6 is perhaps most common for ordinary work. In the construction of the Mississippi jetties, the block concrete was made with the following proportion: Portland cement, 1, sand 2½, clean gravel 1½, broken stone 5; the resulting mass was about 1¼ yds. of concrete per yd. of broken stone. Common lime may be added to cement to increase the bulk of the "cement" paste, and consequently of the mortar, where concrete is not to be placed under water and where it is not subjected to excessive stress; but as common lime has no hydraulic properties, its addition to cement in mortar is simply the addition of so much inert matter, like sand, when the concrete is deposited under water.

A common custom of late is to designate concrete by the proportion of cement to sand assuming that the volume of broken stone shall be twice that of the sand. Thus, 1 to 2 matrix would mean 1 cement, 2 sand, and 4 broken stone, or 1 : 2 : 4 mix; 1 to 2½ matrix, a 1 : 2½ : 5 mix, etc. This abbreviation, unless explained (which explanation would tend to nullify the advantage of abbreviation itself) is apt to lead to error. The proportion of broken stone to sand need not necessarily be in the ratio 2 : 1. In the case of broken limestone, where the crushed rock is graded, and also where gravel is used, especially with broken stone, the ratio may be, say, 2¼ : 1, 2½ : 1, etc. In all large work the percentage of voids, and consequently the proportions, should be determined by experiment for the classes of materials used. See page 417, where the method of determining voids, etc., is given.

In Mixing by hand, the cement and sand are thoroughly mixed dry on a clean platform, enough water is added to make a stiff paste, the broken stone (wet and previously washed clean) is then spread over the mortar and the whole thoroughly mixed by shoveling into another pile, and re-shoveling as often as necessary.* Concrete should be mixed in small batches and placed immediately, as the cement may set appreciably within 30 to 40 minutes after water is applied, and all subsequent disturbance may tend to weaken the mass.

Placing, Spreading and **Ramming** are operations which should closely follow one another rapidly. The concrete may be delivered in barrows, cable buckets, carts or chutes, the last named being the cheapest under favorable conditions. Spreading consists in pushing over the top of each batch dumped, so as to present a fairly level bed for the next layer. The layers are usually specified to be from 6 to 9 ins. in thickness. Ramming compacts concrete from 5 to 25% and makes it stronger by bringing the ingredients more intimately together, so that crystallization takes place more firmly. Rammers are round wooden blocks or logs about 4 ft. long, ring-shod at the lower end and provided at the upper end with handles for one-man or two-man manipulation.

Ramming should bring the concrete to a rather firm state with a flush of water at the top, but should not continue for an undue length of time so as to weaken its setting. "Dry" concrete naturally requires more ramming than a "wet" mixture. Before depositing a new layer, the top of the preceding layer should be wetted, after allowing not less than 12 hrs. for it to set. Where a layer is not wholly completed at the end of a day's work, its edge should be left rough (not smooth) for the next day's joining. "Medium" concrete is between "dry" and "wet." The advantage of medium concrete is that it can be rammed better than "wet," while at the same time possessing some of the good qualities of the latter.

Subaqueous Concrete has seldom been deposited with entire satisfaction even in still water to any great depth, but in moderate depths this has been accomplished with quite favorable results. The essential principle in depositing concrete under water is to convey it properly through the water so that the mix will not be disturbed by "wash." Examinations by divers

* Some engineers and most builders prefer to spread the (wet) gravel and broken stone on the dry cement-sand mix before water is added.—See German Specifications, pages 442, 443.

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have in many cases disclosed "streaky concrete," with cement, sand and matrix more or less separated by wash, owing to their different specific gravities and fineness. The result of this lack of homogeneity is a reduction of strength, hence great care should be taken to insure as little disturbance as possible in placing the material. Under no circumstances should concrete be dumped loosely into water and allowed to settle in place. The three principal methods most commonly used are:

(1) Depositing through closed chutes or tubes.
(2) Depositing by means of specially arranged buckets.
(3) Depositing in sacks or in bags.

The first method, by chute, is seldom used. The tube may be of any section, gradually enlarging toward the bottom to avoid clogging of the concrete fed continuously at top of tube. The tube is suspended vertically with the lower end at the bottom, and is moved laterally as the concrete falls into place. The principal objection to this method is the washing which the concrete receives in its descent.

The second method, by buckets, has been used with greater or less success. One of the most recent uses of this method was in the construction of a 3000-ft. pier at Superior Entry, Wis., by the Government, in about 23 ft. of water.* The steel buckets (Fig. 10) about 4 ft. cubes, open at the top, were provided with canvas covers, quilted with strips of sheet lead, to fold over the concrete and prevent wash. The buckets when lowered were tripped by a latch. Mr. Clarence Coleman, Asst. Engr. in charge, states that the examinations of concrete lowered 23 ft. and raised again in

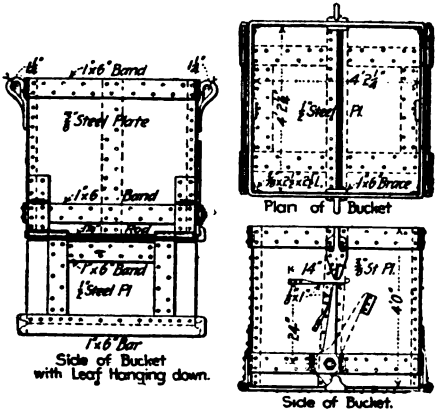

Fig. 10.

the bucket showed the concrete to be in good condition; and that discoloration of the water from cement was seldom noticed during the descent of the bucket.

The third method, by bags, has been used extensively in both this country and in Europe. Mr. W. M. Patton, in his "Treatise on Foundations,"† page 102, says: "Perhaps the best mode of depositing concrete under water is to fill open sacks or gunny sacks about two-thirds to three-fourths full of the concrete or mortar, and deposit these in place, arranging them in courses, where practicable, header and stretcher system, and ramming each course as laid; the bagging is close enough not to allow the cement to be washed out, but at the same time open enough to allow the whole mass to be united and to become as compact as concrete itself. The writer used

* See Vol. 8, Part 4, of Report of Chief of Engineers, U. S. A.
† Published by John Wiley & Sons, New York.

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this method in the foundation of a pier over 100 feet high, and has also adopted this plan in other works of less magnitude, but never has the result been satisfactory when deposited under water in any other manner."

Sub-Foundations are prepared by dredging, if necessary, and driving piles and cutting them off near the river bottom. Before concrete is deposited, molds are constructed of timber and sunk in place to give form to the concrete pier. The inside faces of the molds are of course smooth lined, the timber frames being outside. Long adjustable bolts or turnbuckle rods are convenient to use with collapsible sides, where the same molds are to be used over again. See Sec. 50, Foundations.

Cement Grout (either pure cement, or 1 cement and say 1 sand) may be injected into a quick-sand or gravel foundation bed to form a sub foundation. The grout is pumped into vertical iron pipes perforated with holes at the bottom to allow it to ooze through into the natural bed material. The pipes should extend downward through the soft material to bed rock or other firm sub-stratum.

German Specifications for Concrete.—The following is a digest of portions of the report of standing committee of the German Concrete Society as adopted by that Society and entitled "Specifications for Designing, Constructing and Testing Concrete Structures." These specifications were brought to the attention of *Eng. News* by L. S. Moisseiff, and published under date of Nov. 9, 1905.

I. General.—Specifications apply to concrete construction in general, and to use of Portland cement in particular. Concrete is termed "wet" or "dry;" shall be capable of being tamped to acquire the necessary density to develop the required resistance. *II. Planning and Designing. III. Construction.* (*A*) General. (*B*) Superintendence and workmen. (*C*) Building Materials and their Working. (*a*) Materials: Cement which fulfils the requirements of the standard specifications for Portland cement shall be used exclusively. Quick setting cement shall not be used for concrete except in special cases. (The setting of the concrete is affected by the temperature and moisture of the air and the temperature of the water used. High temperature accelerates setting; low temperature retards it. In the presence of water pressure the use of quick setting cement is frequently necessary.) "Sand" includes land-, river- and sea-sand, as well as broken or crushed products, from fine grains to 0.28 in. in diameter (includes granulated furnace slag of proper consistency). "Gravel," from 0.28 in. up. "Gravel-sand," the natural mixture from excavations or beds of streams. The sand, gravel and broken stone shall be suitable (loam, clay and similar admixtures have an injurious effect if they adhere to the sand and stone; but if they are finely distributed in the sand, without adhering to the grains, they are, as a rule, harmless and may even sometimes increase the resistance), and shall not contain vegetable matter or other impurities. (The stone or gravel used shall, as a rule, have at least the same resistance as the hardened mortar. Stone not weather-resisting, soft sandstone, underburnt brick shall not be used for concrete. Slag is variable and should be tested). According to the thickness of the concrete body, gravel up to 2 ins. in dia. may be used. (Clean stones of large size of good compressive resistances and weathering qualities may be embedded in the concrete up to 40% of the total volume if the character and dimensions of the member allow it and provision be made for the proper distribution of such stones in the concrete as well as for the use of a sufficiently wet concrete to surround them completely.) For "broken stone," only hard rocks, unaffected by the weather, shall be used. As a rule, the broken stone shall also be of different sizes of stones as the concrete mass can then be worked easier and better, and gives a denser and stronger concrete. The sizes of the stones vary with the thickness of the concrete mass. The largest stones shall, according the their use, pass in any direction through a ring of 2.4 to 2.8 ins. diameter, or through a square of 2 to 2.4 on a side. Particles of sizes smaller than 0.28 in. down to stone dust shall be considered as sand. The "water" used shall be clean, etc. Marshy water is injurious. (*b*) Preparing the concrete: In proportioning the materials, if the cement be measured by volume it is understood that it is emptied into the measuring vessel without dropping and the latter is not shaken. To convert volumes into weights, a cubic foot of Portland cement shall be taken as 87.5 lbs. Gravel-sand and mixed broken stone may in many cases be used without being separated. Tests shall then be made by sieving to determine the proportions of the sand in the gravel or the stone dust in

"Mixing by hand" shall be in a proper mixing box. el-sand shall first be mixed dry with the cement to a color, then only shall the previously wetted gravel or the whole, together with the water added to it, shall ntil a uniformly wet mass is produced. "Mixing by lly superior to hand-mixing. The time required varies chine and the character and quantity of the mix. The d dry ½ to 1 minute, and the mixing is then continued tion of water until a thoroughly mixed, uniformly wet Generally, the mixing is complete when the stones are ith the mortar. The "quantity of water" to be added a "dry" or "wet" mix is required, the character tions, moisture, and absorptive capacity. (More water nd hot weather. When laying concrete in wet excava- f water will have to be reduced, while a porous, absorb- be wetted before laying the concrete.) When making nly enough water should be added that the concrete into a ball by the closing of the hand, leaving moisture making "wet" concrete the proportion of water shall as to make the concrete wet during the tamping. (*c*) : The concrete shall be laid in place (excavation or and in such depths that the thickness of the completed g shall not, as a rule, exceed the following amounts: , according to the stress, 6 to 8 ins.; for "wet" concrete, ess, 8 to 12 ins. The individual layers shall be laid, t right angles to the direction of pressure, and where- ble the layers shall be laid parallel with the direction ing in the direction of the pressure insures a uniform concrete in the whole section, and is to be desired pri- is impracticable or where the quality of the tamping on would suffer, as for instance in a flat arch, care shall mping shall not tend to displace the individual layers.) a rule, be laid on fresh concrete to obtain a sufficiently mooth surfaces after tamping shall be roughened; and face shall be roughened by sweeping with steel brooms. rich in stone, spread on a thin layer of wet mortar. or rectangular, 4 to 6½ ins. on a side and 22 to 27 lbs. d. The "amount of tamping" required depends on obtained and the character of the concrete, "dry" or oncrete requires more effort in tamping and greater n, but it generally produces for the same amount of istance than the "wet" concrete. Wet concrete may s to be injured by separation of the ingredients). Con- "freezing weather" if the injurious effect of freezing oper means. Frozen materials shall not be used.

IV. REINFORCED CONCRETE.

rete is now used in various classes of construction of is a partial list: Abutments, aqueducts, arches, bins etc.), bridges, buildings (foundations, walls, columns, neys, columns, cisterns, cross-ties, dams, docks, footings, tions, girders, graving docks, piers, piles, pipes, poles, taining walls, sewers, sidewalks (supported), stand- ks, ties (railroad), tunnels, etc., etc. The cheapening manufacture, due to the improved rotary kiln, the e recent types of machines for preparing broken stone v cost of manufacture of the simple forms of steel used, vith which the materials may be fabricated at the site, n labor, are factors which have enetred largely into cent rapid development of this class of construction, competition with what are ordinarily known as steel- uctions. The added strength due to the steel rein- a more scientific and economical distribution of material, ompetitor in many structures formerly built of plain- asonry, as bridges, building walls, retaining walls, etc. its merits are its preservative- and fire-resisting quali- actically permanent construction.

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The Preservative Qualities of Cement, with regard to iron or steel imbedded in the concrete, are pretty clearly established. Iron exposed to pure, dry air, or in moist air free from carbonic oxide, and at ordinary temperatures, will not rust. Rust is formed through the combined agency of carbonic acid and moisture, and as carbonic acid has a greater affinity for the cement mortar, the iron will not be attacked while the former is present, and encases it in a thorough manner. The few isolated cases recorded where rust has been found must be explained on the theory of some defect in workmanship or materials, as they stand out in marked contrast with practically all the accepted results of investigation on this subject. Perhaps the most notable instance of what may indirectly be called a long-time test was the finding of a small piece of "bright" iron in mortar taken from the base of the Egyptian obelisk (of granite, 70 ft. long, weighing 200 tons,) re-erected in Central Park, New York City, 1881. Numerous instances may be cited of iron being found in perfect condition after from 10 to 25 years' service in mortar or concrete (both stone and cinder) entering into various kinds of construction, both in and out of water (salt and fresh). To insure absolute protection, the bright rods may be coated with fresh cement when placed; and it is well to remember that they should be embedded thoroughly in the concrete, the latter to be a wet mixture, or a medium, well rammed, (the former preferred) to guard against voids.

The Fire-Resisting Qualities of Concrete and concrete-steel construction in buildings were illustrated in the Baltimore fire, in 1904. Although no construction was found to be really fireproof, it was noticeable that concrete made from steam-boiler cinders and Portland cement seemed to act the best; and that stone concrete stood fully as well as, if not better than, terra cotta. Results of other tests, specially made for comparison, point to the same conclusion. We may safely say, then, that reinforced concrete has all the fire-resisting qualities which may be expected in the best materials of construction of the present day.

The **Proportions** used in mixing concrete for reinforced-concrete construction almost invariably range from 1 cement, 2 sand, and 4 broken stone or gravel or cinders, to a 1 : 3 : 6 mix. Portland cement should be used. Clean, sharp sand is usually specified, but some engineers claim that sand well worn and rounded is best, and also cheapest as the voids are less, requiring less cement. A 1 : 3 : 6 mix is suitable for column- and wall construction; a 1 : 2½ : 5 mix, for girders; and a 1 : 2 : 4 mix for the lighter construction as floor slabs and small, shallow beams subject to direct impact from live load. The broken stone is also graded, say, from ¼ in. upward to conform somewhat with the above proportions, the finer stone being used with the 1 : 2 : 4 mix, for the thinner masses.

Calculation of Beams.—The following analysis assumes that the resisting moment of the beam is made up of two moments: (1) that above the neutral axis, due to the concrete area in compression; and (2) that below the neutral axis, due to the steel area in tension, no account being taken of the concrete in tension. This is in accordance with the best practice at the present time.

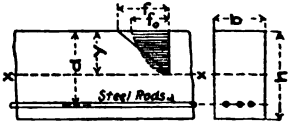

Fig. 11.

Notation.

Let h = height of beam, in ins. (Fig. 11);
b = breadth of beam, in ins.;
$X - X$ = neutral axis;
d = depth to center of reinforcement, below top of beam, in ins.;
$h - d$ = distance from bottom of beam to center of reinforcement, in ins. (either d or $h - d$ may be fixed arbitrarily);
f = allowable compressive stress in lbs. per sq. in. on concrete to be used with the straight-line equivalent (instead of the stress f_0 due to the actual stress-strain diagram—shaded):

*f_2 mos. = 720 lbs. per sq. in. for good	1 : 2 : 4 concrete.	Factor safety = 4.
*f_2 mos. = 660 " " " " "	1 : 2½ : 5 "	
*f_2 mos. = 600 " " " " " "	1 : 3 : 6 "	

* These values are about 10% greater than the values of f_0 which would be used with the exact stress-strain curve.

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F = allowable tensile stress in lbs. per sq. in. on the steel rods (15000 for buildings);
a = sectional area of the steel rods, in sq. ins.;
k = ratio of modulii of elasticity of steel and concrete:
$k = 13$ for steel and good 1 : 2 : 4 concrete.
$k = 14$ " " " " 1 : 2½ : 5 "
$k = 15$ " " " " 1 : 3 : 6 "
y = distance, in ins., from top of beam to neutral axis.
M'' = bending moment = resisting moment, inch-lbs.
M' = " " = " " ft.-lbs.

General Formulas:

Resisting moment, $M'' = \frac{1}{2} f b y^2 + aF(d-y)$ ..... (1)

For conditions of equilibrium, the algebraic sum of the horizontal forces at any section must equal zero ($\Sigma H = O$), hence we have, neglecting the tension in concrete, below the neutral axis, $a F = \frac{1}{2} f b y$ ..... (2)

From the ratio k of the modulii of elasticity of the materials, and the relative position of the neutral axis, we have, $k f (d-y) = Fy \therefore y = d\frac{kf}{F+kf}$ .. (3)

Combining (2) and (3), we have:

Distance to neutral axis, $y = \frac{ak}{b}\left(\sqrt{1+\frac{2bd}{ak}}-1\right)$* ..... (4)

Breadth of beam, $b = \frac{2ak}{y^2}(d-y)$ ..... (4a)

Area of steel rods, $a = \frac{by^2}{2k(d-y)}$ ..... (4b)

Also, from (2), $a = y\frac{fb}{2F}$ ..... (4c)

Depth to center of rods, $d = y\left(1+\frac{by}{2ak}\right)$ ..... (4d)

Combining (1) and (2) we have:

Stress in the concrete, $f = \frac{6M''}{by(3d-y)}$ lbs. per sq. in ..... (5)

Stress in the steel,..... $F = \frac{3M''}{a(3d-y)}$ " " " " ..... (6)

Also, from (2), $F = \frac{fby}{2a}$ " " " " ..... (6a)

The following values of f† may be assumed for the three standard concrete mixes: (See Table 4, next page.)

Concrete. Mix.	Time.	Ultimate.	Factor 4.	Factor 5.	Factor 6.
1 : 2 : 4	1 mo.	$f = 2400$.	$f = 600$.	$f = 480$.	$f = 400$.
"	2 mos.	2880.	720.	576.	480.
"	3 mos.	3000.	750.	600.	500.
"	6 mos.	3600.	900.	720.	600.
1 : 2½ : 5	1 mo.	2200.	550.	440.	367.
"	2 mos.	2640.	660.	528.	440.
"	3 mos.	2750.	688.	550.	458.
"	6 mos.	3300.	825.	660.	550.
1 : 3 : 6	1 mo.	2000.	500.	400.	333.
"	2 mos.	2400.	600.	480.	400.
"	3 mos.	2500.	625.	500.	417.
"	6 mos.	3000.	750.	600.	500.

* Equation (3) gives y when a is unknown; it is directly proportional to d.
† For use with straight-line formulas adopted above. The values of f_0, or actual maximum compression on the outer element of the concrete, will be about 9% less.

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4.—PROPERTIES OF REIN.-CONC. BEAMS 1" WIDE ($b=1$) AND OF VARIOUS DEPTHS, d OR h. (Fig. 11, page 444.)

Data: Concrete, 1 : 3 : 6; age, 2 mos.; $f=600$. Steel, $F=15000$. $k=15$. Factor of safety, 4. For factor of 5, mult. M' of table by $\frac{4}{5}$; for 6, by $\frac{2}{3}$.

Depth d (Fig. 11.) Ins.	Resisting Moment M'. Ft.-Lbs.	Dist. y to Neutral Axis. Ins.	Area a of Steel Rods. Sq. Ins.	Ratio $\frac{a}{d}$	Maximum Depth h. Ins.	Approx. Ratio $a \div h$. Ins.	Wt. per Lin. Ft. of Beam, at 150 Lbs. per Cu. Ft. Lbs.
1	8.2	.375	.0075	.0075	1.5	.0050	1.56
2	32.8	.75	.0150	"	2.71	.0055	2.82
3	73.8	1.125	.0225	"	3.87	.0058	4.03
4	131.2	1.50	.03	"	5.	.0060	5.21
5	205.8	1.875	.0375	"	6.12	.0061	6.38
6	295.	2.25	.045	"	7.22	.0062	7.52
7	402.	2.625	.0525	"	8.32	.0063	8.67
8	525.	3.00	.06	"	9.41	.0064	9.80
9	664.	3.375	.0675	"	10.5	.0064	10.94
10	820.	3.75	.075	"	11.58	.0065	12.06
11	902.	4.125	.0825	"	12.66	.0065	13.19
12	1181.	4.50	.09	"	13.73	.0066	14.30
14	1608.	5.25	.105	"	15.87	.0066	16.53
16	2100.	6.00	.12	"	18.	.0067	18.75
18	2658.	6.75	.135	"	20.12	.0067	20.96
20	3281.	7.50	.15	"	22.24	.0067	23.17
22	3970.	8.25	.165	"	24.35	.0068	25.36
24	4725.	9.00	.18	"	26.45	.0068	27.55

Formulas for above table, reduced from General Formulas, preceding, are: $y=\frac{2}{3}d$; $a=.02\times\frac{2}{3}y=.0075d$; $M'=8.203\,d^2$. These values will vary, of course, with variations in the values of f, F, and k.

Suggested Formulas for Reinforced Concrete Construction (From Majority Report of Special Committee of Am. Soc. C. E. on Concrete and Reinforced Concrete. Proc. Am. Soc. C. E., Feb., 1909).—These formulas are based upon the assumptions and principles given in the chapter on Design (see Trans. A. S. C. E., Vol. LXVI). For WORKING STRESSES, see: Sec. 31, Beams, page 585; Sec. 32, Columns, page 609.

A. STANDARD NOTATION.

a. Rectangular Beams.

f_s = tensile unit stress in steel.
f_c = compressive unit stress in concrete.
E_s = modulus of elasticity of steel.
E_c = modulus of elasticity of concrete.
$n=E_s \div E_c$.
M = moment of resistance, or bending moment in general.
A = steel area.
b = breadth of beam.
d = depth of beam to center of steel.
k = ratio of depth of neutral axis to effective depth, d.
z = depth of resultant compression below top.
j = ratio of lever arm of resisting couple to depth, d.
$jd=d-z$ = arm of resisting couple.
p = steel ratio (not percentage).

I-Beams.

b = width of flange.
b' = width of stem.
t = thickness of flange.

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Beams Reinforced for Compression.

A' = area of compressive steel.
p' = steel ratio for compressive steel.
f_c' = unit compressive stress in steel.
C = total compressive stress in concrete.
C' = total compressive stress in steel.
d' = depth to center of compressive steel.
z = depth to resultant of C and C'.

Shear and Bond.

V = total shear.
v = shearing unit stress.
u = bond stress per unit area of bar.
o = circumference or perimeter of bar.
Σ_0 = sum of the perimeters of all bars.

Columns.

A = total net area.
A_s = area of longitudinal steel.
A_c = area of concrete.
P = total safe load.

B. FORMULAS.

a. Rectangular Beams.

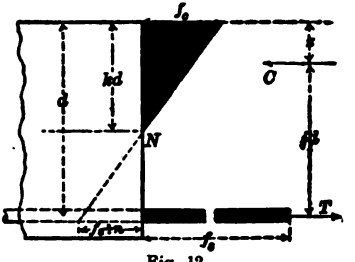

Fig. 12.

Position of neutral axis,

$$k = \sqrt{2pn + (pn)^2} - pn.$$

Arm of resisting couple, $\quad j = 1 - \frac{1}{3}k.$

For $f_s = 15\,000$ to $16\,000$ and $f_c = 600$ to 650, j may be taken at $\frac{7}{8}$.]

Fiber stresses, $\quad f_s = \frac{M}{Ajd} = \frac{M}{pjbd^2}. \qquad f_c = \frac{2M}{jkbd^2} = \frac{2pf_s}{k}.$

Steel ratio, $\quad p = \frac{1}{2}\frac{1}{\frac{f_c}{f_s}\left(\frac{f_s}{nf_c}+1\right)}.$

b. T-Beams.

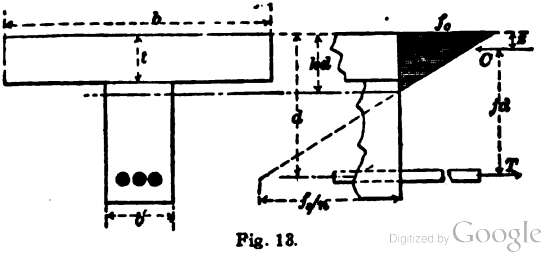

Fig. 13.

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Case I. When the neutral axis lies in the flange, use the formulas for rectangular beams.

Case II. When the neutral axis lies in the stem.

The following formulas neglect the compression in the stem:

Position of neutral axis,

$$kd = \frac{2ndA + bt^2}{2nA + 2bt}.$$

Position of resultant compression,

$$z = \frac{3kd - 2t}{2kd - t} \cdot \frac{t}{3}.$$

Arm of resisting couple,

$$jd = d - z.$$

Fiber stresses,

$$f_s = \frac{M}{Ajd}. \quad f_c = \frac{Mkd}{bt(kd - \frac{1}{2}t)jd} = \frac{f_s}{n} \cdot \frac{k}{k-1}.$$

(For approximate results, the formulas for rectangular beams may be used.)

The following formulas take into account the compression in the stem; they are recommended where the flange is small compared with the stem.

Position of neutral axis,

$$kd = \sqrt{\frac{2ndA + (b-b')t^2}{b'} + \left(\frac{nA + (b-b')t}{b'}\right)^2} - \frac{nA + (b-b')t}{b'}.$$

Position of resultant compression,

$$z = \frac{(kdt^2 - \frac{2}{3}t^3)b + [(kd-t)^2(t + \frac{1}{3})(kd - t)]b'}{t(2kd - t)b + (kd - t)^2 b'}.$$

Arm of resisting couple,

$$jd = d - z.$$

Fiber stresses,

$$f_s = \frac{M}{Ajd}. \quad f_c = \frac{2\,Mkd}{[(2kd - t)bt + (kd - t)^2 b']jd}.$$

c. Beams Reinforced for Compression.

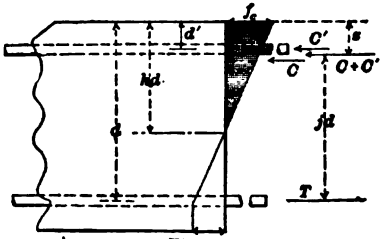

Fig. 14.

Position of neutral axis,

$$k = \sqrt{2n\left(p + p'\frac{d'}{d}\right) + n^2(p + p')^2} - n(p + p').$$

Position of resultant compression,

$$z = \frac{\frac{1}{3}k^3 d + 2p'nd'\left(k - \frac{d'}{d}\right)}{k^2 + 2p'n\left(k - \frac{d'}{d}\right)}$$

Arm of resisting couple,

$$jd = d - z.$$

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Fiber stresses

$$f_c = \frac{6Mk}{bd^2\left[3k - k^2 + \frac{6p'n}{k}\left(k - \frac{d'}{d}\right)\left(1 - \frac{d'}{d}\right)\right]}$$

$$f_s = \frac{M}{pjbd^2} = nf_c\frac{1-k}{k}.$$

$$f_s' = nf_c\frac{k - \frac{d'}{d}}{k}.$$

d. Shear, Bond, and Web Reinforcement.

In the following formula, Σ_0 refers only to the bars constituting the tension reinforcement at the section in question and $j\,d$ is the lever arm of the resisting couple at the section.

For rectangular beams,

$$v = \frac{V}{bjd}.$$

$$u = \frac{V}{jd\,.\,\Sigma_0}.$$

[For approximate results, j may be taken at $\frac{7}{8}$.]

The stresses in web reinforcement may be estimated by using the following formulas:

Vertical reinforcement,

$$P = \frac{Vs}{jd}.$$

Reinforcement inclined at 75°,

$$P = 0.7\frac{Vs}{jd},$$

in which P = stress in single reinforcing member, V = proportion of total shear assumed as carried by the reinforcement, and s = horizontal spacing of the reinforcing members.

The same formulas apply to beams reinforced for compression as regards shear and bond stress for tensile steel.

For T-beams,

$$v = \frac{V}{b'jd}, \quad u = \frac{V}{jd\,.\,\Sigma_0}.$$

[For approximate results, j may be taken at $\frac{7}{8}$.]

e. Columns.

Total safe load,

$$P = f_c(A_c + nA_s) = f_cA(1 + (n-1)p).$$

Unit stresses,

$$f_c = \frac{P}{A(1 + (n-1)p)}$$

$$f_s = nf_c.$$

V. MIXED MASONRY.

The object of using mixed masonry is to give a substantial looking, finished face, using a better class of masonry for this purpose, and using a cheaper quality for interior and back of wall. Strictly speaking, most masonry is more or less mixed—ashlar backed with rubble, face-brick backed with common brick, etc.; but the term "mixed masonry" is generally restricted to stone facing with brick backing.

The *bond* between stone and brick, or stone and concrete, is often a source of weakness. When interior brick walls are joined to stone face-walls, iron "cramps" are generally employed.

VI.—CONCRETE-BLOCK MASONRY.

Solid concrete blocks are often used in submarine work, as fo in breakwater construction, in preference to laying the concret

Hollow concrete blocks are made in many shapes and are usec ing construction. The completed wall should generally be not two-thirds solid, although in some places as much as 40- to 50 pe voids is allowed. The surface of the blocks should be rich in ce have a finer aggregate than the interior. The best blocks are machine under heavy (usually hydraulic) pressure. The writer many blocks turned out by the small, portable machines in wl tamping is required, and the results were invariably inferior. T are laid in the wall in cement mortar, and sometimes iron cramp for bonding them together. Outside cement plaster or stucco of quality may be laid directly on the blocks, and gives a finished ap Inside plaster should not be laid without furring and lathing, if t are porous or inferior in quality on account of moisture and fro the blocks have been waterproofed.

Specifications for Hollow Concrete Building Blocks.—The fo from the Rules and Regulations governing the use and manu Hollow Concrete Building Blocks in the City of Philadelphia— Building Inspection.

Rules and Regulations.

1. Hollow concrete building blocks may be used for building or less in height, where said use is approved by the Bureau of Inspection; provided, however, that such blocks shall be compo least 1 part standard Portland cement, and not to exceed 5 pa coarse, sharp sand or gravel, or a mixture of at least 1 part Portla to 5 parts crushed rock or other suitable aggregate. Provided that this section shall not permit the use of hollow blocks in pa Said party walls must be built solid.

2. All material to be of such fineness as to pass a ¼-in. ring a from dirt or foreign matter. The material composing such bl be properly mixed and manipulated, and the hollow space in s shall not exceed the percentage given in the following table fo height walls, and in no case shall the walls or webs of the block thickness than ¼ of the height. The figures given in the tal represent the percentage of such hollow space for different heig

Stories.	1st.	2nd.	3rd.	4th.	5th.	6th.
1 and 2.....	33	33	....	....	....	....
3 and 4 ...	25	33	33	33	....	....
5 and 6 ...	20	25	25	33	33	33

3. The thickness of walls for any building where hollow concr are used shall not be less than is required by law for brick walls.

4. Where the face only is of hollow concrete building bloc backing is of brick, the facing of hollow concrete blocks must b bonded to the brick either with headers projecting 4 ins. into work, every fourth course being a heading course, or with appr no brick backing to be less than 8 ins. Where the walls are mad of hollow concrete blocks, but where said blocks have not the s as the wall, every fifth course shall extend through the wall, secure bond. All walls, where blocks are used, shall be laid up i cement mortar.

5. All hollow concrete building blocks, before being used i struction of any building in the City of Philadelphia, shall hav the age of at least 3 weeks.

6. Wherever girders or joists rest upon walls so that there is trated load on the block of over 2 tons, the blocks supporting th joists must be made solid. Where such concentrated load shal tons, the blocks for 2 courses below, and for a distance extendi 18 ins. each side of said girder, shall be made solid. Where the l wall from the girder exceeds 5 tons, the blocks for 3 courses bene be made solid with similar material as in the blocks. Wherever decreased in thickness, the top course of the thicker wall to be

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7. Provided always, that no wall, or any part thereof, composed of hollow concrete blocks shall be loaded to an excess of 8 tons per superficial foot of the area of such blocks, including the weight of the wall, and no blocks shall be used that have an average crushing at less than 1000 pounds per square inch of area at the age of 28 days; no deduction to be made in figuring the area for the hollow spaces.

8. All piers and buttresses that support loads in excess of 5 tons, shall be built of solid concrete blocks for such distance below as may be required by the Bureau of Building Inspection. Concrete lintels and sills shall be reinforced by iron or steel rods in a manner satisfactory to the Bureau of Building Inspection, and any lintels spanning over 4 feet six inches in the clear shall rest on solid concrete blocks.

9. Provided, that no hollow concrete building blocks shall be used in the construction of any building in the City of Philadelphia, unless the maker of said blocks has submitted his product to the full test required by the Bureau of Building Inspection, and placed on file with said B. of B. I. a certificate from a reliable testing laboratory showing that samples from the lot of blocks to be used have successfully passed the requirements of the B. of B. I., and filing a full copy of the test with the Bureau.

10. A brand or mark of identification must be impressed in, or otherwise permanently attached to, each block for purpose of identification.

11. No certificate of approval shall be considered in force for more than four months, unless there be filed with the B. of B. I., in the City of Phila., at least once every four months following, a certificate from some reliable physical testing laboratory showing that the average of three (3) specimens tested for compression, and three (3) specimens tested for transverse strength, comply with the requirements of the B. of B. I.; samples to be selected either by a Building Inspector or by the laboratory, from blocks actually going into construction work. Samples must not be furnished by the contractors or builders.

12. The manufacturer and user of any such hollow concrete blocks as are mentioned in this regulation, or either of them, shall, at any and all times, have made such tests of the cements used in making such blocks, or such further tests of the completed blocks, or of each of these, at their own expense, and under the supervision of the B. of B. I., as the Chief of said Bureau shall require.

13. The cement used in making said blocks shall be Portland cement, and must be capable of passing the minimum requirements as set forth in the "Standard Specifications for Cement" by the American Society for Testing Materials.

14. Any and all blocks, samples of which, on being tested under the direction of the B. of B. I., fail to stand at 28 days the tests required by this regulation, shall be marked "condemned" by the manufacturer or user, and shall be destroyed.

15. No concrete blocks shall be used in the construction of any building within the City of Phila. until they shall have been inspected, and average samples of the lot tested, approved and accepted by the Chief of Building Inspectors.

Method of Testing Hollow Blocks.

1. These regulations shall apply to all such new materials as are used in building construction, in the same manner and for the same purposes as stones, brick, concrete are now authorized by the Building Laws, when said new material to be substituted departs from the general shape and dimensions of ordinary building brick, and more particularly to that form of building material known as "Hollow Concrete Block," manufactured from cement and a certain addition of sand, crushed stone, or similar material.

2. Before any such material is used in buildings, an application for its use and for a test of the same must be filed with the Chief of the B. of B. I. A description of the material and a brief outline of its manufacture and proportions of the materials used must be embodied in the application.

3. The material must be subjected to the following tests: Transverse, Compression, Absorption, Freezing, and Fire. Additional tests may be called for when, in the judgment of the Chief of the B. of B. I., the same

may be necessary. All such tests must be made in some laboratory of recognized standing, under the supervision of the Engineer of the B. of B. I. The tests will be made at the expense of the applicant.

4. The results of the tests, whether satisfactory or not, must be placed on file in the B. of B. I. They shall be open to inspection upon application to the Chief of the Bureau, but need not necessarily be published.

5. For the purposes of the tests, at least 20 samples of test pieces must be provided. Such samples must represent the ordinary commercial product. They may be selected from stock by the Chief of the B. of B. I., or his representative, or may be made in his presence, at his discretion. The samples must be of the regular size and shape used in construction. In cases where the material is made and used in special shapes and forms, too large for testing in the ordinary machines, smaller sized specimens shall be used as may be directed by the Chief of Building Inspection, to determine the physical characteristic specified in Section 3.

6. The samples may be tested as soon as desired by the applicant, but in no case later than 60 days after manufacture.

7. The weight per cubic foot of the material must be determined.

8. Tests shall be made in series of at least five, except that in the fire tests a series of two (four samples) are sufficient. Transverse tests shall be made on full sized samples. Half samples may be used for the crushing, freezing, and fire tests. The remaining samples are kept in reserve, in case unusual flaws or exceptional or abnormal conditions make it necessary to discard certain of the tests. All samples must be marked for identification and comparison.

9. The Transverse test shall be made as follows: The samples shall be placed flatwise on two rounded knife edge bearings set parallel seven inches apart. A load is then applied on top, midway between the supports, and transmitted through a similar rounded knife edge, until the sample is ruptured. The modulus of rupture shall then be determined by multiplying the total breaking load in pounds by twenty-one (three times the distance between supports in inches) and then dividing the result thus obtained by twice the product of the width in inches by the square of the depth in inches: $R = \frac{3\ w\ l}{2\ b\ d^2}$. No allowance should be made in figuring the modulus of rupture for the hollow spaces.

10. The Compression test shall be made as follows: Samples must be cut from blocks so as to contain a full web section. The sample must be carefully measured, then bedded flat-wise in Plaster of Paris, to secure a uniform bearing in the testing machine, and crushed. The total breaking load is then divided by the area in compression in square inches. No deduction to be made for hollow spaces; the area will be considered as the product of the width by the length.

11. The Absorption test must be made as follows: The sample is first thoroughly dried to a constant weight. The weight must be carefully recorded. It is then placed in a pan or tray of water, face downward, immersing it to a depth of not more than one-half inch. It is again carefully weighed at the following periods: Thirty minutes, four hours, and forty-eight hours, respectively, from the time of immersion, being replaced in the water in each case as soon as the weight is taken. Its compressive strength, while still wet, is then determined at the end of the forty-eight hours' period, in the manner specified in section 10.

12. The Freezing test is made as follows: The sample is immersed, as described in section 11, for at least four hours, and then weighed. It is then placed in a freezing mixture or a refrigerator, or otherwise subjected to a temperature of less than 15 degrees F. for at least 12 hours. It is then removed and placed in water, where it must remain for at least one hour, the temperature of which is at least 150 degrees F. This operation is repeated ten (10) times, after which the sample is again weighed while still wet from the last thawing. Its crushing strength should then be determined as called for in section 10.

13. The Fire test must be made as follows:—Two samples are placed in a cold furnace in which the temperature is gradually raised to 1700 degrees F. The test piece must be subjected to this temperature for at least 30 minutes. One of the samples is then plunged in cold water (about 50

degrees to 60 degrees F.) and the results noted. The second sample is permitted to cool gradually in air, and the results noted.

14. The following requirements must be met to secure the acceptance of the materials: The Modulus of Rupture for concrete blocks at 28 days old must average 150 and must not fall below 100 in any case. The ultimate compressive strength at 28 days must average 1000 lbs. per sq. in., and must not fall below 700 in any case. The percentage of absorption (being the weight of water absorbed divided by the weight of the dry sample), must not average higher than 15% and must not exceed 25% in any case. The reduction of compressive strength must not be more than 33⅓%, except that when the lower figure is still above 1000 lbs. per sq. in., the loss in strength may be neglected. The freezing and thawing process must not cause a loss in weight greater than 10%, nor a loss in strength of more than 33⅓%: except that when the lower figure is still above 1000 lbs. per sq. in., the loss in strength may be neglected. The fire test must cause the material to disintegrate.

15. The approval of any material is given only under the following conditions: *a.* A brand mark for identification must be impressed on, or otherwise attached to, the material. *b.* A plant for the production of the material must be in full operation when the official tests are made. *c.* The name of the firm or corporation and the responsible officers must be placed on file with the Chief of B. of B. I., and changes in same promptly reported. *d.* The chief of the B. of B. I. may require full tests to be repeated on samples selected from the open market when, in his opinion, there is any doubt as to whether the product is up to the standard of these regulations, and the manufacturer must submit to the B. of B. I., once in at least every 4 months, a certificate of tests showing that the average resistance of 3 specimens to cross breaking and crushing are not below the requirements of these regulations. Such tests must be made by some laboratory of recognized standing, on samples selected either by a Building Inspector or the laboratory, from material actually going into construction, and not on ones furnished by the manufacturer. *e.* In case the results of tests made under this condition (*d.*) should show that the standard of these regulations is not maintained, the approval of this bureau to the manufacturer of said blocks will at once be suspended or revoked.

EXCERPTS AND REFERENCES.

Permeability of Concrete Under High Water Pressures (By J. B. McIntyre and A. L. True. Thesis, Thayer School of Civ. Engr., April, 1902; Eng. News, June 26, 1902).—Extensive tables of tests, using different proportions of concrete, and pressures of 20, 40, and 80 lbs., time 2 hours. "Of the various mixtures we may safely choose either 1:2:4 or 1:2.5:4, on account of their simplicity and the ease with which they may be proportioned, either for hand or machine mixing. In extreme cases however, it might be advisable to use one of the richer mixtures."

Notes on Concrete Construction in Government Fortifications—With Waterproofing Data (Report of Chief of Engrs. of U. S. A. for 1902; Eng. News, April 2, 1903).—Subjects treated are: "Tests to show suitability of various kinds of sand for use in concrete," by Capt. Harry Taylor; "Damp proofings for ceilings of gun emplacements," by Maj. G. W. Goethals; "Damp proofing sunken magazines and rooms," by Col. P. C. Hains; "Stoppage of leaks in concrete with linseed oil," by Capt. E. W. Van C. Lucas; "Stoppage of leaks in concrete with asphaltum and oil," by Maj. W. T. Rossell and Capt. S. Crosby; "Asphaltum and alum and lye waterproofing," by Capt. W. C. Langfitt; "A sliding rupture caused by tarred paper waterproofing," by Maj. W. T. Rossell and Capt. S. Crosby.

The Efficiency of Concrete-Mixing Machines (By Clarence Coleman. Eng. News, Aug. 27, 1903).—**The Necessity of Thorough Mixing.**—The amount of cement as determined for any concrete should always be weighed, not measured, as the volume of cement is a variable. The entire amount of cement should be added before mixing is commenced, and then mixed dry before water is added. The proper amount of water to be added requires judgment after visual inspection, as the hygrometric condition of the sand is a variable; hence the entire mass of the materials should be plainly visible to the person adding the water. **Proportion of Water in**

Concrete.—Gives a table showing the strength of mortar due to varying proportions of water. **Classification of Machines.**—Batch Mixers (7 types described); Continuous Mixers (5 types described). **Comparison of Concrete Mixers.**—Descriptions with illustrations.

The Method of Finishing the Concrete Surfaces of Philadelphia Bridges (By H. H. Quimby. Eng. News, Feb. 4, 1904).—Remove the forms while the concrete is still "green," and simply wash the surface with water, squirting it on with a nozzle if the work is soft enough, or, if harder, using a scrubbing brush. If the cement is too hard to wash off it must be cut by hard rubbing with a brick or a wooden float and sand, using plenty of water. The removal of the cement exposes the sand and grit or pebbles or stone—whatever the aggregate may be—leaving a surface that is mostly stone, and is probably as little subject to discoloration and cracks as stone, and as durable as any plastic material can be made.

Tests of Adhesion and Initial Stress of Steel in Concrete (By S. W. Emerson. Eng. News, Mar. 10, 1904).—Diagrams showing results of tests.

Materials Required to Make Different Classes of Concrete for Connecticut Ave. Bridge, Wash., D. C. (By W. J. Douglas and A. W. Dow. Eng. News, Mar. 10, 1904).—

Class	A.	B.	B.	C.
Mixture (cem., sand, gravel, stone)	1:2:0:4½	1:2¼:0:6	1:2½:3:3	1:3:10:0
Cement, cu. ft	4.5	4.5	4.5	4.5
Sand, cu. ft	9.0	11.25	11.25	13.5
Gravel, cu. ft	0.0	0.0	18.5	45.0
Broken Stone, cu. ft	20.25	27.0	13.5	0.0
Yielded, rammed concrete, cu. ft	21.4	27.66	27.66	45.0

Note that 4.5 cu. ft. (Vulcanite) cement = 1 bbl. = 4 bags = 378.25 lbs.

French Governmen. Rules for the Design and Construction of Reinforced Concrete (Eng. News, Mar. 21, 1907).—Discussion of formulas, etc.

Expansion Joints in Concrete Structures, with Special Reference to Block Construction in Drydocks and Reservoirs (L. F. Bellinger. Eng. News, May 2, 1897).—**Brightwood Reservoir, Wash., D. C.**—Dimensions, 415 x 300 x 20 ft. deep; plain concrete with a vertical outer face and a sloping inner face of concrete; the vertical outer face having a heavy earth embankment against it. The concrete wall was built in alternate sections not exceeding 50 ft. long. Vertical key ways 6 ins. square, ½ of which was in each section, were built for expansion joints and first filled with aphalt. The concrete proportions were 1:2½:5. In placing the fresh concrete on that which was already set, the surface was carefully washed and rich mortar was placed in order to make a joint. The forms were left on from one to two days to harden; surfaces which were to be exposed to the air or water had a face of 1:2 mortar, 6 ins. thick, placed at the same time the concrete backing was placed. The concrete floor was laid in two layers, each 5 ins. thick, and laid in blocks 15 ft. square. In the joints was placed a 3-ply felt, up to bevels, which topped off each joint. After the shrinkage cracks appeared, these bevels were run full of asphalt and then 2 ins. of 1:2 mortar was troweled over everything. The experience with this reservoir is interesting since the asphalt in the key ways cracked and leaked in cold weather, and in warm weather ran out beneath the water surface. The asphalt was finally taken out and replaced with carefully selected puddling clay and rammed into the key ways. The contraction of the concrete permitted this material also to run out through the expansion joints into the reservoir and caused leaks to appear through the embankment. After this occurred, the puddling clay was taken out and plain loam, with clay, sand, grass roots, dirt and fibrous material, were rammed into the key ways. This material has successfully prevented leaks up to the present time. **Other Data.**—About a dozen other works described; also Expansion Joints in Drydocks (illus.); Block Construction; Key Ways, etc.

The Use of Reinforced Concrete in Engineering Structures (Trans. A. S. C. E., Vol. LXI).—Discussions by E. P. Goodrich, Edwin Thacher, S. E. Thompson, W. H. Burr, T. K. Thompson, and others.

Traveling Concrete Mixers (Eng. News, Aug. 5. 1909).—Illustrated: U. S. Steel Mixer Co.

The Bonding of New to Old Concrete (By E. P. Goodrich. Trans. A. S. C. E., Vol. LXIV., Sept., 1909).—Article contains: Literature on the subject; patented processes; published experiments; Goodrich's experiments.

Making Concrete Waterproof (By Ira O. Baker. The Univ. of Ill. "Technograph," No. 23, 1908-09; Eng. News, Oct. 7, 1909).—Description of alum-and-soap waterproofing compound, etc.

The Compressive Strength of Coke Concrete (By J. M. Lewis. Eng. Rec., Oct. 30, 1909).—Comparison of tests of concrete columns made with stone and with coke cinder. Weight of coke-cinder concrete, 97 lbs. per cu. ft.; stone concrete, 150 lbs. per cu. ft.

Impurities in Sand for Concrete (Informal Discussion—Trans. A. S. C. E., Vol. LXV., Dec., 1909).—Sand tests, washing, etc.

Office Methods in a Concrete Designing Office (Eng. Rec., June 11, 1910). —Illustrations: Beam design sheet, half of column design sheet, typical details for T-stirrups, upper part of splice rod detail sheet, upper part of beam detail sheet.

The Effect of Alkali on Concrete (By G. G. Anderson. Trans. A. S. C. E., Vol. LXVII., June, 1910).

Corrosion of Iron Embedded in Concrete (By G. F. Shaffer. Eng. Rec., July 30, 1910).—Gives results of tests at Mass. Inst. Tech., obtaining some data on the effect of currents of low potential on embedded steel.

Computation of Reinforced Concrete Flat Slabs (By L. F. Brayton. Eng. Rec., Aug. 27, 1910).—Comparison with the McMillan method.

Exterior Treatment of Concrete Surfaces: Committee Report to the Nat'l Ass. Cement Users (Eng. News, Sept. 15, 1910).—(a) Effect of material and workmanship on surface. (b) Removal of surface in various ways. (c) Coating surfaces in various ways. (d) Defects, blemishes of various sorts, and remedies. (e) Costs.

Investigations on the Slip of Rods Imbedded in Concrete Beams ("Armitier Beton," Sept., 1910; Eng. Rec., Nov. 12, 1910).—Gives the slip at four points of the beams under incremental twin loadings. Beams are 12 x 12-ins., 74-in. span. Rod, 0.94 in. dia.

Oil-Mixed Concrete as a Waterproofing Material (By T. W. Symons, Eng. News, Dec 15, 1910).—"The peculiarly waterproof and water-repellant quality of this oil-concrete, combined with its strength and greater density renders it a material remarkably adapted to such canal structures as culverts, locks, the canal prism troughs crossing the Irondequoit Valley and the Medina Gorge, to the core walls of earthen dams like that at Hinckley, etc." Oil to the amount of about 10% of the weight of the cement gives very satisfactory results. The oil costs about 6 to 7 cents per gallon, or about 40 to 50 cents per cu. yd. of concrete; or about 60 to 70 cents more per cu. yd. of concrete (including handling and incorporation) than plain concrete, in place.

Specifications for Scrubbed Concrete Surface (By H. H. Quimby. Paper before Annual Conv. of Nat'l Cement Users, New York, Dec. 10, 1910; Eng. News, Dec. 22, 1910).—Sent by that Convention to a letter ballot of the Association to be adopted as a standard of the Association.

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Some important Illustrations.

Description.	
	Eng. News.
Spec. for plain and rein.-conc. and steel rein., A. R. E. & M.W.A	Apr. 14, '10
Stand. Spec. (Assn. of Am. Steel Mf'rs) conc. rein. bars.	June 16, '10
	Eng. Rec.
Standard forms for a reinforced-concrete viaduct	Feb. 13, '09
Plant for washing and screening concrete aggregates	June 26, '09
A traveling shed for cold-weather viaduct concreting	July 10, '09
Tests of bond between steel and concrete.—H. C. Berry	Sept. 4, '09
Analysis of concrete-bridge failures.—C. R. Young	Apr. 16, '10
Clips and rods for tying concrete to steel members	Sept. 3, '10
Diagram of bending moments in concrete columns and beams	Oct. 15, '10

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26.—STEREOTOMY.

This term in its broadest sense includes the subject of Stone Cutting (see page 426.), but it is here restricted to the preparation of the drawings and sketches of the dimension stones, which enter into the masonry structure, previously designed.

The drawings of the simpler shapes may be ordinary sketches in projection, showing the principal faces with dimensions thereon, or descriptions of same; but isometric views should be shown of the more complex shapes. The dimensions of the shapes may be obtained (1) by analytic calculation, (2) by projection and development, (3) by Descriptive Geometry. All these methods commonly enter into the case of a single structure. Only a few hints can be given here, but these, it is hoped, will serve to illustrate a more extended application.

Wall of Building.—Fig. 1 illustrates the simple method of showing the dimensions of stones on the Elevation plan. These dimensions are usually given on the drawing *to center of joint* with a note on plan to that effect, as

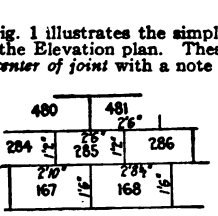

Fig. 1.

"Allow for $\frac{3}{16}$" joints." The thickness of joint may vary from about ⅛" upward, depending upon the quality of the masonry; $\frac{3}{16}$" is common for public buildings. In engineering structures ¼" is about the minimum, and from that up to ½" for dimension work. Note that in Fig. 1 each stone is num-

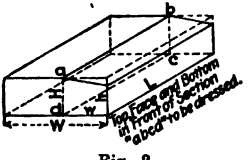

Fig. 2.

bered. Fig. 2 is an isometric view of stone No. 63 (not shown in elevation) of the water-table. All the dimensions L, H, h, W and w should be placed directly on the drawing. L and H may be "finished dimensions" like the others, or they may be distances to center of joints. Proper notes should be made showing which system is used.

Stone Arch.—Fig. 3 illustrates, briefly, two methods of showing the dimensions of the voussoirs for the stone cutter. Those shown in the end view (face of arch) are in planes perp to axis of arch, whether for right- or skew arch. The measurements shown may be (1) to center of joints as per the "numbered" vouissoirs, or (2) actual dimensions as per the "lettered" voussoirs (Fig. 3). Isometric views of stones "4" and "I" assume the arch to be skewed or oblique, with coursing joints parallel with axis of arch. (Such a construction is allowable only for slight skew, for small arches, or culverts. When the skew is considerable, especially for large spans, the

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construction should be as per Figs. 6 or 7, page 764, Section 44.) obliquity of faces of end voussoirs may be shown directly on the end v arch by measurements in the corners *a*, *b*, *c*, showing the *projection o corners beyond a right section* passing through the corners *o*, assun zero. These projections may be obtained graphically as illustrate

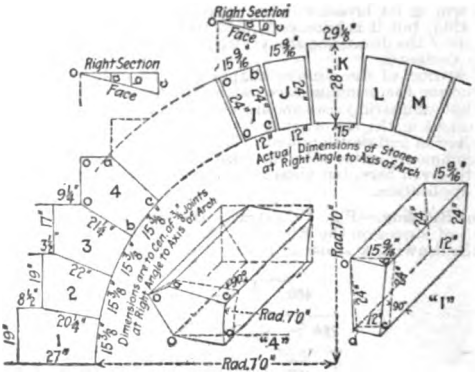

Fig. 3.

should appear also on the isometric views. A plan of soffit and also o of arch should also be submitted showing the coursing joints, len voussoirs (at least near ends of arch), and other information. The finish desired should be marked plainly on plans.

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27.—WEIGHTS AND SPECIFIC GRAVITIES OF MATERIALS.

(For Strength and Resistance of Materials, see Sec. 28.)

DEFINITIONS.

Mass (M) = **Matter.**—If two separate quantities of matter will balance each other in vacuuo they are said to have equal masses; and this regardless of kind of matter, volume, or temperature. The only stipulation is that they shall be affected by the same gravity acceleration g, and this necessarily obtains when they are counterpoised at the same time, at the same elevation above sea level, and at the same parallel of latitude or place.

$$\text{Mass} = \frac{\text{weight}}{\text{gravity acceleration}}; \text{ or } M = \frac{W}{g}, \text{ the usual formula.}$$

The "unit of mass" (M_1) may be a definite and standard quantity of matter. For instance, if a certain quantity of a particular metal occupies one cubic inch (C_1), and weighs one standard pound (W_1) at a point on the earth where the gravity acceleration (g_1) corresponding to W_1, is equal to, say, 32.16,—then its mass (M) is equal to $\frac{W_1}{g_1} = \frac{1}{32.16}$ lb. Hence, the unit of mass (M_1) of the metal would be equal to 32.16 cubic inches, or 32.16 lbs. at that particular locality.

The "unit of mass" is equal to g pounds.

Gravity Acceleration (g).—The value of g increases with the latitude of the place, and decreases with the elevation above sea level. The following formula* gives the value of g for any latitude and elevation:

$$g = 32.172 - 0.082 \cos 2\lambda - 0.000003\,h \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

where g = acceleration in ft. per sec, per sec.;
λ = latitude of the place in degrees;
h = elevation in feet above sea level.

From this formula the values of g and $\sqrt{g}$ (used in Hydraulics) are as follows, for various latitudes, at sea level:

Latitude	0°	10°	20°	30°	40°	45°
g	32.090	32.095	32.109	32.131	32.158	32.172
$\sqrt{2g}$	8.011	8.012	8.014	8.016	8.020	8.021

The value of g may be designated as the *intensity of gravity.*

Weight (W) = Mg.—The weight of a body depends upon its mass (M) and upon the intensity of gravity (g). If the mass is constant the weight will vary directly with g. From the preceding table we see that a mass weighing 32,131 lbs. in latitude 30°, would weigh 32,172 lbs. in latitude 45°; that is, the weight of the same mass would increase 41 lbs. in 15° of latitude, or a little more than 2¾ lbs. per ton. This is so small as to be practically negligible in engineering calculations, and we generally assume for g its value in latitude 40°, namely, $g = 32.16$. It is to be noted, then, that the wieghts per cubic foot of substances given in the subjoined tables are practically constant, in so far as the intensity of gravity alone is concerned, in all portions of the country.

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* In C. G. S. (Centimeter-Gram-Second) System, adopted by the British Association, the value of g in centimeters is sometimes given: $g' = 980.6056 - 2.5028 \cos 2\lambda - .000003h'$. See also formula for g under Simple Circular Pendulum Mechanics, page 287.

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Volume (V).—The unit of volume generally used in the United States is the cubic foot, and weights are given in pounds per cubic foot. The effect of temperature upon any mass is to increase its volume (ice excepted); hence, in giving the weights of those substances which expand materially with heat, the temperature should be stated. This is true especially with gases and, to a much less extent, with liquids. But with ordinary materials of engineering the temperature effect on volume is so slight, within the natural range of the thermometer, as to be negligible.

The volume of any mass is inversely as its density; thus, $V = \frac{M}{D}$. Then for any unit mass the volume and density are reciprocals of each other.

Density (D).—The density of any body is the mass of a unit of its volume; thus in the equation $M = VD$, if $V = 1$, $M = D$. If, now, this unit mass is increased, say by temperature, to two units of volume, then will its density be equal to $\frac{1}{2}M$, or 50 per cent of what it was. If, on the other hand, the unit mass is decreased, say by pressure, to $\frac{4}{5}$ its original volume, then will its density be $1.25\ M$.

The *relative density* of a substance is called its *specific gravity*, when referred to water (at maximum density—4° Centigrade).

Specific Gravity (*s. g.*).—The specific gravity of a substance is its relative density, to a unit standard. More definitely, it is the ratio of the weight of a given volume of the substance to the weight of the same volume of distilled water at 4°C (39.1 F.), its maximum density, equal to 62.424 pounds per cubic foot, and considered as unity. Moreover, the specific gravities of other substances than water are assumed to be taken at 0°C., or if not they are reduced by corrections to that temperature. These are the standards of physicists.

Experimenters, especially in the field of engineering, have not always reduced their results to the above standards, but have variously employed as standard units, uncorrected,

Water at	0° C.	or	32° F.,	equal	to	62.416	lbs.	per	cu. ft.
(" "	4° C.	"	39.1° F.,	"	"	62.424	"	"	")
" "	15.56° C.	"	60° F.,	"	"	62.366	"	"	"
" "	16° C.	"	60.8° F.,	"	"	62.361	"	"	"
" "	16.67° C.	"	62° F.,	"	"	62.355	"	"	"

And for the substances experimented with, the temperatures, when stated, have been as varied as the above. Hence, in the subjoined tables of specific gravities, extreme accuracy cannot be expected in many cases, but they are sufficiently exact for all engineering purposes.

Specific gravity = weight in grams per cubic centimeter.

METHODS FOR DETERMINING SPECIFIC GRAVITY.

Solids Heavier than Water.—A common method for determining the specific gravity of a solid heavier than water is to weigh it *in air* and then weigh it *in water*; that is, while still in the scale it is immersed in water, where its weight will be found to be less. The specific gravity may then be expressed by the following formula, when temperature reduction is not considered:

$$\text{Specific gravity} = \frac{W}{W - w} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

In which W = weight of the substance in air;
w = " " " " when immersed in water;
$W - w$ = "loss of weight" by immersion.

Solids Lighter than Water may be determined by finding the weight (W) in air, as above, and then the buoyancy or minus weight (w') in water, when completely immersed. Formula (2) then reduced to (3), as follows:

$$\text{Specific gravity} = \frac{W}{W + w'} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

In which $w' = -w$ = force required to immerse the body.

Displacement Method.—This consists in immersing the substance to be determined, in a vessel *full* of water. Hence,

$$\text{Specific gravity} = \frac{\text{weight of the substance}}{\text{weight of water displaced}} \quad \ldots\ldots\ldots\ldots (4)$$

Porous substances whose specific gravities are to be determined should be painted with a thin coat of varnish before being immersed, in order to exclude all moisture.

Granular substances may have two specific gravities, namely, *in bulk* and *in granule*. Substances which are affected by water should be weighed in some other liquid which will not affect them and whose specific gravity is known. Alcohol, turpentine and benzine are often used for this purpose. Then, the specific gravity obtained with respect to the particular liquid must be multiplied by the specific gravity of the liquid itself, to find the true specific gravity.

For determining the specific gravity of cement, see description of method, page 407, under Building Stones and Cements, Sec. 22.

Liquids.*—The practical determination of the specific gravities of liquids may be made with instruments called *hydrometers*. They consist usually of a glass tube (or wire) so arranged that it will stand vertically when partly immersed in a liquid. The depth of immersion, registered by a *scale* on the tube, or the weight required to immerse the tube to a certain *fixed mark* upon it, with reference to the surface of the liquid in which it is immersed, is the basis on which the specific gravity is determined. "Scale" hydrometers are of variable immersion and constant weight; "fixed-mark" hydrometers are of constant immersion and variable weight. Some hydrometers are specially adapted to determining liquids *heavier* than water; and some to determine liquids *lighter* than water. Again, some hydrometers are immersed in the liquid whose specific gravity is to be determined; while others are immersed in a standard liquid, and are provided with a cup at the top of the tube to receive the liquid which is to be determined. Other forms of instruments, perhaps not strictly hydrometers, are used to determine the purity or adulteration of various liquids, as spirits, solutions, milk, urine, etc. The relative density to some standard of purity is the basis of the determination.

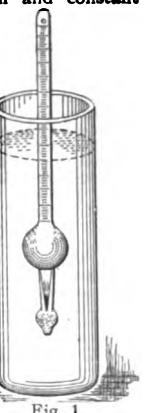

Fig. 1.

Beaumé's hydrometer, Fig. 1, is a "scale" hydrometer of variable immersion, which is immersed in the liquid to be determined. The graduation of the scale for liquids lighter than water is different than for those heavier than water. The graduation is standardized by the depth of immersion (1) in pure water for one point on the scale, and (2) in a saline solution of known strength for another point. The distance between the two points is then graduated, and the graduation extended beyond either point when necessary. The lower end of the tube is loaded with mercury, and a bulb is blown above it.

The following relations exist between Beaumé's Hydrometer scale and the corresponding *Specific Gravity* desired:

Beaumé (deg.)	0	10	20	30	40	50	60
Liquid heavier than water	1.000	1.070	1.152	1.246	1.357	1.490	1.652
Liquid lighter than water	.....	1.000	.936	.880	.830	.785	.745

Many hydrometers now record the specific gravities directly.

Tweddell's hydrometer is a scale hydrometer for determining liquids heavier than water. It is graduated in degrees $D°$ such that

$$\text{Specific gravity} = \frac{5\,D° + 1000}{1000} \quad \ldots\ldots\ldots\ldots\ldots (5)$$

Rousseau's densimeter is for variable immersion in a standard liquid. It is constructed somewhat similar to Beaume's (Fig. 1) but has in addition a tube or cup at the top of the stem which contains the liquid to be determined. Hence, it is specially adapted to determining specific gravities of small quantities of liquids.

* The specific gravity of a liquid may be obtained directly by weighing equal volumes of the liquid and of water, dividing the weight of the former by that of the latter.

Nicholson's and Fahrenheit's hydrometers are hydrometers of constant immersion and variable weight. They differ from the above. For instance, Nicholson's hydrometer consists of a hollow metal float, always submerged, below which is suspended a dish loaded with weights. Above the float is supported a shallow dish on a thin vertical wire. On this wire is a mark which is brought to the surface of the liquid by the weights in the upper and lower dishes. These weights determine the specific gravity of the liquid in which the hydrometer is immersed.

Refinements.—In laboratory work where great refinement is necessary the exact determination of specific gravities involves quite intricate formulas and extremely careful observations. The formulas include reduction of weighings to vacuuo, and various temperature reductions.

Gases.—When we speak of the specific gravity of a gas we must have clearly in mind its *pressure* and *temperature*, and to what *standard* it is referred. The specific gravity of a unit mass of gas varies directly as its density and inversely as its volume. For constant temperature the density varies directly as the pressure, for constant pressure the density varies inversely as the temperature, meaning of course the temperature above absolute zero.*

The "standard" pressure of the gas determined is taken at, or reduced to, one "standard" atmosphere. Now a standard atmospheric pressure is *about* 14.7 lbs. per square inch, near enough for all practical engineering work. But it is not a fixed quantity. In *French units* it is assumed as a pressure equivalent to a column of mercury at 0° C. (32° F.), 760 *m m* (= 0.76 meter = 29.921 ins.) in height, and acted upon by an intensity of gravity, *g*, equal to that of Paris. This value of *g* has been determined as equal to an acceleration of 980.94 *c m* (= 9.8094 meters = 32.183 ft.) per sec. Now the specific gravity of mercury at 0° C. is taken at 13.596 (Regnault's determination, 13.5959; commonly assumed at 13.6); hence the pressure of a standard atmosphere at Paris = 76 × 13.596 = 1033.3 grams per sq. centimeter = 10.333 kilograms per sq. meter (= 14.697 lbs. per sq. in. = 2116.37 lbs. per sq. ft.). It is thus seen that the atmospheric pressure may be stated in terms of a column of mercury, or as a pressure per unit of area; and that these values are dependent on the temperature (mainly affecting the density of the mercury, and very slightly the air), the specific gravity of the mercury (affected by temperature and value of *g*), the value of *g* (affected by latitude, and elevation above sea level), the latitude, and the distance above sea level. In view of this, English units are often used, with round units of 16° C., 30 ins. of mercury, and 14.7 lbs. per sq. ft. Thus:

In *English units* the "standard" pressure is often assumed equivalent to a column of mercury at 60° to 62° F. (16° C.), 30 inches (762 millimeters) in height, and acted upon by an intensity of gravity *g* equal to that at 45° latitude. This gives a pressure practically the same as that derived from the French standard, namely, 14.7 lbs. per sq. in.

The standard temperature of the gas whose specific gravity is determined should be taken at or reduced to 0° C. (32° F.) for French units, or 60° to 62° F. (16° C.) for English units. The temperature reduction should be stated in all cases.

The standard substance referred to in determining the specific gravities of gases is air at 0° C. (32° F.), with the barometer at 760 m m (29.921 ins.) or one standard atmospheric pressure. Sometimes air at 60° or 62° F. is used. The specific gravity of the air multiplied by the sp grav of the gas referred to it = the sp grav of the gas with reference to water. Water at its maximum density, 4° C., is the standard, but sometimes 60° or 62° F. is used.

One cubic foot of air at				0° C.	(32° F.)	weighs	0.0807 lb.
"	"	"	" "	4° C.	(39.1° F.)	"	0.0796 "
"	"	"	" "	15.56° C.	(60° F.)	"	0.0764 "
"	"	"	" "	16° C.	(60.8° F.)	"	0.0763 "
"	"	"	" "	16.67° C.	(62° F.)	"	0.0761 "
"	"	"	water "	0° C.	(32° F.)	"	62.416 "
"	"	"	" "	4° C.	(39.1° F.)	"	62.424 "
"	"	"	" "	15.56° C.	(60° F.)	"	62.366 "
"	"	"	" "	16° C.	(60.8° F.)	"	62.361 "
"	"	"	" "	16.67° C.	(62° F.)	"	62.355 "

* Absolute zero (no heat) is 273.7° C. below 0° C., or 460.7° F. below 0° F.

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GASES.

1.—Weight of a Cubic Foot of Dry Air at Various Temperatures.*

(At Atmospheric Pressure — 14.7 lbs. per sq. in.)

Temperature (Deg. F.)	Weight (Lbs.)	Temperature (Deg. F.)	Weight (Lbs.)	Temperature (Deg. F.)	Weight (Lbs.)	Temperature (Deg. F.)	Weight (Lbs.)
0	.0863	50	.0779	100	.0710	150	.0652
1	.0862	51	.0777	101	.0708	151	.0650
2	.0860	52	.0776	102	.0707	152	.0649
3	.0858	53	.0774	103	.0706	153	.0648
4	.0856	54	.0773	104	.0705	154	.0647
5	.0854	55	.0771	105	.0703	155	.0646
6	.0852	56	.0770	106	.0702	156	.0645
7	.0851	57	.0768	107	.0701	157	.0644
8	.0849	58	.0767	108	.0700	158	.0643
9	.0847	59	.0766	109	.0698	159	.0642
10	.0845	60	.0764	110	.0697	160	.0641
11	.0843	61	.0763	111	.0696	161	.0640
12	.0842	62	.0761	112	.0695	162	.0639
13	.0840	63	.0760	113	.0694	163	.0638
14	.0838	64	.0758	114	.0692	164	.0637
15	.0836	65	.0757	115	.0691	165	.0636
16	.0834	66	.0755	116	.0690	166	.0635
17	.0833	67	.0754	117	.0689	167	.0634
18	.0831	68	.0752	118	.0688	168	.0633
19	.0829	69	.0751	119	.0686	169	.0632
20	.0828	70	.0750	120	.0685	170	.0631
21	.0826	71	.0748	121	.0684	171	.0630
22	.0824	72	.0747	122	.0683	172	.0629
23	.0822	73	.0745	123	.0682	173	.0628
24	.0821	74	.0744	124	.0680	174	.0627
25	.0819	75	.0743	125	.0679	175	.0626
26	.0817	76	.0741	126	.0678	176	.0625
27	.0816	77	.0740	127	.0677	177	.0624
28	.0814	78	.0739	128	.0676	178	.0623
29	.0812	79	.0737	129	.0675	179	.0622
30	.0811	80	.0736	130	.0674	180	.0621
31	.0809	81	.0734	131	.0672	181	.0620
32	.0807	82	.0733	132	.0671	182	.0619
33	.0806	83	.0732	133	.0670	183	.0618
34	.0804	84	.0730	134	.0669	184	.0617
35	.0803	85	.0729	135	.0668	185	.0616
36	.0801	86	.0728	136	.0667	186	.0615
37	.0799	87	.0726	137	.0666	187	.0614
38	.0798	88	.0725	138	.0665	188	.0613
39	.0796	89	.0724	139	.0663	189	.0612
40	.0795	90	.0722	140	.0662	190	.0612
41	.0793	91	.0721	141	.0661	191	.0611
42	.0791	92	.0720	142	.0660	192	.0610
43	.0790	93	.0719	143	.0659	193	.0609
44	.0788	94	.0717	144	.0658	194	.0608
45	.0787	95	.0716	145	.0657	195	.0607
46	.0785	96	.0715	146	.0656	196	.0606
47	.0784	97	.0713	147	.0655	197	.0605
48	.0782	98	.0712	148	.0654	198	.0604
49	.0781	99	.0711	149	.0653	199	.0603
50	.0779	100	.0710	150	.0652	200	.0602

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* For comparison of Fahrenheit with Centigrade scale see Table 3.

2.—WEIGHTS AND SPECIFIC GRAVITIES OF GASES.

Substance under 1 atmosphere.		Relative Density to Air. *	Relative Density to Water. †Sp. Grav.	Weight per Cubic Foot. Lbs.	Coef. of Expansion per Degree ‡Cent.
Name.	Temp.				
Air, dry (see Table 1).....	0° C.	1.0000	.001,293,2	.08071	.003666
Ammonia.....	"	.5967	.000,769,7	.04805	.....
Binoxide of Nitrogen.....	"	1.0388	.001,343,4	.08387	.....
Carbonic acid.....	"	1.52901	.001,977,4	.12345	.003699
Carbonic oxide.....	"	.9569	.001,234,4	.07706	.003667
Chlorine.....	"	2.4216	.003,132,8	.19558	.....
Cyanogen.....	"	1.8064	.002,330,2	.14547	.003877
Hydrogen.....	"	.06926	.000,089,57	.00559	.003664
Marsh gas.....	"	.559	.000,727,0	.04539	.....
Nitrogen.....	"	.97137	.001,256,15	.07842	.003665
Olefiant gas.....	"	.985	.001,274,0	.07953	.....
Oxygen.....	"	1.10563	.001,429,8	.08926	.....
Protoxide of nitrogen.....	"	1.5269	.001,969,7	.12297	.....
Steam (ideal).....	.....	.6221	.000,804,5	.05022	.....
Sulphurous acid.....	0° C.	2.1930	.002,728,9	.17036	.003900

* Air at 0° C. under 760 *mm* of mercury at Paris, unless otherwise stated.

† Water at 4° C. under 760 *mm* of mercury at Paris, unless otherwise stated.

Specific gravity of a substance is its weight in grams per cubic centimeter.

‡ Coef. of expansion for low temperatures approaches $\frac{1}{273.7}$ per degree C., or $\frac{1}{492.7}$ per degree F.

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LIQUIDS.

3.—Weight of a Cubic Foot of Water at Various Temperatures.
(Also comparison of Fahr. and Cent. Scales.)

Deg. C.	Add:	.0	.06	.11	.17	.22	.28	.33	.39	.44	.50
	Deg. F.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	32	62.416	62.416	62.416	62.417	62.417	62.417	62.417	62.417	62.418	62.418
.56	33	.418	.418	.418	.419	.419	.419	.419	.419	.420	.420
1.11	34	.420	.420	.420	.420	.420	.421	.421	.421	.421	.421
1.67	35	62.421	62.421	62.421	62.421	62.421	62.422	62.422	62.422	62.422	62.422
2.22	36	.422	.422	.422	.422	.422	.423	.423	.423	.423	.423
2.78	37	.423	.423	.423	.423	.423	.423	.423	.423	.423	.423
3.33	38	.423	.423	.423	.423	.423	.423	.423	.424	.424	.424
3.89	39	.424	*.424	*.424	.424	.424	.424	.424	.424	.424	.423
4.44	40	62.423	62.423	62.423	62.423	62.423	62.423	62.423	62.423	62.423	62.423
5.00	41	.423	.423	.423	.423	.423	.423	.423	.423	.423	.423
5.56	42	.423	.423	.423	.423	.423	.423	.423	.422	.422	.422
6.11	43	.422	.422	.422	.421	.421	.421	.421	.421	.420	.420
6.67	44	.420	.420	.420	.420	.420	.420	.419	.419	.419	.419
7.22	45	62.419	62.419	62.419	62.419	62.419	62.419	62.418	62.418	62.418	62.418
7.78	46	.418	.418	.418	.417	.417	.417	.417	.417	.416	.416
8.33	47	.416	.416	.415	.415	.415	.415	.414	.414	.414	.413
8.89	48	.413	.413	.413	.412	.412	.412	.412	.412	.411	.411
9.44	49	.411	.411	.410	.410	.410	.410	.409	.409	.409	.408
10.00	50	62.408	62.408	62.407	62.407	62.407	62.407	62.406	62.406	62.406	62.405
10.56	51	.405	.405	.404	.404	.404	.404	.403	.403	.403	.402
11.11	52	.402	.402	.401	.401	.400	.400	.400	.399	.399	.398
11.67	53	.398	.398	.397	.397	.396	.396	.396	.395	.395	.394
12.22	54	.394	.394	.393	.393	.392	.392	.392	.391	.391	.390
12.78	55	62.390	62.390	62.389	62.389	62.388	62.388	62.388	62.387	62.387	62.386
13.33	56	.386	.386	.385	.385	.384	.384	.383	.383	.382	.382
12.89	57	.381	.381	.380	.380	.379	.379	.379	.378	.378	.377
14.44	58	.377	.377	.376	.376	.375	.375	.374	.374	.373	.373
15.00	59	.372	.371	.371	.370	.370	.369	.368	.368	.367	.367
15.56	†60	62.366	62.365	62.365	62.364	62.364	62.363	62.362	62.362	62.361	62.361
†16.11	61	.360	.360	.359	.359	.358	.358	.358	.358	.357	.357
16.67	†62	.355	.354	.354	.353	.353	.352	.351	.351	.350	.350
17.22	63	.349	.348	.348	.347	.346	.346	.345	.344	.343	.343
17.78	64	.342	.341	.341	.340	.340	.339	.338	.338	.337	.337
18.33	65	62.336	62.335	62.335	62.334	62.333	62.333	62.332	62.331	62.330	62.330
18.89	66	.329	.328	.328	.327	.326	.326	.325	.324	.323	.323
19.44	67	.322	.321	.321	.320	.319	.319	.318	.317	.316	.316
20.00	68	.315	.314	.314	.313	.312	.312	.311	.310	.309	.309
20.56	69	.308	.307	.306	.306	.305	.304	.303	.302	.302	.301
21.11	70	62.300	62.299	62.299	62.298	62.297	62.297	62.296	62.295	62.294	62.294
21.67	71	.293	.292	.291	.291	.290	.289	.288	.287	.287	.286
22.22	72	.285	.284	.283	.283	.282	.281	.280	.279	.279	.278
22.78	73	.277	.276	.275	.275	.274	.273	.272	.271	.271	.270
23.33	74	.269	.268	.267	.267	.266	.265	.264	.263	.263	.262
23.89	75	62.261	62.260	62.259	62.259	62.258	62.257	62.256	62.255	62.255	62.254
24.44	76	.253	.252	.251	.250	.249	.249	.248	.247	.246	.245
25.00	77	.244	.243	.242	.241	.240	.240	.239	.238	.237	.236
25.56	78	.235	.234	.233	.233	.232	.231	.230	.229	.229	.228
26.11	79	.227	.226	.225	.224	.223	.222	.221	.220	.219	.218
26.67	80	62.217	62.216	62.215	62.214	62.213	62.213	62.212	62.211	62.210	62.209
27.22	81	.208	.207	.206	.205	.204	.204	.203	.202	.201	.200
27.78	82	.199	.198	.197	.196	.195	.194	.193	.192	.191	.190
28.33	83	.189	.188	.187	.186	.185	.184	.183	.182	.181	.180
28.89	84	.179	.178	.177	.176	.175	.174	.173	.172	.171	.170

* Maximum density, at about 39.1 F. or 4° C.

† "Ordinary" temperatures used for determining specific gravities, about 60° to 62° F. or 16° C. For scientific determinations, 0° C. is referred to as standard.

3.—WEIGHT OF A CUBIC FOOT OF WATER AT VARIOUS TEMPERATURES—Continued.

Deg. C.	Add:	.0	.06	.11	.17	.22	.28	.33	.39	.44	
	Deg. F.	.0	.1	.2	.3	.4	.5	.6	.7	.8	
29.44	85	62.169	62.168	62.167	62.166	62.165	62.164	62.163	62.162	62.161	62.
30.00	86	.159	.158	.157	.156	.155	.154	.153	.152	.151	
30.56	87	.149	.148	.147	.146	.145	.144	.143	.142	.141	
31.11	88	.139	.138	137	.136	.135	.134	.133	.132	.131	
31.67	89	.129	.128	.127	.126	.125	.124	.122	.121	.120	
32.22	90	62.118	62.117	62.116	62.115	62.114	62.113	62.111	62.110	62.109	62.
32.78	91	.107	.106	.105	.104	.103	.102	.100	.099	.098	
33.33	92	.096	.095	.094	.092	.091	.090	.089	.088	.086	
33.89	93	.084	.083	.082	.081	.080	.079	.077	.076	.075	
34.44	94	.073	.072	.071	.069	.068	.067	.066	.065	.063	
35.00	95	62.061	62.060	62.059	62.057	62.056	62.055	62.054	62.053	62.051	62.
35 56	96	.049	.048	.046	.045	.044	.043	.041	.040	.038	
36.67	97	.036	.035	.034	.032	.031	.030	.029	.027	.026	
36.22	98	.024	.023	.021	.020	.019	.018	.016	.015	.013	
37.11	99	.011	.010	.008	.007	.006	.005	.003	.002	.000	61.
37.78	100	61.998	61.997	61.996	61.994	61.993	61.992	61.991	61.989	61.988	61.
38.33	101	.986	.985	.983	.982	.981	.980	.978	.977	.975	
38.89	102	.973	.972	.970	.969	.968	.967	.965	.964	.962	
39.44	103	.960	.959	.957	.956	.955	.954	.952	.951	.949	
40.00	104	.947	.946	.944	.943	.941	.940	.939	.937	.936	
40.56	105	61.933	61.932	61.930	61.929	61.928	61.926	61.925	61.924	61.922	61.
41.11	106	.920	.919	.917	.916	.915	.913	.912	.911	.910	
41.67	107	.907	.906	.904	.903	.901	.900	.899	.897	.896	
42.22	108	.893	.892	.890	.889	.887	.886	.885	.883	.882	
42.78	109	.879	.878	.876	.875	.873	.872	871	.869	.868	
43.33	110	61.865	61.864	61.862	61.861	61.859	61.858	61.857	61.855	61.854	61.
43.89	111	.851	.850	.848	.847	.845	.844	.843	.841	.840	
44.44	112	.837	.836	.834	.833	.831	.830	.829	.827	.826	
45.00	113	.823	.822	.820	.819	.817	.816	.815	.813	.812	
45.56	114	.809	.808	806	.805	.803	.802	.800	.799	.797	
46.11	115	61.794	61.793	64.791	61.790	61.788	61.787	61.786	61.784	61.783	61.
46.67	116	.780	.779	.777	.776	.774	.773	.771	.770	.768	
47.22	117	.765	.764	762	.761	.759	.758	.756	.755	.753	
47.78	118	.750	.748	747	.745	.744	.742	.740	.739	.737	
48.33	119	.734	.733	731	.730	.728	.727	.725	.724	.722	
48.89	120	61.719	61.717	61.716	61.714	61.713	61.711	61.709	61.708	61.706	61.
49.44	121	.703	.701	.700	.698	.697	.695	.693	.692	.690	
50.00	122	.687	.685	.684	.682	.681	.679	.677	.676	.674	
50.56	123	.671	.669	.668	.666	.665	.663	.661	.660	.658	
51.11	124	.655	.653	.652	.650	.648	.646	.645	.643	.641	
51.67	125	61.638	61.636	61.635	61.633	61.631	61.629	61.628	61.626	61.624	61.
52.22	126	.621	.619	.618	.616	.615	.613	.611	.610	.608	
52.78	127	.605	.603	.602	.600	.598	.596	.595	.593	.591	
53.33	128	.588	.586	.585	.583	.581	.579	.578	.576	.574	
53.89	129	.571	.569	.568	.566	.565	.563	.561	.560	.558	
54.44	130	61.555	61.553	61.552	61.550	61.549	61.547	61.545	61.544	61.542	61.
55.00	131	.539	.537	.536	.534	.533	.531	.529	.528	.526	
55.56	132	.523	.521	.520	.518	.516	.514	.513	.511	.509	
56.11	133	.506	.504	.503	.501	.500	.498	.496	.495	.493	
56.67	134	.490	.488	.487	.485	.483	.481	.480	.478	.476	
57.22	135	61.473	61.471	61.470	61.468	61.466	61.464	61.463	61.461	61.459	61.
57.78	136	.456	.454	.453	451	.449	.447	.446	.444	.442	
58.33	137	.439	.437	.436	.434	.432	.430	.429	.427	.425	
58.89	138	.422	.420	.418	.417	.415	.413	.411	.409	.408	
59.44	139	.404	.402	.400	.399	.397	.395	.393	.391	.390	

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UBIC FOOT OF WATER AT VARIOUS TEMPERATURES.
—Concluded.

.06	.11	.17	.22	.28	.33	.39	.44	.50
.1	.2	.3	.4	.5	.6	.7	.8	.9
.384	61.382	61.381	61.379	61.377	61.375	61.373	61.372	61.370
.366	.364	.363	.361	.359	.357	.355	.354	.352
.348	.346	.345	.343	.341	.339	.337	.336	.334
.330	.328	.327	.325	.323	.321	.319	.318	.316
.312	.310	.309	.307	.305	.303	.301	.300	.298
.294	61.292	61.291	61.289	61.287	61.285	61.283	61.282	61.280
.276	.274	.272	.270	.268	.267	.265	.263	.261
.257	.255	.254	.252	.250	.248	.246	.245	.243
.239	.237	.235	.233	.231	.230	.228	.226	.224
.220	.218	.216	.214	.212	.211	.209	.207	.205
.201	61.199	61.197	61.195	61.193	61.192	61.190	61.188	61.186
.182	.180	.178	.176	.174	.173	.171	.169	.167
.163	.161	.159	.157	.155	.154	.152	.150	.148
.144	.142	.140	.138	.136	.134	.132	.130	.128
.124	.122	.120	.118	.116	.114	.112	.110	.108
.104	61.102	61.100	61.098	61.096	61.094	61.092	61.090	61.088
.084	.082	.080	.078	.076	.075	.073	.071	.069
.065	.063	.061	.059	.057	.055	.053	.051	.049
.045	.043	.041	.039	.037	.035	.033	.031	.029
.025	.023	.021	.019	.017	.014	.012	.010	.008
.004	61.002	61.000	60.998	60.996	60.994	60.992	60.990	60.888
0.984	60.982	60.980	.978	.976	.974	.972	.970	.968
.964	.962	.960	.958	.956	.953	.951	.949	.947
.943	.941	.939	.937	.935	.933	.931	.929	.927
.923	.921	.919	.917	.915	.912	.910	.908	.906
0.902	60.900	60.898	60.896	60.894	60.891	60.889	60.887	60.885
.880	.878	.876	.874	.872	.870	.868	.868	.864
.860	.858	.856	.854	.852	.850	.848	.846	.843
.839	.837	.835	.833	.831	.829	.827	.825	.823
.819	.817	.815	.813	.811	.809	.807	.805	.802

00	Difference for 1 degree F.
87	.0213
66	.0221
36	.0230
94	.0242
41	.0253
72	.0269
96	.0276
12	.0284
17	.0295
14	.0303
03	.0311
585	.0318
259	.0326
925	.0334
584	.0341
236	.0348
883	.0353
523	.0360
158	.0365
787	.0371
411	.0376
030	.0381
635	.0395

REFERENCES:

Water.	Table.	Page.
Critical temperature	11	514
Critical pressure.....	11	514
Boiling point.....	11	514
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Freezing point	11	514
Melting point (ice).....	13	515
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Specific gravity	4	468
Specific gravity (sea-) ...	4	468
Weight in pipes.....	19	246

See, also,

Hydrostatics.....Section 61.
Hydraulics..... " 62.

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4—Weights and Specific Gravities of Liquids.
(Authoritative. See also Table 5.)

Substance under 1 Atmosphere. Name.	Temp.	Relative Density to Water at 4° C. Sp. Gr.	Relative Density to Water at 60° to 62° F. (= 16° C.).	Weight per Cubic Foot. Lbs.	* Coef. of Expansion per Degree. Cent.
Acid, acetic	60° F.	.....	1.065	66.41	.....
arsenous	4° C.	.....	.....	.....	.....
carbonic (liquid)	0° C.	0.830	.....	51.810	.....
fluoric	60° F.	.....	1.500	93.54	.....
hydrochloric	0° C.	1.240	.....	77.406	.....
muriatic	60° F.	.....	1.200	74.83	.....
nitric	0° C.	1.420	.....	88.642	.00110
phosphoric	60° F.	.....	1.558	97.16	.....
sulphuric	0° C.	1.841	.....	114.923	.00063
Alcohol, pure (absolute)	0° C.	0.793	.....	49.502	.00104
95 per cent	60° F.	.....	.815	50.82	.....
commercial	60° F.	.....	.833	51.95	.....
proof spirits	60° F.	.....	.920	57.37	.....
50 per cent	60° F.	.....	.934	58.24	.....
Ammonia	60° F.	.....	.894	55.81	.....
Benzine	60° F.	.69	.....	.....	.....
Benzole	0° C.	.90	.....	.....	.....
Blood	0° C.	1.060	.....	66.169	.....
Bromine	0° C.	2.960	.....	184.775	.00104
Carbon bisulphide	0° C.	1.293	.....	80.714	.00114
Chloroform	0° C.	1.525	.....	95.197	.00111
Cider	60° F.	.....	1.018	63.48	.....
Claret	0° C.	0.994	.....	62.049	.....
Ether	0° C.	0.713	.....	44.508	.00015
Ethylic iodide	0° C.	1.946	.....	121.477	.....
Gasoline	.....	.....	.....	.....	.....
Glycerine	0° C.	1.260	.....	78.654	.....
Mercury	0° C.	13.598	.....	848.84	.00018
Methylene iodide	0° C.	3.342	.....	208.62	.....
Milk	0° C.	1.029	.....	64.224	.....
Naphtha (oil)	60° F.	.....	.848	52.88	.....
Oil, castor	60° F.	.....	.961	59.93	.....
hemp-seed	60° F.	.....	.928	57.87	.....
lemon	0° C.	0.852	.....	53.185	.....
linseed	60° F.	.....	.935	58.31	.....
naphtha	60° F.	.....	.848	52.88	.....
olive	0° C.	0.915	.....	57.118	.....
petroleum	0° C.	0.836	.....	52.186	.....
poppy	60° F.	.....	.924	57.62	.....
rape-seed	60° F.	.....	.913	56.93	.....
sesame	60° F.	.....	.932	58.12	.....
spindle	60° F.	.....	.936	58.37	.....
turpentine	0° C.	0.870	.....	54.309	.00090
walnut	60° F.	.....	.926	58.12	.....
wood	60° F.	.....	.936	58.37	.....
whale (sperm)	60° F.	.....	.920	57.37	.....
Oxygen (liquid)	0° C.	0.899	.....	56.119	.....
Pentane	0° C.	0.626	.....	39.077	.....
Petroleum (oil)	0° C.	0.836	.....	52.186	.....
Urine	0° C.	1.020	.....	63.672	.....
Water, distilled	4° C.	1.000	.....	62.424	.....
distilled	0° C.	0.999	.....	62.416	.....
sea	60° F	.....	1.026	63.98	.....
Dead sea	60° F.	.....	1.240	77.33	.....

* For moderate temperatures.

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5.—WEIGHTS AND SPECIFIC GRAVITIES OF MISCELLANEOUS LIQUIDS.*
(See also Table 4.)

Name.	Specific Gravity.	Wt. Lbs. per Cu. Ft.	Name.	Specific Gravity.	Wt. Lbs. per Cu. Ft.
Acid, Benzoic	.667	41.59	Ether, Acetic	.866	54.00
Citric	1.034	64.48	Muriatic	.845	52.69
Concentrated	1.521	94.85	Sulphuric	.715	44.59
Phosphoric (solid)	2.800	174.61	Honey	1.450	90.42
Alcohol, Pure, 60°	.794	49.51	Oil, Anise-seed	.986	61.49
80%	.863	53.82	Codfish	.923	57.56
40%	.951	59.30	Palm	.969	60.43
25%	.970	60.49	Sunflower	.926	57.75
10%	.986	61.49	Spirit, rectified	.824	51.38
5%	.992	61.86	Tar	1.015	63.30
Ammonia, 28%	.891	55.56	Vinegar	1.080	67.35
Aqua fortis, double	1.300	81.07	Water (See Table 3)	1.000	62.36
single	1.200	74.83	Wine, Burgundy	.992	61.86
Beer	1.034	64.48	Champagne	.997	62.17
Bitumen, liquid	.848	52.88	Madeira	1.038	64.73
Brandy	.924	57.62	Port	.997	62.17

* Based on weight of water at 62.36 lbs. per cubic foot, i.e., at 16° C. (= 60.8° F.)

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SOLIDS AND MISCELLANEOUS.

6.—Weights and Specific Gravities of Woods.*

(From Circular No. 15, U. S. Dep't of Agric.—Div. of Forestry.)

(See also Table 7.)

No. *	Name of species.	Number of trees.	Number of mechanical tests.	Average specific gravity of dry wood.	Average weight per cu. ft. † Lbs.	Localities and number of trees from each.
1	Longleaf Pine.....	68	6,478	0.61	38.0	Alabama, coast plain (22) *a*; uplands (6); hill district (6); Georgia, undulating uplands (6); South Carolina, coast plain (7); Mississippi, low coast plain (2) Louisiana, low coast plain, gravelly soil (7); sandy loam (6); Texas, low coast plain (6).
2	Cuban Pine.....	12	2,113	.63	39.3	Alabama, coast plain (6); Georgia, uplands (1); South Carolina, coast (5).
3	Shortleaf Pine.....	22	1,831	.51	31.8	Alabama, uplands (4); Missouri, low hilly uplands (6); Arkansas, low hilly uplands (6); Texas, uplands (6).
4	Loblolly Pine.....	32	3,335	.53	33.1	Alabama, mountainous plateau (8); low coast plain (6); Arkansas, level flood plain (5); Georgia, level coast plain (6); South Carolina, low coast plain (7).
5	White Pine.....	17	540	.38	23.7	Wisconsin, clay uplands (5); sandy soils (4); sandy loam (5); Michigan, level drift lands (3).
6	Red Pine.....	8	412	.50	31.2	Wisconsin, drift (5); Michigan (3).
7	Spruce Pine.....	4	696	.44	27.4	Alabama, low coast plain.
8	Bald Cypress.....	20	3.396	.46	28.7	South Carolina, pine barren (6); river bottom (4); Louisiana, coast plain, border of lake (4); Mississippi, Yazoo bottom (3); upland (3).
9	White Cedar.....	4	354	.37	23.1	Mississippi, low plain.
10	Douglas Spruce.....	.....	225	.51	31.8	(From lumber yard.)
11	White Oak.....	12	1,009	.80	49.9	Alabama, ridges of Tennessee Valley (5); Mississippi, low plain (7).
12	Overcup Oak.....	10	911	.74	46.1	Mississippi, low plain (7); Arkansas, Mississippi bottoms (3).
13	Post Oak.....	8	256	.80	49.9	Alabama, Tennessee Valley (5); Arkansas, Mississippi bottom (3).
14	Cow Oak.....	11	935	.74	46.1	Alabama, Tennessee Valley (4); Arkansas, Mississippi bottoms (3); Mississippi, low plain (4).
15	Red Oak.....	7	299	.73	45.5	Alabama, Tennessee Valley (5); Arkansas, Mississippi bottom (2). *b*
16	Texan Oak.....	3	479	.73	45.5	Arkansas, Mississippi bottom.
17	Yellow Oak.....	5	222	.72	44.9	Alabama, Tennessee Valley (5).
18	Water Oak.....	4	132	.73	45.5	Mississippi, low plain (4).
19	Willow Oak.....	12	649	.72	44.9	Alabama, Tennessee Valley (5); Arkansas, Mississippi bottom (3); Mississippi low plain (4).

20	Spanish Oak.....	11	1,035	.73	45.5	Alabama, Tennessee Valley (5); Arkansas, Mississippi bottom (3); Mississippi, low plain (3).
21	Shagbark Hickory..	6	794	.81	50.5	Mississippi, alluvial plain (3); limestone (3).
22	Mockernut Hickory.	4	300	.85	53.0	Mississippi, low plain.
23	Water Hickory.....	2	197	.73	45.5	Do.
24	Bitternut Hickory..	4	100	.77	48.0	Do.
25	Nutmeg Hickory...	3	294	.78	48.6	Do.
26	Pecan Hickory.....	2	172	.78	48.6	Do.
27	Pignut Hickory.....	3	84	.89	55.5	Do.
28	White Elm.....	2	91	.54	33.7	Mississippi, bottom.
29	Cedar Elm.....	3	201	.74	46.1	Arkansas, bottom.
30	White Ash.....	3	476	.62	38.7	Mississippi, bottom.
31	Green Ash.....	1	45	.62	38.7	Do.
32	Sweet Gum.....	7	508	.59	36.8	Arkansas, bottom (3); Mississippi, low plain (4).

* Nos. correspond with similar numbers in Tables 1 to 6, Sec. 28, Strength and Resistance of Materials.

† This column supplied by author, assuming specific gravity at 62° F.; thus, weight = spec grav × 62.36.

a Sixteen of these were bled trees to study the effects of boxing.

b These two should probably be classed as Southern Red Oak. They were collected before the distinction was finally decided upon.

Note.—The values for specific gravity here given refer to "dry" wood of test material, i. e., wood containing variable amounts of moisture below 15 per cent; the moisture effect has therefore not been taken into account, but more careful experiments indicate that its influence on specific gravity at such low per cent is so small that it may be neglected for practical purposes.

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7.—Average Weights and Specific Gravities of Woods.

General Table.

[See also Table 6.]

Name of Species.		* Dry. Spec. Grav.	* Dry. Wt. in Lbs. per Cu. Ft.	* Dry. Wt. in Lbs. per Ft. B. M.	Green. Spec. Grav.	Green. Wt. in Lbs. per Cu. Ft.	Green. Wt. in Lbs. per Ft. B. M.
Alder	.42—1.01	.55	34.3	2.86	.82	51.2	4.27
Apple	.66—1.25	.75	46.8	3.90	1.10	68.7	5.72
Ash, Green		.62	38.7	3.23	.....	.....	.....
Mountain		.55	34.3	2.86	.....	.....	.....
White		.62	38.7	3.23	.84	52.4	4.37
Bamboo	.31—	.36	22.5	1.87	.....	.....	.....
Beech	.65—1.12	.74	46.2	3.85	.98	61.2	5.10
Birch	.52—1.08	.65	40.6	3.38	.94	58.7	4.89
Box, Brazilian		1.03	64.3	5.36	.....	.....	.....
Dutch	.92—1.25	1.04	64.9	5.41	1.22	76.2	6.35
French		1.33	83.0	6.92	.....	.....	.....
Cedar, Red	.48—	.53	33.1	2.76	.75	46.8	39.0
White	.32—	.37	23.1	1.92	.....	.....	.....
Cherry	.61—	.66	41.2	3.43			
Chestnut	.53—	.63	39.3	3.28			
Cork		.24	15.0	1.25			
Cypress, Bald		.46	28.7	2.39			
Douglas "Fir" (See Spruce, Douglas)							
Ebony		1.24	77.4	6.45			
Elm, Cedar		.74	46.2	3.85			
White		.54	33.7	2.81			
"Fir" Washington (See Spruce, Douglas)							
Gum, Sweet		.59	36.8	3.07			
Hackmatack		.61	38.1	3.17			
Hemlock		.42	26.2	2.19			
Hickory, Bitternut	From Circular 15, U. S. Dpt. of Agric. (See Table 6.)	.77	48.1	4.01		Green timbers 16% to 50% heavier than dry; usually 25% to 40%; Average 33%.	
Mockernut		.85	53.1	4.42			
Nutmeg		.78	48.7	4.06			
Pecan		.78	48.7	4.06			
Pignut		.89	55.6	4.63			
Shagbark		.81	50.6	4.21			
Water		.73	45.6	3.79			
(Average of above Hickories)		.80	49.9	4.16			
Juniper		.57	35.6	2.97			
Larch		.55	34.3	2.86	.81	50.6	4.21
Lignum-Vitae	1.18—1.38	1.28	79.9	6.66	.....	.....	.....
Logwood		.91	56.8	4.73	.....	.....	.....
Mahogany, Honduras		.56	35.0	2.91	.....	.....	.....
Spanish	.56—	.85	53.1	4.42	1.06	66.2	5.51
St. Domingo		.72	44.9	3.75	.....	.....	.....
(Average of above Mahoganies)		.71	44.3	3.69	.....	.....	.....
Maple	.53—1.05	.75	46.8	3.90	.94	58.7	4.89
Oak, Cow		.74	46.2	3.85	.....	.....	.....
Overcup	From Circular 15, U. S. Dept. of Agric. (See Table 6.)	.74	46.2	3 85	.....	.....	.....
Post		.80	49.9	4.16	.....	.....	.....
Red		.73	45.6	3.79	.....	.....	.....
Spanish		.73	45.6	3.79	.....	.....	.....
Texan		.73	45.6	3.79	.....	.....	.....
Water		.73	45.6	3.79	.....	.....	.....
White		.80	49.9	4.16	.....	.....	.....
Willow		.72	44.9	3.75	.....	.....	.....
Yellow		.72	44.9	3.75	.....	.....	.....
(Average of above Oaks, say)		.75	46.8	3.90	1.10	68.7	5.72
African		.82	51.2	4.27	.....	.....	.....
Canadian		.87	54.3	4.53	.....	.....	.....
Dantzic		.76	47.4	3.95	.....	.....	.....

* A wood is considered "dry" when it contains not more than 15 per cent of moisture.

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7.—AVERAGE WEIGHTS AND SPECIFIC GRAVITIES OF WOODS—Concluded.
GENERAL TABLE.
[See also Table 6.]

Name of Species.	Dry.			Green.		
	Spec. Grav.	Wt. in Lbs. per Cu. Ft.	Wt. in Lbs. per Ft. B. M.	Spec. Grav.	Wt. in Lbs. per Cu. Ft.	Wt. in Lbs. per Ft. B. M.
Oak—Continued.						
English	.93	58.1	4.84	1.15	71.8	5.98
" (heart of oak)	1.17	73.0	6.09	.....	.....	.....
James River	.76	47.4	3.95	.....	.....	.....
Live	1.07	66.8	5.57	1.26	78.7	6.55
Oregon "Pine" (See Spruce, Douglas)						
Pear .61—1.07	.67	41.8	3.49	1.01	63.0	5.25
Pine, Cuban	.63	39.3	3.28	.....	.....	.....
Loblolly (From Circular 15, U. S. Dpt. of Agric. (See Table 6.))	.53	33.1	2.76	.....	.....	.....
Longleaf (Yellow)	.61	38.1	3.17	.....	.....	.....
Red (Norway)	.50	31.2	2.60	.....	.....	.....
Shortleaf	.51	31.8	2.65	.....	.....	.....
Spruce	.44	27.5	2.29	.....	.....	.....
White	.38	23.7	1.98	.60	37.5	3.12
(Average of above Pines, say)	.52	32.5	2.71	.75	46.8	3.90
Pitch	.52	32.5	2.71	.....	.....	.....
Sugar	.45	28.1	2.34	.58	36.2	3.02
Poplar	.38	23.7	1.98	.....	.....	.....
White	.53	33.1	2.76	.....	.....	.....
Redwood .41—.87	.44	27.5	2.29	.82	51.1	4.27
Spruce, California	.40	25.0	2.08	.....	.....	.....
Douglas { Oregon "Pine" / Washington "Fir" }	.51	31.8	2.65	.63	39.3	3.28
Sycamore	.59	36.8	3.07	.....	.....	.....
Teak	.70	43.7	3.64	.....	.....	.....
Walnut, Black	.58	36.2	3.02	.....	.....	.....
Willow	.49	30.6	2.55	.....	.....	.....

Note that treated timbers will weigh from 5 to 20 lbs. per cu. ft. more than the untreated timbers, well seasoned. Beech, well treated, will receive 18.5 lbs. per cu. ft. of zinc chloride solution or about $\frac{3}{10}$ that quantity of creosote oil; pine, about the same quantities, usually a little less; and oaks about $\frac{4}{5}$ the above quantities. In general, the harder woods increase in weight less than the softer; exact quantities depend upon the specifications.

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8.—Weights and Specific Gravities of Building Stones, Masonry and Cements.
(Average Values.)

Name of Material.	Specific Gravity. *	Wt. per Cu. Ft. Lbs.	Wt. in Lbs. per – Cu. Yd.	Wt. in Lbs. per – Cu. In.
Brick, Chrome	2.803	175.	4725.	.1013
Magnesia 160—170	2.643	165.	4455.	.0955
Pressed (hard)	2.403	150.	4050.	.0868
Paving and Fire	2.403	150.	4050.	.0868
Lime—sand 130—140	2.163	135.	3645.	.0781
Pressed	2.163	135.	3645.	.0781
Common, building	1.922	120.	3240.	.0694
Soft, building	1.602	100.	2700.	.0579
Light, inferior 85— 95	1.442	90.	2430.	.0521
Cement (loose or granular)				
Natural (Rosendale) 55— 75	(1.04)	65.	1755.	.....
Portland 75— 95	(1.36)	85.	2295.	.....
" shaken, usually taken at	.....	100.	.....	.....
Cement (solid)				
Natural, Illinois, Utica	2.70	.....	.....	.....
Kansas, Fort Scott	2.79	.....	.....	.....
Maryland, Cumberland	2.88	.....	.....	.....
Round Top	2.88	.....	.....	.....
Minnesota, Austin	3.15	.....	.....	.....
Mankato	2.87	.....	.....	.....
New York, Akron	3.12	.....	.....	.....
Rosendale	3.04	.....	.....	.....
Pennsylvania, Lehigh	2.98	.....	.....	.....
(General average, say)	2.95	(184.1)	(4971.)	.....
(Specifications A. S. T. M., 1904) 2.8 min.	.....	.....	.....	.....
(Report U. S. Engrs., 1902) 2.5—2.8	.....	.....	.....	.....
(Extreme limits for good) 2.7—3.2	.....	.....	.....	.....
Portland				
(General average, say)	3.15	(196.6)	(5308.)	.....
(Specifications A. S. T. M., 1904) 3.1 min.	.....	.....	.....	.....
(Extreme limits for good) 3.0—3.2	.....	.....	.....	.....
Puzzolan or slag				
(General average, say)	2.85	(177.9)	(4803.)	.....
(Report U. S. Engrs., 1902) 2.7—2.8	.....	.....	.....	.....
(Extreme limits for good) 2.7—2.9	.....	.....	.....	.....
Blended, Natural (50) and Portland (50), say	3.05	(190.4)	(5141.)	.....
Cement (in barrels), weight per bbl., net:				
Spec. U. S. Engineers, 1902:				
Natural 300 lbs., min.	.....	.....	.....	.....
West of Allegheny Mts. may be 265 " "	.....	.....	.....	.....
Portland 4 sacks @ 93.75 lbs. 375 " "	.....	.....	.....	.....
Puzzolan 4 sacks @ 82.50 " 333 " "	.....	.....	.....	.....
Spec. Am. Soc. T. M., 1904:				
Natural 3 bags @ 94 lbs. 282 lbs. min.	.....	.....	.....	.....
Portland 4 bags @ 94 " 376 " "	.....	.....	.....	.....
Foreign:				
English Portland 400 to 430 lbs	.....	.....	.....	.....
German Portland, gross 400 to 440 "	.....	.....	.....	.....
net 375 "	.....	.....	.....	.....

*Specific Gravity Equivalent for any Weight.
Water at 4° C.

Weight.	Spec. Grav.	Weight.	Spec. Grav.	Weight.	Spec. Grav.
1	.016 02	4	.064 08	7	.112 14
2	.032 04	5	.080 10	8	.128 16
3	.048 06	6	.096 12	9	.144 18

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8.—WEIGHTS AND SPECIFIC GRAVITIES OF BUILDING STONES, MASONRY AND CEMENTS—Continued.
(Average Values.)

Name of Material.	Specific Gravity.	Wt. per Cu. Ft. Lbs.	Wt. in Lbs. per— Cu. Yd.	Cu. In.
Cement Mortar (cement-sand-water-mix)				
Portland, say	1.68	105.	2835.	.0608
Resulting Volume of Mix:				
1 cement, 1 sand =1.5; 1 cement, 2 sand=2.3;				
1 cement, 3 sand =3.1; 1 cement, 4 sand=3.8.				
Concrete, Cinder 100—110	1.68	105.	2835.	.0608
Stone 130—150	2.33	145.	3915.	.0839
Earth, Clayey, moistened and rolled	(2.00)	125.	3375.	.....
dry 100—120	(1.76)	110.	2970.	.....
Loose, dry 80— 90	(1.36)	85.	2295.	.....
Muddy, say 105—120	(1.79)	112.	3024.	.....
Loam, dry, loose	(1.2)	75.	.....	.....
packed	(1.6)	100.	.....	.....
Gneiss and				
Granite, California, Penryn (hornblende)	2.77	172.9	.....	.....
Rocklin (muscovite)	2.68	167.3	.....	.....
Colorado, Georgetown (biotite)	2.63	164.2	.....	.....
Connecticut, Greenwich	2.84	177.3	.....	.....
New London	2.66	166.0	.....	.....
Georgia, Lithonia and Stone Mountain	2.69	167.9	.....	.....
Maine, Fox Island	2.63	164.2	.....	.....
Hallowell	2.65	165.4	.....	.....
Maryland, Port Deposit	2.72	171.0	.....	.....
Massachusetts, Quincy (hornblende)	2.70	168.5	.....	.....
Rockport "	2.61	162.9	.....	.....
Minnesota 2.66—2.69	2.68	167.3	.....	.....
New Hampshire, Concord	2.65	165.4	.....	.....
Keene	2.66	166.0	.....	.....
New York 2.71—2.76	2.74	171.0	.....	.....
Rhode Island, Westerley	2.67	166.7	.....	.....
Vermont, Barre	2.65	165.4	.....	.....
Wisconsin, Athelstane	2.70	168.5	.....	.....
Montello	2.64	164.8	.....	.....
(Average of above Granites, say)	2.683	167.5	4523.	.0969
Gravel, Dry, say	(1.79)	112.	3024.	.....
Wet, say	(2.00)	125.	3375.	.....
Lime (loose or granular)				
Freshly burned (quicklime)	(.96)	60.	1620.	.....
Slaked	(2.08)	130.	3510.	.....
Lime (solid) 3.09—3.15	3.12	(195.)	(5265.)	.....
Lime Mortar	1.60	100.	2700.	.0579
Limestone, Illinois, Joliet	2.56	159.8	.....	.....
Lemont 2.51—2.65	2.58	161.1	.....	.....
Quincy	2.57	160.4	.....	.....
Indiana, Bedford	2.48	154.8	.....	.....
Salem	2.51	156.7	.....	.....
Kentucky, Bardstown	2.67	166.7	.....	.....
Bowling Green	2.70	168.5	.....	.....
Michigan, Marquette	2.34	146.1	.....	.....
Lime Island	2.54	158.6	.....	.....
Minnesota, Frontenac 2.42—2.63	2.53	157.9	.....	.....
Stillwater	2.76	172.3	.....	.....
Winona	2.67	166.7	.....	.....
Missouri, Canton	2.32	144.8	.....	.....
New York, Canajoharie 2.69—2.73	2.71	169.2	.....	.....
Cobelskill	2.71	169.2	.....	.....
Glens Falls 2.70—2.72	2.71	169.2	.....	.....

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8.—WEIGHTS AND SPECIFIC GRAVITIES OF BUILDING STONES, MASONRY AND CEMENTS—Continued.
(Average Values.)

Name of Material.	Specific Gravity.	Wt. per Cu. Ft. Lbs.	Wt. in Lbs. per - Cu. Yd.	Cu. In.
Limestone—Continued.				
New York—Continued.				
Kingston	2.69	167.9	.....	.....
Lake Champlain	2.75	171.7	.....	.....
(Average of above Limestones, say)	2.6	162.5	4388.	.0940
Marble, California, Colton	2.75	171.7	.....	.....
Georgia, Tate	2.73	170.4	.....	.....
New York, Gouverneur	2.76	172.3	.....	.....
Pleasantville	2.87	179.2	.....	.....
Tuckahoe	2.87	179.2	.....	.....
Vermont, Dorset	2.66	166.0	.....	.....
(Average of above Marbles, say)	2.77	173.	4671.	.1001
African	2.80	174.8	.....	.....
Biscayan	2.71	169.2	.....	.....
British	2.71	169.2	.....	.....
Carrara	2.72	171.0	.....	.....
Egyptian	2.67	166.7	.....	.....
French	2.65	165.4	.....	.....
Italian (white)	2.71	169.2	.....	.....
Parian	2.84	177.3	.....	.....
Masonry, Brick, Pressed	2.24	140.	3780	.0810
Medium	2.00	125.	3375.	.0723
Soft	1.60	100.	2700.	.0579
Concrete, Cinder	1.68	105.	2835.	.0608
Stone..... 130—150	2.33	145.	3915.	.0839
Granite, Dressed, for Buildings	2.64	165.	4455.	.0955
Bridges	2.56	160.	4320.	.....
Dams 2.50—2.56	2.53	158.	4266.	.....
Rubble in cement	2.48	155.	4185.	.....
dry, say	2.24	140.	3780.	.....
Limestone, Dressed, for Buildings	2.60	162.	4374.	.0938
Bridges	2.56	160.	4320.	.....
Dams 2.47—2.53	2.50	156.	4212.	.....
Marble, Dressed, for Buildings	2.72	170.	4590.	.0984
Sandstone, Dressed, for Buildings	2.40	150.	4050.	.0868
Sand, Fine, dry..... 1.40—1.70	(1.60)	100.	2700	.....
wet..... 1.90—2.05	(1.98)	124.	3348.	.....
Coarse..... 1.40—1.65	(1.52)	95.	2565.	.....
Mixed, coarse and fine	(1.85)	115.	3105.	.....
Sandstone, California, Angel Island	2.73	170.4	.....	.....
Colorado, Fort Collins	2.43	151.7	.....	.....
Manitou	2.23	139.2	.....	.....
Trinidad	2.34	146.1	.....	.....
Connecticut, Portland 2.36—2.63	2.50	156.1	.....	.....
Massachusetts, Long Meadow	2.49	155.4	.....	.....
Michigan, Marquette	2.17	135.5	.....	.....
Portage Entry	2.54	158.6	.....	.....
Minnesota, Fond du Lac	2.25	140.5	.....	.....
New Jersey, Belleville 2.26—2.56	2.41	150.4	.....	.....
New York, Albion	2.60	163.3	.....	.....
Medina...2.40—2.58	2.49	155.4	.....	.....
Hulberton	2.41	150.4	.....	.....
Potsdam	2.60	162.3	.....	.....
Oswego	2.62	163.6	.....	.....
Oxford	2.71	169.2	.....	.....
Fortage	2.70	168.5	.....	.....
Warsaw	2.68	167.3	.....	.....
Ohio, Berea..... 2.11—2.57	2.34	146.1	.....	.....
Cleveland	2.21	138.0	.....	.....
Massillon	2.11	131.7	.....	.....

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8.—WEIGHTS AND SPECIFIC GRAVITIES OF BUILDING STONES, MASONRY AND CEMENTS—Concluded.
(Average Values.)

Name of Material.	Specific Gravity.	Wt. per Cu. Ft. Lbs.	Wt. in Lbs. per –	
			Cu. Yd.	Cu. In.
Sandstone—Continued.				
Pennsylvania, Hummelstown	2.66	166.0	.....	.....
Lumberville	2.60	162.3	.....	.....
Wisconsin, Fond du Lac	2.22	138.6	.....	.....
Virginia, Bristow	2.61	162.9	.....	.....
(Average of above Sandstones, say)	2.47	154.	4158.	.0891
Slate, New York, Granville 2.78—2.84	2.81	175.4	.....	.....
Pennsylvania, Slatington	2.80	174.8	.....	.....
Vermont, Rutland 2.76—2.80	2.78	173.5	.....	.....
(Average of above Slates, say) 2.75—2.85	2.8	175.	4725.	.1013
Austria, Silesia	2.75	171.7	.....	.....
England, Cornwall	2.81	175.4	.....	.....
Welsh	2.88	180.	.....	.....
Trap, Minnesota, Duluth	2.95	184.2	.....	.....
Taylors' Falls	3.00	187.3	.....	.....
New Jersey, Jersey City	3.03	189.1	.....	.....
New York, Staten Island	2.86	178.5	.....	.....
(Average of above Trap rocks, say)	2.96	185.	4995.	.0171

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9.—General Table of Weights and Specific Gravities of Materials.

(Average Values.)

Name of Substance.	Spec. Grav.	Wt. per Cu. Ft. Lbs.
Acid (See Table 4)	.....	.....
Agate..... 2.5—2.8	2.59	161.7
Air (See Table 2)	.....	.....
Alabaster, Calcareous —2.8	2.76	172.3
Gypseous 2.3—	2.61	162.9
Alcohol (See Table 4)	.....	.....
Alder (See Table 7)	.55	34.3
Alloys (See Brass, Bronze, etc.)	.....	.....
Alum	1.72	107.4
Aluminum, Cast	2.56	160.
Hammered	2.75	171.7
Drawn wire	2.68	167.3
Pure	2.67	166.7
Sheet	2.67	166.7
Aluminum bronze	7.7	480.
Amalgam... 13.7—14.1	13.9	868.
Amber	1.08	67.4
Ambergris	.87	54.3
Ammonia (See Table 4)	.894	55.8
Anthracite (solid) 1.4 —1.7	1.55	96.8
Antimony, Cast 6.67—6.74	6.71	419.
Pure	6.80	424.5
Apatite..... 3.16—3.22	3.19	199.
Apple-tree (See Table 7)	.75	46.8
Aqua fortis (See Table 5)	.....	.....
Aragonite	3.0	18.7
Arsenic..... 5.7— 5.8	5.76	360.
Asbestos.... 2.1— 3.1	2.8	175.
paper	1.2	75.
Ash (See Table 7)	.....	.....
Ashes, Coal packed .6—.8	(.7)	44.
Asphalt, Paving	1.6	100.
Asphaltum, Natural 1.1— 1.8	1.44	90.
Atmospheric air (See Table 2)	.....	.....
Ballast, brick and gravel	(1.79)	112.
Bamboo (See Table 7)	.36	22.5
Barium	.47	29.3
Barytes	4.45	278.
Basalt (See Trap)	2.96	185.
Beech (See Table 7)	.74	46.2
Beef fat	.92	57.4
Beeswax	.96	60.
Benzine	.69	43.1
Beer	1.034	64.48
Beton (See Concrete)	.....	.....
Birch (See Table 7)	.65	40.6
Bismuth, Cast 9.76—9.90	9.82	613.
Bitumen (See Ashphaltum)	.....	.....
Blood	1.06	66.2
Bone..... 1.8— 2.0	1.9	118.6
Borax..... 1.7— 1.8	1.75	109.2
Boxwood (See Table 7)	.....	.....
Brass, Cast 7.8— 8.8	8.4	524.
Rolled	8.5	530.

Name of Substance.	Spec. Grav.	Wt. per Cu. Ft. Lbs.
Brass, Sheet	8.5	530.
Wire	8.54	533.
Brick (See Table 8)	.....	.....
Brickwork (See Masonry, Table 8)	.....	.....
Bromine	2.96	185.
Bronze, Aluminum	7.7	480.
Coinage	8.66	540.6
Gun Metal 8.15—8.95	8.6	537.
Ordinary	8.4	524.
Bushel of Produce, etc.		
1 pound per U. S. bu. =	.....	.80356
1.24445 do =	.....	1.00000
24 do =	.....	19.29
32 do =	.....	25.71
40 do =	.....	32.14
48 do =	.....	38.57
56 do =	.....	45.00
Butter..... .94—.95	.94	58.7
Butternut tree	.38	23.7
Cadmium... 8.6— 8.7	8.65	540.
Calcite..... 2.6—2.8	2.7	168.5
Calcium	1.58	98.6
Camphor	.99	61.8
Caoutchouc	.93	58.
Carbon (See Diamond)	.....	.....
Carbon bisulphide	1.293	80.7
Carbonic acid (liquid)	.830	51.8
Castor oil	.961	59.9
Cedar (See Table 7)	.....	.....
Cement (See Table 8)	.....	.....
Chalk..... 2.2—2.8	2.6	162.3
Champagne	.997	62.2
Charcoal, Birch	(.54)	34.
Oak and Fir	(.45)	28.
Pine	(.30)	19.
Powdered	(1.38)	86.
Cherry (See Table 7)	.66	41.2
Chestnut (See Table 7)	.63	39.3
Chloroform	1.525	95.2
Chromium	5.	312.
Cider	1.018	63.5
Cinnabar	8.1	506.
Claret	.994	62.2
Clay, Dry	1.52	95.
Moist, loose 1.7—2.1	1.9	119.
Moist, packed 2.0—2.4	2.2	137.
with gravel	2.5	156.
Coal, Anthracite (solid)	1.55	96.8
Lump	(1.04)	65.
Broken	(1.02)	64.
Egg	(.99)	62.
Stove (average)	(.96)	60.
Nut	(.93)	58.
Pea	(.90)	56.
Buckwheat	(.88)	55.
Bituminous (solid)	1.33	83.
Loose	(.83)	52.
Cobalt..... 8.51—8.54	8.52	532.
Coke, Solid, Natural	(1.0)	62.
Pressed	1.4	87.
Loose..... 28—36	.5	32.

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9.—General Table of Weights and Specific Gravities of Materials—Continued.

(Average Values.)

Name of Substance.	Spec. Grav.	Wt. per Cu. Ft. Lbs.	Name of Substance.	Spec. Grav.	Wt. per Cu. Ft. Lbs.
Concrete (See Table 8)	.....	.....	Elder pith	.076	4.7
Copper, Cast. 8.6—8.9	8.8	550.	Elm (See Table 8)	.....	.....
Drawn wire 8.8—9.0	8.89	555.	Emerald	2.7	(168.5)
Hammered 8.9—9.0	8.95	559.	Emery	4.0	250.
Melted	8.22	513.	Ether (See Table 5)	.713	44.5
Rolled 8.9—9.0	8.95	559.	Ethylic iodide (Table 4)	1.946	121.5
Sheet	8.89	555.	Fat, Beef	.92	57.4
Cork	.24	15.0	Hog	.94	58.7
Creosote oil. 1.04—1.10	1.07	66.8	Mutton	.93	58.
Cypress (See Table 7)	.46	28.7	Feldspar	2.60	(162.)
Delta Metal (Copper 60, zinc, 34-44, iron 2-4, tin 1-2)	8.6	537.	Filbert tree	.60	37.5
Diamond... 3.45—3.60	3.52	(220.)	Fir (See Table 7)	.....	.....
Dogwood	.75	46.8	Flint	2.59	(162.)
Dolomite (See Limestone)	.....	.....	Gallon, of liquid 1 pound per U.S. gallon=	.120	7.4805
Douglas "Fir" (See Table 7)	.51	31.8	0.13368 pound per U. S. gallon=	.016	1.0000
Earth (See Table 8)	.....	.....	Gamboge	1.2	75.
Ebony (See Table 7)	1.24	77.4	Garnet.... 3.75—4.20	4.2	(262.)
Egg	1.09	68.0	German silver 8.4—8.7	8.55	(534.)
			Glass, Crown	2.50	156.
			Common Window	2.50	1.56

Window Glass.

(a.) Official Prices Current American Pittsburg Plate Glass Co.

Prices are per box of about 50 square feet of glass.

United Ins.	Sizes and No. of Lights per Box.			Double.			Single.			
	(No.)	Sizes.	(No.)	AA	A	B	AA	A	B	C
25	(150)	6x 8 to 10 x 15	(48)	$42.75	$37.50	$35.50	$32.00	$26.75	$25.50	$25.00
34	(47) (46)	11 x 14 11 x 13 to 14 x 20	(26)	46.75	41.50	38.75	33.50	28.00	26.75	26.00
40	(28)	10 x 26 to 16 x 24	(19)	52.00	45.50	41.50	36.00	30.00	28.00	27.00
50	(18) (18)	18 x 22 20 x 20 to 20 x 30	(12)	56.00	49.50	46.00	37.50	31.75	29.50	.....
54	(13)	15 x 36 to 24 x 30	(10)	57.50	50.75	46.75	38.75	32.75	30.00	.....
60	(10)	26 x 28 to 24 x 36	(9)	58.75	52.00	47.50	40.00	34.75	31.00	.....
70	(8) (8) (8)	26 x 34 28 x 32 30 x 30 to 30 x 40	(6)	62.75	56.00	50.75	42.75	38.50	33.75	.....
80	(6) (6)	32 x 38 34 x 36 to 30 x 50	(5)	6.800	61.50	55.50	48.75	44.50	38.50	.....
84	(5)	30 x 52 to 30 x 54	(4)	69.50	62.75	56.75	52.00	47.50	41.75	.....
90	(4)	30 x 56 to 34 x 56	(4)	73.50	66.75	61.50	.....	.....	.....	.....
94	(4)	34 x 58 to 34 x 60	(4)	74.75	68.00	62.75	.....	.....	.....	.....
100	(3)	36 x 60 to 40 x 60	(3)	88.00	80.00	74.75	.....	.....	.....	.....
105	(3)	40 x 62 to 40 x 64	(3)	94.75	86.75	80.00	.....	.....	.....	.....
110	(3)	40 x 66 to 40 x 70	(3)	105.50	97.50	90.75	.....	.....	.....	.....
115	(3)	40 x 72 to 40 x 74	(3)	118.75	108.00	101.50	.....	.....	.....	.....
120	(3)	40 x 76 to 40 x 80	(3)	140.00	126.75	120.00	.....	.....	.....	.....
125	(3)	40 x 82 to 40 x 84	(3)	153.50	140.25	133.50	.....	.....	.....	.....
130	.....	40 x 86 to 40 x 90	....	167.00	153.75	147.00	.....	.....	.....	.....

Note.—An additional 10 per cent will be charged for all glass more than 40 inches wide. All sizes over 52 inches in length, and not making more than 81 united inches, will be charged in the 84 united inches bracket. *All glass 54 inches or wider, not making more than* 116 united inches, will be charged in the 120 united inches bracket. Prices are subject to discount in quantities.

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9.—GENERAL TABLE OF WEIGHTS AND SPECIFIC GRAVITIES OF MATERIALS—Continued.

(Average Values.)

Name of Substance.	Spec. Grav.	Wt. per Cu. Ft. Lbs.
Glass—Continued.		
Flint	3.00	187.
Flooring, thick	2.53	158.
Green	2.67	167.
Optical	3.45	215.
Plate	2.8	175.
White	2.89	180.
Window	2.50	156.
Gneiss (See Table 8)		
Gold, Cast	19.258	1203.
Native, hammered	19.4	1211.
Pure "	19.50	1217.
Granite (See Table 8)		
Graphite	2.26	141.
Gravel (See Table 8)		
Greenstone trap	2.96	185.
Grindstone	2.14	134.
Gum Arabic 1.32—1.44	1.38	86.
goods	1.50	94.
Raw (Caoutchouc =	93	58.
India rubber)	.93	58.
Gun Metal (bronze)	8.6	537.
Gunpowder (granular)	1.0	62.4
Gutta percha	.98	61.
Gypsum, Pure, unburned	2.30	144.
Calcined, 1 lump	1.80	112.
Powder, solid	2.55	159.
loose	(.97)	60.
shaken	(1.03)	64.
Plaster of paris 2.1—2.4	2.25	140.
(See Plaster)		
Hackmatack	.61	38.1
Hemlock	.42	26.2
Hickory (See Table 7)		
Holly	.76	47.4
Honey	1.45	90.5
Horn	1.69	105.5
Hornblende 3.0—3.5	3.25	203.
Human body	1.07	66.8
Ice .88—.92	.92	57.4
melting	.92	57.4
India rubber	.93	58.
Iodine	4.95	309.
Iridium, pure	21.15	1320.
Iron, Cast		450.
Wrought, purest		485.
average		480.
Molten		433.
Ivory	1.87	117.
Juniper tree	.57	35.6
Kaolin	2.2	137.
Lava, Basaltic	2.9	181.
Trachytic	2.4	150.
Larch	.55	34.3
Lard	.94	58.7
Lead, Commercial, Cast	11.38	710.
Sheet	11.40	712.
Pure	11.42	713.
Molten	10.40	649.
Lignite, perfect	1.29	80.5
Lignum-Vitae 1.18—1.38	1.28	79.9
Lime (See Table 8)		

Name of Substance.	Spec. Grav.	W C
Limestone (See Table 8)		
Linden tree	.60	
Linseed oil (See Table 5)	.935	
Lithium	.585	
Locust	.71	
Logwood	.91	
Magnesia, solid	3.2	
loose	(1.74)	1
Magnesite	3.0	1
Magnesium, pure	1.75	1
Magnetic iron ore	(5.0)	3
Mahogany (See Table 7)		
Manganese, pure	8.00	4
ore, black	3.45	2
red	4.0	2
Maple (See Table 7)	.75	
Marble (See Table 8)	2.77	1
Marl 1.7—2.5	2.1	1
Masonry (See Table 8)		
Mastic (resin)	.85	
Mercury, Solid. −40° F.	15.632	9
Liquid +32° F.	13.598	8
+60° F.	13.580	8
212° F.	13.370	8
Methylene iodide	3.342	2
Mica 2.65—3.15	2.90	1
Milk	1.029	
Molybdenum, pure	8.63	5
Mortar, Cement	1.68	1
Lime	1.60	1
Mud 105—120	(1.79)	1
Mulberry tree	.75	
Nickel 8.3—9.2	8.8	5
Nitric acid (See Table 4)	1.420	
Commercial	1.22	
Oak (See Table 7)		
Ochre	3.5	2
Oil (See Table 4)		
Olive oil (See Table 4)	.915	
Oolitic stones 1.9—2.6	2.25	1
Opal 2.1—2.2	2.15	1
Opium	1.34	
Orange tree	.71	
Ore, Brown iron	3.9	2
Chrome dust, shaken	2.56	1
Magnetic iron	5.0	3
Red iron (specular)	5.2	3
Spathic	3.9	2
Palladium	11.8	7
Paper .75—1.15	.95	
Paraffin	.88	
Pear tree (See Table 7)	.67	
Peat, pressed .60—.85	.72	
Pentane	.626	
Petroleum (See Table 4)	.836	
Pewter	11.6	
Phosphorus	1.77	1
Pine (See Table 7)		
Pitch	1.12	
Plaster (burned Gypsum)		
Cement		
Keene's cement		
of Paris	2.25	

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9.—General Table of Weights and Specific Gravities of Materials—Continued.

(Average Values.)

Name of Substance.	Spec. Grav.	Wt. per Cu. Ft. Lbs.
Plaster—Continued.		
Parian cement	.....	.....
Stucco	.....	.....
Average for above:		
Gypsum, unburned	2.30	144.
Calcined, 1 lump	1.80	112.
powder, loose	.97	60.
shaken	1.03	64.
Sand (1) plaster (2), dry	1.52	95.
Ordinary plaster (See Gypsum)	1.76	110.
Platinum, Cast		
19.6—20.3	19.95	1245.
Native	16.0	1000.
Pure	21.50	1342.
Rolled	22.07	1378.
(Average, use)	21.50	1342.
Plum tree	.78	48.7
Plumbago	2.1	131.
Poplar	.38	23.7
Poppy oil (See Table 4)	.924	57.6
Porcelain China	2.3	244.
Porphyry	2.76	172.3
Portland cement (See Cement)	.....	.....
Potash	2.05	128.
Potassium	.865	54.
Pumice stone	.92	57.4
Quartz crystal, pure 2.61—2.71	2.66	166.
Quince tree	.71	44.3
Rape-seed oil	.913	56.9
Red lead	8.94	558.
*Resin	1.089	68.
Rock crystal (halite)	2.69	168.
Rock Elm	.80	50.
Rosendale cement (See Cement)	.....	.....
Rosewood	.73	45.6
*Rosin	1.1	68.7
Ruby	3.9	(243.)
Salt, Common, solid	2.20	137.
loose, packed	(.96)	60.
Coarse, Syracuse	(.72)	45.
Turk's Island 78—80	(1.25)	78.
Fine, Liverpool	(.78)	49.
Saltpetre, Chili	2.26	141.
Kali	2.05	128.
Sand (See Table 8)	.....	.....
Sandstone (See Table 8)	.....	.....
Sapphire	3.95	(246.6)
Selenium	4.5	281.
Serpentine 2.5—2.8	2.65	165.
Shale	2.6	162.
Silica	2.66	166.
Silicic acid, crystalline	2.6	162.
powder	(2.2)	137.
Silver	10.5	655.
Slag	.....	40.
Slate (See Table 8)	.....	.....
Smalt	2.44	152.
Snow, freshly fallen	.....	8.
wet	.....	50.
Soapstone	2.73	170.
Sodium	.978	61.
Spar, Calcareous	2.74	171.
Feld-	2.70	168.5
Fluor	3.40	212.
Heavy 4.4—4.5	4.43	277.
Spelter	7.1	443.
Spindle oil	.936	58.4
Spirit, rectified	.824	51.4
Spruce (See Table 7)	.....	.....
Steel, average	7.85	490.
(Handbook calculations)†	7.843	489.6
Wire	7.856	490.4
Strontium	2.54	158.6
Sulphur	2.0	125.
Sulphuric Acid	1.841	114.9
Sycamore	.59	36.8
Talc (steatite)	2.73	170.
Black	2.9	181.
Tallow	.93	58.
Tamarack	.38	23.7
Tar, average	1.	62.4
Teak	.70	43.7
Tellurium, pure	6.11	381.
Tiles, solid 1.9—2.5	2.2	137.
hollow (variable)	.....	.....
Tin, Cast	7.3	456.
Rolled 7.3—7.5	7.4	462.
Molten	7.03	439.
Topaz	3.52	(220.)
Trap rock (See Table 8)	2.96	185.
Tungsten	17.62	1100.
Turpentine	.87	54.3
Type metal, cast	10.45	652.
Uranium	18.3	1142.
Urine	1.02	63.7
Vinegar	1.08	67.4
Walnut, Black	.58	36.2
Walnut oil	.926	58.1
Water (See Tables 3 and 4)	.....	.....
Dead Sea	1.24	77.33
Distilled 4° C.	1.000	62.42
0° C.	.999	62.42
Mediterranean	1.029	64.23

* Resin is the liquid sap of the Longleaf Yellow Pine; rosin is the hard, brittle substance which remains after the turpentine has been extracted.

† Based on bar 1 in. sq. and 1 ft. long weighing 3.4 lbs.; or plate 12 ins. sq. and 1 in. thick weighing 40.8 lbs.; that is, 2 per cent heavier than iron at 480 lbs. per cu. ft.

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9.—General Table of Weights and Specific Gravities of Materials—Concluded.

(Average Values.)

Name of Substance.	Spec. Grav.	Wt. per Cu. Ft. Lbs.	Name of Substance.	Spec. Grav.	Wt. per Cu. Ft. Lbs.
Water—Continued.			Wine, Madeira	1.038	64.80
Rain 4° C	1.000	62.42	Port	.997	62.24
Sea (ocean)	1.026	63.98	Wolfram	7.13	445.
Wax, Bees	.96	60.	Wood Oil	.936	58.4
Willow	.49	30.6	Zinc, Cast	6.86	428.
Wine	.99—1.04	.....	Pure	7.1	443.
Burgundy	.992	61.92	Sheet	7.24	452.
Champagne	.997	62.24	(Average, say)	7.1	443.

10.—Weights of Produce.*

The following are *minimum weights* according to the laws of the United States, and adopted by a majority of States.:

	Per bushel.			Per bushel.
a Apples (dried)	26 lbs.		Lime (unslacked)	30 lbs.
b Barley	48 "		Malt	38 "
c Beans (white)	60 "		Millet seed	50 "
Beans (castor)	46 "		*j* Oats	32 "
Blue grass seed	44 "		*k* Onions	57 "
Bran	20 "		Peaches (dried)	33 "
d Buckwheat	48 "		*l* Peas	60 "
e Clover seed	60 "		Peas (ground)	24 "
Coal	80 "		Potatoes (sweet)	55 "
f Corn (on cob)	70 "		*m* Potatoes (white)	60 "
g Corn meal	48 "		*n* Rye	56 "
h Corn (shelled)	56 "		*o* Salt (fine)	55 "
i Flaxseed	56 "		*o* Salt (coarse)	50 "
Hair (plastering)	8 "		*p* Timothy seed	45 "
Hemp seed	44 "		*q* Turnips	55 "
Hungarian grass seed	50 "		*r* Wheat	60 "

* The following are the greatest and least *minimum* weights adopted by the various States: *Apples* (*not dried*), 24 to 57 lbs. *Anthracite coal*, 76 to 80 lbs. *a*, 22 to 28 lbs. *b*, 47 to 50 lbs. *c*, 60 to 62 lbs. *d*, 42 to 56 lbs. *e*, 60 to 64. *f*, 68 to 70. *g*, 46 to 50. *h*, 52 to 58. *i*, 44 to 56. *j*, 26 to 32. *k*, 48 to 57. *l*, 46 to 60. *m*, 56 to 60. *n*, 54 to 56. *o*, from 50 to 80 lbs. *Coarse salt*, in Penn., 80 lbs.; in Ill., 50 lbs. *Fine salt*, in Penn., 62 lbs.; in Ky., and Ill., 55 lbs. *p*, 42 to 60. *q*, 42 to 60. *r*, 60 lbs. per bu. in all the States.

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REDUCTION TABLES.

11.—Weight Equivalent for any Specific Gravity.

Water at 4° C. and 16° C.

Specific Gravity of the Substance.	Weight of any Substance Compared with Distilled Water at—					
	4° C. = 39.1° F.			16° C. = 60.8° F.		
	Per Cu. Ft. Lbs.	Per Ft. B. M. Lbs.	Per Cu. In. Lbs.	Per Cu. Ft. Lbs.	Per Ft. B. M. Lbs.	Per Cu. In. Lbs.
.1	6.2424	.5202	.0036 125	6.236	.5197	.0036 089
.2	12.4848	1.0404	.0072 250	12.472	1.0393	.0072 178
.3	18.7272	1.5606	.0108 375	18.708	1.5590	.0108 267
.4	24.9696	2.0808	.0144 500	24.944	2.0787	.0144 356
.5	31.2120	2.6010	.0180 625	31.180	2.5983	.0180 444
.6	37.4544	3.1212	.0216 750	37.416	3.1180	.0216 533
.7	43.6968	3.6414	.0252 875	43.652	3.6377	.0252 622
.8	49.9392	4.1616	.0289 000	49.888	4.1573	.0288 711
.9	56.1816	4.6818	.0325 125	56.124	4.6770	.0324 800
1.0	62.4240	5.2020	.0361 250	62.360	5.1967	.0360 889

Example.—What are the weights per cu. ft., per ft. B. M., and per cu. in., of a wood whose specific gravity is 0.61, water at 16° C.?

Solution.—	Cu. Ft.	Ft. B. M.	Cu. In.
.60	37.416	3.118	.02165
.01	.624	.052	.00036
.61	38.040 lbs.	3.170 lbs.	.02201 lb.

Note that for the same specific gravity compared with water at 4° C. the resultant weight is 1 part in 1000 greater than when compared with water at 16° C.; that is, a volume of water at the lower temperature is $\frac{1}{10}$ of 1% heavier than an equal volume at the higher temperature. As the specific gravity of many substances varies greatly, depending upon the particular specimens selected for test, it will be noted that water at either temperature may generally be used as a basis for calculating the weight of the material. This is especially allowable for woods, building stones and other natural materials subject to variable composition, texture, and degrees of moisture. With liquids the variation of the substance is less marked; and with gases the temperature of the water with which it is compared should always be stated, as well as the temperature and pressure of the gas.

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12.—Weight of Sheets, Bars and Wire, of any Material From its Specific Gravity.

(Water at 4° C.)

Specific Gravity of the Material.	Weight of 12"x12" Sheet 1/16" Thick. Lbs.	Weight of 1" Square Bar 1 Ft. Long. Lbs.	Weight of 1" Round Bar 1 Ft. Long. Lbs.	Weight of 1000 Lineal Ft. of		
				Wire 1/16" in Dia. Lbs.	Wire .01" in Dia. Lbs.	Wire .001" in Dia. (1 Mil.) Lbs.
.1	.0325 125	.04335	.034 047	.133	.00340	.0000 340
.2	.0650 250	.08670	.068 094	.266	.00681	.0000 681
.3	.0975 375	.13005	.102 141	.399	.01021	.0001 021
.4	.1300 500	.17340	.136 188	.532	.01362	.0001 362
.5	.1625 625	.21675	.170 235	.665	.01702	.0001 702
.6	.1950 750	.26010	.204 282	.798	.02043	.0002 043
.7	.2275 875	.30345	.238 329	.931	.02383	.0002 383
.8	.2601 000	.34680	.272 376	1.064	.02724	.0002 724
.9	.2926 125	.39015	.306 423	1.197	.03064	.0003 064
1.0	.3251 250	.43350	.340 470	1.330	.03405	.0003 405

Example.—What is the weight of 1000 ft. of wire .02 in. in dia., if the specific gravity of the drawn metal is 8.9?

Solution:
Wt. is proportional to (diam.)^2
For wire .01" dia., 8.0 = .2724
.9 = .03064
8.9 = .30304
4
For wire .02" dia, wt. = 1.21216 lbs.

13.—Weight per Cubic Yard of any Material From its Specific Gravity.

Specific gravity of material × 1685.4 = weight in pounds per cu. yd.
" " " × .8427 = " " short tons " "
" " " × .7524 = " " long tons " "

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14.—Comparison of Various Weights, Capacities and Volumes. From 1 to 9 Units.

*Weight in pounds per —				†Number of Cu. Ft. per –	
Cubic Foot. Lbs.	U. S. Liquid Gallon (231 Cu. Ins.) Lbs.	U. S. Bushel (2150.42 Cu. Ins.) Lbs.	Square Yard 1 In. Thick (1296 Cu. Ins.) Lbs.	Short Ton (2000 Lbs.) No.	Long Ton (2240 Lbs.) No.
.803 564	.107 421	1	.602 673	2 488.912	2 787.581
1	.133 681	1.244 456	.75	2 000.	2 240.
1.333 333	.178 241	1.659 275	1	1 500.	1 680.
1.607 128	.214 842	2	1 205 346	1 244.456	1 393.791
2	.267 362	2.488 912	1.50	1 000	1 120.
2.410 692	.322 263	3	1.806 019	829.637	929.194
2 666 667	.356 482	3.318 550	2	750	840.
3	.401 043	3.733 368	2.25	666.667	746.667
3.214 256	.429 684	4	2.410 692	622.228	696.895
4	.534 724	4.977 824	3	500.	560.
4.017 820	.537 105	5	3.013 365	497.782	557.516
4.821 384	.644 526	6	3.616 038	414.819	464.597
5	.668 405	6.222 280	3.75	400.	448.
5.333 333	.712 964	6.637 100	4	375.	420.
5.624 948	.751 947	7	4.218 711	355.559	398.226
6	.802 086	7.466 736	4.50	333.333	373.333
6.428 512	.859 368	8	4.821 384	311.114	348.448
6.666 667	.891 205	8.296 375	5	300.	336.
7	.935 767	8.711 192	5.25	285.714	320.
7.232 076	.966 789	9	5.424 057	276.546	309.731
7.480 519	1	9.309 178	5.610 39	267.362	299.445
8	1.069 448	9.955 648	6	250.	280.
9	1.203 129	11.200	6.75	222.222	248.889
9.333 333	1.247 687	11.615	7	214.286	240.
10.667	1.425 928	13.274	8	187.500	210.
12.000	1.604 169	14.933	9	166.667	186.667
14.961	2	18.618	11.221	133.681	149.722
22.442	3	27.928	16.831	89.121	99.815
29.922	4	37.237	22.442	66.840	74.861
37.403	5	46.546	28.052	53.472	59.889
44.883	6	55.855	33.662	44.560	49.908
52.364	7	65.164	39.273	38.195	42.778
59.844	8	74.473	44.883	33.420	37.431
62.360	8.336 328	77.604	46.770	32.072	35.920
62.424	8.344 875	77.684	46.818	32.039	35.884
62.500	8.355 035	77.779	46.875	32.	35.840
67.225	9	83.783	50.494	29.707	33.272
75.	10.026	93.334	56.25	26.667	29.867
100.	13.368	124.446	75.	20.	22.400
125.	16.710	155.557	93.75	16.	17.920
150.	20.052	186.668	112.50	13.333	14.933
175.	23.394	217.780	131.25	11.429	12.800
200.	26.736	248.891	150.	10.	11.200
222.222	29.707	276.546	166.667	9	10.080
248.889	33.272	309.731	186.667	8.036	9
250.	33.420	311.114	187.50	8	8.960
280.	37.431	348.448	210.	7.143	8
285.714	38.195	355.559	214 286	7	7.840
320.	42.778	398.226	240.	6.25	7
332.333	44.560	414.819	250.	6	6.720
372.233	49.908	464.597	280.	5.357	6
400.	53.472	497.782	300.	5	5.600
448	59.889	557.516	336.	4.464	5
500.	66.840	622.228	375.	4	4.480
560.	74.861	696.895	420.	3.571	4
666.667	89.121	829.637	500.	3	3.360
746.667	99.815	929.194	560.	2.679	3
1 000.	133.681	1 244.456	750.	2	2.240
1 120.	149.722	1 393.791	840.	1.786	2
2 000.	267.362	2 488.912	1 500.	1	1.120
2 240.	299.445	2 787.581	1 680.	.893	1
Weight per Cubic Foot	Above values are directly proportional to weight per cubic foot.			Above values, inversely proportional to wt. per cu. ft.	

* Inversely proportional to Capacity or Volume per given Weight.
† Inversely proportional to Weight per given Capacity or Volume.

28.—STRENGTH AND RESISTANCE OF MATERIALS.

I. GENERAL PRINCIPLES.

The theory of the resistance of beams is explained in Sec. 15, Mechanics, page 298. The following discussion is pertinent to the subjoined tables, and to general working formulas.

a. Stresses and Resistance.

Stress f = *force* acting on any plane section of a material, produced either directly or by transmission; as, for instance, by leverage. It is measured usually in lbs. per sq. in. (or lbs. per sq. ft.), that is, in units of force per unit of area. The three primary stresses are *tension, compression* and *shear*. Transverse *bending* is accompanied by all three of the primary stresses. *Torsion* or twisting is usually treated as shear.

Strain e = *percentage of distortion* produced by stress. All strains can be reduced primarily to tension, compression of shearing, although torsion (mainly shear) is sometimes classed separately.

Modulus of elasticity $E = \frac{\text{Stress } f}{\text{Strain } e}$ (within the "elastic limit") = a constant. This is equivalent to stating that "Stress is proportional to strain" (Hooke's law), the stress f being equal to the applied load per square inch of cross-section, and the strain e, the percentage of resulting deformation of the material in the direction in which the force acts. If the material is in tension there obtains $E_t = \frac{f_t}{e}$, in which f_t equals tensile stress per sq. in.; if in compression, $E_c = \frac{f_c}{e}$; if in shear, $E = \frac{f_s}{e}$; if in torsion, $E_{tor} = \frac{f_{tor}}{e}$. Within practical limits, the moduli of elasticity for tension and compression are regarded by most engineers as equal for the same class of material.* If this is true the neutral axis in the beam under flexure (and also in the ideal column under concentric loading) will pass through the center of gravity of the section and remain stationary for all safe loadings. This is fundamental to the present theories of beams and columns.

Modulus of elasticity E is assumed to mean the tension or compression modulus unless otherwise stated or characterized, and will be so considered hereafter. If a weight is suspended at the lower end of a rod one inch in cross-section, producing an equal stress f in the rod, then will $f = \frac{E}{1000}$ when the rod is extended $\frac{1}{1000}$ part of its length,† i. e., when $e = 0.001$; $f = \frac{E}{500}$ when the extension is $\frac{1}{500}$ or $e = 0.002$; etc. In general, $f = Ee$. It is easy to see, then, that if these ideal conditions of elasticity could continue to the point where e = unity, f would equal E. Hence, the modulus or "coefficient" of elasticity is sometimes defined as that force which will stretch a rod of unit cross-section to *double* its length, based, of course, on the original length of rod and on the cross-section remaining constant. Long before this 100 per cent deformation can obtain, however, we reach the "elastic limit" or limit of elasticity of the material.

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* The modulus of elasticity for most materials varies more or less with the intensity of stress, and is different for tension and compression of the same material, the difference becoming more apparent as the stresses increase. Hence, the neutral axis of beams varies in position with the amount of loading.

† The *original* length is used by the engineer for simplicity, although the *final*, or some distorted, length might be more applicable.

Elastic limit, and **yield point.**—Within the elastic limit of the material the strain is proportional to the stress or load, and when the latter is removed the material will return to its original length, form or position. (In other words, a perfectly elastic material or a material strained within the elastic limit is capable of transmitting all the energy that it receives, none of it being absorbed in internal work.) If, however, the material has been strained (by stress) in excess of or "beyond" the elastic limit it will begin to "yield" and the resulting deformation will increase *more rapidly* than the increase of stress or load. Hence the term "yield point" used to denote a point just beyond the elastic limit of the material. In Fig. 1, which illustrates the method of plotting the relation between stress and strain for any material, *y* is the yield point; *o–e.l.* is a straight line showing "stress proportional to strain" within the elastic limit; and *u* is the position of ultimate strength, showing the relation between stress and strain at the "breaking point." The yield point is often almost coincident with the elastic limit.

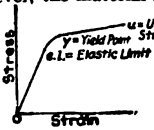

Fig. 1.

The elastic limit is affected more or less by static-, repeated-, and alternating stresses.

Ultimate strength—ultimate stress.—The ultimate strength of material is the ultimate stress per sq. in. which the material is capable of resisting, up to the point of breaking. This is shown in Fig. 1 as point *u*. In testing material the stress is increased gradually by incremental loading, and the breaking point is always above the yield point.

The ultimate strength of material may be affected by the kind of stress or stresses to which it is subjected.

Static stress, or stress in one direction, tends to raise the elastic limit.

Repeated stresses, or stresses varying in intensity in one direction (either in tension or in compression, etc., and never reversing or passing through zero), tend to raise the elastic limit, sometimes materially, even when the stresses are well below it; and at the same time they tend to lower the ultimate strength. The latter will be the more marked the greater the number of repetitions and the wider their range.

Alternating stresses, or stresses passing through zero from tension to compression or vice versa, or from positive to negative shear, torsion, etc., in a vibratory manner, as often occurs in actual structures, tend to act injuriously in lowering both the elastic limit and the ultimate strength. Hence, working alternating stresses should be kept low.

Working stress and factor of safety.—The working stress is the maximum (safe) stress allowed by the designing engineer in his calculations, and from the previous discussion it will be seen that its value may vary greatly. In selecting the proper working stresses we must consider: (1) the kind of material, its physical qualities, and its durability; (2) the kind of loading, and the nature of the stresses; (3) the probable life of the structure, whether permanent or temporary. In general, the working stresses must be well within the elastic limit of the material, for any possible loading, and during any age of the completed* structure. The ratio of the ultimate strength to the working strength is called the factor of safety; thus,

$$\text{Factor of safety} = \frac{\text{ultimate strength}}{\text{working stress}}.$$

The reason for basing the safety factor on the ultimate strength instead of on the elastic limit is that the former is more definitely ascertained; but the elastic limit is also considered to a certain extent, perhaps indirectly, in fixing the working stress.

With reference to the same kinds of loading or stresses, we select larger factors of safety for natural materials, as wood and stone, than we do for

* Concrete and reinforced concrete structures are not considered "completed" until the cement has set to that degree of hardness consistent with the working stresses adopted and based on the "age" of the concrete. Concrete structures grow stronger with age; wooden structures, weaker.

manufactured materials, as steel and iron. The obvious reason fo that the ultimate strength of the manufactured material may re be expected to vary but little from the specification, say 8 to metals, whereas the ultimate strength of wood and stone, even o quality, may vary 100% or more; furthermore, tests of metals same heat will determine the general character of the material f heat, while the physical properties of natural materials are not e termined, excepting for the particular piece under test. Again, substances, as stone, are more uncertain than fibrous substances, weaknesses are less easily detected; and hence stone should ha factors of safety than wood.

Factors of safety may be considered to range from 3, for sta on steel, under favorable conditions, up to 20 or 30, for impact masonry, due to moving machinery.

Resilience = work.—If a weight W is applied *gradually* to sectional area A, and length l, equal to volume $Al = V$, the elo will be equal to $\frac{W}{A} \times \frac{l}{E}$; and as W is applied gradually, the *work* in stretching the rod $= \frac{W}{2} \times s = \frac{W^2 l}{2AE}$. If f represents the stress p then $\frac{W}{A} = f$, and the work $= \frac{f^2}{2E} \times Al$; but $Al =$ the volume V, h resilience or work $= \frac{f^2 V}{2E}$. This is called the *elastic resilience* when the elastic limit of the material.

Similarly, if a *beam* is loaded in the middle with a *concentr* W applied gradually, the deflection at the center will equal $\frac{W l^3}{48EI}$ the average load $= \frac{W}{2}$, the total *work performed* $= \frac{W}{2} \times \frac{W l^3}{48EI}$. But theory of beams, the bending moment $M_B = \frac{W}{2} \times \frac{l}{2}$, equals $M_B = \frac{f}{}$ $\frac{4fI}{ly}$; and, substituting this value of W in the above, we have

Resilience or work $= \frac{W^2 l^3}{96EI} = \frac{f^2 l l}{6Ey^2} = \frac{f^2 A r^2 l}{6Ey^2} = \frac{f^2}{2E} \cdot \frac{r^2}{3y^2} \cdot Al$;

or, in terms of volume it is equal to $\frac{f^2 r^2}{6Ey^2} \cdot V = \frac{f^2}{2E} \cdot \frac{r^2}{3y^2} \cdot V$.

By the last equation it can be compared directly with the res a rod. The above equations will represent the *elastic resilience* of t for concentrated loading at the middle, when f (the outer fiber s sq. in.) is equal to the elastic limit of the material. In general, the of the beam is obtained by multiplying the greatest allowable applied load, by one-half the deflection.

The resilience† of a material may be defined as the capacit material to absorb and give out energy, measured in units of inch-pounds.

For different kinds of stress we have the following values for or work of deformation of any material. By proper substitution for the particular kind and shape of the material the resilience obtained in definite units; and the result will be the *elastic resilie* value of f represents the elastic limit of the material:

Stress.	*Resil*
Tension, direct, - - - - - - - - - -	$\frac{f}{2}$
Compression, direct, - - - - - - - - -	$\frac{f}{2}$

* See Table 1, Sec. 31, Properties and Tables of Beams and Girde

† The *modulus of resilience* is the resilience of a unit volume (c inch) of the material It is obtained by making $V =$ unity, in the e

Stress.	*Resilience.*
Bending, in beam, from concentrated load, . - -	$\frac{f^2r^2}{6Ey^2}V$
" " " " uniform load, - - -	$\frac{4f^2r^2}{15Ey^2}V$
" " rectangular beam ($b \times h$), *concen.* load, -	$\frac{f^2}{18E}V$
" " " " " uniform " -	$\frac{4f^2}{45E}V$
" " beam with *unif.* longitudinal bending moment,	$\frac{f^2}{6E}V$
Torsion, in solid rod, - - - - - - - - -	$\frac{5f^2}{8E_{rot}}V$

where V = volume of material under action, in cubic inches;
E = modulus of elasticity of the material,
r = radius of gyration, in inches;
y = distance from neutral axis to extreme outer fiber, in inches;
f = extreme outer fiber stress, in lbs. per sq. in.

b. Loading and Impact.

Sudden Loading.—In the discussion of resilience, the loading is considered as applied gradually, the resultant being a static load; and at no time is the distortion greater than that due to the load statically applied. The intensity of stress increases constantly from zero to a maximum, and consequently the average stress equals one-half the maximum stress. If, however, a load just "touching" a rod or beam, so as to produce the least amount of stress, is applied *suddenly* but without impact, that is, "let go," the distortion will practically be *doubled*, and hence the fiber stress, deflection, and work performed will be *double* that produced by resilience as when the load is applied gradually. For this reason "live loads" on structures, and especially train loads on bridges, are considered as "sudden loads," the working stresses for which are usually but *one-half* the working stresses for dead or static loads.

Impact is the energy of the blow when one mass, possessing momentum, comes in contact with another mass. At the instant of contact each gives energy to and absorbs energy from the other, and if they were perfectly elastic (an ideal condition never realized in practice) none of the energy would be dissipated into heat.

The mathematical analyses of many operations, as pile driving, water ram in pipes, etc., are dependent upon reducing the *energy* of the blow to an equivalent static *force*; but no formula yet devised correctly represents the relation. To do so, it must take into consideration the element of distortion or *distance* (energy is a measure of force × distance), the inertia of the resisting mass, the loss of energy in heat, in deformation of material, etc. Partial analyses of some of these conditions have been attempted, upon certain assumptions, but they are of doubtful utility unless accompanied by data from practical experiments. The term "impact" is sometimes wrongly applied to "sudden loading."

For discussion of pure Impact, see Sec. 15, Mechanics, pages 303, 304.

II. TABLES OF STRENGTH OF MATERIALS.

Comprising also Standard Specifications.

Treated under the following heads:

A. Woods, page 490. C. Building Stones, Cements, etc., page 507.
B. Metals, page 496. D. Miscellaneous Materials, page 512.

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A. Woods.

1.—Compression (End) Tests of Timber.

(From Circular No. 15, U. S. Dep't of Agric.—Div. of Forestry.

[Pounds per square inch.]

No *	Species.	Number of tests.	Highest single test.	Lowest single test.	Average highest 10 per cent of tests.	Average lowest 10 per cent of tests.	Average of all tests. (8)	Proportion of tests within 10 per cent of average.	Proportion of tests within 25 per cent of average.
	Reduced to 15 per cent moisture. (See Table 2).							*Per cent.*	*Per cent.*
1	Longleaf Pine.....	1,230	11,900	3,400	8,600	5,700	6,900	53	90
2	Cuban Pine.....	410	10,600	2,800	9,500	6,500	7,900	61	93
3	Shortleaf Pine.....	330	8,500	4,500	7,600	4,800	5,900	47	90
4	Loblolly Pine.....	660	11,200	3,900	8,700	5,400	6,500	49	84
	Reduced to 12 per cent moisture.								
5	White Pine.....	130	8,500	3,200	6,800	4,000	5,400	49	93
6	Red Pine.....	100	8,200	4,300	8,100	4,900	6,700	54	96
7	Spruce Pine.....	170	10,000	4,400	8,800	5,600	7,300	66	95
8	Bald Cypress.....	655	9,900	2,900	8,500	4,200	6,000	31	74
9	White Cedar.....	87	6,200	3,200	6.000	4,400	5,200	79	99
10	Douglas Spruce *a*....	41	8,900	4,100	8,100	4,200	5,700	28	65
11	White Oak.....	218	12,500	5,100	11,300	6,300	8,500	40	81
12	Overcup Oak.....	216	9,100	3,700	8,600	6,000	7,300	70	95
13	Post Oak.....	49	8,200	5,900	8,100	6,000	7,100	58	100
14	Cow Oak.....	256	11,500	4,600	9,800	5,600	7,400	51	89
15	Red Oak.....	57	9,700	5,400	9,200	5,500	7,200	36	94
16	Texan Oak.....	117	11,300	5,800	9,800	6,900	8,100	62	98
17	Yellow Oak.....	40	8,600	5,500	8,300	5,800	7,300	58	100
18	Water Oak.....	31	9,200	6,200	9,000	6,300	7,800	75	100
19	Willow Oak.....	153	11,000	4,200	8,700	5,500	7,200	51	88
20	Spanish Oak.....	251	10,600	3,700	9,500	5,100	7,700	61	94
21	Shagbark Hickory...	137	13,700	5,800	10,900	7,500	9,500	79	97
22	Mockernut Hickory..	75	12,200	6,200	11,600	8,000	10,100	65	99
23	Water Hickory.....	14	10,000	6,700	9,600	7,000	8,400	71	100
24	Bitternut Hickory...	25	11,500	7,300	11,200	7,800	9,600	60	100
25	Nutmeg Hickory	72	12,300	6,400	11,000	7,100	8,800	79	97
26	Pecan Hickory.....	37	10,500	5,800	10,400	7,300	9,100	51	95
27	Pignut Hickory	30	13,000	8,700	12,700	8,900	10,900	72	100
28	White Elm.....	18	8,800	4,900	8,800	5,000	6,500	28	88
29	Cedar Elm.....	44	10,600	6,200	10,100	6,500	8,000	66	95
30	White Ash.....	87	9,600	5,000	8,700	5,700	7,200	48	96
31	Green Ash.....	10	9 800	6,600	9,800	6,600	8,000	29	100
32	Sweet Gum.....	118	8,900	4,600	8,500	5,600	7,100	60	87

* Nos. correspond with similar numbers in Sec. 27, Weights and Specific Gravities of Materials, Table 6, page 470; also in the five following tables.

a Actual tests on "dry" material not reduced for moisture.

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2.—FACTORS TO BE ADDED to the strength factors of Southern Pines at 15 per cent moisture in order to reduce them to 12 per cent:

No.	Species.	Crushing endwise.	Bending— At elastic limit.	Bending— At rupture.	Modulus of elasticity.	Crushing across grain.
		Lbs. per sq. in.	*Lbs. per sq. in.*	*Lbs. per sq. in.*	*Lbs. per sq. in.*	*Lbs. per sq. in.*
1	Longleaf Pine (*Pinus palustris*)	1,100	1,500	1,700	180,000	180
2	Cuban Pine (*Pinus heterophylla*)	800	1,500	1,700	70,000	220
3	Shortleaf Pine (*Pinus echinata*)	600	600	900	80,000	60
4	Loblolly Pine (*Pinus taeda*)	900	1,000	1,200	100,000	150
	Reference to Table	Tab. 1.	Tab. 5.	Tab. 4.	Tab. 5.	.....
	and name of Column	Col. (8).	Col. (8).	Col. (8).	Col. (11).	.....

3.—COMPRESSION (END) TESTS OF GREEN TIMBER.
(Above 40 per cent moisture; not reduced.)
[Pounds per square inch.]

No.	Species.	Number of tests.	Highest single test.	Lowest single test.	Average of all tests.
1	Longleaf Pine	86	7,300	2,800	4,300
2	Cuban Pine	38	6,100	3,500	4,800
3	Shortleaf Pine	8	4,000	3,000	3,300
4	Loblolly Pine	69	5,500	2,600	4,100
7	Spruce Pine	71	4,700	2,800	3,900
8	Bald Cypress	280	8,200	1,800	4,200
9	White Cedar	34	3,400	2,300	2,900
11	White Oak	25	7,000	3,200	5,300
12	Overcup Oak	45	4,900	2,800	3,800
14	Cow Oak	58	4,900	2,300	3,800
16	Texan Oak	39	6,000	3,100	5,200
19	Willow Oak	49	5,500	2,300	3,800
20	Spanish Oak	52	5,100	2,500	3,900
21	Shagbark Hickory	22	6,900	3,500	5,700
22	Mockernut Hickory	18	7,200	4,500	6,100
23	Water Hickory	4	5,600	4,700	5,200
25	Nutmeg Hickory	26	5,500	3,700	4,500
26	Pecan Hickory	4	3,800	3,300	3,600
27	Pignut Hickory	5	6,200	4,700	5,400
32	Sweet Gum	6	3,600	3,000	3,300

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4.—Bending Tests of Timber at Rupture.

[Pounds per square inch.]

No.	Species.	Number of tests.	Highest single test.	Lowest single test.	Average highest 10 per cent of tests.	Average lowest 10 per cent of tests.	Average of all tests. (8)	Proportion of tests within 10 per cent of average.	Proportion of tests within 25 per cent of average.
	Reduced to 15 per cent moisture. (See Table 2).							*Per cent.*	*Per cent.*
1	Longleaf Pine.....	1,160	17,800	3,300	14,200	8,800	10,900	41	84
2	Cuban Pine.....	390	17,000	2,900	14,600	8,800	11,900	46	83
3	Shortleaf Pine.....	330	15,300	5,000	12,400	7,000	9,200	40	79
4	Loblolly Pine.....	650	14,800	3,900	13,100	8,100	10,100	44	84
	Reduced to 12 per cent moisture.								
5	White Pine..	120	11,100	4,600	10,100	5,000	7,900	43	81
6	Red Pine.....	95	12,900	3,100	12,300	4,900	9,100	28	60
7	Spruce Pine.....	170	16,300	3,100	13,600	5,800	10,000	43	81
8	Bald Cypress.....	655	14,800	2,300	11,700	5,000	7,900	25	69
9	White Cedar.....	87	9,100	3,500	8,400	4,000	6,300	32	78
10	Douglas Spruce *a*. .	41	13,000	3,300	12,000	4,100	7,900	22	58
11	White Oak.....	218	20,300	5,700	18,500	7,600	13,100	39	75
12	Overcup Oak.....	216	19,600	4,900	14,900	6,300	11,300	47	81
13	Post Oak.....	49	16,400	5,100	15,300	7,400	12,300	47	92
14	Cow Oak.....	256	23,000	3,300	12,500	6,500	11,500	32	68
15	Red Oak.....	57	16,500	5,700	15,400	9,100	11,400	46	84
16	Texan Oak.	117	19,500	8,200	16,900	10,000	13,100	64	86
17	Yellow Oak.....	40	15,000	5,100	14,600	5,700	10,800	28	65
18	Water Oak	31	16,000	5,800	15,700	7,200	12,400	40	76
19	Willow Oak.....	153	16,000	3,200	13,800	5,400	10,400	33	70
20	Spanish Oak.....	257	17,300	5,000	15,600	6,900	12,000	40	72
21	Shagbark Hickory..	187	23,300	5,700	20,300	9,400	16,000	46	84
22	Mockernut Hickory.	75	20,700	5,300	19,700	7,900	15,200	45	79
23	Water Hickory.....	14	18,000	5,300	17,300	5,400	12,500	21	64
24	Bitternut Hickory..	25	19,500	7,000	19,300	8,700	15,000	28	60
25	Nutmeg Hickory ...	72	16,600	6,700	15,600	8,100	12,500	40	88
26	Pecan Hickory.....	37	18,300	5,600	18,100	10,300	15,300	38	95
27	Pignut Hickory	30	25,000	11,100	24,300	11,500	18,700	43	77
28	White Elm.	18	14,000	7,300	13,600	7,300	10,300	44	72
29	Cedar Elm.....	44	19,200	6,600	17,300	8,500	13,500	50	86
30	White Ash.....	87	15,000	5,000	14,200	6,300	10,800	37	77
31	Green Ash.....	10	16,000	5,100	16,000	5,100	11,000	20	60
32	Sweet Gum.....	118	14,400	5,100	12,700	6,000	9,500	39	70

a Actual tests on "dry" material not reduced for moisture.

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5.—Bending Tests of Timber at Relative Elastic Limit.

[Pounds per square inch.]

No.	Species.	Number of tests.	Highest single test.	Lowest single test.	Average of highest 10 per cent of tests.	Average of lowest 10 per cent of tests.	Average of all tests. (8)	Proportion of tests within 10 per cent of average.	Proportion of tests within 25 per cent of average.	Modulus of elasticity (average of all tests) (11)
	Reduced to 15 per cent moisture. (See Table 2.)							*Per cent.*	*Per cent.*	
1	Longleaf Pine	1 160	13,500	2,400	11,100	5,400	8,500	43	81	1,890,000
2	Cuban Pine	390	12,900	2,200	11,500	5,600	9,500	42	83	2,300,000
3	Shortleaf Pine	330	11,900	2,900	9,700	4,800	7,200	48	81	1,600,000
4	Loblolly Pine	650	12,700	3,100	10,800	5,400	8,200	46	85	1,950,000
	Reduced to 12 per cent moisture.									
5	White Pine	130	10,000	4,100	8,200	4,500	6,400	58	85	1,390,000
6	Red Pine	95	11,300	3,100	10,300	4,500	7,700	38	73	1,620,000
7	Spruce Pine	170	13,700	3,000	11,200	5,000	8,400	51	82	1,640,000
8	Bald Cypress	655	12,000	2,200	9,900	4,200	6,600	25	66	1,290,000
9	White Cedar	87	8,200	3,400	7,390	4,000	5,800	44	86	910,000
10	Douglas Spruce *a*	41	13,700	2,800	9,600	3,400	6,400	32	56	1,680,000
11	White Oak	218	15,700	4,400	14,100	6,100	9,600	37	73	2,090,000
12	Overcup Oak	216	11,600	4,000	9,500	5,400	7,500	47	91	1,620,000
13	Post Oak	49	10,600	5,100	9,600	6,000	8,400	34	76	2,030,000
14	Cow Oak	256	14,200	3,400	11,600	5,000	7,600	50	95	1,610,000
15	Red Oak	57	14,500	5,100	13,600	5,600	9,200	15	49	1,970,000
16	Texan Oak	117	12,000	5,900	11,400	7,800	9,400	62	94	1,860,000
17	Yellow Oak	40	11,800	4,900	11,100	5,100	8,100	35	75	1,740,000
18	Water Oak	31	11,800	4,500	11,400	5,500	8,800	40	84	2,000,000
19	Willow Oak	153	13,100	2,700	10,000	4,300	7,400	42	81	1,750,000
20	Spanish Oak	257	13,500	5,100	11,600	6,600	8,600	41	80	1,930,000
21	Shagbark Hickory	187	16,100	5,400	14,200	7,700	11,200	50	89	2,390,000
22	Mockernut Hickory	75	15,400	4,300	14,600	7,800	11,700	39	83	2,320,000
23	Water Hickory	14	11,900	4,100	11,800	4,800	9,800	21	86	2,080,000
24	Bitternut Hickory	25	14,300	7,500	14,000	7,600	11,100	44	84	2,280,000
25	Nutmeg Hickory	72	12,200	4,200	11,200	6,400	9,300	46	93	1,940,000
26	Pecan Hickory	37	15,000	5 800	14,400	7,900	11,500	65	89	2,530,000
27	Pignut Hickory	30	17,500	7,400	16,400	8,300	12,600	40	83	2,730,000
28	White Elm	18	9,700	5,300	9,600	5,400	7,300	33	71	1,540,000
29	Cedar Elm	44	10,700	4,700	10,100	5,800	8,000	57	91	1,700,000
30	White Ash	87	11,500	3,600	10,400	5,200	7,900	43	83	1,640,000
31	Green Ash	10	13,200	3,200	13,200	3,200	8,900	40	70	2,050,000
32	Sweet Gum	118	11,000	3,500	10,100	5,100	7,800	46	82	1,700,000

a Actual tests on "dry" material not reduced for moisture.

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6.—COMPRESSION TESTS OF TIMBER ACROSS GRAIN,* AND SHEARING TESTS OF TIMBER WITH GRAIN.

[Pounds per square inch.]

No.	Species.	Number of tests.	Compression across grain. (4)	Shearin with grain n reduce for moistur
	Reduced to 15 *per cent. moisture.* (*See Table* 2).			
1	Longleaf Pine	1,210	1,000	700
2	Cuban Pine	400	1,000	700
3	Shortleaf Pine	330	900	700
4	Loblolly Pine	690	1,000	700
	Reduced to 12 *per cent. moisture.*			
5	White Pine	130	700	400
6	Red Pine	100	1,000	500
7	Spruce Pine	175	1,200	800
8	Bald Cypress	650	800	500
9	White Cedar	87	700	400
10	Douglas Spruce †	41	800	500
11	White Oak	218	2,200	1,000
12	Overcup Oak	216	1,900	1,000
13	Post Oak	49	3,000	1,100
14	Cow Oak	256	1,900	900
15	Red Oak	57	2,300	1,100
16	Southern Red Oak	117	2,000	900
17	Black Oak	40	1,800	1,100
18	Water Oak	30	2,000	1,100
19	Willow Oak	153	1,600	900
20	Spanish Oak	255	1,800	900
21	Shagbark Hickory	135	2,700	1,100
22	White Hickory	75	3,100	1,100
23	Water Hickory	14	2,400	1,000
24	Bitternut Hickory	25	2,200	1,000
25	Nutmeg Hickory	72	2,700	1,100
26	Pecan Hickory	37	2,800	1,200
27	Pignut Hickory	30	3,200	1,200
28	White Elm	18	1,200	800
29	Cedar Elm	44	2,100	1,300
30	White Ash	87	1,900	1,100
31	Green Ash	10	1,700	1,000
32	Sweet Gum	118	1,400	800

* To an indentation of 3 per cent of the height of the specimen.
† Actual tests on "dry" material not reduced for moisture.

The above tests made under the direction of J. B. Johnson, for t Division of Forestry, are given in full because, aside from the actual val of the results, they show clearly the wide variation which may be expect in submitting timber to actual test.

Relation Between Weight and Strength of Timber.—For want of space t author omits two diagrams which exhibit close relations of *weight breaking strength* of timber, but the relations may be shown analyticall as follows:

Compression, endwise; pounds per square inch:

For pines and hickories = weight (dry) in pounds per cu. ft. × 195.
For oaks = weight (dry) in pounds per cu. ft. × 170.

Bending, transverse; pounds per square inch (fiber stress):

For pines and hickories = weight (dry) in pounds per cu. ft. × 300.
For oaks = weight (dry) in pounds per cu. ft. × 255.

It is to be noted that the *dry* weights per cubic foot are to be used. (S Foot-note to Table 6, Sec. 27, Weights and Specific Gravities, pages 470, 471

7.—Timber in Tension, Compression, Bearing, Bending and Shear.

Practical Working Units for Columns, Beams, etc.*

(See Author's Column Formula, Sec. 82, Columns, page 500.)

[Note.—Values in table are in thousand pounds per square inch; hence, multiply by 1000 to reduce to pounds per square inch. Thus 12,0=12000.]

Timber	Safety Factor.	Tens.	Compression.			Bearing.		Bending.		Shear
			Zero Length.							
		With Grain.	For straight-line formula.	For curved-line formula	Length under 15 Diameter	End of Column.	Across Grain.	Extreme Fiber.	Modulus of Elasticity.	With Grain.
(1)		(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Douglas Spruce (Oreg. and Wash. "Pine" or "Fir").	1	12.0	7.5	7.0	5.5	8.0	1.2	6.0		0.6
	5	2.4	1.5	1.4	1.1	1.6	....	1.2	1500.	.....
	6	2.0	....	....	....	1.3	....	1.0		.....
Longleaf Pine	1	12.0	7.0	6.5	5.0	8.0	1.4	7.0		0.6
	5	2.4	1.4	1.3	1.0	1.6	....	1.4	1600.	.....
	6	2.0	....	....	....	1.3	....	1.2		.....
Shortleaf Pine	1	10.0	5.5	5.0	4.0	6.0	1.0	6.0		0.4
	5	2.0	1.1	1.0	0.8	1.2	....	1.2	1100.	.....
	6	1.7	....	....	....	1.0	....	1.0		.....
White Oak	1	10.0	6.5	6.0	4.5	7.0	2.0	6.5		0.8
	5	2.0	1.3	1.2	0.9	1.4	....	1.3	1400.	.....
	6	1.7	....	....	....	1.2	....	1.1		.....
White Pine	1	8.0	5.5	5.0	4.0	5.5	0.8	4.5		0.4
	5	1.6	1.1	1.0	0.8	1.1	....	0.9	1000.	.....
	6	1.3	....	....	....	0.9	....	0.7		.....
Red Pine	1	9.0	5.5	5.0	4.0	6.0	0.8	5.0		0.4
	5	1.8	1.1	1.0	0.8	1.2	....	1.0	1100.	.....
	6	1.5	....	....	....	1.0	....	0.8		.....
Norway Pine	1	9.0	5.5	5.0	4.0	6.0	0.8	4.5		0.4
	5	1.8	1.1	1.0	0.8	1.2	....	0.9	1100.	.....
	6	1.5	....	....	....	1.0	....	0.7		.....
Canadian White Pine (Ottawa)	1	10.0	7.0	6.5	5.0	7.0	0.8	5.0		0.4
	5	2.0	1.4	1.3	1.0	1.4	....	1.0	1400.	.....
	6	1.7	....	....	....	1.2	....	0.8		.....
Canadian Red Pine (Ontario)	1	10.0	7.0	6.5	5.0	7.0	0.8	5.0		0.4
	5	2.0	1.4	1.3	1.0	1.4	....	1.0	1400.	.....
	6	1.7	....	....	....	1.2	....	0.8		.....
Spruce and Eastern Fir	1	8.0	5.5	5.0	4.0	6.0	0.7	4.0		0.4
	5	1.6	1.1	1.0	0.8	1.2	....	0.8	1100.	.....
	6	1.3	....	....	....	1.0	....	0.7		.....
California Spruce	1	8.0	5.5	5.0	4.0	....	0.7	5.0		0.4
	5	1.6	1.1	1.0	0.8	....	....	1.0	1100.	.....
	6	1.3	....	....	....	....	....	0.8		.....
California Redwood	1	7.0	5.5	5.0	4.0	....	0.8	4.5		0.4
	5	1.4	1.1	1.0	0.8	....	....	0.9	1000.	.....
	6	1.2	....	....	....	....	....	0.7		.....
Hemlock	1	6.0	5.5	5.0	4.0	....	0.6	3.5		0.4
	5	1.2	1.1	1.0	0.8	....	....	0.7	1000.	.....
	6	1.0	....	....	....	....	....	0.6		.....
White Cedar	1	7.0	5.5	5.0	4.0	6.0	0.7	5.0		0.4
	5	1.4	1.1	1.0	0.8	1.2	....	1.0	900.	.....
	6	1.2	....	....	....	1.0	....	0.8		.....
Bald Cypress	1	6.0	5.5	5.0	4.0	6.0	0.7	5.0		0.4
	5	1.2	1.1	1.0	0.8	1.2	....	1.0	1000.	.....
	6	1.0	....	....	....	1.0	....	0.8		.....
Chestnut	1	9.0	7.0	6.5	5.0	....	0.9	5.0		0.6
	5	1.8	1.4	1.3	1.0	....	....	1.0	1200.	.....
	6	1.5	....	....	....	....	....	0.8		....

*See Recommendations and Remarks on following page.

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RECOMMENDATIONS AND REMARKS ON PRECEDING TABLE:

Tension.—Use safety factor of 8 for tension, in bridges and buildings.

Compression.—Use safety factor of 5 for compression, in bridges and bui ings, under ordinary circumstances.

Bearing.—Use safety factor of 5 for end bearing, with grain; and safe factor of 4 for side bearing, across grain.

Bending.—Use safety factor of 6 for extreme fiber of railroad stringers a for joists; use factor of 5 for floorbeams and for large girders.

Shear.—Use safety factor of 5 for shearing, with grain. Note that t longitudinal shearing force H per unit of length of beam is equal to t total vertical shear V multiplied by the statistical moment Q of a above plane of shear, about neutral axis, and divided by the moment inertia I of total section. Thus, $H = \frac{VQ}{I}$. (See Sec. 31, Beams a Girders, page 565.)

B. Metals.

8.—TENSION, COMPRESSION, BENDING, SHEARING, ETC., OF METALS.

[Pounds per square inch.]

Metal.	Tension (Ult.)	Elastic Limit.	Compression (Ult.)	Bending (Extr. Fiber.)	Shearing (Ult.)	Modulus of Elasticity
*Aluminum, Commercial						
Castings	15 000	6 500	12 000	.....	12000	11 000
Forgings	.....	.....	.....	.....	16000	.....
Sheets	24 000	12 000	.....	.....	.....	.....
Bars	28 000	14 000	.....	.....	.....	.....
Wire (hard)	(30-65) 48 000	(16-30) 23 000	.....	.....	.....	.....
Wire (Elec. cond.)	(20-35) 28 000	14 000	.....	.....	.....	.....
*Aluminum-bronze 5% to 7½% Al. with copper	(70-80) 75 000	40 000	120 000	.....	.....	.....
10% Al. with copper, forged eye-bars	100 000	60 000	.....	.....	.....	.....
*Aluminum, "Nickel," 2 to 7% of nickel, copper, iron, etc.	(40-50) 45 000	25 000	.....	.....	.....	.....
Antimony, cast	1 000	.....	.....	.....	.....	.....
Bismuth	6 000	.....	.....	.....	.....	.....
Brass, cast	24 000	6 000	(say) 30 000	(say) 20 000	36 000	9 000
wire, unannealed (hard)	80 000	16 000	.....	.....	.....	14 000
annealed	50 000	.....	.....	.....	.....	.....

* Mainly on the authority of Alfred E. Hunt, and the Pittsburg Red tion Co., principal manufacturers of aluminum in the United States. Assu ing the conductivity of copper wire at .97 and aluminum wire at .62, area of aluminum wire must be 1.565 times that of copper to carry the sa electrical current; and on this basis the aluminum wire, for any giv length, say per mile, will weigh .47 as much as copper wire. Hence, wh the price of aluminum wire per pound is .47 that of copper, or when cop is 2.13 that of aluminum, they will be on about an equal footing for econor on transmission lines. The relative cross-sections of wire of different c ductivities, for equal electric transmission, are about as follows: Copper .97 conductivity equals area 1.00; aluminum at .60 conductivity = a 1.617, at .61 *cond* = area 1.59, at 62 *cond* = area 1.565, at .63 *cond* = area 1. The physical properties of pure aluminum are increased by cold rolling a forging. There are two principal alloys with copper, viz., aluminum-bro and copper-hardened aluminum, the latter containing from 2 to 15 per c copper.

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8.—TENSION, COMPRESSION, BENDING, SHEARING, ETC. OF METALS—Cont'd.

[Pounds per square inch.]

Metal.	Tension (Ult.)	Elastic Limit.	Compression (Ult.)	Bending (Extr. Fiber.)	Shearing (Ult.)	Modulus of Elasticity.
Bronze, Aluminum (See Aluminum-bronze)	.....	.....	.....	.....	.....	.....
* Gun (metal), U. S. Ordnance, cop. 9, tin 1	25-55	.....	.....	.....	.....	.....
	40 000	10 000	.....	52 000	.....	10 000 000
" same greatly compressed	72-78	.....	.....	.....	.....	.....
	75 000	.....	.....	.....	.....	.....
Manganese, cast	60 000	30 000	125 000	.....	.....	.....
rolled	100 000	80 000	.....	.....	.....	.....
Phosphor	50 000	24 000	.....	.....	.....	.....
" wire	100 000	.....	.....	.....	.....	.....
Silicon, cast, 3% Si	55 000	.....	.....	.....	.....	.....
" " 5 % "	75 000	.....	.....	.....	.....	.....
" hard wire†	108 000	.....	.....	.....	.....	.....
Tobin, cast	66 000	.....	.....	.....	.....	.....
rolled	80 000	40 000	.....	.....	.....	4 500 000
cold rolled	100 000	.....	.....	.....	.....	.....
Copper bolts	32 000	.....	32 000	.....	.....	.....
cast	25 000	6 000	40 000	22 000	30 000	10 000 000
plates	32 000	10 000	.....	.....	.....	.....
rods (drawn)	35 000	.....	.....	.....	.....	.....
wire, unannealed	(55-65)	.....	.....	.....	.....	.....
(hard)	60 000	.....	.....	.....	.....	18 000 000
" annealed	36 000	.....	.....	.....	.....	15 000 000
‡ Delta metal, cast	45 000	.....	.....	.....	.....	.....
rolled plates	68 000	.....	.....	.....	.....	.....
small bars	85 000	.....	.....	.....	.....	.....
wire (hard)	100 000	.....	.....	.....	.....	.....
Gold, cast	20 000	4 000	.....	.....	.....	8 000 000
wire (hard)	30 000	.....	.....	.....	.....	.....
Gold (5), copper (1) part	50 000	.....	.....	.....	.....	.....
Gun metal (see Bronze)	.....	.....	.....	.....	.....	.....
Iron, cast (ordinary)	.....	.....	(say)			
	15 000	6 000	80 000	30 000	18 000	12 000 000
a Gray cast	.....	.....	.....	.....	.....	.....
*Malleable Cast	(27-35)	.....	.....	.....	.....	.....
Common	32 000	20 000	.....	.....	.....	.....
Stayboltz	48000+	.....	.....	.....	.....	.....
¶ Wrought (see Steel)	.....	.....	.....	.....	.....	.....

* The physical properties of gun-metal vary greatly with the method of casting, and also with the position of the metal in the cast. The tenacity increases with the specific gravity and pressure; it is greater at the bottom of the melt than at the top.

† The relative conductivity of silicon-bronze wire to pure copper at 1.00 varies from .95 for the soft wire down to .35 for the hard.

‡ Composed of about 60 parts copper, 38 to 40 parts zinc, 2 to 4 iron, 1 to 2 tin.

a See Proposed Standard Specifications for Gray Iron Castings, p. 498.

° The Am. Soc. for Testing Materials adopted (by letter-ballot on Nov. 15, 1904—Vol. IV, page 96) the following tests for tensile and transverse strength of malleable castings: *Tensile Test.* The tensile strength of a standard test bar (a bar 1 in. square and 14 ins. long, without chills and with ends perfectly free in the mold) for castings under specification shall not be less than 40000 lbs. per sq. in.; and the elongation measured in 2 ins. shall be not less than 2½ per cent. *Transverse Test.* The transverse strength of a standard test bar, on supports 12 ins. apart, pressure being applied at center, shall be not less than 3000 lbs., deflection being at least ¼ inch.

¶ Wrought iron is now seldom manufactured except for blacksmith iron and water pipes. Steel which often goes under the double name of "Iron and Steel," in specifications, has entirely superseded iron for structural work because it is stronger and more cheaply manufactured.

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8.—TENSION, COMPRESSION, BENDING, SHEARING, ETC. OF METALS—Cont'd.
[Pounds per square inch.]

Metal.	Tension (Ult.)	Elastic Limit.	Compression (Ult.)	Bending (Extr. Fiber.)	Shearing (Ult.)	Modulus of Elasticity
Iron—Cont'd.						
Wrought shapes	48 000	26 000	46 000	44 000	40 000	.....
" bars	50 000	27 000	48 000	48 000	40 000	28 000 0
" bolts	52 000	.....	.....	.....	.....	.....
Wire, annealed	60 000	.....	.....	.....	.....	15 000 0
" unannealed	80 000	27 000	.....	.....	.....	25 000 0
Lead, cast	1 800	.....	.....	.....	.....	1 000 0
milled	3 200	.....	.....	.....	.....	.....
wire	2 500	.....	.....	.....	.....	.....
Malleable castings (see Iron)	.....	.....	.....	.....	.....	.....
* Nickel	.....	.....	.....	.....	.....	.....
Platinum wire, unannealed	53 000	.....	.....	.....	.....	.....
annealed	32 000	.....	.....	.....	.....	.....
Silver, cast	40 000	.....	.....	.....	.....	.....

* Alloys with copper and steel. (See Steel, Nickel.)

Gray Iron Castings.—Proposed Standard Specifications, adopted b letter-ballot of the Am. Soc. for Testing Materials, September 1, 190 Digest:—1. Unless furnace iron is specified, all gray castings are to made by the cupola process. 2. The sulphur contents to be as follow Light castings, not >.08%; medium castings, not >.10%; heavy casting not >.12%. 3. "Light" castings are those having any section less tha ½ in. thick; "heavy" castings are those in which no section is less than 2 in thick; "medium" castings are those not included under light or heav 4. *Transverse Test.* The minimum breaking strength of the "Arbitratio Bar" (a round bar 1¼ ins. dia. and 15 ins. long) under transverse load sh not be under: Light castings, 2500 lbs.; medium castings, 2900 lbs heavy castings, 3300 lbs. In no case shall the deflection be under .10 inc (The loads to be applied at middle of bars resting on supports 12 ins. apar The deflection at rupture. Two sets of 2 bars each from each heat; one s from the first, and the other set from the last iron going into the castin Where the heat exceeds 20 tons, an additional set of two bars shall be ca for each 20 tons or fraction thereof above this amount. In case of change of mixture during the heat, one set of two bars shall also be cast for every mixture other than the regular one. Each set of 2 bars is to go into a single mold. The bars shall not be rumbled or otherwise treated, being simply bushed off before testing.) *Tensile Test.* Where specified, this shall not run less than: Light castings, 18000 lbs. per sq. in.; medium castings, 21000 lbs. per sq. in.; heavy castings, 24000 lbs. per sq. in. The tensile test shall be made on the "Tensile Test Piece*" (3½ ins. long with sectional diameter 1 in. round). 5. The quality of the iron going into castings under specification shall be determined by means of the "Arbitration Bar." The tensile test is not recommended, but in case it is called for, the "Tensile Test Piece" turned up from any of the pieces of the transverse te shall be used. The expense of the tensile test shall fall on the purchaser.

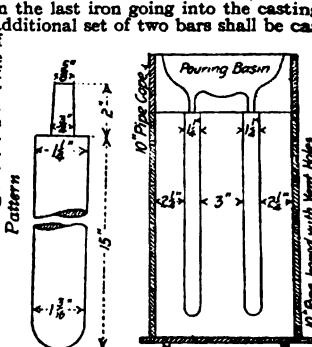

Fig. 2.—Mold for "Arbitration Bar

* See Fig. 3, page 501.

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8.—TENSION, COMPRESSION, BENDING, SHEARING, ETC., OF METALS—Cont'd.
[Pounds per square inch.]

Metal.	Tension (Ult.)	Elastic Limit.	Compression (Ult.)	Bending (Extr. Fiber.)	Shearing (Ult.)	Modulus of Elasticity.
Steel *						
Steel, castings, max	142 000	.....	.....	.....	.....	.....
" hard	85 000	.....	.....	.....	.....	.....
" medium	70 000	40 000	70 000	70 000	60 000	30 000 000
" soft	60 000	.....	.....	.....	.....	.....
†forgings	.....	.....	.....	.....	.....	.....
springs, (untempered)	(65-115)	(40-70)	.....	.....	.....	.....
	90 000	55 000	.....	.....	.....	.....
wire, annealed	80 000	40 000	.....	.....	.....	.....
" unannealed	120 000	60 000	.....	.....	.....	.....
" crucible	180 000	80 000	.....	.....	.....	.....
" susp. bridges	200 000	95 000	.....	.....	.....	.....
" extra, tempered	280 000	.....	.....	.....	.....	.....
Tin, cast	3 500	1 800	(say) 6 000	4 000	.....	4 000 000
Tin 10, antimony 1	11 000	.....	.....	.....	.....	.....
Zinc, cast	(4-6)	.....	(say)	.....	.....	.....
	5 000	4 000	18 000	7 000	.....	13 000 000
rolled	16 000	.....	.....	.....	.....	.....

* High ult tens str of **steel** was employed in the following structures: St. Louis Br., 100000; Plattsmouth Br. and Niagara Centilever, 80 000; Bismarck Br., 80-90000 comp., 70-80000 tens; Susquehanna Br. (B. & O.), Ky. and Ind. Cantilever, and Van Buren Br., 80000 comp, 70000 tens. The Manufacturers' Standard (1903) employs three grades: Rivet (48-58000), Railway Bridge (55-65000), Medium (60-70000); elastic limit not less than ½ ult str; percentage of elongation, 1400000 ÷ ultimate strength.

† The Bethlehem Steel Co. guarantees the following physical properties in solid and hollow **steel forgings** of different sizes; the tensile test specimen to be used being the U. S. Standard as adopted by the Am. Soc. for Testing Materials (see Fig. 5, page 501).

Class of Steel Forging.		Tensile Strength. Lbs. per Sq. In.	Elastic Limit. Lbs. per Sq. In.	Elongation. Per Cent.	Contraction of Area. Per Cent.	Dimensions of solid and hollow forgings in which the physical qualities mentioned to the left are guaranteed.
Nickel Steel Oil-Tempered.	(1)	95 000	65 000	21	50	Solid or Hollow forgings, no diameter or thickness of section to exceed 3 ins.
	(2)	90 000	60 000	22	50	Solid forgings of rectangular sections not exceeding 6 ins. thick; or Hollow forgings, the walls of which do not exceed 6 ins. in thickness.
	(3)	85 000	55 000	24	45	Solid forgings of rectangular sections not exceeding 10 ins. thick, or Hollow forgings, the walls of which do not exceed 10 ins. in thickness.
Nickel Steel Annealed.	(1)	80 000	50 000	25	45	Solid or Hollow forgings, no diameter or thickness of section to exceed 10 ins.
	(2)	80 000	45 000	25	45	Solid forgings, no diameter to exceed 20 ins., or thickness of section 15 ins.
	(3)	80 000	45 000	24	40	Solid forgings over 20 ins. diam.

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Class of Steel Forging.		Tensile Strength Lbs. per Sq. In.	Elastic Limit. Lbs. per Sq. In.	Elongation. Per Cent.	Contraction of Area. Per Cent.	Dimensions of solid a hollow forgings in whi the physical qualit mentioned to the left a guaranteed.
Carbon Steel Oil-Tempered.	(1)	90 000	55 000	20	45	Solid or Hollow forgi no diameter or thickn of section to exceed 3
	(2)	85 000	50 000	22	45	Solid forgings of rect gular sections not exce ing 6 ins. in thickness Hollow forgings, the w of which do not exc 6 ins. in thickness.
	(3)	80 000	45 000	23	40	Solid forgings or rect gular section not exce ing 10 ins. in thickness Hollow forgings, the w of which do not exc 10 ins. in thickness.
Carbon Steel Annealed.	(1)	80 000	40 000	22	35	Solid or Hollow forgi no diam. or thickness section to exceed 10
	(2)	75 000	37 500	23	35	Solid forgings, no di to exceed 20 ins., thickness of section 15
	(3)	70 000	35 000	24	30	Solid forgings over ins. diam.

Digest of Standard Specifications for Steel, adopted by the Am. S for Testing Materials. (See Proceedings, Vols. 1, V, etc.)

	Ref.	Adopted
i. Stand. Spec. for Structural Steel for Bridges.	Vol. V.	Sept. 1, 19
ii. Stand. Spec. for Open-hearth Boiler Plate and Rivet Steel. - - - - - - - -	Vol. 1.	
iii. Standard Specifications for Steel Rails. -	Vol. VI.	Proposed.
iv. Standard Specifications for Steel Castings. -	Vol. V.	Sept. 1, 19
v. Standard Specifications for Steel Axles. -	Vol. V.	Sept. 1, 19
vi. Standard Specifications for Steel Forgings. -	Vol. V.	Sept. 1, 19

i. Structural Steel for Bridges.

1. *Steel* shall be made by the open-hearth process. 2. The *chemical physical properties* shall conform to the following limits:

Elements Considered.	Structural Steel.	Rivet Steel.	Steel Casting
Phosphorus Max. Basic	0.04 per cent.	0.04 per cent.	0.05 per cen
Phosphorus Max. Acid	0.08 "	0.04 "	0.08 "
Sulphur Max.....	0.05 "	0.04 "	0.05 "
Ult. tensile strength Lbs. per sq. in.	Desired 60 000	Desired 50 000	Not less tha 65 000
Elong.: Min. per cent in 8 ins. (Fig. 3)	1 500 000* / Ult. tens. str.	1 500 000 / Ult. tens. str.	
Elong.: Min. per cent in 2 ins. (Fig. 4)	22		18
Character of fracture	Silky	Silky	Silky or fine gr ular
Cold bend without fracture	180 degrees flat †	180 degrees flat‡	90° on dia. = t × the thickn ($d = 3t$)

* See par. 11. † See par. 12, 13 and 14. ‡ See par. 15.

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The yield point, as indicated by the drop of beam, shall be recorded in the test reports. 3. If the *ult str* varies more than 4000 lbs. from that desired, a *re-test* may be made, at the discretion of the inspector, on the same gauge, which, to be acceptable, shall be within 5000 lbs. of the desired ultimate. 4. *Chemical determinations* for percentages of carbon, phosphorus, sulphur and managnese to be made from test ingot at time of pouring; check analyses from finished material may show 25 per cent above required limits. 5. Specimens for tensile and bending tests for *plates, shapes and bars* shall be made by cutting coupons from the finished product, which shall have both faces rolled and both edges milled to the form shown by Fig. 4; or with both edges parallel; or they may be turned to a dia of ¾ in. for a length of at least 9 ins., with enlarged ends. 6. *Rivet* rods shall be tested as rolled. 7. For *pins and rollers*, specimens shall be cut from the finished rolled or forged bars in such a manner that the center of the specimen shall be 1 inch from surface of bar. Specimen for tensile test shall be turned to the form shown by Fig. 5; specimens for bending test shall be 1 × ½ in. in section. 8. *Steel Castings*. The number of tests will depend on the character and importance of the castings. Specimens shall be cut cold from coupons molded and cast on some portion of one or more castings from each melt or from the sink-heads, if the heads are of sufficient size. The coupon or sink-head, so used, shall be annealed with the casting before it is cut off. Test specimens to be of the form prescribed for pins and rollers. 9. *Material* which is to be used without annealing or further treatment shall be tested in the condition in which it comes from the rolls. When material is to be annealed or otherwise treated before use, the specimens for tensile test, representing such material, shall be cut from properly annealed or similarly treated short lengths of the full section of the bar. 10. *Number of tests*. At least one tensile and one bending test shall be made from each melt of steel as rolled. In case steel differing ⅛ inch and more in thickness is rolled from one melt, a test shall be made from the thickest and thinnest material rolled. 11. *Elongation*. For material less than $\frac{5}{16}$ inch and more than ⅜ inch in thickness the following modifications will be allowed in the requirements for elongation:

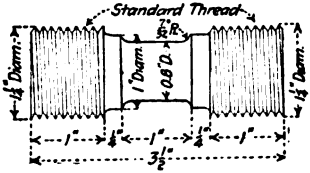

Fig. 3.

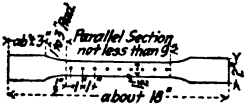

Fig. 4.

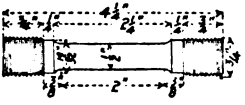

Fig. 5.

(a) For each $\frac{1}{16}$ in. in thickness below $\frac{5}{16}$ in., a deduction of 2½ from the specified percentage.

(b) For each ⅛ in. in thickness above ⅜ in., a deduction of 1 from the specified percentage.

12. *Bending tests* may be made by pressure or by blows. Plates, shapes and bars less than 1 in. thick shall bend as called for in par. 2. 13. *Full-sized material* for eye-bars and other steel 1 inch thick and over, tested as rolled, shall bend cold 180° around a pin whose diam is twice the thickness of the bar, without fracture on outside of bend. 14. *Angles* ¾ inch and less in thickness shall open flat, and angles ½ inch and less in thickness shall bend shut, cold, under blows of a hammer, without sign of fracture. This test to be made only when required by inspector. 15. *Rivet steel*, when nicked and bent around a bar of the same diam as the rivet rod, shall give a gradual break and a fine, silky, uniform fracture.

ii. Open-Hearth Boiler Plate and Rivet Steel.

1. *Steel* shall be made by the open-hearth process. 2. *Chemical properties* of the three classes shall conform to the following limits:

		Plange or boiler steel.	Fire-box steel.	Extra soft steel.
Phosphorus shall not exceed	Acid	0.06 per cent.	0.04 per cent.	0.04 per cent.
	Basic	0.04 "	0.03 "	0.04 "
Sulphur shall not exceed		0.05 "	0.04 "	0.04 "
Manganese		0.30 to 0.60	0.30 to 0.50	0.30 to 0.50

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3. *Steel for boiler rivets* shall be of the extra soft class as specified in pa
and 4. 4. *Physical properties* of the three classes shall be as follows:

	Flange or boiler steel.	Fire-box steel.	Extra soft st
Tensile strength, lbs. per sq. in.	55000 to 65000	52000 to 62000	45000 to 550
Yield point, in lbs. per sq. in. shall not be less than	½ tens str	½ tens str	½ tens str
Elongation, per cent in 8 ins., shall not be less than	25 (see par. 5)	26 (see par. 5)	28 (see par. 5

5. *Elongation.* For material less than $\frac{5}{16}$ inch and more than ¾ inch thickness the following modifications will be allowed in the requireme for elongation:

(a) For each ⅛ in. in thickness above ¾ in., a deduction of 1 from specified percentage.

(b) For each $\frac{1}{16}$ in. in thickness below $\frac{5}{16}$ in., a deduction of 2½ from specified percentage.

6. *Bending tests.* The three classes shall conform to the following bend tests; the bending-test specimen to be 1½ ins. wide if possible, and for material ¾ in. in thickness or less, it shall be of the same thickness as t of the material from which it is cut; but it may be ½ in. thick for mate over ¾ in. thick. (Round rods shall be tested of full size as rolled.): (c) T specimens cut from the rolled material as specified above, shall be subjec to a cold bending test, and also to a quenched bending test. The cold be ing test shall be made on the material in the condition in which it is to used; and prior to the quenched bending test, the specimen shall be hea to a light cherry-red as seen in the dark and quenched in water, the t perature of which is between 80 and 90° F. (d) Flange or boiler steel, f box steel and rivet steel (all three classes), both before and after quench shall bend cold 180° flat on itself without fracture on outside of bend. 7. *Ho geneity.* For fire-box steel a sample taken from a broken tensile test sp men shall not show any single seam or cavity more than ¼ in. long in ei of the three fractures obtained on the test for homogeneity as descri below in paragraph 12. 8. *Test pieces and testing.* The standard specimen of 8 in. gauged length, shall be used to determine the phys properties specified in par. 4 and 5. The standard shape of the test sp men for sheared plates shall be as shown in Fig. 4, preceding, the piece to of same thickness as the plate. For other material the test specimen m be the same as for sheared plates, or it may be planed or turned para throughout its entire length, and in all cases where possible two oppo sides of the test specimens shall be the rolled surfaces. Rivet rounds small rolled bars shall be tested of full size as rolled. 9. *One tensile specimen* will be furnished from each plate as it is rolled, and two ten test specimens will be furnished from each melt of rivet rounds. In c any one of these develops flaws or breaks outside of the middle third of gauged length, it may be discarded and another test specimen substitu therefor. 10. For material ¾ in. or less in thickness the *bending test specim* shall have the natural rolled surface on two opposite sides. The bend test specimens cut from plates shall be 1½ ins. wide, and for material m than ¼ in. thick they may be ½ in. thick; the sheared edges may be mi or planed. The bending test specimens for rivet rounds shall be of full as rolled. The bending test may be made by pressure or by blows. 11. cold and one quenched bending specimen will be furnished from e plate as it is rolled. Two cold and two quenched bending specimens be furnished from each melt of rivet rounds. The homogeneity test for f box steel shall be made on one of the broken tensile test specimens. 12. homogeneity test for fire-box steel is made as follows: A portion of broken tensile test specimen is either nicked with a chisel or grooved o machine, transversely about $\frac{1}{16}$ in. deep, in three places about 2 apart. The first groove should be made on one side, 2 ins. from the squ end of the specimen; the second, 2 ins. from it on the opposite side; the third, 2 ins. from the last, and on the opposite side from it. The specimen is then put in a vice, with the first groove about ¼ in. above jaws, care being taken to hold it firmly. The projecting end of the specimen is then broken off by means of a hammer, a number of light bl being used, and the bending being away from the groove. The specime

broken at the other two grooves in the same way. The object of this treatment is to open and render visible any seams due to failure to weld up, or to foreign interposed matter, or cavities due to gas bubbles in the ingot. After rupture, one side of each fracture is examined, a pocket lens being used if necessary, and the lengths of seams and cavities determined.

iii. Steel Rails.

1. *Manufacture.* (a) The entire process of manufacture and testing shall be in accordance with the best current practice, and conform carefully to the following instructions: (b) Ingots shall be kept in a vertical position in the pit heating furnaces until ready to be rolled, or until the metal in the interior has time to solidify. (c) No bled ingots shall be used. (d) Sufficient material shall be discarded from the top of ingot to insure sound rails. 2. *Chemical composition.* Rails of the various weights per yard specified below shall conform to the following limits in chemical composition:

	50 to 59 pounds. Per cent.	60 to 69 pounds. Per cent.	70 to 79 pounds. Per cent.	80 to 89 pounds. Per cent.	90 to 100 pounds. Per cent.
Carbon	.35–.45	.38–48.	.40–.50	.43–.53	.45–.55
Phosphorus, shall not exceed	.10	.10	.10	.10	.10
Silicon, shall not exceed	.20	.20	.20	.20	.20
Manganese	.70–1.00	.70–1.00	.75–1.05	.80–1.10	.80–1.10

3. One *drop test* shall be made on a piece of rail not less than 4 ft. and not more than 6 ft. long, selected from every fifth blow of steel. The test shall be taken from the top of the ingot. The rail shall be placed head upwards on the supports, and the various sections shall be subjected to the following impact tests under a free falling weight:

Weight of Rail—lbs. per yard,			45 to 55;	height of drop,		15 ft.
"	"	"	55+ to 65	"	"	16 "
"	"	"	65+ to 75	"	"	17 "
"	"	"	75+ to 85	"	"	18 "
"	"	"	85+ to 100	"	"	19 "

If any rail break when subject to the drop test, two additional tests taken from the top of the ingot will be made of other rails from the same blow of steel, and if either of these latter tests fail, all the rails of the blow which they represent will be rejected, but if both of these additional test pieces meet the requirements, all the rails of the blow which they represent will be accepted. 4. *Finishing temperature.* The number of passes and speed of train shall be so regulated that on leaving the rolls at the final pass the temperature of the rail will not exceed that which requires a shrinkage allowance at the hot-saws, for a 30-ft. rail of 100-lb. section, of $6\frac{11}{16}$ ins., and $\frac{1}{16}$ in. less for each 5-lb. decrease of section. These allowances to be decreased at the rate of $\frac{1}{160}$ in. for each second of time elapsed between the rail leaving the finishing rolls and being sawn. No artificial means of cooling the rails shall be used between the finishing pass and the hot-saws. 5. The *drop testing machine* shall have a tup of 2000 lbs. weight, the striking face of which shall have a radius of not more than 5 ins., and the test rail shall be placed head upwards on solid supports 3 ft. apart. The anvil block shall weigh at least 20000 lbs., and the supports shall be part of, or firmly secured to, the anvil. The report of the drop test shall state the atmospheric temperature at the time the test was made. (The Am. Soc. C. E. standard rail section is recommended. See Sec. 30, page 560, and Sec. 59, page 1060.)

iv. Steel Castings.

1. Steel for castings may be made by the open-hearth, crucible or Bessemer process. Castings to be annealed unless otherwise specified. 2. *Ordinary castings*, those in which no physical requirements are specified, shall not contain over 0.40 per cent of carbon, nor over 0.08 per cent of phosphorus. 3. Castings which are *subjected to physical test* shall not contain over 0.05 per cent of phosphorus, nor over 0.05 per cent of sulphur.

4. Tested castings shall be of three classes: "Hard," "Medium," an "Soft." The *minimum physical qualities* required in each class shall be a follows:

	Hard Castings.	Medium C.	Soft C.
Tensile strength, lbs. per sq. in - -	85 000	70 000	60 000
Yield point, lbs. per sq. in. [.45 T.] -	38 250	31 500	27 000
Elongation, per cent in two ins. - -	15	18	22
Contraction of area, per cent - - -	20	25	30

5. *Drop test.* A test to destruction may be substituted for the tensile tes in the case of small or unimportant castings, by selecting three castin from a lot. This test shall show the material to be ductile and free fro injurious defects, and suitable for the purposes intended. A lot shall consi of all castings from the same melt or blow. annealed in the same furnac charge. 6. *Percussive test.* Large castings are to be suspended and han mered all over. No cracks, flaws, defects, nor weakness shall appear aft such treatment. 7. *Bending test.* A specimen $1 \times \frac{1}{2}$ in. shall bend co around a diam. of 1 in. without fracture on outside of bent portion, throug an angle of 120° for "soft" castings, and 90° for "medium" castings. 8. *Te sile test piece.* The standard turned test specimen (Fig. 5, p. 501), $\frac{1}{2}$ in. dia and 2-in. gauged length, shall be used to determine the physical properti specified in par. 4. 9. The number of standard test specimens shall depen upon the character and importance of the castings. A test piece shall be c cold from a coupon to be molded and cast on some portion of one or mo castings from each melt or blow or from the sink heads (in case heads sufficient size are used). The coupon or sink-head must receive the sar treatment as the casting or castings, before the specimen is cut out, a before the coupon or sink-head is removed from the casting. 10. *Test pie for bending.* One specimen for bending test, $1'' \times \frac{1}{2}''$, shall be cut cold fro the coupon or sink-head of the casting or castings as specified in par. The bending test may be made by pressure or by blows.

V. Steel Axles.

1. *Manufacture.* Steel for axles shall be made by the open-hearth pro ess. 2. *Chemical properties.* There will be three classes of steel ax which shall conform to the following limits in chemical composition:

	Car and Tender Truck Axles. Per cent.	Driving and Engine Truck Axles.	
		(CarbonSteel) Per cent.	(NickelStee Per cent.
Phosphorus shall not exceed	0.06	0.06	0.04
Sulphur " " "	0.06	0.06	0.04
Manganese " " "	.....	0.06	.....
Nickel.....	.....	.....	3. to 4.

3. *Physical properties.* For car and tender truck axles no tensile test sh be required. 4. The minimum physical qualities required in the two clas of driving and engine truck axles shall be as follows:

	Driving and Engine Truck Ax	
	(Carbon Steel)	(Nickel Stee
Tensile strength, pounds per sq. in.....	80 000	80 000
Yield point, pounds per sq. in.....	40 000	50 000
Elongation..per cent in 2 ins.....	20	25
Concentration of area, per cent.....	25	45

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5. *Drop test.* One axle selected from each melt, when tested by the drop test described in par. 9, shall stand the number of blows at the height specified in the following table without rupture and without exceeding, as the result of the first blow, the deflection given. Any melt failing to meet these requirements will be rejected:

Diam. of axle at center, inches...	4¼	4⅜	4 7/16	4½	4¾	5⅛	5¼
Number of blows.....	5	5	5	5	5	5	7
Height of drop, feet.....	24	26	28½	31	34	43	43
Deflection (max., 1st blow), inches.	8½	8¼	8¼	8	8	7	5½

6. Carbon steel and nickel steel driving and engine truck axles shall not be subject to the above drop test. 7. *Test pieces and testing.* The standard turned test specimen (Fig. 5, page 501), ½ in. diam. and 2-in. gauged length, shall be used to determine the physical properties specified in par. 4. 8. For driving and engine truck axles one longitudinal test specimen shall be cut from one axle of each melt. The center of this test specimen shall be half-way between the center and outside of the axle. 9. The points of supports on which the axle rests during tests must be 3 ft. *c.* to *c.*; the tup must weigh 1640 lbs.; the anvil, which is supported on springs, must weigh 17500 lbs.; it must be free to move in a vertical direction; the springs upon which it rests must be 12 in number, of the kind described on drawing; and the radius of supports and of the striking face on the tup in the direction of the axis of the axle must be 5 ins. When an axle is tested it must be so placed in the machine that the tup will strike it midway between the ends, and it must be turned over after the first and third blows, and when required, after the fifth blow. To measure the deflection after the first blow prepare a straight edge as long as the axle, by reinforcing it on one side, equally at each end, so that when it is laid on the axle, the reinforced parts will rest on the collars or ends of the axle, and the balance of the straight edge not touch the axle at any place. Next place the axle in position for test, lay the straight edge on it, and measure the distance from the straight edge to the axle at the middle point of the latter. Then after the first blow, place the straight edge on the now bent axle in the same manner as before, and measure the distance from it to that side of the axle next to the straight edge at the point farthest away from the latter. The difference between the two measurements is the deflection. The report of the drop test shall state the atmospheric temperature at the time the tests were made.

vi. Steel Forgings.

1. Steel forgings may be made by the open-hearth, crucible or Bessemer process. 2. *Chemical properties.* There will be four classes of steel forgings which shall conform to the following limits in chemical composition:

	Forgings of Soft or Low Carbon Steel.	Forgings of Carbon Steel not Annealed.	Forgings of Carbon Steel, Oil Tempered or Aneal'd.	Locomotive Forgings	Forgings of Nickel Steel, Oil Tempered or Aneal'd.
	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.
Phosphorus shall not exceed	0.10	0.06	0.04	0.05	0.04
Sulphur " " "	0.10	0.06	0.04	0.05	0.04
Manganese " " "	.....	.....	.....	0.60	.....
Nickel.....	.....	.....	.....	.....	3. to 4.

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3. *Physical properties.* The minimum physical qualities required of t different sized forgings of each class shall be as follows:

	Tensile Strength.	Yield Point.	Elongation in 2 ins.	Contraction
Soft Steel or Low Carbon Steel.	Lbs. per sq. in.		Per ce	
For solid or hollow forgings, no diam. or thickness of section to exceed 10″	58 000	29 000	28	
Carbon Steel Not Annealed.				
For solid or hollow forgings, no diam. or thickness of section to exceed 10″	75 000	37 500	18	
Carbon Steel Annealed.		Elastic Limit.		
For solid or hollow forgings, no diam. or thickness of section to exceed 10″	80 000	40 000	22	
For solid forgings, no diam. to exceed 20″ or thickness of section 15″	75 000	37 500	23	
For solid Forgings, over 20″ diam.	70 000	35 000	24	
Carbon Steel, Oil Tempered.				
For solid or hollow forgings, no diam. or thickness of section to exceed 3″	90 000	55 000	20	
For solid forgings of rect. sections not exceeding 6″ in thickness; or hollow forgings the walls of which do not exceed 6″ in thickness	85 000	50 000	22	
For solid forgings of rect. sections not exceeding 10″ in thickness; or hollow forgings, the walls of which do not exceed 10″ in thickness	80 000	45 000	23	
Locomotive Forgings	80 000	40 000	20	
Nickel Steel Annealed.				
For solid or hollow forgings, no diam. or thickness of section to exceed 10″	80 000	50 000	25	
For solid forgings, no diam. to exceed 20″ or thickness of section 15″	80 000	45 000	25	
For solid forgings, over 20″ diam.	80 000	45 000	24	
Nickel Steel, Oil Tempered.				
For solid or hollow forgings, no diam. or thickness of section to exceed 3″	95 000	65 000	21	
For solid forgings of rect. sections not exceeding 6″ in thickness; or hollow forgings, the walls of which do not exceed 6″ in thickness	90 000	60 000	22	
For solid forgings of rect. sections not exceeding 10″ in thickness; or hollow forgings, the walls of which do not exceed 10″ in thickness	85 000	55 000	24	

4. *Bending test.* A specimen 1×½ in. shall bend cold 180° around t following diameters without fracture on outside of bent portion: ½″ diam. forgings of soft steel, and for forgings of nickel steel annealed; 1″ diam. forgings of carbon steel annealed, if under 20″ diam. and for forgings carbon steel oil-tempered, and for forgings of nickel steel oil-temper 1½″ diam. for forgings of carbon steel not annealed, and for forgings of carb steel annealed, if 20″ diam. or over. For locomotive forgings no bend test will be required. 5. The *standard turned test specimen* (Fig. 5, pa 501), ½ in. diam. and 2-in. gauged length, shall be used to determine t physical properties specified in par. 3. 6. The number and location of t specimens to be taken from a melt, blow, or a forging shall depend upon character and importance and must therefore be regulated by individ cases. The test specimens shall be cut cold from the forging or full-si prolongation of same parallel to the axis of the forging and half-way betwe the center and outside, the specimens to be longitudinal, i. e., the length the specimen to correspond with the direction in which the metal is m drawn out or worked. When forgings have large ends or collars, the t

specimens shall be taken from a prolongation of the same diameter or section as that of the forging back of the large end or collar. In the case of hollow shafting, either forged or bored, the specimen shall be taken within the finished section prolonged, half-way between the inner and outer surface of the wall of the forging. 7. The *specimen for bending test*, 1×½ in., shall be cut as specified in par. 6. The bending test may be made by pressure or by blows.

C. Building Stones, Cements, Etc.

9.—Compression, Tension, Bending, etc., of Above Materials.

[Pounds per square inch.]

Material.	Compression. (Ult.)	Tension. (Ult.)	Bending. (Extr. Fiber.)
*Bluestone (New York).....Use	12 000	1200	2500
Brick, soft, inferior.....	1 000	60	.....
good common.....	10 000	200	600
† Bay State, medium burned.....	11 000	.....	.....
† Face, hard burned.....	13 000	.....	.....
† Common, hard burned.....	15 000	.....	.....
‡ Paving, 5000 to 7000.....	6 000	.....	.....
Pressed 4000 to 11000.....	6 000	.....	.....
Vitrified, tests on 2" cubes.....	38 000	Extra high record.	

*High Records for Compressive Strength of Bluestone (Hudson R.).—41422 lbs. per sq. in.; 38000 lbs. per sq. in., at capacity of machine without failing. Test made on 2-in. cube.

† Tests at Watertown Arsenal (1883): Bay State, medium burned, 10390 to 12709; Face, hard burned, 11050 to 16734; Common, hard burned, 12995 to 22351 lbs. per sq. in. The direction of pressure was at right angle to the largest face. (For tests of Brick Piers, see page 522.)

‡ A commission appointed by the Nat'l Brick Man'f'rs' Ass'n (1895) to study the best methods for testing paving brick concluded that the "Rattler" test was the most valuable; that the absorption and cross-bearing tests were of little use; and the crushing test was useless. A standard rattler test was formulated as follows: The rattler barrel is 28" diam.×20" long; it is a 14-sided polygon, supported on trunnions, the shaft not being continuous through the barrel; it is built of cast iron heads and either cast or wrought iron staves 6" wide; the space between the staves, for the escape of dust, not to exceed ⅜" in width; rate of rotation 30 revs. per min., with 28 and 32 as limits; each test requiring 1800 revs., or 60 mins. at standard speed; number of brick per test charge, 9 to 12; an official test to be the average loss in per cent of initial charge from two separate charges; the brick to be absolutely dry when tested; each charge to contain, in addition to the brick, 225 lbs. of 1½ in. cast iron cubes at 0.88 lb. each when new, and also 75 lbs. cast iron shot 2½×2½×4½ ins. at about 7 lbs. each (to be renewed when the loss in weight is 10%). The losses in the brick vary from 12 to 25%. Mr. Edward Orton, Jr., (see Proc. Am. Soc. for Test'g Mat'ls, Vol. V, page 296) considers 17% loss for heavy traffic streets and 20% loss for light residence streets to constitute reasonable limits for standard tests.

[Pounds per square inch.]

Material.		Compression. (Ult.)	Tension. (Ult.)	Bending. (Extr. Fiber.)	Reference.
			Tensile Strength.		
		1 day.	7 days.	28 days.	
Cement, Natural, neat.....	Am. Soc. standard.	50-100	100-200	200-300	See pages 412, 413.
" 1 cement, 3 sand		.....	25- 75	75-150	
Portland, neat.....		150-200	450-550	550-650	
" 1 cement, 3 sand ...		.....	150-200	200-300	

9.—STRENGTH OF BUILDING STONES, CEMENTS, ETC.—Continued.

* TESTS BY W. PURVES TAYLOR. (Rate of stress, 600 lbs. per min.)	7 days.	28 days.	3 mos.	6 mos
	Tensile Strength, *t*.			
Cement, Natural, neat. (24 specimens)....	253	355	441	453
" 1 cem., 2 std. quartz sand (24 specimens).....	196	301	426	508
Portland, neat. (24 specimens)....	703	741	744	705
" 1 cem., 3 st'd quartz sand. (96 specimens) .	238	316	343	349
1 cem., 3 Ottawa sand. (24 specimens).....	320	374	435	400
	Compressive Strength, *c*.			
Cement, Natural, neat. (24 specimens)....	2010	2689	3643	5052
" 1 cem., 2 st'd quartz sand. (24 specimens) .	940	1390	1730	2012
Portland, neat. (24 specimens)....	5915	7041	7347	9760
" 1 cem., 3 st'd quartz sand. (96 specimens) .	941	1290	1490	1529
" 1 cem., 3 Ottawa sand. (24 specimens).....	1199	1796	1887	2181

	Ratios of *t* to *c*, and *c* to *t*.							
	$\frac{c}{t}$	$\frac{t}{c}$	$\frac{c}{t}$	$\frac{t}{c}$	$\frac{c}{t}$	$\frac{t}{c}$	$\frac{c}{t}$	[illegible]
Cement, Natural, neat. (24 specimens)...	7.9	.13	7.6	.13	8.5	.12	11.1	[illegible]
" 1 cem., 2 st'd quartz sand. (24 specimens).	4.7	.21	4.6	.19	4.1	.25	4.0	[illegible]
Portland, neat. (24 specimens)...	8.4	.12	9.5	.11	9.9	.10	13.8	[illegible]
" 1 cem., 3 st'd quartz sand. (96 specimens).	3.9	.25	4.1	.24	4.3	.23	4.4	[illegible]
" 1 cem., 3 Ottawa sand. (24 specimens).....	3.7	.27	4.8	.21	4.3	.23	5.5	[illegible]

* Compression tests made on cylindrical specimens one inch in heig and with a diam. ($1\frac{1}{8}$+in.) that will give an area of one sq. in. No the dropping off in tensile strength of neat Portland, from 3 to 6 mo also the same characteristic when used with Ottawa sand. Long time tes extending up to a year or more, frequently show a dropping off in tens strength in a few weeks or months, and later increased strength. This doubtless due to some changed condition in the process of hardening. age of two years the tensile strength of neat Portland may be assumed 14 to 18% greater than 6 mos.; and the tensile strength of neat Natur 18 to 22% greater. Portland mortars increase about 18%, and Natur mortars about 6%, in the period between 6 mos. and two years.

Concrete, Portland. Compression.

Varying Mixtures, Columns, 60 *days.*

The following formula for strength of short, round, plain (unrei forced) stone concrete columns, is deduced from tests reported by Pr A. N. Talbot (see *Eng. News*, Sept. 26, 1907). The columns we generally 12 ins. in diam and 10 ft. long. The concrete was a wet mi ture of proportions 1 : $1\frac{1}{2}$: 3, 1 : 2 : 4, 1 : 3 : 6, and 1 : 4 : 8. The for were removed in 10 days and the columns were tested in *60 days*. T straight-line formula is:

$$\textit{Ultimate compressive strength}, \text{ lbs. per sq. in.} = \frac{12000}{S_s + S_t} - 400 \ldots\ldots\ldots$$

in which S_s = ratio of sand to cement, and S_t = ratio of stone to cement. } Thus, in 1 : 2 : 4 concrete $S_s = 2$, and $S_t = 4$.

The following values are deduced from equation (1):

Mixture.....	1:1:2	1:1½:3	1:2:4	1:2½:5	1:3:6	1:3½:7	1:4:8
Ult. comp. str., lbs. per sq. in..	(3600)	2267	1600	1200	933	743	600

The value 3600, for 1:1:2 concrete, is beyond the range of the tes but it is believed to be fairly reliable.

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6" Stone Concrete Cubes, 1:2:4 mix, 30 days.

Compression tests on 6-inch cubes, by Prof. Edgar Marburg (Proc. Am. Soc. Test'g Mat'ls, 1904), give mean average values as follows: Wet, or rather medium, mixtures (amount of water 16% of combined weight of cement and sand, the water flushing freely to top during tamping), *av.* 1650 lbs. per sq. in., with 1364 min. and 2503 max.; two cubes mixed "dry" with 10% water, instead of 16%, developed a compressive strength, without crushing, of 2778 lbs. per sq. in., the limit of the testing machine being 100000 lbs.

8" and 12" Trap-rock Concrete Cubes, 1:2.3:4.6 mix.

(Watertown Arsenal Tests.)

These were wet, or rather medium, mixtures (amount of water 15.2% of combined weight of cement and sand):

2- 8" cubes	developed	(2630 and 2440),	av. 2535,	lbs. per sq. in.	at 30dys
2- 8" "	"	(3830 and 3300),	av. 3565,	lbs. " "	60 "
2-12" cubes	"	(3360 and 3100),	av. 3230,	lbs. " "	30 "
2-12" "	"	(3940 and 4780),	av. 4360,	lbs. " "	60 "
1-12" cube	"		5170	lbs. " "	6 mos

Age, months	1,	2,	6.	1,	2,	6,	1,	2,	6,
Ratio of strength, 8" cubes	1.0	1.40	...	0.71	1.0	...	....	....	....
" " 12" "	1.0	1.35	1.6	0.74	1.0	1.19	0.62	0.84	1.0

Note that the *size* of the cube seems to be a function of the crushing strength of concrete, as it is with natural stone—granite, limestone, etc.

The elastic limit of concrete under compression is about ½ to ⅔ (say ⅝) the ultimate strength.

Concrete, Portland. Tension.

(Tests by Prof. W. K. Hatt, 1901-2.)

No.	Kind Stone.	Age in Days.	Modulus of Elasticity, lbs. per sq. in.	Elongation at Rupture 1 part in.	Strength lbs. per sq. in.	Where Broken.
1	1:2:4	35	2 700 000	11 660	300	At pin.
2	1:2:4	33	2 400 000	8 750	305	"
3	1:2:4	28	1 400 000	4 400	360	"
4	1:2:4	26	1 900 000	7 700	280	"
	Average		2 100 000	7 000	311	....
5	1:2:4	28	.....	910	281	Body

The tension specimens were 4" square in cross-section, with gauged length of 24", and 30" between centers of pins, through heads 8" diam. The concrete was fairly dry concrete intended to be plastic after a thorough ramming. About 5.5% of water by weight was added to the mortar. Prof. Hatt says: These tension tests were not satisfactory for determination, since the heads of the bars, with some exceptions, pulled off; but it is believed that the strength of the body of the bars did not differ greatly from the loads recorded at the point of rupture of the heads. They served, however, very well for the determination of modulus of elasticity.

The elastic limit of concrete under tension is very near the ult. str.

Concrete, Portland. Compression, Tension, Bending, and Shearing.*

That some broad relations exist between the compressive strength, tensile strength, modulus of rupture (extreme fiber stress due to bending) and shearing strength of concrete, is quite probable; but it is certain that no constant ratios can be assigned. Even for the same proportions of mixture the above mentioned ratios would vary with the consistency of the mix and the age of the concrete. We have seen also that compressive strength per sq. in. increases with the size of the cube or mass,

*For Shearing values of stone and concrete, see Proc. A. S. T. M., Vol. VIII., 1908.—By H. H. Quimby.

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but we have every reason to believe that a like increment would for tension, bending and shearing, although the compressive st cubes is influenced greatly by the shearing resistence of the

The following ratios are approximate only:

Compressive Strength.	Tensile Strength.	Mod. of Rupture.	Shearin Streng
1.00	0.08 to 0.12	.....	.....
....	1.00	1.4 to 1.8	1.2 to

Concrete, Portland. Modulus of Elasticity (E) of concrete increases richness of the mixture and with the age; it decreases (usuall stress increases; and is greater (usually) when the concrete compression than when under tension, although for practical of design the latter distinction is seldom recognized. Some authorities assume the ratio of the modulus of elasticity of co that of steel as 1:9 or 230 000 (kilograms per sq. meter), mal English units = 3 270 000 (lbs. per sq. in.), this value being us calculations of reinforced concrete beams. In American pra high value of E would be assigned to a rich mixture of concrete 3 to 6 months. (See Sec. 25, Masonry, page 445.)

Concrete, Portland. **Heat Effect.** The coefficient of linear exp concrete may be assumed, for all practical purposes, to equ steel, namely, .000006½ per degree Fahrenheit, or a change in .01 ft. per 100 ft. in length for each 15 degrees variation in ten Fahr. To this fact, the use of reinforced-concrete is made pos

Heating tests on concrete prisms, by Prof. Ira H. Woolson, University, 1905–6, indicate a marked decrease in strength an of elasticity after the specimens had been subjected to a temp 750° F., and a decided decrease in both of these physical char after exposure to 1500° F. It is to be noted that broken concrete stood the heating effect far better than the smooth gravel concrete, which latter practically disintegrated at t temperature, maintained for 2 to 3 hours. The specimens 4" to 7" in size which naturally exposed, in the furnace, a p ately large "surface" to "mass," and hence no direct conclusi drawn from these tests as to the fire-resisting qualities of c actual construction. The tests, however, showed the infe gravel concrete under the conditions imposed.

Concrete, Natural. The compression, tension, modulus of ru shearing of natural (Rosendale) cement concrete may be a about 50% of the values for Portland cement concrete, unl ular brands of known value are used.

Concrete, Cinder.

Watertown Arsenal Tests (1903) *for Eastern Expanded Metal*

(12-in. cubes, Lehigh Portland Cement.)

Proportions. Cem. Sand. Cinder.	Age, Days.	Modulus of Elasticity— At 500 to 1000 lbs.	At ult. str.	Lb
1 : 2 : 4	38	1 786 000	1 136 000	
	38	1 923 000	1 136 000	
	224	1 471 000	1 087 000	
	224	1 563 000	463 000	
1 : 2½ : 5	38	1 250 000	.....	
	38	893 000	.....	
	224	1 136 000	893 000	
	224	1 250 000	694 000	
1 : 3 : 6	34	781 000	.....	
	34	1 000 000	.....	
	220	1 000 000	694 000	
	220	735 000	463 000	

Glass. (See D. Miscellaneous Materials, Table 10, page 512.)

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9.—STRENGTH OF BUILDING STONES, CEMENTS, ETC.—Continued.

Material.	Color & Grain.	Compression. (Ult.)	Tension. (Ult.)	Bending. (Extr. Fiber.)
Gneiss and Granite [tested in small cubes].				
California, Penryn,	Gray; fine.	6 000	.....	.....
Rocklin,	White; fine.	5 200	.....	.....
Colorado, Georgetown,	Gray; fine.	17 500	.....	.....
Connecticut, Stony Creek,	Pink; fine.	16 000	(NOTE.—High Records for Compressive Strength of Granite (Conn. Val.).—43300 and 40840 lbs. per sq. in.; average of 23 tests from same quarries gave 35000 lbs. per sq. in. Tests made on nominally 2-in. cubes, prepared by sawing and rubbing, but not polished.)	
Georgia, Lithonia,	Gray; fine.	18 000		
Maine, Fox Island,	Gray; coarse.	15 000		
Waldo County,	Gray; fine.	23 000		
Maryland, Port Deposit,	Bluish-gray.	21 000		
Massachusetts, Quincy,	Bl.-Gr.; coarse.	17 000		
Rockport,	Greenish.	19 000		
Minnesota, E. St. Cloud,	Gray.	27 000		
E. St. Cloud,	Red.	24 000		
Sauk Rapids,	Gray.	23 000		
Missouri, Graniteville,	Red; coarse.	13 000		
N. Hampshire, Concord,	White; fine.	16 000		
Virginia, Richmond,	Gray; fine.	25 000	.....	.....
Wisconsin, Athelstane,		20 000	.....	.....
Montello,	Rd.-Gr.; fine.	13 500	.....	.....
(Average of above Gneisses and Granites)		17 733	.....	.....
Unless tested, use, for good stone		12 000	.....	1 600

Material.	Compression. (Ult.)	Tension. (Ult.)	Bending. (Extr. Fiber.)
Limestone (*L*) **and Marble** (*M*) [tested in small cubes].			
California, Colton (*M*)	17 000	.....	.....
Connecticut, Canaan, (*M*)	5 800	.....	.....
Illinois, Lemont, (Dolomite *L*)	13 000	.....	.....
Indiana, Greensborough, (*L*)	16 500	.....	.....
Putnamville, (*L*)	11 500	.....	.....
Kentucky, Bardstown, (*L*)	15 500	.....	.....
Massachusetts, Lee, (*M*)	22 000	.....	.....
Michigan, Lime Island, (*L*)	20 000	.....	.....
Marquette, (*L*)	7 500	(NOTE.—For Tensile Strengths of Granites and Limestones, see table of tests in "Baumaterialienkunde," Aug. 1, 1905; Eng. News, Oct. 12, 1905.)	
Minnesota, Frontenac, (Dolomite *L*)	11 500		
Stillwater, (Dolomite *L*)	25 000		
Missouri, Canton, (*L*)	7 000		
New York, Canajoharie, (*L*)	20 000		
Glens Falls, (*L*)	11 000		
Kingston, (*L*)	12 000	.....	.....
Lake Champlain, (*L*)	23 000	.....	.....
Ohio, Marblehead, (*L*)	12 000	.....	.....
Pennsylvania, Conshohocken, (*L*)	15 000	.....	.....
Montgomery Co., (*M*)	10 500	.....	.....
Vermont, Dorset, (*M*)	8 000	.....	.....
Wisconsin, Big Sturgeon Bay, (*L*)	18 000	.....	.....
Door County, (*L*)	16 000	.....	.....
(Av. of above Limestones and Marbles)	14 445	.....	.....
Unless tested, use, for good limestone	8 000	800	1 500
Masonry. Brickwork, Cement mortar			
Small Brick Piers.			
Face brick; Port. cem. 1, sand 2	2 500	.....	.....
Nat. cem., 1 sand 2	2 000	.....	.....
Lime 1, sand 3	1 500	.....	.....
Com. brick; Port. cem. 1, sand 2	2 000	.....	.....
Nat. cem. 1, sand 2	1 500	.....	.....
Lime 1, sand 3	1 000	.....	.....
Concrete (see Concrete)			
Stonework. Squared stone work from ¼ to ½ the strength of the stone. Use about 4-10 for good ashlar in cement	.....	.....	.....

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9.—STRENGTH OF BUILDING STONES, CEMENTS, ETC.—Co

Material.	Color & Grain.	Compression. (Ult.)	Tension. (Ult.)
Sandstone [tested in small cubes].			
California, Angel Island,	Gr'n-gr'y; fine.	4 500	.....
Colorado, Manitou,	L't red.	9 500	
Connecticut, Portland,	R'd-brown.	13 000	(NOTE
Massachusetts, Long Meadow,	Red; fine.	8 500	ords for C
Michigan, Marquette,	Br.-red.	6 000	Strength o
Minnesota, Fond du Lac,	Red.	6 500	(Pottsdam,
New Jersey, Belleville,	R'd-brown.	10 000	lbs. per sq. i
New York, Medina,	R'd-gr'y; fine	15 000	of machine
North Carolina, Wadesborough,	R'd-br.; fine.	8 000	ing. Test
Ohio, Berea,	Gray; fine.	9 000	nominally
Pennsylvania, Hummelstown,	R'd.-br.; med.	12 500	prepared by
Wisconsin, Fond du Lac,	Brownstone.	5 500	rubbing, bu
(Average of above Sandstones)		9 000	ed.)
Unless tested, use, for good stone		5 000	150
Slate		10 000	3 000
Terra Cotta		5 000	.....

D. Miscellaneous Materials.

10.—ULTIMATE TENSION, COMPRESSION, ETC., OF MISCELL MATERIALS.

[Pounds per square inch.]

Canvas. Tensile strength: lengthwise, 260; crosswise, 330.

Cotton. Tensile strength: belting, solid, woven, 7260; bel stitched, 6160.

Flax. Tensile strength: yarn fiber, 25000; belting, solid, woven ing, folded, stitched, 6400.

Glass, Common green. Tension: thin plates, 4800; bars ½" Compression: small cubes, 20000; small cylinders, 40000. 3000. Mod. of rupture (extreme fiber) 4000. Mod. of elastic

Glass, Flint (best). Tension: thin plates, 4200; bars ½" diam., pression: small cubes, 13000; small cylinders, 27000.

Glass, Flooring. Tension, 3000; compression, 10000; mod. of ru

Ice, Hard. Compression, 250.

Leather, Ox. Tension, 3600. Modulus of elasticity, 250000.

Plaster of Paris. Tension, 70; compression, 700.

III. HEAT EFFECT ON VARIOUS SUBSTANCES.

General Discussion.—Substances are usually classed from standpoint as gases, liquids and solids, according to their con natural atmospheres (with regard to both temperature and pres

A *Gas* may be defined as a substance which when enclose vessel will assume the shape and volume of the vessel; a Liquid the shape of that portion of the vessel which corresponds to the v liquid itself; while a Solid will assume neither the shape nor of an arbitrary vessel.

All gases may be reduced to liquids by lowering the temper gas sufficiently, and subjecting it to the required pressure.

The *Critical Point* or *Critical Temperature* of a gas may b that temperature above which it cannot be liquefied, no matte the pressure exerted upon it. If the temperature of a gas is al the critical temperature it is known as *vapor*, and pressure alon to reduce it to a liquid. Every gas has its critical temperatur this point a sudden change from gas to liquid is accompanied change of volume—with a clear line of demarcation when onl of the gas is so transformed.

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sure is that pressure at which condensation takes place
erature; i. e., it is the vapor tension of a liquid at its
. The *Critical Volume* is the specific volume of the
he critical temperature, and under the critical pressure.

it depends upon the temperature and pressure, and may
emperature above which the body of the liquid passes
The *Absolute Boiling Point* of a liquid is identical with
ure; i. e., it is that temperature above which the body
into a gaseous state no matter what pressure is exerted

of Vaporization is the quantity of heat absorbed in boiling,
e liquid to a gaseous condition merely, without increase
d, conversely, it is the quantity of heat given out in
e of vapor to that of a liquid merely, without decrease
e latent heat of vapor may be expressed (1) in British
T. U.), one B. T. U. being equivalent to 778 ft.-lbs. of
y of heat required to raise the temperature of one pound
r near its temperature of maximum density (39.1° F.),
eabody, at 62° F.; (2) in calories, one calorie being
meter-kilograms of work, or the quantity of heat re-
temperature of one kilogram of water 1° C. at or near
in pounds of the substance which can be raised, by the
n 1° C. from 0° C.; (4), equivalent to (3), the tempera-
pplying the latent heat to one pound of the substance
lso, Section 69, Steam and Gas Power.)

int of a solid (=practically the *freezing point* of the liquid)
rature of fusion. In some substances, as glass and iron,
melting point, while in others it is quite definite. The
body may be raised by increase of pressure, but in most
s relatively small and may be neglected. The laws of
ery substance begins to fuse at a certain temperature
or constant pressure; and (2) during fusion the tempera-
mains constant.

Fusion is analogous to latent heat of vaporization, and
he quantity of heat absorbed in liquefaction, i. e., trans-
o a liquid condition merely, without increase in tempera-
ly, since the laws of solidification are similar to those of
e process.

Gases may be accomplished:

ion. If a gas under pressure is allowed to expand sud-
d if under sufficiently high pressure and allowed to ex-
re will fall to such an extent that it will liquefy. The ice
by cylinder and piston, produces cold by the expansion

ration. The process of evaporation requires heat which
rom the substance being evaporated and from its sur-
lowering of temperature.

g-mixtures. Certain substances when properly mixed in
ons and to the required fineness will produce a decided
ature. The lowest temperature ever attained by mixture
sulphuric ether and solid carbon dioxide, known as
re." The most common mixture is that of salt and ice,
rature of $-22°$ C. may be attained.

erative Method. This method is by far the most general
e gas, which is under pressure, is allowed to expand and,
le to circulate around the vessel containing the compound
emperature of the latter is lowered incrementally to the
on.

Zero of Temperature is that temperature at which heat
On the centigrade scale it is about $-273.7°$ C., usually
° C. All the known gases except helium have been lique-
en found that this gas does not liquefy at the low tempera-
The absolute temperature scale is based on the absolute
re.

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11.—Critical Temperature, Critical Pressure, Boiling Point and Freezing Point.
Gases and Liquids.

Substance.	Critical Temperature		Critical Pressure in Atmosph's	Boiling Point at Atmos. Pressure.		Freezing Point.	
	Cent.	Fahr.		Cent.	Fahr.	Cent.	Fah
Acetic acid	+322°	+612°	57	+117°	+243°	+ 17°	+ 6
Acetylene	+ 37	+ 99	68	.....	.....	.....	...
Air	−140	−220	39	.....	.....	.....	...
Alcohol	+244	+471	64	+ 78	+172	−130	−20
Ammonia	+130	+266	115	− 34	− 29	− 75	−10
Argon	−121	−186	.....	−187	−305	−190	−31
Carbonic acid	+ 31	+ 88	77	− 80	−112	− 78	−10
Carbonic oxide	−141	−222	35	.....	.....	.....	...
Chlorine	+146	+295	94	.....	.....	.....	...
Chloroform	+268	+514	55	+ 60	+140	.....	...
Ether	+194	+381	36	+ 37	+ 99	.....	...
Fluorine	−120	−184	.....	−187	−305	.....	...
Hydrochloric acid	+ 52	+126	86	.....	.....	.....	...
Hydrogen	−235	−391	20	−253	−423	−258	−43
Laughing Gas	+ 36	+ 97	73	.....	.....	.....	...
Marsh gas	− 82	−116	55	.....	.....	.....	...
Nitrogen	−146	−231	35	−192	−314	−214	−35
Oxygen	−115	−175	50	−181	−294	.....	...
Sulphurous acid	+155	+311	79	− 10	+ 14	−100	−14
Water	+365	+689	196	+100	+212	0	+ 3
Wood alcohol	+233	+451	73	+ 66	+150	.....	...

Note.— $F^\circ = \frac{9}{5}C^\circ + 32^\circ$; $C^\circ = \frac{5}{9}(F^\circ - 32^\circ)$.

12.—Boiling Points at Atmospheric Pressure.

Substance.	Cent.	Fahr.	Substance.	Cent.	Fah
Acetic acid	+117°	+243°	Mercury	+358°	+67
Acetone	+ 56	+133	Methylic alcohol	+ 66	+15
Alcohol	+ 78	+172	Naphthalin	+217	+42
Ammonia	− 34	− 29	Nitric acid	+120	+24
Amylic alcohol	+131	+268	Nitrogen	−192	−31
Aldehyde	+ 21	+ 70	Nitrous oxide	− 92	−13
Aniline	+182	+360	Oxygen	−181	−29
Arsenic	+437	+819	Phenanthrene	+340	+64
Benzine	+ 80	+176	Phosphate of phenyl	+407	+76
Benzoic acid	+261	+502	Phosphorus	+290	+54
Benzole	+ 80	+176	Propionic acid	+137	+27
Bisulphide of carbon	+ 47	+117	Saturated brine	+108	+22
Bromine	+ 60	+140	Selenium	+665	+12
Cadmium	+746	+1375	Sulphur	+448	+83
Carbon bisulphide	+ 48	+118	Sulphuric acid	+310	+59
Carbonic acid	− 80	−112	" " (strong)	+318	+60
Chlorform	+ 60	+140	Sulphurous acid	− 10	+
Cyanogen	− 20	− 4	Turpentine (oil)	+157	+31
Ether	+ 37	+ 99	Water (distilled)	+100	+21
Fluorine	−187	−305	" (sea)	+101	+21
Hydrogen	−253	−423	Wood alcohol	+ 66	+15
Iodine	+200	+392	Zinc	+940	+17
Linseed oil	+314	+597			

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13.—MELTING POINTS OF VARIOUS SUBSTANCES.

Substance.	Cent.	Fahr.	Substance.	Cent.	Fahr.
Acetic acid	+ 17°	+ 63°	Lead	+ 325°	+ 617°
Alcohol	− 130	− 202	Magnesium	+ 700	+ 1292
Aluminum	+ 625	+ 1157	Margaric acid	+ 57	+ 135
Ammonia	− 75	− 103	Mercury	− 39	− 38
Antimony	+ 500	+ 932	Nitrobenzine	+ 3	+ 37
Argon	− 190	− 310	Nitrogen	− 214	− 353
Arsenic	+ 500	+ 932	Nitroglycerin	+ 7	+ 45
Benzoic acid	+ 120	+ 248	Palladium	+ 1500	+ 2732
Bismuth	+ 262	+ 504	Phosphorus	+ 44	+ 111
Brass	+ 1025	+ 1877	Platinum	+ 1775	+ 3227
Bromine	− 12	+ 10	Potassium	+ 60	+ 140
Bronze	+ 910	+ 1670	Potassium sulphate	+ 1015	+ 1859
Butter	+ 33	+ 91	Rose's fusible metal	+ 94	+ 201
Cadmium	+ 275	+ 525	Rubidium	+ 39	+ 102
Calcium	at red	heat.	Silver	+ 982	+ 1800
Carbonic acid	− 78	− 108	Sodium	+ 90	+ 194
Cast iron, white	+ 1135	+ 2075	Spermaceti	+ 49	+ 120
" gray	+ 1220	+ 2228	Stearic acid	+ 70	+ 158
Copper	+ 1054	+ 1929	Stearine	+ 55	+ 131
Ethylene	− 169	− 272	Steel	+ 1350	+ 2462
Formic acid	− 13	+ 9	Sulphur	+ 114	+ 237
Gold	+ 1045	+ 1913	Sulphurous acid	− 100	− 148
Hydrogen	− 258	− 432	Tallow	+ 33	+ 92
Hyponitric acid	− 9	+ 16	Tin	+ 229	+ 444
Ice	0	+ 32	Turpentine, oil of	− 27	− 17
Indium	+ 176	+ 349	Wax	+ 65	+ 149
Iodine	+ 107	+ 225	Wood's fusible metal	+ 68	+ 154
Iridium	+ 1950	+ 3542	Wrought iron	+ 1550	+ 2822
Iron, wrought	+ 1550	+ 2822	Zinc	+ 412	+ 774
Lard	+ 33	+ 91			

Note.—Some substances, as glass and iron, have no definite melting point, passing gradually from the solid to the liquid state, by what is known as *vitreous fusion*.

THERMODYNAMICS—REFERENCES.

In Sec. 69, Steam and Gas Power, the following subjects are defined and discussed, and may be considered somewhat pertinent to the present matter in this Section:

British Thermal Unit (B.T.U.)
Density.
Entropy.
Entropy Diagrams.
External and Internal Work.
First Law of Thermodynamics.
Heat.
Heat Equivalent of External Work.
Heat Equivalent of Internal Work.
Heat of Combustion.
Heat of the Liquid.
Heat of Vaporization.
Heating Power of Fuels.
Mechanical Equivalent of Heat.
Mechanical Equivalents of Heat (Table).
Pressure of Saturated Steam.
Specific Heat of the Liquid.
Specific Volume.
Specific Volume of Saturated Steam.
Specific Volume of Water.
Steam.
Steam Tables.
Thermal Energy.
Total Heat.
Thermal Units.
Thermal Unit Equivalents (Table).

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Coefficient of Expansion.—The coefficient of expansion of a body is th rate of increase per deg. of temperature, usually based on the Fahrenheit scal

Let us take a cube of metal, and denote the length of each edge by L when the temperature is t°. At the higher temperature of T° the length o each edge expands to $L+x$. Then we have—

	At T°.	At t°.	Expansion for $T^\circ - t^\circ$.
Length of each edge =	$L+x$;	$=L$.	x
Surface of each face =	$(L+x)^2$;	$=L^2$.	$2Lx+x^2$
Volume of the cube =	$(L+x)^3$;	$=L^3$.	$3L^2x+3Lx^2+x^3$.

Hence (for one degree), we have—

	(Exact.)	(Approx.)
Coef. of Linear Expansion	$=\frac{1}{T^\circ-t^\circ}\times\frac{x}{L}$	$=\frac{1}{T^\circ-t^\circ}\left(\frac{x}{L}\right)$.
Coef. of Surface Expansion	$=\frac{1}{T^\circ-t^\circ}\times\frac{2Lx+x^2}{L^2}$	$=\frac{1}{T^\circ-t^\circ}\left(\frac{2x}{L}\right)$.
Coef. of Volumetric Expansion	$=\frac{1}{T^\circ-t^\circ}\times\frac{3L^2x+3Lx^2+x^3}{L^3}$	$=\frac{1}{T^\circ-t^\circ}\left(\frac{3x}{L}\right)$.

The approximate results are obtained from the assumption that, as itself is so extremely small, any power of x greater than unity (as x^2 an x^3) would be practically *zero*.

It is thus seen that the Surface expansion and the Volumetric expansio are, respectively, *two* and *three* times the Linear expansion.

14.—Coefficients of Linear Expansion of Solids.*
Average Values.

Substance.	*Coef. of Linear Expansion. For 1° Cent.	For 1° Fahr.	Substance.	*Coef. of Linear Expansion. For 1° Cent.	For 1° Fahr.
Aluminum, cast	.0000222	.0000123	Masonry, brick	.0000067	.00000
" wro'ht	.0000231	.0000128	" concrete	.0000120	.00000
Antimony	.0000113	.0000063	" granite		
Bismuth	.0000175	.0000097	" limestone		
Brass	.0000188	.0000104	" sandstone		
Brick	.0000055	.0000031	Nickel	.0000126	.00000
Bronze	.0000181	.0000101	Oak		
Carbon, graphite	.0000079	.0000044	Palladium	.0000100	.00000
Cast iron, gray	.0000106	.0000059	Paraffin	.0002785	.00015
" white			Pewter	.0000203	.00001
Cast steel	.0000110	.0000061	Pine	.0000050	.00000
			Plaster	.0000166	.00000
Cement, Portland, neat	.0000107	.0000059	Platinum	.0000090	.00000
Concrete	.0000143	.0000079	Platinum-Iridium (85–15%)	.0000081	.00000
Copper	.0000168	.0000093	Porcelain	.0000036	.00000
Diamond	.0000012	.0000007	Potassium	.0000830	.00004
Ebonite	.0000770	.0000428	Sal ammoniac	.0000630	.00003
Glass	.0000085	.0000047	Sandstone	.0000110	.00000
Gold	.0000150	.0000083	Silver	.0000192	.00001
Granite	.0000084	.0000047	Slate	.0000104	.00000
Graphite	.0000079	.0000044	Sodium	.0000720	.00004
Guttapercha	.0005980	.0003322	Sodium chloride	.0000404	.00002
Ice	.0000640	.0000356	Steel, cast	.0000110	.00000
Iridium	.0000064	.0000036	" hard	.0000132	.00000
Iron, cast (see above)			" medium	.0000120	.00000
			" soft	.0000110	.00000
wrought	.0000120	.0000067	" tempered	.0000124	.00000
Lead	.0000286	.0000159	Tin	.0000210	.00001
Marble	.0000100	.0000056	Zinc	.0000290	.00001

* Values in table are for Linear expansion. For Surface expansion a Volumetric expansion, multiply by 2 and 3, respectively. To change fro Cent. to Fahr., mult by $\frac{9}{5}$; from Fahr. to Cent., mult by $\frac{5}{9}=1.8$. Note th a range of temperature of 180° F = 100° C.

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IV. FRICTIONAL RESISTANCE OF MATERIALS.

In 1831-4 General Arthur Morin, of the French Artillery, conducted a series of experiments on friction, which were published in his "Lecons de Mecanique Pratique." The results of these experiments have been reprinted extensively in various works because of the care with which the tests were conducted. Tables 15, 16 and 17 are reproduced here from Joseph Bennett's translation of the above work.* Table 16, from Rankine's *Applied Mechanics*, is also added in order to show that author's version of Morin's tests. Later experiments, notably those of Beauchamp Tower,† have added to our knowledge of the laws of friction.

The three fundamental laws of friction deduced by Morin, who experimented with pressure up to about 30 lbs. per sq. in., are as follows:

1st. Friction is directly proportional to the pressure; the coefficient of friction [natural tang. of the angle of repose] being constant at all pressures.

2nd. The total friction and the coefficient of friction are independent of the areas in contact; the total pressure remaining the same. [This naturally follows from the 1st law.]

3rd. The coefficient of friction is independent of the velocity of one surface on the other.

It is perhaps well to suggest here that Table 15 be not used with undue reliability, as any jarring of the structure, the conditions of the atmosphere, etc., might affect the tabular results. Table 16 would be safer to use where stability is desired.

Mr. Tower found in experimenting with revolving journals at high speeds and under heavy pressures, that the coefficient of friction, f, varied directly with the square root of the linear velocity, and inversely with the pressure. With good oil the value of f with revolving journals may be assumed at: $f = \frac{\sqrt{\text{Velocity in ft. per sec.}}}{4 \times \text{Pres. in lbs. per sq. in.}}$.

* "Fundamental Ideas of Mechanics," revised and translated by Joseph Bennett; D. Appleton & Co., New York, 1860. Published by permission.

† See Proceedings of the Institution of Mechanical Engineers, 1883.

15.—FRICTION OF PLANE SURFACES WHICH HAVE BEEN SOME TIME IN CONTACT.

(Morin.)

Kind of Surfaces in Contact.	Disposition of the Fibres.	Condition of the Surfaces.	Ratio of Friction to Pressure, f.	Angle of Friction. ° ′
Oak on oak	Parallel	Without unguent.	0.62	31-48
	"	Rubbed with dry soap	0.44	23-45
	Perpendicular	Without unguent.	0.54	28-22
	"	Moistened with water.	0.71	35-22
	Wood upright on wood flatwise	Without unguent.	0.43	23-16
Oak on elm	Parallel	"	0.38	20-48
Elm on oak	"	"	0.69	34-36
	"	Rubbed with dry soap	0.41	22-18
	Perpendicular	Without unguent.	0.57	29-41
Ash, pine, beech, sorb, on oak	Parallel	"	0 53	27-55
Tanned leather on oak	The leather flatwise	"	0.61	31-23
	The leather on edge.	"	0.43	23-16
		Moistened with water.	0.79	38-19
Black curried leather or belt: On plane oak surface	Parallel	Without unguent.	0.74	36-30
Black curried leather or belt: On oak drum.	Perpendicular	"	0.47	25-10

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15.—FRICTION OF PLANE SURFACES WHICH HAVE BEEN SOME TIME IN CONTACT.—Concl'd.

Kind of Surfaces in Contact.	Disposition of the Fibres.	Condition of the Surfaces.	Ratio of Friction to Pressure, f.	° Angle of Friction
Hemp matting on oak.	Parallel.....	Without unguent.	0.50	26-
	"	Moistened with water.	0.87	41-
Hemp cord on oak	Parallel.....	Without unguent.	0.80	38-
Iron on oak.....	"	"	0.62	31-
	"	Moistened with water.	0.65	33-
Cast iron on oak.....	"	"	0.65	33-
Brass on oak.....	"	Without unguent.	0.62	31-
Ox hide for piston packing on cast iron.	Flatwise.....	Moistened with water.	0.62	31-
	On edge.....	With oil, lard, tallow.	0.12	6-
Block curried leather, or belt upon cast iron pulley.....	Flatwise.....	Without unguent.	0.28	15-
	"	Moistened with water.	0.38	20-
Cast iron upon cast iron.....	"	Without unguent.	0.16*	9-
Iron upon cast iron.....	"	"	0.19	10-
Oak, elm, yoke elm, iron, cast iron, and brass, sliding two and two one upon the other.....	"	Spread with tallow.	0.10†	5-
	"	With oil or lard.	0.15‡	8-
Calcareous oolite upon oolite limestone.....	"	Without unguent.	0.74	36-
Hard calcareous stone called Muschelkalk upon oolite limestone	"	"	0.75	36-
Brick on calcareous oolite	"	"	0.67	33-
Oak " " "	Wood upright	"	0.63	32-
Iron " " "	"	"	0.49	26-
Hard muschelkalk on muschelkalk.....	"	"	0.70	35-
Calcareous oolite upon muschelkalk.....	"	"	0.75	36-
Brick on muschelkalk...	"	"	0.67	33-
Iron upon "	"	"	0.42	22-
Oak on "	"	"	0.64	32-
Calcareous oolite on calcareous oolite.....	"	With mortar: hyd. cem., 1; fine sand, 3	0.74¶	36-

* The surfaces being somewhat unctuous (oily or greasy).

† When the contact had not been long enough to press out the unguen (ointment).

‡When the contact had been long enough to press out the unguen and bring the surfaces to an unctuous state.

¶ After a contact of from 10 to 15 minutes.

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16.—FRICTION OF PLANE SURFACES IN MOTION UPON EACH OTHER. (Morin.)

Surfaces in Contact.	Position of Fibres.	State of Surfaces.	Ratio of Friction to Pressure. f.	Angle of Friction. ° ′
Oak on oak	Parallel	Without unguent.	0.48	25-38
	"	Rubbed with dry soap	0.1	9-05
	Perpendicular	Without unguent.	0.34	18-47
	"	Wet with water.	0.25	14-02
	Upright on Flatwise	Without unguent.	0.19	10-45
Elm on oak	Parallel	"	0.43	23-16
	Perpendicular	"	0.45	24-14
	Parallel	"	0.25	14-02
Ash, pine, beach, wild pear and sorb, on oak	"	"	0.36 to 0.40	19-48 to 21-48
Iron on oak	"	"	0.62	31-48
		Wet with water.	0.26	14-34
		Rubbed with dry soap	0.21	11-52
Cast iron on oak	"	Without unguent.	0.49	26-06
		Wet with water.	0.22	12-24
		Rubbed with dry soap	0.19	10-45
Copper on oak	"	Without unguent.	0.62	31-48
Iron on elm	"	"	0.25	14-02
Cast iron on elm	"	"	0.20	11-19
Black curried leather on oak	"	"	0.27	15-07
Tanned leather on oak	Flatwise on edge		.30 to .35	16-42 to 19-17
	"	Wet with water.	0.29	16-10
Tanned leather on cast iron and brass	Flatwise and on edge	Without unguent.	0.56	29-15
		Wet with water.	0.36	19-48
		Unctuous and wet with water.	0.23	12-57
		Spread with oil.	0.15	8-32
Hemp strips or cords upon oak	Parallel	Without unguent.	0.52	27-28
	Perpendicular	Wet with water.	0.33	18-16
Oak and elm on cast iron	Parallel	Without unguent.	0.38	20-48
Wild pear on cast iron	"	"	0.44	23-45
Iron upon iron	"	"	*	.....
Iron upon cast iron and brass	"	"	0.18†	10-12
Cast iron upon cast iron and brass	"	"	0.15†	8-32
Cast iron on cast iron	"	Wet with water.	0.31	17-13
Brass on brass	"	Without unguent.	0.20	11-19
Brass on cast iron	"	"	0.22	12-24
Brass on iron	"	"	0.16‡	9-05
Oak, elm, yoke elm, wild pear, cast iron, iron, steel, steel brass sliding upon each other or themselves	"	Lubricated in the usual way with tallow, lard, soft coom, etc.	.7 to .8¶	35° to 38-40
Calcareous oolite on calc. oolite	"	Slightly unctuous to the touch.	0.15	8-32
		Without unguent.	0.64	32-37
Muschelkalk on calc. oolite	"	"	0.67	33-49
Common brick on calc. oolite	"	"	0.65	33-01

* Surfaces worn when there was no unguent (ointment).
† The surfaces still being slightly unctuous (oily or greasy).
‡ The surfaces slightly unctuous.
¶ When the unguent is constantly supplied, and uniformly laid on, this ratio may be lowered to 0.05.

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16.—FRICTION OF PLANE SURFACES IN MOTION UPON EACH OTHER—Concl

Surfaces in Contact.	Position of Fibres.	State of Surfaces.	Ratio of Friction to Pressure. f.	° Angle of
Oak on oolitic limestone	Wood upright	Without unguent	0.38	20
Forged iron on oolitic limestone	Parallel	"	0.69	34
Muschelkalk upon muschelkalk	.....	"	0.38	20
Oolitic limestone upon muschelkalk	.....	"	0.65	33
Common brick on muschelkalk	.....	"	0.60	30
Oak on muschelkalk	Wood upright	"	0.38	20
Iron on muschelkalk	Parallel	"	0.24	13
	"	Wet with water.	0.30	16

17.—FRICTION OF JOURNALS IN MOTION UPON THEIR PILLOWS. (Morin.)

Surfaces in Contact.	State of Surfaces.	Ratio of Friction to Pressure when the Unguent is Renewed— In the Common way.	Continuously.
Cast iron journals in cast iron bearings.	Unguents of olive oil, of lard, of tallow, or of soft coom.	0.07 to 0.08	0.030 to 0.
	With the same unguents and moistened with water.	0.08	.....
	Asphalt	0.054	.....
	Unctuous	0.14	.....
	Unctuous and wet with water	0.14	.....
Cast iron journals on brass cushions	Unguents of olive oil, of lard, of tallow, and of soft coom.	0.07 to 0.08	0.030 to 0.
	Unctuous	0.16	.....
	Unctuous and wet with water	0.16	.....
	Slightly unctuous	0.19	*
	Without unguent	0.18	†
Cast iron journals on lignum vitae bearings	Unguents of oil or lard	.....	0.090
	Unctuous with oil or lard	0.10	.....
	Unctuous, with a mixture of lard and black lead	0.14	.....
Wrought iron journals on cast iron bearings	Unguents of olive oil, tallow, lard, or soft coom	0.07 to 0.08	0.030 to 0.
Wrought iron journals on brass bearings	Unguents of olive oil, tallow, lard	0.07 to 0.08	0.030 to 0.
	Unguents of soft coom	0.09	.....
	Unctuous, and wet with water	0.19	.....
	Slightly unctuous	0.25	‡
Iron journals on lignum vitae bearings	Unguents of oil or lard	0.11	.....
	Unctuous	0.19	.....
Brass journals on brass bearings	Unguents of oil	0.10	.....
	Unguents of lard	0.09	.....
Brass journals on cast iron cushions	Unguents of oil or tallow	.....	0.030 to 0.
Lignum vitae journals on cast iron cushions	Unguents of lard	0.12	.....
	Unctuous	0.15	.....
Lignum vitae journals on lignum vitae cushions	Unguents of lard	.....	0.07

* The surfaces began to wear. ‡ The surfaces began to wear away.
† The wood being slightly unctuous.

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—COEFFICIENTS OF FRICTION AND ANGLES OF REPOSE OR FRICTION.
(Condensed and adapted from Morin and others, by Rankine.*)

Materials.	Ratio of Friction to Pressure. f.	Angle of Repose or Friction. ϕ	Cot. ϕ $=\frac{1}{f}$
Structures:			
masonry and brickwork	0.6 to 0.7	31° to 35°	1.67 to 1.43
onry and brickwork, with damp mortar	0.74	36½°	1.35
ber on stone	abt. 0.4	22°	2.5
on stone	0.7 to 0.3	35° to 16⅔°	1.43 to 3.33
ber on timber	0.5 to 0.2	36½° to 11½°	2 to 5
ber on metals	0.6 to 0.2	31° to 11½°	1.67 to 5
als on metals	0.25 to 0.15	14° to 8½°	4 to 6.67
onry on dry clay	0.51	27°	1.96
onry on moist clay	0.33	18½°	3
th on earth	0.25 to 1.0	14° to 45°	4 to 1
th on earth, dry sand, clay, and mixed rth	0.38 to 0.75	21° to 37°	2.63 to 1.33
th on earth, damp clay	1.0	45°	1
th on earth, wet clay	0.31	17°	3.23
th on earth, shingle and gravel	0.81 to 1.11	39° to 48°	1.23 to 0.9
Machines:			
d on wood, dry	.25 to .5	14° to 26½°	4 to 2
" " " soapy	.2	11½°	5
als on oak, dry	.5 to .6	26½° to 31°	2 to 1.67
" " " wet	.24 to .26	13½° to 14½°	4.17 to 3.85
" " " soapy	.2	11½°	5
als on elm, dry	.2 to .25	11½° to 14°	5 to 4
p on oak, dry	.53	28°	1.89
" " " wet	.33	18½°	3
ther on oak	.27 to .38	15° to 19½°	3.7 to 2.86
ther on metals, dry	.56	29½°	1.79
" " wet	.36	20°	2.78
" " greasy	.23	13°	4.35
" " oily	.15	8½°	6.67
als on metals, dry	.15 to .2	8½° to 11½°	6.67 to 5
" " wet	.3	16½°	3.33
oth surfaces, occasionally greased	.07 to .08	4° to 4½°	14.3 to 12.5
" " continually greased	.05	3°	20
" " " best results	.03 to .036	1¾° to 2°	33.3 to 27.6

* *Applied Mechanics*, published by Charles Griffin & Co., London.

Rolling Friction (Dry).—In the above discussion we have dealt with ing friction only, which of course includes journal friction.—(See Tables o 18).

Rolling friction is the resistance at the line of contact only, when a el or cylinder is rolled over a "smooth," (horizontal) surface, by a force acting parallel to the surface) applied at the axle of the wheel or roller adius r inches, in moving the total load (W).

The general equation of rolling friction is,

$$F=\frac{CW}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

hich C, of equation (1), is a constant determined by experiment, for a kind of material rolling on each kind of material.

For iron or steel, rolling on iron or steel, $C=0.02$, about.

Example.—A train weighing 1200 tons, with steel car-wheels 24 ins. in , is drawn on a level track of steel rails, by an engine exerting a pulling e P. What part of P is required to overcome the rolling friction of the n?

Solution.—From equation (1), we have, by substitution,

Rolling friction $F=(0.02\times1200)\div12=2$ tons. Ans.

Rolling friction is a factor in estimating the power required to swing ntables and drawbridges.

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EXCERPTS AND REFERENCES.

Results of Tests of Materials Subjected to Combined Stresses (By E. Hancock. Proc. A. S. T. M., Vol. VIII., 1908).—Maximum stress a shear theories. Diagrams of tests.

Crushing Tests on Water-Soaked Timbers (By E. C. Sherman. E News, July 1, 1909).—Diagrams of tests on water-soaked oak and yell pine blocks, transverse to grain, proposed for supporting tracks for rolli caisson lock gates for Charles River dam. Result of tests showed tra verse tests so low that cast iron chairs were substituted.

Crushing Tests of Brick and Terra-Cotta Piers (By A. N. Talbot a D. A. Abrams. Bulletin No. 27, Ill. Eng'g Exper. Sta.; Eng. News, Aug. 1909).—Summaries of the brick and terra-cotta tests are given below:

SUMMARY OF *BRICK COLUMN TESTS.

Column Characteristics. (Mortar, Age, Loading, Workmanship, Kind of Brick.)	Ultimate Crushing Strength			
	Lbs. per sq. in.	Ratio to first of series.	Ratio to strength c	
			Single brick.	Mort cube
1 : 3 Portland, *well laid* (3 tests).....	3365	St'd	0.31	1.1
1 : 3 Portland, *hurriedly laid* (2 tests)....	2920	0.87	0.27	1.0
1 : 3 Portland, *six months*† (2 tests).....	3950	1.18	0.37	...
1 : 3 Portland, *eccentric loading* (2 tests)..	2800	0.83	0.26	...
1 : 3 Portland, *soft brick* (2 tests).....	1060	0.31	0.27	0.3
1 : 5 *Portland*, (2 tests).....	2225	0.66	0.21	1.3
1 : 3 *Natural* (1 test).....	1750	0.52	0.16	5.7
1 : 2 Lime (2 tests).....	1450	0.43	0.14	...

*Hard bricks: size 2.35 x 3.95 x 8.25 ins.; crush. strength, flatwi bedded in plaster, av. 10700 lbs. per sq. in.; cross-breaking, edgewise, 6- span, av. mod. of rupture, 1670 lbs. per sq. in. Soft bricks: 2.40 x 4 x 8.20 ins.; crush. strength, as above, 3900; mod. of rupture, as above, 4 Size of piers: 12½″ x 12½″ x 10 ft. high, with a 1¼″ C. I. plate at bott. and t

†One column failed at 7th repetition of 4110-lb. load.

SUMMARY OF TERRA-COTTA COLUMN* LOAD.

Column Characteristics. (Mortar, Loading, Workmanship, Kind of Blocks.)	Ultimate Crushing Strength.			
	Lbs. per sq. in.	Ratio to first of series.	Ratio to strength	
			Single block.	Mort cube
1 : 3 Portland, *well laid* (2 tests).....	4300†	St'd	0.83	1.1
1 : 3 Portland, *hurriedly laid* (1 test).....	3305	0.76	0.64	1.0
1 : 3 Portland, *eccentric loading*, well laid (1 test).....	3470	0.81	0.65	1.1
1 : 3 Portland, *hurriedly laid* (1 test).....	3110	0.75	0.60	1.0
1 : 3 Portland, *underburned blocks*‡ (1 test).....	3050	0.71	0.59	0.9
1 : 5 Portland (2 tests).....	3350	0.78	0.65	..
Concave-end blocks, 1 : 2 Port. (6 tests)..	2970	0.69	0.86	..

*All columns 12½ x 12½ ins. nominal, about 9¾ ft. long; made from blo of second shipment except where noted "Concave-end blocks;" all tes in two months.

†Estimated.

‡Blocks from second shipment, all of which were of good quality; enou blocks that appeared somewhat underburned or otherwise inferior w selected to make up one column.

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The Effect of Fire on Building Materials (By Jas. E. Howard. Paper l before the Natl. Assn. of Mutual Insurance Co's.; Eng. Rec., Dec. 3, 0).—The following stresses in lbs. per sq. in. for various materials, when ined, correspond to a rise in temperature of 160° F.: Steel, 30000 lbs. sq. in.; cast iron, 17000 to 13590 lbs. per sq. in.; granite, 6270 to 3200 ; marble, 10800 to 2880 lbs.; slate, 13480 lbs.; sandstone, 3200 to 960 hard burned brick, 4960 lbs.; light hard brick, 2230 lbs.; soft (salmon) k, 248 lbs.; neat portland cement, 3710 lbs. per sq. in. Other experi- ts show that stones when even moderately heated do not return exactly heir primitive dimensions, but retain as a permanent set some of the ex- sion which they acquired when hot, although the permanent sets are paratively small. Experiments on 12″ x 12″ sticks of Douglas fir, ned over a wood fire for 2½ hours, at about 1380° F., until they were from 7 ins., showed increased compressive strength in lbs. per sq. in. for the urned portion.

Gypsum as a Fireproofing Material (By H. G. Perring. Paper, Natl. Protection Assn.; Eng. Rec., Dec. 3, 1910).—Advantages in the use of plaster: (1) Low heat conductivity: the heat penetrates the plaster at a slow rate that in fires of ordinary duration the metal would hardly warm. (2) No appreciable expansion of plaster under heat action. Hence, no contraction and destructive cracking when water is applied fire. (4) Does not burn or support combustion. (5) Plaster is the test practical fireproofing material. (6) Plaster has the required ngth for use in partitions and column coverings, and against fire streams. It is adaptable to any form of construction: even when used in slabs they readily cut or sawed to fit any desired location. (8) Plaster is low in . (9) Trim can be nailed to plaster fireproofing without the use of wood ks or grounds. (10) Plaster blocks or boards, made of pure gypsum wood fiber, are true and even and reduce the cost of plastering to a imum.

Some Tests of Old Timber (By C. P. Buchanan. Eng. News, Dec. 8, 0).—Crushing strength of 25-year-old and of new white pine, compared:

Area, q. ins.	Crushing load, lbs. per sq. in.	Remarks.
		SPECIMENS FROM BRIDGE.
9.25	5780	Clear, bolt-hole mark ¼″ deep on narrow side.
8.50	4795	Clear. Hart. Slightly decayed.
6.31	5950	Sound. One sap corner.
0.00	4920	Clear. Hart. Well defined season cracks.
9.25	5330	Large black knot in longer side and end.
0.00	5100	Clear.
6.00	5440	Clear and slightly decayed.
9.00	5060	Clear. Hart. Well defined season crack.
....	5300	Average of above 8 tests.
		SPECIMENS FROM STOCK.
0.00	4510	Clear.
8.50	3210	Clear.
7.50	4570	Clear. Slight season cracks.
0.00	3900	Clear. Hart. Well defined season cracks.
0.00	5420	Clear. Slight season cracks.
0.00	4420	Clear. Hart. Slight season cracks.
3.89	4070	Clear. Hart. Large season cracks.
6.52	3970	Clear. Hart. Large season cracks.
....	4260	Average of above 8 tests.

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29.—PROPERTIES AND TABLES OF PLAN SURFACES.

I.—GEOMETRICAL FIGURES.

Center of gravity indicated by heavy dot.
(Values are exact.)

NOTATION.

A = area of figure. $I = Ar^2$ = moment of inertia about axis $X-X$; $\sqrt{I \div A}$ = radius of gyration about axis $X-X$. $S = \frac{I}{y}$ = section modu $S_1 = \frac{I}{y_1}$; $S_2 = \frac{I}{y_2}$. a or y_0 = distance from axis considered to parallel through center of gravity.; x = length of strip of infinitesimal width distant y from neutral axis. All in inches.

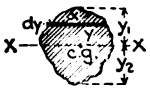

Fig. 1.—Any figure; neutral axis $X-X$ through center of gravity.

$$A = \int_{-y_2}^{y_1} x\,dy \qquad I = \int_{-y_2}^{y_1} xy^2\,dy$$

$$r^2 = I \div A \qquad r = \sqrt{\frac{I}{A}}$$

$$S_1 = \frac{I}{y_1} \qquad S_2 = \frac{I}{y_2}$$

Fig. 2.—Any figure; axis at base.

$$A = \int_0^{y_1} x\,dy \qquad I = \int_0^{y_1} xy^2\,dy$$

$$y_0 = \frac{\int_0^{y_1} y\,dA}{\int_0^{y_1} dA} = \frac{\int_0^{y_1} xy\,dy}{\int_0^{y_0} x\,dy}$$

$$r = \sqrt{\frac{I}{A}} \qquad S = \frac{I}{y_1}$$

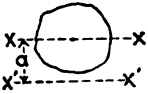

Fig. 3.—Any figure; axis $X'-$ parallel with neutral axis $X-X$.

I' (about $X'-X'$) $= I + a^2A$

$$= \int_0^{y_1} xy^2\,dy +$$

(I is for neutral axis thro cen of grav as in Fig. 1; a = p dist between the two axes.)

Fig. 4.—Triangle; axis thro cen of grav.

$$A = \frac{bd}{2} \qquad I = \frac{bd^3}{36}$$

$$y_1 = \tfrac{2}{3}d \qquad y_2 = \frac{d}{3}$$

$$S_1 = \frac{bd^2}{24} \qquad S_2 = \frac{bd^2}{12}$$

$$r = \frac{d}{\sqrt{18}} = 0.236\,d$$

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e; axis at base.

$$S_1 = \frac{bd^2}{12}$$

8 d

; axis through apex.

$$S_1 = \frac{bd^2}{4}$$

7 d

gle or parallelo-
y_2.

9 d

rectangle; $y_1 = y_2$.

d_1^3

d_1^3
$b_1 d_1)$

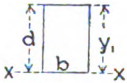

Fig. 9.—Rectangle; axis at base.

$$I = \frac{bd^3}{3} \qquad S_1 = \frac{bd^2}{3}$$

$$r = \frac{d}{\sqrt{3}} = 0.577\,d$$

Fig. 10.—Rectangle; diagonal axis.

$$y_1 = y_2 = \frac{bd}{\sqrt{b^2+d^2}}$$

$$I = \frac{b^3d^3}{6\,(b^2+d^2)}$$

$$S_1 = S_2 = \frac{b^2d^2}{6\sqrt{b^2+d^2}}$$

$$r = \frac{bd}{\sqrt{6(b^2+d^2)}}$$

Fig. 11.—Rectangle; angular axis.

$$y_1 = y_2 = \frac{d\cos\alpha + b\sin\alpha}{2}$$

$$I = \frac{bd}{12}\,(d^2\cos^2\alpha + b^2\sin^2\alpha)$$

$$S_1 = S_2 = \frac{bd}{6}\left(\frac{d^2\cos^2\alpha + b^2\sin^2\alpha}{d\cos\alpha + b\sin\alpha}\right)$$

$$r = \sqrt{\frac{d^2\cos^2\alpha + b^2\sin^2\alpha}{12}}$$

Fig. 12.—Square or rhomboid; $y_1 = y_2$.

$$A = d^2 \qquad I = \frac{d^4}{12}$$

$$S_1 = S_2 = \frac{d^3}{6} \qquad r = \frac{d}{\sqrt{12}} = 0.289\,d$$

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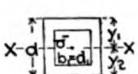

Fig. 13.—Hollow square; $y_1 = y_2$.

$$A = d^2 - d_1^2$$

$$I = \frac{d^4 - d_1^4}{12}$$

$$S_1 = S_2 = \frac{d^4 - d_1^4}{6d}$$

$$r = \sqrt{\frac{d^2 + d_1^2}{12}}$$

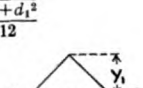

Fig. 14.—Square; diagonal axis.

$$y_1 = y_2 = \frac{d}{\sqrt{2}} = 0.707\,d$$

$$I = \frac{d^4}{12} \text{ (same as for 12)}$$

$$S_1 = S_2 = \frac{d^3}{6\sqrt{2}} = 0.118\,d^3$$

$$r = \frac{d}{\sqrt{12}} = 0.289\,d \text{ (same as for 12)}$$

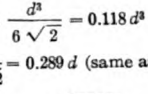

Fig. 15.—Hollow square; diag axis.

$$y_1 = y_2 = \frac{d}{\sqrt{2}} = 0.707\,d$$

$$A = d^2 - d_1^2$$

$$I = \frac{d^4 - d_1^4}{12}$$

$$S_1 = S_2 = \frac{d^3 - d_1^3}{6\sqrt{2}}$$

$$= 0.118\,(d^3 - d_1^3)$$

$$r = \frac{d - d_1}{\sqrt{12}} = 0.289\,(d - d_1)$$

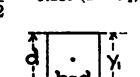

Fig. 16.—Square; axis at base.

$$I = \frac{d^4}{3} \qquad S_1 = \frac{d^3}{3}$$

$$r = \frac{d}{\sqrt{3}} = 0.577\,d$$

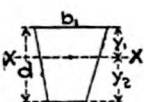

Fig. 17.—Trapezoid; axis
cen of grav.

$$y_1 = \frac{d}{3}\left(\frac{b_1 + 2b_2}{b_1 + b_2}\right)$$

$$y_2 = \frac{d}{3}\left(\frac{2b_1 + b_2}{b_1 + b_2}\right)$$

$$A = \frac{d}{2}\,(b_1 + b_2)$$

$$I = \frac{d^3}{36}\left(\frac{b_1^2 + 4b_1b_2 + b_2^2}{b_1 + b_2}\right)$$

$$S_1 = \frac{d^2}{12}\left(\frac{b_1^2 + 4b_1b_2 + b_2^2}{b_1 + 2b_2}\right)$$

$$S_2 = \frac{d^2}{12}\left(\frac{b_1^2 + 4b_1b_2 + b_2^2}{2b_1 + b^2}\right)$$

$$r = \frac{d}{6(b_1 + b_2)}\sqrt{2(b_1^2 + 4b}$$

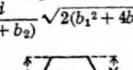

Fig. 18.—Regular hexago

$$y_1 = y_2 = \frac{d}{2} = \frac{b\sqrt{3}}{2} =$$

$$A = \frac{3b^2\sqrt{3}}{2} = 2.598\,b^2$$

$$= \tfrac{3}{2}d^2 \tan 30° = 0.866\,d^2$$

(hence, $d^2 = 3b^2$)

$$I = \frac{5b^4\sqrt{3}}{16} = 0.541\,b^4$$

$$= 0.060\,d^4$$

$$S_1 = S_2 = \frac{5b^3}{3} = 0.120\,d^3$$

$$r = b\sqrt{\frac{5}{24}} = 0.456\,b$$

$$= 0.264\,d$$

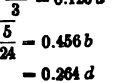

Fig. 19.—Regular hexago

$y_1 = y_2 = 0.433\,d = b$

A (same as for 18)

I (same as for 18)

$$S_1 = S_2 = \frac{5b^3\sqrt{3}}{16} = 0.54$$

$$= 0.10$$

r (same as for 18)

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hexagon;

$2b_1 = d_1$.

$_1{}^2) = 2.598\,(b^2 - b_1{}^2)$

$b_1{}^4) = 0.541\,(b^4 - b_1{}^4)$

$\frac{{}^4 - b_1{}^4)}{b}$

$= 0.456\,(b - b_1)$

r octagon.

$+ \frac{b}{\sqrt{2}}$

$.207\,b$

$.414\,b)$

$= 0.828\,d^2$

$2.828\,c^2$

$= 4.828\,b^2$

$\overline{2} - 2)\,d^2 =$

$\sqrt{2} + 2)\,b^2;$

$)\,d^2 = 1.707\,b^2)$

$\overline{2}) = 0.638\,c^4$

$= 0.055\,d^4$

$= 1.859\,b^4$

^{3}

$257\,d = 0.621\,b$

ysis of properties of
hus,
on) $= I_s$ (for square)
corner triangles, t)
s $X-X$.
ext column.

Fig. 22.—*Continued.*

I_s (Fig 12) $= \frac{d^4}{12} = 0.0833\,d^4$

I_t about $X'-X'$ (see Fig. 4)

$= \frac{s^4}{36}$; and

I_t about $X-X$ (see Fig. 3)

$= \frac{s^4}{36} + \frac{a^2s^2}{2}$. Hence,

$4I_t = \frac{s^4}{9} + 2a^2s^2$

$= \frac{b^4}{6}\,(3 + 2\sqrt{2}) = 0.9714\,b^4$

$= 0.0287\,d^4$

$I_o = I_s - 4I_t$

$= (0.0833 - 0.0287)\,d^4 = 0.055\,d^4$

as given for Fig. 21.

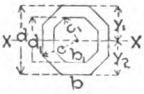

Fig. 23.—Hollow octagon.

$A = 0.828\,(d^2 - d_1{}^2)$

$= 2.828\,(c^2 - c_1{}^2)$

$= 4.828\,(b^2 - b_1{}^2)$

$I = 0.055\,(d^4 - d_1{}^4)$

$= 0.638\,(c^4 - c_1{}^4)$

$= 1.859\,(b^4 - b_1{}^4)$

$S_1 = \frac{I}{y_1}$ $\qquad S_2 = \frac{I}{y_2}$

$r = \sqrt{\frac{I}{A}}$

Fig. 24.—Regular octagon.

$y_1 = y_2 = b\,(\sin 22\frac{1}{2}° + \cos 22\frac{1}{2}°)$

$= 1.3066\,b$

$= 0.5412\,d$

A (same as for 21)

I (same as for 21)

$S_1 = \frac{I}{y_1} = S_2 = \frac{I}{y_2}$

r (same as for 21)

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Fig. 25.—Circle.

$$y_1 = y_2 = r_1 = \frac{d}{2}$$

$$A = \pi r_1^2 = \frac{\pi d^2}{4} = 0.7854\,d^2$$

$$I = \frac{\pi r_1^4}{4} = \frac{\pi d^4}{64} = 0.0491\,d^4$$

$$S_1 = S_2 = \frac{\pi d^3}{32} = 0.0982\,d^4$$

$$r = \frac{r_1}{2} = \frac{d}{4} = 0.25\,d$$

Fig. 26.—Hollow circle.

$$A = \pi(r_1^2 - r_2^2) = 0.7854\,(d^2 - d_1^2)$$

$$I = \frac{\pi}{4}(r_1^4 - r_2^4) = 0.0491\,(d^4 - d_1^4)$$

$$S_1 = S_2 = \frac{I}{y_1} = \frac{I}{y_2}$$

$$r = \frac{\sqrt{r_1^2 + r_2^2}}{2} = \frac{\sqrt{d^2 + d_1^2}}{4}$$

Fig. 27.—Semicircle; axis at base.

$$A = \frac{\pi r_1^2}{2} = 1.5708\,r_1^2$$

$$I = \frac{\pi r_1^4}{8} = 0.3927\,r_1^4$$

$$S_1 = \frac{\pi r_1^3}{8} = 0.3927\,r_1^3$$

$$r = \frac{r_1}{2} = 0.5\,r_1$$

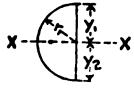
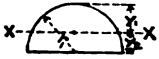


Fig. 28.—Semicircle; axis through cen of grav.

A, I, S and r same as for 27.



Ffg. 29.—Semicircle; axis cen of grav.

$$y_1 = \frac{(3\pi - 4)r_1}{3\pi} = 0.5756\,r_1$$

$$y_2 = \frac{4r_1}{3\pi} = 0.4244\,r_1$$

$$I = \frac{(9\pi^2 - 64)r_1^4}{72\pi} = 0.1098\,r_1$$

$$S_1 = \frac{I}{y_1} \qquad S_2 = \frac{I}{y_2}$$

$$r = \frac{r_1\sqrt{9\pi^2 - 64}}{6\pi} = 0.264\,r_1$$

Fig. 30.—Circular sector through cen of grav.

$$y_2 = \tfrac{2}{3}\,r_1\frac{\sin\alpha}{\alpha}$$

$$y_1 = r_1 - y_2$$

$$A = r_1^2\alpha$$

[Note. 180° = π = 3.1416.]

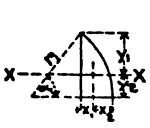


Fig. 31.—Circular half-se axes through cen of grav

$$y_2 = \tfrac{2}{3}r_1\frac{4\sin^2\tfrac{1}{2}\alpha - \sin^2\alpha\cos\alpha}{\alpha - \sin\alpha\cos\alpha}$$

$$y_1 = r_1\sin\alpha - y_2$$

$$x_1 = \tfrac{2}{3}r_1\frac{\sin^3\alpha}{\alpha - \sin\alpha\cos\alpha} - r_1\cos$$

$$x_2 = r_1(1 - \cos\alpha) - x_1$$

$$A = \frac{r_1^2}{2}(\alpha - \sin\alpha\cos\alpha)$$

Note.—For transposed a Fig. 3. For skeleton figur Figs. 49 to 54. For Mensurat Sec. 11, page 214.

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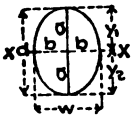


ig. 32.—Ellipse.

= semi major axis
= semi minor axis

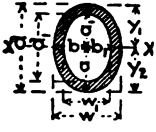
$_1 = y_2 = a = \frac{d}{2}$

$= \pi ab = \frac{\pi wd}{4} = 0.7854\, wd$

$= \frac{\pi a^3 b}{4} = \frac{\pi wd^3}{64} = 0.049\, wd^3$

$_1 = S_2 = \frac{\pi a^2 b}{4} = \frac{\pi wd^2}{32}$
$= 0.098\, wd^2$

$= \frac{a}{2} = \frac{d}{4}$



ig. 33.—Hollow ellipse.

and b = outer semi-axes
and b_1 = inner semi-axes
$= \pi (ab - a_1 b_1)$

$= \frac{\pi}{4} (a^3 b - a_1^3 b_1)$

$= S_2 = \frac{I}{y_1} = \frac{I}{y_2}$

$= \sqrt{\frac{I}{A}}$

Note.—For other properties of the Ellipse, see Sec. 11, Mensuration, page 238.

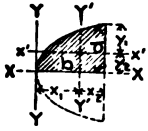


Fig. 34.—Parabolic half-segment; axes $X'-X'$ and $Y'-Y'$ passing through cen of grav.

$x_1 = \frac{3}{5} b;\ x_2 = \frac{2}{5} b.$

$y_2 = \frac{3}{8} d;\ y_1 = \frac{5}{8} d.$

$A = \frac{2}{3} bd.$

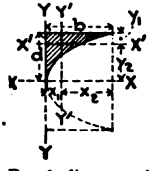


Fig. 35.—Parabolic spandril; axes $X'-X'$ and $Y'-Y'$ passing through cen of grav.

$x_1 = \frac{3}{10} b;\ x_2 = \frac{7}{10} b.$

$y_2 = \frac{3}{4} d;\ y_1 = \frac{1}{4} d.$

$A = \frac{1}{3} bd.$

Note.—For other properties of the Parabola, see Sec. 11, Mensuration, page 237.

2.—SKELETON FIGURES WITH THIN LINES OF WIDTH t.

For Notation, See Page 524.

Center of gravity indicated by heavy dot.

(Values are generally more or less approx., depending on thickness of lines.)

g. 36.—Vertical web plate.

$= dt$

$= \frac{d^3 t}{12} = \frac{Ad^2}{12}$

$= \frac{d}{\sqrt{12}} = 0.289\, d$

(Values exact.)

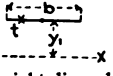


Fig. 37.—Straight line about parallel axis.

$A = bt$

$I = Ay_1^2 = bty_1^2$

$r = \sqrt{\frac{I}{A}} = y_1$

(I and r practically correct for small values of t).

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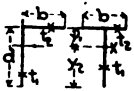

Fig. 38.—Angle. Tee.

$A = dt_1 + bt_2$

$y_1 = \frac{d^2t_1}{2(dt_1 + bt_2)}$

$y_2 = \frac{2bt_2 + dt_1}{2(bt_2 + dt_1)}d$

$I = \frac{d^3t_1}{12}\left(\frac{4bt_2 + dt_1}{bt_2 + dt_1}\right)$

$r = \sqrt{\frac{I}{A}}$

(Errors in y_1, y_2, I and r decrease as b approaches zero).

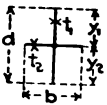

Fig. 39.—Cross; $y_1 = y_2 = \frac{d}{2}$.

$A = dt_1 + bt_2$

$I = \frac{d^3t_1}{12}$ (line b neglected)

$r = \sqrt{\frac{I}{A}}$

(Note that line b is included in A and not in I).

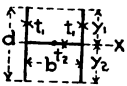

Fig. 40.—H-section; $y_1 = y_2 = \frac{d}{2}$

$A = 2dt_1 + bt_2$

$I = \frac{d^3t_1}{6}$ (line b neglected)

$r = \sqrt{\frac{I}{A}}$

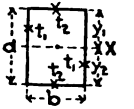

Fig. 41.—Rectangular cell.

$y_1 = y_2 = \frac{d}{2} = y$

$A = 2(dt_1 + bt_2)$

$I = \frac{d^2}{2}\left(\frac{dt_1}{3} + bt_2\right)$

$r = \sqrt{\frac{I}{A}}$

Note.—For *square* cell, $t_2 = t_1 = t$, we have,

$A = 4dt$

$I = \frac{2}{3}d^3t$

$r = \frac{d}{\sqrt{6}} = 0.408d$

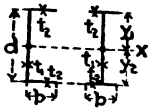

Fig. 42.—Channel. I-beam.

$y_1 = y_2 = \frac{d}{2} = y$

$A = dt_1 + 2bt_2$

$I = \frac{d^2}{2}\left(\frac{dt_1}{6} + bt_2\right)$

$r = \sqrt{\frac{I}{A}}$

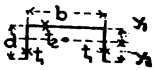

Fig. 43.—Channel.

$A = 2dt_1 + bt_2$

$y_1 = \frac{d^2t_1}{2dt_1 + bt_2}$

$y_2 = d\left(\frac{dt_1 + bt_2}{2dt_1 + bt_2}\right)$

$I = \frac{d^3}{2}\left(\frac{t_1}{3} + \frac{bt_1t_2}{2dt_1 + bt_2}\right)$

$r = \sqrt{\frac{I}{A}}$

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Fig. 47.—*Continued.*

Note.—For angle with *equal* legs, $b = d$, and

$$y_1 = y_2 = \frac{b\sqrt{2}}{4} = 0.3535\,d$$

$$A = 2\,dt$$

$$I = \frac{d^3t}{12}$$

$$r = \frac{d}{\sqrt{24}} = 0.204\,d$$

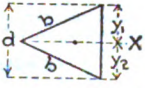

Fig. 48.—Triangular cell.*

$$y_1 = y_2 = \frac{d}{2}$$

$$A = (d + 2b)\,t$$

$$I = \frac{d^2t}{12}\,(d + 2b)$$

$$r = \frac{d}{\sqrt{12}} = 0.289\,d$$

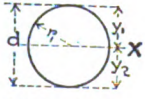

Fig. 49.—Circular cell.*

$$y_1 = y_2 = r_1 = \frac{d}{2}$$

$$A = 2\pi r_1 t = 6.2832\,r_1 t$$
$$= 3.1416\,dt$$

$$I = \pi r_1^3 t = 3.1416\,r_1^3 t$$
$$= 0.3927\,d^3 t$$

$$r = \frac{r_1}{\sqrt{2}} = 0.707\,r_1$$
$$= 0.354\,d$$

Fig. 50.—Semi-circular arc.*

$$A = \pi r_1 t = 3.1416\,r_1 t$$

$$I = \frac{\pi r_1^3 t}{3} = 1.5708\,r_1^3 t$$

$$r = \frac{r_1}{\sqrt{2}} = 0.707\,r_1$$

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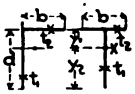

Fig. 38.—Angle. Tee.

$$A = dt_1 + bt_2$$

$$y_1 = \frac{d^2t_1}{2(dt_1 + bt_2)}$$

$$y_2 = \frac{2bt_2 + dt_1}{2(bt_2 + dt_1)}d$$

$$I = \frac{d^3t_1}{12}\left(\frac{4bt_2 + dt_1}{bt_2 + dt_1}\right)$$

$$r = \sqrt{\frac{I}{A}}$$

(Errors in y_1, y_2, I and r decrease as b approaches zero).

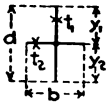

Fig. 39.—Cross; $y_1 = y_2 = \frac{d}{2}$.

$$A = dt_1 + bt_2$$

$$I = \frac{d^3t_1}{12} \text{ (line } b \text{ neglected)}$$

$$r = \sqrt{\frac{I}{A}}$$

(Note that line b is included in A and not in I).

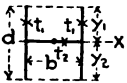

Fig. 40.—H-section; $y_1 = y_2 = \frac{d}{2}$

$$A = 2dt_1 + bt_2$$

$$I = \frac{d^3t_1}{6} \text{ (line } b \text{ neglected)}$$

$$r = \sqrt{\frac{I}{A}}$$

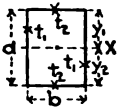

Fig. 41.—Rectangular cell.

$$y_1 = y_2 = \frac{d}{2} = y$$

$$A = 2(dt_1 + bt_2)$$

$$I = \frac{d^2}{2}\left(\frac{dt_1}{3} + bt_2\right)$$

$$r = \sqrt{\frac{I}{A}}$$

Note.—For *square* cell, $t_2 = t_1 = t$, we have,

$$A = 4dt$$

$$I = \tfrac{2}{3}d^3t$$

$$r = \frac{d}{\sqrt{6}} = 0.408d$$

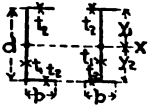

Fig. 42.—Channel. I-beam

$$y_1 = y_2 = \frac{d}{2} = y$$

$$A = dt_1 + 2bt_2$$

$$I = \frac{d^2}{2}\left(\frac{dt_1}{6} + bt_2\right)$$

$$r = \sqrt{\frac{I}{A}}$$

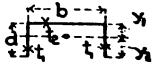

Fig. 43.—Channel.

$$A = 2dt_1 + bt_2$$

$$y_1 = \frac{d^2t_1}{2dt_1 + bt_2}$$

$$y_2 = d\left(\frac{dt_1 + bt_2}{2dt_1 + bt_2}\right)$$

$$I = \frac{d^3}{2}\left(\frac{t_1}{3} + \frac{bt_1t_2}{2dt_1 + bt_2}\right)$$

$$r = \sqrt{\frac{I}{A}}$$

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Fig. 47.—*Continued.*

Note.—For angle with *equal* legs, $b = d$, and

$$y_1 = y_2 = \frac{b\sqrt{2}}{4} = 0.3535\,d$$

$$A = 2\,dt$$

$$I = \frac{d^3 t}{12}$$

$$r = \frac{d}{\sqrt{24}} = 0.204\,d$$

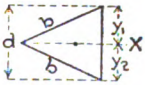

Fig. 48.—Triangular cell.*

$$y_1 = y_2 = \frac{d}{2}$$

$$A = (d + 2b)\,t$$

$$I = \frac{d^2 t}{12}\,(d + 2b)$$

$$r = \frac{d}{\sqrt{12}} = 0.289\,d$$

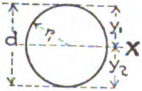

Fig. 49.—Circular cell.*

$$y_1 = y_2 = r_1 = \frac{d}{2}$$

$$A = 2\pi r_1 t = 6.2832\,r_1 t$$
$$= 3.1416\,dt$$

$$I = \pi r_1^3 t = 3.1416\,r_1^3 t$$
$$= 0.3927\,d^3 t$$

$$r = \frac{r_1}{\sqrt{2}} = 0.707\,r_1$$
$$= 0.354\,d$$

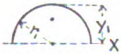

Fig. 50.—Semi-circular arc.*

$$A = \pi r_1 t = 3.1416\,r_1 t$$

$$I = \frac{\pi r_1^3 t}{3} = 1.5708\,r_1^3 t$$

$$r = \frac{r_1}{\sqrt{2}} = 0.707\,r_1$$

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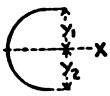

Fig. 51.—Semi-circular arc.*

$y_1 = y^2$.

A, I and r same as for 50.

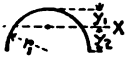

Fig. 52.—Semi-circular arc; axis through cen of grav.*

$$y_1 = r_1\left(1 - \frac{2}{\pi}\right) = 0.3634\,r_1$$

$$y_2 = \frac{2r_1}{\pi} = 0.6366\,r_1$$

$$A = \pi r_1 t = 3.1416\,r_1 t$$

$$I = r_1^3 t\left(\frac{\pi}{2} - \frac{4}{\pi}\right) = 0.2976\,r_1^3 t$$

$$r = r_1\sqrt{\tfrac{1}{2} - \frac{4}{\pi^2}} = 0.308\,r_1$$

(Note that I_{52} can be obtained from I_{50} by method explained under Fig. 3; using the minus sign, as $I_{52} < I_{50}$).

Fig. 53.—Circular arc.*

$$y_1 = r_1\left(1 - \frac{\sin\alpha}{\alpha}\right)$$

$$y_2 = r_1\left(\frac{\sin\alpha}{\alpha} - \cos\alpha\right)$$

$$A = 2r_1\alpha t$$

$$I = r_1^3 t\left(\alpha + \sin\alpha\cos\alpha - \frac{2\sin^2\alpha}{\alpha}\right)$$

$$r = \sqrt{\frac{I}{A}}$$

(Note.—The angle α may be expressed in terms of π; thus, $180° = \pi$, $90° = \frac{\pi}{2}$, etc.)

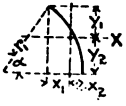

Fig. 54.—Circular arc.*

$$y_2 = 2r_1\frac{\sin^2\frac{1}{2}\alpha}{\alpha} \qquad y_1 = r_1\sin\alpha - y_2$$

*Thickness of lines $= t$.

Fig. 54—*Continued.*

$$x_1 = r_1\left(\frac{\sin\alpha}{\alpha} - \cos\alpha\right)$$

$$x_2 = r_1(1 - \cos\alpha) - x_1$$

$$A = r_1\alpha t$$

$$I = r_1^3 t\left(\frac{\alpha - \sin\alpha\cos\alpha}{2} - 4s\right.$$

$$r = \sqrt{\frac{I}{A}}$$

Special Case.

For axis X' at base:

$$I = \frac{r_1^3 t}{2}(\alpha - \sin\alpha\cos\alpha)$$

$$r = r_1\sqrt{\tfrac{1}{2} - \frac{\sin\alpha\cos\alpha}{2\alpha}}$$

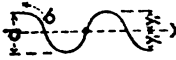

Fig. 55.—Corrugated sheet. ing corrugations to be c curves, we have,*

b = Breadth of sheet befor gating.

$$y_1 = \frac{d}{2}$$

$$A = bt$$

$$I = \tfrac{1}{30}d^2bt = 0.1333\,d^2bt$$

$$r = d\sqrt{\tfrac{1}{30}} = 0.365\,d$$

(Compare this with 49, 50, that semi-circular arcs wou $r = 0.354\,d$.)

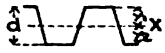

Fig. 56.—Corrugated sheet.

$y_1 = y_2 = \frac{d}{2}$. Let

F = area top flanges,

= area bottom flanges,

W = area of webs. Then

$$A = 2F + W$$

$$I = \frac{d^2}{2}\left(F + \frac{W}{6}\right)$$

$$r = \sqrt{\frac{I}{A}}$$

(Note.—The above valu holds true for any inclinatio webs, from vertical to hor the upper and lower flanges equal areas).

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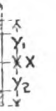
3.—BLOCK SHAPES.

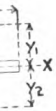
For Notation, see page 524.

r of gravity indicated by heavy dot.
(Values are exact.)



y.

)



^{3}

$- t_1) t_2^3$

$t_1) t_2^3$

$- t_1) t_2]$

as for 58.

as for 58.

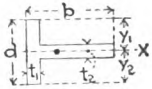


Fig. 61.—Tee.
Properties same as for 58.

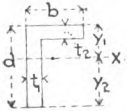


Fig. 62.—Angle.

$$A = dt_1 + t_2 (b - t_1)$$

$$y_1 = \frac{d^2 t_1 + t_2^2 (b - t_1)}{2A}$$

$$y_2 = d - y_1$$

$$I = \frac{b y_1^3 + t_1 y_2^3}{3} - \frac{(b - t_1)(y_1 - t_2)^3}{3}$$

$$S_1 = \frac{I}{y_1} \qquad S_2 = \frac{I}{y_2}$$

$$r = \sqrt{\frac{I}{A}}$$

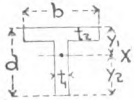


Fig. 63.—Tee.
Properties same as for 62.

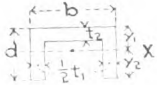


Fig. 64.—Channel.
Properties same as for 62.

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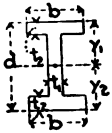

Fig. 65.—I-beam.

$$y_1 = y_2 = \frac{d}{2}$$

$$A = dt_1 + 2t_2(b - t_1)$$

$$I = \frac{bd^3 - (b - t_1)(d - 2t_2)^3}{12}$$

$$S_1 = S_2 = \frac{I}{y_1} = \frac{I}{y_2}$$

$$r = \sqrt{\frac{I}{A}}$$

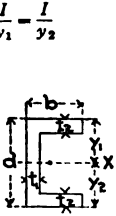

Fig. 66.—Channel.
Properties same as for 65.

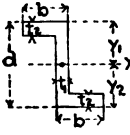

Fig. 67.—Z-bar.
Properties same as for 65.

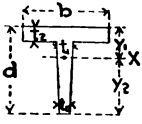

Fig. 68.—Tee; tapered stem.

$$A = bt_2 + \frac{(d - t_2)(t_0 + t_1)}{2}$$

$$y_1 = \frac{bt_2^2 + t_0(d^2 - t_2^2)}{2A} + \frac{(t_1 - t_0)(d - t_2)(d + 2t_2)}{6A}$$

$$y_2 = d - y_1$$

$$I = \frac{bt_2^3}{3} + \frac{(d - t_2)^3(3t_0 + t_1)}{12} - A(y_1 - t_2)^2$$

Continued in next column.

Fig. 67.—*Continued.*

$$S_1 = \frac{I}{y_1} \qquad S_2 = \frac{I}{y_2} \qquad r =$$

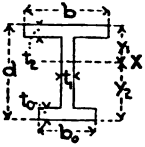

Fig. 69.—I-beam; unequal f
with unequal thicknesses.

$$A = bt_2 + (d - t_2 - t_0)t_1 + b_0t_0$$

$$y_1 = \left[\frac{bt_2^2}{2} + t_1(d - t_2 - t_0)\left(t_2 + \frac{d -}{}\right.\right.$$

$$\left. + b_0t_0\left(d - \frac{t_0}{2}\right)\right] \div A$$

$$y_2 = d - y_1$$

$$I = \frac{by_1^3 + b_0y_2^3}{3} - \frac{(b - t_1)(y_1 - t_2)^3}{3} - \frac{(b_0 - t_1)}{}$$

$$S_1 = \frac{I}{y_1} \qquad S_2 = \frac{I}{y_2}$$

$$r = \sqrt{\frac{I}{A}}$$

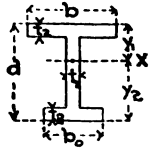

Fig. 70.—I-beam; unequal
with equal thicknesses.

$$A = (b + b_0)t_2 + (d - 2t_2)t_1$$

$$y_1 = \left[\frac{t_2^2}{2}(b - b_0) + t_2d(b_0 - t_1) + \frac{t_1d^2}{2}\right] \div A$$

$$y_2 = d - y_1$$

$$I = \frac{by_1^3 + b_0y_2^3}{3} - \frac{(b - t_1)(y_1 - t_2)^3}{3} - \frac{(b_0 - t_1)}{}$$

$$S_1 = \frac{I}{y_1} \qquad S_2 = \frac{I}{y_2}$$

$$r = \sqrt{\frac{I}{A}}$$

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4.—ROLLED SHAPES.

(See, also, Sections 30, 31, 32, following.)

In the following illustrations the center of gravity is indicated by a heavy dot.

Values are exact for shapes as outlined. These shapes do not show the actual rounded "fillets" because the latter are disregarded in structural calculations, except in special cases.

The flanges of I-beams and standard channels slope at the rate of 2 : 12 on their inner faces, equivalent to an angle of 9° 28′. The heavier sections of I-beams, channels and Z-bars are made by spreading the rolls, thereby increasing the width of flange; but most angles are rolled in "finishing" grooves, which maintain a standard width of flange for various thicknesses of metal.

Moment of inertia ($I\alpha$) about inclined axis.—In the preceding illustrations (Figs. 1-70) we have confined our attention to moments of inertia about the coordinate axes $X-X$ and $Y-Y$, one of which is usually drawn parallel with a principal line of the figure, and the other at right angle to the first. It very often happens, however, that we wish to know the value $I\alpha$, the moment of inertia about an inclined axis making an angle α with the axis $X-X$, Fig. 71, about which latter axis I_x is the moment of inertia. This may be obtained from the following formula:

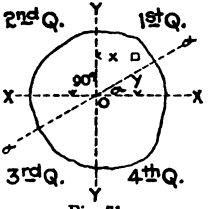

Fig. 71.

$$I\alpha = I_x \cos^2 \alpha + Iy \sin^2 \alpha - 2\,K \cos \alpha \sin \alpha \qquad (1)$$

In which $I_x = \iint y^2 dx dy$ = moment of inertia about axis $X-X$;

$Iy = \iint x^2 dx\, dy$ = moment of inertia about axis $Y-Y$;

$I\alpha$ = moment of inertia about axis $\alpha-\alpha$;

α = angle between the axes $\alpha-\alpha$ and $X-X$, the functions of α to be considered algebraically; thus, $\sin \alpha$ is + in the 1st and 2nd quadrants, and − in the 3rd and 4th; while $\cos \alpha$ is + in the 1st and 4th, and negative in the 2nd and 3rd. Hence, $\cos^2\alpha$ and $\sin^2\alpha$ are always positive (+), while $\sin \alpha \cos \alpha$ becomes positive (+) when the axis $\alpha-\alpha$ lies in the 1st and 3rd quadrants, and negative (−) when it lies in the 2nd and 4th.

$K = \iint xy\ dx\ dy$ = double integration (summation) of dA ($= dx\, dy$, or □) multiplied by its axial distances x and y (see Fig. 71). Those portions of the figure or surface considered which lie in the 1st and 3rd quadrants tend to make K positive (+), while those portions which lie in the 2nd and 4th quadrants tend to make K negative (−).

In solving equation (1) we find the values of I_x, Iy and K from the shape of the plane figure and with the coordinate axes $X-X$ and $Y-Y$ intersecting at the origin O. Then by assuming the proper value for the angle α we may solve equation (1) for $I\alpha$. Values of I_x and Iy for many figures are given under the preceding illustrations. We will now explain how K is derived.

Values of K in Equation (1).—The value of K is

$$K = \iint xydxdy \qquad (2)$$

bearing in mind what has previously been said regarding positive and negative values.

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Secondly, the preceding formula (2) may also be expressed:

$$K = A\,x_0\,y_0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

In which A = area of the figure or plane surface,

x_0 = + or − distance from axis of $Y-Y$ to cen of grav o

y_0 = + or − distance from axis of $X-X$ to cen of grav o

all in inches. It is evident that when the cen of grav lies at the orig (Fig. 71) the value of K is zero; when in the 1st or 3rd quadrants, it is and when in the 2nd and 4th, it is −. But the minus sign in equation must also be considered, as well as the algebraic value of $\cos\alpha \sin$ solving for $I\,\alpha$.

Thirdly, the figure may be cut up into smaller areas, each area b multiplied by its respective coordinate distances x and y to the cen of of each area; thus,

$$K = \Sigma\, A_1\, x_1\, y_1 + A_2\, x_2\, y_2 + A_3 x_3\, y_3 + \text{—etc.} \ldots\ldots\ldots\ldots$$

The four following examples will illustrate the methods of finding value of K in Equation (1):—

(1) $K = \int_0^d \int_0^b xy\,dx\,dy = {}_0^b\left[\frac{x^2}{2}\right]\int_0^d y\,dy = \frac{b^2}{2}\cdot{}_0^d\left[\frac{y^2}{2}\right. = \frac{b^2}{2}\cdot\frac{d^2}{2} = \frac{b^2d^2}{4}.$

Or, $K = bd \cdot \frac{b}{2}\cdot\frac{d}{2} = \frac{b^2d^2}{4}$ (second method).

Fig.

(2) $K = bd\left(b_1 + \frac{b}{2}\right)\left(-d_1 - \frac{d}{2}\right)$

$= -bd\left(\frac{bd}{4} + \frac{bd_1 + b_1d + b_1d_1}{2}\right).$

If $d_1 = 0$, $K = -bd\left(\frac{bd}{4} + \frac{b_1d}{2}\right)$.

If, also, $b_1 = 0$, $K = -\frac{b^2d^2}{4}$. (Compare with Fig. 72)

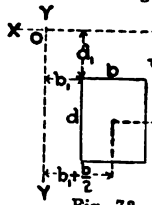

Fig. 73.

(3) $K = \underset{\text{1st Q.}}{\frac{b^2d^2}{4}} - \underset{\text{2nd Q.}}{\frac{b_1^2d^2}{4}} + \underset{\text{3rd Q.}}{\frac{b_1^2d_1^2}{4}} - \underset{\text{4th Q.}}{\frac{b^2d_1^2}{4}}$

If $b_1 = b$ and $d_1 = d$, $K = 0$.

But if $b = 0$ and $d_1 = 0$, there will remain for the 2nd quadrant, $K = -\frac{b_1^2d^2}{4}$.

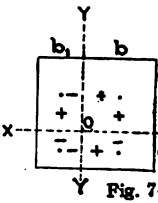

Fig. 7

(4) $K = K_1 + K_2 + K_3$

$= t\,(d-t)\,x_1y_1 + bt \times 0 + t\,(d-t)\,(-x_3)\,(-y_3).$

But $x_1 = x_3 = \frac{b-t}{2}$, and $y_1 = y_3 = \frac{d}{2}$; hence

$K = \frac{1}{4}\,(bt - t^2)\,(d^2 - dt).$

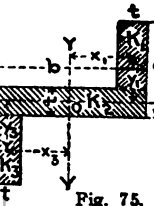

Fig. 75.

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Problem.—(1) Find the moment of inertia $I\alpha$, about the axis $\alpha_1-\alpha_1$ of a square beam of cross-section $d\times d$, α_1 making an *upward* angle of 45° with the axis $X-X$; (2) find $I\alpha_2$ about the axis $\alpha_2-\alpha_2$ making a *downward* angle of 45° with the axis $X-X$.

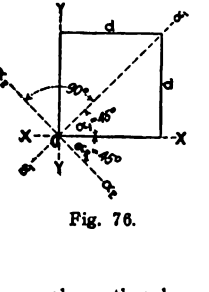

Fig. 76.

Solution.—$I_x=I_y=\frac{d^4}{3}$ (Fig. 9).

$K=\frac{d^4}{4}$ (Fig. 72). Then from equation (1):

$$(1^\circ)\ I\alpha_1=\frac{d^4}{3}\ (\cos^2\alpha+\sin^2\alpha)-\frac{d^4}{2}\cos\alpha\sin\alpha$$
$$=\frac{d^4}{3}-\frac{d^4}{2}\cdot\tfrac{1}{2}=\frac{d^4}{12}\quad\text{(See Fig. 14).}$$
$$(2^\circ)\ I\alpha_2=\frac{d^4}{3}\ (\cos^2\alpha+\sin^2\alpha)-\frac{d^4}{2}\cos\alpha\ (-\sin\alpha)$$
$$=\frac{d^4}{3}+\frac{d^4}{2}\cdot\tfrac{1}{2}=\frac{7d^4}{12}.$$

Maximum and minimum values of $I\alpha$.—If a Z-bar, angle or other shape is used as a column or strut it is essential that we know the least radius of gyration of the section, and this can be obtained from the minimum moment of inertia, as $r=\sqrt{\frac{I}{A}}$; also $I\ min$ and $I\ max$ are valuable in connection with the resisting moments of beams. The following formulas give the value of α for minimum or maximum value of $I\alpha$:

$$\text{Tan } 2\alpha=\frac{2K}{Iy-Ix}\qquad(5)$$

Moreover, it can be stated that the axis say $\alpha_1-\alpha_1$ giving $I\alpha_1$ a minimum value will be at right angle to the axis say $\alpha_2-\alpha_2$, giving $I\alpha_2$ a maximum value. That is, "maximum" and "minimum" axes intersect at the origin, O, at an angle of 90°. In practice it is easy to distinguish one from the other; thus, in Fig. 76, $\alpha_1-\alpha_1$ is the "minimum" axis, and $\alpha_2-\alpha_2$ the "maximum." The maxima and minima axes are called the "principal" axes, and in this particular case they happen to lie at an angle of 45° with the axes of X and Y. Figs. 80 and 81, following, show positions of axes for minimum values of I for the Z-bar and angle.

For a more complete discussion of moments of inertia about inclined axes see Lanza's "Applied Mechanics"; also Paper No. 1020, Trans. Am. Soc. C. E., Vol. LVI, p. 169. See also Handbook of Cambria Steel Co.

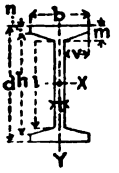

Fig. 77.—*T*-beam.

$A=td+2v\,(m+n)$

Slope $s=\frac{1}{t}=\frac{m-n}{v}=\frac{h-l}{b-t}$

$I_x=\frac{1}{12}[bd^3+\frac{1}{4s}\,(h^4-l^4)]$

$Iy=\frac{1}{12}[b^3\,(d-h)+l^3+\frac{s}{4}(b^4-t^4)]$

Section modulus $S=\frac{2I_x}{d}$ or $\frac{2Iy}{b}$

Rad of gyration $r=\sqrt{\frac{I_x}{A}}$ or $\sqrt{\frac{Iy}{A}}$

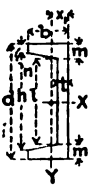

Fig. 78.—Channel.

$A=td+(b-t)\ (m-n)$

Slope $s=\frac{1}{t}=\frac{h-l}{2(b-t)}$

$x_1=\left[b^2n+\frac{hl^2}{2}+\frac{s(b-t)^2(b+2t)}{3}\right]\div A$

$I_x=\frac{1}{12}[bd^3-\frac{1}{8s}\,(h^4-l^4)]$

$Iy-\frac{1}{3}\,[2b^3n+l^3+\frac{s}{2}\,(b^4-t^4)]-Ax_1^2$

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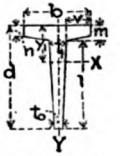

Fig. 79.—Tee.

$$A = \frac{l}{2}(t_0+t_1) + v\,(m+n) + mt_1$$

$$y_1 = [2v\,(m-n)\,(m+2n) + 3n^2\,(b-t_1) + 3t_1d^2 - l\,(t_1-t_0)\,(3d-l)] \div 6A$$

$$I_x = \tfrac{1}{12}[l^3\,(3t_0+t_1) + 4bm^3 - 2v\,(m-n)^3] - A\,(y_1-m)^2$$

$$Iy = \frac{nb^3 + (m-n)\,t_1^3 + lt_0^3}{12} + \frac{v\,(m-n)\,[2v^2+(2v+3t_1)^2}{36} + \frac{l(t_1-t_0)\,[(t_1-t_0)^2+2\,(t_1+2t_0)^2]}{144}$$

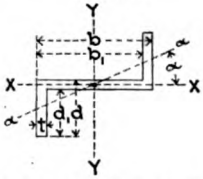

Fig. 80.—*Z*-bar; uniform thickness *t*

$$A = t\,(b+2d_1)$$

$$I_x = \frac{b\,(d+d_1)^3 - 2\,b_1d_1^3 - 6d_1d^2b_1}{12}$$

Continued in next column.

Fig. 80.—*Continued.*

$$Iy = \frac{db^3 - d_1\,(b-2t)^3}{12}$$

$$I\alpha = \frac{I_x\cos^2\alpha - Iy\sin^2\alpha}{\cos 2\alpha}$$

In which $\tan 2\alpha = \dfrac{(bt-t^2)\,(d^2-}{Iy-I_x}$

(Note.—$I\alpha$ = minimum I.)

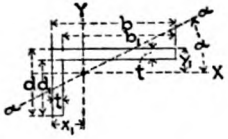

Fig. 81.—Angle.

$$A = t\,(b+d_1)$$

$$x_1 = \frac{t\,(2\,b_1+d) + b_1^2}{2\,(b_1+d)}$$

$$y_1 = \frac{t\,(2\,d_1+b) + d_1^2}{2\,(d_1+b)}$$

$$I_x = \frac{t\,(d-y_1)^3 + by_1^3 - (b-t)\,(y_1}{3}$$

$$Iy = \frac{t\,(b-x_1)^3 + dx_1^3 - (d-t)\,(x_1}{3}$$

$$I\alpha = \frac{I_x\cos^2\alpha - Iy\sin^2\alpha}{\cos 2\alpha}$$

In which $\tan 2\alpha = t\,[(2y_1-t)\,(b^2-2$
$+ (2x_1-t)\,(d-t)\,(d+t-2y_1)] \div 2\,(Iy$

(Note.—$I\alpha$ = minimum I.)

Reference.

The Polar Moment of Inertia, and Its Graphical Application Riveted Joints (C. F. Blake and R. W. Runge. Eng. News, May 21, 1903 Formulas and illustrations.

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5.—* MOMENTS OF INERTIA (I) OF b × d RECTANGLES.

Depth in Inches. d	b.—Width of Rectangle in Inches.						
	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$
2	.17	.21	.25	.29	.33	.38	.42
3	.56	.70	.84	.98	1.13	1.27	1.41
4	1.33	1.67	2.00	2.33	2.67	3.00	3.33
5	2.60	3.26	3.91	4.56	5.21	5.86	6.51
6	4.50	5.63	6.75	7.88	9.00	10.13	11.25
7	7.15	8.93	10.72	12.51	14.29	16.08	17.86
8	10.67	13.33	16.00	18.67	21.33	24.00	26.67
9	15.19	18.98	22.78	26.58	30.38	34.17	37.97
10	20.83	26.04	31.25	36.46	41.67	46.87	52.08
11	27.73	34.66	41.59	48.53	55.46	62.39	69.32
12	36.00	45.00	54.00	63.00	72.00	81.00	90.00
13	45.77	57.21	68.66	80.10	91.54	102.98	114.43
14	57.17	71.46	85.75	100.04	114.33	128.63	142.92
15	70.31	87.89	105.47	123.05	140.63	158.20	175.78
16	85.33	106.67	128.00	149.33	170.67	192.00	213.33
17	102.35	127.94	153.53	179.12	204.71	230.30	255.89
18	121.50	151.88	182.25	212.63	243.00	273.38	303.75
19	142.90	178.62	214.34	250.07	285.79	321.52	357.24
20	166.67	208.33	250.00	291.67	333.33	375.00	416.67
21	192.94	241.17	289.41	337.64	385.88	434.11	482.34
22	221.83	277.29	332.75	388.21	443.67	499.13	554.58
23	253.48	316.85	380.22	443.59	506.96	570.33	633.70
24	288.00	360.00	432.00	504.00	576.00	648.00	720.00
25	325.52	406.90	488.28	569.66	651.04	732.42	813.80
26	366.17	457.71	549.25	640.79	732.33	823.88	915.42
27	410.06	512.58	615.09	717.61	820.13	922.64	1025.16
28	457.33	571.67	686.00	800.33	914.67	1029.00	1143.33
29	508.10	635.13	762.16	889.18	1016.21	1143.23	1270.26
30	562.50	703.13	843.75	984.38	1125.00	1265.63	1406.25
32	682.67	853.33	1024.00	1194.67	1365.33	1536.00	1706.67
34	818.83	1023.54	1228.25	1432.96	1637.67	1842.38	2047.08
36	972.00	1215.00	1458.00	1701.00	1944.00	2187.00	2430.00
38	1143.17	1428 96	1714.75	2000.54	2286.33	2572.13	2857.92
40	1333.33	1666.67	2000.00	2333.33	2666.67	3000.00	3333.33
42	1543.50	1929.38	2315.25	2701.13	3087.00	3472.88	3858.75
44	1774.67	2218.33	2662.00	3105.67	3549.33	3993.00	4436.67
46	2027.83	2534.79	3041.75	3548.71	4055.67	4562.63	5069.58
48	2304.00	2880.00	3456.00	4032.00	4608.00	5184.00	5760.00
50	2604.17	3255.21	3906.25	4557.29	5208.33	5859.38	6510.42
52	2929.33	3661.67	4394.00	5126.33	5858.67	6591.00	7323.33
54	3280.50	4100.63	4920.75	5740.88	6561.00	7381.13	8201.25
56	3658.67	4573.33	5488.00	6403.67	7317.33	8232.00	9146.67
58	4064.83	5081.04	6097.25	7113.46	8129.67	9145.87	10162.08
60	4500.00	5625.00	6750.00	7875.00	9000.00	10125.00	11250.00

* $I = \frac{bd^3}{12}$. Section modulus $S = \frac{2I}{d} = \frac{1}{6} bd^2$.

Resisting moment $M'' = \frac{fI}{y} = \frac{2fI}{d} = fS = \frac{1}{6} fbd^2$, in which

I = the value in above table;
f = the outer fiber stress in beam, in lbs. per sq. in.;
M'' = moment in inch-lbs.; M' = moment in ft.-lbs. $= M'' \div 12$.

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5.—* MOMENTS OF INERTIA (I) OF $b \times d$ RECTANGLES.—Concluded.

Depth in Inches. d	b.—Width of Rectangle in Inches.					
	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$	$\frac{15}{16}$	1
2	.46	.50	.54	.58	.63	[illegible]
3	1.55	1.69	1.83	1.97	2.11	2.[illegible]
4	3.67	4.00	4.33	4.67	5.00	5.[illegible]
5	7.16	7.81	8.46	9.11	9.77	10.[illegible]
6	12.38	13.50	14.63	15.75	16.88	18.[illegible]
7	19.65	21.44	23.22	25.01	26.80	28.[illegible]
8	29.33	32.00	34.67	37.33	40.00	42.[illegible]
9	41.77	45.56	49.36	53.16	56.95	60.[illegible]
10	57.29	62.50	67.71	72.92	78.13	83.[illegible]
11	76.26	83.19	90.12	97.05	103.98	110.[illegible]
12	99.00	108.00	117.00	126.00	135.00	144.[illegible]
13	125.87	137.31	148.75	160.20	171.64	183.[illegible]
14	157.21	171.50	185.79	200.08	214.38	228.[illegible]
15	193.36	210.94	228.52	246.09	263.67	281.[illegible]
16	234.67	256.00	277.33	298.67	320.00	341.[illegible]
17	281.47	307.06	332.65	358.24	383.83	409.[illegible]
18	334.13	364.50	394.88	425.25	455.63	486.[illegible]
19	392.96	428.69	464.41	500.14	535.86	571.[illegible]
20	458.33	500.00	541.67	583.33	625.00	666.[illegible]
21	530.58	578.81	627.05	675.28	723.52	771.[illegible]
22	610.04	665.50	720.96	776.42	831.87	887.[illegible]
23	697.07	760.44	823.81	887.18	950.55	1013.[illegible]
24	792.00	864.00	936.00	1008.00	1080.00	1152.[illegible]
25	895.18	976.56	1057.94	1139.32	1220.70	1302.[illegible]
26	1006.96	1098.50	1190.04	1281.58	1373.13	1464.[illegible]
27	1127.67	1230.19	1332.70	1435.22	1537.73	1640.[illegible]
28	1257.67	1372.00	1486.33	1600.67	1715.00	1829.[illegible]
29	1397.29	1524.31	1651.34	1778.36	1905.39	2032.[illegible]
30	1546.88	1687.50	1828.13	1968.75	2109.38	2250.[illegible]
32	1877.33	2048.00	2218.67	2389.33	2560.00	2730.[illegible]
34	2251.79	2456.50	2661.21	2865.92	3070.63	3275.[illegible]
36	2673.00	2916.00	3159.00	3402.00	3645.00	3888.[illegible]
38	3143.71	3429.50	3715.29	4001.08	4286.88	4572.[illegible]
40	3666.67	4000.00	4333.33	4666.67	5000.00	5333.[illegible]
42	4244.63	4630.50	5016.38	5402.25	5788.13	6174.[illegible]
44	4880.33	5324.00	5767.67	6211.33	6655.00	7098.[illegible]
46	5576.54	6083.50	6590.46	7097.42	7604.38	8111.[illegible]
48	6336.00	6912.00	7488.00	8064.00	8640.00	9216.[illegible]
50	7161.46	7812.50	8463.54	9114.58	9765.63	10416.[illegible]
52	8055.57	8788.00	9520.33	10252.67	10985.00	11717.[illegible]
54	9021.38	9841.50	10661.63	11481.75	12301.88	13122.[illegible]
56	10061.33	10976.00	11890.67	12805.33	13720.00	14634.[illegible]
58	11178.00	12194.50	13210.71	14226.92	15243.12	16259.[illegible]
60	12375.00	13500.00	14625.00	15750.00	16875.00	18000.[illegible]

Ex.—What resisting moment has a beam $4\frac{1}{2}$ ins. wide by 15 ins depth?

Ans.—$I = 281.25 \times 4 + 140.63 = 1265.63$; hence,

$M = \frac{2I}{d} f = 168.75 f$ inch-lbs. $= 14.06 f$ ft.-lbs.

If $f = 1000$, $M' = 14060$ ft.-lbs.

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1.—Steel Rods—Weights and Areas.

Weights at 489.6 lbs. per cu. ft., or 2% more than Iron at 480 lbs.

Thickness or Diam. in ins.	Weight of ■ Bar 1 Foot Long. Lbs.	Weight of ● Bar 1 Foot Long. Lbs.	Area of □ Bar in Square Ins.	Area of ○ Bar in Square Ins.	Thickness or Diam. in ins.	Weight of ■ Bar 1 Foot Long. Lbs.	Weight of ● Bar 1 Foot Long. Lbs.	Area of □ Bar in Square Ins.	Area of ○ Bar in Square Ins.
0					3	30.60	24.03	9.0000	7.0
1/16	.013	.010	.0039	.0031	1/16	31.89	25.04	9.3789	7.3
1/8	.053	.042	.0156	.0123	1/8	33.20	26.08	9.7656	7.6
3/16	.119	.094	.0352	.0276	3/16	34.55	27.13	10.160	7.9
1/4	.212	.167	.0625	.0491	1/4	35.92	28.20	10.563	8.2
5/16	.333	.261	.0977	.0767	5/16	37.31	29.30	10.973	8.6
3/8	.478	.375	.1406	.1104	3/8	38.73	30.42	11.391	8.9
7/16	.651	.511	.1914	.1503	7/16	40.18	31.56	11.816	9.2
1/2	.850	.667	.2500	.1963	1/2	41.65	32.71	12.250	9.6
9/16	1.076	.845	.3164	.2485	9/16	43.14	33.90	12.691	9.9
5/8	1.328	1.043	.3906	.3068	5/8	44.68	35.09	13.141	10.
11/16	1.608	1.262	.4727	.3712	11/16	46.24	36.31	13.598	10.
3/4	1.913	1.502	.5625	.4418	3/4	47.82	37.56	14.063	11.
13/16	2.245	1.763	.6602	.5185	13/16	49.42	38.81	14.535	11.
7/8	2.603	2.044	.7656	.6013	7/8	51.05	40.10	15.016	11.
15/16	2.989	2.347	.8789	.6903	15/16	52.71	41.40	15.504	12.
1	3.400	2.670	1.0000	.7854	4	54.40	42.73	16.000	12.
1/16	3.838	3.014	1.1289	.8866	1/16	56.11	44.07	16.504	12.
1/8	4.303	3.379	1.2656	.9940	1/8	57.85	45.44	17.016	13.
3/16	4.795	3.766	1.4102	1.1075	3/16	59.62	46.83	17.535	13.
1/4	5.312	4.173	1.5625	1.2272	1/4	61.41	48.24	18.063	14.
5/16	5.857	4.600	1.7227	1.3530	5/16	63.23	49.66	18.598	14.
3/8	6.428	5.049	1.8906	1.4849	3/8	65.08	51.11	19.141	15.
7/16	7.026	5.518	2.0664	1.6230	7/16	66.95	52.58	19.691	15.
1/2	7.650	6.008	2.2500	1.7671	1/2	68.85	54.07	20.250	15.
9/16	8.301	6.520	2.4414	1.9175	9/16	70.78	55.59	20.816	16.
5/8	8.978	7.051	2.6406	2.0739	5/8	72.73	57.12	21.391	16.
11/16	9.682	7.604	2.8477	2.2365	11/16	74.70	58.67	21.973	17.
3/4	10.41	8.178	3.0625	2.4053	3/4	76.71	60.25	22.563	17.
13/16	11.17	8.773	3.2852	2.5802	13/16	78.74	61.84	23.160	18.
7/8	11.95	9.388	3.5156	2.7612	7/8	80.81	63.46	23.766	18.
15/16	12.76	10.02	3.7539	2.9483	15/16	82.89	65.10	24.379	19.
2	13.60	10.68	4.0000	3.1416	5	85.00	66.76	25.000	19.
1/16	14.46	11.36	4.2539	3.3410	1/16	87.14	68.44	25.629	20.
1/8	15.35	12.06	4.5156	3.5466	1/8	89.30	70.14	26.266	20.
3/16	16.27	12.78	4.7852	3.7583	3/16	91.49	71.86	26.910	21.
1/4	17.22	13.52	5.0625	3.9761	1/4	93.72	73.60	27.563	21.
5/16	18.19	14.28	5.3477	4.2000	5/16	95.96	75.37	28.223	22.
3/8	19.18	15.07	5.6406	4.4301	3/8	98.23	77.15	28.891	22.
7/16	20.20	15.86	5.9414	4.6664	7/16	100.5	78.95	29.566	23.
1/2	21.25	16.69	6.2500	4.9087	1/2	102.8	80.77	30.250	23.
9/16	22.33	17.53	6.5664	5.1572	9/16	105.2	82.62	30.941	24.
5/8	23.43	18.40	6.8906	5.4119	5/8	107.6	84.49	31.641	24.
11/16	24.56	19.29	7.2227	5.6727	11/16	110.0	86.38	32.348	25.
3/4	25.71	20.20	7.5625	5.9396	3/4	112.4	88.29	33.063	25.
13/16	26.90	21.12	7.9102	6.2126	13/16	114.9	90.22	33.785	26.
7/8	28.10	22.07	8.2656	6.4918	7/8	117.4	92.17	34.516	27.
15/16	29.34	22.04	8.6289	6.7771	15/16	119.9	94.14	35.254	27.
3	30.60	24.03	9.0000	7.0686	6	122.4	96.14	36.000	28.

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1.—STEEL RODS—WEIGHTS AND AREAS.—Concluded.
Weights at 489.6 lbs. per cu. ft., or 2% more than Iron at 480 lbs.

Thickness or Diam. in ins.	Weight of ■ Bar 1 Foot Long. Lbs.	Weight of ● Bar 1 Foot Long. Lbs.	Area of □ Bar in Square Ins.	Area of ○ Bar in Square Ins.	Thickness or Diam. in ins.	Weight of ■ Bar 1 Foot Long. Lbs.	Weight of ● Bar 1 Foot Long. Lbs.	Area of □ Bar in Square Ins.	Area of ○ Bar in Square Ins.
6	122.4	96.14	36.000	28.274	9	275.4	216.3	81.000	63.617
1/16	125.0	98.14	36.754	28.866	1/16	279.3	219.3	82.129	64.504
1/8	127.6	100.2	37.516	29.465	1/8	283.2	222.4	83.266	65.397
3/16	130.2	102.2	38.285	30.069	3/16	287.0	225.4	84.410	66.296
1/4	132.8	104.3	39.063	30.680	1/4	290.9	228.5	85.563	67.201
5/16	135.5	106.4	39.848	31.296	5/16	294.9	231.5	86.723	68.112
3/8	138.2	108.5	40.641	31.919	3/8	298.9	234.7	87.891	69.029
7/16	140.9	110.7	41.441	32.548	7/16	302.8	237.9	89.066	69.953
1/2	143.6	112.8	42.250	33.183	1/2	306.8	241.0	90.250	70.882
9/16	146.5	114.9	43.066	33.824	9/16	310.9	244.2	91.441	71.818
5/8	149.2	117.2	43.891	34.472	5/8	315.0	247.4	92.641	72.760
11/16	152.1	119.4	44.723	35.125	11/16	319.1	250.6	93.848	73.708
3/4	154.9	121.7	45.563	35.785	3/4	323.2	253.9	95.063	74.662
13/16	157.8	123.9	46.410	36.450	13/16	327.4	257.1	96.285	75.622
7/8	160.8	126.2	47.266	37.122	7/8	331.6	260.4	97.516	76.589
15/16	163.6	128.5	48.129	37.800	15/16	335.8	263.7	98.754	77.561
7	166.6	130.9	49.000	38.485	10	340.0	267.0	100.00	78.540
1/16	169.6	133.2	49.879	39.175	1/16	344.3	270.4	101.25	79.525
1/8	172.6	135.6	50.766	39.871	1/8	348.5	273.8	102.52	80.516
3/16	175.6	137.9	51.660	40.574	3/16	352.9	277.1	103.79	81.513
1/4	178.7	140.4	52.563	41.282	1/4	357.2	280.6	105.06	82.516
5/16	181.8	142.8	53.473	41.997	5/16	361.6	284.0	106.35	83.525
3/8	184.9	145.3	54.391	42.718	3/8	366.0	287.4	107.64	84.541
7/16	188.1	147.7	55.316	43.445	7/16	370.4	290.9	108.94	85.562
1/2	191.3	150.2	56.250	44.179	1/2	374.9	294.4	110.25	86.590
9/16	194.4	152.7	57.191	44.918	9/16	379.4	297.9	111.57	87.624
5/8	197.7	155.2	58.141	45.664	5/8	383.8	301.4	112.89	88.664
11/16	200.9	157.8	59.098	46.415	11/16	388.3	305.0	114.22	89.710
3/4	204.2	160.3	60.063	47.173	3/4	392.9	308.6	115.56	90.763
13/16	207.6	163.0	61.035	47.937	13/16	397.5	312.2	116.91	91.821
7/8	210.8	165.6	62.016	48.707	7/8	402.1	315.8	118.27	92.886
15/16	214.2	168.2	63.004	49.483	15/16	406.8	319.5	119.63	93.956
8	217.6	171.0	64.000	50.265	11	411.4	323.1	121.00	95.033
1/16	221.0	173.6	65.004	51.054	1/16	416.1	326.8	122.38	96.116
1/8	224.5	176.3	66.016	51.849	1/8	420.9	330.5	123.77	97.205
3/16	228.0	179.0	67.035	52.649	3/16	425.5	334.3	125.16	98.301
1/4	231.4	181.8	68.063	53.456	1/4	430.3	337.9	126.56	99.402
5/16	234.9	184.5	69.098	54.269	5/16	435.1	341.7	127.97	100.51
3/8	238.5	187.3	70.141	55.088	3/8	439.9	345.5	129.39	101.62
7/16	242.0	190.1	71.191	55.914	7/16	444.8	349.4	130.82	102.74
1/2	245.6	193.0	72.250	56.745	1/2	449.6	353.1	132.25	103.87
9/16	249.3	195.7	73.316	57.583	9/16	454.5	357.0	133.69	105.00
5/8	252.9	198.7	74.391	58.426	5/8	459.5	360.9	135.14	106.14
11/16	256.6	201.6	75.473	59.276	11/16	464.4	364.8	136.60	107.28
3/4	260.3	204.4	76.563	60.132	3/4	469.4	368.6	138.06	108.43
13/16	264.1	207.4	77.660	60.994	13/16	474.4	372.6	139.54	109.59
7/8	267.9	210.3	78.766	61.862	7/8	479.5	376.6	141.02	110.75
15/16	271.6	213.3	79.879	62.737	15/16	484.5	380.6	142.50	111.92
9	275.4	216.3	81.000	63.617	12	489.6	384.6	144.00	113.10

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2.—Steel Plates—Weights and Areas.

(Weights at 489.6 lbs. per cubic foot.)

Thickness in Inches.	Width of Plate in Inches.										
	1″	1¼″	1½″	1¾″	2″	2¼″	2½″	2¾″	3″	3¼″	3½″
	WEIGHT lbs. per lin. ft.										
1/16	.212	.266	.319	.372	.425	.48	.53	.58	.64	.69	.74
1/8	.425	.531	.637	.744	.850	.96	1.06	1.17	1.28	1.38	1.49
3/16	.638	.797	.957	1.11	1.28	1.44	1.59	1.75	1.91	2.07	2.23
1/4	.850	1.06	1.28	1.49	1.70	1.91	2.12	2.34	2.55	2.76	2.98
5/16	1.06	1.33	1.59	1.86	2.12	2.39	2.65	2.92	3.19	3.45	3.72
3/8	1.28	1.59	1.92	2.23	2.55	2.87	3.19	3.51	3.83	4.15	4.47
7/16	1.49	1.86	2.23	2.60	2.98	3.35	3.72	4.09	4.46	4.83	5.20
1/2	1.70	2.12	2.55	2.98	3.40	3.83	4.25	4.67	5.10	5.53	5.95
9/16	1.92	2.39	2.87	3.35	3.83	4.30	4.78	5.26	5.74	6.22	6.70
5/8	2.12	2.65	3.19	3.72	4.25	4.78	5.31	5.84	6.38	6.91	7.44
11/16	2.34	2.92	3.51	4.09	4.67	5.26	5.84	6.43	7.02	7.60	8.18
3/4	2.55	3.19	3.83	4.47	5.10	5.75	6.38	7.02	7.65	8.29	8.93
13/16	2.76	3.45	4.14	4.84	5.53	6.21	6.90	7.60	8.29	8.98	9.67
7/8	2.98	3.72	4.47	5.20	5.95	6.69	7.44	8.18	8.93	9.67	10.41
15/16	3.19	3.99	4.78	5.58	6.38	7.18	7.97	8.77	9.57	10.36	11.16
1	3.40	4.25	5.10	5.95	6.80	7.65	8.50	9.35	10.20	11.05	11.90
1/16	3.61	4.52	5.42	6.32	7.22	8.13	9.03	9.93	10.84	11.74	12.65
1/8	3.83	4.78	5.74	6.70	7.65	8.61	9.57	10.52	11.48	12.43	13.39
3/16	4.04	5.05	6.06	7.07	8.08	9.09	10.10	11.11	12.12	13.12	14.13
1/4	4.25	5.31	6.38	7.44	8.50	9.57	10.63	11.69	12.75	13.81	14.87
5/16	4.46	5.58	6.69	7.81	8.93	10.04	11.16	12.27	13.39	14.50	15.62
3/8	4.67	5.84	7.02	8.18	9.35	10.52	11.69	12.85	14.03	15.20	16.36
7/16	4.89	6.11	7.34	8.56	9.78	11.10	12.22	13.44	14.66	15.88	17.10
1/2	5.10	6.38	7.65	8.93	10.20	11.48	12.75	14.03	15.30	16.58	17.85
	AREA of section.										
1/16	.063	.078	.094	.109	.125	.141	.156	.172	.188	.203	.219
1/8	.125	.156	.188	.219	.250	.281	.313	.344	.375	.406	.438
3/16	.188	.234	.281	.328	.375	.422	.469	.516	.563	.609	.656
1/4	.250	.313	.375	.438	.500	.563	.625	.688	.750	.813	.875
5/16	.313	.391	.469	.547	.625	.703	.781	.859	.938	1.02	1.09
3/8	.375	.469	.563	.656	.750	.844	.938	1.03	1.13	1.22	1.31
7/16	.438	.547	.656	.766	.875	.984	1.09	1.20	1.31	1.42	1.53
1/2	.500	.625	.750	.875	1.00	1.13	1.25	1.38	1.50	1.63	1.75
9/16	.563	.703	.844	.984	1.13	1.27	1.41	1.55	1.69	1.83	1.97
5/8	.625	.781	.938	1.09	1.25	1.41	1.56	1.72	1.88	2.03	2.19
11/16	.688	.859	1.03	1.20	1.38	1.55	1.72	1.89	2.06	2.23	2.41
3/4	.750	.938	1.13	1.31	1.50	1.69	1.88	2.06	2.25	2.44	2.63
13/16	.813	1.02	1.22	1.42	1.63	1.83	2.03	2.23	2.44	2.64	2.84
7/8	.875	1.09	1.31	1.53	1.75	1.97	2.19	2.41	2.63	2.84	3.06
15/16	.938	1.17	1.41	1.64	1.88	2.11	2.34	2.58	2.81	3.05	3.28
1	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50
1/16	1.06	1.33	1.59	1.86	2.13	2.39	2.66	2.92	3.19	3.45	3.72
1/8	1.13	1.41	1.69	1.97	2.25	2.53	2.81	3.09	3.38	3.66	3.94
3/16	1.19	1.48	1.78	2.08	2.38	2.67	2.97	3.27	3.56	3.86	4.16
1/4	1.25	1.56	1.88	2.19	2.50	2.81	3.13	3.44	3.75	4.06	4.38
5/16	1.31	1.64	1.97	2.30	2.63	2.95	3.28	3.61	3.94	4.27	4.59
3/8	1.38	1.72	2.06	2.41	2.75	3.09	3.44	3.78	4.13	4.47	4.81
7/16	1.44	1.80	2.16	2.52	2.88	3.23	3.59	3.95	4.31	4.67	5.03
1/2	1.50	1.88	2.25	2.63	3.00	3.38	3.75	4.13	4.50	4.88	5.25

2.—Steel Plates—Weights and Areas.—Continued.

(Weights at 489.6 lbs. per cubic foot.)

Thickness in Inches.	Width of Plate in inches.											
	4"	4¼"	4½"	4¾"	5"	5¼"	5½"	5¾"	6"	6¼"	6½"	6¾"
	WEIGHT lbs. per lin. ft.											
1/16	.85	.90	.96	1.01	1.06	1.12	1.17	1.22	1.28	1.33	1.38	1.43
1/8	1.70	1.81	1.91	2.02	2.13	2.23	2.34	2.44	2.55	2.66	2.76	2.87
3/16	2.55	2.71	2.87	3.03	3.19	3.35	3.51	3.67	3.83	3.99	4.14	4.30
1/4	3.40	3.61	3.83	4.04	4.25	4.46	4.67	4.89	5.10	5.31	5.53	5.74
5/16	4.25	4.52	4.78	5.05	5.31	5.58	5.84	6.11	6.38	6.64	6.90	7.17
3/8	5.10	5.42	5.74	6.06	6.38	6.69	7.02	7.34	7.65	7.97	8.29	8.61
7/16	5.95	6.32	6.70	7.07	7.44	7.81	8.18	8.56	8.93	9.29	9.67	10.04
1/2	6.80	7.22	7.65	8.08	8.50	8.93	9.35	9.77	10.20	10.63	11.05	11.48
9/16	7.65	8.13	8.61	9.09	9.57	10.04	10.52	11.00	11.48	11.95	12.43	12.91
5/8	8.50	9.03	9.57	10.10	10.63	11.16	11.69	12.22	12.75	13.28	13.81	14.34
11/16	9.35	9.93	10.52	11.11	11.69	12.27	12.85	13.44	14.03	14.61	15.20	15.78
3/4	10.20	10.84	11.48	12.12	12.75	13.39	14.03	14.67	15.30	15.94	16.58	17.22
13/16	11.05	11.74	12.43	13.12	13.81	14.50	15.19	15.88	16.58	17.27	17.95	18.65
7/8	11.90	12.65	13.39	14.13	14.87	15.62	16.36	17.10	17.85	18.60	19.34	20.08
15/16	12.75	13.55	14.34	15.14	15.94	16.74	17.53	18.33	19.13	19.92	20.72	21.51
1	13.60	14.45	15.30	16.15	17.00	17.85	18.70	19.55	20.40	21.25	22.10	22.95
1 1/16	14.45	15.35	16.26	17.16	18.06	18.96	19.87	20.77	21.68	22.58	23.48	24.39
1 1/8	15.30	16.26	17.22	18.17	19.13	20.08	21.04	21.99	22.95	23.91	24.87	25.82
1 3/16	16.15	17.16	18.17	19.18	20.19	21.20	22.21	23.22	24.23	25.23	26.24	27.25
1 1/4	17.00	18.06	19.13	20.19	21.25	22.32	23.38	24.44	25.50	26.56	27.62	28.69
1 5/16	17.85	18.96	20.08	21.20	22.32	23.43	24.54	25.66	26.78	27.90	29.01	30.12
1 3/8	18.70	19.87	21.04	22.21	23.38	24.54	25.71	26.88	28.05	29.22	30.39	31.56
1 7/16	19.55	20.77	21.99	23.22	24.44	25.66	26.88	28.10	29.33	30.55	31.77	32.99
1 1/2	20.40	21.68	22.95	24.23	25.50	26.78	28.05	29.33	30.60	31.88	33.15	34.43
	AREA of section.											
1/16	.250	.266	.281	.297	.313	.328	.344	.359	.375	.391	.406	.422
1/8	.500	.531	.563	.594	.625	.656	.688	.719	.751	.781	.813	.844
3/16	.750	.797	.844	.891	.938	.984	1.03	1.08	1.13	.117	1.22	1.27
1/4	1.00	1.06	1.13	1.19	1.25	1.31	1.38	1.44	1.50	1.56	1.63	1.69
5/16	1.25	1.33	1.41	1.48	1.56	1.64	1.72	1.80	1.88	1.95	2.03	2.11
3/8	1.50	1.59	1.69	1.78	1.88	1.97	2.06	2.16	2.25	2.34	2.44	2.53
7/16	1.75	1.86	1.97	2.08	2.19	2.30	2.41	2.52	2.63	2.73	2.84	2.95
1/2	2.00	2.13	2.25	2.38	2.50	2.63	2.75	2.88	3.00	3.13	3.25	3.38
9/16	2.25	2.39	2.53	2.67	2.81	2.95	3.09	3.23	3.38	3.52	3.66	3.80
5/8	2.50	2.66	2.81	2.97	3.13	3.28	3.44	3.59	3.75	3.91	4.06	4.22
11/16	2.75	2.92	3.09	3.27	3.44	3.61	3.78	3.95	4.13	4.30	4.47	4.64
3/4	3.00	3.19	3.38	3.56	3.75	3.94	4.13	4.31	4.50	4.69	4.88	5.06
13/16	3.25	3.45	3.66	3.86	4.06	4.27	4.47	4.67	4.88	5.08	5.28	5.48
7/8	3.50	3.72	3.94	4.16	4.38	4.59	4.81	5.03	5.25	5.47	5.69	5.91
15/16	3.75	3.98	4.22	4.45	4.69	4.92	5.16	5.39	5.63	5.86	6.09	6.33
1	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00	6.25	6.50	6.75
1 1/16	4.25	4.52	4.78	5.05	5.31	5.58	5.84	6.11	6.38	6.64	6.91	7.17
1 1/8	4.50	4.78	5.06	5.34	5.63	5.91	6.19	6.47	6.75	7.03	7.31	7.59
1 3/16	4.75	5.05	5.34	5.64	5.94	6.23	6.53	6.83	7.13	7.42	7.72	8.02
1 1/4	5.00	5.31	5.63	5.94	6.25	6.56	6.88	7.19	7.50	7.81	8.13	8.44
1 5/16	5.25	5.58	5.91	6.23	6.56	6.89	7.22	7.55	7.88	8.20	8.53	8.86
1 3/8	5.50	5.84	6.19	6.53	6.88	7.22	7.56	7.91	8.25	8.59	8.94	9.28
1 7/16	5.75	6.11	6.47	6.83	7.19	7.55	7.91	8.27	8.63	8.98	9.34	9.70
1 1/2	6.00	6.38	6.75	7.13	7.50	7.88	8.25	8.63	9.00	9.38	9.75	10.13

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2.—Steel Plates—Weights and Areas.—Continued.
(Weights at 489.6 lbs. per cubic foot.)

Thickness in Inches.	Width of Plate in Inches.											
	7"	7¼"	7½"	7¾"	8"	8¼"	8½"	8¾"	9"	9¼"	9½"	9
	WEIGHT lbs. per lin. ft.											
1/16	1.49	1.54	1.59	1.65	1.70	1.75	1.81	1.86	1.91	1.97	2.02	2
1/8	2.98	3.08	3.19	3.29	3.40	3.51	3.61	3.72	3.83	3.93	4.04	4
3/16	4.46	4.62	4.78	4.94	5.10	5.26	5.42	5.58	5.74	5.90	6.06	6
1/4	5.95	6.16	6.36	6.58	6.80	7.01	7.22	7.43	7.65	8.86	8.08	8
5/16	7.44	7.70	7.97	8.23	8.50	8.76	9.03	9.29	9.56	9.83	10.10	10
3/8	8.93	9.25	9.57	9.88	10.20	10.52	10.84	11.16	11.48	11.80	12.12	12
7/16	10.41	10.78	11.16	11.53	11.90	12.27	12.64	13.02	13.40	13.76	14.14	14
1/2	11.90	12.32	12.75	13.18	13.60	14.03	14.44	14.87	15.30	15.73	16.16	16
9/16	13.39	13.86	14.34	14.82	15.30	15.78	16.26	16.74	17.22	17.69	18.18	18
5/8	14.87	15.40	15.94	16.47	17.00	17.53	18.06	18.59	19.13	19.65	20.19	20
11/16	16.36	16.94	17.53	18.12	18.70	19.28	19.86	20.45	21.04	21.62	22.21	22
3/4	17.85	18.49	19.13	19.77	20.40	21.04	21.68	22.32	22.96	23.59	24.23	24
13/16	19.34	20.03	20.72	21.41	22.10	22.79	23.48	24.17	24.86	25.55	26.24	26
7/8	20.83	21.57	22.32	23.05	23.80	24.55	25.30	26.04	26.78	27.52	28.26	29
15/16	22.32	23.11	23.91	24.70	25.50	26.30	27.10	27.89	28.69	29.49	30.28	31
1	23.80	24.65	25.50	26.35	27.20	28.05	28.90	29.75	30.60	31.45	32.30	33
1/16	25.29	26.19	27.10	28.00	28.90	29.80	30.70	31.61	32.52	33.41	34.32	35
1/8	26.78	27.73	28.68	29.64	30.60	31.56	32.52	33.47	34.43	35.38	36.34	37
3/16	28.26	29.27	30.28	31.29	32.30	33.31	34.32	35.33	36.34	37.35	38.36	39
1/4	29.75	30.81	31.88	32.94	34.00	35.06	36.12	37.20	38.26	39.31	40.37	41
5/16	31.23	32.35	33.48	34.59	35.70	36.81	37.93	39.05	40.16	41.28	42.40	43
3/8	32.72	33.89	35.06	36.23	37.40	38.57	39.74	40.91	42.08	43.25	44.41	45
7/16	34.21	35.44	36.66	37.88	39.10	40.32	41.54	42.77	44.00	45.22	46.44	47
1/2	35.70	36.98	38.26	39.53	40.80	42.08	43.35	44.63	45.90	47.18	48.45	49
	AREA of section.											
1/16	.438	.453	.469	.484	.500	.516	.531	.547	.563	.578	.594	
1/8	.875	.906	.938	.969	1.00	1.03	1.06	1.09	1.13	1.16	1.19	1
3/16	1.31	1.36	1.41	1.45	1.50	1.55	1.59	1.64	1.69	1.73	1.78	1
1/4	1.75	1.81	1.88	1.94	2.00	2.06	2.13	2.19	2.25	2.31	2.38	2
5/16	2.19	2.27	2.34	2.42	2.50	2.58	2.66	2.73	2.81	2.89	2.97	3
3/8	2.63	2.72	2.81	2.91	3.00	3.09	3.19	3.28	3.38	3.47	3.56	3
7/16	3.06	3.17	3.28	3.39	3.50	3.60	3.72	3.83	3.94	4.05	4.16	4
1/2	3.50	3.63	3.75	3.88	4.00	4.13	4.25	4.38	4.50	4.63	4.75	4
9/16	3.94	4.08	4.22	4.36	4.50	4.64	4.78	4.92	5.06	5.20	3.34	5
5/8	4.38	4.53	4.69	4.84	5.00	5.16	5.31	5.47	5.63	5.78	5.94	6
11/16	4.81	4.98	5.16	5.33	5.50	5.67	5.84	6.02	6.19	6.36	6.53	6
3/4	5.25	5.44	5.63	5.81	6.00	6.19	6.38	6.56	6.75	6.94	7.13	7
13/16	5.69	5.89	6.09	6.30	6.50	6.70	6.91	7.11	7.31	7.52	7.72	7
7/8	6.13	6.34	6.56	6.78	7.00	7.22	7.44	7.66	7.88	8.09	8.31	8
15/16	6.56	6.80	7.03	7.27	7.50	7.73	7.97	8.20	8.44	8.67	8.91	9
1	7.00	7.25	7.50	7.75	8.00	8.25	8.50	8.75	9.00	9.25	9.50	9
1/16	7.44	7.70	7.97	8.23	8.50	8.77	9.03	9.30	9.56	9.83	10.09	10
1/8	7.88	8.16	8.44	8.72	9.00	9.28	9.56	9.84	10.13	10.41	10.69	10
3/16	8.31	8.61	8.91	9.20	9.50	9.80	10.09	10.39	10.69	10.98	11.28	11
1/4	8.75	9.06	9.38	9.69	10.00	10.31	10.63	10.94	11.25	11.56	11.88	12
5/16	9.19	9.52	9.84	10.17	10.50	10.83	11.16	11.48	11.81	12.14	12.47	12
3/8	9.63	9.97	10.31	10.66	11.00	11.34	11.69	12.03	12.38	12.72	13.06	13
7/16	10.06	10.42	10.78	11.14	11.50	11.86	12.22	12.58	12.94	13.30	13.66	14
1/2	10.50	10.88	11.25	11.63	12.00	12.38	12.75	13.13	13.50	13.88	14.25	14

2.—Steel Plates—Weights and Areas.—Concluded.

(Weights at 489.6 lbs. per cubic foot.)

Thickness in Inches.	Width of Plate in Inches.											
	10"	10¼"	10½"	10¾"	11"	11¼"	11½"	11¾"	12"	12¼"	12½"	12¾"
	WEIGHT lbs. per lin. ft.											
1/16	2.13	2.18	2.23	2.28	2.34	2.39	2.44	2.50	2.55	2.60	2.66	2.71
1/8	4.25	4.36	4.46	4.57	4.68	4.78	4.89	4.99	5.10	5.21	5.31	5.42
3/16	6.38	6.54	6.70	6.86	7.02	7.17	7.32	7.49	7.65	7.82	7.98	8.13
1/4	8.50	8.71	8.92	9.14	9.34	9.57	9.78	10.00	10.20	10.42	10.63	10.84
5/16	10.62	10.89	11.16	11.42	11.68	11.95	12.22	12.49	12 75	13.01	13.28	13.55
3/8	12.75	13.07	13.39	13.71	14.03	14.35	14 68	14 99	15.30	15.62	15.94	16.26
7/16	14.88	15 25	15.62	15.99	16.36	16.74	17 12	17.49	17.85	18.23	18.60	18.97
1/2	17.00	17.42	17.85	18.28	18.70	19.13	19.55	19.97	20.40	20.82	21.25	21.67
9/16	19.14	19.61	20.08	20.56	21.02	21.51	22.00	22.48	22 95	23.43	23.90	24.39
5/8	21.25	21.78	22.32	22.85	23.38	23.91	24.44	24.97	25.50	26.03	26.56	27.09
11/16	23.38	23.96	24.54	25.13	25.70	26.30	26.88	27.47	28.05	28.64	29.22	29.80
3/4	25.50	26.14	26.78	27.42	28.05	28.68	29.33	29.97	30.60	31.25	31.88	32.52
13/16	27.62	28.32	29.00	29.69	30.40	31.08	31.76	32.46	33.15	33.83	34.53	35.22
7/8	29.75	30.50	31.24	31.98	32.72	33.47	34.21	34.95	35.70	36.44	37.19	37.93
15/16	31.88	32.67	33.48	34.28	35.06	35.86	36.66	37.46	38.25	39.05	39.84	40.64
1	34.00	34.85	35.70	36.55	37.40	38.25	39.10	39.95	40.80	41.65	42.50	43.35
1 1/16	36.12	37.03	37.92	38.83	39.74	40.64	41.54	42.45	43.35	44.25	45.16	46.06
1 1/8	38.25	39.21	40.17	41.12	42.08	43.04	44.00	44.94	45.90	46.86	47.82	48.77
1 3/16	40.38	41.39	42.40	43.40	44.42	45.42	46.44	47.45	48.45	49.46	50.46	51.48
1 1/4	42.50	43.56	44.63	45.69	46.76	47.82	48.88	49.94	51.00	52.06	53.12	54.19
1 5/16	44.64	45.75	46.86	47.97	49.08	50.20	51.32	52.44	53.55	54.67	55.78	56.90
1 3/8	46.75	47.92	49.08	50.25	51.42	52.59	53.76	54.93	56.10	57.27	58.44	59.60
1 7/16	48.88	50.10	51.32	52.54	53.76	54.99	56.21	57.43	58.65	59.87	61.10	62.32
1 1/2	51.00	52.28	53.55	54.83	56.10	57.37	58.65	59.93	61.20	62.48	63.75	65.03
	AREA of section.											
1/16	.625	.641	.656	.672	.688	.703	.719	.734	.750	.766	.781	.797
1/8	1.25	1.28	1.31	1.34	1.38	1.41	1.44	1.47	1.50	1.53	1.56	1.59
3/16	1.88	1.92	1.97	2.02	2.06	2.11	2.16	2.20	2.25	2.30	2.34	2.39
1/4	2.50	2.56	2.63	2.69	2.75	2.81	2.88	2.94	3.00	3.06	3.13	3.19
5/16	3.13	3.20	3.28	3.36	3.44	3.52	3.59	3.67	3.75	3.83	3.91	3.98
3/8	3.75	3.84	3.94	4.03	4.13	4.22	4.31	4.41	4.50	4.59	4.69	4.78
7/16	4.38	4.48	4.59	4.70	4.81	4.92	5.03	5.14	5.25	5.36	5.47	5.58
1/2	5.00	5.13	5.25	5.38	5.50	5.63	5.75	5.88	6.00	6.13	6.25	6.38
9/16	5.63	5.77	5.91	6.05	6.19	6.33	6.47	6.61	6.75	6.89	7.03	7.17
5/8	6.25	6.41	6.56	6.72	6.88	7.03	7.19	7.34	7.50	7.66	7.81	7.97
11/16	6.88	7.05	7.22	7.39	7.56	7.73	7.91	8.08	8.25	8.42	8.59	8.77
3/4	7.50	7.69	7.88	8.06	8.25	8.44	8.63	8.81	9.00	9.19	9.38	9.56
13/16	8.13	8.33	8.53	8.73	8.94	9.14	9.34	9.55	9.75	9.95	10.16	10.36
7/8	8.75	8.97	9.19	9.41	9.63	9.84	10.06	10 28	10.50	10.72	10.94	11.16
15/16	9.38	9.61	9.84	10.08	10.31	10.55	10.78	11.02	11.25	11.48	11.72	11.95
1	10.00	10.25	10.50	10.75	11.00	11.25	11.50	11.75	12.00	12.25	12.50	12.75
1 1/16	10.63	10.89	11.16	11.42	11.69	11.95	12.22	12.48	12.75	13.02	13.28	13.55
1 1/8	11.25	11.53	11.81	12.09	12.38	12.66	12.94	13.22	13.50	13.78	14.06	14.34
1 3/16	11.88	12.17	12.47	12.77	13.06	13.36	13.66	13.95	14.25	14.55	14.84	15.14
1 1/4	12.50	12.81	13.13	13.44	13.75	14.06	14.38	14.69	15.00	15.31	15.63	15.94
1 5/16	13.13	13.45	13.78	14.11	14.44	14.77	15.09	15.42	15.75	16.08	16.41	16.73
1 3/8	13.75	14.09	14.44	14.78	15.13	15.47	15.81	16.16	16.50	16.84	17.19	17.53
1 7/16	14.38	14.73	15.09	15.45	15.81	16.17	16.53	16.89	17.25	17.61	17.97	18.33
1 1/2	15.00	15.38	15.75	16.13	16.50	16.88	17.25	17.63	18.00	18.38	18.75	19.13

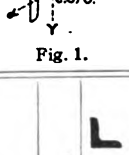

Fig. 1.

3.—Properties of Angles (Steel). Unequal Legs.

(Weights at 489.6 lbs. per cubic foot.)

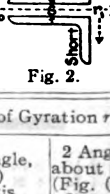

Fig. 2.

Size. Ins.	Thickness. t Ins.	Wt. per Foot. W Lbs.	Area of Sec. A Sq. Ins.	Dist. from b of Angle to c of g. Fig. 1. x_1	y_1	Moment of Inertia I, (Fig. 1) about X—X I_x	Y—Y I_y	Radius of Gyration r: Single Angle, (Fig. 1) about axis X—X r_x	Y—Y r_y	$\alpha-\alpha$ ¶r_α	2 Ang about (Fig. 2) X_0-X_0 For $s=0.5$ †r_1
*8 x3½	11/16	20.5	6.02	3.00	0.75	4.92	39.96	0.90	2.58	0.74	1.35
*7 x3½	1	32.3	9.50	2.71	0.96	7.53	45.37	0.89	2.19	0.88	1.50
* "	15/16	30.5	8.97	2.69	0.94	7.18	43.13	0.89	2.19	0.88	1.49
* "	7/8	28.7	8.42	2.67	0.92	6.83	40.82	0.90	2.20	0.88	1.47
* "	13/16	26.8	7.87	2.64	0.89	6.46	38.45	0.91	2.21	0.88	1.46
* "	3/4	24.9	7.31	2.62	0.87	6.08	35.99	0.91	2.22	0.88	1.44
* "	11/16	23.0	6.75	2.60	0.85	5.69	33.47	0.92	2.23	0.89	1.43
* "	5/8	21.0	6.17	2.57	0.82	5.28	30.86	0.93	2.24	0.89	1.41
* "	9/16	19.1	5.59	2.55	0.80	4.86	28.18	0.93	2.25	0.89	1.40
* "	1/2	17.0	5.00	2.53	0.78	4.41	25.41	0.94	2.25	0.89	1.39
* "	7/16	15.0	4.40	2.50	0.75	3.95	22.56	0.95	2.26	0.89	1.39
6 x4	1	30.6	9.00	2.17	1.17	10.75	30.75	1.09	1.85	0.85	1.79
"	15/16	28.9	8.50	2.14	1.14	10.26	29.26	1.10	1.86	0.85	1.78
"	7/8	27.2	7.99	2.12	1.12	9.75	27.73	1.11	1.86	0.86	1.77
"	13/16	25.4	7.47	2.10	1.10	9.23	26.15	1.11	1.87	0.86	1.75
"	3/4	23.6	6.94	2.08	1.08	8.68	24.51	1.12	1.88	0.86	1.74
"	11/16	21.8	6.41	2.06	1.06	8.11	22.82	1.13	1.89	0.86	1.73
"	5/8	20.0	5.86	2.03	1.03	7.52	21.07	1.13	1.90	0.86	1.72
"	9/16	18.1	5.31	2.01	1.01	6.91	19.26	1.14	1.90	0.87	1.71
"	1/2	16.2	4.75	1.99	0.99	6.27	17.40	1.15	1.91	0.87	1.69
"	7/16	14.3	4.18	1.96	0.96	5.60	15.46	1.16	1.92	0.87	1.68
"	3/8	12.3	3.61	1.94	0.94	4.90	13.47	1.17	1.93	0.88	1.67
6 x3½	1	28.9	8.50	2.26	1.01	7.21	29.24	0.92	1.85	0.74	1.56
"	15/16	27.3	8.03	2.24	0.99	6.88	27.84	0.93	1.86	0.74	1.55
"	7/8	25.7	7.55	2.22	0.97	6.55	26.38	0.93	1.87	0.75	1.54
"	13/16	24.0	7.06	2.20	0.95	6.20	24.89	0.94	1.88	0.75	1.52
"	3/4	22.4	6.56	2.18	0.93	5.84	23.34	0.94	1.89	0.75	1.51
"	11/16	20.6	6.06	2.15	0.90	5.47	21.74	0.95	1.89	0.75	1.50
"	5/8	18.9	5.55	2.13	0.88	5.08	20.08	0.96	1.90	0.75	1.48
"	9/16	17.1	5.03	2.11	0.86	4.67	18.37	0.96	1.91	0.75	1.47
"	1/2	15.3	4.50	2.08	0.83	4.25	16.59	0.97	1.92	0.76	1.46
"	7/16	13.5	3.97	2.06	0.81	3.81	14.76	0.98	1.93	0.76	1.44
"	3/8	11.7	3.42	2.04	0.79	3.34	12.86	0.99	1.94	0.77	1.43
*5 x4	7/8	24.2	7.11	1.71	1.21	9.23	16.42	1.14	1.52	0.84	1.85
* "	13/16	22.7	6.65	1.68	1.18	8.74	15.54	1.15	1.53	0.84	1.84
* "	3/4	21.1	6.19	1.66	1.16	8.23	14.60	1.15	1.54	0.84	1.83
* "	11/16	19.5	5.72	1.64	1.14	7.70	13.62	1.16	1.54	0.84	1.81

*Carnegie special angles. ¶r_α = minimum radius.

$$\dagger r_1 = \sqrt{r_x^2 + \left(y_1 + \frac{s}{2}\right)^2} \quad \ddagger r_2 = \sqrt{r_y^2 + \left(x_1 + \frac{s}{2}\right)^2}$$

Note.—$r_x^2 = I_x$ ... $r_y^2 = I_y$ ...

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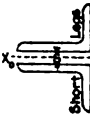

Fig. 2.

3.—Properties of Angles (Steel). Unequal Legs. —Continued.

(Weights at 489.6 lbs. per cubic foot.)

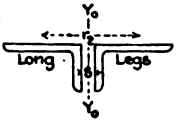

Fig. 3.

Size. Ins.	Thickness. t Ins.	Wt. per Foot. W Lbs.	Area of Sec. A Sq. Ins.	Dist. from b of Angle to c of g. *Fig. 1. x_1	y_1	Moment of Inertia I, (Fig. 1) about X—X I_x	Y—Y I_y	Radius of Gyration r, Single Angle, (Fig. 1) about axis X—X r_x	Y—Y r_y	a—a ¶r_a	2 Angles about axis (Fig. 2) X_o—X_o For s=0.5 †r_1	(Fig. 3) Y_o—Y_o For s=0.5 ‡r_2
*5 x4	5/8	17.8	5.23	1.62	1.12	7.14	12.61	1.17	1.55	0.84	1.80	2.43
* "	9/16	16.2	4.75	1.60	1.10	6.56	11.55	1.18	1.56	0.85	1.79	2.42
* "	1/2	14.5	4.25	1.57	1.07	5.96	10.46	1.18	1.57	0.85	1.78	2.41
* "	7/16	12.8	3.75	1.55	1.05	5.32	9.32	1.19	1.58	0.85	1.76	2.39
* "	3/8	11.0	3.23	1.53	1.03	4.67	8.14	1.20	1.59	0.86	1.75	2.38
5 x3½	7/8	22.7	6.67	1.79	1.04	6.21	15.67	0.96	1.53	0.75	1.61	2.55
"	13/16	21.3	6.25	1.77	1.02	5.89	14.81	0.97	1.54	0.75	1.60	2.54
"	3/4	19.8	5.81	1.75	1.00	5.55	13.92	0.98	1.55	0.75	1.59	2.53
"	11/16	18.3	5.37	1.72	0.97	5.20	12.99	0.98	1.56	0.75	1.57	2.51
"	5/8	16.8	4.92	1.70	0.95	4.83	12.03	0.99	1.56	0.75	1.56	2.50
"	9/16	15.2	4.47	1.68	0.93	4.45	11.03	1.00	1.57	0.75	1.55	2.49
"	1/2	13.6	4.00	1.66	0.91	4.05	9.99	1.01	1.58	0.75	1.54	2.48
"	7/16	12.0	3.53	1.63	0.88	3.63	8.90	1.01	1.59	0.76	1.52	2.46
"	3/8	10.4	3.06	1.61	0.86	3.18	7.78	1.02	1.60	0.76	1.51	2.45
"	5/16	8.7	2.56	1.59	0.84	2.72	6.60	1.03	1.61	0.76	1.50	2.44
5 x3	13/16	19.9	5.84	1.86	0.86	3.71	13.98	0.80	1.55	0.64	1.37	2.62
"	3/4	18.5	5.44	1.84	0.84	3.51	13.15	0.80	1.55	0.64	1.36	2.61
"	11/16	17.1	5.03	1.82	0.82	3.29	12.28	0.81	1.56	0.64	1.34	2.59
"	5/8	15.7	4.61	1.80	0.80	3.06	11.37	0.82	1.57	0.64	1.33	2.58
"	9/16	14.3	4.18	1.77	0.77	2.83	10.43	0.82	1.58	0.65	1.32	2.57
"	1/2	12.8	3.75	1.75	0.75	2.58	9.45	0.83	1.59	0.65	1.30	2.55
"	7/16	11.3	3.31	1.73	0.73	2.32	8.43	0.84	1.60	0.65	1.29	2.54
"	3/8	9.8	2.86	1.70	0.70	2.04	7.37	0.84	1.61	0.65	1.27	2.52
"	5/16	8.2	2.40	1.68	0.68	1.75	6.26	0.85	1.61	0.66	1.26	2.51
*4½x3	13/16	18.5	5.43	1.66	0.90	3.60	10.33	0.81	1.38	0.64	1.46	2.35
* "	3/4	17.3	5.06	1.63	0.88	3.40	9.73	0.82	1.39	0.64	1.44	2.34
* "	11/16	16.0	4.68	1.60	0.85	3.19	9.10	0.83	1.39	0.64	1.42	2.32
* "	5/8	14.7	4.30	1.58	0.83	2.98	8.44	0.83	1.40	0.64	1.40	2.31
* "	9/16	13.3	3.90	1.56	0.81	2.75	7.75	0.85	1.41	0.64	1.38	2.30
* "	1/2	11.9	3.50	1.54	0.79	2.51	7.04	0.85	1.42	0.65	1.37	2.28
* "	7/16	10.6	3.09	1.51	0.76	2.25	6.29	0.85	1.43	0.65	1.35	2.27
* "	3/8	9.1	2.67	1.49	0.74	1.98	5.50	0.86	1.44	0.66	1.33	2.25
* "	5/16	7.7	2.25	1.47	0.72	1.73	4.69	0.88	1.44	0.66	1.31	2.24
*4 x2½	13/16	18.5	5.43	1.36	1.11	5.49	7.77	1.01	1.19	0.72	1.69	2.01
* "	3/4	17.3	5.06	1.34	1.09	5.18	7.32	1.01	1.20	0.72	1.68	2.00
* "	11/16	16.0	4.68	1.32	1.07	4.86	6.86	1.02	1.21	0.72	1.67	1.99
* "	5/8	14.7	4.30	1.29	1.04	4.52	6.37	1.03	1.22	0.72	1.66	1.97

*Carnegie special angles. ¶r_a = minimum radius.

†$r_1 = \sqrt{r_x^2 + \left(y_1 + \frac{s}{2}\right)^2}$ ‡$r_2 = \sqrt{r_y^2 + \left(x_1 + \frac{s}{2}\right)^2}$ Note.—$r_x^2 = I_x + A$. $r_y^2 = I_y + A$.

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Fig. 1.

3.—PROPERTIES OF ANGLES (STEEL).

UNEQUAL LEGS.

—Continued.

(Weights at 489.6 lbs. per cubic foot.)

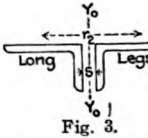

Fig. 3.

Size. Ins.	Thickness. t Ins.	Wt. per Foot. W Lbs.	Area of Sec. A Sq. Ins.	Dist. from b of Angle to c of g. Fig. 1. x_1	y_1	Moment of Inertia I, (Fig. 1) about X—X I_x	Y—Y I_y	Radius of Gyration r. Single Angle, (Fig. 1) about axis X—X r_x	Y—Y r_y	$\alpha-\alpha$ ¶ r_α	2 Angles about axis (Fig. 2) X_0-X_0 For $s=0.5$ † r_1
*4 x3½	9/16	13.3	3.90	1.27	1.02	4.17	5.86	1.03	1.23	0.72	1.64
* "	½	11.9	3.50	1.25	1.00	3.79	5.32	1.04	1.23	0.72	1.63
* "	7/16	10.6	3.09	1.23	0.98	3.40	4.76	1.05	1.24	0.72	1.62
* "	⅜	9.1	2.67	1.21	0.96	2.99	4.18	1.06	1.25	0.73	1.61
* "	5/16	7.7	2.25	1.18	0.93	2.59	3.56	1.07	1.26	0.73	1.60
4 x3	13/16	17.1	5.03	1.44	0.94	3.47	7.34	0.83	1.21	0.64	1.45
"	¾	16.0	4.69	1.42	0.92	3.28	6.93	0.84	1.22	0.64	1.44
"	11/16	14.8	4.34	1.39	0.89	3.08	6.49	0.84	1.22	0.64	1.43
"	⅝	13.6	3.98	1.37	0.87	2.87	6.03	0.85	1.23	0.64	1.41
"	9/16	12.4	3.62	1.35	0.85	2.66	5.55	0.86	1.24	0.64	1.40
"	½	11.1	3.25	1.33	0.83	2.42	5.05	0.86	1.25	0.64	1.39
"	7/16	9.8	2.87	1.30	0.80	2.18	4.52	0.87	1.25	0.64	1.38
"	⅜	8.5	2.48	1.28	0.78	1.92	3.96	0.88	1.26	0.64	1.36
"	5/16	7.2	2.09	1.26	0.76	1.65	3.38	0.89	1.27	0.65	1.35
3½ x3	13/16	15.8	4.62	1.23	0.98	3.33	4.98	0.85	1.04	0.62	1.50
"	¾	14.7	4.31	1.21	0.96	3.15	4.70	0.85	1.04	0.62	1.49
"	11/16	13.6	4.00	1.19	0.94	2.96	4.41	0.86	1.05	0.62	1.48
"	⅝	12.5	3.67	1.17	0.92	2.76	4.11	0.87	1.06	0.62	1.46
"	9/16	11.4	3.34	1.15	0.90	2.55	3.79	0.87	1.07	0.62	1.45
"	½	10.2	3.00	1.13	0.88	2.33	3.45	0.88	1.07	0.62	1.44
"	7/16	9.1	2.65	1.10	0.85	2.09	3.10	0.89	1.08	0.62	1.43
"	⅜	7.9	2.30	1.08	0.83	1.85	2.72	0.90	1.09	0.62	1.41
"	5/16	6.6	1.93	1.06	0.81	1.58	2.33	0.90	1.10	0.63	1.40
3½ x2½	11/16	12.5	3.65	1.27	0.77	1.72	4.13	0.67	1.06	0.53	1.23
"	⅝	11.5	3.36	1.25	0.75	1.61	3.85	0.69	1.07	0.53	1.22
"	9/16	10.4	3.06	1.23	0.73	1.49	3.55	0.70	1.08	0.53	1.20
"	½	9.4	2.75	1.20	0.70	1.36	3.24	0.70	1.09	0.53	1.19
"	7/16	8.3	2.43	1.18	0.68	1.23	2.91	0.71	1.09	0.54	1.17
"	⅜	7.2	2.11	1.16	0.66	1.09	2.56	0.72	1.10	0.54	1.16
"	5/16	6.1	1.78	1.14	0.64	0.94	2.19	0.73	1.11	0.54	1.14
"	¼	4.9	1.44	1.11	0.61	0.78	1.80	0.74	1.12	0.54	1.13
*3¼ x2	9/16	9.0	2.64	1.21	0.59	0.75	2.64	0.53	1.00	0.44	0.99
* "	½	8.1	2.38	1.19	0.57	0.69	2.42	0.54	1.01	0.44	0.98
* "	7/16	7.2	2.11	1.17	0.54	0.62	2.18	0.54	1.02	0.44	0.96
* "	⅜	6.3	1.83	1.15	0.52	0.55	1.92	0.55	1.02	0.44	0.95
* "	5/16	5.3	1.54	1.12	0.50	0.48	1.65	0.56	1.03	0.45	0.93
* "	¼	4.3	1.25	1.09	0.48	0.40	1.36	0.57	1.04	0.45	0.92

*Carnegie special angles. ¶ r_α = minimum radius.

$$\dagger r_1 = \sqrt{r_x^2 + \left(y_1 + \frac{s}{2}\right)^2} \qquad \ddagger r_2 = \sqrt{r_y^2 + \left(x_1 + \frac{s}{2}\right)^2}$$

Note.—$r_x^2 = I_x \div$
$r_y^2 = I_y \div$

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3.—PROPERTIES OF ANGLES (STEEL).
UNEQUAL LEGS.
—Concluded.
(Weight at 489.6 lbs. per cubic foot.)

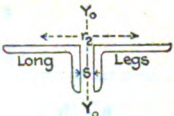

Fig. 3.

Wt. per Foot. W Lbs.	Area of Sec. A Sq. Ins.	Dist. from b of Angle to c of g. Fig. 1. x_1	y_1	Moment of Inertia I, (Fig. 1) about X—X I_x	Y—Y I_y	Radius of Gyration r, Single Angle, (Fig. 1) about axis X—X r_x	Y—Y r_y	α—α ¶ r_α	2 Angles about axis (Fig. 2) X_0-X_0 For s = 0.5 † r_1	(Fig. 3) Y_0-Y_0 For s = 0.5 ‡ r_2
9.5	2.78	1.02	0.77	1.42	2.28	0.72	0.91	0.52	1.25	1.56
8.5	2.50	1.00	0.75	1.30	2.08	0.72	0.91	0.52	1.24	1.55
7.6	2.22	0.98	0.73	1.18	1.88	0.73	0.92	0.52	1.23	1.54
6.6	1.92	0.96	0.71	1.04	1.66	0.74	0.93	0.52	1.21	1.52
5.6	1.62	0.93	0.68	0.90	1.42	0.74	0.94	0.52	1.19	1.51
4.5	1.31	0.91	0.66	0.74	1.17	0.75	0.95	0.53	1.18	1.50
7.7	2.25	1.08	0.58	0.67	1.92	0.55	0.92	0.43	1.00	1.62
6.8	2.00	1.06	0.56	0.61	1.73	0.55	0.93	0.43	0.98	1.60
5.9	1.73	1.04	0.54	0.54	1.53	0.56	0.94	0.43	0.96	1.59
5.0	1.47	1.02	0.52	0.47	1.32	0.57	0.95	0.43	0.95	1.57
4.1	1.19	0.99	0.49	0.39	1.09	0.57	0.95	0.43	0.93	1.56
6.8	2.00	0.88	0.63	0.64	1.14	0.56	0.75	0.42	1.04	1.35
6.1	1.78	0.85	0.60	0.58	1.03	0.57	0.76	0.42	1.03	1.34
5.3	1.55	0.83	0.58	0.51	0.91	0.58	0.77	0.42	1.01	1.32
4.5	1.31	0.81	0.56	0.45	0.79	0.58	0.78	0.42	1.00	1.31
3.7	1.06	0.79	0.54	0.37	0.65	0.59	0.78	0.42	0.98	1.29
2.8	0.81	0.76	0.51	0.29	0.51	0.60	0.79	0.43	0.97	1.28
5.6	1.63	0.86	0.48	0.26	0.75	0.40	0.68	0.39	0.83	1.30
5.0	1.45	0.83	0.46	0.24	0.68	0.41	0.69	0.39	0.82	1.29
4.4	1.27	0.81	0.44	0.21	0.61	0.41	0.69	0.39	0.80	1.27
3.7	1.07	0.79	0.42	0.19	0.53	0.42	0.70	0.40	0.79	1.26
3.0	0.88	0.77	0.39	0.16	0.44	0.42	0.71	0.40	0.77	1.24
2.3	0.67	0.75	0.37	0.12	0.34	0.43	0.72	0.40	0.76	1.23
2.7	0.78	0.69	0.37	0.12	0.37	0.39	0.63	0.30	0.73	1.13
2.1	0.60	0.66	0.35	0.09	0.24	0.40	0.63	0.31	0.72	1.11
1.9	0.53	0.48	0.29	0.04	0.09	0.27	0.41	0.22	0.60	0.84
1.0	0.28	0.44	0.26	0.02	0.05	0.29	0.44	0.22	0.59	0.82

special angles. ¶ r_α = minimum radius.

$+ \left(y_1 + \frac{s}{2}\right)^2$ ‡ $r_2 = \sqrt{ry^2 + \left(x_1 + \frac{s}{2}\right)^2}$ Note.—$rx^2 = Ix \div A$.
$ry^2 = Iy \div A$.

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4.—Properties of Angles (Steel).

Equal Legs.

(Weights at 489.6 lbs. per cubic foot.)

Each Leg. Ins.	Thickness. t Ins.	Wt. per Ft. W Lbs.	Area of Sec. A Sq. Ins.	Dist. from b. of L to c. of g. $x_1 = y_1$	Mom. of Inertia thro' c. of g. ‖ to Flange. $I_x = I_y$	Radius of Gyration. Ordinary. $r_x = r_y$	Radius of Gyration. Least. r_α
8	1⅛	56.9	16.73	2.41	97.97	2.42	1.55
"	1 1/16	54.0	15.87	2.39	93.53	2.43	1.56
"	1	51.0	15.00	2.37	88.98	2.44	1.56
"	15/16	48.1	14.12	2.34	84.33	2.44	1.56
"	⅞	45.0	13.23	2.32	79.58	2.45	1.57
"	13/16	42.0	12.34	2.30	74.71	2.46	1.57
"	¾	38.9	11.44	2.28	69.74	2.47	1.57
"	11/16	35.8	10.53	2.25	64.64	2.48	1.58
"	⅝	32.7	9.61	2.23	59.42	2.49	1.58
"	9/16	29.6	8.68	2.21	54.09	2.50	1.58
"	½	26.4	7.75	2.19	48.63	2.50	1.58
6	1	37.4	11.00	1.86	35.46	1.80	1.16
"	15/16	35.3	10.37	1.84	33.72	1.80	1.16
"	⅞	33.1	9.74	1.82	31.92	1.81	1.17
"	13/16	31.0	9.09	1.80	30.06	1.82	1.17
"	¾	28.7	8.44	1.78	28.15	1.83	1.17
"	11/16	26.5	7.78	1.75	26.19	1.83	1.17
"	⅝	24.2	7.11	1.73	24.16	1.84	1.18
"	9/16	21.9	6.43	1.71	22.07	1.85	1.18
"	½	19.6	5.75	1.68	19.91	1.86	1.18
"	7/16	17.2	5.06	1.66	17.68	1.87	1.19
"	⅜	14.9	4.36	1.64	15.39	1.88	1.19
*5	1	30.6	9.00	1.61	19.64	1.48	0.96
*"	15/16	28.9	8.50	1.59	18.71	1.48	0.96
*"	⅞	27.2	7.99	1.57	17.75	1.49	0.96
*"	13/16	25.4	7.46	1.55	16.77	1.50	0.97
*"	¾	23.6	6.94	1.52	15.74	1.51	0.97
*"	11/16	21.8	6.42	1.50	14.68	1.51	0.97
*"	⅝	20.0	5.86	1.48	13.58	1.52	0.97
*"	9/16	18.1	5.31	1.46	12.44	1.53	0.98
*"	½	16.2	4.75	1.43	11.25	1.54	0.98
*"	7/16	14.3	4.18	1.41	10.02	1.55	0.98
*"	⅜	12.3	3.61	1.39	8.74	1.56	0.99
4	13/16	19.9	5.84	1.29	8.14	1.18	0.77
"	¾	18.5	5.44	1.27	7.67	1.19	0.77
"	11/16	17.1	5.03	1.25	7.17	1.19	0.77
"	⅝	15.7	4.61	1.23	6.66	1.20	0.77
"	9/16	14.3	4.18	1.21	6.12	1.21	0.78
"	½	12.8	3.75	1.18	5.56	1.22	0.78
"	7/16	11.3	3.31	1.16	4.97	1.23	0.78
4	⅜	9.8	2.86	1.14	4.36	1.23	0
"	5/16	8.2	2.40	1.12	3.71	1.24	0
3½	13/16	17.1	5.03	1.17	5.25	1.02	0
"	¾	16.0	4.69	1.15	4.96	1.03	0
"	11/16	14.8	4.34	1.12	4.65	1.04	0
"	⅝	13.6	3.98	1.10	4.33	1.04	0
"	9/16	12.4	3.62	1.08	3.99	1.05	0
"	½	11.1	3.25	1.06	3.64	1.06	0
"	7/16	9.8	2.87	1.04	3.26	1.07	0
"	⅜	8.5	2.48	1.01	2.87	1.07	0
"	5/16	7.2	2.09	0.99	2.45	1.08	0
3	5/8	11.5	3.36	0.98	2.62	0.88	0
"	9/16	10.4	3.06	0.95	2.43	0.89	0
"	½	9.4	2.75	0.93	2.22	0.90	0
"	7/16	8.3	2.43	0.91	1.99	0.91	0
"	⅜	7.2	2.11	0.89	1.76	0.91	0
"	5/16	6.1	1.78	0.87	1.51	0.92	0
"	¼	4.9	1.44	0.84	1.24	0.93	0
*2½	½	8.5	2.50	0.87	1.67	0.82	0
*"	7/16	7.6	2.22	0.85	1.51	0.82	0
*"	⅜	6.6	1.92	0.82	1.33	0.83	0
*"	5/16	5.6	1.62	0.80	1.15	0.84	0
*"	¼	4.5	1.31	0.78	0.93	0.85	0
2½	½	7.7	2.25	0.81	1.23	0.74	0
"	7/16	6.8	2.00	0.78	1.11	0.74	0
"	⅜	5.9	1.73	0.76	0.98	0.75	0
"	5/16	5.0	1.47	0.74	0.85	0.76	0
"	¼	4.1	1.19	0.72	0.70	0.77	0
"	3/16	3.1	0.90	0.69	0.55	0.78	0
*2¼	½	6.8	2.00	0.74	0.87	0.66	0
*"	7/16	6.1	1.78	0.72	0.79	0.67	0
*"	⅜	5.3	1.55	0.70	0.70	0.67	0
*"	5/16	4.5	1.31	0.68	0.61	0.68	0
*"	¼	3.7	1.06	0.66	0.51	0.69	0
*"	3/16	2.8	0.81	0.63	0.39	0.70	0
2	7/16	5.3	1.56	0.66	0.54	0.59	0
"	⅜	4.7	1.36	0.64	0.48	0.59	0

* Carnegie Special Angles. Note.—See Figs. 1, 2 and 3, Table 3; a
foot-notes to same.

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—PROPERTIES OF ANGLES (STEEL).—Concluded.

EQUAL LEGS.

(Weights at 489.6 lbs. per cubic foot.)

Dist. from b. of L to c. of g. $x_1=y_1$	Mom. of Inertia thro' c. of g. ‖ to Flange. $I_t=I_f$	Radius of Gyration. Ordinary. $r_x=r_y$	Radius of Gyration. Least. r_a	Each Leg. Ins.	Thickness. Ins. t	Wt. per Ft. W Lbs.	Area of Sec. A Sq. Ins.	Dist. from b. of L to c. of g. $x_1=y_1$	Mom. of Inertia thro' c. of g. ‖ to Flange. $I_t=I_f$	Radius of Gyration. Ordinary. $r_x=r_y$	Radius of Gyration. Least. r_a
0.61	0.42	0.60	0.39	1¼	5/16	2.4	0.69	0.42	0.09	0.36	0.23
0.59	0.35	0.61	0.39	"	¼	2.0	0.56	0.40	.077	0.37	0.24
0.57	0.28	0.62	0.40	"	3/16	1.5	0.43	0.38	.061	0.38	0.24
				"	⅛	1.1	0.30	0.35	.044	0.38	0.25
0.59	0.35	0.51	0.33								
0.57	0.31	0.51	0.34	1	¼	1.5	0.44	0.34	.037	0.29	0.19
0.55	0.27	0.52	0.34	"	3/16	1.2	0.34	0.32	.030	0.30	0.19
0.53	0.23	0.53	0.34	"	⅛	0.8	0.24	0.30	.022	0.31	0.20
0.51	0.18	0.54	0.35								
				* ¾	3/16	1.0	0.29	0.29	.019	0.26	0.18
0.51	0.19	0.44	0.29	* "	⅛	0.7	0.21	0.26	.014	0.26	0.19
0.49	0.16	0.44	0.29								
0.47	0.14	0.45	0.29	¾	3/16	0.9	0.25	0.26	.012	0.22	0.16
0.44	0.11	0.46	0.29	"	⅛	0.6	0.17	0.23	.009	0.23	0.17
0.42	0.08	0.46	0.30								

Special Angles. Note.—See Figs. 1, 2 and 3, Table 3; also same.

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5.—PROPERTIES OF I-BEAMS (STEEL).
(Weights at 489.6 lbs. per cubic foot.)

Depth of Beam. Inches.	Weight per Foot. Pounds.	Area of Section. Square Inches.	Thickness of Web. Inches.	Width of Flange. Inches.	Mom. of Inertia Neutral Axis Perpendicular to Web at Center. I	Mom. of Inertia Neutral Axis Coincident with Center Line of Web. I'	Radius of Gyration Neutral Axis Perpendicular to Web at Center. r	Radius of Gyration Neutral Axis Coincident
	100.00	29.41	0.754	7.254	2380.3	48.56	9.00	1.
	95.00	27.94	0.692	7.192	2309.6	47.10	9.09	1.
24	90.00	26.47	0.631	7.131	2239.1	45.70	9.20	1.
	85.00	25.00	0.570	7.070	2168.5	44.35	9.31	1.
	80.00	**23.32**	**0.500**	**7.000**	**2087.9**	**42.86**	**9.46**	**1.**
	100.00	29.41	0.884	7.284	1655.8	52.65	7.50	1.
	95.00	27.94	0.810	7.210	1606.8	50.78	7.58	1.
20	90.00	26.47	0.737	7.137	1557.8	48.98	7.67	1.
	85.00	25.00	0.663	7.063	1508.7	47.25	7.77	1.
	80.00	**23.73**	**0.600**	**7.000**	**1466.5**	**45.81**	**7.86**	**1.**
	75.00	22.06	0.649	6.399	1268.9	30.25	7.58	1.
20	70.00	20.59	0.575	6.325	1219.9	29.04	7.70	1.
	65.00	**19.08**	**0.500**	**6.250**	**1169.6**	**27.86**	**7.83**	**1.**
	70.00	20.59	0.719	6.259	921.3	24.62	6.69	1.
18	65.00	19.12	0.637	6.177	881.5	23.47	6.79	1.
	60.00	17.65	0.555	6.095	841.8	22.38	6.91	1.
	55.00	**15.93**	**0.460**	**6.000**	**795.6**	**21.19**	**7.07**	**1.**
	100.00	29.41	1.184	6.774	900.5	50.98	5.53	1.
	95.00	27.94	1.085	6.675	872.9	48.37	5.59	1.
15	90.00	26.47	0.987	6.577	845.4	45.91	5.65	1.
	85.00	25.00	0.889	6.479	817.8	43.57	5.72	1.
	80.00	**23.81**	**0.810**	**6.400**	**795.5**	**41.76**	**5.78**	**1.**
	75.00	22.06	0.882	6.292	691.2	30.68	5.60	1.
15	70.00	20.59	0.784	6.194	663.6	29.00	5.68	1.
	65.00	19.12	0.686	6.096	636.0	27.42	5.77	1.
	60.00	**17.67**	**0.590**	**6.000**	**609.0**	**25.96**	**5.87**	**1.**
	55.00	16.18	0.656	5.746	511.0	17.06	5.62	1.
15	50.00	14.71	0.558	5.648	483.4	16.04	5.73	1.
	45.00	13.24	0.460	5.550	455.8	15.00	5.87	1.
	42.00	**12.48**	**0.410**	**5.500**	**441.7**	**14.62**	**5.95**	**1.**
	55.00	16.18	0.822	5.612	321.0	17.46	4.45	1.
12	50.00	14.71	0.699	5.489	303.3	16.12	4.54	1.
	45.00	13.24	0.576	5.366	285.7	14.89	4.65	1.
	40.00	**11.84**	**0.460**	**5.250**	**268.9**	**13.81**	**4.77**	**1.**
12	35.00	10.29	0.436	5.086	228.3	10.07	4.71	0.
	31.50	**9.26**	**0.350**	**5.000**	**215.8**	**9.50**	**4.83**	**1.**
	40.00	11.76	0.749	5.099	158.7	9.50	3.67	0.
10	35.00	10.29	0.602	4.952	146.4	8.52	3.77	0.

Note.—Weights in heavy type are standard; others are spec

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5.—PROPERTIES OF I-BEAMS (STEEL).—Concluded.
(Weights at 489.6 lbs. per cubic foot.)

Depth of Beam. Inches.	Weight per Foot. Pounds.	Area of Section. Square Inches.	Thickness of Web. Inches.	Width of Flange. Inches.	Mom. of Inertia Neutral Axis Perpendicular to Web at Center. I	Mom. of Inertia Neutral Axis Coincident with Center Line of Web. I'	Radius of Gyration Neutral Axis Perpendicular to Web at Center. r	Radius of Gyration Neutral Axis Coincident with Center Line of Web. r'	Distance Center to Center Required to make Radii of Gyration equal. D
10	**30.00**	8.82	0.455	4.805	134.2	7.65	3.90	0.93	7.57
	25.00	7.37	0.310	4.660	122.1	6.89	4.07	0.97	7.91
9	**35.00**	10.29	0.732	4.772	111.8	7.31	3.29	0.84	6.36
	30.00	8.82	0.569	4.609	101.9	6.42	3.40	0.85	7.58
	25.00	7.35	0.406	4.446	91.9	5.65	3.54	0.88	6.86
	21.00	6.31	0.290	4.330	84.9	5.16	3.67	0.90	7.12
8	**25.50**	7.50	0.541	4.271	68.4	4.75	3.02	0.80	5.82
	23.00	6.76	0.449	4.179	64.5	4.39	3.09	0.81	5.96
	20.50	6.03	0.357	4.087	60.6	4.07	3.17	0.82	6.12
	18.00	5.33	0.270	4.000	56.9	3.78	3.27	0.84	6.32
7	**20.00**	5.88	0.458	3.868	42.2	3.24	2.68	0.74	5.15
	17.50	5.15	0.353	3.763	39.2	2.94	2.76	0.76	5.31
	15.00	4.42	0.250	3.660	36.2	2.67	2.86	0.78	5.50
6	**17.25**	5.07	0.475	3.575	26.2	2.36	2.27	0.68	4.33
	14.75	4.34	0.352	3.452	24.0	2.09	2.35	0.69	4.49
	12.25	3.61	0.230	3.330	21.8	1.85	2.46	0.72	4.70
5	**14.75**	4.34	0.504	3.294	15.2	1.70	1.87	0.63	.....
	12.25	3.60	0.357	3.147	13.6	1.45	1.94	0.63	.....
	9.75	2.87	0.210	3.000	12.1	1.23	2.05	0.65	.....
4	**10.50**	3.09	0.410	2.880	7.1	1.01	1.52	0.57	.....
	9.50	2.79	0.337	2.807	6.7	0.93	1.55	0.58	.....
	8.50	2.50	0.263	2.733	6.4	0.85	1.59	0.58	.....
	7.50	2.21	0.190	2.660	6.0	0.77	1.64	0.59	.....
3	**7.50**	2.21	0.361	2.521	2.9	0.60	1.15	0.52	.....
	6.50	1.91	0.263	2.423	2.7	0.53	1.19	0.52	.....
	5.50	1.63	0.170	2.330	2.5	0.46	1.23	0.53	.....

Note.—Weights in heavy type are standard; others are special.

For Bethlehem girder (single I) beams and Bethlehem special I-beams, see Tables 8 and 9, Sec. 31, pages 583 and 584.

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6.—Properties of Channels (Steel).

(Weights at 489.6 lbs. per cubic foot.)

Depth of Channel. Inches.	Weight per Foot. Pounds.	Area of Section. Square Inches.	Thickness of Web. Inches.	Width of Flange. Inches.	Mom. of Inertia Neutral Axis Perpendicular to Web at Center. I	Mom. of Inertia Neutral Axis Parallel with Center Line of Web. I′	Radius of Gyration Neutral Axis Perpendicular to Web at Center. r	Radius of Gyration Neutral Axis Parallel with Center Line of Web. r′	Distance Required to make Radii of Gyration equal.]D[	Distance of Center of
	55.00	16.18	0.818	3.818	430.2	12.19	5.16	.868	8.53	
	50.00	14.71	0.720	3.720	402.7	11.22	5.23	.873	8.71	
15	45.00	13.24	0.622	3.622	375.1	10.29	5.32	.882	8.92	
	40.00	11.76	0.524	3.524	347.5	9.39	5.43	.893	9.15	
	35.00	10.29	0.426	3.426	320.0	8.48	5.58	.908	9.43	
	33.00	**9.90**	**0.400**	**3.400**	**312.6**	**8.23**	**5.62**	**.912**	**9.50**	
	40.00	11.76	0.758	3.418	197.0	6.63	4.09	.751	6.60	
	35.00	10.29	0.636	3.296	179.3	5.90	4.17	.757	6.81	
12	30.00	8.82	0.513	3.173	161.7	5.21	4.28	.768	7.07	
	25.00	7.35	0.390	3.050	144.0	4.53	4.43	.785	7.36	
	20.50	**6.03**	**0.280**	**2.940**	**128.1**	**3.91**	**4.61**	**.805**	**7.67**	
	35.00	10.29	0.823	3.183	115.5	4.66	3.35	.672	5.17	
	30.00	8.82	0.676	3.036	103.2	3.90	3.42	.672	5.40	
10	25.00	7.35	0.529	2.889	91.0	3.40	3.52	.680	5.67	
	20.00	5.88	0.382	2.742	78.7	2.85	3.66	.696	5.97	
	15.00	**4.46**	**0.240**	**2.600**	**66.9**	**2.30**	**3.87**	**.718**	**6.33**	
	25.00	7.35	0.615	2.815	70.7	2.98	3.10	.637	4.84	
9	20.00	5.88	0.452	2.652	60.8	2.45	3.21	.646	5.12	
	15.00	4.41	0.288	2.488	50.9	1.95	3.40	.665	5.49	
	13.25	**3.89**	**0.230**	**2.430**	**47.3**	**1.77**	**3.49**	**.674**	**5.63**	
	21.25	6.25	0.582	2.622	47.8	2.25	2.77	.600	4.23	
	18.75	5.51	0.490	2.530	43.8	2.01	2.82	.603	4.38	
8	16.25	4.78	0.399	2.439	39.9	1.78	2.89	.610	4.54	
	13.75	4.04	0.307	2.347	36.0	1.55	2.98	.619	4.72	
	11.25	**3.35**	**0.220**	**2.260**	**32.3**	**1.33**	**3.11**	**.630**	**4.94**	
	19.75	5.81	0.633	2.513	33.2	1.85	2.39	.565	3.48	
	17.25	5.07	0.528	2.408	30.2	1.62	2.44	.564	3.64	
7	14.75	4.34	0.423	2.303	27.2	1.40	2.50	.568	3.80	
	12.25	3.60	0.318	2.198	24.2	1.19	2.59	.575	3.99	
	9.75	**2.85**	**0.210**	**2.090**	**21.1**	**0.98**	**2.72**	**.586**	**4.22**	
	15.50	4.56	0.563	2.283	19.5	1.28	2.07	.529	2.91	
6	13.00	3.82	0.440	2.160	17.3	1.07	2.13	.529	3.09	
	10.50	3.09	0.318	2.038	15.1	0.88	2.21	.534	3.28	
	8.00	**2.38**	**0.200**	**1.920**	**13.0**	**0.70**	**2.34**	**.542**	**3.52**	
	11.50	3.38	0.477	2.037	10.4	0.82	1.75	.493	2.34	
5	9.00	2.65	0.330	1.890	8.9	0.64	1.83	.493	2.56	
	6.50	**1.95**	**0.190**	**1.750**	**7.4**	**0.48**	**1.95**	**.498**	**2.79**	
	7.25	2.13	0.325	1.725	4.6	0.44	1.46	.455	1.85	
4	6.25	1.84	0.252	1.652	4.2	0.38	1.51	.454	1.96	
	5.25	**1.55**	**0.180**	**1.580**	**3.8**	**0.32**	**1.56**	**.453**	**2.06**	
	6.00	1.76	0.362	1.602	2.1	0.31	1.08	.421	1.07	
3	5.00	1.47	0.264	1.504	1.8	0.25	1.12	.415	1.19	
	4.00	**1.19**	**0.170**	**1.410**	**1.6**	**0.20**	**1.17**	**.409**	**1.31**	

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7.—PROPERTIES OF Z-BARS (STEEL).
(Weights at 489.6 lbs. per cubic foot.)

Depth of Web. Inches.	Width of Flange. Inches.	Thickness of Metal. Inches.	Weight per Foot. Pounds.	Area of Section. Square Inches.	Moment of Inertia. I		Radii of Gyration. r		
					Neutral Axis Through Center of Gravity Perpendicular to Web.	Neutral Axis Through Center of Gravity Coincident with Web.	Neutral Axis Through Center of Gravity Perpendicular to Web.	Neutral Axis Through Center of Gravity Coincident with Web.	Least Radius, Neutral Axis Diagonal.
6	3½	⅜	15.6	4.59	25.32	9.11	2.35	1.41	0.83
6 1/16	3 5/16	7/16	18.3	5.39	29.80	10.95	2.35	1.43	0.84
6⅛	3⅜	½	21.0	6.19	34.36	12.87	2.36	1.44	0.84
6	3½	5/16	22.7	6.68	34.64	12.59	2.28	1.37	0.81
6 1/16	3 5/16	⅝	25.4	7.46	38.86	14.42	2.28	1.39	0.82
6⅛	3⅜	11/16	28.0	8.25	43.18	16.34	2.29	1.41	0.84
6	3½	¾	29.3	8.63	42.12	15.44	2.21	1.34	0.81
6 1/16	3 5/16	13/16	31.9	9.40	46.13	17.27	2.22	1.36	0.82
6⅛	3⅜	⅞	34.6	10.17	50.22	19.18	2.22	1.37	0.83
5	3¼	5/16	11.6	3.40	13.36	6.18	1.98	1.35	0.75
5 1/16	3 3/16	⅜	13.9	4.10	16.18	7.65	1.99	1.37	0.76
5⅛	3⅛	7/16	16.4	4.81	19.07	9.20	1.99	1.38	0.77
5	3¼	½	17.9	5.25	19.19	9.05	1.91	1.31	0.74
5 1/16	3 3/16	9/16	20.2	5.94	21.83	10.51	1.91	1.33	0.75
5⅛	3⅛	⅝	22.6	6.64	24.53	12.06	1.92	1.35	0.76
5	3¼	11/16	23.7	6.96	23.68	11.37	1.84	1.28	0.73
5 1/16	3 3/16	¾	26.0	7.64	26.16	12.83	1.85	1.30	0.75
5⅛	3⅛	13/16	28.3	8.33	28.70	14.36	1.86	1.31	0.76
4	3 1/16	¼	8.2	2.41	6.28	4.23	1.62	1.33	0.67
4 1/16	3 1/16	5/16	10.3	3.03	7.94	5.46	1.62	1.34	0.68
4⅛	3 1/16	⅜	12.4	3.66	9.63	6.77	1.62	1.36	0.69
4	3 1/16	7/16	13.8	4.05	9.66	6.73	1.55	1.29	0.66
4 1/16	3 1/16	½	15.8	4.66	11.18	7.96	1.55	1.31	0.67
4⅛	3 1/16	9/16	17.9	5.27	12.74	9.26	1.55	1.33	0.69
4	3 1/16	⅝	18.9	5.55	12.11	8.73	1.48	1.25	0.66
4 1/16	3 1/16	11/16	20.9	6.14	13.52	9.95	1.48	1.27	0.67
4⅛	3 1/16	¾	23.0	6.75	14.97	11.24	1.49	1.29	0.69
3	2 11/16	¼	6.7	1.97	2.87	2.81	1.21	1.19	0.55
3 1/16	2¾	5/16	8.4	2.48	3.64	3.64	1.21	1.21	0.56
3	2 11/16	3/16	9.7	2.86	3.85	3.92	1.16	1.17	0.55
3 1/16	2¾	7/16	11.4	3.36	4.57	4.75	1.17	1.19	0.56
3	2 11/16	½	12.5	3.69	4.59	4.85	1.12	1.15	0.55
3 1/16	2¾	9/16	14.2	4.18	5.26	5.70	1.12	1.17	0.56

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8.—PROPERTIES OF CARNEGIE T-SHAPES (STEEL).
(Weights at 489.6 lbs. per cubic foot.)

Size, Flange by Stem. Inches.	Weight per Foot Pounds.	Area of Section. Square Inches.	Distance of Center of Gravity from Outside of Flange. Inches.	Moment of Inertia, Neutral Axis through Center of Gravity Parallel to Flange. I	Radius of Gyration, Neutral Axis as before. r	Moment of Inertia, Neutral Axis through Center of Gravity Coincident with Center Line of Stem. I′	Radius of Gyration, Neutral Axis as before. r′
5 x3	13.6	3.99	0.75	2.6	0.82	5.6	1.19
5 x2½	11.0	3.24	0.65	1.6	0.71	4.3	1.16
4½x3½	15.9	4.65	1.11	5.1	1.04	3.7	0.90
4½x3	8.6	2.55	0.73	1.8	0.87	2.6	1.03
4½x3	10.0	3.00	0.75	2.1	0.86	3.1	1.04
4½x2½	8.0	2.40	0.58	1.1	0.69	2.6	1.07
4½x2½	9.3	2.79	0.60	1.2	0.68	3.1	1.08
4 x5	15.7	4.56	1.56	10.7	1.54	2.8	0.79
4 x5	12.3	3.54	1.51	8.5	1.56	2.1	0.78
4 x4½	14.8	4.29	1.37	8.0	1.37	2.8	0.81
4 x4½	11.6	3.36	1.31	6.3	1.38	2.1	0.80
4 x4	13.9	4.02	1.18	5.7	1.20	2.8	0.84
4 x4	10.9	3.21	1.15	4.7	1.23	2.2	0.84
4 x3	9.3	2.73	0.78	2.0	0.86	2.1	0.88
4 x2½	8.7	2.52	0.63	1.2	0.69	2.1	0.92
4 x2½	7.4	2.16	0.60	1.0	0.70	1.8	0.91
4 x2	7.9	2.31	0.48	0.6	0.52	2.1	0.96
4 x2	6.7	1.95	0.51	0.54	0.51	1.8	0.95
3½x4	12.8	3.75	1.25	5.5	1.21	1.89	0.72
3½x4	10.0	2.91	1.19	4.3	1.22	1.42	0.70
3½x3½	11.9	3.45	1.06	3.7	1.04	1.89	0.74
3½x3½	9.3	2.70	1.01	3.0	1.05	1.42	0.73
3½x3	11.0	3.21	0.88	2.4	0.87	1.88	0.77
3½x3	8.7	2.49	0.83	1.9	0.88	1.41	0.75
3½x3	7.7	2.28	0.78	1.6	0.89	1.18	0.76
3 x4	11.9	4.38	1.32	5.2	1.23	1.21	0.59
3 x4	10.6	3.12	1.32	4.8	1.25	1.09	0.60
3 x4	9.3	2.73	1.29	4.3	1.26	0.93	0.59
3 x3½	11.0	3.21	1.12	3.5	1.06	1.20	0.62
3 x3½	9.8	2.88	1.11	3.3	1.08	1.31	0.68
3 x3½	8.6	2.49	1.09	2.9	1.09	0.93	0.61

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IES OF CARNEGIE T-SHAPES (STEEL).—Concluded.
Weights at 489.6 lbs. per cubic foot.)

rea of ction. uare ches.	Distance of Center of Gravity from Outside of Flange. Inches.	Moment of Inertia, Neutral Axis through Center of Gravity Parallel to Flange. I	Radius of Gyration. Neutral Axis as before. r	Moment of Inertia, Neutral Axis through Center of Gravity Coincident with Center Line of Stem. I′	Radius of Gyration, Neutral Axis as before. r′
.94	0.93	2.3	0.88	1.20	0.64
.67	0.92	2.1	0.90	1.08	0.64
.28	0.88	1.8	0.90	0.90	0.63
.95	0.86	1.6	0.90	0.75	0.62
.10	0.71	1.1	0.72	0.89	0.66
.80	0.68	0.94	0.73	0.75	0.65
.16	0.53	1.1	0.71	0.62	0.54
.10	0.97	1.8	0.92	0.54	0.51
.80	0.92	1.6	0.94	0.44	0.51
.98	0.87	1.4	0.84	0.66	0.58
.71	0.83	1.2	0.83	0.44	0.51
.89	0.76	1.0	0.74	0.52	0.53
.62	0.74	0.87	0.74	0.44	0.52
.84	0.29	0.094	0.31	0.29	0.58
.44	0.69	0.66	0.68	0.33	0.48
.20	0.66	0.51	0.67	0.25	0.47
.26	0.63	0.45	0.60	0.23	0.43
.08	0.59	0.36	0.60	0.18	0.42
.90	0.42	0.16	0.42	0.18	0.45
.90	0.54	0.23	0.51	0.12	0.37
.75	0.42	0.15	0.49	0.08	0.34
.54	0.44	0.11	0.45	0.06	0.31
.60	0.40	0.08	0.36	0.05	0.27
.45	0.38	0.06	0.37	0.03	0.26
.36	0.32	0.03	0.29	0.02	0.21
.26	0.29	0.02	0.29	0.01	0.21

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9.—* STANDARD STEEL RAIL SECTIONS.
(Cambria Steel Co.).

Weight per Yard.	Area.	b	d	w	t	y_1	Axis X–X	
							Moment of Inertia	Sec Mod
Pounds.	Sq. Ins.	Inches	Inches	Inches	Inch	Inches	I_x	
8	0.78	1½	1½	11/16	5/32	0.75	0.23	0.
12	1.18	1⅞	1⅞	1 1/16	3/16	0.92	0.55	0.
16	1.57	2¼	2¼	1¼	15/64	1.1	1.1	0.
20	2.00	2½	2½	1⅜	17/64	1.2	1.7	1.
25	2.5	2¾	2¾	1½	19/64	1.3	2.6	1.
30	2.9	3	3	1⅝	21/64	1.4	3.6	2.
35	3.4	3¼	3¼	1¾	23/64	1.6	4.9	2.
40	3.9	3½	3½	1⅞	25/64	1.7	6.6	3.
45	4.4	3 11/16	3 11/16	2	27/64	1.8	8.1	4.
50	4.9	3⅞	3⅞	2⅛	7/16	1.9	9.8	4.
55	5.4	4 1/16	4 1/16	2¼	15/32	2.0	12.2	5.
60	5.9	4¼	4¼	2⅜	31/64	2.1	14.7	6.
65	6.4	4 7/16	4 7/16	2 13/32	½	2.2	17.0	7.
70	6.9	4⅝	4⅝	2 7/16	33/64	2.2	20.0	8.
75	7.4	4 13/16	4 13/16	2 15/32	17/32	2.3	23.0	9.
80	7.8	5	5	2½	35/64	2.4	26.7	10.
85	8.3	5 3/16	5 3/16	2 9/16	9/16	2.5	30.5	11.
90	8.8	5⅜	5⅜	2⅝	9/16	2.6	34.4	12.
95	9.3	5 9/16	5 9/16	2 11/16	9/16	2.7	38.6	13.
100	9.8	5¾	5¾	2¾	9/16	2.8	43.4	14.
150	14.7	6	6	4¼	1	3.0	69.3	23.

* Sections at 40 to 100 lbs. per yard are Am. Soc. C. E. Standard.
Note.—See page 541, for reference to Tables in Sec. 59.

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For Table of
BETHLEHEM H-SHAPES (STEEL)
See Table 14, Sec. 32, Columns, page 608.

EXCERPTS AND REFERENCES.

th of Corrugated Sheeting (By G. H. Blakely. Eng. News, —Discussion of formulas.

READERS' MEMORANDA.

ng skeleton arrangement is for the use of the reader in making ables and items of special interest which may be found in her works and to which reference may be convenient.

pecifications for Reinforced-Concrete Bars is printed in a published by the Steel Manufacturers' Association and may e upon application.

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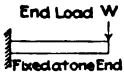
31.—PROPERTIES AND TABLES OF BEAMS AND GIRDERS.

1.—Working Stress, Load, Moment, Slope and Deflection of Beams.

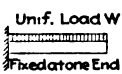
Notation.

f = outer fiber stress in lbs. per sq. in.;
W = total load on beam, in lbs.;
M'' = moment in in.-lbs. } Resisting moment
M' = moment in ft.-lbs. } = bending moment;
i = tangent of angle of maximum slope of beam with the horizontal;
v = maximum deflection of beam, in ins.;
y = distance from neutral axis to extreme fiber, in ins.;
E = modulus of elasticity of material;
I = moment of inertia of section;
L = length of beam, in feet;
l = length of beam, in inches;
b = breadth of rectangular beam, in ins.;
d = depth of rectangular beam, in ins.

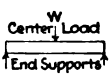
Case A.



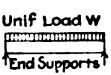
Case B.



Case C.



Case D.



(1)	(2)	(3)	(4)	(5)
Case.	Working Load W, in lbs.	Max. Moment M'', in.-lbs.; M', ft.-lbs. $M' = M'' \div 12$.	Maximum Slope i = tang of ang.	Maximum Deflection v, in inches.
a.—Beam of any Uniform Section; l = length of beam, in Inches = 12 L.				
Aa	$\frac{fI}{ly} = \frac{3\,EIv}{l^3}$	$M'' = \frac{Wl}{2} = \frac{fI}{y}$	$\frac{Wl^2}{2\,EI} = \frac{fl}{2\,Ey}$	$\frac{Wl^3}{3\,EI} = \frac{fI}{ly} \cdot \frac{l^3}{3\,EI} = \frac{fl^2}{3\,Ey}$
Ba	$\frac{2\,fI}{ly} = \frac{8\,EIv}{l^3}$	$M'' = \frac{Wl}{2} = \frac{fI}{y}$	$\frac{Wl^2}{6\,EI} = \frac{fl}{3\,Ey}$	$\frac{Wl^3}{8\,EI} = \frac{2\,fI}{ly} \cdot \frac{l^3}{8\,EI} = \frac{fl^2}{4\,Ey}$
Ca	$\frac{4\,fI}{ly} = \frac{48\,EIv}{l^3}$	$M'' = \frac{Wl}{4} = \frac{fI}{y}$	$\frac{Wl^2}{16\,EI} = \frac{fl}{4\,Ey}$	$\frac{Wl^3}{48\,EI} = \frac{4\,fI}{ly} \cdot \frac{l^3}{48\,EI} = \frac{fl^2}{12\,Ey}$
Da	$\frac{8\,fI}{ly} = \frac{384\,EIv}{l^3}$	$M'' = \frac{Wl}{8} = \frac{fI}{y}$	$\frac{Wl^2}{24\,EI} = \frac{fl}{8\,Ey}$	$\frac{5\,Wl^3}{384\,EI} = \frac{8\,fI}{ly} \cdot \frac{5\,l^3}{384\,EI} = \frac{5\,fl^2}{48\,Ey}$

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Db	$\frac{2\,fI}{3\,Ly} = \frac{2\,E\,I\,v}{45\,L^3}$	$M'' = \frac{3WL}{2} = \frac{fI}{y}$	$\frac{6\,WL^2}{E\,I} = \frac{4\,fL}{Ey}$	$\frac{45\,WL^3}{2\,E\,I} = \frac{2\,fI}{3\,Ly} \cdot \frac{45\,L^3}{2\,E\,I} = \frac{15\,fL^2}{Ey}$

c.—Rectangular Beam; l = length in Inches = 12L.

Ac	$\frac{fbd^2}{6\,l} = \frac{Ebd^3v}{4\,l^3}$	$M'' = Wl = \frac{fbd^2}{6}$	$\frac{6\,Wl^2}{Ebd^3} = \frac{fl}{Ed}$	$\frac{4\,Wl^3}{Ebd^3} = \frac{fbd^2}{6\,l} \cdot \frac{4\,l^3}{Ebd^3} = \frac{2\,fl^2}{3\,Ed}$
Bc	$\frac{fbd^2}{3\,l} = \frac{2\,Ebd^3v}{3\,l^3}$	$M'' = \frac{Wl}{2} = \frac{fbd^2}{6}$	$\frac{2\,Wl^2}{Ebd^3} = \frac{2\,fl}{3\,Ed}$	$\frac{3\,Wl^3}{2\,Ebd^3} = \frac{fbd^2}{3\,l} \cdot \frac{3\,l^3}{2\,Ebd^3} = \frac{fl^2}{2\,Ed}$
Cc	$\frac{2\,fbd^2}{3\,l} = \frac{4\,Ebd^3v}{l^3}$	$M'' = \frac{Wl}{4} = \frac{fbd^2}{6}$	$\frac{3\,Wl^2}{4\,Ebd^3} = \frac{fl}{2\,Ed}$	$\frac{Wl^3}{4\,Ebd^3} = \frac{2\,fbd^2}{3\,l} \cdot \frac{l^3}{4\,Ebd^3} = \frac{fl^2}{6\,Ed}$
Dc	$\frac{4\,fbd^2}{3\,l} = \frac{32\,Ebd^3v}{5\,l^3}$	$M'' = \frac{Wl}{8} = \frac{fbd^2}{6}$	$\frac{Wl^2}{2\,Ebd^3} = \frac{2\,fl}{3\,Ed}$	$\frac{5\,Wl^3}{32\,Ebd^3} = \frac{4\,fbd^2}{3\,l} \cdot \frac{5\,l^3}{64\,Ebd^3} = \frac{5\,fl^2}{24\,Ed}$

d.—Rectangular Beam; L = length in Feet = $l \div 12$.

Ad	$\frac{fbd^2}{72\,L} = \frac{Ebd^3v}{6912\,L^3}$	$M'' = 12\,WL = \frac{fbd^2}{6}$	$\frac{864\,WL^2}{Ebd^3} = \frac{12\,fL}{Ed}$	$\frac{6912\,WL^3}{Ebd^3} = \frac{fbd^2}{72\,L} \cdot \frac{6912\,L^3}{Ebd^3} = \frac{96\,fL^2}{Ed}$
Bd	$\frac{fbd^2}{36\,L} = \frac{Ebd^3v}{2592\,L^3}$	$M'' = 6\,WL = \frac{fbd^2}{6}$	$\frac{288\,WL^2}{Ebd^3} = \frac{8\,fL}{Ed}$	$\frac{2592\,WL^3}{Ebd^3} = \frac{fbd^2}{36\,L} \cdot \frac{2592\,L^3}{Ebd^3} = \frac{72\,fL^2}{Ed}$
Cd	$\frac{fbd^2}{18\,L} = \frac{Ebd^3v}{432\,L^3}$	$M'' = 3\,WL = \frac{fbd^2}{6}$	$\frac{108\,WL^2}{Ebd^3} = \frac{6\,fL}{Ed}$	$\frac{432\,WL^3}{Ebd^3} = \frac{fbd^2}{18\,L} \cdot \frac{432\,L^3}{Ebd^3} = \frac{24\,fL^2}{Ed}$
Dd	$\frac{fbd^2}{9\,L} = \frac{Ebd^3v}{270\,L^3}$	$M'' = \frac{3\,WL}{2} = \frac{fbd^2}{6}$	$\frac{72\,WL^2}{Ebd^3} = \frac{8\,fL}{Ed}$	$\frac{270\,WL^3}{Ebd^3} = \frac{fbd^2}{9\,L} \cdot \frac{270\,L^3}{Ebd^3} = \frac{30\,fL^2}{Ed}$

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Examples in the Use of Table 1.

Ex. 1. (*Cases A and B combined*).—A 15-in. steel I-beam weighing 60 per ft., and 10 ft. long, is fixed at one end and loaded with 10000 lb the other. Find the max. fiber stress f, max. moment M'', max. slope i, max. deflection v?

Solution.—From "Properties of I-Beams," Table 5, page 554, $I =$

Also, $E = 29\ 000\ 000$; $y = \frac{15}{2}$. Then, from preceding table,

	f	M''	i	v
Case Ab ($W = 10000$)	(3) $f = \frac{12\,WLy}{I} = 14778$	(3) $M'' = 12\,WL = 1\,200\,000$	(4) $i = \frac{72\,WL^2}{EI} = .00408$	(5) $v = \frac{576}{I} = .3$
Case Bb ($W = 600$)	(3) $f = \frac{6\,WLy}{I} = 443$	(3) $M'' = 6\,WL = 36\,000$	(4) $i = \frac{24\,WL^2}{EI} = .00008$	(5) $v = \frac{216}{I} = .0$
Totals	$f = 15221$ lbs. per sq. in.	$M'' = 1\,236\,000$ in.-lbs.	$i = .00416$	$v = 0.3$

Ex. 2. (*Cases C and D combined*).—A wooden beam 10″ x 16″, and span, is loaded at the middle so as to produce a *total* max. outer fiber s $f = 1000$ lbs. per sq. in. Assuming the weight of the beam at 4 lbs. p B. M., find the superimposed load, max. moment M'', max. slope i, max. deflection v?

Solution.—Weight of beam = 800 lbs. Assume $E = 1\ 400\ 000$.

		M''	i	v
Case Dd ($W = 800$)	(3) $f = \frac{9\,WL}{bd^2} = 42$	(3) $M'' = \frac{3\,WL}{2} = 18000$	(4) $i = \frac{8\,fL}{Ed} = .00022$	(5) $v = \frac{30}{E} = .01$
Case Cd ($f = 1000-42$)	(3) $W = \frac{fbd^2}{18L} = 9083$	(3) $M'' = 3\,WL = 408735$	(4) $i = \frac{6\,fL}{Ed} = .00385$	(5) $v = \frac{24}{E} = .23$
Totals	.....	$M'' = 426735$ in.-lbs.	$i = .00407$	$v = 0.2$

The weight of beam produces a max. fiber stress of 42 lbs. per sq and this deducted from 1000 = 958, produced by the center load of 908 Note that twice the center load or 18166 lbs. if uniformly distributed (ing about 1211 lbs. per lin. ft.) would produce the same fiber stress o lbs. per sq. in.

Ex. 3. (*Span and Deflection for Plastered Ceiling*).—The max. allo deflection v of beams for plastered ceiling is $\frac{1}{30}$ of an in. per ft. of spa $v = \frac{L}{30} = \frac{l}{360}$. What general formulas may be used for wooden b ($E = 1\ 000\ 000$) and for steel girders ($E = 29\ 000\ 000$) to represent limiting relation? (See Note (*) for Wooden Beams on page 565.)

Solution (from Table 1).

Loading.	*Wooden Beams.**	*Steel Girders.*
I. Uniform load W	Case Dd, (2): $W = \frac{10000\,bd^3}{81\,L^2}$	Case Db, (2): $W = \frac{1\,160}{27}$
II. "	" (5): $f = \frac{10000\,d}{9\,L}$	" (5): $f = \frac{580}{9}$
III. Center load W	Case Cd, (2): $W = \frac{6250\,bd^3}{81\,L^2}$	Case Cb, (2): $W = \frac{725}{27}$
IV. "	" (5): $f = \frac{12500\,d}{9\,L}$	" (5): $f = \frac{725}{9}$

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Furthermore, if the allowable outer fiber stress f for timber is 1000 lbs. per sq. in., and f for steel is 15000, we have, from II and IV:—

Loading.	*Wooden Beams.**	*Steel Girders.*
V. Uniform load W	$L=\frac{10}{9}d=1.55\sqrt{5}\,d$	$L=\frac{116}{27}y=4.296y$
VI. Center load W	$L=\frac{25}{18}d=1.94\sqrt{4}\,d$	$L=\frac{145}{27}y=5.370y$

which are the limiting spans for plastered ceiling, and which enable us to design these spans economically.

Longitudinal Shear in Beams.—While *long* beams, even of sufficient strength to resist bending, are often limited by the deflection, as previously noted, *short* beams may fail by horizontal (longitudinal) shear. The general formula for longitudinal shear is

$$H=\frac{VQ}{I}=\frac{VA_1y_1}{I}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

In which H = the longitudinal shearing force in lbs. per unit (lin. in.) length of beam;
V = the total vertical shear in lbs. at section considered;
Q = the statical moment in in.-lbs. of the area A_1 above the plane of shear $=A_1y_1$, in which y_1 = distance from neutral axis to cen. of grav. of A_1;
I = the moment of inertia of beam section.

For a rectangular beam, the longitudinal shear at the neutral axis would be, if b = breadth and d = depth in inches:

$$H=V\,\frac{bd}{2}\cdot\frac{d}{4}\div\frac{bd^3}{12}=\frac{3}{2}\cdot\frac{V}{d}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

and the intensity of shear per square inch would be

$$\frac{H}{b}=\frac{3}{2}\cdot\frac{V}{bd}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Note that the vertical shear V at any point of a beam is equal to the differential coefficient of the bending moment at that point; thus,

$$V=\frac{dM}{dx}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

x being the distance from left-hand end of beam to section considered.

Problem:—A wooden beam 12 x 12 ins., weighing 600 lbs. ($=w$), supports a uniform load of W lbs. What will be the maximum value of W, so that the maximum intensity of shear, $H \div b$, (at the neutral axis,) shall not exceed 50 lbs. per sq. in.?

Solution:—The maximum vertical shear (at the end of the beam) equals $V=\frac{W+w}{2}$. Substituting this value of V in equation (3), we have,

$$\text{Intensity of shear}=\frac{H}{b}=\frac{3}{2}\cdot\frac{W+w}{2bd}=50$$

Hence, by substitution, $W=9600-600=9000$ lbs. *Ans.* (See also, Example 1, bottom of page 567.)

Note that the length of span, in the above problem, is not a factor in longitudinal shear.

---

* Note that W and f are directly proportional to the modulus of elasticity, E, of the material, which, in the present instance, is assumed at 1 000 000. For any other modulus, as E_1, multiply above values by $\frac{E_1}{1\,000\,000}$. See Sec. 28, Strength of Materials, Table 7, page 495.

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2.—Uniformly Distributed Loads W in Pounds On Rectangular Beams 1 Inch Wide Producing Extreme Fiber Stress $f = 1000$ Lbs. per Sq.

$$\left(\text{By Formula, Case } Dd\ (2): W = \frac{fbd^2}{9L} = \frac{1000d^2}{9L}\right).$$

For any other fiber stress, as f_1, multiply values in table by $\frac{}{1}$

[Total load in Pounds, including weight of beam.]

Span. Feet.	Depth of Beam.										
	1″	2″	3″	4″	5″	6″	7″	8″	9″	10″	
1	111	444	1000	1778	2778	4000	.....	.....	.....	.....	...
2	56	222	500	889	1389	2000	2722	3556	.....	.....	...
3	37	148	333	593	926	1333	1815	2370	3000	3704	...
4	28	111	250	444	694	1000	1361	1778	2250	2778	3
5	22	89	200	356	556	800	1089	1422	1800	2222	2
6	19	74	167	296	463	667	907	1185	1500	1852	2
7	16	64	143	254	397	571	778	1016	1286	1587	1
8	14	56	125	222	347	500	681	889	1125	1389	1
9	12	49	111	198	309	444	605	790	1000	1235	1
10	11	44	100	178	278	400	544	711	900	1111	1
11	10	40	91	162	253	364	495	646	818	1010	1
12	.....	37	83	148	231	333	454	593	750	926	1
13	.....	34	77	137	214	308	419	547	692	855	1
14	.....	32	71	127	198	286	389	508	643	794	
15	.....	30	67	119	185	267	363	474	600	741	
16	.....	28	62	111	174	250	340	444	563	694	
17	.....	26	59	105	163	235	320	418	529	654	
18	.....	25	56	99	154	222	302	395	500	617	
19	.....	23	53	94	146	211	287	374	474	585	
20	.....	.....	50	89	139	200	272	356	450	556	
21	.....	.....	48	85	132	190	259	339	429	529	
22	.....	.....	45	81	126	182	247	323	409	505	
23	.....	.....	43	77	121	174	237	309	391	483	
24	.....	.....	42	74	116	162	227	296	375	463	

Span. Feet.	Depth of Beam.										
	13″	14″	15″	16″	17″	18″	19″	20″	21″	22″	
8	2347	2722	3125	3556	4014	4500	5014	5556	6125	6722	7
9	2086	2420	2778	3160	3568	4000	4457	4938	5444	5975	6
10	1878	2178	2500	2844	3211	3600	4011	4444	4900	5378	5
11	1707	1980	2273	2586	2919	3273	3646	4040	4455	4889	5
12	1565	1815	2083	2370	2676	3000	3343	3704	4083	4481	4
13	1444	1675	1923	2188	2470	2769	3085	3419	3769	4137	4
14	1341	1556	1786	2032	2294	2571	2865	3175	3500	3841	4
15	1252	1452	1667	1896	2141	2400	2674	2963	3267	3585	3
16	1174	1361	1563	1778	2007	2250	2507	2778	3062	3361	3
17	1105	1281	1471	1673	1889	2118	2359	2614	2882	3163	3
18	1043	1210	1389	1580	1789	2000	2228	2469	2722	2988	3
19	988	1146	1316	1497	1690	1895	2111	2339	2579	2830	3

Note.—For allowable fiber stresses in Wooden Beams, se
Strength of Materials, Table 7, column 8, page 495. For bridge
use safety factor 6; for floorbeams, 5.

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2.—Uniformly Distributed Loads W in Pounds On Rectangular Beams 1 Inch Wide.
—Concluded.

[Total load in Pounds, including weight of beam.]

Span, Feet.	Depth of Beam.											
	13"	14"	15"	16"	17"	18"	19"	20"	21"	22"	23"	24"
20	939	1089	1250	1422	1606	1800	2006	2222	2450	2689	2939	3200
21	894	1037	1190	1354	1529	1714	1910	2116	2333	2561	2799	3048
22	854	990	1136	1293	1460	1636	1823	2020	2227	2444	2672	2909
23	816	947	1087	1237	1396	1565	1744	1932	2130	2338	2556	2783
24	782	907	1042	1185	1338	1500	1671	1852	2042	2241	2449	2667
25	751	871	1000	1138	1284	1440	1604	1778	1960	2131	2351	2560
26	722	838	962	1094	1235	1385	1543	1709	1885	2068	2261	2462
27	695	807	926	1053	1189	1333	1486	1646	1815	1992	2177	2370
28	671	778	893	1016	1147	1286	1433	1587	1750	1921	2099	2286
29	648	751	862	981	1107	1241	1383	1533	1690	1854	2027	2207
30	626	726	833	948	1070	1200	1337	1481	1633	1793	1959	2133
31	606	703	806	918	1036	1161	1294	1434	1581	1735	1896	2065
32	587	681	781	889	1003	1125	1253	1389	1531	1681	1837	2000
33	569	660	758	862	973	1091	1215	1347	1485	1630	1781	1939
34	552	641	735	837	944	1059	1180	1307	1441	1582	1728	1882
35	537	622	714	813	917	1029	1146	1270	1400	1537	1677	1829
36	522	605	694	790	894	1000	1114	1235	1361	1494	1633	1778
37	507	589	676	769	868	973	1084	1201	1324	1453	1589	1730
38	494	573	658	749	845	947	1056	1169	1289	1415	1547	1684
39	481	558	641	729	823	923	1028	1140	1256	1379	1507	1641
40	469	544	625	711	803	900	1003	1111	1225	1344	1469	1600

Note.—For allowable fiber stresses in Wooden Beams, see Sec. 28, Strength of Materials, Table 7, column 8, page 495. For bridge stringers use safety factor 6; for floor beams, 5.

Remember that W is proportional to d^2 and inversely proportional to L; thus the range of the table may be extended greatly.

Examples in Use of Table 2.

Example 1.—In the problem at bottom of page 565, longitudinal shear, we find that a 12 x 12-in. wooden beam will support a total load (including weight of beam = 600 lbs.) of 9600 lbs. Find the maximum length of span so that the outer fiber stress f shall not exceed 1000 lbs. per square inch.

Solution.—From the data given, a beam 12 ins. wide and 12 ins. deep will support a total load of 9600 lbs.; hence, for the same depth of beam, each one inch in width will support 9600÷12=800 lbs. Now, in the last column (the 12" column) on preceding page we find that the load 800 corresponds to a span of 20 ft. Ans.

Example 2.—In the preceding Example, find the maximum length of span so that the outer fiber stress f shall not exceed 600 lbs. per square inch.

Solution.—Corresponding length of span,

$$L = 20\frac{f_1}{f} = 20\frac{600}{1000} = 12 \text{ ft. Ans.}$$

Example 3.—A beam supports a load W = 9600 lbs., producing an outer fiber stress f=1000. What load W_1 will it support if the allowable fiber stress f_1 is increased to 1200 lbs. per square inch?

Solution.—W is proportional to f; hence

$$W_1 = W\frac{f_1}{f} = 9600\frac{1200}{1000} = 11520 \text{ lbs. Ans.}$$

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3.—Beam Box Girders (Steel).

(Adapted from Carnegie and Cambria.)

Fig. 1.

Note.—I-beams are the sections as rolled by principal mills. Finished weight includes rivet he Section modulus, S, allows for $\frac{13}{16}$" rivet holes deduc both flanges. Safe load W includes weight of gir therefore, the allowable superimposed load will be than W. For *Deflection*, see Table 1, preceding, Example 3.

Section.		Finished weight per foot.	Section modulus $S=\frac{I_x}{y}$	*Safe total load W Unif. Distrib't'd for 10-ft span.
Two I-Beams	Two Plates	Lbs.	Ins.	Lbs.
10"–25#	12x ½	94.6	90.1	90,100
" " " "	x 9/16	99.8	96.3	96,300
" " " "	x ⅝	104.8	102.4	102,400
" " " "	x 11/16	110.0	108.6	108,600
" " " "	x ¾	115.0	114.8	114,800
" " " "	x 13/16	120.1	121.0	121,000
" " " "	x ⅞	125.2	127.2	127,200
" " " "	x 15/16	130.3	133.5	133,500
" " " "	x1	135.4	139.8	139,800
12"–31½#	14x ½	114.4	132.1	132,100
" " " "	x 9/16	120.4	141.0	141,000
" " " "	x ⅝	126.3	149.7	149,700
" " " "	x 11/16	132.3	158.5	158,500
" " " "	x ¾	138.3	167.4	167,400
" " " "	x 13/16	144.2	176.3	176,300
" " " "	x ⅞	150.1	185.3	185,300
" " " "	x 15/16	156.1	194.2	194,200
" " " "	x1	162.0	203.2	203,200
12"–40#	14x ½	131.4	146.6	146,600
" " " "	x 9/16	137.4	155.3	155,300
" " " "	x ⅝	143.3	164.0	164,000
" " " "	x 11/16	149.3	172.7	172,700
" " " "	x ¾	155.3	181.4	181,400
" " " "	x 13/16	161.2	190.2	190,200
" " " "	x ⅞	167.1	199.0	199,000
" " " "	x 15/16	173.1	207.8	207,800
" " " "	x1	179.0	216.7	216,700
15"–42#	14x ⅝	147.3	212.1	212,100
" " " "	x 11/16	153.3	223.0	223,000
" " " "	x ¾	159.3	234.0	234,000
" " " "	x 13/16	165.2	245.0	245,000
" " " "	x ⅞	171.1	256.0	256,000
" " " "	x 15/16	177.1	267.0	267,000
" " " "	x1	183.0	278.0	278,000
" " " "	x1 1/16	189.0	289.3	289,300
" " " "	x1⅛	194.9	300.5	300,500
" " " "	x1 3/16	200.9	311.6	311,600
" " " "	x1¼	206.8	322.8	322,800

Section.		Finished weight per foot.	Section modulus $S=\frac{I_x}{y}$	*Safe total load W Unif. Distrib't'd for 10-ft span.
Two I-Beams	Two Plates	Lbs.	Ins.	Lbs.
15"–60#	14x ⅝	180.8	246.8	246,
"	x 11/16	187.2	258.4	258,
"	x ¾	193.6	270.0	270,
"	x 13/16	199.9	281.7	281,
"	x ⅞	206.3	293.4	293,
"	x 15/16	212.7	305.1	305,
"	x1	219.0	316.9	316,
"	x1 1/16	225.4	328.7	328,
"	x1⅛	231.8	340.6	340,
"	x1 3/16	238.2	352.5	352,
"	x1¼	244.6	364.4	364,
15"–60#	15x ⅝	187.6	259.2	259,
"	x 11/16	194.0	270.8	270,
"	x ¾	200.4	282.4	282,
"	x 13/16	206.7	294.1	294,
"	x ⅞	213.1	305.8	305,
"	x 15/16	219.5	317.5	317,
"	x1	225.8	329.3	329,
"	x1 1/16	232.2	341.1	341,
"	x1⅛	238.6	353.0	353,
"	x1 3/16	245.0	365.0	365,
"	x1¼	251.4	376.8	376,
15"–80#	14x ⅝	220.8	287.5	287,
"	x 11/16	227.2	299.8	299,
"	x ¾	233.6	310.2	310,
"	x 13/16	239.9	321.5	321,
"	x ⅞	246.3	332.9	332,
"	x 15/16	252.7	344.4	344,
"	x1	259.0	355.9	355,
"	x1 1/16	265.4	367.4	367,
"	x1⅛	271.8	379.2	379
"	x1 3/16	278.2	390.6	390
"	x1¼	284.6	402.2	402

* Based on allowable fiber stress of 15000 lbs. per sq. in. For 16-ft. s divide total load by 1.6; 20-ft. span, divide by 2; 30-ft. span, divide by 3;

Note that total load W includes weight of girder, which must be ducted to obtain superimposed load.

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3.—Beam Box Girders (Steel)—Concluded.
(Adapted from Carnegie and Cambria.)

Section.		Finished weight per foot.	Section modulus $S=\frac{I_x}{y}$	*Safe total load W Unif. Distrib't'd for 10-ft span.
Two I-Beams	Two Plates	Lbs.	Ins.	Lbs.
15″–80#	15x 5/8	227.6	299.7	299,700
"	x 11/16	234.0	311.0	311,000
"	x 3/4	240.4	322.4	322,400
"	x 13/16	246.7	333.7	333,700
"	x 7/8	253.1	345.1	345,100
"	x 15/16	259.5	356.6	356,600
"	x 1	265.8	368.1	368,100
"	x 1 1/16	272.2	379.6	379,600
"	x 1 1/8	278.6	391.2	391,200
"	x 1 3/16	285.0	402.8	402,800
"	x 1 1/4	291.4	414.4	414,400
18″–55#	16x 3/4	195.5	340.5	340,500
"	x 13/16	202.2	355.8	355,800
"	x 7/8	209.0	371.2	371,200
"	x 15/16	215.8	386.6	386,600
"	x 1	222.6	402.0	402,000
"	x 1 1/16	299.4	417.5	417,500
"	x 1 1/8	236.2	433.0	433,000
"	x 1 3/16	243.1	448.6	448,600
"	x 1 1/4	249.8	464.2	464,200
"	x 1 5/16	256.7	479.8	479,800
"	x 1 3/8	263.4	495.4	495,400
20″–65#	16x 3/4	215.5	411.8	411,800
"	x 13/16	222.2	428.7	428,700
"	x 7/8	229.0	445.7	445,700
"	x 15/16	235.8	462.7	462,700
"	x 1	242.6	479.7	479,700
"	x 1 1/16	249.4	496.7	496,700
"	x 1 1/8	256.2	513.8	513,800
"	x 1 3/16	263.1	531.2	531,200
$a=7\frac{1}{4}$, $b=10\frac{1}{4}$; to $a=8\frac{1}{4}$, $b=11\frac{1}{4}$.				
$a=6\frac{3}{4}$, $b=10$; to $a=8\frac{1}{4}$, $b=11\frac{1}{4}$.				
$a=7$, $b=10\frac{1}{2}$; to $a=8$, $b=11\frac{1}{2}$.				

Section.		Finished weight per foot.	Section modulus $S=\frac{I_x}{y}$	*Safe total load W Unif. Distrib't'd for 10-ft span.
Two I-Beams	Two Plates	Lbs.	Ins.	Lbs.
20″–65#	16x1 1/4	269.8	548.1	548,100
"	x1 3/16	276.7	565.3	565,300
"	x1 5/8	283.4	582.5	582,500
20″–80#	16x 3/4	245.5	463.8	463,800
"	x 13/16	252.2	480.4	480,400
"	x 7/8	259.0	497.1	497,100
"	x 15/16	265.8	513.8	513,800
"	x1	272.6	530.6	530,600
"	x1 1/16	279.4	547.3	547,300
"	x1 1/8	286.2	564.1	564,100
"	x1 3/16	293.1	581.2	581,200
"	x1 1/4	299.8	597.8	597,800
"	x1 5/16	306.7	614.7	614,700
"	x1 5/8	313.4	631.7	631,700
24″–80#	18x 3/4	255.7	593.7	593,700
"	x 13/16	263.3	616.9	616,900
"	x 7/8	271.0	640.1	640,100
"	x 15/16	278.6	663.4	636,400
"	x1	286.2	686.7	686,700
"	x1 1/16	293.9	710.0	710,000
"	x1 1/8	301.5	733.3	733,300
"	x1 3/16	309.2	757.1	757,100
"	x1 1/4	316.8	780.2	780,200
"	x1 5/16	324.5	803.6	803,600
"	x1 3/8	332.1	827.1	827,100
"	x1 7/16	339.8	850.5	850,500
$a=8$, $b=12$; to $a=9\frac{1}{2}$, $b=13\frac{1}{2}$.				

* Based on allowable fiber stress of 15 000 lbs. per sq. in. For 16-ft. span, divide total load by 1.6; 20-ft. span, divide by 2; 30-ft. span, divide by 3; etc.

Note that total load W includes weight of girder, which must be deducted to obtain superimposed load.

Problem.—Required to design a beam box girder of 16-ft. clear span, for a superimposed load which produces a maximum bending moment $M''=3,150,000$ in.-lbs.; with a total allowable fiber stress $f=14400$ lbs. per square inch.

Solution.—(See Table 1, page 562, for formulas.) From the general formulas (3) we have,

(1.) For the superimposed load: Sec. mod. $S=\frac{I}{y}=\frac{M''}{f}=\frac{3150000}{14400}=218.75$

From the above table this value of S calls for 15″–42# beams with plates.

(2.) For the weight of the girder: Sec. mod. $S=\frac{I}{y}=\frac{M''}{f}=\frac{Wl}{8f}=\frac{wl^2}{8f}$,

in which w = the weight in lbs. per lin. ft. of girder. From the above table, we may assume $w=183$; then $S=\frac{183\times256\times12\times12}{8\times14400}=$ 58.55

(1 and 2.) *Ans.*—Use two 15″ – 42# beams and two 14 × 1″ plates: For required $S=$ 277.30

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4.—Plate-Girder Tables* (Steel).
Properties of Plate Girders Complete.

(Compiled from Tables 5, 6 and 7, following.)
[Resisting moment (=bending moment) in 1000 Ft.-Lbs.]

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Four Angles, Each	Cover Plates.†	Web Plate.	Wt. per ft. Gross section.	Resisting moments in 1000 ft.-lbs. including in flanges:			Web Plate.	Wt. per ft. Gross section.	Resisting moments in 1000 ft.-lbs. including in flanges:			Web Plate.	Wt. per ft. Gross section.	Resis moment 1000 ft inclu in flan	
			W Lbs.	⅛ of web.	1/10 of web.	No part of web.		W Lbs.	⅛ of web.	1/10 of web.	No part of web.		W Lbs.	⅛ of web.	1/10 of wel
2½x2½x¼	.....	16 x ¼	30.0	30.6	29.3	23.9	18 x ¼	31.7	35.6	33.9	27.2	20 x ¼	33.4	40.8	38.
" x⅜	.....		37.2	41.0	39.7	34.4		38.9	47.5	45.8	39.0		40.6	54.1	52.
" x½	.....		44.4	50.9	49.6	44.3		46.1	58.7	57.1	50.3		47.8	66.8	64.
" x¼	6x¼		40.2	45.2	43.9	38.5		41.9	52.0	50.3	43.5		43.6	58.9	56.
" x½	6x½		64.8	80.7	79.3	74.0		66.5	92.0	90.3	83.6		68.2	103.6	101.
2½x2½x¼	.....	22 x ¼	35.1	46.3	44.4	33.6	24 x ¼	36.8	51.9	48.9	36.9	26 x ¼	38.5	57.7	54.
" x⅜	.....		42.3	61.0	59.2	48.4		44.0	68.0	65.0	53.0		45.7	75.3	71.
" x½	.....		49.5	75.0	73.2	62.4		51.2	83.5	80.5	68.5		52.9	92.1	88.
" x¼	6x¼		45.3	66.1	64.4	53.5		47.0	73.5	70.5	58.5		48.7	81.1	77
" x½	6x½		69.9	115.4	113.5	102.8		71.6	127.4	124.4	112.4		73.3	139.6	136.
3x2½x¼	.....	18 x ¼	33.3	39	38	31	20 x ¼	35.0	45	43	34	22 x ¼	36.7	51	4
" x⅜	.....		41.7	53	51	45		43.4	60	58	50		45.1	68	6
" x½	.....		49.3	66	64	58		51.0	75	73	65		52.7	84	8
" x¼	7x¼		45.2	59	58	51		46.9	67	65	56		48.6	75	7
" x½	7x½		73.1	107	105	99		74.8	120	118	110		76.5	134	13
3x2½x¼	.....	24 x ¼	38.4	57	54	42	26 x ¼	40.1	63	59	45	28 x ¼	41.8	69	6
" x⅜	.....		46.8	76	73	61		48.5	83	80	66		50.2	92	8
" x½	.....		54.4	93	90	78		56.1	103	99	85		57.8	112	10
" x¼	7x¼		50.3	83	80	68		52.0	92	88	74		53.7	100	9
" x½	7x½		78.2	148	145	133		79.9	161	158	144		81.6	175	17
3x3 x¼	.....	18 x ¼	34.9	42	40	34	24 x ¼	40.0	61	58	46	30 x ¼	45.1	82	7
" x½	.....		52.9	72	70	63		58.0	101	98	86		63.1	133	12
" x⅝	.....		65.3	85	83	76		70.4	120	117	105		75.5	156	15
" x¼	7x¼		45.9	62	61	54		51.9	88	85	73		57.0	118	11
" x⅝	7x⅝		95.1	136	135	128		100.2	188	185	173		105.3	241	23
3x3 x¼	.....	18 x 5/16	38.7	44	42	34	24 x 5/16	45.1	65	61	46	30 x 5/16	51.5	87	8
" x½	.....		56.7	74	72	63		63.1	105	101	86		69.5	139	13
" x⅝	.....		69.1	87	85	76		75.5	123	120	105		81.9	162	15
" x¼	7x¼		50.6	64	62	54		57.0	91	88	73		63.4	124	11
" x⅝	7x⅝		98.9	139	136	128		105.3	191	188	173		111.6	246	24
3½x2½x¼	.....	18 x 5/16	38.7	45	43	35	24 x 5/16	45.1	66	62	47	30 x 5/16	51.5	88	8
" x½	.....		56.7	77	73	65		63.1	107	103	88		69.5	142	13
" x⅝	.....		69.1	89	87	79		75.5	125	122	107		81.9	164	15
" x¼	8x¼		52.3	69	67	59		58.7	97	94	79		65.1	128	12
" x⅝	8x⅝		103.1	151	149	140		109.5	206	203	188		115.9	265	24
3½x3 x 5/16	.....	18 x 5/16	45.5	56	54	46	24 x 5/16	51.9	81	77	62	30 x 5/16	58.3	108	10
" x⅝	.....		69.1	96	94	85		75.5	135	132	117		81.9	177	17
" x 5/16	8x 5/16		62.5	87	84	76		68.9	121	117	102		75.3	158	15
" x⅝	8x⅝		103.1	158	155	147		109.5	217	213	198		115.9	278	27

* Table 4 will cover most cases in practice; but Tables 5, 6 and 7 further be combined to meet almost any requirement.
† One top and one bottom.

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LATE-GIRDER TABLES (STEEL)—Continued.
OPERTIES OF PLATE GIRDERS COMPLETE.
ompiled from Tables 5, 6 and 7, following.)
g moment (= bending moment) in 1000 ft.-lbs.]

5	6	7	8	9	10	11	12	13	14	15	16	17
Resisting moments in 1000 ft.-lbs. including in flanges:			Web Plate.	Wt. per ft. Gross section.	Resisting moments in 1000 ft.-lbs. including in flanges:			Web Plate.	Wt. per ft. Gross section.	Resisting moments in 1000 ft.-lbs. including in flanges:		
⅛ of web.	1/16 of web.	No part of web.		W Lbs.	⅛ of web.	1/16 of web.	No part of web.		W Lbs.	⅛ of web.	1/16 of web.	No part of web.
88	84	69	30 x 5/16	60.7	116	110	87	36 x 5/16	67.1	147	139	105
148	144	129		86.3	193	187	163		92.7	240	231	198
134	130	115		79.8	174	168	145		86.2	216	208	174
242	238	223		124.6	309	303	280		130.9	379	371	337
97	93	78	30 x 5/16	64.7	129	123	99	36 x 5/16	71.1	163	154	121
166	163	148		94.7	218	212	188		101.1	271	263	229
143	140	125		83.8	186	181	157		90.2	232	224	190
260	257	242		133.0	334	328	305		139.3	410	402	368
119	114	96	36 x 3/8	85	198	187	147	48 x 3/8	100	288	270	198
203	198	180		120	326	316	276		135	461	443	371
190	186	168		113	303	293	253		128	428	410	338
347	343	325		176	540	530	490		191	745	727	655
125	120	102	36 x 3/8	88	207	197	157	48 x 3/8	103	301	283	211
214	209	191		125	345	335	294		140	487	469	397
196	191	173		116	313	303	262		131	442	424	352
358	354	336		181	559	549	508		197	771	753	681
127	122	104	36 x 3/8	93	210	200	160	48 x 3/8	108	305	287	215
246	242	224		149	395	385	344		164	555	537	465
214	209	191		126	339	329	289		141	476	458	386
452	448	430		226	699	689	649		241	957	939	867
133	128	110	36 x 3/8	95	220	210	169	48 x 3/8	110	318	300	228
259	255	237		155	417	407	366		170	585	567	495
219	215	197		128	349	339	298		144	489	471	399
466	461	443		232	721	711	671		247	988	970	898
355	342	292	50 x 3/8	142	469	449	371	60 x 3/8	155	591	563	450
550	538	488		196	718	699	621		209	894	866	754
558	546	496		190	722	703	625		203	895	866	754
963	950	900		291	1231	1211	1133		304	1507	1479	1366
753	708	530	80 x 7/16	197	900	842	609	90 x 7/16	212	1057	983	688
1110	1066	887		251	1312	1254	1020		266	1523	1449	1153
1107	1062	883		245	1304	1246	1012		260	1511	1437	1142
1823	1778	1600		347	2124	2066	1833		362	2435	2361	2066
816	779	629	70 x ½	225	996	945	741	80 x ½	242	1187	1120	854
1251	1214	1064		299	1511	1459	1255		316	1780	1713	1447
1221	1184	1034		286	1468	1417	1213		303	1725	1659	1392
2486	2449	2299		483	2945	2894	2690		500	3415	3348	3082
1388	1304	966	100 x ½	276	1599	1495	1079	120 x ½	310	2054	1904	1304
2060	1975	1638		350	2350	2246	1829		384	2962	2812	2212
1993	1909	1571		337	2271	2167	1750		371	2859	2709	2109
3895	3810	3474		534	4385	4281	3864		568	5397	5247	4647

nd one bottom.

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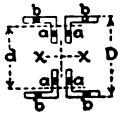

Fig. 2.

5.—Plate Girders (Steel).
Properties of Flange Angles Only.

1	2	3	4	5	6	7	8	9
4 Flange Angles. Size. Ins.	Weight per Ft. Gross Section. W Lbs.	Depth b. to b. of Angles. D Ins.	Dist. c. to c. of Gravity. d Ins.	Net Area Each Flange (2 Angles) A Sq. Ins.	Section Modulus $S=\frac{I}{y}=\frac{M''}{f}$ $(=Ad)$ Ins.	Increase of Section Modulus for Each 12" Increase of D.	Resisting Moment $M'=\frac{fI}{12y}$ for $f=10{,}000$. Ft.-Lbs.	Increase of Resisting Moment for Each 12" Increase
(a) Use with or without Cover Plates. Rivets ¾". One ⅞" rivet-hole deduc from Each Angle for A.								
2½x2½x3/16	12.4	12¼	10.87	1.47	15.98	17.64	13 320	14
" x¼	16.4	"	10.81	1.94	20.97	23.28	17 490	19
" x5/16	20.0	"	10.77	2.39	25.74	28.68	21 450	23
" x⅜	23.6	"	10.73	2.80	30.04	33.60	25 040	28
" x7/16	27.2	"	10.69	3.23	34.53	38.76	28 770	32
" x½	30.8	"	10.63	3.63	38.59	43.56	32 160	36
3 x2½x¼	18.0	12¼	10.93	2.18	23.83	26.16	19 860	21
" x5/16	22.4	"	10.89	2.69	29.29	32.28	24 410	26
" x⅜	26.4	"	10.83	3.18	34.44	38.16	28 700	31
" x7/16	30.4	"	10.79	3.67	39.60	44.04	33 000	36
" x½	34.0	"	10.75	4.13	44.40	49.56	37 000	41
3 x3 x¼	19.6	18¼	16.57	2.44	40.43	29.28	33 690	24
" x5/16	24.4	"	16.51	3.01	49.70	36.12	41 410	30
" x⅜	28.8	"	16.47	3.56	58.63	42.72	48 860	35
" x7/16	33.2	"	16.43	4.09	67.20	49.08	56 000	40
" x½	37.6	"	16.39	4.63	75.89	55.56	63 240	46
" x9/16	41.6	"	16.35	5.14	84.04	61.68	70 030	51
" x⅝	50.0	"	16.29	5.63	91.71	67.56	76 430	56
3½x2½x¼	19.6	18¼	17.03	2.44	41.55	29.28	34 630	24
" x5/16	24.4	"	16.97	3.01	51.08	36.12	42 570	30
" x⅜	28.8	"	16.93	3.56	60.27	42.72	50 230	35
" x7/16	33.2	"	16.89	4.09	69.08	49.08	57 570	40
" x½	37.6	"	16.85	4.63	78.02	55.56	65 010	46
" x9/16	41.6	"	16.79	5.14	86.30	61.68	71 920	51
" x⅝	50.0	"	16.75	5.63	94.30	67.56	78 590	56
3½x3 x5/16	26.4	18¼	16.63	3.31	55.05	39.72	45 870	33
" x⅜	31.6	"	16.59	3.94	65.36	47.28	54 470	39
" x7/16	36.4	"	16.55	4.53	74.97	54.36	62 480	45
" x½	40.8	"	16.49	5.13	84.59	61.56	70 490	51
" x9/16	45.6	"	16.45	5.70	93.77	68.40	78 140	57
" x⅝	50.0	"	16.41	6.25	102.56	75.00	85 470	62
" x11/16	54.4	"	16.37	6.80	111.32	81.60	92 760	68
" x¾	58.8	"	16.33	7.31	119.37	87.72	99 480	73

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—PLATE GIRDERS (STEEL)—Continued.
PROPERTIES OF FLANGE ANGLES ONLY.

3	4	5	6	7	8	9
Depth b to b. of Angles. D Ins.	Dist. c to c. of Gravity. d Ins.	Net Area Each Flange (2 Angles) A Sq. Ins.	Section Modulus $S = \frac{I}{y} = \frac{M''}{f}$ $(= Ad)$ Ins.	Increase of Section Modulus for Each 12″ Increase of D.	Resisting Moment $M' = \frac{fI}{12y}$ for f=10,000. Ft.-Lbs.	Increase of Resisting Moment for Each 12″ Increase of D.
ithout Cover Plates. Rivets ¾″. One ⅞″ rivet-hole deducted from Each Angle for A.						
18¼	16.27	3.63	59.06	43.56	49 220	36 300
"	16.23	4.30	69.79	51.60	58 160	43 000
"	16.17	4.97	80.36	59.64	66 970	49 700
"	16.13	5.63	90.81	67.56	75 680	56 300
"	16.09	6.26	100.72	75.12	83 940	62 600
"	16.05	6.87	110.26	82.44	91 890	68 700
"	16.01	7.48	119.75	89.76	99 800	74 800
"	15.95	8.07	128.72	96.84	107 260	80 700
24¼	22.73	3.63	82.51	43.56	68 760	36 300
"	22.69	4.30	97.57	51.60	81 310	43 000
"	22.65	4.97	112.57	59.64	93 810	49 700
"	22.59	5.63	127.18	67.56	105 980	56 300
"	22.55	6.26	141.16	75.12	117 640	62 600
"	22.51	6.87	154.64	82.44	128 870	68 700
"	22.47	7.48	168.08	89.76	140 060	74 800
"	22.41	8.07	180.85	96.84	150 710	80 700
24¼	22.01	4.26	93.76	51.12	78 140	42 600
"	21.97	5.06	111.17	60.72	92 640	50 600
"	21.93	5.85	128.29	70.20	106 910	58 500
"	21.89	6.63	145.13	79.56	120 940	66 300
"	21.83	7.38	161.11	88.56	134 260	73 800
"	21.79	8.13	177.15	97.56	147 630	81 300
"	21.75	8.86	192.71	106.32	160 590	88 600
"	21.71	9.57	207.76	114.84	173 140	95 700
24¼	22.85	5.06	115.62	60.72	96 350	50 600
"	22.79	5.85	133.32	70.20	111 100	58 500
"	22.75	6.63	150.83	79.56	125 690	66 300
"	22.71	7.38	167.60	88.56	139 670	73 800
"	22.65	8.13	184.14	97.56	153 450	81 300
"	22.61	8.86	200.32	106.32	166 940	88 600
"	22.57	9.57	215.99	114.84	180 000	95 700
24¼	22.53	5.44	122.56	65.28	102 140	54 400
"	22.49	6.29	141.46	75.48	117 890	62 900
"	22.43	7.13	159.93	85.56	133 270	71 300
"	22.39	7.96	178.22	95.52	148 520	79 600
"	22.35	8.75	195.56	105.00	162 970	87 500
"	22.31	9.54	212.84	114.48	177 360	95 400
"	22.25	10.31	229.40	123.72	191 160	103 100

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5.—Plate Girders (Steel)—Concluded.

Properties of Flange Angles Only.*

1	2	3	4	5	6	7	8	9
4 Flange Angles. Size. Ins.	Weight per Ft. Gross Section. W Lbs.	Depth b. to b. of Angles. D Ins.	Dist. c. to c. of Gravity. d Ins.	Net Area Each Flange (2 Angles) A Sq. Ins.	Section Modulus $S=\frac{I}{y}=\frac{M''}{f}$ $(=Ad)$ Ins.	Increase of Section Modulus for Each 12" Increase of D.	Resisting moment $M'=\frac{fI}{12y}$ for $f=10{,}000$. Ft.-Lbs.	Increase of Resisting Moment for Each 12" Increase of D.
(b) Use with or without Cover Plates. Rivets ¾". Two ⅞" rivet-holes deducted from Each Angle for A.								
6 x3½x⅜	46.8	30¼	28.67	5.53	158.55	66.36	132 120	55 300
" x7/16	54.0	"	28.63	6.41	183.52	76.92	152 930	64 100
" x½	61.2	"	28.59	7.25	207.28	87.00	172 730	72 500
" x9/16	68.4	"	28.53	8.09	230.81	97.08	192 340	80 900
" x⅝	75.6	"	28.49	8.91	253.85	106.92	211 540	89 100
" x11/16	82.4	"	28.45	9.71	276.25	116.52	230 210	97 100
" x¾	89.6	"	28.39	10.50	298.10	126.00	248 410	105 000
" x13/16	96.0	"	28.35	11.28	319.79	135.36	266 660	112 800
" x⅞	102.8	"	28.31	12.04	340.85	144.48	284 040	120 400
6 x4 x⅜	49.2	30¼	28.37	5.91	167.67	70.82	139 720	59 100
" x7/16	57.2	"	28.33	6.83	193.49	81.96	161 240	68 300
" x½	64.8	"	28.27	7.75	219.10	93.00	182 580	77 500
" x9/16	72.4	"	28.23	8.65	244.19	103.80	203 490	86 500
" x⅝	80.0	"	28.19	9.53	268.65	114.36	223 880	95 300
" x11/16	87.2	"	28.13	10.41	292.83	124.92	244 030	104 100
" x¾	94.4	"	28.09	11.26	316.29	135.12	263 580	112 600
" x13/16	101.6	"	28.05	12.10	339.41	145.20	282 840	121 000
" x⅞	108.8	"	28.01	12.92	361.89	155.04	301 570	129 200
(c) Use with or without Cover Plates. Rivets ⅞". Two 1" rivet-holes deducted from Each Angle for A.								
6 x6 x7/16	68.8	30¼	26.93	8.37	225.40	100.44	187 840	83 700
" x½	78.4	"	26.89	9.50	255.46	114.00	212 880	95 000
" x9/16	87.6	"	26.83	10.61	284.67	127.32	237 220	106 100
" x⅝	96.8	"	26.79	11.72	313.98	140.64	261 650	117 200
" x11/16	106.0	"	26.75	12.81	342.67	153.72	285 560	128 100
" x¾	114.8	"	26.69	13.88	370.46	166.56	308 710	138 800
" x13/16	124.0	"	26.65	14.93	397.88	179.16	331 570	149 300
" x⅞	132.4	"	26.61	15.98	425.23	191.76	354 360	159 800
8 x8 x½	105.6	36¼	31.87	13.50	430.25	162.00	358 540	135 000
" x9/16	118.4	"	31.83	15.11	480.95	181.32	400 790	151 100
" x⅝	130.8	"	31.79	16.72	531.53	200.64	442 940	167 200
" x11/16	143.2	"	31.75	18.31	581.34	219.72	484 450	183 100
" x¾	155.6	"	31.69	19.88	630.00	238.56	525 000	198 800
" x13/16	168.0	"	31.65	21.43	678.26	257.16	565 220	214 300
" x⅞	180.0	"	31.61	22.96	725.77	275.52	604 800	229 600

*See Fig. 2, page 572.

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Fig. 3.

6.—Plate Girders (Steel)

Properties of Web Plates Only.

(M' = moment in ft.-lbs.; M'' = moment in in.-lbs.)

1	2	3	4	5	6	7	8	9
Web Plate.	Weight per Foot Gross Section.	Area of Gross Section.	Full Efficiency of Web; ⅙ Area Considered at Upper and Lower Edges; No Deduction for Rivet Holes, etc.		75% Efficiency of Web; ⅓ Area Considered at Upper and Lower Edges; 25% Deduct'n for Rivet Holes, etc.		60% Efficiency of Web; $\frac{1}{10}$ Area Considered at Upper and Lower Edges; 40% Deduct'n for Rivet Holes, etc.	
Depth. Width. d b Ins.	W Lbs.	A Sq. Ins	Section Modulus $S = \frac{Ad}{6}$	Moment of Resistance $M' = \frac{fS}{12}$ (f = 10000) Ft.-Lbs.	Section Modulus $S = \frac{Ad}{8}$	Moment of Resistance $M' = \frac{fS}{12}$ (f = 10000) Ft.-Lbs.	Section Modulus $S = \frac{Ad}{10}$	Moment of Resistance $M' = \frac{fS}{12}$ (f = 10000) Ft.-Lbs.
1 x $\frac{1}{16}$	.212	.062	.010	9	.008	7	.006	5
" x ¼	.85	.25	.042	35	.031	26	.025	21
" x1	3.40	1.00	.167	139	.125	104	.100	83
2 x $\frac{1}{16}$	.425	.125	.042	35	.031	26	.025	21
" x ¼	1.70	.50	.167	139	.125	104	.100	83
" x1	6.80	2.00	.667	556	.500	417	.400	333
3 x $\frac{1}{16}$	.637	.187	.094	78				
" x ¼	2.55	.75	.375	312				
" x1	10.20	3.00	1.500	1250				
4 x $\frac{1}{16}$	.85	.25	.167	139				
" x ¼	3.40	1.00	.667	556				
" x1	13.60	4.00	2.667	2222				
5 x $\frac{1}{16}$	1.062	.312	.260	217				
" x ¼	4.25	1.25	1.042	868				
" x1	17.00	5.00	4.167	3472				
6 x $\frac{1}{16}$	1.275	.375	.375	312				
" x ¼	5.10	1.50	1.500	1250				
" x1	20.40	6.00	6.000	5000				
7 x $\frac{1}{16}$	1.487	.437	.510	425	.383	319	.306	255
" x ¼	5.95	1.75	2.042	1701	1.531	1276	1.225	1021
" x1	23.80	7.00	8.167	6806	6.125	5104	4.900	4083
8 x $\frac{1}{16}$	1.70	.50	.667	556	.500	417	.400	333
" x ¼	6.80	2.00	2.667	2222	2.000	1667	1.600	1333
" x1	27.20	8.00	10.667	8889	8.000	6667	6.400	5333
9 x $\frac{1}{16}$	1.912	.562	.844	703	.633	527	.506	422
" x ¼	7.65	2.25	3.375	2812	2.531	2109	2.025	1687
" x1	30.60	9.00	13.500	11250	10.125	8437	8.100	6750

Note.—Columns 4 and 5 are useful also in the calculation of wooden beams:

Problem.—What bending moment in ft.-lbs. will a beam 3"x8" sustain safely at an allowable fibre stress of 1100 lbs. per sq. in.?

Solution 1.—$M' = \frac{1100\,S}{12} = \frac{1100}{12} \times 3 \times 10.667 = 2933$ ft. lbs.

Solution 2.—$M' = \frac{3 \times 8889 \times 1100}{10000} = 3 \times 8889 \times .11 = 2933$ ft.-lbs.

Hence, the problem can be solved by the use of either S or M'.

Note that properties in table are directly proportional to width b; and that the Sectional Moduli and Moments of Resistance are proportional to the *square* of depth d. Hence the range of the table may be increased indefinitely. See first column on following page.

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6.—Plate Girders (Steel)—Continued.

Properties of Web Plates Only.*

(M' = moment in ft.-lbs.; M'' = moment in in.-lbs.)

1	2	3	4	5	6	7	8	9
Web Plate.			Full Efficiency of Web; ⅛ Area Considered at Upper and Lower Edges: No Deduction for Rivet Holes, etc.		75% Efficiency of Web; ⅛ Area Considered at Upper and Lower Edges: 25% Deduct'n for Rivet Holes, etc.		60% Efficiency of Web; ⅛ Area Considered at Upper and Lower Edges: 40% Deduct'n for Rivet Holes, etc.	
Depth. d Ins.	Width. b Ins.	Weight per Foot Gross Section. W Lbs.	Area of Gross Section. A Sq. Ins.	Section Modulus $S=\frac{Ad}{6}$	Moment of Resistance $M'=\frac{fS}{12}$ (f = 10000) Ft.-Lbs.	Section Modulus $S=\frac{Ad}{8}$	Moment of Resistance $M'=\frac{fS}{12}$ (f = 10000) Ft.-Lbs.	Section Modulus $S=\frac{Ad}{10}$	Moment of Resistance $M'=\frac{fS}{12}$ (f = 10000) Ft.-Lbs.
Note.—S and M' are proportional to sq. of d in inches.									
10	x ¼	2.125	.625	1.042	868	.781	651	.625	521
	x ⅜	4.25	1.25	2.083	1736	1.562	1302	1.25	1042
	x ½	6.375	1.875	3.125	2604	2.344	1953	1.875	1562
	x ¾	8.50	2.50	4.167	3472	3.125	2604	2.50	2083
	x 1	10.625	3.125	5.208	4340	3.906	3255	3.125	2604
	x 1⅛	12.75	3.75	6.250	5208	4.687	3906	3.75	3125
	x 1¼	14.875	4.375	7.292	6076	5.469	4557	4.375	3646
	x 1½	17.00	5.00	8.333	6944	6.250	5208	5.00	4167
	x 1¾	19.125	5.625	9.375	7812	7.031	5859	5.625	4687
	x 2	21.25	6.25	10.417	8681	7.812	6510	6.25	5208
	x 2¼	23.375	6.875	11.458	9549	8.594	7161	6.875	5729
	x 2½	25.50	7.50	12.500	10417	9.375	7812	7.50	6250
	x 2¾	27.625	8.125	13.542	11285	10.156	8463	8.125	6771
	x 3	29.75	8.75	14.583	12153	10.937	9115	8.75	7292
	x 3½	31.875	9.375	15.625	13021	11.719	9766	9.375	7812
	x 1	34.00	10.00	16.667	13889	12.500	10417	10.00	8333
Note.—S and M' are proportional to sq. of depth in ft.									
12	x ¼	2.55	.750	1.500	1250	1.125	937	.9	750
	x ⅜	5.10	1.50	3.000	2500	2.250	1875	1.8	1500
	x ½	7.65	2.25	4.500	3750	3.375	2812	2.7	2250
	x ¾	10.20	3.00	6.000	5000	4.500	3750	3.6	3000
	x 1	12.75	3.75	7.500	6250	5.625	4687	4.5	3750
	x 1⅛	15.30	4.50	9.000	7500	6.750	5625	5.4	4500
	x 1¼	17.85	5.25	10.500	8750	7.875	6562	6.3	5250
	x 1½	20.40	6.00	12.000	10000	9.000	7500	7.2	6000
	x 1⅞	22.95	6.75	13.500	11250	10.125	8437	8.1	6750
	x 2	25.50	7.50	15.000	12500	11.250	9375	9.0	7500
	x 2¼	28.05	8.25	16.500	13750	12.375	10312	9.9	8250
	x 2½	30.60	9.00	18.000	15000	13.500	11250	10.8	9000
	x 2¾	33.15	9.75	19.500	16250	14.625	12187	11.7	9750
	x 3	35.70	10.50	21.000	17500	15.750	13125	12.6	10500
	x 3½	38.25	11.25	22.500	18750	16.875	14062	13.5	11250
	x 1	40.80	12.00	24.000	20000	18.000	15000	14.4	12000
14	x ¼	11.90	3.50	8.167	6806	6.125	5104	4.90	4083
"	x ⅜	14.88	4.38	10.208	8507	7.656	6380	6.125	5104
"	x ½	17.85	5.25	12.250	10208	9.187	7656	7.35	6125
"	x ⅝	20.83	6.13	14.292	11910	10.719	8932	8.575	7146
"	x ½	23.80	7.00	16.333	13611	12.250	10208	9.80	8167
"	x 1	47.60	14.00	32.667	27222	24.500	20417	19.600	16333

*See Fig. 3, page 575.

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6.—Plate Girders (Steel)—Continued.

Properties of Web Plates Only.*

(M' = moment in ft.-lbs.; M'' = moment in in.-lbs.)

1	2	3	4	5	6	7	8	9
Web Plate. Depth. Width.	Weight per Foot Gross Section.	Area of Gross Section.	Full Efficiency of Web; ⅛ Area Considered at Upper and Lower Edges, No Deduction for Rivet Holes, etc.		75% Efficiency of Web; ⅛ Area Considered at Upper and Lower Edges; 25% Deduct'n for Rivet Holes, etc.		60% Efficiency of Web; ⅛ Area Considered at Upper and Lower Edges: 40% Deduct'n for Rivet Holes, etc.	
			Section Modulus $S=\frac{Ad}{6}$	Moment of Resistance $M'=\frac{fS}{12}$ (f = 10000)	Section Modulus $S=\frac{Ad}{8}$	Moment of Resistance $M'=\frac{fS}{12}$ (f = 10000)	Section Modulus $S=\frac{Ad}{10}$	Moment of Resistance $M'=\frac{fS}{12}$ (f = 10000)
d b Ins.	W Lbs.	A Sq. Ins		Ft.-Lbs.		Ft.-Lbs.		Ft.-Lbs.
15 x ¼	12.75	3.75	9.375	7812	7.031	5859	5.625	4687
" x 5/16	15.94	4.687	11.719	9766	8.789	7324	7.031	5859
" x ⅜	19.13	5.625	14.062	11719	10.547	8789	8.437	7031
" x 7/16	22.31	6.562	16.406	13672	12.305	10254	9.844	8203
" x ½	25.50	7.500	18.75	15625	14.062	11719	11.250	9375
" x1	51.00	15.000	37.50	31250	28.125	23438	22.500	18750
16 x ¼	13.60	4.00	10.667	8889	8.000	6667	6.400	5333
" x 5/16	17.00	5.00	13.333	11111	10.000	8333	8.000	6667
" x ⅜	20.40	6.00	16.000	13333	12.000	10000	9.600	8000
" x 7/16	23.80	7.00	18.667	15556	14.000	11667	11.200	9333
" x ½	27.20	8.00	21.333	17778	16.000	13333	12.800	10667
" x1	54.40	16.00	42.667	35556	32.000	26667	25.600	21333
18 x ¼	15.30	4.50	13.500	11250	10.125	8437	8.100	6750
" x 5/16	19.13	5.625	16.875	14062	12.656	10547	10.125	8437
" x ⅜	22.95	6.75	20.250	16875	15.187	12656	12.150	10125
" x 7/16	26.78	7.875	23.625	19687	17.719	14766	14.175	11812
" x ½	30.60	9.00	27.000	22500	20.250	16875	16.200	13500
" x1	61.20	18.00	54.000	45000	40.500	33750	32.400	27000
20 x ¼	17.00	5.00	16.667	13889	12.500	10417	10.000	8333
" x 5/16	21.25	6.25	20.833	17361	15.625	13021	12.500	10417
" x ⅜	25.50	7.50	25.000	20833	18.750	15625	15.000	12500
" x 7/16	29.75	8.75	29.167	24306	21.875	18229	17.500	14583
" x ½	34.00	10.00	33.333	27778	25.000	20833	20.000	16667
" x1	68.00	20.00	66.667	55556	50.000	41667	40.000	33333
22 x ¼	18.70	5.50	20.167	16806	15.125	12604	12.100	10833
" x 5/16	23.38	6.875	25.208	21007	18.906	15755	15.125	13542
" x ⅜	28.05	8.25	30.250	25208	22.688	18906	18.150	16250
" x 7/16	32.73	9.625	35.291	29410	26.469	22057	21.175	18958
" x ½	37.40	11.000	40.333	33611	30.250	25208	24.200	21667
" x1	74.80	22.000	80.667	67222	60.500	50417	48.400	43333
24 x ¼	20.40	6.00	24.000	20000	18.000	15000	14.400	12000
" x 5/16	25.50	7.50	30.000	25000	22.500	18750	18.000	15000
" x ⅜	30.60	9.00	36.000	30000	27.000	22500	21.600	18000
" x 7/16	35.70	10.50	42.000	35000	31.500	26250	25.200	21000
" x ½	40.80	12.00	48.000	40000	36.000	30000	28.800	24000
" x1	81.60	24.00	96.000	80000	72.000	60000	57.600	48000

* See Fig. 3, page 575.

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6.—Plate Girders (Steel)—Continued.

Properties of Web Plates Only.*

(M' = moment in ft.-lbs.; M'' = moment in in.-lbs.)

1	2	3	4	5	6	7	8	9
Web Plate.			Full Efficiency of Web; ⅛ Area Considered at Upper and Lower Edges; no Deduction for Rivet Holes, etc.		75% Efficiency of Web; ⅛ Area Considered at Upper and Lower Edges; 25% Deduct'n for Rivet Holes, etc.		60% Efficiency of Web; ⅛ Area Considered at Upper and Lower Edges; 40% Deduct'n for Rivet Holes, etc.	
Depth. Width d b Ins.	Weight per Foot Gross Section. W Lbs.	Area of Gross Section. A Sq. Ins	Section Modulus $S=\frac{Ad}{6}$	Moment of Resistance $M'=\frac{fS}{12}$ (f = 10000) Ft.-Lbs.	Section Modulus $S=\frac{Ad}{8}$	Moment of Resistance $M'=\frac{fS}{12}$ (f = 10000) Ft.-Lbs.	Section Modulus $S=\frac{Ad}{10}$	Moment of Resistance $M'=\frac{fS}{12}$ (f = 10000) Ft.-Lbs.
26 x ¼	22.10	6.50	28.167	23472	21.125	17604	16.900	14083
" x 5/16	27.63	8.125	35.208	29340	26.406	22005	21.125	17604
" x ⅜	33.15	9.75	42.250	35208	31.687	26406	25.350	21125
" x 7/16	38.68	11.375	49.292	41076	36.969	30807	29.575	24646
" x ½	44.20	13.00	56.333	46944	42.250	35208	33.800	28167
28 x ¼	23.80	7.00	32.667	27222	24.500	20417	19.600	16333
" x 5/16	29.75	8.75	40.833	34028	30.625	25521	24.500	20417
" x ⅜	35.70	10.50	49.000	40833	36.750	30625	29.400	24500
" x 7/16	41.65	12.25	57.167	47639	42.875	35729	34.300	28583
" x ½	47.60	14.00	65.333	54444	49.000	40833	39.200	32667
30 x ¼	25.50	7.50	37.500	31250	28.125	23437	22.500	18750
" x 5/16	31.88	9.375	46.875	39062	35.156	29297	28.125	23437
" x ⅜	38.25	11.25	56.250	46875	42.187	35156	33.750	28125
" x 7/16	44.63	13.125	65.625	54687	49.219	41016	39.375	32812
" x ½	51.00	15.00	75.000	62500	56.250	46875	45.000	37500
32 x ¼	27.20	8.00	42.667	35556	32.000	26667	25.600	21333
" x 5/16	34.00	10.00	53.333	44444	40.000	33333	32.000	26667
" x ⅜	40.80	12.00	64.000	53333	48.000	40000	38.400	32000
" x 7/16	47.60	14.00	74.667	62222	56.000	46667	44.800	37333
" x ½	54.40	16.00	85.333	71111	64.000	53333	51.200	42667
36 x ¼	30.60	9.00	54.000	45000	40.500	33750	32.400	27000
" x 5/16	38.25	11.25	67.500	56250	50.625	42187	40.500	33750
" x ⅜	45.90	13.50	81.000	67500	60.750	50625	48.600	40500
" x 7/16	53.55	15.75	94.500	78750	70.875	59062	56.700	47250
" x ½	61.20	18.00	108.000	90000	81.000	67500	64.800	54000
40 x ¼	34.00	10.00	66.667	55556	50.000	41667	40.000	33333
" x 5/16	42.50	12.50	83.333	69444	62.500	52083	50.000	41667
" x ⅜	51.00	15.00	100.000	83333	75.000	62500	60.000	50000
" x 7/16	59.50	17.50	116.667	97222	87.500	72917	70.000	58333
" x ½	68.00	20.00	133.333	111111	100.000	83333	80.000	66667
42 x ¼	35.70	10.50	73.500	61250	55.125	45937	44.100	36750
" x 5/16	44.63	13.125	91.875	76562	68.906	57422	55.125	45937
" x ⅜	53.55	15.75	110.250	91875	82.687	68906	66.150	55125
" x 7/16	62.48	18.375	128.625	107187	96.469	80391	77.175	64312
" x ½	71.40	21.00	147.000	122500	110.250	91875	88.200	73500

* See Fig. 3, page 575.

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6.—Plate Girders (Steel)—Concluded.

Properties of Web Plates Only.*

(M' = moment in ft.-lbs.; M'' = moment in in.-lbs.)

1	2	3	4	5	6	7	8	9
Web Plate. Depth. Width. d b Ins.	Weight per Foot Gross Section. W Lbs.	Area of Gross Section. A Sq. Ins.	Full Efficiency of Web; ⅙ Area Considered at Upper and Lower Edges; No Deduction for Rivet-Holes, etc.		75% Efficiency of web; ⅙ Area Considered at Upper and Lower Edges; 25% Deduct'n for Rivet Holes, etc.		60% Efficiency of Web; $\frac{1}{10}$ Area Considered at Upper and Lower Edges; 40% Deduct'n for Rivet Holes, etc.	
			Section Modulus $S = \frac{Ad}{6}$	Moment of Resistance $M' = \frac{fS}{12}$ (f = 10000) Ft.-Lbs.	Section Modulus $S = \frac{Ad}{8}$	Moment of Resistance $M' = \frac{fS}{12}$ (f = 10000) Ft.-Lbs.	Section Modulus $S = \frac{Ad}{10}$	Moment of Resistance $M' = \frac{fS}{12}$ (f = 10000) Ft.-Lbs.
44 x 5/16	46.75	13.75	100.83	84028	75.62	63021	60.50	50417
" x ⅜	56.10	16.50	121.00	100833	90.75	75625	72.60	60500
" x 7/16	65.45	19.25	141.17	117639	105.87	88229	84.70	70583
" x ½	74.80	22.00	161.33	134444	121.00	100833	96.80	80667
48 x 5/16	51.00	15.00	120.00	100000	90.00	75000	72.00	60000
" x ⅜	61.20	18.00	144.00	120000	108.00	90000	86.40	72000
" x 7/16	71.40	21.00	168.00	140000	126.00	105000	100.80	84000
" x ½	81.60	24.00	192.00	160000	144.00	120000	115.20	96000
50 x 5/16	53.13	15.625	130.21	108507	97.53	81276	78.12	65104
" x ⅜	63.75	18.75	156.25	130208	117.04	97531	93.75	78125
" x 7/16	74.38	21.875	182.29	151910	136.54	113786	109.37	91146
" x ½	85.00	25.00	208.33	173611	156.05	130042	125.00	104167
54 x 5/16	57.38	16.875	151.87	126562	113.91	94922	91.12	75937
" x ⅜	68.85	20.25	182.25	151875	136.69	113906	109.35	91125
" x 7/16	80.33	23.625	212.62	177187	159.47	132891	127.57	106312
" x ½	91.80	27.00	243.00	202500	182.25	151875	145.80	121500
60 x ⅜	76.50	22.50	225.00	187500	168.75	140625	135.00	112500
" x 7/16	89.25	26.25	262.50	218750	196.87	164062	157.50	131250
" x ½	102.00	30.00	300.00	250000	225.00	187500	180.00	150000
70 x ⅜	89.25	26.25	306.25	255208	229.69	191406	183.75	153125
" x 7/16	104.13	30.625	357.29	297743	267.97	223307	214.37	178646
" x ½	119.00	35.00	408.33	340278	306.25	255208	245.00	204167
80 x 7/16	119.00	35.00	466.67	388889	350.00	291667	280.00	233333
" x ½	136.00	40.00	533.33	444444	400.00	333333	320.00	266667
90 x 7/16	133.88	39.375	590.62	492187	442.97	369141	354.37	295312
" x ½	153.00	45.00	675.01	562500	506.25	421875	405.00	337500
100 x 7/16	148.75	43.75	729.17	607639	546.87	455729	437.50	364583
" x ½	170.00	50.00	833.33	694444	625.00	520833	500.00	416667
120 x ½	204.00	60.00	1200.00	1000000	900.00	750000	720.00	600000

* See Fig. 3, page 575.

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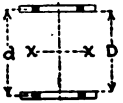

Fig. 4.

7.—Plate Girders (Steel).

Properties of Flange Plates only.

M' = moment in ft.-lbs.;
M'' = moment in in.-lbs.

1	2	3	4	5	6	7	8	9	10	11
2 Flange (Cover) Plates. Size. Ins.	Weight per Ft. Gross Section. W Lbs.	Depth b. to b. of Angles. D Ins.	Distance c. to c. of Gravity. d Ins.	Net Area Each Flange (1 Plate) A Sq. Ins	Section Modulus $S = \frac{I}{y} = \frac{M''}{f} (= Ad)$ Ins.	Increase of S for Each Additional 1 in. Width of Plate.	Increase of S for Each Additional 12 ins. of Depth *(D).	Resisting Moment $M' = \frac{fI}{12y}$ (For f = 10000). Ft.-Lbs.	Increase of M' for Each Additional 1 in. Width of Plate.	Increase of M' for Each Additional 12 ins. of Depth †(D).
(a) Use with smaller Flange Angles. Rivets, ¾". Two ⅞" rivet-holes deducted from Each Plate for A.										
6 x ¼	10.20	12¼	12½	1.06	13.25	3.12	12.72	11 040	2 600	96.0*
" x ⅜	15.30	"	12⅝	1.59	20.07	4.73	19.08	16 730	3 950	95.0 "
" x ½	20.40	"	12¾	2.13	27.16	6.37	25.56	22 630	5 310	94.1 "
" x ⅝	25.50	"	12⅞	2.66	34.25	8.05	31.92	28 540	6 710	93.2 "
7 x ¼	11.90	12¼	12½	1.31	16.37	3.12	15.72	13 650	2 600	96.0*
" x ⅜	17.85	"	12⅝	1.97	24.87	4.73	23.64	20 730	3 950	95.0 "
" x ½	23.80	"	12¾	2.63	33.53	6.37	31.56	27 940	5 310	94.1 "
" x ⅝	29.75	"	12⅞	3.28	42.23	8.05	39.36	35 190	6 710	93.2 "
8 x ¼	13.60	18¼	18½	1.56	28.86	4.62	18.72	24 050	3 850	64.9*
" x ⅜	17.00	"	18⅝	1.95	36.20	5.80	23.40	30 160	4 830	64.6 "
" x ⅝	20.40	"	18⅞	2.34	43.58	6.98	28.08	36 320	5 820	64.4 "
" x 7/16	23.80	"	18 11/16	2.73	51.02	8.18	32.76	42 520	6 810	64.2 "
" x ½	27.20	"	18¾	3.13	58.69	9.37	37.56	48 910	7 810	64.0 "
" x 9/16	30.60	"	18 13/16	3.52	66.22	10.58	42.24	55 180	8 820	63.8 "
" x ⅝	34.00	"	18⅞	3.91	73.80	11.80	46.92	61 500	9 830	63.6 "
" x 11/16	37.40	"	18 15/16	4.30	81.43	13.02	51.60	67 860	10 850	63.4 "
" x ¾	40.80	"	19	4.69	89.11	14.25	56.28	74 260	11 870	63.2 "
" x 13/16	44.20	"	19 1/16	5.08	96.84	15.49	60.96	80 700	12 910	63.0 "
" x ⅞	47.60	"	19 1/8	5.47	104.61	16.73	65.64	87 180	13 950	62.7 "
" x 15/16	51.00	"	19 3/16	5.86	112.44	17.99	70.32	93 700	14 990	62.5 "
" x1	54.40	"	19¼	6.25	120.31	19.25	75.00	100 260	16 040	62.3 "
9 x ¼	15.30	24¼	24½	1.81	44.35	6.12	21.72	36 950	5 100	49.0*
" x ⅜	22.95	"	24⅝	2.72	66.98	9.23	32.64	55 820	7 700	48.7 "
" x ½	30.60	"	24¾	3.63	89.84	12.37	43.56	74 870	10 310	48.5 "
" x ⅝	38.25	"	24⅞	4.53	112.68	15.55	54.36	93 900	12 960	48.2 "
" x ¾	45.90	"	25	5.44	136.00	18.75	65.28	113 330	15 620	48.0 "
10x ¼	17.00	24¼	24½	2.06	50.47	6.12	24.72	42 060	5 100	49.0*
" x ⅜	25.50	"	24⅝	3.09	76.09	9.23	37.08	63 410	7 700	48.7 "
" x ½	34.00	"	24¾	4.13	102.22	12.37	49.56	85 180	10 310	48.5 "
" x ⅝	42.50	"	24⅞	5.16	128.36	15.55	61.92	106 960	12 960	48.2 "
" x ¾	51.00	"	25	6.19	154.75	18.75	74.28	129 000	15 620	48.0 "
11x ¼	18.70	24¼	24½	2.31	56.60	6.12	27.72	47 160	5 100	49.0*
" x ⅜	28.05	"	24⅝	3.47	85.45	9.23	41.64	71 210	7 700	48.7 "
" x ½	37.40	"	24¾	4.63	114.59	12.37	55.56	95 490	10 310	48.5 "
" x ⅝	46.75	"	24⅞	5.78	143.78	15.55	69.36	119 810	12 960	48.2 "
" x ¾	36.10	"	25	6.94	173.50	18.75	83.28	144 580	15 620	48.0 "

* Percent of increase is same as for column 11.
† Actual increase in ft.-lbs. = 10 000 A (col. 5).

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7.—Plate Girders (Steel)—Continued.

Properties of Flange Plates only.

M' = moment in ft.-lbs.;
M'' = moment in in.-lbs.

1	2	3	4	5	6	7	8	9	10	11
2 Flange (Cover) Plates.	Weight per Ft. Cross Section.	Depth b. to b. of Angles.	Distance c. to c. of Gravity.	Net Area Each Flange (1 Plate)	Section Modulus $S = \frac{I}{y} = \frac{M''}{f} (= Ad)$	Increase of S for Each Additional		Resisting Moment $M' = \frac{fI}{12y}$	Increase of M' for Each Additional	
Size. Ins.	W Lbs.	D Ins.	d Ins.	A Sq. Ins	Ins.	1 in. Width of Plate.	12 in. of Depth *(D).	(For f = 10000). Ft.-Lbs.	1 in. Width of Plate.	12 ins. of Depth †(D).
(a) Use with smaller Flange Angles. Rivets, ¾". Two ⅞" rivet-holes deducted from Each Plate for A.										
12x ¼	20.40	24¼	24½	2.56	62.72	6.12	30.72	52 270	5 100	49.0%
" x ⅜	30.60	"	24⅜	3.84	94.56	9.23	46.08	78 800	7 700	48.7 "
" x ½	40.80	"	24¾	5.13	126.97	12.37	61.56	105 810	10 310	48.5 "
" x ⅝	51.00	"	24⅞	6.41	159.45	15.55	76.92	132 870	12 960	48.2 "
" x ¾	61.20	"	25	7.69	192.25	18.75	92.28	160 210	15 620	48.0 "
(b) Use with 6 x 3½ and 6 x 4 Angles. Rivets, ¾", Two ⅞" rivet-holes deducted from Each Plate for A.										
13x ⅜	33.15	30¼	30⅝	4.22	129.24	11.48	50.64	107 700	9 570	39.2%
" x ½	44.20	"	30¾	5.63	173.12	15.37	67.56	144 270	12 810	39.0 "
" x ⅝	55.25	"	30⅞	7.03	217.05	19.30	84.36	180 880	16 080	38.9 "
" x ¾	66.30	"	31	8.44	261.64	23.25	101.28	218 030	19 370	38.7 "
" x ⅞	77.35	"	31⅛	9.84	306.27	27.23	118.08	255 230	22 700	38.6 "
" x1	88.40	"	31¼	11.25	351.56	31.25	135.00	292 970	26 040	38.4 "
14x ⅜	35.70	30¼	30⅝	4.59	140.57	11.48	55.08	117 140	9 570	39.2%
" x ½	47.60	"	30¾	6.13	188.50	15.37	73.56	157 080	12 810	39.0 "
" x ⅝	59.49	"	30⅞	7.66	236.50	19.30	91.92	197 090	16 080	38.9 "
" x ¾	71.40	"	31	9.19	284.89	23.25	110.28	237 410	19 370	38.7 "
" x ⅞	83.30	"	31⅛	10.72	333.66	27.23	128.64	278 050	22 700	38.6 "
" x1	95.20	"	31¼	12.25	382.81	31.25	147.00	319 010	26 040	38.4 "
(c) Use with 6 x 6 and 8 x 8 Angles. Rivets, ⅞". Two 1" rivet-holes deducted from Each Plate for A.										
14x ½	47.60	30¼	30¾	6.00	184.50	15.37	72.00	153 750	12 810	39.0%
" x ⅝	59.50	"	30⅞	7.50	231.56	19.30	90.00	192 970	16 080	38.9 "
" x ¾	71.42	"	31	9.00	279.00	23.25	108.00	232 500	19 370	38.7 "
" x ⅞	83.30	"	31⅛	10.50	326.81	27.23	126.00	272 340	22 700	38.6 "
" x1	95.20	"	31¼	12.00	375.00	31.25	144.00	312 500	26 040	38.4 "
15x ½	51.00	30¼	30¾	6.50	199.88	15.37	78.00	166 560	12 810	39.0%
" x ⅝	63.75	"	30⅞	8.13	251.01	19.30	97.56	209 180	16 080	38.9 "
" x ¾	76.50	"	31	9.75	302.25	23.25	117.00	251 870	19 370	38.7 "
" x ⅞	89.25	"	31⅛	11.38	354.20	27.23	136.56	295 170	22 700	38.6 "
" x1	102.00	"	31¼	13.00	406.25	31.25	156.00	338 540	26 040	38.4 "
" x1⅛	114.75	"	31⅜	14.63	459.02	35.30	175.56	382 510	29 410	38.2 "
" x1¼	127.50	"	31½	16.25	511.88	39.37	195.00	426 560	32 810	38.1 "

* Percent of increase is same as for column 11.
† Actual increase in ft.-lbs. = 10 000 A (col. 5).

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7.—Plate Girders (Steel)—Concluded.

Properties of Flange Plates only.

(See Fig. 4, page 580.)

M' = moment in ft.-lbs.;
M'' = moment in in -lbs.

1	2	3	4	5	6	7	8	9	10	11
2 Flange (Cover) Plates.	Weight per Ft. Gross Section.	Depth b. to b. of Angles.	Distance c. to c. of Gravity.	Net Area Each Flange (1 Plate)	Section Modulus $S = \frac{I}{y} = \frac{M''}{f} (= Ad)$	Increase of S for Each Additional		Resisting Moment $M' = \frac{fI}{12y}$ (For f = 10000).	Increase of M' for Each Additional	
Size. Ins.	W Lbs.	D Ins.	d Ins.	A Sq. Ins.	Ins.	1 in. Width of Plate.	12 ins. of Depth *(D).	Ft.-Lbs.	1 in. Width of Plate.	12 ins. of Depth †(D).
(c) Use with 6 x 6 and 8 x 8 Angles. Rivets, ⅞". Two 1" rivet-holes deducted from Each Plate for A.										
16x ½	54.40	30¼	30¾	7.00	215.25	15.37	84.00	179 370	12 810	39.0 ¢
" x ⅝	68.00	"	30⅞	8.75	270.16	19.30	105.00	225 130	16 080	38.9 "
" x ¾	81.60	"	31	10.50	325.50	23.25	126.00	271 250	19 370	38.7 "
" x ⅞	95.20	"	31⅛	12.25	381.28	27.23	147.00	317 730	22 700	38.6 "
" x1	108.80	"	31¼	14.00	437.50	31.25	168.00	364 580	26 040	38.4 "
" x1⅛	122.40	"	31⅜	15.75	494.16	35.30	189.00	411 800	29 410	38.2 "
" x1¼	136.00	"	31½	17.50	551.25	39.37	210.00	459 380	32 810	38.1 "
" x1⅜	149.60	"	31⅝	19.25	608.78	43.48	231.00	507 320	36 240	37.9 "
" x1½	163.20	"	31¾	21.00	666.75	47.62	252.00	555 620	39 690	37.8 "
18x ½	61.20	36¼	36¾	8.00	294.00	18.37	96.00	245 000	15 310	32.7 ¢
" x ⅝	76.50	"	36⅞	10.00	368.75	23.05	120.00	307 290	19 210	32.6 "
" x ¾	91.80	"	37	12.00	444.00	27.75	144.00	370 000	23 120	32.4 "
" x ⅞	107.10	"	37⅛	14.00	519.75	32.48	168.00	433 120	27 070	32.3 "
" x1	122.40	"	37¼	16.00	596.00	37.25	192.00	496 670	31 040	32.2 "
" x1⅛	137.70	"	37⅜	18.00	672.75	42.05	216.00	560 620	35 040	32.1 "
" x1¼	153.00	"	37½	20.00	750.00	46.87	240.00	625 000	39 060	32.0 "
" x1⅜	168.30	"	37⅝	22.00	827.75	51.73	264.00	689 790	43 110	31.9 "
" x1½	183.60	"	37¾	24.00	906.00	56.62	288.00	755 000	47 190	31.8 "

* Percent of increase is same as for column 11.
† Actual increase in ft.-lbs. = 10 000 A (col. 5).

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Depth of Beam Ins.	Wt. per Ft., Lbs.	Area of Sec., Sq. Ins.	Thickness of Web, Ins.	Width of Flange, Ins.	Moment of Inertia. I	Section Modulus. S	For f= 16000 Lbs. per Sq. Ins. Buildings. C	For f= 12500 Lbs. per Sq. Ins. Moving Lds. C'	Load in Tons of 2000 Lbs.	Number of Beams.	Depth, Ins.	Wt. of Each Beam, Lbs. per Ft.	Section Modulus of Two Beams.	Girder Beams. Lbs. per Ft.	Remarks
30	200.0	58.85	.75	15.00	9154.7	610.3	6,510,000	5,085,900	95.2	.....	.....	.....	.....	.....	The diff. in weights in preceding column does not include separators for assembling the Am. Standard beams into girders.
30	175.0	51.35	.68	12.00	7851.8	523.5	5,583,500	4,362,100	81.1	.....	.....	.....	.....	.....	
28	180.0	52.98	.69	14.35	7269.0	519.2	5,538,200	5,326,700	81.3	.....	.....	.....	.....	.....	
28	162.5	47.81	.65	12.00	6465.1	461.8	4,925,800	3,848,200	73.8	.....	.....	.....	.....	.....	
26	160.0	47.00	.63	13.60	5618.7	432.2	4,610,200	3,601,700	68.3	.....	.....	.....	.....	.....	
26	150.0	44.13	.62	12.00	5200.4	400.0	4,267,000	3,333,600	66.6	2	24	100	396.8	50	
24	140.0	41.03	.56	13.00	4241.9	353.5	3,770,700	2,945,800	54.9	2	24	80	348.0	20	
24	120.0	35.31	.51	12.00	3630.7	302.6	3,227,200	2,521,200	46.5	2	20	85	301.8	50	
20	140.0	41.28	.64	12.50	2938.3	293.8	3,134,200	2,448,600	62.4	2	20	80	293.4	20	
20	112.0	32.88	.52	12.00	2368.9	236.9	2,526,700	1,974,000	45.6	2	20	65	234.0	18	
18	92.0	27.09	.47	11.50	1595.3	177.3	1,890,800	1,477,200	37.1	2	18	55	176.8	18	
15	140.0	41.28	.80	11.75	1591.5	212.2	2,263,500	1,768,300	67.3	2	15	80	212.2	20	
15	104.0	30.58	.60	11.25	1219.7	162.6	1,734,700	1,355,200	47.4	2	15	60	162.4	16	
15	73.0	21.52	.42	10.50	886.5	118.2	1,260,900	985,100	28.8	2	15	42	117.8	11	
12	70.0	20.60	.445	10.00	540.9	90.2	961,600	751,200	28.0	2	12	40	89.6	10	
12	55.0	16.12	.35	9.75	432.0	72.0	768,000	600,000	19.7	2	12	31.5	72.0	8	
10	44.0	12.95	.30	9.00	244.3	48.9	521,200	407,200	14.3	2	10	25	48.8	6	
9	38.0	11.18	.29	8.50	169.8	37.7	402,500	314,400	12.8	2	9	21	37.8	4	
8	32.5	9.52	.28	8.00	113.9	28.5	303,800	237,400	11.4	2	8	18	28.4	3.5	

Note.—C or $C' = WL = 8M = \frac{8}{3}fS$; W = total unif. ld. (inc. wt. of beam), L = span in ft., M = moment of forces in ft.-lbs., f = safe fiber stress in lbs. per sq. in., $S = I \div$ half depth of beam.

Reference.—For Standard I-Beams, see Table 5, Sec. 30, pages 554, 555.

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9.—BETHLEHEM SPECIAL I BEAMS.
And Comparison with ordinary Standard I Beams.

Depth of Beam. Ins.	Wt. per Ft., Lbs.	Area of Sec., Sq. Ins.	Thickness of Web, Ins.	Width of Flange, Ins.	Neutral Axis perp. to Web at Center. Moment of Inertia I	Neutral Axis perp. to Web at Center. Section Modulus. S	Coefficients of Strength. For $f=$ 16000 Lbs. per Sq. In. Buildings. C	Coefficients of Strength. For $f=$ 12500 Lbs. per Sq. In. Moving Lds. C'	Max. Safe Shear on Web, in Tons of 2000 Lbs.	Equivalent American Standard Beams. Number of Beams.	Equivalent American Standard Beams. Depth, Ins.	Equivalent American Standard Beams. Wt. of Each Beam, Lbs. per Ft.	Equivalent American Standard Beams. Section Modulus	Economy of Special Beams. Lbs. per Ft.	Remarks.
30	120.0	35.30	.540	10 50	5239.6	349.3	3,726,000	2,910,900	50.6	2	24	80.	348.0	40.*	The diff. in weights in preceding column indicated by a star(*) does not include separators for assembling the Am. standard beams into girders.
28	105.0	30.88	.500	10.00	4014.0	286.7	3,058,400	2,389,400	43.2	2	20	80.	293.4	55.*	
26	90.0	26.49	.460	9.50	2977.1	229.0	2,442,800	1,908,400	36.5	2	20	65.	234.0	40.*	
24	84.0	24.80	.460	9.25	2382.0	198.5	2,117,300	1,654,100	37.2	1	24	100.	198.4	16.0	
24	83.0	24.59	.520	9.13	2240.6	186.7	1,991,900	1,556,200	45.6	1	24	90.	186.6	7.0	
24	73.0	21.47	.390	9.00	2091.3	174.3	1,858,700	1,452,100	25.7	1	24	80.	174.0	7.0	
20	82.0	24.17	.570	8.89	1560.0	156.0	1,663,800	1,299,800	51.5	1	20	90.	155.8	8.0	
20	72.0	21.37	.430	8.75	1466.7	146.7	1,564,300	1,222,100	32.7	1	20	80.	146.7	8.0	
20	69.0	20.26	.520	8.145	1269.0	126.9	1,353,500	1,057,400	42.9	1	20	75.	126.9	6.0	
20	64.0	18.86	.450	8.075	1221.9	122.2	1,303,600	1,018,400	34.2	1	20	70.	122.0	6.0	
20	59.0	17.36	.375	8.000	1172.1	117.2	1,250,300	976,800	25.3	1	20	65.	117.0	6.0	
18	59.0	17.40	.495	7.675	883.1	98.1	1,046,900	817,900	38.6	1	18	65.	97.9	6.0	
18	54.0	15.87	.410	7.590	842.2	93.6	997,900	779,600	29.2	1	18	59.	92.6	5.0	
18	52.0	15.24	.375	7.555	825.1	91.7	977,700	763,800	24.8	.....	.....	.....	.....	.....	
18	48.5	14.25	.320	7.500	798.3	88.7	946,100	739,100	18.0	1	18	55.	88.4	6.5	
15	71.0	20.95	.520	7.500	796.0	106.1	1,132,400	884,700	39.7	1	15	80.	106.1	9.0	
15	64.0	18.81	.605	7.195	664.7	88.6	945,600	738,700	47.0	1	15	70.	88.5	6.0	
15	54.0	15.88	.410	7.000	609.9	81.3	867,600	677,800	27.2	1	15	60.	81.2	6.0	
15	46.0	13.52	.440	6.810	484.6	64.6	689,500	538,700	29.8	1	15	50.	64.5	4.0	
15	41.0	12.02	.340	6.710	456.7	60.9	649,500	507,400	20.9	.....	.....	.....	.....	.....	
15	38.0	11.27	.290	6.666	442.5	59.0	629,500	491,800	[illegible]	.....	.....	.....	.....	.....	

Reinforced Concrete Beams.—The following Working Stresses for Static Loads are recommended by the Special Committee of the Am. Soc. C. E., on Concrete and Reinforced Concrete. (See Trans. A. S. C. E., Vol. LXVII., page 452.) For Notation and Formulas, see Sec. 25, Masonry, page 446. For working stresses for columns, see Sec. 32, Columns, page 609.

Compressive Strength of Stone Concrete.—Average, 2000 lbs. per sq. in. at 28 days, when tested in cylinders 8 ins in dia. and 16 ins. long, under laboratory conditions of manufacture and storage.

Bearing.—650 lbs. per sq. in. on 2000-lb. concrete.

Bending.—Compression on extreme fiber of beam, 650 lbs. per sq. in. for 2000-lb. concrete; adjacent to support of continuous beams, 750 lbs. per sq. in.

Pure Shear.—When uncombined with compression normal to the shearing surface, and with all tension normal to the shearing plane provided for by metal reinforcement, 120 lbs. per sq. in. on 2000-lb. concrete.

Shear with Diagonal Tension.—In beams without diagonal reinforcement, 40 lbs. per sq. in. for 2000-lb. concrete.

Bond.—Between concrete and plain reinforcing bars, 80 lbs. per sq. in. for 2000-lb. concrete; between concrete and drawn wire, 40 lbs. per sq. in.

Reinforcement.—Tensile stress in steel shall not exceed 16000 lbs. per sq. in.

EXCERPTS AND REFERENCES.

Tests of Reinforced Concrete Beams (By W. K. Hatt. Proc. A. S. T. M., 1902; Eng. News, July 17, 1902).—This is a very complete Paper and contains numerous diagrams and tables of the tests, with discussion of same; also includes Hatt's formula for reinforced concrete beams.

Computing the Strength of Reinforced Concrete Beams (By Edwin Thacher. Eng. News, Feb. 12, 1903).—Discussion of above. "It is the practice of the writer to give the concrete a certain factor of safety at the end of one month, and to give the steel the same factor of safety as the concrete at the end of six months, as it is evident that if there was not an excess of steel in one month, there would be a deficiency after the concrete gained its full strength. For example, to design a slab of length $l = 6$ ft., using 60000-lb. steel, for a total load w of 400 lbs. per sq. ft., that shall have a factor of safety of 4 in one month, but in which the tensile strength of the steel shall be equal to the compressive strength of the concrete after 6 months:—Then, the distance from top of beam to center of reinforcement $= d = \sqrt{Pw + 883} = \sqrt{6 \times 6 \times 400 + 883} = 3.92$ ins.; or, if the center of bars is 1.08 ins. from bottom of concrete, the total height $h = 3.92 + 1.08 = 5$ ins., and the area of metal for 1" width of beam $= A = d + 90 = 3.92 + 90 = .0435$ sq. in. Using Thacher bars, having original diam. of $\frac{3}{8}$ in., area $= .276$, weight per ft. $= .94$ lb., dist. c. to c. bars $= .276 + .0435 = 6.35$ ins." Gives tabulated formulas for calculating rein. conc. beams ($E_c = 30,000,000$).

Table of Tests on the Union Between Concrete and Steel (At Mass. Inst. Tech. Eng. News, May 5, 1904).—Types of rods used in the tests were: Thacher, Ransome, Johnson, and plain round and square rods.

Tests of Reinforced Concrete Beams (By Arthur N. Talbot. Proc. A. S. T. M., 1904. Eng. News, Aug. 11, 1904).—Types of rods used in the tests were: Thacher, Ransome, Johnson, Kahn, and plain round and square rods. Table and Diagrams.

Tests of Reinforced Concrete Beams and Floors (At St. Louis Exposition. By R. L. Humphrey. Eng. News, Sept. 21, 1905).—Illustrated diagrams of beams tested.

The Design of Reinforced Concrete Structures (By F. W. Keyser and E. L. Heidenreich; and C. A. P. Turner).—Discussion.

Notes on the Stress-Deformation Curve in Concrete Beams (By N. Werenskiold. Eng. News, April 5, 1906.)—Diagrams and formulas.

Economic Design of Reinforced Concrete (By F. W. Hanna. Eng. News, Feb. 21, 1907).—Gives a table of the economical working stresses and percentages of steel reinforcement for varying relative costs of steel and concrete—calculated for unit compressive working stress in outer fiber of concrete $= 600$ lbs. per sq. in., and for $r = E_s \div E_c = 10$. (See, also, Eng. News, June 20, 1907.)

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Diagram for Proportioning Reinforced Concrete Beams (By A. H. Perkins. Eng. News, April 25, 1907).—It is a compound diagram, comprising (1) a set of lines giving the bending moment in a simple beam for any span and any load per foot of length and per inch of width; (2) a set of lines giving the corresponding values of breadth and depth of reinforced concrete beam to resist this moment. This latter part of the diagram is based on the following unit values:—Max. comp. in concrete $c = 600$ lbs. per sq. in.; tension in steel $s = 12{,}000$ lbs. per sq. in.; $Es \div Ec = n = 10$; percentage of steel $p = 0.95\%$; moment capacity of beam b inches wide and d inches deep $= M = 100\ bd^2$. The stress-strain curve of concrete is assumed to be between the straight line and the parabola. (See, also, Tiechman's formula and diagrams in Eng. News, July 11, 1907.)

Reinforced Concrete T-Beam and Column Tests (At Univ. of Ill. By A. N. Talbot. Bulletins Nos. 10 and 12 of the Eng. Exper. Station, Univ. of Ill.; Eng. News, July 11, 1907).—Diagrams of beams and columns tested, and tables of tests.

Tests of Adhesion of Steel to Concrete in Beams (By L. J. Johnson. Jl. of Assn. of Eng. Soc. for June, 1907; Eng. News, Aug. 15, 1907).—Tables and diagrams.

Effect of Time Element in Loading Reinforced Concrete Beams (By W. K. Hatt. Proc. A. S. T. M., 1907; Eng. News, Oct. 24, 1907).—Tables and diagrams.

Section Modulus Diagram for Plate and Lattice Girders (By L. R. Shellenberger. Eng. News, Nov. 26, 1908).—The diagram is for flange angles 6"x 6"; and for section moduli up to 3400 ins., and depth of web up to 85 ins.

Tests of Standard I Beams and Special I Beams and Girder Beams (By Edgar Marburg. Proc. A. S. T. M., Vol. IX., 1909; Eng. News, Aug. 12, 1909).—Illustrations, diagrams and tables. Generally speaking, the modulus of rupture in lbs. per sq. in. decreases as the depth of the girder increases. In actual practice, however, the beams would be supported laterally, at intervals, which might affect the comparative results of the experiments.

Moments in Continuous Concrete Beams Under Uniform Loading (By R. E. Spaulding. Eng. News, Sept. 30, 1909).—Numerous formulas and diagrams.

Slide-Rule for Reinforced-Concrete Slabs (By A. W. French. Eng. News, Feb. 3, 1910).—Illustrated.

Table of Moments of Inertia of Flat Rectangles (By H. Loewenhern. Eng. News, Feb. 24, 1910).—For widths b (advancing by one inch from 6 to 40 ins.) and depths d (advancing by sixteenths of an inch from ¼ to 3 inches).

Some Deductions from Warburg's I-Beam Tests (By C. J. Tilden. Eng. News, Feb. 24, 1910).—With discussion by Edgar Marburg.

Comparison of Methods of Computing the Strength of Flat Reinforced Concrete Plates (By A. B. MacMillan. Paper, Natl. Assn. Cement Users, Feb. 21-25, 1910; Eng. News, Mar. 31, 1910).—Illustrated. Discussions: Cantilever method; Turneaure and Maurer method; Grashof's Analysis; Mensch's method; Turner's method; Macmillan method; Mushroom floor.

Interesting Illustrations.

Description.	Eng. Rec.
Diagrams for reinforced-concrete beams.—Schermerhorn	Mar. 6, '09
Diagrams for disigning reinforced-concrete beams—Carter	June 18, '10
Instruction sheet for placing floor reinforcement	June 25, '10

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32.—PROPERTIES AND TABLES OF COLUMNS.

General Stresses.—A column is a pillar or strut acting mainly in compression; but in addition to the direct compressive stress, it has also to resist, to a greater or less extent, shearing and bending. To what degree the shearing stresses enter into and affect a column we are unable to say, and no column formula has yet been devised which includes the shearing effect of the stresses. Indeed, omitting the effect of shear altogether, it is a mooted question whether we are able even to combine rationally in a single formula the effects of direct compression and bending.

Shearing Effect.—Although the effect of shear is seldom noticeable in an ordinary column, yet it becomes quite apparent in a short block or cube of stone, cement, or other granular material when tested in compression. The sides of the block flake away, often leaving the broken specimen like a shattered pyramid or cone. The same shearing stresses which rupture the block of stone are undoubtedly present in complex form in any column, but our means of analysis are limited. If, instead of the stone, we select a short block of wood, with the grain in the direction of the pressure, the shearing will be along the grain, or planes of least resistance, although the tendency to shear will be along sloping, pyramidal planes.

If, now, a sheet of lead is interposed between the stone cube and one of the platforms of the testing machine the flaking of the stone will be more pronounced, due to the spreading or lateral expansion of the lead as it is flattened against the stone.* This illustrates in an exaggerated way how the material in any column tends to spread out under pressure, and weaken the resistance to shear.

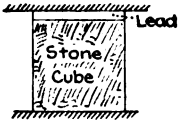

Fig. 1.

Notation for Column Formulas:

A = area of column section, in sq. ins.;
I = moment of inertia of section in ins. $= Ar^2$;
r = radius of gyration of section in ins. $= \sqrt{I \div A}$;
d = least diam. of rectangular section, in ins.;
d_0 = diameter of round column, in ins.;
P = total load on column, in lbs.;
p = load per sq. in. on column $= P \div A$;
y = distance, in ins., of most strained fiber, from neutral axis;
v = lateral deflection of column in ins., under load P;
e = eccentricity of loading on columns, in ins.;
fc = compressive stress in lbs. per sq. in. $= P \div A = p$;
fb = bending stress, outer fiber, in lbs. per sq. in. $= Pvy \div I = pvy \div r^2$;
f = max. outer fiber stress, comp. + bending, in lbs. per sq. in. $= fc + fb$;
fe = f at the elastic limit of the material;
fu = f at the ultimate strength of the material;
f' = f at any assignable value, for distinction;
E = modulus of elasticity of material, in lbs. per sq. in.;
l = effective length of column, in inches
L = effective length of column, in feet;

$$a = \text{value of } p \text{ in Euler's formula for long columns} = \frac{n\pi^2 E r^2}{l^2} = \frac{n\pi^2 E r^2}{144 L^2}:$$

In which $n = 1$ for 2 pivot ends — established by Euler,
$n = 5/2$ for 2 pin ends — established by Johnson,
$n = 1\frac{1}{4}$ for 1 pivot, 1 fixed — established by Euler,
$n = \frac{2}{3}$ for 1 pin, 1 flat — established by Johnson,
$n = 5/2$ for 2 flat ends — established by Johnson,
$n = 4$ for 2 fixed ends — established by Euler.

* For this reason sheets of lead or other soft material are now considered objectionable in testing machines; and, by some, on masonry abutments under bridge shoes or pedestals.

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Short Strut—Direct Compression Only.—By a short strut whose length is not over, say, 12 or 15 times its diameter or le For such a column we have the formula:

$$f = \frac{P}{A} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Whence $P = fA$ and $A = \frac{P}{f}$ .....

Short Strut—Eccentric Loading—The usual formula for t stress f in a short column eccentrically loaded, is

$$f = \frac{P}{A} + \frac{P\,e\,y}{I} = \frac{P}{A}\left(1 + \frac{ey}{r^2}\right)$$

it being assumed that the column is braced laterally, as in building, against falling over; and furthermore that there is appreciable lateral deflection v. If lateral deflection were a taken into account we would have

$$f = \frac{P}{A} + \frac{P(e+v)y}{I} = \frac{P}{A}\left[1 + \frac{(e+v)y}{r^2}\right] \ldots\ldots\ldots\ldots$$

Long Columns—Bending Only.—The following formulas end conditions of long columns, bending only, are adapt from Euler and T. H. Johnson.

Euler's Formulas:

2 Pivot Ends ● ●

$$P = \frac{\pi^2 EI}{l^2} = \frac{\pi^2 EI}{144\,L^2}$$

$$p = \frac{\pi^2 E r^2}{l^2} = \left(\frac{\pi r}{l}\right)^2 E = \left(\frac{\pi r}{12\,L}\right)^2 E$$

1 Pivot, 1 Fixed ● ▨

$$P = \frac{16\pi^2 EI}{9\,l^2} = \frac{\pi^2 EI}{81\,L^2}$$

$$p = \frac{16\pi^2 E r^2}{9\,l^2} = \left(\frac{4\pi r}{3l}\right)^2 E = \left(\frac{\pi r}{9L}\right)^2 E$$

2 Fixed Ends ▨ ▨

$$P = \frac{4\pi^2 EI}{l^2} = \frac{\pi^2 EI}{36\,L^2}$$

$$p = \frac{4\pi^2 E r^2}{l^2} = \left(\frac{2\pi r}{l}\right)^2 E = \left(\frac{\pi r}{6L}\right)^2 E$$

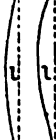

Fig.

Johnson's Formulas:

2 Pin Ends ○ ○

$$P = \frac{5\pi^2 EI}{3\,l^2} = \frac{5\pi^2 EI}{432\,L^2}$$

$$p = \frac{5\pi^2 E r^2}{3\,l^2} = \frac{5}{3}\left(\frac{\pi r}{l}\right)^2 E = \frac{5}{432}\left(\frac{\pi r}{L}\right)^2 E$$

1 Pin, 1 Flat ○ □

$$P = \frac{25\pi^2 EI}{12\,l^2} = \frac{25\pi^2 EI}{1728\,L^2}$$

$$p = \frac{25\pi^2 E r^2}{12\,l^2} = \frac{25}{12}\left(\frac{\pi r}{l}\right)^2 E = \frac{25}{1728}\left(\frac{\pi r}{L}\right)^2 E$$

2 Flat Ends □ □

$$P = \frac{5\pi^2 EI}{2\,l^2} = \frac{5\pi^2 EI}{288\,L^2}$$

$$p = \frac{5\pi^2 E r^2}{2\,l^2} = \frac{5}{2}\left(\frac{\pi r}{l}\right)^2 E = \frac{5}{288}\left(\frac{\pi r}{L}\right)^2 E$$

Fig

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There is one peculiar feature about the preceding formulas, namely, that they do not take into consideration the compressive (nor the shearing) resistance of the material, but simply the bending resistance or stiffness of the column. Hence they give excessive values for the loads P or p. Euler attempted to remedy this defect by placing an upper limit to the value of p, equal to the compressive resistance f_c of the material. This is illustrated in Fig. 5, a diagram showing the (elastic) strength of steel columns with pivoted ends.

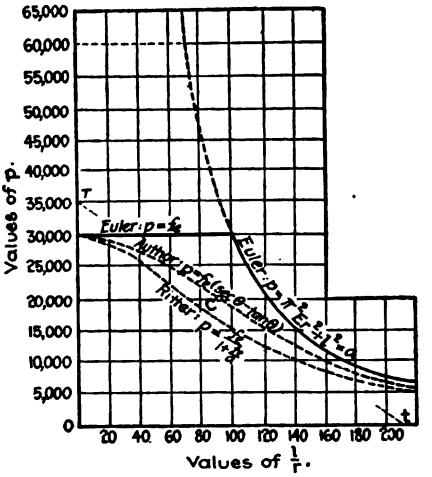

Fig. 5.

Ritter's Formula for Columns.—The following formula was proposed by Ritter in 1873 and has been worked out by others later. Although very ingenious and possessing considerable merit the reasoning by which it is deduced is not considered strictly rational and it takes its place with all other formulas in this respect. The equation of the elastic strength of columns is

$$p = \frac{P}{A} = \frac{fe}{1 + \frac{fel^2}{n\pi^2 E r^2}} = \frac{fe}{1 + \frac{fe}{na}} \quad \ldots\ldots\ldots\ldots (11)$$

The curve of this formula, for pivoted ends, is platted in Fig. 5. The method of applying a factor of safety to formula (11) is

$$p = \frac{f}{1 + \frac{f'}{na}} \quad \ldots\ldots\ldots\ldots (12)$$

in which f is considered to be the working stress of the material, and f' any value between the working stress and the ultimate stress. It is to be noted that with f constant, the factor of safety increases as f' increases, but not proportionately. Strictly speaking, the formula applies only within the elastic limit of the material, i. e., when f' (and f) do not exceed f_e. The value of f' should be above any possible value of f, and should always be greater than $2f$ for maximum *static* loading, or its equivalent. Again, with $f' > f$ the factor of safety is greater for long columns than for short.

Author's Formula for Columns.—From mechanics, $f = fc + fb = \frac{P}{A} + \frac{Pvy}{I} = p\left(1 + \frac{vy}{r^2}\right)$, whence

$$p = \frac{P}{A} = \frac{f}{1 + \frac{vy}{r^2}} \quad \text{(13)}$$

Again, the bending moment at center of column is

$$M = Pv \quad \text{(14)}$$

But according to Euler, $P = \frac{n\pi^2 EI}{l^2}$, whence (14) reduces to

$$v = \frac{M}{P} = \frac{Ml^2}{n\pi^2 EI} \quad \text{(15)}$$

Substituting this value of v in equation (13) we have

$$p = \frac{f}{1 + \frac{My}{I}\left(\frac{l^2}{n\pi^2 Er^2}\right)} = \frac{f}{1 + \frac{My}{nal}} \quad \text{(16)}$$

From the theory of flexure, we have,

$$M = \frac{I}{y} fb = \frac{I}{y}(f - fc) = \frac{I}{y}\left(f - \frac{P}{A}\right) = \frac{I}{y}(f - p) \quad \text{(17)}$$

Whence, by substitution, equation (16) reduces to

$$p = \frac{f}{1 + \frac{f - p}{na}} = \frac{f}{\left(1 - \frac{p}{na}\right) + \frac{f}{na}} \quad \text{(18)}$$

So far, our reasoning seems to be correct, but when we attempt to solve equation (18) we find that $p = f$ or na. From an examination of the last form of equation (18) it will be seen that the first term p, equal to the load per sq. in. on column, would be decreased by substituting unity for the quantity $\left(1 - \frac{p}{na}\right)$ in the denominator, and hence would be on the side of safety. Moreover, it would reduce to Ritter's formula (12), by making $f' = f$; thus, $p = \frac{f}{1 + \frac{f}{na}}$. But from the above discussion, we are certain that, for any column of given length l, $p < na$, and $\left(1 - \frac{p}{na}\right) <$ unity. Assuming tentatively the value of Ritter, $p = \frac{f}{1 + \frac{f}{na}}$, in order to arrive at an approximate value of $\left(1 - \frac{p}{na}\right)$ in the second form of equation (18), we have $p + \frac{pf}{na} = f$; whence, by transposition, $p = f - \frac{fp}{na}$, or $1 - \frac{p}{na} = \frac{p}{f}$; and this substituted in equation (18) gives

$$p = \frac{f}{\frac{p}{f} + \frac{f}{na}} \quad \text{(19)}$$

This reduces still further to

$$p = f(\sec\theta - \tan\theta) \quad \text{(20)}$$

the desired formula; in which $\tan\theta = \frac{fe}{2na}$.

In using formula (20), find

1st. The value of $\sqrt{\tan\theta}$, from Table 1, on the following page.
2nd. The corresponding value of $\sec\theta - \tan\theta$, from Table 2.
3rd. Multiply this last value by f to obtain the allowable load p per sq. in. on column; f being the allowable working stress per square inch for a short column.

Thus, by the use of Tables 1 and 2, the value (sec. $\theta - \tan\theta$) in equation (20) can be transformed into a numerical quantity, or decimal factor. See Examples, page 592.

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ES OF $\sqrt{\tan\theta}$ IN COLUMN FORMULA, EQUATION (20)
IOUS MATERIALS AND END CONDITIONS OF COLUMNS.
[Use this table in connection with Table 2.]

f. With Factor of Safety.	$\frac{fe}{E}$.	Values of $\sqrt{\tan\theta}$. End Conditions of Column.					
		Pivot Ends.		Pivot & Fix'd		Fixed Ends.	
.....	.....	$2.70\sqrt{\frac{fe}{E}}\cdot\frac{L}{r}$		$2.025\sqrt{\frac{fe}{E}}\cdot\frac{L}{r}$		$1.35\sqrt{\frac{fe}{E}}\cdot\frac{L}{r}$	
8 000(5) 10 000(4)	$\frac{25\,000}{28\,000\,000}$	$.081\frac{L}{r}$		$.061\frac{L}{r}$		$.040\frac{L}{r}$	
10 000(5) 12 500(4)	$\frac{30\,000}{29\,000\,000}$	$.087\frac{L}{r}$		$.065\frac{L}{r}$		$.043\frac{L}{r}$	
12 000(5) 15 000(4)	$\frac{35\,000}{30\,000\,000}$	$.092\frac{L}{r}$		$.069\frac{L}{r}$		$.046\frac{L}{r}$	
14 000(5) 17 500(4)	$\frac{40\,000}{31\,000\,000}$	$.097\frac{L}{r}$		$.073\frac{L}{r}$		$.049\frac{L}{r}$	
		Rect.	*Round*	*Rect.*	*Round*	*Rect.*	*Round*
10 000(8)	$\left(\frac{1}{250}\right)$	$\left(0.46\frac{L}{d}\right)$	$\left(0.51\frac{L}{d_0}\right)$	$\left(0.34\frac{L}{d}\right)$	$\left(0.38\frac{L}{d_0}\right)$	$\left(0.23\frac{L}{d}\right)$	$\left(0.26\frac{L}{d_0}\right)$
*	$\frac{1}{333}$	$0.51\frac{L}{d}$	$0.59\frac{L}{d_0}$	$0.38\frac{L}{d}$	$0.44\frac{L}{d_0}$	$0.26\frac{L}{d}$	$0.30\frac{L}{d_0}$
		Pin Ends.		Pin & Flat.		Flat Ends.	
.....	.....	$2.09\sqrt{\frac{fe}{E}}\cdot\frac{L}{r}$		$1.87\sqrt{\frac{fe}{E}}\cdot\frac{L}{r}$		$1.71\sqrt{\frac{fe}{E}}\cdot\frac{L}{r}$	
8 000(5) 10 000(4)	$\frac{1}{1120}$	$.062\frac{L}{r}$		$.056\frac{L}{r}$		$.051\frac{L}{r}$	
10 000(5) 12 500(4)	$\frac{1}{967}$	$.067\frac{L}{r}$		$.060\frac{L}{r}$		$.055\frac{L}{r}$	
12 000(5) 15 000(4)	$\frac{1}{857}$	$.071\frac{L}{r}$		$.064\frac{L}{r}$		$.058\frac{L}{r}$	
14 000(5) 17 500(4)	$\frac{1}{775}$	$.075\frac{L}{r}$		$.067\frac{L}{r}$		$.061\frac{L}{r}$	
		Rect.	*Round*	*Rect.*	*Round*	*Rect.*	*Round*
0 000(8)	$\left(\frac{1}{250}\right)$	$\left(0.35\frac{L}{d}\right)$	$\left(0.40\frac{L}{d_0}\right)$	$\left(0.32\frac{L}{d}\right)$	$\left(0.36\frac{L}{d_0}\right)$	$\left(0.29\frac{L}{d}\right)$	$\left(0.32\frac{L}{d_0}\right)$
*	$\frac{1}{333}$	$0.40\frac{L}{d}$	$0.46\frac{L}{d_0}$	$0.35\frac{L}{d}$	$0.41\frac{L}{d_0}$	$0.32\frac{L}{d_0}$	$0.37\frac{L}{d_0}$

4, Table 7, Strength and Resistance of Materials, page 495,
ee also Table 3, following.

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2.—Values of (Sec θ − tan θ) in Column Formula, Equation (20) for Successive Values of $\sqrt{\tan\theta}$, Table 1.

[Values of $\sqrt{\tan\theta}$ may be derived from Table 1.]

$\sqrt{\text{Tan}\ \theta}$	Sec θ − tan θ	Diff. (Minus)	$\sqrt{\text{Tan}\ \theta}$	Sec θ − tan θ	Diff. (Minus)	$\sqrt{\text{Tan}\ \theta}$	Sec θ − tan θ	Diff. (Minus)	$\sqrt{\text{Tan}\ \theta}$	Sec θ − tan θ	Diff. (Minus)
0.00	1.000	.010	0.50	.781	.078	1.00	.414	.054	1.50	.212	.024
0.10	.990	.029	0.60	.703	.079	1.10	.360	.047	1.60	.188	.020
0.20	.961	.047	0.70	.624	.077	1.20	.313	.039	1.70	.168	.017
0.30	.914	.061	0.80	.547	.070	1.30	.274	.034	1.80	.151	.015
0.40	.853	.072	0.90	.477	.063	1.40	.240	.028	1.90	.136	.013
0.50	.781		1.00	.414		1.50	.212		2.00	.123	

Examples in the Use of Tables 1 and 2.

[From Column Formula, $p = f$ (sec θ − tan θ), equation (20).]

Ex. 1.—Find the safe load, factor 4, of a medium-steel z-bar column with flat ends, whose length is 26 ft., sectional area 24.8 sq. ins., and radius of gyration 2.6.

Solution.—From Table 1, $\sqrt{\tan\theta} = .058\frac{L}{r} = .58$; and from Table 2, the corresponding value of (sec θ − tan θ) = .72; hence the safe working load $p = .72 \times 15\,000 = 10{,}800$ lbs. per sq. in., and the total safe load $= 10\,800 \times 24.8 = 267\,840$ lbs.

Ex. 2.—Find the safe load, factor 5, of a vertical post of Douglas spruce in a Pratt steel combination highway bridge, said post being 12×14 ins., 28 ft. long, and with pin end bearings.

Solution.—The value of f for Douglas spruce (col. 4, Table 7, page 495) is 1400. From Table 1, $\sqrt{\tan\theta} = \frac{4}{10} \cdot \frac{32}{12} = .933$; and from Table 2, the corresponding value of (sec θ − tan θ) is .456; hence total allowable load $P = .456 \times 1400 \times 12 \times 14 = 107\,251$ lbs. (See also Table 3.)

Ex. 3.—What load would be carried by a column similar to that in Ex. 2, but with flat ends?

Solution.—Table 1: $\frac{32}{100} \cdot \frac{28}{12} = .747$; Table 2 equivalent $= .588$; hence $P = .588 \times 1400 \times 12 \times 14 = 138\,298$ lbs.

Gordon's Formula for Columns.—Probably no other formula has been so universally accepted as that of Gordon—sometimes called Rankine's formula. Adopting the previous notation, it is deduced as follows: The maximum outer fiber stress due to both compression and bending is

$$f = \frac{P}{A} + \frac{Pvy}{I} = \frac{P}{A}\left(1 + \frac{vy}{r^2}\right) \qquad (21)$$

whence

$$\frac{P}{A} = p = \frac{f}{1 + \frac{vy}{r^2}} \qquad (22)$$

Now the deflection v of the column is an unknown quantity, but Gordon assumed it to be proportional to $\frac{l^2}{y}$, which assumption would be allowable if the total fiber stress f were due to *bending only*. Substituting this proportional equivalent of v in equation (21) and supplying the coefficient c whose value is to be determined by experiment, we have the general form:

$$p = \frac{f}{1 + \frac{l^2}{cr^2}} \qquad (23)$$

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For the working formula, the value assigned to f is usually the ultimate strength per sq. in. of a short column of the material, divided by factor of safety, as 4, 5, 6, etc. The coefficient c is an arbitrary constant determined by trials in fitting the curves of the formula to plattings from actual tests of strengths of columns up to the points of failure, and making f = ult. strength ÷ factor of safety. (See Secs. on *Bridges* and *Buildings* for special application of Gordon's formula; also the tables following under this section.)

C. Shaler Smith's Formula for Wooden Columns.—This formula gives values much too low for long columns. It is reproduced here more for its historical, than for its actual working value. The formula is as follows:

$$p = \frac{f}{1 + .004\frac{l^2}{d^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

the ultimate strength of white pine being assumed at $f_u = 5000$. The ends of the column are to be flat and firmly fixed; with concentric loading.

Straight-Line Formulas.—Curved-line formulas, previously explained, may be reduced to straight-line formulas by finding the equation of the tangent to the curve at the point of contra-flexure. Thus, in Fig. 5, page 589, c is the point of contra-flexure of the curve, and its tangent at that point cuts the coordinate axes of the diagram at T and t; the resulting straight-line formula for the elastic strength of the steel column with pivot ends thus reducing to

$$p = 35\,400 - 169\frac{l}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

in which $35\,400 = 1.18\,fe = 1.18 \times 30\,000$; E being assumed at 30 000 000 in the present instance. If now this tangent is swung slightly on the point c so that the point T is lowered to 34 000, it will practically fit the curve for a long distance, with the resulting equation

$$p = 34\,000 - 150\frac{l}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

STRAIGHT-LINE FORMULAS FOR STEEL AND WROUGHT IRON COLUMNS.
(Reduced from data in Tables 1 and 2.)

Material.	Ultimate Stren'th fu	Elastic Limit. fe	Elastic Strength of Column. Pounds per sq. in.		
			Pin Ends.	Pin & Flat.	Flat Ends.
Wrought iron....	40000	25000	$28500 - 95\frac{l}{r}$	$28500 - 85\frac{l}{r}$	$28500 - 75\frac{l}{r}$
Soft steel.....	50000	30000	$35000 - 130\frac{l}{r}$	$35000 - 115\frac{l}{r}$	$35000 - 100\frac{l}{r}$
Medium steel....	60000	35000	$41000 - 160\frac{l}{r}$	$41000 - 145\frac{l}{r}$	$41000 - 130\frac{l}{r}$
Hard steel.....	70000	40000	$46500 - 185\frac{l}{r}$	$46500 - 170\frac{l}{r}$	$46500 - 155\frac{l}{r}$

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3.—Safe Loads in Lbs. per Sq. In., Factor 5, for Wooden Columns, Pin Bearing.

[From Column Formula, $p = f\ (\sec\theta - \tan\theta)$, Equation (20).]

Values of p in above formula.

Caution.—When interpolating, *subtract* the difference.

¶ Sq. Rnd. $\frac{L}{d}$ or $\frac{L}{d_0}$	Square and Rectangular Columns. Safe load (5) per sq. in. for Zero Length— 1000 Lbs. (1)	1100 Lbs. (2)	1200 Lbs. (3)	1300 Lbs. (4)	1400 Lbs. (5)	Round Columns. Safe load (5) per sq. in. for Zero Length— 1000 Lbs. (1)	1100 Lbs. (2)	1200 Lbs. (3)	1300 Lbs. (4)	1400 Lbs. (5)
1.0	853	938	1024	1109	1194	810	891	972	1053	1134
1.1	824	906	989	1071	1154	776	854	931	1009	1086
1.2	795	874	954	1033	1113	*741	*815	*889	*963	*1037
1.3	765	841	918	994	1071	705	775	846	916	987
1.4	734	807	881	954	1028	668	735	802	868	935
1.5	703	773	844	914	984	632	695	758	822	885
1.6	671	738	805	872	939	596	656	715	775	834
1.7	*640	*704	*768	*832	*896	561	617	673	729	785
1.8	609	670	731	792	853	527	580	632	685	738
1.9	578	636	692	751	809	495	544	594	643	693
2.0	547	602	656	711	766	465	511	558	604	651
2.1	519	571	623	675	727	†435	†478	†522	†565	†609
2.2	491	540	589	638	687	408	449	490	530	571
2.3	464	510	557	603	650	383	421	460	498	536
2.4	438	482	526	569	613	358	394	430	465	501
2.5	†414	455	497	538	580	336	370	403	437	470
2.6	392	431	470	510	549	315	346	378	409	441
2.7	371	408	445	482	519	*297	*327	*356	*386	*416
2.8	351	386	421	456	491	279	307	335	363	391
2.9	332	365	398	432	465	262	288	314	341	367

4.3	163	181	199	217	235	127	139	151	163	175	159	173	187	201	215	139	151	163	175	187	151	163	175	187
4.4	158	174	190	205	221	121	133	145	157	169	153	167	181	195	209	133	145	157	169	181	145	157	169	181
4.5	151	166	181	196	211	116	128	139	151	162	146	159	173	186	199	128	139	151	162	173	139	151	162	173
4.6	145	159	174	188	203	111	122	133	144	155	140	153	166	179	192	122	133	144	155	166	133	144	155	166
4.7	139	153	167	181	195	106	117	127	138	148	135	147	159	172	185	117	127	138	148	159	127	138	148	159
4.8	133	146	160	173	186	102	112	122	133	143	130	142	154	166	178	112	122	133	143	153	122	133	143	153
4.9	128	141	154	166	179	97	107	116	126	136	125	137	148	160	172	107	116	126	136	146	116	126	136	146
5.0	123	135	148	160	172	93	102	112	121	130	120	132	143	154	165	102	112	121	130	140	112	121	130	140
Approximate Equivalent Straight-Line Formulas.	$\frac{L}{d}$ 1050—240	$\frac{l}{d}$ 1050—20	$\frac{L}{d}$ 1155—264	$\frac{l}{d}$ 1155—22	$\frac{L}{d}$ 1260—288	$\frac{l}{d}$ 1260—24	$\frac{L}{d}$ 1365—312	$\frac{l}{d}$ 1365—26	$\frac{L}{d}$ 1470—336	$\frac{l}{d}$ 1470—28	$\frac{L}{d_0}$ 1100—300	$\frac{l}{d_0}$ 1100—25	$\frac{L}{d_0}$ 1210—330	$\frac{l}{d_0}$ 1210—27.5	$\frac{L}{d_0}$ 1320—360	$\frac{l}{d_0}$ 1320—30	$\frac{L}{d_0}$ 1430—390	$\frac{l}{d_0}$ 1430—32.5	$\frac{L}{d_0}$ 1540—420	$\frac{l}{d_0}$ 1540—35				
Shortleaf Pine																								
White Pine																								
Red Pine																								
Norway Pine																								
Spruce																								
Eastern Fir																								
California Spruce																								
California Redwood																								
Hemlock																								
White Cedar																								
Bald Cypress																								
White Oak																								
Longleaf Pine																								
Canadian White P.																								
Canadian Red P.																								
Chestnut																								
Douglas Spruce																								

Same kinds of timber apply to round columns as to square—see to left.

Notation:

L = length of column in feet;
l = length of column in inches;
d = least width of rectangular column in ins.;
d_0 = diameter of round column in ins.

* Points of equality of values in table with values deduced from approximate straight-line formula.
† Maximum excess of values from straight-line formulas over those of table; 8.7% for square, 8% for round columns.
¶ For Rectangular Columns, use d for least dimensions, in ins.

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Steel Columns.—From the preceding discussion of Column Forr might readily conclude that, for a column of given sectional are length l, that column is the strongest which has the greatest r gyration r. But this is not always true. To illustrate: On p Fig. 26, we find the properties of the Hollow Circle, of outside dia

and inside diameter d_1, giving $r = \frac{\sqrt{d^2 + d_1^2}}{4}$ and $A = 0.7834\ (d^2 - d_1$

if we let A remain constant and gradually increase d_1 from zero until it approach d, which must also gradually increase, but less ra then r likewise increases in value from $r = 0.25\ d$ (when the column i circular cylinder) up to its maximum limit $r = 0.3535\ d$ (when the becomes a cylindrical shell whose thickness approaches zero). quite apparent that before the maximum limit of r is reached, th column will fail through *auxiliary* or *secondary stresses*, set up in metal shell, causing it to buckle and collapse.

The Secondary Stresses, above described, may be provided ag either (1) Making the metal thick enough, throughout, to withstan (2) Providing longitudinal ribs, as in the case with the Phœnix col Table 11, page 604) and other standard built-up columns, in (3) Inserting (transverse) diaphrams at intervals along the colu done with very large sections.

Where Columns are Latticed, or where stay-plates are simply independent (unsupported) portion should be calculated as a column with its own radius of gyration.

Column Sections may be made up as follows:—

(A). —Two angles riveted back to back, forming a T-section.

(Aa).—Same as (A), but with round fillers or a plate riveted bet angles, giving a greater radius of gyration.

(B). —Four angles and a web plate, forming an H-section, simil rolled H-Column of Table 14, page 608. The distance bac of angles should be in whole or half inches, and from greater than width of web plate.

(Ba).—Same as (B), but with the addition of two channels rivet backs of the angles, with flanges of channels projecting in

(Bb).—Same as (B), but with lattice bars instead of web plate.

(C). —Two channels and two plates, as per Fig. 6, below. See al 8, 9, and 10, pages 661, etc.

(Ca).—Same as (C), but with latticing instead of plates—one or bot

(Cb).—Same as (C), but with flanges of channels projecting inw latticing or stay plates instead of cover plates.

(D). —Same as (C), (Ca) and (Cb), but with plate and two angl of channel.

(E). —Z-bar column. See Tables 6 and 7, pages 598, etc.

Also other sections made up of a combination of these shapes.

To find the movement of inertia, I, and the radius of gyration, column section, see Example, page 627, in connection with Tables 3 pages 639, etc.

The Channel Column, Fig. 6, is standard for all classes of con The following table gives the standard dimensions:

4.—CHANNEL COLUMNS—STANDARD DIMENSIONS IN INCHE

Depth of Channel C	Width of Plate P	W	R	$W+R$	E
15	18	5½	1¾	7¼	1¾
15	16	4½	1¾	6¼	1¾
12	16	5	1½	6½	1½
12	14	4	1½	5½	1½
10	14	4¼	1¼	5½	1½
10	12	3¼	1¼	4½	1½
9	12	3⅜	1⅛	4½	1½
9	10	2⅝	1⅛	3¾	1¼
8	12	3½	1	4½	1½
8	10	2¾	1	3¾	1¼

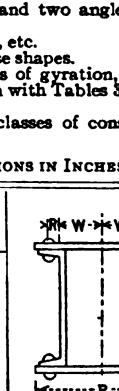

Fig. 6

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5.—Ultimate Strength of Steel Columns.*

FLAT ENDS		PIN AND FLAT		PIN ENDS
$\frac{50\,000}{1+\frac{(12L)^2}{36000r^2}}$	$=\frac{50\,000}{1+.004\frac{L^2}{r^2}}$	$=\frac{50\,000}{1+\frac{(12L)^2}{24000r^2}}$	$=\frac{50\,000}{1+.006\frac{L^2}{r^2}}$	$=\frac{50\,000}{1+\frac{(12L)^2}{18000r^2}}$ $=\frac{50\,000}{1+.008\frac{L^2}{r^2}}$
$\frac{45\,000}{1+\frac{(12L)^2}{36000r^2}}$	$=\frac{45\,000}{1+.004\frac{L^2}{r^2}}$	$=\frac{45\,000}{1+\frac{(12L)^2}{24000r^2}}$	$=\frac{45\,000}{1+.006\frac{L^2}{r^2}}$	$=\frac{45\,000}{1+\frac{(12L)^2}{18000r^2}}$ $=\frac{45\,000}{1+.008\frac{L^2}{r^2}}$

ate Str'gth in 1000 lbs. pr sq. in.					$\frac{L}{r}$	†Ultimate Str'gth in 1000 lbs. pr sq. in.					
um Steel.		Soft Steel.				Medium Steel.			Soft Steel.		
Pin & Sqr.	Pin.	Sqr.	Pin & Sqr.	Pin.		Sqr.	Pin & Sqr.	Pin.	Sqr.	Pin & Sqr.	Pin.
47.44	46.64	43.44	42.69	41.98	12.0	31.73	26.82	23.23	28.55	24.14	20.91
47.11	46.21	43.23	42.39	41.59	12.2	31.34	26.41	22.82	28.21	23.77	20.54
46.76	45.77	43.01	42.08	41.19	12.4	30.96	26.01	24.42	27.86	23.41	20.18
46.39	45.30	42.78	41.75	40.77	12.6	30.58	25.61	22.03	27.52	23.05	19.82
46.01	44.82	42.54	41.41	40.34	12.8	30.21	25.21	21.64	27.18	22.69	19.47
45.62	44.33	42.29	41.06	39.89	13.0	29.83	24.83	21.26	26.85	22.34	19.13
45.21	43.82	42.03	40.69	39.43	13.2	29.47	24.45	20.89	26.52	22.00	18.80
44.80	43.30	41.76	40.32	38.97	13.4	29.10	24.07	20.52	26.19	21.66	18.47
44.37	42.76	41.49	39.93	38.48	13.6	28.74	23.70	20.16	25.86	21.33	18.15
43.93	42.22	41.20	39.53	38.00	13.8	28.38	23.34	19.81	25.54	21.00	17.83
43.48	41.67	40.91	39.13	37.50	14.0	28.03	22.98	19.47	25.22	20.68	17.52
43.12	41.11	40.61	38.81	37.00	14.2	27.68	22.63	19.13	24.91	20.36	17.22
42.56	40.54	40.30	38.30	36.49	14.4	27.33	22.28	18.81	24.60	20.05	16.92
42.08	39.97	39.98	37.87	35.98	14.6	26.99	21.94	18.48	24.29	19.75	16.63
41.60	39.40	39.66	37.44	35.46	14.8	26.65	21.60	18.17	23.98	19.44	16.35
41.12	38.82	39.33	37.01	34.94	15.0	26.32	21.28	17.86	23.68	19.15	16.07
40.63	38.24	39.00	36.57	34.42	15.2	25.98	20.95	17.55	23.39	18.86	15.80
40.14	37.66	38.66	36.12	33.89	15.4	25.66	20.64	17.26	23.09	18.57	15.53
39.64	37.08	38.32	35.68	33.37	15.6	25.34	20.32	16.97	22.80	18.29	15.27
39.13	36.50	37.98	35.22	32.85	15.8	25.02	20.02	16.68	22.52	18.01	15.10
38.64	35.92	37.62	34.78	32.33	16.0	24.70	19.72	16.40	22.23	17.74	14.76
38.14	35.34	37.27	34.32	31.81	16.2	24.39	19.42	16.13	21.95	17.48	14.52
37.64	34.77	36.91	33.87	31.29	16.4	24.09	19.13	15.86	21.68	17.22	14.28
37.13	34.20	36.55	33.42	30.78	16.6	23.78	18.84	15.60	21.41	16.96	14.04
36.63	33.63	36.19	32.97	30.27	16.8	23.49	18.56	15.35	21.14	16.71	13.81
36.13	33.07	35.83	32.51	29.76	17.0	23.19	18.29	15.09	20.87	16.46	13.58
35.63	32.51	35.46	32.06	29.26	17.2	22.90	18.02	14.85	20.61	16.22	13.37
35.13	31.96	35.09	31.61	28.76	17.4	22.61	17.75	14.61	20.35	15.98	13.15
34.63	31.41	34.73	31.17	28.27	17.6	22.33	17.49	14.38	20.10	15.74	12.94
34.14	30.87	34.36	30.72	27.79	17.8	22.05	17.23	14.14	19.85	15.51	12.73
33.65	30.34	33.99	30.28	27 31	18.0	21.78	16.98	13.92	19.60	15.29	12.53
33.16	29.81	33.61	29.84	26.83	18.2	21.50	16.74	13.70	19.35	15.06	12.33
32.68	29.29	32.25	29.41	26.36	18.4	21.24	16.49	13.48	19.11	14.84	12.13
32.20	28.78	32.88	28.98	25.90	18.6	20.97	16.26	13.27	18.88	14.63	11.94
31.72	28.28	32.51	28.55	25.45	18.8	20.71	16.02	13.06	18.64	14.42	11.76
31.25	27.78	32.14	28.12	25.00	19.0	20.46	15.80	12.86	18.42	14.22	11.58
30.78	27.29	31.78	27.71	24.56	19.2	20.21	15.57	12.66	18.18	14.01	11.39
30.32	26.81	31.41	27.29	24.12	19.4	19.96	15.35	12.47	17.96	13.81	11.22
29.87	26.33	31.05	26.88	23.70	19.6	19.71	15.13	12.27	17.74	13.62	11.05
29.42	25.87	30.68	26.47	23.28	19.8	19.47	14.91	12.09	17.52	13.42	10.88
28.97	25.41	30.32	26.07	22.87	20.0	19.23	14.71	11.90	17.31	13.23	10.71
28.53	24.96	29.96	25.67	22.46	20.2	19.00	14.50	11.72	17.10	13.05	10.55
28.09	24.51	29.61	25.28	22.06	20.4	18.76	14.30	11.55	16.89	12.87	10.43
27.67	24.08	29.25	24.90	21.67	20.6	18.54	14.10	11.38	16.68	12.69	10.25
27.24	23.65	28.90	24.52	21.29	20.8	18.31	13.90	11.21	16.48	12.51	10.09

following notation is used in the above formulas: p = ultimate
n lbs. per sq. in.; L = length of column in ft.; r = radius of gyra-
ches.
loads as in buildings, divide by 4; for loads as in bridges, divide by 5.

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6.—*Dimensions of Carnegie Z-Bar Columns—and—
(Without Side Plates.)

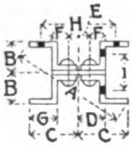

Fig. 7.

Note.—Diameter of Bolt or Rivet, ¾ in
ness of Web Plate = thickness of Z-Bar m
dimensions in inches. See opposite page
weights, safe loads, etc.

(For Columns with Side Plates, see Tab

Column.	Thickness of Metal. t	Notation in Cut, above. A	B	C	D	E	F	G
6-inch Col. 4 Z-bars 3-3½ in. deep. Web pl. 6" by thickness of bars	¼	12¾	3⅛	5 9/16	3⅛	3	1⅝	2 11/16
	5/16	12⅞	3 3/32	5 9/16	3⅛	3	1⅝	2¾
	⅜	12⅝	3 1/16	5 9/16	3⅛	3	1⅝	2 11/16
	7/16	12 11/16	3 9/32	5 9/16	3⅛	3	1⅝	2¾
	½	12 7/16	3¼	5 9/16	3⅛	3	1⅝	2 11/16
	9/16	12 9/16	3 11/32	5 9/16	3⅛	3	1⅝	2¾
8-in. Column. 4 Z-bars 4-4⅛ in. deep. Web pl. 7" by thickness of bars.	¼	15 5/16	4⅛	6 7/16	3⅝	3½	1⅞	3 1/16
	5/16	15⅜	4 7/32	6 7/16	3⅝	3½	1⅞	3⅛
	⅜	15½	4 9/16	6 7/16	3⅝	3½	1⅞	3 3/16
	7/16	15 9/16	4 7/32	6¼	3⅝	3½	1⅞	3 1/16
	½	15 3/16	4 9/16	6¼	3⅝	3½	1⅞	3⅛
	9/16	15 5/16	4 13/32	6¼	3⅝	3½	1⅞	3 3/16
	⅝	14⅞	4 9/16	6 1/16	3⅝	3½	1⅞	3 1/16
	11/16	15	4 13/32	6 1/16	3⅝	3½	1⅞	3⅛
	¾	15⅛	4½	6 1/16	3⅝	3½	1⅞	3 3/16
10-in. Column. 4 Z-bars 5-5⅛ in. deep. Web pl. 7" by thickness of bars.	5/16	16 11/16	5 5/32	6 9/16	3⅝	3½	1⅞	3¼
	⅜	16 13/16	5¼	6 9/16	3⅝	3½	1⅞	3 5/16
	7/16	16 15/16	5 11/32	6 9/16	3⅝	3½	1⅞	3⅜
	½	16½	5¼	6⅜	3⅝	3½	1⅞	3¼
	9/16	16⅝	5 11/32	6⅜	3⅝	3½	1⅞	3 5/16
	⅝	16¾	5 7/16	6⅜	3⅝	3½	1⅞	3⅜
	11/16	16⅜	5 13/32	6 3/16	3⅝	3½	1⅞	3¼
	¾	16½	5 7/16	6 3/16	3⅝	3½	1⅞	3 5/16
	13/16	16⅝	5 13/32	6 3/16	3⅝	3½	1⅞	3⅜
12-in Column. 4 Z-bars 6-6⅛ in. deep. Web pl. 8" by thickness of bars.	⅜	19 1/16	6 3/16	7¼	4⅛	4	2⅛	3½
	7/16	19 3/16	6 9/32	7¼	4⅛	4	2⅛	3 9/16
	½	19 5/16	6⅜	7¼	4⅛	4	2⅛	3⅝
	9/16	18⅞	6 9/32	7 1/16	4⅛	4	2⅛	3½
	⅝	19	6⅜	7 1/16	4⅛	4	2⅛	3 9/16
	11/16	19⅛	6 15/32	7 1/16	4⅛	4	2⅛	3⅝
	¾	18¾	6⅜	6⅞	4⅛	4	2⅛	3½
	13/16	18⅞	6 13/32	6⅞	4⅛	4	2⅛	3 9/16
	⅞	19	6 7/16	6⅞	4⅛	4	2⅛	3⅝

* All dimensions in inches.

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—TOTAL SAFE LOAD IN THOUSAND POUNDS FOR Z-BAR COLUMNS (WITHOUT SIDE PLATES.)

t = thickness of metal;
A = area of cross-section of column;
w = weight of column in lbs. per lin. ft., not including weight of rivets;
r = least radius of gyration.

Allowable stresses per sq. in.; safety factor 4: $\begin{cases} 12{,}000 \text{ lbs. for lengths of 90 radii or under;} \\ 17{,}100 - 57\frac{l}{r} \text{ for lengths over 90 radii.} \end{cases}$

(See opposite page for Dimensions.)

t Ins	A Sq. Ins.	w* Lbs.	r (Min)	Length of Columns in Feet. 12	14	18	22	26	30	34	38	42	46	50
1/8	9.31	31.7	1.86	111	111	98	84	70	57	*6-in. Col.* For columns less than 12 ft. long use given loads for 12 ft.		NOTE.—Additional metal from side plates on columns tends to increase the radius of gyration of the section; hence the resulting increase in sectional area by adding side-plate metal will not be proportionate. Increase in the tabulated safe loads. But see next page for columns with side plates.		
1/16	11.7	39.8	1.90	141	141	125	103	91	74					
3/16	13.6	46.2	1.88	163	163	143	124	104	84					
1/4	16.0	54.3	1.93	192	192	171	149	126	103					
1/16	17.6	59.9	1.90	211	211	187	162	136	111					
5/16	20.0	67.9	1.95	240	240	216	188	160	132					
1/8	11.3	38.3	2.47	*8-in. Column.* For columns less than 18 ft. long use given loads for 18 ft.		135	124	111	99	86	74			
1/16	14.1	48.1	2.52			170	157	142	127	111	96			
3/16	17.1	58.0	2.57			205	192	174	155	137	119			
1/4	19.0	64.7	2.49			228	211	190	169	148	127			
1/16	21.9	74.4	2.55			262	245	221	198	174	151			
5/16	24.8	84.1	2.60			297	280	254	228	202	176			
3/8	26.3	89.2	2.52			315	292	264	235	207	179			
7/16	29.0	98.8	2.58			349	327	296	265	235	204			
1/2	31.9	108.4	2.63			382	363	329	296	263	230			
1/8	15.8	53.7	3.08	*10-in. Column.* For columns less than 22 ft. long use given loads for 22 ft.		.....	189	179	165	151	137	123	108	94
3/16	19.0	64.7	3.13			.....	228	217	200	184	167	151	134	117
1/4	22.3	75.8	3.18			.....	268	257	237	218	199	180	161	141
5/16	24.5	83.3	3.10			.....	294	278	257	235	214	192	170	149
3/8	27.7	94.2	3.15			.....	332	317	293	269	245	221	197	173
7/16	30.9	105.2	3.21			.....	371	357	331	305	278	252	225	199
1/2	32.7	111.0	3.13			.....	392	373	344	316	287	259	230	202
9/16	35.8	121.8	3.18			.....	430	412	382	351	320	289	258	228
5/8	39.0	132.6	3.25			.....	468	453	420	388	355	322	289	256
1/8	21.4	72.7	3.67	*12-in. Column.* For columns less than 26 ft. long use given loads for 26 ft.		.....	.....	257	246	230	214	198	182	166
3/16	25.0	85.2	3.72			.....	.....	301	290	272	253	235	217	198
1/4	28.8	97.8	3.77			.....	.....	345	335	314	294	273	252	231
5/16	31.2	106.2	3.70			.....	.....	375	360	337	314	291	268	245
3/8	34.8	118.5	3.75			.....	.....	418	405	380	354	329	303	278
7/16	38.5	130.9	3.73			.....	.....	462	447	418	390	362	334	305
1/2	40.5	137.8	3.68			.....	.....	486	466	436	406	376	346	316
9/16	44.1	149.9	3.66			.....	.....	529	506	473	440	407	374	341
5/8	47.7	162.1	3.64			.....	.....	572	546	510	475	439	403	367

* Add weight of rivet heads.

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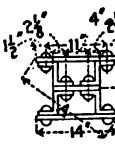

Fig. 8.

7.—Carnegie 14-inch Z-Bar Columns. With Side Plates.

Note.—Diameter of Bolt in Rivet, ⅞ in.

For notation, A, w, r, safety factor, working formula, etc., see top of page 1044.

For increase of safe load over tabulated values, the area A may be increased proportionately by increasing metal in side plates.

Dimensions and Properties.								Safe Load in Thousand Pounds.							
Two Side Pl'ts	Z-Brs & Web	Dim'n's A	Dim'n's B	A Sq. Ins.	w* Lbs.	r (min)		Length of Column in Feet. 26†	28	30	34	38	42	46	50
14x⅜	4 Z-bars 6½x¼ in. 1 web pl. 8x¼ in.	19 3/16	6 3/32	49.0	166.6	3.80		588	588	573	538	503	467	432	397
14x 7/16		19 11/16	6 29/32	50.8	172.6	3.81		609	609	594	558	521	485	449	412
14x½		19¾	6 31/32	52.5	178.5	3.82		630	630	615	578	540	503	465	427
14x 9/16		19⅞	7 3/32	54.3	184.5	3.82		651	651	637	598	559	520	481	443
14x⅝		19 15/16	7 5/32	56.0	190.4	3.83		672	672	658	618	578	538	498	458
14x 11/16		20 1/16	7 9/32	57.8	196.4	3.84		693	693	679	638	597	556	514	473
14x¾		20⅛	7 7/32	59.5	202.3	3.85		714	714	700	658	615	573	530	488
14x 13/16		20¼	7 11/32	61.3	208.4	3.85		735	735	721	677	634	590	547	503
14x⅞		20 5/16	7 13/32	63.0	214.2	3.85		756	756	742	697	652	608	563	518
14x⅜	4 Z-bars 6½x¼ in. 1 web pl. 8x¼ in.	19 7/16	6¾	51.0	173.4	3.75		612	612	593	556	519	482	445	407
14x 7/16		19½	6 11/16	52.8	179.4	3.76		633	633	614	576	538	499	461	423
14x½		19⅝	6 7/8	54.5	185.3	3.77		654	654	636	596	557	517	477	438
14x 9/16		19¾	6 15/16	56.3	191.4	3.78		675	675	657	616	575	535	494	453
14x⅝		19 13/16	7	58.0	197.2	3.79		696	696	678	636	594	552	510	468
14x 11/16		19⅞	7 1/16	59.8	203.2	3.80		717	717	699	656	613	570	527	484
14x¾		20	7 1/8	61.5	209.1	3.80		738	738	720	676	631	587	543	499
14x 13/16		20 1/16	7 3/16	63.3	215.1	3.81		759	759	741	696	650	605	559	514
14x⅞		20⅛	7¼	65.0	221.0	3.82		780	780	762	716	669	622	576	529
14x⅜	4 Z-bars 6½x¼ in. 1 web pl. 8x 11/16 in.	19 7/16	6 23/32	54.6	185.6	3.73		655	653	633	593	553	513	473	433
14x 7/16		19⅝	6 29/32	56.3	191.5	3.74		676	675	654	613	572	531	490	449
14x½		19¾	6 31/32	58.1	197.5	3.75		697	697	675	633	591	549	506	464
14x 9/16		19⅞	7 5/32	59.8	203.4	3.76		718	718	697	653	610	566	523	479
14x⅝		19 15/16	7 9/32	61.6	209.4	3.77		739	739	718	673	628	584	539	494
14x 11/16		20 1/16	7 7/32	63.3	215.3	3.78		760	760	739	693	647	601	555	510
14x¾		20⅛	7 11/32	65.1	221.3	3.78		781	781	760	713	666	619	572	525
14x 13/16		20¼	7 15/32	66.8	227.2	3.79		802	802	781	733	685	636	588	540
14x⅞		20 5/16	7 13/32	68.6	233.2	3.80		823	823	802	753	703	654	605	555
14x⅜	4 Z-bars 6½x¼ in. 1 web pl. 8x 13/16 in.	19¾	6 13/16	58.2	197.8	3.71		698	695	673	631	588	545	502	459
14x 7/16		19 11/16	7	59.9	203.8	3.72		719	717	694	650	606	562	518	474
14x½		19⅞	7 1/16	61.7	209.7	3.73		740	738	716	670	625	580	535	490
14x 9/16		20	7 1/8	63.4	215.7	3.74		761	760	737	690	644	598	551	506
14x⅝		20 1/16	7 3/16	65.2	221.6	3.75		782	782	758	710	663	615	568	520
14x 11/16		20⅛	7¼	66.9	227.6	3.76		803	803	779	730	682	633	584	535
14x¾		20¼	7 5/16	68.7	233.5	3.77		824	824	800	750	700	650	601	551
14x 13/16		20 5/16	7 7/16	70.4	239.5	3.77		845	845	821	770	719	668	617	566
14x⅞		20 7/16	7 9/16	72.2	245.4	3.78		866	866	842	790	738	686	633	581

* Add weight of rivet heads. † 26 feet or less.

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Fig. 9.

8—Carnegie Channel Columns—Flat Ends. 7″

Safe Loads* and Properties.

(Safe Loads are in Thousand Pounds.)

2 6-inch Chan. Lat.† or with 2 Side Plates 8 inches wide $\left\{ B=3\frac{7}{8}'' \atop C=5\frac{3}{4}'' \right.$

Wt. per foot ea. Channel.	Thickness of Plate.	Wt. per foot of Column.	Area of Section.	Rad of Gyr.	Short Col. Max. L'gth.	Short Col. Safe Load.	Decr. in S per ea. foot Incr. in L.
Lbs.	Ins.	Lbs.	Sq. Ins.	r (min)	L Ft.	S 1000#	1000#
8 lbs.	Lat.	16.0	4.76	2.33	17.4	57.1	1.40
	¼	29.6	8.76	2.32	17.3	105.1	2.58
	5/16	33.0	9.76	2.32	17.3	117.1	2.88
	⅜	36.4	10.76	2.32	17.3	129.1	3.17
	7/16	39.8	11.76	2.32	17.3	141.1	3.47
	½	43.2	12.76	2.32	17.3	153.1	3.76
	9/16	46.6	13.76	2.32	17.3	165.1	4.06
	⅝	50.0	14.76	2.32	17.3	177.1	4.35
	11/16	53.4	15.76	2.32	17.3	189.1	4.65
10.5 lbs.	Lat.	21.0	6.18	2.20	16.4	74.2	1.92
	¼	34.6	10.18	2.24	16.7	122.2	3.11
	5/16	38.0	11.18	2.25	16.8	134.2	3.40
	⅜	41.4	12.18	2.25	16.8	146.2	3.70
	7/16	44.8	13.18	2.26	16.9	158.2	3.99
	½	48.2	14.18	2.26	16.9	170.2	4.29
	9/16	51.6	15.18	2.27	16.9	182.2	4.57
	⅝	55.0	16.18	2.27	16.9	194.2	4.87
	11/16	58.4	17.18	2.27	16.9	206.2	5.17
13 lbs.	Lat.	26.0	7.64	2.09	15.6	91.7	2.50
	5/16	43.0	12.64	2.18	16.3	151.7	3.96
	⅜	46.4	13.64	2.19	16.3	163.7	4.26
	7/16	49.8	14.64	2.20	16.4	175.7	4.55
	½	53.2	15.64	2.21	16.5	187.7	4.84
	9/16	56.6	16.64	2.21	16.5	199.7	5.15
	⅝	60.0	17.64	2.22	16.6	211.7	5.44
	11/16	63.4	18.64	2.22	16.6	223.7	5.75
	¾	66.8	19.64	2.23	16.6	235.7	6.03
15.5 lbs.	Lat.	31.0	9.12	2.00	14.9	109.4	3.12
	5/16	48.0	14.12	2.12	15.8	169.4	4.56
	⅜	51.4	15.12	2.13	15.9	181.4	4.86
	7/16	54.8	16.12	2.14	16.0	193.4	5.15
	½	58.2	17.12	2.15	16.0	205.4	5.45
	9/16	61.6	18.12	2.16	16.1	217.4	5.74
	⅝	65.0	19.12	2.17	16.2	229.4	6.03
	11/16	68.4	20.12	2.18	16.2	241.4	6.31
	¾	71.8	21.12	2.18	16.2	253.4	6.63

2 7-inch Chan. Lat.‡ or with 2 Side Plates 9 inches wide $\left\{ B=4\frac{5}{8}'' \atop C=6\frac{3}{4}'' \right.$

Wt. per foot ea. Channel.	Thickness of Plate.	Wt. per foot of Column.	Area of Section.	Rad of Gyr.	Short Col. Max. L'gth.	Short Col. Safe Load.	Decr. in S per ea. foot Incr. in L.
Lbs.	Ins.	Lbs.	Sq. Ins.	r (min)	L Ft.	S 1000#	1000#
9.75 lbs.	Lat.	19.5	5.70	2.72	21.3	68.4	1.43
	¼	34.8	10.20	2.67	19.9	122.4	2.61
	5/16	38.6	11.32	2.67	19.9	135.8	2.90
	⅜	42.5	12.45	2.66	19.8	149.4	3.20
	7/16	46.3	13.58	2.66	19.8	163.0	3.49
	½	50.1	14.70	2.65	19.7	176.4	3.79
	9/16	53.9	15.82	2.65	19.7	189.8	4.08
	⅝	57.8	16.95	2.64	19.7	203.4	4.39
	11/16	61.6	18.07	2.64	19.7	216.8	4.68
12.25 lbs.	Lat.	24.5	7.20	2.59	19.3	86.4	1.90
	¼	39.8	11.70	2.60	19.4	140.4	3.08
	5/16	43.6	12.82	2.60	19.4	153.8	3.37
	⅜	47.5	13.95	2.60	19.4	167.4	3.67
	7/16	51.3	15.08	2.60	19.4	181.0	3.97
	½	55.1	16.20	2.60	19.4	194.4	4.26
	9/16	58.9	17.32	2.60	19.4	207.8	4.56
	⅝	62.8	18.45	2.60	19.4	221.4	4.85
	11/16	66.6	19.57	2.60	19.4	234.8	5.15
14.75 lbs.	Lat.	29.5	8.68	2.50	18.6	104.2	2.38
	¼	44.8	13.18	2.54	18.9	158.2	3.55
	5/16	48.6	14.30	2.55	19.0	171.6	3.84
	⅜	52.5	15.43	2.55	19.0	185.2	4.14
	7/16	56.3	16.56	2.55	19.0	198.7	4.44
	½	60.1	17.68	2.56	19.1	213.2	4.72
	9/16	63.9	18.80	2.56	19.1	225.6	5.02
	⅝	67.8	19.93	2.56	19.1	239.2	5.33
	11/16	71.6	21.05	2.56	19.1	252.6	5.62
17.25 lbs.	Lat.	34.5	10.14	2.43	18.1	121.7	2.85
	5/16	53.6	15.76	2.49	18.6	189.1	4.33
	⅜	57.5	16.89	2.50	18.6	202.7	4.62
	7/16	61.3	18.02	2.50	18.6	216.2	4.93
	½	65.1	19.14	2.51	18.7	229.7	5.22
	9/16	68.9	20.26	2.51	18.7	243.1	5.52
	⅝	72.8	21.39	2.52	18.8	256.7	5.81
	11/16	76.6	22.51	2.52	18.8	270.1	6.11
	¾	80.4	23.64	2.53	18.9	283.7	6.39
19.75 lbs.	Lat.	39.5	11.62	2.35	17.5	139.4	3.38
	5/16	58.6	17.24	2.44	18.2	206.9	4.82
	⅜	62.5	18.37	2.45	18.3	220.4	5.13
	7/16	66.3	19.50	2.45	18.3	234.0	5.44
	½	70.1	20.62	2.46	18.3	247.4	5.73
	9/16	73.9	21.74	2.47	18.4	260.9	6.02
	⅝	77.8	22.87	2.48	18.5	274.4	6.31
	11/16	81.6	23.99	2.48	18.5	287.9	6.62
	¾	85.4	25.12	2.49	18.6	301.4	6.90

* Allowed stresses per sq. in.; safety factor 4:
12,000 lbs. for length of 90 radii or under;
$17{,}100 - 57\,\frac{l}{r}$ for lengths over 90 radii and up to 125. l and r in inches.
Note.—Above weights do not include rivet heads or lattice bars.

† Lattice bars not less than $1\frac{1}{2}'' \times \frac{5}{16}''$. ‡ Lattice bars not less than $1\frac{3}{4}'' \times \frac{5}{16}''$

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Fig. 9.

9.—Carnegie Channel Columns—Flat Ends.

8″ 9″

Safe Loads and Properties.

(Safe Loads are in Thousand Pounds.)

2 8-in. Channels Lat.† or with 2 Side Plates 10 inches wide. $B=5\frac{1}{2}''$, $C=7\frac{1}{2}''$

Wt. per foot ea. Channel. Lbs.	Thickness of Plate. Ins.	Wt. per foot of Column. Lbs.	Area of Section. Sq. Ins.	Rad of Gyr r (min)	Short Col. Max. L'gth. L Ft.	Short Col. Safe Load. S 1000#	Decr. in S per ea. foot incr. in L. 1000#
11.25 lbs.	Lat.	22.5	6.70	3.11	23.2	80.4	1.47
	¼	39.5	11.70	3.03	22.6	140.4	2.64
	5/16	43.7	12.95	3.02	22.5	155.4	2.93
	⅜	48.0	14.20	3.01	22.4	170.4	3.23
	7/16	52.3	15.45	3.00	22.4	185.4	3.52
	½	56.5	16.70	2.99	22.3	200.4	3.82
	9/16	60.8	17.95	2.98	22.2	215.4	4.12
	⅝	65.0	19.20	2.98	22.2	230.4	4.42
	11/16	69.2	20.45	2.97	22.2	245.4	4.71
13.75 lbs.	Lat.	27.5	8.08	2.98	22.2	97.0	1.86
	5/16	48.7	14.33	2.97	22.2	172.0	3.30
	⅜	53.0	15.58	2.96	22.1	187.0	3.60
	7/16	57.3	16.83	2.96	22.1	202.0	3.89
	½	61.5	18.08	2.95	22.0	217.0	4.19
	9/16	65.8	19.33	2.95	22.0	232.0	4.48
	⅝	70.0	20.58	2.95	22.0	247.0	4.78
	11/16	74.2	21.83	2.94	21.9	262.0	5.08
	¾	78.5	23.08	2.94	21.9	277.0	5.37
16.25 lbs.	Lat.	32.5	9.56	2.89	21.6	114.7	2.26
	⅜	58.0	17.06	2.92	21.8	204.7	3.99
	7/16	62.3	18.31	2.91	21.7	219.7	4.30
	½	66.5	19.56	2.91	21.7	234.7	4.60
	9/16	70.8	20.81	2.91	21.7	249.7	4.89
	⅝	75.0	22.06	2.91	21.7	264.7	5.18
	11/16	79.2	23.31	2.91	21.7	279.7	5.48
	¾	83.5	24.56	2.91	21.7	294.7	5.78
	13/16	87.8	25.81	2.91	21.7	309.7	6.07
18.75 lbs.	Lat.	37.5	11.02	2.82	21.0	132.2	2.67
	7/16	67.3	19.77	2.87	21.4	237.2	4.71
	½	71.5	21.02	2.87	21.4	252.2	5.01
	9/16	75.8	22.27	2.87	21.4	267.2	5.31
	⅝	80.0	23.52	2.87	21.4	282.2	5.61
	11/16	84.2	24.77	2.87	21.4	297.2	5.91
	¾	88.5	26.02	2.88	21.5	312.2	6.18
	13/16	92.8	27.27	2.88	21.5	327.2	6.48
	⅞	97.0	28.52	2.88	21.5	342.2	6.78
21.25 lbs.	Lat.	42.5	12.50	2.77	20.7	150.0	3.09
	½	76.5	22.50	2.83	21.1	270.0	5.44
	9/16	80.8	23.75	2.84	21.2	285.0	5.73
	⅝	85.0	25.00	2.84	21.2	300.0	6.02
	11/16	89.2	26.25	2.84	21.2	315.0	6.33
	¾	93.5	27.50	2.84	21.2	330.0	6.63
	13/16	97.8	28.75	2.84	21.2	345.0	6.93
	⅞	102.0	30.00	2.85	21.3	360.0	7.21
	15/16	106.2	31.25	2.85	21.3	375.0	7.50

2 9-in. Channels Lat.‡ or with 2 Side Plates 11 inches wide. $B=6\frac{1}{4}''$, $C=8\frac{1}{2}''$

Wt. per foot ea. Channel. Lbs.	Thickness of Plate. Ins.	Wt. per foot of Column. Lbs.	Area of Section. Sq. Ins.	Rad of Gyr r (min)	Short Col. Max. L'gth. L Ft.	Short Col. Safe Load. S 1000#	Decr. in S per ea. foot incr. in L. 1000#
13.25 lbs.	Lat.	26.5	7.78	3.49	26.0	93.4	1.53
	¼	45.2	13.28	3.40	25.4	159.4	2.67
	5/16	49.9	14.66	3.38	25.2	175.9	2.97
	⅜	54.6	16.03	3.36	25.1	192.4	3.27
	7/16	59.2	17.40	3.35	25.0	208.8	3.56
	½	63.9	18.78	3.33	24.8	225.4	3.86
	9/16	68.5	20.16	3.32	24.8	241.9	4.15
	⅝	73.3	21.53	3.31	24.7	258.4	4.45
	11/16	77.9	22.90	3.31	24.7	274.8	4.73
15 lbs.	Lat.	30.0	8.82	3.40	25.4	105.8	1.78
	¼	48.7	14.32	3.36	25.1	171.8	2.92
	5/16	53.4	15.70	3.34	24.9	188.4	3.22
	⅜	58.1	17.07	3.33	24.8	204.8	3.51
	7/16	62.7	18.44	3.32	24.8	221.3	3.80
	½	67.4	19.82	3.31	24.7	237.8	4.09
	9/16	72.0	21.20	3.30	24.6	254.4	4.39
	⅝	76.8	22.57	3.29	24.5	270.8	4.69
	11/16	81.4	23.94	3.29	24.5	287.3	4.98
20 lbs.	Lat.	40.0	11.76	3.21	23.9	141.1	2.51
	⅜	68.1	20.01	3.25	24.2	240.1	4.22
	7/16	72.7	21.38	3.25	24.2	256.6	4.50
	½	77.4	22.76	3.24	24.2	273.1	4.81
	9/16	82.0	24.14	3.24	24.2	289.7	5.10
	⅝	86.8	25.51	3.23	24.1	306.1	5.41
	11/16	91.4	26.88	3.23	24.1	322.6	5.69
	¾	96.1	28.26	3.23	24.1	339.1	5.99
	13/16	100.8	29.63	3.23	24.1	355.6	6.28
25 lbs.	Lat.	50.0	14.70	3.10	23.1	176.4	3.24
	9/16	92.0	27.08	3.17	23.6	325.0	5.84
	⅝	96.8	28.45	3.17	23.6	341.4	6.13
	11/16	101.4	29.82	3.17	23.6	357.8	6.43
	¾	106.1	31.20	3.17	23.6	374.4	6.73
	13/16	110.8	32.57	3.17	23.6	390.8	7.03
	⅞	115.4	33.95	3.17	23.6	407.4	7.33
	15/16	120.1	35.32	3.17	23.6	423.8	7.63
	1	124.8	36.70	3.17	23.6	440.4	7.92

Problem.—A column composed of 2 8-in. channels at 16.25 lbs. and 2 plates $10\times\frac{9}{16}$ ins. will support a load of 249,700 lbs. up to 21.7 ft. in length. Find the load that it will support if 30 ft. long?

Solution.—

	249,700 lbs.
$4{,}890\times(30.0-21.7)=$	40,600 "
Answer	209,100 "

† Lattice bars not less than $1\frac{3}{4}''\times\frac{5}{16}''$. ‡ Lattice bars not less than $2''\times\frac{5}{16}''$.

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Fig. 9.

10″ **10.—Carnegie Channel Columns—Flat Ends.** 12″

Safe Loads and Properties.

(Safe Loads are in Thousand Pounds.)

2 10-in. Chan. Lat. † or with 2 Side Plates 12 inches wide. } $B=7$″, $C=9\frac{1}{2}$″

Wt. per foot ea. Channel.	Thickness of Plate.	Wt. per foot of Column.	Area of Section.	Rad of Gyr	Short Col.		Decr. in S per ea. foot incr. in L.
					Max. L'gth.	Safe Load.	
Lbs.	Ins.	Lbs.	Sq. Ins.	r (min)	L Ft.	S 1000#	1000#
15 lbs.	Lat.	30.0	8.92	3.87	28.8	107.0	1.57
	$\frac{5}{16}$	55.5	16.42	3.74	27.8	197.0	3.00
	$\frac{3}{8}$	60.6	17.92	3.72	27.7	215.0	3.30
	$\frac{7}{16}$	65.7	19.42	3.70	27.6	233.0	3.59
	$\frac{1}{2}$	70.8	20.92	3.68	27.4	251.0	3.89
	$\frac{9}{16}$	75.9	22.42	3.67	27.4	269.0	4.18
	$\frac{5}{8}$	81.0	23.92	3.65	27.2	287.0	4.48
	$\frac{11}{16}$	86.1	25.42	3.64	27.1	305.0	4.78
	$\frac{3}{4}$	91.2	26.92	3.63	27.0	323.0	5.07
20 lbs.	Lat.	40.0	11.76	3.66	27.3	141.1	2.20
	$\frac{7}{16}$	75.7	22.26	3.64	27.1	267.1	4.18
	$\frac{1}{2}$	80.8	23.76	3.63	27.0	285.1	4.48
	$\frac{9}{16}$	85.9	25.26	3.62	27.0	303.1	4.77
	$\frac{5}{8}$	91.0	26.76	3.61	26.9	321.1	5.07
	$\frac{11}{16}$	96.1	28.26	3.60	26.8	339.1	5.37
	$\frac{3}{4}$	101.2	29.76	3.59	26.8	357.1	5.67
	$\frac{13}{16}$	106.3	31.26	3.59	26.8	375.1	5.96
	$\frac{7}{8}$	111.4	32.76	3.58	26.7	393.1	6.26
25 lbs.	Lat.	50.0	14.70	3.52	26.2	176.4	2.86
	$\frac{9}{16}$	95.9	28.20	3.56	26.5	338.4	5.42
	$\frac{5}{8}$	101.0	29.70	3.55	26.4	356.4	5.72
	$\frac{11}{16}$	106.1	31.20	3.55	26.4	374.4	6.02
	$\frac{3}{4}$	111.2	32.70	3.55	26.4	392.4	6.32
	$\frac{13}{16}$	116.3	34.20	3.54	26.4	410.4	6.62
	$\frac{7}{8}$	121.4	35.70	3.54	26.4	428.4	6.91
	$\frac{15}{16}$	126.5	37.20	3.54	26.4	446.4	7.20
	1	131.6	38.70	3.53	26.3	464.4	7.50
30 lbs.	Lat.	60.0	17.64	3.42	25.5	211.7	3.53
	$\frac{11}{16}$	116.1	34.14	3.50	26.1	409.7	6.67
	$\frac{3}{4}$	121.2	35.64	3.50	26.1	427.7	6.97
	$\frac{13}{16}$	126.3	37.14	3.49	26.0	445.7	7.28
	$\frac{7}{8}$	131.4	38.64	3.49	26.0	463.7	7.58
	$\frac{15}{16}$	136.5	40.14	3.49	26.0	481.7	7.88
	1	141.6	41.64	3.49	26.0	499.7	8.17
	$1\frac{1}{8}$	151.8	44.64	3.49	26.0	535.7	8.75
	$1\frac{1}{4}$	162.0	47.64	3.49	26.0	571.7	9.33
35 lbs.	Lat.	70.0	20.58	3.35	25.0	247.0	4.20
	$\frac{13}{16}$	136.3	40.08	3.45	25.7	481.0	7.94
	$\frac{7}{8}$	141.4	41.58	3.45	25.7	499.0	8.24
	$\frac{15}{16}$	146.5	43.08	3.45	25.7	517.0	8.54
	1	151.6	44.58	3.45	25.7	535.0	8.83
	$1\frac{1}{8}$	161.8	47.58	3.45	25.7	571.0	9.43
	$1\frac{1}{4}$	172.0	50.58	3.45	25.7	607.0	10.0
	$1\frac{3}{8}$	182.2	53.58	3.45	25.7	643.0	10.6
	$1\frac{1}{2}$	192.4	56.58	3.45	25.7	679.0	11.2

† Lattice bars not less than $2'' \times \frac{5}{16}''$.

2 12-in. Chan Lat.‡ or with 2 Side Plates 14 inches wide } $B=8\frac{1}{4}$″, $C=11\frac{1}{4}$″

Wt. per foot ea. Channel.	Thickness of Plate.	Wt. per foot of Column.	Area of Section.	Rad of Gyr	Short Col.		Decr. in S per ea. foot incr. in L.
					Max. L'gth.	Safe Load.	
Lbs.	Ins.	Lbs.	Sq. Ins.	r (min.)	L Ft.	S 1000#	1000#
20.5 lbs.	Lat.	41.0	12.06	4.61	34.3	144.7	1.79
	$\frac{5}{16}$	70.8	20.81	4.40	32.8	249.7	3.23
	$\frac{3}{8}$	76.7	22.56	4.38	32.6	270.7	3.52
	$\frac{7}{16}$	82.6	24.31	4.35	32.4	291.7	3.82
	$\frac{1}{2}$	88.6	26.06	4.33	32.3	312.7	4.12
	$\frac{9}{16}$	94.6	27.81	4.32	32.2	333.7	4.41
	$\frac{5}{8}$	100.5	29.56	4.30	32.1	354.7	4.71
	$\frac{11}{16}$	106.4	31.31	4.29	32.0	375.7	4.98
	$\frac{3}{4}$	112.4	33.06	4.27	31.8	396.7	5.30
25 lbs.	Lat.	50.0	14.70	4.43	33.0	176.4	2.27
	$\frac{7}{16}$	91.6	26.95	4.30	32.1	323.4	4.28
	$\frac{1}{2}$	97.6	28.70	4.29	32.0	344.4	4.58
	$\frac{9}{16}$	103.6	30.45	4.27	31.8	365.4	4.88
	$\frac{5}{8}$	109.5	32.20	4.26	31.7	386.4	5.18
	$\frac{11}{16}$	115.4	33.95	4.25	31.6	407.4	5.47
	$\frac{3}{4}$	121.4	35.70	4.24	31.6	428.4	5.77
	$\frac{13}{16}$	127.3	37.45	4.23	31.6	449.4	6.06
	$\frac{7}{8}$	133.3	39.20	4.22	31.4	470.4	6.36
30 lbs.	Lat.	60.0	17.64	4.28	31.9	211.7	2.82
	$\frac{9}{16}$	113.6	33.39	4.23	31.6	400.7	6.39
	$\frac{5}{8}$	119.5	35.14	4.22	31.4	421.7	5.69
	$\frac{11}{16}$	125.4	36.89	4.21	31.4	442.7	6.00
	$\frac{3}{4}$	131.4	38.64	4.20	31.3	463.7	6.29
	$\frac{13}{16}$	137.3	40.39	4.20	31.3	484.7	6.57
	$\frac{7}{8}$	143.3	42.14	4.19	31.2	505.7	6.87
	$\frac{15}{16}$	149.3	43.89	4.18	31.2	526.7	7.17
	1	155.2	45.64	4.18	31.2	547.7	7.47
35 lbs.	Lat.	70.0	20.58	4.17	31.1	247.0	3.37
	$\frac{11}{16}$	135.4	39.83	4.17	31.1	478.0	6.54
	$\frac{3}{4}$	141.4	41.58	4.16	31.0	499.0	6.84
	$\frac{13}{16}$	147.3	43.33	4.16	31.0	520.0	7.13
	$\frac{7}{8}$	153.3	45.08	4.15	30.9	541.0	7.43
	$\frac{15}{16}$	159.3	46.83	4.15	30.9	562.0	7.72
	1	165.2	48.58	4.14	30.8	583.0	8.02
	$1\frac{1}{8}$	177.1	52.08	4.14	30.8	625.0	8.61
	$1\frac{1}{4}$	189.0	55.58	4.13	30.8	667.0	9.20
40 lbs.	Lat.	80.0	23.52	4.09	30.5	282.2	3.93
	$\frac{13}{16}$	157.3	46.27	4.12	30.7	555.2	7.68
	$\frac{7}{8}$	163.3	48.02	4.11	30.6	576.2	7.98
	$\frac{15}{16}$	169.3	49.77	4.11	30.6	597.2	8.28
	1	175.2	51.52	4.11	30.6	618.2	8.58
	$1\frac{1}{8}$	187.1	55.02	4.10	30.5	660.2	9.18
	$1\frac{1}{4}$	199.0	58.52	4.10	30.5	702.2	9.76
	$1\frac{3}{8}$	210.9	62.02	4.10	30.5	744.2	10.3
	$1\frac{1}{2}$	222.8	65.52	4.09	30.5	786.2	10.9

‡ Lattice bars not less than $2'' \times \frac{3}{8}''$.

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11.—* Phœnix Steel Columns.

Dimensions, Properties and Loads.

"The dimensions given in the following table are subject to such slight variations as are unavoidable in the manufacture of these shapes.

The weights given are those of the segments composing the columns, and from 2 to 5 per cent must be added for weight of the rivet heads.

The safe loads specified are computed as being one-fourth of the ultimate, or breaking loads, and as producing a strain, or pressure, in an axial direction on square-end columns, of not more than 12,000 lbs. per square inch for lengths of 90 radii and under.

The A, B1, B2, and C columns have each 4 segments, the E have 6, and the G have 8 segments.

Any desired thickness between the minimum and maximum can be furnished.

Least Radius of Gyration $= D_0 \times 0.3636$.

One Segment.				Diameters, in Ins.			One Column.			
Section.		Thickness in Inches	Wt. per Foot in Lbs.	D Inside	D_0 Outside.	D_1 Over Flanges.	Area of Cross Section. Sq. In.	Wt. per Foot in Lbs.	Least Rad. of Gyration in Ins.	Safe Ld. for 16-ft. Col. in 1000 lbs.
	4 segments.	3/16	3.2	A 3 5/8	4	6 1/16	3.8	12.9	1.45	36
		1/4	4.1		4 1/8	6 3/16	4.8	16.3	1.50	47
		5/16	4.9		4 1/4	6 5/16	5.8	19.7	1.55	58
		3/8	5.8		4 3/8	6 7/16	6.8	23.1	1.59	69
5.33 to 8.33 lbs.	4 segments.	1/4	5.4	B1 4 7/8	5 3/8	8 1/8	6.4	21.8	1.95	74
		5/16	6.6		5 1/2	8 3/16	7.8	26.5	2.00	90
		3/8	7.8		5 5/8	8 5/16	9.2	31.3	2.04	108
		7/16	9.0		5 3/4	8 7/16	10.6	36.0	2.09	126
		1/2	10.2		5 7/8	8 1/2	12.0	40.8	2.13	144
		9/16	11.4		6	8 9/16	13.4	45.6	2.18	161
		5/8	12.6		6 1/8	8 5/8	14.8	50.3	2.23	178
6.16 to 14.16 lbs.	4 segments.	1/4	6.3	B2 6 1/16	6 3/16	9 1/4	7.4	25.2	2.39	89
		5/16	7.6		6 5/16	9 3/8	9.0	30.6	2.43	108
		3/8	9.0		6 11/16	9 7/16	10.6	36.0	2.48	127
		7/16	10.4		6 11/16	9 1/2	12.2	41.5	2.52	146
		1/2	11.7		7 1/16	9 5/8	13.8	46.9	2.57	166
		9/16	13.1		7 3/16	9 3/4	15.4	52.4	2.61	185
		5/8	14.4		7 5/16	9 11/16	17.0	57.8	2.66	204
8.33 to 34.33 lbs.	4 segments.	1/4	8.5	C 7 5/16	7 13/16	11 11/16	10.0	34.0	2.84	120
		5/16	10.3		7 15/16	11 3/4	12.1	41.3	2.88	145
		3/8	12.0		8 1/16	11 13/16	14.1	48.0	2.93	169
		7/16	13.7		8 3/16	11 7/8	16.0	54.6	2.97	192
		1/2	15.3		8 5/16	11 15/16	18.0	61.3	3.01	216
		9/16	17.0		8 7/16	12	20.0	68.0	3.06	239
		5/8	18.3		8 9/16	12 1/16	21.9	74.6	3.11	263
		11/16	20.7		8 11/16	12 3/16	24.3	82.6	3.16	292
		3/4	22.7		8 13/16	12 5/16	26.6	90.6	3.20	319
		13/16	24.3		8 15/16	12 7/16	28.6	97.3	3.24	343
		7/8	26.0		9 1/16	12 1/2	30.6	104.0	3.29	367
		1	29.7		9 3/16	12 5/8	34.9	118.6	3.34	418
		1 1/8	33.0		9 5/16	12 11/16	38.8	132.0	3.48	466
		1 1/4	36.3		9 11/16	13	42.7	145.3	3.57	512

Figs. 10–13.

*By permission of Mr. D. W. Bowman, Chief Engineer Phœnix Iron Works.

11.—PHŒNIX STEEL COLUMNS.—Concluded.

One Segment.			Diameters, in Ins.			One Column.			
Section.	Thickness in Inches	Wt. per Foot in Lbs.	D Inside	D_0 Outside	D_1 Over Flanges.	Area of Cross Section. Sq. In.	Wt. per Foot in Lbs.	Least Rad. of Gyration in Ins.	Safe Ld. for 16-ft. Col. in 1000 lbs.
Fig. 14.	6 segments.								
	1/4	9.3	E $11\frac{1}{16}$	$11\frac{9}{16}$	$15\frac{1}{2}$	16.5	56.0	4.20	198
	5/16	10.8		$11\frac{11}{16}$	$15\frac{5}{8}$	19.1	65.0	4.25	229
	3/8	12.3		$11\frac{13}{16}$	$15\frac{3}{4}$	21.7	74.0	4.29	260
	7/16	14.0		$11\frac{15}{16}$	$15\frac{7}{8}$	24.7	84.0	4.34	296
	1/2	15.7		$12\frac{1}{16}$	$15\frac{15}{16}$	27.6	94.0	4.38	331
	9/16	17.3		$12\frac{3}{16}$	$16\frac{1}{16}$	30.6	104.0	4.43	367
	5/8	19.0		$12\frac{5}{16}$	$16\frac{3}{16}$	33.5	114.0	4.48	402
	11/16	20.7		$12\frac{7}{16}$	$16\frac{5}{16}$	36.4	124.0	4.52	437
	3/4	22.7		$12\frac{9}{16}$	$16\frac{7}{16}$	40.0	136.0	4.56	480
	13/16	24.3		$12\frac{11}{16}$	$16\frac{9}{16}$	43.0	146.0	4.61	516
	7/8	26.0		$12\frac{13}{16}$	$16\frac{11}{16}$	45.9	156.0	4.66	551
	1	29.3		$13\frac{1}{16}$	$16\frac{13}{16}$	51.7	176.0	4.73	620
	1 1/8	32.7		$13\frac{3}{16}$	$17\frac{1}{16}$	57.6	196.0	4.84	691
	1 1/4	36.0		$13\frac{5}{16}$	$17\frac{3}{16}$	63.5	216.0	4.93	762
Fig. 15.	8 segments.								
	1/4	10.3	G $14\frac{5}{8}$	$15\frac{1}{4}$	$19\frac{3}{8}$	24.3	82.6	5.54	290
	5/16	12.0		$15\frac{3}{8}$	$19\frac{1}{2}$	28.2	96.0	5.59	337
	3/8	13.7		$15\frac{1}{2}$	$19\frac{5}{8}$	32.1	109.3	5.64	384
	7/16	15.3		$15\frac{5}{8}$	$19\frac{11}{16}$	36.0	122.6	5.68	432
	1/2	17.0		$15\frac{3}{4}$	$19\frac{3}{4}$	40.0	136.0	5.73	479
	9/16	18.7		$15\frac{7}{8}$	$19\frac{7}{8}$	43.9	149.3	5.77	526
	5/8	20.3		16	20	47.8	162.6	5.82	572
	11/16	22.0		$16\frac{1}{8}$	$20\frac{1}{8}$	51.7	176.0	5.88	620
	3/4	23.7		$16\frac{1}{4}$	$20\frac{1}{4}$	55.7	189.3	5.91	667
	13/16	25.3		$16\frac{3}{8}$	$20\frac{3}{8}$	59.6	202.6	5.95	715
	7/8	28.7		$16\frac{5}{8}$	$20\frac{5}{8}$	67.4	229.3	6.04	809
	1	32.0		$16\frac{7}{8}$	$20\frac{7}{8}$	75.3	256.0	6.13	904
	1 1/8	35.3		$17\frac{1}{8}$	21	83.1	282.6	6.27	997
	1 1/4	38.7		$17\frac{3}{8}$	$21\frac{1}{4}$	90.9	309.3	6.32	1091

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12.—Ultimate and Safe (1/8) Strengths of Hollow Round and Hollow Rectangular Cast Iron Columns.

In the following formulas, $U = 80\,000$ and $S = 10\,000$ lbs. per sq. in.:

Round Columns. Rectangular Columns.

Square Ends. Square & Pin. Pin Ends. Square Ends. Square & Pin. Pin Ends.

$$p = \frac{U \text{ or } S}{1 + \frac{(12L)^2}{800d^2}} = \frac{U \text{ or } S}{1 + \frac{3(12L)^2}{1600d^2}} = \frac{U \text{ or } S}{1 + \frac{(12L)^2}{400\,d^2}} = \frac{U \text{ or } S}{1 + \frac{3(12L)^2}{3200d^2}} = \frac{U \text{ or } S}{1 + \frac{9(12L)^2}{6400d^2}} = \frac{U \text{ or } S}{1 + \frac{3(12L)^2}{1600d^2}}$$

L = length of column in feet; d = (least) outside diameter in inches;
p = load in lbs. per square inch on column to produce U or S.

$\frac{L}{d}$	Round Columns. Loads in 1000 lbs. per square inch.						Rectangular Columns. Loads in 1000 lbs. per square inch.					
	Square Ends		Sq. and Pin		Pin Ends		Square Ends		Sq. and Pin		Pin Ends	
	Ult. Load.	Safe Load.	Ult. Load.	Safe Load.	Ult. Load.	Safe Load.	Ult. Load.	Safe Load.	Ult. Load.	Safe Load.	Ult. Load.	Safe Load.
1.0	67.80	8.47	62.99	7.87	58.82	7.35	70.48	8.81	66.52	8.31	62.99	7.87
1.1	65.69	8.21	60.30	7.54	55.73	6.97	68.79	8.60	64.26	8.03	60.30	7.54
1.2	63.53	7.94	57.60	7.20	52.69	6.59	67.00	8.37	61.94	7.74	57.60	7.20
1.3	61.34	7.67	54.93	6.87	49.74	6.22	65.14	8.14	59.60	7.45	54.96	6.87
1.4	59.14	7.39	52.31	6.54	46.90	5.86	63.26	7.91	57.27	7.16	52.32	6.54
1.5	56.94	7.12	49.77	6.22	44.20	5.52	61.35	7.67	54.96	6.87	49.76	6.22
1.6	54.76	6.84	47.30	5.91	41.63	5.20	59.45	7.43	52.68	6.58	47.30	5.91
1.7	52.62	6.58	44.94	5.62	39.21	4.90	57.55	7.19	50.46	6.31	44.96	5.62
1.8	50.53	6.32	42.67	5.33	36.93	4.62	55.67	6.96	48.30	6.04	42.67	5.33
1.9	48.49	6.06	40.51	5.06	34.79	4.35	53.80	6.72	46.23	5.78	40.51	5.06
2.0	46.51	5.81	38.46	4.81	32.79	4.10	51.94	6.49	44.20	5.52	38.46	4.81
2.1	44.60	5.57	36.52	4.56	30.92	3.86	50.16	6.27	42.26	5.28	36.52	4.56
2.2	42.75	5.34	34.68	4.33	29.18	3.65	48.40	6.05	40.40	5.05	34.68	4.33
2.3	40.98	5.12	32.94	4.12	27.54	3.44	46.67	5.83	38.63	4.83	32.95	4.12
2.4	39.28	4.91	31.31	3.91	26.03	3.25	44.99	5.62	36.93	4.62	31.31	3.91
2.5	37.65	4.71	29.77	3.72	24.62	3.08	43.39	5.42	35.31	4.41	29.76	3.72
2.6	36.09	4.51	28.32	3.54	23.30	2.91	41.82	5.23	33.77	4.22	28.32	3.54
2.7	34.60	4.32	26.95	3.37	22.07	2.76	40.32	5.04	32.31	4.04	26.95	3.37
2.8	33.18	4.15	25.67	3.21	20.93	2.62	38.87	4.86	30.92	3.86	25.67	3.21
2.9	31.82	3.98	24.46	3.06	19.86	2.48	37.47	4.68	29.60	3.70	24.46	3.06
3.0	30.53	3.82	23.32	2.91	18.87	2.36	36.12	4.51	28.34	3.54	23.32	2.91
3.1	29.31	3.66	22.25	2.78	17.94	2.24	34.83	4.35	27.15	3.39	22.25	2.78
3.2	28.14	3.52	21.25	2.66	17.07	2.13	33.58	4.20	26.03	3.25	21.25	2.66
3.3	27.03	3.38	20.30	2.54	16.26	2.03	32.39	4.05	24.96	3.12	20.30	2.54
3.4	25.97	3.25	19.41	2.43	15.50	1.94	31.25	3.91	23.94	2.99	19.41	2.43
3.5	24.96	3.12	18.58	2.32	.....	.....	30.15	3.77	22.97	2.87	.....	.....
3.6	24.00	3.00	17.71	2.21	.....	.....	29.10	3.64	22.05	2.76	.....	.....
3.7	23.09	2.89	.....	.....	.....	.....	28.09	3.51	.....	.....	.....	.....
3.8	22.23	2.78	.....	.....	.....	.....	27.13	3.39	.....	.....	.....	.....
3.9	21.40	2.67	.....	.....	.....	.....	26.21	3.28	.....	.....	.....	.....

Safe loads given in the table are equal to one-eighth the ultimate loads, that is, using safety factor 8. If the safety factor 10 is preferred the new safe load may be found from the *ultimate* load by moving the decimal point one place to the left.

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13.—SAFE (1/8) LOADS ON HOLLOW ROUND CAST IRON COLUMNS WITH FLAT ENDS

By Formula $P = \dfrac{10\,000A}{1 + \dfrac{(12L)^2}{800d^2}}$; in which $\begin{cases} P = \text{total load on column in 1000 lbs.} \\ A = \text{sectional area of column in sq.in.} \\ L = \text{length of column in feet.} \\ d = \text{outside diam. of column in ins.} \end{cases}$

Outside Diam.	Thickn's of Metal.	Sectional Area.	Weight of Col. per Lin. Ft.	Length of Column in Feet.									
				6	8	10	12	14	16	18	20	22	24
Ins.	Ins.	Ins.	Lbs.	Total *Safe Load on Column in 1000 lbs.									
6	1/8	8.6	27.0	73	65	57	50	44	38	33	29	25	22
	5/16	10.6	33.0	90	80	71	62	54	46	40	35	31	27
	3/8	12.4	38.7	105	94	82	72	62	54	47	41	36	32
	7/16	14.1	44.0	119	107	94	82	71	62	54	47	41	36
	1	15.7	49.1	133	119	105	91	79	69	60	52	46	40
7	5/16	12.5	39.1	111	101	91	82	73	64	57	51	45	40
	3/8	14.7	46.0	130	119	108	96	86	76	67	60	53	47
	7/16	16.8	52.6	149	136	123	110	98	87	77	68	61	54
	1	18.9	58.9	167	153	138	123	109	97	86	76	68	60
	1 1/8	20.8	64.9	184	168	152	136	121	107	95	84	75	67
8	3/8	17.1	53.4	155	145	133	122	110	99	89	80	72	65
	7/16	19.6	61.2	178	166	153	139	126	114	104	92	83	75
	1	22.0	68.7	200	186	172	158	142	128	115	103	93	84
	1 1/8	24.3	75.9	220	206	190	173	157	142	127	114	103	93
	1 1/4	26.5	82.8	239	225	207	189	171	154	139	125	112	101
9	7/16	22.3	69.8	207	196	183	169	159	142	130	118	108	98
	1	25.1	78.5	233	220	206	190	179	160	146	133	121	110
	1 1/8	27.8	87.0	258	244	228	211	198	177	162	147	134	122
	1 1/4	30.4	95.1	281	266	249	230	212	194	177	161	147	133
	1 3/8	32.9	102.9	304	288	269	249	229	210	191	174	159	145
10	7/16	25.1	78.4	235	225	212	199	185	172	158	146	134	123
	1	28.3	88.4	265	254	240	224	209	194	178	164	151	139
	1 1/4	34.4	107.4	323	308	291	273	254	235	217	200	184	169
	1 1/2	41.1	125.2	376	359	339	318	296	274	253	233	214	197
	1 3/4	45.4	141.7	426	407	384	360	335	310	286	264	242	223
11	1	31.4	98.2	298	287	273	259	243	227	212	197	183	169
	1 1/4	38.3	119.7	363	350	333	315	296	277	258	240	223	206
	1 1/2	44.8	139.9	425	409	390	369	346	322	302	286	260	241
	1 3/4	50.9	158.9	483	464	443	419	394	368	343	324	296	274
	2	56.6	176.7	537	516	492	466	438	409	381	354	329	306
12	1	34.6	108.0	331	320	307	293	277	262	246	230	215	201
	1 1/4	42.2	131.9	404	391	375	358	339	320	300	281	263	245
	1 1/2	49.5	154.6	473	458	440	419	397	375	352	330	308	288
	1 3/4	56.4	176.1	540	522	501	477	453	427	401	376	351	328
	2	62.8	196.4	602	582	558	532	505	476	447	419	391	365
13	1	37.7	117.8	363	353	341	327	312	296	280	264	249	234
	1 1/4	46.1	144.2	444	432	417	400	382	363	343	323	304	286
	1 1/2	54.2	169.4	522	507	490	470	448	426	403	380	358	336
	1 3/4	61.9	193.3	596	579	559	536	512	486	460	434	408	383
	2	69.1	216.0	665	647	625	599	572	543	514	485	456	428
14	1	40.8	127.6	395	386	374	361	346	331	315	299	283	267
	1 1/4	50.1	156.5	485	473	459	442	424	405	386	366	347	327
	1 1/2	58.9	184.1	570	556	540	520	499	477	454	431	408	385
	1 3/4	67.4	210.5	652	636	617	595	571	545	519	492	466	440
	2	75.4	235.6	730	712	690	666	639	610	581	551	522	493
15	1	44.0	137.4	427	418	407	394	380	365	349	333	317	301
	1 1/4	54.0	168.7	525	514	500	484	467	448	429	409	389	370
	1 1/2	63.6	203.4	618	605	589	570	550	528	505	482	459	439
	1 3/4	72.9	227.6	708	694	675	653	630	605	577	552	525	502
	2	81.7	255.2	795	777	756	732	706	678	649	619	589	559

* This table is for safety factor 8; for safety factor 10 multiply the tabular safe loads by 4/5.

14.—Rolled Steel H Columns.
(Bethlehem Steel Co.)

For all sections.

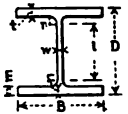

Fig. 16.

W = wt. of section in lbs. per lin. ft.
A = area of section in sq. ins.
r' = least radius of gyration.

8" H Columns.
(Section Number = H8.)

W	A	Dimen. in Ins.[1] D	t	w	r'
31.5	9.17	7⅞	7/16	.31	1.98
34.5	10.17	8	½	.31	2.01
39.0	11.50	8⅛	9/16	.35	2.03
43.5	12.83	8¼	5/8	.39	2.04
48.0	14.18	8⅜	11/16	.43	2.05
53.0	15.53	8½	¾	.47	2.07
57.5	16.90	8⅝	13/16	.51	2.08
62.0	18.27	8¾	7/8	.55	2.09
67.0	19.66	8⅞	15/16	.59	2.11
71.5	21.05	9	1	.63	2.12
76.5	22.46	9⅛	1 1/16	.67	2.13
81.0	23.78	9¼	1⅛	.70	2.14
85.5	25.20	9⅜	1 3/16	.74	2.16
90.5	26.64	9½	1¼	.78	2.17

[1] l = 6.14 = tang. dist. bet. fillets; r = 0.40 = rad. of fillets; B = 7.69 + w; m = t − 0.038; n = t + 0.038.

10" H Columns.
(Section Number = H10.)

W	A	Dimen. in Ins.[2] D	t	w	r'
49.0	14.37	9⅞	9/16	.36	2.49
54.0	15.91	10	⅝	.39	2.51
59.5	17.57	10⅛	11/16	.43	2.53
65.5	19.23	10¼	¾	.47	2.54
71.0	20.91	10⅜	13/16	.51	2.56
77.0	22.59	10½	⅞	.55	2.57
82.5	24.29	10⅝	15/16	.59	2.58
88.5	25.99	10¾	1	.63	2.60
94.0	27.71	10⅞	1 1/16	.67	2.61
99.5	29.32	11	1⅛	.70	2.62
105.5	31.06	11⅛	1 3/16	.74	2.64
111.5	32.80	11¼	1¼	.78	2.65
117.5	34.55	11⅜	1 5/16	.82	2.66
123.5	36.32	11½	1⅜	.86	2.67

[2] l = 7.67 = tang. dist. bet. fillets; r = 0.50 = rad. of fillets; B = 9.61 + w; m = t − 0.048; n = t + 0.048.

12" H Columns.
(Section Number = H12.)

W	A	Dimen. in Ins.[3] D	t	w	r'
64.5	19.00	11¾	5/8	.39	2.98
71.5	20.96	11⅞	11/16	.43	3.00
78.0	22.94	12	¾	.47	3.01
84.5	24.92	12⅛	13/16	.51	3.03
91.5	26.92	12¼	⅞	.55	3.04
98.5	28.93	12⅜	15/16	.59	3.06
105.0	30.94	12½	1	.63	3.07
112.0	32.96	12⅝	1 1/16	.67	3.08
118.5	34.87	12¾	1⅛	.70	3.10
125.5	36.91	12⅞	1 3/16	.74	3.11
132.5	38.97	13	1¼	.78	3.13
139.5	41.03	13⅛	1 5/16	.82	3.14
146.5	43.10	13¼	1⅜	.86	3.15
153.5	45.19	13⅜	1 7/16	.90	3.16
161.0	47.28	13½	1½	.94	3.18

[3] l = 9.21 = tang. dist. bet. fillets; r = 0.60 = rad. of fillets; B = 11.53 + w; m = t − 0.058; n = t + 0.058.

14" H Columns.
(Section Number = H14.)

W	A	Dimen. in Ins.[4] D	t	w	r'
83.5	24.46	13¾	11/16	.43	3.47
91.0	26.76	13⅞	¾	.47	3.49
99.0	29.06	14	13/16	.51	3.50
106.5	31.38	14⅛	⅞	.55	3.52
114.5	33.70	14¼	15/16	.59	3.53
122.5	36.04	14⅜	1	.63	3.55
130.5	38.38	14½	1 1/16	.67	3.56
138.0	40.59	14⅝	1⅛	.70	3.58
146.0	42.95	14¾	1 3/16	.74	3.59
154.0	45.33	14⅞	1¼	.78	3.61
162.0	47.71	15	1 5/16	.82	3.63
170.5	50.11	15⅛	1⅜	.86	3.64
178.5	52.51	15¼	1 7/16	.90	3.65
186.5	54.92	15⅜	1½	.94	3.66
195.0	57.35	15½	1 9/16	.98	3.68
203.5	59.78	15⅝	1⅝	1.02	3.69
211.0	62.07	15¾	1 11/16	1.05	3.70
219.5	64.52	15⅞	1¾	1.09	3.71
227.5	66.98	16	1 13/16	1.13	3.72
236.0	69.45	16⅛	1⅞	1.17	3.74
244.5	71.94	16¼	1 15/16	1.21	3.75
253.0	74.43	16⅜	2	1.25	3.76
261.5	76.93	16½	2 1/16	1.29	3.77
270.0	79.44	16⅝	2⅛	1.33	3.79
278.5	81.97	16¾	2 3/16	1.37	3.80
287.5	84.50	16⅞	2¼	1.41	3.81

[4] l = 11.06 = tang. dist. bet. fillets; r = 0.60 = rad. of fillets; B = 13.49 + w; m = t − 0.067; n = t + 0.067.

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Reinforced Concrete Columns.—The following working stresses for static loads are recommended by the Special Committee of the Am. Soc. C. E., on Concrete and Reinforced Concrete. See Trans. A. S. C. E., Vol. LXVI., page 452. For Notation and Formulas, see Sec. 25, Masonry, page 446. For working stresses for Beams, see Sec. 31, page 585.

Average compressive strength of concrete—2000 lbs. per sq. in. at 28 days, when tested in cylinders 8 ins. in dia. and 16 ins. long, under laboratory conditions of manufacture and storage.

Bearing.—See page 585.

Axial Compressions.—(A). For concentric compression on a plain concrete column or pier, when the length does not exceed 12 diameters, 450 lbs. per sq. in. on 2000-lb. concrete may be allowed. (B). Columns with longitudinal reinforcement only, 450 lbs. per sq. in. on 2000-lb. concrete. (C). Columns with reinforcement of bands or hoops, 540 lbs. per sq. in. on 2000-lb. concrete may be allowed. (D). Columns reinforced with not less than 1% and not more than 4% of longitudinal bars and with bands or hoops, 650 lbs. per sq. in. for 2000-lb. concrete. (E). Columns reinforced with structural steel column units which thoroughly encase the concrete core, 650 lbs. per sq. in. for 2000-lb. concrete.

Reinforcement.—In all cases, longitudinal steel is assumed to carry its proportion of stress; and the compressive stress shall not exceed 16000 lbs. per sq. in. or 15 times the working compressive stress in the concrete. Hoops or bands are not to be counted upon directly as adding to the strength of the column. Bars composing longitudinal reinforcement shall be straight, and shall have sufficent lateral support to be securely held in place until the concrete has set. When bands or hoops are used, the total amount of reinforcement shall not be less than 1% of the volume of the column enclosed. The clear spacing of such bands or hoops shall not be greater than one-fourth the diameter of the enclosed column. Adequate means must be provided to hold bands or hoops in place so as to form a column with a straight and well-centered core. Bending stresses due to eccentric loads must be provided for by increasing the section until the maximum stress does not exceed the values above specified.

EXCERPTS AND REFERENCES.

Reinforcement of Concrete Columns (By E. P. Goodrich. Eng. News, July 19, 1906).—"Considere reports that spiral steel is 2.4 times as effective as longitudinal reinforcement of equal weight, and this figure was closely checked by v. Bach, of Stuttgart."

Table of Weights of Lacing for Steel Compression Members (By C. T. Lewis. Eng. News, Aug. 2, 1906).—Weights are in lbs. per lin. ft. of single- or double-laced member on *one side;* the depths of member ranging from 5" to 23", and the size of lacing bars from 1½"x¼" to 3"x⅜". Rivets, ½" to ¾".

Table of the Various Column Formulas in Use (Eng. News, Jan. 3, 1907).—Comprises formulas of the Rankin type for steel and cast iron; of the straight-line type for steel, cast iron and timber; and of miscellaneous type for timber: with an equivalent reduction, for all formulas, to a formula having a factor of eccentricity e. Interesting as a study.

Detachable Form for Concrete Columns (By W. S. Coulter. Eng. News, Mar. 28, 1907).—Illustration and description. When building reinforced concrete columns, having flexure rods united at intervals by ties, forms are often used having one side open, which is built up in sections as the concrete is deposited. These forms usually consist of four or more uprights enclosed on three sides, the fourth being open to allow access to the interior, and closed as the work proceeds by horizontal boards nailed to the uprights. Through the open side all the operations of depositing, spading and tying are conducted. The present device is intended to facilitate the work of erection and expedite the placing of the concrete.

Table of Tests of Carbon-Steel and Nickel-Steel Columns, and Comparison with Formulas (By C. P. Buchanan. Eng. News, Feb. 13, 1908).—Table gives actual strength and computed strength of columns. The computed strengths are from the following formulas:

(1) Dagron's tests (steel cols.) compared with..... $P = 61000 - 263\ l/r$
(2) Waddell's tests (nickel-steel) compared with..... $P = 47000 - 178\ l/r$
(3) Buchanan's tests (steel) compared with..... $P = 79000 - 388\ l/r$

In which P = computed strength in lbs. per sq. in.; and l and r the length and radius of gyration, in ins. (See, also, Eng. News of April 9, 1908.)

Safe Stresses in Steel Columns (By J. R. Worcester. Trans A. Vol. LXI).

Tests of Reinforced Concrete Columns at Minneapolis, Mi J. G. Houghton and W. P. Cowles. Eng. News, Dec. 3, 1908).—The tested had the following kinds of reinforcement: Spiral wire circular flat-bar bands; wire bands, square; wire bands, circular. columns were made of 1:2:3½ concrete, using bank sand and blue li one-half of the stone being of ½-in. size and one-half of pea size. inforced columns averaged about 25% stronger than the plain though 5 of the 17 failed at lower loads than the plain columns.

Preliminary Program of Tests of Steel Columns (Proc. A. S. T. VIII., 1908).—Types of sections selected for testing, at Watertown (a) Annular section (welded tubes); (b) New wide flange H sectio section of four angles and central web-plate; (d) Double-channel latticed in two planes. Also other shapes. Illustrated. "Some r the tests," by J. E. Howard, may be found on pages 336 to 344, s of Proc.; also Vol. IX., 1909, page 413.

Tests of Plain and Reinforced Concrete Columns (By M. O. Proc. A. S. T. M., Vol. IX., 1909).—Tables and diagrams.

Tests of Plain and Reinforced Concrete Columns (By M. O. Paper A. S. T. M., July 1, 1909; Eng. Rec., July 10, 1909).—Co derived from the tests: 1. A small amount, ½ to 1%, of closely space reinforcement, such as the spirals used, will greatly increase the t and ultimate strength of a concrete column, but does not material the yield point. More than 1% of lateral reinforcement does not a be necessary. The use of lateral reinforcement alone does not see able. 2. Vertical steel in combination with such a lateral reinf raises the yield point and ultimate strength of the column and inc stiffness. Columns reinforced with vertical steel only are brittle suddenly when the yield point of the steel is reached, but are cons stronger than plain columns made from the same grade of conc Increasing the amount of cement in a spirally reinforced column the strength and stiffness of the column. A column made of rich co mortar and containing small percentages of longitudinal and late forcement is without doubt fully as stiff and strong and more ec than one made from a leaner mix reinforced with considerably m In these tests doubling the amount of cement increased the yield p ultimate strength of the columns without vertical steel about 100% a about 50% to the strength of those with 6.1% vertical steel. 4. behavior of the columns reinforced with spirals and vertical steel, u and the results computed, it would seem that a static load equal to 3 of the yield point would be a safe working load. As the ultimate of the concrete and the yield point of the steel are generally know be assumed with fair accuracy, formula (A) can be readily used to d the working load. In Fig. 7 the dotted lines represent working $P+A$ equal to 40% of the yield point load. (See original article for A and Fig. 7). 5. The results obtained from tests of columns re with structural steel indicate that such columns have considerable and toughness and that the steel and concrete core act in unison u yield point of the former. The shell concrete will remain intact yield point of the steel is reached, but no allowance should be mad strength or stiffness. 6. As many of the blotters on the tops and of columns bore imprints of the vertical steel after failure, it woul safe precaution to use bed plates at the foundations for such colu thus prevent any possibility of the steel punching through the concre an excessive load.

Tests of Nickel-Steel Models of Compression Members in the Design of the New Quebec Bridge (Eng. Rec., Nov. 19, 1910).—Ill description with table of results of tests.

Illustrations and Diagrams.

Description.	E
Rein.-terra-cotta col. tested to 4109 lbs. per sq. in., uninjured	Fe
Economy diagrams of plain and reinforced concrete columns	Ja
Repeated and eccentric load tests on rein.-conc. columns	Ju

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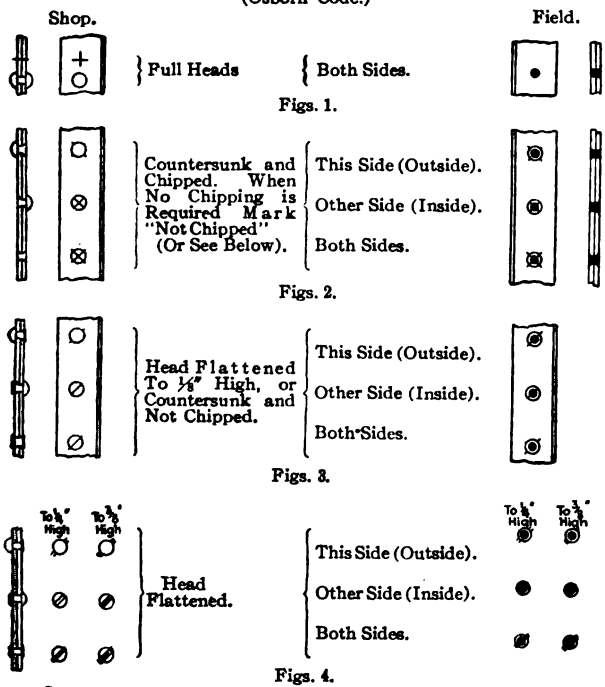
33.—STRUCTURAL DETAILS.

The handbooks published by the various steel manufacturers are now pretty well standardized, and are indispensable to constant designers and detailers of structural work. The writer aims to keep this volume abreast of the most approved practice in the design of ordinary structures.

Rivets.—In the '80's, iron rivets began to give way to steel rivets in engineering structures, and now the latter are universally employed. The best rivet steel is a soft steel whose tensile strength varies not more than a few thousand pounds from 53 000 lbs. per sq. in.; the manufacturer's standard specifying 48 000 and 58 000 as the lower and upper limits. Higher grades of steel are more liable to fracture both in driving and afterward when subjected to repeated stresses in the structure.

CONVENTIONAL RIVET SIGNS.

(Osborn Code.)



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1.—Rivets—Shearing and Bearing Value

In Pounds.

(Dimensions are in Inches.)

Diam. of Rivet.	Area. Sq. Ins.	Single Shear		Bearing Value for Plates of Thickness—								
				¼	5/16	⅜	7/16	½	9/16	⅝	11/16	¾
⅜	.1104	At 6,000 lbs.	660	At 12,000 lbs.	1130	1410	1690					
½	.1963		1180		1500	1880	2250	2630	3000			
⅝	.3068		1840		1880	2340	2810	3280	3750	4220	4690	
¾	.4418		2650		2250	2810	3380	3940	4500	5060	5630	6190	6750
⅞	.6013		3610		2630	3280	3940	4590	5250	5910	6560	7220	7880
⅜	.1104	At 7,500 lbs.	830	At 15,000 lbs.	1410	1760	2110					
½	.1963		1470		1880	2340	2810	3280	3750			
⅝	.3068		2300		2340	2930	3520	4100	4690	5280	5860	
¾	.4418		3310		2810	3520	4220	4920	5630	6330	7030	7720	8440
⅞	.6013		4510		3280	4100	4920	5740	6560	7380	8200	9030	9850
⅜	.1104	At 10,000 lbs.	1100	At 20,000 lbs.	1880	2340	2810					
½	.1963		1960		2500	3130	3750	4380	5000			
⅝	.3068		3070		3130	3910	4690	5470	6250	7030	7810	
¾	.4418		4420		3750	4690	5630	6560	7500	8440	9380	10310	11250
⅞	.6013		6010		4380	5470	6570	7660	8750	9840	10940	12030	13130
⅜	.1104	At 12,000 lbs.	1320	At 25,000 lbs.	2350	2930	3520					
½	.1963		2360		3130	3910	4690	5470	6250			
⅝	.3068		3680		3910	4880	5860	6840	7810	8790	9770	
¾	.4418		5300		4690	5860	7030	8210	9380	10550	11720	12890	14060
⅞	.6013		7220		5470	6840	8210	9580	10940	12310	13670	15040	16410

Note.—Bearing values above or to right of upper zigzag line of pair are greater than double shear. Values between upper and zigzag lines of each pair are less than double and greater than single Values below or to left of lower zigzag line of each pair are less than shear.

Problem in Riveted Joints.

(Reference to Tables 1, 2, 6; and to Figs. 5, 6, 29.)

Example.—Let it be required to design a flat steel bar, spliced at m with a chain-riveted joint (Fig. 29, page 617), to resist a tensile str 69600 lbs.; as per following data.

Data.—Assume rivets in single shear at 10000 lbs. per sq. in. and be value for plates at 20000 lbs. per sq. in. (see Table 1, above). Assume able tension on net area of splice plates and bars, at 17500 lbs. per s (see Figs. 5 and 6, in Table 2, for pitch, etc., of rivets.)

Solution.—

Allowable total stress on bar and joint = 69600

Rivets in shear (Table 1): Eight ¾-in. rivets in double shear

$= 8 \times 2 \times 4420 = 70720$

Rivets in bearing on splice plates (Table 1): Two 5/16-in. plates

$= 8 \times 2 \times 4690 = 75040$

Rivets in bearing on main bar (Table 1): One ⅝-in. bar $= 8 \times 9380 = 75040$

Tension on main bar: $\frac{5}{8} \times 17500 \times 6\frac{3}{8}$ (net width of bar in ins.) $= 69730$

Transverse width in bar occupied by holes (see Note to Table 6):

$3(\frac{3}{4} + \frac{1}{8}) = 2\frac{5}{8}$ in.

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Total width of bar (or splice plate) required for stress
$= 6\frac{3}{8}$(net) $+ 2\frac{5}{8}$(holes) $= 9$ ins.

Natural spacing of rivets on transverse section of bar and splice
$= 9 + 3 = 3$ ins.

(This pitch is allowable, from Table 2, second column.)

Distance from center of rivet to edge of plate $= (9+2) - 3 = 1\frac{1}{2}$ ins.

(This is allowable, from Table 2, and Fig. 6.)

Distance between centers of rows of rivets (Table 2): $3 \times \cos 30° = 2\frac{5}{8}$ ins.

Distance from centers of rivets to ends of bars or plates:

not $< \frac{1}{2}$ pitch $\frac{P}{2} = 1\frac{1}{2}$ ins.

Length of splice plates: $1\frac{1}{2} + 2\frac{5}{8} + 2\frac{5}{8} + 1\frac{1}{2} + 1\frac{1}{2} + 2\frac{5}{8} + 2\frac{5}{8} + 1\frac{1}{2}$ $= 16\frac{1}{2}$ ins.

2.—General Rivet Spacing, Clearances, etc.

(All dimensions in Inches.)

Diameter of Rivet.	Fig. 5. Min. Pitch. p	Min. Pitch in Flanges of Chords and Girders.	Max. Pitch at Ends of Compression Members.	Ordinary Pitch in Columns Except at Ends.	Min. Dist. e to Edge of Plate. Fig. 6. Usual. e	Plates Over $\frac{5}{8}$" Thick. Sheared Edge. e	Plates Over $\frac{5}{8}$" Thick. Rolled Edge. e	Plates Not Over $\frac{5}{8}$" Thick. Sheared Edge. e	Plates Not Over $\frac{5}{8}$" Thick. Rolled Edge. e	Min. Clear. From Center of Rivet. Fig. 7. A	Min. Clear. From Center of Rivet. Fig. 8. C
	Best					Not including Fillers or Lattice Bars.				A (min) $= \frac{1}{4} + \frac{H}{2}$	
$\frac{1}{4}$	$\frac{3}{4}$ 1	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
$\frac{3}{8}$	$1\frac{1}{8}$ $1\frac{1}{4}$	.....	.....	.....	.....	.....	.....	.....	.....	$\frac{11}{16}$	$1\frac{1}{8}$
$\frac{1}{2}$	$1\frac{1}{2}$ $1\frac{3}{4}$	.....	.....	.....	.....	.....	.....	.....	.....	$\frac{13}{16}$	$1\frac{1}{8}$
$\frac{5}{8}$	$1\frac{7}{8}$ 2	4	$2\frac{1}{2}$	.....	$1\frac{1}{4}$	.....	.....	.....	.....	$\frac{15}{16}$	$1\frac{1}{8}$
$\frac{3}{4}$	$2\frac{1}{4}$	4	3	6	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{4}$	$1\frac{1}{4}$	$1\frac{1}{8}$	$\frac{7}{8}$	$1\frac{1}{8}$
$\frac{7}{8}$	$2\frac{1}{2}$ $2\frac{5}{8}$	4	$3\frac{1}{2}$	6	$1\frac{3}{4}$	$1\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{4}$	1	$1\frac{1}{4}$
1	3	4	4	.....	2	.....	.....	.....	.....	$1\frac{1}{16}$	$1\frac{1}{2}$

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3.—Rivet Gages for Standard Shapes.

(All dimensions in Inches.)

Angles.

Fig. 9.

I-Beams.

Fig. 10.

Channels.

Fig. 11.

Z-Bars.

Fig. 12.

T-Bars.

Fig. 13.

Length of Leg.	Single Gage. G	Double Gage. G_1	Double Gage. G_2	Depth of Beam.	Wt. per Ft., Lbs.	G Gage.	Max. Rivet.	Depth of Channel.	Wt. per Ft., Lbs.	G Gage.	Max. Rivet.	Depth of Bar.	G Gage.	Max. Rivet.	Width of Flange or Depth of Stem.	Gage. G	Gage. G_1	Max. Rivet. Flange.	Max. Rivet. Stem.
8	4½	3	3	24	100	4	⅞	15	55	2¼	¾				Ada	pt	ed	fro	
7	4	2½	3	24	80	4	⅞	15	45	2¼	¾		Car	neg	ie	Ca	m	br	ia
6	3½	2¼	2¼	20	100	4	⅞	15	40	1⅞	¾	6 to 6⅛	2⅛	..	5	2⅜	...	¾	..
5	3	2	1¾	20	80	4	⅞	15	33	1⅞	¾	5 to 5⅛	1⅞	..	4½	2¼	...	¾	..
4½	2½	2	1¼	20	75	3½	⅞	12	40	2	¾	4 to 4⅛	1⅞	..	4	2	2¼	⅝	1
4	2¼	Max.		20	65	3½	⅞	12	30	2	¾	3 to 3 1/16	1⅝	..	3½	1¾	2	⅝	1
3½	2	Rivet.		18	70	3¼	⅞	12	25	1¾	¾				3	1½	1¾	½	⅞
3¼	1¾	⅞		18	55	3¼	⅞	12	20.5	1¾	¾	Cam	brr	ia	2¾	1⅜	...	½	⅞
3	1¾	⅞		15	100	3¼	⅞	10	35	2	¾	6 to 6⅛	2	1	2½	1¼	1⅜	½	⅞
2¾	1½	¾		15	80	3¼	⅞	10	25	2	¾	5 to 5⅛	1⅞	⅞	2¼	1⅛	1¼	½	⅞
2½	1⅜	¾		15	75	3¼	¾	10	20	1½	¾	4 to 4⅛	1¾	⅞	2	1	1⅛	½	⅞
2¼	1¼	¾		15	60	3¼	¾	10	15	1½	¾	3 to 3 1/16	1⅝	¾	1¾	..	1	..	⅞
2	1⅛	⅝		15	55	3	¾	9	25	1¾	¾				1⅜	⅝	¾	¼	⅞
1¾	1	⅝		15	42	3	¾	9	20	1¾	¾				1¼	⅝	11/16	¼	⅞
1½	⅞	½		12	55	3	¾	9	15	1⅜	¾				1 1/16	⅝	⅝	¼	⅞
1⅜	¾	½		12	40	3	¾	9	13.25	1⅜	¾				1⅛	⅝	9/16	¼	⅞
1¼	¾	½		12	35	2¾	¾	8	21.25	1½	¾				1 1/16	..	9/16	..	⅞
1⅛	⅝	½		12	31.5	2¾	¾	8	16.25	1½	¾				1	⅝	9/16	¼	⅞
1/16	⅝	⅜		10	.....	2⅝	¾	8	13.75	1¼	¾								
1	9/16	⅜		9	.....	2½	¾	8	11.25	1¼	¾								
⅞	½	⅜		8	.....	2¼	¾	7	19.75	1½	⅝								
¾	7/16	¼		7	.....	2¼	⅝	7	17.25	1½	⅝								
⅝	⅜	9/16		6	.....	2	⅝	7	14.75	1¼	⅝								
				5	.....	1¾	½	7	9.75	1¼	⅝								
				4	.....	1½	½	6	15.5	1⅜	⅝								
				3	.....	1 7/16	⅜	6	13	1⅜	⅝								
								6	10.5	1⅛	⅝								
								6	8	1⅛	⅝								
								5	11.5	1¼	½								
								5	9	1¼	½								
								5	6.5	1	½								
								4	.....	1	½								
								3	.....	13/16	⅜								

Remarks on Tables 3 and 4.

The advantage of using standard rivet gages and connections is the sa
ing of time in both the drafting room and in the shop. In detailing a st
building, the first step is to get out the erection drawings, showing diagra
of columns, floor plans, sectional elevations of complicated and special co
nections, and all sizes and dimensions from previous calculations of t
design. Each member of the structure is numbered for the convenience
the detailer, the shop and the erection gangs. The detailing begins with t
columns and then proceeds with the girders, beams, etc.

Caution.—For *special heavy* and *short* beams the erection drawi
should show special connections, if required.

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4.—Standard Connection Angles for I-Beams and Channels.

For 24″ Beams.
Ls 4″x4″x$\frac{7}{16}$″x1′ 6″ *Ls* 4″x4″x$\frac{3}{8}$″x1′ 6″

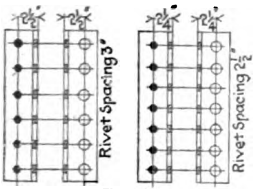


Fig. 14.

For 20″ Beams.
Ls 4″x4″x$\frac{7}{16}$″x1′ 3″ *Ls* 4″x4″x$\frac{3}{8}$″x1′ 3″

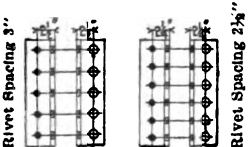


Fig. 15.

Ls 4″x4″x$\frac{3}{8}$″x1′ 1″

For 18″ Beams. The Carnegie Steel Company usessame connections as for 20″ Beams.

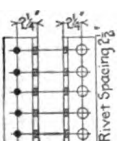


Fig. 16.

For 15″ Beams and Channels.
Ls 6″x4″x$\frac{7}{16}$″x0′ 10″ *Ls* 6″x4″x$\frac{3}{8}$″x0′10″

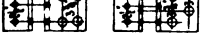


Fig. 17.

For 12″ Beams and Channels.
Ls 6″x4″x$\frac{7}{16}$″x0′ 7½″
Ls 6″x4″x$\frac{3}{8}$″x0′ 7½″

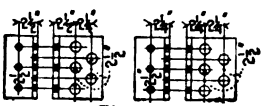


Fig. 18.

For 10″, 9″, 8″ and 7″ Beams and Channels.
Ls 6″x4″x$\frac{7}{16}$″x0′ 5″ *Ls* 6″x4″x$\frac{3}{8}$″x0′ 5″

Rivet Spacing 2½″.

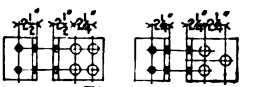


Fig. 19.

For 6″ and 5″ Beams and Channels.
6 in.:
Ls 6″x4″x$\frac{7}{16}$″x0′ 3″ *Ls* 6″x4″x$\frac{3}{8}$″x0′ 2½″
5 in.: do. x0′ 2½″

Fig. 20.

For 4″ and 3″ Beams and Channels.
Ls 6″x4″x$\frac{7}{16}$″x0′ 2″ *Ls* 6″x4″x$\frac{3}{8}$″x0′ 1¾″

Fig. 21.

Note.—In the above illustrations, two systems of connection angles are shown, that of the Carnegie Steel Co. being on the left, and Cambria on the right, in each case. Legs showing shop rivets are riveted directly to ends of I-beam or channel, while those showing field rivets are for field connection. Angles are riveted on in pairs. A play of $\frac{1}{16}$ of an inch is allowed at *each end* of the built member between the back of angles and the girder to which it is to be connected in the field.

5.—Weights and Dimensions of Rivets.

Button Head. Fig. 22.

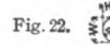


Fig. 23. Counte Hea

Dimensions (in inches) of Heads after Driving.

Kind of Rivet.		Diameter of Rivet, in Inch						
		¼	⅜	½	⅝	¾	⅞	
Button Head Rivets.	Values of H (= $\frac{6}{10}$ diam.)	.15	.225	.3	.375	.45	.525	
	Values of R (= ¾ diam. + $\frac{1}{16}$)	¼	$\frac{11}{32}$	$\frac{7}{16}$	$\frac{17}{32}$	⅝	$\frac{23}{32}$	
	Values of W	$\frac{15}{32}$	$\frac{21}{32}$	$\frac{27}{32}$	$1\frac{1}{32}$	$1\frac{7}{32}$	$1\frac{3}{32}$	1
Countersunk Rivets.	Values of h	$\frac{3}{16}$	$\frac{7}{32}$	$\frac{9}{32}$	$\frac{11}{32}$	⅜	$\frac{13}{32}$	
	Values of w	$\frac{15}{32}$	⅝	$\frac{3}{16}$	1	$1\frac{3}{16}$	$1\frac{3}{8}$	1

Lengths and Weights of Rivets.

*Lengths of Rivets for Various Grips. (Dimensions in Inches.)

Grip.	Diameter of Rivets.						
	¼	⅜	½	⅝	¾	⅞	1
½	1	1¼	1½	1⅜	1⅞	2	2⅛
⅝	1⅛	1⅜	1⅝	1⅞	2	2⅛	2¼
¾	1¼	1½	1¾	2	2⅛	2¼	2⅜
⅞	1⅜	1⅝	1⅞	2⅛	2¼	2⅜	2½
1	1½	1¾	2	2¼	2⅜	2½	2⅝
1⅛	1⅝	1⅞	2⅛	2⅜	2½	2⅝	2¾
1¼	1¾	2	2¼	2½	2⅝	2¾	2⅞
1⅜	1⅞	2⅛	2⅜	2⅝	2⅞	3	3
1½	2	2¼	2½	2¾	3	3⅛	3⅛
1⅝	2⅛	2⅜	2⅝	2⅞	3⅛	3¼	3¼
1¾	2¼	2½	2¾	3	3¼	3⅜	3½
1⅞	2⅜	2⅝	2⅞	3¼	3⅜	3½	3⅝
2	2½	2¾	3⅛	3⅜	3½	3⅝	3¾
2⅛	2⅝	2⅞	3¼	3½	3⅝	3¾	3⅞
2¼	2¾	3	3⅜	3⅝	3¾	3⅞	4
2⅜	2⅞	3⅛	3½	3¾	3⅞	4	4⅛
2½	3	3¼	3⅝	3⅞	4	4⅛	4¼
2¾	3¼	3⅝	3⅞	4⅛	4¼	4⅜	4½
3	3½	3⅞	4⅛	4⅜	4½	4⅝	4¾
3¼	3¾	4⅛	4⅜	4¾	4⅞	5	5⅛
3½	4	4⅜	4⅝	5	5⅛	5¼	5⅜
3¾	4¼	4⅝	4⅞	5¼	5⅜	5½	5⅝
4	4½	4⅞	5⅛	5½	5⅝	5¾	5⅞
4¼	4¾	5⅛	5½	5¾	5⅞	6	6⅛
4½	5	5⅜	5¾	6	6⅛	6¼	6⅝
4¾	5¼	5⅞	6	6¼	6½	6⅝	6¾
5	5½	5⅞	6¼	6⅝	6¾	6⅞	7
5¼	5¾	6⅛	6½	6⅞	7	7⅛	7¼
5½	6	6⅜	6¾	7⅛	7¼	7⅜	7½
5¾	6¼	6¾	7	7⅜	7⅝	7⅝	7¾
6	6½	7	7¼	7⅝	7⅞	7⅞	8

‡Weight of 100 Steel Rivets† of V Lengths. (Weight in Lbs.; Di sions in Inches.)

Length	Diameter of Rivets.				
	⅜	½	⅝	¾	⅞
1¼	5.5	12.8	22.0	29.3	43.
1½	6.3	14.2	24.1	32.4	48.
1¾	7.0	15.5	26.3	35.5	52.
2	7.9	16.9	28.5	38.7	56.
2¼	8.7	18.3	30.7	41.8	61.
2½	9.4	19.7	32.8	44.9	65.
2¾	10.2	21.1	35.0	48.0	69.
3	11.0	22.5	37.2	51.1	73.
3¼	11.7	23.9	39.3	54.3	78.
3½	12.6	25.3	41.5	57.4	82.
3¾	13.4	26.7	43.7	60.5	86.
4	14.1	28.1	45.9	63.6	90.
4¼	14.9	29.4	48.0	66.7	95.
4½	15.7	30.8	50.2	69.9	99.
4¾	16.5	32.2	52.4	73.0	104.
5	17.2	33.6	54.5	76.1	108.
5¼	18.1	35.0	56.7	79.2	112.
5½	18.8	36.4	58.9	82.3	116.
5¾	19.6	37.8	61.1	85.5	120.
6	20.4	39.2	63.2	88.6	124.
6½	21.9	42.0	67.6	95.1	133.
7	23.5	44.7	71.9	101.	142.
7½	25.1	47.5	76.1	108.	150.
8	26.6	50.3	80.6	114.	159.
8½	28.2	53.1	85.0	120.	167.
9	29.8	55.9	89.3	126.	176.
9½	31.3	58.7	93.7	133.	185.
10	32.8	61.4	98.0	139.	193.

	Weight of One Button He				
**...	.018	.058	.111	.136	.2
||...	.016	.042	.090	.125	.2

Deduction in Inches from Above Lengths if One Head is Countersunk.

	⅛	¼	½	½	⅝	¾	⅞

Weight of One Inch of Sha					
	.031	.056	.087	.125	.1

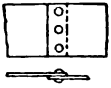
* For ⅝" to 1" diam., $L = 1.08$ (Grip + diam. + ½).
For ½" rivets, $L = 1.08$ (Grip + ⅞).
‡ For iron rivets deduct 2 per cent.

† Also round-headed bolts with
** Before driving. || After dri

6.—Riveted Joints.

Kinds of Riveted Joints.

Lap Joints. **Butt Joints.**



Single Riveted.

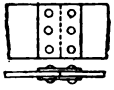
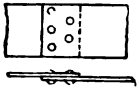


Fig. 24. **Fig. 25.**



Double Riveted.

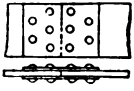
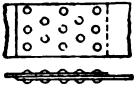


Fig. 26. **Fig. 27.**



Chain Riveted.

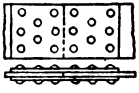


Fig. 28. **Fig. 29.**

Table for Finding Net Areas of Riveted Joints.

Areas in square inches to be deducted for rivet holes in plates of various thicknesses to obtain net area of joint for tension.

Thickness of Plate.	* Size of Hole.																	
	3/4	9/16	5/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1″	1 1/16	1 1/8	1 3/16	1 1/4
1/4″	.06	.08	.09	.11	.12	.14	.16	.17	.19	.20	.22	.23	.25	.26	.28	.30	.31
5/16″	.08	.10	.12	.14	.16	.17	.18	.21	.23	.25	.27	.29	.31	.33	.35	.37	.39
3/8″	.09	.12	.14	.16	.19	.21	.23	.26	.28	.30	.33	.35	.37	.40	.42	.44	.47
7/16″	.11	.14	.16	.19	.22	.25	.27	.30	.33	.35	.38	.41	.44	.46	.49	.52	.55
1/2″	.12	.16	.19	.22	.25	.28	.31	.34	.37	.41	.44	.47	.50	.53	.56	.59	.62
9/16″	.14	.17	.21	.25	.28	.32	.35	.39	.42	.46	.49	.53	.56	.60	.63	.66	.70
5/8″	.16	.19	.23	.27	.31	.35	.39	.43	.47	.51	.55	.58	.62	.66	.70	.74	.78
11/16″	.17	.21	.26	.30	.34	.39	.43	.47	.51	.55	.60	.64	.69	.73	.77	.81	.86
3/4″	.19	.23	.28	.33	.37	.42	.47	.51	.56	.61	.66	.70	.75	.80	.84	.89	.94
13/16″	.20	.25	.30	.35	.41	.46	.51	.55	.61	.66	.71	.76	.81	.86	.91	.96	1.01
7/8″	.22	.27	.33	.38	.44	.49	.55	.60	.66	.71	.76	.82	.87	.93	.98	1.04	1.09
15/16″	.23	.29	.35	.41	.47	.53	.58	.64	.70	.76	.82	.88	.94	1.00	1.05	1.13	1.17
1 ″	.25	.31	.37	.44	.50	.56	.62	.69	.75	.81	.87	.94	1.00	1.06	1.12	1.19	1.26
1 1/16″	.26	.33	.40	.46	.53	.60	.66	.73	.80	.86	.93	1.00	1.06	1.13	1.19	1.26	1.33
1 1/8″	.28	.35	.42	.47	.56	.63	.70	.77	.84	.91	.98	1.05	1.12	1.19	1.26	1.33	1.41
1 3/16″	.30	.37	.44	.52	.59	.66	.74	.81	.89	.96	1.04	1.13	1.19	1.26	1.33	1.41	1.48
1 1/4″	.31	.39	.47	.55	.62	.70	.78	.86	.94	1.01	1.09	1.17	1.25	1.33	1.41	1.48	1.56

* Note that diam. of hole is greater than diam. of rivet; usually assumed at 1/16 to 1/8 in. greater.

For Problem in Riveted Joints see page 612.

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7.—Standard Bolts for Fastenings.

Drift Bolts for Timber.

Headed and Pointed.

Plain.

Fig. 30. Fig. 31.

Expans'n Bolts for Timber and Stone.

Before Expansion.

In Place.

Figs. 32, 33.

Screw Bolts for Timber and Metal.

Square Head and Nut.

Hexagonal Head and Nut.

Figs. 34, 35.

Hook Bolt for Bridge Work.

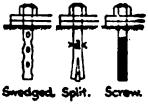
Fastening Guard Rail and Tie to Girder.

Fig. 36.

Stone Bolts for Bridge Work.

Diam. of hole = diam. of bolt + ¼″.

Length of wedge for split bolt = $3d$; width = d; thickness at head = ½d; point rounded.



Figs. 37, 38, 39.

Swedge Bolts. (Fig. 37.)
- ¾″x 9″, Wt. = 2#.
- ⅞″x12″, Wt. = 3#.
- 1 ″x12″, Wt. = 4#.
- 1¼″x15″, Wt. = 7#.

Anchor Bolts for Concrete: Screw bolts (Figs. 34, 35) with flat washers; middle of bolt deflected 1 dia. before setting.

8.—Standard Screw Threads.

(Sellers; Franklin Institute, Dec., 1864; United States, 1868.)

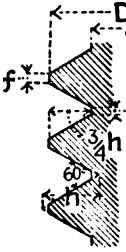


Fig. 40.

Diam. D, Screw.	Diam. d, Thread.	No. of Threads, N, per Inch.	Diam. D, Screw.	Diam. d, Thread.	No. of Threads, N, per Inch.	Diam. D, Screw.	Diam. d, Thread.	No. of Threads, N, per Inch.	Diam. D, Screw.	Diam. d, Thread.	No. of Threads, N, per Inch.
¼	.185	20	1	.837	8	2	1.712	4¼	4	3.567	3
5/16	.240	18	1¼	.940	7	2¼	1.962	4½	4¼	3.798	2¾
⅜	.294	16	1¼	1.065	7	2½	2.175	4	4½	4.028	2¾
7/16	.344	14	1¾	1.160	6	2¾	2.425	4	4¾	4.255	2½
½	.400	13	1½	1.284	6	3	2.629	3½	5	4.480	2½
9/16	.454	12	1⅝	1.389	5½	3¼	2.879	3½	5¼	4.730	2½
⅝	.507	11	1¾	1.490	5	3½	3.100	3¼	5½	4.953	2½
¾	.620	10	1⅞	1.615	5	3¾	3.317	3	5¾	5.203	2½
⅞	.731	9	.....	.....	.....	.....	.....	.....	6	5.423	2¼

Note.—The Pitch is $\frac{1}{N}$; and the flat f, at top and bottom, is $\frac{1}{8N}$.

In the Whitworth or English standard the angle of thread is 55° instead of 60°; the top and bottom of threads are rounded; and N is 12 for D = ½, otherwise N is the same as above for D up to 3 inches.

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9.—Dimensions and Weights of Hot Pressed Nuts.

The sizes are the usual manufacturers', not the Franklin Institute Standard. Both weights and sizes are for the unfinished nut.

(Dimensions in ins.; weights in lbs.)

Size of Bolt.	Rough Hole.	* Square Nuts.				† Hexagon Nuts.				
		Side of Square.	Diagonal.	Wt. of 100 Nuts.	No. of Nuts in 100 lbs.	Thickness of Nut.	Short Diam.	Long Diam.	Wt. of 100 Nuts.	No. of Nuts in 100 lbs.
1/4	7/32	1/2	.71	1.5	6800	1/4	1/2	.58	1.3	8000
5/16	9/32	5/8	.88	2.9	3480	5/16	5/8	.72	2.4	4170
3/8	11/32	3/4	1.06	4.9	2050	3/8	3/4	.87	4.1	2410
7/16	13/32	7/8	1.24	7.7	1290	7/16	7/8	1.01	6.8	1460
1/2	9/16	7/8	1.24	8.6	1170	1/2	7/8	1.01	7.1	1410
1/2	9/16	1	1.41	11.8	850	1/2	1	1.15	9.8	1020
9/16	5/8	1 1/8	1.59	16.7	600	9/16	1 1/8	1.30	14.0	710
5/8	11/16	1 1/8	1.59	17.7	570	5/8	1 1/8	1.30	14.7	680
5/8	11/16	1 1/4	1.77	22.8	440	5/8	1 1/4	1.44	19.1	520
5/8	11/16	.....	.....	.....	.....	3/4	1 1/4	1.44	22.9	440
3/4	13/16	1 3/8	1.94	32.3	310	3/4	1 3/8	1.59	27.2	370
3/4	13/16	1 1/2	2.12	39.8	251	7/8	1 1/2	1.73	39.	256
7/8	15/16	1 5/8	2.30	53.	190	7/8	1 5/8	1.88	44.	226
7/8	15/16	1 3/4	2.47	63.	159	1	1 5/8	1.88	50.	198
1	7/8	1 3/4	2.47	68.	146	1	1 3/4	2.02	57.	176
1	7/8	2	2.83	94.	106	1 1/8	1 3/4	2.02	64.	156
1 1/8	1 1/16	2	2.83	103.	97	1 1/4	2	2.31	96.	104
1 1/8	1 1/16	2 1/4	3.18	137.	73	.....	.....	.....	.....	.....
1 1/4	1 1/16	2 1/4	3.18	145.	69	.....	.....	.....	.....	.....
1 1/4	1 3/16	2 1/2	3.54	186.	54	1 3/8	2 1/4	2.60	134.	75
1 3/8	1 3/16	2 3/4	3.89	247.	41	1 1/2	2 1/2	2.89	180.	56
1 1/2	1 5/16	3	4.24	319.	31.3	1 5/8	2 3/4	3.18	235.	42
1 5/8	1 7/16	3 1/4	4.60	400.	24.8	1 3/4	3	3.46	300.	33.4
1 3/4	1 9/16	3 1/2	4.95	500.	19.9	1 7/8	3 1/4	3.75	370.	26.7
1 7/8	1 11/16	3 3/4	5.30	620.	16.2	2	3 1/2	4.04	460.	21.5
2	1 13/16	4	5.66	750.	13.4	2	3 1/2	4.04	450.	22.4
2 1/8	1 7/8	4	5.66	780.	12.8	2 1/8	3 3/4	4.33	560.	18.0
2 1/4	2	4 1/4	6.01	930.	10.7	2 1/4	3 3/4	4.33	560.	17.7
2 3/8	2 1/8	4 1/4	6.01	960.	10.4	2 3/8	4	4.62	680.	14.7
2 1/2	2 1/4	4 1/2	6.36	1130.	8.9	2 1/2	4 1/4	4.91	810.	12.3
2 3/4	2 7/16	4 3/4	6.72	1370.	7.3	2 3/4	4 1/2	5.20	980.	10.2
3	2 11/16	5	7.07	1610.	6.2	3	4 3/4	5.48	1150.	8.7
3 1/4	2 15/16	5 1/2	7.78	2110.	4.7	3 1/4	5	5.77	1340.	7.5
3 1/2	3 1/8	6	8.49	2750.	3.6	3 1/2	5 1/4	6.06	1580.	6.3

*Thickness of *square* nut is equal to diameter of bolt.
†Thickness of *hexagon* nut not always equal to diameter of bolt.

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10.—Bolts.

Dimensions for Heads, Nuts, etc., etc.
(Am. Bridge Co. Standard.)

Diameter of Bolt.	Round.				Head.					Nut.			
					Hexagonal.	Hex. or Square.	Square.			Square.	Hex. or Square.	H c	
	Fig. 41.				Figs. 42.					Figs. 43.			
Ins.	Sq. Ins.	Diameter at Root of Thread. Ins.	Area at Root of Thread. Sq. Ins.	Threads per Inch. No.	Long Diam. Ins.	Short Diam. Ins.	Height. Ins.	Short Diam. Ins.	Long Diam. Ins.	Long Diam. Ins.	Short Diam. Ins.	Height, Ins.	Short Diam. Ins
1/4	.049	.185	.027	20	7/16	3/8	....	3/8	1/2	33/32	1/2	1/4	1/4
3/8	.110	.294	.068	16	9/16	1/4	....	9/32	3/4	1	11/16	3/8	11/16
1/2	.196	.400	.126	13	5/8	3/4	....	9/16	1	1 1/4	7/8	1/2	7/8
5/8	.307	.507	.202	11	1 3/32	7/16	....	9/16	1 3/32	1 1/2	1 1/16	5/8	1 1/16
3/4	.442	.620	.302	10	1 5/16	1 1/8	5/8	1 3/32	1 1/2	1 13/16	1 1/4	3/4	1 1/4
7/8	.601	.731	.420	9	1 17/32	1 3/16	3/4	1 3/32	1 11/16	2 1/32	1 7/16	7/8	1 7/16
1	.785	.837	.550	8	1 23/32	1 1/2	7/8	1 13/32	2 1/16	2 3/16	1 5/8	1	1 5/8
1 1/8	.994	.940	.694	7	1 31/32	1 11/16	1	1 5/8	2 5/16	2 9/16	1 13/16	1 1/8	1 13/16
1 1/4	1.227	1.065	.893	7	2 3/16	1 7/8	1 1/8	1 11/16	2 7/16	2 13/16	2	1 1/4	2
1 3/8	1.485	1.160	1.057	6	2 7/16	2	1 1/4	2	2 11/16	3 3/16	2 3/16	1 3/8	2 3/16
1 1/2	1.767	1.284	1.295	6	2 5/8	2 1/4	1 3/8	2 3/16	3 1/16	3 3/8	2 3/8	1 1/2	2 3/8
1 5/8	2.074	1.389	1.515	5 1/2	2 13/16	2 7/16	1 1/2	2 3/8	3 11/16	3 5/8	2 9/16	1 5/8	2 9/16
1 3/4	2.405	1.491	1.746	5	3 1/32	2 5/8	1 5/8	2 17/32	3 19/32	3 29/32	2 3/4	1 3/4	2 3/4
1 7/8	2.761	1.616	2.051	5	3 1/4	2 11/16	1 3/4	2 23/32	3 27/32	4 3/32	2 15/16	1 7/8	2 15/16
2	3.142	1.712	2.302	4 1/2	3 11/32	3	1 7/8	2 29/32	4 1/8	4 7/16	3 1/8	2	3 1/8
2 1/4	3.976	1.962	3.023	4 1/2	3 29/32	3 3/8	2 1/8	3 1/4	4 5/8	4 23/32	3 1/2	2 1/4	3 1/2
2 1/2	4.909	2.176	3.719	4	4 11/32	3 3/4	2 3/8	3 5/8	5 1/8	5 1/2	3 7/8	2 1/2	3 7/8
2 3/4	5.940	2.426	4.620	4	4 29/32	4 1/8	2 5/8	4	5 11/32	6	4 1/4	2 3/4	4 1/4
3	7.069	2.629	5.428	3 1/2	5 7/32	4 1/2	2 7/8	4 11/32	6 5/32	6 9/16	4 5/8	3	4 5/8
3 1/4	8.296	2.879	6.510	3 1/2	5 5/8	4 7/8	3 1/8	4 23/32	6 21/32	7 1/16	5	3 1/4	5
3 1/2	9.621	3.100	7.548	3 1/4	6 1/8	5 1/4	3 3/8	5 1/16	7 1/16	7 5/8	5 3/8	3 1/2	5 3/8
3 3/4	11.045	3.317	8.641	3	6 1/2	5 5/8	3 5/8	5 7/16	7 11/16	8 1/8	5 3/4	3 3/4	5 3/4
4	12.566	3.567	9.963	3	6 15/16	6	3 7/8	5 13/16	8 7/32	8 3/4	6 1/8	4	6 1/8
4 1/4	14.186	3.798	11.329	2 7/8	7 11/32	6 3/8	4 1/8	6 5/32	8 23/32	9 3/16	6 1/2	4 1/4	6 1/2
4 1/2	15.904	4.028	12.753	2 3/4	7 23/32	6 3/4	4 3/8	6 1/2	9 7/32	9 5/8	6 7/8	4 1/2	6 7/8
4 3/4	17.721	4.256	14.226	2 5/8	8 3/32	7 1/8	4 5/8	6 7/8	9 3/4	10 1/4	7 1/4	4 3/4	7 1/4
5	19.635	4.480	15.763	2 1/2	8 21/32	7 1/2	4 7/8	7 1/4	10 1/4	10 13/32	7 5/8	5	7 5/8
5 1/4	21.648	4.730	17.572	2 1/2	9 3/32	7 7/8	5 1/8	7 5/8	10 3/4	11 3/8	8	5 1/4	8
5 1/2	23.758	4.953	19.267	2 3/8	9 1/2	8 1/4	5 3/8	7 31/32	11 9/32	11 7/8	8 3/8	5 1/2	8 3/8
5 3/4	25.967	5.203	21.262	2 3/8	9 15/16	8 5/8	5 5/8	8 13/32	11 13/16	12 3/8	8 3/4	5 3/4	8 3/4
6	28.274	5.423	23.098	2 1/4	10 3/8	9	5 7/8	8 23/32	12 7/16	12 15/16	9 1/8	6	9 1/8

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11.—WEIGHT OF 100 BOLTS WITH SQUARE HEADS AND NUTS.

Length under head to point.	Diameter of Bolts.								
	¼ in.	5/16 in.	⅜ in.	7/16 in.	½ in.	⅝ in.	¾ in.	⅞ in.	1 in.
	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
1½	4.0	7.0	10.5	15.2	22.5	39.5	63.0	.....	.....
1¾	4.4	7.5	11.3	16.3	23.8	41.6	66.0	.....	.....
2	4.8	8.0	12.0	17.4	25.2	43.8	69.0	109.0	163
2¼	5.2	8.5	12.8	18.5	26.5	45.8	72.0	113.3	169
2½	5.5	9.0	13.5	19.6	27.8	48.0	75.0	117.5	174
2¾	5.8	9.5	14.3	20.7	29.1	50.1	78.0	121.8	180
3	6.3	10.0	15.0	21.8	30.5	52.3	81.0	126.0	185
3½	7.0	11.0	16.5	24.0	33.1	56.5	87.0	134.3	196
4	7.8	12.0	18.0	26.2	35.8	60.8	93.1	142.5	207
4½	8.5	13.0	19.5	28.4	38.4	65.0	99.1	151.0	218
5	9.3	14.0	21.0	30.6	41.1	69.3	105.2	159.6	229
5½	10.0	15.0	22.5	32.8	43.7	73.5	111.3	168.0	240
6	10.8	16.0	24.0	35.0	46.4	77.8	117.3	176.6	251
6½	.....	.....	25.5	37.2	49.0	82.0	123.4	185.0	262
7	.....	.....	27.0	39.4	51.7	86.3	129.4	193.7	273
7½	.....	.....	28.5	41.6	54.3	90.5	135.0	202.0	284
8	.....	.....	30.0	43.8	59.6	94.8	141.5	210.7	295
9	.....	.....	.....	46.0	64.9	103.3	153.6	227.8	317
10	.....	.....	.....	48.2	70.2	111.8	165.7	244.8	339
11	.....	.....	.....	50.4	75.5	120.3	177.8	261.9	360
12	.....	.....	.....	52.6	80.8	128.8	189.9	278.9	382
13	.....	.....	.....	.....	86.1	137.3	202.0	296.0	404
14	.....	.....	.....	.....	91.4	145.8	214.1	313.0	426
15	.....	.....	.....	.....	96.7	154.3	226.2	330.1	448
16	.....	.....	.....	.....	102.0	162.8	238.3	347.1	470
17	.....	.....	.....	.....	107.3	171.0	250.4	364.2	492
18	.....	.....	.....	.....	112.6	179.5	262.6	381.2	514
19	.....	.....	.....	.....	117.9	188.0	274.7	398.3	536
20	.....	.....	.....	.....	123.2	206.5	286.8	415.3	558
Per Inch additional.	1.4	2.1	3.1	4.2	5.5	8.5	12.3	16.7	21.8

WEIGHTS OF NUTS AND BOLT-HEADS IN POUNDS.
For calculating the weight of longer Bolts.

Diameter of Bolt in Inches.		¼	⅜	½	⅝	¾	⅞
Weight of Hexagon Nut and Head	.....	.017	.057	.128	.267	.43	.73
Weight of Square Nut and Head	.....	.021	.069	.164	.320	.55	.88

Diameter of Bolt in Inches.	1	1¼	1½	1¾	2	2½	3
Weight of Hexagon Nut and Head	1.10	2.14	3.78	5.6	8.75	17.0	28.8
Weight of Square Nut and Head	1.31	2.56	4.42	7.0	10.5	21.0	36.4

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12.—Lag Screws—Weight in Lbs. of 100.

Fig. 44.

Diam. in Ins.	Length of Screw, in Inches.															
	1½	1¾	2	2¼	2½	3	3½	4	4½	5	5½	6	7	8	9	10
5/16	6.88	7.50	8.25	9.25	9.6	10.8	11.5	13.3	14.8	16.5	17.4	18.8	....	....	....	....
7/16	....	11.75	12.62	12.88	13.28	16.62	18.18	18.9	19.5	21.3	23.6	25.3	....	....	....	....
1/2	....	16.88	17.18	18.07	19.18	22.	24.	26.82	28.25	30.37	33.8	35.4	38.9	44.4	....	....
5/8	....	....	....	....	....	34.1	35.9	39.3	42.6	47.8	51.6	55.1	61.9	68.8	77.	90.
3/4	....	....	....	....	....	....	....	64.	67.9	71.4	79.4	86.6	92.8	97.5	108.8	124.3

The Use of Lag Screws.

The principal use of lag screws is for bridge work—in fastening one timber down on another. They are better than spikes, but not as good as screw bolts (with nut and head.)

The lag screw has a square head and is screwed in place with a wrench. A plate washer is used under the head to distribute the bearing stress on the timber.

A hole is first bored in the timber, somewhat smaller in diameter than the screw, the lag screw is inserted in the hole and tapped on the head a few times with a sledge hammer, and then screwed to a firm bearing with the wrench.

Caution. The writer has used lag screws for fastening small wooden guard rails (4"x6") to wooden ties. When such work is being done, especially by contract, there should be an inspector on the work to see that the lag screws are not hammered into the timber too far with the sledge hammer before the wrench is used. To the contractor, hammering is much the cheaper and is a great temptation.

13.—Wood Screws—Sizes.

(Diameter in Inches = Number x 0.01325 + 0.056.)

No.	Diam.	No.	Diam.	No.	Diam.	No.	Diam.	No.	Diam.	No.	Diam.	No.	Diam.	No.	Diam.
0	.056	4	.109	8	.162	12	.215	16	.268	20	.321	24	.374	28	.427
1	.069	5	.122	9	.175	13	.228	17	.281	21	.334	25	.387	29	.440
2	.082	6	.135	10	.188	14	.241	18	.293	22	.347	26	.401	30	.453
3	.096	7	.149	11	.201	15	.255	19	.308	23	.361	27	.414	....	....

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Fig. 45.

14.—Standard Cast Iron Separators for I-Beams.

Fig. 46.

Beam.		Distances.		Bolts.			Weights.			
Depth, Inches.	Weight, Pounds.	Out to Out of Flanges of Beams, Inches.	Center to Center of Beams, Inches.	Size, Inches.	Distance Center to Center, Inches.	Length, Inches.	Bolts and Nuts, Pounds.	Increase in Weight of Separator Bolts for 1 in. Additional Spread of Beams, Pounds.	Separator, Pounds.	Increase in Weight of Separator for 1 in. Additional Spread of Beams, Pounds.
Separators With Two Bolts. (Fig. 45.)										
24	80	14¾	7¾	¾	12	9¼	3.41	.250	32	5.50
20	80	14¾	7¾	¾	12	9¼	3.41	.250	28	3.10
20	65	13¼	7	¾	12	8½	3.23	.250	25	3.10
18	55	12¾	6¾	¾	9	8¼	3.16	.250	16	2.75
15	80	13⅝	7¼	¾	7½	9	3.55	.250	15	1.75
15	60	12¾	6¾	¾	7½	8½	3.23	.250	15	1.75
15	42	11¾	6¼	¾	7½	7½	2.98	.250	15	1.75
12	40	11¼	6	¾	5	7½	2.98	.250	11	1.50
12	31.5	10¾	5¾	¾	5	7¼	2.92	.250	11	1.50
Separators With One Bolt. (Fig. 46.)										
12	40.0	11¼	6	¾	.....	7½	1.49	.125	10	1.50
12	31.5	10¾	5¾	¾	.....	7¼	1.46	.125	10	1.50
10	25.0	10⅛	5½	¾	.....	6¾	1.40	.125	8	1.25
9	21.0	9 1/16	5	¾	.....	6¼	1.34	.125	7	1.20
8	18.0	8½	4½	¾	.....	5¾	1.28	.125	6	1.00
7	15.0	7⅞	4¼	¾	.....	5½	1.25	.125	4	.75
6	12.25	7 1/16	4	¾	.....	5¼	1.22	.125	4	.60
5	9.75	6½	3½	¾	.....	4¾	1.16	.125	3	.50
4	7.50	5⅞	3¼	¾	.....	4½	1.13	.125	3	.40
3	5.50	5 1/16	3	⅝	.....	4¼	0.70	.09	2	.25

Separators for 18, 20 and 24 in. beams are made of ⅝ in. metal.
Separators for 6 to 15 in. beams are made of ½ in. metal.
Separators for 5 in. beams and under are made of ⅜ in. metal.

Remarks on Cast Iron Separators.

The use of cast iron separators and bolts for fastening two or more I-beams together, side by side, is gradually giving place to steel diaphrams, composed of angles riveted to a connecting web plate. The latter is much the better, and is now preferred in important steel building construction. Cast iron separators, however, are still used in connecting steel I-beams in grillage foundations, where spaces between the beams are filled with concrete.

Cast separators for timber work are round, of cylindrical or spool shape, with a hole through which the screw bolt passes. They are used principally for packing, between lines of wooden bridge-stringers, spacing the wooden stringers about one inch apart. This spacing provides for the circulation of air, which dries the moisture from rain and retards rotting of the wood. Cast spool separators are shaped like a spool, while the cylindrical separators are perfectly cylindrical on the outside. In order to save metal, in the separator, the hole expands towards the ends, being a little larger than the bolt, only, at middle of separator.

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15.—Dimensions and Weights of Cast Iron Washers.

(Dimensions in inches; weight in lbs. each.)

Fig. 47.—Diametric section of round washer.

Bolt.	W	w	h	H	e	R	r	T	Wt.
									#
1/2	2 1/2	1 1/8	5/8	11/16	1/4	1/2	1/8	1/2	0.32
5/8	3	1 3/8	3/4	13/16	1/4	1/2	3/32	11/16	0.61
3/4	3 1/2	1 5/8	7/8	15/16	1/4	5/8	1/16	11/16	0.78
7/8	3 3/4	1 7/8	1	1 1/16	1/4	11/16	7/64	3/4	0.89
1	4 1/4	2 1/8	1 1/8	1 3/16	1/4	3/4	1/4	7/8	1.75
1 1/8	4 3/4	2 3/8	1 1/4	1 5/16	1/4	13/16	5/32	1	2.30
1 1/4	5 1/4	2 5/8	1 3/8	1 7/16	1/4	7/8	1/8	1 1/8	3.00
1 3/8	5 3/4	2 3/4	1 1/2	1 9/16	1/4	7/8	9/32	1 1/4	4.20
1 1/2	6 1/4	3	1 5/8	1 11/16	1/4	7/8	5/16	1 3/8	5.20
1 5/8	6 3/4	3 1/4	1 3/4	1 13/16	1/4	7/8	11/32	1 1/2	7.00
1 3/4	7 1/4	3 1/2	1 7/8	1 15/16	1/4	1	3/8	1 5/8	8.30
1 7/8	7 3/4	3 3/4	2	2 1/16	5/16	1	13/32	1 3/4	10.40
2	8 1/4	4	2 1/8	2 3/16	5/16	1 1/16	1/2	1 7/8	12.40
2 1/8	8 3/4	4 1/4	2 1/4	2 5/16	5/16	1 1/8	17/32	2	13.40
2 1/4	9 1/8	4 1/2	2 3/8	2 7/16	5/16	1 1/4	9/16	2 1/8	15.80
2 3/8	9 3/8	4 3/4	2 1/2	2 9/16	5/16	1 3/8	19/32	2 1/4	17.50
2 1/2	9 1/2	5	2 5/8	2 11/16	5/16	1 3/4	5/8	2 3/8	20.00

16.—Plate (Flat) Washers.

Dimensions, and number per pound.

Diam. Bolt. Ins.	Diam. Washer in Inches. Outside.	Diam. Washer in Inches. Hole.	Thickness of Washer. B.W.G.	Thickness of Washer. In.	No. per Pound.	Diam. Bolt. Ins.	Diam. Washer in Inches. Outside.	Diam. Washer in Inches. Hole.	Thickness of Washer. B.W.G.	Thickness of Washer. In.	No. per Pound.
3/16	1/2	1/4	18	.05	450	5/8	1 1/2	11/16	11	.125	23.
1/4	5/8	5/16	16	.062	293	11/16	1 3/4	3/4	11	1/8	15.
1/4	3/4	5/16	16	.062	138	3/4	2	7/8	10	.141	11.
5/16	7/8	3/8	16	1/16	110	7/8	2 1/4	1	9	.156	8.5
3/8	1	7/16	14	.078	76	1	2 1/2	1 1/8	9	5/64	6.3
7/16	1 1/4	1/2	14	5/64	43	1 1/8	2 3/4	1 1/4	8	.172	4.7
1/2	1 3/8	9/16	12	.109	26	1 1/4	3	1 3/8	7	3/16	3.6
1/2	1 1/2	5/8	12	7/64	23	1 1/2	3 1/2	1 5/8	6	.203	.....

Remarks on Cast and Plate Washers.

Cast iron washers are used principally with screw bolts in connecton with timber work. They are round and have a diametric section as shown in Fig. 47. These washers are sometimes called *O. G.* washers, or *S* washers, because the outline curve has an *O. G.* (architectural term) or *S*, shape, being a reversed curve.

Large, square, cast iron washers are often used for anchorage at the ends of rods in concrete.

Plate, or flat, washers are stamped from sheet metal, being round with a hole in the center for the bolt. Large bolts require the thicker washers, that will not bend when the bolt is tightened.

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17.—Miscellaneous Steel Wire Nails.

Approximate Number per Pound.

Lengths—Inches.

3/16	1/4	3/8	1/2	5/8	3/4	7/8	1	1 1/8	1 1/4	1 1/2
.....	.....	.....	.....	.....	.....	.....	.....	.....	33	27
.....	.....	.....	.....	.....	.....	.....	.....	.....	33	27
.....	.....	.....	.....	.....	.....	.....	.....	.....	34	29
.....	.....	.....	.....	.....	.....	.....	57	50	45	38
.....	.....	.....	.....	.....	.....	.....	65	58	52	44
.....	.....	.....	.....	.....	100	87	76	67	60	50
.....	.....	.....	.....	.....	120	104	90	80	72	60
.....	.....	.....	211	169	141	121	106	94	85	71
.....	.....	.....	247	197	164	141	123	111	99	82
.....	.....	.....	299	239	200	171	149	133	120	100
.....	.....	.....	345	275	229	197	172	153	137	115
.....	.....	.....	414	331	276	236	207	184	165	138
.....	.....	663	496	397	333	283	248	220	198	165
.....	.....	837	628	502	418	359	314	279	251	209
.....	.....	1096	822	658	548	469	411	365	329	274
.....	.....	1429	1072	857	714	613	536	476	429	357
.....	2840	1893	1420	1136	947	811	710	631	568	473
.....	3504	2336	1752	1402	1168	1001	876	778	701	584
.....	4571	3048	2280	1828	1523	1305	1143	1015	913	761
.....	6233	4156	3116	2495	2077	1781	1558	1385	1246	1038
.....	8276	5517	4138	3310	2758	2364	2069	1839	1655	1379
.....	10668	7112	5334	4267	3556	2933	2667	2370	2133	1778
20000	15000	10000	7500	6000	5000	4400	3750	3333	3000	
23702	17777	11850	8888	7111	5926	5079	4444			
30476	22856	15237	11428	9143	7618					

Lengths—Inches.

1 3/4	2	2 1/4	2 1/2	2 3/4	3	3 1/2	4	4 1/2	5	6	7	8	9	10
23	20	18	16	15	14	12	10	9	8	7	6	5	4 1/2	4
23	20	18	16	15	14	12	10	9	8	7	6	5	4 1/2	4
25	21	19	17	16	15	13	11	10	9	8	7	5 1/2	5	4 1/2
32	28	25	23	21	19	16	14	13	11	10	8	7	6	5 1/2
37	32	29	26	24	22	19	16	14	13	11	9	8	7	6 1/2
43	3[illegible]	34	30	28	25	22	19	17	15	13	11	10	8	7 1/2
51	45	40	36	33	30	26	23	20	18	15	13	11	10	9
60	5[illegible]	47	42	39	35	30	26	24	21	18	15			
71	62	55	50	45	41	35	31	28	25	21	18			
85	75	67	60	54	50	43	37	33	30	25				
98	86	76	69	62	57	49	43	39	35	29				
118	103	92	82	75	69	59	52	46	41					
142	124	110	99	90	83	71	62	55	50					
179	157	139	125	114	105	90	79	70						
235	204	182	164	149	137	117	103							
306	268	238	214	195	178	153								
406	350	315	284	258	236									
500	438	389	350											
653	571	508												
890	779													
1182														

Wire Gage.	Lengths.	
	11	12
00	3 3/4	3 1/4
3/8	3 3/4	3 1/4
0	4	3 1/2
15/16	5	4 1/2
2	6	5 1/2

roximate numbers are an average only, and the figures given l either way, by changes in the dimensions of heads or points. -head nails will run more to the pound than table shows, and headed nails will run less.

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18.—Standard Steel Wire Nails.

Sizes, Lengths and Approximate Number per Pound.

Size.	Length.	Common. Diam. B. W. G.	Common. Diam. Inches.	Common. No. per Lb.	Barbed Common.	Clinch.	Fence.	Finishing.	Barbed Finishing.	Fine.	Barrel.	Casing.	Smooth Box.	Barbed Box.	Flooring Brads.	Barbed Oval Head Car Nails. Light.	Barbed Oval Head Car Nails. Heavy.	Slating.	Barbed Roofing.
....	¾	....	....	....	....	....	....	....	....	....	940	....	....	....	....	....	....	....	648
....	⅞	....	....	....	....	....	....	....	....	....	804	....	....	....	....	....	....	....	413
2d	1	15	.072	900	860	622	....	1558	1558	1440	620	1140	1000	1000	....	....	....	385	384
*3d	1⅛	....	....	....	....	....	....	....	....	810	590	....	....	....	....	....	....	....	339
†3d	1¼	14	.083	615	594	412	....	884	884	....	542	675	660	660	....	....	....	230	231
....	1⅜	....	....	....	....	....	....	....	....	....	365	....	....	....	....	....	....	....	....
4d	1½	12½	.102	322	339	267	....	767	767	550	332	567	550	550	....	260	164	198	154
5d	1¾	12½	.102	250	230	230	127	491	491	....	....	396	366	366	....	134	103	125	135
6d	2	11½	.113	200	205	156	114	359	359	....	....	260	250	250	151	119	91	112	90
7d	2¼	11½	.113	154	135	110	88	317	317	....	....	239	236	236	136	85	73	....	....
8d	2½	10¼	.124	106	96	98	74	214	214	....	....	160	157	157	98	75	65	....	....
9d	2¾	10¼	.124	85	92	86	58	195	195	....	....	148	145	145	86	58	51	....	....
12d	3	9	.148	74	63	66	42	134	134	....	....	108	107	107	66	55	45	....	....
10d	3¼	9	.148	57	52	57	36	120	120	....	....	99	98	98	51	43	38	....	....
16d	3½	8	.165	46	38	46	28	91	91	....	....	69	65	65	40	39	34	....	....
20d	4	6	.203	29	30	35	22	61	61	....	....	50	45	45	29	31	26	....	....
30d	4½	5	.220	23	23	....	....	....	....	....	....	45	40	40	....	27	23	....	....
40d	5	4	.238	17	17	....	....	....	....	....	....	35	30	30	....	21	17	....	....
50d	5½	3	.259	13½	13½	....	....	....	....	....	....	....	....	....	....	18	14	....	....
60d	6	2	.284	10½	10½	....	....	....	....	....	....	....	....	....	....	15	13	....	....

* Fine. † Common.

19.—Standard Steel Wire Spikes.

Diameter	B. W. G.	6	5	4	3	2	1	1	....	....	....	....	....
	Inch....	.203	.220	.238	.259	.284	.300	.300	5/16	3/8	3/8	3/8	....
Length, Inches. ...		3	3½	4	4½	5	5½	6	6½	7	8	9	....
No. per Pound.....		37	29	23	18	13	10	9	7½	6½	4¼	3⅔	....

Remarks on Spikes and Nails.

Steel wire spikes and nails are made from steel wire, being cut, and pointed by machine. They may be either plain (smooth) or bar added holding power).

Cut spikes and nails have greater holding power than the smoo wire, because they have a rougher surface to give the greater friction.

Tests on the holding power of railway spikes were made by Mr Webber, Instructor of Civ. Eng., Univ. of Ill., and the results are g Bulletin No. 6, issued by the Experiment Station. The tests were o spikes and plain spikes: direct pull and lateral displacement.

It is to be noted that the variation in weight of spikes and nails siderable, even for the same sizes.

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20.—Spikes and Nails.

Wrought Spikes. Number to a keg of 150 lbs.						Steel Wire Spikes.			Standard Steel Wire Nails.					
											Common.		Finishing.	
Length in Inches.	1/4" No.	5/16" No.	3/8" No.	7/16" No.	1/2" No.	Length, Inches.	Diam. Inches.	No. per Pound.	Size.	Length, Inches.	Diam. Inches.	No. per Pound.	Diam. Inches.	No. per Pound.
3	2250	.....	.....	.....	.....	3	.1620	41	2d	1	.0524	1060	.0453	1558
3½	1890	1208	.....	.....	.....	3½	.1819	30	3d	1¼	.0588	640	.0508	913
4	1650	1135	.....	.....	.....	4	.2043	23	4d	1½	.0720	380	.0508	761
4½	1464	1064	.....	.....	.....	4½	.2294	17	5d	1¾	.0764	275	.0571	500
5	1380	930	742	.....	.....	5	.2576	13	6d	2	.0808	210	.0641	350
6	1292	868	570	.....	.....	5½	.2893	11	7d	2¼	.0858	160	.0641	315
7	1161	662	482	445	306	6	.2893	10	8d	2½	.0935	115	.0720	214
8	.....	635	455	834	256	6½	.2249	7½	9d	2¾	.0963	93	.0720	195
9	.....	573	424	300	240	7	.2249	7	10d	3	.1082	77	.0808	137
10	.....	.....	391	270	222	8	.3648	5	12d	3¼	.1144	60	.0808	127
11	.....	.....	.....	249	203	9	.3648	4½	16d	3½	.1285	48	.0907	90
12	.....	.....	.....	236	180				20d	4	.1620	31	.1019	62
									30d	4½	.1819	22		
									40d	5	.2043	17		
									50d	5½	.2294	13		
									60d	6	.2576	11		

21.—Tacks.

Title Ounce.	Length Inches.	Number per Pound.	Title Ounce.	Length Inches.	Number per Pound.	Title Ounce.	Length Inches.	Number per Pound.	Title Ounce.	Length Inches.	Number per Pound.	Title Ounce.	Length Inches.	Number per Pound.
1	1/8	16000	3	3/8	5333	10	11/16	1600	18	11/16	888			
1½	3/16	10666	4	7/16	4000	12	3/4	1333	20	1	800			
2	1/4	8000	6	1/2	2666	14	13/16	1143	22	1 1/16	727			
2½	5/16	6400	8	5/8	2000	16	7/8	1000	24	1 1/8	666			

22.—Miscellaneous Spikes.

Railroad Spikes.

Size Measured Under Head. Inches.	Average Number per Keg of 200 Pounds.	Quantity of Spikes per Mile of Single Track. Ties 2 Feet c. to c. 4 Spikes per Tie.		Rail Used. Weight per Yard. Pounds.
		Pounds.	Kegs.	
5½x5/8	300	7040	35 1/3	75 to 100
5½x7/16	375	5870	29½	45 " 75
5 x7/16	400	5170	26	40 " 56
5 x½	450	4660	23⅓	35 " 40
4½x½	530	3960	20	30 " 35
4 x½	600	3520	17⅔	25 " 35
4½x9/16	680	3110	15½	20 " 30
4 x9/16	720	2910	14¾	20 " 30
3½x9/16	900	2350	11	16 " 25
4 x5/8	1000	2090	10½	16 " 25
3½x5/8	1190	1780	9	16 " 20
3 x5/8	1240	1710	8½	16 " 20
2½x5/8	1342	1575	7⅞	8 " 16
2½x9/16	1600	1292	5	8 " 10

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22.—Miscellaneous Spikes.—Concluded.

Square Boat Spikes.

Approximate Number in a Keg of 200 Pounds.

Size, Inches.	Length of Spike, Inches.									
	3	4	5	6	7	8	9	10	11	12
1/4	3000	2375	2050	1825						
5/16	1660	1360	1230	1175	990	880				
3/8	1320	1140	940	800	650	600	525	475		
7/16	.....	.....	.....	600	590	510	400	360	320	280
1/2	.....	.....	.....	450	375	335	300	275	230	240
5/8	.....	.....	.....	.....	.....	260	240	220	205	190

Street Railway Spikes.

Spikes.	Number per Keg of 200 Pounds.	Kegs per Ties 2 feet
5 1/2 x 9/16	400	30
5 x 1/2	575	19
4 1/2 x 7/16	800	13

23.—Cut Steel Nails and Spikes.

Sizes, Lengths, and Approximate Number per Pour

Sizes.	Length. Inches.	Common.	Clinch.	Finishing.	Casing and Box.	Fencing.	Spikes.	Barrel.	Light Barrel.	Slating.	Sizes.	Length. Inches.
....	5/8	.....	.....	.....	.....	.....	.....	750	.....	.....	....	3/4
....	3/4	.....	.....	.....	.....	.....	.....	600	.....	.....	....	7/8
....	7/8	.....	.....	.....	.....	.....	.....	500	.....	.....	2d	1
2d	1	740	400	1100	.....	.....	.....	450	.....	340	3d	1 1/8
....	1 1/8	.....	.....	.....	.....	.....	.....	310	400	.....	4d	1 3/8
3d	1 1/4	460	260	880	.....	.....	.....	280	304	280		
....	1 3/8	.....	.....	.....	.....	.....	.....	210	.....	.....	Tobacco	
4d	1 1/2	280	180	530	420	.....	.....	190	224	220		
5d	1 3/4	210	125	350	300	100	.....	.....	.....	180	130	
6d	2	160	100	300	210	80	.....	.....	.....	.....	97	
7d	2 1/4	120	80	210	180	60	.....	.....	.....	.....	85	
8d	2 1/2	88	68	168	130	52	.....	.....	.....	.....	68	
9d	2 3/4	73	52	130	107	38	.....	.....	.....	.....	58	
10d	3	60	48	104	88	26	.....	.....	.....	.....	48	
12d	3 1/4	46	40	96	70	20	.....	.....	.....	.....	.....	
16d	3 1/2	33	34	86	52	18	17.	.....	.....	.....	.....	
	4	23	24	76	38	16	14	.....	.....	.....	.....	
	4 1/4	20	.....	.....	.....	.....	.....	.....	.....	.....	.....	
	4 1/2	16 1/2	.....	.....	30	.....	11	.....	.....	.....	.....	
	5	12	.....	.....	26	.....	9	.....	.....	.....	.....	
	5 1/2	10	.....	.....	20	.....	7 1/2	.....	.....	.....	.....	
	6	8	.....	.....	16	.....	6	.....	.....	.....	.....	
	6 1/2	.....	.....	.....	.....	.....	5 1/2	.....	.....	.....	.....	
	7	.....	.....	.....	.....	.....	5	.....	.....	.....	.....	

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24.—Carnegie Standard Pin-Nuts.
With Weight of Pins per Lineal Inch.

Pin-Nuts.					Pins.			Pin-Nuts.				
per Inch.	Short Diameter.	Long Diameter.	Thickness.	*Weight.	Diameter of Pin.	†Wt. in Lbs. per Lin. In.	Diameter of Screw.	Threads per Inch.	Short Diameter.	Long Diameter.	Thickness.	*Weight.
8	2¼	2⅝	⅞	0.85	3⅞	3.342	3¼	6	5	5¾	1¼	4.74
8	2½	2⅞	31/32	1.03	4⅛	3.787	3½	6	5½	6⅜	1¼	6.19
8	2½	2⅞	1	0.97	4⅜	4.259	3½	6	5½	6⅜	1¼	6.19
8	3	3½	1	1.50	4⅝	4.760	3¾	6	5½	6⅜	1¼	5.37
8	3	3½	1	1.37	4⅞	5.288	4	6	6	6 13/16	1¼	6.63
8	3½	4	1	2.06	5⅛	5.845	4	6	6	6 13/16	1¼	6.63
8	3½	4	1	1.96	5⅜	6.429	4¼	6	6	6 13/16	1¼	5.82
8	4	4⅝	1¼	3.38	5⅝	7.041	4½	6	6¾	7 13/16	1¼	8.53
8	4	4⅝	1¼	3.22	5⅞	7.681	4¾	6	6¾	7 13/16	1¼	7.59
8	4¼	4⅞	1¼	3.63	6⅛	8.483	4¾	6	6¾	7 13/16	1¼	7.59
8	4¼	4⅞	1¼	3.41	6⅜	9.042	5	6	8	9¼	1¼	13.06
6	4½	5 3/16	1¼	4.09	6⅝	9.767	5¼	6	8	9¼	1½	14.86
6	4¾	5½	1¼	4.63	6⅞	10.517	5½	6	8	9¼	1½	14.00
6	5	5¾	1¼	5.25	7⅛	11.300	5¾	6	8	9¼	1½	13.10

ns given above are in inches.
fer to untapped nuts.
for normal diameter before being turned.

'HOENIX Standard Pin Nuts and Pilots.

diam. of pins for turning:
sive allow ⅛" to 1/16" to 2' 0"
4" to 1/16" above 2' 0" long.
ot less than 1/16" to 2' 0" long,
ove 2' 0" long.
ots: 4 lbs. for 2 7/16" diam. of
3 7/16"; 18 lbs. for 4 5/16"; 34 lbs.
for 6 11/16"; 116 lbs. for 8⅜";
in.

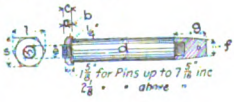


Fig. 48.

IBRIA Bridge Pins, Nuts and Pilot Nuts.

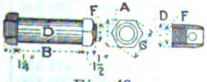
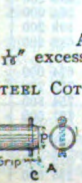


Fig. 49.

All Threads 8 per inch.
1/16" excess for each eye bar packed on the pin.

TEEL Cotter Pins.



'ig. 50.

Lateral Pins.

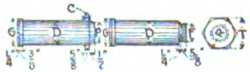


Fig. 51.

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25.—Pins—Bending Moments; and Bearing Values on Plates 1 In. Thick.

(Bending moment = .0982×fiber stress×(dia.)³=⅛×fiber stress×area×dia.)

Diam. of Pins. Ins.	Area of Pin. Sq. Ins.	Moments in Inch-Pounds for Fiber Stresses of					Bearing in lbs. on Plate 1 in. thick, at	
		15,000 lbs. per sq. in.	18,000 lbs. per sq. in.	20,000 lbs. per sq. in.	22,500 lbs. per sq. in.	25,000 lbs. per sq. in.	12,000 lbs. per sq. in.	15,000 lbs. per sq. in.
1	0.785	1 470	1 770	1 960	2 210	2 450	12 000	15 000
1⅛	0.994	2 100	2 520	2 800	3 140	3 500	13 500	16 900
1¼	1.227	2 880	3 450	3 830	4 310	4 790	15 000	18 800
1⅜	1.485	3 830	4 590	5 100	5 740	6 380	16 500	20 600
1½	1.767	4 970	5 960	6 630	7 460	8 280	18 000	22 500
1⅝	2.074	6 320	7 580	8 430	9 480	10 500	19 500	24 400
1¾	2.405	7 890	9 470	10 500	11 800	13 200	21 000	26 300
1⅞	2.761	9 710	11 600	12 900	14 600	16 200	22 500	28 100
2	3.142	11 800	14 100	15 700	17 700	19 600	24 000	30 000
2⅛	3.547	14 100	17 000	18 800	21 200	23 600	25 500	31 900
2¼	3.976	16 800	20 100	22 400	25 200	28 000	27 000	33 800
2⅜	4.430	19 700	23 700	26 300	29 600	32 900	28 500	35 600
2½	4.909	23 000	27 600	30 700	34 500	38 400	30 000	37 500
2⅝	5.412	26 600	32 000	35 500	40 000	44 400	31 500	39 400
2¾	5.940	30 600	36 800	40 800	45 900	51 000	33 000	41 300
2⅞	6.492	35 000	42 000	46 700	52 500	58 300	34 500	43 100
3	7.069	39 800	47 700	53 000	59 600	66 300	36 000	45 000
3⅛	7.670	44 900	53 900	59 900	67 400	74 900	37 500	46 900
3¼	8.296	50 600	60 700	67 400	75 800	84 300	39 000	48 800
3⅜	8.946	56 600	67 900	75 500	84 900	94 400	40 500	50 600
3½	9.621	63 100	75 800	84 200	94 700	105 200	42 000	52 500
3⅝	10.321	70 100	84 200	93 500	105 200	116 900	43 500	54 400
3¾	11.045	77 700	93 200	103 500	116 500	129 400	45 000	56 300
3⅞	11.793	85 700	102 800	114 200	128 500	142 800	46 500	58 100
4	12.566	94 200	113 100	125 700	141 400	157 100	48 000	60 000
4⅛	13.364	103 400	124 000	137 800	155 000	172 300	49 500	61 900
4¼	14.186	113 000	135 700	150 700	169 600	188 400	51 000	63 800
4⅜	15.033	123 300	148 000	164 400	185 000	205 500	52 500	65 600
4½	15.904	134 200	161 000	178 900	201 300	223 700	54 000	67 500
4⅝	16.800	145 700	174 800	194 300	218 500	242 800	55 500	69 400
4¾	17.721	157 800	189 400	210 400	236 700	263 000	57 000	71 300
4⅞	18.665	170 600	204 700	227 500	255 900	284 400	58 500	73 100
5	19.635	184 100	220 900	245 400	276 100	306 800	60 000	75 000
5⅛	20.629	198 200	237 900	264 300	297 300	330 400	61 500	76 900
5¼	21.648	213 100	255 700	284 100	319 600	355 200	63 000	78 800
5⅜	22.691	228 700	274 400	304 900	343 000	381 100	64 500	80 600
5½	23.758	245 000	294 000	326 700	367 500	408 300	66 000	82 500
5⅝	24.850	262 100	314 500	349 500	393 100	436 800	67 500	84 400
5¾	25.967	280 000	335 900	373 300	419 900	466 600	69 000	86 300
5⅞	27.109	298 600	358 300	398 200	447 900	497 700	70 500	88 100
6	28.274	318 100	381 700	424 100	477 100	530 200	72 000	90 000
6⅛	29.465	338 400	406 100	451 200	507 600	564 000	73 500	91 900
6¼	30.680	359 500	431 400	479 400	539 300	599 200	75 000	93 800
6⅜	31.919	381 500	457 800	508 700	572 300	635 900	76 500	95 600
6½	33.183	404 400	485 300	539 200	606 600	674 000	78 000	97 500
6⅝	34.472	428 200	513 800	570 900	642 300	713 700	79 500	99 400
6¾	35.785	452 900	543 500	603 900	679 400	754 800	81 000	101 300
6⅞	37.122	478 500	574 200	638 000	717 800	797 500	82 500	103 100
7	38.485	505 100	606 100	673 500	757 700	841 900	84 000	105 000
7½	44.179	621 300	745 500	828 400	931 900	1 035 400	90 000	112 500
8	50.265	754 000	904 800	1 005 300	1 131 000	1 256 600	96 000	120 000

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26.—Eye Bars.

American Bridge Company's Standards.

dinary.

ig. 52.

Adjustable.

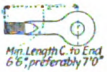

Fig. 53.

Head.			Screw End.			Thickness of Bar, Inches.	Width of Bar, Inches.
Diameter, Inches	Max. Pin, Inches.	Additional Material for 1 Head. Ft. and Ins.	Additional Material for Upset. Ft. and Ins.	Diameter, Inches.	Length, Inches.		
4½	1¾	0—10	0— 7	2	5	13/16	2
5½	2¾	1— 2					
5½	2¼	0—11	1— 1	2¼	5	13/16 to 1 1/16	2½
6½	3¼	1— 2					
7	3	1— 1	1— 5	2½	5½	1 to 1 1/16	3
8	4	1— 5	1— 5	2¾	6	1⅛ to 1¼	
9½	4¼	1— 6	1— 8	3	6	1 to 1⅛	4
10½	5¼	1— 9	1— 8	3¼	6½	1 3/16 to 1⅜	
11½	6¼	2— 3					
11½	5	1— 8	1— 9	3¼	6½	1 to 1 1/16	5
13	6½	2— 3	1— 9	3½	7	1⅛ to 1¼	
13½	5½	1—10	1—11	3¾	8	1⅛ to 1 3/16	6
14½	6½	2— 4	1—11	4	8	1¼ to 1⅜	
16½	7¼	2— 4	2— 3	4¼	9	1¼ to 1 5/16	7
17½	8¼	2— 8	2— 3	4½	9	1⅜ to 1½	
17½	7	2— 3	.....	.....	.....	.....	8
18½	8	2— 8	.....	.....	.....	.....	
19½	9	3— 0	.....	.....	.....	.....	
22	9	2—11	.....	.....	.....	.....	10
23½	10½	3— 4	.....	.....	.....	.....	
24½	11½	3— 8	.....	.....	.....	.....	
26	10	3— 3	.....	.....	.....	.....	12
27½	11½	3— 9	.....	.....	.....	.....	
29	13	4— 2	.....	.....	.....	.....	
30½	12	3—10	.....	.....	.....	.....	14
32½	14	4— 5	.....	.....	.....	.....	
33½	15	4— 8	.....	.....	.....	.....	

d * should only be used when absolutely unavoidable.
bars are hydraulic forged and are guaranteed to develop
h of the bar, under conditions given in the above table,
destruction.

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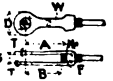
27.—Clevises.

American Bridge Company's Standards.

All dimensions in inches.



Grip G can be made to suit connections.

Fig 54.

Diameter of Clevis D.	Max. Pin P.	Clevis						
		Fork F	Nut N	Width W	Thickness T	A	B	Ap'rx. Wt.
3	1½	1½	1½	1½	⅜	6	5	5
4	2¼	1¾	1¾	1¾	½	9	8	9
5	3	2¼	2¼	2¼	⅝	9	8	14
6	3½	2¾	2¾	2¾	¾	9	8	25
7	4	3¼	3¼	3¼	⅞	9	8	39

Table giving Diameter of Clevis for given Rod and Pin.

Rod.			Pins.													Rod.		
Round.	Square.	Upset.	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼	3½	3¾	4	Upset.	Square.	Round.
¾	⅝	1	3	3	3											1	⅝	¾
	¾	1⅛	3	3	3	4	4	4								1⅛	¾	
⅞	⅞	1¼		4	4	4	4	4								1¼	⅞	⅞
1		1⅜		4	4	4	4	4								1⅜		1
1⅛	1	1½		4	4	4	4	4	5	5	5					1½	1	1⅛
1¼	1⅛	1⅝			4	4	5	5	5	5	5					1⅝	1⅛	1¼
1⅜		1¾			5	5	5	5	5	5	5					1¾		1⅜
	1¼	1⅞			5	5	5	5	5	5	5					1⅞	1¼	
1½	1⅜	2				5	5	5	5	5	6	6	6			2	1⅜	1½
1⅝		2⅛				5	5	5	5	6	6	6	6			2⅛		1⅝
1¾	1½	2¼					6	6	6	6	6	6	7	7	7	2¼	1½	1¾
1⅞	1⅝	2⅜					6	6	6	6	7	7	7	7	7	2⅜	1⅝	1⅞
2	1¾	2½						6	6	7	7	7	7	7	7	2½	1¾	2
2⅛		2⅝							7	7	7	7	7			2⅝		2⅛
	1⅞	2¾							7	7	7	7	7			2¾	1⅞	
2¼	2	2⅞								7	7					2⅞	2	2¼
Round.	Square.	Upset.	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼	3½	3¾	4	Upset.	Square.	Round.
Rod.			Pins.													Rod.		

Clevises above and to right of heavy zigzag line may be used with forks straight.

Clevises below and to left of same line should have forks closed in until pin is not overstrained.

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28.—Sleeve Nuts and Turnbuckles.

American Bridge Company's Standards.
All Dimensions in Inches.

eve Nuts.

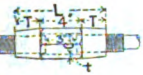

Fig. 55.

Short Diam. A	Long Diam. B	Inside Diam. C	Thickness t	Weight in Pounds.
1⅝	1⅞	1⅛	¼	2¾
1⅝	1⅞	1⅛	¼	3
2	2 1/16	1⅜	5/16	3½
2	2 1/16	1⅜	5/16	4
2⅜	2¾	1⅝	⅜	4½
2⅜	2¾	1⅝	⅜	6½
2¾	3 1/16	1⅞	7/16	8
2¾	3 3/16	1⅞	7/16	8½
3⅛	3⅝	2⅛	½	10
3⅛	3⅝	2⅛	½	11
3½	4 1/16	2⅜	9/16	14
3½	4 1/16	2⅜	9/16	15
3⅞	4½	2⅝	⅝	18
3⅞	4½	2⅝	⅝	19
4¼	4 15/16	2⅞	11/16	22
4¼	4 15/16	2⅞	11/16	23
4⅝	5⅜	3⅛	¾	27
4⅝	5⅜	3⅛	¾	28
5	5 13/16	3⅜	13/16	34
5	5 13/16	3⅜	13/16	35
5⅜	6⅛	3⅝	⅞	39
5⅜	6⅛	3⅝	⅞	40
5¾	6 11/16	3⅞	15/16	45
5¾	6 11/16	3⅞	15/16	47
6⅛	7⅛	4⅛	1	52
6⅛	7⅛	4⅛	1	55
6½	7 5/16	4⅜	1 1/16	65
6⅞	8	4¾	1 1/16	75

Turnbuckles.
Cleveland City Forge and Iron Co.

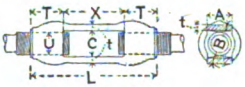

Fig. 56.
Standard Length, X = 6 in.
Extra Lengths, 9, 12, 18, 24, 36, 48 and 72 in. (Special Prices.)

Weight in Pounds.	Standard Dimensions. t	A	B	C	L	T	Diameter of Screw U.
2½	⅜	1¼	2¼	1¼	8⅝	1 5/16	⅞
3½	7/16	1¼	2 7/16	1 5/16	9	1½	1
4	½	1¼	2 9/16	1 7/16	9⅜	1 11/16	1⅛
5¼	½	1½	2¾	1 9/16	9¾	1⅞	1¼
6	½	1⅝	3 1/16	1 11/16	10⅛	2 1/16	1⅜
7	⅝	1⅝	3 3/16	1¾	10½	2¼	1½
8½	⅝	1¾	3½	2	10⅞	2 7/16	1⅝
10	⅝	2	3¾	2⅛	11¼	2⅝	1¾
11½	11/16	2	3⅞	2 3/16	11⅝	2 13/16	1⅞
13	11/16	2¼	4¼	2⅜	12	3	2
14	13/32	2½	4½	2½	12⅜	3 3/16	2⅛
15	13/16	2½	4¾	2 11/16	12¾	3⅜	2¼
18	13/16	2¾	4⅞	2¾	13⅛	3 9/16	2⅜
19	27/32	3	5⅜	3 1/16	13½	3¾	2½
22	13/16	3	5 9/16	3⅛	13⅞	3 13/16	2⅝
23	13/16	3¼	5¾	3¼	14¼	4⅛	2¾
27	1 3/32	3¼	6 1/16	3 7/16	14⅝	4 5/16	2⅞
28	1 3/32	3½	6⅜	3⅝	15	4½	3
.....	.....	.....	.....	.....	.....	.....	3⅛
35	1 1/16	4	6¾	3⅞	15¾	4⅞	3¼
.....	.....	.....	.....	.....	.....	.....	3⅜
40	65	1 7/32	4	7¼	4¼	16½	5¼	3½
45	.....	.....	.....	.....	.....	.....	.....	3⅝
47	.....	1 5/16	5	8¼	4 7/16	18	6	3¾
52	.....	.....	.....	.....	.....	.....	.....	3⅞
55	.....	1 7/16	5	8¾	4⅝	18	6	4
65	.....	.....	.....	.....	.....	.....	.....	4¼
75	.....	.....	.....	.....	.....	.....	.....	4½

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29.—Upset Screw Ends for Round and Square Bars.

Screw threads are the Franklin Institute Standard.

Allow 6 inches additional length of rod for 5 in. length of thread.

Side of Square Bar. Inches.	Diameter of Round Bar. Inches.	Diameter of Upset Screw End. Inches.	Diameter of Screw at Root of Thread. Inches.	Threads per Inch. No.	Length of Upset. Inches. *	Length of Upset. Inches. †	Side of Square Bar. Inches.	Diameter of Round Bar. Inches.	Diameter of Upset Screw End. Inches.	Diameter of Screw at Root of Thread. Inches.	Threads per Inch. No.	Length of Upset. Inches. *	Length of Upset. Inches. †
½	½	¾	.620	10	.....	4¼	1¾	2	2½	2.175	4	6½	6
.....	9/16	¾	.620	10	.....	4¼	1 11/16	2 1/16	2 5/8	2.300	4	6¾	6¼
9/16	⅝	⅞	.731	9	2½	4½	1⅞	2⅛	2¾	2.300	4	6¾	6¼
⅝	11/16	1	.837	8	2¾	4½	1 11/16	2 3/16	2¾	2.425	4	7¼	6¼
11/16	¾	1	.837	8	2¾	4½	2	2¼	2⅞	2.550	4	7½	6½
¾	13/16	1⅛	.940	7	3	4¾	2 1/16	2 5/16	2⅞	2.550	4	7½	6½
.....	⅞	1¼	1.065	7	3¼	4¾	2⅛	2⅜	3	2.629	3½	7¾	6½
13/16	15/16	1¼	1.065	7	3¼	4¾	2 3/16	2 7/16	3⅛	2.754	3½	8	6¾
⅞	1	1⅜	1.160	6	3¾	5	2¼	2½	3⅛	2.754	3½	8	6¾
15/16	1 1/16	1⅜	1.160	6	3¾	5	.....	2 5/16	3¼	2.879	3½	8¼	6¾
.....	1⅛	1½	1.284	6	4	5	2 5/16	2⅝	3¼	2.879	3½	8¼	6¾
1	1 1/16	1½	1.284	6	4	5	2⅜	2 11/16	3⅜	3.004	3½	8½	7
1 1/16	.....	1⅝	1.389	5½	4¼	5¼	2 7/16	2¾	3⅜	3.004	3½	8½	7
1⅛	1¼	1⅝	1.389	5½	4¼	5¼	2½	2 13/16	3½	3.100	3¼	9	7
.....	1 5/16	1¾	1.490	5	4¾	5¼	2 9/16	2⅞	3⅝	3.225	3¼	.....	7¼
1 3/16	1⅜	1¾	1.490	5	4¾	5¼	2⅝	2 15/16	3⅝	3.225	3¼	.....	7¼
1¼	.....	1⅞	1.615	5	5	5½	2 11/16	3	3¾	3.317	3	10	7¼
1 5/16	1 1/16	1⅞	1.615	5	5	5½	2¾	.....	3⅞	3.442	3	10	7½
.....	1½	2	1.712	4½	5¼	5½	2 13/16	3⅛	3⅞	3.442	3	10	7½
1⅜	1 9/16	2	1.712	4½	5¼	5½	2⅞	3¼	4	3.567	3	10	7½
1 7/16	1⅝	2⅛	1.837	4½	5½	5¾	2 15/16	.....	4⅛	3.692	3	10½	7½
1½	1 11/16	2⅛	1.837	4½	5½	5¾	3	3⅜	4⅛	3.692	3	10½	7¾
.....	1¾	2¼	1.962	4½	5¾	5¾	.....	3½	4¼	3.798	2⅞	10½	8
1 9/16	1 13/16	2¼	1.962	4½	5¾	5¾	3⅛	.....	4⅜	3.923	2⅞	10½	8
1⅝	.....	2⅜	2.087	4½	6¼	6	3¼	3⅝	4½	4.028	2¾	10½	8
1 13/16	1⅞	2⅜	2.087	4½	6¼	6	3⅜	3¾	4⅝	4.153	2¾	11	8¼
.....	1 15/16	2½	2.175	4	6½	6	.....	3⅞	4¾	4.255	2⅝	11½	8½

* Phoenix Iron Company.

† Cambria Steel Company upset lengths for use with Standard turnbuckles (6 inches between heads) and with clevises. Make upset one inch shorter for use with ordinary right and left nuts. For other uses length of upset will vary to suit the particular case.

Right and Left Nuts.
(Sleeve Nuts.)

Fig. 57.

Counter and Lateral Rods.
Solid or Upset Eyes.

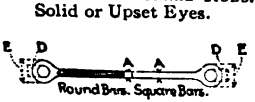

Fig. 58.

Counter and Lateral Rods.
Loop Welded Eyes.

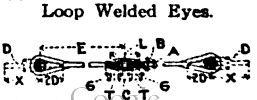

Fig. 59.

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30.—Segmental Rollers.

(For Bridge Expansion.)

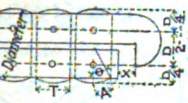

Fig. 60.

Theoretical Values.

$$\text{Angle } \theta = \frac{180 \times \text{arc } A}{\pi \times \text{diam. } D}$$

$$w = \frac{D}{2} \cos \theta$$

$$x = \frac{D}{2} \sin \theta$$

$$y = \frac{T}{\cos \theta}$$

* Practical Values.

(All dimensions in Inches.)

6" 3"	Diam. of rollers 8" Thickness $T = 4''$				Diam. of rollers 10" Thickness $T = 4''$				Diam. of rollers 12" Thickness $T = 5''$			
y	Expansion of Bridge.	w	x	y	Expansion of Bridge.	w	x	y	Expansion of Bridge.	w	x	y
1/16	2	3¾	1¼	4⅛	2 to 2⅜	4¾	1¼	4⅛	3 to 3¼	5⅝	1¾	5 3/16
⅛	2⅛ to 2⅜	"	"	4 3/16	2½ to 2⅝	"	1½	"	3⅜	"	"	5¼
3/16	2½	"	1½	"	2¾ to 2⅞	4⅝	"	4 3/16	3½ to 3⅞	"	2	"
¼	2⅝ to 2⅞	3⅝	"	4¼	3	"	1¾	"	4 to 4¼	5½	2¼	5 5/16
	3 to 3⅛	"	1¾	4 5/16	3⅛ to 3⅜	"	"	4¼	4⅜	"	"	5⅜
	3¼	"	"	4⅜	3½	"	2	"	4½	5⅜	"	"
	3⅜	3½	"	"	3⅝ to 4	4½	"	4 5/16	4⅝	"	2½	"
	3½ to 3⅝	"	2	4 7/16					4¾ to 4⅞	"	"	5 7/16
	3¾ to 3⅞	"	"	4½					5	"	"	5½
	4	"	"	4 9/16								

t 1 inch expansion per 100 ft. of bridge. The rollers should hown in above cut) for mean temperatures; and hence the qual to, and even a little greater than, half the expansion.

Brace Problem.—The following solution of the "Howe blem" is one which the writer devised and has used exten-

Fig. 61, let the length de of panel (generally s between chords) = $2h$, rter side (generally the $2p$. Let x and y be the center of end (point width = $2d$. With h and d constant the cribe a circle of radius he origin o.

$= d^2$ .....(1)

p constant and d varia- will trace the curve $o\,s\,o'$, which is derived as fol- etry the angles a and a_1

t $\tan a = \frac{y}{x}$, and tan

quating,

$= \frac{p-x}{h-y}$;

$= p\,x - x^2$,

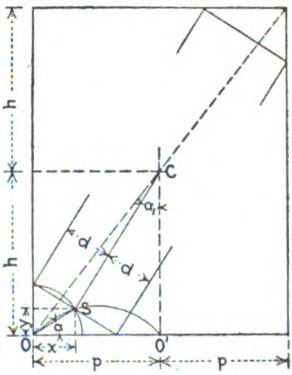

Fig. 61.

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and if p is less than h,

$$y = \frac{h}{2} - \sqrt{\left(\frac{h}{2}\right)^2 - x(p-x)} \quad \ldots\ldots (2)$$

Combining the equation of the circle (1) and of the equilateral hyperbola (2) analytically is a complex process and unnecessary in solving the values of x and y.

By inspection, Fig. 61, it will be seen that as the point s approaches the origin o (as the width of brace decreases) the angles a and a_1 increase, and when s reaches o $\tan a = \tan a_1 = \frac{p}{h}$. That is to say, the equation of a straight line tangent to the hyperbola at the origin o is

$$y' = \frac{p}{h} x' \quad \ldots\ldots (3)$$

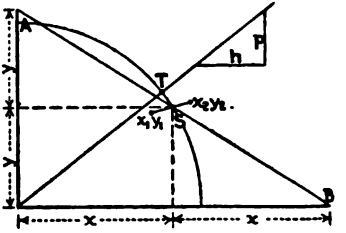

Fig. 62.

Plotting this line (see Fig. 62), we know very nearly where the point s is on the circle (1). It will be just below and close to it. Then solving equation (3) for two values of x, on either side of and near s will give, without appreciable error, the tangent to the hyperbola at s, and the intersection of this tangent with the circle will be the point s.

Example: Let height of truss = 20 ft. clear between chords; panel length = 12 ft.; width of brace = 14 ins.; then, reducing to inches, h = 120, p = 72, d = 7. With o as origin plot circle (1) with d as radius, full size; also lay off tangent, equation (3), intersecting the circle at T. The point s will lie a little below T on the circle. Assume x_1 = 6, then, equation (2), $y_1 = 60 - \sqrt{3{,}600 - 396} = 3.40$. Assume $x_2 = 6\frac{1}{4}$, then equation (2), $y_2 = 60 - \sqrt{3{,}600 - 410.94} = 3.52$.

Plot $x_1\, y_1$ and $x_2\, y_2$; connect with line intersecting circle at s. Then, by scale, x = 6.09 ins., y = 3.45 ins. $A\ s\ B$ is face of angle block. Length of brace is

$$2\sqrt{(h-y)^2 + (p-x)^2} = 22 \text{ ft. } 3\tfrac{3}{4} \text{ ins.} + . \quad \text{Ans.}$$

***Tables of Cubes and Squares.**—The subjoined tables of cubes and squares are very useful in designing and detailing. For convenient reference they are listed as follows:

Table 31.	—Cubes of Inches,	0″ to 9″,	advancing by	64ths and 32ds.
" 31a.—	" " "	9″ to 29″,	" "	16ths.
" 31b.—	" " "	29″ to 109″	" "	8ths.
" 32.	—Squares of Inches,	0″ to 12″ (0′ to 1′),	advancing by	64ths.
" 32a.—	" " "	12″ to 120″ (1′ to 10′)	" "	32nds.
" 32b.—	" " "	120″ to 624″ (10′ to 52′)	" "	16ths.

Note that the last column of Table 32b, ($+\frac{1}{32}$), is the average amount (the amount at the center of the respective line—the half-inch column) to be added for each 32nd of an inch. Thus, the square for 13′ $0\frac{7}{16}$″ is 24472.69; and for 13′ $0\frac{15}{32}$″ is 24472.69 + 9.78 = 24482.47. The same result can also be obtained by adding the squares of 13′ $0\frac{7}{16}$″ and 13′ $0\frac{1}{2}$″ together, and dividing the sum by 2; thus, (24472.69 + 24492.25) ÷ 2 = 24482.47. Remember that this difference for ($\frac{1}{32}$) is for the $\frac{1}{2}$″-column, in middle of line; and that for the beginning of the line the tabular difference in the last column should be decreased by 0.03, and increased by 0.03 for end of same line. This refinement, however, of modifying the amounts given in the last column, to conform to different parts of the line, may generally be neglected.

Explanation of Uses of Tables 31 *to* 32 *b*.—These tables may be used in finding bending moments, moments of resistance, moments of inertia and radii of gyration; and in the solution of right angle triangles when two sides

*These tables are abridged from those in the writer's forthcoming structural handbook.

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are given, and of any triangle when three sides are given. The following examples will illustrate.

Finding Bending Moments. (Table 32b.)

Example.—A girder having a span (L) of 14′ $5\frac{1}{16}$″ supports a uniform load (W) of 400 lbs. per lin. ft. What is the bending moment (M'') in in.-lbs.?

Solution.—From the well-known formula $M'' = \frac{12WL^2}{8}$, we have, if l = span in ins. = $12L$, $M'' = \frac{Wl^2}{12 \times 8} = \frac{400 \times 30080.6}{12 \times 8} = 125336$ in.-lbs. Ans.

Finding Resisting Moments. (Table 32a.)

Example.—What is the resisting moment in inch-lbs. (M'') of a rectangular beam 6″ wide (b) and $12\frac{7}{8}$″ deep (d), assuming the allowable fiber stress per sq. in. (f) = 1000 lbs.?

Solution.—From the well-known formula $M'' = \frac{1}{6} f b d^2$, we have, by substitution, $M'' = \frac{1000 \times 6 \times 165.766}{6} = 165766$ in.-lbs. Ans.

Finding Radii of Gyration.

The square of the radius of gyration (r^2) = the moment of inertia (I) divided by the area (A) of the section. Thus $r = \sqrt{I \div A}$.

Finding Moments of Inertia.

Example. — Fig. 63 represents the section of a steel column latticed on two of the sides. Find the moments of inertia about the axes X and Y.

Solution.—The moment of inertia (I) of a rectangle about its base, is $I = bh^3 \div 3$; in which b = width and h = height of rectangle. Hence, $3I = bh^3$. Now, Fig. 63 is symmetrical about each of its axes, hence if we find the value of $3I$ for one-quarter of the section and then increase this value by one-third, we obtain the value $4I$ for one-fourth of the section = I for the whole section. The calculation is tabulated below; the values of bh^3 being made up from the various rectangles in one-quarter of the section.

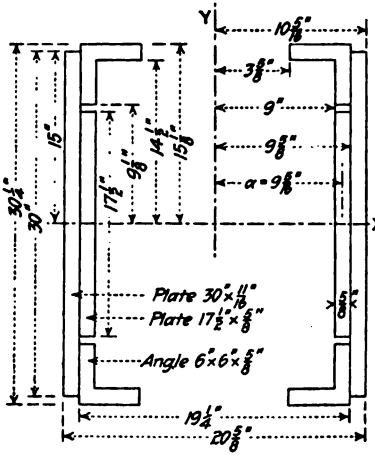

Fig. 63.

	bh^3 about axis X.		bh^3 about axis Y.	
30 ″ plate.	$\frac{11}{16} \times (15\ \ )^3$ =	+ 2320.31	15 $\times (10\frac{5}{16})^3$ =	+ 16450.65
" "			− 15 $\times (9\frac{1}{8})^3$ =	− 13375.00
$17\frac{1}{2}$″ plate.	$\frac{5}{8} \times (8\frac{3}{4})^3$ =	+ 418.70	$8\frac{3}{4} \times (9\frac{5}{8})^3$ =	+ 7802.08
" "			− $8\frac{3}{4} \times (9\ \ )^3$ =	− 6378.75
6 ″ angle.	6 $\times (15\frac{1}{8})^3$ =	+ 20760.48	6 $\times (9\frac{5}{8})^3$ =	+ 5350.00
" "	− $5\frac{3}{8} \times (14\frac{1}{2})^3$ =	− 16386.36	− $5\frac{3}{4} \times (9\ \ )^3$ =	− 3918.38
" "	− $\frac{5}{8} \times (9\frac{1}{8})^3$ =	− 474.88	− $\frac{5}{8} \times (3\frac{5}{8})^3$ =	− 29.77
		+ 6638.25		+ 5900.83
Add one-third—		2212.75		1966.94
For the whole section,	Ix =	8851.00	Iy =	7867.77

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Another Method.—Instead of using the "rectangle method" (preceding case), we may use the method of "transferrence of neutral axis." Thus, to find the moment of inertia of the $17\frac{1}{2}'' \times \frac{5}{8}''$ plate about the axis Y we proceed as follows: Find Io about its own neutral axis ($Io = bh^3 \div 12 = 17\frac{1}{2} \times (\frac{5}{8})^3 \div 12 = 0.356$) and then use the formula $Iy = Io + Aa^2$, in which A = area of plate $= 17\frac{1}{2} \times \frac{5}{8}$, and a = distance between the two axes (see Fig. 63). Hence, the moment of inertia of the plate about the axis $Y = Iy = Io + Aa^2 = 0.356 + 17\frac{1}{2} \times \frac{5}{8} \times (9\frac{1}{4})^2 = 948.88$. Proceed in like manner with the other plates, and the angles, etc. The sum of the several moments of inertia will be the total moment of inertia of the section.

Corollary.—In the last method, above, we have the equation, $Iy = Io + Aa^2$. When the section of the column is unsymmetrical, it is often necessary to assume an axis Y and find the value of Iy about this axis, and then, from the said equation, to find the vaules of a and Io.

Solving Right Angle Triangles. (Tables 32, 32a, 32b.)

Example.—What is the hypothenuse of a right angle triangle whose base is $21'\ 7\frac{1}{2}''$ and perpendicular $38'\ 9\frac{1}{16}''$? ($h^2 = b^2 + p^2$.)

Solution.—From Table 32b we have:

For square corresponding to	$21'\ 7\frac{1}{2}''$	=	67340.25
	+ $\frac{1}{2}''$	=	16.22
" " " "	$38'\ 9\frac{1}{16}''$	=	216515.70
		=	283872.17
Ans.	$44'\ 4\frac{11}{16}''$	=	283855.9

Corollary.—From the above equation, $h^2 = b^2 + p^2$, it is evident that $b^2 = h^2 - p^2$ and $p^2 = h^2 - b^2$.

Solving Any Triangle—3 Sides Given.

Example.—In Fig. 64 let there be given the sides H, h and $B + b$. Solve the triangle.

Solution.—Drop the perpendicular p upon the base $B + b$. Then, $p^2 = H^2 - B^2 = h^2 - b^2$; therefore, $H^2 - h^2 = B^2 - b^2$.

Whence, $B - b = \dfrac{H^2 - h^2}{B + b}$,

And $B = \dfrac{B + b}{2} + \dfrac{B - b}{2}$,

And $b = \dfrac{B + b}{2} - \dfrac{B - b}{2}$.

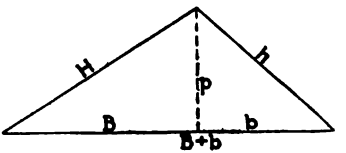

Fig. 64.

Problem.—Given the three sides of a triangle (Fig. 64): $h = 14'\ 3\frac{3}{16}''$, $H = 28'\ 7\frac{5}{8}''$, and $B + b = 35'\ 6\frac{1}{2}''$. Solve for the left-hand angle at the base (which call θ)?

Solution.—Using the formulas in the solution to the above Example, we have,

$H = 28'\ 7\frac{5}{8}''$; H^2 (ins.) $= 118078.1$
$h = 14'\ 3\frac{3}{16}''$; h^2 (ins.) $= 29433.7$

$\therefore H^2 - h^2$ (ins.) $= 88644.4$; log $= 4.9476513$
$B + b$ (ins.) $= 426.50$; log $= 2.6289190$

diff. $= 2.3177323$

$\therefore B - b$ (ins.) $= 207.84$; log $= 2.3177291$

$2\overline{)634.34}$

$\therefore B$ (ins.) $= 317.17$; log $= 2.5012921$
H (ins.) $= 343.625$; log $= 2.5360848$

diff. $= 9.9652073$

Ans.—$\theta = 22°\ 37'\ 46''$, from log cos $\theta = 9.9652076$

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Cubes of Inches, 0″ to 9″, Advancing by 64ths and 32nds.

be.	No.	Cube.	No.	Cube.	No.	Cube.	No.	Cube.
.....	1	1.000000	3	27.00000	5	125.00000	7	343.00000
8147*	1/32	1.096710	1/32	27.85257	1/32	127.35843	1/32	347.61429
0518	1/16	1.199463	1/16	28.72290	1/16	129.74634	1/16	352.26978
0300	3/32	1.308441	3/32	29.61118	3/32	132.16391	3/32	356.96664
4414	1/8	1.423828	1/8	30.51758	1/8	134.61133	1/8	361.70508
7684	5/32	1.545807	5/32	31.44229	5/32	137.08878	5/32	366.48526
2397	3/16	1.674561	3/16	32.38550	3/16	139.59644	3/16	371.30737
3084	7/32	1.810272	7/32	33.34738	7/32	142.13449	7/32	376.17160
9531	1/4	1.953125	1/4	34.32813	1/4	144.70313	1/4	381.07813
7809	9/32	2.103302	9/32	35.32791	9/32	147.30252	9/32	386.02713
8147	5/16	2.260986	5/16	36.34692	5/16	149.93286	5/16	391.01880
0774	11/32	2.426361	11/32	37.38535	11/32	152.59433	11/32	396.05331
5918	3/8	2.599609	3/8	38.44336	3/8	155.28711	3/8	401.13086
3809	13/32	2.780914	13/32	39.52115	13/32	158.01138	13/32	406.25162
4675	7/16	2.970459	7/16	40.61890	7/16	160.76733	7/16	411.41577
8746	15/32	3.168427	15/32	41.73679	15/32	163.55515	15/32	416.62350
6250	1/2	3.375000	1/2	42.87500	1/2	166.37500	1/2	421.87500
7416	17/32	3.590363	17/32	44.03372	17/32	169.22708	17/32	427.17044
2473	9/16	3.814697	9/16	45.21313	9/16	172.11157	9/16	432.51001
1650	19/32	4.048187	19/32	46.41342	19/32	175.02866	19/32	437.89389
5176	5/8	4.291016	5/8	47.63477	5/8	177.97852	5/8	443.32227
3279	21/32	4.543365	21/32	48.87735	21/32	180.96133	21/32	448.79532
5189	11/16	4.805420	11/16	50.14136	11/16	183.97729	11/16	454.31323
1134	23/32	5.077362	23/32	51.42697	23/32	187.02658	23/32	459.87619
7344	3/4	5.359375	3/4	52.73438	3/4	190.10938	3/4	465.48438
5046	25/32	5.651642	25/32	54.06375	25/32	193.22586	25/32	471.13797
0471	13/16	5.954346	13/16	55.41528	13/16	196.37622	13/16	476.83716
9847	27/32	6.267670	27/32	56.78915	27/32	199.56064	27/32	482.58212
7402	7/8	6.591797	7/8	58.18555	7/8	202.77930	7/8	488.37305
9367	29/32	6.926910	29/32	59.60464	29/32	206.03238	29/32	494.21011
0968	15/16	7.273193	15/16	61.04663	15/16	209.32007	15/16	500.09351
5426	31/32	7.630829	31/32	62.51169	31/32	212.64255	31/32	506.02341
0000	2	8.000000	4	64.00000	6	216.00000	8	512.00000
9888	1/32	8.380890	1/32	65.51175	1/32	219.39261	1/32	518.02347
9329	1/16	8.773682	1/16	67.04712	1/16	222.82056	1/16	524.09399
5551	3/32	9.178558	3/32	68.60629	3/32	226.28403	3/32	530.21176
9785	1/8	9.595703	1/8	70.18945	1/8	229.78320	1/8	536.37695
9259	5/32	10.025299	5/32	71.79678	5/32	233.31827	5/32	542.58975
8201	3/16	10.467529	3/16	73.42847	3/16	236.88940	3/16	548.85034
9840	7/32	10.922577	7/32	75.08469	7/32	240.49680	7/32	555.15891
9406	1/4	11.390625	1/4	76.76563	1/4	244.14063	1/4	561.51563
9128	9/32	11.871857	9/32	78.47147	9/32	247.82108	9/32	567.92068
9223	5/16	12.366455	5/16	80.20239	5/16	251.53833	5/16	574.37427
9951	11/32	12.874603	11/32	81.95859	11/32	255.29257	11/32	580.87656
9512	3/8	13.396484	3/8	83.74023	3/8	259.08398	3/8	587.42773
9143	13/32	13.932281	13/32	85.54752	13/32	262.91275	13/32	594.02798
9074	7/16	14.482178	7/16	87.38062	7/16	266.77905	7/16	600.67749
9533	15/32	15.046356	15/32	89.23972	15/32	270.68307	15/32	607.37643
9750	1/2	15.625000	1/2	91.12500	1/2	274.62500	1/2	614.12500
9953	17/32	16.218292	17/32	93.03665	17/32	278.60501	17/32	620.92337
9372	9/16	16.826416	9/16	94.97485	9/16	282.62329	9/16	627.77173
9234	19/32	17.449554	19/32	96.93979	19/32	286.68002	19/32	634.67026
9770	5/8	18.087891	5/8	98.93164	5/8	290.77539	5/8	641.61914
9207	21/32	18.741608	21/32	100.95059	21/32	294.90958	21/32	648.61856
9775	11/16	19.410889	11/16	102.99683	11/16	299.08276	11/16	655.66870
9703	23/32	20.095917	23/32	105.07053	23/32	303.29514	23/32	662.76974
9219	3/4	20.796875	3/4	107.17188	3/4	307.54688	3/4	669.92188
9552	25/32	21.513947	25/32	109.30106	25/32	311.83817	25/32	677.12527
9932	13/16	22.247314	13/16	111.45825	13/16	316.16919	13/16	684.38013
9587	27/32	22.997162	27/32	113.64365	27/32	320.54013	27/32	691.68661
9746	7/8	23.763672	7/8	115.85742	7/8	324.95117	7/8	699.04492
9838	29/32	24.547028	29/32	118.09976	29/32	329.40250	29/32	706.45523
9492	15/16	25.347412	15/16	120.37085	15/16	333.89429	15/16	713.91772
9536	31/32	26.165009	31/32	122.67087	31/32	338.42673	31/32	721.43259

47 = .0000038147.

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31a.—Cubes of Inches, 9" to 29", Advancing by 16ths.

No.	Cube.	No.	Cube.	No.	Cube.	No.	Cube.	No.	Cube.
9	729.0000	**13**	2197.0000	**17**	4913.0000	**21**	9261.000	**25**	15625.000
1/16	744.2932	1/16	2228.8401	1/16	4967.3870	1/16	9343.934	1/16	15742.481
1/8	759.7988	1/8	2260.9863	1/8	5022.1738	1/8	9427.361	1/8	15860.549
3/16	775.5183	3/16	2293.4402	3/16	5077.3621	3/16	9511.284	3/16	15979.206
1/4	971.4531	1/4	2326.2031	1/4	5132.9531	1/4	9595.703	1/4	16098.453
5/16	807.6047	5/16	2359.2766	5/16	5188.9485	5/16	9680.620	5/16	16218.292
3/8	823.9746	3/8	2392.6621	3/8	5245.3496	3/8	9766.037	3/8	16338.725
7/16	840.5642	7/16	2426.3611	7/16	5302.1580	7/16	9851.955	7/16	16459.752
1/2	857.3750	1/2	2460.3750	1/2	5359.3750	1/2	9938.375	1/2	16581.375
9/16	874.4084	9/16	2494.7053	9/16	5417.0022	9/16	10025.299	9/16	16703.596
5/8	891.6660	5/8	2529.3535	5/8	5475.0410	5/8	10112.729	5/8	16826.416
11/16	909.1491	11/16	2564.3210	11/16	5533.4929	11/16	10200.665	11/16	16949.837
3/4	926.8594	3/4	2599.6094	3/4	5592.3594	3/4	10289.109	3/4	17073.859
13/16	944.7981	13/16	2635.2200	13/16	5651.6418	13/16	10378.064	13/16	17198.486
7/8	962.9668	7/8	2671.1543	7/8	5711.3418	7/8	10467.529	7/8	17323.717
15/16	981.3669	15/16	2707.4138	15/16	5771.4607	15/16	10557.508	15/16	17449.554
10	1000.0000	**14**	2744.0000	**18**	5832.0000	**22**	10648.000	**26**	17576.000
1/16	1018.8674	1/16	2780.9143	1/16	5892.9612	1/16	10739.008	1/16	17703.055
1/8	1037.9707	1/8	2818.1582	1/8	5954.3457	1/8	10830.533	1/8	17830.721
3/16	1057.3113	3/16	2855.7332	3/16	6016.1550	3/16	10922.577	3/16	17958.999
1/4	1076.8906	1/4	2893.6406	1/4	6078.3906	1/4	11015.141	1/4	18087.891
5/16	1096.7102	5/16	2931.8821	5/16	6141.0540	5/16	11108.226	5/16	18217.398
3/8	1116.7715	3/8	2970.4590	3/8	6204.1465	3/8	11201.834	3/8	18347.521
7/16	1137.0759	7/16	3009.3728	7/16	6267.6697	7/16	11295.967	7/16	18478.263
1/2	1157.6250	1/2	3048.6250	1/2	6331.6250	1/2	11390.625	1/2	18609.625
9/16	1178.4202	9/16	3088.2170	9/16	6396.0139	9/16	11485.811	9/16	18741.608
5/8	1199.4629	5/8	3128.1504	5/8	6460.8379	5/8	11581.525	5/8	18874.213
11/16	1220.7546	11/16	3168.4265	11/16	6526.0984	11/16	11677.770	11/16	19007.441
3/4	1242.2969	3/4	3209.0469	3/4	6591.7969	3/4	11774.547	3/4	19141.297
13/16	1264.0911	13/16	3250.0129	13/16	6657.9348	13/16	11871.857	13/16	19275.779
7/8	1286.1387	7/8	3291.3262	7/8	6724.5137	7/8	11969.701	7/8	19410.889
15/16	1308.4412	15/16	3332.9880	15/16	6791.5349	15/16	12068.082	15/16	19546.629
11	1331.0000	**15**	3375.0000	**19**	6859.0000	**23**	12167.000	**27**	19683.000
1/16	1353.8167	1/16	3417.3635	1/16	6926.9104	1/16	12266.457	1/16	19820.004
1/8	1376.8926	1/8	3460.0801	1/8	6995.2676	1/8	12366.455	1/8	19957.643
3/16	1400.2292	3/16	3503.1511	3/16	7064.0730	3/16	12466.995	3/16	20095.917
1/4	1423.8281	1/4	3546.5781	1/4	7133.3281	1/4	12568.078	1/4	20234.828
5/16	1447.6907	5/16	3590.3625	5/16	7203.0344	5/16	12669.706	5/16	20374.375
3/8	1471.8184	3/8	3634.5059	3/8	7273.1934	3/8	12771.881	3/8	20514.563
7/16	1496.2126	7/16	3679.0095	7/16	7343.8064	7/16	12874.603	7/16	20655.400
1/2	1520.8750	1/2	3723.8750	1/2	7414.8750	1/2	12977.875	1/2	20796.875
9/16	1545.8069	9/16	3769.1038	9/16	7486.4006	9/16	13081.698	9/16	20938.994
5/8	1571.0098	5/8	3814.6973	5/8	7558.3848	5/8	13186.072	5/8	21081.760
11/16	1596.4851	11/16	3860.6570	11/16	7630.8289	11/16	13291.001	11/16	21225.172
3/4	1622.2344	3/4	3906.9844	3/4	7703.7344	3/4	13396.484	3/4	21369.234
13/16	1648.2590	13/16	3953.6809	13/16	7777.1028	13/16	13502.525	13/16	21513.947
7/8	1674.5605	7/8	4000.7480	7/8	7850.9355	7/8	13609.123	7/8	21659.311
15/16	1701.1404	15/16	4048.1873	15/16	7925.2341	15/16	13716.281	15/16	21805.328
12	1728.0000	**16**	4096.0000	**20**	8000.0000	**24**	13824.000	**28**	21952.000
1/16	1755.1409	1/16	4144.1877	1/16	8075.2346	1/16	13932.281	1/16	22099.328
1/8	1782.5645	1/8	4192 7520	1/8	8150.9395	1/8	14041.127	1/8	22247.314
3/16	1810.2722	3/16	4241.6941	3/16	8227.1160	3/16	14150.538	3/16	22395.960
1/4	1838.2656	1/4	4291.0156	1/4	8303.7656	1/4	14260.516	1/4	22545.266
5/16	1866.5461	5/16	4340.7180	5/16	8380.8899	5/16	14371.062	5/16	22695.234
3/8	1895.1152	3/8	4390.8027	3/8	8458.4902	3/8	14482.178	3/8	22845.865
7/16	1923.9744	7/16	4441.2712	7/16	8536.5681	7/16	14593.865	7/16	22997.162
1/2	1953.1250	1/2	4492.1250	1/2	8615.1250	1/2	14706.125	1/2	23149.125
9/16	1982.5686	9/16	4543.3655	9/16	8694.1624	9/16	14818.959	9/16	23301.756
5/8	2012.3066	5/8	4594.9941	5/8	8773.6816	5/8	14932.369	5/8	23455.057
11/16	2042.3406	11/16	4647.0125	11/16	8853.6843	11/16	15046.356	11/16	23609.028
3/4	2072.6719	3/4	4699.4219	3/4	8934.1719	3/4	15160.922	3/4	23763.672
13/16	2103.3020	13/16	4752.2239	13/16	9015.1458	13/16	15276.068	13/16	23918.989
7/8	2134.2324	7/8	4805.4199	7/8	9096.6074	7/8	15391.795	7/8	24074.982
15/16	2165.4646	15/16	4859.0115	15/16	9178.5583	15/16	15508.105	15/16	24231.652

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Cubes of Inches, 29″ to 69″, ADVANCING BY 8THS.

No.	Cube.	No.	Cube.	No.	Cube.	No.	Cube.
37	50653.000	45	91125.00	53	148877.00	61	226981.00
1/8	51168.111	1/8	91886.49	1/8	149932.86	1/8	228379.24
1/4	55686.703	1/4	92652.20	1/4	150993.70	1/4	229783.20
3/8	56208.787	3/8	93422.16	3/8	152059.54	3/8	231192.91
1/2	52734.375	1/2	94196.38	1/2	153130.38	1/2	232608.38
5/8	53263.479	5/8	94974.85	5/8	154206.23	5/8	234029.60
3/4	53796.109	3/4	95757.61	3/4	155287.11	3/4	235456.61
7/8	54332.279	7/8	96544.65	7/8	156373.03	7/8	236889.40
38	54872.000	46	97336.00	54	157464.00	62	238328.00
1/8	55415.283	1/8	98131.66	1/8	158560.03	1/8	239772.41
1/4	55962.141	1/4	98931.64	1/4	159661.14	1/4	241222.64
3/8	56512.584	3/8	99735.96	3/8	160767.33	3/8	242678.71
1/2	57066.625	1/2	100544.63	1/2	161878.63	1/2	244140.63
5/8	57624.275	5/8	101357.65	5/8	162995.03	5/8	245608.40
3/4	58185.547	3/4	102175.05	3/4	164116.55	3/4	247082.05
7/8	58750.451	7/8	102996.83	7/8	165243.20	7/8	248561.58
39	59319.000	47	103823.00	55	166375.00	63	250047.00
1/8	59891.205	1/8	104653.58	1/8	167511.96	1/8	251538.33
1/4	60467.078	1/4	105488.58	1/4	168654.08	1/4	253035.58
3/8	61046.631	3/8	106328.01	3/8	169801.38	3/8	254538.76
1/2	61629.875	1/2	107171.88	1/2	170953.88	1/2	256047.88
5/8	62216.822	5/8	108020.20	5/8	172111.57	5/8	257562.95
3/4	62807.484	3/4	108872.98	3/4	173274.48	3/4	259083.98
7/8	63401.873	7/8	109730.25	7/8	174442.62	7/8	260611.00
40	64000.000	48	110592.00	56	175616.00	64	262144.00
1/8	64601.877	1/8	111458.25	1/8	176794.63	1/8	263683.00
1/4	65207.516	1/4	112329.02	1/4	177978.52	1/4	265228.02
3/8	65816.928	3/8	113204.30	3/8	179167.68	3/8	266779.05
1/2	66430.125	1/2	114084.13	1/2	180362.13	1/2	268336.13
5/8	67047.119	5/8	114968.49	5/8	181561.87	5/8	269899.24
3/4	67667.922	3/4	115857.42	3/4	182766.92	3/4	271468.42
7/8	68292.545	7/8	116750.92	7/8	183977.29	7/8	273043.67
41	68921.000	49	117649.00	57	185193.00	65	274625.00
1/8	69553.299	1/8	118551.67	1/8	186414.05	1/8	276212.42
1/4	70189.453	1/4	119458.95	1/4	187640.45	1/4	277805.95
3/8	70829.475	3/8	120370.85	3/8	188872.22	3/8	279405.60
1/2	71473.375	1/2	121287.38	1/2	190109.38	1/2	281011.38
5/8	72121.166	5/8	122208.54	5/8	191351.92	5/8	282623.29
3/4	72772.859	3/4	123134.36	3/4	192599.86	3/4	284241.36
7/8	73428.467	7/8	124064.84	7/8	193853.22	7/8	285865.59
42	74088.000	50	125000.00	58	195112.00	66	287496.00
1/8	74751.471	1/8	125939.85	1/8	196376.22	1/8	289132.60
1/4	75418.891	1/4	126884.39	1/4	197645.89	1/4	290775.39
3/8	76090.271	3/8	127833.65	3/8	198921.02	3/8	292424.40
1/2	76765.625	1/2	128787.63	1/2	200201.63	1/2	294079.63
5/8	77444.963	5/8	129746.34	5/8	201487.71	5/8	295741.09
3/4	78128.297	3/4	130709.80	3/4	202779.30	3/4	297408.80
7/8	78815.639	7/8	131678.01	7/8	204076.39	7/8	299082.76
43	79507.000	51	132651.00	59	205379.00	67	300763.00
1/8	80202.393	1/8	133628.77	1/8	206687.14	1/8	302449.52
1/4	80901.828	1/4	134611.33	1/4	208000.83	1/4	304142.33
3/8	81605.318	3/8	135598.69	3/8	209320.07	3/8	305841.44
1/2	82312.875	1/2	136590.88	1/2	210644.88	1/2	307546.88
5/8	83024.510	5/8	137587.88	5/8	211975.26	5/8	309258.63
3/4	83740.234	3/4	138589.73	3/4	213311.23	3/4	310976.73
7/8	84460.061	7/8	139596.44	7/8	214652.81	7/8	312701.19
44	85184.000	52	140608.00	60	216000.00	68	314432.00
1/8	85912.064	1/8	141624.44	1/8	217352.81	1/8	316169.19
1/4	86644.266	1/4	142645.77	1/4	218711.27	1/4	317912.77
3/8	87380.615	3/8	143671.99	3/8	220075.37	3/8	319662.74
1/2	88121.125	1/2	144703.13	1/2	221445.13	1/2	321419.13
5/8	88865.807	5/8	145739.18	5/8	222820.56	5/8	323181.93
3/4	89614.672	3/4	146780.17	3/4	224201.67	3/4	324951.17
7/8	90367.732	7/8	147826.11	7/8	225588.48	7/8	326726.86

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31b.—Cubes of Inches, 69" to 109", Advancing by 8ths.—Concluded.

No.	Cube.	No.	Cube.	No.	Cube.	No.	Cube.	No.	Cube.
69	328509.0	77	456533.0	85	614125.0	93	804357.0	101	1030301.0
1/8	330297.6	1/8	458760.0	1/8	616838.4	1/8	807604.7	1/8	1034131.1
1/4	332092.7	1/4	460994.2	1/4	619559.7	1/4	810861.2	1/4	1037970.7
3/8	333894.3	3/8	463235.7	3/8	622289.0	3/8	814126.4	3/8	1041819.8
1/2	335702.4	1/2	465484.4	1/2	625026.4	1/2	817400.4	1/2	1045678.4
5/8	337567.0	5/8	467740.4	5/8	627771.7	5/8	820683.1	5/8	1049546.5
3/4	339338.1	3/4	470003.6	3/4	630525.1	3/4	823974.6	3/4	1053424.1
7/8	341165.8	7/8	472274.2	7/8	633286.5	7/8	827274.9	7/8	1057311.3
70	343000.0	78	474552.0	86	636056.0	94	830584.0	102	1061208.0
1/8	344840.8	1/8	476837.2	1/8	638833.5	1/8	833901.9	1/8	1065114.3
1/4	346688.1	1/4	479129.6	1/4	641619.1	1/4	837228.6	1/4	1069030.1
3/8	348542.1	3/8	481429.5	3/8	644412.8	3/8	840564.2	3/8	1072955.6
1/2	350402.6	1/2	483736.6	1/2	647214.6	1/2	843908.6	1/2	1076890.6
5/8	352269.8	5/8	486051.2	5/8	650024.5	5/8	847261.9	5/8	1080835.3
3/4	354143.5	3/4	488373.0	3/4	652842.5	3/4	850624.0	3/4	1084789.5
7/8	356024.0	7/8	490702.3	7/8	655668.7	7/8	853995.1	7/8	1088753.5
71	357911.0	79	493039.0	87	658503.0	95	857375.0	103	1092727.0
1/8	359804.7	1/8	495383.1	1/8	661345.5	1/8	860763.8	1/8	1096710.2
1/4	361705.1	1/4	497734.6	1/4	664196.1	1/4	864161.6	1/4	1100703.1
3/8	363612.1	3/8	500093.5	3/8	667054.9	3/8	867568.3	3/8	1104705.6
1/2	365525.9	1/2	502459.9	1/2	669921.9	1/2	870983.9	1/2	1108717.9
5/8	367446.3	5/8	504833.7	5/8	672797.1	5/8	874408.5	5/8	1112739.8
3/4	369373.5	3/4	507215.0	3/4	675680.5	3/4	877842.0	3/4	1116771.5
7/8	371307.4	7/8	509603.8	7/8	678572.1	7/8	881284.5	7/8	1120812.9
72	373248.0	80	512000.0	88	681472.0	96	884736.0	104	1124864.0
1/8	375195.4	1/8	514403.8	1/8	684380.1	1/8	888196.5	1/8	1128924.9
1/4	377149.5	1/4	516815.0	1/4	687296.5	1/4	891666.0	1/4	1132995.5
3/8	379110.4	3/8	519233.8	3/8	690221.2	3/8	895144.6	3/8	1137075.9
1/2	381078.1	1/2	521660.1	1/2	693154.1	1/2	898632.1	1/2	1141166.1
5/8	383052.6	5/8	524094.0	5/8	696095.4	5/8	902128.8	5/8	1145266.1
3/4	385033.9	3/4	526535.4	3/4	699044.9	3/4	905634.4	3/4	1149375.9
7/8	387022.1	7/8	528984.4	7/8	702002.8	7/8	909149.2	7/8	1153495.5
73	389017.0	81	531441.0	89	704969.0	97	912673.0	105	1157625.0
1/8	391018.8	1/8	533905.2	1/8	707943.6	1/8	916205.9	1/8	1161764.3
1/4	393027.5	1/4	536377.0	1/4	710926.5	1/4	919748.0	1/4	1165913.5
3/8	395043.0	3/8	538856.4	3/8	713917.7	3/8	923299.1	3/8	1170072.5
1/2	397065.4	1/2	541343.4	1/2	716917.4	1/2	926859.4	1/2	1174241.4
5/8	399094.7	5/8	543838.0	5/8	719925.4	5/8	930428.8	5/8	1178420.2
3/4	401130.9	3/4	546340.4	3/4	722941.9	3/4	934007.4	3/4	1182608.9
7/8	403174.0	7/8	548850.3	7/8	725966.7	7/8	937595.1	7/8	1186807.5
74	405224.0	82	551368.0	90	729000.0	98	941192.0	106	1191016.0
1/8	407281.0	1/8	553893.4	1/8	732041.7	1/8	944798.1	1/8	1195234.5
1/4	409344.9	1/4	556426.4	1/4	735091.9	1/4	948413.4	1/4	1199462.9
3/8	411415.8	3/8	558967.2	3/8	738150.5	3/8	952037.9	3/8	1203701.3
1/2	413493.6	1/2	561515.6	1/2	741217.6	1/2	955671.6	1/2	1207949.6
5/8	415578.5	5/8	564071.8	5/8	744293.2	5/8	959314.6	5/8	1212208.0
3/4	417670.3	3/4	566635.8	3/4	747377.3	3/4	962966.8	3/4	1216476.3
7/8	419769.1	7/8	569207.5	7/8	750469.9	7/8	966628.3	7/8	1220754.6
75	421875.0	83	571787.0	91	753571.0	99	970299.0	107	1225043.0
1/8	423987.9	1/8	574374.3	1/8	756680.6	1/8	973979.0	1/8	1229341.4
1/4	426107.8	1/4	576969.3	1/4	759798.8	1/4	977668.3	1/4	1233649.5
3/8	428234.8	3/8	579572.2	3/8	762925.6	3/8	981367.0	3/8	1237968.3
1/2	430368.9	1/2	582182.9	1/2	766060.9	1/2	985074.9	1/2	1242296.9
5/8	432510.0	5/8	584801.4	5/8	769204.8	5/8	988792.1	5/8	1246635.5
3/4	434658.2	3/4	587427.7	3/4	772357.2	3/4	992518.7	3/4	1250984.2
7/8	436813.6	7/8	590061.9	7/8	775518.3	7/8	996254.7	7/8	1255343.1
76	438976.0	84	592704.0	92	778688.0	100	1000000.0	108	1259712.0
1/8	441145.6	1/8	595353.9	1/8	781866.3	1/8	1003754.7	1/8	1264091.1
1/4	443322.3	1/4	598011.8	1/4	785053.3	1/4	1007518.8	1/4	1268480.3
3/8	445506.1	3/8	600677.5	3/8	788248.9	3/8	1011292.2	3/8	1272879.6
1/2	447697.1	1/2	603351.1	1/2	791453.1	1/2	1015075.1	1/2	1277289.1
5/8	449895.3	5/8	606032.7	5/8	794666.1	5/8	1018867.4	5/8	1281708.8
3/4	452100.7	3/4	608722.2	3/4	797887.7	3/4	1022669.2	3/4	1286138.7
7/8	454313.2	7/8	611419.6	7/8	801118.0	7/8	1026480.4	7/8	1290578.7

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32.—Squares of Inches, 0″ to 6″ (0′ 0″ to 0′ 6″), Advancing by 64ths.

In.	0	1	2	3	4	5	In.
.....	.....	1.0000000	4.0000000	9.000000	16.000000	25.000000	.....
1/64	.00624414	1.0314941	4.0627441	9.093994	16.125244	25.156494	1/64
1/32	.00097656	1.0634766	4.1259766	9.188477	16.250977	25.313477	1/32
3/64	.00219727	1.0959473	4.1896973	9.283447	16.377197	25.470947	3/64
1/16	.00390625	1.1289063	4.2539063	9.378906	16.503906	25.628906	1/16
5/64	.00610352	1.1623535	4.3186035	9.474854	16.631104	25.787354	5/64
3/32	.00878906	1.1962891	4.3837891	9.571289	16.758789	25.946289	3/32
7/64	.01196289	1.2307129	4.4494629	9.668213	16.886963	26.105713	7/64
1/8	.01562500	1.2656250	4.5156250	9.765625	17.015625	26.265625	1/8
9/64	.01977539	1.3010254	4.5822754	9.863552	17.144775	26.426025	9/64
5/32	.02441406	1.3369141	4.6494141	9.961914	17.274414	26.586914	5/32
11/64	.02954102	1.3732910	4.7170410	10.060791	17.404541	26.748291	11/64
3/16	.03515625	1.4101563	4.7851563	10.160156	17.535156	26.910156	3/16
13/64	.04125977	1.4475098	4.8537598	10.260010	17.666260	27.072510	13/64
7/32	.04785156	1.4853516	4.9228516	10.360352	17.797852	27.235352	7/32
15/64	.05493164	1.5236816	4.9924316	10.461182	17.929932	27.398682	15/64
1/4	.06250000	1.5625000	5.0625000	10.562500	18.062500	27.562500	1/4
17/64	.07055664	1.6018066	5.1330566	10.664307	18.195557	27.726807	17/64
9/32	.07910156	1.6416016	5.2041016	10.766602	18.329102	27.891602	9/32
19/64	.08813477	1.6818848	5.2756348	10.869385	18.463135	28.056885	19/64
5/16	.09765625	1.7226563	5.3476563	10.972656	18.597656	28.222656	5/16
21/64	.10766602	1.7639160	5.4201660	11.076416	18.732666	28.388916	21/64
11/32	.11816406	1.8056641	5.4931641	11.180664	18.868164	28.555664	11/32
23/64	.12915039	1.8479004	5.5666504	11.285400	19.004150	28.722900	23/64
3/8	.14062500	1.8906250	5.6406250	11.390625	19.140625	28.890625	3/8
25/64	.15258789	1.9338379	5.7150879	11.496338	19.277588	29.058838	25/64
13/32	.16503906	1.9775391	5.7900391	11.602539	19.415039	29.227539	13/32
27/64	.17797852	2.0217285	5.8654785	11.709229	19.552979	29.396729	27/64
7/16	.19140625	2.0664063	5.9414063	11.816406	19.691406	29.566406	7/16
29/64	.20532227	2.1115723	6.0178223	11.924072	19.830322	29.736572	29/64
15/32	.21972656	2.1572266	6.0947266	12.032227	19.969727	29.907227	15/32
31/64	.23461914	2.2033691	6.1721191	12.140869	20.109619	30.078369	31/64
1/2	.25000000	2.2500000	6.2500000	12.250000	20.250000	30.250000	1/2
33/64	.26586914	2.2971191	6.3283691	12.359619	20.390869	30.422119	33/64
17/32	.28222656	2.3447266	6.4072266	12.469727	20.532227	30.594727	17/32
35/64	.29907227	2.3928223	6.4865723	12.580322	20.674072	30.767822	35/64
9/16	.31640625	2.4414063	6.5664063	12.691406	20.816406	30.941406	9/16
37/64	.33422852	2.4904785	6.6467285	12.802979	20.959229	31.115479	37/64
19/32	.35253906	2.5400391	6.7275391	12.915039	21.102539	31.290039	19/32
39/64	.37133789	2.5900879	6.8088379	13.027588	21.246338	31.465088	39/64
5/8	.39062500	2.6406250	6.8906250	13.140625	21.390625	31.640625	5/8
41/64	.41040039	2.6916504	6.9729004	13.254150	21.535400	31.816650	41/64
21/32	.43066406	2.7431641	7.0556641	13.368164	21.680664	31.993164	21/32
43/64	.45141602	2.7951660	7.1389160	13.482666	21.826416	32.170166	43/64
11/16	.47265625	2.8476563	7.2226563	13.597656	21.972656	32.347656	11/16
45/64	.49438477	2.9006348	7.3068848	13.713135	22.119385	32.525635	45/64
23/32	.51660156	2.9541016	7.3916016	13.829102	22.266602	32.704102	23/32
47/64	.53930664	3.0080566	7.4768066	13.945557	22.414307	32 883057	47/64
3/4	.56250000	3.0625000	7.5625000	14.062500	22.562500	33.062500	3/4
49/64	.58618164	3.1174316	7.6486816	14.179932	22.711182	33.242432	49/64
25/32	.61035156	3.1728516	7.7353516	14.297852	22.860352	33.422852	25/32
51/64	.63500977	3.2287598	7.8225098	14.416260	23.010010	33.603760	51/64
13/16	.66015625	3.2851563	7.9101563	14.535156	23.160156	33.785156	13/16
53/64	.68579102	3.3420410	7.9982910	14.654541	23.310791	33.967041	53/64
27/32	.71191406	3.3994141	8.0869141	14.774414	23.461914	34.149414	27/32
55/64	.73852539	3.4572754	8.1760254	14.894775	23.613525	34.332275	55/64
7/8	.76562500	3.5156250	8.2656250	15.015625	23.765625	34.515625	7/8
57/64	.79321289	3.5744629	8.3557129	15.136963	23.918213	34.699463	57/64
29/32	.82128906	3.6337891	8.4462891	15.258789	24.071289	34.883789	29/32
59/64	.84985352	3.6936035	8.5373535	15.381104	24.224854	35.068604	59/64
15/16	.87890625	3.7539063	8.6289063	15.503906	24.378906	35.253906	15/16
61/64	.90844727	3.8146973	8.7209473	15.627197	24.533447	35.439697	61/64
31/32	.93847656	3.8759766	8.8134766	15.750977	24.688477	35.625977	31/32
63/64	.96899414	3.9377441	8.9064941	15.875244	24.843994	35.812744	63/64

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32.—Squares of Inches, 6" to 12" (0' 6" to 1' 0"), Advancing by 64ths.—Concluded.

In.	6	7	8	9	10	11	In.
.....	36.000000	49.000000	64.000000	81.000000	100.00000	121.00000	.....
1/64	36.187744	49.218994	64.250244	81.281494	100.31274	121.34399	1/64
1/32	36.375977	49.438477	64.500977	81.563477	100.62598	121.68848	1/32
3/64	36.564697	49.658447	64.752197	81.845947	100.93970	122.03345	3/64
1/16	36.753906	49.878906	65.003906	82.128906	101.25391	122.37891	1/16
5/64	36.943604	50.099854	65.256104	82.412354	101.56860	122.72485	5/64
3/32	37.133789	50.321289	65.508789	82.696289	101.88379	123.07129	3/32
7/64	37.324463	50.543213	65.761963	82.980713	102.19946	123.41821	7/64
1/8	37.515625	50.765625	66.015625	83.265625	102.51563	123.76563	1/8
9/64	37.707275	50.988525	66.269775	83.551025	102.83228	124.11353	9/64
5/32	37.899414	51.211914	66.524414	83.836914	103.14941	124.46191	5/32
11/64	38.092041	51.435791	66.779541	84.123291	103.46704	124.81079	11/64
3/16	38.285156	51.660156	67.035156	84.410156	103.78516	125.16016	3/16
13/64	38.478760	51.885010	67.291260	84.697510	104.10376	125.51001	13/64
7/32	38.672852	52.110352	67.547852	84.985352	104.42285	125.86035	7/32
15/64	38.867432	52.336182	67.804932	85.273682	104.74243	126.21118	15/64
1/4	39.062500	52.562500	68.062500	85.562500	105.06250	126.56250	1/4
17/64	39.258057	52.789307	68.320557	85.851807	105.38306	126.91431	17/64
9/32	39.454102	53.016602	68.579102	86.141602	105.70410	127.26660	9/32
19/64	39.650635	53.244385	68.838135	86.431885	106.02563	127.61938	19/64
5/16	39.847656	53.472656	69.097656	86.722656	106.34766	127.97266	5/16
21/64	40.045166	53.701416	69.357666	87.013916	106.67017	128.32642	21/64
11/32	40.243164	53.930664	69.618164	87.305664	106.99316	128.68066	11/32
23/64	40.441650	54.160400	69.879150	87.597900	107.31665	129.03540	23/64
3/8	40.640625	54.390625	70.140625	87.890625	107.64063	129.39063	3/8
25/64	40.840088	54.621338	70.402588	88.183838	107.96509	129.74634	25/64
13/32	41.040039	54.852539	70.665039	88.477539	108.29004	130.10254	13/32
27/64	41.240479	55.084229	70.927979	88.771729	108.61548	130.45923	27/64
7/16	41.441406	55.316406	71.191406	89.066406	108.94141	130.81641	7/16
29/64	41.642822	55.549072	71.455322	89.361572	109.26782	131.17407	29/64
15/64	41.844727	55.782227	71.719727	89.657227	109.59473	131.53223	15/32
31/64	42.047119	56.015869	71.984619	89.953369	109.92212	131.89087	31/64
1/2	42.250000	56.250000	72.250000	90.250000	110.25000	132.25000	1/2
33/64	42.453369	56.484619	72.515869	90.547119	110.57837	132.60962	33/64
17/32	42.657227	56.719727	72.782227	90.844727	110.90723	132.96973	17/32
35/64	42.861572	56.955322	73.049072	91.142822	111.23657	133.33032	35/64
9/16	43.066406	57.191406	73.316406	91.441406	111.56641	133.69141	9/16
37/64	43.271729	57.427979	73.584229	91.740479	111.89673	134.05298	37/64
19/32	43.477539	57.665039	73.852539	92.040039	112.22754	134.41504	19/32
39/64	43.683838	57.902588	74.121338	92.340088	112.55884	134.77759	39/64
5/8	43.890625	58.140625	74.390625	92.640625	112.89063	135.14063	5/8
41/64	44.097900	58.379150	74.660400	92.941650	113.22290	135.50415	41/64
21/32	44.305664	58.618164	74.930664	93.243164	113.55566	135.86816	21/32
43/64	44.513916	58.857666	75.201416	93.545166	113.88892	136.23267	43/64
11/16	44.722656	59.097656	75.472656	93.847656	114.22266	136.59766	11/16
45/64	44.931885	59.338135	75.744385	94.150635	114.55688	136.96313	45/64
23/32	45.141602	59.579102	76.016602	94.454102	114.89160	137.32910	23/32
47/64	45.351807	59.820557	76.289307	94.758057	115.22681	137.69556	47/64
3/4	45.562500	60.062500	76.562500	95.062500	115.56250	138.06250	3/4
49/64	45.773682	60.304932	76.836182	95.367432	115.89868	138.42993	49/64
25/32	45.985352	60.547852	77.110352	95.672852	116.23535	138.79785	25/32
51/64	46.197510	60.791260	77.385010	95.978760	116.57251	139.16626	51/64
13/16	46.410156	61.053156	77.660156	96.285156	116.91016	139.53516	13/16
53/64	46.623291	61.279541	77.935791	96.592041	117.24829	139.90454	53/64
27/32	46.836914	61.524414	78.211914	96.899414	117.58691	140.27441	27/32
55/64	47.051025	61.769775	78.488525	97.207275	117.92603	140.64478	55/64
7/8	47.265625	62.015625	78.765625	97.515625	118.26563	141.01563	7/8
57/64	47.480713	62.261963	79.043213	97.824463	118.60571	141.38696	57/64
29/32	47.696289	62.508789	79.321289	98.133789	118.94629	141.75879	29/32
59/64	47.912354	62.756104	79.599854	98.443604	119.28735	142.13110	59/64
15/16	48.128906	63.003906	79.878906	98.753906	119.62891	142.50391	15/16
61/64	48.345947	63.252197	80.158447	99.064697	119.97095	142.87720	61/64
31/32	48.563477	63.500977	80.438477	99.375977	120.31348	143.25098	31/32
63/64	48.781494	63.750244	80.718994	99.687744	120.65649	143.62524	63/64

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32a.—Squares of Inches, 12″ to 30″ (1′ 0″ to 2′ 6″), Advancing by 32nds.

In.	12	14	16	18	20	22	24	26	28
.....	144.0000	196.0000	256.0000	324.0000	400.0000	484.0000	576 0000	676.0000	784.0000
1/32	44.7510	96.8760	57.0010	25.1260	01.2510	85.3760	77.5100	77.6260	85.7510
1/16	45.5039	97.7539	58.0039	26.2539	02.5039	86.7539	79.0039	79.2539	87.5039
3/32	46.2588	98.6338	59.0088	27.3838	03.7588	88.1338	80.5088	80.8838	89.2588
1/8	147.0156	199.5156	260.0156	328.5156	405.0156	489.5156	582.0156	682.5156	791.0156
5/32	47.7744	200.3994	61.0244	29.6494	06.2744	90.8994	83.5244	84.1494	92.7744
3/16	48.5352	01.2852	62.0352	30.7852	07.5352	92.2852	85.0352	85.7852	94.5352
7/32	49.2979	02.1729	63.0479	31.9229	08.7979	93.6729	86.5479	87.4229	96.2979
1/4	150.0625	203.0625	264.0625	333.0625	410.0625	495.0625	588.0625	689.0625	798.0625
9/32	50.8291	03.9541	65.0791	34.2041	11.3291	96.4541	89.5791	90.7041	99.8291
5/16	51.5977	04.8477	66 0977	35.3477	12.5977	97.8477	91.0977	92.3477	801.5977
11/32	52.3682	05.7432	67.1182	36.4932	13.8682	99.2432	92.6182	93.9932	03.3682
3/8	153.1406	206.6406	268.1406	337.6406	415.1406	500.6406	594.1406	695.6406	805.1406
13/32	53.9150	07.5400	69.1650	38.7900	16.4150	02.0400	95.6650	97.2900	06.9150
7/16	54.6914	08.4414	70.1914	39.9414	17.6914	03.4414	97.1914	98.9414	08.6914
15/32	55.4697	09.3447	71.2197	41.0947	18.9697	04.8447	98.7197	700.5947	10.4697
1/2	156.2500	210.2500	272.2500	342.2500	420.2500	506.2500	600.2500	702.2500	812.2500
17/32	57.0322	11.1572	73.2822	43.4072	21.5322	07.6572	01.7822	03.9072	14.0322
9/16	57.8164	12.0664	74.3164	44.5664	22 8164	09.0664	03.3164	05.5664	15.8164
19/32	58.6025	12.9775	75.3525	45.7275	24.1025	10.4775	04.8525	07.2275	17.6025
5/8	159.3906	213.8906	276.3906	346.8906	425.3906	511.8906	606.3906	708.8906	819.3906
21/32	60.1807	14.8057	77.4307	48.0557	26.6807	13.3057	07.9307	10.5557	21.1807
11/16	60.9727	15.7227	78.4727	49.2227	27.9727	14.7227	09.4727	12.2227	22.9727
23/32	61.7666	16.6416	79.5166	50.3916	29.2666	16.1416	11.0166	13.8916	24.7666
3/4	162.5625	217.5625	280.5625	351.5625	430.5625	517.5625	612.5625	715.5625	826.5625
25/32	63.3604	18.4854	81.6104	52.7354	31.8604	18.9854	14.1104	17.2354	28.3604
13/16	64.1602	19.4102	82.6602	53 9102	33.1602	20.4102	15.6602	18.9102	30.1602
27/32	64.9619	20.3369	83.7119	55.0869	34.4619	21.8369	17.2119	20.5869	31.9619
7/8	165.7656	221.2656	284.7656	356.2656	435.7656	523.2656	618.7656	722.2656	833.7656
29/32	66.5713	22.1963	85.8213	57.4463	37.0713	24.6963	20.3213	23.9463	35.5713
15/16	67.3789	223.1289	86.8789	58.6289	38.3789	26.1289	21.8789	25.6289	37.3789
31/32	68.1885	24.0635	87.9385	59.8135	39.6885	27.5635	23.4385	27.3135	39.1885

In.	13	15	17	19	21	23	25	27	29
.....	169.0000	225.0000	289.0000	361.0000	441.0000	529.0000	625.0000	729.0000	841.0000
1/32	69.8135	25.9385	90.0635	62.1885	42.3135	30.4385	26.5635	30.6885	42.8135
1/16	70.6289	26.8789	91.1289	63.3789	43.6289	31.8789	28.1289	32.3789	44.6289
3/32	71.4463	27.8213	92.1963	64.5713	44.9463	33.3213	29.6963	34.0713	46.4463
1/8	172.2656	228.7656	293.2656	365.7656	446.2656	534.7656	631.2656	735.7656	848.2656
5/32	73.0869	29.7119	94.3369	66.9619	47.5869	36.2119	32.8369	37.4619	50.0869
3/16	73.9102	30.6602	95.4102	68.1602	48.9102	37.6602	34.4102	39.1602	51.9102
7/32	74.7354	31.6104	96.4854	69.3604	50.2354	39.1104	35.9854	40.8604	53.7354
1/4	175.5625	232.5625	297.5625	370.5625	451.5625	540.5625	637.5625	742.5625	855 5625
9/32	76.3916	33.5166	98.6416	71.7666	52.8916	42.0166	39.1416	44.2666	57.3916
5/16	77.2227	34.4727	99.7227	72.9727	54.2227	43.4727	40.7227	45.9727	59.2227
11/32	78.0557	35.4307	300.8057	74.1807	55.5557	44.9307	42.3057	47.6807	61 0157
3/8	178.8906	236.3906	301.8906	375.3906	456.8906	546.3906	643.8906	749.3906	862.8906
13/32	79.7275	37.3525	02.9775	76.6025	58.2275	47.8525	45.4775	51.1025	64.7275
7/16	80.5664	38.3164	04.0664	77.8164	59.5664	49.3164	47.0664	52.8164	66.5664
15/32	81.4072	39.2822	05.1572	79.0322	60.9072	50.7822	48.6572	54.5322	68.4072
1/2	182.2500	240.2500	306.2500	380.2500	462.2500	552.2500	650.2500	756.2500	870.2500
17/32	83.0947	41.2197	07.3447	81.4697	63.5947	53.7197	51.8447	57.9697	72.0947
9/16	83.9414	42.1914	08.4414	82.6914	64.9414	55.1914	53.4414	59.6914	73.9414
19/32	84.7900	43.1650	09.5400	83.9150	66.2900	56.6650	55.0400	61.4150	75.7900
5/8	185.6406	244.1406	310.6406	385.1406	467.6406	558.1406	656.6406	763.1406	877.6406
21/32	86.4932	45.1182	11.7432	86.3682	68.9932	59.6182	58.2432	64.8682	79.4932
11/16	87.3477	46.0977	12.8477	87.5977	70.3477	61.0977	59.8477	66.5977	81.3477
23/32	88.2041	47.0791	13 9541	88.8291	71.7041	62.5791	61.4541	68.3291	83.2041
3/4	189.0625	248.0625	315.0625	390.0625	473.0625	564.0625	663.0625	770.0625	885.0625
25/32	89.9229	49.0479	16.1729	91.2979	74 4229	65.5479	64.6729	71.7979	86.9229
13/16	90.7852	50.0352	17.2852	92.5352	75.7852	67.0352	66.2852	73.5352	88.7852
27/32	91.6494	51.0244	18.3994	93.7744	77.1494	68.5244	67.8994	75.2744	90.6494
7/8	192.5156	252.0156	319.5156	395.0156	478.5156	570.0156	669.5156	777.0156	892.5156
29/32	93.3838	53.0088	20.6338	96.2588	79.8838	71.5088	71.1338	78.7588	94.3838
15/16	94.2539	54.0039	21.7539	97.5039	81.2539	73.0039	72.7539	80.5039	96.2539
31/32	95.1260	55.0010	22.8760	98.7510	82.6260	74.5010	74.3760	82.2510	98.1260

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32a.—Squares of Inches, 30″ to 48″ (2′ 6″ to 4′ 0″), Advancing by 32nds.—Continued.

In.	30	32	34	36	38	40	42	44	46
.....	900.000	1024.000	1156.000	1296.000	1444.000	1600.000	1764.000	1936.000	2116.000
1/32	01.876	026.001	158.126	298.251	446.376	602.501	766.626	938.751	118.876
1/16	03.754	028.004	160.254	300.504	448.754	605.004	769.254	941.504	121.754
3/32	05.634	130.009	162.384	302.759	451.134	607.509	771.884	944.259	124.634
1/8	907.516	1032.016	1164.516	1305.016	1453.516	1610.016	1774.516	1947.016	2127.516
5/32	09.399	034.024	166.649	307.274	455.899	612.524	777.149	949.774	130.399
3/16	11.285	036.035	168.785	309.535	458.285	615.035	779.785	952.535	133.285
7/32	13.173	038.048	170.923	311.798	460.673	617.548	782.423	955.298	136.173
1/4	915.063	1040.063	1173.063	1314.063	1463.063	1620.063	1785.063	1958.063	2139.063
9/32	16.954	042.079	175.204	316.329	465.454	622.579	787.704	960.829	141.954
5/16	18.848	044.098	177.348	318.598	467.848	625.098	790.348	963.598	144.848
11/32	20.743	046.118	179.493	320.868	470.243	627.618	792.993	966.368	147.743
3/8	922.641	1048.141	1181.641	1323.141	1472.641	1630.141	1795.641	1969.141	2150.641
13/32	24.540	050.165	183.790	325.415	475.040	632.665	798.290	971.915	153.540
7/16	26.441	052.191	185.941	327.691	477.441	635.191	800.941	974.691	156.441
15/32	28.345	054.220	188.095	329.970	479.845	637.720	803.595	977.470	159.345
1/2	930.250	1056.250	1190.250	1332.250	1482.250	1640.250	1806.250	1980.250	2162.250
17/32	32.157	058.282	192.407	334.532	484.657	642.782	808.907	983.032	165.157
9/16	34.066	060.316	194.566	336.816	487.066	645.316	811.566	985.816	168.066
19/32	35.978	062.353	196.728	339.103	489.478	647.853	814.228	988.603	170.978
5/8	937.891	1064.391	1198.891	1341.391	1491.891	1650.391	1816.891	1991.391	2173.891
21/32	39.806	066.431	201.056	343.681	494.306	652.931	819.556	994.181	176.806
11/16	41.723	068.473	203.223	345.973	496.723	655.473	822.223	996.973	179.723
23/32	43.642	070.517	205.392	348.267	499.142	658.017	824.892	999.767	182.641
3/4	945.563	1072.563	1207.563	1350.563	1501.563	1660.563	1827.563	2002.563	2185.563
25/32	47.485	074.610	209.735	352.860	503.985	663.110	830.235	005.360	188.485
13/16	49.410	076.660	211.910	355.160	506.410	665.660	832.910	008.160	191.410
27/32	51.337	078.712	214.087	357.462	508.837	668.212	835.587	010.962	194.337
7/8	953.266	1080.766	1216.266	1359.766	1511.266	1670.766	1838.266	2013.766	2197.266
29/32	55.196	082.821	218.446	362.071	513.696	673.321	840.946	016.571	200.196
15/16	57.129	084.879	220.629	364.379	516.129	675.879	843.629	019.379	203.129
31/32	59.063	086.938	222.813	366.688	518.563	678.438	846.313	022.188	206.063

In.	31	33	35	37	39	41	43	45	47
.....	961.000	1089.000	1225.000	1369.000	1521.000	1681.000	1849.000	2025.000	2209.000
1/32	62.938	091.063	227.188	371.313	523.438	683.563	851.688	027.813	211.938
1/16	64.879	093.129	229.379	373.629	525.879	686.129	854.379	030.629	214.879
3/32	66.821	095.196	231.571	375.946	528.321	688.696	857.071	033.446	217.821
1/8	968.766	1097.266	1233.766	1378.266	1530.766	1691.266	1859.766	2036.266	2220.766
5/32	70.712	099.337	235.962	380.587	533.212	693.837	862.462	039.087	223.712
3/16	72.660	101.410	238.160	382.910	535.660	696.410	865.160	041.910	226.660
7/32	74.610	103.485	240.360	385.235	538.110	698.985	867.860	044.735	229.610
1/4	976.563	1105.563	1242.563	1387.563	1540.563	1701.563	1870.563	2047.563	2232.563
9/32	78.517	107.642	244.767	389.892	543.017	704.142	873.267	050.392	235.517
5/16	80.473	109.723	246.972	392.223	545.473	706.723	875.973	053.223	238.473
11/32	82.431	111.806	249.181	394.556	547.931	709.306	878.681	056.056	241.431
3/8	84.391	1113.891	1251.391	1396.891	1550.391	1711.891	1881.391	2058.891	2244.391
13/32	86.353	115.978	253.603	399.228	552.853	714.478	884.103	061.728	247.353
7/16	88.316	118.066	255.816	401.566	555.316	717.066	886.816	064.566	250.316
15/32	90.282	120.157	258.032	403.907	557.782	719.657	889.532	067.407	253.282
1/2	992.250	1122.250	1260.250	1406.250	1560.250	1722.250	1892.250	2070.250	2256.250
17/32	94.220	124.345	262.470	408.595	562.720	724.845	894.970	073.095	259.220
9/16	96.191	126.441	264.691	410.941	565.191	727.441	897.691	075.941	262.191
19/32	98.165	128.540	266.915	413.290	567.665	730.040	900.415	078.790	265.165
5/8	1000.141	1130.641	1269.141	1415.641	1570.141	1732.641	1903.141	2081.641	2268.141
21/32	002.118	132.743	271.368	417.993	572.618	735.243	905.868	084.493	271.118
11/16	004.098	134.848	273.598	420.348	575.098	737.848	908.598	087.348	274.098
23/32	006.079	136.954	275.829	422.704	577.579	740.454	911.329	090.204	277.079
3/4	1008.063	1139.063	1278.063	1425.063	1580.063	1743.063	1914.063	2093.063	2280.063
25/32	010.048	141.173	280.298	427.423	582.548	745.673	916.798	095.923	283.048
13/16	012.035	143.285	282.535	429.785	585.035	748.285	919.535	098.785	286.035
27/32	014.024	145.399	284.774	432.149	587.524	750.899	922.274	101.649	289.024
7/8	1016.016	1147.516	1287.016	1434.516	1590.016	1753.516	1925.016	2104.516	2292.016
29/32	018.009	149.634	289.259	436.884	592.509	756.134	927.759	107.384	295.009
15/16	020.004	151.754	291.504	439.254	595.004	758.754	930.504	110.254	298.004
31/32	022.001	153.876	293.751	441.626	597.501	761.376	933.251	113.126	301.001

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32a.—Squares of Inches, 48″ to 66″ (4′ 0″ to 5′ 6″), Advancing by 32nds.—Continued.

In.	48	50	52	54	56	58	60	62	64
.....	2304.000	2500.000	2704.000	2916.000	3136.000	3364.000	3600.000	3844.000	4096.000
1/32	307.001	503.126	707.251	919.376	139.501	367.626	603.751	847.876	100.001
1/16	310.004	506.254	710.504	922.754	143.004	371.254	607.504	851.754	104.004
3/32	313.009	509.384	713.759	926.134	146.509	374.884	611.259	855.634	108.009
1/8	2316.016	2512.516	2717.016	2929.516	3150.016	3378.516	3615.016	3859.516	4112.016
5/32	319.024	515.649	720.274	932.899	153.524	382.149	618.774	863.399	116.024
3/16	322.035	518.785	723.535	936.285	157.035	385.785	622.535	867.285	120.035
7/32	325.048	521.923	726.798	939.673	160.548	389.423	626.298	871.173	124.048
1/4	2328.063	2525.063	2730.063	2943.063	3164.063	3393.063	3630.063	3875.063	4128.063
9/32	331.079	528.204	733.329	946.454	167.579	396.704	633.829	878.954	132.079
5/16	334.098	531.348	736.598	949.848	171.098	400.348	637.598	882.848	136.098
11/32	337.118	534.493	739.868	953.243	174.618	403.993	641.368	886.743	140.118
3/8	2340.141	2537.641	2743.141	2956.641	3178.141	3407.641	3645.141	3890.641	4144.141
13/32	343.156	540.790	746.415	960.040	181.665	411.290	648.915	894.540	148.165
7/16	346.191	543.941	749.691	963.441	185.191	414.941	652.691	898.441	152.191
15/32	349.220	547.095	752.970	966.845	188.720	418.595	656.470	902.345	156.220
1/2	2352.250	2550.250	2756.250	2970.250	3192.250	3422.250	3660.250	3906.250	4160.250
17/32	355.282	553.407	759.532	973.657	195.782	425.907	664.032	910.157	164.282
9/16	358.316	556.566	762.816	977.066	199.316	429.566	667.816	914.066	168.316
19/32	361.353	559.728	766.103	980.478	202.853	433.228	671.603	917.978	172.353
5/8	2364.391	2562.891	2769.391	2983.891	3206.391	3436.891	3675.391	3921.891	4176.391
21/32	367.431	566.056	772.681	987.306	209.931	440.556	679.181	925.806	180.431
11/16	370.473	569.223	775.973	990.723	213.473	444.223	682.973	929.723	184.473
23/32	373.517	572.392	779.267	994.142	217.017	447.892	686.767	933.642	188.517
3/4	2376.563	2575.563	2782.563	2997.563	3220.563	3451.563	3690.563	3937.563	4192.565
25/32	379.610	578.735	785.860	3000.985	224.110	455.235	694.360	941.485	196.610
13/16	382.660	581.910	789.160	004.410	227.660	458.910	698.160	945.410	200.660
27/32	385.712	585.087	792.462	007.837	231.212	462.587	701.962	949.337	204.712
7/8	2388.766	2588.266	2795.766	3011.266	3234.766	3466.266	3705.766	3953.266	4208.766
29/32	391.821	591.446	799.071	014.696	238.321	469.946	709.571	957.196	212.821
15/16	394.879	594.629	802.379	018.129	241.879	473.629	713.379	961.129	216.879
31/32	397.938	597.813	805.688	021.563	245.438	477.313	717.188	965.063	220.938

In.	49	51	53	55	57	59	61	63	65
.....	2401.000	2601.000	2809.000	3025.000	3249.000	3481.000	3721.000	3969.000	4225.000
1/32	404.063	604.188	812.313	028.438	252.563	484.688	724.813	972.938	229.063
1/16	407.129	607.379	815.629	031.879	256.129	488.379	728.629	976.879	233.129
3/32	410.196	610.571	818.946	035.321	259.696	492.071	732.446	980.821	237.196
1/8	2413.266	2613.766	2822.266	3038.766	3263.266	3495.766	3736.266	3984.766	4241.266
5/32	416.337	616.962	825.587	042.212	266.837	499.462	740.087	988.712	245.337
3/16	419.410	620.160	828.910	045.660	270.410	503.160	743.910	992.660	249.410
7/32	422.485	623.360	832.235	049.110	273.985	506.860	747.735	996.610	253.485
1/4	2425.563	2626.563	2835.563	3052.563	3277.563	3510.563	3751.563	4000.563	4257.563
9/32	428.642	629.767	838.892	056.017	281.142	514.267	755.392	004.517	261.642
5/16	431.723	632.973	842.223	059.473	284.723	517.973	759.223	008.473	265.723
11/32	434.806	636.181	845.556	062.931	288.306	521.681	763.056	012.431	269.806
3/8	2437.891	2639.391	2848.891	3066.391	3291.891	3525.391	3766.891	4016.391	4273.891
13/32	440.978	642.603	852.228	069.853	295.478	529.103	770.728	020.353	277.978
7/16	444.066	645.816	855.566	073.316	299.066	532.816	774.566	024.316	282.066
15/32	447.157	649.032	858.907	076.782	302.657	536.532	778.407	028.282	286.157
1/2	2450.250	2652.250	2862.250	3080.250	3306.250	3540.250	3782.250	4032.250	4290.250
17/32	453.345	655.470	865.595	083.720	309.845	543.970	786.095	036.220	294.345
9/16	456.441	658.691	868.941	087.191	313.441	547.691	789.941	040.191	298.441
19/32	459.540	661.915	872.290	090.665	317.040	551.415	793.790	044.165	302.540
5/8	2462.641	2665.141	2875.641	3094.141	3320.641	3555.141	3797.641	4048.141	4306.641
21/32	465.743	668.368	878.993	097.618	324.243	558.868	801.493	052.118	310.742
11/16	468.848	671.598	882.348	101.098	327.848	562.598	805.348	056.098	314.848
23/32	471.954	674.829	885.704	104.579	331.454	566.329	809.204	060.079	318.954
3/4	2475.063	2678.063	2889.063	3108.063	3335.063	3570.063	3813.063	4064.063	4323.063
25/32	478.173	681.298	892.423	111.548	338.673	573.798	816.923	068.048	327.173
13/16	481.285	684.535	895.785	115.035	342.285	577.535	820.785	072.035	331.285
27/32	484.399	687.774	899.149	118.524	345.899	581.274	824.649	076.024	335.399
7/8	2487.516	2691.016	2902.516	3122.016	3349.516	3585.016	3828.516	4080.016	4339.516
29/32	490.634	694.259	905.884	125.509	353.134	588.759	832.384	084.009	343.634
15/16	493.754	697.504	909.254	129.004	356.754	592.504	836.254	088.004	347.754
31/32	496.876	700.751	912.626	132.501	360.376	596.251	840 126	092.001	351.876

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32a.—Squares of Inches, 66″ to 84″ (5′ 6″ to 7′ 0″), Advancing by 32nds.—Continued.

In.	66	68	70	72	74	76	78	80	82
.....	4356.000	4624.000	4900.000	5184.000	5476.000	5776.000	6084.000	6400.000	6724.000
1/32	360.126	628.251	904.376	188.501	480.626	780.751	088.876	405.001	729.126
1/16	364.254	632.504	908.754	193.004	485.254	785.504	093.754	410.004	734.254
3/32	368.384	636.759	913.134	197.509	489.884	790.259	098.634	415.009	739.384
1/8	4372.516	4641.016	4917.516	5202.016	5494.516	5795.016	6103.516	6420.016	6744.516
5/32	376.649	645 274	921.899	206.524	499.149	799 774	108.399	425.024	749.649
3/16	380.785	649.535	926.285	211.035	503.785	804.535	113.285	430.035	754.785
7/32	384.923	653.798	930.673	215.548	508.423	809.298	118.173	435.048	759.923
1/4	4389.063	4658.063	4935.063	5220.063	5513.063	5814.063	6123.063	6440.063	6765.063
9/32	393.204	662.329	939.454	224.579	517.704	818.829	127.954	445.079	770.204
5/16	397.348	666.598	943.848	229.098	522.348	823.598	132.848	450.098	775.348
11/32	401.493	670.868	948.243	233.618	526.993	828.368	137.743	455.118	780.493
3/8	4405.641	4675.141	4952.641	5238.141	5531.641	5833.141	6142.641	6460.141	6785.641
13/32	409.790	679.415	957.040	242.665	536.290	837.915	147.540	465.165	790.790
7/16	413.941	683.691	961.441	247.191	540.941	842.691	152.441	470.191	795.941
15/32	418.095	687.970	965.845	251.720	545.595	847.470	157.345	475.220	801.095
1/2	4422.250	4692.250	4970.250	5256.250	5550.250	5852.250	6162.250	6480.250	6806.250
17/32	426.407	696.532	974.657	260.782	554.907	857.032	167.157	485.282	811.407
9/16	430.566	700.816	979.066	265.316	559.566	861.816	172.066	490.316	816.566
19/32	434.728	705.103	983.478	269.853	564.228	866.603	176.978	495.353	821.728
5/8	4438.891	4709.391	4987.891	5274.391	5568.891	5871.391	6181.891	6500.391	6826.891
21/32	443.056	713.681	992.306	278.931	573.556	876.181	186.806	505.431	832.056
11/16	447.223	717.973	996.723	283.473	578.223	880.973	191.723	510.473	837.223
23/32	451.392	722.267	5001.142	288.017	582.892	885.767	196.642	515.517	842.392
3/4	4455.563	4726.563	5005.563	5292.563	5587.563	5890.563	6201.563	6520.563	6847.563
25/32	459.735	730.860	009.985	297.110	592.235	895.360	206.485	525.610	852.735
13/16	463.910	735.160	014.410	301.660	596.910	900.160	211.410	530.660	857.910
27/32	468.087	739.462	018.837	306.212	601.587	904.962	216.337	535.712	863.087
7/8	4472.266	4743.766	5023.266	5310.766	5606.266	5909.766	6221.266	6540.766	6868.266
29/32	476.446	748.071	027.696	315.321	610.946	914.571	226.196	545.821	873.446
15/16	480.629	752.379	032.129	319.879	615.629	919.379	231.129	550.879	878.629
31/32	484.813	756.688	036.563	324.438	620.313	924.188	236.063	555.938	883.813

In.	67	69	71	73	75	77	79	81	83
.....	4489.000	4761.000	5041.000	5329.000	5625.000	5929.000	6241.000	6561.000	6889.000
1/32	493.188	765.313	045.438	333.563	629.688	933.813	245.938	566.063	894.188
1/16	497.379	769.629	049.879	338.129	634.379	938.629	250.879	571.129	899.379
3/32	501.571	773.946	054.321	342.696	639.071	943.446	255.821	576.196	904.571
1/8	4505.766	4778.266	5058.766	5347.266	5643.766	5948.266	6260.766	6581.266	6909.766
5/32	509.962	782.587	063.212	351.837	648.462	953.087	265.712	586.337	914.962
3/16	514.160	786.910	067.660	356.410	653.160	957.910	270.660	591.410	920.160
7/32	518.360	791.235	072.110	360.985	657.860	962.735	275.610	596.485	925.360
1/4	4522.563	4795.563	5076.563	5365.563	5662.563	5967.563	6280.563	6601.563	6930.563
9/32	526.767	799.892	081.017	370.142	667.267	972.392	285.517	606.642	935.767
5/16	530.973	804.223	085.473	374.723	671.973	977.223	290.473	611.723	940.973
11/32	535.181	808.556	089.931	379.306	676.681	982.056	295.431	616.806	946.181
3/8	4539.391	4812.891	5094.391	5383.891	5681.391	5986.891	6300.391	6621.891	6951.391
13/32	543.603	817.228	098.853	388.478	686.103	991.728	305.353	626.978	956.603
7/16	547.816	821.566	103.316	393.066	690.816	996.566	310.316	632.066	961.816
15/32	552.032	825.907	107.782	397.657	695.532	6001.407	315.282	637.157	967.032
1/2	4556.250	4830.250	5112.250	5402.250	5700.250	6006.250	6320.250	6642.250	6972.250
17/32	560.470	834.595	116.720	406.845	704.970	011.095	325.220	647.345	977.470
9/16	564.691	838.941	121.191	411.441	709.691	015.941	330.191	652.441	982.691
19/32	568.915	843.290	125.665	416.040	714.415	020.790	335.165	657.540	987.915
5/8	4573.141	4847.641	5130.141	5420.641	5719.141	6025.641	6340.141	6662.641	6993.141
21/32	577.368	851.993	134.618	425.243	723.868	030.493	345.118	667.743	998.368
11/16	581.598	856.348	139.098	429.848	728.598	035.348	350.098	672.848	7003.598
23/32	585.829	860.704	143.579	434.354	733.329	040.204	355.079	677.954	008.829
3/4	4590.063	4865.063	5148.063	5439.063	5738.063	6045.063	6360.063	6683.063	014.063
25/32	594.298	869.423	152.548	443.673	742.798	049.923	365.048	688.173	019.298
13/16	598.535	873.785	157.035	448.285	747.535	054.785	370.035	693.285	024.535
27/32	602.774	878.149	161.524	452.899	752.274	059.649	375.024	698.399	029.774
7/8	4607.016	4882.516	5166.016	5457.516	5757.016	6064.516	6380.016	6703.516	7035.016
29/32	611.259	886.884	170.509	462.134	761.759	069.384	385.009	708.634	040.259
15/16	615.504	891.254	175.004	466.754	766.504	074.254	390.004	713.754	045.504
31/32	619.751	895.626	179.501	471.376	771.251	079.126	395.001	718.876	050.751

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32a.—Squares of Inches, 84″ to 102″ (7′ 0″ to 8′ 6″), Advancing by 32nds.—Continued.

In.	84	86	88	90	92	94	96	98	100
.....	7056.000	7396.000	7744.000	8100.000	8464.000	8836.000	9216.000	9604.000	10000.00
1/32	061.251	401.376	749.501	105.626	469.751	841.876	222.001	610.126	0006.25
1/16	066.504	406.754	755.004	111.254	475.504	847.754	228.004	616.254	0012.50
3/32	071.759	412.134	760.509	116.884	481.259	853.634	234.009	622.384	0018.76
1/8	7077.016	7417.516	7766.016	8122.516	8487.016	8859.516	9240.016	9628.516	10025.02
5/32	082.274	422.899	771.524	128.149	492.774	865.399	246.024	634.649	0031.27
3/16	087.535	428.285	777.035	133.785	498.535	871.285	252.035	640.785	0037.54
7/32	092.798	433.673	782.548	139.423	504.298	877.173	258.048	646.923	0043.80
1/4	7098.063	7439.063	7788.063	8145.063	8510.063	8883.063	9264.063	9653.063	10050.06
9/32	103.329	444.454	793.579	150.704	515.829	888.954	270.079	659.204	0056.33
5/16	108.598	449.848	799.098	156.348	521.598	894.848	276.098	665.348	0062.60
11/32	113.868	455.243	804.618	161.993	527.368	900.743	282.118	671.493	0068.87
3/8	7119.141	7460.641	7810.141	8167.641	8533.141	8906.641	9288.141	9677.641	10075.14
13/32	124.415	466.040	815.665	173.290	538.915	912.540	294.165	683.790	0081.42
7/16	129.691	471.441	821.191	178.941	544.691	918.441	300.191	689.941	0087.69
15/32	134.970	476.845	826.720	184.595	550.470	924.345	306.220	696.095	0093.97
1/2	7140.250	7482.250	7832.250	8190.250	8556.250	8930.250	9312.250	9702.250	10100.25
17/32	145.532	487.657	837.782	195.907	562.032	936.157	318.282	708.407	0106.53
9/16	150.816	493.066	843.316	201.566	567.816	942.066	324.316	714.566	0112.82
19/32	156.103	498.478	848.853	207.228	573.603	947.978	330.353	720.728	0119.10
5/8	7161.391	7503.891	7854.391	8212.891	8579.391	8953.891	9336.391	9726.891	10125.39
21/32	166.681	509.306	859.931	218.556	585.181	959.806	342.431	733.056	0131.68
11/16	171.973	514.723	865.473	224.223	590.973	965.723	348.473	739.223	0137.97
23/32	177.267	520.142	871.017	229.892	596.767	971.642	354.517	745.392	0144.27
3/4	7182.563	7525.563	7876.563	8235.563	8602.563	8977.563	9360.563	9751.563	10150.56
25/32	187.860	530.985	882.110	241.235	608.360	983.485	366.610	757.735	0156.86
13/16	193.160	536.410	887.660	246.910	614.160	989.410	372.660	763.910	0163.16
27/32	198.462	541.837	893.212	252.587	619.962	995.337	378.712	770.087	0169.46
7/8	7203.766	7547.266	7898.766	8258.266	8625.766	9001.266	9384.766	9776.266	10175.77
29/32	209.071	552.696	904.321	263.946	631.571	007.196	390.821	782.446	0182.07
15/16	214.379	558.129	909.879	269.629	637.379	013.129	396.879	788.629	0188.38
31/32	219.688	563.563	915.438	275.313	643.188	019.063	402.938	794.813	0194.69

In.	85	87	89	91	93	95	97	99	101
.....	7225.000	7569.000	7921.000	8281.000	8645.000	9025.000	9409.000	9801.000	10201.00
1/32	230.313	574.438	926.563	286.688	654.813	030.938	415.063	807.188	0207.31
1/16	235.629	579.879	932.129	292.379	660.629	036.879	421.129	813.379	0213.63
3/32	240.946	585.321	937.696	298.071	666.446	042.821	427.196	819.571	0219.95
3/8	7246.266	7590.766	7943.266	8303.766	8672.266	9048.766	9433.266	9825.766	10226.27
5/32	251.587	596.212	948.837	309.462	678.087	054.712	439.337	831.962	0232.59
3/16	256.910	601.660	954.410	315.160	683.910	060.660	445.410	838.160	0238.91
7/32	262.235	607.110	959.985	320.860	689.735	066.610	451.485	844.360	0245.24
1/4	7267.563	7612.563	7965.563	8326.563	8695.563	9072.563	9457.563	9850.563	10251.56
9/32	272.892	618.017	971.142	332.267	701.392	078.517	463.642	856.767	0257.89
5/16	278.223	623.473	976.723	337.973	707.223	084.473	469.723	862.973	0264.22
11/32	283.556	628.931	982.306	343.681	713.056	090.431	475.806	869.181	0270.56
3/8	7288.891	7634.391	7987.891	8349.391	8718.891	9096.391	9481.891	9875.391	10276.89
13/32	294.228	639.853	993.478	355.103	724.728	102.353	487.978	881.603	0283.23
7/16	299.566	645.316	999.066	360.816	730.566	108.316	494.066	887.816	0289.57
15/32	304.907	650.782	8004.657	366.532	736.407	114.282	500.157	894.032	0295.91
1/2	7310.250	7656.250	8010.250	8372.250	8742.250	9120.250	9506.250	9900.250	10302.25
17/32	315.595	661.720	015.845	377.970	748.095	126.220	512.345	906.470	0308.59
9/16	320.941	667.191	021.441	383.691	753.941	132.191	518.441	912.691	0314.94
19/32	326.290	672.665	027.040	389.415	759.790	138.165	524.540	918.915	0321.29
5/8	7331.641	7678.141	8032.641	8395.141	8765.641	9144.141	9530.641	9925.141	10327.64
21/32	336.993	683.618	038.243	400.868	771.493	150.118	536.743	931.368	0333.99
11/16	342.348	689.098	043.848	406.598	777.348	156.098	542.848	937.598	0340.35
23/32	347.704	694.579	049.454	412.329	783.204	162.079	548.954	943.829	0346.70
3/4	7353.063	7700.063	8055.063	8418.063	8789.063	9168.063	9555.063	9950.063	10353.06
25/32	358.423	705.548	060.673	423.798	794.923	174.048	561.173	956.298	0359.42
13/16	363.785	711.035	066.285	429.535	800.785	180.035	567.285	962.535	0365.79
27/32	369.149	716.524	071.899	435.274	806.649	186.024	573.399	968.774	0372.15
7/8	7374.516	7722.016	8077.516	8441.016	8812.516	9192.016	9579.516	9975.016	10378.52
29/32	379.884	727.509	083.134	446.759	818.384	198.009	585.634	981.259	0384.88
15/16	385.254	733.004	088.754	452.504	824.254	204.004	591.754	987.504	0391.25
31/32	390.626	738.501	094.376	458.251	830.126	210.001	597.876	993.751	0397.63

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32a.—Squares of Inches, 102″ to 120″ (8′ 6″ to 10′ 0″), Advance by 32nds.—Concluded.

In.	102	104	106	108	110	112	114	1
.....	10404.00	10816.00	11236.00	11664.00	12100.00	12544.00	12996.00	1345
1/32	0410.38	0822.50	1242.63	1670.75	2106.88	2551.00	3003.13	346
1/16	0416.75	0829.00	1249.25	1677.50	2113.75	2558.00	3010.25	347
3/32	0423.13	0835.51	1255.88	1684.26	2120.63	2565.01	3017.38	347
1/8	10429.52	10842.02	11262.52	11691.02	12127.52	12572.02	13024.52	1348
5/32	0435.90	0848.52	1269.15	1697.77	2134.40	2579.02	3031.65	349
3/16	0442.29	0855.04	1275.79	1704.54	2141.29	2586.04	3038.79	349
7/32	0448.67	0861.55	1282.42	1711.30	2148.17	2593.05	3045.92	350
1/4	10455.06	10868.06	11289.06	11718.06	12155.06	12600.06	13053.06	1351
9/32	0461.45	0874.58	1295.70	1724.83	2161.95	2607.08	3060.20	352
5/16	0467.85	0881.10	1302.35	1731.60	2168.85	2614.10	3067.35	352
11/32	0474.24	0887.62	1308.99	1738.37	2175.74	2621.12	3074.49	353
3/8	10480.64	10894.14	11315.64	11745.14	12182.64	12628.14	13081.64	1354
13/32	0487.04	0900.67	1322.29	1751.92	2189.54	2635.17	3088.79	355
7/16	0493.44	0907.19	1328.94	1758.69	2196.44	2642.19	3095.94	355
15/32	0499.84	0913.72	1335.59	1765.47	2203.34	2649.22	3103.09	356
1/2	10506.25	10920.25	11342.25	11772.25	12210.25	12656.25	13110.25	1357
17/32	0512.66	0926.78	1348.91	1779.03	2217.16	2663.28	3117.41	357
9/16	0519.07	0933.32	1355.57	1785.82	2224.07	2670.32	3124.57	358
19/32	0525.48	0939.85	1362.23	1792.60	2230.98	2677.35	3131.73	358
5/8	10531.89	10946.39	11368.89	11799.39	12237.89	12684.39	13138.89	1360
21/32	0538.31	0952.93	1375.56	1806.18	2244.81	2691.43	3146.06	360
11/16	0544.72	0959.47	1382.22	1812.97	2251.72	2698.47	3153.22	361
23/32	0551.14	0966.02	1388.89	1819.77	2258.64	2705.52	3160.39	362
3/4	10557.56	10972.56	11395.56	11826.56	12265.56	12712.56	13167.56	1363
25/32	0563.99	0979.11	1402.24	1833.36	2272.49	2719.61	3174.74	363
13/16	0570.41	0985.66	1048.91	1840.16	2279.41	2726.66	3181.91	364
27/32	0576.84	0992.21	1415.59	1846.96	2286.34	2733.71	3189.09	365
7/8	10583.27	10998.77	11422.27	11853.77	12293.27	12740.77	13196.27	1365
29/32	0589.70	1005.32	1428.95	1860.57	2300.20	2747.82	3203.45	366
15/16	0596.13	1011.88	1435.63	1867.38	2307.13	2754.88	3210.63	367
31/32	0602.56	1018.44	1442.31	1874.19	2314.06	2761.94	3217.81	368

In.	103	105	107	109	111	113	115	1
.....	10609.00	11025.00	11449.00	11881.00	12321.00	12769.00	13225.00	1368
1/32	0615.44	1031.56	1455.69	1887.81	2327.94	2776.06	3232.19	369
1/16	0621.88	1038.13	1462.38	1894.63	2334.88	2783.13	3239.38	370
3/32	0628.32	1044.70	1469.07	1901.45	2341.82	2790.20	3246.57	371
1/8	10634.77	11051.27	11475.77	11908.27	12348.77	12797.27	13253.77	1371
5/32	0641.21	1057.84	1482.46	1915.09	2355.71	2804.34	3260.96	372
3/16	0647.66	1064.41	1489.16	1921.91	2362.66	2811.41	3268.16	373
7/32	0654.11	1070.99	1495.86	1928.74	2369.61	2818.49	3275.36	374
1/4	10660.56	11077.56	11502.56	11935.56	12376.56	12825.56	13282.56	1374
9/32	0667.02	1084.14	1509.27	1942.39	2383.52	2832.64	3289.77	375
5/16	0673.47	1090.72	1515.97	1949.22	2390.47	2839.72	3296.97	376
11/32	0679.93	1097.31	1522.68	1956.06	2397.43	2846.81	3304.18	376
3/8	10686.39	11103.89	11529.39	11962.89	12404.39	12853.89	13311.39	1377
13/32	0692.85	1110.48	1536.10	1969.73	2411.35	2860.98	3318.60	378
7/16	0699.32	1117.07	1542.82	1976.57	2418.32	2868.07	3325.82	379
15/32	0705.78	1123.66	1549.53	1983.41	2425.28	2875.16	3333.03	379
1/2	10712.25	11130.25	11556.25	11990.25	12432.25	12882.25	13340.25	1380
17/32	0718.72	1136.84	1562.97	1997.09	2439.22	2889.34	3347.47	381
9/16	0725.19	1143.44	1569.69	2003.94	2446.19	2896.44	3354.69	382
19/32	0731.67	1150.04	1576.42	2010.79	2453.17	2903.54	3361.92	383
5/8	10738.14	11156.64	11583.14	12017.64	12460.14	12910.64	13369.14	1383
21/32	0744.62	1163.24	1589.87	2024.49	2467.12	2917.74	3376.37	384
11/16	0751.10	1169.85	1596.60	2031.35	2474.10	2924.85	3383.60	385
23/32	0757.58	1176.45	1603.33	2038.20	2481.08	2931.95	3390.83	385
3/4	10764.06	11183.06	11610.06	12045.06	12488.06	12939.06	13398.06	1386
25/32	0770.55	1189.67	1616.80	2051.92	2495.05	2946.17	3405.30	387
13/16	0777.04	1196.29	1623.54	2058.79	2502.04	2953.29	3412.54	387
27/32	0783.52	1202.90	1630.27	2065.65	2509.02	2960.40	3419.77	388
7/8	10790.02	11209.52	11637.02	12072.52	12516.02	12967.52	13427.02	1388
29/32	0796.51	1216.13	1643.76	2079.38	2523.01	2974.63	3434.26	390
15/16	0803.00	1222.75	1650.50	2086.25	2530.00	2981.75	3441.50	390
31/32	0809.50	1229.38	1657.25	2093.13	2537.00	2988.88	3448.75	391

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3	123	5129.	5144.38	5159.77	5175.16	5190.56	5205.97	5221.39	5236.82	5252.25	5267.69	5283.14	5298.60	5314.06	5329.54	5345.02	5360.50	7.72
4	124	15376.	15391.50	15407.02	15422.54	15438.06	15453.60	15469.14	15484.69	15500.25	15515.82	15531.39	15546.97	15562.56	15578.16	15593.77	15609.38	7.78
5	125	5625.	5640.63	5656.27	5671.91	5687.56	5703.22	5718.89	5734.57	5750.25	5765.94	5781.64	5797.35	5813.06	5828.79	5844.52	5860.25	7.84
6	126	15876.	15891.75	15907.52	15923.29	15939.06	15954.85	15970.64	15986.44	16002.25	16018.07	16033.89	16049.72	16065.56	16081.41	16097.27	16113.13	7.91
7	127	6129.	6144.88	6160.77	6176.66	6192.56	6208.47	6224.39	6240.32	6256.25	6272.19	6288.14	6304.10	6320.06	6336.04	6352.02	6368.00	7.97
8	128	16384.	16400.00	16416.02	16432.04	16448.06	16464.10	16480.14	16496.19	16512.25	16528.32	16544.39	16560.47	16576.56	16592.66	16608.77	16624.88	8.03
9	129	6641.	6657.13	6673.27	6689.41	6705.56	6721.72	6737.89	6754.07	6770.25	6786.44	6802.64	6818.85	6835.06	6851.29	6867.52	6883.75	8.09
10	130	16900.	16916.25	16932.52	16948.79	16965.06	16981.35	16997.64	17013.94	17030.25	17046.57	17062.89	17079.22	17095.56	17111.91	17128.27	17144.63	8.16
11	131	7161.	7177.38	7193.77	7210.16	7226.56	7242.97	7259.39	7275.82	7292.25	7308.69	7325.14	7341.60	7358.06	7374.54	7391.02	7407.50	8.22
11 0	132	17424.	17440.50	17457.02	17473.54	17490.06	17506.60	17523.14	17539.69	17556.25	17572.82	17589.39	17605.97	17622.56	17639.16	17655.77	17672.38	8.28
1	133	7689.	7705.63	7722.27	7738.91	7755.56	7772.22	7788.89	7805.57	7822.25	7838.94	7855.64	7872.35	7889.06	7905.79	7922.52	7939.25	8.34
2	134	17956.	17972.75	17989.52	18006.29	18023.06	18039.85	18056.64	18073.44	18090.25	18107.07	18123.89	18140.72	18157.56	18174.41	18191.27	18208.13	8.41
3	135	8225.	8241.88	8258.77	8275.66	8292.56	8309.47	8326.39	8343.32	8360.25	8377.19	8394.14	8411.10	8428.06	8445.04	8462.02	8479.00	8.47
4	136	18496.	18513.00	18530.02	18547.04	18564.06	18581.10	18598.14	18615.19	18632.25	18649.32	18666.39	18683.47	18700.56	18717.66	18734.77	18751.88	8.53
5	137	8769.	8786.13	8803.27	8820.41	8837.56	8854.72	8871.89	8889.07	8906.25	8923.44	8940.64	8957.85	8975.06	8992.29	19009.52	19026.75	8.59
6	138	19044.	19061.25	19078.52	19095.79	19113.06	19130.35	19147.64	19164.94	19182.25	19199.57	19216.89	19234.22	19251.56	19268.91	19286.27	19303.63	8.66
7	139	9321.	9338.38	9355.77	9373.16	9390.56	9407.97	9425.39	9442.82	9460.25	9477.69	9495.14	9512.60	9530.06	9547.54	9565.02	9582.50	8.72
8	140	19600.	19617.50	19635.02	19652.54	19670.06	19687.60	19705.14	19722.69	19740.25	19757.82	19775.39	19792.97	19810.56	19828.16	19845.77	19863.38	8.78
9	141	9881.	9898.63	9916.27	9933.91	9951.56	9969.22	9986.89	20004.57	20022.25	20039.94	20057.64	20075.35	20093.06	20110.79	20128.52	20146.25	8.84
10	142	20164.	20181.75	20199.52	20217.29	20235.06	20252.85	20270.64	20288.44	20306.25	20324.07	20341.89	20359.72	20377.56	20395.41	20413.27	20431.13	8.91
11	143	0449.	0466.88	0484.77	0502.66	0520.56	0538.47	0556.39	0574.32	0592.25	0610.19	0628.14	0646.10	0664.06	0682.04	0700.02	0718.00	8.97
12 0	144	20736.	20754.00	20772.02	20790.04	20808.06	20826.10	20844.14	20862.19	20880.25	20898.32	20916.39	20934.47	20952.56	20970.66	20988.77	21006.88	9.03
1	145	1025.	1043.13	1061.27	1079.41	1097.56	1115.72	1133.89	1152.07	1170.25	1188.44	1206.64	1224.85	1243.06	1261.29	1279.52	1297.75	9.09
2	146	21316.	21334.25	21352.52	21370.79	21389.06	21407.35	21425.64	21443.94	21462.25	21480.57	21498.89	21517.22	21535.56	21553.91	21572.27	21590.63	9.16
3	147	1609.	1627.38	1645.77	1664.16	1682.56	1700.97	1719.39	1737.82	1756.25	1774.69	1793.14	1811.60	1830.06	1848.54	1867.02	1885.50	9.22
4	148	21904.	21922.50	21941.02	21959.54	21978.06	21996.60	22015.14	22033.69	22052.25	22070.82	22089.39	22107.97	22126.56	22145.16	22163.77	22182.38	9.28
5	149	2201.	2219.63	2238.27	2256.91	2275.56	2294.22	2312.89	2331.57	2350.25	2368.94	2387.64	2406.35	2425.06	2443.79	2462.52	2481.25	9.34
6	150	22500.	22518.75	22537.52	22556.29	22575.06	22593.85	22612.64	22631.44	22650.25	22669.07	22687.89	22706.72	22725.56	22744.41	22763.27	22782.13	9.41
7	151	2801.	2819.88	2838.77	2857.66	2876.56	2895.47	2914.39	2933.32	2952.25	2971.19	2990.14	3009.10	3028.06	3047.04	3066.02	3085.00	9.47
8	152	23104.	23123.00	23142.02	23161.04	23180.06	23199.10	23218.14	23237.19	23256.25	23275.32	23294.39	23313.47	23332.56	23351.66	23370.77	23389.88	9.53
9	153	3409.	3428.13	3447.27	3466.41	3485.56	3504.72	3523.89	3543.07	3562.25	3581.44	3600.64	3619.85	3639.06	3658.29	3677.52	3696.75	9.59
10	154	23716.	23735.25	23754.52	23773.79	23793.06	23812.35	23831.64	23850.94	23870.25	23889.57	23908.89	23928.22	23947.56	23966.91	23986.27	24005.63	9.66
11	155	4025.	4044.38	4063.77	4083.16	4102.56	4121.97	4141.39	4160.82	4180.25	4199.69	4219.14	4238.60	4258.06	4277.54	4297.02	4316.50	9.72

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32b.—Squares of Inches, 156″ to 192″ (13′ 0″ to 16′ 0″), Advancing by 16ths.—Continued.

′	″	Ins.	1/16	1/8	3/16	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	+ 1/32
13	0	156	24336.	24355.50	24375.02	24394.54	24414.06	24433.60	24453.14	24472.69	24492.25	24511.82	24531.39	24550.97	24570.56	24590.16	24609.77	24629.38	9.78
	1	157	4649.	4668.63	4688.27	4707.91	4727.56	4747.22	4766.89	4786.57	4806.25	4825.94	4845.64	4865.35	4885.06	4904.79	4924.52	4944.25	9.84
	2	158	24964.	24983.75	25003.52	25023.29	25043.06	25062.85	25082.64	25102.44	25122.25	25142.07	25161.89	25181.72	25201.56	25221.41	25241.27	25261.13	9.91
	3	159	5281.	5300.88	5320.77	5340.66	5360.56	5380.47	5400.39	5420.32	5440.25	5460.19	5480.14	5500.10	5520.06	5540.04	5560.02	5580.00	9.97
	4	160	25600.	26520.00	25640.02	25660.04	25680.06	25700.10	25720.14	25740.19	25760.25	25780.32	25800.39	25820.47	25840.56	25860.66	25880.77	25900.88	10.03
	5	161	5921.	5941.13	5961.27	5981.41	6001.56	6021.72	6041.89	6062.07	6082.25	6102.44	6122.64	6142.85	6163.06	6183.29	6203.52	6223.75	10.09
	6	162	26244.	26264.25	26284.52	26304.79	26325.06	26345.35	26365.64	26385.94	26406.25	26426.57	26446.89	26467.22	26487.56	26507.91	26528.27	26548.63	10.16
	7	163	6569.	6589.38	6609.77	6630.16	6650.56	6670.97	6691.39	6711.82	6732.25	6752.69	6773.14	6793.60	6814.06	6834.54	6855.02	6875.50	10.22
	8	164	26896.	26916.50	26937.02	26957.54	26978.06	26998.60	27019.14	27039.69	27060.25	27080.82	27101.39	27121.97	27142.56	27163.16	27183.77	27204.38	10.28
	9	165	7225.	7245.63	7266.27	7286.91	7307.56	7328.22	7348.89	7369.57	7390.25	7410.94	7431.64	7452.35	7473.06	7493.79	7514.52	7535.25	10.34
	10	166	27556.	27576.75	27597.52	27618.29	27639.06	27659.85	27680.64	27701.44	27722.25	27743.07	27763.89	27784.72	27805.56	27826.41	27847.27	27868.13	10.41
	11	167	7889.	7909.88	7930.77	7951.66	7972.56	7993.47	8014.39	8035.32	8056.25	8077.19	8098.14	8119.10	8140.06	8161.04	8182.02	8203.00	10.47
14	0	168	28224.	28245.00	28266.02	28287.04	28308.06	28329.10	28350.14	28371.19	28392.25	28413.32	28434.39	28455.47	28476.56	28497.66	28518.77	28539.88	10.53
	1	169	8561.	8582.13	8603.27	8624.41	8645.56	8666.72	8687.89	8709.07	8730.25	8751.44	8772.64	8793.85	8815.06	8836.29	8857.52	8878.75	10.59
	2	170	28900.	28921.25	28942.52	28963.79	28985.06	29006.35	29027.64	29048.94	29070.25	29091.57	29112.89	29134.22	29155.56	29176.91	29198.27	29219.63	10.66
	3	171	9241.	9262.38	9283.77	9305.16	9326.56	9347.97	9369.39	9390.82	9412.25	9433.69	9455.14	9476.60	9498.06	9519.54	9541.02	9562.50	10.72
	4	172	29584.	29605.50	29627.02	29648.54	29670.06	29691.60	29713.14	29734.69	29756.25	29777.82	29799.39	29820.97	29842.56	29864.16	29885.77	29907.38	10.78
	5	173	9929.	9950.63	9972.27	9993.91	30015.56	30037.22	30058.89	30080.57	30102.25	30123.94	30145.64	30167.35	30189.06	30210.79	30232.52	30254.25	10.84
	6	174	30276.	30297.75	30319.52	30341.29	30363.06	30384.85	30406.64	30428.44	30450.25	30472.07	30493.89	30515.72	30537.56	30559.41	30581.27	30603.13	10.91
	7	175	0625.	0646.88	0668.77	0690.66	0712.56	0734.47	0756.39	0778.32	0800.25	0822.19	0844.14	0866.10	0888.06	0910.04	0932.02	0954.00	10.97
	8	176	30976.	30998.00	31020.02	31042.04	31064.06	31086.10	31108.14	31130.19	31152.25	31174.32	31196.39	31218.47	31240.56	31262.66	31284.77	31306.88	11.03
	9	177	1329.	1351.13	1373.27	1395.41	1417.56	1439.72	1461.89	1484.07	1506.25	1528.44	1550.64	1572.85	1595.06	1617.29	1639.52	1661.75	11.09
	10	178	31684.	31706.25	31728.52	31750.79	31773.06	31795.35	31817.64	31839.94	31862.25	31884.57	31906.89	31929.22	31951.56	31973.91	31996.27	32018.63	11.16
	11	179	2041.	2063.38	2085.77	2108.16	2130.56	2152.97	2175.39	2197.82	2220.25	2242.69	2265.14	2287.60	2310.06	2332.54	2355.02	2377.50	11.22
15	0	180	32400.	32422.50	32445.02	32467.54	32490.06	32512.60	32535.14	32557.69	32580.25	32602.82	32625.39	32647.97	32670.56	32693.16	32715.77	32738.38	11.28
	1	181	2761.	2783.63	2806.27	2828.91	2851.56	2874.22	2896.89	2919.57	2942.25	2964.94	2987.64	3010.35	3033.06	3055.79	3078.52	3101.25	11.34
	2	182	33124.	33146.75	33169.52	33192.29	33215.06	33237.85	33260.64	33283.44	33306.25	33329.07	33351.89	33374.72	33397.56	33420.41	33443.27	33466.13	11.41
	3	183	3489.	3511.88	3534.77	3557.66	3580.56	3603.47	3626.39	3649.32	3672.25	3695.19	3718.14	3741.10	3764.06	3787.04	3810.02	3833.00	11.47
	4	184	33856.	33879.00	33902.02	33925.04	33948.06	33971.10	33994.14	34017.19	34040.25	34063.32	34086.39	34109.47	34132.56	34155.66	34178.77	34201.88	11.53
	5	185	4225.	4248.13	4271.27	4294.41	4317.56	4340.72	4363.89	4387.07	4410.25	4433.44	4456.64	4479.85	4503.06	4526.29	4549.52	4572.75	11.59
	6	186	34596.	34619.25	34642.52	34665.79	34689.06	34712.35	34735.64	34758.94	34782.25	34805.57	34828.89	34852.22	34875.56	34898.91	34922.27	34945.63	11.66
	7	187	4969.	4992.38	5015.77	5039.16	5062.56	5085.97	5109.39	5132.82	5156.25	5179.69	5203.14	5226.60	5250.06	5273.54	5297.02	5320.50	11.72
	8	188	35344.	35367.50	35391.02	35414.54	35438.06	35461.60	35485.14	35508.69	35532.25	35555.82	35579.39	35602.97	35626.56	35650.16	35673.77	35697.38	11.78
	9	189	5721.	5744.63	5768.27	5791.91	5815.56	5839.22	5862.89	5886.57	5901.25	5933.94	5957.64	5981.35	6005.06	6028.79	6052.52	6076.25	11.84
	10	190	36100.	36123.75	36147.52	36171.29	36195.06	36218.85	36242.64	36266.44	36290.25	36314.07	36337.89	36361.72	36385.56	36409.41	36433.27	36457.13	11.91
	11	191	6481.	6504.88	6528.77	6552.66	6576.56	6600.47	6624.39	6648.32	6672.25	6696.19	6720.14	6744.10	6768.06	6792.04	6816.02	6840.00	11.97

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32b.—Squares of Inches, 192″ to 228″ (16′ 0″ to 19′ 0″), Advancing by 16ths.—Continued.

′	″	Ins.		1/16	1/8	3/16	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	+1/32
16	0	192	36864.	36888.00	36912.02	36936.04	36960.06	36984.10	37008.14	37032.19	37056.25	37080.32	37104.39	37128.47	37152.56	37176.66	37200.77	37224.88	12.03
	1	193	7249.	7273.13	7297.27	7321.41	7345.56	7369.72	7393.89	7418.07	7442.25	7466.44	7490.64	7514.85	7539.06	7563.29	7587.52	7611.75	12.09
	2	194	37636.	37660.25	37684.52	37708.79	37733.06	37757.35	37781.64	37805.94	37830.25	37854.57	37878.89	37903.22	37927.56	37951.91	37976.27	38000.63	12.16
	3	195	8025.	8049.38	8073.77	8098.16	8122.56	8146.97	8171.39	8195.82	8220.25	8244.69	8269.14	8293.60	8318.06	8342.54	8367.02	8391.50	12.22
	4	196	38416.	38440.50	38465.02	38489.54	38514.06	38538.60	38563.14	38587.69	38612.25	38636.82	38661.39	38685.97	38710.56	38735.16	38759.77	38784.38	12.28
	5	197	8809.	8833.63	8858.27	8882.91	8907.56	8932.22	8956.89	8981.57	9006.25	9030.94	9055.64	9080.35	9105.06	9129.79	9154.52	9179.25	12.34
	6	198	39204.	39228.75	39253.52	39278.29	39303.06	39327.85	39352.64	39377.44	39402.25	39427.07	39451.89	39476.72	39501.56	39526.41	39551.27	39576.13	12.41
	7	199	9601.	9625.88	9650.77	9675.66	9700.56	9725.47	9750.39	9775.32	9800.25	9825.19	9850.14	9875.10	9900.06	9925.04	9950.02	9975.00	12.47
	8	200	40000.	40025.00	40050.02	40075.04	40100.06	40125.10	40150.14	40175.19	40200.25	40225.32	40250.39	40275.47	40300.56	40325.66	40350.77	40375.88	12.53
	9	201	0401.	0426.13	0451.27	0476.41	0501.56	0526.72	0551.89	0577.07	0602.25	0627.44	0652.64	0677.85	0703.06	0728.29	0753.52	0778.75	12.59
	10	202	40804.	40829.25	40854.52	40879.79	40905.06	40930.35	40955.64	40980.94	41006.25	41031.57	41056.89	41082.22	41107.56	41132.91	41158.27	41183.63	12.66
	11	203	1209.	1234.38	1259.77	1285.16	1310.56	1335.97	1361.39	1386.82	1412.25	1437.69	1463.14	1488.60	1514.06	1539.54	1565.02	1590.50	12.72
17	0	204	41616.	41641.50	41667.02	41692.54	41718.06	41743.60	41769.14	41794.69	41820.25	41845.82	41871.39	41896.97	41922.56	41948.16	41973.77	41999.38	12.78
	1	205	2025.	2050.63	2076.27	2101.91	2127.56	2153.22	2178.89	2204.57	2230.25	2255.94	2281.64	2307.35	2333.06	2358.79	2384.52	2410.25	12.84
	2	206	42436.	42461.75	42487.52	42513.29	42539.06	42564.85	42590.64	42616.44	42642.25	42668.07	42693.89	42719.72	42745.56	42771.41	42797.27	42823.13	12.91
	3	207	2849.	2874.88	2900.77	2926.66	2952.56	2978.47	3004.39	3030.32	3056.25	3082.19	3108.14	3134.10	3160.06	3186.04	3212.02	3238.00	12.97
	4	208	43264.	43290.00	43316.02	43342.04	43368.06	43394.10	43420.14	43446.19	43472.25	43498.32	43524.39	43550.47	43576.56	43602.66	43628.77	43654.88	13.03
	5	209	3681.	3707.13	3733.27	3759.41	3785.56	3811.72	3837.89	3864.07	3890.25	3916.44	3942.64	3968.85	3995.06	4021.29	4047.52	4073.75	13.09
	6	210	44100.	44126.25	44152.52	44178.79	44205.06	44231.35	44257.64	44283.94	44310.25	44336.57	44362.89	44389.22	44415.56	44441.91	44468.27	44494.63	13.16
	7	211	4521.	4547.38	4573.77	4600.16	4626.56	4652.97	4679.39	4705.82	4732.25	4758.69	4785.14	4811.60	4838.06	4864.54	4891.02	4917.50	13.22
	8	212	44944.	44970.50	44997.02	45023.54	45050.06	45076.60	45103.14	45129.69	45156.25	45182.82	45209.39	45235.97	45262.56	45289.16	45315.77	45342.38	13.28
	9	213	5369.	5395.63	5422.27	5448.91	5475.56	5502.22	5528.89	5555.57	5582.25	5608.94	5635.64	5662.35	5689.06	5715.79	5742.52	5769.25	13.34
	10	214	45796.	45822.75	45849.52	45876.29	45903.06	45929.85	45956.64	45983.44	46010.25	46037.07	46063.89	46090.72	46117.56	46144.41	46171.27	46198.13	13.41
	11	215	6225.	6251.88	6278.77	6305.66	6332.56	6359.47	6386.39	6413.32	6440.25	6467.19	6494.14	6521.10	6548.06	6575.04	6602.02	6629.00	13.47
18	0	216	46656.	46683.00	46710.02	46737.04	46764.06	46791.10	46818.14	46845.19	46872.25	46899.32	46926.39	46953.47	46980.56	47007.66	47034.77	47061.88	13.53
	1	217	7089.	7116.13	7143.27	7170.41	7197.56	7224.72	7251.89	7279.07	7306.25	7333.44	7360.64	7387.85	7415.06	7442.29	7469.52	7496.75	13.59
	2	218	47524.	47551.25	47578.52	47605.79	47633.06	47660.35	47687.64	47714.94	47742.25	47769.57	47796.89	47824.22	47851.56	47878.91	47906.27	47933.63	13.66
	3	219	7961.	7988.38	8015.77	8043.16	8070.56	8097.97	8125.39	8152.82	8180.25	8207.69	8235.14	8262.60	8290.06	8317.54	8345.02	8372.50	13.72
	4	220	48400.	48427.50	48455.02	48482.54	48510.06	48537.60	48565.14	48592.69	48620.25	48647.82	48675.39	48702.97	48730.56	48758.16	48785.77	48813.38	13.78
	5	221	8841.	8868.63	8896.27	8923.91	8951.56	8979.22	9006.89	9034.57	9062.25	9089.94	9117.64	9145.35	9173.06	9200.79	9228.52	9256.25	13.84
	6	222	49284.	49311.75	49339.52	49367.29	49395.06	49422.85	49450.64	49478.44	49506.25	49534.07	49561.89	49589.72	49617.56	49645.41	49673.27	49701.13	13.91
	7	223	9729.	9756.88	9784.77	9812.66	9840.56	9868.47	9896.39	9924.32	9952.25	9980.19	50008.14	50036.10	50064.06	50092.04	50120.02	50148.00	13.97
	8	224	50176.	50204.00	50232.02	50260.04	50288.06	50316.10	50344.14	50372.19	50400.25	50428.32	50456.39	50484.47	50512.56	50540.66	50568.77	50596.88	14.03
	9	225	0625.	0653.13	0681.27	0709.41	0737.56	0765.72	0793.89	0822.07	0850.25	0878.44	0906.64	0934.85	0963.06	0991.29	1019.52	1047.75	14.09
	10	226	51076.	51104.25	51132.52	51160.79	51189.06	51217.35	51245.64	51273.94	51302.25	51330.57	51358.89	51387.22	51415.56	51443.91	51472.27	51500.63	14.16
	11	227	1529.	1557.38	1585.77	1614.16	1642.56	1670.97	1699.39	1727.82	1756.25	1784.69	1813.14	1841.60	1870.06	1898.54	1927.02	1955.50	14.22

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82b.—Squares of Inches, 228″ to 264″ (19′ 0″ to 22′ 0″), Advancing by 16ths.—Continued.

′	″	Ins.		1/16	1/8	3/16	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	+1/32
19	0	228	51984.	52012.50	52041.02	52069.54	52098.06	52126.60	52155.14	52183.69	52212.25	52240.82	52269.39	52297.97	52326.56	52355.16	52383.77	52412.38	14.28
	1	229	2441.	2469.63	2498.27	2526.91	2555.56	2584.22	2612.89	2641.57	2670.25	2698.94	2727.64	2756.35	2785.06	2813.79	2842.52	2871.25	14.34
	2	230	52900.	52928.75	52957.52	52986.29	53015.06	53043.85	53072.64	53101.44	53130.25	53159.07	53187.89	53216.72	53245.56	53274.41	53303.27	53332.13	14.41
	3	231	3361.	3389.88	3418.77	3447.66	3476.56	3505.47	3534.39	3563.32	3592.25	3621.19	3650.14	3679.10	3708.06	3737.04	3766.02	3795.00	14.47
	4	232	53824.	53853.00	53882.02	53911.04	53940.06	53969.10	53998.14	54027.19	54056.25	54085.32	54114.39	54143.47	54172.56	54201.66	54230.77	54259.88	14.53
	5	233	4289.	4318.13	4347.27	4376.41	4405.56	4434.72	4463.89	4493.07	4522.25	4551.44	4580.64	4609.85	4639.06	4668.29	4697.52	4726.75	14.59
	6	234	54756.	54785.25	54814.52	54843.79	54873.06	54902.35	54931.64	54960.94	54990.25	55019.57	55048.89	55078.22	55107.56	55136.91	55166.27	55195.63	14.66
	7	235	5225.	5254.38	5283.77	5313.16	5342.56	5371.97	5401.39	5430.82	5460.25	5489.69	5519.14	5548.60	5578.06	5607.54	5637.02	5666.50	14.72
	8	236	55696.	55725.50	55755.02	55784.54	55814.06	55843.60	55873.14	55902.69	55932.25	55961.82	55991.39	56020.97	56050.56	56080.16	56109.77	56139.38	14.78
	9	237	6169.	6198.63	6228.27	6257.91	6287.56	6317.22	6346.89	6376.57	6406.25	6435.94	6465.64	6495.35	6525.06	6554.79	6584.52	6614.25	14.84
	10	238	56644.	56673.75	56703.52	56733.29	56763.06	56792.85	56822.64	56852.44	56882.25	56912.07	56941.89	56971.72	57001.56	57031.41	57061.27	57091.13	14.91
	11	239	7121.	7150.88	7180.77	7210.66	7240.56	7270.47	7300.39	7330.32	7360.25	7390.19	7420.14	7450.10	7480.06	7510.04	7540.02	7570.00	14.97
20	0	240	57600.	57630.00	57660.02	57690.04	57720.06	57750.10	57780.14	57810.19	57840.25	57870.32	57900.39	57930.47	57960.56	57990.66	58020.77	58050.88	15.03
	1	241	8081.	8111.13	8141.27	8171.41	8201.56	8231.72	8261.89	8292.07	8322.25	8352.44	8382.64	8412.85	8443.06	8473.29	8503.52	8533.75	15.09
	2	242	58564.	58594.25	58624.52	58654.79	58685.06	58715.35	58745.64	58775.94	58806.25	58836.57	58866.89	58897.22	58927.56	58957.91	58988.27	59018.63	15.16
	3	243	9049.	9079.38	9109.77	9140.16	9170.56	9200.97	9231.39	9261.82	9292.25	9322.69	9353.14	9383.60	9414.06	9444.54	9475.02	9505.50	15.22
	4	244	59536.	59566.50	59597.02	59627.54	59658.06	59688.60	59719.14	59749.69	59780.25	59810.82	59841.39	59871.97	59902.56	59933.16	59963.77	59994.38	15.28
	5	245	60025.	60055.63	60086.27	60016.91	60147.56	60178.22	60208.89	60239.57	60270.25	60300.94	60331.64	60362.35	60393.06	60423.79	60454.52	60485.25	15.34
	6	246	60516.	60546.75	60577.52	60608.29	60639.06	60669.85	60700.64	60731.44	60762.25	60793.07	60823.89	60854.72	60885.56	60916.41	60947.27	60978.13	15.41
	7	247	1009.	1039.88	1070.77	1101.66	1132.56	1163.47	1194.39	1225.32	1256.25	1287.19	1318.14	1349.10	1380.06	1411.04	1442.02	1473.00	15.47
	8	248	61504.	61535.00	61566.02	61597.04	61628.06	61659.10	61690.14	61721.19	61752.25	61783.32	61814.39	61845.47	61876.56	61907.66	61938.77	61969.88	15.53
	9	249	2001.	2032.13	2063.27	2094.41	2125.56	2156.72	2187.89	2219.07	2250.25	2281.44	2312.64	2343.85	2375.06	2406.29	2437.52	2468.75	15.59
	10	250	62500.	62531.25	62562.52	62593.79	62625.06	62656.35	62687.64	62718.94	62750.25	62781.57	62812.89	62844.22	62875.56	62906.91	62938.27	62969.63	15.66
	11	251	3001.	3032.38	3063.77	3095.16	3126.56	3157.97	3189.39	3220.82	3252.25	3283.69	3315.14	3346.60	3378.06	3409.54	3441.02	3472.50	15.72
21	0	252	63504.	63535.50	63567.02	63598.54	63630.06	63661.60	63693.14	63724.69	63756.25	63787.82	63819.39	63850.97	63882.56	63914.16	63945.77	63977.38	15.78
	1	253	4009.	4040.63	4072.27	4103.91	4135.56	4167.22	4198.89	4230.57	4262.25	4293.94	4325.64	4357.35	4389.06	4420.79	4452.52	4484.25	15.84
	2	254	64516.	64547.75	64579.52	64611.29	64643.06	64674.85	64706.64	64738.44	64770.25	64802.07	64833.89	64865.72	64897.56	64929.41	64961.26	64993.13	15.91
	3	255	5025.	5056.88	5088.77	5120.66	5152.56	5184.47	5216.39	5248.32	5280.25	5312.19	5344.14	5376.10	5408.06	5440.04	5472.02	5504.00	15.97
	4	256	65536.	65568.00	65600.02	65632.04	65664.06	65696.10	65728.14	65760.19	65792.25	65824.32	65856.39	65888.47	65920.56	65952.66	65984.77	66016.88	16.03
	5	257	6049.	6081.13	6113.27	6145.41	6177.56	6209.72	6241.89	6274.07	6306.25	6338.44	6370.64	6402.85	6435.06	6467.29	6499.52	6531.75	16.09
	6	258	66564.	66596.25	66628.52	66660.79	66693.06	66725.35	66757.64	66789.94	66822.25	66854.57	66886.89	66919.22	66951.56	66983.91	67016.27	67048.63	16.16
	7	259	7081.	7113.38	7145.77	7178.16	7210.56	7242.97	7275.39	7307.82	7340.25	7372.69	7405.14	7437.60	7470.06	7502.54	7535.02	7567.50	16.22
	8	260	67600.	67632.50	67665.02	67697.54	67730.06	67762.60	67795.14	67827.69	67860.25	67892.82	67925.39	67957.97	67990.56	68023.16	68055.77	68088.38	16.28
	9	261	8121.	8153.63	8186.27	8218.91	8251.56	8284.22	8316.89	8349.57	8382.25	8414.94	8447.64	8480.35	8513.06	8545.79	8578.52	8611.25	16.34
	10	262	68644.	68676.75	68709.52	68742.29	68775.06	68807.85	68840.64	68873.44	68906.25	68939.07	68971.89	69004.72	69037.56	69070.41	69103.27	69136.13	16.41
	11	263	9169.	9201.88	9234.77	9267.66	9300.56	9333.47	9366.39	9399.32	9432.25	9465.19	9498.14	9531.10	9564.06	9597.04	9630.02	9663.00	16.47

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32b.—Squares of Inches, 264″ to 300″ (22′ 0″ to 25′ 0″), Advancing by 16ths.—Continued.

′	″	Ins.		1/16	1/8	3/16	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	+1/32
22	0	264	69696.	69729.00	69762.02	69795.04	69828.06	69861.10	69894.14	69927.19	69960.25	69993.32	70026.39	70059.47	70092.56	70125.66	70158.77	70191.88	16.53
	1	265	70225.	70258.13	70291.27	70324.41	70357.56	70390.72	70423.89	70457.07	70490.25	70523.44	70556.64	70589.85	70623.06	70656.29	70689.52	70722.75	16.59
	2	266	70756.	70789.25	70822.52	70855.79	70889.06	70922.35	70955.64	70988.94	71022.25	71055.57	71088.89	71122.22	71155.56	71188.91	71222.27	71255.63	16.66
	3	267	1289.	1322.38	1355.77	1389.16	1422.56	1455.97	1489.39	1522.82	1556.25	1589.69	1623.14	1656.60	1690.06	1723.54	1757.02	1790.50	16.72
	4	268	71824.	71857.50	71891.02	71924.54	71958.06	71991.60	72025.14	72058.69	72092.25	72125.82	72159.39	72192.97	72226.56	72260.16	72293.77	72327.38	16.78
	5	269	2361.	2394.63	2428.27	2461.91	2495.56	2529.22	2562.89	2596.57	2630.25	2663.94	2697.64	2731.35	2765.06	2798.79	2832.52	2866.25	16.84
	6	270	72900.	72933.75	72967.52	73001.29	73035.06	73068.85	73102.64	73136.44	73170.25	73204.07	73237.89	73271.72	73305.56	73339.41	73373.27	73407.13	16.91
	7	271	3441.	3474.88	3508.77	3542.66	3576.56	3610.47	3644.39	3678.32	3712.25	3746.19	3780.14	3814.10	3848.06	3882.04	3916.02	3950.00	16.97
	8	272	73984.	74018.00	74052.02	74086.04	74120.06	74154.10	74188.14	74222.19	74256.25	74290.32	74324.39	74358.47	74392.56	74426.66	74460.77	74494.88	17.03
	9	273	4529.	4563.13	4597.27	4631.41	4665.56	4699.72	4733.89	4768.07	4802.25	4836.44	4870.64	4904.85	4939.06	4973.29	5007.52	5041.75	17.09
	10	274	75076.	75110.25	75144.52	75178.79	75213.06	75247.35	75281.64	75315.94	75350.25	75384.57	75418.89	75453.22	75487.56	75521.91	75556.27	75590.63	17.16
	11	275	5625.	5659.38	5693.77	5728.16	5762.56	5796.97	5831.39	5865.82	5900.25	5934.69	5969.14	6003.60	6038.06	6072.54	6107.02	6141.50	17.22
23	0	276	76176.	76210.50	76245.02	76279.54	76314.06	76348.60	76383.14	76417.69	76452.25	76486.82	76521.39	76555.97	76590.56	76625.16	76659.77	76694.38	17.28
	1	277	6729.	6763.63	6798.27	6832.91	6867.56	6902.22	6936.89	6971.57	7006.25	7040.94	7075.64	7110.35	7145.06	7179.79	7214.52	7249.25	17.34
	2	278	77284.	77318.75	77353.52	77388.29	77423.06	77457.85	77492.64	77527.44	77562.25	77597.07	77631.89	77666.72	77701.56	77736.41	77771.27	77806.13	17.41
	3	279	7841.	7875.88	7910.77	7945.66	7980.56	8015.47	8050.39	8085.32	8120.25	8155.19	8190.14	8225.10	8260.06	8295.04	8330.02	8365.00	17.47
	4	280	78400.	78435.00	78470.02	78505.04	78540.06	78575.10	78610.14	78645.19	78680.25	78715.32	78750.39	78785.47	78820.56	78855.66	78890.77	78925.88	17.53
	5	281	8961.	8996.13	9031.27	9066.41	9101.56	9136.72	9171.89	9207.07	9242.25	9277.44	9312.64	9347.85	9383.06	9418.29	9453.52	9488.75	17.59
	6	282	79524.	79559.25	79594.52	79629.79	79665.06	79700.35	79735.64	79770.94	79806.25	79841.57	79876.89	79912.22	79947.56	79982.91	80018.27	80053.63	17.66
	7	283	80089.	80124.38	80159.77	80195.16	80230.56	80265.97	80301.39	80336.82	80372.25	80407.69	80443.14	80478.60	80514.06	80549.54	80585.02	80620.50	17.72
	8	284	80656.	80691.50	80727.02	80762.54	80798.06	80833.60	80869.14	80904.69	80940.25	80975.82	81011.39	81046.97	81082.56	81118.16	81153.77	81189.38	17.78
	9	285	1225.	1260.63	1296.27	1331.91	1367.56	1403.22	1438.89	1474.57	1510.25	1545.94	1581.64	1617.35	1653.06	1688.79	1724.52	1760.25	17.84
	10	286	81796.	81831.75	81867.52	81903.29	81939.06	81974.85	82010.64	82046.44	82082.25	82118.07	82153.89	82189.72	82225.56	82261.41	82297.27	82333.13	17.91
	11	287	2369.	2404.88	2440.77	2476.66	2512.56	2548.47	2584.39	2620.32	2656.25	2692.19	2728.14	2764.10	2800.06	2836.04	2872.02	2908.00	17.97
24	0	288	82944.	82980.00	83016.02	83052.04	83088.06	83124.10	83160.14	83196.19	83232.25	83268.32	83304.39	83340.47	83376.56	83412.66	83448.77	83484.88	18.03
	1	289	3521.	3557.13	3593.27	3629.41	3665.56	3701.72	3737.89	3774.07	3810.25	3846.44	3882.64	3918.85	3955.06	3991.29	4027.52	4063.75	18.09
	2	290	84100.	84136.25	84172.52	84208.79	84245.06	84281.35	84317.64	84353.94	84390.25	84426.57	84462.89	84499.22	84535.56	84571.91	84608.27	84644.63	18.16
	3	291	4681.	4717.38	4753.77	4790.16	4826.56	4862.97	4899.39	4935.82	4972.25	5008.69	5045.14	5081.60	5118.06	5154.54	5191.02	5227.50	18.22
	4	292	85264.	85300.50	85337.02	85373.54	85410.06	85446.60	85483.14	85519.69	85556.25	85592.82	85629.39	85665.97	85702.56	85739.16	85775.77	85812.38	18.28
	5	293	5849.	5885.63	5922.27	5958.91	5995.56	6032.22	6068.89	6105.57	6142.25	6178.94	6215.64	6252.35	6289.06	6325.79	6362.52	6399.25	18.34
	6	294	86436.	86472.75	86509.52	86546.29	86583.06	86619.85	86656.64	86693.44	86730.25	86767.07	86803.89	86840.72	86877.56	86914.41	86951.27	86988.13	18.41
	7	295	7025.	7061.88	7098.77	7135.66	7172.56	7209.47	7246.39	7283.32	7320.25	7357.19	7394.14	7431.10	7468.06	7505.04	7542.02	7579.00	18.47
	8	296	87616.	87653.00	87690.02	87727.04	87764.06	87801.10	87838.14	87875.19	87912.25	87949.32	87986.39	88023.47	88060.56	88097.66	88134.77	88171.88	18.53
	9	297	8209.	8246.13	8283.27	8320.41	8357.56	8394.72	8431.89	8469.07	8506.25	8543.44	8580.64	8617.85	8655.06	8692.29	8729.52	8766.75	18.59
	10	298	88804.	88841.25	88878.52	88915.79	88953.06	88990.35	89027.64	89064.94	89102.25	89139.57	89176.89	89214.22	89251.56	89288.91	89326.27	89363.63	18.66
	11	299	9401.	9438.38	9475.77	9513.16	9550.56	9587.97	9625.39	9662.82	9700.25	9737.69	9775.14	9812.60	9850.06	9887.54	9925.02	9962.50	18.72

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32b.—Squares of Inches, 300″ to 336″ (25′ 0″ to 28′ 0″), Advancing by 16ths.—Continued.

′	″	Ins.		1/16	1/8	3/16	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	+1/32
25	0	300	90000.	90037.50	90075.02	90112.54	90150.06	90187.60	90225.14	90262.69	90300.25	90337.82	90375.39	90412.97	90450.56	90488.16	90525.77	90563.38	18.78
	1	301	0601.	0638.63	0676.27	0713.91	0751.56	0789.22	0826.89	0864.57	0902.25	0939.94	0977.64	1015.35	1053.06	1090.79	1128.52	1166.25	18.84
	2	302	91204.	91241.75	91279.52	91317.29	91355.06	91392.85	91430.64	91468.44	91506.25	91544.07	91581.89	91619.72	91657.56	91695.41	91733.27	91771.13	18.91
	3	303	1809.	1846.88	1884.77	1922.66	1960.56	1998.47	2036.39	2074.32	2112.25	2150.19	2188.14	2226.10	2264.06	2302.04	2340.02	2378.00	18.97
	4	304	92416.	92454.00	92492.02	92530.04	92568.06	92606.10	92644.14	92682.19	92720.25	92758.32	92796.39	92834.47	92872.56	92910.66	92948.77	92986.88	19.03
	5	305	3025.	3063.13	3101.27	3139.41	3177.56	3215.72	3253.89	3292.07	3330.25	3368.44	3406.64	3444.85	3483.06	3521.29	3559.52	3597.75	19.09
	6	306	93636.	93674.25	93712.52	93750.79	93789.06	93827.35	93865.64	93903.94	93942.25	93980.57	94018.89	94057.22	94095.56	94133.91	94172.27	94210.63	19.16
	7	307	4249.	4287.38	4325.77	4364.16	4402.56	4440.97	4479.39	4517.82	4556.25	4594.69	4633.14	4671.60	4710.06	4748.54	4787.02	4825.50	19.22
	8	308	94864.	94902.50	94941.02	94979.54	95018.06	95056.60	95095.14	95133.69	95172.25	95210.82	95249.39	95287.97	95326.56	95365.16	95403.77	95442.38	19.28
	9	309	5481.	5519.63	5558.27	5596.91	5635.56	5674.22	5712.89	5751.57	5790.25	5828.94	5867.64	5906.35	5945.06	5983.79	6022.52	6061.25	19.34
	10	310	96100.	96138.75	96177.52	96216.29	96255.06	96293.85	96332.64	96371.44	96410.25	96449.07	96487.89	96526.72	96565.56	96604.41	96643.27	96682.13	19.41
	11	311	6721.	6759.88	6798.77	6837.66	6876.56	6915.47	6954.39	6993.32	7032.25	7071.19	7110.14	7149.10	7188.06	7227.04	7266.02	7305.00	19.47
26	0	312	97344.	97383.00	97422.02	97461.04	97500.06	97539.10	97578.14	97617.19	97656.25	97695.32	97734.39	97773.47	97812.56	97851.66	97890.77	97929.88	19.53
	1	313	7969.	8008.13	8047.27	8086.41	8125.56	8164.72	8203.89	8243.07	8282.25	8321.44	8360.64	8399.85	8439.06	8478.29	8517.52	8556.75	19.59
	2	314	98596.	98635.25	98674.52	98713.79	98753.06	98792.35	98831.64	98870.94	98910.25	98949.57	98988.89	99028.22	99067.56	99106.91	99146.27	99185.63	19.66
	3	315	9225.	9264.38	9303.77	9343.16	9382.56	9421.97	9461.39	9500.82	9540.25	9579.69	9619.14	9658.60	9698.06	9737.54	9777.02	9816.50	19.72
	4	316	99856.	99895.50	99935.02	99974.54	100014.1	100053.6	100093.1	100132.7	100172.3	100211.8	100251.4	100291.0	100330.6	100370.2	100409.8	100449.4	19.78
	5	317	100489.	100528.6	100568.3	100607.9	00647.6	00687.2	00726.9	00766.6	00806.3	00845.9	00885.6	00925.3	00965.1	01004.8	01044.5	01084.3	19.84
	6	318	01124.	01163.8	01203.5	01243.3	01283.1	01322.8	01362.6	01402.4	01442.3	01482.1	01521.9	01561.7	01601.6	01641.4	01681.3	01721.1	19.91
	7	319	01761.	01800.9	01840.8	01880.7	01920.6	01960.5	02000.4	02040.3	02080.3	02120.2	02160.1	02200.1	02240.1	02280.0	02320.0	02360.0	19.97
	8	320	102400.	102440.0	102480.0	102520.0	102560.1	102600.1	102640.1	102680.2	102720.3	102760.3	102800.4	102840.5	102880.6	102920.7	102960.8	103000.9	20.03
	9	321	03041.	03081.1	03121.3	03161.4	03201.6	03241.7	03281.9	03322.1	03362.3	03402.4	03442.6	03482.8	03523.1	03563.3	03603.5	03643.8	20.09
	10	322	103684.	103724.3	103764.5	103804.8	103845.1	103885.3	103925.6	103965.9	104006.3	104046.6	104086.9	104127.2	104167.6	104207.9	104248.3	104288.6	20.16
	11	323	04329.	04369.4	04409.8	04450.2	04490.6	04531.0	04571.4	04611.8	04652.3	04692.7	04733.1	04773.6	04814.1	04854.5	04895.0	04935.5	20.22
27	0	324	104976.	105016.5	105057.0	105097.5	105138.1	105178.6	105219.1	105259.7	105300.3	105340.8	105381.4	105422.0	105462.6	105503.2	105543.8	105584.4	20.28
	1	325	05625.	05665.6	05706.3	05746.9	05787.6	05828.2	05868.9	05909.6	05950.3	05990.9	06031.6	06072.3	06113.1	06153.8	06194.5	06235.3	20.34
	2	326	106276.	106316.8	106357.5	106398.3	106439.1	106479.8	106520.6	106561.4	106602.3	106643.1	106683.9	106724.7	106765.6	106806.4	106847.3	106888.1	20.41
	3	327	06929.	06969.9	07010.8	07051.7	07092.6	07133.5	07174.4	07215.3	07256.3	07297.2	07338.1	07379.1	07420.1	07461.0	07502.0	07543.0	20.47
	4	328	107584.	107625.0	107666.0	107707.0	107748.1	107789.1	107830.1	107871.2	107912.3	107953.3	107994.4	108035.5	108076.6	108117.7	108158.8	108199.9	20.53
	5	329	08241.	08282.1	08323.3	08364.4	08405.6	08446.7	08487.9	08529.1	08570.3	08611.4	08652.6	08693.8	08735.1	08776.3	08817.5	08858.8	20.59
	6	330	108900.	108941.3	108982.5	109023.8	109065.1	109106.3	109147.6	109188.9	109230.3	109271.6	109312.9	109354.2	109395.6	109436.9	109478.3	109519.6	20.66
	7	331	09561.	09602.4	09643.8	09685.2	09726.6	09768.0	09809.4	09850.8	09892.3	09933.7	09975.1	10016.6	10058.1	10099.5	10141.0	10182.5	20.72
	8	332	110224.	110265.5	110307.0	110348.5	110390.1	110431.6	110473.1	110514.7	110556.3	110597.8	110639.4	110681.0	110722.6	110764.2	110805.8	110847.4	20.78
	9	333	10889.	10930.6	10972.3	11013.9	11055.6	11097.2	11138.9	11180.6	11222.3	11263.9	11305.6	11347.4	11389.1	11430.8	11472.5	11514.3	20.84
	10	334	111556.	111597.8	111639.5	111681.3	111723.1	111764.8	111806.6	111848.4	111890.3	111932.1	111973.9	112015.7	112057.6	112099.4	112141.3	112183.1	20.91
	11	335	12225.	12266.9	12308.8	12350.7	12392.6	12434.5	12476.4	12518.3	12560.3	12602.2	12644.1	12686.1	12728.1	12770.0	12812.0	12854.0	20.97

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32b.—Squares of Inches, 336″ to 372″ (28′ 0″ to 31′ 0″), Advancing by 16ths.—Continued.

′	″	Ins.		1/16	1/8	3/16	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	+1/32
28	0	336	112896.	112938.0	112980.0	113022.0	113064.1	113106.1	113148.1	113190.2	113232.3	113274.3	113316.4	113358.5	113400.6	113442.7	113484.8	113526.9	21.03
	1	337	13569.	13611.1	13653.3	13695.4	13737.6	13779.7	13821.9	13864.1	13906.3	13948.4	13990.6	14032.8	14075.1	14117.3	14159.5	14201.8	21.09
	2	338	114244.	114286.3	114328.5	114370.8	114413.1	114455.3	114497.6	114539.9	114582.3	114624.6	114666.9	114709.2	114751.6	114793.9	114836.3	114878.6	21.16
	3	339	14921.	14963.4	15005.8	15048.2	15090.6	15133.0	15175.4	15217.8	15260.3	15302.7	15345.1	15387.6	15430.1	15472.5	15515.0	15557.5	21.22
	4	340	115600.	115642.5	115685.0	115727.5	115770.1	115812.6	115855.1	115897.7	115940.3	115982.8	116025.4	116068.0	116110.6	116153.2	116195.8	116238.4	21.28
	5	341	16281.	16323.6	16366.3	16408.9	16451.6	16494.2	16536.9	16579.6	16622.3	16664.9	16707.6	16750.3	16793.1	16835.8	16878.5	16921.3	21.34
	6	342	116964.	117006.8	117049.5	117092.3	117135.1	117177.8	117220.6	117263.4	117306.3	117349.1	117391.9	117434.7	117477.6	117520.4	117563.3	117606.1	21.41
	7	343	17649.	17691.9	17734.8	17777.7	17820.6	17863.5	17906.4	17949.3	17992.3	18035.2	18078.1	18121.1	18164.1	18207.0	18250.0	18293.0	21.47
	8	344	118336.	118379.0	118422.0	118465.0	118508.1	118551.1	118594.1	118637.2	118680.3	118723.3	118766.4	118809.5	118852.6	118895.7	118938.8	118981.9	21.53
	9	345	19025.	19068.1	19111.3	19154.4	19197.6	19240.7	19283.9	19327.1	19370.3	19413.4	19456.6	19499.8	19543.1	19586.3	19629.5	19672.8	21.59
	10	346	119716.	119759.3	119802.5	119845.8	119889.1	119932.3	119975.6	120018.9	120062.3	120105.6	120148.9	120192.2	120235.6	120278.9	120322.3	120365.6	21.66
	11	347	20409.	20452.4	20495.8	20539.2	20582.6	20626.0	20669.4	20712.8	20756.3	20799.7	20843.1	20886.6	20930.1	20973.5	21017.0	21060.5	21.72
29	0	348	121104.	121147.5	121191.0	121234.5	121278.1	121321.6	121365.1	121408.7	121452.3	121495.8	121539.4	121583.0	121626.6	121670.2	121713.8	121757.4	21.78
	1	349	21801.	21844.6	21888.3	21931.9	21975.6	22019.2	22062.9	22106.6	22150.3	22193.9	22237.6	22281.3	22325.1	22368.8	22412.5	22456.3	21.84
	2	350	122500.	122543.8	122587.5	122631.3	122675.1	122718.8	122762.6	122806.4	122850.3	122894.1	122937.9	122981.7	123025.6	123069.4	123123.3	123167.1	21.91
	3	351	23201.	23244.9	23288.8	23332.7	23376.6	23420.5	23464.4	23508.3	23552.3	23596.2	23640.1	23684.1	23728.1	23772.0	23816.0	23860.0	21.97
	4	352	123904.	123948.0	123992.0	124036.0	124080.1	124124.1	124168.1	124212.2	124256.3	124300.3	124344.4	124388.5	124432.6	124476.7	124520.8	124564.9	22.03
	5	353	24609.	24653.1	24697.3	24741.4	24785.6	24829.7	24873.9	24918.1	24962.3	25006.4	25050.6	25094.8	25139.1	25183.3	25227.5	25271.8	22.09
	6	354	125316.	125360.3	125404.5	125448.8	125493.1	125537.3	125581.6	125625.9	125670.3	125714.6	125758.9	125803.2	125847.6	125891.9	125936.3	125980.6	22.16
	7	355	26025.	26069.4	26113.8	26158.2	26202.6	26247.0	26291.4	26335.8	26380.3	26424.7	26469.1	26513.6	26558.1	26602.5	26647.0	26691.5	22.22
	8	356	126736.	126780.5	126825.0	126869.5	126914.1	126958.6	127003.1	127047.7	127092.3	127136.8	127181.4	127226.0	127270.6	127315.2	127359.8	127404.4	22.28
	9	357	27449.	27493.6	27538.3	27582.9	27627.6	27672.2	27716.9	27761.6	27806.3	27850.9	27895.6	27940.3	27985.1	28029.8	28074.5	28119.3	22.34
	10	358	128164.	128208.8	128253.5	128298.3	128343.1	128387.8	128432.6	128477.4	128522.3	128567.1	128611.9	128656.7	128701.6	128746.4	128791.3	128836.1	22.41
	11	359	28881.	28925.9	28970.8	29015.7	29060.6	29105.5	29150.4	29195.3	29240.3	29285.2	29330.1	29375.1	29420.1	29465.0	29510.0	29555.0	22.47
30	0	360	129600.	129645.0	129690.0	129735.0	129780.1	129825.1	129870.1	129915.2	129960.3	130005.3	130050.4	130095.5	130140.6	130185.7	130230.8	130275.9	22.53
	1	361	30321.	30366.1	30411.3	30456.4	30501.6	30546.7	30591.9	30637.1	30682.3	30727.4	30772.6	30817.8	30863.1	30908.3	30953.5	30998.8	22.59
	2	362	131044.	131089.3	131134.5	131179.8	131225.1	131270.3	131315.6	131360.9	131406.3	131451.6	131496.9	131542.2	131587.6	131632.9	131678.3	131723.6	22.66
	3	363	31769.	31814.4	31859.8	31905.2	31950.6	31996.0	32041.4	32086.8	32132.3	32177.7	32223.1	32268.6	32314.1	32359.5	32405.0	32450.5	22.72
	4	364	132496.	132541.5	132587.0	132632.5	132678.1	132723.6	132769.1	132814.7	132860.3	132905.8	132951.4	132997.0	133042.6	133088.2	133133.8	133179.4	22.78
	5	365	33225.	33270.6	33316.3	33361.9	33407.6	33453.2	33498.9	33544.6	33590.3	33635.9	33681.6	33727.3	33773.1	33818.8	33864.5	33910.3	22.84
	6	366	133956.	134001.8	134047.5	134093.3	134139.1	134184.8	134230.6	134276.4	134322.3	134368.1	134413.9	134459.7	134505.6	134551.4	134597.3	134643.1	22.91
	7	367	34689.	34734.9	34780.8	34826.7	34872.6	34918.5	34964.4	35010.3	35056.3	35102.2	35148.1	35194.1	35240.1	35286.0	35332.0	35378.0	22.97
	8	368	135424.	135470.0	135516.0	135562.0	135608.1	135654.1	135700.1	135746.2	135792.3	135838.3	135884.4	135930.5	135976.6	136022.7	136068.8	136114.9	23.03
	9	369	36161.	36207.1	36253.3	36299.4	36345.6	36391.7	36437.9	36484.1	36530.3	36576.4	36622.6	36668.8	36715.1	36761.3	36807.5	36853.8	23.09
	10	370	136900.	136946.3	136992.5	137038.8	137085.1	137131.3	137177.6	137223.9	137270.3	137316.6	137362.9	137409.2	137455.6	137501.9	137548.3	137594.6	23.16
	11	371	37641.	37687.4	37733.8	37780.2	37826.6	37873.0	37919.4	37965.8	38012.3	38058.7	38105.1	38151.6	38198.1	38224.5	38291.0	38337.5	23.22

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32b.—Squares of Inches, 372″ to 408″ (31′ 0″ to 34′ 0″), Advancing by 16ths.—Continued.

′	″	Ins.	1/16	1/8	3/16	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	+1/32
31	0	372	138384.	138430.5	138477.0	138523.5	138570.1	138616.6	138663.1	138709.7	138756.3	138802.8	138849.4	138896.0	138942.6	138989.2	139035.8	139082.4	23.28
	1	373	39129.	39175.6	39222.3	39268.9	39315.6	39362.2	39408.9	39455.6	39502.3	39548.9	39595.6	39642.3	39689.1	39735.8	39782.5	39829.3	23.34
	2	374	139876.	139922.8	139969.5	140016.3	140063.1	140109.8	140156.6	140203.4	140250.3	140297.1	140343.9	140390.7	140437.6	140484.4	140531.3	140578.1	23.41
	3	375	40625.	40671.9	40718.8	40765.7	40812.6	40859.5	40906.4	40953.3	41000.3	41047.2	41094.1	41141.1	41188.1	41235.0	41282.0	41329.0	23.47
	4	376	141376.	141423.0	141470.0	141517.0	141564.1	141611.1	141658.1	141705.2	141752.3	141799.3	141846.4	141893.5	141940.6	141987.7	142034.8	142081.9	23.53
	5	377	42129.	42176.1	42223.3	42270.4	42317.6	42364.7	42411.9	42459.1	42506.3	42553.4	42600.6	42647.8	42695.1	42742.3	42789.5	42836.8	23.59
	6	378	142884.	142931.3	142978.5	143025.8	143073.1	143120.3	143167.6	143214.9	143262.3	143309.6	143356.9	143404.2	143451.6	143498.9	143546.3	143593.6	23.66
	7	379	43641.	43688.4	43735.8	43783.2	43830.6	43878.0	43925.4	43972.8	44020.3	44067.7	44115.1	44162.6	44210.1	44257.5	44305.0	44352.5	23.72
	8	380	144400.	144447.5	144495.0	144542.5	144590.1	144637.6	144685.1	144732.7	144780.3	144827.8	144875.4	144923.0	144970.6	145018.2	145065.8	145113.4	23.78
	9	381	45161.	45208.6	45256.3	45303.9	45351.6	45399.2	45446.9	45494.6	45542.3	45589.9	45637.6	45685.3	45733.1	45780.8	45828.5	45876.3	23.84
	10	382	145924.	145971.8	146019.5	146067.3	146115.1	146162.8	146210.6	146258.4	146306.3	146354.1	146401.9	146449.7	146497.6	146545.4	146593.3	146641.1	23.91
	11	383	46689.	46736.9	46784.8	46832.7	46880.6	46928.5	46976.4	47024.3	47072.3	47120.2	47168.1	47216.1	47264.1	47312.0	47360.0	47408.0	23.97
32	0	384	147456.	147504.0	147552.0	147600.0	147648.1	147696.1	147744.1	147792.2	147840.3	147888.3	147936.4	147984.5	148032.6	148080.7	148128.8	148176.9	24.03
	1	385	48225.	48273.1	48321.3	48369.4	48417.6	48465.7	48513.9	48562.1	48610.3	48658.4	48706.6	48754.8	48803.1	48851.3	48899.5	48947.8	24.09
	2	386	148996.	149044.3	149092.5	149140.8	149189.1	149237.3	149285.6	149333.9	149382.3	149430.6	149478.9	149527.2	149575.6	149623.9	149672.3	149720.6	24.16
	3	387	49769.	49817.4	49865.8	49914.2	49962.6	50011.0	50059.4	50107.8	50156.3	50204.7	50253.1	50301.6	50350.1	50398.5	50447.0	50495.5	24.22
	4	388	150544.	150592.5	150641.0	150689.5	150738.1	150786.6	150835.1	150883.7	150932.3	150980.8	151029.4	151078.0	151126.6	151175.2	151223.8	151272.4	24.28
	5	389	51321.	51369.6	51418.3	51466.9	51515.6	51564.2	51612.9	51661.6	51710.3	51758.9	51807.6	51856.3	51905.1	51953.8	52002.5	52051.3	24.34
	6	390	152100.	152148.8	152197.5	152246.3	152295.1	152343.8	152392.6	152441.4	152490.3	152539.1	152587.9	152636.7	152685.6	152734.4	152783.3	152832.1	24.41
	7	391	52881.	52929.9	52978.8	53027.7	53076.6	53125.5	53174.4	53223.3	53272.3	53321.2	53370.1	53419.1	53468.1	53517.0	53566.0	53615.0	24.47
	8	392	153664.	153713.0	153762.0	153811.0	153860.1	153909.1	153958.1	154007.2	154056.3	154105.3	154154.4	154203.5	154252.6	154301.7	154350.8	154399.9	24.53
	9	393	54449.	54498.1	54547.3	54596.4	54645.6	54694.7	54743.9	54793.1	54842.3	54891.4	54940.6	54989.8	55039.1	55088.3	55137.5	55186.8	24.59
	10	394	155236.	155285.3	155334.5	155383.8	155433.1	155482.3	155531.6	155580.9	155630.3	155679.6	155728.9	155778.2	155827.6	155876.9	155926.3	155975.6	24.66
	11	395	56065.	56074.4	56123.8	56173.2	56222.6	56272.0	56321.4	56370.8	56420.3	56469.7	56519.1	56568.6	56618.1	56667.5	56717.0	56766.5	24.72
33	0	396	156816.	156865.5	156915.0	156964.5	157014.1	157063.6	157113.1	157162.7	157212.3	157261.8	157311.4	157361.0	157410.6	157460.2	157509.8	157559.4	24.78
	1	397	57609.	57658.6	57708.3	57757.9	57807.6	57857.2	57906.9	57956.6	58006.3	58055.9	58105.6	58155.3	58205.1	58254.8	58304.5	58354.3	24.84
	2	398	158404.	158453.8	158503.5	158553.3	158603.1	158652.8	158702.6	158752.4	158802.3	158852.1	158901.9	158951.7	159001.6	159051.4	159101.3	159151.1	24.91
	3	399	59201.	59250.9	59300.8	59350.7	59400.6	59450.5	59500.4	59550.3	59600.3	59650.2	59700.1	59750.1	59800.1	59850.0	59900.0	59950.0	24.97
	4	400	160000.	160050.0	160100.0	160150.0	160200.1	160250.1	160300.1	160350.2	160400.3	160450.3	160500.4	160550.5	160600.6	160650.7	160700.8	160750.9	25.03
	5	401	60801.	60851.1	60901.3	60951.4	61001.6	61051.7	61101.9	61152.1	61202.3	61252.4	61302.6	61352.8	61403.1	61453.3	61503.5	61553.8	25.09
	6	402	161604.	161654.3	161704.5	161754.8	161805.1	161855.3	161905.6	161955.9	162006.3	162056.6	162106.9	162157.2	162207.6	162257.9	162308.3	162358.6	25.16
	7	403	62409.	62459.4	62509.8	62560.2	62610.6	62661.0	62711.4	62761.8	62812.3	62862.7	62913.1	62963.6	63014.1	63064.5	63115.0	63165.5	25.22
	8	404	163216.	163266.5	163317.0	163367.5	163418.1	163468.6	163519.1	163569.7	163620.3	163670.8	163721.4	163772.0	163822.6	163873.2	163923.8	163974.4	25.28
	9	405	64025.	64075.6	64126.3	64176.9	64227.6	64278.2	64328.9	64379.6	64430.3	64480.9	64531.6	64582.3	64633.1	64683.8	64734.5	64785.3	25.34
	10	406	164836.	164886.8	164937.5	164988.3	165039.1	165089.8	165140.6	165191.4	165242.3	165293.1	165343.9	165394.7	165445.6	165496.4	165547.3	165598.1	25.41
	11	407	65649.	65699.9	65750.8	65801.7	65852.6	65903.5	65954.4	66005.3	66056.3	66107.2	66158.1	66209.1	66260.1	66311.0	66362.0	66413.0	25.47

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32b.—**Squares of Inches, 408″ to 444″ (34′ 0″ to 37′ 0″), Advancing by 16ths.—Continued.**

′	″	Ins.		1/16	1/8	3/16	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	+ 1/32
34	0	408	166464.	166515.0	166566.0	166617.0	166668.1	166719.1	166770.1	166821.2	166872.3	166923.3	166974.4	167025.5	167076.6	167127.7	167178.8	167229.9	25.53
	1	409	67281.	67332.1	67383.3	67434.4	67485.6	67536.7	67587.9	67639.1	67690.3	67741.4	67792.6	67843.8	67895.1	67946.3	67997.5	68048.8	25.59
	2	410	168100.	168151.3	168202.5	168253.8	168305.1	168356.3	168407.6	168458.9	168510.3	168561.6	168612.9	168664.2	168715.6	168766.9	168818.3	168869.6	25.66
	3	411	68921.	68972.4	69023.8	69075.2	69126.6	69178.0	69229.4	69280.8	69332.3	69383.7	69435.1	69486.6	69538.1	69589.5	69641.0	69692.5	25.72
	4	412	169744.	169795.5	169847.0	169898.5	169950.1	170001.6	170053.1	170104.7	170156.3	170207.8	170259.4	170311.0	170362.6	170414.2	170465.8	170517.4	25.78
	5	413	70569.	70620.6	70672.3	70723.9	70775.6	70827.2	70878.9	70930.6	70982.3	71033.9	71085.6	71137.3	71189.1	71240.8	71292.5	71344.3	25.84
	6	414	171396.	171447.8	171499.5	171551.3	171603.1	171654.8	171706.6	171758.4	171810.3	171862.1	171913.9	171965.7	172017.6	172069.4	172121.3	172173.1	25.91
	7	415	72225.	72276.9	72328.8	72380.7	72432.6	72484.5	72536.4	72588.3	72640.3	72692.2	72744.1	72796.1	72848.1	72900.0	72952.0	73004.0	25.97
	8	416	173056.	173108.0	173160.0	173212.0	173264.1	173316.1	173368.1	173420.2	173472.3	173524.3	173576.4	173628.5	173680.6	173732.7	173784.8	173836.9	26.03
	9	417	73889.	73941.1	73993.3	74045.4	74097.6	74149.7	74201.9	74254.1	74306.3	74358.4	74410.6	74462.8	74515.1	74567.3	74619.5	74671.8	26.09
	10	418	174724.	174776.3	174828.5	174880.8	174933.1	174985.3	175037.6	175089.9	175142.3	175194.6	175246.9	175299.2	175351.6	175403.9	175456.3	175508.6	26.16
	11	419	75561.	75613.4	75665.8	75718.2	75770.6	75823.0	75875.4	75927.8	75980.3	76032.7	76085.1	76137.6	76190.1	76242.5	76295.0	76347.5	26.22
35	0	420	176400.	176452.5	176505.0	176557.5	176610.1	176662.6	176715.1	176767.7	176820.3	176872.8	176925.4	176978.0	177030.6	177083.2	177135.8	177188.4	26.28
	1	421	77241.	77293.6	77346.3	77398.9	77451.6	77504.2	77556.9	77609.6	77662.3	77714.9	77767.6	77820.3	77873.1	77925.8	77978.5	78031.3	26.34
	2	422	178084.	178136.8	178189.5	178242.3	178295.1	178347.8	178400.6	178453.4	178506.3	178559.1	178611.9	178664.7	178717.6	178770.4	178823.3	178876.1	26.41
	3	423	78929.	78981.9	79034.8	79087.7	79140.6	79193.5	79246.4	79299.3	79352.3	79405.2	79458.1	79511.1	79564.1	79617.0	79670.0	79723.0	26.47
	4	424	179776.	179829.0	179882.0	179935.0	179988.1	180041.1	180094.1	180147.2	180200.3	180253.3	180306.4	180359.5	180412.6	180465.7	180518.8	180571.9	26.53
	5	425	80625.	80678.1	80731.3	80784.4	80837.6	80890.7	80943.9	80997.1	81050.3	81103.4	81156.6	81209.8	81263.1	81316.3	81369.5	81422.8	26.59
	6	426	181476.	181529.3	181582.5	181635.8	181689.1	181742.3	181795.6	181848.9	181902.3	181955.6	182008.9	182062.2	182115.6	182168.9	182222.3	182275.6	26.66
	7	427	82329.	82382.4	82435.8	82489.2	82542.6	82596.0	82649.4	82702.8	82756.3	82809.7	82863.1	82916.6	82970.1	83023.5	83077.0	83130.5	26.72
	8	428	183184.	183237.5	183291.0	183344.5	183398.1	183451.6	183505.1	183558.7	183612.3	183665.8	183719.4	183773.0	183826.6	183880.2	183933.8	183987.4	26.78
	9	429	84041.	84094.6	84148.3	84201.9	84255.6	84309.2	84362.9	84416.6	84470.3	84523.9	84577.6	84631.3	84685.1	84738.8	84792.5	84846.3	26.84
	10	430	184900.	184953.8	185007.5	185061.3	185115.1	185168.8	185222.6	185276.4	185330.3	185384.1	185437.9	185491.7	185545.6	185599.4	185653.3	185707.1	26.91
	11	431	85761.	85814.9	85868.8	85922.7	85976.6	86030.5	86084.4	86138.3	86192.3	86246.2	86300.1	86354.1	86408.1	86462.0	86516.0	86570.0	26.97
36	0	432	186624.	186678.0	186732.0	186786.0	186840.1	186894.1	186948.1	187002.2	187056.3	187110.3	187164.4	187218.5	187272.6	187326.7	187380.8	187434.9	27.03
	1	433	87489.	87543.1	87597.3	87651.4	87705.6	87759.7	87813.9	87868.1	87922.3	87976.4	88030.6	88084.8	88139.1	88193.3	88247.5	88301.8	27.09
	2	434	188356.	188410.3	188464.5	188518.8	188573.1	188627.3	188681.6	188735.9	188790.3	188844.6	188898.9	188953.2	189007.6	189061.9	189116.3	189170.6	27.16
	3	435	89225.	89279.4	89333.8	89388.2	89442.6	89497.0	89551.4	89605.8	89660.3	89714.7	89769.1	89823.6	89878.1	89932.5	89987.0	90041.5	27.22
	4	436	190096.	190150.5	190205.0	190259.5	190314.1	190368.6	190423.1	190477.7	190532.3	190586.8	190641.4	190696.0	190750.6	190805.2	190859.8	190914.4	27.28
	5	437	90969.	91023.6	91078.3	91132.9	91187.6	91242.2	91296.9	91351.6	91406.3	91460.9	91515.6	91570.3	91625.1	91679.8	91734.5	91789.3	27.34
	6	438	191844.	191898.8	191953.5	192008.3	192063.1	192117.8	192172.6	192227.4	192282.3	192337.1	192391.9	192446.7	192501.6	192556.4	192611.3	192666.1	27.41
	7	439	92721.	92775.9	92830.8	92885.7	92940.6	92995.5	93050.4	93105.3	93160.3	93215.2	93270.1	93325.1	93380.1	93435.0	93490.0	93545.0	27.47
	8	440	193600.	193655.0	193710.0	193765.0	193820.1	193875.1	193930.1	193985.2	194040.3	194095.3	194150.4	194205.5	194260.6	194315.7	194370.8	194425.9	27.53
	9	441	94481.	94536.1	94591.3	94646.4	94701.6	94756.7	94811.9	94867.1	94922.3	94977.4	95032.6	95087.8	95143.1	95198.3	95253.5	95308.8	27.59
	10	442	195364.	195419.3	195474.5	195529.8	195585.1	195640.3	195695.6	195750.9	195806.3	195861.6	195916.9	195972.2	196027.6	196082.9	196138.3	196193.6	27.66
	11	443	96249.	96304.4	96359.8	96415.2	96470.6	96526.0	96581.4	96636.8	96692.3	96747.7	96803.1	96858.6	96914.1	96969.5	97025.0	97080.5	27.72

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′	″	Ins.	1/16	1/8	3/16	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	+ 1/32
37	0	444	197136.1	197191.5	197247.0	197302.5	197358.1	197413.6	197469.1	197524.7	197580.3	197635.8	197691.4	197747.0	197802.6	197858.2	197913.8	197969.4	27.78
	1	445	98025.1	98080.6	98136.3	98191.9	98247.6	98303.2	98358.9	98414.6	98470.3	98525.9	98581.6	98637.3	98693.1	98748.8	98804.5	98860.3	27.84
	2	446	198916.1	198971.8	199027.5	199083.3	199139.1	199194.8	199250.6	199306.4	199362.3	199418.1	199473.9	199529.7	199585.6	199641.4	199697.3	199753.1	27.91
	3	447	99809.1	99864.9	99920.8	99976.7	200032.6	200088.5	200144.4	200200.3	200256.3	200312.2	200368.1	200424.1	200480.1	200536.0	200592.0	200648.0	27.97
	4	448	200704.0	200760.0	200816.0	200872.0	200928.1	200984.1	201040.1	201096.2	201152.3	201208.3	201264.4	201320.5	201376.6	201432.7	201488.8	201544.9	28.03
	5	449	01601.0	01657.1	01713.3	01769.4	01825.6	01881.7	01937.9	01994.1	02050.3	02106.4	02162.6	02218.8	02275.1	02331.3	02387.5	02443.8	28.09
	6	450	202500.2	202556.3	202612.5	202668.8	202725.1	202781.3	202837.6	202893.9	202950.3	203006.6	203062.9	203119.2	203175.6	203231.9	203288.3	203344.6	28.16
	7	451	03401.0	03457.4	03513.8	03570.2	03626.6	03683.0	03739.4	03795.8	03852.3	03908.7	03965.1	04021.6	04078.1	04134.5	04191.0	04247.5	28.22
	8	452	204304.0	204360.5	204417.0	204473.5	204530.1	204586.6	204643.1	204699.7	204756.3	204812.8	204869.4	204926.0	204982.6	205039.2	205095.8	205152.4	28.28
	9	453	05209.0	05265.6	05322.3	05378.9	05435.6	05492.2	05548.9	05605.6	05662.3	05718.9	05775.6	05832.3	05889.1	05945.8	06002.5	06059.3	28.34
	10	454	206116.0	206172.8	206229.5	206286.3	206343.1	206399.8	206456.6	206513.4	206570.3	206627.1	206683.9	206740.7	206797.6	206854.4	206911.3	206968.1	28.41
	11	455	07025.0	07081.9	07138.8	07195.7	07252.6	07309.5	07366.4	07423.3	07480.3	07537.2	07594.1	07651.1	07708.1	07765.0	07822.0	07879.0	28.47
38	0	456	207936.0	207993.0	208050.0	208107.0	208164.1	208221.1	208278.1	208335.2	208392.3	208449.3	208506.4	208563.5	208620.6	208677.7	208734.8	208791.9	28.53
	1	457	08849.0	08906.1	08963.3	09020.4	09077.6	09134.7	09191.9	09249.1	09306.3	09363.4	09420.6	09477.8	09535.1	09592.3	09649.5	09706.8	28.59
	2	458	209764.0	209821.3	209878.5	209935.8	209993.1	210050.3	210107.6	210164.9	210222.3	210279.6	210336.9	210394.2	210451.6	210508.9	210566.3	210623.6	28.66
	3	459	10681.0	10738.4	10795.8	10853.2	10910.6	10968.0	11025.4	11082.8	11140.3	11197.7	11255.1	11312.6	11370.1	11427.5	11485.0	11542.5	28.72
	4	460	211600.0	211657.5	211715.0	211772.5	211830.1	211887.6	211945.1	212002.7	212060.3	212117.8	212175.4	212233.0	212290.6	212348.2	212405.8	212463.4	28.78
	5	461	12521.0	12578.6	12636.3	12693.9	12751.6	12809.2	12866.9	12924.6	12982.3	13039.9	13097.6	13155.3	13213.1	13270.8	13328.5	13386.3	28.84
	6	462	213444.0	213501.8	213559.5	213617.3	213675.1	213732.8	213790.6	213848.4	213906.3	213964.1	214021.9	214079.7	214137.6	214195.4	214253.3	214311.1	28.91
	7	463	14369.0	14426.9	14484.8	14542.7	14600.6	14658.5	14716.4	14774.3	14832.3	14890.2	14948.1	15006.1	15064.1	15122.0	15180.0	15238.0	28.97
	8	464	215296.0	215354.0	215412.0	215470.0	215528.1	215586.1	215644.1	215702.2	215760.3	215818.3	215876.4	215934.5	215992.6	216050.7	216108.8	216166.9	29.03
	9	465	16225.1	16283.1	16341.3	16399.4	16457.6	16515.7	16573.9	16632.1	16690.3	16748.4	16806.6	16864.8	16923.1	16981.3	17039.5	17097.8	29.09
	10	466	217156.1	217214.3	217272.5	217330.8	217389.1	217447.3	217505.6	217563.9	217622.3	217680.6	217738.9	217797.2	217855.6	217913.9	217972.3	218030.6	29.16
	11	467	18089.1	18147.4	18205.8	18264.2	18322.6	18381.0	18439.4	18497.8	18556.3	18614.7	18673.1	18731.6	18790.1	18848.5	18907.0	18965.5	29.22
39	0	468	219024.0	219082.5	219141.0	219199.5	219258.1	219316.6	219375.1	219433.7	219492.3	219550.8	219609.4	219668.0	219726.6	219785.2	219843.8	219902.4	29.28
	1	469	19961.0	20019.6	20078.3	20136.9	20195.6	20254.2	20312.9	20371.6	20430.3	20488.9	20547.6	20606.3	20665.1	20723.8	20782.5	20841.3	29.34
	2	470	220900.0	220958.8	221017.5	221076.3	221135.1	221193.8	221252.6	221311.4	221370.3	221429.1	221487.9	221546.7	221605.6	221664.4	221723.3	221782.1	29.41
	3	471	21841.0	21899.9	21958.8	22017.7	22076.6	22135.5	22194.4	22253.3	22312.3	22371.2	22430.1	22489.1	22548.1	22607.0	22666.0	22725.0	29.47
	4	472	222784.0	222843.0	222902.0	222961.0	223020.1	223079.1	223138.1	223197.2	223256.3	223315.3	223374.4	223433.5	223492.6	223551.7	223610.8	223669.9	29.53
	5	473	23729.	23788.1	23847.3	23906.4	23965.6	24024.7	24083.9	24143.1	24202.3	24261.4	42320.6	24379.8	24439.1	24498.3	24557.5	24616.8	29.59
	6	474	224676.0	224735.3	224794.5	224853.8	224913.1	224972.3	225031.6	225090.9	225150.3	225209.6	225268.9	225328.2	225387.6	225446.9	225506.3	225565.6	29.66
	7	475	25625.	25684.4	25743.8	25803.2	25862.6	25922.0	25981.4	26040.8	26100.3	26159.7	26219.1	26278.6	26338.1	26397.5	26457.0	26516.5	29.72
	8	476	226576.	226635.5	226695.0	226754.5	226814.1	226873.6	226933.1	226992.7	227052.3	227111.8	227171.4	227231.0	227290.6	227350.2	227409.8	227469.4	29.78
	9	477	27529.	27588.6	27648.3	27707.9	27767.6	27827.2	27886.9	27946.6	28006.3	28065.9	28125.6	28185.3	28245.1	28304.8	28364.5	28424.3	29.84
	10	478	228484.	228543.8	228603.5	228663.3	228723.1	228782.8	228842.6	228902.4	228962.3	229022.1	229081.9	229141.7	229201.6	229261.4	229321.3	229381.1	29.91
	11	479	29441.	29500.9	29506.8	29620.7	29680.6	29740.5	29800.4	29860.3	29920.3	29980.2	30040.1	30100.1	30160.1	30220.0	30280.0	30340.0	29.97

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22b.—**Squares of Inches, 480″ to 516″ (40′ 0″ to 43′ 0″), Advancing by 16ths.—Continued.**

′	″	Ins.		1/16	1/8	3/16	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	+1/32
40	0	480	230400.	230460.0	230520.0	230580.0	230640.1	230700.1	230760.1	230820.2	230880.3	230940.3	231000.4	231060.5	231120.6	231180.7	231240.8	231300.9	30.03
	1	481	31361.	31421.1	31481.3	31541.4	31601.6	31661.7	31721.9	31782.1	31842.3	31902.4	31962.6	32022.8	32083.1	32143.3	32230.5	32263.8	30.09
	2	482	232324.	232384.3	232444.5	232504.8	232565.1	232625.3	232685.6	232745.9	232806.3	232866.6	232926.9	232987.2	233047.6	233107.9	233168.3	233228.6	30.16
	3	483	33289.	33349.4	33409.8	33470.2	33530.6	33591.0	33651.4	33711.8	33772.3	33832.7	33893.1	33953.6	34014.1	34074.5	34135.0	34195.5	30.22
	4	484	234256.	234316.5	234377.0	234437.5	234498.1	234558.6	234619.1	234679.7	234740.3	234800.8	234861.4	234922.0	234982.6	235043.2	235103.8	235164.4	30.28
	5	485	35225.	35285.6	35346.3	35406.9	35467.6	35528.2	35588.9	35649.6	35710.3	35770.9	35831.6	35892.3	35953.1	36013.8	36074.5	36135.3	30.34
	6	486	236196.	236256.8	236317.5	236378.3	236439.1	236499.8	236560.6	236621.4	236682.3	236743.1	236803.9	236864.7	236925.6	236986.4	237047.3	237108.1	30.41
	7	487	37169.	37229.9	37290.8	37351.7	37412.6	37473.5	37534.4	37595.3	37656.3	37717.2	37778.1	37839.1	37900.1	37961.0	38022.0	38083.0	30.47
	8	488	238144.	238205.0	238266.0	238327.0	238388.1	238449.1	238510.1	238571.2	238632.3	238693.3	238754.4	238815.5	238876.6	238937.7	238998.8	239059.9	30.53
	9	489	39121.	39182.1	39243.3	39304.4	39365.6	39426.7	39487.9	39549.1	39610.3	39671.4	39732.6	39793.8	39855.1	39916.3	39977.5	40038.8	30.59
	10	490	240100.	240161.3	240222.5	240283.8	240345.1	240406.3	240467.6	240528.9	240590.3	240651.6	240712.9	240774.2	240835.6	240896.9	240958.3	241019.6	30.66
	11	491	41081.	41142.4	41203.8	41265.2	41326.6	41388.0	41449.4	41510.8	41572.3	41633.7	41695.1	41756.6	41818.1	41879.5	41941.0	42002.5	30.72
41	0	492	242064.	242125.5	242187.0	242248.5	242310.1	242371.6	242433.1	242494.7	242556.3	242617.8	242679.4	242741.0	242802.6	242864.2	242925.8	242987.4	30.78
	1	493	43049.	43110.6	43172.3	43233.9	43295.6	43357.2	43418.9	43480.6	43542.3	43603.9	43665.6	43727.3	43789.1	43850.8	43912.5	43974.3	30.84
	2	494	244036.	244097.8	244159.5	244221.3	244283.1	244344.8	244406.6	244468.4	244530.3	244592.1	244653.9	244715.7	244777.6	244839.4	244901.3	244963.1	30.91
	3	495	45025.	45086.9	45148.8	45210.7	45272.6	45334.5	45396.4	45458.3	45520.3	45582.2	45644.1	45706.1	45768.1	45830.0	45892.0	45954.0	30.97
	4	496	246016.	246078.0	246140.0	246202.0	246264.1	246326.1	246388.1	246450.2	246512.3	246574.3	246636.4	246698.5	246760.6	246822.7	246884.8	246946.9	31.03
	5	497	47009.	47071.1	47133.3	47195.4	47257.6	47319.7	47381.9	47444.1	47506.3	47568.4	47630.6	47692.8	47755.1	47817.3	47879.5	47941.8	31.09
	6	498	248004.	248066.3	248128.5	248190.8	248253.1	248315.3	248377.6	248439.9	248502.3	248564.6	248626.9	248689.2	248751.6	248813.9	248876.3	248938.6	31.16
	7	499	49001.	49063.4	49125.8	49188.2	49250.6	49313.0	49375.4	49437.8	49500.3	49562.7	49625.1	49687.6	49750.1	49812.5	49875.0	49937.5	31.22
	8	500	250000.	250062.5	250125.0	250187.5	250250.1	250312.6	250375.1	250437.7	250500.3	250562.8	250625.4	250688.0	250750.6	250813.2	250875.8	250938.4	31.28
	9	501	51001.	51063.6	51126.3	51188.9	51251.6	51314.2	51376.9	51439.6	51502.3	51564.9	51627.6	51690.3	51753.1	51815.8	51878.5	51941.3	31.34
	10	502	252004.	252066.8	252129.5	252192.3	252255.1	252317.8	252380.6	252443.4	252506.3	252569.1	252631.9	252694.7	252757.6	252820.4	252883.3	252946.1	31.41
	11	503	53009.	53071.9	53134.8	53197.7	53260.6	53323.5	53386.4	53449.3	53512.3	53575.2	53638.1	53701.1	53764.1	53827.0	53890.0	53953.0	31.47
42	0	504	254016.	254079.0	254142.0	254205.0	254268.1	254331.1	254394.1	254457.2	254520.3	254583.3	254646.4	254709.5	254772.6	254835.7	254898.8	254961.9	31.53
	1	505	55025.	55088.1	55151.3	55214.4	55277.6	55340.7	55403.9	55467.1	55530.3	55593.4	55656.6	55719.8	55783.1	55846.3	55909.5	55972.8	31.59
	2	506	256036.	256099.3	256162.5	256225.8	256289.1	256352.3	256415.6	256478.9	256542.3	256605.6	256668.9	256732.2	256795.6	256858.9	256922.3	256985.6	31.66
	3	507	57049.	57112.4	57175.8	57239.2	57302.6	57366.0	57429.4	57492.8	57556.3	57619.7	57683.1	57746.6	57810.1	57873.5	57937.0	58000.5	31.72
	4	508	258064.	258127.5	258191.0	258254.5	258318.1	258381.6	258445.1	258508.7	258572.3	258635.8	258699.4	258763.0	258826.6	258890.2	258953.8	259017.4	31.78
	5	509	59081.	59144.6	59208.3	59271.9	59335.6	59399.2	59462.9	59526.6	59590.3	59653.9	59717.6	59781.3	59845.1	59908.8	59972.5	60036.3	31.84
	6	510	260100.	260163.8	260227.5	260291.3	260355.1	260418.8	260482.6	260546.4	260610.3	260674.1	260737.9	260801.7	260865.6	260929.4	260993.3	261057.1	31.91
	7	511	61121.	61184.9	61248.8	61312.7	61376.6	61440.5	61504.4	61568.3	61632.3	61696.2	61760.1	61824.1	61888.1	61952.0	62016.0	62080.0	31.97
	8	512	262144.	262208.0	262272.0	262336.0	262400.1	262464.1	262528.1	262592.2	262656.3	262720.3	262784.4	262848.5	262912.6	262976.7	263040.8	263104.9	32.03
	9	513	63169.	63233.1	63297.3	63361.4	63425.6	63489.7	63553.9	63618.1	63682.3	63746.4	63810.6	63874.8	63939.1	64003.3	64067.5	64131.8	32.09
	10	514	264196.	264260.3	264324.5	264388.8	264453.1	264517.3	264581.6	264645.9	264710.3	264774.6	264838.9	264903.2	264967.6	265031.9	265096.3	265160.6	32.16
	11	515	65225.	65289.4	65358.8	65418.2	65482.6	65547.0	65611.4	65675.8	65740.3	65804.7	65869.1	65933.6	65998.1	66062.5	66127.0	66191.5	32.22

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32b.—**Squares of Inches, 516″ to 552″ (43′ 0″ to 46′ 0″), Advancing by 16ths.—Continued.**

′	″	Ins.		1/16	1/8	3/16	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	+1/32
43	0	516	266256.	266320.5	266385.0	266449.5	266514.1	266578.6	266643.1	266707.7	266772.3	266836.8	266901.4	266966.0	267030.6	267095.2	267159.8	267224.4	32.28
	1	517	67289.	67353.6	67418.3	67482.9	67547.6	67612.2	67676.9	67741.6	67806.3	67870.9	67935.6	68000.3	68065.1	68129.8	68194.5	68259.3	32.34
	2	518	268324.	268388.8	268453.5	268518.3	268583.1	268647.8	268712.6	268777.4	268842.3	268907.1	268971.9	269036.7	269101.6	269166.4	269231.3	269296.1	32.41
	3	519	69361.	69425.9	69490.8	69555.7	69620.6	69685.5	69750.4	69815.3	69880.3	69945.2	70010.1	70075.1	70140.1	70205.0	70270.0	70335.0	32.47
	4	520	270400.	270465.0	270530.0	270595.0	270660.1	270725.1	270790.1	270855.2	270920.3	270985.3	271050.4	271115.5	271180.6	271245.7	271310.8	271375.9	32.53
	5	521	71441.	71506.1	71571.3	71636.4	71701.6	71766.7	71831.9	71897.1	71962.3	72027.4	72092.6	72157.8	72223.1	72288.3	72353.5	72418.8	32.59
	6	522	272484.	272549.3	272614.5	272679.8	272745.1	272810.3	272875.6	272940.9	273006.3	273071.6	273136.9	273202.2	273267.6	273332.9	273398.3	273463.6	32.66
	7	523	73529.	73594.4	73659.8	73725.2	73790.6	73856.0	73921.4	73986.8	74052.3	74117.7	74183.1	74248.6	74314.1	74379.5	74445.0	74510.5	32.72
	8	524	274576.	274641.5	274707.0	274772.5	274838.1	274903.6	274969.1	275034.7	275100.3	275165.8	275231.4	275297.0	275362.6	275428.2	275493.8	275559.4	32.78
	9	525	75625.	75690.6	75756.3	75821.9	75887.6	75953.2	76018.9	76084.6	76150.3	76215.9	76281.6	76347.3	76413.1	76478.8	76544.5	76610.3	32.84
	10	526	276676.	276741.8	276807.5	276873.3	276939.1	277004.8	277070.6	277136.4	277202.3	277268.1	277333.9	277399.7	277465.6	277531.4	277597.3	277663.1	32.91
	11	527	77729.	77794.9	77860.8	77926.7	77992.6	78058.5	78124.4	78190.3	78256.3	78322.2	78388.1	78454.1	78520.1	78586.0	78652.0	78718.0	32.97
44	0	528	278784.	278850.0	278916.0	278982.0	279048.1	279114.1	279180.1	279246.2	279312.3	279378.3	279444.4	279510.5	279576.6	279642.7	279708.8	279774.9	33.03
	1	529	79841.	79907.1	79973.3	80039.4	80105.6	80171.7	80237.9	80304.1	80370.3	80436.4	80502.6	80568.8	80635.1	80701.3	80767.5	80833.8	33.09
	2	530	280900.	280966.3	281032.5	281098.8	281165.1	281231.3	281297.6	281363.9	281430.3	281496.6	281562.9	281629.2	281695.6	281761.9	281828.3	281894.6	33.16
	3	531	81961.	82027.4	82093.8	82159.2	82226.6	82293.0	82359.4	82425.8	82492.3	82558.7	82625.1	82691.6	82758.1	82824.5	82891.0	82957.5	33.22
	4	532	283024.	283090.5	283157.0	283223.5	283290.1	283356.6	283423.1	283489.7	283556.3	283622.8	283689.4	283756.0	283822.6	283889.2	283955.8	284022.4	33.28
	5	533	84089.	84155.6	84222.3	84288.9	84355.6	84422.2	84488.9	84555.6	84622.3	84688.9	84755.6	84822.3	84889.1	84955.8	85022.5	85089.3	33.34
	6	534	285156.	285222.8	285289.5	285356.3	285423.1	285489.8	285556.6	285623.4	285690.3	285757.1	285823.9	285900.7	285967.6	286034.4	286101.3	286168.1	33.41
	7	535	86225.	86291.9	86358.8	86425.7	86492.6	86559.5	86626.4	86693.3	86760.3	86827.2	86894.1	86961.1	87028.1	87095.0	87162.0	87229.0	33.47
	8	536	287296.	287363.0	287430.0	287497.0	287564.1	287631.1	287698.1	287765.2	287832.3	287899.3	287966.4	288033.5	288100.6	288167.7	288234.8	288301.9	33.53
	9	537	88369.	88436.1	88503.3	88570.4	88637.6	88704.7	88771.9	88839.1	88906.3	88973.4	89040.6	89107.8	89175.1	89242.3	89309.5	89376.8	33.59
	10	538	289444.	289511.3	289578.5	289645.8	289713.1	289780.3	289847.6	289914.9	289982.3	290049.6	290116.9	290184.2	290251.6	290318.9	290386.3	290453.6	33.66
	11	539	90521.	90588.4	90655.8	90723.2	90790.6	90858.0	90925.4	90992.8	91060.3	91127.7	91195.1	91262.6	91330.1	91397.5	91465.0	91532.5	33.72
45	0	540	291600.	291667.5	291753.0	291802.5	291870.1	291937.6	292005.1	292072.7	292140.3	292207.8	292275.4	292343.0	292410.6	292478.2	292545.8	292613.4	33.78
	1	541	92681.	92748.6	92816.3	92883.9	92951.6	93019.2	93086.9	93154.6	93222.3	93289.9	93357.6	93425.3	93493.1	93560.8	93628.5	93696.3	33.84
	2	542	293764.	293831.8	293899.5	293967.3	294035.1	294102.8	294170.6	294238.4	294306.3	294374.1	294441.9	294509.7	294577.6	294645.4	294713.3	294781.1	33.91
	3	543	94849.	94916.9	94984.8	95052.7	95120.6	95188.5	95256.4	95324.3	95392.3	95460.2	95528.1	95596.1	95664.1	95732.0	95800.0	95868.0	33.97
	4	544	295936.	296004.0	296072.0	296140.0	296208.1	296276.1	296344.1	296412.2	296480.3	296548.3	296616.4	296684.5	296752.6	296820.7	296888.8	296956.9	34.03
	5	545	97025.	97093.1	97161.3	97229.4	97297.6	97365.7	97433.9	97502.1	97570.3	97638.4	97706.6	97774.8	97843.1	97911.3	97979.5	98047.8	34.09
	6	546	298116.	298184.3	298252.5	298320.8	298389.1	298457.3	298525.6	298593.9	298662.3	298730.6	298798.9	298867.2	298935.6	299003.9	299072.3	299140.6	34.16
	7	547	99209.	99277.4	99345.8	99414.2	99482.6	99551.0	99619.4	99687.8	99756.3	99824.7	99893.1	99961.6	300030.1	300098.5	300167.0	300235.5	34.22
	8	548	300304.	300372.5	300441.0	300509.5	300578.1	300646.6	300715.1	300783.7	300852.3	300920.8	300989.4	301058.0	301126.6	301195.2	301263.8	301332.4	34.28
	9	549	01401.	01469.6	01538.3	01606.9	01675.6	01744.2	01812.9	01881.6	01950.3	02018.9	02087.6	02156.3	02225.1	02293.8	02362.5	02431.3	34.34
	10	550	302500.	302568.8	302637.5	302706.3	302775.1	302843.8	302912.6	302981.4	303050.3	303119.1	303187.9	303256.7	303325.6	303394.4	303463.3	303532.1	34.41
	11	551	03601.	03669.9	03738.8	03807.7	03876.6	03945.5	04014.4	04083.3	04152.3	04221.2	04290.1	04359.1	04428.1	04497.0	04566.0	04635.0	34.47

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32B.—Squares of Inches, 552″ to 588″ (46′ 0″ to 49′ 0″), Advancing by 16ths.—Continued.

′	″	Ins.	1/16	1/8	3/16	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	+1/32
46	0	552	304704.0	304773.0	304842.0	304911.0	304980.1	305049.1	305118.1	305187.2	305256.3	305325.3	305394.4	305463.5	305532.6	305601.7	305670.8	305739.9	34.53
	1	553	05809.	05878.1	05947.3	06016.4	06085.6	06154.7	06223.9	06293.1	06362.3	06431.4	06500.6	06569.8	06639.1	06708.3	06777.5	06846.8	34.59
	2	554	306916.	306985.3	307054.5	307123.8	307193.1	307262.3	307331.6	307400.9	307470.3	307539.6	307608.9	307678.2	307747.6	307816.9	307886.3	307955.6	34.66
	3	555	08025.	08094.4	08163.8	08233.2	08302.6	08372.0	08441.4	08510.8	08580.3	08649.7	08719.1	08788.6	08858.1	08927.5	08997.0	09066.5	34.72
	4	556	309136.	309205.5	309275.0	309344.5	309414.1	309483.6	309553.1	309662.7	309692.3	309761.8	309831.4	309901.0	309970.6	310040.2	310109.8	310179.4	34.78
	5	557	10249.	10318.6	10388.3	10457.9	10527.6	10597.2	10666.9	10736.6	10806.3	10875.9	10945.6	11015.3	11085.1	11154.8	11224.5	11294.3	34.84
	6	558	311364.	311433.8	311503.5	311573.3	311643.1	311712.8	311782.6	311852.4	311922.3	311992.1	312061.9	312131.7	312201.6	312271.4	312341.3	312411.1	34.91
	7	559	12481.	12550.9	12620.8	12690.7	12760.6	12830.5	12900.4	12970.3	13040.3	13110.2	13180.1	13250.1	13320.1	13390.0	13460.0	13530.0	34.97
	8	560	313600.	313670.0	313740.0	313810.0	313880.1	313950.1	314020.1	314090.2	314160.3	314230.3	314300.4	314370.5	314440.6	314510.7	314580.8	314650.9	35.03
	9	561	14721.	14791.1	14861.3	14931.4	15001.6	15071.7	15141.9	15212.1	15282.3	15352.4	15422.6	15492.8	15563.1	15633.3	15703.5	15773.8	35.09
	10	562	315844.	315914.3	315984.5	316054.8	316125.1	316195.3	316265.6	316335.9	316406.3	316476.6	316546.9	316617.2	316687.6	316757.9	316828.3	316898.6	35.16
	11	563	16969.	17039.4	17109.8	17180.2	17250.6	17321.0	17391.4	17461.8	17532.3	17602.7	17673.1	17743.6	17814.1	17884.5	17955.0	18025.5	35.22
47	0	564	318096.	318166.5	318237.0	318307.5	318378.1	318448.6	318519.1	318589.7	318660.3	318730.8	318801.4	318872.0	318942.6	319013.2	319083.8	319154.4	35.28
	1	565	19225.	19295.6	19366.3	19436.9	19507.6	19578.2	19648.9	19719.6	19790.3	19860.9	19931.6	20002.3	20073.1	20143.8	20214.5	20285.3	35.34
	2	566	320356.	320426.8	320497.5	320568.3	320639.1	320709.8	320780.6	320851.4	320922.3	320993.1	321063.9	321134.7	321205.6	321276.4	321347.3	321418.1	35.41
	3	567	21489.	21559.9	21630.8	21701.7	21772.6	21843.5	21914.4	21985.3	22056.3	22127.2	22198.1	22269.1	22340.1	22411.0	22482.0	22553.0	35.47
	4	568	322624.	322695.0	322766.0	322837.0	322908.1	322979.1	323050.1	323121.2	323192.3	323263.3	323334.4	323405.5	323476.6	323547.7	323618.8	323689.9	35.53
	5	569	23761.	23832.1	23903.3	23974.4	24045.6	24116.7	24187.9	24259.1	24330.3	24401.4	24472.6	24543.8	24615.1	24686.3	24757.5	24828.8	35.59
	6	570	324900.	324971.3	325042.5	325113.8	325185.1	325256.3	325327.6	325398.9	325470.3	325541.6	325612.9	325684.2	325755.6	325826.9	325898.3	325969.6	35.66
	7	571	26041.	26112.4	26183.8	26255.2	26326.6	26398.0	26469.4	26540.8	26612.3	26683.7	26755.1	26826.6	26898.1	26969.5	27041.0	27112.5	35.72
	8	572	327184.	327255.5	327327.0	327398.5	327470.1	327541.6	327613.1	327684.7	327756.3	327827.8	327899.4	327971.0	328042.6	328114.2	328185.8	328257.4	35.78
	9	573	28329.	28400.6	28472.3	28543.9	28615.6	28687.2	28758.9	28830.6	28902.3	28973.9	29045.6	29117.3	29189.1	29260.8	29332.5	29404.3	35.84
	10	574	329476.	329547.8	329619.5	329691.3	329763.1	329834.8	329906.6	329978.4	330050.3	330122.1	330193.9	330265.7	330337.6	330409.4	330481.3	330553.1	35.91
	11	575	30625.	30696.9	30768.8	30840.7	30912.6	30984.5	31056.4	31128.3	31200.3	31272.2	31344.1	31416.1	31488.1	31560.0	31632.0	31704.0	35.97
48	0	576	331776.	331848.0	331920.0	331992.0	332064.1	332136.1	332208.1	332280.2	332352.3	332424.3	332496.4	332568.5	332640.6	332712.7	332784.8	332856.9	36.03
	1	577	32929	33001.1	33073.3	33145.4	33217.6	33289.7	33361.9	33434.1	33506.3	33578.4	33650.6	33722.8	33795.1	33867.3	33939.5	34011.8	36.09
	2	578	334084.	334156.3	334228.5	334300.8	334373.1	334445.3	334517.6	334589.9	334662.3	334734.6	334806.9	334879.2	334951.6	335023.9	335096.3	335168.6	36.16
	3	579	35241.	35313.4	35385.8	35458.2	35530.6	35603.0	35675.4	35747.8	35820.3	35892.7	35965.1	36037.6	36110.1	36182.5	36255.0	36327.5	36.22
	4	580	336400.	336472.5	336545.0	336617.5	336690.1	336762.6	336835.1	336907.7	336980.3	337052.8	337125.4	337198.0	337270.6	337343.2	337415.8	337488.4	36.28
	5	581	37561.	37633.6	37706.3	37778.9	37851.6	37924.2	37996.9	38069.6	38142.3	38214.9	38287.6	38360.3	38433.1	38505.8	38578.5	38651.3	36.34
	6	582	338724.	338796.8	338869.5	338942.3	339015.1	339087.8	339160.6	339233.4	339306.3	339379.1	339451.9	339524.7	339597.6	339670.4	339743.3	339816.1	36.41
	7	583	39889.	39961.9	40034.8	40107.7	40180.6	40253.5	40326.4	40399.3	40472.3	40545.2	40618.1	40691.1	40764.1	40837.0	40910.0	40983.0	36.47
	8	584	341056.	341129.0	341202.0	341275.0	341348.1	341421.1	341494.1	341567.2	341640.3	341713.3	341786.4	341859.5	341932.6	342005.7	342078.8	342151.9	36.53
	9	585	42225.	42298.1	42371.3	42444.4	42517.6	42590.7	42663.9	42737.1	42810.3	42883.4	42956.6	43029.8	43103.1	43176.3	43249.5	43322.8	36.59
	10	586	343396.	343469.3	343542.5	343615.8	343689.1	343762.3	343835.6	343908.9	343982.3	344055.6	344128.9	344202.2	344275.6	344348.9	344422.3	344495.6	36.66
	11	587	44569.	44642.4	44715.8	44789.2	44862.6	44936.0	45009.4	45082.8	45156.3	45229.7	45303.1	45376.6	45450.1	45523.5	45597.0	45670.5	36.72

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32b.—Squares of Inches, 588″ to 624″ (49′ 0″ to 52′ 0″), Advancing by 16ths.—Concluded.

′	″	Ins.		1/16	1/8	3/16	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	+ 1/32
49	0	588	345744.	345817.5	345891.0	345964.5	346038.1	346111.6	346185.1	346258.7	346332.3	346405.8	346479.4	346553.0	346626.6	346700.2	346773.8	346847.4	36.78
	1	589	46921.	46994.6	47068.3	47141.9	47215.6	47289.2	47362.9	47436.6	47510.3	47583.9	47657.6	47731.3	47805.1	47878.8	47952.5	48026.3	36.84
	2	590	348100.	348173.8	348247.5	348321.3	348395.1	348468.8	348542.6	348616.4	348690.3	348764.1	348837.9	348911.7	348985.6	349059.4	349133.3	349207.1	36.91
	3	591	49251.	49324.8	49398.7	49472.6	49546.5	49620.5	49694.4	49768.3	49842.3	49916.2	49990.2	50064.1	50138.1	50212.0	50286.0	50359.9	36.97
	4	592	350464.	350538.0	350612.0	350686.0	350760.1	350834.1	350908.1	350982.2	351056.3	351130.3	351204.4	351278.5	351352.6	351426.7	351500.8	351574.9	37.03
	5	593	51649.	51723.1	51797.3	51871.4	51945.6	52019.7	52093.9	52168.1	52242.3	52316.4	52390.6	52464.8	52539.1	52613.3	52687.5	52761.8	37.09
	6	594	352836.	352910.3	352984.5	353058.8	353133.1	353207.3	353281.6	353355.9	353430.3	353504.6	353578.9	353653.2	353727.6	353801.9	353876.3	353950.6	37.16
	7	595	54025.	54099.4	54173.8	54248.2	54322.6	54397.0	54471.4	54545.8	54620.3	54694.7	54769.1	54843.6	54918.1	54992.5	55067.0	55141.5	37.22
	8	596	355216.	355290.5	355365.0	355439.5	355514.1	355588.6	355663.1	355737.7	355812.3	355886.8	355961.4	356036.0	356110.6	356185.2	356259.8	356334.4	37.28
	9	597	56409.	56483.6	56558.3	56632.9	56707.6	56782.2	56856.9	56931.6	57006.3	57080.9	57155.6	57230.3	57305.1	57379.8	57454.5	57529.3	37.34
	10	598	357604.	357678.8	357753.5	357828.3	357903.1	357977.8	358052.6	358127.4	358202.3	358277.1	358351.9	358426.7	358501.6	358576.4	358651.3	358726.1	37.41
	11	599	58801.	58875.9	58950.8	59025.7	59100.6	59175.5	59250.4	59325.3	59400.3	59475.2	59550.1	59625.1	59700.1	59775.0	59850.0	59925.0	37.47
50	0	600	360000.	360075.0	360150.0	360225.0	360300.1	360375.1	360450.1	360525.2	360600.3	360675.3	360750.4	360825.5	360900.6	360975.7	361050.8	361125.9	37.53
	1	601	61201.	61276.1	61351.3	61426.4	61501.6	61576.7	61651.9	61727.1	61802.3	61877.4	61952.6	62027.8	62103.1	62178.3	62253.5	62328.8	37.59
	2	602	362404.	362479.3	362554.5	362629.8	362705.1	362780.3	362855.6	362930.9	363006.3	363081.6	363156.9	363232.2	363307.6	363382.9	363458.3	363533.6	37.66
	3	603	63609.	63684.4	63759.8	63835.2	63910.6	63986.0	64061.4	64136.8	64212.3	64287.7	64363.1	64438.6	64514.1	64589.5	64665.0	64740.5	37.72
	4	604	364816.	364891.5	364967.0	365042.5	365118.1	365193.6	365269.1	365344.7	365420.3	365495.8	365571.4	365647.0	365722.6	365798.2	365873.8	365949.4	37.78
	5	605	66025.	66100.6	66176.3	66251.9	66327.6	66403.2	66478.9	66554.6	66630.3	66705.9	66781.6	66857.3	66933.1	67008.8	67084.5	67160.3	37.84
	6	606	367236.	367311.8	367387.5	367463.3	367539.1	367614.8	367690.6	367766.4	367842.3	367918.1	367993.9	368069.7	368145.6	368221.4	368297.3	368373.1	37.91
	7	607	68449.	68524.9	68600.8	68676.7	68752.6	68828.5	68904.4	68980.3	69056.3	69132.2	69208.1	69284.1	69360.1	69436.0	69512.0	69588.0	37.97
	8	608	369664.	369740.0	369816.0	369892.0	369968.1	370044.1	370120.1	370196.2	370272.3	370348.3	370424.4	370500.5	370576.6	370652.7	370728.8	370804.9	38.03
	9	609	70881.	70957.1	71033.3	71109.4	71185.6	71261.7	71337.9	71414.1	71490.3	71566.4	71642.6	71718.8	71795.1	71871.3	71947.5	72023.8	38.09
	10	610	372100.	372176.3	372252.5	372328.8	372405.1	372481.3	372557.6	372633.9	372710.3	372786.6	372862.9	372939.2	373015.6	373091.9	373168.3	373244.6	38.16
	11	611	73321.	73397.4	73473.8	73550.2	73626.6	73703.0	73779.4	73855.8	73932.3	74008.7	74085.1	74161.6	74238.1	74314.5	74391.0	74467.5	38.22
51	0	612	374544.	374620.5	374697.0	374773.5	374850.1	374926.6	375003.1	375079.7	375156.3	375232.8	375309.4	375386.0	375462.6	375539.2	375615.8	375692.4	38.28
	1	613	75769.	75845.6	75922.3	75998.9	76075.6	76152.2	76228.9	76305.6	76382.3	76458.9	76535.6	76612.3	76689.1	76765.8	76842.5	76919.3	38.34
	2	614	376996.	377072.8	377149.5	377226.3	377303.1	377379.8	377456.6	377533.4	377610.3	377687.1	377763.9	377840.7	377917.6	377994.4	378071.3	378148.1	38.41
	3	615	78225.	78301.9	78378.8	78455.7	78532.6	78609.5	78686.4	78763.3	78840.3	78917.2	78994.1	79071.1	79148.1	79225.0	79302.0	79379.0	38.47
	4	616	379456.	379533.0	379610.0	379687.0	379764.1	379841.1	379918.1	379995.2	380072.3	380149.3	380226.4	380303.5	380380.6	380457.7	380534.8	380611.9	38.53
	5	617	80689.	80766.1	80843.3	80920.4	80997.6	81074.7	81151.9	81229.1	81306.3	81383.4	81460.6	81537.8	81615.1	81692.3	81769.5	81846.8	38.59

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EXCERPTS AND REFERENCES.

Direct Method of Spacing Rivets and Finding Position, etc., of Stiffeners in Plate Girders (By E. Schmitt. Trans. A. S. C. E., Vol. XLV).

Diagrams for Determining Minimum Alternate Spacing of Rivets (By F. L. Batchelder. Eng. News, Oct. 31, 1901).—This diagram is very simple to construct for any diameter of rivets, and is based on the general equation, $a^2 = x^2 + y^2$.

A Table for Pitch and Efficiency of Riveted Joints (By P. B. Hill. Eng. News, July 16, 1903).—For various thicknesses of plates and diameters of rivets.

Standard Heads for Machine Screws (By H. G. Reist. Eng. News, June 29, 1905).—Illustrations and table of dimensions of screws with round head, fillister head, flat head and hexagon head. (Also see Eng. News, Dec. 28, 1905, for table of standard threads for machine screws and taps, for ⅜" diameter and less; also, Eng. News, June 20, 1907, for A. S. M. E. standards.)

Tension-Tests of Steel Angles with Various Types of End-Connection (By F. P. McKibben. Proc. A. S. T. M., 1906; Eng. News, July 5, 1906).—Table shows percent strength of material developed: usually from 75 to 80%. (See, also, Eng. News, Aug. 22, 1907.)

Cost of Shop Drawings for Structural Iron and Steel (By R. H. Gage. "The Technograph," Univ. of Ill., No. 21, 1906–7; Eng. News, Aug. 8, 1907).—Condensed as follows:

Type.	Character of Building.	Av. Cost per Ton.
A.	Entire skel. cons.; loads all carried to found'n by steel columns	$1.45
B.	Exterior supported on steel cols.; floor loads carried by exter. walls	1.22
C.	Inter. portion sup. by cast iron cols.; fl. lds. by exter. walls	.70
D.	No cols.; floor beams resting on masonry walls throughout	.85
E.	Structure consisting mostly of roof trusses resting on columns	2.47
F.	Structure consisting mostly of roof trusses resting on mas. walls	1.25
G.	Mill buildings	2.56
H.	Flat one-story shop or manufacturing buildings	.74
I.	Tipples, mining- or other complicated structures	4.88
J.	Malt or grain bins and hoppers	2.47
K.	Remodeling and additions where measurements are necessary before details can be made	1.87

The Detailing of Skew Portals (By J. P. Davies. Eng. News, Feb. 11, 1909).—Formulas, diagrams and shop detail drawings.

Diagrams for Rivet Pitch in Loaded Girder Flanges (By P. L. Pratley. Eng. News, Feb. 18, 1909).—Formulas and diagrams.

Tests of Nickel Steel Riveted Joints (By A. N. Talbot. Eng. Rec., Aug. 20, 1910).—Gives details and results of tests made for the Board of Engineers of the Quebec Bridge. Illustrations of the joints tested, and tables of the results of the tests. Table 3 (not reproduced here) gives the ultimate strength of the nickel steel joints, and of the carbon steel joints (tested in 1905 by Am. Ry. Eng. & M. W. Assn.) reported in lbs. per sq. in. of the shearing area of the rivets. The average for the nickel steel joints is seen to be about 16% greater than for the carbon steel joints. It should be noted that the rivets of the nickel steel joints are considerably weaker than the plates and that the failure of these joints was due in all cases to shear of rivets, while in the carbon steel joints there was evidently less inequality between the strength of rivets and of plates, a number of joints failing by tearing the plate.

Formulas for Use in Detailing Steel Structures (By H. Vance. Eng. News, Sept. 1, 1910).—Examples: Hoppers, towers and oblique bends in riveted pipe.

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34.—METAL GAGES.

1.—Standard Gages.*

† No. of Gage	Thickness in Decimals of an Inch.						
	Birmingham (B.W.G.)	Brown & Sharpe B. & S.	1893 United States Standard Plate Iron and Steel	1894 British Imperial	American Steel & Wire Co.	Trenton Iron Co.	Stubs Steel Wire
7°	.....	.....	.500	.500	.49	.....	.....
6°	.....	.....	.46875	.464	.46	.....	.....
5°	.....	.....	.4375	.432	.43	.45	.....
4°	.454	.46	.40625	.400	.3938	.40	.....
3°	.425	.40964	.375	.372	.3625	.36	.....
2°	.380	.3648	.34375	.348	.3310	.33	.....
0	.340	.32486	.3125	.324	.3065	.305	.....
1	.300	.2893	.28125	.300	.2830	.285	.227
2	.284	.25763	.265625	.276	.2625	.265	.219
3	.259	.22942	.25	.252	.2437	.245	.212
4	.238	.20431	.234375	.232	.2253	.225	.207
5	.220	.18194	.21875	.212	.2070	.205	.204
6	.203	.16202	.203125	.192	.1920	.190	.201
7	.180	.14428	.1875	.176	.1770	.175	.199
8	.165	.12849	.171875	.160	.1620	.160	.197
9	.148	.11443	.15625	.144	.1483	.145	.194
10	.134	.10189	.140625	.128	.1350	.130	.191
11	.120	.090742	.125	.116	.1205	.1175	.188
12	.109	.080808	.109375	.104	.1055	.1050	.185
13	.095	.071961	.09375	.092	.0915	.0925	.182
14	.083	.064084	.078125	.080	.0800	.0800	.180
15	.072	.057068	.0703125	.072	.0720	.0700	.178
16	.065	.05082	.0625	.064	.0625	.0610	.175
17	.058	.045257	.05625	.056	.0540	.0525	.172
18	.049	.040303	.05	.048	.0475	.0450	.168
19	.042	.03589	.04375	.040	.0410	.0400	.164
20	.035	.031961	.0375	.036	.0348	.0350	.161
21	.032	.028462	.034375	.032	.03175	.0310	.157
22	.028	.025347	.03125	.028	.0286	.0280	.155
23	.025	.022571	.028125	.024	.0258	.0250	.153
24	.022	.0201	.025	.022	.0230	.0225	.151
25	.020	.0179	.021875	.020	.0204	.0200	.148
26	.018	.01594	.01875	.018	.0181	.0180	.146
27	.016	.014195	.0171875	.0164	.0173	.0170	.143
28	.014	.012641	.015625	.0148	.0162	.0160	.139
29	.013	.011257	.0140625	.0136	.0150	.0150	.134
30	.012	.010025	.0125	.0124	.0140	.0140	.127
31	.010	.008928	.0109375	.0116	.0132	.0130	.120
32	.009	.00795	.01015625	.0108	.0128	.0120	.115
33	.008	.00708	.009375	.0100	.0118	.0110	.112
34	.007	.006304	.00859375	.0092	.0104	.0100	.110
35	.005	.005614	.0078125	.0084	.0095	.0095	.108
36	.004	.005	.00703125	.0076	.0090	.0090	.106
37	.....	.004453	.006640625	.0068	.0085	.0085	.103
38	.....	.003965	.00625	.0060	.008	.0080	.101
39	.....	.003531	.....	.0052	.0075	.0075	.099
40	.....	.003144	.....	.0048	.007	.0070	.097
	Stubs' Iron Wire	American			Roebling, Washburn & Moen		

* See also Table 3, Sec. 35, Cordage, Wire and Cables, page (includes the Edison Gage. † 7° means 0000000, 2° means

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—UNITED STATES STANDARD (JULY 1, 1893) GAGE
For Sheet and Plate Iron and Steel.
as Standard by American Railway Master Mechanics Asso-
ssociation of American Steel Manufacturers.)

4	5	6	7	8	9	10	11	12	13
e Thickness	Weight in Ounces per Sq. Ft.	Weight in Pounds per Square Foot		Weight in Kilograms per Square Meter		Weight in Kilograms per Sq. Foot		Weight in Pounds per Square Meter	
Millimeters	Iron	*Iron	Steel	*Iron	Steel	*Iron	Steel	*Iron	Steel
12.7	320	20	20.4	97.648	99.601	9.072	9.253	215.28	219.58
11.90625	300	18.75	19.125	91.545	93.376	8.505	8.675	201.82	205.86
11.1125	280	17.50	17.85	85.442	87.151	7.938	8.097	188.37	192.14
10.31875	260	16.25	16.575	79.339	80.926	7.371	7.518	174.91	178.41
9.525	240	15	15.3	73.236	74.701	6.804	6.940	161.46	167.69
8 73125	220	13.75	14.025	67.133	68.476	6.237	6.362	148.00	150.96
7.9375	200	12.50	12.75	61.030	62.251	5.670	5.783	134.55	137.24
7.14375	180	11.25	11.475	54.927	56.026	5.103	5.205	121.09	123.52
6.746875	170	10.625	10.8375	51.876	52.913	4.819	4.916	114.37	116.65
6.35	160	10.	10.2	48.824	49.800	4.536	4.627	107.64	109.79
5.953125	150	9.375	9.5625	45.773	46.688	4.252	4.338	100.91	102.93
5.55625	140	8.75	8.925	42.721	43.575	3.969	4.048	94.18	96.07
5.159375	130	8.125	8.2875	39.670	40.463	3.685	3.759	87.45	89.21
4.7625	120	7.5	7.65	36.618	37.350	3.402	3.470	80.72	82.34
4.365625	110	6.875	7.0125	33.567	34.238	3.118	3.181	74.00	75.48
3.96875	100	6.25	6.375	30.515	31.125	2.835	2.892	67.27	68.62
3.571875	90	5.625	5.7375	27.464	28.013	2.552	2.603	60.55	61.76
3.175	80	5.	5.1	24.412	24.900	2.268	2.313	53.82	54.90
2.778125	70	4.375	4.4625	21.361	21.788	1.984	2.024	47.09	48.03
2.38125	60	3.75	3.825	18.309	18.675	1.701	1.735	40.36	41.17
1.984375	50	3.125	3.1875	15.258	15.563	1.417	1.446	33.64	34.31
1.7859375	45	2.8125	2.86875	13.732	14.006	1.276	1.301	30.27	30.88
1.5875	40	2.5	2.55	12.206	12.450	1.134	1.157	26.91	27.45
1.42875	36	2.25	2.295	10.985	11.205	1.021	1.041	24.22	24.70
1.27	32	2.	2.04	9.765	9.960	.907	.925	21.53	21.96
1.11125	28	1.75	1.785	8.544	8.715	.794	.810	18.84	19.21
0.9525	24	1.50	1.53	7.324	7.470	.680	.694	16.15	16.47
0.873125	22	1.375	1.4025	6.713	6.848	.624	.636	14.80	15.10
0.79375	20	1.25	1.275	6.103	6.225	.567	.578	13.46	13.72
0.714375	18	1.125	1.1475	5.493	5.603	.510	.520	12.11	12.35
0.635	16	1.	1.02	4.882	4.980	.454	.463	10.76	10.98
0.555625	14	.875	.8925	4.272	4.358	.397	.405	9.42	9.61
0.47625	12	.75	.765	3.662	3.735	.340	.347	8.07	8.23
0.4365625	11	.6875	.70125	3.357	3.424	.312	.318	7.40	7.55
0.396875	10	.625	.6375	3.052	3.113	.284	.289	6.73	6.86
0.3571875	9	.5625	.57375	2.746	2.801	.255	.260	6.05	6.18
0.3175	8	.5	.51	2.441	2.490	.227	.231	5.38	5.49
0.2778125	7	.4375	.44625	2.136	2.179	.198	.202	4.71	4.80
0.25796875	6.5	.40625	.414375	1.983	2.023	.184	.188	4.37	4.46
0.238125	6	.375	3825	1.831	1.868	.170	.174	4.04	4.12
0.21828125	5.5	.34375	.350625	1.678	1.712	.156	.159	3.70	3.77
0.1984375	5	.3125	.31875	1.526	1.556	.142	.145	3.36	3.43
0.17859375	4.5	.28125	.286875	1.373	1.401	.128	.130	3.03	3.09
0.168671875	4.25	.265625	.2709375	1.297	1.323	.120	.123	2.87	2.92
0.15875	4	.25	.255	1.221	1.245	.113	.116	2.69	2.74

olumn 1, 7° means 0000000, 2° means 00, etc. The thickness of metal
2, 3 and 4 is based on the weight of *iron* in avoirdupois ounces per
in column 5, assuming the weight of iron at 480 lbs. per cubic foot.
self-explanatory, steel being assumed 2 per cent heavier than iron.
per cent either way from the above weights is usually allowed.
ds per square foot to kilograms per square meter, mult. by 4.8824.
ds per square foot to kilograms per square foot, mult. by 0.4536.
ds per square foot to pounds per square meter, mult. by 10.7639.
ltiply the tabulated weight of iron by 1.133; for brass, mult. by 1.07;
9; for lead, mult. by 1.47; for aluminum, mult. by 0.347. All some-
See also Standard Gages, Table 1, preceding.

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35.—CORDAGE, WIRE AND CABLES.

Technical Cordage Terms.

Make Up (in Manufacture):

Yarn.—Fibers twisted together.

Marline.—Two yarns twisted together.

Thread.—Two or more small yarns twisted together.

Cord.—Several threads twisted together.

String.—Two or more slightly larger yarns twisted together.

Strand.—Three (by some authorities two) or more large yarns twisted together.

Rope.—Several strands twisted together.

Hawser.—Large rope of three strands.

Shroud laid.—Rope of four strands (with a heart).

Cable.—Three hawsers twisted together (left handed).

A Rope Is:

Laid—By twisting strands together in making the rope.
Spliced—By joining to another rope by interweaving the strands.
Whipped—By winding yarn or small stuff around the end to prevent unstranding.
Served—When wound tightly or continuously with yarn or small stuff.
Parceled—When served or wrapped tightly with canvas.
Seized—When two parts are bound tightly together by yarn or small stuff.
Payed—When painted, tarred or greased to resist wet.

Practical Operation:

Haul.—To pull on a rope.
Taut.—Drawn tight or strained.
Bight.—A loop in the rope.
Fall.—The rope in a hoisting tackle.
Tackle.—An assemblage of ropes and blocks.

Hitch.—Attaching a rope to an object.
Bend.—Attaching two ropes together or to an object.
Knot.—A loop or fastening with a rope.

Knots, Hitches, etc.—(See *Manila Rope*, next page.)

(Note that Ends are whipped to prevent unstranding.)

Fig. 1, Bight.

Fig. 2, Simple Knot.

Fig. 3, Figure 8 Knot.

Fig. 4, Double Knot.

Fig. 5, Boat Knot.

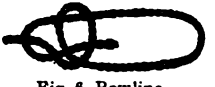

Fig. 6, Bowline.

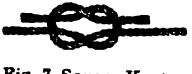

Fig. 7, Square Knot.

Fig. 8, Weaver's Knot.

Fig. 9, Toggle.

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Fig. 10, Carrick Bend.

Fig. 11, Stevedore Knot.

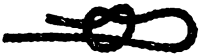

Fig. 12, Slip Knot.

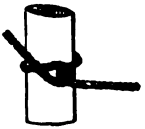

Fig. 13, Half Hitch.

Fig. 14, Timber Hitch.

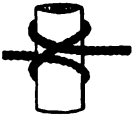

Fig. 15, Clove Hitch.

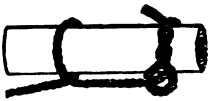

Fig. 16, Timber Hitch and Half Hitch.

Fig. 17, Round Turn and Half Hitch.

Fig. 18, Blackwall Hitch.

Fig. 19, Fisherman's Bend.

Splices.—To splice an ordinary transmission rope, wind twine around the rope the length of the proposed splice, say 6 ft. or more, from each end, and unlay the strands back to the twine. Then butt the ropes together so that the untwisted strands will meet opposite each other in pairs. Next, cut the twine and unlay one strand from one rope end, following it up with a strand from the other rope end, and leaving about 18 or 20 ins. loose end on each strand at the *meeting point*, for sub-splicing. Make the points of meeting of other pairs of strands staggered regularly so no two points will be opposite. For sub-splicing of each pair of strands, split each strand, unlay and interweave, passing the ends through the rope, or tie with ordinary knots. Hammer smooth.

Manila Rope.—(Adapted from C. W. Hunt.*) Manila rope is made from Manila hemp fibers (inferior in strength to the Italian hemp). In manufacturing rope, the fibers are first spun into a yarn about ¼ in. in diam. this yarn being twisted in a "right-hand" direction. From 20 to 80 of these yarns, depending on size of rope, are then put together and twisted in a "left-hand" direction, into a strand. In a 3-strand rope three, and in a 4-strand rope four, of these strands are then twisted together, again in a "right-hand" direction. Note that when *each strand* is twisted it tends to untwist the threads, but later when the strands are twisted *together* into a rope, each strand tends to untwist, but to twist up the threads. It is this opposite twist of the threads and strands that keeps the rope in its proper form.

The *durability* of Manila rope is quite variable under different uses. Experience has shown that 4-strand rope is more serviceable than 3-strand; it is stronger for the same diameter, wears rounder and smoother, and the section is much nearer a circle.

The Strength and Weight of Manila Rope are given by Mr. Hunt in the following formulas:

Breaking strength, in pounds $= 7160 \times (\text{diam in inches})^2 \ldots\ldots (1)$

$= 725 \times (\text{circum in inches})^2 \ldots (1a)$

Weight per lineal foot, in pounds $= 0.34 \times (\text{diam in inches})^2 \ldots\ldots (2)$

$= 0.0344 \times (\text{circum in inches})^2 \ldots (2a)$

* See *Manila Rope* by C. W. Hunt; also Trans. Am. Soc. M. E., Vol. xii, p. 230, and Vol. xxiii (1901).

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Hence, from (1) and (2) we have:

Breaking strength (lbs.) = 21060 × weight per lineal foot (lbs.) ..... (3)

It is to be noted from formula (3) that pound for pound, manila rope is as strong as steel which has an ultimate tensile strength of 71600 lbs. per square inch: as a square inch bar of steel weighs 3.4 lbs. per lineal foot, and 21060 × 3.4 = 71600 lbs.

1.—Weight and Strength of Manila Rope. (By Slide Rule.)

Nom. Diam.	Circumference.	*Weight of 100 ft. of Rope.	†Strength of Rope. Breaking Str'gth.	†Strength of Rope. Safe (1/10) Str'ngth.	Nom. Diam.	Circumference.	*Weight of 100 ft. of Rope.	†Strength of Rope. Breaking Str'gth.	†Strength of Rope. Safe (1/10) Str'ngth.
Ins.	Ins.	Lbs.	Lbs.	Lbs.	Ins.	Ins.	Lbs.	Lbs.	Lbs.
5/16	1 1/16	1.1+1.0	230	11	1 1/2	4 1/2	69.6− 4.5	14600	730
1/4	3/4	1.9+0.8	410	20	1 9/16	4 3/4	77.5− 5.0	16300	815
5/16	1	3.4+0.6	730	36	1 5/8	5	85.9− 5.5	18200	910
3/8	1 1/8	4.4+0.4	920	46	1 3/4	5 1/2	104.0− 6.5	21800	1090
7/16	1 1/4	5.4+0.2	1130	57	2	6	124.0− 8.0	26100	1300
1/2	1 1/2	7.7±0.0	1630	81	2 1/8	6 1/2	145.0− 9.0	30600	1530
9/16	1 3/4	10.5±0.0	2220	111	2 1/4	7	168.0−10.0	35500	1780
5/8	2	13.8−0.3	2900	145	2 1/2	7 1/2	193.0−11.0	40800	2040
3/4	2 1/4	17.4−0.6	3660	183	2 5/8	8	220.0−12.0	46400	2320
13/16	2 1/2	21.5−0.9	4530	226	2 7/8	8 1/2	247.0−13.0	52200	2610
7/8	2 3/4	25.9−1.2	5480	274	3	9	278.0−14.0	58600	2930
1	3	30.9−1.6	6520	326	3 1/8	9 1/2	310.0−15.0	65400	3270
1 1/16	3 1/4	36.3−2.0	7630	382	3 1/4	10	344.0−16.0	72600	3630
1 1/8	3 1/2	42.1−2.5	8880	444	3 1/2	11	416.0−18.0	87800	4390
1 1/4	3 3/4	48.4−3.0	10200	510	3 3/4	12	495.0−20.0	104300	5210
1 3/16	4	55.0−3.5	11600	580	4 1/8	13	580.0−22.0	122400	6120
1 3/8	4 1/4	60.4−4.0	13100	655	4 1/2	14	670.0−24.0	142000	7100

* The left-hand figures in the column are the weights according to Mr. Hunt's formula (2), and the right-hand figures are corrections which give resulting weights in accordance with some of the manufacturers' tables. The true weights are somewhat approximate and probably lie within the two limits.

† Calculated from Mr. Hunt's formula. See also table of Breaking Strength of Manila Rope by Spencer Miller in *Eng. News*, Dec. 6, 1890.

The *safe strength* of manila rope given in the above table is based on a safety factor of 20, which Mr. Hunt recommends for rope driving. The following working loads are recommended for *rapid* (400 to 800 feet per minute), *medium* (wharf and cargo, hoisting 150 to 300 feet per minute), and *slow* (derrick, crane and quarry, speed at 50 to 100 feet per minute) work:

2.—Working Load for Manila Rope.

Diameter of Rope, Inches.	Ultimate Strength, Pounds.	Working Load in Pounds. Rapid.	Working Load in Pounds. Medium.	Working Load in Pounds. Slow.	Minimum Diameter of Sheaves in Inches. Rapid.	Minimum Diameter of Sheaves in Inches. Med'm.	Minimum Diameter of Sheaves in Inches. Slow.
1	7,100	200	400	1,000	40	12	8
1 1/8	9,000	250	500	1,250	45	13	9
1 1/4	11,000	300	600	1,500	50	14	10
1 3/8	13,400	380	750	1,900	55	15	11
1 1/2	15,800	450	900	2,200	60	16	12
1 5/8	18,800	530	1,100	2,600	65	17	13
1 3/4	21,800	620	1,250	3,000	70	18	14

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3.—Comparison of Wire Gages.

n (B. W. G.), Browne & Sharpe (B. & S.), and Edison.
=.001 inch. (Diam in mils of round bar)^2=area of section
and this multiplied by 0.78540=area of section in square
ivided by 1 000 000=area of section in square inches.

Area of Cross-Section, in		*Weight of 1000 Ft. of Soft Iron Wire. Pounds.	Gages.			Diam. in Mills.	Area of Cross-Section, in		*Weight of 1000 Ft. of Soft Iron Wire. Pounds.
Circular ○ Mils.	Square □ Mils.		B. W. G.	B. & S.	Edison.		Circular ○ Mils.	Square □ Mils.	
360 000	282 743	942.5	11			120.	14400.	11310.	37.70
340 000	267 035	890.1		9		114.43	13094.	10284.	34.28
320 000	251 327	837.8			12	109.55	12000.	9425.	31.42
300 000	235 619	785.4	12			109.	11881.	9331.	31.10
280 000	211 911	706.4		10		101.89	10382.	8154.	27.18
260 000	204 203	680.7	13			95.	9025.	7088.	23.63
240 000	188 496	628.3		11		90.742	8234.	6467.	21.56
220 000	172 788	576.0			8	89.45	8000.	6283.	20.94
211 600	166 190	554.0	14			83.	6889.	5411.	18.04
206 116	161 883	539.6		12		80.808	6529.	5128.	17.09
200 000	157 080	523.6	15			72.	5184.	4072.	13.57
190 000	149 226	497.4		13		71.961	5178.	4067.	13.56
180 625	141 863	472.9			5	70.72	5000.	3927.	13.09
180 000	141 372	471.2	16			65.	4225.	3318.	11.06
170 000	133 518	445.1		14		64.084	4107.	3225.	10.75
167 805	131 790	439.3	17			58.	3364.	2642.	8.807
160 000	125 664	418.9		15		57.068	3257.	2558.	8.527
150 000	117 810	392.7			3	54.78	3000.	2356.	7.853
144 400	113 411	378.0		16		50.82	2583.	2029.	6.763
140 000	109 956	366.5	18			49.	2401.	1886.	6.287
133 079	104 518	348.4		17		45.257	2048.	1609.	5.363
130 000	102 102	340.3	19			42.	1764.	1385.	4.617
120 000	94 248	314.1		18		40.303	1624.	1276.	4.253
115 600	90 792	302.6		19		35.89	1288.	1012.	3.373
110 000	86 394	288.0	20			35.	1225.	962.1	3.207
105 534	82 887	276.3	21			32.	1024.	804.2	2.681
100 000	78 540	261.8		20		31.961	1022.	802.3	2.674
95 000	74 613	248.7		21		28.462	810.1	636.2	2.121
90 000	70 686	235.6	22			28.	784.0	615.8	2.053
85 000	66 759	222.5		22		25.347	642.5	504.6	1.682
83 694	65 732	219.1	23			25.	625.0	490.9	1.636
80 656	63 347	211.1		23		22.571	509.4	400.1	1.334
80 000	62 832	209.4	24			22.	484.0	380.1	1.267
75 000	58 905	196.4		24		20.1	404.1	317.3	1.058
70 000	54 978	183.3	25			20.	400.0	314.2	1.047
67 081	52 685	175.6	26			18.	324.0	254.5	.848
66 373	52 128	173.8		25		17.9	320.4	251.7	.839
65 000	51 051	170.2	27			16.	256.0	201.1	.670
60 000	47 124	157.1		26		15.94	254.1	199.6	.665
56 644	44 488	148.3		27		14.195	201.5	158.3	.528
55 000	43 197	144.0	28			14.	196.0	153.9	.513
52 634	41 339	137.8	29			13.	169.0	132.7	.442
50 000	39 270	130.9		28		12.641	159.8	125.5	.418
48 400	38 013	126.7	30			12.	144.0	113.1	.377
45 000	35 343	117.8		29		11.257	126.7	99.51	.332
41 743	32 784	109.3		30		10.025	100.5	78.93	.263
41 209	32 365	107.9	31			10.	100.0	78.54	.262
40 000	31 416	104.7	32			9.	81.00	63.62	.212
35 000	27 489	91.63		31		8.928	79.71	62.60	.209
33 102	25 999	86.67	33			8.	64.00	50.27	.168
32 400	25 447	84.82		32		7.95	63.20	49.64	.165
30 000	23 562	78.54		33		7.08	50.13	39.37	.131
27 225	21 382	71.27	34			7.	49.00	33.48	.111
26 250	20 618	68.73		34		6.304	39.74	31.21	.104
25 000	19 635	65.45		35		5.614	31.52	24.76	.082
21 904	17 203	57.34	35	36		5.	25.00	19.64	.065
20 817	16 350	54.50		37		4.453	19.83	15.57	.052
20 000	15 708	52.36	36			4.	16.00	12.57	.042
17 956	14 103	47.01		38		3.965	15.72	12.35	.041
16 510	12 967	43.22		39		3.531	12.47	9.79	.033
15 000	11 781	39.27		40		3.144	9.88	7.76	.026

* Based on weight of iron at 480 lbs. per cubic foot. The following is recommended in making estimates: For ordinary iron wire, add one-half per cent to the tabulated weights; for steel wire, add 2 per cent; for copper wire, multiply by 1.16; for brass wire, mult. by 1.11; for aluminum wire, multiply by 0.35. All somewhat approximate.

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4.—Properties of Roebling Steel Wire.

Number, Roebling Gage.	*English System.						†Metric System.				
	Diameter in inches.	Area in square inches.	Breaking strain at rate of 100,000 pounds per square inch.	Weight in pounds. Per 1,000 feet.	Weight in pounds. Per mile.	Number of feet in 1,000 pounds.	Diameter in millimeters.	Area in square millimeters.	Breaking strain at rate of 70.3 kilograms per square millimeter.	Weight in kilograms per kilometer.	Number of meters in 1,000 kilograms.
6°	.460	.166191	16,619	558.4	2,948	1,791	11.683	107.200	7536	830.9	1204
5°	.430	.145221	14,522	487.9	2,576	2,050	10.921	93.673	6585	726.1	1377
4°	.393	.121304	12,130	407.6	2,152	2,453	9.982	78.258	5502	606.5	1649
3°	.362	.102922	10,292	345.8	1,826	2,891	9.195	66.404	4668	514.7	1943
2°	.331	.086049	8,605	289.1	1,527	3,458	8.407	55.510	3902	430.4	2323
0	.307	.074023	7,402	248.7	1,313	4,020	7.798	47.759	3357	370.1	2702
1	.283	.062902	6,290	211.4	1,116	4,731	7.188	40.580	2853	314.5	3180
2	.263	.054325	5,433	182.5	964	5,478	6.680	35.046	2464	271.7	3681
3	.244	.046760	4,676	157.1	830	6,365	6.198	30.171	2121	233.9	4275
4	.225	.039761	3,976	133.6	705	7,485	5.715	25.652	1803	198.7	5033
5	.207	.033654	3,365	113.1	597	8,843	5.257	21.705	1526	168.3	5942
6	.192	.028953	2,895	97.3	514	10,279	4.877	18.668	1314	144.9	6901
7	.177	.024606	2,461	82.7	437	12,095	4.496	15.876	1116	123.2	8117
8	.162	.020612	2,061	69.3	366	14,439	4.115	13.299	935	103.2	9690
9	.148	.017203	1,720	57.8	305	17,300	3.759	11.098	780	85.96	11633
10	.135	.014314	1,431	48.1	254	20,792	3.429	9.2347	649	71.59	13968
11	.120	.011310	1,131	38.0	201	26,315	3.048	7.2966	513	56.65	17652
12	.105	.008659	866	29.1	154	34,376	2.667	5.5865	393	43.41	23036
13	.092	.006648	665	22.3	118	44,762	2.337	4.2895	302	33.26	30066
14	.080	.005027	503	16.9	89.2	59,206	2.032	3.2422	230	25.14	39777
15	.072	.004071	407	13.7	72.2	73,099	1.829	2.6274	184.7	20.35	49140
16	.063	.003117	312	10.5	55.3	95,511	1.600	2.0106	141.3	15.59	64144
17	.054	.002290	229	7.70	40.6	129,954	1.372	1.4784	103.9	11.44	87413
18	.047	.001735	174	5.83	30.8	171,556	1.194	1.1197	78.71	8.68	115208
19	.041	.001320	132	4.44	23.4	225,428	1.041	0.8511	59.83	6.60	151515
20	.035	.000962	96	3.23	17.1	309,310	.8890	0.6207	43.64	4.82	207469
21	.032	.000804	80	2.70	14.3	370,096	.8128	0.5189	36.48	4.03	248139
22	.028	.000616	62	2.07	10.9	483,325	.7112	0.3973	27.93	3.07	325733
23	.025	.000491	49	1.65	8.71	.....	.6350	0.3167	22.26	2.45	408163
24	.023	.000415	42	1.40	7.37	.....	.5842	0.2680	18.84	2.08	480770
25	.020	.000314	31	1.06	5.58	.....	.5080	0.2027	14.25	1.57	636943
26	.018	.000254	25	.855	4.51	.....	.4572	0.1642	11.54	1.27	787402
27	.017	.000227	23	.763	4.03	.....	.4318	0.1464	10.29	1.14	877194
28	.016	.000201	20	.676	3.57	.....	.4064	0.1297	9.12	1.01	990100
29	.015	.000177	18	.594	3.14	.....	.3810	0.1140	8.01	0.885	.....
30	.014	.000154	15	.517	2.73	.....	.5556	0.0993	6.98	0.769	.....
31	.0135	.000143	14	.481	2.54	.....	.3429	0.0923	6.49	0.716	.....
32	.013	.000133	13	.446	2.36	.....	.3302	0.0856	6.02	0.665	.....
33	.011	.000095	9.5	.319	1.69	.....	.2794	0.0613	4.31	0.476	.....
34	.010	.000079	7.9	.264	1.39	.....	.2540	0.0507	3.56	0.392	.....
35	.0095	.000071	7.1	.238	1.26	.....	.2413	0.04573	3.21	0.355	.....
36	.009	.000064	6.4	.214	1.13	.....	.2286	0.04104	2.90	0.318	.....

*This table was calculated on a basis of 483.84 lbs. per cubic foot for steel wire. Iron wire is a trifle lighter.

The breaking strains were calculated for 100,000 lbs. per sq. in. throughout, simply for convenience, so that the breaking strains of wires of any strength per square inch may be quickly determined by multiplying the values given in the table by the ratio between the strength per square inch and 100,000. Thus, a No. 15 wire, with a strength per square inch of 150,000 lbs., has a breaking strain of $407 \times \frac{150,000}{100,000} = 610.5$ lbs.

It must not be thought from this table that steel wire invariably has a strength of 100,000 lbs. per sq. in. As a matter of fact it ranges from 45,000 lbs. for soft annealed to over 400,000 lbs. per sq. in. for hard wire.

†This table gives the strength of wire at the rate of 70.3 kilograms per square millimeter, which is equivalent to 100,000 lbs. per sq. in. and was calculated on this basis simply for convenience. Steel wire may have a tensile strength from 30 to 300 kilograms per square millimeter, according to treatment, composition, etc.

—(Adapted from Roebling.) Wire rope is made of wires
l together, which is the most common form, or (2) *laid*
other, as employed in cables of large suspension bridges.

or stranded rope may be either (a) *round* or (b) *flat*. The
each commonly composed of 4, 7, 12, 19 or 37 wires twisted
ly in the opposite direction to the twists of the strands into

rope is composed of a number of wire strands twisted around
r around a wire strand or wire rope. Standard wire rope is
strands and a hemp core practically of the same size. But
ses 4, 5, 7, 8, 9 or any reasonable number of strands may be
strands are twisted around the core to the right the rope is
," and if to the left "left lay." When wires and strands are
me direction the resulting rope is called "lang" rope. The
orm the more flexible rope, and for great flexibility the
es may consist of wire ropes, as in tiller ropes. Generally,
saturated with tar, providing little additional strength but
on to preserve the shape of the rope and helping to lubricate
re-strand or wire-rope core will usually add from 7 to 10 per
gth of the rope, but, excepting in stationary ropes, the wear
n is as rapid as the outside wear of the rope. For ships'
ning ropes, where great pliability is demanded, as well as
side friction, the strands are made of 12 or 18 wires twisted
enter.

e consists of a number of strands laid side by side in alter-
ft lay, and sewed together with annealed wire so as to form
n usually from ⅜ to ½ inch in thickness, and from 1 inch
h. (The sewing wires wear out and have to be renewed
This kind of wire rope is specially adapted for long and
s in deep shafts, under which conditions it possesses advan-
l rope which has a tendency to twist and untwist.

mposing wire ropes and cables are usually of steel or iron;
ecial purposes they are sometimes made of copper, bronze,
th of iron wire ranges from 45,000 to 100,000 lbs. per sq. in.
s of steel employed are open-hearth steel with an ultimate
50,000 to 130,000 lbs. per sq. in.; crucible steel from 130,000
er sq. in.; and plough steel from 190,000 to 350,000 lbs. per

h of a wire rope is seldom more than 90 per cent of the
gth of all of its wires in a testing machine; and the *average*
about 82½ per cent. This is due to the difficulty in making
ents at the ends of the test piece.

ngth of wire rope, that is, the working load, is usually assumed
te strength, for ordinary work. For severe or hazardous
of safety is increased from 5 up to even 10 or 15, as for

drum and sheave should be large enough that the wires of
t be strained beyond the elastic limit in passing over them.
diameters increase with the size of the rope and its speed.
are too small the life of the rope will be shortened.

as linseed oil, tar or other similar materials free from acids
stances, should be used freely and frequently in preventing
ng wear, thus increasing the life of the rope. In order to
ction against the action of salt air, rust, etc., wire is often
ned, as for ships' rigging, etc.; but ropes subject to constant
drums and sheaves are not usually so treated.

ust not be coiled or uncoiled like hemp rope. When it is
reel, the latter should be mounted upon a spindle or turn-
pe then run off. When shipped in a coil it should be rolled
like a wheel. All untwisting and kinking must be avoided.
pe is to be cut, soft iron wire should be served on each side
ere the division is to be made to prevent the rope from

ter" of a rope is the diameter of a circumscribing circle and
gest diameter.

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5.—Roebling Round Wire-Rope.

(Swedish Iron, Cast Steel, Extra Strong Crucible Cast Steel, and Plough Steel.)

Table of dimensions, weight, breaking strength, safe (1/5) strength. Also minimum diameter of drum or sheave.

Diameter in Inches.	Approx. Circ. in Ins.	Weight per Foot in Pounds.	Breaking Strength in 1,000 lbs.				Safe (1/5) Strength in 1,000 Pounds.				Min. Diam. of Drum or Sheaves in Ft. for			
			Swedish Iron. A	Cast Steel. B	Extra Strong Crucible Cast Steel. C	Plough Steel. D	Swedish Iron. A	Cast Steel. B	Extra Strong Crucible Cast Steel. C	Plough Steel. D	Swedish Iron. A	Cast Steel. B	Extra Strong Crucible Cast Steel. C	Plough Steel. D
Rope Composed of 6 Strands and a Hemp Center, 19 Wires to the Strand.														
2¾	8⅝	11.95	228.	456.	532.	610.	45.6	91.2	106.4	122.0	16.	10.	10.	11.
2½	7⅞	9.85	189.	379.	444.	508.	37.9	75.8	88.8	101.6	17.	9.5	9.5	10.
2¼	7⅛	8.00	156.	312.	364.	416.	31.2	62.4	72.8	83.2	13.	8.5	8.5	9.
2	6¼	6.30	124.	248.	288.	330.	24.8	49.6	57.6	66.0	12.	8.	8.	8.
1¾	5½	4.85	96.	192.	224.	256.	19.2	38.4	44.8	51.2	10.	7.25	7.25	7.5
1⅝	5	4.15	84.	168.	194.	222.	16.8	33.6	38.8	44.4	8.5	6.25	6.25	6.
1½	4¾	3.55	72.	144.	168.	192.	14.4	28.8	33.6	38.4	7.5	5.75	5.75	5.5
1⅜	4¼	3.00	62.	124.	144.	164.	12.4	24.8	28.8	32.8	7.	5.5	5.5	5.25
1¼	4	2.45	50.	100.	116.	134.	10.0	20.0	23.2	26.8	6.5	5.	5.	5.
1⅛	3½	2.00	42.	84.	98.	112.	8.4	16.8	19.6	22.4	6.	4.5	4.5	4.5
1	3	1.58	34.	68.	78.	88.	6.8	13.6	15.6	17.6	5.25	4.	4.	4.25
⅞	2¾	1.20	26.	52.	60.	68.	5.2	10.4	12.0	13.6	4.5	3.5	3.5	3.75
¾	2¼	0.89	19.4	38.8	44.	50.	3.88	7.76	8.8	10.0	4.	3.	3.	3.5
⅝	2	0.62	13.6	27.2	31.6	36.	2.72	5.44	6.32	7.2	3.5	2.25	2.25	3.
½	1¾	0.50	11.0	22.0	25.4	29.	2.20	4.40	5.08	5.8	2.75	1.75	1.75	2.5
½	1½	0.39	8.8	17.6	20.2	22.8	1.76	3.52	4.04	4.56	2.25	1.5	1.5	2.
⅜	1¼	0.30	6.8	13.6	15.6	17.7	1.36	2.72	3.12	3.54	2.	1.25	1.25	1.5
⅜	1⅛	0.22	5.0	10.0	11.5	13.1	1.00	2.00	2.31	2.62	1.5	1.	1.	1.
¼	1	0.15	3.4	6.8	8.1	9.0	.68	1.36	1.62	1.80	1.	.67	.67	.88
¼	¾	0.10	2.4	4.8	5.4	6.0	.48	.96	1.08	1.20	.75	.50	.50	.67
Rope Composed of 6 Strands and a Hemp Center, 7 Wires to the Strand.														
1½	4¾	3.55	68.	136.	158.	182.	13.6	27.2	31.6	36.4	13.	8.5	8.5	8.5
1⅜	4¼	3.00	58.	116.	136.	156.	11.6	23.2	27.2	31.2	12.	8.	8.	8.
1¼	4	2.45	48.	96.	112.	128.	9.6	19.2	22.4	25.6	10.75	7.25	7.25	7.25
1⅛	3½	2.00	40.	80.	92.	106.	8.0	16.0	18.4	21.2	9.5	6.25	6.25	6.25
1	3	1.58	32.	64.	74.	84.	6.4	12.8	14.8	16.8	8.5	5.75	5.75	5.5
⅞	2¾	1.20	24.	48.	56.	64.	4.8	9.6	11.2	12.8	7.5	5.	5.	5.
¾	2¼	0.89	08.6	37.2	42.	48.	3.72	7.44	8.4	9.6	6.75	4.5	4.5	4.
⅝	2	0.75	15.8	31.6	36.8	42.	3.16	6.32	7.36	8.4	6.	4.	4.	3.5
⅝	2	0.62	13.2	26.4	30.2	34.	2.64	5.28	6.04	6.8	5.25	3.5	3.5	3.
½	1¾	0.50	10.6	21.2	24.6	28.	2.12	4.24	4.92	5.6	4.5	3.	3.	2.75
½	1½	0.39	8.4	16.8	19.4	22.	1.68	3.36	3.88	4.4	4.	2.5	2.5	2.5
⅜	1¼	0.30	6.6	13.2	15.0	17.1	1.32	2.64	3.00	3.42	3.25	2.25	2.25	2.
⅜	1⅛	0.22	4.8	9.6	11.1	12.7	.96	1.92	2.22	2.54	2.75	2.	2.	1.5
¼	1	0.15	3.4	6.8	7.7	8.7	.68	1.36	1.54	1.74	2.5	1.75	1.75	1.25
¼	⅞	0.125	2.8	5.6	6.4	7.3	.56	1.12	1.28	1.46	2.25	1.5	1.5	1.

Note.—The above rope is furnished either galvanized or tinned; also with wire center—at an extra cost of 10 per cent for each. For standard hoisting rope the Swedish iron (A) and cast steel (B) with 19 wires to the strand, are used; while for transmission or haulage, the same, but with 7 wires to the strand, are used. Before ordering, consult the manufacturers in regard to the best size of rope, grade of steel, etc., to use, if not familiar with same.

WIRE ROPE FASTENINGS.
(Best Forged Steel.)

Fig. 20. Closed Socket.

Fig. 21. Open Socket.

Fig. 22. Socket and Swivel Hook.

Fig. 23. Open Socket and Hook.

Fig. 24. Special Swivel Hook and Socket. (Double Swivel)..

Fig. 25. Hook and Thimble.

Fig. 26. Shackle and Thimble.

Fig. 27. Closed Cast-Iron Socket for Suspension Bridge and Cableway.

Fig. 28. Open Cast-Iron Socket for Suspension Bridge and Cableway.

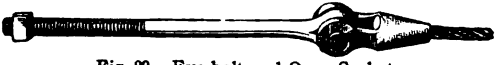

Fig. 29. Eye-bolt and Open Socket.

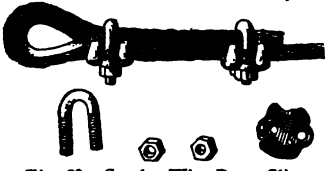

Fig. 30. Crosby Wire-Rope Clip.

Fig. 31. Jupiter Wire-Rope Clip.

Fig. 32. Roebling's Extra Heavy Wire-Rope Clamp with Three Bolts.

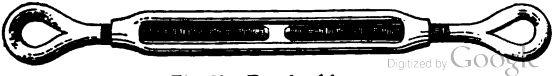

Fig. 33. Turnbuckle.

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EXCERPTS AND REFERENCES.

Telephone Cable in the St. Gotthard Tunnel ("Elektrotechn Zeitschrift" for June 27, 1901; Eng. News, Dec. 5, 1901).—Illustrated. A paper-and-air-insulated cable. It includes 7 two-wire circuits, each wire 1.8 m m. in dia., each set being covered with paper tape to a dia. of 7 m m.; stranded together and covered with a triple envelope of cotton and a double tin-lead sheath; outside is a layer of waterproof compound, a strong armor of interlocking steel wires and an outer coating of jute yarn soaked in a protecting compound. The finished external diameter is 44 m m., or 1.7 ins.

READER'S MEMORANDA.

The following skeleton outline is for the use of the reader in making reference to tables and general items of interest which may be found in this book or in other works.

Cordage.

1.	See		Page
2.	See		Page
3.	See		Page

Steel Wire.

4.	See		Page
5.	See		Page
6.	See		Page

Copper Wire.

7.	See	Table 1, Section 70, Electric Power and Lighting	Page
8.	See		Page
9.	See		Page

Aluminum Wire.

10.	See		Page
11.	See		Page
12.	See		Page

Cables.

13.	See		Page
14.	See		Page
15.	See		Page

Laying Cables.

16.	See		Page
17.	See		Page

Miscellaneous.

18.	See	Table 1, Section 34, Metal Gages	Page
19.	See		Page
20.	See		Page

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36.—PIPES AND TUBES.

(See also various pipes, fittings and specials in Sec. 64, Water Works, page 1207, etc.)

1.—*Standard Wrought Iron Welded Steam, Gas and Water Pipe. (National Tube Works.)

[For Weight of Seamless Brass tubing, iron pipe size (1), following page, multiply tabulated weight by 1.07.]

(a) External Diameters and Properties.

Nom. Diam.	External.			Thre'ds per Inch.	L'th per Sq. Ft. Exter'l Heat'g Surf.	Couplings for (1), next page.			
	Diam.	Circum.	Area.			Inside Diam.	Outside Diam.	Length	Aver'ge Weight
Ins.	Ins.	Ins.	Sq. Ins.	No.	Ft.	Ins.	Ins.	Ins.	Lbs.
1/8	.405	1.272	.1288	27	9.44	11/32	19/32	13/16	.031
1/4	.540	1.696	.2290	18	7.07	15/32	23/32	15/16	.046
3/8	.675	2.121	.3578	18	5.66	37/64	27/32	1 1/16	.078
1/2	.840	2.639	.5542	14	4.55	23/32	1	1 5/16	.124
3/4	1.050	3.299	.8659	14	3.64	63/64	1 21/64	1 9/16	.250
1	1.315	4.131	1.3581	11½	2.90	1 11/64	1 9/16	1 13/16	.455
1¼	1.660	5.215	2.1642	11½	2.30	1 1/2	1 61/64	2¼	.562
1½	1.900	5.969	2.8353	11½	2.01	1 3/4	2 7/32	2⅜	.800
2	2.375	7.461	4.4301	11½	1.61	2 7/32	2 3/4	2⅝	1.250
2½	2.875	9.032	6.4918	8	1.33	2 21/32	3 9/32	2⅞	1.757
3	3.500	10.996	9.6211	8	1.09	3 1/4	3 15/16	3⅛	2.625
3½	4.000	12.566	12.566	8	.955	3 25/32	4 7/16	3⅜	4.000
4	4.500	14.137	15.904	8	.849	4 17/64	5	3⅝	4.125
4½	5.000	15.708	19.635	8	.764	4 3/4	5 1/2	3⅝	4.875
5	5.563	17.477	24.306	8	.687	5 9/32	6 7/32	4⅛	8.437
6	6.625	20.813	34.472	8	.577	6 11/32	7 5/16	4⅛	10.625
7	7.625	23.955	45.664	8	.501	7 3/8	8 5/16	4⅛	11.270
8	8.625	27.096	58.426	8	.443	8 3/8	9 5/16	4⅝	15.150
9	9.625	30.238	72.760	8	.397	9 7/16	10 3/8	5⅛	17.820
10	10.750	33.772	90.763	8	.355	10 7/16	11 21/32	6⅛	27.700
11	11.750	36.913	108.43	8	.325	11 15/32	12 21/32	6⅛	33.250
12	12.750	40.055	127.68	8	.299	12 7/16	13 7/8	6⅛	43.187

*Allow variation of 5 per cent. above and 5 per cent. below standard in weight per foot. Cannot cut to length closer than 1/16 inch.

† Shipped threads and couplings (above) unless otherwise ordered.

‡ Shipped plain ends unless otherwise ordered. Where Extra Strong Pipe is ordered with threads and couplings, regular line pipe couplings (not shown above) will be furnished, unless otherwise specified.

|| Shipped plain ends unless otherwise ordered.

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1.—Standard Wrought Iron Welded Pipe.—Concluded.

(b) Internal Diameters and Properties.

Nom. Diam.	Internal. Diam.	Internal. Circum.	Internal. Area.	Metal. Thickness.	Metal. Area.	Nom. Weight per Foot.	L'th per Sq. Ft. Intern'l Heat'g Surf.	L'th of Pipe Con'tg 1 Cubic Foot.	U. S. Gallons per Ft. of Pipe.
Ins.	Ins.	Ins.	Sq. Ins.	In.	Sq. Ins.	Lbs.	Ft.	Ft.	Gals.
			(1) Black or Galvanized Standard Weight Pipe.†						
Also iron-pipe sizes for seamless brass tubes.									
1/8	.269	.845	.0568	.068	.0720	.241	14.2	2535.	.0038
1/4	.364	1.144	.1041	.088	.1249	.42	10.5	1383.	.0054
3/8	.493	1.549	.1909	.091	.1669	.559	7.76	754.3	.0100
1/2	.622	1.954	.3039	.109	.2503	.837	6.15	473.8	.0159
3/4	.824	2.589	.5333	.113	.3326	1.115	4.64	270.0	.0278
1	1.047	3.289	.8609	.134	.4972	1.668	3.66	167.3	.0440
1 1/4	1.380	4.335	1.4957	.140	.6685	2.244	2.77	96.3	.078
1 1/2	1.610	5.058	2.0358	.145	.7995	2.678	2.38	70.8	.106
2	2.067	6.494	3.3556	.154	1.074	3.609	1.85	42.9	.175
2 1/2	2.467	7.750	4.7800	.204	1.712	5.739	1.55	30.1	.248
3	3.066	9.632	7.3827	.217	2.238	7.536	1.25	19.5	.383
3 1/2	3.548	11.146	9.886	.226	2.680	9.001	1.08	14.56	.514
4	4.026	12.648	12.730	.237	3.174	10 665	.949	11.31	.661
4 1/2	4.508	14.162	15.960	.246	3.675	12.34	.848	9.02	.829
5	5.045	15.849	19.985	.259	4.321	14.502	.757	7.20	1.038
6	6.065	19.054	28.886	.280	5.586	18.762	.630	4.98	1.500
7	7.023	22.063	38.743	.301	6.921	23.271	.544	3.72	2.012
8	7.981	25.073	50.021	.322	8.405	28.177	.478	2.88	2.599
9	8.937	28.076	62.722	.344	10.04	33.701	.427	2.29	3.259
10	10.018	31.472	78.822	.366	11.94	40.065	.381	1.82	4.095
11	11.000	34.558	95.034	.375	13.40	45.95	.348	1.52	4.937
12	12.000	37.699	113.09	.375	14.59	48.985	.319	1.27	5.875
			(2) Standard Extra Strong Pipe.‡						
1/8	.205	.644	.033	.100	.096	.29	18.63	4364.	.0018
1/4	.294	.924	.068	.123	.161	.54	12.99	2118.	.0036
3/8	.421	1.323	.139	.127	.219	.74	9.07	1035.	.0073
1/2	.542	1.703	.231	.149	.323	1.09	7.05	623.	.0120
3/4	.736	2.312	.425	157	.441	1.39	5.11	339.	.0232
1	.951	2.988	.710	.182	.648	2.17	4.02	202.8	.0370
1 1/4	1.272	3.996	1.271	.194	.893	3.00	3.00	113.1	.0663
1 1/2	1.494	4.694	1.753	.203	1.082	3.63	2.56	82.2	.091
2	1.933	6.073	2.935	.221	1.495	5.02	1.97	49.1	.153
2 1/2	2.315	7.273	4.209	.280	2.283	7.67	1.65	34.2	.220
3	2.892	9.086	6.569	.304	3.052	10.25	1.33	21.95	.343
3 1/2	3.358	10.549	8.856	.321	3.710	12.47	1.14	16.25	.463
4	3.818	11.995	11.449	.341	4.455	14.97	1 00	12.57	.596
4 1/2	4.280	13.446	14.387	.360	5.248	18.22	.893	10.01	.752
5	4.813	15.120	18.193	.375	6.113	20.54	.793	7.92	.949
6	5.751	18.067	25.976	.437	8.496	28.58	.664	5.54	1.355
7	6.625	20.813	34.472	.500	11.192	37.67	.598	4.18	1.800
8	7.625	23.955	45.664	.500	12.762	43.00	.502	3.15	2.382
9	8.625	27.096	58.426	.500	14.334	48.25	.443	2.46	3.060
10	9.750	30.631	74.662	.500	16.101	54.25	.399	1.93	3.90
12	11.750	36.914	108.43	.500	19.25	65.00	.325	1.33	5.66
			(3) Standard Double Extra Strong Pipe.‖						
1/8	.244	2.639	.554	.298	.507	1.7	15.67	260.0	.029
3/4	.422	3.299	.866	.314	.726	2.44	9.05	166.3	.045
1	.587	4.131	1.358	.364	1.087	3.65	6.51	106.0	.071
1 1/4	.885	5.215	2.164	.388	1.549	5.2	4.32	66.5	.113
1 1/2	1.088	5.969	2.835	.406	1.905	6.4	3.51	50.8	.148
2	1.491	7.461	4.430	.442	2.686	9.03	2.56	32.5	.231
2 1/2	1.755	9.032	6.492	.560	4.073	13.68	2.18	22.20	.339
3	2.284	10.996	9.621	.608	5.524	18.56	1.67	14.97	.502
3 1/2	2.716	12.566	12.566	.642	6.772	22.75	1.41	11.45	.656
4	3.136	14.137	15.904	.682	8.180	27.48	1.22	9.06	.830
4 1/2	3.564	15.708	19.635	.718	9.659	32.53	1.07	7.33	1.025
5	4.063	17.477	24.306	.750	11.341	38.12	.94	5.93	1.270
6	4.875	20.813	34.472	.875	15.807	53.11	.78	4.18	1.80
7	5.875	23.955	45.664	.875	18.555	62.38	.65	3.15	2.38
8	6.875	27.096	58.426	.875	21.304	71.62	.55	2.47	3.05

† ‡ ‖ Foot-notes, preceding page.

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2.—Lead and Tin Lined Lead Pipe.
(Tatham and Brothers, New York.)

Inner Diam.	Thickness.	Weight per Ft.	Inner Diam.	Thickness.	Weight per Ft.	Inner Diam.	Thickness.	Weight per Ft.	Inner Diam.	Thickness.	Weight per Ft.
Ins.	Ins.	Lbs.	Ins.	Ins.	Lbs.	Ins.	Ins.	Lbs.	Ins.	Ins.	Lbs
⅛	.05	0.42˘42	½	.25	3.	1	.11	2.	1½	.23	6.5
1 "	.06	0.625	⅝	.08	0.72˘72	2 "	.14	2.5	"	.25	7.5
2 "	.08	0.75	1 "	.09	1.	3 "	.17	3.25	* "	.27	8.
3 "	.12	1.	2 "	.13	1.5	4 "	.21	4.	"	.28	8.5
4 "	.16	1.25	3 "	.16	2.	5 "	.24	4.75	1¾	.13	4.
5 "	.19	1.50	4 "	.20	2.5	6 "	.30	6.	"	.17	5.
6 "	.27	1.75	5 "	.22	2.75	1¼	.10	2.	"	.19	6.
7/16	.....	0.8125	6 "	.25	3.5	1 "	.12	2.5	* "	.21	6.5
½	.....	1.	¾	.08	1.	2 "	.14	3.	"	.23	7.
½	.07	0.54˘54	1 "	.10	1.25	3 "	.16	3.75	"	.27	8.5
1 "	.09	0.75	2 "	.12	1.75	4 "	.19	4.75	"	.30	10.
2 "	.11	1.	* "	.14	2.	5 "	.25	5.75	2	.15	4.75
3 "	.13	1.25	3 "	.16	2.25	6 "	.28	6.75	"	.18	6.
* "	.14	1.50	4 "	.20	3.	1½	.12	3.	"	.22	7.
4 "	.16	1.75	5 "	.23	3.5	"	.14	3.5	"	.25	8.
5 "	.19	2.	6 "	.30	4.75	"	.17	4.25	"	.27	9.
* "	.23	2.5	1	.10	1.5	"	.19	5.	"	.30	11.75

˘ Repeating decimal: 0.42˘42 lb. per ft. = 7 lbs. per rod; 0.54˘54 lb. per ft. = 9 lbs. per rod; 0.72˘72 lb. per ft. = 12 lbs. per rod.
* Special.

1 Recommended for pressure of 15 lbs per sq. in., or hydrostatic head of 30 ft.
2 " " " " 25 " " " " " " " 50 "
3 " " " " 38 " " " " " " " 75 "
4 " " " " 50 " " " " " " " 100 "
5 " " " " 75 " " " " " " " 150 "
6 " " " " 100 " " " " " " " 200 "

Weight of Lead Pipe per Lin. Ft.*

Inner Diam.	Thickness.			
	3/16"	¼"	5/16"	⅜"
Ins.	Lbs.	Lbs.	Lbs.	Lbs.
2½	8.	11.	14.	17.
3	9.	12.	16.	20.
3½	9.5	15.	18.	22.
4	12.5	16.	21.	25.
4½	14.	18.	.....	.....
5	15.	20.	25.	31.
6	18.	24.5	30.	37.

* Manufactured in lengths of 10 ft.

Lead Waste Pipe.

1½ in., 2 and 3 lbs. per ft.
2 in., 3 and 4 lbs. per ft.
3 in., 3½, 5 and 6 lbs. per ft.
3½ in., 4 lbs. per ft.
4 in., 5, 6 and 8 lbs. per ft.
4½ in., 6 and 8 lbs. per ft.
5 in., 8, 10 and 12 lbs. per ft.
6 in., 12 lbs. per ft.

Lead and Tin Tubing.

⅛ inch. ¼ inch.

Sheet Lead.

Weight per square foot, 2½, 3, 3½, 4, 4½, 5, 6, 8, 9, 10 lbs. and upwards.

Lighter weights rolled to order at special prices.

Block Tin Pipe.

⅜ in., 4, 5, 6 and 8 oz. per ft.
½ in., 6, 7½ and 10 oz. per ft.
⅝ in., 8 and 10 oz. per ft.
¾ in., 10 and 12 oz. per ft.
1 in., 15 and 18 oz. per ft.
1¼ in., 1¼ and 1½ lbs. per ft.
1½ in., 2 and 2½ lbs. per ft.
2 in., 2½ and 3 lbs. per ft.

Special sizes made to order.

Lead: Wt. per cu. ft., about 710 lbs.; per sq. ft. 1 in. thick, about 59.2 lbs.; bar 1 in. square and 1 ft. long, about 4.93 lbs.; per cu. in., about 0.411 lb.

Cast Tin: Wt. per cu. ft., about 459 lbs.; per sq. ft. 1 in. thick, about 38.25 lbs.; bar 1 in. square and 1 ft. long, about 3.19 lbs.; per cu. in. about 0.266 lbs.

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3.—Spiral Riveted Steel Pipe as Manufactured by American Spiral Pipe Works, Chicago.

Fig. 1.

Fig. 2.

Diameter, Inches.	Standard Weight Pipe. Use Galvanized Pipe for: Exhaust Steam, Pump Suction, Brine Circulation, Refrigerating Coils, Etc.					Standard Weight Pipe. Use Galvanized or Asphalted Pipe for: Paper and Pulp Mills, Irrigation, Pump Discharge, Water Pipe Lines, Etc.					Extra Heavy Weight Pipe. Asphalted for: Intake Mains, Water Works, Hydraulic Mining, Water Sup. Lines.					Extra Heavy Weight Pipe. Galvanized and Flanged for: Compressed Air, Pump Suction, Condenser Pipes, Vacuum Pipes, Etc.				
	Thickness. Gauge.	Galvanized. Price per lineal foot flanged.	Asphalted. Price per lineal foot flanged.	Approximate weight per foot. Pounds.	Approx. bursting pressure. Lbs. per sq. in.	Thickness. Gauge.	Galvanized. Price per lineal foot flanged.	Asphalted. Price per lineal foot flanged.	Approximate weight per foot. Pounds.	Approx. bursting pressure. Lbs. per sq. in.	Thickness. Gauge.	Galvanized. Price per lineal foot flanged.	Asphalted. Price per lineal foot flanged.	Approximate weight per foot. Pounds.	Approx. bursting pressure. Lbs. per sq. in.	Thickness. Gauge.	Galvanized. Price per lineal foot flanged.	Asphalted. Price per lineal foot flanged.	Approximate weight per foot. Pounds.	Approx. bursting pressure. Lbs. per sq. in.
3	No. 20	$0.50	$0.35	2.25	1500	No. 18	$0.55	$0.40	2.60	2000	No. 18	$0.55	$0.40	2.60	2000	No. 18	$0.55	$0.40	2.60	2000
4	"	.70	.45	3.00	1125	"	.80	.55	3.45	1500	"	.80	.55	3.45	1500	"	.80	.55	3.45	1500
5	"	1.00	.55	4.00	900	"	1.10	.65	4.50	1200	"	1.10	.65	4.50	1200	"	1.10	.65	4.50	1200
6	No. 18	1.20	.75	5.00	1000	No. 16	1.30	.90	6.40	1250	No. 16	1.30	.90	6.40	1250	No. 16	1.30	.90	6.40	1250
7	"	1.40	.80	6.00	860	"	1.50	.95	7.50	1070	"	1.50	.95	7.50	1070	"	1.50	.95	7.50	1070
8	"	1.70	.95	7.00	750	"	1.85	1.15	8.90	935	"	1.85	1.15	8.90	935	"	1.85	1.15	8.90	935
9	"	2.00	1.10	8.00	665	"	2.20	1.30	10.25	835	"	2.20	1.30	10.25	835	"	2.20	1.30	10.25	835
10	No. 16	2.60	1.45	11.00	750	No. 14	2.80	1.65	13.25	935	No. 14	2.80	1.65	13.25	935	No. 14	2.80	1.65	13.25	935
11	"	2.85	1.55	12.00	680	"	3.05	1.80	14.75	850	"	3.05	1.80	14.75	850	"	3.05	1.80	14.75	850
12	"	3.15	1.80	14.00	625	"	3.40	2.15	17.00	781	"	3.40	2.15	17.00	781	"	3.40	2.15	17.00	781
13	"	3.60	1.95	15.00	575	"	3.80	2.35	18.25	720	"	3.80	2.35	18.25	720	"	3.80	2.35	18.25	720
14	No. 14	4.00	2.50	20.00	670	No. 12	5.00	3.30	24.50	935	No. 12	5.00	3.30	24.50	935	No. 12	5.00	3.30	24.50	935
15	"	4.40	2.75	22.00	625	"	5.25	3.60	26.85	875	"	5.25	3.60	26.85	875	"	5.25	3.60	26.85	875
16	"	5.00	3.05	24.00	585	"	6.00	3.80	29.20	820	"	6.00	3.80	29.20	820	"	6.00	3.80	29.20	820
18	"	6.00	3.50	29.00	520	"	7.00	4.20	34.70	675	"	7.00	4.20	34.70	675	"	7.00	4.20	34.70	675
20	"	7.00	3.90	34.00	470	"	8.00	4.80	40.30	655	"	8.00	4.80	40.30	655	"	8.00	4.80	40.30	655
22	No. 12	9.00	5.55	40.00	595	No. 10	10.00	6.20	50.10	765	No. 10	10.00	6.20	50.10	765	No. 10	10.00	6.20	50.10	765
24	"	10.50	6.00	50 00	540	"	12.00	7.00	60.20	705	"	12.00	7.00	60.20	705	"	12.00	7.00	60.20	705
26	"	11.80	8.50	58.00	505	"	13.00	9.55	66.00	650	"	13.00	9.55	66.00	650	"	13.00	9.55	66.00	650
28	No. 10	14.60	10.25	27.00	605	No. 8	16.60	11.65	83.00	735	No. 8	16.60	11.65	83.00	735	No. 8	16.60	11.65	83.00	735
30	"	15.70	11.25	79.00	560	"	17.65	12.60	90.00	685	"	17.65	12.60	90.00	685	"	17.65	12.60	90.00	685
32	"	16.70	12.00	85.00	525	"	19.25	13.80	97.00	675	"	19.25	13.80	97.00	675	"	19.25	13.80	97.00	675
36	"	18.45	13.20	94.00	469	"	21.00	15.00	112.00	573	"	21.00	15.00	112.00	573	"	21.00	15.00	112.00	573
40	"	20.80	14.90	106.00	420	"	25.00	17.80	128.00	515	"	25.00	17.80	128.00	515	"	25.00	17.80	128.00	515

The above list is for pipe in standard lengths, with flanges attached or bolted joint connection.

We recommend the use of bolted joints with asphalted pipe for all high pressure water works.

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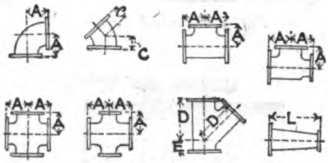
ETED STEEL PIPE AS MANUFACTURED BY AMERICAN SPIRAL PIPE WORKS, CHICAGO.—Concluded.

dard Flanges (Fig. 3) are used with spiral pipe unless other- When other than the standard flanges are required, be ide diameter of flange, number and size of bolts, and diameter All flanges and flanged fittings are drilled in multiples of ngs may be made to face in any quarter, and holes straddle

g. 3.



Figs. 4.—Fittings.

ers and Drilling of Standard Flanges.				* Dimensions of Standard Fittings.				
Bolt Circle B.	Number of Bolts.	Size of Bolts.	Length of Bolts.	Center to Face "A"	Center to Face 45° Ell "C"	Center to Face Branch "D"	Center to Face Y. Branch "E"	Length of Reducer "L"
4¾	4	7/16	1¾	3⅝	2⅙	9	2½	.....
15/16	8	7/16	1¾	4⅜	2 15/16	11	2¾	23
15/16	8	7/16	1¾	5¼	3¼	12	3	23
7⅞	8	½	1¾	6¼	3¾	13½	3¼	23
9	8	½	1¾	7¼	4⅛	15	4¼	22
0	8	½	2	8¼	4½	17	5	22
1¼	8	½	2	9¼	5 1/16	18½	5¼	22
2¼	8	½	2	10¼	5 7/16	21	5½	33
3⅜	12	½	2	11	5¾	22½	5¾	33
4¼	12	½	2	12¼	6 5/16	24	6	33
5¼	12	½	2	13	5½	26	6¼	33
6¼	12	½	2¼	14	6	27	6½	32
7 7/16	12	½	2¼	15	5½	29½	6¾	32
9¼	12	½	2¼	16	6 11/16	31½	7	32
1¼	16	⅝	2½	16½	8½	35	7½	32
3⅛	16	⅝	2½	18	9½	38½	8	32
6	16	⅝	2½	20	10	41	9	32
7¾	16	⅝	2½	22	11	44	10	32
1¾	24	¾	3	23	13	.....	.....	.....
4	28	¾	3	24	14	.....	.....	.....
6	28	¾	3	25	15	.....	.....	.....
8¼	28	¾	3	26	16	.....	.....	.....
2¾	32	¾	3½	28	18	.....	.....	.....
6¾	32	⅞	3½	30	20	.....	.....	.....

nter dimensions are not changed on reducing outlets to tees n increasing outlets, the face to center dimensions are the spective standards.

etails, see following page.

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Spiral Riveted Steel Pipe Details.

Fig. 5. Riveted Lap.

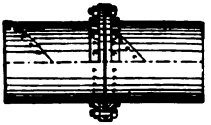

Fig. 6. Flange Connection.

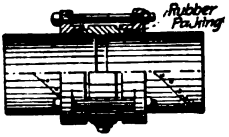

Fig. 7. Bolted Joint Connection.

Fig. 8. Slip Joint Connection.

Fig. 9. Bolts.

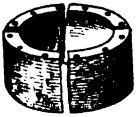

Fig. 10. Gaskets.

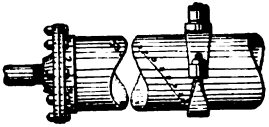

Fig. 11. Threaded Disc. Fig. 12. Clamp Band.

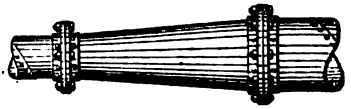

Fig. 13. Reducer.

EXCERPTS AND REFERENCES.

Making Tight Joints in Vitrified Pipe at Atlantic City, N. J. (By Kenneth Allen. Eng. News, Oct. 8, 1903.)

Experiments on Reinforced-Concrete Pipes Made for the U. S. Reclamation Service (By J. H. Quinton. Eng. News, Mar. 9, 1905).—Illustrated

Reinforced-Concrete Pipe with Reinforced Joint (By Lock Joint Pipe Co., N. Y.; Eng. News, Dec. 10, 1908).—Illustrated.

Standard Specifications for Hard-Drawn Copper Wire (Proc. A. S. T. M., Vol. IX., 1909).—Adopted Aug. 16, 1909.

Illustrations.

Description.	Eng. Rec.
Tests of lock-bar pipe 42" dia., Springfield, Mass.	April 17, '09

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37.—BRIDGES.

Economic Lengths of Spans.—If a bridge consisting of any number of spans (to be determined) is to be built over a stream or other crossing of length L, it can be shown that, aside from the (end) abutments,* the *most economic* layout of spans and piers will obtain when the cost of *each pier* is about equal to the cost of that portion of the structure which it supports when stripped of about *one-half* the floor system; that is, equal to the cost of the *supported trusses and laterals* and about *one-half* the floor, the cost *per lin. ft.* of same being assumed as proportional to *length of span.*

Let L = total length of crossing, in ft.;
P = cost in dollars of one pier at any given point of profile;
l = length of economic spans, in ft., based on P;
x = number of spans l in the crossing L;
C = cost in dollars per lin. ft. of trusses, laterals, etc., *for span* L;
c = cost in dollars per lin. ft. of trusses, laterals, etc., *for span* l;
y = total cost of trusses, laterals, etc., piers, and a proportionate cost of foundations in abutments.

Then, since
$$c = \frac{C}{x}, \text{ and } l = \frac{L}{x}$$
we have,
$$y = \frac{CL}{x} + Px, \tag{1}$$

Differentiating, and equating with zero, for minimum,
$$\frac{dy}{dx} = -\frac{CL}{x^2} + P = 0$$
whence,
$$P = \frac{CL}{x^2} = cl \tag{2}$$

Now the value of c for span l may be obtained from any known value c' for span l' as follows:
$$c = \frac{c'l}{l'} \tag{3}$$
it being assumed of course that the same types of bridges are used and the same specifications; also that the lengths of spans l and l' do not vary greatly.

Equation (2) may be applied without serious error to a crossing with irregular profile as in Fig. 1. In such a case we would have, numbering the piers and spans consecutively,
$$P_1 = \frac{c_1l_1 + c_2l_2}{2};\ P_2 = \frac{c_2l_2 + c_3l_3}{2};\ P_3 = \frac{c_3l_3 + c_4l_4}{2}; \text{etc.,} \tag{4}$$
which is the proportion stated in the opening paragraph.

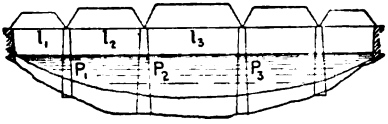

Fig. 1.

Referring to Fig. 1 and to equation (4) it will be noted that for the deeper, or rather more expensive, foundations the longer spans are required. The principles evolved above will apply to spans of plate girders, beams, etc., and also to many other economic problems.

* Abutments usually perform, in part, certain "constant" functions, as retaining walls, etc., not connected with "supporting" the spans, and, if included in the problem, these functions should be considered carefully.

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Equation (2) may be transformed into

$$l = \frac{L}{x} = \sqrt{\frac{lP}{c}} = \sqrt{\frac{l}{w} \cdot \frac{P}{p}} \tag{5}$$

an economic form for general use, in which $\frac{l}{w}$ = ratio of length of span to weight in lbs. per lin. ft. of trusses, laterals, etc., and will be found to be fairly constant within quite wide limits of l; and
p = price in dollars per lb. of w.

Economic Depth of Plate Girders.—Equation (7) will usually give a greater depth of girder than practice warrants, but it will be useful in finding out how much material is sacrificed in any practical limitation of depth.

Let M = total max. bending moment in ft.-lbs. on girder;
A = area of top flange = area of bottom flange, in sq. ins.
f = allowable compressive stress in lbs. per sq. in. in top flange;
x = effective depth of girder = depth of web, in ins.;
t = thickness of web in ins.;
a = total horizontal section of vertical stiffeners and fillers, in sq. ins.;
L = length of span, in ft.;
y = total weight of girder, in lbs.

Then, for a steel girder, assuming no part of web to resist bending, we have, since $A = \frac{12M}{fx}$,

$$y = \frac{3.4 \times 24ML}{fx} + 3.4\ Ltx + \frac{3.4}{12}\ ax. \tag{6}$$

Differentiating and equating with zero, for minimum,

$$\frac{dy}{dx} = -\frac{3.4 \times 24\ ML}{fx^2} + 3.4\ Lt + \frac{3.4}{12}\ a = 0,$$

whence

$$x = 16.97\sqrt{\frac{ML}{f\,(12Lt + a)}} \tag{7}$$

the economic depth of girder.

Ex.—A track stringer, 20-ft. span, is designed to support a total uniform load, including its own weight, of 2500 lbs. per lin. ft. Assuming $f = 10\,000$, $t = \frac{3}{8}$, and $a = 30$, find the economic depth.

Solution.—From equation (7) we have, since $M = \frac{2500\,L^2}{8}$,

$$x = 16{\cdot}97 \times 50 \times 20\sqrt{\frac{20}{8 \times 10\,000\ (240 \times \frac{3}{8} + 30)}} = 24\tfrac{1}{2} \text{ ins. Ans.}$$

Economic Depth of Trusses.—In the preceding case, of the plate girder, it is to be noted that the weight of each part of the girder is given in terms of the variable depth x in order that the value of x may be found for the minimum value of y. This will be done also in the present case, of the truss, but in order to introduce the working stresses per sq. in. in the compression members as "constants" into the fundamental formula (8) for total weight of truss it is convenient to assume that the bridge has been designed using a certain depth of truss (parallel chords) and find whether this depth is the most economic one, which will be the case when the value of x in equation (10) is equal to the assumed depth. If less, the depth will be decreased; if greater, increased. If the difference is slight the economic depth may be assumed equal to the value of x found. One or two trials will be sufficient in most cases. The general equation for total weight of steel truss members is

	Wt. bottom chord.		Wt. top chord.		Wt. end posts.		Wt. int. posts.		Wt. vert. susp.		Wt. diagonals.	
$y =$	B	+	T	+	E	+	P	+	V	+	D	(8)

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In which $B = 3.4\Sigma \frac{M}{fx}(p+e)$

$$T = 3.4p\Sigma \frac{M}{fx}(1+\%)$$

$$E = 3.4\Sigma \frac{S(p^2+x^2)}{fx}(1+\%)$$

$$P = 3.4\Sigma \frac{Sx}{f}(1+\%)$$

$$V = 3.4\Sigma \frac{Sx}{f}(1+\%)$$

$$*D = 3.4\Sigma \frac{S(p^2+x^2)}{fx}(1+\%)$$

Notation:
(May vary for each member.)
M = bending moment, ft.-lbs.;
f = allowable stress per sq. in.;
x = depth of truss, in ft.;
p = panel length, in ft.;
fe = added length for two heads;
$\%$ = percentage added for details;
S = vertical shear, in lbs.

Now, substituting the above values in equation (8) and using summated constants for simplicity, we have the following form:

$$y = 3.4\left[\frac{B'}{x^2} + \frac{T'}{x} + \frac{E'p^2}{x} + E'x + P'x + V'x + \frac{D'p^2}{x} + D'x\right] \quad (9)$$

Differentiating and equating with zero, for minimum,

$$\frac{dy}{dx} = 3.4\left[-\frac{B'}{x^2} - \frac{T'}{x^2} - \frac{E'p^2}{x^2} - \frac{D'p^2}{x^2} + E' + P' + V' + D'\right] = 0$$

whence,

$$x = \sqrt{\frac{B' + T' + E'p^2 + D'p^2}{E' + P' + V' + D'}} \quad (10)$$

In which $B' = \Sigma \frac{M}{f}(p+e)$, for each bottom chord member;

$T' = \Sigma \frac{Mp}{f}(1+\%)$ " top " "

$E' = \Sigma \frac{s}{f}(1+\%)$, " end post;

$P' = \Sigma \frac{s}{f}(1+\%)$, " intermediate post;

$V' = \Sigma \frac{s}{f}(1+\%)$, " vertical suspender;

$D' = \Sigma \frac{s}{f}(1+\%)$, " diag. (incl. counters).

Estimating Weights of Bridges.—The correct weight of a proposed structure can be estimated only from a complete bill of material of the finished design, and young engineers should use this method on all possible occasions. After he has become expert in bridge designing he may resort to quicker methods, more or less approximate, depending on the purpose for which the estimate is to be used.

In designing a structure the live loads, snow load and wind loads must of course be known or assumed. The lengths of spans, if indefinite, may be fixed by the use of formulas (2) and (5); the depths of steel stringers, floorbeams and plate-girders, by equation (7); and the depth of trusses, by equation (10), bearing in mind, of course, that these values are often fixed arbitrarily and that any moderate variation from the economic depth will not add greatly to the weight or cost. For instance, the depth of plate-girders is usually assumed at about $\frac{1}{12}$ the span, and the depth of trusses at about $\frac{1}{8}$ to $\frac{1}{6}$ the span, the larger ratio applying to the shorter spans.

The steps in calculating the design and weights of an ordinary span are as follows: (1) That part of the roadway which directly supports the live load, as the paving, planking and street-car tracks of highway bridges; and the "track" (including rails, guard rails, ties and fastenings) of railroad (steam and electric) bridges, usually assumed at 400 to 450 lbs. per lin. ft. per track. (2) The ballast and corrugated flooring, if used. (3) The stringers or joists. (4) The floorbeams. (5) The top and bottom lateral systems. (6) The portal and vertical sway bracing. (7) The trusses.

* $(1+\%)$ is used instead of $(p+e)$ to simplify equation (9).
† See formula (11) for value of e.

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Each of the above operations, excepting (5) and (6), is dependent on the weights obtained from the preceding operations. Although no "backing up" is required we have to *assume* in some of the operations, as (3), (4) and (7), the weights of the members we are calculating, and sometimes two or three "assumptions" are necessary before the correct one is made.

In estimating the weights of details it is the practice with many engineers to *add certain percentages* to the weights of the *main ribs* of such members, or to *add to their lengths*, or *both*. The method of percentages is always uncertain, and may vary from a few %, for rivet heads, up to 50 or 60 % or more for latticing both sides of light channels. For instance, lacing may be used instead of latticing, or, what is still cheaper, occasional tie plates may be used. Again, a top chord may or may not have a cover plate, and with this uncertainty note what a variation in percentage for details this would imply. Very close estimates have been made on the *total weights of bridges* by adding a certain percentage, say 25%, to the weights of all the main members stripped of their details. As the percentage varies with the type of structure and the specification such a practice would be dangerous for any but the most careful expert.

A good formula for estimating the added length of eye bars to form the heads is the following: For forming two heads,

The added length in ft. $e = \frac{3}{4}$ diam of pin in ins. .....(11)

this to be added to length *c.* to *c.* of pins and estimated as a plain bar.

EXCERPTS AND REFERENCES.

Diagrams and Formulas for Weights of Steel Bridges and Trestles (By H. G. Tyrrell. Eng. News, May and June, 1901).—The following excerpts are noted from the formulas:

Unit stresses in all cases are 10,000 and 12,000 lbs. per sq. in. L = length of span in ft., c. to c. bearings.

Railway Bridges.—Weights are per lin. ft. of single track bridge for steel only; l. l., 2 100-ton engines followed by 4,000 lbs. per lin. ft. of track: Deck plate girder bridge, $100+9L$; deck lattice girder bridge, $100+8L$; half through pl. gird. br. with floor, $300+12L$; same with ties on shelf angle, $200+8\frac{1}{2}L$; same with trough floor, $600+10L$; riveted through truss br., $400+6L$; riveted deck truss bridge (ties on top chord), $200+7L$; pin through truss bridge, $400+5\frac{1}{2}L$; pin deck truss bridge with stringers, $400+6L$; pin deck truss bridge (ties on top chord), $300+6L$.

Railway Trestles.—Assumed loads same as above; weight of spans as above. Weight of bents and bracing is 9 lbs. per sq. ft. of side profile from ground to base of rail.

Electric Railway Bridges.—Live load assumed 25-ton cars, or 2,000 lbs. per lin. ft. of track: Beam bridges, $30+5\frac{1}{2}L$; deck plate girder bridges, $30+5L$; Pony truss bridges, $200+1.8\ L$; through truss bridges, $200+1.6L$.

Highway Bridges with Wooden Floors.—Assumed dead weight of floor is 40 lbs. per sq. ft.; assumed live load is 100 lbs. per sq. ft.; the weights are per sq. ft. of floor, and include that only with joists: Girder bridges and sidewalks, $3+L+4.4$; same without sidewalks, $3+L+3.4$; Truss bridges with sidewalks, $3+L+8$; same without sidewalks, $5+L+7$.

Highway Bridges with Solid Floors.—Assumed dead weight of floor is 150 lbs. per sq. ft.: Deck plate girder bridges, $3+L+2.6$; Half-through girder bridges, $3+L+2.4$; Truss bridges, $3+L+4$.

Graphical Method for Finding Bending Stresses in Eyebars (By W. E. Belcher. Eng. News, July 17, 1902).—From the formula,

$$P = \frac{4\ 900\ 000\ h}{P_1 + 23\ 333\ 000\ \frac{h^2}{l^2}}$$

Where P = stress in lbs. per sq. in. due to bending;
P_1 = working tensile stress in lbs. per sq. in.;
h = depth of bar in inches;
l = length of bar in inches.

The formula is based on the value of E = 28 000 000 and a weight of metal of 0.28 lb. per cu. in.

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Wind Pressure to be Assumed in the Design of Long Bridge Span (By Theodore Cooper. Eng. News, Jan. 5, 1905).

Nickel Steel for Bridges (By J. A. L. Waddell. Trans. A. S. C. E., Vol. LXIII).

Concrete Floors for Railway Bridges (Eng. News, Feb. 16, 1905).—Illustrated.

Tables and Diagrams of the 31 Bridges Over the Missouri River (Eng. News, April 29, 1909).

Standard Specifications for Structural Steel for Bridges (Proc. A. S. T. M., Vol. IX., 1909).—Adopted Aug. 16, 1909.

Diagrams and Illustrations.

Description.	Eng. Rec.
Diagram of compression formulas in long-span bridges	Sept. 3, '10

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38.—RAILROAD BRIDGES.

I.— MOMENTS AND SHEARS—BEAMS OR GIRDERS.

(a) Uniformly Distributed Loads.

w = load in lbs. per lin. foot; l = span in feet.

FULL LOADING—MOMENTS AND SHEARS—FIGS. 1 AND 2.

At any point distant x from left end:

Moment $M_x = \frac{1}{2} wx, (l-x)$, in ft.-lbs.

$= 6\ wx\ (l-x)$, in in.-lbs.

Maximum moment (at center),

$= \frac{1}{8} wl^2$, in ft.-lbs.

$= \frac{3}{4} wl^2$, in in.-lbs.

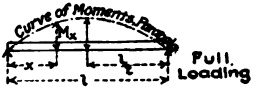

Fig. 1. Moments.

Fig. 2. Shears.

Shear $S_x = w\left(\frac{l}{2} - x\right)$ in lbs.,

(left end positive; right end negative).

Maximum shear (at ends),

$= \frac{1}{2} wl$, in lbs.

Minimum shear (at center) $= 0$.

Note that maximum moment occurs at point of minimum shear; and minimum moment occurs at point of maximum shear.

To Draw a Moment Parabola, divide the span into *any number* of equal parts, odd or even; number the points from one end, beginning with zero; and for ordinates, multiply together the numbers equidistant from center. Afterward, all ordinates so obtained may be multiplied by a constant (one that will give the required middle ordinate). See Figs. 3 and 4.

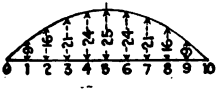

Fig. 3. Even.

Fig. 4. Odd.

PARTIAL LOADING—MOMENTS AND SHEARS—FIGS. 5 AND 6.

At any point distant x ($> a$) from left end:

Moment $M_x = \frac{w}{2}\left[\frac{x}{l}(l-a)^2 - (x-a)^2\right]$, in ft.-lbs.

$= 6w\left[\frac{x}{l}\ (l-a)^2 - (x-a)^2\right]$, in in.-lbs.

Maximum moment is at $x = \frac{l^2+a^2}{2l}$.

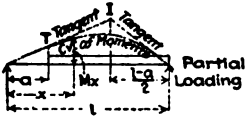

Fig. 5. Moments.

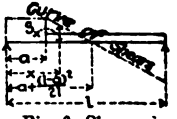

Fig. 6. Shears.

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curve of moments directly above the loading, to the right of T, and to the left of T it is a tangent to the parabola. The point I is directly over the center of gravity of the load, and the

i.e., when $x=a$, is $M_a=\frac{wa}{2l}(l-a)^2$, in ft.-lbs.

$$=\frac{6wa}{l}(l-a)^2, \text{ in in.-lbs.}$$

ar $S_x=\frac{w}{2l}(l-a)^2-w(x-a)$, in lbs.

(left end positive; right end negative).

ximum (+) shear (at $x=a$):

$$=\frac{w}{2l}(l-a)^2, \text{ in lbs.}$$

ximum (−) shear (at $x=l$):

$$=\frac{w}{2l}(l-a)^2-w(l-a), \text{ in lbs.}$$

imum (0) shear is at $x=\frac{(l-a)^2}{2l}+a=\frac{l^2+a^2}{2l}$.

t maximum moment occurs at point of minimum shear; and ent occurs at point of maximum shear.

ED LOADING—MOMENTS AND SHEARS—FIGS. 7 AND 8.

listant x ($>a$ and $<l-c$) from left end:

ment $M_x=\frac{wx}{l}\left(\frac{b^2}{2}+bc\right)-\frac{w}{2}(x-a)^2$, in ft.-lbs.

$$=\frac{12\,wx}{l}\left(\frac{b^2}{2}+bc\right)-6w(x-a)^2, \text{ in in.-lbs.}$$

ment M_a (when $x=a$), at T_1

$$=\frac{wa}{l}\left(\frac{b^2}{2}+bc\right) \text{ ft.-lbs.}=\frac{12\,wa}{l}\left(\frac{b^2}{2}+bc\right) \text{ in.-lbs.}$$

ment M_{a+b} (when $x=a+b$), at T_2

$$=\frac{wc}{l}\left(\frac{b^2}{2}+ab\right) \text{ ft.-lbs.}=\frac{12\,wc}{l}\left(\frac{b^2}{2}+ab\right) \text{ in.-lbs.}$$

ximum moment is at $x=\frac{b}{l}\left(\frac{b}{2}+c\right)+a$.

Isolated Loading

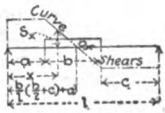

Fig. 7. Moments. Fig. 8. Shears.

ear $S_x=\frac{w}{l}\left(\frac{b^2}{2}+bc\right)-w(x-a)$, in ft.-lbs.

$$=\frac{12w}{l}\left(\frac{b^2}{2}+bc\right)-12w(x-a), \text{ in in.-lbs.}$$

ximum (+) shear (at $x=a$)

$$=\frac{w}{l}\left(\frac{b^2}{2}+bc\right) \text{ ft.-lbs.}=\frac{12w}{l}\left(\frac{b^2}{2}+bc\right) \text{ in.-lbs.}$$

ximum (−) shear (at $x=a+b$)

$$=\frac{w}{l}\left(\frac{b^2}{2}+ab\right) \text{ ft.-lbs.}=\frac{12w}{l}\left(\frac{b^2}{2}+ab\right) \text{ in.-lbs.}$$

nimum (0) shear is at $x=\frac{b}{l}\left(\frac{b}{2}+c\right)+a$.

ension of the principle, the remarks under "Partial loading" to apply to "Isolated loading." Thus, in the above Fig. (7)

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the "curve of moments" comprises the parabola $T_1 - T_2$ and the adjacent tangents, their intersection at I being vertically above the center of gravity of the uniform load.

(b) Concentrated (Engine, Car, Axle, Wheel, etc.) Loads.—During the past 75 years the weight of the locomotive has increased from about 7 to over 200 tons. In approaching the latter mark many engineers predicted that we were reaching the limit, but similar prophesies may be recalled for the 100-ton engine which appeared and disappeared as a standard for bridge calculations during the short period of about ten years. The following may be considered as typical for our heaviest loading—present and prospective—for bridge calculations:

Two engines coupled and followed by uniform load of w *pounds* per lin. ft.

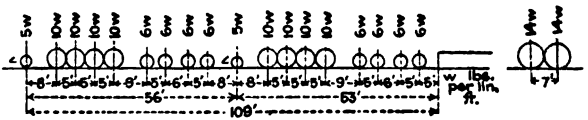
Axle concentrations:	Bogie axle load	$= 5\,w$	1 axle.	
	Driving " "	$= 10\,w$, each;	4, spaced 5 ft.	
	Tender " "	$= 6$ to $7\,w$ " ;	4, spaced 5 and 6 ft.	
Or, for floor loads,	Driving or Car " "	$= 14\,w$ " ;	2, spaced 7 ft.	

as per the following diagrams:

Type R—Engine Diagrams—Axle Loads.

Note.—Wheel* or rail loads w_1 are one-half of axle or track loads w.)

Diagram No. 1. No. 2.



Figs. 9.

1. Types "R" Engine Diagrams—Axle Loads.

[From Figs. 9.]

Name.	Bogie Loads.	Driver Loads.		Tender Loads.		Total Wt. One Engine and Tender.	Uniform Load Following.	Special Pair of Axles, 7' Centres.	
		Each.	Total.	Each.	Total.			Each.	Both.
	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Tons	Lbs.	Lbs.	Lbs.
R	$5\,w$	$10\,w$	$40\,w$	$6\,w$	$24\,w$	$.0345\,w$	w	$14\,w$	$28\,w$
R30	15,000	30,000	120,000	18,000	72,000	103.5	3,000	42,000	84,000
R35	17,500	35,000	140,000	21,000	84,000	120.75	3,500	49,000	98,000
R40	20,000	40,000	160,000	24,000	96,000	138.	4,000	56,000	112,000
R45	22,500	45,000	180,000	27,000	108,000	155.25	4,500	63,000	126,000
R50	25,000	50,000	200,000	30,000	120,000	172.5	5,000	70,000	140,000
R55	27,500	55,000	220,000	33,000	132,000	189.75	5,500	77,000	154,000
R60	30,000	60,000	240,000	36,000	144,000	207.	6,000	84,000	168,000

* By substituting w_1 for w the diagrams will represent "wheel-load" instead of "axle-load" diagrams. Wheel load diagrams are often used in calculations, but axle load diagrams are safer.

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PICAL MOMENT DIAGRAM—"R 50" LOADING.

Moment Diagram of special type "R 50" locomotive (172.5 y 5000 lbs. per lineal foot per track. The loads are given pounds per "axle" and per "track" in order that the diagram convenient for calculation of one truss for double track *afety* in all cases. For practical bridge calculations it will reproduce this diagram on tracing cloth to a scale of say ed over skeleton diagrams of bridges drawn to the same g the engine diagram over the span the moments may be y for any position and hence the maximum moment obtained n the span.

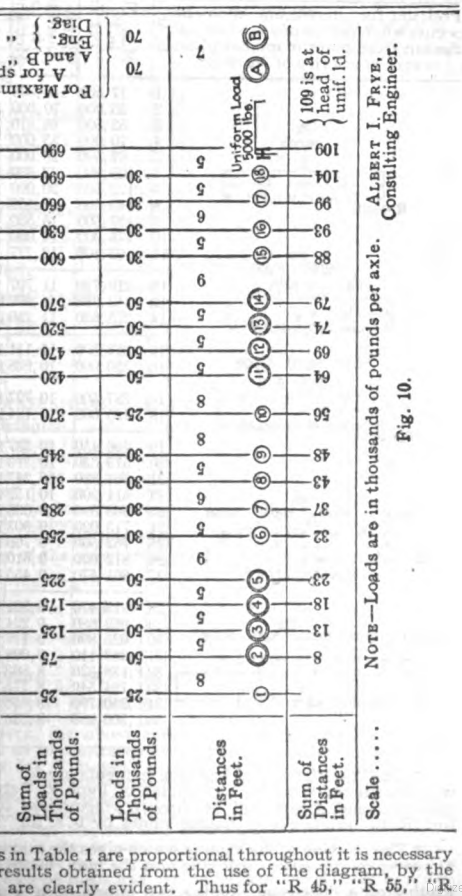

Fig. 10.

grams in Table 1 are proportional throughout it is necessary the results obtained from the use of the diagram, by the which are clearly evident. Thus for "R 45," "R 55," "R y 0.9, 1.1, 1.2, respectively.

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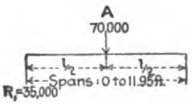
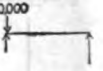
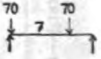
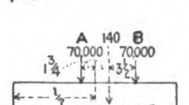
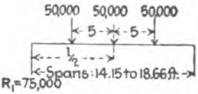
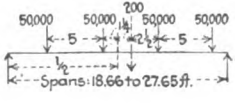
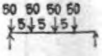
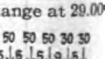
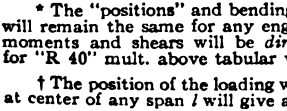
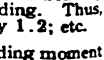
Table 2, following, shows the positions of axle loads, R 50,* giving maximum bending moments on spans up to 55 feet; the maximum bending moments in ft.-lbs. from these positions; and the maximum shears. Useful in the design of girders, stringers, and floorbeams.

2.—Bending Moments and Shears for "R 50" Loading.

†Position of Loading for Maximum Moment. (See Fig. 10.) Position for maximum moment occurs when center of span is equidistant from point of moment and center of gravity of loading.	Span in Ft.	Max. Moment in Ft.-Lbs. per Track.	Equiv. Unif. Load per lin. ft. to give Moment, Lbs.	Maximum Shear, Lbs.	Position of Loading for Maximum Shear.
	1	17,500	140,000	70,000	
	2	35,000	70,000	70,000	
	3	52,500	46,670	70,000	
	4	70,000	35,000	70,000	
	5	87,500	28,000	70,000	
	6	105,000	23,330	70,000	
	7	122,500	20,000	70,000	Change at 7.00
	8	140,000	17,500	78,750	
	9	157,500	15,560	85,560	
	10	175,000	14,000	91,000	
	11	192,500	12,727	95,450	
	12	210,730	11,707	99,170	
	13	242,980	11,502	102,310	
	14	275,630	11,250	105,000	
	15	312,500	11,111	107,330	
	16	350,000	10,938	109,380	Change at 16.33
	17	387,500	10,727	111,760	
	18	425,000	10,494	116,670	
	19	466,450	10,337	121,050	
	20	515,630	10,313	125,000	
	21	564,880	10,247	128,570	
	22	614,200	10,152	131,820	
	23	663,590	10,035	134,780	Change at 23.00
	24	713,020	9,903	138,540	
	25	762,500	9,760	142,000	
	26	812,020	9,610	145,190	
	27	861,570	9,455	148,150	
	28	913,470	9,321	150,890	
	29	969,680	9,224	153,450	Change at 29.00
	30	1,025,890	9,119	157,000	
	31	1,082,110	9,008	160,320	
	32	1,138,320	8,893	163,440	
	33	1,194,540	8,775	166,360	
	34	1,250,780	8,656	169,120	
	35	1,306,980	8,535	171,710	Change at 35.00

* The "positions" and bending moments are for "R 50". The positions will remain the same for any engine of Type R, Table 1; and the bending moments and shears will be *directly proportional* to the loading. Thus, for "R 40" mult. above tabular values by 0.8; for "R 60," by 1.2; etc.

† The position of the loading which gives the maximum bending moment at center of any span l will give also the max. floor-beam reaction, or max. loading on a floor-beam joining two panels each $\frac{l}{2}$ in length.

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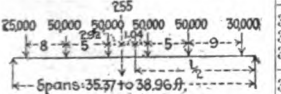
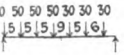
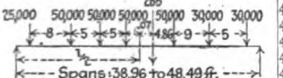
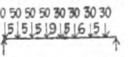
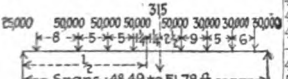
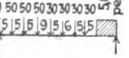
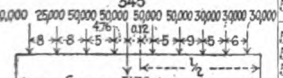
2.—Bending Moments and Shears for "R 50" Loading.—Concluded.

Position of Loading for Maximum Moment. (See Fig. 10.)	Span in Ft.	Max. Moment in Ft.-Lbs. per Track.	Equiv. Unif. Load per lin. ft. to give Moment, Lbs.	Maximum Shear, Lbs.	Position of Loading for Maximum Shear.
	36	1,367,860	8,444	175,000	Change at 35.00
	37	1,431,000	8,362	178,110	
	38	1,494,500	8,280	181,050	
	39	1,558,850	8,199	183,850	
	40	1,630,000	8,150	186,500	Change at 40.00
	41	1,701,330	8,097	189,760	
	42	1,772,580	8,039	192,860	
	43	1,843,830	7,977	195,810	
	44	1,915,090	7,914	198,640	
	45	1,986,330	7,847	201,330	Change at 45.00
	46	2,057,580	7,779	203,970	
	47	2,128,830	7,710	206,600	
	48	2,200,100	7,639	209,220	
	49	2,275,050	7,580	211,840	
	50	2,353,590	7,531	214,450	
	51	2,432,150	7,481	217,060	
	52	2,546,500	7,534	219,660	
	53	2,632,740	7,498	222,260	
	54	2,719,000	7,459	224,860	
	55	2,805,200	7,419	227,450	

II.—MOMENTS AND SHEARS—SPANS WITH FLOOR BEAMS.

(a) Uniformly Distributed Loads.—The maximum moment at any panel point obtains with the span fully loaded from end to end. The maximum shear in any panel obtains with the head of the moving load at or in that panel: specifically, when the length of moving load in the panel + panel length = (the number of the panel from the right end of span, minus 1) + (total number of panels in the span, minus 1). Thus:

Let p = panel length in feet,

n = total number of panels in span,

x = the number of the panel from right end of span; then for maximum shear in panel x, the length of load in "panel x" $= p\frac{x-1}{n-1}$. Thus for a six-panel span the head of load for maximum shear in the 5th panel is $\frac{x-1}{n-1} = \frac{5-1}{6-1} = \frac{4}{5}$ across the panel; for the 4th panel, $\frac{3}{5}$; for the 3rd panel, $\frac{2}{5}$; etc. The maximum shear in the 6th panel obtains with the full panel (and bridge) loaded.

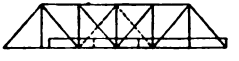


Fig. 11.

In practice, however, the above refinement is eliminated, the panel loads being considered as concentrated at the panel points or joints—upper for "deck" and lower for "through" bridges. This applies also to dead loads, for short spans. For long, through spans a certain portion, say ⅓ to ½, of the dead load per panel is often assumed to be carried at the top chord joints.

In calculating live load stresses, engineers as a rule prefer to use specified engine diagrams, somewhat heavier and more severe, for the structure, than the actual engines in use or immediately contemplated. This is by far the safer, more scientific and more economical when we consider the strength of a structure to be measured by its weakest part. An approximation to actual engine diagrams is the use of equivalent uniform loads which may be used with or without "engine excess."

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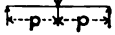
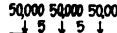
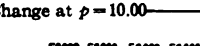
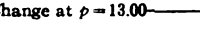
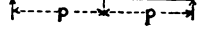
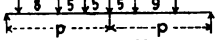
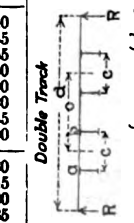
(b) Concentrated Loads—Maximum Floor-Beam Reactions.—The loading which gives the greatest bending moment at *the center* of a span *two panels* in length, will give also the greatest loading at *that point* when supported by a floor-beam, and this loading or its equivalent is called the floor-beam reaction—a necessary factor in the design of the floor-beam itself.

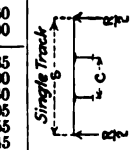
Table No. 3 gives the floor-beam reactions due to "R 50" axle loads, and the positions of the loads, over two panel lengths, which produce these reactions. For any other loading of Type R the floor-beam reaction will be directly proportional to the weights on the drivers.

3.—MAXIMUM FLOORBEAM REACTIONS AND POSITIONS OF LOADING FOR SAME. TYPE "R 50."

Position of Loading for Maximum Floor-beam Reaction.	Span 2 p. Ft.	Panel p. Ft.	Floor-beam Reaction R. Single Track. Lbs.	½ Floor-beam Reaction, R/2. Single Track. Lbs.	Floor-beam Moments for Single and Double Track.
	2	1	70,000	35,000	
	4	2	70,000	35,000	
	6	3	70,000	35,000	
	8	4	70,000	35,000	
	10	5	70,000	35,000	
	12	6	70,000	35,000	
Change at $p = 6.25$	13	6.5	73,080	36,540	
	14	7	78,570	39,285	
	15	7.5	83,330	41,665	
	16	8	87,500	43,750	
	17	8.5	91,180	45,590	
	18	9	94,440	47,220	
	19	9.5	97,370	48,685	
	20	10	100,000	50,000	
Change at $p = 10.00$	21	10.5	104,760	52,380	
	22	11	109,090	54,545	
	23	11.5	113,040	56,520	
	24	12	116,670	58,335	
	25	12.5	120,000	60,000	
	26	13	123,080	61,540	
Change at $p = 13.00$	27	13.5	126,850	63,425	
	28	14	130,360	65,180	
	29	14.5	133,620	66,810	
	30	15	136,670	68,335	
	31	15.5	139,520	69,760	
	32	16	142,190	71,095	
	33	16.5	144,700	72,350	
	34	17	147,060	73,530	
	35	17.5	149,290	74,645	
Change at $p = 18.17$	36	18	151,390	75,695	
	37	18.5	153,920	76,960	
	38	19	156,580	78,290	
Change at $p = 19.00$	39	19.5	159,870	79,935	
	40	20	163,000	81,500	
	41	20.5	165,960	82,980	
	42	21	168,810	84,405	
	43	21.5	171,510	85,755	
	44	22	174,090	87,045	
	45	22.5	176,560	88,280	
	46	23	178,910	89,455	
	47	23.5	181,170	90,585	
	48	24	183,330	91,665	
	49	24.5	185,410	92,705	
	50	25	187,400	93,700	



Floor-beam moment $\left\{ \begin{array}{l} \text{at } a = R\dfrac{(d-o-c)}{2} \text{ ft.-lbs.} \\ \text{at } b = R\dfrac{(d-o)}{2} \text{ ft.-lbs.} \end{array} \right.$



Floor-beam moment $= \dfrac{R}{2}\left(\dfrac{s-c}{2}\right)$ ft. lbs.

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(c) **Concentrated Loads—Positions for Maximum Moment.**—In order to find the stresses in the top and bottom chords of bridge trusses and girders with floor-beams, it is necessary to know the maximum bending moments at the floor-beam or panel points. The main problem consists in finding the "position" of the loading at each panel point, the bending moment being then found by taking moments about that point. The chord stresses are obtained by dividing the bending moments in ft.-lbs. by the respective moment arms of the chord pieces, in feet. For trusses with parallel chords the moment arm is constant, being the height of truss.

The position of loading for maximum bending moment at any panel point* *usually* obtains:

(1) When the heaviest loads are *nearest* the point.
(2) When one of these loads is *at* the point.
(3) When the average loading per lineal foot to the left of the point = the average loading per lineal foot on the whole span (the load *at* the point to be applied in whole or in part to either portion of the span).

Chord Stresses in Pratt Truss.

Problem.—Find the live-load stresses in the top chord member T and in the bottom chord member B of Pratt† truss, Fig. 12, of a single track railroad bridge, 144 ft. span; using loading "R 50" (page 691). Note that in *all cases* the moving load comes on at the right hand end of span.

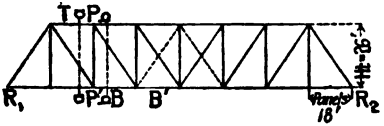

Fig. 12.

Solution.—By "cutting" sections *a-a* and *b-b* it is evident that the bending moment for maximum stress in T will be at the point P', and for B, at the point P, directly above. Hence the same bending moment will do for both stresses. Applying the moment diagram and the rules just given, we place one of the heavy loads at P' so that the loads to the left of P' will equal $\frac{3}{8}$ (or $\frac{1}{4}$) the total loads on the bridge. Hence, if load (5) is placed at P' the load to the left is 175 to 225 thousand lbs., while the whole load on the span is $690 + 5 \times 22 = 800$ thousand lbs. As $\frac{1}{4}$ of 800, or 200, is between 175 and 225 we adopt this position as the *probable* one for maximum moment at P'. By trial we find this to be true and that the moment at $P' = 12{,}045{,}000$ ft.-lbs. As there are two trusses each resisting one-half the moment, and further, as the moment arms are 26 ft. the stresses in T and B are each equal to $12{,}045{,}000 \div 52 = 231{,}600$ lbs. − compression in T and tension in B. If the engine loading is "R 40" instead of "R 50" the stresses will be $\frac{4}{5}$ the above or 185,300 lbs., etc.

The following are methods of calculation with various loads at P', using the engine diagram:

Load (3) *at* P'.	*Load* (4) *at* P'.	*Load* (5) *at* P'.
12 ft. of uniform load.	17 ft. of uniform load.	22 ft. of uniform load.
Mom. at $H = 40{,}090$.	Mom. at $H = 40{,}090$.	Mom. at $H = 40{,}090$.
Add $690 \times 12 = 8{,}280$.	Add $690 \times 17 = 11{,}730$.	Add $690 \times 22 = 15{,}180$.
" $5 \times 12 \times 6 = 360$.	" $5 \times 17 \times 8\frac{1}{2} = 722.5$	" $5 \times 22 \times 11 = 1{,}210$.
48,730.	52,542.5	56,480.
$\frac{48{,}730 \times 36}{144} = 12{,}182.5$	$\times \frac{36}{144} = 13{,}135.6$	$\times \frac{1}{4} = 14{,}120.$
Deduct 575.	1,200.	2,075.
11,607.5	11,935.6	Max. = 12,045.

Hence the maximum moment is 12,045,000 ft.-lbs., as above noted. With load 6 at P' the moment is only 11,870,600 ft.-lbs.

* The moment *at* any floor-beam panel point for any system of loading is the same as if there were no floor-beams.

† In this case the chord stresses will be the same whether the bridge is "deck" or "through."

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Chord Stresses in Warren Truss.

Triangular Web System.—For practical reasons the Post truss and the through triangular or Warren truss types have nearly disappeared. The Warren truss remains principally for deck spans and swing bridges; and for short, fixed, through spans. For deck spans it is economical to erect verticals at the lower chord panel joints to support intermediate floor-beams as in Fig. 13.

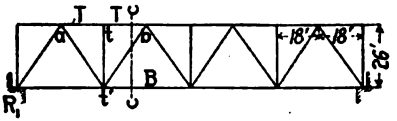

Fig. 13.

Using the same live load, "R 50," as in the preceding case, note that the stress in the top chord members TT is equal to the stress in T of the Pratt truss, Fig. 12, namely, 231,600 lbs. It is obtained by placing load (5) of the engine diagram at *t*, *a floor-beam point*, and taking moments about t'.*

Consider, now, the stress in the upper chord *a b*, Fig. 13, assuming *no floor-beam* at *t*, *in a vertical line with the center of moments*, t'. For this case the loading for maximum will be somewhat modified, and the following rule will usually apply: *The average loading per lineal foot to the left of the center of moments*, t', (*including one-half*† *the load in the panel under consideration*, *ab*) = *the average loading per lineal foot on the whole span*. After the position of loading for maximum has been fixed take moments about t', using the reaction at *a* due to loading in panel *ab*, in deducting the negative moment.

(d) Concentrated Loads—Positions for Maximum Shear.—On page 693, under uniformly distributed loads, we find that for maximum shear in any panel the length of the moving load in that panel will be $p\frac{x-1}{n-1}$, in which p = panel length, x = number of the panel from right hand end of span, and n = the total number of panels. Thus, the head of the moving load‡ in any panel is such that *the average load per lineal foot in the panel equals the average load per lineal foot on the whole span*. This principle may be applied *tentatively* in fixing the position of *concentrated* loads for maximum shear.

The position of loading for maximum shear in any panel of a truss with simple web system‖ *usually* obtains:

(1) When the heaviest loads are *nearest* to and just to the *right* of the panel.

(2) When one of these loads is *at* the panel point at the right hand end of the panel in question.

(3) When the average loading per lineal foot in the panel = the average loading per lineal foot on the whole span (the load at the panel point at the right hand end of the panel *may* be applied in whole or in part to the panel loading).

Problem.—Find the live-load shear in panel *t b*, Fig. 13, of a two-truss double track deck railroad bridge, 144 ft. span; using loading "R 50."

---

* The stress in B of the Warren truss is greater than B of the Pratt, and is obtained by taking moments about *b*, as would naturally be indicated by the cutting section *c-c*, with load (8) at the panel point. This produces a moment of 14,698,750 ft.-lbs. and a corresponding stress in B, for one triangular truss, of 282,670 lbs., equivalent to stress in B' of the Pratt truss.

†Accurately, $at + ab$, whether it is one-half or any other fraction.

‡ The head of the moving load for maximum shear is at the "neutral shear point" because any concentrated load at that point produces zero shear in the panel. A load to the right of the neutral point will produce positive shear, while a load to the left will produce negative shear.

‖ For special case with sub-panel system, see Sec. 40, Highway Bridges, page 724.

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Solution.—The three conditions above named for maximum shear are fulfilled by placing load (3) at b, whence the total load in the panel, 75 to 125, equals the total load on the bridge, 660, divided by the number of panels, 8; thus, $660 \div 8 = 82.5$. Inspection, however, reveals to us the possibility of a maximum with load (2) at b, hence we solve for these two positions, using moment diagram:

Load (2) at b.		Load (3) at b.	
$R_1\, l =$	29,560	$R_1\, l =$	33,340
$+ 630 \times 5 =$	3,150	$+ 660 \times 4 =$	2,640
$=$	32,710		35,980
Dividing by l, $R_1 =$	227,153 lbs.	Dividing by l, $R_1 =$	249,861 lbs.
Deduct reaction at t due to load (1) in panel, $25 \times \frac{5}{18}$ $=$	11,111 "	Deduct reaction at t due to loads (1) and (2) in panel $=$	31,944 "
Shear $=$	216,042 "	Shear $=$	217,917 "

Therefore, maximum shear. 217,917 lbs., is obtained with load (3) at b. With load (4) at b the shear is 206,944 lbs. The compressive stress in the diagonal $t'b$ is $217{,}917 \times \frac{t'b}{t't} = 217{,}917 \times 1.216 = 264{,}990$ lbs.

III.—LATERAL BRACING.—WIND AND CURVE PRESSURE.

Horizontal or lateral bracing is designed to resist the lateral forces due to wind pressure, and the centrifugal pressure of moving trains on curves; to shorten the unsupported "column length" of upper chord, for economy of design; and to give general rigidity to the structure. Stiff bracing is preferable to rods.

Wind Pressure.—The direct wind pressure on any exposed surface is about proportional to the square of the velocity.* A velocity of 100 miles per hour produces a normal pressure of about 50 lbs. per sq. ft.; 90 miles per hour, about 40 lbs.; and 80 miles per hour, about 30 lbs. As rigidity is a very essential feature it is advisable to use the higher figure, 50 lbs., as the pressure per sq. ft. on the exposed surface of all trusses and the floor system when the bridge is unloaded. An *alternative* wind load of 30 lbs. per sq. ft. on the same surface and also on a moving train between elevations 2.5 and 10 ft. above base of rail is also prescribed. Sometimes the wind load per lin. ft. is stated in actual amounts for each chord. Thus, a load (either fixed or moving) of say 150 lbs. per lin. ft. for the unloaded chord and a moving load or say 600 lbs. per lin. ft. for the loaded chord. See Wind Pressure under General Specifications for Steel Railroad Bridges, following.

Problem.—Find the stresses in top lateral system of single track through Pratt truss span of 144 ft., as per following sketch; loading 75 lbs. per lineal foot for each chord ($18 \times 75 = 1{,}350$ lbs. per joint.)

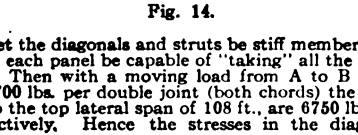

Fig. 14.

Solution.—Let the diagonals and struts be stiff members and let either single member in each panel be capable of "taking" all the shear, in tension or compression. Then with a moving load from A to B of 1350 lbs. per single joint or 2700 lbs. per double joint (both chords) the shears in panels a, b and c, due to the top lateral span of 108 ft., are 6750 lbs., 4500 lbs. and 2700 lbs., respectively. Hence the stresses in the diagonals ($\frac{18}{12} \times$ the

* Pressure in lbs. per sq. ft. $= \frac{(\text{Velocity in miles per hour})^2}{250}$, approximately. (Recent experiments by M. Eiffel, and also by Dr. Stanton, indicate that for velocities of 40 to 90 miles per hour the denominator of the fraction may safely be increased from 250 to 300 or 333.—See *Engineering Digest*, March, 1908.) See also, Section 46, Roofs, pages 794, etc.

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shears) are 10,100 lbs., 6,800 lbs. and 4,100 lbs. In practice, minimum sections of material are specified, either direct or by formula, below which the design would be considered weak no matter how small the stresses.*

IV.—PORTAL AND INTERMEDIATE VERTICAL BRACING.

Portals should be designed to transmit all lateral-bracing stresses at the unloaded or far chords, directly to the abutments through the end posts. For long spans, vertical bracing is inserted at the intermediate posts rather for stiffness than to transmit any wind pressure from one lateral bracing system to another, although it is usually designed sufficiently strong to meet the "local" wind pressure. The same principles which apply to portal- also apply to intermediate bracing, hence our remarks will be confined to the former, and for brevity of explanation, to "through" bridges.

The calculations of portal stresses are much simplified by making certain assumptions which render the framework quite statically determinate. These assumptions are that the bottoms of the end posts are hinged laterally; that all stiff-riveted portal connections are also hinged; and that for a double or multiple system of portal bracing only one simple statically determinate system acts at a time. These assumptions are on the side of safety.

The following simple illustrations are typical. They are tipped to a horizontal position so that the acting forces P may be vertical and perhaps better illustrate the cantilever principle. Note that the horizontal and vertical reactions at the *bottom* of the two end posts are numerically the same for each post and, consequently, the shears and the bending moments in the latter are respectively similar.†

Diagonal d in tension.

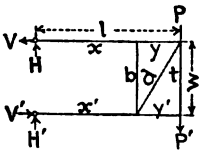

Fig. 15.

Diagonal d in compression.

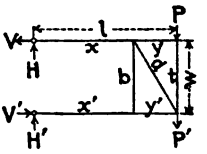

Fig. 16.

Fig. 15 or 16, with either P or P' acting: $H = H' = \frac{P}{2}$ or $\frac{P'}{2}$; $V = V' = \frac{Pl}{w}$ or $\frac{P'l}{w}$; stress in $x = +\frac{Pl}{w}$ or $+\frac{P'l}{w}$; stress in $x' = -\frac{Pl}{w}$ or $-\frac{P'l}{w}$;

Fig. 15, with either P or P' acting:

Stress in $b = -\frac{Pl}{2y}$ or $-\frac{P'l}{2y}$.

" " $t = -\left(P + \frac{Px}{2y}\right)$ or $-\frac{P'x}{2y}$.

" " $d = +\frac{Pl}{y} \cdot \frac{d}{w}$ or $+\frac{P'l}{y} \cdot \frac{d}{w}$.

Fig. 16, with either P or P' acting:

Stress in $b = +\frac{Pl}{2y}$ or $+\frac{P'l}{2y}$.

" " $t = +\frac{Px}{2y}$ or $+\left(P' + \frac{P'x}{2y}\right)$.

" " $d = -\frac{Pl}{y} \cdot \frac{d}{w}$ or $-\frac{P'l}{y} \cdot \frac{d}{w}$.

The following are some of the usual types of portals.

Figs. 17.

* For *highway bridges* adjustable rods are often used instead of stiff members and it is customary to specify an initial stress of, say, 10,000 lbs. to be added to the calculated stress in proportioning the dimensions of the laterals.

† In the design of the posts these wind stresses have to be "considered" in addition to the regular stresses as truss members.

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Fig. *a* is the simplest and the one just analyzed by calculation. By similar methods and with reasonable assumptions the others may be calculated readily.

V.—GENERAL SPECIFICATIONS FOR STEEL RAILROAD BRIDGES.

The main specifications below are adapted from those of the American Bridge Company (Am. Br Co.), 1900, C. C. Schneider, Vice President. The foot-notes are inserted by the writer as showing the practice (in deviation or otherwise) of a few of our leading railroad companies, as follows:

Chicago, R. I. and Pacific (*C. R. I. & P.*), 1906; J. B. Berry, Chief Engineer.
Del., Lack. and Western (*D. L. & W.*), 1903; A. E. Deal, Bridge Engineer.
Lake Shore and Mich. Sou. (*L. S. & M. S.*), 1904; Sam'l Rockwell, Chief Eng.
* Lehigh Valley (*L. V.*), 1906; Walter G. Berg, Chief Engineer.
Philadelphia and Reading (*P. & R.*), 1906; William Hunter, Chief Engineer.
Southern Pacific (*S. P.*), 1906; William Hood, Chief Engineer,

(a.) General Description.

Material.—1. To be of rolled steel† as specified below. ‡Cast iron or cast steel permitted only in machinery of movable bridges and in special cases for shoes and bearings.

Types of Bridges Recommended.—2. Limiting spans, in ft. to be: ||

Clearance.—3. On straight line a clear section shall be provided to conform to given requirements.¶ The width must be increased so as to allow the same minimum clearance on curves§ and on double track.

* "All Railroad Bridges on the Lehigh Valley Railroad System are to be built in accordance with the 'General Specifications for Steel Railroad Bridges and Viaducts; New and Revised Edition, 1901; by Theodore Cooper, Consulting Engineer,' modified as follows:" [Some of the modifications are incorporated in these foot-notes.]—F. E. Schall, Bridge Engineer.

† *P. & R.* allows use of wrought iron for laterals and unimportant members.

‡ *L. S. & M. S.* specifies all castings to be cast steel. *S. P.* specifies rollers for swing bridges to be cast steel; expansion rollers to be bar steel, or cast steel of equal strength.

|| Type.	*Am. B. Co.*	*C. R. I. & P.*	*D. L. & W.*	*L. S. & M. S.*	*L. V.*	*S. P.*
Trough floors (long'l) ...	0–20	.....	.....	.....	0–20	.....
Rolled beams.....	0–20	0–19	0–20	0–23	0–20	0–19
Plate girders (riv.).....	20–100	19–110	16–100	23–100	20–120	19–100
Lattice girders (riv.)	100–140	.....	90–160	.....	.....	.....
Riveted trusses.....	.....	.....	.....	100–180	120–150	.....
with inclined end posts	.....	100–200	.....	.....	.....	100–150
Pin con. trusses.....	140–	.....	150–	180–	150–	.....
with inclined end posts	.....	200–	.....	.....	.....	150–

¶ Road.	*H'*	*H*	*m*	*t*	*b*	*d*	*h*
C R. I. & P.....	23'-6"	.....	7'-0"	3'-6"	5'-6"	5'-0"	4'-0"
D. L. & W......	22'-0"	.....	7'-0"	3'-6"	5'-6"	5'-0"	4'-0"
L. S & M. S....	.....	22' 0"	7'-6"	4'-0"	5'-3"	6'-0"	5'-0"
L. V......	22'-0"	.....	7'-0"	2'-9"	5'-3"	4'-3"	6'-6"
P. & R......	22'-0"	.....	7'-0"	3'-0"	5'-0"	5'-0"	2'-0"
S. P......	.....	24'-0"	7'-6"	3'-0"	5'-6"	5'-0"	4'-0"

H' = to Top of Rail.
H = to Base of Rail.

§ Increase width for curvature and super-elevation for car 80' lg., 14' high and 60' c.-c. trucks—*C. R. I. & P.* Increase *m* 1 in. per deg. of curvature, and increase *m* on inside of curve 2½ ins. additional per each inch super-elevation of track. Width to be increased proportionately for 2 or more tracks.—*L. S. & M. S.*

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Spacing of Trusses.—4. Width between centers of trusses to be not less than $\frac{1}{10}$* of the span.

Spacing of Deck Plate Girders.—5. Generally 6½† ft. centers.

Floor Beams.—6. Shall be riveted between the posts, above or below the pin, in through bridges.

Spacing of Stringers.—7. Generally 6½‡ ft. centers, the tracks being 13 ft. centers, standard.

Wooden Floor.||—8. Cross-ties 8"x8" for stringers spaced 6½ ft. centers. For spacing over 6½ ft., ties to be proportional for fiber strain not to exceed 1000 lbs. per sq. in. on timber, assuming max. wheel load distributed over three ties. Ties to be spaced 6" or less in the clear, notched down ½", and have full bearing on stringers. 9. Every fifth tie to be fastened to stringer by a ¾" bolt.

Guard Rails.°—10. Timbers 6"x8" on each side of each track, with inner faces not less than 3'-3" from cen. of track. To be notched 1" over every tie and fastened to every third tie and at each splice by a ¾-in. bolt. Splices to be over floor timbers, with half-lap joints 6" long. 11. Floor timbers and guards to be continuous over piers and abutments. 12. On curves, outer rails to be elevated as may be required.

(*b*) Loads.

Dead Load.♯—13. In estimating the weight of the structure for the purpose or calculating stresses, timber is assumed at 4½ lbs. per ft. B.M.; and rails, spikes and joints at 100 lbs. per lin. ft. of track.

Live Load.¶—14. A moving load for each track, consisting of two engines coupled at the head of a uniformly distributed train load, placed so as to give the greatest stress in each part of the structure. The load will be as specified by the railroad company. Cooper's standard loading, however, is recommended. (See Tables 4 and 5, pages 707, 708.)

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* And preferably not less than $\frac{1}{15}$ of the span.—*L. S. & M. S.*

† 7 ft. for spans up to 60 ft.; 8 ft. for spans 60 to 110 ft.—*C. R. I. & P.*

‡ 7 ft.—*C. R. I. & P.* and *S. P.* 6 ft. for double track through, and deck plate; 8 ft. for single track through.—*P. & R.* 6½ ft.—*D. L. & W.* and *L. S. & M. S.*

|| 8"x8" by 10 ft., framed to 7½", for stringers spaced 6½ ft. centers; depth of tie to be increased 1" per each 6" increase in stringer spacing. Ties spaced 12" centers, every fourth tie fastened to each stringer by a 1" hook bolt.—*L. S. & M. S.* Ties (yellow pine or white oak) to be proportioned to an extreme fibre stress of 800 lbs. per sq. in. from the loading; spaced not over 6" in clear, notched down ½", and secured to supporting girders by ¾" bolts not over 6 ft. apart.—*L. V.*

° Guards to be 8" wide and 6" deep, framed to 4" over ties, with inner edges 4'-2" from cen. of track. They shall be fastened to the ties by a ¾" bolt through every fourth tie, these bolts to be through the ties which are connected to the stringers by hook bolts.—*L. S. & M. S.* Guards to be 6"x8" southern yellow pine, notched 1½" over every tie, bolted by a ¾" bolt to every third tie and spliced over a tie by half and half joint 8" wide" bolted at splice; the inner face of guard to be 4'-1½" from cen. of track; and all heads or nuts on upper faces of guard timber to be countersunk below surface of wood.—*L. V.*

♯ Timber at 4½ lbs. per ft. B. M.; ballast, 100 lbs. per cu. ft.; rails and fastenings, 150 lbs. per lin. ft. of track.—*C R. I. & P.* The floor, consisting of the ties, rails, guard rails, and all spikes and bolts necessary to fasten same, assumed at 400 lbs. per lin. ft. of track.—*D. L. & W.* Ordinary floor at 400 lbs. per lin. ft. of track. Make due allowance when plank- or ballast floor is used.—*L. S. & M. S.* Ordinary floor assumed at 500 lbs. per lin. ft. of track. For ballast floors allow following weight per cu. ft.: Ballast, 120 lbs., Concrete, 140 lbs., Asphalt, 90 lbs., Lumber 54 lbs.—*P. & R.* Ordinary floor assumed at 500 lbs. per lin. ft. per track. Apply three-fourths of *total dead load* at panel points of loaded chord, and one-fourth at unloaded chord.—*S. P*

¶ The following are Railroad Co. Standards. Diagrams 1 and 2 are shown, and that one which will produce the greatest stress in any member

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mpact.*—15. The effect of impact and vibration due to live determined by the following formula and added to the live

$$I = S\left(\frac{300}{L + 300}\right) \quad \text{(See Table 6, page 709.)}$$

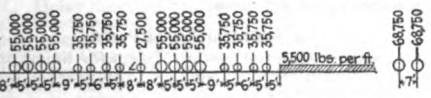
act to be added to live load strees S.

th in ft. of loaded track distance which produces the maxi- member.

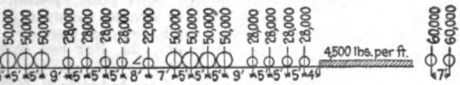
the latter usually for stringers and floor beams. Loads (lbs.) and uniform loads, per lin. ft. of track.

Diagrams: No. 1. *No. 2.*

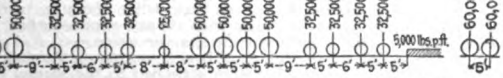
(1906). Two 195¼ ton consolidation. Class E 55 (Cooper):



1903). Two 167 ton consolidation engines.

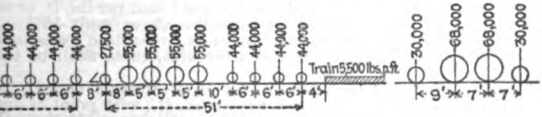


(1904). Two 177½ ton consolidation. Class E 50 (Cooper):

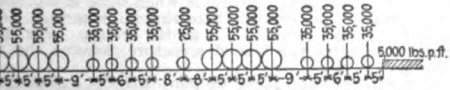


Cooper's Loading; to be specified for each structure.

). Two 211¾ ton consolidation engines. [For trans. trough 0# p. l. f. of track.] (Moving simultaneously on each track e direction):



Two 192½ ton consolidated engines:



ed ck $\Big\}$ $I = S\left(\frac{300}{L + 300} + \frac{100 - L}{500}\right)$; Over 100 ft. $\Big\}$ $I = S\,\frac{300}{L + 300}$. —*C. R. I. & P.*

bridges and other movable structures while in motion only. For counter stresses, $I = L$. For hip and panel suspenders onnections of the floor system, $I = 1.25\ L$. For all other $\frac{L}{+ D}$. In which L = combined stresses from live load and cen- and D = dead load stress.—*L. S. & M. S.*

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Wind Pressure.*—16. Shall be assumed acting in either direction horizontally: *First.* At 30 lbs. per sq. ft. on exposed surface of all trusses and the floor as seen in elevation, in addition to a train of 10 ft. average height, beginning 2'-6" above base of rail, moving across the bridge *Second.* At 50 lbs. per square foot on the exposed surface of all trusses and the floor system. The greatest result to be assumed in proportioning the parts. 17. For determining the requisite anchorage for the loaded structure, the train shall be assumed to weigh 800 lbs. per lin. ft.

Momentum of Train.—18. Coefficient of sliding friction of wheels on rails, in stopping train, to be assumed at 0.2; this to apply to longitudinal bracing of trestle towers and similar structures.

Centrifugal Force of Train.—19. On curves, assume the centrifugal force C, in lbs., due to each train, to be: $C = 0.03$ (Wt. of train in lbs. $\times$ degree of curvature up to 5°); the coefficient (0.03) to be reduced 0.001 for every degree of curvature above 5 degrees.

(c.)—Proportion of Parts.

Least Thickness of Material.†—20. Except for lining or filling vacant spaces, ⅜" thick for main members and their connections, and $\frac{5}{16}$" thick for laterals and their connections.

Permissible Tensile Stresses.‡—21. On all parts of structure, sum of maximum loads, together with impact: Soft steel, 15000; Medium steel, 17000 lbs. per sq. in. 22. Same limiting stresses for wind pressure, centrifugal force, or momentum of train. 23. For net section, deduct size of rivet hole ⅛" larger than diam. of rivet. 24. For pin connected riveted

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* "Lateral Load:" 750 lbs. per ft. of loaded chord; 200, for unloaded; both considered as moving. "Wind load" on viaduct towers: 50 lbs. per sq. ft. on 1½ times the vertical projection of structure unloaded; or 30 lbs. per sq. ft. on same surface plus 400 lbs. per lin. ft. of structure applied 7 ft. above the rail for assumed wind on train when the structure is either fully loaded or loaded on either track with empty cars assumed to weigh 1200 lbs. per lin. ft., whichever gives the greater stress.—*C. R. I. & P.*

For spans, 300 lbs. per lin. ft. of bridge, acting on moving train of loaded or unloaded cars, for lateral system attached to the loaded chord, and 300 lbs. per lin. ft. acting on trusses and divided equally between top and bottom lateral systems. Trestle towers shall be proportioned for above wind forces for spans, and in addition thereto a wind pressure of 100 lbs. per vertical ft. of tower.—*D. L. & W.*

For single track bridges, 300 lbs. live load and 150 lbs. dead load per lin. ft. of loaded chord, and 150 lbs. dead load per lin. ft. of unloaded chord. For double track bridges, increase the above loads 50%. Where 30 lbs. per sq. ft. of exposed surface produces larger dead loads than the above in plate girder bridges or special structures, these shall be taken instead.—*L. S. & M. S.*

For girder bridges, 200 lbs. per lin. ft. for each chord. Also, for loaded chord, a moving load of 400 lbs. per lin. ft., with point of application 7½ ft. above the rail. For viaducts and trestle towers, 50 lbs. per sq. ft. of the projected surface of two trusses and two sides of towers on the vertical plane through the axis of the structure when same is unloaded; with structure loaded, take 30 lbs. per sq. ft. of this same surface, and in addition, the moving wind load specified for girder bridges. For determining the requisite anchorage for the loaded structure, assume the train to weigh 600 lbs. per ft.—*P. & R.*

Lateral wind pressure same as Am. Br. Co. specifications with the following minimum values; Bracing of loaded chord, 500 lbs. per lin. ft., 300 of which is moving- and 200 dead load; unloaded chord, 150 lbs. per lin. ft., uniformly distributed. On viaduct towers, as seen in elevation, use 60 lbs. per sq. ft. on the loaded, and 100 lbs. per sq. ft. on the unloaded, structure.—*S. P.*

† ⅜" except for fillers.—*C. R. I. & P.* ⅜" except for fillers; 1 sq. in. in section for rods or bars; 1½ sq. ins. for counters.—*D. L. & W.* ⅜" except for latticing and fillers; 3×3×⅜ angles.—*L. S. & M. S.*

‡ 16000 for structural steel at 60000 ult.—*C. R. I. & P.*

Soft Steel:—Bottom chords and main diagonals: Eye bars, 9000 *l. l.*, 14000 *d. l.*; built sections, 8500 *l. l.*, 12500 *d. l.*; counters, 8500. Hip sus

ers, make net section through pin hole $1\frac{1}{4}$ of net section ; and net section back of pin hole at least $\frac{3}{4}$ of net section ole.

e Compressive Stresses.*—25. Compression members:

Medium Steel.

$$p = \frac{17000}{1 + \frac{l^2}{11000 r^2}}$$

p = working load in lbs. per sq. in.; l = effective length of column in ins. r = least rad. of gyr. (See Table 7, page 710.)

; 120 for wind bracing; 100 for other members.
th of post in pin connected bridges, 10 ins.

oorbeam hangers, etc., 7500. Tension flanges of built girders ams, 9000. Lateral, vibration and cross bracing, 12000. All ole $\frac{1}{8}''$ + diam. of rivet. *Medium Steel:*—Allow 10% increase l. Holes must be drilled, or punched and reamed. *Double* nain members of trusses, as well as flanges and webs of girders ns designed to carry a double track, the above unit live load usses, and the unit live and dead load stresses for girders and l be increased 10%; but not to apply to details.—*D. L. & W.*
impact, 16000 for 56-64000 steel.—*L. S. & M. S.*
impact, 15000 for 60000 steel.—*P. & R.*

beams and girders, 7000 $\left(1 + \frac{\text{min.}}{\text{max.}}\right)$; bars, forged ends, $\left(\frac{}{}\right)$; plates or shapes (net), 8000 $\left(1 + \frac{\text{min.}}{\text{max.}}\right)$; floorbeam hangers. es (net), 6000. Medium steel, 60-68000.—*S. P.*
0 $\frac{l}{r}$ for 60000 steel.—*C. R. I. & P.*

For *Soft Steel*, 54 – 62000, $p = \frac{a}{1 + \frac{l^2}{br^2}}$.

		a	b
and vertical posts of pin spans	Live load,	8500	18000
members with two pin ends..	Dead load,	12500	18000
nd top chords of lattice spans, f pin spans, and all main mem- e or both ends fixed, not other- d.....	Live load,	8500	24000
	Dead load,	12500	24000
of lattice spans.....	Live load,	8000	24000
	Dead load,	12000	24000
oration and cross bracing.....	Live load,	12000	24000
	Dead load,	12000	24000

eel, 62–70000, above values of p increased 10%.—*D. L. & W.*

Including impact, $p = \frac{16000}{1 + \frac{l^2}{12000\, r^2}}$ for 56-64000 steel;

for laterals, nor 100 for main members (40-60 preferred).
S.

Including impact, $p = \frac{15000}{1 + \frac{l^2}{13500\, r^2}}$ for 60000 steel;

exceed 100, except wind bracing, 120.—*P. & R.*

ns used as girders, and compression members with $\frac{l}{r} < 40$, same see *S. P.* foot-note above). For lengths where $\frac{l}{r} > 40$ use:—
fixed, $p = 8000 \left(1 + \frac{\text{min.}}{\text{max.}}\right) - 25\frac{l}{r}$; for one or both ends hinged, $\left(\frac{\text{min.}}{\text{max.}}\right) - 45\frac{l}{r}$. Steel, 60-68000 ult. No compression member gth greater than 45 times its least width.—*S. P.*

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Alternate Stresses.—28. Make total area in member equal to sum of areas required for each stress.

Combined Stresses.—29. Maximum stresses due to wind and centrifugal force, added to those due to vertical loading (including impact), shall not exceed: 19000 lbs. for soft steel, or 21000 lbs. for medium steel, properly reduced for compression. 30. Reversal of stresses, if any, must be considered.

Transverse Loading of Tension or Compression Members.*—31. When the floor system rests directly on the chord, the chord member must be proportioned so that the algebraic sum of the stresses per sq. in. on outer fibre, resulting from the direct compression or tension, and ¾ of the max. bending moment (considering the chord as a beam of one panel length, supported at ends), shall not exceed the before-mentioned limiting stresses in tension or compression, the proper amount of impact being added to each kind of loading. 32. Bending moment at panel points shall be assumed equal to that at center, but in opposite direction. 33. Other members similarly affected are to be treated likewise.

Shearing and Bearing Stresses†.—34. For rivets, bolts and pins, the shearing stress per sq. in. shall not exceed 11000 for soft steel, and 12000 for medium steel; and the pressure upon the bearing surface of the projected semi-intrados (diam. × thickness) shall not exceed 22000 for soft steel, and 24000 for medium steel. (See Table 1, page 612.) 35. Increase number of rivets thus found if field driven: by hand, 25%; by power, 10%.

Bending Stresses on Pins.‡—36. Extreme fibre stress: Soft steel, 22000; medium steel, 25000 lbs. per sq. in. Use centers of bearings of strained members. (See Table 25, page 630.)

Plate Girders.||—37. Assume ⅛ gross area of web as available flange area. Compression flange to have same sectional area as tension flange;

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* Should the pins be out of the neutral axis, the additional stress thus produced shall be provided for.—*P. & R.*

† Shearing: Shop rivets and pins, 12000; field rivets and turned bolts, 10000; plate girder webs (gross section) 10000. Bearing: Shop rivets and pins, 24000; field rivets and turned bolts, 20000; granite masonry and Portland cement concrete, 600; sandstone and limestone, 400 lbs. per sq. in.—*C. R. I. & P.*

Shearing: Pins and rivets, 7500; web plate, 5000 in direction of rolling, and 6000 across fibre. Bearing: Pins and rivets, 12000; bed plates on masonry, 250. Decrease 25% for hand and field rivets; increase 25% for lateral and vibration riveted connections.—*D. L. & W.*

Shearing: Rivets, bolts and pins, 11000; web plates of stringers, floor beams, and plate girders (net section), 11000. Bearing: Rivets, bolts and pins, 22000; masonry, 400. Deduct 20% for field rivets.—*L. S. & M. S.* Same as preceding excepting, use 10000 for "shear in webs of plate girders."—*P. & R.*

Shearing: Pins and rivets, 7500; webs of plate girders, 6000. Bearing: Pins and rivets, 15000. Increase stresses 50% for knee bracing; decrease 20% for hand rivets.—*S. P.*

‡ 24000.—*C. R. I. & P.* and *L. S. & M. S.* 15000.—*D. L. & W.* 22000.—*P. & R.* 1800.—*S. P.*

|| Proportioned either by moment of inertia of net section; or by assuming ⅛ of gross web section to be added to flange area. Gross section of comp. flange shall not be less than that of tens. flange; nor shall working stress in comp. flange of any beam or girder exceed $16000 - 200\frac{l}{b}$, where l = unsupported distance, and b = width of flange.—*C. R. I. & P.*

Girders and beams must have top or comp. flange braced at intervals of at least 20 times the width of flange. No part of web of plate girder considered as flange area.—*D. L. & W.*

Depth of girder generally ⅛ to $\frac{1}{10}$ of span up to 75 ft. span, and proportionately less up to a minimum of $\frac{1}{15}$ for the largest practicable plate girder. No part of web to be considered as flange area. No cover plate shall have a thickness greater than the angles; ¾ in. to be about the max. thickness. First cover plate to extend full length; other plates, 12" beyond theoretical cut-off. For spans over 70 ft., flange members may be spliced, only one angle at any one point of flange. Stiffener angles to be: 3½x3½x⅜ for spans up to 50 ft.; 4x3½x⅜, 50 to 70 ft. spans; 5x3½x⅜, 70 to 90 ft. spans; 6x3½x⅜, above 90 ft.—*L. S. & M. S.*

but unsupported length shall not exceed 16 times its width. 38. In designing web rivets of plate girders, assume total shear at abutment as transferred into flange angles in a distance equal to depth of girder. 39. Minimum web, $\frac{3}{8}''$. Shear, 9000 for soft steel; 10000 for med. steel. 40. Stiffeners, both sides, close flange bearings, at points of concentrated load; also, when t of web is less than $\frac{1}{60}$ of unsupported distance between flange angles, stiffeners to be spaced generally not farther apart than depth of full web plate, with max. limit of 5 ft.

Provision for Future Increase of Live Load.*—41. When live and dead load stresses are of opposite character, only 70% of the dead load stress shall be considered as effective in counteracting the live load stress.

(d.) Details of Construction.

Camber.†—42. For truss bridges, increase length of top chord $\frac{1}{8}''$ to every 10 ft. [Best: Shorten diagonals, without increasing the chord.—*Author.*]

Adjustable Members.—44. Preferably avoided.

Truss Bridges.—45. Stiff end vertical suspenders for through spans. For end panels of lower chord, preferably stiff members for single track spans.

Lateral and Sway Bracing.—48. To be compression shapes.

Diagonal Bracing.‡—50. Deck bridges shall have diagonal braces at each panel, of sufficient strength to carry half the maximum strain increment due to wind and centrifugal force.

Gusset Plates.—51. At each end of pony trusses and through plate girders, and at floorbeam of same.

Temperature.||—52. Provision for 150° F. variation.

Bolsters and Expansion Rollers.§—53. For bridges exceeding 80 ft. span, hinged bolsters at both ends, with nests of turned friction rollers at one end. Rollers not less than 4" dia.; and pressure p, in lbs. per lin. in. of roller, not to exceed 1200 $\sqrt{d}$ for steel rollers between steel surfaces (d = diam. of roller in ins.).

Bed Plates.⊿—55. Pressure on masonry, including impact, 400 lbs. per sq. in.

Rivets.—56. In direction of strain, max. pitch to be 6" or 16 × thickness of thinnest outside plate, at right angles to strain, max. pitch to be 40Σ × that thickness. At ends of compression members, pitch not to exceed 4 diameters of rivet for a length equal to 2θ times width of member.

Tie Plates.—60. All segments of compression members connected by latticing only, shall have tie plates placed as near the ends as practicable, with length not less than greatest depth or width of member, and thickness not less than $\frac{1}{50}$ of the distance bet. the rivets connecting them to the compressed members.

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* If live load be increased 70% the stress per sq. in. in any member shall not exceed 1.7 × the allowed unit stress, and in case of reversal of stress proper provision shall be made for same.—*C. R. I. & P.*

† Camber for plate girders over 50 ft. span to be $\frac{1}{16}''$ per 10 ft. of length.—*C. R. I. & P.* Camber for all spans, about $\frac{3}{4}''$ in 100 ft.—*D. L. & W.* Camber for movable bridges, such that when end supports are raised to their exact position the base of rail will be at the same level on ends of bridge as at center.—*L. S. & M. S.* Arc of circle; and at least $\frac{1}{1500}$ of the span.—*S. P.*

‡ Overhead diag. bracing at each panel point when height of truss exceeds 25 ft.—*P. & R.*

|| Expansion and contraction, 1" in every 100 ft.—*D. L. & W.* and *L. S. & M. S.* 150° F.—*P. & R.* and *S. P.* $\frac{1}{8}''$ for each 10 ft.—*C. R. I. & P.*

§ Rollers not less than 6" diam.—*C. R. I. & P.* Rollers not less than $2\frac{1}{2}''$ diam.; or $p = 300\ d$.—*D. L. & W.* Rollers: 1200 $\sqrt{d}$.—*P. & R.* For bridges over 75 ft. span, segmental steel friction rollers not less than 6" diam; but cylindrical rollers 4" diam. may be used.—*S. P.*

⊿ See, also, foot-notes to 34, preceding, for bearing on masonry.

Σ 30.—*P. & R.* θ 1½.—*C. R. I. & P.*

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Lacing.*—61. Single lattice bars shall have a thickness of not less than $\frac{1}{40}$, and double bars connected by rivet at intersection, not less than $\frac{1}{60}$, of dist. bet. rivets connecting them to the member; and their width shall be:
2½" (⅞" riv.) for 15" chans., or built sections with 3½" and 4" angles;
2¼" (¾" ") " 12" and 10" " " " " " 3" "
2" (⅝" ") " 9" and 8" " " " " " 2½" "

Pin Plates.—62. Must contain enough rivets to transfer the proportion of pressure upon them, and at least one plate on each side shall extend not less than 6" beyond edge of tie plate.

(e.) Workmanship.

Riveted Work.—65. Hole = rivet + $\frac{1}{16}$"; enlargement, by reaming.

Planing and Reaming.—67. In medium steel over ⅝", sheared edges to be planed, and holes to be drilled or reamed to diam. of rivet + ⅛".

Eye-Bars.—75. Heads of eye-bars to be made by upsetting, rolling, or forging. Welds not allowed. 78. All eye-bars shall be annealed.

Machine Work.—79. Abutting surfaces in compression members shall be truly faced to even bearings. 80. Ends of floor girders shall be faced true and square. 81. No variation of more than $\frac{1}{64}$" for every 20 ft. will be allowed in length between centers of pin holes. 84. Clearance between pin and pin hole shall be $\frac{1}{32}$" for lateral pins; and for truss pins, $\frac{1}{80}$" for pins 3½" diam., gradually increased to $\frac{1}{32}$" for pins 6" diam. and over.

(f.) Steel.

Process of Manufacture.—87. Open hearth. If by acid process, not over .08% phosphorous; if by basic process, not over .05% phosphorous.

Physical Properties.†—92. Three grades:

	Rivet Steel.	*Soft Steel.*	*Medium Steel.*
Ultimate strength, lbs. per sq. in.,	48–58000.	52–62000.	60–70000.
Elastic limit, " " " "	½ ult.	½ ult.	½ ult.
Elongation,	26%.	25%.	22%.
Bending test—without fracture on outside bent portion. 180°.....	Flat on itself.	Flat on itself.	To diam. = thick. of piece.

Pins.—100. Up to 7" diam., rolled. 101. Above 7" diam., forged.

Steel Castings.†—103. Open hearth, containing from 0.25 to 0.40% carbon, and not over .08% phosphorous.

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* Same as Am. Br. Co., 61, above.—*C. R. I. & P.*

Lacing bars shall not be less than 2¼x⅜, and shall form an angle of not less than 60° with axis of member in single lacing, and 45° in double lacing. Double lacing must be riveted at intersections.—*D. L. & W.*

Latticing shall be double, and shall preferably cross at right angles, and be riveted at intersections. Lacing shall be single, and be at angle of about 60° with axis of member. Minimum size of lattice bars shall be as follows: 2¼x$\frac{5}{16}$ for 8" and 9" chans.; 2½x⅜ for 10" and 12" chans. and 3" angles; 2¾x$\frac{7}{16}$ for 15" chans. and 3½" and 4" angles; connected by one rivet at each end. 4x$\frac{7}{16}$ for built sections over 15" wide; connected by two rivets at each end.—*L. S. & M. S.* For chords and posts the lattice bars shall generally be 3x$\frac{7}{16}$; the width may be: 2" for lattice under 10" long, 2¼" under 15" long, and 2½" under 20" long.—*P. & R.*

†Road.	Properties.	Steel.					
		Medium.	R'y Br.	Struct'l.	Soft.	Rivet.	Castings.
C. R. I. & P.	Ult. tens. str..	.....	.....	56–64000	.....	46–54000	65000
D. L. & W.	Ult. tens. str..	62–70000	.....	.....	54–62000	48–56000	66000
	Elas. limit....	.5 ult.	.....	.....	.5 ult.	.5 ult.	.5 ult.
	Elong., %, 8".	22	.....	.....	26	28	.....
L. S. & M. S.	Ult. tens. str..	62–70000	56–64000	.....	.....	48–56000	.....
	Elas. limit....	.6 ult.	.6 ult.	.....	.....	.6 ult.	.....
	Elong., %....	25	28	.....	.....	28	.....
	For bed plates	.....	.....	.....	.....	.....	55–65000
	For gearing...	.....	.....	.....	.....	.....	65–75000
P. & R.	Ult. tens str..	.....	.....	56–64000	.....	46–54000	65000
S. P.	Ult. tens. str..	60–68000	.....	.....	.....	46–54000	.....
	Elas. limit....	33000	.....	.....	.....	26000	.....
	Elong., %, 8".	22	.....	.....	.....	26	.....

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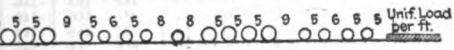
LES USEFUL IN CONNECTION WITH PRECEDING SPECIFICATIONS.

-COOPER'S STANDARD LOADING—AXLE LOADS.
[LOADS ARE IN LBS. PER TRACK.]

DISTANCES IN FEET.



30000	30000	30000	19500	19500	19500	19500	15000	30000	30000	30000	30000	19500	19500	19500	19500	3000 lbs.
40000	40000	40000	26000	26000	26000	26000	20000	40000	40000	40000	40000	26000	26000	26000	26000	4000 lbs.
45000	45000	45000	29250	29250	29250	29250	22500	45000	45000	45000	45000	29250	29250	29250	29250	4500 lbs.
50000	50000	50000	32500	32500	32500	32500	25000	50000	50000	50000	50000	32500	32500	32500	32500	5000 lbs.
55000	55000	55000	35750	35750	35750	35730	27500	55000	55000	55000	55000	35750	35750	35750	35750	5500 lbs.
60000	60000	60000	39000	39000	39000	39000	30000	60000	60000	60000	60000	39000	39000	39000	39000	6000 lbs.

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AXIMUM MOMENTS *M*, END SHEARS *S*, AND FLOORBEAM REACTIONS *R*, 'ER TRACK, PRODUCED BY COOPER'S LOADINGS* E 50 AND E 40.
[Mult. Values in Table by 1000.]

Loading *E* 50.						Loading *E* 40.					
Max. Mom. *M* in 1000 Ft.-Lbs.	Max. End Shear *S* in 1000 Lbs.	Max. Fl.-Bm. Reac. *R* in 1000 Lbs.	Equiv. Unif. Load in 1000 Lbs.			Max. Mom. *M* in 1000 Ft.-Lbs.	Max. End Shear *S* in 1000 Lbs.	Max. Fl.-Bm. Reac. *R* in 1000 Lbs.	Equiv. Unif. Load in 1000 Lbs.		
			M	*S*	*R*				*M*	*S*	*R*
141	75	100	11.25	15.00	10.00	113	60	80	9.00	12.00	8.00
164	82	109	10.86	14.88	9.92	131	66	87	8.69	11.91	7.94
200	88	117	11.11	14.58	9.72	160	70	93	8.89	11.67	7.77
238	92	123	11.24	14.21	9.47	190	74	99	9.00	11.35	7.56
275	96	130	11.22	13.78	9.31	220	77	104	8.98	11.03	7.45
313	100	137	11.11	13.33	9.11	250	80	109	8.89	10.67	7.29
350	106	142	10.94	13.28	8.89	280	85	114	8.75	10.62	7.11
388	112	147	10.73	13.15	8.65	310	90	118	8.58	10.53	6.92
425	117	152	10.49	12.96	8.43	340	93	121	8.40	10.38	6.74
466	121	157	10.34	12.74	8.28	373	97	126	8.27	10.19	6.63
516	125	164	10.31	12.50	8.19	413	100	131	8.25	10.00	6.56
565	129	170	10.25	12.24	8.09	452	103	136	8.20	9.79	6.48
614	132	175	10.15	11 98	7.97	491	106	140	8.12	9.59	6.38
664	135	180	10.04	11.72	7.84	531	108	144	8.03	9.38	6.27
713	139	185	9.90	11.55	7.70	570	111	148	7.92	9.23	6.17
763	142	189	9.76	11.36	7.56	610	114	151	7.81	9.09	6.05
812	145	194	9.61	11.18	7.47	650	116	155	7.69	8.93	5.97
862	148	200	9.46	10.97	7.41	689	119	160	7.56	8.78	5.93
914	151	206	9.32	10.79	7.35	731	121	165	7.46	8.63	5.88
970	154	211	9.23	10.61	7.27	776	123	169	7.37	8.49	5.82
1026	158	216	9.12	10.51	7.19	821	126	173	7.30	8.41	5.75
1082	161	221	9.01	10.39	7.14	866	129	177	7.21	8.31	5.71
1139	164	227	8.90	10.27	7.11	911	132	182	7.12	8.22	5.69
1195	167	233	8.78	10.14	7.07	956	134	187	7.02	8.11	5.66
1251	170	239	8.66	10.01	7.02	1001	136	191	6.92	8.01	5.63
1307	173	244	8.54	9.88	6.97	1046	138	195	6.84	7.91	5.57
1372	176	249	8.47	9.80	6.91	1097	141	199	6.77	7.84	5.53
1436	180	253	8.39	9.71	6.85	1149	144	203	6.71	7.77	5.50
1500	183	259	8.31	9.62	6.82	1200	146	208	6.65	7.70	5.46
1567	186	265	8.24	9.52	6.79	1254	149	212	6.59	7.62	5.43
1639	189	270	8.20	9.43	6.75	1311	151	216	6.56	7.54	5.40
1784	195	280	8.09	9.30	6.67	1427	156	224	6.48	7.46	5.34
1929	201	291	7.97	9.15	6.62	1543	161	233	6.37	7.32	5.30
2074	207	302	7.84	9.00	6.56	1659	166	241	6.28	7.20	5.24
2219	212	312	7.71	8.84	6.49	1776	170	250	6.17	7.07	5.20
2377	218	322	7.61	8.71	6.44	1902	174	257	6.09	6.97	5.14
2538	223	333	7.51	8.58	6.40	2030	179	267	6.01	6.87	5.13
2703	228	345	7.42	8.44	6.39	2162	182	276	5.93	6.76	5.12
2880	233	357	7.35	8.30	6.38	2304	186	286	5.88	6.64	5.11
3058	238	370	7.27	8.22	6.38	2446	191	295	5.82	6.58	5.09
3247	244	383	7.22	8.13	6.38	2599	195	305	5.78	6.51	5.08
3441	250	395	7.16	8.07	6.37	2753	200	315	5.73	6.46	5.08
3639	256	407	7.11	8.01	6.36	2911	205	325	5.69	6.41	5.07
3849	262	419	7.07	7.95	6.35	3079	210	334	5.66	6.36	5.07
4059	270	431	7.02	7.93	6.34	3247	216	344	5.61	6.34	5.06
4269	276	443	6.97	7.89	6.32	3415	221	354	5.58	6.31	5.06
4479	283	454	6.91	7.87	6.30	3584	227	362	5.54	6.30	5.03
4699	291	465	6.86	7.86	6.28	3758	233	371	5.49	6.29	5.01
4925	298	476	6.82	7.83	6.26	3942	238	379	5.46	6.27	4.99
5160	304	487	6.79	7.80	6.24	4129	243	388	5.43	6.24	4.97
5399	311	497	6.75	7.76	6.21	4321	248	396	5.40	6.21	4.96

* Note that values for all classes are proportional; thus, for E 55, mult. es for E 50 by 1.1; for E 45, by 0.9; etc.

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5.—MAXIMUM MOMENTS *M*, END SHEARS *S*, AND FLOORBEAM REACTIONS *R*, *PER TRACK*, PRODUCED BY COOPER'S LOADINGS* E 50 AND E 40.—Concl'd

[Mult. Values in Table by 1000.]

Span *L* Ft.	Loading *E* 50.						Loading *E* 40.					
	Max. Mom. *M* in 1000 Ft.-Lbs.	Max. End Shear *S* in 1000 Lbs.	Max. Fl.-Bm. Reac. *R* in 1000 Lbs.	Equiv. Unif. Load in 1000 Lbs.			Max. Mom. *M* in 1000 Ft.-Lbs.	Max. End Shear *S* in 1000 Lbs.	Max. Fl.-Bm. Reac. *R* in 1000 Lbs.	Equiv. Unif. Load in 1000 Lbs.		
				M	*S*	*R*				*M*	*S*	*R*
82	5638	317	507	6.71	7.74	6.19	4513	254	404	5.37	6.19	4.93
84	5891	324	517	6.68	7.71	6.16	4713	259	412	5.34	6.17	4.91
86	6145	330	527	6.65	7.68	6.13	4919	264	421	5.32	6.15	4.89
88	6406	337	537	6.62	7.65	6.10	5128	269	429	5.30	6.12	4.87
90	6674	343	546	6.59	7.62	6.07	5341	275	437	5.28	6.10	4.86
92	6941	349	556	6.56	7.60	6.04	5552	280	444	5.25	6.08	4.83
94	7209	356	566	6.53	7.57	6.02	5771	285	452	5.23	6.06	4.81
96	7476	362	575	6.49	7.54	5.99	5988	290	459	5.20	6.03	4.78
98	7755	369	584	6.46	7.52	5.96	6213	295	467	5.18	6.02	4.76
100	8048	375	593	6.44	7.50	5.93	6440	300	474	5.15	6.00	4.74
125	12491	449	.....	6.40	7.18	.....	9993	359	.....	5.12	5.74	.....
150	17625	522	.....	6.26	6.96	.....	14100	418	.....	5.01	5.57	.....
175	23400	585	.....	6.11	6.69	.....	18720	468	.....	4.89	5.35	.....
200	29625	656	.....	5.92	6.55	.....	23700	524	.....	4.74	5.24	.....
250	44025	787	.....	5.64	6.29	.....	35220	629	.....	4.51	5.03	.....

* Note that values for all classes are proportional; thus, for E 55, mult. values for E 50 by 1.1; for E 45, by 0.9; etc.

6.—COEFFICIENTS OF IMPACT (*I*).

(See page 701.)

L.	$\frac{300}{L+300}$	*L.*	$\frac{300}{L+300}$	*L.*	$\frac{300}{L+300}$	*L.*	$\frac{300}{L+300}$	*L.*	$\frac{300}{L+300}$
5	0.984	31	0.906	57	0.840	83	0.783	145	0.674
6	0.980	32	0.904	58	0.838	84	0.781	150	0.667
7	0.977	33	0.901	59	0.836	85	0.779	155	0.659
8	0.974	34	0.898	60	0.833	86	0.777	160	0.652
9	0.971	35	0.896	61	0.831	87	0.775	165	0.645
10	0.968	36	0.893	62	0.829	88	0.773	170	0.638
11	0.965	37	0.890	63	0.826	89	0.771	175	0.632
12	0.962	38	0.888	64	0.824	90	0.769	180	0.625
13	0.958	39	0.885	65	0.822	91	0.767	185	0.619
14	0.955	40	0.882	66	0.820	92	0.765	190	0.612
15	0.952	41	0.880	67	0.817	93	0.763	195	0.606
16	0.949	42	0.877	68	0.815	94	0.761	200	0.600
17	0.946	43	0.875	69	0.813	95	0.759	210	0.588
18	0.943	44	0.872	70	0.811	96	0.758	220	0.577
19	0.940	45	0.870	71	0.809	97	0.756	230	0.566
20	0.937	46	0.867	72	0.806	98	0.754	240	0.556
21	0.935	47	0.865	73	0.804	99	0.752	250	0.546
22	0.932	48	0.862	74	0.802	100	0.750	260	0.536
23	0.929	49	0.860	75	0.800	105	0.741	270	0.526
24	0.926	50	0.857	76	0.798	110	0.732	280	0.517
25	0.923	51	0.855	77	0.796	115	0.725	290	0.508
26	0.920	52	0.852	78	0.794	120	0.714	300	0.500
27	0.917	53	0.850	79	0.792	125	0.706	400	0.429
28	0.915	54	0.847	80	0.789	130	0.698	500	0.375
29	0.912	55	0.845	81	0.787	135	0.690	600	0.333
30	0.909	56	0.843	82	0.785	140	0.682		

Note.—For notation and formula, see page 701.

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7.—Permissible Compressive Stresses.

(See page 703.)

p = stress allowed in lbs. per sq. in.; l = length, r = least rad
(both in inches).

$\frac{l}{r}$	Formula. Soft Steel $p=\frac{15,000}{1+\frac{l^2}{13,500r^2}}$	Formula. Medium Steel $p=\frac{17,000}{1+\frac{l^2}{11,000r^2}}$	$\frac{l}{r}$	Formula. Soft Steel $p=\frac{15,000}{1+\frac{l^2}{13,500r^2}}$	Formula. Medium Steel $p=\frac{17,000}{1+\frac{l^2}{11,000r^2}}$	$\frac{l}{r}$	Sof $p=$ $1+$ 1
10	14900	16850	50	12660	13850	90	9
12	14840	16780	52	12500	13650	92	9
14	14780	16710	54	12340	13340	94	9
16	14720	16610	56	12180	13230	96	8
18	14650	16510	58	12010	13020	98	8
20	14560	16410	60	11840	12810	100	8
22	14480	16290	62	11670	12600	102	8
24	14400	16150	64	11500	12390	104	8
26	14280	16020	66	11340	12180	106	8
28	14180	15870	68	11140	11970	108	8
30	14070	15710	70	11010	11760	110	7
32	13940	15550	72	10840	11550	112	7
34	13810	15380	74	10670	11350	114	7
36	13690	15210	76	10500	11510	116	7
38	13550	15030	78	10340	10950	118	7
40	13420	14840	80	10180	10750	120	7
42	13270	14650	82	10010	10550		
44	13120	14460	84	9850	10350		
46	12960	14260	86	9690	10160		
48	12820	14060	88	9530	9970		

VII.—APPROX. WEIGHT OF STEEL IN STEEL RAILROA

By Formula, $w=(xl+y)\sqrt{Loading}$
In which w = weight in lbs. of steel per lin. ft. of bridge (do
rails, guards nor ties),
l = length of span in ft.

8.—Values of x and y, in Above Formula.

Kind of Bridge.	Specifications.	Single Track. x	y
Deck Plate Girder, spans 20–100, ...	Extra.	2.	20
" " " " "	Good.	1.8	20
Through " " " "	"	1.8	70
" " " " (solid floor)	"	1.8	130
Deck Lattice " " 90–150	"	1.2	30
Through " " " "	"	1.2	90
" Pratt Truss, " 130–.....	"	1.1	100
" Baltimore " " 200–.....	"	1.0	120

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9.—VALUES OF $\sqrt{\text{LOADING}}$, IN PRECEDING FORMULA.

Loading (Class):—	E 60	E 55	E 50	E 45	E 40	E 35	E 30
$\sqrt{\text{Loading}}$:—	7.75	7.42	7.07	6.71	6.32	5.92	5.48
Relative Values of $\sqrt{\text{Loading}}$, for Classes E 30 to E 60.	1.00	0.96	0.91	0.86	0.81	0.76	0.71
	1.05	1.00	0.95	0.90	0.85	0.80	0.74
	1.10	1.05	1.00	0.95	0.89	0.84	0.78
	1.15	1.10	1.05	1.00	0.94	0.88	0.82
	1.23	1.17	1.12	1.06	1.00	0.94	0.87
	1.31	1.25	1.19	1.13	1.07	1.00	0.93
	1.41	1.35	1.29	1.22	1.15	1.08	1.00

Example.—Find the approx. weight of steel in a Through Plate Girder Bridge with floor beams and stringers, span 100 ft., loading E 40?

Solution.—$l \times w = 100\ (180 + 70)\ 6.32 = 158000$ lbs.

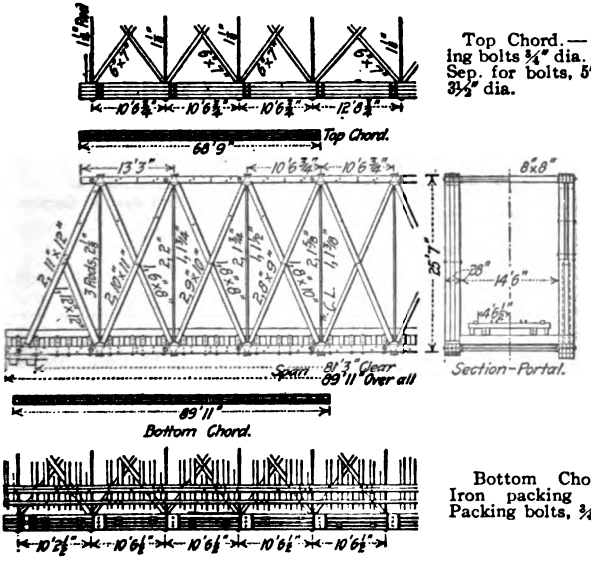
VIII.—PLANS AND DETAILS OF RAILROAD SPANS.

Howe Truss Bridges are being rapidly replaced by steel and concrete structures on all of our American roads, but the following typical plans will be of interest.

N. P. R'Y STANDARD PLAN OF HOWE TRUSS AND DETAILS.

90 Ft. Through Span.

Loads.—Dead Load, 1300 lbs. per lin. ft. Live Load, consolidation engine and tender, 225,000 lbs., followed by 3000 lbs. per lin. ft.



Top Chord.—Packing bolts ¾" dia. C.-I. Sep. for bolts, 5" and 3½" dia.

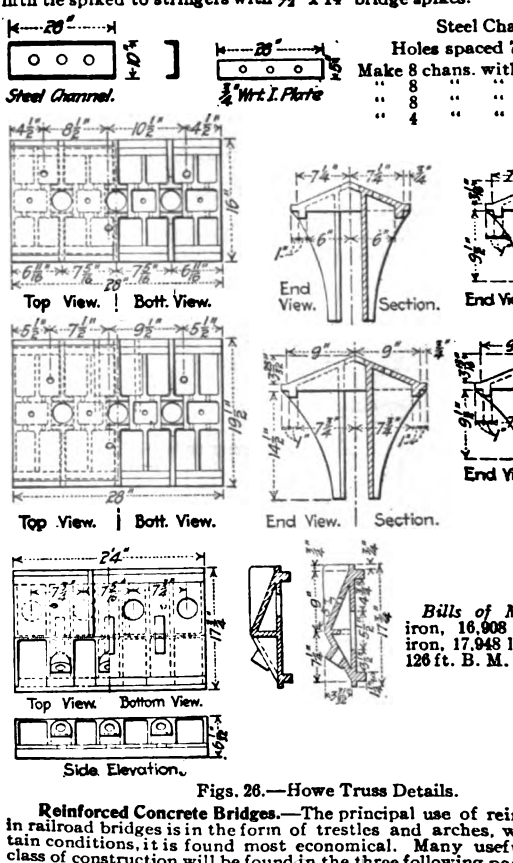
Bottom Chord.—Iron packing keys. Packing bolts, ¾" dia.

Figs. 25.—General Plan and Elevation.

Timber Specifications.—Must be sound, live, straight and close grained red or yellow fir, cut free from wanes, shakes, pitch seams and clear of

knots larger than 1" dia., nor closer than 4 ft. on any side o must be sound and clear of pitch. All timber must be cut f logs free from heart and sap, and sawed true to sizes ordered f. o. b. cars, subject to inspection before delivery.

Notes.—Use red fir cross-ties west of Helena. Frame c sizing stringers into floor-beams. In framing, carpenters panels full size, make templates of angle blocks from the c braces to correct length and shape so determined. Cast iro packing bolts in top chord are 5" dia. except for bolts in ce and at ends of chords, which have separators 3½" dia. All o for packing bolts are 3½" dia. Guard rails bolted to ties middle and spiked to all other ties with ½" x 10" bridge fifth tie spiked to stringers with ½" x 14" bridge spikes.



Figs. 26.—Howe Truss Details.

Reinforced Concrete Bridges.—The principal use of rein in railroad bridges is in the form of trestles and arches, w tain conditions, it is found most economical. Many usef class of construction will be found in the three following pa and References." For formulas and working stresses, see and 32. See, also, other Sections and Index.

EXCERPTS AND REFERENCES.

A Graphical Method of Finding Floor-Beam Concentrations Under Wheel-Loads (By R. H. Bulloch. Eng. News, May 21, 1903).

Standard Plans for Bridges on the Atchison, Topeka & Santa Fe Ry. (By J. Dun. Eng. News, May 28, 1903).—Illustrated.

A Comparison of the Requirements of Recent Railway Bridge Specifications (By A. H. Heller. Eng. News, Nov. 19, 1903).—Tabulated data from 29 specifications.

A New Truss Design (By J. W. Schaub. Eng. News, Mar. 24, 1904).—Warren type, with a combination of pin and riveted connections. Adopted by some of the railroads.

A Moment Table for Wheel Loads (By R. B. Ketchum. Eng. News, May 12, 1904). Table and diagrams.

New Westminster Bridge over Fraser River. B. C. (By Waddell and Herrick. Eng. News, June 15 and 22, 1905).—Foundations, superstructure and erection; illustrated.

Recent Railway Viaducts of Reinforced Concrete (Eng. News, May 31, 1906).—Illustrated.

Diagram Table for Cooper's E-50 Loading (By J. Gibson. Eng. News, June 21, 1906).—Double inset sheet. Very elaborate table.

Wheel Loadings of Mallet Duplex Compound Locomotive for Great Northern Ry. (Eng. News, Nov. 22, 1906).—Corrected diagram.

Rail Expansion Joints on the Thebes Bridge (By Ralph Modjeski. Eng. News, Aug. 22, 1907).—Illustrated.

Proportioning Steel Railway Bridge Members (By H. S. Pritchard. Eng. News, Sept. 19, 1907).

Erection of Long Span Trusses by End Launching (Can. Soc. Civ. Engrs. April 16, 1908; Eng. News, July 23, 1908).—Illustrated.

Safe Unit Stresses in Structural Timber (Eng. Rec., Apr. 24, 1909).—The safe unit stresses in structural timber recommended by the committee on wooden bridges and trestles of the Am. Ry. Eng. and M. of W. Assn., for green condition of timbers and without increasing the live-load stresses for impact, are as follows:—

	Bending.	Shearing.		Compression.		
	Extreme fiber stress.	Parallel to grain.	Long'l in beams.	Perp. to grain.	Parallel to grain.	Cols. under 15 dia's.
Douglas fir.....	1 200	170	110	310	1 200	900
Longleaf pine	1 300	180	120	260	1 300	980
Shortleaf pine....	1 100	170	130	170	1 100	830
White pine.....	900	100	70	150	1 000	750
Spruce.....	1 000	150	70	180	1 100	830
Norway pine.....	800	130	100	150	800	600
Tamarack.....	900	170	100	220	1 000	750
Western hemlock .	1 100	160	100	220	1 200	900
Redwood.....	900	80	....	150	900	680
Bald cypress.....	900	120	....	170	1 100	830
Red Cedar.....	800	....	....	230	900	680
White oak.....	1 100	210	110	450	1 300	980

For long columns exceeding 15 diameters in length the safe stress given for compression parallel to grain are taken and decreased by $60L \div D$, where L=length in ins., and D=least side in ins.

Measurement of Impact Stresses (By B. W. Dunn. Proc. A. S. T. M., Vol. IX., 1909).

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Reinforced-Concrete Bridges for Track Elevation on Ill. Cent R. R.; Failure Test of Very Large Concrete Slabs (Eng. News, Aug. 6, 1908).—Illustrated.

Application of Spiral Hooping to a French Concrete Bridge (Eng. News, April 22, 1909).—Illustrated.

Erection of New River Bridge by the Cantilever Method (By L. L. Jewel. Eng. News, July 8, 1909).—Single-track deck structure, 2155 ft. long, 112 ft. above low water; spans, 125 to 140 ft.; all trusses are riveted Warren trusses, 26 ft. deep c.-c. chords and spaced 12 ft. apart; no floor system provided; the deck of extra heavy ties being carried directly on the top chords. Erected by cantilever traveler; 11 illustrations.

Guard Rail and Deck Construction for Railway Bridges (Eng. News. Sept. 9, 1909).

Illustrations:

Typical arrangement of bridge guards; Lehigh Valley R. R. Deck and guard rail construction (with two and three inside guards); Can. Pac. R. R. Bridge deck with sidewalk; Carolina, Clinchfield and Ohio R. R. Bridge deck construction on curves; Carolina, Clinchfield and Ohio R. R. Details of bridge deck and guards; Lehigh Valley R. R. Bridge deck with timber guards; Louisville and Nashville R. R. Bridge deck and guards (with tangents and curves); Penn. Lines. Bridge guards with re-railing devices; Southern Pacific R. R. Details of re-railing devices; Southern Pacific R. R. Re-railing devices at approaches to jack-knife draws; B. & M. R. R. Bridge guards on concrete bridge on a French railway. Deck and guard rail construction; So. Side Elev. Ry., Chicago. Deck and guard rail construction for curves; So. Side Elev. Ry., Chicago. Examples of bridge guards on English railway bridges.

Lethbridge Viaduct over the Belly River, Canadian Pac. Ry. (By J. E. Schwitzer. Eng. News, Sept. 23, 1909).—Plans of floor and tower.

Four-Track Truss Bridge with Solid Floor; Chicago & Oak Park Elev. Ry. (Eng. News, Dec. 9, 1909).—Numerous plans and details, including falsework.

Long Span Reinforced-Concrete Girder Bridges (By F. W. Scheidenhelm. Eng. News, Jan. 27, 1910).—Illustrations: Girders, abutment and piers; with details of centering and forms used on the 75-ft. girder; also, splice clamp for reinforced-concrete bars.

Reinforced-Concrete vs. Steel for Short Span Railway Bridges (Eng. News, Mar. 24, 1910).—Reinforced-concrete flat slabs or girders not generally advisable for railway loads in spans exceeding 40 feet.

450-Ft. Steel R. R. Span of the Miles Glacier Bridge, Alaska (Eng. Rec., Aug. 6, 1910).—Illustrations: Elevation of bridge (one 450′ span, two 400′ spans, one 300′ span); truss details of 450′ span; fixed and expansion end shoes, with segmental rollers and nest and grillage, 450′ span; erection filler between shoes on pier; erection adjustment devices for top and bottom chords.

The St. Louis Municipal Bridge Superstructure (Eng. Rec., Dec. 3, 1910) —Bridge composed of three 668-ft. steel spans carrying two railroad tracks on the lower floor at bottom chord level, and two electric car tracks, a driveway, and two cantilever sidewalks one on second floor 22 ft. in the clear above the lower floor. The two pin-connected trusses 35 ft. apart on centers are 110 ft. deep, 65 ft. in the clear above high water and will be made of nickel steel, while the floor system and bracing will be made of carbon steel (See Eng. Rec. of Oct. 30, 1909, for diagrams, stress sheets, specifications, etc.; also Eng. Rec. of Oct. 15, 1910, for description of design and construction of the substructure.) The main span trusses are longer than those of any other span with independent trusses; the panel lengths vary with the depths of the trusses in accordance with economical inclinations of the diagonal members; the top chord has a special type of cross-section; the upper deck is made with intermediate floorbeams carried on longitudinal girders. the main truss shoes and pedestals are heavy steel castings; many of the compression members have half-hole pin-bearings without interlocking

pin plates; and the members and details are of standard construction. Illustrations:—Regular cross-section; general diagram of 668-ft. span, giving heights of truss; upper and lower portal bearing; roadway bent over pier; sections of upper and lower decks; expansion shoe and pedestal for same; pedestal for fixed shoe; top chord details of sidewalk and curb girder.

Important Illustrations of Railroad Bridges and Details.

Description.	Eng. News
Large rein.-conc. railway viaduct, Rotterdam, Holland	June 16, '10
Through-truss, 517½-ft. river spans, St. Louis	July 28, '10
Part details 420-ft. truss span and floor	Aug. 25, '10
Stringer connection allowing flexibility	Aug. 25, '10
Erie R. R. viaduct, Penhorn Creek	Oct. 13, '10
	Eng. Rec.
Details of steel viaduct, B. & M. R. R	Feb. 27, '09
Ballasted-floor details and steel construction, Erie R. R	Feb. 27, '09
116-ft. railroad span	Mar. 27, '09
Erection of long span bridges across the Susquehanna	Apr. 3, '09
Falsework for replacing the Cuyahoga Val. viaduct	June 5, '09
6-track, short span, solid-floor bridge, N. Y., N. H. & H. R. R.	July 24, '09
Strain-sheet 668-ft. span—St. Louis Municipal Bridge	Oct. 30, '09
Typical falsework for Poughkeepsie Bridge reinforcement.	Oct. 30, '09
Heavy steel floor for double-track bridge, D. L. & W	Jan. 15, '10
100-ft. span plate-girder bridge, N. Y., N. H. & H. R. R	Mar. 12, '10
C. M. & St. P. R. R. bridge across Missouri River, Mobridge, S. Dak	June 11, '10
Standard 150-ft. Howe truss R. R. span and details	Sept. 3, '10
Standard I-beam R. R. bridges over streets, N. Y. Cent	Sept. 17, '10
Elev. and section of 8-track R. R. bridge over streets	Oct. 1, '10
Structural details, Providence, R. I., station viaduct, N. Y., N. H. & H. R. R	Oct. 22, '10
Construction and reconstruction of the Coteau steel bridge	Dec. 3, '10

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39.—ELECTRIC RAILWAY BRIDGES.

See, also, Sec. 40, Highway Bridges, page 727.

Typical "L" Loading as follows, for electric railway bridges may be used for any structure, heavy or light, by assigning proper values to w.*

(a) For calculation of floor system, use two axle concentrations of 15 w each, spaced 10 ft. centers. (See Fig. 1.)

(b) For calculation of trusses, use for each track a uniform moving load of w lbs. per lin. ft. for spans up to 100 ft.; and 0.8 w for spans 200 ft. and over; with a reduction of loading for intermediate spans of 0.01 w for every 5-ft. increase over 100 ft. The uniform load w per track is assumed to cover a surface 12 ft. wide for single track; 22 ft. wide for double track.

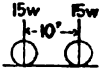

Fig. 1.

Tables 1 and 2 are based on the above typical "L" loading.

1.—Special Type "L" of Electric-Car Loading†—Axle and Uniform.

Name.	* For the floor and its supports. Concentrated Axle Loads.	For the trusses. Uniform live load (12 ft. wide) per lineal foot of track.										
		Up to 100' span	110' span	120' span	130' span	140' span	150' span	160' span	170' span	180' span	190' span	200' span
	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
L	15 w	w	.98w	.96w	.94w	.92w	.90w	.88w	.86w	.84w	.82w	.8w
L 18	18,000	1200	1176	1152	1128	1104	1080	1056	1032	1008	984	960
L 24	24,000	1600	1568	1536	1504	1472	1440	1408	1376	1344	1312	1280
L 30	30,000	2000	1960	1920	1880	1840	1800	1760	1720	1680	1640	1600

Typical "K" Loading, Fig. 2, consists of a train of electric cars with axle spacing, 5'-15'-5'-15', etc., continuous. The loading shown in the subjoined diagram is for 100,000-lb. cars, each covering a length of 40 feet. Tables 3, 4 and 5, following, are calculated from this diagram.

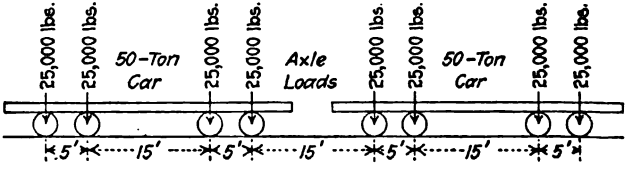

Fig. 2.

* For the Manhattan suspension bridge across the East River, New York, the value of w was originally assumed at 1700 lbs. per lin. ft. for each of four rapid transit trains; and 1000 lbs. per lin. ft. for each of four lines of trolley cars. The revised loading is given on page 756.

† The concentrated loading gives maximum moments for spans under 48.2 feet, and the uniform loading for spans over 48.2 feet. For maximum floorbeam reactions use concentrated loading for panels up to 23.66 feet in length (2 panel span of 47.32 feet). Beyond this length uniform loading gives maximum floorbeam reaction. For maximum end shear use concentrated loading up to 54.5 ft. span, and uniform loading beyond.

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2.—Maximum Moments, End Shears, and Floorbeam Reactions *R per track* for concentrated "*L* 24" loading—24,000 lbs. on each of two axles spaced 10 feet apart.

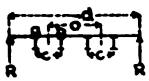
Values for any other "L" loading are directly proportional to the axle loads.

Span Ft.	Moment *M*. Ft.-lbs.	Equivalent Uniform Load *w* for Moment. Lbs. per lin. ft.	End Shear *S*. Lbs.	Equivalent Uniform Load *w* for End Shear. Lbs. per lin. ft.	Floorbeam Reaction *R*. Lbs.	Equivalent Uniform Load *w* for Floorbeam Reaction. Lbs. per lin. ft.
l	For $l > 17.07$ use $\frac{\text{"48"}}{l}\left(\frac{l}{2}-\frac{10}{4}\right)^2$	From equation $M=\frac{wl^2}{8}$ we have $w=\frac{8M}{l^2}$	For $l > 10'$ use $S=\frac{\text{"24"}+\text{"24"}(l-10)}{l}$	From equation $S=\frac{wl}{2}$ we have $w=\frac{2S}{l}$	For $l > 10'$ use $R=\frac{\text{"24"}+\text{"24"}(l-10)}{l}$	From equation $R=wl$ we have $w=\frac{R}{l}$
10	60,000	4,800	24,000	4,800	24,000	2,400
11	66,000	4,364	26,180	4,760	26,180	2,380
12	72,000	4,000	28,000	4,667	28,000	2,333
13	78,000	3,692	29,540	4,545	29,540	2,272
14	84,000	3,429	30,860	4,409	30,860	2,204
15	90,000	3,200	32,000	4,267	32,000	2,133
16	96,000	3,000	33,000	4,125	33,000	2,062
17	102,000	2,824	33,880	3,986	33,880	1,993
18	112,670	2,782	34,670	3,852	34,670	1,926
19	123,790	2,743	35,370	3,723	35,370	1,861
20	135,000	2,700	36,000	3,600	36,000	1,800
21	146,290	2,654	36,570	3,483	36,570	1,741
22	157,640	2,606	37,090	3,372	37,090	1,686
23	169,040	2,556	37,570	3,267	37,570	1,633
24	180,500	2,507	38,000	3,167		
25	192,000	2,458	38,400	3,072		
26	203,540	2,409	38,770	2,982		
27	215,110	2,361	39,110	2,897		
28	226,710	2,313	39,430	2,816		
29	238,340	2,267	39,720	2,740		
30	250,000	2,220	40,000	2,667		
31	261,680	2,178	40,260	2,598		
32	273,380	2,136	40,500	2,531		
33	285,090	2,094	40,730	2,468		
34	296,820	2,054	40,940	2,408		
35	308,570	2,015	41,140	2,351		
36	320,330	1,977	41,330	2,296		
37	332,110	1,941	41,510	2,244		
38	342,900	1,905	41,680	2,194		
39	355,700	1,871	41,850	2,146		
40	367,500	1,838	42,000	2,100		
41	379,320	1,806	42,150	2,056		
42	391,140	1,774	42,290	2,014		
43	402,980	1,744	42,420	1,973		
44	414,820	1,714	42,550	1,934		
45	426,670	1,686	42,670	1,896		
46	438,520	1,658	42,780	1,860		
47	450,380	1,631	42,890	1,825		
48	462,250	1,605	43,000	1,792		
49	.....	.....	43,100	1,759		
50	.....	.....	43,200	1,728		
51	.....	.....	43,290	1,698		
52	.....	.....	43,380	1,668		
53	.....	.....	43,470	1,640		
54	.....	.....	43,560	1,613		

Single Track.

Floorbeam Moment $=\frac{R}{2}\frac{(s-c)}{2}$ ft.-lbs.

Double Track.



Floorbeam Moment:

at a $=R\frac{(d-o-c)}{2}$ ft.-lbs.

at b $=R\frac{(d-o)}{2}$ ft.lbs.

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IAXIMUM MOMENTS *M* PER TRACK FOR CONCENTRATED "K 25" LOAD-ING (FIG. 2)—A TRAIN OF 50-TON ELECTRIC CARS.*

es for any other "K" loading are directly proportional to the axle loads.

Moment *M*. Ft.-Lbs.	Equivalent Unif. Load *w* for Moments. Lbs. per lin. ft.	Span, in Feet	Moment *M*. Ft.-Lbs.	Equivalent Unif. Load *w* for Moments. Lbs. per lin. ft.	Span, in Feet.	Moment *M*. Ft.-Lbs.	Equivalent Unif. Load *w* for Moments. Lbs. per lin. ft.
70,300	5,625	41	559,300	2,662	72	1,653,500	2,550
82,100	5,430	42	583,500	2,646	73	1,697,100	2,548
94,000	5,225	43	607,700	2,629	74	1,740,800	2,543
106,000	5,020	44	632,000	2,611	75	1,787,500	2,542
118,100	4,820	45	659,000	2,604	76	1,837,000	2,544
130,200	4,630	46	690,200	2,609	77	1,886,500	2,545
142,400	4,450	47	721,400	2,613	78	1,936,100	2,546
154,600	4,280	48	752,600	2,613	79	1,986,900	2,547
166,800	4,120	49	783,800	2,612	80	2,035,200	2,544
179,100	3,970	50	835,000	2,609	81	2,084,700	2,542
191,400	3,830	51	854,600	2,628	82	2,134,300	2,539
203,700	3,695	52	892,000	2,639	83	2,183,900	2,536
216,100	3,570	53	929,400	2,647	84	2,233,500	2,532
228,400	3,455	54	966,800	2,653	85	2,283,100	2,528
240,800	3,345	55	1,004,300	2,656	86	2,337,700	2,529
253,100	3,240	56	1,041,700	2,657	87	2,393,900	2,530
265,500	3,140	57	1,079,100	2,657	88	2,450,100	2,531
277,900	3,050	58	1,116,500	2,655	89	2,506,300	2,531
290,300	2,960	59	1,154,000	2,652	90	2,566,800	2,535
302,700	2,880	60	1,191,400	2,648	91	2,629,300	2,540
319,400	2,840	61	1,228,800	2,642	92	2,691,700	2,544
338,000	2,815	62	1,266,300	2,635	93	2,754,200	2,548
356,500	2,785	63	1,303,700	2,628	94	2,816,700	2,550
375,100	2,755	64	1,341,200	2,619	95	2,879,100	2,552
393,600	2,725	65	1,378,600	2,610	96	2,941,600	2,553
415,200	2,710	66	1,416,100	2,601	97	3,004,000	2,554
439,100	2,710	67	1,453,500	2,590	98	3,066,500	2,554
463,000	2,705	68	1,490,900	2,579	99	3,128,900	2,554
487,000	2,700	69	1,528,400	2,568	100	3,191,400	2,553
511,100	2,690	70	1,566,100	2,557	101	3,253,900	2,552
535,200	2,675	71	1,609,800	2,555	102	3,316,300	2,551

For a single car on the span multiply values in table by the following ntages: Spans 0' to 44', mult. by 100%; span 45', mult. by 99.6%; span y 93.2%; span 60', by 85.9%; 70', by 81.1%; 80', by 74.6%; 90', by 68.8%; by 63.1%.

IAXIMUM FLOORBEAM REACTIONS *R*, PER TRACK, FOR CONCENTRATED "K 25" LOADING (FIG. 2).—A TRAIN OF 50-TON ELECTRIC CARS.

(Span = Panel length.)

es for any other "K" loading are directly proportional to the axle loads.

Floor-beam Reaction *R*. Lbs.	Equival't Unif. Load *w* for Floorbe'm Reaction. Lbs. per lin. ft.	Span, in Feet.	Floor-beam Reaction *R*. Lbs.	Equival't Unif. Load *w* for Floorbe'm Reaction. Lbs. per lin. ft.	Span, in Feet.	Floor-beam Reaction *R*. Lbs.	Equival't Unif. Load *w* for Floorbe'm Reaction. Lbs. per lin. ft.
37,500	3,750	15	41,700	2,780	20	50,000	2,500
38,600	3,510	16	42,200	2,635	21	52,400	2,495
39,600	3,300	17	43,100	2,540	22	55,700	2,531
40,400	3,110	18	44,900	2,495	23	58,700	2,552
41,100	2,935	19	47,400	2,490	24	62,800	2,615

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ND SHEARS *S*, PER TRACK, FOR CONCENTRATED "K 25"
(FIG. 2).—A TRAIN OF 50-TON ELECTRIC CARS.*
ther "K" loading are directly proportional to the axle load.

quiva-t Unif. oad *w* r End hear. bs. per in. ft.	Span, in Feet.	End Shear *S*. Lbs.	Equivalent Unif. Load *w* for End Shear. Lbs. per lin. ft.	Span, in Feet.	End Shear *S*. Lbs.	Equivalent Unif. Load *w* for End Shear. Lbs. per lin. ft.
7,500	37	66,200	3,590	64	98,000	3,090
7,020	38	67,100	3,530	65	100,000	3,075
6,580	39	67,900	3,480	66	101,500	3,075
6,220	40	68,800	3 440	67	103,000	3,075
5,870	41	70,100	3 420	68	104,400	3,070
5,560	42	71,400	3,400	69	105,800	3,065
5,270	43	72,700	3,380	70	107,100	3,060
5,020	44	73,600	3 350	71	108,500	3,055
4,790	45	75,000	3,335	72	109,700	3,050
4,570	46	76,600	3,330	73	111,000	3,040
4,380	47	78,200	3,325	74	112,200	3,030
4,310	48	79,700	3,320	75	113,300	3,020
4,240	49	81,100	3,310	76	114,500	3,010
4,160	50	82,500	3,300	77	115,600	3,000
4,080	51	83,800	3,290	78	116,700	2,990
4,000	52	85,100	3,275	79	117,700	2,980
3,990	53	86,400	3,260	80	118,800	2,970
3,980	54	87,500	3,240	81	120,100	2,965
3,960	55	88,600	3,225	82	121,300	2,960
3,920	56	89,700	3,205	83	122,600	2,955
3,890	57	90,800	3,185	84	123,800	2,950
3,850	58	91,800	3,170	85	125,000	2,940
3,810	59	92,800	3,145	86	126,500	2,940
3,760	60	93,800	3,125	87	127,900	2,940
3,720	61	95,100	3,115	88	129,300	2,940
3,670	62	96,400	3,110	89	130,600	2,935
3,630	63	97,600	3,100	90	131,600	2,930

car on the span multiply values in table by the following
ans 0′ to 40′, mult. by 100%; span 45′, mult. by 96.3%;
%; span 60′, by 84.4%; 70′, by 76.8%; 80′, by 71.0%; 90′, by

Illustrations, with Details.

Description.	Eng. Rec.
oorbeam, floor, 515-ft. elec.-highway bridge . . .	June 12,'09
k and 517′ through spans, McKinley Bridge . . .	Apr. 30, '10
uss bridge, carrying street and 2 trolley tracks .	May 7, '10
, stringers, Manhattan Elev. Ry	May 28, '10

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40.—HIGHWAY BRIDGES.

I.—UNIVERSAL STRESS-SHEETS.

Explanation.

The following are types of trusses suitable for bridge spans up to about 260 ft. in length. They are designated by a figure, indicating the number of panels in the truss, and also by a distinguishing letter when more than one type of the same number of panels are used. Thus, Types *4A*, *4B* and *4C* each have 4 panels, but their truss systems are different.

The Diagrams are proportioned by scale and show height of truss in terms of panel length $p=1$. The members of the truss are numbered in each case to correspond with the numbers in the adjoining tables. The practical upper and lower limiting spans, with resulting panel length, are given for each type. The fractions at lower chord joints express the live-load reactions, at left abutment, in terms of a panel load = unity, of panel loads from the right-hand end up to and including that joint; and are useful for finding the shears.

The Tables accompanying the diagrams give the *unit length* of each truss member for a panel length of unity (actual length = unit length × panel length); also the dead- and live-load *unit stresses* in each truss member for unit panel loads (actual stress = unit stress × panel load per truss).

Ex. 1.—What type of truss would be suitable for a 168-ft. span? Find the lengths of the members? Find maximum stresses in members 11 and 16?

Solution.—Type 8; 8 panels @ 21 = 168. Height = $21 \times 1\frac{1}{3} = 28$. Diagonals = $21 \times 1\frac{2}{3} = 35$ ft. Assuming dead-load at 1000, and live-load at 1200 lbs. per lin. ft. of bridge, the respective panel loads per truss are: d. l. = 10.5 and l. l. = 12.6 thousand lbs.; whence the max compressive stress in 11 = $1.5 \times 10.5 + 1.875 \times 12.6 = 39{,}375$ lbs., and the max tensile stress in 16 = $-.625 \times 10.5 + .938 \times 12.6 = 5{,}256$ lbs.

Types of Trusses, and Unit Stress Sheets.

[See Explanation, preceding].
c = compression; t = tension.

Type 2A.
For spans 36′ to 40′.
2 @ 18′ = 36′.
2 @ 20′ = 40′.
Live reaction summations:

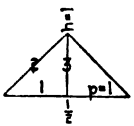

TABLE 2A.

Name of member.....	1	2	3
Unit length of member..	1.	1.414	1.
Dead load unit stress, D	.500 t	.707 c	1.000 t
Live load unit stress, L..	.500 t	.707 c	1.000 t

Type 2B.
For spans 25′ to 32′.
2 @ 12′.5 = 25′.
2 @ 16′ = 32′.
Live reaction summations:

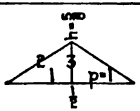

TABLE 2B.

Name of member.....	1	2	3
Unit length of member..	1.	1.179	.625
Dead load unit stress, D.	.800 t	.943 c	1.000 t
Live load unit stress, L..	.800 t	.943 c	1.000 t

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Type 2C.
For spans 25′ to 32′.
2 @ 12′.5 = 25′.
2 @ 16′ = 32′.
Live reaction summations:

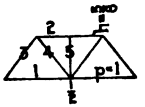


TABLE 2C.

Name of member.....	1	2	3	4	5
Unit length of member..	1.	.5	.8	.8	.625
Dead load unit stress, D.	.400 *t*	.800 *c*	.640 *c*	.640 *t*	0
Live load unit stress, L..	.400 *t*	.800 *c*	.640 *c*	.640 *t*	0

Type 3A.
For spans 48′ to 63′.
3 @ 16′ = 48′.
3 @ 21′ = 63′.
Live reaction summations:

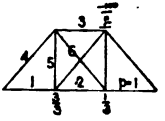


TABLE 3A.

Name of member.....	1, 2	3	4	5	6
Unit length of member..	1.	1.	1.505	1.125	.....
Dead load unit stress, D.	.889 *t*	.889 *t*	1.338 *c*	1. *t*	0
Live load unit stress, L..	.889 *t*	.889 *c*	1.338 *c*	1. *t*	.446 *c*

Type 3B.
For spans 37′.5 to 48′.
3 @ 12′.5 = 37′.5
3 @ 16′ = 48′.
Live reaction summations:

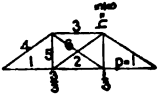
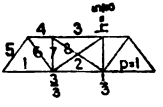


TABLE 3B.

Name of member.....	1, 2	3	4	5	6
Unit length of member..	1.	1.	1.179	.625	1.179
Dead load unit stress, D.	1.600 *t*	1.600 *c*	1.887 *c*	1. *t*	0
Live load unit stress, L..	1.600 *t*	1.600 *c*	1.887 *c*	1. *t*	.629 *t*

Type 3C.
For spans 37′.5 to 48′.
3 @ 12′.5 = 37′.5
3 @ 16′ = 48′.
Live reaction summations:



8
1.179
0
.629 *t*

TABLE 3C.

Name of member.....	1	2	3	4	5	6	7
Unit length of member..	1.	1.	1.	.5	.8	.8	.625
Dead load unit stress, D.	.800 *t*	1.600 *t*	1.600 *c*	1.600 *c*	1.281 *c*	1.281 *t*	0
Live load unit stress, L..	.800 *t*	1.600 *t*	1.600 *c*	1.600 *c*	1.281 *c*	1.281 *t*	.333 *c*

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Type 4A.
For spans 64′ to 80′.
4 @ 16′ = 64′.
4 @ 20′ = 80′.
Live reaction summations:

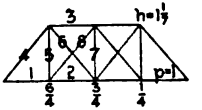


TABLE 4A.

Name of member.....	1, 2	3	4	5	6	7	8
Unit length of member..	1.	1.	1.519	1.143	1.519	1.143	1.519
Dead load unit stress, D..	1.3125 *t*	1.750 *c*	1.994 *c*	1. *t*	.665 *t*	0	.665 *c*
Live load unit stress, L..	1.3125 *t*	1.750 *c*	1.994 *c*	1. *t*	.997 *t*	.250 *c*	.332 *t*

Type 4B.
For spans 50′ to 60′.
4 @ 12′.5 = 50′.
4 @ 15′ = 60′.
Live reaction summations:

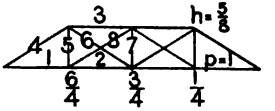
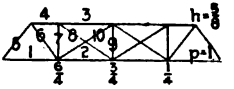


TABLE 4B.

Name of member.....	1, 2	3	4	5	6	7	8
Unit length of member..	1.	1.	1.179	.625	1.179	.625	1.179
Dead load unit stress, D..	2.400 *t*	3.200 *c*	2.831 *c*	1.000 *t*	.943 *t*	0	.943 *c*
Live load unit stress, L..	2.400 *t*	3.200 *c*	2.831 *c*	1.000 *t*	1.414 *t*	.250 *c*	.471 *t*

Type 4C.
For spans 50′ to 60′.
4 @ 12′.5 = 50′.
4 @ 15′ = 60′.
Live reaction summations:

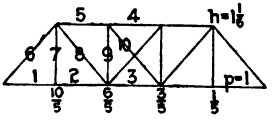


8	9	10
1.179	.625	1.179
.943 *t*	0	.943 *c*
1.414 *t*	.250 *c*	.471 *t*

TABLE 4C.

Name of member.....	1	2	3	4	5	6	7
Unit length of member..	1.	1.	1.	.5	.8	.8	.625
Dead load unit stress, D..	1.200 *t*	2.400 *t*	3.200 *c*	2.400 *c*	1.920 *c*	1.920 *t*	.500 *c*
Live load unit stress, L..	1.200 *t*	2.400 *t*	3.200 *c*	2.400 *c*	1.920 *c*	1.920 *t*	.750 *c*

Type 5A.
For spans 80′ to 105′.
5 @ 16′ = 80′.
5 @ 21′ = 105′.
Live reaction summations:



10
1.537
0
.790 *t*

TABLE 5A.

Name of member.....	1, 2	3	4, 5	6	7	8	9
Unit length of member..	1.	1.	1.	1.537	1.167	1.537	1.167
Dead load unit stress, D..	1.714 *t*	2.571 *t*	2.571 *c*	2.634 *c*	1. *t*	1.317 *t*	0
Live load unit stress, L..	1.714 *t*	2.571 *t*	2.571 *c*	2.634 *c*	1. *t*	1.580 *t*	.600 *c*

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5 80′.
60′.
80′.
mmations:

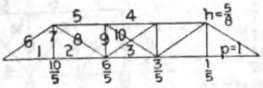

TABLE 5B.

	10
	1.179
	0
	1.132 *t*

er.....	1, 2	3	4, 5	6	7	8	9
member..	1.	1.	1.	1.179	.625	1.179	.625
stress, D.	3.200 *t*	4.800 *t*	4.800 *c*	3.774 *c*	1. *t*	1.187 *t*	0
stress, L..	3.200 *t*	4.800 *t*	4.800 *c*	3.774 *c*	1. *t*	2.264 *t*	.600 *c*

to 129′.
= 102′.
= 129′.
summations:

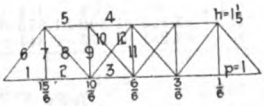

TABLE 6.

ber.....	1, 2	3	4	5	6	7	8
member..	1.	1.	1.	1.	1.562	1.2	1.562
t stress, D.	2.083 *t*	3.333 *t*	3.750 *c*	3.333 *c*	3.255 *c*	1. *t*	1.953 *t*
stress, L..	2.083 *t*	3.333 *t*	3.750 *c*	3.333 *c*	3.255 *c*	1. *t*	2.170 *t*

9	10	11	12
1.2	1.562	1.2	1.562
.500 *c*	.651 *t*	0	.651 *c*
1.000 *c*	1.302 *t*	.500 *c*	.651 *t*

Note.—For economy, use type 5A when practicable.

7.
6′ to 150.′5
= 126′.
.5 = 150′.5
summations:

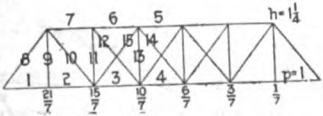

TABLE 7.

nber.....	1, 2	3	4	5	6	7	8
of member..	1.	1	1.	1.	1.	1.	1.601
nit stress, D.	2.400 *t*	4.000 *t*	4.800 *t*	4.800 *c*	4.800 *c*	4.000 *c*	3.843 *c*
it stress, L..	2.400 *t*	4.000 *t*	4.800 *t*	4.800 *c*	4.800 *c*	4.000 *c*	3.843 *c*

9	10	11	12	13	14	15
1.25	1.601	1.25	1.601	1.25	1.601	1.601
1. *t*	2.562 *t*	1. *c*	1.281 *t*	0	0	1.281 *c*
1. *t*	2.745 *t*	1.429 *c*	1.830 *t*	.857 *c*	1.098 *t*	.549 *t*

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Type 8.
For spans 144′ to 172′.
8 @ 18′ = 144′.
8 @ 21′.5 = 172′.
Live reaction summations:

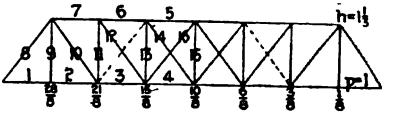

TABLE 8.

Name of member.....	1, 2	3	4	5	6	7	8
Unit length of member..	1.	1.	1.	1.	1.	1.	1⅓
Dead load unit stress, D.	2.625 t	4.500 t	5.625 t	6.000 c	5.625 c	4.500 c	4.375 c
Live load unit stress. L..	2.625 t	4.500 t	5.625 t	6.000 c	5.625 c	4.500 c	4.375 c

9	10	11	12	13	14	15	16
1⅓	1⅔	1⅓	1⅔	1⅓	1⅔	1⅓	1⅔
1.	3.125 t	1.500 c	1.875 t	.500 c	.625 t	0	.625 c
1.	3.281 t	1.875 c	2.344 t	1.250 c	1.563 t	.750 c	.938 t

Type 9A.
For spans 162′ to 193′.5
9 @ 18′ = 162′.
9 @ 21′.5 = 193′.5
Live reaction summations:

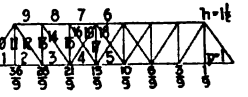

TABLE 9A.

Name of member.....	1, 2	3	4	5	6	7	8
Unit length of member..	1.	1.	1.	1.	1.	1.	1.
Dead load unit stress, D.	2.667 t	4.667 t	6.000 t	6.667 t	6.667 c	6.667 c	6.000 c
Live load unit stress, L..	2.667 t	4.667 t	6.000 t	6.667 t	6.667 c	6.667 c	6.000 c

9	10	11	12	13	14	15	16	17	18	19
1.	1.803	1.5	1.803	1.5	1.803	1.5	1.803	1.5	1.803	1.803
4.667 c	4.808 c	1. t	3.606 t	2.000 c	2.404 t	1.000 c	1.202 t	0	0	1.202 c
4.667 c	4.808 c	1. t	3.740 t	2.333 c	2.805 t	1.667 c	2.003 t	1.111 c	1.336 t	.801 t

Type 9B.
For spans 162′ to 193′.5
9 @ 18′ = 162′.
9 @ 21′.5 = 193′.5
Live reaction summations:

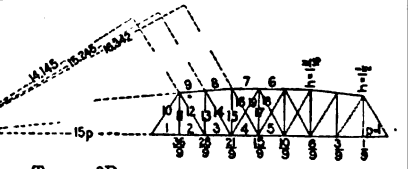

TABLE 9B.

Name of member.....	1, 2	3	4	5	6	7	8
Unit length of member..	1.	1.	1.	1.	1.	1.	1.004
Dead load unit stress, D.	2.667 t	4.391 t	5.333 t	5.926 t	5.926 c	5.926 c	5.357 c
Live load unit stress, L..	2.667 t	4.391 t	5.333 t	5.926 t	5.926 c	5.926 c	5.357 c

9	10	11	12	13	14	15	16	17	18	19
1.004	1.803	1.5	1.803	1.594	1.882	1.688	1.962	1.688	1.962	1.962
4.410 c	4.808 c	1. t	3.111 t	1.588 c	1.771 t	.500 c	1.163 t	0	0	1.163 c
4.410 c	4.808 c	1. t	3.299 t	2.059 c	2.296 t	1.389 c	1.938 t	1.111 c	1.292 t	.775 t

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10.
to 215′.
=180′.
5=215′.
summations:
ds:

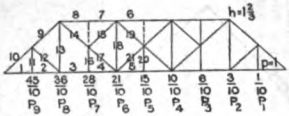
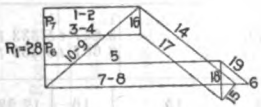


TABLE 10.

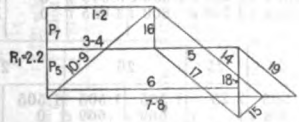
ber.....	1, 2	3, 4	5	6	7, 8	9	10
member.	1.	1.	1.	1.	1.	1.302	1.302
t stress, D	5.400 *t*	4.800 *t*	7.200 *t*	7.800 *c*	7.800 *c*	6.248 *c*	7.029 *c*
stress, L.	5.400 *t*	4.800 *t*	7.200 *t*	7.800 *c*	7.800 *c*	6.248 *c*	7.029 *c*

2	13	14	15		17	18	19	21
2	1.667	1.302	1.302	1.302	1.302	1.667	1.302	1.302
1 *c*	1.500 *t*	3.905 *t*	.781 *t*	2.343 *c*	3.124 *t*	1.000 *c*	.781 *t*	0
1 *c*	1.500 *t*	4.374 *t*	.781 *t*	.937 *t*	3.593 *t*	1.700 *c*	2.343 *t*	1.562 *t*

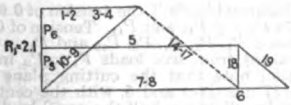
ing graphics illustrate the loadings which give maximum live nbers 14, 17 and 18 of Type 10. (Note the two last cases which xima for these members):



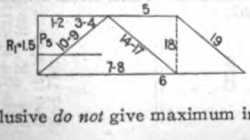
o P_7 inclusive give maxima in members 14 and 17.



o P_7 inclusive, omitting load P_6, give maximum in member



o P_6 inclusive *do not* give maximum in member 17.



o P_5 inclusive *do not* give maximum in member 18.

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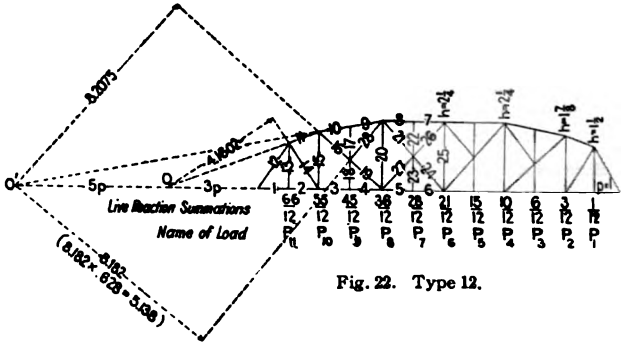

Fig. 22. Type 12.

Type 12.
For spans 216′ to 258′.
12 @ 18′. = 216′.
12 @ 21′.5 = 258′.

TABLE 12.

Name of member	1, 2	3, 4	5, 6	7, 8	9, 10
Unit length of member	1.	1.	1.	1.	1.017
Dead load unit stress, D	3.667 t	5.333 t	7.111 t	8.444 c	7.687 c
Live load unit stress, L	3.667 t	5.333 t	7.111 t	8.444 c	7.687 c

11	12	13	14	15		16	17,22	18	19	20c
1.068	1.803	1.5	1.803	1.875	1.875	1.371	1.125	.938	1.371	2.25
5.696c	6.611 c	1. t	3.005 t	1.500 c	1.500 c	3.046 t	0	1. t	2.332 t	.795c
5.696c	6.611 c	1. t	3.305 t	2.250 c	.600 t	3.655 t	0	1. t	2.942 t	1.567c

20 t	21	23	24	25	26		27		28	
2.25	1.505	1.125	1.505	2.25	1.505	1.505	1.505	1.505	1.65	1.65
.795c	2.007 t	1. t	1.338 t	1. c	.669 t	.669 c	0	1.338 c	.628t	1.711 c
.583t	3.122 t	1. t	2.453 t	1.833 c	.669 t	1.672 t	.....	1.115 t	.628t	1.222 t

Notes.—Using "balanced loads," the tension of 0.6 in member 15 is obtained from live loads P_1, P_2, P_{10} and P_{11}. Tension of 0.583 in member 20 is obtained from live loads P_1, P_2, P_4, P_8, P_{10} and P_{11}. Compression of 1.567 in member 20 is obtained from live loads P_1 to P_9 inclusive, omitting P_8. In the last calculation note that the cutting plane will cut *four* active members, 9, 28, 20 (27 inactive) and 5, with the center of moments at 0′. But the stress in member 28 is one-half the panel load P_9 multiplied by the secant of its angle of inclination = .5 × 1.257 = 0.628, and the moment of this stress about 0′ is 5.138. This enables us to solve the stress in member 20 by cutting the *four* active members. Thus, stress in member $20 = \frac{1}{5.138}(-R_1 \times 8 - 5.138 + P_9 \times 11) = -1.567$.

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Car Tracks.—Loads given in Table 13 are for "L" loading. For special loadings for electric car tracks see "K" loading, Fig. 2 and tables, pages 716, 718 and 719.

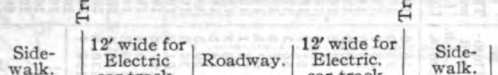

Fig. 23.

Roadway.—For alternative or concentrated "L" loading given in Table 13 see Fig. 1, page 716; also Tables 1 and 2, pages 716 and 717, and Tables 14 and 15, pages 728 and 729.

13.—Classes for Bridges, and Live Loads for Trusses and Floors.

Class.	For the Trusses. From 50' to 100' spans. Roadway. Load per Sq. Ft. Lbs.	Side-walk. Load per Sq. Ft. Lbs.	Track. Load per Lin. Ft. Lbs.	Proportional Loads for Spans 100' to 200'	200-ft. spans and over. Roadway. Load per Sq. Ft. Lbs.	Side-walk. Load per Sq. Ft. Lbs.	Track. Load per Lin. Ft. Lbs.	Minimum Load per Lin. Ft. of Span. Lbs.	For the Floor and its Supports. Roadway. Load per Sq. Ft. Lbs.	Side-walk. Load per Sq. Ft. Lbs.	Track Loading.	Remarks.
$A = H100, L30$	100	100	2000	Or the respective "L" Loading (2 axles, 10-ft. centers, 5-ft. gage) on any portion of roadway.	80	80	1600	1800	125	100	L 30	For "L 24" Loading Data see Tables under Electric Railway Bridges. Values for "L" Loading are directly proportional to Axle Loads. It occupies a width of 12 ft., single track.
$B = H90, L24$	90	90	1600		72	72	1280	1600	112½	90	L 24	
$C = H80, L18$	80	80	1200		64	64	960	1400	100	80	L 18	
$D = H70, L12$	70	70	.....		56	56	.....	1200	87½	70	.....	
$E = H60, L6$	60	60	.....		48	48	.....	1000	75	60	.....	

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14.—Live-Load Data for Designing Floor Systems and Spans Under 50 Ft.

Maximum Moments *M* and End Shears *S* per Track for "L" Loadings.

Note.—For maximum floor-beam reactions use the end shears *S* down to the (*), and below the (*) use the uniform load, covering the two panels.

Span.	Class A "*L* 30" Loading. 30 Tons—2 Axles—10'c.-c. 12' Wide—5' Gage.		Class B "*L* 2' 4' Loading. 24 Tons—2 Axles—10'c.-c. 12' Wide—5' Gage.		Class C "*L* 18" Loading. 18 Tons—2 Axles—10'c.-c. 12' Wide—5' Gage.		Class D "*L* 12" Loading. 12 Tons—2 Axles—10'c.-c. 12' Wide—5' Gage.		Class E "*L* 6" Loading. 6 Tons—2 Axles—10'c.-c. 12' Wide—5' Gage.	
Ft.	M Thousand Ft.-Lb. Units.	S Thousand Lb. Units.	M Thousand Ft.-Lb. Units.	S Thousand Lb. Units.	M Thousand Ft.-Lb. Units.	S Thousand Lb. Units.	M Thousand Ft.-Lb. Units.	S Thousand Lb. Units.	M Thousand Ft.-Lb. Units.	S Thousand Lb. Units.
10	75.0	30.0	60.0	24.0	45.0	18.0	30.0	12.0	15.0	6.0
11	82.5	32.7	66.0	26.2	49.5	19.6	33.0	13.1	16.5	6.5
12	90.0	35.0	72.0	28.0	54.0	21.0	36.0	14.0	18.0	7.0
13	97.5	36.9	78.0	29.5	58.5	22.2	39.0	14.8	19.5	7.4
14	105.0	38.6	84.0	30.9	63.0	23.1	42.0	15.4		7.7
15	112.5	40.0	90.0	32.0	67.5	24.0	45.0	*16.0		8.0
16	120.0	41.3	96.0	33.0	72.0	24.8	48.0	16.5		8.3
17	127.5	42.4	102.0	33.9	76.5	25.4	51.0	16.9		8.5
18	140.8	43.3	112.7	34.7	84.5	26.0	56.3	17.3		8.7
19	154.7	44.2	123.8	35.4	93.9	26.5	61.9	17.7		8.8
20	168.8	45.0	135.0	36.0	101.3	27.0	67.5	18.0		9.0
21	182.9	45.7	146.3	36.6	109.7	27.4	73.1	18.3		
22	197.1	46.4	157.6	37.1	118.2	27.8	78.8	18.5		
23	211.3	47.0	169.0	37.6	126.8	*28.2	84.5	18.8		
24	225.6	47.5	180.5	38.0	135.4	28.5	90.3	19.0		
25	240.0	48.0	192.0	38.4	144.0	28.8	96.0	19.2		
26	254.4	48.5	203.5	38.8	152.7	29.1	101.8	19.4		
27	268.9	48.9	215.1	39.1	161.3	29.3	107.6	19.6		
28	283.4	49.3	226.7	39.4	170.0	29.6	113.4	19.7		
29	297.9	49.7	238.3	*39.7	178.8	29.8	119.2	19.9		
30	312.5	50.0	250.0	*40.0	187.5	30.0	125.0	20.0		
31	327.1	50.3	261.7	40.3	196.3	30.2	130.8	20.1		
32	341.7	50.6	273.4	40.5	205.0	30.4	136.7	20.3		
33	356.4	50.9	285.1	40.7	213.8	30.5		20.4		
34	371.0	*51.2	296.8	40.9	222.6	30.7		20.5		
35	385.7	51.4	308.6	41.1	231.4	30.9		20.6		
36	400.4	51.7	320.3	41.3	240.2	31.0		20.7		
37	415.3	51.9	332.1	41.5	249.1	31.1		20.8		
38	429.9	52.1	343.9	41.7	257.9	31.3		20.8		
39	444.6	52.3	355.7	41.9	266.8	31.4		20.9		
40	459.4	52.5	367.5	42.0	275.6	31.5		21.0		
41	474.2	52.7	379.3	42.2	284.5	31.6				
42	488.9	52.9	391.2	42.3	293.4	31.7				
43	503.7	53.0	403.0	42.4	302.2	31.8				
44	518.5	53.2	414.8	42.6	311.1	31.9				
45	533.3	53.3	426.7	42.7	320.0	32.0				
46	548.2	53.5	438.5	42.8	328.9	32.1				
47	563.0	53.6	450.4	42.9	337.8	32.2				
48	577.8	53.8	462.3	43.0	346.7	32.3				
49	592.7	53.9	474.1	43.1		32.3				
50	607.5	54.0	486.0	43.2		32.4				
	Unif. load, 12x125 = 1500		Unif. load, 12x112½ = 1350		Below line, use unif. load of 1200 lbs. per lin. ft.		Below line, use unif. load of 1050 lbs. per lin. ft.		Below the line, use unif. load of 900 lbs. per lin. ft. for max. moment.	Below line, use unif. load of 900 lbs. per lin. ft. for max. shear.

* See note at head of table.

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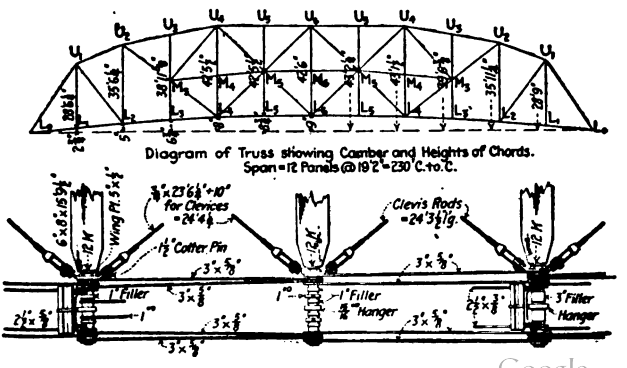
15.—Uniform Live Loads for Trusses of Spans over 50 Ft.

(Each street car track loading occupies width of 12 feet for single track, and 11 feet for double track.)

Span. Ft.	Class A		Class B		Class C		Class D		Class E	
	Vehicular Roadway, and Walks. Lbs. per Sq. Ft.	Each Street car Track. Lbs. per Lin. Ft.	Vehicular Roadway, and Walks. Lbs. per Sq. Ft.	Each Street car Track. Lbs. per Lin. Ft.	Vehicular Roadway, and Walks. Lbs. per Sq. Ft.	Each Street car Track. Lbs. per Lin. Ft.	Vehicular Roadway, and Walks. Lbs. per Sq. Ft.	Each Street car Track. Lbs. per Lin. Ft.	Vehicular Roadway, and Walks. Lbs. per Sq. Ft.	Each Street car Track. Lbs. per Lin. Ft.
50 to 100	100	2000	90	1600	80	1200	70	Special. Not less than load per sq. ft. on roadway and sidewalk.	60	Special. Not less than load per sq. ft. on roadway and sidewalk.
105	99	1980	89	1584	79	1188	69		59	
110	98	1960	88	1568	78	1176	69		59	
115	97	1940	87	1552	78	1164	68		58	
120	96	1920	86	1536	77	1152	67		58	
125	95	1900	86	1520	76	1140	67		57	
130	94	1880	85	1504	75	1128	66		56	
135	93	1860	84	1488	74	1116	65		56	
140	92	1840	83	1472	74	1104	64		55	
145	91	1820	82	1456	73	1092	64		55	
150	90	1800	81	1440	72	1080	63		54	
155	89	1780	80	1424	71	1068	62		53	
160	88	1760	79	1408	70	1056	62		53	
165	87	1740	78	1392	70	1044	61		52	
170	86	1720	77	1376	69	1032	60		52	
175	85	1700	77	1360	68	1020	60		51	
180	84	1680	76	1344	67	1008	59		50	
185	83	1660	75	1328	66	996	58		50	
190	82	1640	74	1312	66	984	57		49	
195	81	1620	73	1296	65	972	57		49	
200 and over	80	1600	72	1280	64	960	56		48	

III.—DETAILS OF COMBINATION BRIDGE, 230 FT. SPAN. TYPE 12.



Figs. 24. Truss Diagram. Bottom Chord and Laterals.

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Timber Details.	Figs.
Chord U_1-L_0,	29

Cast Details.	Figs.
End shoe, 12B,	30
Post shoe, 12G,	31
Bed plate, 12A,	32
Lateral struts, 12J,	34
Lateral struts, 12K,	35
Washers,	48
Separators,	52

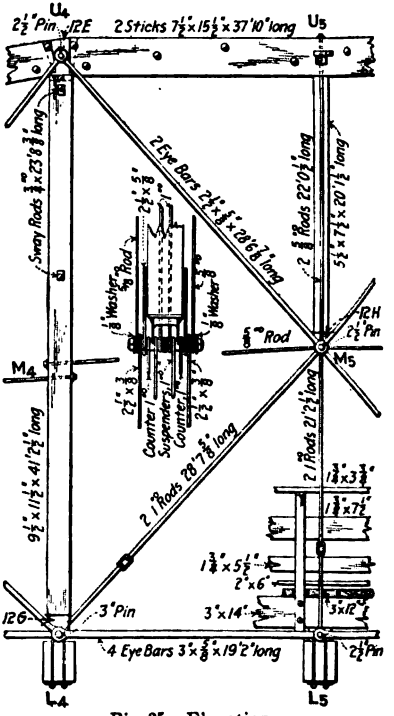
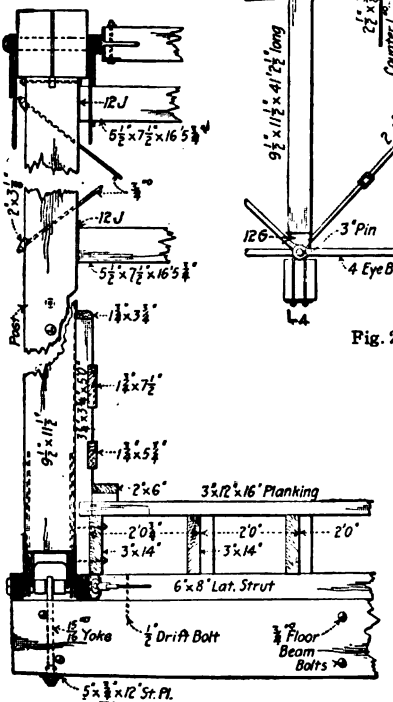


Fig. 25. Elevation.



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Eye-bars,	37
Counters,	38
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Fig. 27. End Panel.

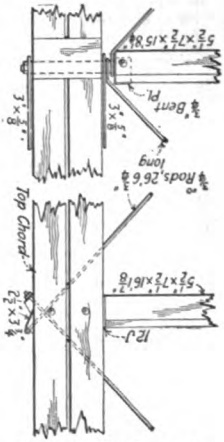

Fig. 28. Top Laterals.

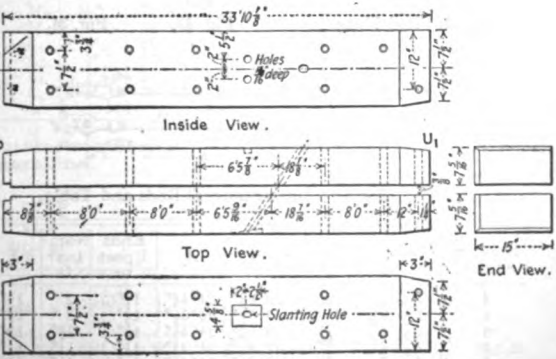

Fig. 29. Top Chord Framing.

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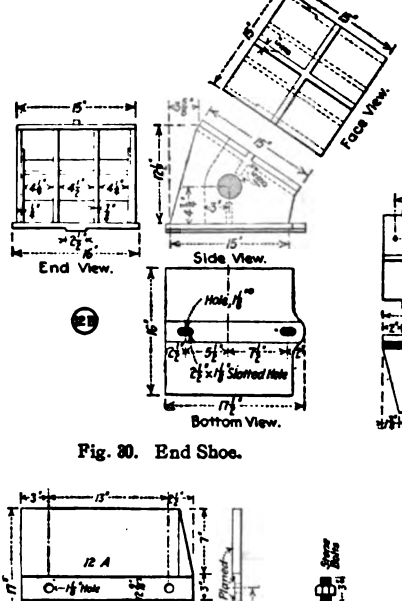

Fig. 30. End Shoe. Fig. 31. Post

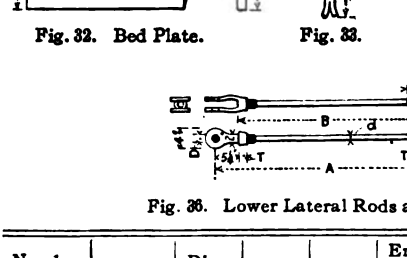

Fig. 32. Bed Plate. Fig. 33. Fig. 34.

Fig. 36. Lower Lateral Rods and Table.

Number Required.	Mark.	Dia. d	B	A	Ends Upset to	Nominal D	T
4	L_0-L_1	1¼"□	23′ 6½"	24′ 4½"	1⅝"○	2 "	1
4	L_1-L_2	1 "□	23′ 7½"	24′ 5½"	1⅜"○	2 "	1
4	L_2-L_3	⅞"□	23′ 6½"	24′ 4½"	1⅛"○	1¾"	1
4	L_3-L_4	⅞"○	23′ 6½"	24′ 4½"	1⅛"○	1¾"	1
4	L_4-L_5	⅞"○	23′ 6½"	24′ 4½"	1 "○	1½"	1
4	L_5-L_6	⅞"○	23′ 5½"	24′ 3½"	1 "○	1½"	1

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STEEL EYE-BARS.

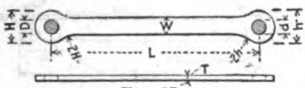

Fig. 37.

Mark.	Dimens'ns. W	Dimens'ns. T	Length L	One Head H	One Head D Hole.	Other Head h	Other Head d Hole.
$-L_1$, $-L_2$, $-L_5$	3″	$\frac{5}{8}$″	19′ 2″	7″	$2\frac{15}{16}$″ + $\frac{1}{32}$″	$6\frac{1}{2}$″	$2\frac{7}{16}$″ + $\frac{1}{32}$″
$-L_6$, $-L_3$, $-L_4$	3″	$\frac{11}{16}$″	19′ 2″	7″	$2\frac{15}{16}$″ + $\frac{1}{32}$″	$6\frac{1}{2}$″	$2\frac{7}{16}$″ + $\frac{1}{32}$″
$-L_2$	3″	$\frac{9}{16}$″	34′ $2\frac{1}{2}$″	7″	$2\frac{15}{16}$″ + $\frac{1}{32}$″	$6\frac{1}{2}$″	$2\frac{7}{16}$″ + $\frac{1}{32}$″
$-M_3$	3″	$\frac{5}{8}$″	26′ $0\frac{1}{2}$″	$6\frac{1}{2}$″	$2\frac{7}{16}$″ + $\frac{1}{32}$″	$6\frac{1}{2}$″	$2\frac{7}{16}$″ + $\frac{1}{32}$″
$-L_4$	$2\frac{1}{2}$″	$\frac{5}{8}$″	26′ $0\frac{1}{2}$″	6″	$2\frac{7}{16}$″ + $\frac{1}{32}$″	$6\frac{1}{2}$″	$2\frac{15}{16}$″ + $\frac{1}{32}$″
$-M_5$	$2\frac{1}{2}$″	$\frac{5}{8}$″	28′ $6\frac{7}{8}$″	6″	$2\frac{7}{16}$″ + $\frac{1}{32}$″	6″	$2\frac{7}{16}$″ + $\frac{1}{32}$″
$-L_6$	$2\frac{1}{2}$″	$\frac{3}{8}$″	28′ $6\frac{7}{8}$″	6″	$2\frac{7}{16}$″ + $\frac{1}{32}$″	$6\frac{1}{2}$″	$2\frac{15}{16}$″ + $\frac{1}{32}$″

COUNTERS AND SUSPENDERS.

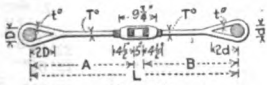

Fig. 38.

″○ upset to $1\frac{1}{4}$″○; 1″○ upset to $1\frac{3}{8}$″○.

ark.	Dia. T○	Length C.-C. L	A	B	Nominal D	Nominal d	t□
$-M_3$	$1\frac{11}{16}$″○	31′ $4\frac{5}{16}$″	25′ $11\frac{5}{16}$″	5′ 0″	$2\frac{1}{2}$″	$2\frac{1}{2}$″	$\frac{7}{8}$″□
$-M_5$	1″○	28′ $7\frac{15}{16}$″	23′ $2\frac{15}{16}$″	5′ 0″	$2\frac{1}{2}$″	$2\frac{1}{2}$″	$\frac{7}{8}$″□
$-L_1$	1″○	28′ $6\frac{1}{2}$″	23′ $1\frac{1}{4}$″	5′ 0″	$2\frac{1}{2}$″	$2\frac{1}{2}$″	$\frac{7}{8}$″□
$-L_3$	1″○	17′ $8\frac{7}{8}$″	12′ $3\frac{7}{8}$″	5′ 0″	$2\frac{1}{2}$″	$2\frac{1}{2}$″	$\frac{7}{8}$″□
$-L_5$	1″○	21′ $2\frac{1}{2}$″	15′ $9\frac{1}{2}$″	5′ 0″	$2\frac{1}{2}$″	$2\frac{1}{2}$″	$\frac{7}{8}$″□
$-L_4$	1″○	28′ $7\frac{5}{8}$″	23′ $2\frac{5}{8}$″	5′ 0″	$2\frac{1}{2}$″	3″	$\frac{7}{8}$″□

ILLERS FOR BOTTOM ORD PINS.

Fig. 39.

ark.	Nominal D	L
$_3$ & L_5	$2\frac{1}{2}$″	1″
L_4	3″	1″
& L_6	3″	3″

TOP LATERAL AND PORTAL RODS.

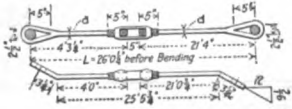

Fig. 40.

Number Required	Mark.	d	L	Upset to
4	$U_1 - U_2$	$\frac{7}{8}$″○	26′ $0\frac{1}{4}$″	$1\frac{1}{4}$″○

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Top Lateral and Portal Rods.—Concluded.

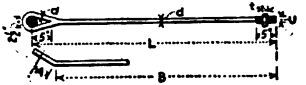

Fig. 41.

Number Required.	Mark.	d	L	A	B	T	U	H
4	Portal	1¼″○	21′ 2″	4″	20′ 10″	1¾″	1¾″○	21½
4	U_2-U_3	⅞″○	26′ 6¾″	3″	26′ 3¾″	1″	1″○	11½
4	U_3-U_4	⅞″○	26′ 5¼″	3″	26′ 2¼″	1″	1″○	11½
4	U_4-U_5	⅞″○	26′ 3″	3″	26′ 0″	1″	1″○	11½
4	U_5-U_6	⅞″○	26′ 2¼″	3″	26′ 11¼″	1″	1″○	11½

Nominal Rods.

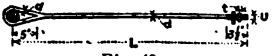

Fig. 42.

Number Required.	Mark.	d	L	U	t
8	M_3-U_3	⅞″○	22′ 0¼″	⅞″○	⅞″
8	M_5-U_5	⅞″○	22′ 0¼″	⅞″○	⅞″
8 { 4 / 4 }	M_3-M_4 / M_4-M_5	⅞″○	19′ 9¼″	⅞″○	⅞″
4	M_5-M_6	⅞″○	19′ 8¼″	⅞″○	⅞″

Sway Rods.

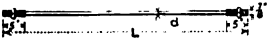

Fig. 43.

Number Required.	Mark.	Dia. d	Length L
4	U_2	⅞″○	23′ 1¼″
6	U_4 and U_6	⅞″○	23′ 8¼″

Cotter Pins for Lower Lateral Rods.

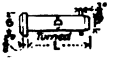

Fig. 44.

Number Required.	Mark.	D	L	G	P	Remarks.
8	L_0 and L_1	1 11/16″○	0′ 4¾″	2¼″	1¾″	Turned
8	L_1 and L_2	1 11/16″○	0′ 3¾″	2¼″	1¾″	"
16	L_2 L_3 & L_4	1 11/16″○	0′ 3¾″	2″	1¾″	"
16	L_4 L_5 & L_6	1 11/16″○	0′ 3¾″	1¾″	1¾″	"

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WING PLATES FOR BOTTOM LATERALS.

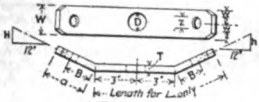

Fig. 45.

rk.	W	T	L	a	A	B	D	H	h	Nom. Pin Dia. x	Nom. Pin Dia. z
0	5 "	¾"	0' 11 "	0 "	5 "	2½"	3 "	...	10"	.....	2 "
1	4½"	¾"	1' 4 "	5 "	5 "	2½"	2½"	10"	10"	2 "	2 "
2	5 "	⅝"	1' 4 "	5 "	5 "	2½"	3 "	10"	10"	2 "	1¾"
3	4½"	⅝"	1' 3½"	5 "	4½"	2½"	2½"	10"	10"	1¾"	1¾"
4	5 "	½"	1' 3 "	4½"	4½"	2½"	3 "	10"	10"	1¾"	1½"
5	4½"	½"	1' 2¾"	4½"	4¼"	2½"	2½"	10"	10"	1½"	1½"
6	5 "	½"	1' 2½"	4¼"	4¼"	2½"	3 "	10"	10"	1½"	1½"

PLATES, BOLTS AND
GHT WASHERS.

Figs. 46.

Mark.	Remarks.
Top aterals	•

TOP CHORD PINS.

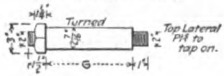

Fig. 47.

Number Required.	Mark.	G
4	U_1	21⅜"
4	U_2	18½"
4	U_4	21 "
2	U_6	21⅞"

Also ship 10 washers ⅜" thick for above pins.

P LATERAL, PORTAL AND SWAY BRACE WASHERS.

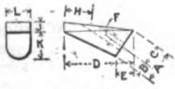

Fig. 48.

Mark.	A	B	C	D	E	F	H	J	K	L
Portal	3¼"	1½"	1¾"	4½"	2⅞"	1⅞"	2¼"	1 "	1⅝"	4½"
Top Lateral	2 "	⅞"	1⅛"	3⅝"	1⅜"	1 3/16"	1⅝"	¾"	1½"	2½"
Sway	2 "	⅞"	1⅛"	3 "	1⅝"	1 3/16"	1 7/16"	½"	1 3/16"	2 "

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Cotter Pins for Mid Truss Connections.

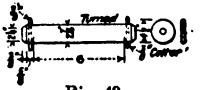

Fig. 49.

Number Required.	Mark.	G
4	M_3	11 "
4	M_5	12¼"

Bottom Chord Pins.

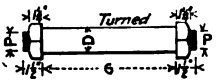

Fig. 51.

Number Required.	Mark.	D	G	P
4	L_0	2 13/16"○	17½"	2"
4	L_1	2 7/16"○	11½"	2"
4	L_2	2 11/16"○	14½"	2"
4	L_3	2 7/16"○	12⅝"	2"
4	L_4	2 13/16"○	16 "	2"
4	L_5	2 7/16"○	13⅞"	2"
2	L_6	2 13/16"○	17½"	2"

Bolts with Nut and 2 Washers Each.

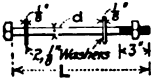

Fig. 53.

Number Required.	Mark.	d	L
90	Floor Beam	⅞"○	15½"
150	Chords	⅞"○	17½"
20	Posts	⅞"○	13½"
100	Railing	⅝"○	8½"

Hangers and Plates.

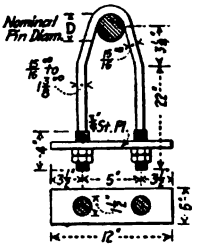

Fig. 50.

Number Required.	Mark.	D
10	$L_2-L_4-L_6$	3 "
12	$L_1-L_3-L_5$	2½"

Cast Separator Spools for Floor Beams.

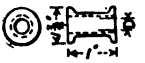

Fig. 52.

Number Required.	Mark.
90	Floor Beam

Wrought Packing Washers for Chords.

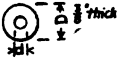

Fig. 54.

Number Required.	Mark.	Remarks.
150	Chords	Standard d and D

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-NICKEL-STEEL AND CARBON-STEEL SPANS.

following, has been condensed from very extensive and red data in the author's forthcoming structural handbook. rs a range of span lengths from 40 to 200 feet, with 5 ft. respective live loads for the same length of span were selected eight of trusses would be about equal. In the nickel-steel al systems are of carbon steel, the floor-beams and trusses steel. Although two different specifications were used—one -steel and one for the nickel-steel spans—the comparative total amount of steel are not materially affected.

ted Costs are based on the following prices:

l, delivered, at 4½ cents per pound;
, delivered, at 6 cents per pound;
livered, at $35 per M. ft., B. M.;
carbon-steel span, cost in dollars = 2 × span in feet + 0.003 t of metal in lbs.;
nickel-steel span, cost in dollars = 2.1 × span in feet + 0.003 t of metal in lbs.;
st in dollars = ½ the span in feet + 0.001 × weight of metal in lbs.

-steel spans have a 3″ wooden floor on wooden joists; the ns have an additional covering of flooring plank, 2 ins. thick. n of the table, below, shows that while the nickel-steel spans to 40% more than the carbon-steel spans, yet their live-load m 60 to 80% greater, thus showing a decided economy in el-steel—based on the above assumed prices. As none of ls are prepared to engage in the general manufacture of structures at the present time, however, the estimated costs ered as being more or less approximate.

cations used in proportioning the principal members of the as follows:

Carbon Steel.

rd, main diagonals, counters and long verticals, tension:
live stress + ½ dead stress) ÷ 12500.
compression:
live stress + ½ dead stress) ÷ $(12000 - 55\frac{l}{r})$.
(inclined), compression:
live stress + ½ dead stress) ÷ $(11000 - 50\frac{l}{r})$.
te posts (vertical), compression:
live stress + ½ dead stress) ÷ $(10000 - 45\frac{l}{r})$.
s—bottom flange of riveted girder, tension:
et) = (live stress + dead stress) ÷ 13,000.

Nickel-Steel.

rd, main diagonals, etc.—eye bars—tension:
live stress $(1+P)$ + dead stress] ÷ 30000.
compression:
live stress $(1+P)$ + dead stress] ÷ $(30000 - 130\frac{l}{r})$.
(inclined), compression:
live stress $(1+P)$ + dead stress] ÷ $(30000 - 140\frac{l}{r})$.
te posts (vertical), compression:
live stress $(1+P)$ + dead stress] ÷ $(27000 - 160\frac{l}{r})$.
s—bottom flange of riveted girder, tension:
et) = [live stress $(1+P)$ + dead stress] ÷ 24000.

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NOTATION.

l and r = length and radius of gyration of member, in ins.
P = percentage of impact for live-load stress,
in formula, $P = 10000 \div (150 + \text{length of span, in feet, or portion of span covered by live load when the member considered is subject to maximum stress.})$

35.—COMPARISON OF CARBON-STEEL AND NICKEL-STEEL SPANS, 40–200 FT. ROADWAY, 20 FEET WIDE.

Span in Feet.	Truss, Type	"Carbon Steel" Spans.					"Nickel Steel" Spans.				
		Loads Per Lin. Ft.		Total Steel.	Total Lumber	Estimated Cost, Erected and Painted	Loads Per Lin. Ft.		Total Steel.	Total Lumber	Estimated Cost, Erected and Painted
		Live. Lbs.	Dead Lbs.	Lbs.	Ft.B.M:		Live. Lbs.	Dead Lbs.	Lbs.	Ft.B.M:	
40	Pony, Pratt, P.-C.	2000	585	7120	4400	$ 585	3200	725	7730	6250	$ 810
45		1900	590	7870	4950	649	3100	730	8410	7000	892
50		1800	595	8610	5500	719	3000	735	8990	8150	983
55		1700	600	9760	6050	803	2900	740	10240	9150	1110
60		1700	615	11940	6600	936	2900	745	12590	9300	1279
65		1650	620	13320	7150	1032	2800	750	14210	10050	1420
70		1600	625	14750	7700	1131	2800	760	15780	11300	1576
75		1550	630	16100	8250	1225	2800	770	17110	12750	1724
80		1550	635	18260	8800	1357	2700	780	19450	12400	1872
80	Through, Pratt, Pin-Connected.	1550	640	19460	8800	1412	2700	790	20130	12400	1872
85		1550	645	21000	9350	1516	2700	800	21860	13750	2043
90		1550	650	21610	9900	1577	2700	810	22650	14900	2176
95		1550	655	23550	10450	1701	2700	820	24040	16100	2290
100		1550	660	25000	11000	1798	2700	830	26320	16650	2463
105		1550	665	27290	11950	1950	2700	840	28300	17950	2646
110		1550	670	30750	12000	2125	2700	850	32260	18050	2905
115		1550	680	32530	12650	2245	2700	860	33840	19450	3060
120		1550	690	34280	13200	2356	2700	870	36020	19950	3234
125		1550	700	36040	14200	2486	2700	880	38130	21500	3433
130		1500	710	41600	14300	2761	2700	890	43180	22200	3775
135		1500	720	44070	14850	2907	2700	900	45440	22900	3958
140		1500	730	45700	15400	3014	2700	910	48150	23250	4152
145		1500	740	48530	16500	3198	2700	920	51530	24950	4441
150		1500	750	53670	16500	3449	2700	930	56530	25400	4777
155		1500	760	57870	17050	3675	2700	940	59540	26250	5007
160		1500	770	60520	17600	3831	2700	950	63330	26600	5272
165		1500	780	65090	18750	4095	2700	960	66830	28400	5545
170		1500	790	68560	19300	4290	2700	970	70380	30050	5830
175		1500	815	77090	19250	4696	2700	985	79450	29150	6378
180		1500	830	80470	19800	4884	2700	1000	82690	29900	6606
185		1500	845	84280	21000	5117	2700	1020	86380	31850	6938
190		1500	860	86890	21600	5272	2700	1040	89830	33650	7231
195		1500	875	95820	21350	5689	2700	1060	100080	32200	7872
200		1500	890	101240	22000	5978	2700	1080	104440	33200	8149

V.—REINFORCED CONCRETE BRIDGES.

Design and Cost of Reinforced-Concrete Highway Bridges (By A.N. Johnson. Paper, Ill. Soc. Eng'rs and Surveyors, Jan. 26, 28, 1910; Eng. News, Feb. 10, 1910).—Summary of cost per cu. yd. of concrete is as follows:

Cement	$2.35	to	$1.25
Stone	3.23	"	1.30
Sand (excluding gravel concrete)	1.47	"	.32
Gravel	2.43	"	.70
Forms	3.95	"	.83
Steel in place	3.10	"	.80
Mixing and placing concrete	2.72	"	.72
Excavation	3.91	"	.21
Total	$17.51	"	$6.54

Spans range from 7 ft. to 60 ft., roadways mostly 16 ft., abutments mostly 10 to 13 ft. high. Extensive cost table not reproduced here.

EXCERPTS AND REFERENCES.

Bascule Bridge at Grand Ave., Milwaukee, Wis. (Eng. News, July 3, 1902).—Illustrated.

Page Bascule Bridge Over Chicago River at Ashland Ave. (Eng. News, Jan. 1, 1903.—Illustrated.

The Wabash River Bridge at Terra Haute, Ind. (By M. A. Howe. Eng. News, Mar. 8, 1906).—Illustrated.

Reinforced-Concrete Viaduct with some Structural Steel Reinforcement (Eng. News, July 1, 1909).—Illustrated: longitudinal section, tower bent, detail of floor system, detail of expansion joint. Consists of three 24-ft., five 30-ft. and two 24-ft. spans. Floor, 3-in. tar-macadam roadway 16-ft. wide, carried on a slab crowned to 9 ins. Designed for l. l. of 100 lbs. per sq. ft., with a concentrated load on the floor system of a 15-ton wagon; future provision for 30-ton car. Cost of viaduct, about $2.30 per sq. ft. of roadway.

Reinforced-Concrete and Steel Spans, Sparkman St. Bridge, Nashville (By Howard M. Jones. Eng. News, Nov. 25, 1909).—Illustrations: Half section of roadway (macadam); reinforced-concrete retaining walls; floor system; typical bent of trestle approach; reinforced-concrete staircase; details of reinforced-concrete hand railing; details of reinforced-concrete trusses; typical channel pier; stress sheet of 318-ft. truss; details of 318-ft. truss.

Illustrations and Diagrams.

Description.	Eng. News.
Truss and floor details of 194-ft. span, Waterford, N. Y.	July 14, '10
	Eng. Rec.
Steel highway bridge with concrete stringers and floor slabs	Mar. 20, '09
Short span bridges, N. Y., N. H. & H. R. R., N. Y. City crossings	July 10, '09
Short span bridges, N. Y., N. H. & H. R. R., N. Y. City Crossings	July 17, '09
Umbrella-column for supporting foot-bridge over street	Aug. 14, '09
Strain-sheet 668-ft. span—St. Louis Municipal Bridge	Oct. 30, '09
Pier type of abutment for highway bridges	Feb. 5, '10
2-span (each 36 ft.) rein.-conc. bridge; cost table	Feb. 19, '10
Overhead street bridge details; concrete-protected floorbeams	Mar. 5, '10
Rein.-conc. girder bridge (30 ft. span), arched bottom chord	Apr. 9, '10
100-ft. concrete-floor plate girder bridge	May 21, '10
Floor construction, Kensington Ave. bridge, Buffalo	Oct. 22, '10

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41.—CANTILEVER BRIDGES.

In True Cantilever Bridges the stresses are statically determinate, and to American engineers this type offers weighty advantages over the continuous-girder* types generally adopted by European engineers. Let us assume an ordinary form of cantilever as per Fig. 1, with a free central span l suspended

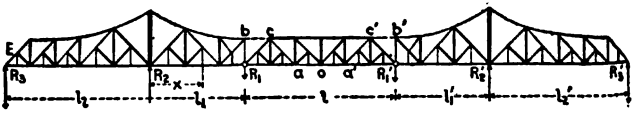

Fig. 1.

at the points b and b', introducing in the cantilever at those points the downward forces R_1 and R_1'. R_1 produces a downward reaction $R_3 = \frac{R_1 l_1}{l_2}$, and an upward reaction $R_2 = \frac{R_1 (l_1 + l_2)}{l_2}$; and similarly with R_3' and R_2'. Reaction R_3 is upward when l_2 is loaded direct, and likewise with R_3' when the loading is on l_2'. Hence to resist both kinds of reaction, upward and downward, at the *ends* of the bridge, *anchorages* as well as piers or *supports* are required (see Fig. 2); while at R_2 and R_2', *supports* only are needed.

In erection, the spans are built outward from the abutments in cantilever fashion to the central point o; hence the lower chords out to the points a and a' must be stiff members to resist compression, while bc and $b'c'$ are introduced as tension members. For any load P on span l_1 the reactions R_3 and R_2 may be obtained as above, but by using Px instead of $R_1 l_1$. Any loading on l_2 can affect only that span. The usual problem is to (1) find the loading which will give the maximum (+ and −) stresses for each member; (2) find the required reaction; and (3) solve as in ordinary trusses. For instance, the maximum compressive stress in the end post E obtains with l_2 full loaded (l_1 and l unloaded); but for maximum tensile stress in that member the reverse loading obtains, l_1 and l loaded (l_2 unloaded).

The position of loading for reactions at supports, and also for maximum stresses in members, may sometimes be studied conveniently by the use of influence diagrams. Fig. 2 is such an influence diagram, and shows the

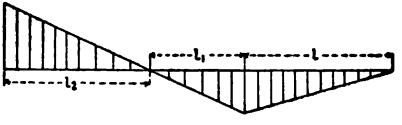

Fig. 2. Reactions R_3 for Left Support.

reaction R_3 for a load moving from the right-hand end of the central span l,

* The writer knows of but one steel truss bridge in America built on the continuous girder principle, excepting of course draw bridges. This is the O. R. & N. R'y Co.'s bridge across the Williamette river at Portland, Oregon. On the other hand, the statically indeterminate types *prevail* in Europe.

toward the left-hand anchorage, over l, l_1 and l_2. The reactions R_3, for successive positions of the concentrated load, are shown by the ordinates at the point *where the load is applied* on the structure (see Fig. 1); downward when load is on l and l_1, and upward when load is on l_2.

Carbon-steel cantilever spans are economical, generally, up to about 1600 to 1700 feet; nickel-steel spans, up to about 1900 or 2000 feet. Beyond these spans, suspension bridges become more economical. Much depends on local conditions.

The limiting length of carbon-steel cantilever spans is practically about 2000 feet; and of nickel-steel cantilever spans, about 2500 feet.

Deck Cantilever Bridges are economical in certain localities where there is sufficient head-room over the street, valley or stream to be bridged. Fig. 3 is typical. The general outline is practically an invert of Fig. 1.

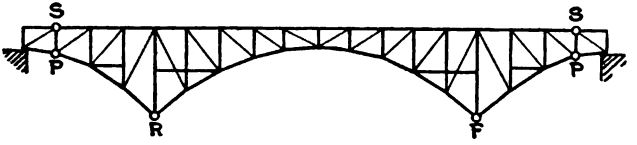

Fig. 3.

The stresses in the truss shown in Fig. 3 are rendered statically determinate by making slotted pin-holes at S, inserting pins at all lettered points, and providing a roller end at R. F is the fixed end.

Camber.—In providing for camber the simplest method, and the best for erection, is to raise the panel points by shortening the diagonals, maintaining all vertical posts mathematically parallel.

EXCERPTS AND REFERENCES.

The Mississippi River Cantilever Bridge at Thebes, Ill. (Eng. News, May 11, 1905).—Illustrated. Railway.

Quebec Cantilever Bridge Disaster and Discussions (Eng. News, Sept., 1907, to Aug., 1908).

Information about Great Cantilever Bridges (Eng. News, April 30, 1908).—Tables.

Stresses in the Blackwell's Island Cantilever Bridge (By Boller and Hodge. Eng. News, Nov. 19, 1908).—Table of calculated stresses.

Illustrations.

Description.	Eng. News.
P. & L. E. cant. bridge (769-ft. span) and foundations	May 5, '10
Outline of trusses for new Quebec bridge	Sept. 8, '10
	Eng. Rec.
Board of Engineers' design for new Quebec bridge	Sept. 10, '10

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42.—MOVABLE BRIDGES.

Movable Bridges are designed to provide temporary openings for one line of traffic (usually a waterway) which is crossed by another (usually a street or railway). There are five distinct types, as follows:

(1). Swing Bridge or "Drawbridge"—balanced and swinging horizontally round an arc (usually a quadrant) of a circle: (a) Equal spans or arms, supported by pivot pier; (b) One span, counterpoised; (c) Two spans counterpoised.

Type (1a) is discussed below.

(2). Traversing Bridge—counterpoised tail-end raised from its supports and span drawn back on rollers along one of its approaches. Used principally for narrow openings.

(3). Bascule Bridge—tail-end ballasted with cast iron and lead and span swinging vertically: (a) One arm; (b) Two arms, either with tail-ends together, or like a jack-knife with two blades hinged at either end. The bascules are raised and lowered by pinions (operated by hydraulic, electric, steam, gasoline or other power) working in segmental racks.

(4). Lift Bridge—whole span raised vertically by chains, at the four corners, suspended from towers. Counterpoised weights are used in order to reduce the required power for operating.

(5). Floating or Pontoon Bridge—iron pontoons (usually rectangular) coupled in pairs for stability and moored at all corners. Bridge is opened to river traffic by the use of running back drawbridges. This type of bridge is used where foundations for piers would be difficult and excessively expensive.

SWING BRIDGES.

Drawbridges may be either rim-bearing or center-bearing.

The *rim-bearing draw*, with each truss supported at two points on the center pier, is the more common type of the two. This is shown in Fig. 1, the center points of support for the trusses being at *a b c d*, irrespective of the diameter of circular drum or turntable. Usually the circular drum is of such diameter as to give support to the center panel of draw at 8 equi-distant or nearly equi-distant points, as shown in the figure. In such a case heavy girders, acting as cantilevers, are introduced in the chords *ab* and *cd* and in the floor-beams *ad* and *bc* in order to distribute the loads more uniformly on the table. These cantilevers may be designed sufficiently strong to carry the live and dead loads, or only sufficient to carry the dead load, in which latter case adjustable supports may be introduced at points *a b c d* to transmit the *live* loads directly from trusses to center pier when draw is closed. In either event the center pier supports are at *a b c d* so far as the calculation of trusses is concerned, and when the draw is closed each truss becomes a continuous girder of 3 spans over 4 supports, for any moving load on the draw.

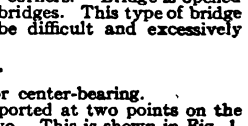

Fig. 1.

The *center-bearing draw* differs from the rim bearing in that the weight is supported at the center either (1) on a vertical, steel pivot pin, or (2) on a nest of conical rollers. The weight is transferred to the center from the trusses by means of transverse superimposed girders between the chords, a light turntable being used to "steady" the span in revolving, but not calculated to give material support. When the draw is closed each truss becomes a continuous girder of two spans over 3 supports, for any moving load on the draw; and such *live* load may be supported, if desirable, by adjustable wedges placed at the middle of draw under the trusses.

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For the *calculation of swing bridges* three cases* or conditions are usually employed in determining the stresses, as follows:

I. Each arm treated as a simple span resting on two supports, with live-, dead- and wind loads acting.

IIa. Bridge swinging and treated as a cantilever, with dead- and wind loads acting, and with reactions at center supports only.

IIb. Bridge closed and treated as a continuous girder over all supports, with live load acting. It is assumed in this case that the vertical reactions at ends of draw are ±0 when no live load is acting. The live load is considered as "balanced," that is, it advances symmetrically on *both* arms from the ends toward center of span.

For maximum stresses in the truss members use Case I, or Cases IIa and IIb combined, whichever gives the maximum. For maximum stresses in the lateral systems use each of the cases separately and select that stress which is a maximum for each member. In case there is a reversal of stress in any member, two maxima will be required, one in tension and one in compression. Some specifications make it necessary to determine minimum as well as maximum stresses, in which case it is evident that, for the trusses, Case IIb cannot be used alone.

Rim-bearing Draw—4 Supports.—For Case I with the draw considered as two simple spans, and Case IIa as a cantilever, the reactions at the supports are obtained easily and the stresses in the members are statically determinate. For Case IIb, however, it is treated as a continuous girder which takes the form of an elastic curve when loaded. The girder is *assumed* to be of homogeneous material with constant modulus of elasticity, E; constant cross-section or rather moment of inertia, I; and resting on level supports. As a matter of fact none of these exact conditions actually obtain in practice. The necessary reactions at the supports due to any loading on the continuous girder are deduced from the "theorem of three moments."

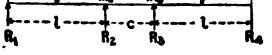

Fig. 2.

Reactions at the Supports—Continuous Girder.—Let P be any concentrated load, Fig. 2, on the left arm, distant a from the end; then will

$$R_1 = P\left(1-\frac{a}{l}\right)\left[1-\left(1+\frac{a}{l}\right)\frac{a}{l}\,\frac{\frac{2l}{c}}{4\left(\frac{l}{c}+1\right)-\frac{1}{\frac{l}{c}+1}}\right] \quad \ldots\ldots\ldots\ldots (1)$$

$$R_4 = P\left(1-\frac{a^2}{l^2}\right)\frac{a}{l}\,\frac{\frac{l}{c}}{4\left(\frac{l}{c}+1\right)^2-1} \quad \ldots\ldots\ldots\ldots (2)$$

$$R_3 = \left[R_1-R_4-P\left(1-\frac{a}{l}\right)\right]\frac{l}{c}-R_4 \quad \ldots\ldots\ldots\ldots (3)$$

$$R_2 = P-(R_1+R_4)-R_3 \text{ (algebraically)} \quad \ldots\ldots\ldots\ldots (4)$$

Of course it is clearly evident that if the load P is on the *right* arm instead of the left, and distant a from the right hand end, the resulting reactions will be interchanged—R_1 with R_4, and R_2 with R_3. Hence, for *balanced loads*, that is, the same loading in position and amount on both arms, each end-support reaction will equal R_1+R_4, and *each* center-support reaction will equal (algebraically) R_2+R_3 *for the specified load on either arm.* In drawbridge calculations P is the panel load and a the distance from end of draw to each consecutive panel point where the load acts. When the reactions are known the stresses in the structure may be solved by the ordinary methods.

**Assumptions for cheap highway drawbridges.*—Various assumptions are made for cheap swing bridges as follows: Case A. Draw swinging and treated as a cantilever, as in Case IIa above. Case B. Draw closed and end raised to support say ⅔ the dead load of one panel; and acting as a continuous girder for the live load.

It is to be noted that the length c of the center panel (over the pier) affects the values of the reactions. Preferably, c is made equal to the distance between the trusses unless this is so great as to require too large a turntable. It is also usually equal to about one panel length of truss. The following Table of reactions is based on c=one panel length of truss, and will be useful for general reference.

1.—REACTIONS R_1, R_4, R_3 AND R_2 FOR DRAWBRIDGE SPANS OF 2 TO 10 PANELS IN EACH ARM; WITH CENTRAL PANEL c EQUAL TO ONE PANEL LENGTH OF TRUSS; AND LOAD P EQUAL TO UNITY.

See Fig. 1, and Formulas (1), (2), (3), and (4).

p=One Panel Length=Unity.

No. of Panels in Each Arm l of Draw.		$\frac{p}{c}=1$	$\frac{2p}{c}=2$	$\frac{3p}{c}=3$	$\frac{4p}{c}=4$	$\frac{5p}{c}=5$	$\frac{6p}{c}=6$	$\frac{7p}{c}=7$	$\frac{8p}{c}=8$	$\frac{9p}{c}=9$
10	$R_1=$	+0.855	+0.713	+0.576	+0.447	+0.329	+0.225	+0.137	+0.069	+0.022
	$R_4=$	+0.002	+0.004	+0.006	+0.007	+0.008	+0.008	+0.007	+0.006	+0.004
	$R_3=$	−0.473	−0.919	−1.307	−1.607	−1.794	−1.836	−1.707	−1.378	−0.818
	$R_2=$	+0.616	+1.202	+1.725	+2.153	+2.457	+2.603	+2.563	+2.303	+1.792
9	$R_1=$	+0.839	+0.682	+0.533	+0.395	+0.271	+0.166	+0.084	+0.027	
	$R_4=$	+0.002	+0.005	+0.007	+0.008	+0.009	+0.008	+0.007	+0.004	
	$R_3=$	−0.470	−0.905	−1.270	−1.531	−1.646	−1.588	−1.316	−0.799	
	$R_2=$	+0.629	+1.218	+1.730	+2.128	+2.366	+2.414	+2.225	+1.768	
8	$R_1=$	+0.820	+0.646	+0.481	+0.333	+0.205	+0.104	+0.034		
	$R_4=$	+0.003	+0.006	+0.008	+0.010	+0.009	+0.008	+0.005		
	$R_3=$	−0.466	−0.888	−1.222	−1.425	−1.444	−1.243	−0.777		
	$R_2=$	+0.643	+1.236	+1.733	+2.082	+2.230	+2.131	+1.738		
7	$R_1=$	+0.796	+0.599	+0.418	+0.260	+0.132	+0.043			
	$R_4=$	+0.004	+0.007	+0.010	+0.011	+0.010	+0.006			
	$R_3=$	−0.460	−0.864	−1.152	−1.268	−1.152	−0.749			
	$R_2=$	+0.660	+1.258	+1.724	+1.997	+2.010	+1.700			
6	$R_1=$	+0.764	+0.539	+0.338	+0.174	+0.057				
	$R_4=$	+0.005	+0.009	+0.012	+0.011	+0.008				
	$R_3=$	−0.454	−0.830	−1.050	−1.037	−0.713				
	$R_2=$	+0.685	+1.282	+1.700	+1.852	+1.648				
5	$R_1=$	+0.719	+0.459	+0.239	+0.079					
	$R_4=$	+0.007	+0.012	+0.013	+0.010					
	$R_3=$	−0.443	−0.775	−0.886	−0.665					
	$R_2=$	+0.717	+1.304	+1.634	+1.576					
4	$R_1=$	+0.655	+0.349	+0.118						
	$R_4=$	+0.009	+0.015	+0.013						
	$R_3=$	−0.426	−0.682	−0.593						
	$R_2=$	+0.762	+1.318	+1.462						
3	$R_1=$	+0.554	+0.192							
	$R_4=$	+0.014	+0.018							
	$R_3=$	−0.395	−0.494							
	$R_2=$	+0.827	+1.284							
2	$R_1=$	+0.371								
	$R_4=$	+0.021								
	$R_3=$	−0.321								
	$R_2=$	+0.929								

Note—The reactions given in the table are for loads P=1 at panel points. Hence, to find the actual reactions, multiply values in table by actual loads P at panel points.

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2.—Practical Data for Drawbridge Calculation, 4 Supports. Case IIb.

Reactions and Moments for Balanced Loads.

Reactions are in terms of unit panel load. Moments are at foot of Tower Posts, and are for Unit Panel loads and Unit Panel Lengths. The Loads are considered as extending from the ends toward the center.

No. of Panels in Each Arm l of Draw.		A (1)	$A\ B$ (2)	$A\ B\ C$ (3)	A to D (4)	A to E (5)	A to F (6)	A to G (7)	A to H (8)	A to I (9)
10	$R_1 = R_4 =$	+0.857	+1.574	+2.156	+2.610	+2.947	+3.180	+3.324	+3.399	+3.425
	$R_2 = R_3 =$	+0.143	+0.426	+0.844	+1.390	+2.053	+2.820	+3.676	+4.601	+5.575
	$M_2 = M_3 =$	−0.43	−1.26	−2.44	−3.90	−5.53	−7.20	−8.76	−10.01	−10.75
9	$R_1 = R_4 =$	+0.841	+1.528	+2.068	+2.471	+2.751	+2.925	+3.016	+3.047	
	$R_2 = R_3 =$	+0.159	+0.472	+0.932	+1.529	+2.249	+3.075	+3.984	+4.953	
	$M_2 = M_3 =$	−0.43	−1.25	−2.39	−3.76	−5.24	−6.68	−7.86	−8.58	
8	$R_1 = R_4 =$	+0.823	+1.475	+1.964	+2.307	+2.521	+2.633	+2.672		
	$R_2 = R_3 =$	+0.177	+0.525	+1.036	+1.693	+2.479	+3.367	+4.328		
	$M_2 = M_3 =$	−0.42	−1.20	−2.29	−3.54	−4.83	−5.94	−6.62		
7	$R_1 = R_4 =$	+0.800	+1.406	+1.834	+2.105	+2.247	+2.296			
	$R_2 = R_3 =$	+0.200	+0.594	+1.166	+1.895	+2.753	+3.704			
	$M_2 = M_3 =$	−0.40	−1.16	−2.16	−3.27	−4.27	−4.93			
6	$R_1 = R_4 =$	+0.769	+1.317	+1.667	+1.852	+1.917				
	$R_2 = R_3 =$	+0.231	+0.683	+1.333	+2.148	+3.083				
	$M_2 = M_3 =$	−0.39	−1.10	−2.00	−2.89	−3.50				
5	$R_1 = R_4 =$	+0.726	+1.197	+1.449	+1.538					
	$R_2 = R_3 =$	+0.274	+0.803	+1.551	+2.462					
	$M_2 = M_3 =$	−0.37	−1.02	−1.76	−2.31					
4	$R_1 = R_4 =$	+0.664	+1.028	+1.159						
	$R_2 = R_3 =$	+0.336	+0.972	+1.841						
	$M_2 = M_3 =$	−0.34	−0.89	−1.36						
3	$R_1 = R_4 =$	+0.568	+0.778							
	$R_2 = R_3 =$	+0.432	+1.222							
	$M_2 = M_3 =$	−0.30	−0.67							
2	$R_1 = R_4 =$	+0.392								
	$R_2 = R_3 =$	+0.608								
	$M_2 = M_3 =$	−0.22								

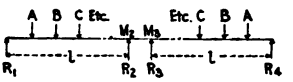

Fig. 3.

Example.—For $l = 9$ (panels), and with Loads at A, B and C on each arm, the end reactions are 2.068 times a panel load; and center reactions, 0.932. Moments M_2 and M_3 are −2.39 times a panel load times a panel length.

Calculation of 315-Ft. Draw. (Fig. 4, page 746.)

Figs. 5, 6, 7 and 8 are stress diagrams.

Fig. 5 (Case I) shows one arm treated as a simple span, fully loaded. Similar diagrams must be drawn for live load retreating, panel by panel, from end of span toward center.

Fig. 6 (Case IIa) is a dead-load stress-diagram of one arm when both arms are swinging clear of end supports.

Fig. 7 (Case IIb) is a stress-diagram for full live load when draw is closed; the condition being that ends of arms are simply touching supports under the dead load only. Similar diagrams must be drawn for live load symmetrically retreating, panel by panel, from the pier ends of arms.

Fig. 8 (Case IIb) is a stress-diagram of a retreating load last mentioned—with symmetrical loads at panel points A, B, C and D, for maximum stress in member 17.

Wind-load stress diagram can similarly be drawn.

See, also, page 742.

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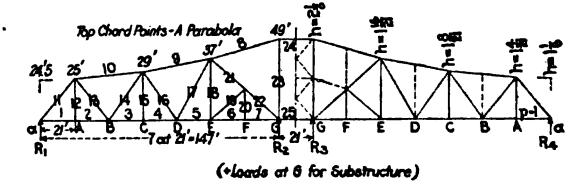

Fig. 4. 315-Ft. Draw.

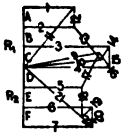

Fig. 5.

CASE I—SIMPLE SPAN—
Full Loaded—Live or Dead.

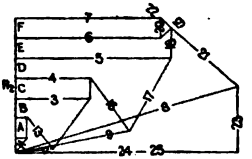

Fig. 6.

CASE IIa—DRAW SWINGING—
Cantilever—Dead Load only.

(Stress in $24\text{-}25 = \frac{M}{h} = 7\frac{1}{2} \div 2\frac{1}{2} = 11.$)

Load at end, a, is $\frac{3}{4}$ of Panel Load.

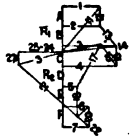

Fig. 7.

CASE IIb—CONTINUOUS SPAN—
Full Loaded—Live Load.
$R_1 = 2.296$; $R_2 = 3.704$;
$M_2 = 4.93 \therefore$ Stress in 24-25 =
$4.93 \div 2\frac{1}{2}$
$= 2.11.$ (See Table 2.)

Fig. 8.

CASE IIb—CONTINUOUS SPAN—
Loaded (A, B, C, D, both arms) for
Max. Live Load in 17.
$R_1 = 2.105$ (See Table 2).

Center-bearing Draw—3 Supports.—This differs from the rim-bearing draw in having one support at the center pier instead of two. Hence, Case IIb (the only variation from the preceding illustrations) will be treated as a continuous girder of *two* equal spans over three supports.

Reactions at the Supports—Continuous girder.—Let P be any concentrated load, Fig. 9, on the left arm, distant a from the end; then will

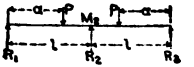

Fig. 9.

$$M_2 = -\frac{Pa}{4}\left[1 - \frac{a^2}{l^2}\right] \qquad [= R_3 l.] \qquad (5)$$

$$R_3 = -\frac{Pa}{4l}\left[1 - \frac{a^2}{l^2}\right] \qquad \left[= \frac{M_2}{l}\right] \qquad (6)$$

$$R_2 = +\frac{Pa}{l} + \frac{Pa}{2l}\left[1 - \frac{a^2}{l^2}\right] \qquad \left[= \frac{Pa}{l} - 2R_3 \text{ (algebraically)}\right] \qquad (7)$$

$$R_1 = +P\left(1 - \frac{a}{l}\right) - \frac{Pa}{4l}\left(1 - \frac{a^2}{l^2}\right) \qquad \left[= P\left(1 - \frac{a}{l}\right) + R_3 \text{ (algeb'y)}\right] \qquad (8)$$

The right hand half of the following table is all that is necessary in Case IIb of drawbridge calculations, the first half being interesting as a study of the continuous girder of two equal spans for any concentrated loading. It is to be noted that for loading on right-hand arm, R_1 and R_3 will be interchanged.

3.—Practical Data for Drawbridge Calculation, 3 Supports. Case IIb.

Reactions and Moments for Balanced Loads.

Reactions are in Terms of Unit Panel Load. Moments are at Center Support and are for Unit Panel Loads and a (Respective Distance from End Support). Balanced loads are Symmetrical Loads, Both Arms.

Concentrated Loads on Left Arm, Distant a From End.					Balanced Concentrated Loads on Both Arms, Distant a From Ends.			
$\frac{a}{l}$	M_2	R_3	R_2	R_1	$\frac{a}{l}$	M_2	$R_1=R_3$	Double Shear. R_2
.04	$-0.2496\ a$	-0.009984	0.059968	0.950016	.04	$-0.499\ a$	0.940	0.120
.08	$-0.2484\ a$	-0.019872	0.119744	0.900128	.08	$-0.497\ a$	0.880	0.239
.12	$-0.2464\ a$	-0.029568	0.179136	0.850432	.12	$-0.493\ a$	0.821	0.358
.16	$-0.2436\ a$	-0.038976	0.237952	0.801024	.16	$-0.487\ a$	0.762	0.476
.20	$-0.2400\ a$	-0.048000	0.296000	0.752000	.20	$-0.480\ a$	0.704	0.592
.24	$-0.2356\ a$	-0.056544	0.353088	0.703456	.24	$-0.471\ a$	0.647	0.706
.28	$-0.2304\ a$	-0.064512	0.409024	0.655488	.28	$-0.461\ a$	0.591	0.818
.32	$-0.2244\ a$	-0.071808	0.463616	0.608192	.32	$-0.449\ a$	0.536	0.927
.36	$-0.2176\ a$	-0.078336	0.516672	0.561664	.36	$-0.435\ a$	0.483	1.033
.40	$-0.2100\ a$	-0.084000	0.568000	0.516000	.40	$-0.420\ a$	0.432	1.136
.44	$-0.2016\ a$	-0.088704	0.617408	0.471296	.44	$-0.403\ a$	0.383	1.235
.48	$-0.1924\ a$	-0.092352	0.664704	0.427648	.48	$-0.385\ a$	0.335	1.329
.52	$-0.1824\ a$	-0.094848	0.709696	0.385152	.52	$-0.365\ a$	0.290	1.419
.56	$-0.1716\ a$	-0.096096	0.752192	0.343904	.56	$-0.343\ a$	0.248	1.504
.60	$-0.1600\ a$	-0.096000	0.792000	0.304000	.60	$-0.320\ a$	0.208	1.584
.64	$-0.1476\ a$	-0.094464	0.828928	0.265536	.64	$-0.295\ a$	0.171	1.658
.68	$-0.1344\ a$	-0.091392	0.862784	0.228608	.68	$-0.269\ a$	0.137	×1.726
.72	$-0.1204\ a$	-0.086688	0.893376	0.193312	.72	$-0.241\ a$	0.107	1.787
.76	$-0.1056\ a$	-0.080256	0.920512	0.159744	.76	$-0.211\ a$	0.079	1.841
.80	$-0.0900\ a$	-0.072000	0.944000	0.128000	.80	$-0.180\ a$	0.056	1.888
.84	$-0.0736\ a$	-0.061824	0.963648	0.098176	.84	$-0.147\ a$	0.036	1.927
.88	$-0.0564\ a$	-0.049632	0.979264	0.070368	.88	$-0.113\ a$	0.021	1.959
.92	$-0.0384\ a$	-0.035328	0.990656	0.044672	.92	$-0.077\ a$	0.009	1.981
.96	$-0.0196\ a$	-0.018816	0.997632	0.021184	.96	$-0.039\ a$	0.002	1.995

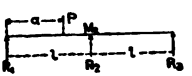

Fig. 10.

Example.—Solve for one load $P=16000$ lbs. on left arm; $l=45$ ft.; $a=17$ ft.

Solution.—$\frac{a}{l}=\frac{17}{45}=.378$; $M_2=$

$-.214\times a\times P\ =-.214\times 17\times$ $16{,}000=-58{,}200$ ft.-lbs.; $R_1=.542\times 16{,}000=8{,}672$ lbs.; $R_2=.539\times 16{,}000=8{,}624$ lbs.; $R_3=-.081\times 16{,}000=-1{,}296$ lbs. (R_3 acting downward.)

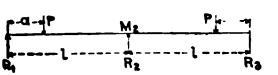

Fig. 11.

Example.—Solve for load $P=16{,}000$ lbs. on each arm; $l=45$ ft.; $a=17$ ft.

Solution.—$M_2=-.428\times 17\times 16{,}000=-116{,}400$ ft.-lbs.; $R_1=R_3=.46\times 16{,}000=7{,}360$ lbs.; $R_2=1.08\times 16{,}000=17{,}280$ lbs. (All reactions acting upward.)

Note.—A minus bending moment ($-M_2$) means tension in top flange or chord, and compression in bottom.

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DECK DRAWBRIDGE—CENTER BEARING.

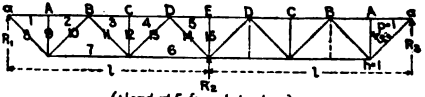

Fig. 12.

Hints for Calculation of Trusses.—In Fig. 12, let P = panel load per truss at A, B, C, D and E; and let $\frac{3}{2}P$ = dead load at a when draw is swinging.

Case I—Simple Span.—Ends at a are raised so there is no stress in DE. For dead load and full live load the shear in panels $a-A$ and $D-E$ are equal. Maximum compressive stress in 10 occurs with live loads at B, C and D.

Case II—Cantilever.—Draw swinging, assuming $\frac{3}{2}$ panel load at a and full panel load at other points. Shear at $D-E$ which is used for R_2 in this case $= 4\frac{1}{2}P$. Stress in $D-E = 13\frac{1}{2}P \times \frac{p}{h} = 13\frac{1}{2}P$.

Case III—Continuous Girder.—Draw closed and just touching supports, with no live load. Balanced live loads are now applied on both arms, extending from ends toward center of draw. The reactions and moments for this loading may be obtained from the preceding table, as it now becomes a continuous girder of two equal spans over three "level" supports.

For loads at A, $\frac{a}{l} = .20$; $R_1 = .704$; $\frac{1}{2}R_2 = .296$; $M_2 = -0.48a = -0.48p$.

For loads at B, $\frac{a}{l} = .40$; $R_1 = .432$; $\frac{1}{2}R_2 = .568$; $M_2 = -0.42a = -0.84p$.

For loads at C, $\frac{a}{l} = .60$; $R_1 = .208$; $\frac{1}{2}R_2 = .792$; $M_2 = -0.32a = -0.96p$.

For loads at D, $\frac{a}{l} = .80$; $R_1 = .056$; $\frac{1}{2}R_2 = .944$; $M_2 = -0.18a = -0.72p$.

For loads at A, B, C, D, $R_1 = 1.400$; $\frac{1}{2}R_2 = 2.600$; $^*M_2 = \quad = -3.00p$.

Note that all calculations can be made for any loading when R_1 is known, for we have only to apply the methods of moments and shears from the *outer forces* acting to the left of the section considered, to obtain the stress in any member.

WEIGHT OF STEEL IN MOVABLE BRIDGES.

Steel Swing Bridges.—The following formula is for single track standard R. R. bridges calculated for live load of two 160-ton engines followed by a uniform load of 4600 lbs. per lineal foot: Total weight of steel in lbs. $= 7.8L^2 + 12(L \times \sqrt{L}) + 100L + 20000$; in which L = extreme length of draw in feet. For double track bridges, multiply results obtained in above formula by 1.85.

Counterweight Jack-knife Draw.—In Fig. 13, let l = the span HE, hinged at H and connected at D_1 (distant l_1 from H) by the chain $C + C_1$ running over the pulley P, with the cylindrical counterweight W_1, suspended at the other end and rolling along the modified cycloidal plane AP. Assume the total weight of the movable leaf to be W acting at the point G. Then we have the following;

Weight of counterweight $W_1 = \frac{Wl\sqrt{2}}{2l_1}$;

Length of chain $C_1 = l_1\sqrt{2}\left(1 + \sin\frac{\theta}{2} - \cos\frac{\theta}{2}\right)$.

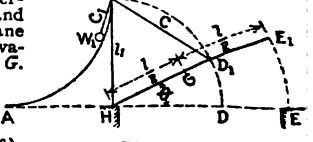

Fig. 13.

*Hence, for full loading, both arms, stress in $D-E = \Sigma M_2 : h = -3\frac{p}{h} = -3 \times 1 = -3$ (panel loads).

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ule Bridges.—The weight of double track bascule bridges for 120 ft. varies from about 2000 to 6000 lbs. per lin. ft. of bridge, on the type and loading. The counterweights often weigh as aves themselves.

EXCERPTS AND REFERENCES.

ridge Over the Chicago River at Clybourne Place (Eng. , 1901).—Illustrated. Highway.

k Two-Truss Swing Bridge, Chicago & Western Indiana ews, Sept. 12, 1901).—Illustrated.

ickinnic River Drawbridge, C. & N.-W. Ry. (Eng. News, —Illustrated.

e of Drawbridge (By Ward Baldwin. Eng. News, Oct. 22, ated. Plate-girder deck structure of 64-ft. span, the swing- rted on a rack circle, laid on a pile foundation, at bottom of

Caisson and Operating Machinery for a Heavy Swing News, Jan. 7, 1904). Illustrated. Machinery calculations.

(26-ft,) Bascule Bridge on the P., Ft. W. & Chi. Ry. (By Eng. News, May 25, 1905).—Illustrated. Double-track,

Rail-Lifts on the Thoroughfare Draw Near Atlantic City, ws, Nov. 15, 1906).—Illustrated.

Bearing Drawbridge with Pneumatic Raising Jack and e (Eng. News, April 4, 1907).—Illustrated. Constructed in many.

End-Lift Device of a German Swingbridge (Eng. News, —Illustrated.

y Lift Bridge (By Eric Swensson. Eng. News, April 2, ated.

or Floating Drawbridge (Eng. News, April, 30, 1908).—

idges Over the East Chicago River (Eng. News, Mar. 18,

Illustrations of Movable Bridges and Details.

Description.	Eng. News.
ift railway span, piers, fender, etc	Apr. 21, '10
nection for drawbridges	Sept. 1, '10
l live ring of drawbridge	Oct. 6, '10
rntable bearing, concrete ball race	Oct. 6, '10
	Eng. Rec.
and operating mechanism of bascule bridge	Mar. 27, '09
for a Scherzer rolling lift bridge	Aug. 21, '09
y pivot pier for 283-ft. drawbridge, Boston	Apr. 16, '10
t. span lift bridge, N. Y., N. H. & H. R. R.	Aug. 20, '10

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43.—SUSPENSION BRIDGES.

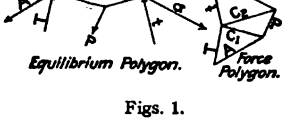
THEORETICAL CONSIDERATIONS.

Curve of Main Cables.—The curve assumed by the main cables of a suspension bridge when the latter is uniformly loaded, is called a *modified* or *transformed catenary*. This curve lies somewhere between the *catenary* and the *parabola*. When the cables are first suspended between the bridge towers (before they support any extraneous load) they form a true *catenary*, due to the weight of the cables alone, assuming of course that they have a uniform weight per lineal foot of cable. But later, when the floor of the bridge is in position, that is, suspended from the main cables, the curve of the cables is "modified" and tends to *approach* the parabola in form. The true parabola, however, is hardly ever realized, the ideal condition for this curve obtaining only when the *horizontal* combined loading per lineal foot on the cables (including weight of cables themselves) is *uniform*.

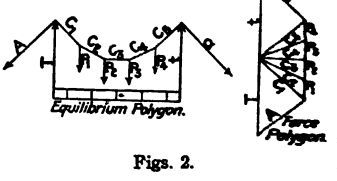
For Short Spans, where weight of cables is small compared with weight of floor, we may assume for all practical purposes that the main cables take the form of a parabola; also bearing in mind that on this assumption the error decreases as the ratio of central deflection to length of span decreases. Indeed, even the arc of a circle may be used in the drafting room and also for the purpose of making estimates, when the central deflection is only $\frac{1}{12}$ or even $\frac{1}{10}$ the span. See also Tables 1 and 2.

In the above discussion of the modified catenary and the parabola it is assumed that the suspenders transmitting the loads to the cables are vertical, uniformly spaced horizontally, and *very close* together, so that the cables form true and continuous curves throughout, between points of supports. But in actual practice the suspenders are not always vertical and are usually spaced quite far apart, tending to further modify these curves.

Force Polygons.—The following force polygons assume that the cables or chords of the equilibrium polygon have no weight, being acted upon by outside forces only.



Figs. 1.



Figs. 2.

The Parabolic Cable.—Horizontal Uniform Load.—Let w = uniform load, in lbs., per lineal foot of bridge; l = span, in feet; c = half span; s = length of cable between towers; θ = angle of inclination of tangent at any point p whose coordinates are x and y; d = center deflection of cable; constant H = horizontal component of tension (lbs.) of cable at any point p. Then with vertical suspenders, we have:

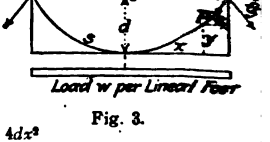


Fig. 3.

General equation, $$y = \frac{wx^2}{2H} = \frac{4dx^2}{l^2} \quad \ldots\ldots\ldots\ldots (1)$$

Horizontal tension (when $x = c$, $y = d$), $$H = \frac{wl^2}{8d} = \frac{wl \cdot l}{8d} \quad \ldots\ldots\ldots\ldots (2)$$

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At any point p, $\tan\theta = \frac{wx}{H} = \frac{8dx}{l^2}$ .....(3)

At top of towers, $\tan\theta_1 = \frac{4d}{l}$ .....(4)

Length of cable, $s = \frac{c}{m}\sqrt{m^2+c^2} + 2.302585m^*\log\left(\frac{c+\sqrt{m^2+c^2}}{m}\right)$ .....(5)

in which $m = \frac{H}{w} = \frac{l^2}{8d}$; $c = \frac{l}{2}$; $\frac{c}{m} = \frac{4d}{l} = \tan\theta_1$.

Tension in cables at any point $p = H\sec\theta = \frac{wl^2}{8d}\sec\theta$ .....(6)

Tension in cables at top of towers $= \frac{wl}{8d}\sqrt{l^2+16d^2}$ .....(7)

Tension in cables at top of towers is greater than at points between, being the least at point of greatest deflection. Hence by the use of equation (7) we may determine the size of cables required; and their weight per lineal foot multiplied by the value of s in equation (5) will give the weight of suspended cable between towers.

For mid-span, equation (6) reduces to $H = \frac{wl^2}{8d}$ .....(8)

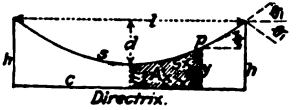

Fig. 4.

The Catenarian Cable—Load Uniform Along Cable.—In Fig. 4 let
l = span, in feet;
c = half span;
d = center deflection;
s = length of suspended cable between towers;
w_1 = weight of cables per lin. ft. of s;
θ = angle of inclination of tangent at point p whose coordinates are x and y;
H = constant horizontal component of tension (lbs.) in cable at any point p:
h = maximum value of ordinate $y = d + m$; $m = \frac{H}{w_1} = \frac{A}{s}$;
A_x = area of shaded portion of length x, above directrix,
$A = m\,s$ = total area for length l, between catenary and directrix;
† e = 2.7182818 = base of Naperian system of logarithms.

Then for equation of the catenary, we have:

General equation, $y = \frac{m}{2}\left(e^{\frac{x}{m}} + e^{-\frac{x}{m}}\right)$ .....(1)

where $m = \frac{A}{s} = \frac{H}{w_1} = \frac{\text{horizontal tension}}{\text{weight per lin. ft.}}$ of cable.....(1a)

But as A, s and H are functions of m itself we have to resort to other methods to find the value of m. Thus:

The approximate value of $m = \frac{x^2}{2(y-m)} = \frac{c^2}{2d} = \frac{l^2}{8d}$ .....(1b)

which may be substituted for value of m in the second member of equation (1c) to obtain its more nearly correct value.

Exact value of $m = \dfrac{0.4342945\,c}{\log\left(\frac{d+m}{m} + \sqrt{\left(\frac{d+m}{m}\right)^2 - 1}\right)}$ .....(1c)

---

*Common logarithm. $2.302585 = \frac{1}{\log e}$; in which e is the Neparian base = 2.7182818. Log 2.302585 is 0.3622157.

† Log e = 0.4342945; log (log e) = 9.6377843 − 10. Also note that $e^{-\frac{x}{m}} = \frac{1}{e^{\frac{x}{m}}}$, in the general equation of the catenary.

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when the *exact* value of m is substituted in the second member; but when m from equation (1b) is substituted, the result from (1c) is *too small* by about its excess over that obtained from equation (1b) alone. In the following table, column c_1 is obtained by adding this excess, i. e., the difference between values in columns b and c, to the values in column c. Column "diff." is a difference column, omitting the decimal point, between c_1 and e, the latter containing exact values of m; and is useful in making corrections to values in column c_1 obtained by the approximate method as above described, for any values of $\frac{d}{l}$.

1.—VALUES OF PARAMETER m FOR THE CATENARY, BASED ON l = UNITY.

For successive values of $\frac{d}{l}$.

(Multiply tabular values of m, below, by length of span l.)

a	b	c	c_1	Diff	e	a	b	c	c_1	Diff	e
Values of $d+l$	From (1b) $\frac{l^2+8d}{l}$ (approx.)	From (1b) and (1c) (approx.)	From $\frac{2c-b}{2}$ (approx.)	Excess of c_1	Exact values of m	Values of $d+l$	From (1b) $\frac{l^2+8d}{l}$ (approx.)	From (1b) and (1c) (approx.)	From $\frac{2c-b}{2}$ (approx.)	Excess of c_1	Exact Values of m.
.01	12.5	12.50084	12.50168	1	12.50167	.11	1.13636	1.14541	1.15446	22	1.15424
.02	6.25	6.25166	6.25333	0	6.25333	.12	1.04167	1.05151	1.06135	27	1.06108
.03	4.16667	4.16917	4.17167	1	4.17166	.13	.96154	.96217	.98280	33	.98247
.04	3.125	3.12833	3.13166	1	3.13165	.14	.89286	.90427	.91569	41	.91528
.05	2.5	2.50415	2.50830	2	2.50828	.15	.83333	.84553	.85773	50	.85723
.06	2.08333	2.08832	2.09330	4	2.09326	.16	.78125	.79422	.80719	60	.80659
.07	1.78571	1.79152	1.79732	6	1.79726	.17	.73529	.74902	.76275	70	.76205
.08	1.5625	1.56912	1.57574	9	1.57565	.18	.69444	.70894	.72344	82	.72262
.09	1.38889	1.39632	1.40375	11	1.40364	.19	.65789	.67312	.68835	94	.68741
.10	1.25	1.25824	1.26648	16	1.26632	.20	.625	.64097	.65694	107	.65587

Remarks.—To find values of m (column e) for values of $\frac{d}{l}$ intermediate to those in the table: Calculate for column c_1 and subtract the interpolated difference in column "diff."

Length of arc $s_x = \frac{m}{2}\left(e^{\frac{x}{m}} - e^{-\frac{x}{m}}\right)$* $= m \tan \theta$ ..... (2)

Substituting c for x in above equation, we have, when towers are of equal height,

Total length of cable $s = m\left(e^{\frac{c}{m}} - e^{-\frac{c}{m}}\right) = 2\,m \tan\theta_1$ ..... (3)

2.—LENGTHS s OF CABLE IN THE CATENARY, BASED ON l = UNITY.

For successive values of $\frac{d}{l}$.

(Multiply tabular values of s, below, by length of span l.)

$\frac{d}{l}$	s	$\frac{d}{l}$	s	$\frac{d}{l}$	s
.01	1.000267	.06	1.009537	.11	1.031569
.02	1.001066	.07	1.012949	.12	1.037421
.03	1.002396	.08	1.016868	.13	1.043729
.04	1.004254	.09	1.021283	.14	1.050484
.05	1.006636	.10	1.026187	.15	1.057674

*See foot-note, following page.

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rve at point p (Fig. 4) is: $\tan\theta = \frac{1}{2}\left(e^{\frac{x}{m}} - e^{-\frac{x}{m}}\right) = \frac{s_x}{m}$ .. (4)

rve at top of tower is: $\tan\theta_1 = \frac{1}{2}\left(e^{\frac{c}{m}} - e^{-\frac{c}{m}}\right) = \frac{s}{2m}$ ...(4a)

ween catenary and directrix (equa. 1a): $A = ms = \frac{sH}{w_1}$ ....(5)

nponent of tension in cable: $H = \frac{Aw_1}{s} = m\,w_1$ .....(6)

tial tension at any point $p = H \sec\theta = H\sqrt{1+\left(\frac{s_x}{m}\right)^2}$ .....(7)

tial tension at top of tower $= H \sec\theta_1 = H\sqrt{1+\left(\frac{s}{2m}\right)^2}$ ...(8)

r *Skeleton Arch*, acted upon by gravity, if constructed on the atenary (inverted), will possess maximum stability because the nt pressure throughout the arch will coincide with the center h ring.

Solution of the Catenary.—Fig. 5 is a graphical representation of the catenary. The theory is correct provided the load line number of divisions—the more divisions the greater accuracy. t 1–6 (Fig. 5) represent the load line half cable (from tower to point of tion) or $\frac{w_1 s}{2}$. Lay off H†, from 6 to 0, horizontal component of tension in vill θ_1 be the angle which the direction s with the horizontal at top of tower; $\frac{s}{H} = \frac{s}{2m}$. Divide the load line into equal parts, the more the better for en from this force polygon the equi- n may be drawn: Draw 1′, equal in ength of cable weighing $\frac{1}{2}$ *part*, paral- w 2′, equal in length to the length of one *part*, parallel to 2–0; similarly equal to 2′, draw 3′, parallel to 3–0; 4–0; 5′, parallel to 5–0; and lastly, draw 6′, equal in length to 6–0. Then the curve $1-E'$, shown imperfectly dotted, urve.

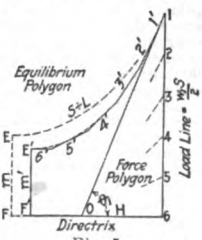

Fig. 5.

Now instead of dividing the load line into five parts as above, ible to divide it into an infinite number of parts, we should a force polygon the true catenary curve $1-E$ for the equili- ; and if $1-E$ represents the half length $\left(\frac{s}{2}\right)$ of the cable s, the middle ordinate m will represent the length of cable $w_1 m$ will be equal to the tension H, at E.

rom the construction of Fig. 5 it is evident that the stress in is 2–0; in 3′ is 3–0; etc.

f the weight w_1 of the cable is assumed as unity then $1-E$ in length, and m will equal H.

ormed Catenary.—This curve applies as well to the arch as ion bridge (see Sec. 44, Arches). The general equation of the atenary is

$$y = \frac{a}{2}\left(e^{\frac{x}{m}} + e^{-\frac{x}{m}}\right) \quad \text{.....(9)}$$

opment, $e^{\frac{x}{m}} = 1 + \frac{x}{m} + \frac{x^2}{2m^2} + \frac{x^3}{2\times 3m^3} + \frac{x_4}{2\times 3\times 4m^4} \times$ - - - - - -;

$e^{-\frac{x}{m}} = 1 - \frac{x}{m} + \frac{x^2}{2m^2} - \frac{x^3}{2\times 3m^3} + \frac{x^4}{2\times 3\times 4m^4}$ - - - - - - - - - - -.

page.

exactly, $= \frac{w_1 l^2}{8d}\left(1 + 0.16\frac{d}{l}\right)$, very nearly, $= \frac{w_1 l^2}{8d}$, approx.

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Comparing equations (1) and (2) we note that any ordinate y of the transformed catenary $= \frac{a}{m}$ times y of the true catenary, for constant values of x and m. In fact the catenary is a special case of the transformed catenary where $a = m$, just as the circle is a special case of the ellipse where the semi axes a and b become equal to each other and are called r, the radius of circle.

In Fig. 6, let $A\ B\ C\ D$ be the diagram of a catenary, similar to one-half of Fig. 4. The curve $A\ B$ is that due to the weight w_1 per lineal foot of the cable itself. Now if there is an additional weight w' per *horizontal* lineal foot imposed upon the cable the latter will assume the exaggerated form AB', the middle ordinate will become a, and the directrix, FE. Let s_x be the length of curve from A to any point p', whose coordinates are x and y. Then will the load on s_x be proportional to the area $Ap'GF$, in the same way that the load on the arc Ap of the true catenary is proportional to the area $ApkD$.

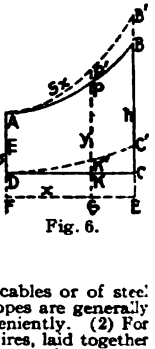

Fig. 6.

PRACTICAL HINTS.

Cables or Chains.—These may be composed of wire cables or of steel eye-bars, as follows: (1) For short spans, twisted wire ropes are generally preferred as they can be manufactured and handled conveniently. (2) For spans of moderate lengths, strands composed of parallel wires, laid together near the bridge site and then hoisted into position, possess an advantage in economy of material over twisted strands, which latter develop only about 90% of the strength of straight wires. (3) For very long spans, the cables are made up of parallel wires as in the second case, but they are built *in place* as it would be impossible to raise them bodily on account of their great weight. (4) Steel eye-bars may be used instead of wire cables, as at first proposed* for the Manhattan suspension bridge over the East River, New York. This bridge to be composed of a central span of 1,470 ft., and two end spans of 725 ft. each. The four main chains to lie in vertical planes 20 and 48 ft. distant from the axis of the bridge. The eye-bars to be of nickel steel, 3¼ to 3½% nickel; not over 0.05% sulphur; not over 0.06% phosphorus if made by acid O-H process, and not over 0.04% phosphorus if made by basic O-H process. The required ultimate strength was 85000 lbs. per sq. in.; actual elastic limit, 48000; percentage of elongation in 18 ft., 9; percentage of reduction at fracture, 40. Among the advantages claimed for the eye-bar cables are: They offer better connections for the vertical suspenders; they are better adapted to form integral parts of the stiffening trusses to equalize moving loads on the bridge; and they can be proportioned economically with varying cross-section to the exact stresses in various parts of the cables, whereas wire cables must have a uniform cross-section. (For revised plans and specifications, see pages 756, etc.)

The parabolic curve may be used in making up preliminary estimates, and also for designing small suspension spans in general. See Fig. 3 and equations (1) to (8).

Example.—What will be the tension in each cable at top of tower, at mid span, and at quarter span, for a clear span of 600 ft., and a center deflection of 50 ft.; assuming total load at 8000 lbs. per lin. ft. of bridge, and supported by two cables?

Solution.—From the preceding equations:

At towers, each cable, equation (7), $t = \frac{1}{2} \cdot \frac{wl}{8d}\sqrt{l^2 + 16d^2} = 3{,}795{,}000$ lbs.

At mid span, each cable, equation (8), $t = \frac{1}{2} \cdot H = \frac{1}{2} \cdot \frac{wl^2}{8d} = 3{,}600{,}000$ lbs.

At quarter span, each cable, equa's (6)(3), $t = \frac{1}{2} \cdot H\sqrt{1 + \tan^2 \theta} = 3{,}650{,}000$ lbs

Towers and Backstays.—Provision for expansion and contraction of the cables may be made at the towers in several ways:

(1) One of the most common methods is to sling the cables over the main piers on Saddles resting on a nest of rollers as in Fig. 7. Note in this

* Eye-bars not adopted; wire cable used in plans finally approved.

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he load and the resultant reaction t necessarily be vertical, as the d eliminate any possibility of a omponent to the loading; also, in cable and the backstay make ngle θ_1 with the horizontal, and alike. Generally, any change in or loading on main span will affect the angle of inclination le at top of towers, and in order that such angle may equal the ckstay the saddle moves automatically to restore equilibrium.

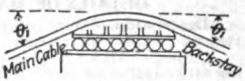

Fig. 7.

Truck, Fig. 8, gives the same move- cable as have the rollers a and b, ig. 7 the cable has double the move- rollers. In Fig. 8, h represents the tress in main cable and in backstay, e stress in link joining rollers a and b; y, stress in main cable $= h \sec \theta_1$, and ackstay $= h \sec \theta_2$. Vertical reaction at , and vertical reaction at $b = h \tan \theta_2$; t the vertical reaction is directly pro- tang of angle of inclination of cable.

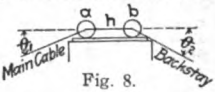

Fig. 8.

Suspended Link (Fig. 9) suspended from at a, and the Toggle (Fig. 10) supported b, give the required lateral motion to ed by temperature changes and moving to be noted, however, in these cases that n of loading on towers and the resultant e not confined to the vertical as in the ng cases, but take the direction of a line ugh pins a and b. The triangle of forces phically in each case. The towers should to take care of these loads for any riation in direction from the vertical, as he vertical loading itself.

Fig. 9.

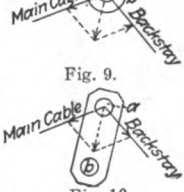

Fig. 10.

Hinged Tower, Fig. 11, is simply an elon- e. The cables may be fastened directly to r, and the latter being hinged at the bottom the direction of the resultant load and re- ive direction to same.

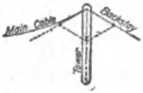

Fig. 11.

ges.—The maximum stress in the backstay, where the latter enters the ground, must e borne by the anchorage, with the proper fety. The anchorage proper consists of the backstays imbedded in masonry having sufficient weight or o overcome the stress. For this purpose the backstays may a straight line into the masonry or they downward as in Fig. 12. The main back- e the point a may be composed of wire le the anchorage chain below a may be f eye-bars in order to provide better con- th the anchors. In some cases the anchor- are imbedded in solid concrete masonry, bably insures permanent protection to the other cases it has been deemed advisable to allow the anchorage ass through open galleries so as to be accessible for inspection. of anchorage for Budapest suspension bridge in *Eng. News*, Aug. 24, 1905; also, Fig. 17, page 759.

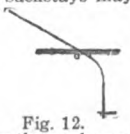
Fig. 12.

Cable Wrappings.—Fig. 13 shows the method of wrapping the cables for the

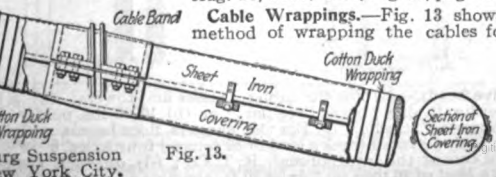

Fig. 13.

rg Suspension
w York City.

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DETAILS OF MANHATTAN SUSPENSION BRIDGE.

Description.—Wire cable bridge of three spans: central span, 1470 ft.; two side spans, 725 ft. each. Four cables.

Cables.—About 20¾″ in dia., each consisting of 37 strands and containing 9472 parallel wires of 0.192 in. dia. before galvanizing, and not more than 0.197 in. dia. after galvanizing.

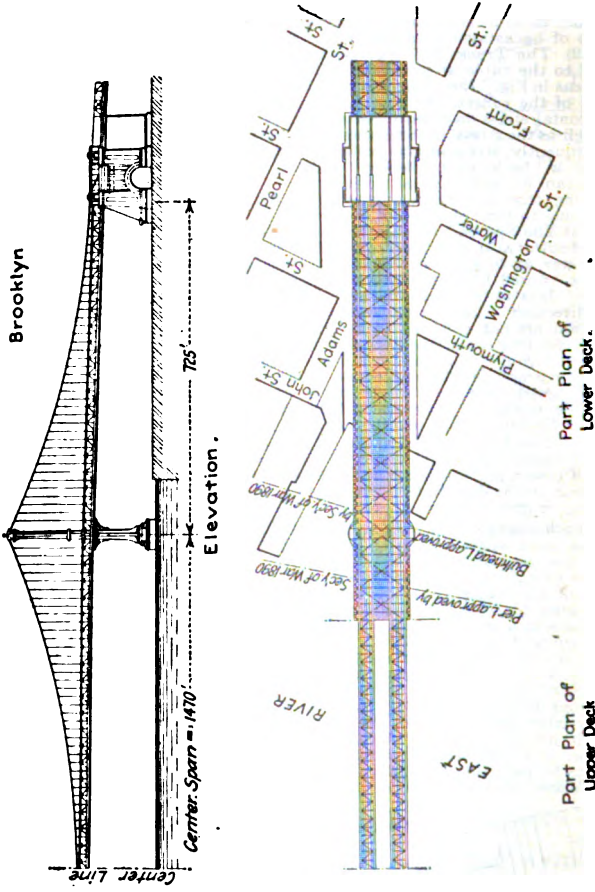

Fig. 14. Side Elevation of Bridge.

Live Loads.—(1) For the cables, trusses and towers: (a) a load of 8000 lbs. per lin. ft. of bridge as "regular," or (b) 16000 lbs. per lin. ft. of bridge as "congested" traffic. (2) For the hangers, floor beams and floor system: (c) on each elevated track a load of 52 tons on four axles, 6′-10′-6′, the motor ends of cars of the Interborough R. T. Co.; (d) on each street car track either a load of 26 tons on 2 axles 10 ft. apart or a load of 1800 lbs. per lin. ft.

on any part of the roadway a load of 24 tons on two axles 10 5 ft. gage (assumed to occupy a width of 12 ft. and a length of pon the remaining portion of the floor a load of 100 lbs. per sq. e footwalks a load of 100 lbs. per sq. ft.

pressure is assumed as a moving load acting in either direc- ally with 2000 lbs. per lin. ft., but due allowance is made by teral deflection into account.

ions for Material.—The wire for cables, hand ropes and sus- have an ultimate strength of not less than 215,000 lbs. per sq. vanizing, and an elongation of not less than 2% in 12 ins. of gth, the stretch to be measured while the specimen is in the ine. The bright wire shall be capable of coiling cold around a its own diameter without sign of fracture. The serving wire gage before galvanizing. After galvanizing, the steel wire ultimate strength of not less than 200,000 lbs. per sq. in. of

(Continued on next page.)

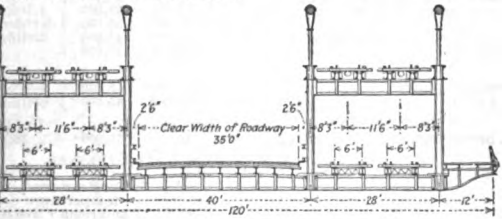

Fig. 15. Cross Section of Bridge.

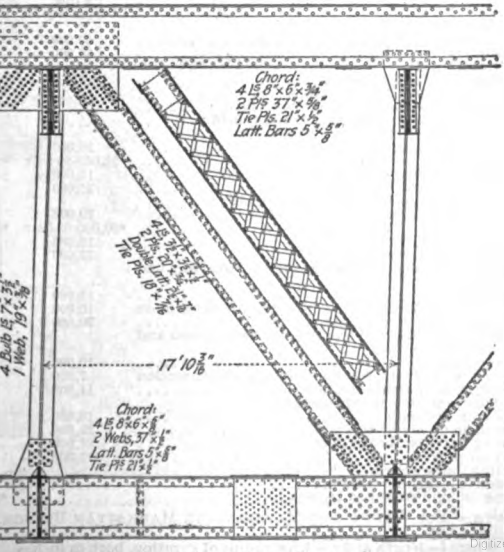

Fig. 16. Details of Floor-Beam Connections.

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The suspenders shall be 1¾ in. in dia. and weigh not less than 5.1 lbs. per lin. ft. They shall be composed of six strands of 19 wires each, laid around an independent wire rope center consisting of 49 wires, left hand lay. The suspenders shall be, preferably of long lay, but the lay must not be long enough to cause trouble in keeping the core in its true position during any of the operations before the suspenders are in their final position and loaded with the superstructure.

REQUIRED PHYSICAL PROPERTIES OF FINISHED MATERIAL.

(Manhattan Bridge.)

Material.	Ultimate Strength. Lbs. per sq. in.	Minimum Elastic Limit. Lbs. per sq. in.	Minimum Elongat'n. Per cent in 8 ins.	Minimum Reducti'n. Per cent of area.
Carbon Steel.				
Shapes and universal mill plates	60-68,000	33,000	1,500,000 divided by ultimate.	44 per ct.
Eyebars, pins and rollers	64-72,000	35,000		40 "
Rivet rods	50-58,000	30,000		50 "
High carbon steel for trusses	85-95,000	45,000		35 "
Sheared plates	60-68,000	33,000		44 "
Nickel Steel.				
Shapes and plates	85-95,000	55,000	1,600,000 ultimate	40 per ct.
Rivet steel	70-80,000	45,000		
Steel Castings.			In 2 ins.	
Test pieces from annealed castings	65,000	35,000	20%	

ALLOWABLE MAXIMUM UNIT STRESSES.

Material, and Parts of Structure.	For dead load, temperature and regul'r live load, or for dead load, temperature and wind. Lbs. per Sq. In.	For dead load, temperat're and congested live load, or for dead load, regular live load, temperature and wind. Lbs. per Sq. In.
Wire:		
Main cables	60,000	73,000
Suspenders	30,000	.....
Nickel Steel:		
Tension in stiffening trusses	.....	40,000
Compression in stiffening trusses	.....	*40,000-150 $l \div r$
Shear on rivets in stiffening trusses; field	.....	20,000
Bearing on rivets in stiffening trusses; field	.....	35,000
Structural Steel in Towers:		
Tension	20,000	†25,000
Compression	*22,000-90 $l \div r$	*†27,000-100 $l \div r$
Shear on shop rivets and bolts	13,000	16,000
Bearing on shop rivets and bolts	25,000	30,000
Structural Steel in Stiffening Trusses:		
Tension	20,000	24,000
Compression	*20,000-90 $l \div r$	*24,000-100 $l \div r$
Shear on shop rivets	13,000	.....
Bearing on shop rivets	25,000	.....
Structural Steel in Floor System of Roadway and Footways:		
Tension chords	15,000	.....
Shear on shop rivets, bolts and web-plate net section	10,000	.....
Bearing on shop rivets and bolts	20,000	.....
Structural Steel in Floor System for Railroad and Trolley Tracks:		
Tension chords	10,000	.....
Shear on shop rivets, bolts and web-plate net section	7,000	.....
Bearing on shop rivets and bolts	14,000	.....
Structural Steel in Anchorages:		
Tension in eye-bars	16,000	20,000
Bearing on diameter of pins	22,000	.....
Bending or outer fibre of pins	22,000	.....
Shear on pins	12,000	.....
High Carbon Steel:		
Tension in stiffening trusses	.....	35,000
Compression in stiffening trusses	.....	*35,000-135 $l \div r$

NOTE.—For WEIGHTS OF MATERIALS IN MANHATTAN BRIDGE, see page 760.

*Where l = length and r = least radius of gyration, both in inches.
†Including secondary stresses.

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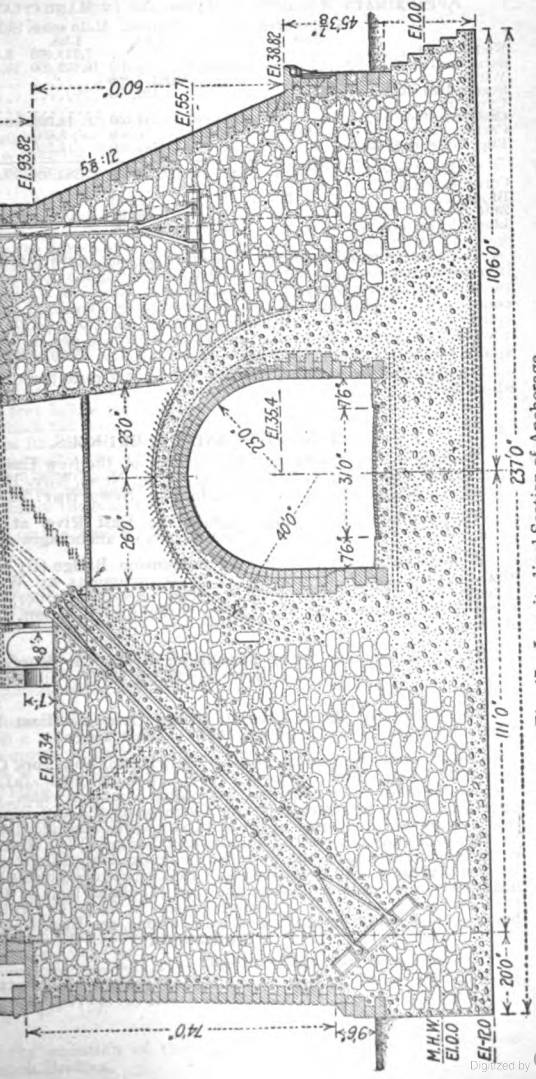

Fig. 17. Longitudinal Section of Anchorage.

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APPROXIMATE WEIGHTS OF MATERIALS IN MANHATTAN BRIDGE.

	Anchorages. Lbs.	Towers. Lbs.	Cables. Lbs.	Main span. Lbs.	Side spans. Lbs.	Totals. Lbs.
Nickel steel	.....	.....	.....	7,349,600	8,897,800	16,247,400
Structural steel	1,335,600	21,333,800	30,200	10,602,600	10,447,200	43,749,400
Wire	.....	.....	12,176,200	.....	.....	12,176,200
Suspenders, etc	.....	.....	1,153,600	.....	.....	1,153,600
Eye-bars	3,731,900	.....	.....	.....	.....	3,731,900
Castings, steel	1,500	3,385,200	1,744,600	13,700	28,200	5,173,200
Castings, iron	18,500	189,100	54,500	7,600	24,600	294,300
Pins, bolts, nuts, etc.	307,500	119,100	383,000	10,000	22,600	842,200
Totals of steel	5,395,000	25,027,200	15,542,000	17,983,500	19,420,400	83,368,200
Concrete (cu. yds.)	.....	930	.....	.....	.....	930
Bronze	400	.....	12,000	2,100	4,200	18,700
Zinc	.....	.....	25,200	.....	.....	25,200
Lead	.....	20,600	7,400	.....	.....	28,000

Economic Considerations.—The approximate costs of materials, erected, in a suspension bridge of about 1200 to 1500-ft. spans designed for combined railway and highway traffic, are as follows: Steel in wire cables, 6 to 6.25 cents per lb. (add about 4% to this for copper covering); riveted steel in center span, 4.25 to 4.5 cents per lb. (nickel steel, 6 to 6.25 cents); steel in towers, side spans and anchorage, 3.4 to 3.6 cents per lb.; steel in viaduct spans, about 3 cents per lb.; anchorage masonry, $5.50 to $6.50 per cu. yd.

Steel in *cantilever* spans, erected, about 4.5 cents per lb. (nickel steel, about 6 cents per lb.).

EXCERPTS AND REFERENCES.

Waterproof Wrapping for the Cables of the New East River Suspension Bridge (By Wilhelm Hildenbrand. Eng. News, Nov. 13, 1902).—Illustrated. Discussed by A. H. Sabin, in Eng. News, Nov. 20, 1902.

The Williamsburg Bridge Across the East River at New York City (Eng. News, Nov. 17, 1903).—Illustrations of anchorages and details.

A Rational Form of Stiffened Suspension Bridge (By Gustav Lindenthal. Trans. A. S. C. E., Vol. LV).—Discussions by W. Hildenbrand, Joseph Mayer, R. S. Buck, W. W. Crehore, Theodore Cooper, C. C. Schnieder, Owsald Erlinghagen, H. W. Hodge, F. Schule, J. Melan, L. S. Moisseiff, A. Rieppel.

The Monongahela River Suspension Bridge at Morgantown, W. Va. (By W. H. Boughton. Eng. News, April 18, 1907).—Illustration of saddle and top of tower.

The Towers of the Manhattan Bridge Over the East River at New York City (Eng. News, April 16, 1908).—Illustrated.

Report on the Manhattan Suspension Bridge at New York City (By Ralph Modjeski. Eng. News, Oct. 14, 1909).—**Calculations.** (a) Extract from specifications for superstructure, with table of unit stresses; (b) Derivation of formulas used in the calculation of stresses in the stiffening trusses, with moment and shear diagrams; (c) Method used in the calculation of stresses in the tower and cable, with formulas and diagrams; (d) Method used in the calculation of stresses in the lower floorbeams and lateral system, with formulas and diagrams. Illustrated: Fig. 1 (not reproduced here) is a diagram of dead-loads and cable ordinates; showing (1) Panel load of susp. structure, (2) Total panel load, (3) Actual ordinates, in feet, (4) Ordinates to parabolas, in feet. For discussions of calculations, see Eng. News, Mar. 3, 1910.

Illustrations.

Description.	Eng. Rec.
Machine for winding wire around main cables—Manhattan bridge	July 31, '09

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44.—ARCHES.

General Discussion.—An arch is a structure so designed that the loading, including the weight of the structure itself, produces a thrust at the abutments, in such a manner that the resultant horizontal reactions at those points tend to relieve, wholly or in part, the bending-moment effect on the span.

The Ideal Arch would naturally take the (inverted) form which the cables of a suspension bridge assume for the particular conditions of loading imposed; for then the horizontal reactions at the abutments will relieve the arch ring of any bending moment whatsoever, the stresses throughout being purely of axial compression. Either of the curves described in Sec. 43, Suspension Bridges, if inverted, will become ideal arches for the *given* loadings. Such arches are called *linear* arches. The circular arch also is a linear arch when the resulting forces are radial, as in the case of a circular dam where the hydrostatic pressure is normal to the up-stream face.

The Circular Arch as a Dam.—Let r, Fig. 1, be the radius in feet of the up-stream face of the dam; h, the depth in feet below the surface of the water to the level at which the pressure is to be considered; p, the pressure in lbs. per sq. ft. at depth h; W, the weight in lbs. of a cu. ft. of water. Then $p = Wh$; and the tangential compressive stress at any point of the circle for one foot vertical, and at depth h, is $T = pr = Whr = 62.5\ hr$ lbs. $= R$. If t is the thickness of the masonry in feet at depth h, the compressive stress in lbs. per sq. ft. on the masonry is $\frac{T}{t} = 62.5\ \frac{hr}{t}$. For overturning effect see Sec. 49, Dams.

Fig. 1.

The Catenary.—Let Fig. 1 represent a vertical arch of uniform thickness t and supporting its own weight only; then in order to be a true linear arch its curvature should be that of a catenary instead of a circle.

The Parabola.—Let Fig. 1 represent a vertical arch supporting, including weight of arch itself, a uniform *horizontal* load; then in order to be a true linear arch its curvature should be that of a parabola.

The Transformed-Catenary Arch.—Let it be required to design a masonry arch of 40 ft. span, 2 ft. in depth at the crown and 8 ft. at the springing line; and supporting a line load of 140 lbs. per sq. ft. Assuming the weight w of masonry at 140 lbs. per cu. ft., and the spandrel filling F to be solid at the same weight; and, further, reducing the live load to equivalent depth of masonry, namely, one foot, we have the outline diagram as shown in Fig. 2. The problem is to find the curve of the intrados (a transformed catenary) so that the line of resultant pressure shall trace the center line of the arch stones (practically) as shown by the middle dotted line. Note that the thinner the arch stones and the flatter the arch, the more nearly will this be true.

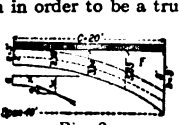

Fig. 2.

Solution.—First, find the value of m in the equation*

$$m = \frac{0.4342945\ c}{\text{com log}\left(\frac{h}{a} + \sqrt{\frac{h^2}{a^2} - 1}\right)} = 11.34+;$$ and substitute the values of $a(=2+1)$

* See also the equation of the catenary and transformed catenary in Sec. 43, Suspension Bridges.

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and m in the general equation *$y = \frac{a}{2}\left(e^{\frac{x}{m}} + e^{-\frac{x}{m}}\right) = \frac{3}{2}\left(e^{\frac{x}{11.34}} + \frac{1}{e^{\frac{x}{11.34}}}\right)$. Then:

For $x =$	0.	5.	10.	15.	20.
$y =$	3.00	3.30	4.24	6.03	9.00
Depth of masonry =	2.00	2.30	3.24	5.03	8.00

The above calculation is very simple, requiring only a few minutes' time, remembering that $\log e^{\frac{1}{m}} = \frac{1}{m} \log e, = 0.4342945 \div 11.34 = 0.03828$ for this particular case; and $\log e^{\frac{x}{m}} = 0.03828\,x$. Also note that $e^{-\frac{x}{m}}$ is the reciprocal of $e^{\frac{x}{m}}$ and hence $\log e^{-\frac{x}{m}} = 1 - \log e^{\frac{x}{m}}$.

The Horizontal Thrust H at any point of the arch for each foot in length (perpendicular to face) is $H = wm^2 = 140 \times (11.34)^2 = 18\,000$ lbs.; and at crown of arch = 9000 lbs. per sq. ft. = 62.5 lbs. per sq. in.

The Vertical Shear P_x, at any point distant x from the center of arch, is $P_x = \frac{H}{m}\sqrt{y^2 - a^2} = wm\sqrt{y^2 - a^2}$; and at the abutment the shear is equal to the vertical reaction $P = \frac{H}{m}\sqrt{h^2 - a^2} = wm\sqrt{h^2 - a^2} = 140 \times 11.34 \times \sqrt{81 - 9} = 13471$ lbs.— per lin. ft. (axial) of bridge.

The Area $\frac{A}{2}$ of Half the catenary, between directrix and soffit, is $\frac{A}{2} = m\sqrt{h^2 - a^2} = \frac{P}{w} = \frac{13471}{140} = 96.22$ sq. ft.

The area of face of masonry = 96.22 − 20 = 76.22 sq. ft.

The depth of arch stones is assumed to be 24 inches, but with a uniform live load as above it may safely be much less. The tangential thrust at the springing line is equal to $\sqrt{H^2 + P^2} = 22482$ lbs. or only 78 lbs. per sq. in. The tangent of the angle which this thrust makes with the horizontal at that point is $\tan\theta_1 = \frac{1}{m}\sqrt{h^2 - a^2} = \frac{P}{H} = 0.7484$; therefore $\theta_1 = 36° - 49'$, the thrust being tangent to the soffit of the arch. At any point distant x from center of span, $\tan\theta = \frac{1}{m}\sqrt{y^2 - a^2}$. With a variable live load on the arch the resultant line of pressure through the middle of the arch stones will change its form and deviate from a true central position. The amount of *deviation* in the present instance cannot be more than $\frac{1}{2} \times \frac{24}{3} = 4$ inches, or one-half the "middle third," without producing tension in the masonry joints, which is not allowable. (See Masonry Arches, following.)

The live load on the span is assumed at 140 lbs. per sq. ft. of floor, equivalent to a depth of one foot of masonry. Now it is plainly evident that the curve of the intrados of the arch (Fig. 2) will not be affected by any relative change of live load to dead load provided the "*loading contour*" *remains the same*. Thus, we may increase the uniform live load to 280 lbs. per sq. ft. by decreasing the depth of masonry one foot, making the depth of arch stones 12 ins. instead of 24 inches. This, of course, would double the thrust on the arch stones per sq. in., making the horizontal thrust at crown 125 lbs. per sq. in., and the tangential thrust at springing 156 lbs. per sq. in. The line of resultant pressure would not change its form but would now conform more nearly with the (new) middle line of the arch stones. They will never exactly coincide unless the arch ring is reduced to a *thin plate*.

The resultant line of pressure as previously determined will evidently be affected by changes in the loading contour: (1) By raising the loading contour, that is, increasing the uniform load per sq. ft.; (2) by lowering the loading contour, that is, decreasing the uniform load; (3) by considering the uniform load at middle of span only; (4) by considering the uniform

* Log $e = 0.4342945$.

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e of the span only; (5) by placing concentrated loads on the center of arch or over the haunches on either side. It is conditions of loading, partly, at least, that render the transry a curve of theory and not of practice, in the masonry arch. Arches, following.)

MASONRY ARCHES.

the Arch (Fig. 3).—*Crown* (*c*), the highest part of the he inner or concave surface of the arch. *Intrados*, curve of soffit with a plane perpendis of arch. The kind of ines the kind of arch, ental (circular), elliptical, of segmented arch), radius s, with center at *c*. *Axis*, the centers *c*. *Face* or the end walls. *Arch-sheeting*, arch masonry, cylindrical pirs, not including the face ck of arch, the upper or e of the arch-sheeting. e of intersection of back plane perpendicular to axis; ning the outer extremities ints between the voussoirs.

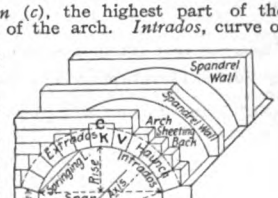

Fig. 3.

rch-stones), the stones radiating from the soffit, and composing r; the arch masonry. *Ring Stones*, the voussoirs or arch-stones face of arch. *Keystone*, the middle ring stone. *Springing line*, n of intrados with pier or abutment; the lower edge of the *wback*, (*a*) the inclined surface or joint from which an arch e stone itself which supports the arch. *Springer*, the lowest rch stone. *Haunch*, that part of the arch about midway n and skewback. *Span*, the perpendicular distance between s. *Rise*, the height of intrados at crown above plane of s. *Abutment*, the skewback and masonry supporting it. s, the walls which rise above the arch-stones or voussoirs. iffen the arch-sheeting and have considerable stiffness themior spandrel walls are placed under railway and street-car erever heavy, concentrated loads are liable to occur. Longibetween the spandrel walls, give added stiffness, and are used . *Spandrel*, the space between spandrel walls; or the space of arch and roadway. *Spandrel-filling*, the filling which he spandril, composed of earth or masonry, or both. The g may support the superimposed loads; or, they may be girders resting on the walls above the arch. *Backing*, masonry e the arch-sheeting and above the skewbacks. *String Course*, oussoirs extending from end to end of arch. *Coursing Joint*,) joint between string courses, extending from end to end *ding Joint*, a (broken) joint at right angle with axis of arch. the voussoirs between two adjacent heading joints.

Arches (from shape of intrados).—*Circular arch*, intrados is ; includes the two following. *Semi-circular* or "*full-centered*" is a semi-circle. *Segmental arch*, intrados is less than a semis not as strong as the preceding, but it is particularly adapted s, having less tendency to "bulge" become "depressed" at the haunches. ed to live concentrated loads. For nings, make rise ÷ span = vers 30°. *ointed*" *arch* (Fig. 4), intrados is two th span as radii; sometimes defined as cs with radius greater than half-span. arly in Architecture, as is also the fol-

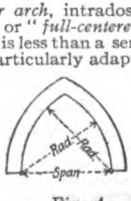

Fig. 4.

adius" of any arch is the radius of intrados at crown.

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Tudor arch, modified Gothic with intrados compounded. The "four-centered" Tudor arch is shown in Fig. 5. The radii are ¼ and 1¼ of the span, respectively. *Elliptical arch*, intrados is part of an ellipse. The major axis may be either horizontal or vertical. (*Any arch* is said to be *surbased* when the rise is less than the half-span; and *surmounted* when the rise is greater than the half-span.) *Oval* or "*basket-handle*" *arch*, intrados composed of arcs or circles approaching the elliptic arc in form. The "three-centered" oval is very common. *Parabolic arch*, intrados is parabolic. *Catenarian arch*, intrados is catenarian.

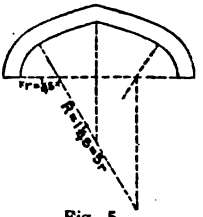

Fig. 5.

Further Classification of Arches.—*Right arch*, one whose ends or faces are perpendicular to axis of arch. *Skew arch*, an arch whose ends or faces are oblique to axis of arch. There are two kinds: *First* (Fig. 6), the skew arch proper, with smooth cylindrical soffit, spiral joints and unbroken axis; *Second* (Fig. 7), the oblique arch being made up of a number of short, right arches called ribs, which are offset transversely and successively in one direction. *Flat arch*, flat soffit with wedge shaped voussoirs; used for doors and windows. *Vault*, surface generated by the intrados of an arch moving in a straight line on the springing lines. *Cloistered vault*, formed when two vaults or arches *meet*; as "vaulted ceiling." *Groined vault*, formed when two vaults or arches *cross* each other. *Dome*, formed by right section of arch revolving around vertical axis through center of keystone. (Also, see Glossary.)

Fig. 6.

Fig. 7.

Brick Arches.—The three principal methods of bonding brick arches are illustrated in Fig. 8, and described as follows:

Rowlock bond (*R*).—All the bricks are laid as stretchers in concentric rings. The average thickness of joint is less under this construction, and hence more bricks and less mortar are required. It is cheaper to lay, and is employed largely in tunnel work, sewers, etc., where architectural effect is not important.

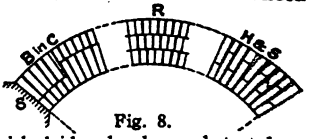

Fig. 8.

Header and Stretcher bond (*H. & S.*).—Radial joints continuous, and bricks laid as headers and stretchers. The average thickness of joint is greater under this method, and hence more mortar and less (the least) number of bricks are required. It is more pleasing to the eye than the rowlock bond, and is used largely in the fronts of buildings.

Block in Course bond (*B. in C.*).—This bond aims to attain with brick the wedge-shaped voussoirs of the stone arch. That is, the bricks are grouped in sections bounded by continuous radial joints. Adjacent sections are of different bond, any good bond being permissible. Bonds with continuous radial joint are usually made narrower than those bonds where radial joints are broken.

The Masonry Arch Is a Statically Indeterminate Structure, and its solution is based on various assumptions more or less approximate. These will be considered in the material order of design. To begin with, we have to *assume the finished arch* and then examine it for stability (resistance to deformation), strength (resistance to crushing), economy, etc.

Curve of Intrados.—The form of curve selected for the intrados will naturally affect the thickness of the arch ring and the economy of the whole structure. We have seen (Fig. 2) that the *transformed catenary* is the ideal curve of soffit where the loading contour is a horizontal line, and this curve would undoubtedly be used were the conditions of loading constant as shown. It will be noted that in the transformed catenary the curve is quite flat at the crown and gradually becomes sharper toward the springing. The *ellipse* is a curve that can be made to approximate the modified catenary.

mple than the latter. It may be used in the form of the
f the semi-ellipse*. If the latter is used the major axis be-
of the arch, and the semi-minor axis the rise. One of the
arches in the world is the ("New") London bridge over the
nd, for street traffic. It is a granite arch of 152 ft. span,
d with a crown radius of 162 ft. The thickness at crown is
springing, 9.00 ft. The *basket-handle* or *many-centered arc* is
of the ellipse, and has certain apparent advantages over
n applied to an arch of cut stone. To this class belongs
constructed stone arch bridge at Plauen, Saxony, with a
295.2 ft., the longest ever built. The total rise is 59 ft., and
of arch stones at crown is 4.92 ft. It is 5-centered, with
ft., 191.8 ft., and 101.7 ft. The *circular* form, using the arc
e most frequently employed on account of its simplicity, in
om, for the stone cutter, and in laying out the centers for
field. The "*segmental*"† arch subtends a central angle at
inly employed in building construction. The *semi-circular*
a central angle of 180° and is particularly adapted to
ugh high embankments where the rise of the arch is
and where the haunches of the arch are called upon
ined forces due to earth pressure. The largest semi-
own to the writer is the Ballochmyle, over the Ayre, Scotland.
arch of 181 ft. span. The thickness of ring at crown is 4.5 ft.,
ng 6 ft. The *parabola* is sometimes used for flat, concrete
ot particularly economical for a masonry arch because of the
the loads; that is, a parabola would call for a uniformly
d over the structure, whereas in a masonry arch this condi-
ven be approximated. In *conclusion*, we may state that,
cing, the basket-handle is doubtless the best form to use for
cut-stone work — 5-centered for extremely long spans, and
medium long spans, say up to 200 feet. For concrete arches
may be used for any length of span; the semi-ellipse, for any
where the rise is considerable; also for short spans in general;
, for moderate and short spans, especially of cut-stone.

olic Oval is a new arch curve designed by Chas. Worthington,
E., and is described very elaborately in *Eng. News* of April
panied by numerous illustrations.

of Arch Ring.—Theoretically, the arch stones should have
ss at the springing than at the crown, because of the greater
ust. For practical reasons, however, the thickness of the
ually made uniform throughout for spans up to 75 ft. For
than 75 ft. the thickness at springing is usually from 25 to
eater than at crown‡, depending upon length of span, rise, etc.

ssuming thickness of arch stones.—Having selected the most
for the intrados the next step is to approximate as closely
thickness of the arch ring at crown and at springing. No

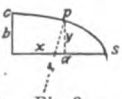

Fig. 9.

s of curvature r for any point p (whose coordi-
d y) of the ellipse, whose semi-major axis is a,
or axis b (Fig. 9), is

. For the semi-ellipse where b = the rise, and

the radius of curvature at crown C is (for

$$= \frac{a^2}{b} = \frac{(\text{half span})^2}{\text{rise}}$$; and the radius of curvature at springing

$$, x = a), r = \frac{b^2}{a} = \frac{(\text{rise})^2}{\text{half span}}.$$

"segmental" is here used in its restricted meaning; but in
se any circular arch is a segmental arch.

exception to this rule is the railway arch bridge of 4 spans
ow river, England. The spans are each 100 ft.; rise, 25 ft.;
ickness of ring, 4 ft. Such a depth at crown, however, is exces-

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absolute, simple rules can be devised, but the writer presents the following as giving very close results for first-class concrete and cut stone work:

Thickness t_c at crown (all dimensions in feet):

For highway bridges t_c at crown $= \sqrt{0.01 \text{ span} \left(\frac{\text{span}}{\text{rise}} + 3\right)} + 0.15$ .....(1)

For high H. W. embankments, or For railroad bridges $\Big\}$ t_c at crown $= \sqrt{0.01 \text{ span} \left(\frac{\text{span}}{\text{rise}} + 4\right)} + 0.20$ .....(2)

For high R. R. embankments, $\Big\}$ t_c at crown $= \sqrt{0.01 \text{ span} \left(\frac{\text{span}}{\text{rise}} + 5\right)} + 0.25$ .....(3)

Thickness t_s at springing (all dimensions in feet):

For all cases, t_s at springing $= t_c\ [1 + 0.002(\text{span} + 2 \times \text{rise})]$.....(4)

1.—CROWN THICKNESS t_c (FEET) FOR MASONRY ARCHES.
[Calculated from Formulas (1), (2) and (3).]

	Rise ÷ Span		Span, in Feet.																	
	Frac.	Dec.	2	5	10	15	25	40	60	80	100	125	150	175	200	240	280	320
For Highway Bridges.	1-10	.10	.66	.96	1.29	1.55	1.95	2.43	2.94	3.37	3.76	4.18	4.57	4.92	5.25	5.74	6.18	6.60
	1-8	.125	.62	.89	1.20	1.43	1.81	2.25	2.72	3.12	3.47	3.86	4.21	4.54	4.85	5.29	5.70	6.08
	1-7	.143	.60	.86	1.15	1.37	1.73	2.15	2.60	2.98	3.31	3.69	4.02	4.33	4.62	5.05	5.44	5.81
	1-6⅔	.15	.59	.85	1.13	1.35	1.70	2.12	2.56	2.93	3.26	3.63	3.96	4.26	4.55	4.97	5.35	5.71
	1-6	.167	.57	.82	1.10	1.31	1.65	2.05	2.47	2.83	3.15	3.50	3.82	4.12	4.39	4.80	5.17	5.52
	1-5	.20	.55	.78	1.04	1.25	1.56	1.94	2.34	2.68	2.98	3.31	3.61	3.89	4.15	4.53	4.88	5.21
	1-4	.25	.52	.74	.99	1.17	1.47	1.82	2.20	2.52	2.80	3.11	3.39	3.65	3.89	4.25	4.58	4.88
	1-3	.333	.50	.70	.92	1.10	1.37	1.70	2.05	2.34	2.60	2.89	3.15	3.39	3.61	3.94	4.25	4.53
	1-2½	.40	.48	.67	.89	1.06	1.32	1.63	1.97	2.25	2.50	2.77	3.02	3.25	3.47	3.78	4.07	4.35
	1-2	.50	.47	.65	.86	1.02	1.27	1.56	1.88	2.15	2.39	2.65	2.89	3.11	3.31	3.61	3.89	4.15
For R. R. Bridges and for High Highway Embankments.	1-10	.10	.73	1.04	1.38	1.65	2.07	2.57	3.10	3.55	3.94	4.38	4.78	5.15	5.49	6.00	6.46	6.89
	1-8	.125	.69	.97	1.30	1.54	1.93	2.39	2.88	3.30	3.66	4.07	4.44	4.78	5.10	5.57	6.00	6.40
	1-7	.143	.67	.94	1.25	1.48	1.86	2.30	2.77	3.17	3.52	3.91	4.26	4.59	4.89	5.34	5.75	6.13
	1-6⅔	.15	.66	.93	1.23	1.46	1.83	2.27	2.73	3.12	3.47	3.85	4.20	4.52	4.82	5.26	5.66	6.04
	1-6	.167	.65	.91	1.20	1.42	1.78	2.20	2.65	3.03	3.36	3.74	4.07	4.38	4.67	5.10	5.49	5.86
	1-5	.20	.62	.87	1.15	1.36	1.70	2.10	2.52	2.88	3.20	3.55	3.87	4.17	4.44	4.85	5.22	5.57
	1-4	.25	.60	.83	1.09	1.30	1.61	1.99	2.39	2.73	3.03	3.36	3.66	3.94	4.20	4.58	4.93	5.26
	1-3	.333	.57	.79	1.04	1.22	1.52	1.87	2.25	2.57	2.85	3.16	3.44	3.70	3.94	4.30	4.63	4.93
	1-2½	.40	.56	.77	1.01	1.19	1.47	1.81	2.17	2.48	2.75	3.05	3.32	3.57	3.81	4.15	4.47	4.76
	1-2	.50	.55	.75	.97	1.15	1.42	1.75	2.10	2.39	2.65	2.94	3.20	3.44	3.66	3.99	4.30	4.58
For High R. R. Embankments.	1-10	.10	.80	1.12	1.47	1.75	2.19	2.70	3.25	3.71	4.12	4.58	4.99	5.37	5.73	6.25	6.73	7.18
	1-8	.125	.76	1.06	1.39	1.65	2.05	2.53	3.04	3.47	3.86	4.28	4.67	5.02	5.35	5.84	6.28	6.70
	1-7	.143	.74	1.02	1.35	1.59	1.98	2.44	2.93	3.35	3.71	4.12	4.49	4.83	5.15	5.62	6.05	6.45
	1-6⅔	.15	.73	1.01	1.33	1.57	1.96	2.41	2.90	3.30	3.67	4.07	4.43	4.77	5.08	5.54	5.97	6.36
	1-6	.167	.72	.99	1.30	1.53	1.91	2.35	2.82	3.22	3.57	3.96	4.31	4.64	4.94	5.39	5.80	6.18
	1-5	.20	.70	.96	1.25	1.47	1.83	2.25	2.70	3.08	3.41	3.79	4.12	4.43	4.72	5.15	5.54	5.91
	1-4	.25	.67	.92	1.20	1.41	1.75	5.15	2.57	2.93	3.25	3.60	3.92	4.22	4.49	4.90	5.27	5.62
	1-3	.333	.65	.88	1.14	1.35	1.66	2.04	2.44	2.78	3.08	3.41	3.71	3.99	4.25	4.63	4.98	5.31
	1-2½	.40	.64	.86	1.12	1.31	1.62	1.98	2.37	2.70	2.99	3.31	3.60	3.87	4.12	4.49	4.83	5.15
	1-2	.50	.62	.84	1.09	1.27	1.57	1.92	2.30	2.62	2.90	3.21	3.49	3.75	3.99	4.35	4.68	4.98

Remarks.—The above table is for solid arch rings. Where arch rings are ribbed the thickness or depth of ribs should be increased. For spans under 75 ft. an arch ring of uniform thickness may be used by increasing the crown thickness obtained from above table by from 5 to 20%. See Table 2.

Tables 5 and 6, pages 774 to 781, contain various data from arches that have been built.

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2.—Table for Obtaining Thickness t, of Arch Stones at Springing.

Values of $[1+0.002\ (\text{span}+2\times\text{rise})]$ in Equation (4).

Thickness at springing = thickness at crown × tabular values below.)

Rise ÷ Span.		Span, in Feet.						Rise ÷ Span.		Span, in Feet.					
Frac.	Dec.	75	100	150	200	250	300	Frac.	Dec.	75	100	150	200	250	300
1-10	.10	1.18	1.24	1.36	1.48	1.60	1.72	1-5	.20	1.21	1.28	1.42	1.56	1.70	1.84
1-8	.125	1.19	1.25	1.38	1.50	1.63	1.75	1-4	.25	1.23	1.30	1.45	1.60	1.75	1.90
1-7	.143	1.19	1.26	1.39	1.51	1.64	1.77	1-3	.333	1.25	1.33	1.50	1.66	1.83	2.00
1-6⅔	.15	1.20	1.26	1.39	1.52	1.65	1.78	1-2½	.40	1.27	1.36	1.54	1.72	1.90	2.08
1-6	.167	1.20	1.27	1.40	1.53	1.67	1.80	1-2	.50	1.30	1.40	1.60	1.80	2.00	2.20

Note from Tables 1 and 2 that for any given span the actual thickness of arch ring at springing remains nearly constant as the rise approaches the half span.

Problem.—In designing a railroad masonry arch of 200 ft. span and 40 ft. rise, what thickness of arch ring shall be assumed, tentatively, at crown? Also at springing?

Solution.—At crown, 4.44 ft. (from Table 1). At springing, 4.44×1.56 (from Table 2) = 6.93 ft. Hence use 4.5 and 7, respectively, for trial.

Forces Acting on a Masonry Arch.—These may be classified as the "outer" and the "inner" forces, the same as in any other structure. The *outer* forces include (1) the moving loads due to trains, vehicles, etc.; (2) the mixed loads, due to weight of track, roadway, embankment, spandrel filling, etc.; (3) the dead loads due to weight of arch ring itself; (4) the reactions. The *inner* forces comprise the stresses in the arch ring, usually calculated at points of imaginary joints. These stresses are examined for compression, shear, and possible tension. The compression is determined from the line of resultant pressure through the arch ring, which also reveals possible tension. The shear at any point of the arch is equal to the algebraic sum of the outer forces at the left of that point, taken in the direction of the cutting plane. Furthermore, we know that the algebraic sum of the outer forces, either of the whole structure or of that portion at the left of any given cutting plane, is equal to zero. The same is true of the inner forces; also of the outer and inner forces combined. Hence, an inner force in a complete structure may be determined by assuming it to be an outer force in maintaining equilibrium in a *portion* of the structure.

Direction of Outer Forces.—Let Fig. 10 represent a masonry arch with arch ring drawn to scale. Let MM represent the mixed static loading above the arch ring due to spandril filling, roadway and track, and with depth reduced to equivalent masonry loading, so that d, in feet, at any point distant x from left abutment, will represent the intensity of the total static loading at that point in terms of the weight of masonry per cubic foot. If, now, we add the moving load m for any particular case of loading, say on the right half of span, and reduce the same to equivalent depth of masonry, we obtain the loading contour for calculating the arch for *that particular case of loading*.

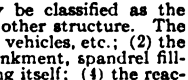

Fig. 10.

Vertical Loading.—In fixing the loading contour as above we have assumed all loading to be vertical, which, although not quite correct, is the usual assumption. Note that the line of resultant pressure (dotted) through the arch ring approaches the upper edge on the more heavily loaded half, and the lower edge on the less heavily loaded half, indicating a tendency to cause tension in joints at t, which must be guarded against, as well as undue compression at c. The thrust T at crown will not be horizontal but will incline upward to the right because of the heavier loading on that side,

producing positive vertical shear V at crown. The horizontal component H of thrust T is, $H = \sqrt{T^2 - V^2}$ = horizontal component of R_1 or of R_2, also.

Inclined Loading.—The loading at the crown may always be assumed as vertical; also at the haunches if the spandril filling is of rough masonry, or of earth or clay well tamped and kept thoroughly drained. But if the filling is of loose earth, sand or gravel, and especially if liable to become saturated with rain, from poor drainage, the material will have a tendency to slide along a low angle of repose, with resultant inclined loading at the haunches, as FFF, Fig. 10. Such inclined loading will produce a different line of resultant pressure in the arch ring than will vertical loading.

Three-Hinged Masonry Arch.—(The following theories are approximate only.) The three hinges, a f k (Fig. 11), placed at crown and at springing, are necessarily points of zero moment and hence the line of resultant pressure must pass through those points. The method of procedure is as follows: Divide the arch ring into imaginary voussoirs with radial joints. From the upper extremities of these joints draw vertical lines up to the loading contour $L.\ C.$ Find the area and the center of gravity of each of these figures, bounded by full lines, between the $L.\ C.$ and the soffit of arch. Then the area multiplied by the weight of the masonry per cu. ft. will give the vertical loads P_1, P_2, P_3, etc., passing through the centers of gravity of the figures. The reactions R_1 and R_2, as well as their horizontal and vertical components, may be found graphically (see Fig. 17, Sec. 16, page 315) by treating each load separately or by using one or more resultant loads. With H, V_1, V_2, and the loads P_1, P_2, etc., the force polygon (Fig. 12) may be constructed, and the *equilibrium* polygon shown in Fig. 11 may be drawn; thus, draw R_1 through a, parallel to R_1 of the force polygon, to meet P_1 produced; from this intersection draw 1′ parallel to 1 to meet P_2 produced, etc. If the work is done carefully the equilibrium polygon will pass through the hinges f and k. See also Fig. 14.

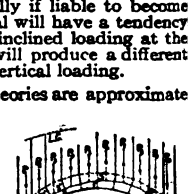

Fig. 11.

Fig. 12.

The Line of Resistance or line of resultant pressure (not shown in Fig. 11) may be drawn by connecting the points a b c d—f—k where the chords of the equilibrium polygon, as shown, intersect the masonry joints. But it must be remembered that if, in the equilibrium polygon, any point as p due to a load as P' falls outside the area from which the loading is derived, then the chords of the equilibrium polygon must be *produced* to meet the joint and give position to points on the line of resistance. Thus, in Fig. 13 the line of resistance will pass through $b'c$ instead of through bc.

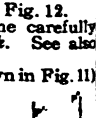

Fig. 13.

It is to be noted that the pressure at each of the joints may be obtained by scaling the rays of the force polygon, using the proper scale; thus, the pressure at c, intersected by 2′ (Fig. 11), is obtained by scaling ray 2; d, by scaling ray 3; f, by scaling ray 5. The force polygon also gives the vertical shear V at crown. As the loading is all vertical the value of H (horizontal thrust) is constant throughout the arch.

Three-hinged masonry arches are usually constructed of concrete or reinforced concrete. The hinges, at crown and springing, usually consist of steel shoes with pin bearings; sometimes lead sheets are used.

The Criterion of Stability of a masonry arch is whether the true line of resistance for any case of assumed loading lies wholly within the middle third. If it does, the arch is stable for that loading. Now, while we can draw practically a true line of resistance for an arch with three hinges (Fig. 11), it is impossible to draw a true line of resistance for an ordinary arch with no hinges, and in order to determine its stability we have to make certain assumptions.

Ordinary Masonry Arch—No Hinges.—Dr. Winklers' theory for the line of resistance of an arch is as follows: "For an arch of constant cross-section, that line of resistance is approximately the true one which lies nearest to the axis of the arch ring, as determined by the method of least squares." Hence, for all practical purposes, if a line of resistance can be drawn within any portion of the arch ring, say the middle third, the true line of resistance

that portion.* Furthermore, in the clear language of Prof. rch and its load being given, a line of resistance can always s through any *two*‡ given points; hence, if any two points of ance are given, the line is determined." In practice, we nts within the portion of the arch ring, say the middle third, e of resistance passing through them. If this line of resist- within the portion of the ring, the arch is stable; if not, again selected (one of which may remain unchanged) and a istance is drawn, etc., until it is determined whether a line of e drawn within the given portion of the arch ring. If such be drawn, the thickness of such ring is increased, or the ch is modified, or both.

l Loading.—Let Fig. 14 represent one half of a symmetrically $_1$, P_2, P_3, P_4 and P_5, the loads on that e resultant of those loads. It is re- mine whether the arch is stable with e middle third, which is bounded by e two points a and b, the former at and the latter at the upper edge, of 1; and pass a line of resistance through The thrust H at b must be horizontal ymmetrical in form and loading, and be determined by taking moments

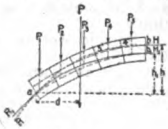

Fig. 14.

$Hh = Pd$, or $H = P\frac{d}{h}$. Now draw the force 5, laying off the value of H thus obtained, om the foot of the load line, to the point O; ays R, 1, 2, 3, etc. Now draw the equilibrium dotted in Fig. 14: producing H to P_5; then el to ray 4, to P_4; 5′ parallel to 5; etc. The $_1$, from P_1, passing through the point a. line of resistance (not shown) will be at the chords of the equilibrium polygon, thus found, with the ; and by inspection we see that the line of resistance, if s outside the middle third of the first three joints to the left owever, does not prove that the arch is unstable, for, by e select b_1 instead of b, and readily obtain a line of resistance ithin the middle third and passing through $a\,b_1$. The new vill have the pole O_1 instead of O, and distant H_1 horizontally f the load line, the value of H_1 being equal to $P\frac{d}{h_1}$. A mere 14 will indicate that the *true* line of resistance, although it etermined definitely, will lie nearer the axis of the arch and inging and crown joints well within the middle third instead as indicated by points a and b_1.

Fig. 15.

ical Loading.—If we analyze the preceding case of sym- g, critically, we cannot fail to see that we have really assumed , of which Fig. 14 represents one-half, to be 3-hinged—one rown and one at each springing; that we have taken the ming these hinges to be at any part of the joint within the portion of the ring; and that, because of the symmetry, it sary to consider one-half the arch only.

t case of unsymmetrical loading calls for but little special en viewed from the above stand- g. 16 be the arch under consideration, o P_{10} derived from a loading contour he Fig.). Assume three points within le portion of the ring to act as hinges 3-hinged arch, and calculate the arch method which will give a line of re- g through the three selected points, h a line of resistance lies wholly within

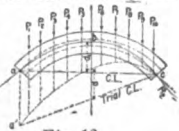

Fig. 16.

ler himself claims that this is not strictly true, but it is ac- y noted authorities.

ed Mechanics, by Gataeno Lanza, page 804.

t in a 3-hinged arch, *three* points are given because they are noment *absolutely determined*.

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the selected middle portion of the arch ring the arch is considered stable; if not, select three points again until, by repeated trial, it is determined whether or not the arch is stable, as explained for Fig. 14. Two methods will be shown for drawing the equilbrium polygon, from which the line of resistance is determined.

First Method.—Time will be saved by a proper selection of points to be assumed for the hinges. In Fig. 16 where the heavier loading is on the right half of span the hinge c on that end is assumed at the inner edge of the middle portion; on the left end, carrying the lighter load, the hinge a is assumed at the outer edge of the middle portion; while at center of span the hinge b is assumed at the middle of the vertical joint. Draw the equilibrium polygon passing through those points, a b c, treating the arch as 3-hinged, as explained for Fig. 11. Examine for stability.

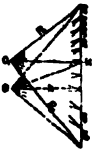

Fig. 17.

Second Method.—Select the hinges as above (Fig. 16). Lay off the load line in the force polygon, Fig. 17. Choose any pole as O, and draw the rays (dotted) of the force polygon. Construct the equilibrium polygon (dotted) in Fig. 16, beginning at c and ending at a'. Draw the trial closing line $a'c$. Draw OM (Fig. 17) parallel to $a'c$ to meet the load line at M. From M draw MO_1 parallel to the true closing line ca. Make the distance $MO_1 = k\frac{d}{d_1}$, in which k = the horizontal distance to the load line. Then will point O_1 be the true pole of the force polygon. Draw the rays (full) from O_1, and construct the equilibrium polygon (full) in Fig. 16, beginning at c. It will be found that the chords will pass through the hinges b and a. Examine for stability. If unstable select new points for hinges, and proceed as before.

The "middle third" of the arch ring is usually selected as the "middle portion" within which to draw the line of resistance because, theoretically, when the line of resistance passes within the middle third there can be no tension in any part of the masonry, nor opening of any of the joints. If the arch ring is stiffened greatly by solid spandril walls the criterion of stability may not require that the line of resistance be confined within the middle third, but, say, within the middle half.

The cases of loading for an arch are: (1) dead load only; (2) full load, including live load over whole span; (3) including live load over one-half of span; (4) including live load in any position; (5) including concentrated loads. It must not be presumed however that for all these cases of loading it is necessary to assume the hinges in any one fixed position.

Centers for Arches.

Definition.—An arch center is a framework for supporting an arch during construction, and hence of temporary character. It is so designed that it can be removed readily after the completion of the arch; and in the removal it has to be *lowered* somewhat in order to give sufficient clearance at the soffit of the arch, as the latter settles and becomes self-supporting.

Parts of the Arch Center.—Fig. 18 shows two half-sections of arch centers with an end view of one of the frames. One half-section illustrates

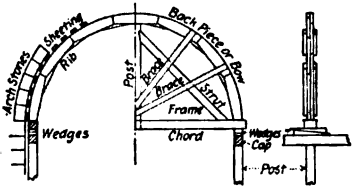

Fig. 18.

the unbraced *rib* which supports the *sheeting* or *lagging* on which the arch stones are laid. The other half-section is a trussed *frame*, suitable for long

spans. When trussed, the rib is called the *back piece*. The truss shown in Fig. 18 is the King post, the inclined *braces* transmitting the stresses from the haunches directly to the vertical post, and then down the inclined *struts* to ends of span. Various forms of trusses are used for the frames, depending upon length of span, rise, etc. The frames are placed perpendicular to the axis, spaced in parallel planes, and supported by *wedges* which are "struck" when the center is removed. Instead of the wedges, *jacks* or *sand cylinders* can be used for this purpose.

Loads.—The loads which an arch center has to support are:

Live loads, including (1) The percentage of weight of placed masonry supported, in different sections of the arch; (2) Same with regard to unplaced masonry and other materials, machines, derricks, men, etc.; (3) Impact due to handling material during construction of arch.

Dead loads, comprising weight of center itself, including sheeting, frames, trestling, etc.

Calculation of live loads.—Three cases will be considered, as noted above:

(1) *Masonry in place.*—In Fig. 19 let the shaded portion show the voussoir V or section of masonry whose weight is W. Then:

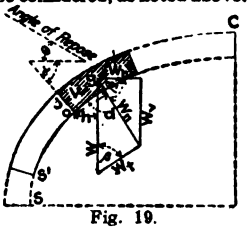

Fig. 19.

(*a*) If we assume V as merely resting *without friction* on the joint J and on the back of the arch center, we have, letting β equal the angle of inclination of the normal pressure W_n with the horizontal,

$$W_n = W \sin \beta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

and the vertical and horizontal components of W_n are $W_v = W \sin^2 \beta$, and $W = W \sin \beta \cos \beta$, respectively. $W_t (= W \cos \beta)$ is the tangential thrust. It is worthy of note that the normal pressure W_n of equation (1) would equal W for any voussoir at the crown c, and *zero* for any voussoir at the hor. springing s. Authorities differ, however, as to the value of the equation for sections between these two points, on account of the unknown effects of friction, which latter would tend to *reduce* the value of W_n.

(*b*) If we assume the voussoir V as hinged at o, then, by taking moments,

$$W . h = W_n' . d, \text{ or } W_n' = W \frac{h}{d} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Now W_n' of equation (2) will equal W_n of equation (1) only when the voussoir V is extremely thin, practically a plate, and when its intrados is a straight line instead of a curve as shown in Fig. 19. Otherwise its value will be less than W_n of equation (1).

(*c*) If we assume friction at the joint J we have, calling α the angle which the joint makes with the horizontal, and θ the angle of repose of the masonry,

$$\text{Normal pressure } W_n'' = W_n - W \tan \theta \cos \alpha, \text{ nearly} \ldots\ldots\ldots\ldots (3)$$

$$\text{or } W_n'' = W_n' - W \tan \theta \cos \alpha, \text{ nearly} \ldots\ldots\ldots\ldots (4)$$

in which $\tan \theta$ may vary from 0.50 to 0.65; that is, the angle of repose θ may vary between, say, 27° to 33°. The problem becomes more difficult if we attempt to include the friction of the soffit of V on the back of the arch center.

(2) *Unplaced masonry, machines, etc.*—These would tend to increase the stresses and must be allowed for liberally, especially over the haunches where the stresses are particularly indeterminate.

(3) *Impact.*—The amount of impact would depend largely on the size of the stones used; it can be taken into account in a general way, if small, by the "factor of safety" adopted.

Practical formula for live loads.—Let W = weight of section or voussoir V (Fig. 19), W_N = normal pressure due to W; and β = angle of W_N with the horizontal. Then

$$W_N = W \sin^2 \beta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

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3.—VALUES OF W_N FOR SUCCESSIVE VALUES OF β.
From $\beta = 90°$ (at crown) to $\beta = 0°$. Equation (5).

Angle β.	Normal Pressure W_N.	Angle β.	Normal Pressure W_N.	Angle β.	Normal Pressure W_N.	Angle β.	Normal Pressure W_N.
90°	1.000 W	70°	0.830 W	50°	0.450 W	30°	0.125 W
88°	.998 "	68°	.797 "	48°	.410 "	28°	.103 "
86°	.993 "	66°	.762 "	46°	.372 "	26°	.084 "
84°	.984 "	64°	.726 "	44°	.335 "	24°	.067 "
82°	.970 "	62°	.688 "	42°	.300 "	22°	.053 "
80°	.955 "	60°	.650 "	40°	.266 "	20°	.040 "
78°	.936 "	58°	.610 "	38°	.233 "	15°	.017 "
76°	.914 "	56°	.570 "	36°	.203 "	10°	.005 "
74°	.888 "	54°	.530 "	34°	.175 "	5°	.001 "
72°	.860 "	52°	.489 "	32°°	.149 "	0°	.000 "

Use of Table: Example.—A section of arch masonry weighing 20 tons is supported radially by a brace of the arch center acting normal to the intrados and making an angle $\beta = 50°$ with the horizontal. Find the compressive stress on the brace due to this load?

Solution: W_N for $\beta = 50°$ is $0.45\ W = 9$ tons. Ans.

Types of arch centers.—Fig. 20 illustrates the most simple form of centering, that for a flat-soffit arch. The illustration is that of a Flat Arch of the recessed type. They are built up to 8 or 10 ft. in span.

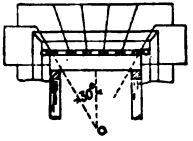

Fig. 20.

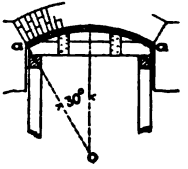

Fig. 21.

The caps supporting the lagging rest on longitudinal stringers, which are in turn supported by posts. Note the economy of material in lagging by increasing the depth of strips at the expense of width. The strength is proportional to width and to (depth)^2.

Fig. 21 is the standard segmental arch used over doors and windows. The "back" of the center consists of templates of thick boards cleated together. The lagging is spaced closer together if the arch is of brick. Note that the points o a a form an equilateral triangle in both Figs. 20 and 21.

Fig. 22 shows the make-up of braced or unbraced wooden rib as illustrated in Fig. 18. The segmental pieces are sawed from plank and (nailed or) bolted together into two or more leaves according to the strength desired. The ribs should, preferably, be braced, although unbraced ribs have been used up to 50 or 60 ft. spans.

Fig. 22.

Fig. 23 represents the leaves of the rib laid flat-wise instead of vertical as in the preceding case. This type is the same as that used in chords of ordinary wooden bowstring truss bridges.

Iron or steel ribs are sometimes used instead of wooden ribs in cases where great strength and stiffness are required. I-beams are the best for this purpose.

Fig. 23.

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Fig. 24 illustrates a supported trussed frame, mainly of the Warren type. The left half-section is shown supported by vertical posts. The right half-section is shown supported by inclined posts, which system is

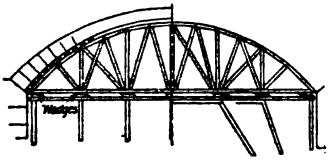

Fig. 24.

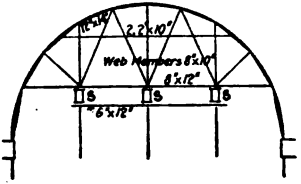

Fig. 25.

sometimes adopted for economy, as in the case of swift currents or ice. The same type may also be used without interior supports. In fact the end supports may also be omitted by allowing the ends of frames to rest on projections or in (artificial) recesses of the abutments.

Fig. 25 is a skeleton outline of center used in the erection of 60-ft. arch of Washington bridge,* New York City. The sand cylinders *s*, are used instead of wedges or jacks for lowering the center after the arch is completed. They consist of 12-in. plate-iron cylinders filled with sand on which rests the plunger, which forms a support to the centering above. By manipulating a plug at the bottom of the cylinder the outflow of sand is regulated and consequently the lowering of the center. In using sand cylinders under centers, care must be used to keep the sand perfectly dry. They have met with varying success.

In Table 6, next to last column, reference is made to the files of *Eng. News*, by date, of descriptions of some recently constructed arches. Many of these descriptions embody the designs of the centers quite as much as of the masonry.

Striking the center.—This consists in lowering the center for removal after the masonry arch is completed, either by striking the wedges, operating the jacks or manipulating the sand-boxes to relieve the pressure at the soffit.

The *time* when this should be done will depend upon the length of span; size and kind of voussoirs, whether brick or stone; character of bond; character and thickness of joint; quality of mortar, etc. For instance, there could be little objection to striking the center almost *immediately* of a first-class stone arch bridge of moderate span with large voussoirs, well bonded, with close-fitting joints. On the other hand, it would be unwise to do so in the case of a brick arch of long span, small rise, thick joints, and bricks poorly bonded, as considerable unhealthy deformation might take place. Again, in the case of arches in building walls, considerable time should elapse before the centers are removed, at least until the bricklaying above the arch is beyond its sphere of influence in causing additional weight or pressure. Three or four months is the outer limit for any case, as that gives ample time for hardening of the mortar. The presence of the centering need in no way interfere with the traffic over the structure.

Camber of center.—In laying out and erecting the center of an arch, it is sometimes advisable to give the frames a slight camber, amounting, at the crown, to about $\frac{1}{8}$ or $\frac{1}{4}$ per cent. of the *radius* of the intrados. Of course the object of this camber is to provide for all settlement so that the intrados of the finished masonry structure shall be the true curve as designed, and on which the calculations of the structure are based.

The total camber to be provided for will equal (1) the deflection of frame due to its own dead weight; (2) the deflection due to weight of masonry of completed arch; (3) settlement when center is removed; (4) further settlement due to superimposed loads on arch.

If the center is rigidly constructed, the arch stones well laid with close joints, and the arch itself not of the pronounced "flat" type, very little, if any, camber is required.

* Wm. R. Hutton, chief engineer.

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Tables of Arches.

Table 5, following, is a list of some notable arches that have been built, with principal dimensions and important remarks.

Table 6 comprises some typical modern arches. Reference is given to the files of *Engineering News*, where fuller descriptions are given.

5.—Some Notable Arches that Have Been Built.

Note.—*Ris* = rise; *Rad* = radius at crown; *Cr* = thickness of ring at crown; *Sp* = thickness at springing; all dimensions in feet and decimals. *Rein.-conc.* = reinforced concrete; *3-cen.* = 3-centered, etc.

No.	Span.	(Dimensions.)	Name and Location.	"Kind."	[Engineer and Date.]	Remarks.
1*	295.27	(Ris = 60±, Rad = 344.48, Cr = 4.92, Sp = 11.15).	*Plauen*, Saxony.	"Stone; 3-cen."	[1905.]	Flattest stone arch ever built, and longest span. Solid arch (without hinges). Arched abutments rise from solid rock foundation so that static (elastic) arch proper has span of 213.2 and rise of 21.2 ft. Live loads: (1) A 15-ton vehicle 9.84 ft. bet. axles and a unif. load of 114.6 lbs. per sq. ft.; (2) 3 steel rollers total weight 25.16 tons and a unif. load of 114.6 lbs. per sq. ft. Calc'd pressure, including temp., was 981.4 lbs. per sq. in. on top edges of joints at crown, and 745.1 lbs. on lower edges of joints. Max. pres. on foundations 355.6 lbs. per sq. in., the foundation rock having crushing strength of 23760 lbs. per sq. in. Mortar for foundations, 1 Portland cement to 4 sand; for main and secondary arch, 1 Port. cem. to 3 sand. Large spandrel openings.
2*	275.6	(Ris = 100±, Rad = , Cr = , Sp =).	*Luxemburg*.	"Stone, -cen."	[1901.]	
3	251.	(Ris = 88, Rad = 133, Cr = 4).	*Trezzo*, over Adda, Italy.	"Granite; circ."	[Built, 1380; destroyed intentionally, 1420 ±.]	
4*	233.	(Ris = 70.25, Rad = 118.75, Cr = 5.5, Sp = 9.5).	*Walnut Lane*, Phila.	"Concrete, 3-centered."	[Webster, 1907.]	Twin arch rings. Spandrel arches; and arch approaches. Actual deflection on removing centers was $\frac{1}{16}$ in. at crown and 1-32 in. at quarter points, the calculated deflection at crown being $\frac{3}{8}$", the difference being accounted for partly to rise in temp.
5	220.	(Ris = 57, Rad = 134, Cr = 4.2 + 4, Sp = 6 + 15).	*Cabin John*, Washington, D. C., aqueduct.	"Stone, circ."	[Meigs, 1853-9.]	The voussoirs of the arch ring are of Quincy granite making depth at crown 4.2 ft., and thickness at springing 6 ft. The spandrel filling of sandstone is laid partly with radial joints so as to increase effective depth of ring to 8.2 ft. at crown and 21 ft. at springing. The arch carries a 20-ft. roadway and a 9-ft. conduit.
6	213.	(Ris = 59.6, Cr = 6.9, Sp = 10.2).	*Jaremcze*, Austria.	"Circ."	[1892.]	Spandrel openings. Railroad.
7*	211.5	(Ris = 87.6, Rad = 89.25, Cr = 4.33, Sp = 6.5).	*Kempten*, Bavaria.	"Concrete; basket-handle."	[1907.]	4-track railroad. Twin arch rings.
8*	210.	(Ris = 52.5).	*Gutach* Riv., Bavaria.	"Stone."	[1905?]	Spandrel walls. Railroad.
9*	209.9	(Cr = 3.4).	*Bogenhausen*, over Isar R., Bavaria.	"Stone."	[1902.]	3-hinged arch, 21.4 ft. rise. Highway.
10	200.	(Ris = 42, Rad = 140, Cr = 4.6, Sp = 7).	*Grosvenor*, over Dee, Chester, Eng.	"Circular."	[Hartley, 1833.]	

* Described also in Table 6.

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5.—SOME NOTABLE ARCHES THAT HAVE BEEN BUILT.—Continued.

No.	Span.	(Dimensions.) Name and Location.	"Kind." [Engineer and Date.]	Remarks.
11	196.8	(Ris=52.8, Cr=5.6, Sp=13.8). *Gour Noir*, France.	"Circular." [Draux, 1888.]	Railroad.
12*	187.5	(Ris=32.2). *Lautrach*, over Iller, Bavaria.	"Concrete." [1907.]	
13*	187.	(Ris=55.8, Cr=6.56, Sp=9.18). *Schwaenderhols*, on N. & D. Ry	"Stone."	Railroad.
14	181.	(Ris=90.5, Rad=90, Cr=4.5, Sp=6). *Ballochmyle*, Ayre, Scotland.	"Circular." [Miller.]	Railroad.
15	164.	(Ris=16.4, Cr=3.3, Sp=3.6). *Munderkingen*, over Danube.	"Circular." [Bois, 1893.]	Hinged arch. Railroad.
16*	164.	(Ris=15.75, Cr=1.77, Sp=2.98). *Chattellerault*, France.	"Reinforced concrete." [1902-.]	Four arched ribs. Hennebique system. Highway.
17	159.	(Ris=28, Cr=4.5, Sp=6). *Wheeling*, W. Va.	"Circular." [Hogue and White, 1893.]	
18	152.	(Ris=37, Rad=162, Cr=4.9, Sp=10). *London*, over Thames, Eng.	"Ellip." [Rennie, 1831.]	5 spans.
19	150.	(Ris=35, Rad=98, Cr=4.5). *Gloucester*, over Severn, Eng.	"Ellip." [Telford.]	
20	150.	(Ris=27, Cr=3.8, Sp=4.5). *Elyria*, Ohio.	"Sandstone; circ." [Kinney, 1886.]	Highway.
21	148.	(Ris=18, Rad=160, Cr=4.92). *Turin*, Italy.	"Circular." [Mosca.]	
22	144.	(Ris=19.3, Cr=4.2). *Putney*, over Thames, Eng.	[Bazalzette, 1886.]	4 other spans.
23*	143.87	(Ris=19.03, Cr=4.10, Sp=4.78). *Orleans*, over the Loire, France.	"Stone; catenoid." [Renardier, 1906.]	Highway. 70 spans.
24	141.	(Ris=28, Rad=103, Cr=4.92). *Alma*, Paris.	"Small, cement rubble; ellip." [Darcel.]	Railroad.
25	140.	(Ris=35, Rad=88, Cr=1.5+1, Sp=2.5+). *Pont-y-Prydd*, over Taff, So. Wales.	"Rough rubble in lime mortar; circular." [Built by a stone mason in 1750, to replace a former bridge of the same general design which fell on striking centers; present bridge, however, has spandrel openings which former bridge did not.]	
26*	140.	(Ris=30, Cr=5+2). *Ill. Cent. R. R.*, over Big Muddy.	"Concrete; ellip." [1903.]	
27	131.2	(Ris=18.2, Cr=3, Sp=3). *Coulouvreniere*, France.	"Circ." [Bois, 1896.]	
28	128.2	(Ris=32, Cr=5.1). *Neuilly*, Seine, France.	"Ellip." [Perronet, 1773.]	Arch settled 2 ft. on removing center, and radius at crown increased from 160 to about 250 ft.
29	128.	(Ris=24.2, Rad=169, Cr=5.25, Sp=7.2). *Maidenhead*, Eng.	"Brick in cement; Ellip," [Brunel, 1837.]	Railroad.
30*	125.	(Ris=39, Rad=67.75, Cr=5, Sp=7.67). *Piney Branch*, Wash., D. C.	"Concrete; parabolic." [Douglas and Darwin, 1906.]	Highway.
31	124.	(Ris=6.92, Rad=281, Cr=2.67, Sp=3.60). *Experimental Arch at Souppes*, France—See No. 32.	"Cut granite in Portland cement; circular." [Vaudrey, 1866.]	Width, 12 ft. Ratio span to rise, 17.83. Centers struck in 4 mos.; deflection 1¼ ins. Tested without injury by distributed load of about 500 lbs. per sq. ft.; also by weight of about 5 tons falling 18 ins. on key.

* Described also in Table 6.

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5.—Some Notable Arches that Have Been Built.—Continued.

No.	Span.	(Dimensions.) Name and Location.	"Kind." [Engineer and Date.]	Remarks.
32	124.	(Ris = 6.92, Rad = 281, Cr = 2.67, Sp = 3.60). *Bourbonnais*, France.	"Cut granite; circ." [Vaudrey.]	Very bold: built after experiment at Souppes, preceding. Railroad.
33	120.	(Ris = 31, Rad = 112, Cr = 4.5, Sp = 8.) *Waterloo*, over Thames, London.	"Granite; ellip." [Rennie, 1816.]	8 other spans.
34*	120.	(Ris = 12, Cr = 2.33). *Jacaguas Riv.*, Porto Rico.	"Rein-concrete." [Thacher, 1901.]	Highway. Two other spans, each 100 ft.
35	118.	(Ris = 38, Rad = 65, Cr = 3.6, Sp = 3.6). *Tongueland*, over Dee, Eng.	"Circular." [Telford, 1801.]	Highway.
36	116.	(Ris = 21.2, Cr = 3.5, Sp = 4). *Cresheim*, Fairmount Park, Phila.	"Sandstone; circ." [Webster, 1893.]	Sewer.
37	116.	(Ris = 14.8, Rad = 120, Cr = 4). *Napoleon*, Paris.	"Small rough rubble in cement; circ." [Couche.]	Railroad.
38*	112.	(Ris = 17.7, Cr = 2.46, Sp = 2.79). *Miltonburg*, over the Main, Germany.	"Concrete." [Fleischman, 1899.]	Highway. 5 other spans.
39	100.	(Ris = 25, Cr = 4, Sp = 4). *Etherow Riv.*, Eng.	"Circular." [Haskoll.]	Railroad. 3 other spans.
40	100.	(Ris = 22, Cr = 1.83, Sp = 1.83.) *Bishop-Auckland*, Eng.	"Circular." [1388.]	Highway.
41	100.	(Ris = 15, Cr = 3, Sp = 7). *Wellington*, over Aire, Leeds, Eng.	[Rennie, 1819.]	
42	90.	(Ris = 30, Rad = 49, Cr = 3). *Dean*, near Edinburgh, Scot.	"Circular." [Telford.]	Highway.
43	90.	(Ris = 15, Rad = 75, Cr = 2.83). *Licking Aqueduct*, Ches. & O. Canal.	"Circular." [Fisk.]	
44	84.	(Ris = 27.9, Cr = 3, Sp = 4). *Elkader*, Ia.	"Limestone; circ." [Tschirgi, 1888.]	One other span.
45	83.	(Ris = 11.75, Cr = 4.6). Over the *Oise*, France.	"Circular."	Railroad.
46	81.	(Ris = 28, Cr = 4.5). *Trilport*, France.	"Ellip."	Railroad.
47	80.	(Ris = 40, Rad = 40, Cr = 3, Sp = 3.50). *Conemaugh Viaduct*, Penn. R. R.	"Sandstone in lime without sand; circ."	
48	80.	(Ris = 40, Rad = 40, Cr = 2.66). *Royal Border Viaduct*, Eng.	"Brick in cement; circ."	
49	80.	(Ris = 16, Rad = 58, Cr = 4.66). *Posen Viaduct*, Germany.	"Brick in cement; circ."	
50*	80.	(Ris = 12, Rad = 88, Cr = 1.33). *Clifty Creek*, Greensburg, Ind.	"Rein.-con.; 5-cen." [Luten, 1906.]	Highway.
51	79.	(Ris = 26.3, Cr = 4). *Orleans*, France.	"Ellip."	Railroad.
52	79.	(Ris = 13, Cr = 3.5, Sp = 4.5). *Hutcheson*, Glasgow, Scot.	[Stephenson.]	
53*	79.	(Ris = 11, Rad = 88, Cr = 1.5, Sp = 2.5). *Grand Rapids*, Mich.	"Rein.-con.; 3-cen." [Anderson.]	Highway.
54	78.	(Ris = 25, Rad = 43, Cr = 3). *Falls*, P. & R. Ry.	"Circ."	
55	76.	(Ris = 38, Rad = 38, Cr = 7.6, Sp = 14). *Westminster*, over Thames, London.	"Circ." [Labelye, 1747.]	12 + 2 other spans.
56	75.	(Ris = 15, Cr = 2.4, Sp = 3.4). *Albany St.*, New Brunswick, N. J.	"Brick ring; circ." [1893.]	Small skew; 6 other spans.

* Described also in Table 6.

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5.—SOME NOTABLE ARCHES THAT HAVE BEEN BUILT.—Concluded.

No.	Span.	(Dimensions.)	Name and Location.	"Kind." [Engineer and Date.]	Remarks.
57	75.	(Ris=11.5, Cr=2.5, Sp=3).	*Allentown*, Eng.	"Circ." [Stephenson.]	Highway.
58	72.	(Ris=16.5, Rad=47, Cr=2.75, Sp=2.75).	*Black Rock Tunnel Br.*, P. & R. Ry.	"Circular." [Robinson.]	
59*	70.	(Ris=19.75, Cr=3, Sp=3).	*Rockville Br.*, Penn. R. R.	"Stone; circ." [Brown, 1901.]	47 other spans.
60	70.	(Ris=25, Cr=3.5, Sp=3.5).	*Swatara*, P. & R. Ry.	"Brick; circ." [Osborn.]	
61	70.	(Ris=17.6, Rad=44, Cr=3).	*Brent* Viaduct, Eng.	"Brick in cement; ellip." [Brunel, 1837.]	Railroad. 7 other spans.
62	70.	(Ris=17.5, Cr=2+.5).	*Wellesley*, Limerick.	"Ellip."	4 other spans.
63	66.	(Ris=13.8, Rad=47, Cr=2.5).	*Bow*, over Lea, Eng.	"Granite; ellip." [Walker, 1837.]	Highway.
64	65.	(Ris=32.5, Cr=2.75, Sp=2.75).	*Houghton Riv.*, Eng.	"Circular." [Haskoll.]	Railroad.
65	64.	(Ris=16.5, Rad=39.28, Cr=3, Sp=3).	*Watertown*, Wis.; C. M. & St. P. R. R.	"Stone; circular." [Loweth, 1903.]	Double track; 3 other spans.
66	60.	(Ris=30, Cr=2.7, Sp=2.7).	*Conemaugh Viaduct*, Penn. R. R.	"Stone; circ." [Brown, 1890.]	One other span; 3 tracks, on curve.
67	60.	(Ris=20, Rad=33, Cr=2.2).	*Bewdley*, Eng.	"Circular." [Telford.]	Highway.
68	60.	(Ris=18, Rad=34, Cr=2.5).	*Chestnut St.*, Phila.	"Brick in cement; circ." [Kneass.]	
69	60.	(Ris=8.5, Rad=42.33, Cr=1.75, Sp=2.5).	*Sandy Hill*, over Hudson.	"Rein.-conc.; circ." [Kasson, 1906.]	Highway & Elec. Ry.
70	58.	(Ris=29, Rad=29, Cr=2.5, Sp=2.5).	*Carrollton*, near Baltimore.	"Granite; circ."	Railroad.
71	58.	(Ris=17, Rad=33, Cr=1.5).	*Llanrwast*, Wales.	"Circ." [Jones, 1636.]	Highway.
72*	56.	(Ris=28, Rad=28, Cr=3.17, Sp=3.17).	*Raritan Riv.*; P. R. R.	"Stone; circ." [Bowles, 1903.]	
73	54.	(Ris=9, Rad=45, Cr=2.5).	*Monacacy Viaduct*, Ches. & O. Canal.	"Ellip." [Fisk.]	
74	53.	(Ris=10.3, Cr=2.75).	*Sterling*, over Fourth.	"Circ."	
75	53.	(Ris=3.8, Cr=3.16).	*Nemours*, France.	"Circ." [Perronet.]	
76	53.	(Ris=5.1, Cr=3).	*Abbatoir St.*, Paris.	"Circ."	Railroad.
77	50.	(Ris=15, Rad=28.3, Cr=2).	*Avon Viaduct*, Eng.	"Brick in cement; ellip." [Vignoles.]	
78	50.	(Ris=7, Cr=2).	*Filbert St.*, Phila.; Penn. R. R.	"Brick in lime mortar; circ."	
79	50.	(Ris=7, Rad=47, Cr=2+.67).	*James Riv.* Aqueduct, Va.	"Circ." [Ellet.]	
80	45.	(Ris=3.83, Cr=3.83).	*Pesmes*, France.	"Circ." [Bertrand.]	
81	43.	(Ris=6.1, Cr=3).	*Conturette*, France.	"Circ."	
82	40.	(Ris=15, Rad=21, Cr=2).	*Touoloway* Culvert, under Ches. & O. Canal.	"Rubble in cement; circ." [Fisk.]	

* Described also in Table 6.

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6.—Some Typical—

Note.—The next to the last column in table makes reference. by

No.	Name and Location.	Material.	Bridge Ft.	One Arch Span.	Rise.	Rad. at Crown	Curve of Intrados.	Thickness at Crown	Thickness at Sp'g'g.	Foundations.
1	Plauen Arch B., Plauen, Saxony.	Stone	492.	295.27 (213.2)	60.± (21.2)	344.48	3-cen.	4.92	11.15	.....
2	Luxemburg B. (Valley of the Pretrusse.)	Stone	.....	275.6	100.±	.....	.....	.....	.....	Conc.
3	Walnut Lane B., Philadelphia.	Conc.	585.	233.	70′-3″	118′-9″	3-cen.	5′-6″	9′-6″	Conc.
4	Kempton B., Bavaria (Across the Iller R.)	Conc.	.....	211′-6″ (166)	87.6 (29)	89.25	Basket-h'ndle	4′-4″	6′-6″ (6.1)	Conc.
5	Gutach Riv. B. (On Neustadt & Donaeuschingen Ry).	Stone	.....	210.	52.5	.....	.....	.....	.....	.....
6	Bogenhausen B., Bavaria (Across Isar R.).	Stone	.....	209.9	.....	.....	.....	3.4	.....	.....
7	Lautrach B., Bavaria (Across the Iller R.).	Conc.	.....	187.5	32.2	.....	.....	.....	.....	Conc.
8	Schwaenderholz B. (On Neustadt & Donaeuschingen Ry.).	Stone.	.....	187.	55.8	.....	.....	6.56	9.18	.....
9	Chatellerault B., France.	Rein-conc.	442.8 (bet. abuts.)	164.	15.75	.....	.....	1.77	2.98	Conc.
10	Orleans B., France (Across the Loire R.).	Stone	1088.8	143.87 (70 spans)	19.03	.....	Cate-noid	4.10	4.78	Conc.
11	Illinois Cent. R. R. B. (Across Big Muddy R.).	Conc.	.....	140.	30.	.....	Ellip.	5.+2.	.....	Pile.
12	Piney Branch B., Washington, D. C.	Conc.	272.	125.	39.	67′-9″	Parab.	5′-0″	7′-8″	Conc.
13	Jacaguas R. Br., Porto Rico.	Rein-Conc.	404.	120. (3 spans)	12.	.....	.....	2′-4″	.....	Pile & Conc.
14	Miltenburg B., Germany (Over the River Main)	Conc.	733.	112. (6 spans)	17.7	.....	.....	2.46	2.79	.....
15	Laibach B., Austria.	Rein.-Conc.	Skew.	102.8	14.6	.....	.....	.....	.....	Pile & Conc.
16	Venice, Cal.	R.-C.	200.	96.	13′-10″	.....	Ellip	2′-2″	3′-0″	Pile
17	Yorktown B., Indiana.	Rein.-Conc.	.....	95.	11.1	.....	.....	.....	.....	Conc.
18	Dayton B., Ohio (Across Miami R.).	Rein-Conc.	588.	88.	8.8	132.	3-cen.	1′-8″	.....	Conc.

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—MODERN ARCHES.

Eng. News, where more complete descriptions may be found.

Width	Engineers.	Ref. Eng. News.	Remarks.
6.1+ 9.68 2.48	.....	8-17-05	The longest single span masonry arch in the world (1907).
	.....	10-10-01 2-27-02 3-5-03	Two separate parallel arches 19.68 ft apart in clear, with intervening space spanned with reinforced-concrete slabs.
40+ 20	Webster	1-31-07 8-15-07	Twin arch rings each 21.5 ft. wide by 9.5 ft. thick at haunch, and 18 ft. wide by 5.5 ft. thick at crown. Abuts. reinforced, 1" sq. rods. Rubble concrete 1 : 3 : 6 for abuts.; concrete 1 : 2 : 5 for arch rings.
4. max.)	.....	5-2-07	Twin arches spaced 4 ins. Concrete: Large arch ring, 1 : 2½ : 5; abut piers 1 : 3 : 6; small piers, 1 : 4 : 8; foundations, 1 : 5 : 9; backing for steel hinges, 1 : 2 : 2; with computed stresses respectively 498, 370, 370, 270, and 938 lbs. per sq. in.
.....	.....	8-17-05	
9.+ 0.	.....	9-12-02	3-hinged arch; rise between centers of hinges, 21.4 ft. Limestone arch ring with granite blocks at steel hinges.
3.8	.....	5-2-07	
4.4	.....	8-17-05 12-26-01	Arch ring built in two thicknesses or layers, toothed.
6.24	.....	4-10-02	Hennebique system. 4 arched ribs reinforced with round iron rods. Live load, 82 lbs. per sq. ft.
4.13	Renardier.	3-28-07	Ratio of span to rise = 7.5. Special provision for expansion.
.....	.....	11-12-03	Traffic maintained during erection.
.....	Douglas and Darwin.	11-16-05 4-19-06 1-20-07	Half bridge erected on one side of center line; in future when other half is erected roadway will be 61 ft.
8. (20)	Thacher.	8-1-01	Two end spans each 100 ft., rise 11.28 ft. Bridge tested under light loads after 30 days, for deflection. Sand boxes.
14.4 + 9.8	Fleischmann.	2-25-01	Arches, 23 ft. wide, 3-hinged, with narrow strips of lead (5 to 6 ins.) 0.78 in. thick. Max. pres. in masonry 360 lbs. per sq. in. Factor safety, 8. Concrete in arches, 1 : ¼ : 3.
2. (50)	Melan.	7-16-03	3-hinged, 109.4 ft. c.-c. pins. Live load 96 lbs. per sq. ft., and 2 13-ton wagons. Concrete, arch ring, 1 : 3 : 3; top of foundations, 1 : 3 : 5.
6.	Ehlers.	8-29-07	1 cement to 5 ocean gravel; light construction throughout.
6.	Luten.	5-11-05	3 ribs; middle rib 12 ft. wide, 2 side ribs, each 3.5 ft. wide. Settlement at crown ½ in. after striking centers, 30 days.
4. clear)	.....	5-19-04	Concrete for piers, 1 : 3 : 6; for abuts. 1 : 3 : 7; for arch ring, 1 : 2 : 4. Melan system; 2½x2½x5-16 angles. Live load 150 lbs. per sq. ft. and 2 lines 40-ton elec. cars.

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6.—Some Typical—

Note.—The next to the last column in table makes reference, by

No.	Name and Location.	Materials.	Bridge Ft.	One Arch Span.	Rise.	Rad. at Crown	Curve of Intrados.	Thickness at Crown	Thickness at Spg'g.	Foundations.
19	Santa Ana Viaduct; San Pedro, Los Ang. & St. Lake R. R.	Conc.	984.	86. (8 spans)	36.9	43.5	Circ.	3'-6"	.....	.....
20	Belvidere B., Ill. (Elec. Ry.) (Across Kishwaukee R.).	Rein.-Conc.	.....	81. (4 spans)	10.5	83.36	Circ.	3.	4'-6½"	Conc.
21	Clifty Creek B., Greensburg, Ind.	Rein.-Conc.	.....	80.	12.	88.	5-cen.	1'-4"	.....	Conc.
22	Grand Rapids B. Mich. (Across Grand R.).	Rein.-Conc.	.....	79. (5 spans)	11.	88.	3-cen.	1'-6"	2'-6"	Conc.
23	Krosno B., Galicia, Austria.	Rein.-Conc.	257.	75.	.....	.....	.....	1'-0"	2'-8"	.....
24	Rockville B., Penn., Penn. R. R.	Stone	3820.	70. (48 spans)	19.75	.....	Circ.	3.	3.	Conc.
25	Guayo River B., Porto Rico.	Rein.-Conc.	270.	70. (3 spans)	7.50	.....	.....	.....	.....	Pile & Conc.
26	C. M. & St. Paul R. R. Br., Watertown, Wis.	Stone	360.	64. (4 spans)	16'-6"	39'-3½"	Circ.	3.	3.	Pile & Conc.
27	Sandy Hill B., N. Y. (Across the Hudson R.)	Rein.-Conc.	1025.	60. (6 spans)	8'-6"	42'-4"	Circ.	1'-9"	2'-6"	Conc.
28	Decatur B., Ill. (Wabash R. R.) (Across Sangammon R.).	Rein.-Conc.	Skew (4 spans)	59.	.....	.....	.....	3'-9"	.....	Conc.
29	Raritan R. Br. (Penn. R.R.) New Brunswick N. J.	Stone	1500.	56. (21 spans)	28.	28.	Circ.	3'-2"	3'-2"	.....
30	Como Park B., St. Paul, Minn.	Rein.-Conc.	.....	50.	12'-6"	.....	.....	0'-10"	2'-6"	.....
31	Standard Overhead B's. (Local) (N.Y.N.H. & H.R.R.)	Rein.-Conc.	.....	31.	6'-8"	.....	Ellip.	1'-2"	.....	Conc. (Piers)
32	L.S. & M.S.R. R. Arch B's. (Local Standard).	Rein.-Conc.	.....	30.	9.	25.	3-cen.	2'-9"	6'-5"	Conc.

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—MODERN ARCHES—Concluded.

Eng. News, where more complete descriptions may be found.

Width	Engineers.	Ref. Eng. News.	Remarks.
17.	.....	10-22-03	Concrete arch rings, 1 : 2 : 4½; foundations, piers and spandrel walls, 1 : 11.
14.	Strauss.	8-30-06	Two arched ribs 2.5 ft. wide spaced 8′-10″ c.-c. Concrete, without temperature, 500 lbs. comp., 50 lbs. tens.; with temperature, 650 and 75 lbs. Steel 10,000 lbs. without temp.; 13,000 lbs. with temp. variation. E of concrete, 1 400 000; E or steel, 28 000 000. Factor of safety, 4, for 40-ton double-truck cars. Novel mode of erection, without centering.
16.	Luten.	6-28-06	Live load 200 lbs. per sq. ft., and 20-ton roller. Mr. Luten gives general formulas for design of R.-C. bridges—curve of intrados, crown thickness, curve of extrados, abutment ties, arch rods—in *Eng. News*, June 28, 1906.
.....	Anderson.	12-1-04	Concrete for arch rings, 1 : 2 : 4; for piers, 1 : 3½ : 7; for abuts., 1 : 2½ : 4. Thacher bars 1¼″ dia. at intrados.
.....	.....	.....	Arch: 4 ribs 18 ins. thick. Concrete 420 lbs. per sq. in.; wt. at 140 lbs. Live load 20 lbs. per sq. ft. and a continuous string of 16-ton road cars.
51.	Brown	12-12-01	Arch rings, spandrel walls and pier facings, stone; filling and backing, concrete. One half longitudinal section constructed at a time, re-using centering for other half. Longest stone-arch bridge in the world (1907).
18. (20)	Thacher.	8-1-01	Two end spans each 70 ft., rise 7 ft. Sand boxes 10 ins. dia., 12 ins. high, with 6 to 8 ins. sand—not satisfactory
30. (o to o)	Loweth.	3-26-03	Traffic maintained during construction.
32′-4″ (clear)	Kasson.	5-9-07	Live loads; 12+100 lbs. per sq. ft.; elec. cars, 60-ton; for floor plates, 12-ton truck on base 6x10 ft. Concrete, 1 : 2 : 4 and 1 : 3 : 5.
34.	Cunningham.	3-21-07	Span 58′-3½″ meas. square. Concrete: For arches, slabs, carrying tracks, and footings for piers, 1 : 2½ : 5; for piers, spandrels, abuts., and lower slabs, 1 : 3 : 6. Live loads 9000 lbs. per lin. ft. per track. Working stresses, 600 lbs. for conc. in comp., and 12000 lbs. for steel in tens.
57.	Bowles.	6-18-03	Sandstone of excellent quality from Cambria and Clearfield Counties, Pa. (Conemaugh Stone Co.).
17′-2 (15)	Wilson.	4-6-05	Concrete, 1 : 2 : 4; mortar facing, 1 : 2.
.....	.....	4-11-07	Old-rail reinforcement—economical
Variable (Culverts)	.....	7-14-04	Concrete, 1 : 3 : 6. Johnson steel rods—Monier plan.

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Concrete Culverts.

Fig. 26.

Fig. 26 is a section of a small railroad culvert, 3 ft. wide and 3 ft. high, which has been used considerably on the Penn. R. R. as a standard. The amount of concrete required is about 0.41 cu. yd. per lin. ft. of culvert.

STEEL AND COMBINATION ARCHES.

A few hints will be given to illustrate the method employed in the calculation of some of the forms of steel arches most commonly used in practice. Steel arches differ from stone arches in that they are designed to resist bending, as well as compression and shear. In other words, the line of resultant pressure is not confined within any given limits but may pass anywhere outside the middle third or even outside the rib of the arch altogether. Hence the form of arch selected, and depth of rib or truss, are matters of economy, adaptability and appearance, rather than of mere gravitational stability of the respective parts.

In the following discussion it is assumed that the forms of the structures have been determined, the problems being to find the stresses in the various members or parts under certain given loadings. The first thing to do in each case is to find (one or more of) the *reactions* at the points of support; and when these are known the remaining solution reduces to the simple case of "finding the inner forces or stresses when the outer forces are given;" that is, to the case of any simple structure.

The methods used in finding the reactions may be wholly analytical or partly graphical. The latter is chosen here for simplicity. The first step is to draw the arch and its "position line," sometimes called the "locus line."

Arch with No Hinges.—This arch (Fig. 27) is seldom used in practice, the 2-hinged and 3-hinged arches being mainly employed. The position line is laid off by means of an equation containing x and y. From any point p the direction of the reactions for any load P may be obtained by either one of several methods: (1) by drawing lines from any point as p tangent to the reaction curves; (2) by calculating the vertical ordinates y_1 and y_2 from the points a and b, to points on the line of direction of the reactions; (3) similarly, by calculating x_1 and y_1; (4) by calculating the right angle offsets from a and b; (5) by calculating the horizontal thrust H; etc. The force polygon, above, gives the value and direction of R_1 and R_2. The equation for giving these values vary with the form of arch, direction of loading, etc., and will not be given here. The temperature stresses have also to be considered.*

Fig. 27.

Two-Hinged Parabolic Arched Rib.—Let the curve ab (Fig. 28) be the neutral axis of the rib with end hinges at a and b. The "position line" may be determined from the formula $y = \frac{6.4\,c^2\,h}{5c^2 - x^2}$, for any value of x on either side of the vertical axis; c being the half span, and h the rise. The reactions R_1 and R_2 are obtained for any force P at point p by drawing the triangle of forces as above. These reactions must pass through or act at the hinges a and b. H_1 and V_1 are the horizontal and vertical components of R_1; H_2 and V_2 of R_2. The bending moment at any point m is equal to R_2 multiplied by the offset d. The axial thrust due to R_2 must also be considered in designing the flanges of the girder; and the shear, in designing the web. A table may be made showing the bending, compressive and shearing stresses at various points m along the girder due to loads P at successive positions, and summarized, being careful to use the proper + and − signs.

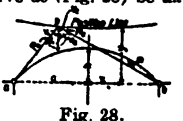
Fig. 28.

The above method is exact only for shallow, solid ribs of constant cross-section; but it is used also for plate girders and open ribs even when the flange plates do not extend to the hinges.

* See A Treatise on Arches by Malverd A. Howe; also Trusses and Arches, Part III Arches, by Charles E. Green.

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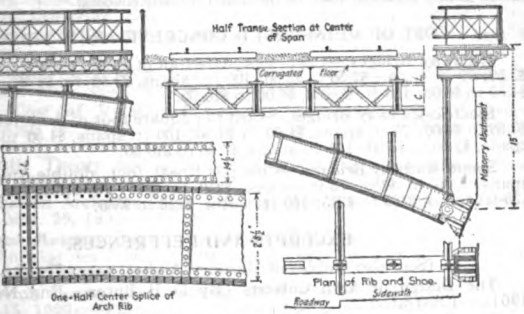
t of temperature is to increase the stresses very considerably, eason many engineers prefer the 3-hinged arch, not affected by and statically determinate like any simple structure.
and 30 illustrate a design of a 100-ft. span, 4-track railroad, l arch bridge:



Figs. 29. Half-Elevation, Half-Section and Details.

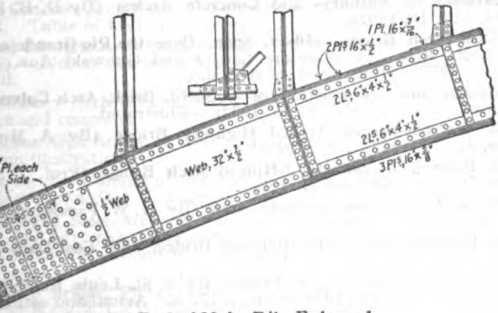


Fig. 30. End of Main Rib, Enlarged.

inged Arch.—Let Fig. 31 be a braced arch with hinges at crown ng; and let it be required to find the stress in the diagonal *cd*

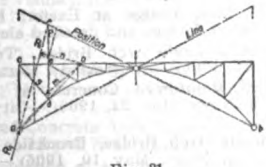


Fig. 31.

panel load P at e. Draw the Position Line as shown, and from the rectly above e, lay off by scale the load P, vertically. Then by of forces find R_1 acting in the direction ap. To find the stress s a cutting plane through the panel and find the origin O at the

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intersection of the other two active members *cv* and *ad*. Taking moments about *O*, of the forces acting at the left of the cutting plane, we have, using the lever arms *m* and *n*, $mS = nR_1$, or $S = \frac{n}{m} R_1$ (tension). Similarly, the stress in any member may be obtained for any loading.

COST OF REINFORCED CONCRETE ARCH BRIDGES.

Highway Bridges.—Cost per square foot of floor for 50-ft. spans, $2.00 to $2.50; 75-ft. spans, $2.50 to $3.50; 100-ft. spans, $3.50 to $4.50; 125-ft. spans, $4.50 to $6.00; 150-ft. spans, $6.00 to $10.00.

Electric-Railway Bridges.—Cost per square foot of floor for 50-ft. spans, $3.50 to $4.00; 75-ft. spans, $4.00 to $4.50; 100-ft. spans, $4.50 to $5.00; 125-ft. spans, $5.00 to $6.50; 150-ft. spans, $7.00 to $10.00.

Steam-Railway Bridges.—Cost per lineal foot, double track, for 50-ft. spans, $200 to $250; 75-ft. spans, $250 to $275; 100-ft. spans, $275 to $300; 125-ft. spans, $300 to $325; 150-ft. spans, $325 to $350.

EXCERPTS AND REFERENCES.

(See, also, Tables of Masonry Arches, pages 774, etc.)

The Design of Arch Culverts (By D. B. Luten. Eng. News, June 13, 1901).—Illustrated.

The Design of a Reinforced-Concrete Arch Bridge (By D. B. Luten. Eng. News, May 8, 1902).—Illustrated.

Stresses in Masonry- and Concrete Arches (By D. B. Luten. Eng. News, June 12, 1902).

Steel Arch Bridge, 450-Ft. Span, Over the Rio Grande on the Pacific Ry., Costa Rica (By Theodore Cooper and Gunwald Aus. Eng. News, Oct. 23, 1902).—Illustration of steel shoe.

Design and Construction of a 50-Ft. Brick Arch Culvert (By W. J. Douglas. Eng. News, Dec. 25, 1902).—Illustrated.

A Wooden-Braced Arched Highway Bridge (By A. Munster. Eng. News, Jan. 8, 1903).—Illustrated.

A Reinforced-Concrete 3-Hinged Arch Bridge (Prof. J. Melan. Eng. News, July 16, 1903).—Illustrated.

Stress Diagram of Concrete Arch (By H. W. Parkhurst. Eng. News, Nov. 12, 1903).—Illustrated.

A Reinforced-Concrete Highway Bridge, With Cost Data (P. A. Courtright. Eng. News, May 12, 1904).

Three-Hinged Steel Arch Trusses for a St. Louis Exhibition Building (Eng. News, Sept. 29, 1904).—Span, 172 ft. Actual and estimated weights are given.

A New Graphical Method for Stresses in 3-Hinged Arches (By J. W. Balet. Eng. News, Oct. 20, 1904).

The Connecticut Ave. Concrete Arch Bridge (By Geo. S. Morison. Eng. News, June 1, 1905).—Illustrated.

Three-Hinged Steel Arch Bridge at Exeter, England (Eng. News, July 20, 1905).—Illustrated details, and detailed elevation of half-rib.

Parabolic Reinforced-Concrete Arch Bridge (Trussed Concrete Steel Co. Eng. News, Nov. 15, 1906).—Illustrated; 75-ft. span.

Arch Rib Bridge of Reinforced Concrete at Grand Rapids, Mich. (L. W. Anderson. Eng. News, Mar. 22, 1906).—City Bridge; illustrated; cost data.

Three-Hinged Concrete Arch Bridge, Brookside Park, Cleveland (By H. F. Hackerdorn. Eng. News, May 10, 1906).—Illustrated details of hinges.

Graphical Method of Laying Out a 5-Centered Arch (By A. Swartz. Eng. News, May 10, 1906).

Empirical Formulas for Reinforced Concrete Arches (By D. B. Luten. Eng. News, June 28, 1906).—Illustrated.

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Low-Cost Concrete Culverts (By W. H. Whorley. Eng. News, July 5, 1906).—Tables.

Special Form of Arch Centering (By J. H. Milburn. Eng. News, Aug. 23, 1906).—Illustrated form for 50-ft. arch.

Reinforced Concrete Arch Bridge Built in Reinforced-Concrete Forms Without Centering (Eng. News, Aug. 30, 1906).—Illustrated.

Some 3-Hinged Concrete Arches in Germany (Eng. News, May 2, 1907).—Illustrated.

The Oakland Steel Arch Bridge Without Hinges, at Pittsburg (By Willis Whited. Eng. News, May 16, 1907).—Illustrated.

Itemized Cost of Reinforced Concrete Arches (By G. P. Carver. Eng. News, Aug. 22, 1907).—Table and diagram of costs. Illustration of a 100-ft. arch.

The Elastic Theory and a Faulty Arch (By H. S. Pritchard. Eng. News, Jan. 9, 1908).

A Combination Arch-Cantilever Concrete and Steel Bridge in France (Eng. News, Mar. 26, 1908).—Illustrated.

A 3-Hinged Masonry Arch with Metal Joints and Concrete Superstructure ("Annales des Ponts et Chaussees," Vol. 29, part 5, 1907; Eng. News, Sept. 10, 1908).—Illustrated.

A New Arch Curve, the Parabolic Oval (By C. Worthington. Eng. News, April 15, 1909).—Formulas and illustrations.

Subdividing An Arch Ring for Stress Analysis (By F. E. Turneaure. Eng. News, April 22, 1909).

A 259-Ft. Concrete Arch Bridge in Switzerland (Eng. News, Aug. 5, 1909).—Illustrated. Table of load and temperature stresses. Sand boxes used.

Reinforced Concrete Arch Bridges, Spans 281 ft. and 120 ft. (Eng. News, Sept. 2, 1909).—Illustrated, with plans of floor system, abutment and high retaining wall.

List of Masonry Arch Bridges over 175-Ft. Span. (Eng. News, Sept. 2, 1909).—Stone and concrete.

180-Ft. Stone Arch Bridge at Wiesen, Switzerland (Eng. News, Sept. 16, 1909).—Eleven illustrations.

Formulas for the Volume of Material in Groined Arches (By Chas. B. Buerger. Eng. News, Oct. 7, 1909).—Illustrated.

Novel 3-Hinged Steel Arch, in Greece (Eng. News, Nov. 4, 1909).—Railway arch, 193.5-ft. span. Illustrations:—Plan and elevation of viaduct; Details of half arch rib; Details of hinges at crown and springing; Scheme of erection.

Walnut Lane Bridge, Phila. (By G. S. Webster and H. H. Quimby. Trans. A. S. C. E., Vol. LXV., Dec., 1909).—Plans, including central concrete arch of 233-ft. span. Total cost of bridge, including electrical conduits and lamp standards, bush-hammering, and all extra work, was $267,000, which gives a rate of $7.60 per sq. ft. of floor surface and $0.880 per cu. ft. of space—area of profile by width of bridge.

Reinforced-Concrete Viaduct, Harrisburg, Pa. (Eng. News, Jan. 13, 1910).—The viaduct proper is 1841 ft. long, 78 ft. above ground at its highest point, and carries on 19 arches a 28-ft. roadway and two 8-ft. sidewalks. Full description, illustrated.

Tests of Model Concrete Arches by the New York State Engineer's Office ("Barge Canal Bulletin" for February, 1910; Eng. News, Mar. 10 and July 7, 1910).—Illustrated.

Failure of Reinforced-Concrete Arch Highway Bridge (Eng. News, Mar. 17, 1910).—Illustrated. 75 and 90-ft. spans.

The Analytical Calculation of a Concrete Arch (By Malverd A. Howe. Eng News, May 12, 1910).—Illustrated. Discussions: Horizontal thrust; Bending moments at the supports; Vertical reactions; Dead load; Live load; Temperature; Effect of direct stress; Changes in dimensions. There are nine tables for use in making calculations. Article continued in Eng. News, June 2, 1910.

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The Meadow St. Reinforced-Concrete Arch Bridge, Pittsburg, Pa. (By N. S. Sprague. Eng. News, Dec. 1, 1910).—Description with 8 illustrations. The length of the main arch span is 209 ft. with a rise of 46.14 ft. and consists of three arch ribs, the two outside ribs being uniformly 3 ft. 9 ins. in thickness and the central rib 5 ft. and all three ribs having a depth varying from 5 ft. at the crown to 6 ft. 2 ins. at the springing line.

The New Charles River Bridge, Boston Elevated Railway (Eng. Rec., Dec. 17, 1910).—A reinforced-concrete arch bridge of five arches of 122-ft. 4-in. clear span, four of 98-ft. 4-in. clear span, a steel lift bridge at the river lock and a special span of 125 ft. 4 ins. at the small boat lock. The special illustrations of the 98-ft. span, accompanying this descriptive article are:—Details of hinge, including arch reinforcement and plan of skewback; longitudinal section of half span, part plan and details; reinforcement around stringers and of floorbeam.

Illustrations of Recent Arch Spans.

Description.	Eng. News.
Steel A-arch, 3-hinged, 180-ft. span, 51-ft. rise	Apr. 21, '10
Arch dam design for the site of Shoshone dam	June 9, '10
Wooden arch centering for 144-ft. masonry arch, Norway	July 7, '10
3-hinged rein-conc. arch, 88-ft. span, Paris	Sept. 15, '10
	Eng. Rec.
Details of centering trusses for cathedral stone arches	Jan. 23, '09
Table of stresses in 280-ft. span, concrete arch bridge	Jan. 23, '09
Rein.-conc. arch bridge, Grand River, L. S. & M. S. Ry	Apr. 24, '09
3-hinged centers for building 150-ft. concrete arches	Apr. 24, '09
A combination steel and concrete arch bridge, 250-ft. span	May 22, '09
Reinforcement and erection of concrete arch	June 12, '09
139-ft. rein.-conc. arch, Edmondson Ave. bridge, Baltimore	June 19, '09
364-ft. steel spandrel-braced arches and details	June 26, '09
Elastic rein.-conc. railway arch span, 97½ ft., rise 36¾ ft.	June 26, '09
Engineer's and contractor's falsework, Edmondson Ave. bridge	Aug. 14, '09
Short-span (34 to 44 ft.) rein.-conc. R. R. bridges	Mar. 19, '10
Determination of wind stresses in 3-hinged arches—Eusink	May 21, '10
Rein.-conc. R. R. viaduct, five 120-ft. and two 100-ft. spans	July 16, '10
Details three 87.5 ft. rein.-conc. arch highway spans, Los Angeles	Aug. 13, '10
Details of plate girder and rein.-conc. arch (70-ft. span) R. R. bridge	Aug. 20, '10
Highway viaduct—28 70-ft. rein. conc. arch spans	Oct. 15, '10
Proposed 864-ft. arch across the Kentucky River	Nov. 26, '10
149-ft. rein.-conc. arch for electric R. R., Maritime Alps	Dec. 3, '10
Centerings for 80-ft., 90-ft., 100-ft. stone arches, B. & O. viaduct, Brand. Cr	Dec. 17, '10

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45.—TRESTLES.

A Trestle is a bridge composed of a series of relatively short beam- or girder spans resting on "bents," which take the place of ordinary piers.

Pile Trestles.—Where the bents are composed of piles it is called a pile trestle or pile bridge. Each bent may consist of any number of piles driven in line transversely with the axis of the bridge, the number required depending upon the kind of bridge, the width, loading, height of trestle, kind of soil, lateral stiffness required, whether on tangent or curve, etc. Generally, the piles are driven vertically but where great stiffness is required the end piles of each bent are driven commonly on a batter. In fact, this latter practice is usual with some railroads even for low trestles on tangents. The cost of driving such batter piles is not excessive, as the gins of the driver may be arranged easily so as to swing laterally on a pivot like a pendulum. For a foot bridge, two or more piles are used to the bent; for a highway bridge, three or more; and for a railroad bridge, four or more.

The maximum load on a pile should be limited to about 40,000 lbs., and a load of 25,000 to 30,000 lbs. is preferable, usually. If driven in soft material a less load should be used. Again, the loading on pile may be limited to the allowable compression "across grain" of the wooden cap resting upon it. Piles should be peeled before driving unless they are driven in salt water.

Wooden caps are usually 12 x 14 ins. for railroad bridges; 12 x 12, 10 x 12, 10 x 10, etc., for highway bridges—with greater dimension vertical. The piles when driven are sawed off on a plane, either level or inclined (to give proper elevation to outer rail if on a curve), and the caps are drift-bolted to them. The usual size of drift bolt is ¾ to ⅞ in. in diam., and 18 to 22 ins. long. Holes are first bored with an auger about ⅛ in. smaller in dia. than the bolt.

Diagonal bracing for each bent consists usually of two planks, one on either side of bent and crossing at the middle line, with upper ends bolted to caps near their outer ends, and with lower ends bolted to the outer piles —using ¾-in. screw bolts and cast washers. At intersection with the inner piles the braces are fastened to same with two wrought spikes with length at least twice the thickness of the planks. Similarly, horizontal "sash" braces may be fastened to the piles on both sides of the bent at foot of diagonals. For railroad trestles, 4 x 10 in. bracing is common; and for highway bridges, 3 x 10, 3 x 8, 2 x 10, 2 x 8, etc. High trestles are double braced.

Standard Plans for Pile Trestles.

Each railroad has its own standards, differing in certain essential details from those of other roads, such, for instance, as spacing of stringers and jack stringers, arrangement and size of guard rails, and the design of the floor in general.

Fig. 1 is an elevation of bent of single track trestle of the Oregon Short Line R. R., showing ballast floor.

Timber Trestles.—The term "frame trestles" is applied to those trestles constructed of framed timbers, usually sawed to "dimension," but sometimes consisting of straight, round poles or piling timbers denuded of the bark. The latter construction has been used considerably in the Pacific-Northwest. Where such poles are used the posts are commonly in one length, even for very high trestles.

Ordinarily, however, the bents of high trestles are constructed in sections, vertically: A single-deck trestle is one in which the bents are in one section; double-deck, in two sections; 3-deck, in three sections; etc. Fig. 2 is an illustration of a 3-deck trestle showing the right half resting on piling, and the left half resting on mud-sills, the latter being used in cases where piles cannot be driven or where the cost of driving is prohibitive. The mud-sills should be of cedar, about 8 x 12 ins. by 4 ft. long, and laid side by side on a well tamped foundation thoroughly drained and free from

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wash. Instead of *driving* piles, "false" piles, *set* by hand, are sometimes employed. Concrete or rubble masonry piers are desirable where a suitable sub-foundation is presented cheaply.

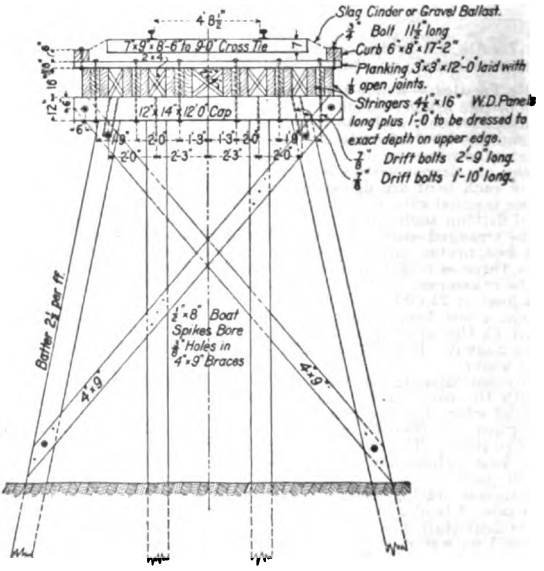

Fig. 1. (See page 787.)

Fig. 2 illustrates a more or less typical railroad trestle which will be described briefly: The main posts, 12 x 12 ins., have a continuous batter of 3:12 and 1:12. They are "dapped" into the caps and sills about ½ in., drift-bolted at tops, and doweled (with 1-in. round iron) and toe-nailed (with wrought spikes) at feet. (Mortise and tenon joints are no longer used.) The "batter" posts, used below the top deck, are likewise framed, drift-bolted, doweled and toe-nailed. They give lateral stiffness as well as vertical support. The main caps and sills are 12 x 14 ins., while the intermediate caps and sills are 12 x 12 ins., extending 12 ins. or more beyond the outer edge of posts. Each section of the bent is sway-braced with 4 x 10 in. plank, screw-bottled at ends, and spiked at intersections. The bracing between the bents consists of 12 x 12 in. longitudinal girts *G*, framed into the intermediate caps and sills, with lap joints, and thoroughly drift-bottled to them; and the longitudinal diagonal braces *L* consist of 4 x 10 in. plank, screw-bottled at ends. The floor of the bridge rests on two lines of main stringers and the two lines of jack stringers, the latter being used as safety stringers in case of derailment. Main stringers are ordered in 2-span lengths, that is, for bents 15 ft. centers they are ordered 30 ft. long, laid two, three or four to each line with alternate and butt joints, dapped over the caps and drift-bolted thereto. They are separated about 1 inch laterally by cast spool separators through which the screw bolts are inserted (using cast washers), four to each joint over the caps. Between the lines of stringers a 4-in. plank of proper length is spiked to top of caps to preserve the required stringer spacing. The jack stringers are ordered one foot longer than the main stringers (or say 32 ft. long) so that the joints can be "halved" and drift-bolted to caps with one

bolt. The practical sizes of main stringers are 8 x 14, 9 x 16, 10 x 18, using two or three (or more) stringers under each rail. The width of jack stringer should be not less than 6 ins., and it should not be less than about one-third the width of one line of main stringers. High trestles are provided usually with outer guard rails or "bull" rails, which have, in certain cases, prevented trains from plunging over the side of the bridge. These bull rails are preferably about 12 ins. high, and 10 x 12 in. timbers are commonly used. They may be ordered in any convenient lengths, are halved like the jack stringers, and secured firmly by screw bolts passing vertically through bull rail, tie and jack stringer. In addition to the bull rails or "outer" guard rails, there should be "inner" guard rails. The inner guard rails are usually common rails placed inside the main rails, but wooden guard rails, say 5 x 8 ins. (5 ins. vertical), dapped over the ties to preserve proper spacing of the

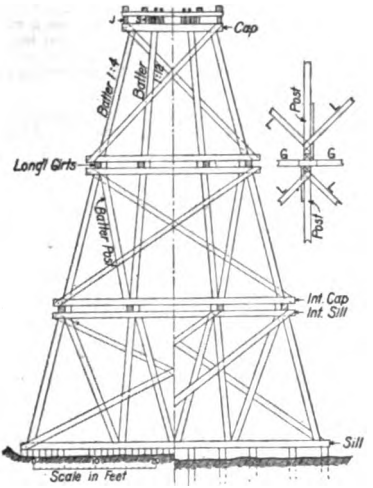

Fig. 2.

latter, are frequently used. They are secured to the ties by wood or lag screws (8 or 9 in.) with heads flush with top of guard and bearing on flat washers. Sections of bents are from 20 to 24 ft. in height and uniform from top of bridge downward so that the various decks are on planes parallel with the track. Longitudinal diagonal bracing may generally be omitted between alternate pairs of bents.

Many modifications may be made of Fig. 2. For instance, the inner main posts may be vertical; all the main posts may be continuous (each composed of one stick, or of two sticks bolted together with alternate joints) and the transverse diagonal bracing divided by horizontal sash braces spaced vertically about the depth of one deck, with longitudinal girts bolted at intersections of sash braces and posts; instead of having separate intermediate caps and sills, one piece may serve to act as the cap of the section below as well as the sill of the section above; etc.

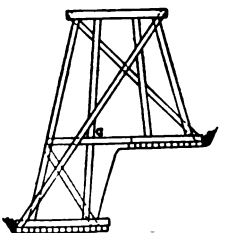

Fig. 3.

Grass-hopper bents, Fig. 3, are often used with economy on side-hill work; either to save cost of excavation or to avoid encroachment on ad-

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joining property. The writer has used as many as three broken sills to the bent with perfect safety and even without concrete backing to resist thrust. At *a*, the upper broken sill is dapped into the post one inch and sash braces are spiked and bolted on as shown in Fig. 3. Careful attention should be paid to drainage.

The bents of wooden trestles are spaced usually from 12 to 16 ft. centers, the ordinary spacing being, perhaps, 15 ft., calling for 30-ft. stringers which are shipped conveniently on the average flat car. Unlike steel trestles the spacing is constant or nearly so throughout the bridge, regardless of the variation in height of bents. Where long spans are introduced in a wooden trestle, as for spanning a creek, the supporting bents are usually doubled or tripled.

On curves, the caps are inclined in order to give proper elevation to the outer rail. This method is preferable to using level caps, with shins under the stringers or on the ties, etc., as is sometimes done. The writer generally supplies the foreman of the framing crews with tables giving the increased lengths of posts on outside of curve and the decreased lengths on inside of curve for each bent, to provide for the proper elevation of the outer rail and a like depression of the inner rail, each being one-half of the required "elevation."

Pile and Timber Trestles.—A pile and timber trestle is one in which the piling of the foundation is cut off high enough above the ground to

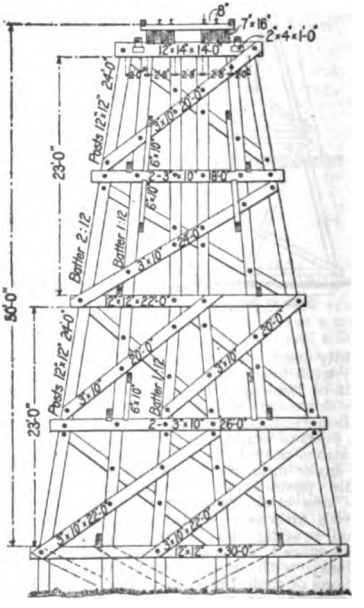

Fig. 4.

constitute the lower deck, and on which the timber trestle proper is erected. Sometimes, however, the term is applied to a frame or timber trestle with simply a pile foundation.

Fig. 4 is a section of a pile and timber trestle of the A., T. & S. F. R. R.

LE BENDING MOMENTS IN FT.-LBS. ON WOODEN STRINGERS.

By Formula, $M = \frac{1}{6} f b d^2 \div 12$

f = allowable outer fiber stress in lbs. per sq. in.;
b = breadth of stringer, in ins.;
d = depth of stringer, in ins.

[Values of M in Ft.-Lbs.]

Stringer.		2 Stringers.		3 Stringers.		4 Stringers.	
900.	f=1000.	f=900.	f=1000.	f=900.	f=1000.	f=900.	f=1000.
200	3 556	6 400	7 111	9 600	10 667	12 800	14 222
200	21 333	38 400	42 667	57 600	64 000	76 800	85 333
400	24 889	44 800	49 778	67 200	74 667	89 600	99 556
600	28 444	51 200	56 889	76 800	85 333	102 400	113 778
800	32 000	57 600	64 000	86 400	96 000	115 200	128 000
000	35 556	64 000	71 111	96 000	106 667	128 000	142 222
050	4 500	8 100	9 000	12 150	13 500	16 200	18 000
300	27 000	48 600	54 000	72 900	81 000	97 200	108 000
350	31 500	56 700	63 000	85 050	94 500	113 400	126 000
400	36 000	64 800	72 000	97 200	108 000	129 600	144 000
450	40 500	72 900	81 000	109 350	121 500	145 800	162 000
500	45 000	81 000	90 000	121 500	135 000	162 000	180 000
600	54 000	97 200	108 000	145 800	162 000	194 400	216 000
000	5 556	10 000	11 111	15 000	16 667	20 000	22 222
000	44 444	80 000	88 889	120 000	133 333	160 000	177 778
000	50 000	90 000	100 000	135 000	150 000	180 000	200 000
000	55 556	100 000	111 111	150 000	166 667	200 000	222 222
000	66 667	120 000	133 333	180 000	200 000	240 000	266 667

are directly proportional to width of stringer, and to (depth)^2.
.—Find the size of wooden stringers required for 15-ft. spans,
engine load to give a maximum bending moment of 298500
rack, and assuming the floor to weigh 450 lbs. per lin. ft. per
he stringers 420 lbs. per lin. ft. of track).

—Total bending moment $= 298500 + \frac{870 \times (15)^2}{8} = 322969$ ft.-lbs.

61485 ft.-lbs. per rail. From the above table this calls for 4—
gers under each rail, if the outer fiber stress is assumed at 900.

stles.—The proper and economical design of a steel trestle for
ty calls for a thorough examination of the site. In addition to
nd contour map, borings may be required to determine the
the sub-foundation. The spans are longer than for wooden
are made to increase with the height of the structure so that
rule "The cost of piers should equal cost of spans†" will apply

nctive feature of steel trestles is that there are few posts to the
t, two posts are sufficient for single track railroad trestles.
gh trestles the bents are usually arranged in pairs, each pair
horizontally and diagonally, both longitudinally and trans-
sometimes laterally, really forming a steel pier.
consideration in design it may be noted that, as far as the floor
he spans) alone is concerned, that floor is the most economical
ns are of equal length; that is, for any given number of supports
ts should be spaced equidistant throughout to give the least cost
t taking the structure as a whole we have *second* that the cost of
ether with its foundation, should equal the cost of the portion
which it supports (Fig. 5). Hence the profile, character of
etc., have considerable influence in varying the lengths of the
applies as well to bents as to piers. In testing the economy
n let us assume any portion as DG to be fixed and equal
$+ l_6$. Now any small *increase* in l_5 will require an increase in

imately; see Sec. 37, Bridges, page 683.

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the amount of material in both the span and bracing between bents B_5 and B_6, and a decrease in material in spans S_4 and S_6. Conversely, any *decrease* in l_5 will produce the opposite effect. Hence, l_5 should be such that for any small change in its length the increase in material on the one

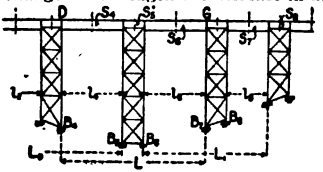

Fig. 5.

hand will just equal the decrease on the other. But note that in the similar adjustment of adjacent portions the above spans may require re-adjustment; and likewise, all other sections. We have also to take into consideration the best *relative* lengths of l_2, l_4, l_6, etc. Tables or diagrams showing the weights of spans, bents, and bracing for various lengths and heights for the particular loading will greatly facilitate the above adjustments.

The floor system is arranged similar to that of an ordinary steel bridge, that is, with floor beams and stringers, the former being supported directly by the bents. For short spans, I-beams may be used; for longer spans, plate girders. For very long spans both floor beams and stringers may be trussed, using deck girders for this purpose.

The posts of the bents (Fig. 6.) are battered about 1 : 6 — considerably less than for wooden trestles. This narrowing of the space between posts tends to call for less metal in the transverse bracing; for more metal in the posts, from wind stresses; but for slightly less metal in the posts from direct loads. The best designs have stiff diagonal bracing instead of rods, and large members with few connections are preferable. The feet of columns should be anchored. The wind stresses may be calculated by treating the bent as a cantilever arm, assuming all the joints to be hinged; but if the framework is stiffened by heavy gusset plates, and riveted connections are used, it becomes statically indeterminate.

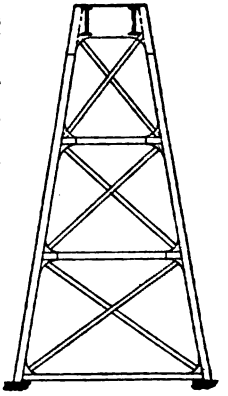

Fig. 6.

Elevated Railroad Trestles.—For a full discussion of this subject the reader is referred to Paper No. 806, Trans. Am. Soc. C. E., June, 1897, by Mr. J. A. L. Waddell.

Reinforced Concrete Trestles.—The outline shown in Fig. 6, above, for steel trestles, is a good design for a high trestle bent of reinforced concrete. It can also be modified as follows: (a) By omitting the horizontal braces, using the diagonal bracing only; (b) by omitting the diagonal braces, using the horizontal bracing only; (c) by omitting all bracing. In case (b), however, the mushroom connections of horizontal braces to posts must be of the large gusset type, in order to introduce bending resistance at ends of braces; and similarly at the floor-beam connections. Case (c) should be used for short bents only.

The posts and bracing should be calculated for live-, dead- and wind loads; for centrifugal force due to moving load if trestle is on a curve (see page 702); and for longitudinal thrust or momentum due to stopping of trains (page 702).

For proportioning the members, see pages 585 and 609; also page 446 for beam and column formulas.

References to numerous designs and details may be found on the following page.

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COST OF RAILROAD TRESTLES.

Timber Trestles.—(a) *Single Track.* Cost in dollars per lin. ft. (approx.) $=8+0.2H+0.002H^2$. *Double Track.* Cost in dollars per lin. ft. (approx.) $=16+0.4H+0.003H^2$. In which H = height of trestle bent, in feet.

Steel Trestles.—About three to five times the cost of timber trestles.

Reinforced Concrete Trestles.—Low trestles cost about the same as small arch spans. See page 784.

EXCERPTS AND REFERENCES.

Wooden Trestles, Utah Central Ry. (Eng. News, Jan. 17, 1901).—Illustrated.

Steel Trestle Viaduct, C. & N.-W. Ry. (Eng. News April 22, 1901).—Illustrated.

Railway Trestle Bents of Reinforced Concrete (By W. A. Allen. Eng. News., Mar. 12, 1903).—Illustrated.

Reinforced-Concrete Trestlework Viaduct for a Spanish Mineral Ry. (Eng. News, May 17, 1906).

Reinforced-Concrete Viaduct on the Richmond & Chesapeake Bay Ry. (Eng. News, Dec. 12, 1907).—Illustrated.

A Traveler for Viaduct Erection (By L. L. Jewel. Eng. News, Oct. 8, 1908).—Illustrated.

The Bear-River Steel Viaduct, Cal. (Eng. News, Mar. 11, 1909).—Illustrated details.

Table for Estimating Quantities in Timber Trestles (By Emile Low. Eng. News, April 22, 1909).

Reinforced-Concrete Viaduct with Some Structural Steel Reinforcement (Eng. News, July 1, 1909).—Tower bents composed of two posts, 18 x 18 ins. square at the high bents and 15 x 15 ins. at the low, spaced 12 ft. c. to c. at the girders and extending on an outward batter of 1 : 6 to spread foundations on solid rock. Posts are concrete reinforced at the four corners by straight round rods, encircled every 12 ins. with ¼-in. wire. Longitudinal and transverse braces are 9-in. and 12-in. I-beams encased in concrete. Illustrated.

Reinforced-Concrete Trestle, Pasadena, Cal. (Eng. News, Nov. 18, 1909).—Consists of six girder spans resting on five tower bents; whole length divided into panels of 17 ft. 3 ins., each bent being of that length, and each span divided into three of such spaces by the floor beams, making a span length for each girder of 51 ft. 9 ins. Towers consist of four columns each; columns 18 x 18 ins. and reinforced with eight 1¼-in. round steel bars extending into the pier footings to within one foot of their base. Nearly all the columns are over 50 ft. long and have longitudinal and transverse struts framing into their third points to stiffen them. Struts vary in size from 10 x 18 ins. to 12 x 24 ins. and are reinforced with four ¾-in. or four 1-in. twisted steel bars laced as a column. Illustrations include details of expansion joint.

Important Illustrations of Trestles and Details.

Description.	Eng. News.
A 5-mile railway trestle across Albemarle Sound	Apr. 21, '10
Willow Creek Viaduct, Des Chutes Railway	Aug. 11, '10
	Eng. Rec.
Stresses in typical tower of a railway trestle	Jan. 9, '09
Design, construction and cost of a rein.-conc. trestle	Feb. 20, '09
Details reinforcement, rein.-conc. R. R. trestle	Apr. 3, '09
The Sligo reinforced-concrete highway viaduct	May 15, '09
Typical tower bent, manuf'rs railway steel viaduct	May 15, '09
A substitute for drift bolts on wooden trestles	June 5, '09
Details of steel trestle, Norfolk & Western Ry	Mar. 19, '10
Details 6-post and 4-post pile R. R. trestle, Albemarle Sound	Apr. 30, '10
Standard solid floor railroad trestles	Oct. 29, '10

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46.—ROOFS.

Wind Pressure.—The problem of wind pressure which indirectly presents itself to the engineer consists in discovering the existing relation between the *velocity* of the wind and its *pressure* against any surface—right, oblique, plane, curved, etc. Then, knowing the maximum velocity of the wind in the particular locality in which a structure is to be erected, the probable pressure on its surface is deduced within a reasonable degree of accuracy by this ratio. The subject never has been treated satisfactorily on the grounds of pure theory, while the few practical experiments recorded seem to give results not entirely in accord with each other nor with any theory yet advanced.

Velocities Attained.—The velocity of a volume of air moving along the surface of the earth increases with its distance above the average surface, hence high structures, or those in exposed positions, should be designed to resist the greater wind pressures in any locality. Prof. Henry claims that on Mt. Washington, N. H., 150 miles per hour has been recorded. This probably exceeds by over 50% the maximum velocity ever attained at the average surface of the ground in that State. The tornado which tore up a portion of the St. Louis Bridge floor is credited with but 120 miles per hour. A hurricane such as occasionally visits the Atlantic coast may attain a velocity of 60 miles per hour upward, depending upon exposure. From 90 to 100 miles per hour is probably the maximum velocity ever attained in New York City in the most exposed positions. The Pacific coast is never visited by the violent hurricanes incident to other sections of the country.

Direct Wind Pressure.—When the atmosphere is at rest it exerts a pressure at sea level of about 14.7 lbs. per square inch; while a cubic foot of dry air under one atmospheric pressure (760 millimeters of mercury) weighs about 0.081 lb. per cubic foot. Wind is air in motion caused by a tendency to restore equilibrium in atmospheric pressure, at about the same level, by air rushing in to replace a heated and rising atmosphere in another locality. Hence, the directing force is a *tension* or tendency to partial vacuum "ahead" of the wind, as well as a compression from behind. It is well to bear this in mind, as explaining in part at least the uplifting or overturning power exerted on roofs, due to *suction*. This suctional power is not well known, and its effect should be considered more in future experiments and investigations. Some attempts have been made to deduce a rule for pressure, based on the weight of the volume of air moving against the exposed surface, taking into account temperature, humidity and barometric pressure, but the results have been more or less unsatisfactory.

Smeaton, 150 years ago, made some crude experiments on wind pressure in connection with the power of windmills, and constructed a table from the formula

$$P = .005\ V^2 \qquad (1)$$

in which P = horizontal normal pressure in lbs. per sq. ft.,
and V = velocity of wind in miles per hour.

This formula seems to agree fairly well with many experiments on small surfaces. Another formula, of the same form as Smeaton's, giving results 20% less, is used considerably:*

$$P = .004\ V^2 \qquad (2)$$

In both of the above the pressure is assumed to be proportional to the *square* of the velocity. Lieut. Crosby's experiments near Baltimore, Md. (*Engineering*, June 13, 1890) to determine the resistance of the air to fast moving trains seem to indicate that the pressure P is directly proportional to the velocity V, and not to V^2. But this conclusion is generally discredited.

The conclusion from Baker's experiments in connection with the construction of the Forth Bridge is that the pressures given by Smeaton's formula (1) are too great for high velocities. In the light, or perhaps better

* This formula is now used by the U. S. Signal Service.

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"darkness," of modern experimental data, the pressure P may be assumed to lie somewhere between the values given by formulas (1) and (2)—near the former for low velocities, and near the latter for high velocities. These values are deduced in the following table; and two columns are added giving results from the experiments of Eiffel and Stanton.

1.—DIRECT NORMAL WIND PRESSURES.

Velocity V of Wind in Miles per Hour.	$P = .005\ V^2$ (1) Pressure P Lbs. per Sq. Ft.	$P = .004\ V^2$ (2) Pressure P Lbs. per Sq. Ft.	$P = .003\frac{1}{2}\ V^2$ (sm. areas) Pressure P Lbs. per Sq. Ft.*	$P = .003\ V^2$ (lg. areas) Pressure P Lbs. per Sq. Ft.*	Remarks.
10	.50	.40	.33	.30	
20	2.00	1.60	1.33	1.20	Brisk wind.
30	4.50	3.60	3.00	2.70	
40	8.00	6.40	5.33	4.80	High wind.
50	12.50	10.00	8.33	7.50	
60	18.00	14.40	12.00	10.80	Violent storm.
70	24.50	19.60	16.33	14.70	
80	32.00	25.60	21.33	19.20	Hurricane.
90	40.50	32.40	27.00	24.30	
100	50.00	40.00	33.33	30.00	Violent hurricane.
110	60.50	48.40	40.33	36.30	
120	72.00	57.60	48.00	43.20	Tornado.

Before considering the resolution of a direct wind pressure into its normal, vertical and horizontal components, as practically applied to roofs and other structures, it will be well to emphasize the above hints regarding pressure and tension (suction) on any exposed *body*. If the surface of a thin flat sheet is exposed to the direct force of the wind, there will be tension or suction on the leeward face, due to partial vacuum, thus producing apparently additional pressure on the windward face and as these forces act in the same direction, the resultant "wind pressure" is thereby increased. The tension may be reduced by placing a long tapering projection on the leeward face of the plate, flush with the edge, to prevent the formation of air eddies. Further, if the windward face is convex, the pressure also will decrease, while if it is concave the pressure will increase. The thickness of the plate within certain limits is also a factor, as well as the density and humidity of the atmosphere.

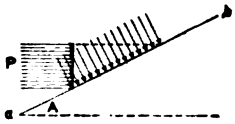

Fig. 1.

Normal and Component Wind Pressures.—Let P, Fig. 1, be the direct horizontal pressure per square foot on any vertical surface, and P_n the normal pressure on the *same unit* of inclined surface, sloping at angle A with the horizontal. Then—

By Abstract theory, $P_n = P \sin^2 A$ ..... (3)

By Hutton's experiments, $P_n = P\ (\sin A)^{1.84 \cos A - 1}$ ..... (4)

By Duchemin's formula, $P_n = P \dfrac{2 \sin A}{1 + \sin^2 A}$ ..... (5)

The following table gives values of P_n deduced from these three formulas, assuming $P = 50$, 40 and 30 lbs. Use Hutton or Duchemin.

*From experiments by M. Eiffel and Dr. Stanton; on velocities of 40 to 90 miles per hour. See, also, remarks under Railroad Bridges, Section 38, page 697.

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2.—NORMAL WIND PRESSURES P_n ON INCLINED SURFACES.*
For horizontal pressures of 50, 40 and 30 lbs.

Angle, A.	Pitch	50 Lbs.			40 Lbs.			30 Lbs.		
		(3) Theory.	(4) Hutton.	(5) Duchemin.	(3) Theory.	(4) Hutton.	(5) Duchemin.	(3) Theory.	(4) Hutton.	(5) Duchemin.
5°		0.4	6.5	8.61	0.3	5.2	6.89	0.2	3.9	5.17
10°		1.5	12.1	17.00	1.2	9.7	13.59	0.9	7.2	10.19
15°		3.3	17.8	24.15	2.7	14.2	19.82	2.0	10.7	14.49
18°-26′	1-6	5.0	21.2	28.75	4.0	17.0	23.00	3.0	12.7	17.25
20°		5.8	23.0	30.30	4.7	18.4	24.24	3.5	13.8	18.18
21°-48′	1-5	6.9	24.8	32.66	5.5	19.8	26.13	4.1	14.9	19.60
25°		8.9	28.3	35.96	7.1	22.6	28.77	5.4	17.0	21.58
26°-34′	1-4	10.0	29.7	37.27	8.0	23.8	29.82	6.0	17.8	22.36
30°		12.5	33.1	40.00	10.0	26.5	32.00	7.5	19.9	24.00
33°-41′	1-3	15.4	36.6	42.42	12.3	29.2	33.93	9.2	21.9	25.45
35°		16.5	37.6	43.15	13.2	30.1	34.52	9.9	22.6	25.89
40°		20.7	41.6	45.50	16.5	33.3	36.40	12.4	25.0	27.30
45°-00′	1-2	25.0	45.0	47.16	20.0	36.0	37.73	15.0	27.0	28.30
50°		29.3	47.6	48.30	23.5	38.1	38.64	17.6	28.6	28.98
55°		33.6	49.3	49.01	26.8	39.4	39.21	20.1	29.6	29.41
60°	1-1	37.5	50.0	49.68	30.0	40.0	39.74	22.7	30.0	29.81
65°		41.1	50.0	49.78	32.9	40.0	39.82	24.6	30.0	29.87
70°		44.2	50.0	49.89	35.3	40.0	39.91	26.5	30.0	29.93
75°		46.7	50.0	49.95	37.5	40.0	39.96	28.0	30.0	29.97
80°		48.5	50.0	50.00	38.8	40.0	40.00	29.1	30.0	30.00
85°		49.6	50.0	50.00	39.7	40.0	40.00	29.8	30.0	30.00
90°		50.0	50.0	50.00	40.0	40.0	40.00	30.0	30.0	30.00

*The normal pressures given in the table are in lbs. per square foot of inclined surface of exposed roof—from horizontal winds producing pressures of 50, 40 and 30 lbs. per square foot on vertical surfaces.

Example.—Assuming direct wind pressure to be 50 lbs. per square foot of vertical surface, and using Hutton's formula, find from the above table the normal pressure in lbs. per square foot on a roof with a pitch of one in two.

Answer—45 lbs. per square foot.

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In designing roofs and buildings it is convenient to use the normal pressure against the roof surface, and to know also its vertical and horizontal components. The subjoined table gives these values for the 5 standard pitches* of roofs, and based on Hutton's and Duchemin's formulas at 50, 40 and 30 lbs. direct wind pressure. See table 2, preceding.

3.—WIND PRESSURES IN LBS. PER SQUARE FOOT ON ROOFS.

Formula.	Pitch.*	Angle *A*.	50.			40.			30.		
			Normal.	Horizont'l	Vertical.	Normal.	Horizont'l	Vertical.	Normal.	Horizont'l	Vertical.
Hutton.	1-6	18°-26′	21.2	6.7	20.1	17.0	5.4	16.1	12.7	4.0	12.0
	1-5	21°-48′	24.8	9.2	23.0	19.8	7.4	18.4	14.9	5.5	13.8
	1-4	26°-34′	29.7	13.3	26.5	23.8	10.6	21.3	17.8	8.0	15.9
	1-3	33°-41′	36.6	20.3	30.5	29.2	16.2	24.3	21.9	12.1	18.2
	1-2	45°-00′	45.0	31.8	31.8	36.0	25.5	25.5	27.0	19.1	19.1
Duchemin.	1-6	18°-26′	28.8	9.1	27.3	23.0	7.3	21.8	17.3	5.5	16.4
	1-5	21°-48′	32.7	12.1	30.4	26.1	9.7	24.2	19.6	7.3	18.2
	1-4	26°-34′	37.3	16.7	33.4	29.8	13.3	26.7	22.4	10.0	20.0
	1-3	33°-41′	42.4	23.5	35.3	33.9	18.8	28.2	25.5	14.1	21.2
	1-2	45°-00′	47.2	33.4	33.4	37.7	26.7	26.7	28.3	20.0	20.0

Probably the low direct pressure value of 30 lbs. per square foot, reduced by Hutton's† (or Duchemin's) formula will be sufficient for general cases; the next higher value, 40 lbs., for particularly exposed positions; and the highest value, 50 lbs., for special cases as in the tornado belts.

In open sheds the maximum direct pressure may be assumed as acting normal to the inside leeward surface, and the "lifting" force may be obtained by multiplying the total direct pressure by cos *A*, the angle of inclination of roof with the horizontal.

On a *cylinder* the theoretical pressure is ⅔ that on a corresponding plane diametrical section or rectangular plate; but Borda, by experiment, found it to be only 0.57. Likewise, he found the pressure on a sphere to be but 0.41 of that on a corresponding circular plate of the same diameter, while theory gives ½. In general, the pressure on a concavity, measured by the diametrical plane surface, is greater than unity; while on a convexity it is less than unity. The intensity of pressure may be increased by the deflection of the wind from an adjacent structure.

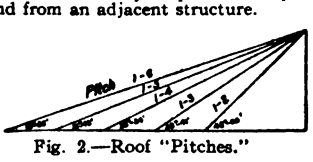

Fig. 2.—Roof "Pitches."

Snow Loads.—The snow loads which may come on a roof will vary with the latitude of the place; its altitude above sea level; the general humidity of the atmosphere; the winter temperature; the location with respect to mountain ranges; the pitch of the roof (Fig. 2); the character of roofing. The writer is familiar with the character of the snow fall in nearly every State in the Union and in Canada For the Pacific slope west of the Coast and Cascade mountain ranges, there is no need to provide for any snow-load, but up *in* these ranges and in Eastern Washington, Eastern Oregon, Northern California, and in all sections eastward to the Atlantic coast, provision must be made. The heaviest snow-falls in the United

* The pitch of a roof is one-half the natural tangent of inclination with the horizontal.

† The writer believes Hutton's formula, founded on practical experimental data, to be quite well established and fairly reliable. Duchemin's formula gives values about 25% higher for the ordinary 1-4 pitch.

States are in the Central Northwest, New England and the Rocky Mountain regions. The following diagram, giving the snow load per horizontal square foot of roofs for different localities and for standard roof pitches, will be found practically reliable. The latitude of the place is considered as increased one degree for each thousand feet in altitude above sea level.

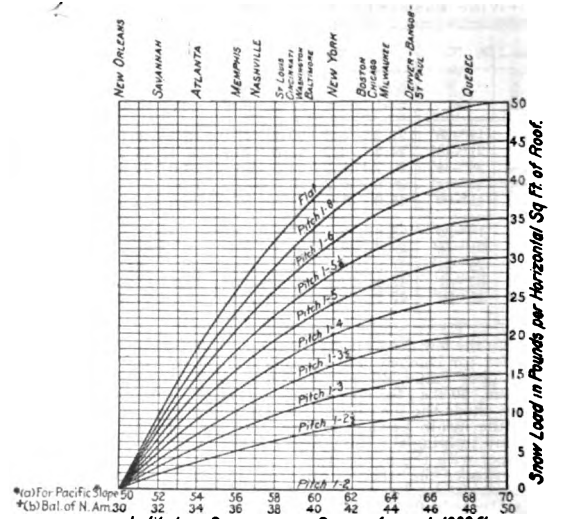

Fig. 3.

Example.—To find the snow load in pounds per horizontal square foot at Denver for a pitch of 1 in 5? The latitude of Denver, 40, plus $\frac{5}{1000}$ of its altitude in feet above sea level, 5, gives 45. Using line (b) of above diagram this is equivalent to a snow load of 28.2 lbs. per horizontal square foot for a pitch of 1–5.

Investigations by S. de Perrot, in Switzerland, according to the "*Engineer*" (London), show that where a heavy fall of snow is followed by thawing and freezing and then more snow, in repeated cycles, the laminar mass of snow and ice will have a weight of 36 to 38 lbs. per cu. ft.; and the thickness of the mass on the roof, from 24 to 32 ins., will produce a load of 70 to 100 lbs. per sq. ft., about 2 to 4 times the weight ordinarily assumed in calculations.

Roof Coverings.—Materials for roof covering are selected for protection against rain, snow and other natural agencies. They should be light, durable, economical and more or less artistic; and their selection will be dependent on the character of the building, its location with respect to climate, the amount of acids and injurious gases in the atmosphere, and the pitch of the roof. Among the most commonly used materials for "pitch" roofs are shingles, slate and tile for residences; slate, gravel and tile for railway structures; corrugated steel for warehouses. For temporary structures, as exposition buildings, the patented roofings are generally used, and then relaid elsewhere. For "flat" roofs, tin, tar and gravel, asphalt, and other compositions are preferred.

*West of the Coast and Cascade Mt. ranges. †In general.

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Shingle Roofing.—Shingles are made of white cedar, red cedar, spruce, pine, fir and cypress.* In the United States the life, in years, of cedar shingles will correspond to about the latitude of the place in degrees; pine will last about one-third to one-half as long as cedar, depending upon the climate. The following table is based on shingles 4″ wide, and with an average thickness of 1/5″. The weight of cedar is assumed at 36 lbs. and pine at 40 lbs. per cubic foot. The length of shingle is a little over 3 times the "weather."

4.—Weight of Shingles 0.2 Inch Thick, Laid on Roofs.
(Weight is proportional to thickness of shingles.)

Length. Ins.	Assumed Width. Ins.	Weather or Gage. Ins.	Shingles per Square of 100 Sq. Ft. Number.	Weight per Square of 100 Sq. Ft.		Nails per Square. Number.	Weight of Nails per Square. Lbs.
				Cedar. Lbs.	Pine. Lbs.		
14	4	4	900	210	233	1800	4.50
15	4	4½	800	200	222	1600	4.00
16	4	5	720	192	213	1440	3.60
18	4	5½	655	197	218	1310	3.28
20	4	6	600	200	222	1200	3.00
22	4	6½	554	203	226	1108	2.77
24	4	7	515	206	229	1030	2.58

Shingles are nailed directly on shingle laths or on solid 1″ sheathing covered with tarred paper, using two 1¼″ nails to each shingle. The laths are from two inches wide upward, spaced a few inches apart, and nailed horizontally to the jack-rafters.

Slate Roofing.—Slates are laid shingle fashion. The length of slate is usually 2 times the "weather" + 3 inches. The following table is based on the above. The number of slates per square of 100 sq. ft. = 14,400 ÷ (width × weather).

5.—Number of Slates per Square, Laid on Roof.
(See below for weight per sq. ft. of slate unlaid.)

Size. Ins.	Weather or Gauge. Ins.	Slates per 100 Sq. Ft. Number.	Size. Ins.	Weather or Gauge. Ins.	Slates per 100 Sq. Ft. Number.	Size. Ins.	Weather or Gauge. Ins.	Slates per 100 Sq. Ft. Number.
6x12	4½	533	8x16	6½	277	10x20	8½	170
7 12	"	457	9 16	"	246	12 20	"	141
8 12	"	400	10 16	"	222	14 20	"	121
9 12	"	356	11 16	"	202	12x22	9½	126
10 12	"	320	12 16	"	185	14 22	"	108
7x14	5½	374	9x18	7½	213	12x24	10½	114
8 14	"	327	10 18	"	192	14 24	"	98
9 14	"	291	12 18	"	160	16 24	"	86
10 14	"	262	14 18	"	137	14x26	11½	89

Note.—Sizes range up to 24″x44″.

At 174 lbs. per cubic foot the weight of one square foot of slate at various thicknesses is as follows:

Thickness, in inches	¼″	3/16″	⅛″	⅜″	½″	⅝″	¾″	1″
Weight, in pounds	1.81	2.72	3.62	5.44	7.25	9.06	10.88	14.50

* Good heart-cypress shingles are now almost unprocurable.

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6.—Total Weight of Slate per Square of Roof.
(Weather or gage as per Table 5.)

Length of Slate. Ins.	Weight in Lbs. per Thickness.							
	$\frac{1}{8}''$	$\frac{3}{16}''$	$\frac{1}{4}''$	$\frac{3}{8}''$	$\frac{1}{2}''$	$\frac{5}{8}''$	$\frac{3}{4}''$	$1''$
12	483	725	967	1450	1934	2417	2900	3867
14	461	692	923	1384	1845	2307	2768	3691
16	446	669	892	1338	1785	2231	2677	3569
18	435	652	870	1305	1740	2175	2610	3480
20	427	640	853	1279	1706	2133	2559	3412
22	420	630	840	1259	1679	2099	2518	3358
24	414	621	828	1242	1657	2071	2485	3313
26	410	615	820	1230	1639	2048	2459	3278

Note.—$\frac{3}{16}''$ slates are the most common.

Slate may be nailed on wooden sheathing, laths, porous terra cotta, reinforced concrete sheathing, etc., with or without felt between. The nails may be of malleable iron, copper, zinc, or composition metal. Iron or steel nails should be tinned or galvanized. The slates are often fastened directly to the sub-purlins by copper wire. Slater's cement makes a good tight bond and is recommended for flat pitch, which should not be less than 1-4 for slate roof.

Tile Roofing.—Tiles are made of terra cotta (baked clay), glass, and metal.

Clay tiles come in various shapes and under different names, as plain or flat, groove-and-fillet, pan, Spanish, etc. The plain clay tiles, say $6\frac{1}{2}''$x$10\frac{1}{2}''$ x$\frac{5}{8}''$ thick, will weigh from 15 to 18 lbs. per sq. ft. of roof when laid $5\frac{1}{2}''$ to weather. The weight of porous terra cotta roofing in lbs. per square foot = 4 (thickness in inches + 1).

The M. W. Powell Co.'s* Specifications for a Tile Roof are: First cover the roof foundation with 6 thicknesses of No. 1 wool roofing felt, weighing not less than 15 lbs. (single thickness) per 100 sq. ft.; the felt to be laid smoothly and evenly, and well cemented together, not less than 9 ins. between each layer, with roofing cement. All joinings along the walls and around the openings to be made carefully. The roof then to be covered with actinolite cement, and vitrified tile to be laid on this surface, the joints of the tile to be made with marmolite cement. The tile to be $6''$x$9''$x$\frac{3}{4}''$ thick. All walls and openings should be flashed with copper. The surface of the roof foundation should be perfectly smooth before the felt is laid.

Glass tiles are used principally for skylights.

Metallic tiles, of copper, zinc, iron, tin, etc., are made up in artistic forms and laid as shingles.

Tin Roofing.—Tin plate proper consists of thin sheets of iron or steel coated with *tin* by dipping and rolling. When the molten tin is adulterated with lead the product is *terne* plate, which is much cheaper. For the base, "charcoal" plates are better than "coke."

Roofing tin comes commonly in sheets of two sizes,† $14''$x$20''$ and $20''$x $28''$. As manufactured by the American Sheet and Tin Plate Co. of Pittsburg, there are 112 14x20 sheets, or 56 20x28 sheets, in a box. The sheets are graded as "Primes" and "Wasters." The Primes or perfect sheets are branded according to thickness or weight of iron body, as IC ($\frac{1}{8}$ lb. per sq. ft.) and IX ($\frac{1}{4}$ lb. per sq. ft.). The IC 20x28 plates manufactured by Merchant and Evans, of Philadelphia, weigh about 215 lbs. net per box of 112 sheets, and the IX 20x28 plates weigh about 270 lbs. net. The IC and IX plates are supposed to have the same thickness of coating.

* Manufacturers of patent roofing materials, cements, etc., 204 Dearborn St., Chicago, Ill.

† Sheets may be obtained in sizes 10x14 and multiples thereof.

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The tin sheets are laid on the roof in two ways, viz., with *flat* seam (Fig. 4), or with *standing* seam (Fig. 5). The flat seam is preferred for roofs of small pitch, using 14x20 plates; the standing seam for steep roofs, using 20x28 plates, generally. When laid with flat seam, a box of 112 sheets 20x28, containing 435 sq. ft. of plate, will cover about 384 sq. ft. of roof, about 12% being used in seams; and laid with standing seam it will cover about 370 sq. ft., with a loss of about 15%. If 14x20 sheets are used, the percentage of loss is slightly greater. Inversely, to cover 1000 sq. ft. of roof will require 583 sheets of 14x20 if laid with flat seam, or 303 sheets of 20x28 if laid with standing seam.

Fig. 4.

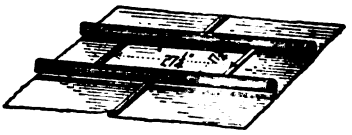


Fig. 5.

For fastening flat-seam roofing, use 1" barbed and tinned roofing nails about 6" apart and well under edges of seams. In soldering use rosin (not acid) as a flux. In laying standing-seam roofing the sheets are locked and soldered together in long rolls from ridge to eaves. The standing seams are not soldered, but are locked together and held in place with tin cleats spaced 15 to 18 ins. apart, through which nails are driven.

Sheet Steel Roofing.—Sheet steel, "black" or galvanized, and of Nos. 26, 27 and 28 gauge, is used for roofing. It comes in sheets, or in rolls up to 50 ft. in length. The sheets are laid with horizontal flat seams, or by lapping, and with vertical standing seams, crimped, with tin fastenings.

Corrugated Steel Roofing.—Corrugated steel, "black" or galvanized, and of Nos. 16, 18, 20, 22, 24, 26 and 27 gage is usually laid directly on the purlins. To these sheets are fastened clips of various kinds by means of clinch rivets or bolts. The clips may hook under some edge of the purlin or pass completely around it, as in Fig. 6. They are usually spaced 12 inches apart. Corrugated sheets are rolled from plain sheets 30" wide and in lengths up to 10 feet. The standard corrugation is about 2½" wide and ¾" deep, which narrows the 30" sheet down to 27½". Allowing for side lap, the net "weather" width is reduced to about 24". Lengthwise, the sheets should preferably rest on three purlins to give continuous-girder strength, although the strength is calculated for a simple span. The horizontal or "end" lap should be from 4" to 8", the latter for the flatter pitches and where the use of slater's cement is desirable. The *average* end lap is say 6", hence for sheets 4-ft. long the "weather" area is but 70% of the original flat sheet; 5-ft. long, 72%; 6-ft. long, 73.3%; 7-ft. long, 74.3%; 8-ft. long, 75%; 9-ft. long, 75.6%; 10-ft. long, 76%. In other words the weight of corrugated sheets per square foot of roof, *laid*, is respectively 43, 39, 36½, 35, 33½, 32½ and 31½ per cent. *greater* than that of the *plain* sheet metal. Galvanizing adds about one-third of a pound per square foot to flat metal, or, say, three-eighths of a pound per square foot to corrugated.

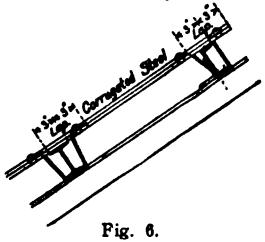


Fig. 6.

The strength of corrugated steel roofing may be obtained from the formula,

$$M = \frac{1}{9 \times 5} f b d t$$

In which M = bending moment or resisting moment, in *ft.-lbs.*;
f = allowable stress in lbs. per sq. in. in the metal;
d = depth of corrugations, in ins.;
b = breadth in ins. of loaded sheet *before corrugating.*

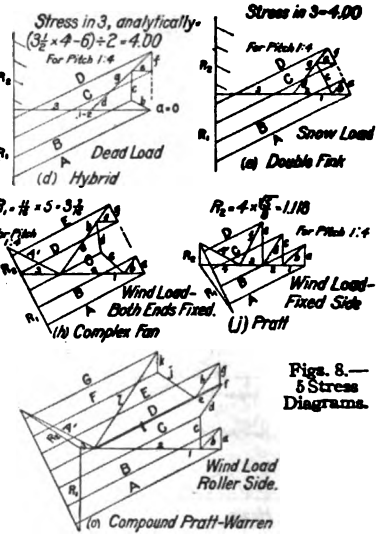
Tar-Gravel Roofing.—Gravel roofing is usually laid on wooden sheathing covered with roofing felt. The felt is laid in several thicknesses, with lap

(*C*).—The only correct method is to make the assumptions as near the true conditions as possible. Thus:

For short spans with both ends fixed, the wind load and reactions will be considered normal to one face of the roof as shown in truss and stress diagrams (*h*).

For long spans where one end of truss is fixed and the other end is a roller or sliding end, it should be calculated for that condition; namely, for wind pressure on the fixed side, and, again, for wind pressure on the roller side. See truss- and stress diagrams (*j*) and (*o*).

Generally, in all cases, it is good practice to design both halves of the truss symmetrically, using the maximum stresses obtainable by considering the wind pressure on either side of the roof, and either end of truss roller or fixed.



Figs. 8.—5 Stress Diagrams.

8.—Unit Stresses in Pratt Roof Trusses for Unit Loads *P*.
(See Figs. 9, Next Page.)

[+ = tension; − = compression. For character of stress see 1 to 4 pitch.]

Member.	*A* 10-Panel Pratt.			*B* 8-Panel Pratt.			*C* 6-Panel Pratt.			*D* 4-Panel Pratt.		
	Pitch 1 to 3.	Pitch 1 to 4.	Pitch 1 to 5.	Pitch 1 to 3.	Pitch 1 to 4.	Pitch 1 to 5.	Pitch 1 to 3.	Pitch 1 to 4.	Pitch 1 to 5.	Pitch 1 to 3.	Pitch 1 to 4.	Pitch 1 to 5.
1	6.75	+9.00	11.25	5.25	+7.00	8.75	3.75	+5.00	6.25	2.25	+3.00	3.75
2	6.00	+8.00	10.00	4.50	+6.00	7.50	3.00	+4.00	5.00	1.50	+2.00	2.50
3	5.25	+7.00	8.75	3.75	+5.00	6.25	2.25	+3.00	3.75	2.70	−3.35	4.04
4	4.50	+6.00	7.50	3.00	+4.00	5.00	3.61	−4.47	5.39	2.70	−3.35	4.04
5	3.75	+5.00	6.25	4.51	−5.59	6.73	4.51	−5.59	6.73	1.00	−1.00	1.00
6	5.41	−6.71	8.08	5.41	−6.71	8.08	4.51	−5.59	6.73	1.25	+1.41	1.60
7	6.31	−7.83	9.42	6.31	−7.83	9.42	1.00	−1.00	1.00			
8	7.21	−8.94	10.77	6.31	−7.83	9.42	1.25	+1.41	1.60			
9	8.11	−10.06	12.12	1.00	−1.00	1.00	1.50	−1.50	1.50			
10	8.11	−10.06	12.12	1.25	+1.41	1.60	1.68	+1.80	1.95			
11	1.00	−1.00	1.00	1.50	−1.50	1.50						
12	1.25	+1.41	1.60	1.68	+1.80	1.95						
13	1.50	−1.50	1.50	2.00	−2.00	2.00						
14	1.68	+1.80	1.95	2.14	+2.24	2.36						
15	2.00	−2.00	2.00									
16	2.14	+2.24	2.36									
17	2.50	−2.50	2.50									
18	2.61	+2.69	2.80									

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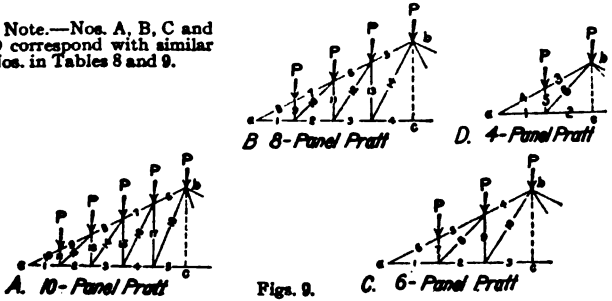
Note.—Nos. A, B, C and D correspond with similar Nos. in Tables 8 and 9.



Figs. 9.

9.—Unit Deductions for One-Half Truss (Lean-to). Supplementary to Table 8.

(No deductions for web members.)

For unit stresses in *one-half* of each of the above trusses, considered as supported at points *a* and *b*, or *a* and *c*, make the following deductions:

Bottom chord members.—Deduct from the unit stress in each bottom chord member, as shown in Table 8, preceding, the unit stress in the center panel member of truss (as shown in black face type). The remainders are the unit stresses to be used.

Top chord members.—Deduction for each top chord member will be the bottom chord deduction multiplied by secant of angle of inclination of roof with the horizontal. For 1 to 3 pitch, secant = 1.202; 1 to 4 pitch, sec. = 1.118; 1 to 5 pitch, sec. = 1.077. The following data in connection with Table 8 will be found useful:

		A.	*B.*	*C.*	*D.*
Deduct for	Pitch 1-3:	1.202×3.75 = 4.51;	×3.00 = 3.61;	×2.25 = 2.70;	×1.50 = 1.80.
	" 1-4:	1.118×5.00 = 5.59;	×4.00 = 4.47;	×3.00 = 3.35;	×2.00 = 2.24.
	" 1-5:	1.077×6.25 = 6.73;	×5.00 = 5.39;	×3.75 = 4.04;	×2.50 = 2.69.

10.—Unit Stresses in Fan and Fink Roof Trusses for Unit Loads *P*. (See Figs. 10, next page.)

[+ = tension; − = compression. For character of stress see 1 to 4 pitch.]

Member.	*E* Compound Fan.			*F* Compound Fink.			*G* Simple Fan.			*H* Simple Fink.		
	Pitch 1 to 3.	Pitch 1 to 4.	Pitch 1 to 5.	Pitch 1 to 3.	Pitch 1 to 4.	Pitch 1 to 5.	Pitch 1 to 3.	Pitch 1 to 4.	Pitch 1 to 5.	Pitch 1 to 3.	Pitch 1 to 4.	Pitch 1 to 5.
1	8.25	+11.00	13.75	5.25	+7.00	8.75	3.75	+5.00	6.25	2.25	+3.00	3.75
2	6.75	+9.00	11.25	4.50	+6.00	7.50	2.25	+3.00	3.75	1.50	+2.00	2.50
3	4.50	+6.00	7.50	3.00	+4.00	5.00	3.39	−4.70	5.98	2.15	−2.91	3.66
4	7.14	−10.06	12.95	4.65	−6.48	8.31	3.53	−4.55	5.58	2.71	−3.35	4.04
5	7.28	−9.93	12.54	5.20	−6.93	8.68	4.50	−5.59	6.73	0.83	−0.89	0.93
6	8.25	−10.96	13.69	5.76	−7.38	9.05	0.93	−1.08	1.21	0.75	+1.00	1.25
7	8.80	−11.40	14.07	6.31	−7.83	9.42	0.93	−1.08	1.21			
8	8.94	−11.25	13.66	0.83	−0.89	0.93	1.50	+2.00	2.50			
9	9.91	−12.30	14.80	0.75	+1.00	1.25						
10	0.93	−1.08	1.21	1.66	−1.79	1.86						
11	0.93	−1.08	1.21	1.50	+2.00	2.50						
12	1.50	+2.00	2.50	0.75	+1.00	1.25						
13	2.50	−2.69	2.79	0.83	−0.89	0.93						
14	2.25	+3.00	3.75	2.25	+3.00	3.75						
15	1.50	+2.00	2.50									
16	0.93	−1.08	1.21									
17	0.93	−1.00	1.21									
18	3.75	+5.06	6.25									

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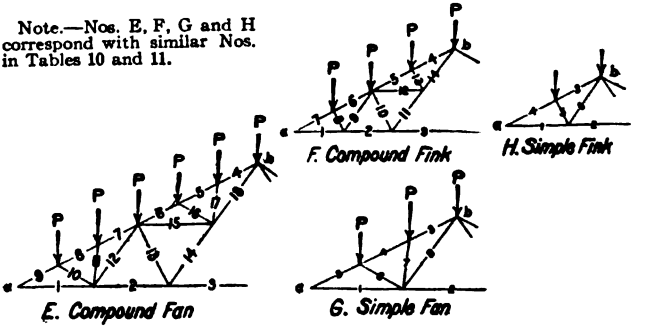
Note.—Nos. E, F, G and H correspond with similar Nos. in Tables 10 and 11.



Figs. 10.

11.—Unit Deductions for One-Half Truss (Lean-to). Supplementary to Table 10. (No deductions for web members.)

Bottom chord members.—Deduct from the unit stress in each bottom chord member as shown in Table 10, preceding, the unit stress in the center panel member of truss (as shown in black face type). The remainders are the unit stresses to be used.

Top chord members.—Deduction for each top chord member will be the bottom chord deduction multiplied by secant of angle of inclination of roof with the horizontal. Following are unit deductions for top chords:

Pitch	*E*	*F*	*G*	*H*
1-3	5.41	3.61	2.70	1.80
1-4	6.71	4.47	3.35	2.24
1-5	8.08	5.39	4.04	2.69

DESIGN OF COMBINATION ROOF TRUSSES.

Problem.—Trusses, 8 panels @ 9 ft. = 72 ft., spaced 14 ft. centers; pitch 1 to 4; height, 72 ÷ 4 = 18 ft. Covering, slate laid on felt and 1-inch spruce sheathing. *Loads*:—Spruce sheathing, 3 lbs. per ft. B. M.; slate and felt, 9 lbs. per sq. ft.; snow, 20 lbs. per horizontal sq. ft.; wind, 30 lbs. per sq. ft. against vertical surface.

Solution.—For lack of space, hints only can be given. In this calculation it is assumed that *the wind load and snow load do not act on the same face of roof at the same time*; but may act separately on either side, or simultaneously on opposite sides: For maximum stresses in sheathing, jack-rafters and purlins, assume the wind load or snow load to act with the dead loads; for maximum stresses in the trusses, the wind load is assumed to act on one side of the roof and the snow load on the other at the same time, the wind load being considered as acting (1) on the fixed side, and (2) on the roller side; also (3) the snow load may be considered as acting on the whole roof without any wind load. The trusses are then designed symmetrically, using the maximum dimensions of twin members in either half of truss. The solution in detail is as follows:

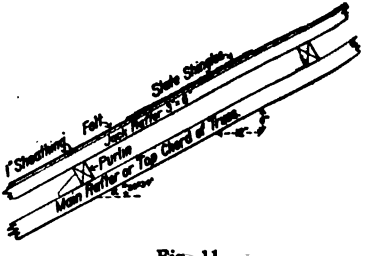


Fig. 11.

Spacing of Jack-Rafters.—The horizontal sheathing is nailed to the vertically inclined jack-rafters, and these are laid directly on the horizontal purlins, which in turn rest on the top chord joists of the roof trusses.

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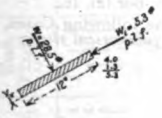
ength of the sheathing will determine, within certain limits, of the jack rafters, which spacing will be constant and some 14 ft., the spacing of the trusses. For the purpose of reference, and tangential pressures on roof surface for pitch of 1 to 4 (slope re given below:

	Pressure in Lbs. per Sq. Ft. of Roof.		
	Vert.	Normal.	Tangential.
e 3, from Hutton's rule) 30.0	15.9	17.8	...
rmal $= 20 \times \cos^2$ 26°-34′; 0 $\times \cos \alpha \sin \alpha$)	17.9	16.0	8.0
mal $= 9 \times \cos \alpha$; tangen- $\times \sin \alpha$)	9.0	8.0	4.0
Normal $= 3 \times \cos \alpha$; tan- $= 3 \times \sin \alpha$)	3.0	2.7	1.3

be noted in the above table that the normal wind pressure is considered the "direct" pressure, and 15.9 lbs. its vertical t." On the other hand, the vertical loads due to snow, slate and re the "direct" loads, and the normal and tangential forces are ponents," The snow load per square foot of roof $= 20 \times \cos$ 26°- ne sheathing, the normal pressure due to wind is greater than that r, but with the latter there is a tangential pressure of 8 lbs. to ed, which is lacking in wind pressure. Assuming the sheathing wide, the following two alternative conditions obtain as showing er lin. ft. acting on the sheathing:



Vind and dead loads on sheathing.

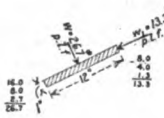


Fig. 13.—Snow and dead loads on sheathing.

lowable outer fiber stress per square inch (= 800 lbs. for spruce and x = length of span in ft., we have:

$$\frac{28.5 \times x^2 \times 6}{12 \times 8 \times 1} \times 12$$

$$\frac{5.3 \times x^2 \times 6}{144 \times 8 \times 1} \times 12$$

$$\tfrac{3}{4}\left(28.5 + \frac{5.3}{12}\right) x^2$$

6.07 ft.

Fig. 13, $f = \dfrac{26.7 \times x^2 \times 6}{12 \times 8 \times 1} \times 12$

$$+ \frac{13.3 \times x^2 \times 6}{144 \times 8 \times 1} \times 12$$

$$\therefore 800 = \tfrac{3}{4}\left(26.7 + \frac{13.3}{12}\right) x^2$$

Or $x = 6.19$ ft.

ack rafters might be spaced 6 ft. apart, but for practical reasons e spaced $\frac{1}{3}^4 = 4'$-8″ apart, centers.

Jack-Rafters (yellow pine).—The unsupported length = panel ant 26°-34′ $= 9 \times 1.118 = 10$ ft. The jack-rafter must resist e to the maximum normal forces on roof, which are $17.8 + 8.0$ 5 lbs. per square ft. (wind acting—no snow load), + the normal o the weight of the jack rafter itself. Neglecting the latter for we have, W = total normal load $= 28.5 \times 4\frac{2}{3} \times 10$, and bending

$$= 28.5 \times \frac{14}{3} \times \frac{10 \times 10 \times 12}{8} = \tfrac{1}{6} f b d^2$$

= allowable fiber stress per sq. in. = 1300 lbs.,
= breadth of rafter in inches,
= depth of rafter in inches;

$$= \frac{6 M_b}{f} = \frac{6 \times 28.5}{1300} \cdot \frac{14}{3} \cdot \frac{100 \times 12}{8} = 92.1.$$

$d = 6$, $b = 2.56$; hence use rafters size 3″ x 6″ to allow liberally for fter itself. At 4 lbs. per ft. B. M., weight of jack rafter = 6 lbs. normal component = 5.4 lbs.; tangential component = 2.7 lbs.

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Purlins (white pine).—The span of the purlins is 14 feet, the distance center to center of trusses. They are laid horizontally, resting on the top chord joints, and hence spaced 10 ft. apart, centers. There are two concentrated loads on each purlin, namely, where the jack rafters rest, at points 4'-8" apart (Fig. 14). Let P and P', Fig. 5, represent respectively the normal and tangential components of each of these loads on the purlin. Neglecting the weight of the purlin itself, the acting forces are:

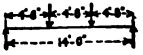

Fig. 14.

	Wind acting.		Snow acting.	
	P	P'	P	P'
Wind.....	831	...	...	...
Snow.....	...	...	747	373
Slate.....	373	187	373	187
Sheathing....	126	63	126	63
Jack-rafters..	54	27	54	27
Total, lbs..	1384	277	1300	650

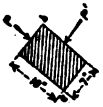

Fig. 15.—Purlin.

Assuming the allowable outer fiber stress $f = 1200$ lbs. per sq. in. at a and b, and the condition "snow-acting" for maximum, we have:

For 7" x 10", $f = \frac{1300 \times 14 \times 12 \times 6}{7 \times 3 \times 10 \times 10} + \frac{650 \times 14 \times 12 \times 6}{10 \times 3 \times 7 \times 7} = 1070$ lbs.; therefore use purlins 7" x 10", which allows liberally for weight of purlins themselves. At 3 lbs. per ft. B. M. these will weigh $17\frac{1}{2}$ lbs. per lin. ft.

Trusses.—The trusses are designed for the following Cases, *considering only the members in the left hand half of the symmetrical truss*:

Case I.—Dead load over all.
Case II.—Snow load over all.
Case III.—Snow load on right half only.
Case IV.—Snow load on left half only.
Case V.—Wind on left; left end "fixed."
Case VI.—Wind on left; left end "roller."
Case VII.—Wind on right; left end "fixed."
Case VIII.—Wind on right; left end "roller."

For maximum stresses—

Combine I with II.
" I with VI or VII.
" I and III with V or VI.
" I and IV with VII or VIII.

Assuming the weight of the truss at 60 lbs. per lin. ft., the loads per joint of truss, for calculating stresses, are as follows:

Dead load per joint.—Slate, felt and sheathing, $12 \times 14 \times 10 = 1680$ lbs.; jack-rafters, $3 \times 6 \times 10 = 180$ lbs.; purlins, $14 \times 17\frac{1}{2} = 245$ lbs.; truss 540 lbs.; total, 2645 lbs.

Snow load per joint (vertical).—$20 \times 14 \times 9 = 2520$ lbs.

Wind load per joint (normal).—$17.8 \times 14 \times 10 = 2492$ lbs.

Table 12, following page, gives a summary of the stresses.

Remarks.—The preceding principles used in the design of jack-rafters, purlins, trusses, etc., of the combination truss, can readily be applied to the design of steel roofs.

For weight of steel trusses and purlins, see pages 810 and 811.

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12.—TABLE OF STRESSES FOR SEPARATE CASES, AND COMBINED CASES FOR MAXIMUM. TRUSS 72′ SPAN.

Members.		Dead Load. I.	Snow. Over All. II.	Snow. On Right. III.	Snow. On Left. IV.	Wind on Left. Left End Fixed. V.	Wind on Left. Left End Roller. VI.	Wind on Right. Left End Fixed. VII.	Wind on Right. Left End Roller. VIII.	Cases Combined for Maximum.	Maximum Stress.	Size of Members.
Bottom Chord.	1	+18.5	+17.6	+5.0	+12.6	+14.0	+9.4	+1.1	+5.6	I, III, V.	+37.5	8″x8″
	2	+18.5	+17.6	+5.0	+12.6	+14.0	+9.4	+1.1	+5.6	I, III, V.	+37.5	"
	3	+15.9	+15.1	+5.0	+10.1	+11.2	+6.6	+1.1	+5.6	I, III, V.	+32.1	"
	4	+13.2	+12.5	+5.0	+7.5	+8.4	+3.9	+1.1	+5.6	I, III, V.	+26.6	"
Top Chord.	5	−11.8	−11.2	−5.6	−5.6	−5.6	−5.6	−6.3	−6.3	I, IV, VII (or VIII).	−23.7	8″x8″
	6	−14.8	−14.1	−5.6	−8.5	−7.5	−7.5	−6.3	−6.3	I, IV, VII (or VIII).	−29.6	"
	7	−17.7	−16.8	−5.6	−11.2	−9.4	−9.4	−6.3	−6.3	I, IV, VII (or VIII).	−35.2	"
	8	−20.7	−19.7	−5.6	−14.1	−11.3	−11.3	−6.3	−6.3	I, IV, VII (or VIII).	−41.1	"
Web Members.	9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	I, III, V (or VI).	0.0	$\frac{3}{8}$″ ○
	10	−3.0	−2.9	0.0	−2.9	−3.1	−3.1	0.0	0.0	I, III, V (or VI).	−6.1	4″x6″
	11	+1.3	+1.2	0.0	+1.2	+1.4	+1.4	0.0	0.0	I, III, V (or VI).	+2.7	$1\frac{1}{8}$″○upset.
	12	−3.7	−3.5	0.0	−3.5	−4.0	−4.0	0.0	0.0	I, III, V (or VI).	−7.7	4″x6″
	13	+2.6	+2.5	0.0	+2.5	+2.8	+2.8	0.0	0.0	I, III, V (or VI).	+5.4	1″○upset.
	14	−4.8	−4.6	0.0	−4.6	−5.0	−5.0	0.0	0.0	I, III, V (or VI).	−9.8	6″x6″
	15	+7.9	+7.5	+3.8	+3.8	+4.2	+4.2	+4.2	+4.2	I, III, V (or VI).	+15.9	$1\frac{3}{8}$″○ upset.

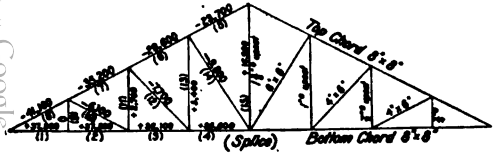

Fig. 16.

Fig. 16.—Stress Sheet 72′ Span, 18′ Rise.

(From above Table.)

The formula used in proportioning the compression members is: Allowable stress in lbs. per sq. in. $= 1200 - \frac{12L}{d}$; in which L = unsupported length in ft., and d = least depth in ins. For soft steel tension members, Area = (stress in lbs. + 5000 lbs. initial stress) ÷ 15,000.

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Details of Design.—Before leaving the subject of the design for combination roof truss, three points in the details of design will be considered, namely, (*a*) the center lower chord splice, (*b*) the end corbel, and (*c*) the center lower chord block.

(*a*).—The writer can conceive of no case in practice where the tensile strength of a full main wooden member is used in proportioning that member in tension. But in the case of a splice, as shown in Fig. 17, the tensile strength of the *net sections* of main member and of splice have to be considered, as well as the shearing values along the grain, and the end fiber bearing values. Fig. 17 is the splice designed for center of lower chord of roof truss shown in Fig. 16.

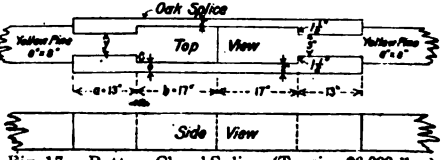

Fig. 17.—Bottom Chord Splice. (Tension 26,000 lbs.)

	Yellow pine.	Oak.
Data.—Shearing along grain, lbs. per sq. in.....	100	130
Bearing.....	1500	1500
Tension.....	1500	1500

For shear: Oak keys, $a = \frac{26600}{2\times8\times130} = 13''$; Yellow pine, $b = \frac{26600}{2\times8\times100} = 17''$.

For bearing: $c = \frac{26600}{2\times8\times1500} = 1\frac{1}{4}''$; call $1\frac{1}{2}''$.

For tension: Oak keys, $o = \frac{26600}{2\times8\times1500} = 1\frac{1}{4}''$; Yellow pine, $y = 2\frac{1}{4}''$.

(*b*.)—Fig. 18 shows the detail at end of truss. When a corbel is used it has to be long enough to give proper shearing surface at the right of each key, while the shear on the bottom chord itself is at the left of each key. Various devices are used to resist the thrust of the rafter. In addition to the $1\frac{1}{2}''$ notch at its toe, there may be a bolt or strap *a b* used in connection with a corbel; or a shorter bolt *a p* without the corbel; or (still without the corbel) straps *a p* and *p s* may be joined by a common pin *p*.

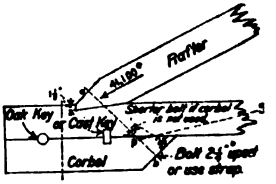

Fig. 18.

(*c*.)—Detail of lower chord block (cast iron) is shown in Fig. 19. Where the block is not used the lower chord is simply notched as shown in Fig. 20.

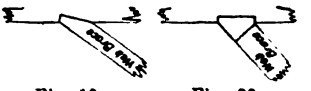

Fig. 19. Fig. 20.

Framing Table or Table of Squares.—For calculating lengths of roof-truss members, use Tables of Squares, Sec. 33, pages 643 to 664. (See problems on page 638.) For calculating Howe truss braces and blocks see page 635.

Weight of Steel Construction in Roofs.—Where trusses, purlins, etc., are of steel the following formulas may be used in obtaining the approximate weights prior to actual calculation:

Weight in lbs. of metal in Trusses, per hor. sq. ft. of roof..... $= \frac{1}{5}\left(\sqrt{S} + \frac{S}{10}\right)$ ..... (1)

Weight in lbs. of metal in Trusses, Purlins and Bracing, per hor. sq. ft. of roof..... $= \frac{1}{10} S \left\{ \begin{array}{l} \text{With minimum} \\ \text{value of about} \\ \text{4 or 5.} \end{array} \right\}$ ..... (2)

In which S = span of trusses, in feet.

Weight of Steel Trusses and Purlins.—The above formulas, (1) and (2), are, of course, only approximate.

Table 13, following, gives more exact weights of trusses and of purlins for the specified horizontal load of 50 lbs. per square foot of building.

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13.—STEEL ROOFS—APPROXIMATE WEIGHT OF TRUSSES AND PURLINS.
(Based on Uniform Load of 50 Lbs. per sq. ft. of Building.)

Span in Feet.	Distance Center to Center of Trusses, in Feet.									
	6	8	10	12	14	16	18	20	22	24
	Weight of Trusses in Lbs. per sq. ft. of Building.									
16	1.61	1.51	1.42	1.34	1.27	1.21	.....	.....	.....	.....
18	1.77	1.66	1.57	1.48	1.40	1.33	1.27	.....	.....	.....
20	1.92	1.81	1.70	1.61	1.53	1.46	1.39	1.33	.....	.....
22	2.07	1.95	1.84	1.74	1.65	1.58	1.50	1.44	1.38	.....
24	2.21	2.08	1.96	1.86	1.77	1.69	1.61	1.54	1.48	1.42
26	2.34	2.21	2.09	1.98	1.89	1.80	1.72	1.65	1.58	1.52
28	2.46	2.33	2.20	2.10	2.00	1.90	1.82	1.75	1.68	1.61
30	2.59	2.45	2.32	2.21	2.10	2.01	1.92	1.85	1.77	1.70
35	2.87	2.72	2.58	2.46	2.35	2.25	2.16	2.08	2.00	1.92
40	3.13	2.97	2.83	2.70	2.59	2.48	2.38	2.29	2.21	2.13
45	3.36	3.20	3.05	2.92	2.80	2.69	2.59	2.49	2.40	2.32
50	3.57	3.41	3.25	3.13	3.00	2.88	2.78	2.68	2.59	2.50
55	3.77	3.60	3.45	3.30	3.19	3.07	2.96	2.86	2.76	2.67
60	3.95	3.78	3.63	3.49	3.36	3.24	3.13	3.02	2.92	2.83
65	4.11	3.95	3.79	3.65	3.52	3.40	3.28	3.18	3.08	2.98
70	4.26	4.10	3.95	3.80	3.67	3.55	3.43	3.32	3.22	3.13
75	4.41	4.25	4.09	3.95	3.81	3.69	3.57	3.46	3.36	3.26
80	4.55	4.38	4.22	4.08	3.95	3.82	3.70	3.59	3.49	3.39
85	.....	4.51	4.35	4.21	4.07	3.95	3.83	3.72	3.61	3.51
90	.....	4.62	4.47	4.33	4.19	4.07	3.95	3.84	3.73	3.63
100	.....	4.84	4.69	4.55	4.41	4.28	4.17	4.06	3.95	3.85
110	.....	.....	4.88	4.74	4.61	4.48	4.37	4.25	4.14	4.04
120	.....	.....	5.05	4.92	4.79	4.66	4.55	4.43	4.33	4.23
	Weight of Purlins in Lbs. per sq. ft. of Building.									
	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60

Reference.—See, also, Sec. 47, Buildings.

EXCERPTS AND REFERENCES.

Concrete Platform and Umbrella Roof of Union Station, at Dayton, O. (Eng. News, Aug. 8, 1901).—Illustrated.

Howe Truss Roofs for Transportation Building, at St. Louis Exposition (Eng. News, May 19, 1904).—Illustrated. Also cost data of the principal buildings.

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Method of Erecting the Roof Trusses of the 71st Regiment Armory, N. Y. City (By W. T. McCarthy. Eng. News, June 16, 1904).—Illustrated.

Wind Stresses in Knee-Braced Mill Buildings (By W. H. Dunham. Eng. News, Oct. 6, 1904).—Graphically illustrated. Discussions: Eng. News, Nov. 10, 1904; Jan. 25, 1905.

Reinforced-Concrete Slab Roof for a Small Warehouse (Eng. News, June 22 1905).—Illustrated.

Modified Saw-Tooth Roof (By M. S. Ketchum. Eng. News, Nov. 23, 1905).—Illustrated.

Reinforced-Concrete Shingles for Roofing (Eng. News, Aug. 30, 1906).—Illustration of hand molding machine for concrete shingles.

Saw-Tooth Roofs for Factories (By K. C. Richmond. Eng. News, Dec. 13, 1906).—Illustrated.

Steel Dome for Emporium Building, San Francisco (Eng. News, May 14, 1908).—Illustrated.

An Improved Method of Saw-Tooth Roof Construction (By S. M. Green. Eng. News, Sept. 3, 1908).—Illustrations of gutter construction and ventilator design for use on weave sheds.

Illustrations.

Description.	Eng. Rec.
Roof of the Standard Steel Car Co., Butler, Pa.....	Nov. 26,'10

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47.—BUILDINGS.

Plastering.—Plastering usually consists of three coats, viz., (1) the rough or "scratch" coat which is applied directly to the wood- or metal laths; (2) the "brown" coat which is floated either on the scratch coat (the latter having previously been scratched with a comb in order to roughen it so the brown coat will adhere better), or sometimes directly on the wall; and (3) the finishing or "skim" coat which is applied to the brown coat after it has been finely scratched or roughened. The skim coat may be either "stucco" or "hard finish" (gage stuff).

(1.)—The scratch coat is composed of a mixture of slaked lime, clear river or pit sand (essentially free from salt) and cattle hair* (preferably goat or cow). These are mixed in the proportion of one part lime paste to two parts sand, with 1½ bushels of hair to each barrel of unslaked lime. Less hair is required for walls than for ceilings. A barrel of Rockland, Me., lime weighs 220 lbs. net, contains about 3½ cubic feet, and will make about 2.6 barrels or 9 cubic feet of paste. A barrel of 200 lbs. will make about 8 cubic feet of paste. Approximately, 9 cubic feet of lime paste, 18 cubic feet of sand and 4 bushels of hair will cover 40 square yards about ⅝" thick on wooden laths (Fig. 1), and about 30 square yards on metal laths.

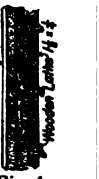

Fig. 1.

(2.)—The brown coat is sometimes leaner in cement than the scratch coat and contains usually but half the quantity of hair. It is generally ¼" thick, sometimes ⅜". Plaster prepared in sheets and shipped ready for nailing is a common substitute for the scratch and brown coats.

(3.)—The skim coat, usually ⅛", contains no hair. Stucco is composed of one part pure lime and two parts clear sand of the purest kind, white preferred. Hard finish is made from any of the patent plasters on the market. They are composed principally of plaster of Paris or gypsum, which gives the hard finish, and are recommended for general use, being more satisfactory in many ways than the ordinary lime mixture. A mixture of 2½ cubic feet each of lime, plaster of Paris and white sand or marble dust will skim-coat about 100 square yards from $\frac{1}{16}$" to ⅛" thick.

Lathing and plastering is commonly estimated to weigh about 10 lbs. per square foot.

Lathing.—These may be of wood or metal. Wooden laths are usually 1½" wide, ¾" thick and 4 ft. long. They are made of pine, spruce or hemlock. The straight-grained split lath is preferable to the sawed. A bundle of 100 laths (50 sq. ft., solid—12½ ft. B. M.—37½ lbs.),

spaced	¾ inch,	will cover	6.48	sq. yds.:	equal to	1543	laths per	100	sq. yds.
"	⅞ "	" "	6.94	"	" "	1441	"	"	"
"	1 "	" "	7.41	"	" "	1350	"	"	"

About 10 lbs. of nails are required per 100 sq. yds. of lathing. From the above it is to be noted that lathing weighs about ⅜ lb. per square foot in place.

Metal lathing, either wire or expanded metal (see Figs. 7 to 10), is now universally used in fire-proof buildings. The weight ranges from 2¾ to 4½ lbs. per sq. yd., or ⅓ to ½ lbs. per sq. ft. Generally, the weight of the expanded metal per unit of area is about one-half or less than the weight of the original sheet, or, in other words, a sheet is expanded so as to cover twice or more its original area, depending upon the mesh. Thus, Diamond gage 24 covers 2.2; gage 26 covers 2.4; "A" gage 24 covers 1.9; "B" gage 27 covers 2.06, etc. 10 lbs. of staples will fasten 100 sq. yds. of expanded metal lathing.

Partitions are either permanent (fixed) or temporary (movable). In addition to the ordinary wooden partition, hollow tile and expanded metal are very largely used.

* Wood fiber is often used instead of hair, for cheap work.

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Wooden Partitions.—Fig. 2 represents the average partition used in a frame dwelling. The studding is spaced 16 ins. on centers. It will be noted that those pieces marked with a cross (×) would be superfluous if the door opening were omitted: *a, c, c* would complete the studding, and *b* the

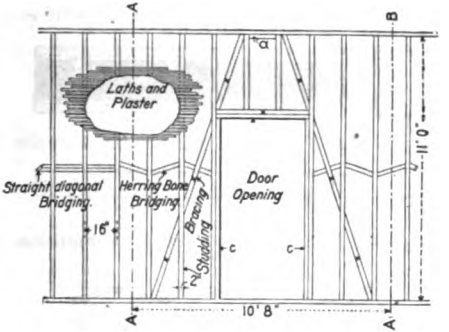

Fig. 2.—Stud Partition; dimension stuff either all 2"x 6" or all 2"x 4".

bridging, across the door opening. In the above illustration, about 26½ lineal feet of extra scantling is required for each door opening, over that for a plain partition. A partition of this kind will weigh 3 or 4 lbs. per square foot, exclusive of laths and plaster.

Hollow-Tile Partitions.—Solid terra cotta weighs from 120 to 125 lbs. per cubic foot, 122½ lbs. being a good average; but the hollow blocks for partitions will usually not exceed two-thirds that amount, and may weigh somewhat less. The following types of blocks are used:

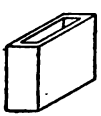

Fig. 3.—Plain Hollow Block.

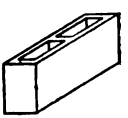

Fig. 4.—Webbed Block.

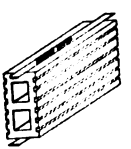

Fig. 5.—Patent Block for plastering.

Figs. 3 and 4 are ordinary blocks such as are used in a partition with I-beam studding (see Fig. 6). The patent blocks illustrated in Fig. 5 are sustained laterally by means of horizontal metal strips or bands of steel between vertical studding.

Other Partitions than the hollow tile which may be used in connection with I-beam studding (Fig. 6) are (*a*) plaster boards, which are laid in between the beams and flushed for plastering; (*b*) wire lathing strung between the beams flush with the flanges and fastened to them, and also supported intermediately with angles about 2-ft. centers; (*c*) expanded metal lathing.

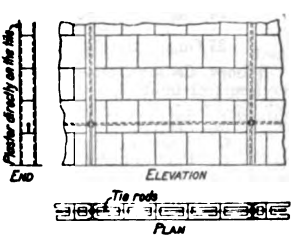

Fig. 6.—Ordinary Hollow-tile wall with steel I-beam studs.

These three classes of material may also be used with other skeleton designs, provided the principle of rigidity is maintained. Figs. 7 and 8 are examples of hollow and of solid partition construction by the expanded metal system, and Figs. 9 and 10 are cuts of the metal lath used. The lathing can be plastered with ordinary lime plaster, but cement plaster is better.

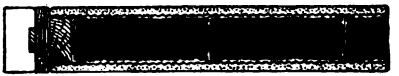

Fig. 7.—Plan of Expanded Metal Hollow Partition

Fig. 8.—Plan of Expanded Metal Solid Partition.

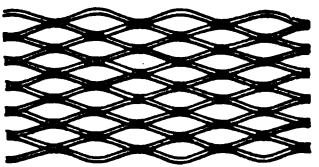

Fig. 9.—"Diamond" Lath (Expanded Metal).

24 Gage. Sheets, 18 ins. x 96 ins. 20 sq. yds. per bundle.
26 Gage. Sheets, 24 ins. x 96 ins. 16 sq. yds. per bundle.

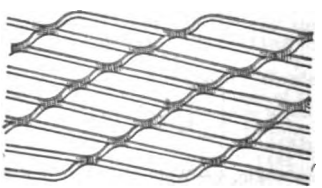

Fig. 10.—"A" Lath (Expanded Metal).

24 Gage. Sheets, 18 ins. x 96 ins. 12 sq. yds. per bundle.

"B" Lath.

27 Gage. Sheets, 18 ins. x 96 ins. 20 sq. yds. per bundle.

Floors, Ceilings, etc.—The following loads are considered in designing the floors of buildings:

(1.) Live loads, due to—
- (*a*) People;
- (*b*) Safes, merchandise, furniture, machinery, etc.;
- (*c*) Partitions which are subject to change of position.

(2.) Dead loads, due to—
- (*d*) Flooring or tiling;
- (*e*) Fireproof arches between the beams;
- (*f*) Ceiling, under the floor;
- (*g*) Beams directly supporting the above;
- (*h*) Girders, supporting the beams and in turn directly supported by the columns or walls.

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Live Loads.—There is great diversity of opinion among engineers regarding the live loads to be assumed for each class of buildings, and this is aggravated by the decided lack of harmony of the various city building codes. For instance, the requirements of the ten leading cities of the United States are shown in the following table:

1.—MINIMUM LIVE LOADS FOR FLOORS AND ROOFS—TEN LEADING CITIES.

Structure.	Pounds per Square Foot.										
	New York. (1906)	Chicago. (1905)	Philadelphia (1907)	Boston. (1907)	Buffalo. (1905)	Minneapolis. (1907)	Milwaukee. (1901)	D. of C. (1902)	Baltimore. (1908)	St. Louis. (1907)	Average.
Dwellings—one or two families	60	40	70	50	40	50	40	50	60	60	52
Lodging houses, apartment houses, tenement houses, hotels, etc	60	50	70	50	70	50	40	50	60	60	56
Halls, dining-rooms, cafés, offices, etc., in hotels and apartment-houses	....	....	....	....	....	....	....	75	160	....	68
Office buildings, first floor	150	50	100	100	70	150	60	75	50	150	97
Office buildings, above first floor	75	50	100	100	70	75	60	75	75	70	75
Halls and lobbies in office buildings	....	....	....	....	....	....	....	110	....	....	110
Public assembly rooms: churches, theaters, etc	90	100	120	200	100	125	80	110	....	100	103
Schools	75	75	120	60	100	100	50	75	75	100	83
Machine shops, armories, drill-rooms, etc	....	....	....	....	....	....	250	....	....	....	250
Light manufacturing and retail stores and storehouses	120	100	120	125	120	100	100	110	125	150	117
Heavy storehouses, warehouses, livery stables, etc	150	100	150	250	150	200	....	200	250	150	160
Stairways	....	....	....	70	....	....	100*	....	....	....	85
Sidewalks	300	....	....	....	....	300	....	....	....	....	300
Roofs, per square foot of superficial surface	50†	....	30	40¶	....	....	30	25	....	40¶	.....
Roofs, per square foot of horizontal projection	30‡	25	....	....	40	30	....	....	....	....	.....

* The lower supports to carry two-thirds of the total weight.
† Pitch less than 20 degrees.
‡ Pitch more than 20 degrees.
¶ For flat roofs.

It has been found by actual test that forty selected men with an average weight of 163.2 lbs. may be *packed* into a floor area of 6-ft. square, when each man tries to occupy as little space as possible. This is equivalent to an occupied area of $\frac{9}{10}$ of a sq. ft. per man, an average load of 181.3 lbs. per sq. ft. of floor, and a total load of 6528 lbs. Perhaps the passenger elevator would instance a load approaching the above ideal more closely than would any other case in practice, but even there a load of 120 lbs. per sq. ft. makes a very compact and uncomfortable mass, and is probably very rarely obtained. For a "mixed" crowd covering a considerable area as in public halls or corridors, 100 lbs. per sq. ft. may be taken as the extreme loading, even when the ordinary crowding takes place. People in a crowd do not stand perfectly erect and allow themselves to be packed together as in the ideal case above cited. Occasionally during a panic the people in the *center* of a crowd may be packed so as to produce a concentrated loading nearly equivalent to the ideal load of 180 lbs. per sq. ft. over a *limited area*, but by

no means over the whole floor space or even over any considerable portion of it. Hence in designing floors where people congregate it is good practice to adopt heavier loadings for small floor areas in a building than for large ones. In other words the live load per sq. ft. of floor area may be allowed to decrease, in tall buildings, from the maximum loading for floor beams and arches, down to the minimum for columns and foundations—main girders taking an intermediate position.

Loads from Safes.—One of the most important factors to be considered in the design of floors for office buildings and offices in general is the effect due to the weight of safes. Hence the following table is inserted as comprising the heaviest safes in use (the 900's and 800's) and also those commonly used for upper floor offices (the 500's). Numbers "A", "B" and "C" are very heavy compared with their dimensions and are liable to be placed in any office. The 600's and 700's, omitted in this table, range in weight between the 500's and 800's. The 2500-lb. safe, No. 513, is the lightest one considered. For any desired calculation columns 6 and 7, showing the distance apart of the supporting wheels, may be used in connection with either column 8, or with column 2 in connection with 10, 11 or any other assumed weight of the contents of the safe. In general, the use of column 8 is considered good practice.

2.—Table of Weights and Dimensions of Heavy Safes.
(Hall's Safe Company of Cincinnati, Ohio.)

No.	Weight of Safe (Empty.) Lbs.	Outside Dimensions in Inches. H'ht	W'th	D'th	Wheel Base in Ft. (Approx.) W'th	D'th	Dead Weight on Each Wheel, + 10%. Lbs.	Inside Capacity. Cu. Ft.	Wt. in Lbs. of Contents at 100 Lbs. per Cu. Ft.	25 Lbs. per Cu. Ft.	Kind of Safe.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
921	16080	86.5	68.5	36	4.7	2.7	4422	29.2	2920	730	Double-door, fire-proof, lined with steel, and steel inside doors. For Bankers.:
920	12530	77.5	59.5	35	4.0	2.6	3445	20.2	2020	505	
923	11100	79	51	34	3.4	2.5	3052	16.0	1600	400	
918	9650	69.25	51	34	3.4	2.5	2653	13.2	1320	330	
922	8000	73.5	45.5	34	2.8	2.5	2200	12.2	1220	305	
916	7270	61	44	34	2.7	2.5	2000	9.7	970	243	
915	6200	57	40	34	2.5	2.5	1705	7.6	760	190	
914	5380	53	38	34	2.4	2.5	1480	6.3	630	158	
821	11000	86.5	68.5	30	4.7	2.3	3025	28.4	2840	710	Double-door, fire-proof, lined with steel; no inside doors. For Jewelers.
820	8900	77.5	59.5	30	4.0	2.3	2448	21.2	2120	530	
823	7900	79	51	30	3.4	2.3	2173	18.3	1830	458	
818	6400	69	51	30	3.4	2.3	1760	15.1	1510	378	
822	6300	73.5	45.5	30	2.8	2.3	1733	14.1	1410	353	
819	5250	69	38	30	2.4	2.3	1443	11.0	1100	275	
816	4900	61	44	30	2.7	2.3	1348	11.3	1130	283	
815	4200	56.5	39.5	30	2.5	2.3	1155	8.5	850	213	
817	4025	61	34	30	2.0	2.3	1107	8.0	800	200	
814	3700	53	38	30	2.4	2.3	1018	7.4	740	185	
521	8000	86.5	68.5	34	4.7	2.5	2200	38.2	3820	955	Double-door, fire-proof, with inside doors; containing Banker's steel chest. For general office use.
520	6000	77.5	59.5	32	4.0	2.4	1650	27.0	2700	675	
523	5350	79	51	32	3.4	2.4	1471	23.2	2320	580	
518	5150	69	51	30	3.4	2.3	1416	17.0	1700	425	
522	4700	73.5	45.5	30	2.8	2.3	1292	15.9	1590	398	
516	3650	61	44	30	2.7	2.3	1003	12.7	1270	318	
519	3550	69	38	30	2.4	2.3	976	12.2	1220	305	
517	3200	61	34	30	2.0	2.3	880	8.9	890	223	
515	3150	57	40	29	2.5	2.2	866	9.4	940	235	
514	2850	53	38	29	2.4	2.2	783	7.8	780	195	
512	2800	55.5	34.75	29	2.0	2.2	770	7.2	720	180	
513	2500	49	36	28	2.2	2.1	685	6.0	600	150	
"C"	5750	52	30	26	1.9	1.9	1582	9.2	920	230	Square door. For Bankers.
"B"	4500	45	28	25	1.5	1.8	1238	6.4	640	160	
"A"	3500	41	25	24	1.7	1.7	962	4.5	450	113	

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Fixed Vaults with their contents may be considered as static loads and must be specially considered, but Movable Safes are liable to be placed in almost any position on the floor.

Problem 1.—On the first floor of a building find what single concentrated load will give the same bending moment as safe No. 921, Table 2, to one of a system of parallel beams spaced 5 ft. centers and 16 ft. long?

Solution.—Let A, B and C, Fig. 11, represent the beams of which B is the one under consideration; D and E the supporting girders; and 1, 2, 3 and 4 the four wheels supporting the safe, of which c is the center. From the principle of the maximum floor-beam reaction, page 692, B will support the greatest load when it bisects the normal distance between the center of gravity c and the line 1–2; that is, with maximum resultant at r. Likewise, the maximum moment on B will obtain at m, equidistant with r from the line Y–Y which bisects the beams. If each of the corner weights of the safe is 4422 lbs., we have, for resultant at r, on beam B,

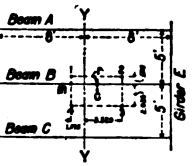


Fig. 11.

$$r = 8844\left(\frac{10-2.7}{5}\right) = 12912 \text{ lbs.},$$

and maximum moment, at m, $= \frac{r}{16}(8-1.175)^2$.

The moment due to a load P at the center of the beam $= \frac{P}{2} \times 8$; whence, equating with the above,

$$\frac{P}{2} \times 8 = \frac{r}{16}(6.825)^2 \text{ or } P = .7278\, r = 9400 \text{ lbs.}$$

Hence, in the above case it will be noted that a concentrated load of 9400 lbs. applied at the center of the beam B will produce a bending moment equal to that of the safe weighing 17,688 lbs., placed *rectangularly* in the most critical position. Furthermore, this is equivalent to a running load of 1175 lbs. per lineal foot of beam, or to a distributed load of 235 lbs. per square foot of floor. If safe is placed *diagonally* on beam, the bending moment will be increased about 5 per cent.

Examples of Floor and Ceiling Construction.

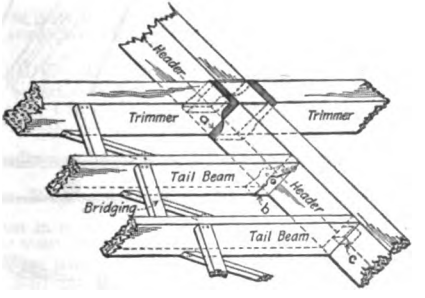


Fig. 12.—Floor Framing.—*a* is a double hanger or stirrup; *b* is a patent hanger; *c* is a common mortise.

The under flooring may be spruce or hemlock, ⅞" thick and laid diagonally in order to brace or stiffen the floor. On top of this and at right angle with the tail beams is laid the finished flooring which may be white pine ⅞" thick or any other flooring material. The ceiling may consist of lathing and plastering below the beams in the usual manner.

Fig. 13.—Centering for Brick Arch.

Fig. 14.—4-inch Brick Arch.—c i
wooden screed on which is nailed th
the rod; w, washer; a, angle iron
thrust of the arch; l, steel beam.

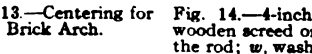


Fig. 15.—Hollow Bricks.

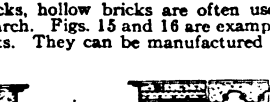
Fig. 16.—S

Instead of the solid bricks, hollow bricks are often us
reduce the dead load of the arch. Figs. 15 and 16 are examp
hollow bricks and skew-backs. They can be manufactured
pattern.

Fig. 17.—Corrugated Steel and Concrete Floor.

Fig. 18.—Steel T
Concrete F

Fig. 19.—Typical Terra-Cotta Floor.



Figs. 20.—End-Construction
Tile Floor.

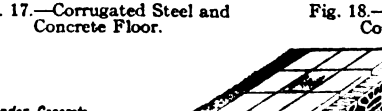


Fig. 21.

Fi

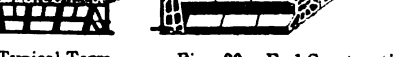


Fig. 23.

Fig. 24

Fig. 25.

Fig. 26.

Fig. 27.

Fig. 28.

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DIGEST OF THE NEW YORK CITY BUILDING CODE (1906).

[Also District of Columbia, practically the same.]

QUALITY OF MATERIALS.

1. **Lime mortar.**—1 part lime, not > 4 parts sand; lime properly slaked before being mixed with the sand.

2. **Cement mortar.**—1 part cement, not > 3 parts sand; mixed before adding water. *Portland cement*=cement that, when tested neat, will resist tension of at least 120 lbs. per sq. in., after 1 day air setting; and 300 lbs., after 1 day in air and 6 days in water. *Other cements*, 60 lbs. and 120 lbs., respectively.

3. **Cement and lime mortar.**—1 part cement, 1 part lime, not > 3 parts of sand to each.

4. **Concrete.**—At least 1 cement; 2 sand; 5 clean broken stone (2 in. ring), or 5 clean gravel.

5. **Wrought iron.**—Ultimate strength 48,000 lbs. per sq. in.; elastic limit not < 24,000; elongation 20% in 8 ins. (small specimens).

6. **Steel.**—Ult str, 54–64,000; elastic limit, not < 32,000; elongation, not < 0.20. Rivet steel, 50–58,000 ult str.

7. **Cast iron.**—One-inch square bar, 54-in. span, shall support central load of 450 lbs. before breaking. Tensile strength, not < 16,000 lbs. per sq. in. (small specimens).

EXCAVATIONS AND FOUNDATIONS.

8. **Bearing capacity of soil.**—Where no tests are made allow for soft clay, 1 ton per sq. ft.; ordinary clay and sand together, in layers, wet and springy, 2 tons; loam, clay or fine sand, firm and dry, 3 tons; very firm, coarse sand, stiff gravel or hard clay, 4 tons.

9. **Pressure under footings of foundations.**—For warehouses and factories, full dead and full live loads; for stores, light factories, churches, school houses, and places of public amusement or assembly, full dead and 75 per cent of live loads; for office buildings, hotels, dwellings, apartment houses, tenement houses, lodging houses, and stables, full dead load and 60% of live load.

10. **Foundations.**—Piles 20 ft. or less in length, not < 5 ins. at small end and 10 ins. at butt. Piles over 20 ft., not < 12 ins. at butt. Max load per pile, 40,000 lbs. Use Engineering News formula when pile is not driven to refusal: $P=\frac{2wh}{s+1}$*. Safe load for stone, brick or concrete piers in caissons: to rock, not > 15 tons per sq. ft.; to firm gravel or hard clay, 10 tons; in open caissons or sheet pile trenches, 8 tons.

WOODEN BEAMS, GIRDERS AND COLUMNS.

11. **Wood beams.**—Minimum thickness, 3 ins. Every wood header or trimmer more than 4 ft. long shall be hung in stirrup irons. All wood floor and wood roof beams bridged with cross bridging spaced not > 8 ft. Safe uniform load in lbs. per lineal foot for long spans: hemlock, $70\frac{bd^2}{L}$; spruce and white pine, $90\frac{bd^2}{L}$; oak, $120\frac{bd^2}{L}$; yellow pine, $140\frac{bd^2}{L}$; in which b = breadth of beam in inches., d=depth in ins., L=length in ft. For short spans the shear must be considered.

12. **Timber for trusses.**—Working stresses in timber struts of pin-connected trusses shall not exceed 75% of the working stresses established in Par. 27–30.

FIREPROOF BUILDINGS.

13. **Fireproof buildings.**—For buildings exceeding 12 stories or 150 ft. the floors shall be of stone, cement, rock asphalt, tiling, etc., or the sleepers and floors may be of wood treated by some approved process to render them fireproof.

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* See Sec. 50, Foundations, page 871.

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14. **Fireproof floors.**—Shall be constructed with wrought iron or steel floor beams calculated to deflect no more than $\frac{1}{360}$-inch per foot of span under total (live and dead) load; and tie rods shall be spaced not > 8 times depth of beam. (1) Brick arches, springing from the lower flange of the steel beams, shall be designed with a rise not < $1\frac{1}{4}$ ins. for each foot of span, and with a thickness not < 4 ins. for spans of 5 ft. or less, and not < 8 ins. for spans over 5 ft. They shall be composed of good hard brick or hollow brick of ordinary dimensions laid to a line on the centers, properly and solidly bonded, each longitudinal line of brick breaking joints with the adjoining lines in the same ring and with the ring under it when more than 4-in. arch is used; cement mortar joints. (2) Hollow tile arches of hard-burned clay or porous terra-cotta shall have an effective depth not < $1\frac{1}{4}$ ins. per ft. of span, some allowance (not over 6 ins.) being made if soffit of arch is straight; if segmental, the depth of tile shall be not < 6 ins. if rise is not < $1\frac{1}{4}$ ins. times span in ft.; cement mortar joints. (3) Portland cement concrete arches, segmental, with rise not < $1\frac{1}{4}$ ins. times span in ft. Thickness at crown not < 4 ins. Mixed as per par. 4. Arches shall be reinforced and protected on under side with corrugated or sheet steel, steel ribs, or metal in other forms weighing not less than 1 lb. per sq. ft., and having no openings larger than 3 sq. ins. (4) Reinforced floors of solid or hollow burned clay, stone, brick, or concrete slabs in flat or curved shapes, in combination with wire cloth, expanded metal, wire strands, or wrought iron or steel bars, may be used, but proper tests shall be made as provided in the Code.

IRON AND STEEL CONSTRUCTION.

15. **Skeleton construction.**—Where columns are used to support iron or steel girders carrying inclosure walls, the said columns shall be of cast iron, wrought iron, or rolled steel, and on their exposed outer and inner surfaces be constructed to resist fire by having a casing of brickwork not less than 8 ins. thick on the outer surfaces, nor less than 4 ins. thick on the inner surfaces, all bonded into the brickwork of the inclosure walls. Exposed sides of girders protected with 4 ins. of brickwork; outer edges of flanges, 2 ins.

16. **Steel and wrought iron columns.**—Minimum thickness of metal, $\frac{1}{4}$ inch. Least lateral dimension $\frac{1}{40}$ length of column, except as allowed in par. 26.

17. **Cast iron columns.**—Minimum diameter, 5 ins.; minimum thickness of metal, $\frac{3}{4}$ in. Least lateral dimension $\frac{1}{30}$ length of column, except as allowed in par. 26.

18. **Steel and iron girders.**—Stiffeners shall be used at intervals not exceeding 120 times thickness of web if the unsupported depth of the web plate exceeds 60 times its thickness.

19. **Rolled beams used as girders.**—Beams in pairs to form girders shall be connected together by bolts and separators at intervals of not more than 5 ft. Beams 12 ins. or more in depth shall have 2 bolts to each separator.

20. **Cast iron lintels.**—Maximum span, 16 ft.; minimum thickness of metal, $\frac{3}{4}$ in.

21. **Painting of structural metal work.**—After erection all work shall be painted at least one additional coat. All iron or steel used under water shall be inclosed with concrete.

FLOOR LOADS.

22. **Floor loads.**—Dead loads = weight of walls, floors, roofs, partitions and all permanent construction. Live loads = variable loads = all loads other than dead loads. Live loads per sq. ft. of floor shall be assumed as follows:

For dwelling, apartment, tenement, hotel or lodging house.	not < 60	lbs.
For office purpose, first floor,	" 75	"
" " all floors above the first,	" 150	"
" school or place of instruction,	" 75	"
" stable and carriage house purposes,	" 75	"
" place of public assembly,	" 90	"
" ordinary stores, light manufacturing, light storage,	" 120	"
" warehouse, factory, heavy stores,	" 150	"
" roofs with pitch less than 20°, per area of roof,	" 50	"
" " " more " 20°, per horizontal area,	" 30	"
" sidewalks, between the curb and area lines,	" 300	"

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For the purpose of determining the carrying capacity of columns , office buildings, stores, stables and public buildings when over height, a reduction of the live loads shall be permissible as fol- he roof and top floor the full live loads shall be used; for each ower floor it shall be permissible to reduce the live load by 5% f the live loads fixed by this section is reached, when such re- shall be used for all remaining floors.

ALCULATIONS.—STRENGTH OF MATERIALS.

e load for masonry work.—Safe load per sq. ft.: Brickwork in , 16,000 lbs.; in lime and cement mortar mixed, 23,000 lbs.; in tar, 30,000 lbs. Rubble-stone work in Portland cement, 20,000 er cement, 16,000 lbs., in lime and cement mixed, 14,000 lbs.; tar, 10,000 lbs. Portland cement concrete, 30,000 lbs. Other rete, 16,000 lbs.

ights of certain materials.—In computing weight of walls, one ckwork shall be deemed to weigh 115 lbs.; sandstone, white nite and other kinds of building stone, 170 lbs. per cubic foot.

tors of safety.—While working stresses are not prescribed netals, 6 for timber, 10 for natural or artificial stones and brick sonry.

ngth of columns.—Safe load in lbs. per sq. in.: Cast iron, * ; steel, $15{,}200-58\frac{l}{r}$; wrought iron, $14{,}000-80\frac{l}{r}$. Yellow

eaf), $1000-18\frac{l}{d}$; white pine, Norway pine, spruce, $800-15\frac{l}{d}$;

$\frac{l}{d}$; chestnut and hemlock, $\frac{5}{8}\left(800-15\frac{l}{d}\right)$; locust, $\frac{3}{2}\left(800-15\frac{l}{d}\right)$.

entrically loaded shall have such stresses computed, and the resses shall not exceed the allowable working stresses. The d of a column shall be considered to be distributed equally over ea of that column at the next point below at which the column raced laterally in the direction of the eccentricity.

ıpression (Direct).—Safe load in lbs. per sq. in.: Rolled steel, steel, 16,000; wrought iron, 12,000; cast iron (in short blocks), el pins and rivets (bearing), 20,000; wrought iron pins and ng), 15,000.

	With Grain.	Across Grain.		With Grain.	Across Grain.
-	900	800	Locust, - -	1,200	1,000
-	1,000	600	Hemlock, -	500	500
spruce	800	400	Chestnut, -	500	1,000

, Portland: cement, 1; sand, 2; stone, 4 - - -	230
" " 1; " 2; " 5 - - -	208
sendale, or equal: " 1; " 2; " 4 - - -	125
" " 1; " 2; " 5 - - -	111
tonework in Portland cement mortar - - -	140
" " Rosendale " " - - - -	111
" " lime and " " - - - -	97
" " lime mortar - - - - - -	70
k in Portland cement mortar: cement, 1; sand, 3	250
" Rosendale, or equal " " " 1; " 3	208
" lime and cement " " 1; lime, 1; sand, 6	160
" lime mortar: lime, 1; sand, 4 - - - -	111
acc. to test, 1,000 to 2,400.	
es, " " 700 " 2,300.	
" " 600 " 1,200.	
es, " " 400 " 1,600.	
averstraw, flatwise) 300.	

Greenwich stone -	1,200
Greiss (New York City)	1,300
Bluestone, North River,	2,000
Slate, - -	1,000

th of column, in ins.; r = radius of gyration, in ins.; d = least n ins.

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28. **Tension (Direct).**—Safe stress in lbs. per sq. in.: Rolled steel, 16,000; cast steel, 16,000; wrought iron, 12,000; cast iron, 3,000; yellow pine, 1,200; white pine, spruce, 800; oak, 1,000; hemlock, 600.

29. **Shear.**—Safe stress in lbs. per sq. in.: Cast iron, 3,000. Web plates: Steel, 9,000; wrought iron, 6,000. Shop rivets and pins: Steel, 10,000; wrought iron, 7,500. Filed rivets: Steel, 8,000; wrought iron, 6,000. Field bolts: Steel, 7,000; wrought iron, 5,500.

	With Fiber.	Across Fiber.		With Fiber.	Across Fiber.
Yellow pine -	70	500	Locust, - -	100	720
White pine, -	40	250	Hemlock, - -	40	275
Spruce, - -	50	320	Chestnut, - -	...	150
Oak, - - -	100	600			

30. **Bending.**—Safe extreme fiber stess in lbs. per sq. in.: Rolled beams, Steel, 16,000; wrought iron, 12,000. Rolled pins, rivets and bolts: Steel, 20,000; wrought iron, 15,000. Riveted beams (not flange section): Steel, 14,000; wrought iron, 12,000. Cast iron: Compression side, 16,000; tension side, 3,000. Yellow pine, 1,200; white pine, spruce, 800; oak, 1,000; locust, 1,200; hemlock, 600; chestnut, 800. Granite, 180; Greenwich stone, 150; gneiss (New York City), 150; limestone, 150; slate, 400; marble, 120; sandstone, 100; bluestone (North River), 300; brick (common), 50; brickwork (in cement), 30. Concrete: (Portland), 1:2:4, 30 lbs.; 1:2:5, 20 lbs.; (Rosendale, or equal), 1:2:4, 16 lbs.; 1:2:5, 10 lbs.

31. **Wind pressure.**—All structures exposed to wind shall be designed to resist a horizontal wind pressure of 30 lbs. for every sq. ft. of surface thus exposed, from the ground to the top of same, including roof, in any direction. In no case shall the overturning moment due to wind pressure exceed 75% of the moment of stability of the structure. In all structures exposed to wind, if the resisting moments of the ordinary materials of construction such as masonry, partitions, floors and connections are not sufficient to resist the moment of distortion due to wind pressure, taken in any direction on any part of the structure, additional bracing shall be introduced sufficient to make up the difference in the moments. In calculations for wind bracing, the working stresses set forth above may be *increased by* 50%. In buildings under 100 ft. high, where the height does not exceed four times the average width of the base, the wind pressure may be disregarded.

CONCRETE-STEEL CONSTRUCTION.

1. The term "concrete-steel" shall mean an approved concrete mixture reinforced by steel of any shape, the steel to take up the tensional stresses and assist in the resistance to shear.

2. Concrete-steel construction will be approved only for buildings which are not required to be fireproof by the building code, unless satisfactory fire and water tests shall have been made under the supervision of this bureau.

3. Complete drawings and specifications must be filed with the supt. of buildings, showing all details of the construction, the size and position of all reinforcing rods, stirrups, etc., and giving the composition of the concrete.

4. Execution of work shall be under the control of a competent foreman or superintendent.

5. Concrete to be mixed in the proportions of 1 cement, 2 sand, and 4 stone or gravel; or the proportions may be such that the resistance of the concrete to crushing shall not be less than 2000 lbs. per sq. in. after hardening for 28 days. The concrete is to be what is usually known as a "wet" mixture.

6. Only high-grade Portland cements shall be permitted; and shall develop a tensile strength of at least 300 lbs. per sq. in. after 1 day in water; at least 500 lbs. per sq. in. after 1 day in air and 6 days in water; and at least 600 lbs. per sq. in. after 1 day in air and 27 days in water.

7. The sand must be clean and sharp, free from loam or dirt, and not finer than the standard sample submitted.

8. The stone shall be clean, broken trap rock, or gravel, of a size that will pass through a three-quarter inch ring.

9. The steel shall have an ult. str. of 54-64,000 lbs. per sq. in.; an elastic limit of not less than 32,000 lbs. per sq. in.; and a min. elong. of not less than 20% in 8 ins.

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oncrete-steel design the following stresses in lbs. per square t be exceeded:

er stress on concrete in compression	500 (650*)
ess in concrete	50
direct compression	350 (450*)
s in steel	16,000
ess in steel	10,000

adhesion of concrete to steel shall be assumed to be not greater aring strength of the concrete.

ratio of modulus of elasticity of concrete and steel shall be 12.

following assumptions shall guide in the determination of the nents due to the external forces: (a) Beams and girders shall l as simply supported at the ends, no allowance being made for over supports. (b) Floor plates, when constructed continuous ovided with reinforcement at top of plate over the supports, ed as continuous beams, the bending moment for uniformly oads being taken at not less than $WL \div 10$; the bending moment n as $WL \div 20$ in the case of square floor plates which are rein- h directions and supported on all sides. The floor plates to the more than ten times the width of any beam or girder may be rt of that beam or girder in computing its moment of resistance. lculating the moment of resistance it is assumed that the fiber is directly proportional to its distance from the neutral into consideration the proportionate moduli of elasticity of crete). The tensile strength of the concrete shall not be con-

n the shearing stresses developed in any part of a concrete- ction exceed the safe working strength of concrete, a sufficient eel shall be introduced in such a position that the deficiency nce to shear is overcome.

n the safe limit of adhesion between the concrete and steel is ne provision must be made for transmitting the strength of the oncrete.

rete-steel may be used for columns in which the ratio of length or diameter does not exceed 12. The reinforcing rods must be at intervals of not more than the least side or dia. of column.

contractor must be prepared to make load tests on any portion steel construction, within a reasonable time after erection, as be required by the supt. of buildings. The tests must show truction will sustain a load of 3 times that for which it is de- at any sign of failure.

ACT FROM CHICAGO BUILDING ORDINANCE (1910).

REINFORCED CONCRETE.

Moduli of Elasticity—Adhesion—Bond.—(a) The calculations th of reinforced concrete shall be based on the assumed ulti- ssive strength U, in pounds per square inch, given in the table mixture to be used. (b) The ratio of the modulus of elasticity t of concrete is designated by the letter R:

cement, 1 sand, 2 broken stone, gravel or slag) $U=2{,}900$; $R=10$.
cement, 1½ sand, 3 broken stone, gravel or slag) $U=2{,}400$; $R=12$.
cement, 2 sand, 4 broken stone, gravel or slag) $U=2{,}000$; $R=15$.
cement, 2½ sand, 5 broken stone, gravel or slag) $U=1{,}750$; $R=18$.
cement, 3 sand, 7 broken stone, gravel or slag) $U=1{,}500$; $R=20$.

aynor's views expressed under date of July 21, 1911, regarding revised code. He also says: As near as I can make out from een explained to me the cost of cinder concrete should not be is not necessary that the cinders should be screened. The he arches should not be increased from the minimum of 4½ to f 6½. That is unnecessary. The weight of the reinforcing in should not be raised to a minimum of one pound per square arter of a pound per square foot may suffice. A precise and e should be made to govern.

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Adhesion—Bond.—For a concrete of 1:2:4 mix, the allowable adhesion in lbs. per sq. in. of surface of embedment shall not exceed the following: On plain bars of structural steel, 70; on plain bars of high carbon steel, 50; on plain flat bars in which width to thickness is not >2 to 1, 50; on twisted bars when twisting is not < one complete twist in eight diameters, 100.

EXTRACT FROM BUILDING LAWS AND ORDINANCES OF PHILADELPHIA (1907).

Live Loads for Floors.—*Lbs. per sq. ft.*: Dwellings, tenement houses, apartment houses, hotels, hospitals and asylums, 70 lbs.; office buildings, 100 lbs.; places of public assembly, light manufacturing and retail stores, 120 lbs.; storehouses, warehouses and manufactories, 150 lbs. and upward in proportion to the loads they have to carry.

Roofs shall be constructed to bear a safe weight of 30 lbs. per superficial foot.

Ultimate Stresses in lbs. per sq. in.:

	Cast Iron.	Wrt. Iron.	Mild Steel.	Medium Steel.	Hemlock.	Spruce.	Long Leaf Yellow Pine.
Tension (direct)	.....	50,000	58,000	65,000	4,000	5,000	7,200
Compression (direct)	70,000	50,000	58,000	65,000	2,100	3,000	4,500
Bending—extreme fiber (tension)	15,000	.....	.....	.....	3,600	4,400	6,400
Shear	.....	30,000	35,000	40,000	.....	.....	.....
Shear—perp. to grain	.....	.....	.....	.....	2,500	3,000	4,500
Shear—parallel with grain	.....	.....	.....	.....	250	300	400

Working Stresses* in lbs. per sq. in.:

	Cast Iron.	Wrt. Iron.	Mild Steel.	Medium Steel.	Hemlock.	Spruce.	Long Leaf Yellow Pine.
Tension	.....	12,500	14,500	16,250	1,000	1,250	1,800
Compression	11,667	12,500	14,500	16,250	350	500	750
Compression—perp. to grain	.....	.....	.....	.....	250	300	550
Bending—extreme fiber (tension)	3,750	.....	.....	.....	900	1,100	1,600
Shear	.....	7,500	8,750	10,000	.....	.....	.....
Shear—perp. to grain	.....	.....	.....	.....	416⅔	500	750
Shear—parallel to grain	.....	.....	.....	.....	41⅔	50	66⅔

* For columns, the safe working loads p in lbs. per sq. in. may be reduced by the following formulas:—

Cast iron, $p = \dfrac{11667}{1 + \dfrac{l^2}{400d^2}}$; Wrought iron, $p = \dfrac{12500}{1 + \dfrac{l^2}{15000r^2}}$;

Mild steel, $p = \dfrac{14500}{1 + \dfrac{l^2}{13500r^2}}$; Medium steel, $p = \dfrac{16250}{1 + \dfrac{l^2}{11000r^2}}$;

Hemlock, $p = 350 - 3.5\dfrac{l}{d}$; Spruce, $p = 500 - 5\dfrac{l}{d}$; Yellow pine, $p = 750 - 7.5\dfrac{l}{d}$.

In which l = length, r = least radius of gyration, d = least diameter, all in inches. The allowable reduction of live load on columns and girders shall be as follows: "For all tenement houses, hotels, apartment houses, hospitals and office buildings the live loads on columns, girders and foundations may be estimated by the formula $X = 100 - \frac{2}{3}\sqrt{A}$, and for light manufacturing buildings by the formula $X = 100 - \frac{3}{4}\sqrt{A}$, in which "$X$" equals the percentage of live load to be used, and "A" equals area carried by any girder, column or foundation.

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Allowable Pressures in lbs. per sq. ft.—Concrete, 15 tons; brickwork in lime mortar, 8 tons; brickwork in lime and cement mortar, 12 tons; brickwork in cement mortar, 15 tons; stonework (rubble) in lime mortar, 5 tons; stonework (rubble) in lime and cement mortar, 8 tons; stonework (rubble) in cement mortar, 10 tons.

Reinforced Concrete.—Reinforced concrete construction will be accepted for fireproof buildings of the first class, if designed as hereinafter prescribed; provided, that the aggregate for such concrete shall be clean broken hard stone, or clean graded gravel, together with clean siliceous sand or fine grained gravel; should the concrete be used for flooring between rolled steel beams, clean furnace clinkers entirely free of combustible matter, or suitable seasoned furnace slag may be used; when stone is used with sand or gravel it must be of a size to pass through a one-inch ring, and 25% of the whole must not be more than one-half the maximum size; and provided further, that the minimum thickness of concrete surrounding the reinforcing members of reinforced concrete beams and girders shall be 2 ins. on the bottom and one-half inch on the sides of said beams and girders. The minimum thickness of concrete under slab rods shall be one inch. All reinforcement in columns to have a minimum protection of 2 ins. of concrete.

For walls, reinforced concrete may be used in place of brick and stone walls, in which case the thickness may be two-thirds of that required for brick walls. Concrete walls in such cases must be reinforced in both directions in an approved manner.

All steel reinforcement shall be of standard grade of structural steel or iron of either grade to meet the "Manufacturers' Standard Specifications," revised Feb. 3, 1903.

Slabs, beams and girders shall be designed on the assumption of a load four times as great as the total load (ordinary dead load plus ordinary live load). The steel to take all the tensile stresses. The stress-strain curve of concrete in compression is a straight line.

Ratio of moduli of elasticity of concrete to steel: Stone or gravel concrete, 1 to 12; slag concrete, 1 to 15; cinder concrete, 1 to 30.

Allowable unit transverse stress (lbs. per sq. in.) upon concrete in compression: Stone or gravel concrete, 600; slag concrete, 400; cinder concrete, 250.

Allowable unit transverse stress (lbs. per sq. in.) in tension: Iron, 12,000; steel, 16,000.

Allowable unit shearing stress (lbs. per sq. in.) upon concrete: Stone or gravel concrete, 75; slag concrete, 50; cinder concrete, 25.

Allowable unit adhesive strength (lbs. per sq. in.) of concrete: Stone or gravel concrete, 50; slag concrete, 40; cinder concrete, 15.

Allowable unit stresses (lbs. per sq. in.) upon concrete in direct compression in columns: Stone or gravel concrete, 500; slag concrete, 300; cinder concrete, 150.

Allowable unit stress upon hoop columns composed of stone or gravel concrete shall not be over 1000 lbs. per sq. in., figuring the net area of the circle within the hooping. The percentage of longitudinal rods and the spacing of the hoops to be such as to permit the concrete to safely develop the above unit stress with a factor of safety of 4.

Floor slabs, when constructed continuously, and when provided with reinforcement at top of slab over the supports, may be treated as continuous beams, the bending moment for uniformly distributed loads being taken at not less than $WL \div 10$. In case of square floor slabs which are reinforced in both directions and supported on all sides, the bending moment may be taken at $WL \div 20$; provided, that in floor slabs in juxtaposition to the walls of the building the bending moment shall be considered as $WL \div 8$, when reinforced in one direction, and if the floor slab is square and reinforced in both directions the bending moment shall be taken as $WL \div 16$.

In columns the longitudinal rods will not be considered as taking any direct compression.

EXTRACT FROM THE BUILDING LAW OF BOSTON (1909).

MATERIALS—ALLOWABLE FIBER STRESSES.

[Lbs. per square inch.]

Timber.—**Extreme fiber (bending)**: White pine and spruce, 1000; white oak, 1000; yellow pine (long leaf), 1500. **Shearing along grain**: White pine, 100; white oak, 150; yellow pine (long leaf), 100. **Compression perpendicular to grain**: White pine and spruce, 250; white oak, 600; yellow pine (long leaf), 300. **Modulus of elasticity**: White pine, 750,000; spruce, 900,000; white oak,

850,000; yellow pine (long leaf), 1,300,000. **Columns (centrally loaded and flat ends)**: White pine and spruce, 630 for $L+D=0$ to 10, 595 for $L+D=10$ to 15, 560 for $L+D=15$ to 20, 525 for $L+D=20$ to 25, 490 for $L+D=25$ to 30; white oak, 810 for $L+D=0$ to 10, 765 for $L+D=10$ to 15, 720 for $L+D=15$ to 20, 675 for $L+D=20$ to 25, 630 for $L+D=25$ to 30; yellow pine (long leaf), 900 for $L+D=0$ to 10, 850 for $L+D=10$ to 15, 800 for $L+D=15$ to 20, 750 for $L+D=20$ to 25, 700 for $L+D=25$ to 30. No column shall be used with greater value than $L+D=30$. For eccentric loads, see Methods of Computation, p. 827.

Wrought Iron and Steel (steel at 55-65,000).—Extreme fiber of rolled beams or shapes: Wrought iron, 12,000; steel, 16,000. Tension: Wrought iron, 12,000; steel, 16,000. Compression in flanges of built beams: Wrought iron, 12,000; steel, 16,000. Shearing (including pins and rivets, but not bolts): Wrought iron, 9,000; steel, 10,000. Shearing (bolts): 80% of preceding values. Direct bearing (including pins and rivets, but not bolts): Wrought iron, 15,000; steel, 18,000. Direct bearing (bolts): 80% of preceding values. Bending on pins: Wrought iron, 18,000; steel, 22,500. Modulus of elasticity: Wrought iron, 27,000,000; steel, 29,000,000. For compression members, use the formula: [12,000 for iron or 16,000 for steel] *divided by* $[1+(L^2+20000r^2)]$, in which L = length and r = radius of gyration in inches. Compression flanges of beams shall be proportioned to resist lateral flexure; if the ratio of unsupported length of flange to width of flange does not exceed 20, no allowance need be made; if the ratio is 70, the above specified allowable fiber stress shall be reduced by one-half; and proportionate for values between 20 and 70.

Cast Iron.—Extreme fiber stress: Tension, 3,000; compression, 16,000. **Columns** (centrally loaded and unsupported laterally): Average stress, 11,000 for $L+r=10$, 10,700 for $L+r=20$, 10,400 for $L+r=30$, 10,000 for $L+r=40$, 9,800 for $L+r=50$, 9,500 for $L+r=60$, 9,200 for $L+r=70$. L and r in inches. Cast iron shall not be used for columns in buildings of more than 75 feet in height, nor in cases where $L+r$ exceeds 70.

[Tons (2000 lbs.) per square foot.]

Stone Work, in Compression.—First quality dressed beds and builds, laid solid in mortar of one part Portland cement to three parts sand, or one part natural cement to two parts sand. Granite, 60; marble and limestone, 40; sandstone, 30. When poorer mortar is used the above stresses shall be lowered (as approved).

Brickwork, in Compression.—(1.) For first class hard-burned bricks, including piers in which the height does not exceed six times the least dimension, laid in: (a) One part Portland cement, three parts sand, by volume, dry, 20; (b) One part natural cement, two parts sand, by volume, dry, 18; (c) One part natural cement, one part lime and six parts sand, by volume, dry, 12; (d) Lime mortar, one part lime, six parts sand, by volume, dry, 8. (2.) For brick piers of hard-burned bricks, in which the height is from six to twelve times the least dimension: Mortar (a), 18; mortar (b), 15; mortar (c), 10; mortar (d), 7. (3.) For brickwork made of "light-hard" bricks, the stresses shall not exceed two-thirds of the stresses for like work of hard-burned bricks.

CONCRETE AND REINFORCED CONCRETE.

Cement shall conform to the specifications of the American Society for Testing Materials, as modified from time to time by that association.

Concrete.—When the structural use of concrete is proposed, a specification, stating the quality and proportions of materials, and the methods of mixing the same, shall be submitted to the building commissioner, who may issue a permit at his discretion and under such further conditions, in addition to those stated below, as he sees fit to impose.

A. In first class Portland cement concrete, containing one part cement to not more than six parts mixed properly graded aggregate, except in piers or columns of which the height exceeds six times the least dimension, the compressive stress shall not exceed 30 tons of 2000 lbs. per sq. ft.

B. In piers and columns of first class Portland cement concrete, containing one part cement to not more than five parts mixed properly graded aggregate, where the height of the pier or column is more than six times and does not exceed twelve times its least dimension, the compressive stress shall not exceed 25 tons of 2000 lbs. per sq. ft.

By "aggregate" shall be understood all the materials in the concrete except the cement. Cinder concrete shall be used constructively only for floors, roofs and for filling.

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Rules for the computation of reinforced concrete columns may be formulated from time to time by the building commissioner with the approval of the board of appeal.

In reinforced concrete beams or slabs subjected to bending stresses, the entire tensile stress shall be assumed to be carried by the steel, which shall not be stressed above the limits allowed for this material. First class Portland cement concrete in such beams or slabs, containing one part cement to not more than five parts mixed properly graded aggregate, may be stressed in compression to not more than 500 lbs. per sq. in. In case a richer concrete is used, this stress may be increased with the approval of the commissioner to not more than 600 lbs. per sq. in.

In reinforced concrete the maximum shearing force upon the concrete, when uncombined with compression upon the same plane shall not exceed 60 lbs. per sq. in., unless the building commissioner with the consent of the board of appeal shall fix some other value.

If the embedded steel has no mechanical bond with the concrete, its holding power shall not exceed the allowable shearing strength of the concrete.

Reinforced Concrete.—Reinforced concrete slabs, beams or girders, if rendered continuous over supports by being unbroken in section, shall be provided with proper metal reinforcement at the top over said supports and may be computed as continuous beams, as hereinafter described.

The modulus of elasticity of the concrete, if not shown by direct tests, may for beams and slabs be taken as one-fifteenth that of steel, and for columns one-tenth that of steel.

The reinforcing metal shall be covered by not less than three-fourths inch of concrete in slabs, and by not less than one and one-half inches of concrete in beams and columns.

Methods of Computation.—Beams or girders of metal or reinforced concrete shall be considered as simply supported at their ends, except when they extend with unbroken cross-section over the supports, in which case they may be considered as continuous.

The span of a beam shall be considered as the distance from center to center of the bed plates or surfaces upon which it rests. If it is fastened to the side of a column, the span will be measured to the center of the column.

In slabs, beams or girders continuous over supports, provision shall be made for a negative bending moment at such supports equal to four-fifths of the positive bending moment that would exist at the center of the span if the piece were simply supported; and the positive bending moment at the center of the span may be taken equal to the negative bending moment at the support.

In the case of a slab of reinforced concrete with parallel ribs or girders beneath, the rib or girder may be considered to include a portion of the slab between the ribs, forming a T-beam. The width of the T-beam on top shall not exceed one-third of the span of the rib nor the distance from center to center of the ribs.

Reinforced concrete columns shall be proportioned on the assumption that the concrete and the steel are shortened in length in the same proportion. The steel members shall be tied together at intervals sufficiently short to prevent buckling.

If a column is loaded eccentrically or transversely, the maximum fiber stress, taking account of the direct compression, the bending which it causes, its eccentricity and the transverse load, shall not exceed the maximum allowable stress in compression.

If a tension piece is loaded eccentrically or transversely, the maximum fiber stress, taking account of the direct tension, its eccentricity and the transverse load, shall not exceed the maximum allowable stress in tension.

An eccentric load upon a column shall be considered to affect eccentrically only the length of column extending to the next point below at which the column is held securely in the direction of the eccentricity.

If a piece is exposed to tension and compression at different times, it shall be proportioned to resist the maximum of each kind, but the unit stresses shall be less than those used for stress of one kind, depending upon the ratio and the relative frequence of the two maxima.

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EXTRACT FROM BUILDING LAWS OF CITY OF BUFFALO (1909).

CONCRETE CONSTRUCTION.

Concrete may be used in buildings of all classes when such construction is approved by the Deputy Building Commissioner.

Regulations regarding the use of concrete in hollow blocks and in reinforced steel construction:

Height of buildings.—Buildings whose exterior walls are of *hollow concrete blocks* may be erected not to exceed 3 stories in height, and the thickness of such walls shall be as given below for brick walls; provided, however, that the materials of construction are not strained beyond the safe limits:

Stories.	Buildings of Class I: Sale, storage or manufacture of merchandise, and public livery, boarding or sale stables.					Buildings of Classes II, III, IV: all other buildings, as hotels, hospitals, office buildings, halls, theaters, etc.				
	Basement.		Story.			Basement.		Story.		
	Stone.	Brick.	Gro'nd	2	3	Stone.	Brick.	Gro'nd	2	3
1	18"	12"	12"			18"	12"	12"		
2	18"	16"	12"	12"		18"	16"	12"	12"	
3	20"	16"	16"	12"	12"	20"	16"	12"	12"	12"

Buildings whose exterior walls are of reinforced concrete steel may be erected 3 stories in height, and the thickness of such walls shall be as given in the following table; provided, that the materials of construction are not strained beyond the safe limits:

Stories.	Basement.	1st Story.	2nd Story.	3rd Story.
1	8"	6"	..	..
2	10"	6"	6"	..
3	12"	8"	6"	6"

Concrete must be mixed in the proportions of 1 of Portland cement, 2 of sand, and 5 of stone or gravel; or the proportions may be such that the resistance of the concrete to crushing shall not be less than 2,000 lbs. per sq. in. after hardening for 28 days, by approved test. The concrete used in reinforced concrete steel construction must be what is usually known as a "wet mixture."

"Reinforced concrete steel" shall be understood to mean an approved concrete mixture reinforced by steel of any shape, so combined that the steel will take up the tensional stresses and assist in the resistance to shear.

Concrete construction will be approved only for buildings which are not required to be fireproof by the building ordinances, unless fire and water tests shall have been made under the supervision and to the satisfaction of the Deputy Building Commissioner. Each company offering a system of concrete construction for fireproof buildings must submit such construction to a fire and water test.

Inspection and tests.—The execution of concrete work shall be confided to workmen who shall be under the control of a competent foreman or superintendent, and persons erecting buildings of concrete shall provide for expert inspection of the cement and inerts and a daily record shall be kept of the tests, the temperature in which the concrete was worked, and all other conditions which may be of importance in the construction, and a certified copy of such record shall be filed twice each week, or oftener if required.

Quality of materials.—Only high grade Portland cement shall be permitted in concrete construction. Such cement when tested, after 1 day in air and 6 days in water, shall develop a tensile strength of at least 500 lbs. per sq. in.; and after 1 day in air and 27 days in water shall develop a tensile strength of at least 600 lbs. per sq. in. Other tests as to fineness, constancy, volume, etc., shall be made in accordance with the standard

method prescribed by the Committee of the Am. Soc. C. E., as may from time to time be directed.

The sand to be used must be clean, sharp grit sand, free from loam or dirt.

The stone used in the concrete must be clean broken stone or gravel, of a size that will pass through a ¾-in. ring. In case it is desired to use other materials or other kinds of stone, samples of same must be submitted for approval.

Reinforced concrete steel must be so designed that the stresses shall not exceed the following limits:

Extreme fiber stress on concrete in compression, 500 lbs. per sq. in.
Shearing stress in concrete, 50 lbs. per sq. in.
Concrete in direct compression, 350 lbs. per sq. in.
Tensile stress in steel, 16,000 lbs. per sq. in.
Shearing stress in steel, 10,000 lbs. per sq. in.
Adhesion of concrete to steel, not greater than shearing strength of concrete.
Modulus of elasticity of concrete to steel, 1 to 12.

Bending moments.—The following assumption shall guide in the determination of the bending moments due to the external forces: Beams and girders shall be considered as simply supported at the ends, no allowance being made for continuous construction over the supports. Floor plates when constructed continuous and when provided with reinforcement at top of plate over the supports, may be treated as continuous beams, the bending moment for uniformly distributed loads being taken at not less than $WL \div 10$; the bending moment may be taken $WL \div 20$ in the case of square floor plates which are reinforced in both directions and supported on all sides. The floor plate may be taken as part of the beam or girder in computing its moment of resistance to the extent of not more than 10 times the width of that beam or girder.

Resisting moments.—The moment of resistance of any reinforced concrete steel construction under transverse loads shall be determined by formulas based on the following assumptions:

The bond between concrete and steel is sufficient to make the two materials act together as a homogeneous solid.
The strain in any fiber is directly proportionate to the distance of that fiber from the neutral axis.
The modulus of elasticity of the concrete remains constant within the limits of the working stresses.
The tensile strength of the concrete shall not be considered.

When the shearing stresses developed in any part of the construction exceeds the safe working strength of the concrete, a sufficient amount of steel shall be introduced in such a position that the deficiency in the resistance to shear is overcome.

When the safe limit of adhesion between the concrete and steel is exceeded, some provision must be made for transmitting the strength of the steel to the concrete.

Columns.—Reinforced concrete steel may be used for columns in which the ratio of length to least side or diameter does not exceed 16. The reinforcing rods must be tied together at intervals of not more than the least side or diameter of the column.

Tests.—Tests must show that the construction will sustain a load of 3 times that for which that portion of the building is designed, without any sign of failure.

***Hollow concrete blocks** used for outside walls and partitions shall not be loaded to more than 150 lbs. per sq. in. of available or effective section, and the hollow spaces shall not exceed ⅓ the area of the blocks when using the tables for thickness of walls.

Untried methods of construction may first require preliminary trial tests.

Frost.—The influence of frost must be excluded when concrete work is done.

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* For specifications for hollow concrete building blocks in the city of Philadelphia, see Sec. 25, Masonry, page 450.

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EXCERPTS AND REFERENCES.

Reinforced-Concrete Work at the Atlanta Railway Terminal Station (Eng. News, April 12, 1906).—Illustrated details.

A System of Reinforced-Concrete Construction Without Wooden Forms (Eng. News, July 12, 1906).—Illustrated.

Practical Hints for Concrete Constructors (By W. J. Douglas. Eng. News, Dec. 20, 1906, and Jan. 24, 1907).—Illustrated.

A Reinforced-Concrete Shop with Steel Roof Trusses and Crane-Girders (By W. F. Tubesing. Eng. News, Jan. 10, 1907).—Illustrated.

The 48-Story Tower of the Metropolitan Life Building, N. Y. City (By Purdy & Henderson. Eng. News, Jan. 31, 1907).—Illustrated details of column shoes and column connections.

Table Showing Proportions of Value in the Various Items of Construction of Fireproof Buildings (By F. J. T. Stewart. Eng. News, Feb. 7, 1907).—The table embraces various kinds of buildings in New York, Chicago, Boston, Baltimore and St. Louis, and gives the percentage of cost of about 50 items of construction arranged under the following headings: Foundations, steel frame, mason work, equipment, trim and finish, and general expenses. The cost per cubic foot is also given. Cost of foundations varies from 2.3 to 24.5% of the total cost of building.

A Reinforced-Concrete Mill Building With Separately-Molded Members (Eng. News, July 4, 1907).—Ten illustrations, showing details of construction.

Stresses in Gas-Holder Girder Frames (By H. Stoffels. Eng. News, Aug. 15, 1907).—Illustrated diagrams of stresses in a single-lift gas holder, with three different arrangements of guide rollers.

The Singer Building and the City Investment Building, of New York City (Eng. News, Dec. 5, 1907).—Sixteen illustrations, including: Typical floor plan, foundation plan, cast steel shoe for column base, elevation of cupola, diagram of wind bracing, details of wind bracing, typical column, column anchorage, of Singer tower; foundation plan, details of cast-steel bases of columns, foundation girders, floor plan, typical column sections, details of portal girders and wind bracing, of City Investment Building.

A Reinforced-Concrete Building With Concrete Domes: Cincinnati Zoological Garden (Eng. News, Feb. 20, 1908).—Illustrated details of typical column, girder and dome construction.

Adjustable and Portable Forms for Concrete Building Construction (By L. G. Hallberg. Eng. News, Mar. 5, 1908).—Illustrated details of post and form for girder, with adjustable and portable centering.

Steel Construction for Long Span Floors in the Chicago Athletic Assn. Building (Eng. News, Mar. 19, 1908).—Illustrations of steel framing for floor, and plan of steel floor girder 43-ft. long.

Reinforced-Concrete Cantilever Girders in the Bogertown Building, Phila. (Eng. News, April 23, 1908).—Illustration of the side-wall-bearing cantilevers, and saw-tooth roof construction.

The 10-Story Reinforced Concrete Hostetter Building, Pittsburg (Eng. News, May 14, 1908).—Illustrated details of column reinforcement and cast-iron base.

A Reinforced-Concrete Cold Storage Building (By W. F. Tubesing. Eng. News, July 11, 1908).—Illustrated details of wall and footings, tank, and reinforced-concrete covered bridge between buildings.

The Reinforced-Concrete Court House at New Orleans (Eng. News, July 2, 1908).—Illustrated details of floor construction.

A Steel Frame Grand Stand at Dallas, Tex. (By Howard Arthur. Eng. News, Aug. 20, 1908).—Illustrated: Side elevation, showing dimensions of members.

Conservatory Buildings of Steel Construction in Garfield Park, Chicago (Eng. News, Aug. 27, 1908).—Illustrated: Stress sheet of arched ribs of Palm House; details of steel ribs.

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Structural Steel Work in the La Salle Hotel, Chicago (Eng. 3, 1908).—Illustrated: Diagrams and details of trusses and

Concrete Construction as Applied to Buildings (Paper pre- Nat'l Assn. of Cement Users, Cleveland, Jan. 11 to 16, 1909. Jan. 14, 1909).—Tables.

in Reinforced-Concrete Construction; An Argument for y Reinforcement in Floor-Slabs (By C. A. P. Turner. Eng. 18, 1909).

eration of Passenger Elevators (By R. P. Bolton. Trans. Vol. LXIV).

Concrete Construction as Applied to Buildings (By L. C. Wason. Jan. 16, 1909).—Numerous tables of cost.

Concrete Construction as Applied to Buildings (By L. C. Wason. l Assn. Cement Users. Eng. Rec., Jan. 16, 1909).—Summary everal buildings.

ced-Concrete Building for a Large Manufacturing Plant (Eng. 26, 1909).—Nine illustrations. A table of reinforcing rods is ing the shop data required when rods are ordered. Floor loads 100 to 700 lbs. per sq. ft. In many places there was heavy achinery. Unit stresses were decided upon for different con- formulas made up as shown in the following table:—

TABLE OF FORMULAS FOR UNIT STRESSES.

	Fiber Stress: lbs. per sq. in.		Per Cent of Steel.	Resisting moment. in.-lbs.
	Concrete.	Steel.		
ditions.....	700	16000	0.85	120 bd^2
rating loads.....	500	15000	0.75	100 bd^2
brating loads.....	550	14000	0.70	90 bd^2
rating loads.....	450	13000	0.62	70 bd^2
ining liquid.....	650	13000	1.10	120 bd^2
t reinforced.....	.....	.....	.....	14 bd^2

d Tar Base for Plank Flooring on Reinforced-Concrete Floor- News, Nov. 18, 1909).—For second-story floor of factory. aying the plank flooring directly on sleepers embedded in the or slab (ordinary method), in this case (for fear that dry rot destroy the sleepers and the flooring immediately next to the layer of sand mixed with coal tar was filled between the ground story and its flooring: First, a mixture of 50 gals. of coal-tar o. 5) to each cu. yd. of sand was spread 1½ ins. thick on the floor eled while still warm and soft with a straight-edge. Second, on ere was then placed a layer of 2-in. plank, then ⅞-in. rough pine finally a wearing surface of 1½-in. square edge maple; the differ- f planking being placed in different directions, compacting the er to about 1 in. in thickness. The floor is solid but resilient atively noiseless.

ed-Concrete Car Barns; Separately Molded vs. Monolithic By M. D. Pratt. Eng. News, Dec. 9, 1909).—Illustrations: details, separately-molded construction. Tables: (1) Cost of molded construction; (2) Comparative cost between separately- l cast-in-place construction. Conclusion: Separately-molded is cheaper.

l Building Regulations for the Use of Reinforced Concrete Cement Users, Chicago convention amendment, 1910; Eng. 5, 1910).—Condensed digest:—**Walls**—9. Rein.-conc. walls, e thickness called for brick or stone walls, may be used. Curtain ss than 4 ins. thick. 10. Conc. walls must be reinforced in both Max. spacing of bars to be 18 in. centers, reinforcement in both ll to be considered. Total reinforcement not less than ¼%. **rete.**—19. Shall not be used for reinforced-concrete structures;

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it may be used for fireproofing. **Assumptions in Design.**—43. The span length for beams and slabs shall be the dist. c.-c. of supports, but not to exceed the clear span plus the depth of beam or slab; brackets shall not be considered as reducing the clear span. 44. Length of columns shall be the max. unsupported length. 45. Where slabs and beams are figured as simple beams the length shall be the clear dist. between supports excluding brackets. **Loads.**—47. Weight of rein.-conc. to be taken as 150 lbs. per cu. ft. 49. The roof shall be figured to carry 30 lbs. live load per sq. ft. unless otherwise noted. 50. A reduction of live load coming to the column supporting the floor below the roof of 5% to be allowed and a further reduction of 5% of the live load of each story below until the total reduction shall amount to 50% of the live load of any floor, after which all loads shall be figured net to the foundations. These reductions shall not apply to storage warehouses. 51. No reduction of loads shall be allowed for figuring floor slabs. 52. Nor none for figuring beams. 53. A reduction of 15% live load may be allowed in figuring the girders, except in buildings used for storage purposes. 54. In assuming the load coming to the columns all beams and girders shall be considered as carrying a net load consisting of 100% each of live load, subject to the above reductions. **Bending Moments.**—55. Slabs.—The bending moment of slabs uniformly loaded and supported at two sides only shall be taken as $wl^2 \div 8$, where w=unit load and l=span. 56. Continuous Slabs.—For interior slabs overhanging two or more supports the bending moment shall be taken as $wl^2 \div 12$. The reinforcement at the top of the slab over supports must equal that used at the center. 57. Slabs Reinforced in Both Directions.—Slabs reinforced in both directions and supported on four sides and fully reinforced over the supports (the reinforcement passing into the adjoining slabs) may be figured on the basis of bending moments equivalent to $wl^2 \div F$ for load in each direction. When span under consideration is not continuous, $F=8$; when continuous over one support, $F=10$; when continuous over both supports, $F=12$. The distribution of the loads to be determined by the formula: $r=L^4 \div (L^4-b^4)$, in which r=proportion of load carried by the transverse reinforcement, L=span, b=breadth of slab. 58. The slab area may be reduced by one-half as above figured, when the reinforcement is parallel to and not further from the supports than $\frac{1}{4}$ of the shortest side. The reinforcement spanning the shortest direction shall be below the reinforcement spanning the longer direction, and shall not be further apart than $2\frac{1}{2}$ times the thickness of the floor including the finish. 59. Simple Beams.—The bending moment of beams supported at the ends only shall be figured as of simple beams. 60. Partially Restrained Beams.—Beams supported at one end and continuous at the other to be figured partially restrained with a bending moment of $\frac{2}{3}$ that of a simple beam. When the over-all vertical distance of the tension members is greater than $\frac{1}{8}$ of the total depth of the beam the stresses in each member shall be computed in proportion to the distance from the neutral axis. Beams supporting rectangular slabs reinforced in both directions shall be assumed to take the following load: The beams on which the shortest sides of the slab rest shall take the load of that portion of the slab formed by the isosceles triangle having this side as its base and half this side as its height. The load from the remaining portion of the slab shall go to the beams on which the long side of the slab rests. 62. Continuous Beams.—When beams or girders are continuous over two or more supports, the interior beams may be considered as partially restrained, and the bending moments at the center and support figured as $\frac{2}{3}$ that of a simple beam, unless the concrete at the bottom of the beam at the support shall by this consideration receive excess compression. 63. T-Beams.—In beam and slab construction, an effective metallic bond should be provided at the junction of the beam and slab. When the principal slab reinforcement is parallel to the girder, transverse reinforcement shall be used extending over the girder and well into the slab. 64.—Where adequate bond between slab and web of beam is provided, the slab may be considered as an integral part of the beam, but its effective width shall not exceed $\frac{1}{4}$ on either side of the beam, nor be greater than 6 times the thickness of the slab on either side of the beam. Measurement from the edge of the web. 65. In the design of T-beams acting as continuous beams, due continuation should be given to the compressive stresses at the supports at the bottom of the beam. **Ratio of Moduli.**—75. The ratio of moduli of elasticity of concrete to steel shall be considered as 1 to 15. 76. The allowable tensile stress in reinforcement to be 16,000 lbs. per sq. in. for medium steel and 20,000 lbs. per sq. in. for high elastic limit steel with adequate mechanical bond. 77. The compres-

steel reinforcement to be 15 times the allowed compression which the steel is embedded. **Fireproofing.**—78. For main columns, 2 ins. of concrete; girders, 1½ ins.; floor slabs,

Cost of Mill Buildings (By C. T. Main. Paper before N. E. Assn., April, 1904, and revised to Jan., 1910; Eng. News. —Diagrams and tables of costs.

gs of the Jones & Laughlin No. 14 Mill (Eng. Rec., May 14, ted details: Runway girder hangers, shipping yard columns, umns, wall columns, 57-ft. riveted roof truss, semi-truss of girders.

for Building Contracts (By W. B. Bamford. Trans. A. S. C. , June, 1910).—Bibliography. Suggested "Agreement and ditions of Contract."

g the Cambridge Building, N. Y. City (By T. K. Thomson. E., Vol. LXVII., June, 1910).—Illustrated.

ulas for Statically Indeterminate Members and Frames (By Eng. News, May 26, 1910).—Formulas and illustrations. (a) Eccentrically-loaded column; base fixed, top hinged. rane-and-roof column. (c) Frame of two columns and cap eral case of two-story framed bent. (e) Kneebraced bent.

teel Details in the Ball Realty Building, New York (Eng. 910).—Details of foundation cantilever, designed to obviate making large excavations and underpinning an adjacent l details of pier shelves, suspended wall beam, depressed ressed wall channel.

ictural Steelwork with the Oxy-Acetylene Flame, in Dis-ding (Eng. Rec., Aug. 27, 1910).—The acetylene flame was 12 columns, four 12-in. I-beams, eight 20-in. I-beams, 2 ins. deep, and for cutting holes in 8 underpinning piles and them in two in the middle; the piles were regular 16-in. steel n concrete and about 70 ft. long. The average speed of s flame is one 24 x ½-in. plate in 1 min., but on heavy solid great as a 4½ x 4½-in. section in 1 min. has been attained. lone with a portable plant including three acetylene safety hose used for car lighting apparatus and other purposes. vas 12 ins. in diameter, and 26 ins. long and contained 225 ne gas at atmospheric pressure, compressed to a pressure of in.

on Mill Buildings (By Albert Smith. Paper before Western Nov. 9, 1910; Eng. News, Nov. 24, 1910).—Mr. Smith ad-lacing the wind loads equally on the two walls, and inward the windward and leeward roofs respectively, as giving ges of stress in members of the roof-truss, as giving less e-braces and columns, and as permitting the rational design 2) An accurate determination of the point of contraflexure lumns. (3) Making the amount of the wind-pressure for a matter to be decided in the light of the exposure of that

Flat Slab Floor in a Reinforced-Concrete Building (By Arthur r before Annual Conv. of Natl. Cement Users, New York, Eng. News, Dec. 22, 1910).—Description of test, with illus-grams.

portant Illustrations of Buildings and Details.

Description.	Eng. News.
for carrying upper front of building	Mar. 10, '10
plaster block floor construction	Mar. 10, '10
Berlin Botan. Gardens, with roof	Mar. 17, '10
steel building, beam conn. with brackets	May 26, '10
crete cold storage warehouse, 72 ft. x 100 ft	Nov. 3, '10
	Eng. Rec.
rehouse insulation, rein.-conc. construction	Jan. 30, '09
ls of the Philadelphia Opera House	Jan. 30, '09
rced-concrete warehouse, Pittsburg	Feb. 6, '09

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Description.	Eng. Rec.
Alaska Commercial Bldg., San Francisco, eng'g features	Feb. 6, '09
Columns with connections for wind bracing and cantilevers	Feb. 20, '09
Methods of hanging shafting, etc., in rein.-conc. buildings	Mar. 13, '09
Reinforced-concrete church, with dome, in Los Angeles	Mar. 20, '09
A light steel pier shed, roof 53-ft. span	Mar. 27, '09
Construction of the Baxter Bldg. (rein.-conc.), Portland, Me.	Apr. 10, '09
Methods of hanging wires and shafting to conc. beams	Apr. 24, '09
Reinforced-concrete dome of the Porto Rico Capitol	May 1, '09
Types of hangers used for piping in a power-house	May 15, '09
The Keewatin rein.-conc. flour mill	May 29, '09
Principal roof trusses and banquet hall girders, La Salle Hotel	June 5, '09
Steel details, floor, girders, columns, cornice of Trust Building	July 3, '09
Engine house: sep.-molded roof members; engine and drop pits, etc	July 10, '09
Cement shed, mixing building, measuring tank, concrete plant	July 17, '09
Section of framework, Copper Queen smelter building	July 31, '09
Details steel framework of large coal storage shed	Sept. 4, '09
Plans of sedimentation basin, Goderich, Ontario	Sept. 4, '09
Plan of wood-framed machine shop	Sept. 4, '09
Details concrete and steel work, Met. Life Bldg., San Francisco	Sept. 11, '09
Details safety equipment, Singer Building elevators	Sept. 11, '09
A reinforced-concrete sawmill	Sept. 11, '09
Floor beam plans, typical columns, Bank Bldg., N. Y.	Oct. 2, '09
Structural steel frames of open hearth bldgs., Gary, Ind	Oct. 9, '09
Rein.-conc. joists with hollow tile fillers. Tables	Oct. 9, '09
Rein.-conc. (Quincy market) cold storage warehouse, Boston	Nov. 13, '09
General plans of Balloon house, U. S. Signal Corps	Dec. 4, '09
Details of forms for concrete floor beams and slabs	Dec. 11, '09
Details, steel truss supporting column, Martinique Hotel	Jan. 1, '10
Rein.-conc. grand stand at Minn. State Fair Grounds	Jan. 15, '10
Approx. cost of mill buildings; diagrams, tables	Jan. 29, '10
Details, columns, etc., large steel frame rolling mill	Jan. 29, '10
Cross-section rein.-conc. warehouse; details machinery	Mar. 12, '10
Steel and architectural details, N. Y. Municipal Building	Mar. 19, '10
Cast-steel column-pedestals (4 to 6½ ft. sq.), N. Y. Munic. Bldg	Mar. 19, '10
Rein.-conc. girder beam, 53 ft. clear span	Mar. 5, '10
12-story rein.-conc. bldg., 48 ft. x 110 ft., without inter. columns	Mar. 5, '10
Steel freight sheds at Winnepeg, C. N. & G. T. P. Ry	Apr. 16, '10
Arrangement of reinforcing at elevator and stairway	Apr. 16, '10
Framework 22d reg't armory; 3-hinged arch; cantilever	May 5, '10
Rein.-conc. grandstand, Minn. State Fair race track	June 4, '10
Formula for determining the elevation of grandstand seats	June 4, '10
Concrete and tile floors with 2-way reinforcement	June 16, '10
Steel and architectural details of Chicago station, C. & N. W. Ry	June 18, '10
Structural details of Columbia Theatre, San Francisco	June 23, '10
Column sections, bases and splices, Curtis bldg., Phila	July 9, '10
Concrete building with steel columns in lower stories	July 30, '10
Saw-tooth-roof machine shop for Georgia Ry	July 30, '10
Heating and ventilation of Union Passenger Sta., Wash., D. C.	Jan. 2, '08
Reinforced-concrete construction in the Hartford Armory	Jan. 9, '08
54-ft. rein.-conc. arch for supporting warehouse floor	Jan. 16, '08
Wall insulation of a cooling room	Aug. 6, '10
Half of steel roof truss for the Doe Memorial Library	Aug. 27, '10
Diagram from column formula: $15000-60(l+r)$	Sept. 3, '10
Steel and rein.-conc. grand-stand, baseball park, Chicago	Sept. 3, '10
Structural steel details in the Wick Bldg., Youngstown	Sept. 24, '10
Details of cantilever beam in rein.-conc. storage warehouse	Oct. 15, '10
Deep underpinning (12-story steel bldg.) through sand	Oct. 22, '10
Depositing concrete by gravity in a 7-story building	Oct. 22, '10
The Tacoma High School Stadium (L. D. Howell)	Oct. 29, '10
Structural details of the Curtis Bldg., Phila., Pa	Nov. 5, '10
Structural details in the Soldan High School, St. Louis	Nov. 5, '10
Metal wall forms for concrete houses	Nov. 19, '10
A large concrete coal breaker and washery building	Dec. 3, '10
Underpinning the Manhasset Building, New York	Dec. 10, '10
Typical beam, columns and floor reinforcement, conc. bldg	Dec. 10, '10
Grandstand of reinforced concrete, Cleveland (O.) B. B. Club	Dec. 17, '10

48.—RETAINING WALLS.

The forces acting on a retaining wall, due to the pressure of the earth behind it, are not susceptible of exact determination. Several theories have been advanced from time to time, based on assumptions more or less at variance with practical conditions, and from these theories formulas have been deduced, but they are not relied upon with any degree of certainty. We depend rather upon the proportions of existing structures for our designs. Some data have been obtained from tests of models, but the conditions under which the tests were made are not considered sufficiently reliable upon which to base a general working formula.

Theory of Earth Pressure (No. 1).—Any theory of earth pressure should be founded on assumptions clearly and carefully made, and leaning rather on the side of safety than otherwise. It is believed that in the present discussion a clearer conception may be had by taking a concrete example for an illustration.

Let us consider an earth fill 20 ft. high, level, and of indefinite extent. Imagine this fill to be cut by the vertical plane *ab*; then will there be on either side of the plane a set of equal and symmetrical forces acting on the plane, in equilibrium. What is the nature of these forces, their intensities and directions, and their resultants?

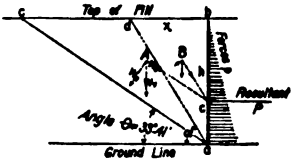

Fig. 1.

Firstly, it is assumed that the earth fill is dry and granular, in fact sand, as that will probably produce about the greatest pressure*; also that there is no cohesion among the grains of sand and hence there can be no tension in any part of the mass. If now the fill to the right of the plane *ab* is removed, it is clearly evident that certain forces as p, whose resultant is P, may be applied on the right face of the plane to hold the fill to the left of it in place, and maintain equilibrium as before. In Fig. 1 these forces are represented as acting horizontally, but no assumption is being made at present as to the direction of the *original* forces on the plane *ab* due to the earth fill removed, nor to the direction of the *existing* forces acting to the *left* of the plane due to the earth fill in place. It is assumed merely that the p forces represent in intensity the horizontal components of these forces, in the direction of the former and opposite in direction to the latter.

Let us next remove the plane *ab*, the forces acting to the right of it, and also that portion of the ground line to the right of *a*, if we can so stretch our imagination. Immediately, the sand will begin to slide over fan-like planes radiating from *a*: There will be a tendency for the whole triangular prism as *abd* to slide *en masse* on some plane *ad*, and the general movement of the earth will not cease until some plane as *ac* is exposed and all the material above is removed. It will further be found that this plane *ac* makes an angle $\phi = 33°\text{-}41'$ (about) with the horizontal. This angle is called the *angle of repose*, angle of friction,† or natural slope for that material. It is based on the engineer's slope of 1½ horizontal to 1 vertical, which earth fill in general assumes, and is the slope at which the material will just barely remain at rest. For instance, if we consider the mass of earth *abc* restored above the plane *ac* and assume it for the moment to have sufficient cohesion so that no sliding plane *above* the plane *ac* can develop through it, that is, no part of the mass *abc* can slide on the other part, then will the mass be at rest, as the tendency to slide on the plane *ac* will just equal the resistance due to friction. Clearly, then, considering the prism *abc* as a *united mass*, it is evident from the foregoing that no *horizontal* pressure would be exerted

* The exception to this is clean, coarse, uncemented gravel; see page 838.
† The coefficient of friction = tan ϕ.

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by it, and hence no forces p, acting on the face ab, would be needed to keep it in place. As such it would exert a vertical force $= W_v$, a normal force $W_v \cos \phi = W_n$, and a tangential force on the plane ac equal to $W_v \sin \phi = W_t = W_n \tan \phi = (W_n \times 1) + 1\frac{1}{2} = \frac{3}{2} W_n$. The tangent of the natural slope, or tan ϕ, equal to $\frac{3}{2}$ for earthwork, is commonly termed the *coefficient of friction* because it requires a force a little greater than $\frac{3}{2}$ of the normal pressure on the natural slope, to move the mass. Hence the united prism abc is just stationary on the $1\frac{1}{2}$ to 1 slope ac, because the total friction, $\frac{3}{2} W_n = W_t$, the tangential force ..... (1)

Slope of Maximum Pressure.—Consider any triangular prism abd, Fig. 1, resting on the sliding plane ad whose base is x, height h, and length perpendicular to the paper is one—all in feet. The weight of the material composing the fill is taken at 100 lbs. per cu. ft. Then, with the center of gravity at B, we have:

Vertical force or weight, $W_v = \frac{100hx}{2}$ lbs.;

Normal force (to plane ad), $W_n = W_v \cos \alpha = \frac{100hx}{2} \cdot \frac{x}{\sqrt{h^2+x^2}}$;

Tangential force (on plane ad), $W_t = W_v \sin \alpha = \frac{100hx}{2} \cdot \frac{h}{\sqrt{h^2+x^2}}$;

Total friction (on plane ad), $f = \frac{3}{2} W_n = \frac{100hx}{3} \cdot \frac{x}{\sqrt{h^2+x^2}}$;

and the resultant force R, parallel with the sliding plane ad, will equal the tangential force minus the total friction, or

$$R = \frac{100hx}{2} \cdot \frac{h}{\sqrt{h^2+x^2}} - \frac{100hx}{3} \cdot \frac{x}{\sqrt{h^2+x^2}} \quad \text{..... (2)}$$

By placing the first differential coefficient equal to zero, and solving for maximum,

$$\frac{dR}{dx} = \frac{(h^2+x^2)^{\frac{1}{2}} (50\,h^2 - 66\frac{2}{3}\,hx) - (50\,h^2x - 33\frac{1}{3}\,hx^2)x\,(h^2+x^2)^{-\frac{1}{2}}}{h^2+x^2} = 0$$

whence, $(h^2+x^2)\,(50\,h^2 - 66\frac{2}{3}\,hx) = 50\,h^2x^2 - 33\frac{1}{3}\,h\,x^3$.

$$\text{or, } x^3 + 2\,h^2\,x = \frac{3h^3}{2} \text{ (General equation.)} \quad \text{..... (3)}$$

Solving this cubic equation there is obtained

$x = .627\,h$; whence $\alpha = 57°\text{-}55'$.

Hence, the *maximum pressure in a direction parallel with the slope*, against the vertical plane ab, obtains when we consider that the sliding plane makes an angle $\alpha = 57°\text{-}55'$ with the horizontal; and when $h = 20$, $x = 12.54$. Substituting the value of x in equation (2), or the values of α and x in the following:

$$R = 50\,h\,x \sin \alpha - 33\frac{1}{3}\,h\,x \cos \alpha \quad \text{..... (4)}$$

we have, $R = 12540 \times .847 - 8360 \times .531 = 6182$ lbs. Hence, from the above analysis, the resultant pressure R, due to the earth sliding on the plane of maximum *slope pressure*, is 6182 lbs., which force is assumed to act parallel with the direction of the sliding plane, through the center of gravity B of the mass, and intersecting the vertical plane ab at a point distant $\frac{2}{3}\,h$ below the top of fill (Fig. 1).

In a similar manner the total *horizontal pressure* H ($= R \cos \alpha$), against the wall, may be found; thus,

$$H = R \cos \alpha = \frac{50h^2x^2 - 33\frac{1}{3}hx^3}{h^2+x^2} \quad \text{..... (5)}$$

Placing the first differential coefficient $\frac{dH}{dx}$ equal to 0 and solving, we have, $x^3 + 3\,h^2x = 3\,h^3$, and for maximum value of H we have $x = 0.8178\,h = 16.36$ ft.; whence $\alpha = 50°\text{-}43'$, and $H = 16360\,(\sin \alpha - \frac{2}{3} \cos \alpha) \cos \alpha = 3645$ lbs., applied horizontally at c, Fig. 1. Neglecting friction on slope, $H = 8016$ lbs., making the lateral pressure about 40% of the vertical pressure, which may be considered a maximum for any earthy material.

Assumptions Regarding Friction.—Reverting to Fig. 1, there are two planes where friction may be considered, namely, *ad* and *ab*. We have found (equation 4) that the possible friction on the plane *ad* is $33\frac{1}{3}\ hx \cos \alpha = 4439$ lbs. which, deducted from the tangential force, $50\ hx \sin \alpha$ (or 10621 lbs.), gives 6182 lbs., the resultant. Hence, if the friction is neglected our resultant will be increased from 6182 to 10621 lbs., an increase of nearly 72%. Should this friction, for safety, be wholly or partly neglected? Before arriving at any conclusion in regard to this, let us consider the possible friction on plane *ab*. If *a b* represents the back of the retaining wall, the amount of possible friction will depend upon the construction of the wall itself.

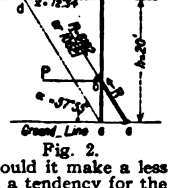

Fig. 2.

Let Fig. 2 represent a simple frame construction to better analyze the acting forces. The resultant force R is resisted by the strut *c e*, and hence the stress in the latter is equal and opposite to R. The point *c* is the center of gravity of the distributed earth pressure acting on the facing, which is supported by the stud *ab*, which facing is stiff enough to resist bending, and which is balanced on the pivot or point of support *c*. As long as the resultant pressure acts parallel to the plane *a d*, as indicated, there will be, apparently, unstable equilibrium; but should it make a less angle than α with the horizontal, then there would be a tendency for the strut *c e* to revolve about *e*. In doing so it would tend to raise the facing *a b*, thereby causing friction of the latter against the earth fill. Indeed, as the forces now act, the resultant R can be transmitted to the strut *c e* only on the assumption that there is sufficient frictional force downward on the "fill" face of the wall to resist the vertical component of the stress in the strut *c e*. Let us examine this: Assuming the coefficient of friction of the earth on the face of the wall as equal to $\frac{2}{3}$ of the normal pressure, there is obtained,

$$\text{Total friction} = \tfrac{2}{3} R \cos \alpha$$
$$= 2189 \text{ lbs. for } R = 6182;$$
$$= 3760 \text{ lbs. for } R = 10621.$$

This friction on the "fill" face of the wall will be opposed by the vertical component of the stress in the strut *c e*, equal to

$$R \sin \alpha$$
$$= 5236 \text{ lbs. for } R = 6182;$$
$$= 8996 \text{ lbs. for } R = 10621.$$

Hence it is evident that the wall may have to be anchored, at *a*, an amount equal to

Vertical component of R − (total friction + wt. of wall)

of else the strut *c e* will have to have a less inclination with the horizontal.

Similarly, the preceding form of analysis may be repeated, but using the maximum value of H obtained from equation (5) with the angle of slope 50°-43'.

Practical Deductions.—The uncertainty of some of the foregoing assumptions makes the problem a difficult one to solve. However, we are reasonably certain that the resultant pressure on the wall parallel with the sliding plane *ad* (Fig. 1) is somewhere between 6182 and 10621 lbs. Assuming the latter as correct by neglecting friction on the slope (which can obtain only when the wall begins to tip) we have that the resultant normal pressure acting horizontally at *c* is

$$P = R \cos \alpha = 10621 \times .531 = 5640 \text{ lbs.}$$

which is equivalent to a horizontal pressure per square foot on the vertical wall *a b*, varying uniformly from zero at *b*, to 564 lbs. at *a*. This is equivalent to stating that the horizontal pressure per square foot against the retaining wall at any point below the top is equal to $28\frac{2}{10}$ per cent. of the weight of a vertical column of earth, one square foot in section, extending from the top of the fill to that point: that is, the lateral pressure is $28\frac{2}{10}$ per cent. of the vertical pressure.

For Temporary Shoring, this may be reduced to 25 per cent., or even less, provided of course that the "earth" is not saturated with water so as to be in a muddy condition. Up to a certain point of wetness, moist earth will produce less percentage of lateral pressure than dry, granular earth.

For Permanent Structures, the lateral pressure should be assumed at about 30 per cent. of the vertical, and even up to $33\frac{1}{3}$ per cent. in places subject to considerable jarring, as for retaining walls supporting railway

embankments. Where the weight of a train or of a structure falls within the range of the slope, it can be reduced to an equivalent volume of earth.

If the material is coarse gravel, the ratio of lateral to vertical pressure may reach as high as 40 per cent., for which provision should be made.

Graphical Solution of Preceding Problem.—Fig. 3 is a graphical solution for a masonry retaining wall 20 ft. high to restrain a level earth fill of the same height—the problem which has engaged our attention throughout the discussion of the theory of earth pressure. The weight of the earth fill is assumed at 100 lbs. per cu. ft., and the lateral pressure, 30 per cent. of the vertical. Hence the resultant lateral pressure $P = \frac{30 \times 20^2}{2} = 6000$ lbs., acting horizontally at a point one-third the height from the bottom. Selecting the trapezoidal type of wall, the center of gravity is found by assuming the top and bottom widths; laying off the bottom width on either side of the top, and the top width on either side of the bottom; and finding the point of intersection of the diagonal lines joining the outer points. The weight of one foot section of wall at 150 lbs. per cu. ft. = 16500 lbs., acting vertically through the center of gravity. From the intersection of the two acting forces the triangle of forces is drawn showing the resultant to intersect the base within the middle third, which is good practice.

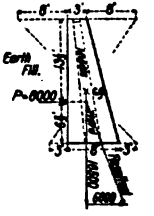

Fig. 3.

Fig. 4 shows another design for the same in which the top width of wall is one ft. instead of three. In this case the resultant falls outside the middle third, and hence there is tension on the face *ab*. In addition to this tension, the factor against overturning at *t* is lessened, and it is not as desirable a type as that shown in Fig. 3.

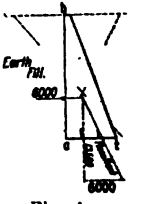

Fig. 4.

Top of Fill, Sloping.—Up to the present, we have considered the top of fill level, and flush with the top of the wall. We will now consider what modification of pressure will be effected in case the surface of the fill should slope either upward or downward from the back or, in fact, show any profile whatever. Fig. 5, in which *ab* is the "fill" face of the retaining wall, shows the slopes of maximum pressure for: (1) a level fill *bd*, with maximum pressure slope *ad* making an angle of 57°-55′ with the horizontal; (2) an upward surface slope *bu*, with maximum pressure slope *au* slightly less than *ad*; and (3) a downward surface slope *bl*, with maximum pressure slope *al* slightly greater than *ad*. It is to be noted that the slopes of maximum pressure *au* and *al* very nearly coincide with *ad*, the slope of maximum pressure which we found for a level fill, and it might also be stated here that this slope, *ad*, may be varied either way several degrees without materially affecting the resultant pressure. Also, as *bu* and *bl* are the upper and lower limits at which earth fill will stand (1½ to 1), then all other possible surface slopes must be less than these, and in approaching a level their slopes of maximum pressure *a u* and *a l* will approach *a d* in direction. *For all practical purposes a d may be assumed as the slope of maximum pressure for any surface slope.* This assumption, containing a small percentage of error, greatly simplifies the method of calculation for general cases. For depressed surface slopes, as *bl*, the assumption is on the side of safety; while for surcharged walls with upward slopes, as *bu*, the assumption involves a slightly opposite effect, to counteract which it would be well to have the resultant of all the forces (earth and wall) cut the base of the retaining wall at least ⅓ of the width from the toe instead of the customary ½.

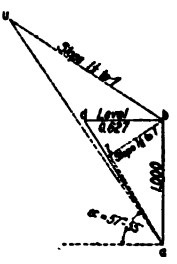

Fig. 5.

General Cases.—By observing the following methods a retaining wall may be designed for any surface slope: (1) Draw the back of the wall, *ab* (Figs. 6, 7, and 8); the slope of greatest pressure, *au* or *al*, making an angle

th the horizontal (tan 57°-55′ = 1.595, cot = 0.627); and the
bu or *bl*. (2) Find the area and the center of gravity of the
, *abu* or *abl*, above the slope of greatest pressure, *au* or *al*.
area by the weight of the fill per cubic foot (average 100 lbs.).
, of one lineal foot of fill is laid off to scale of lbs. vertically
om the center of gravity *c.g.* of the mass. The resultant
ssure *T*, parallel with the slope of maximum pressure, *au* or
and the resultant pressure *P*, normal to the face of the wall,
β. (For a vertical wall, $F \sin \alpha \sin \beta = F \sin \alpha \cos \alpha =$
gs. 6 and 8 show the graphical solution for *P*, in intensity,
point of application, at *c*. (3) Find the point of applica-
enter of gravity of the total lateral earth pressure *P* on the
s point, *c*, is at the intersection of the back of the wall (or
ope) with a line drawn through the center of gravity, *c. g.*,
fill, parallel with the slope of maximum pressure, *au* or *al*.
of *P* is normal to the back of the wall *ab*. (4) Assume
wall, as trapezoidal or rectangular, with a base about $\frac{1}{10}$
nd find its area and center of gravity. Multiply this area
of the masonry per cubic foot (average 150 lbs. for good
). This weight *W*, of one lineal foot of wall is laid off to scale
lly downward from a point *o* at the intersection of a vertical
rough the center of gravity of the masonry, and the force *P*.
d *P* are laid off in sequence and the resultant *R* is drawn. If
se of the wall within the middle third the design is sufficiently
t overturning, with a proper factor of safety. If the wall is
arged, as in Figs. 6 and 7, it is better to have the resultant
of the base from the toe intsead of $\frac{1}{3}$. (5) One or two trial
may be necessary before the desired result is attained. (6)
narks under "Practical Deductions," page 837. The fore-
valent to a lateral pressure equal to $28\frac{3}{10}$ per cent. of the ver-
—for average conditions. If greater or less security is desired,
ncreased or dimished proportionately, depending upon the
f the structure and the quality of the material composing the
ease of *P* of say 20 to 40 per cent. would provide for a per-
ture supporting a gravel fill or a fill liable to become quite
h water. For a temporary wall or shoring, *P* may in some
eased from 5 to 10 per cent. This increase or decrease in *P*
to the dimensioning of the wall so that, in any case, the re-
l fall within the middle third. (7) If any superimposed load,
d, comes on the fill, it should be reduced to an equivalent
of earth, and may be considered as such in finding the weight
ter of gravity, *c.g.*

6.

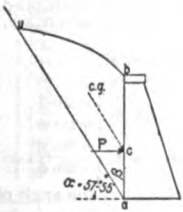

Fig. 7.

Fig. 8.

Theory of Earth Pressure (No. 2).—This theory also assumes
terial to be granular and without power of cohesion, therefore
compressive throughout the mass.

ees with the deduction, page 837, that the lateral pressure for
ow the surface of a *level fill* is $28\frac{3}{10}$ per cent. of the vertical
ough it is not apparent at first glance.

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NOTATION.

w = weight of earth in lbs. per cu. ft.;

x = vert. depth in ft. below surface, of any (conjugate) plane parallel with the surface plane (Fig. 9);

θ = angle of inclination of conjugate plane with the horizontal;

ϕ = angle of repose of earth,* or angle of greatest obliquity;

p_c = intensity of vert. pres. in lbs. per sq. ft. on any *conjugate* plane at depth x below surface;

p_r = intensity of pres. in lbs. per sq. ft., in a direction parallel with the conjugate plane, against a *vertical* plane (as a retaining wall) at depth x below surface.

FORMULAS.

(Surface plane assumed to be of indefinite extent.)

In general, $$p_c = w\,x\cos\theta \qquad (1)$$

and p_r may have any value between

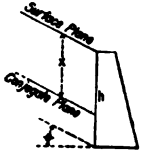

Fig. 9.

$$p_r > w\,x\cos\theta\,\frac{\cos\theta-\sqrt{\cos^2\theta-\cos^2\phi}}{\cos\theta+\sqrt{\cos^2\theta-\cos^2\phi}} \qquad (2a)$$

and $$p_r < w\,x\cos\theta\,\frac{\cos\theta+\sqrt{\cos^2\theta-\cos^2\phi}}{\cos\theta-\sqrt{\cos^2\theta-\cos^2\phi}} \qquad (2b)$$

If h is the height of the retaining wall, the maximum intensity of pressure, at the bottom, may be found by substituting h for x in the preceding equations; and as the intensity of pressure at the top of the wall is zero, the average will be one-half the maximum. Hence the total pressure on the wall will equal the average intensity multiplied by the height h, and this to be applied at a point $\frac{2}{3}h$ from top or surface, and in a direction parallel with the conjugate plane.

If the surface plane is horizontal:

Equation (1) reduces to $$p_c = w\,x \qquad (3)$$

" (2a) " " $$p_r > w\,x\,\frac{1-\sin\phi}{1+\sin\phi} \qquad (4a)$$

" (2b) " " $$p_r < w\,x\,\frac{1+\sin\phi}{1-\sin\phi} \qquad (4b)$$

If the surface plane is inclined at the angle of repose so that $\theta = \phi$, then

$$p_c = w\,x\cos\theta = w\,x\cos\phi \qquad (5)$$

For water, the angle of repose $\phi = 0$, and equations (4a) and (4b) reduce to the hydraulic equation

$$p_r = w\,x \qquad (6)$$

* Usually assumed at 33°-41′ for earth fill, equal to slope of $1\frac{1}{2}$ to 1, making coefficient of friction $(= \tan\phi) = \frac{3}{4}$.

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N. Y. C. & H. R. R. R. STANDARD RETAINING WALL.
(W. J. Wilgus, Chief Engineer.)

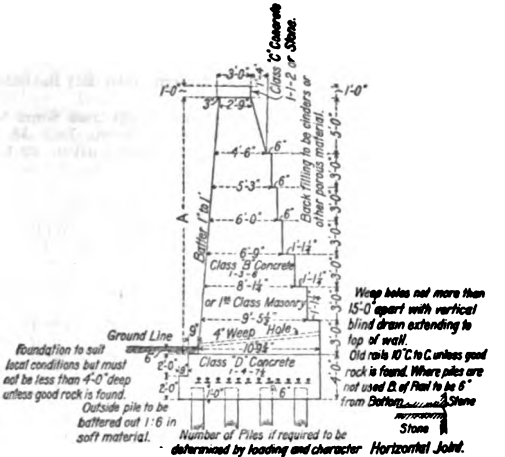

Fig. 10.

1.—CUBIC YARDS OF MASONRY IN RETAINING WALLS (FIG. 10).

Height *A*	Cu. Yds. per Running Ft.			Height *A*	Cu. Yds. per Running Ft.		
	Coping.	Body Wall.	Foundat'n 4′ Deep.		Coping.	Body Wall.	Foundat'n 4′ deep.
5′ 0″	0.111	0.671	0.833	18′ 0″	0.111	3.858	1.544
6′ 0″	"	0.858	0.920	19′ 0″	"	4.204	1.556
7′ 0″	"	1.048	0.932	20′ 0″	"	4.553	1.568
8′ 0″	"	1.241	0.944	21′ 0″	"	4.946	1.744
9′ 0″	"	1.455	1.032	22′ 0″	"	5.341	1.756
10′ 0″	"	1.673	1.044	23′ 0″	"	5.740	1.768
11′ 0″	"	1.894	1.056	24′ 0″	"	6.183	1.946
12′ 0″	"	2.136	1.143	25′ 0″	"	6.629	1.958
13′ 0″	"	2.381	1.155	26′ 0″	"	7.078	1.970
14′ 0″	"	2.630	1.167	27′ 0″	"	7.571	2.146
15′ 0″	"	2.922	1.343	28′ 0″	"	8.068	2.158
16′ 0″	"	3.217	1.355	29′ 0″	"	8.567	2.170
17′ 0″	"	3.516	1.367	30′ 0″	"	9.110	2.346

EXCERPTS AND REFERENCES.

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Comparative Sections of Thirty Retaining Walls and Some Notes on Retaining Wall Design (By F. H. Carter. Eng. News, July 28, 1910).—The following table is compiled from the dimensions given on the cross-sections:—

Retaining Wall.	Height *h.*	Top Width *t.*	Bottom Width *b.*	Ratio *b÷h.*
N. Y., N. H. & H. R. R. (stone masonry)	25′ 9″	3′ 0″	12′ 0″	0.46
Penn. N. Y. & L. I. R. R. (concrete) wet gr'd	23′ 0″	3′ 4″	15′ 9½″	0.69
Penn., N. Y. & L. I. R. R. (concrete)	18′ 0″	3′ 4″	9′ 0″	0.50
Boston subway (conc., granite faced)	13′ 6″	3′ 0″	8′ 0″	0.59
East Boston tunnel (conc., granite faced)	17′ 0″	.....	5′ 0″	0.30
Penn. Ave. subway, Phila. (stone masonry)	28′ 4″	2′ 6″	12′ 4″	0.43
Detroit tunnel (concrete)	28′ 1¼″	3′ 0″	13′ 7″	0.48
Borough of Bronx, N.Y. City (.....)	33′ 2″	2′ 4″	12′ 1½″	0.36
Ill. Cent. R. R., Chicago (concrete)	21′ 0″	1′ 5″	9′ 3½″	0.44
B. & M. R. R. (1st class mas. or concrete)	20′ 0″	1′ 6″	8′ 0″	0.40
B. & A. R. R. (.....) level earth emb'k't	20′ 0″	2′ 0″	9′ 0″	0.45
Penn. R. R., standard (stone masonry)	25′ 0″	3′ 0″	13′ 8″	0.55
N. Y. C. & H. R. R. R. (.....)	28′ 0″	3′ 0″	12′ 3½″	0.44
Sea wall, Lynn shore (concrete)	18′ 0″	3′ 0″	9′ 0″	0.50
Sea wall, Cradock Br. (1 : 3 : 6 concrete)	19′ 9″	2′ 0″	9′ 0″	0.45
At spillway, Wachusett dam (.....)	26′ 4″	3′ 7″	13′ 6″	0.51
Mass. Highway Comm. (stone masonry)	13′ 6″	2′ 4″	7′ 0″	0.52
Board Water Supply, N. Y. (concrete)	20′ 0″	2′ 3″	9′ 5½″	0.47
Board Water Supply, N. Y. (rubble mas.)	18′ 0″	3′ 6″	11′ 3½″	0.63
Board Water Supply, N. Y. (cyclopean mas.)	44′ 6″	3′ 0″	24′ 0½″	0.54
Sea wall, Charlestown (stone masonry)	24′ 0″	2′ 6″	12′ 0″	0.50
Subway wall, Boston Term. Sta. (concrete)	16′ 6″	3′ 0″	8′ 6″	0.52
Harbor wall, Charles river (conc., granite faced)	23′ 4″	4′ 6″	15′ 0″	0.64
Sea wall, Charles River (dry coursed rubble)	15′ 6″	4′ 0″	9′ 6″	0.61
Retaining walls on transverse roads in connection with Boulevard, New York City.—3 examples are given	.....	.....	.....	.....
Retaining walls designed for Cambridge Main Street Subway.—3 examples are given	.....	.....	.....	.....

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The Bracing of Trenches and Tunnels, With Practical Formulas for Earth Pressures (By J. C. Meem. Trans. A. S. C. E., Vol. LX).

A Reinforced-Concrete Retaining-Wall Along the Bank of the Ohio River (By F. A. Bone. Eng. News, June 3, 1909).—Illustrated.

The Design of Retaining Walls (By Comm. on Masonry of the Am. Ry. Eng'g and M. of W. Assn. Eng. Rec., Sept. 11, 1909).—Includes many types of structures in actual use, and contains 32 illustrated sections.

Tables for Determination of Earth Pressure on Retaining Walls (By C. K. Mohler. Eng. News, Nov. 25, 1909).—(1) Rankine's method after Howe; (2) Sliding prism theory.

The Cracking and Partial Failure of Abutments and Retaining Walls (By C. K. Mohler. Eng. News, Oct. 13, 1910).—Criticism of present methods of design and construction.

Illustrations of Various Types of Retaining Walls.

Description.	Eng. News.
Reinforced-concrete retaining walls, bridge approaches	Nov. 25,'09
	Eng. Rec.
32-ft. reinforced-concrete retaining wall	Apr. 3, '09
Section, 31-ft. rein.-conc. retaining wall, C. B. & Q. R. R	Aug. 21,'09
Design of retaining walls for Steptoe smelter	Feb. 19,'10
Combined rein.-conc. fence and retaining wall	Feb. 26,'10
Rein.-conc. retaining wall and roadway bridge and walk	Sept. 24,'10
Types of French railway retaining walls	Nov. 12, '10

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49.—DAMS.

Common Fixed Types.—A dam is a structure designed to hold back a large body of water in an impounding reservoir, at a higher level than would naturally obtain. It should first of all be safe, and the site should be selected with due regard to present and future needs for storage, economy of construction, efficiency of head, outlet, wasteway, etc. With respect to their structural features, dams may be classified as follows:

A *Gravity* dam is a masonry dam designed to resist overturning by the action of gravity alone. It must also resist any tendency to slide horizontally (down-stream) on its base or on any plane above (or below) the base; there must be no tension in the masonry at any point as in the up-stream face where it is most likely to occur; and the compression in any part as the down-stream face, where it is maximum, must not exceed the safe, allowable intensity per square inch.

An *Arched* dam is a horizontally curved (arched) type with ends rigidly braced against the side walls of the canyon—a typical site for this class of dam. The arch is designed to relieve partly—not wholly—gravity requirements although many engineers use the full gravity section that would be required for a straight dam, thereby employing the arch to give greater stability, or in other words, to increase the factor of safety. Although the stresses in an arched dam cannot be determined with accuracy, especially as it acts partly as a gravity dam, being "fixedly" supported throughout its whole length, we know that the arch effect relieves the tendency to overturning, caused by the action of the water pressure on the up-stream face, and this being true, it is evident that the base of the dam can be narrowed materially. This possible reduction of section, however, diminishes as we approach the top of the dam where the combined gravity-, arch- and temperature stresses bear in increased ratio to the gravity stresses alone. Indeed, it has been claimed that, under certain conditions, the arched dam requires a greater section near the top than does the gravity dam, although the writer has never met with such conditions in practice.

A *Buttressed* dam is a gravity dam with buttresses, or thickened sections, spaced at stated intervals for the more economical distribution of the material than one of uniform cross-section. It may be built of concrete, steel-concrete, or stone masonry. The sections at the buttresses will be larger, and the sections between the buttresses will be smaller, than that of an equally safe gravity dam of uniform cross-section. Care should be taken in both the design and construction to avoid any possibility of tension, undue shearing, or excessive compression in any part of the masonry. The facing between the buttresses may be supported by horizontal reinforced concrete beams as used in building construction, or by concrete or stone masonry arches. The facing should practicably be impervious to water.

A *Braced* dam is a dam with braced vertical or sloping face. It differs from a buttressed dam in that the solid masonry buttresses are replaced by open bracing of reinforced concrete, steel or wood. The facing may be of the same character of material, but not connected with the braced legs in such a manner as to form a series of arches between them. A good angle for the face of a braced dam is a slope of 45 degrees with the horizontal, in which case the cost of facing will generally about equal the cost of bracing. But if the facing is very expensive and the bracing cheap, this angle should be increased. The cost of repairs for each should also be considered in determining the economic angle.

A *Cantilever* dam is designed on the cantilever principle and may be braced or buttressed. The facing is supported by beams or trusses projecting beyond the upper limit of the bracing. It may be framed of steel and concrete, reinforced concrete, steel or wood, or a combination of these materials. It should be secured firmly to a natural bed-rock or to a concrete anchorage.

A *Crib* dam is a skeleton, box-like structure weighted with filling and usually faced to prevent undue leakage. The skeleton structure may be composed of logs or of hewn or sawed timbers, framed and bolted into a crib,

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filled with rock, slag, gravel or other suitable material, sunk to the bottom (which has been previously prepared) and preferably bolted thereto. Planking makes a cheap facing.

A *Composite* dam is one composed of two or more radically different kinds of material, composite but not structural in character. A good example of a composite dam is the common brush and rock dam, the brush to prevent excessive leakage and the rock to give stability.

A *Filled* dam is one not strictly structural in character, containing little or no cohesion and hence not capable of being "overturned," but which, if flooded (overtopped) with water, will disintegrate and wash away. It may be composed of any durable material of specific gravity greater than unity, as clay, earth, sand, gravel or loose rock. If it is of the finer materials it is called an Earth dam; if of the coarser (rock), a Rock-fill dam.

(*a*) An Earth dam should be composed of material or materials which will pack well and allow but small voids, if any, and which, under the action of water, will not wash, leaving large holes or pockets. "Cement" gravel is probably the best material as it contains the proper natural binder for a solid mass. If materials are mixed, such as earth, sand, clay, etc., they are better if mixed thoroughly and uniformly—if not, there are liable to be seams of strata allowing the water to percolate and wash the materials. The ratio of 3 horizontal to 1 vertical for the wet slope and 2 to 1 for the dry slope is ordinarily good practice and generally should not be exceeded.

(*b*) A Rock-fill dam is composed of loose rock dumped—not carefully laid—in place, with proper up-stream and down-stream slopes. The up-stream slope may sometimes be as steep as ½ horizontal to 1 vertical if the rock *facing* is laid by hand, otherwise 1 to 1 is the usual practice. The dry- or down-stream slope is usually broken, being steeper, say 1 to 1, for the upper and 1½ to 1 or 1¼ to 1 for the lower section. The facing may be of double planking, caulked and asphalted, and nailed to wooden stringers, say 6"x6", imbedded in the face rock. The thickness of the planking will of course be greater for the greater head of water.

STABILITY OF GRAVITY DAMS.

A gravity dam to be safe must be built on the natural, hard bed-rock not liable to disintegrate, and capable of withstanding the maximum intensity of pressure at the toe of the dam, to resist overturning. The *outlet*, for drawing off the water for domestic or commercial uses, is usually by (cast iron) pipes extending through the masonry wall of the dam itself, but preferably by tunnel construction through the solid rock at one side of and apart from the dam. Likewise, the *wasteway*, for carrying off the flood waters, should preferably be at some other point than at the dam itself. In many cases, however, there is no alternative but to let the surplus or waste water from the full reservoir flow over the entire crest of the dam, or through a specially provided wasteway in a certain limited part of the crest.

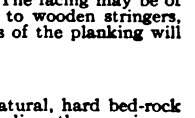

Fig. 1.

When the dam acts as a waste-weir, i.e., with the water flowing over the crest, additional forces must be considered as tending to overturn the structure. There are (1) the additional head of water h_1 above the top of the dam; (2) the tension or suction at the rear face of the dam, due to a partial vacuum at *a*, Fig. 1, caused by the falling water taking up the particles of air between it and the rear face of the dam; and (3) the pressure of ice and logs against the upper face. The vacuum effect may be lessened materially by rounding off the rear upper corner of the dam, as shown in Fig. 2.

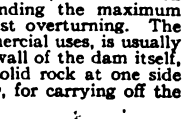

Fig. 2.

Hydrostatic Pressure.—In the discussion of dams and the forces acting against them it is convenient to assume a section of the dam and of the column of water, etc. acting, as one foot thick. The weight of a cubic foot of water is generally assumed at 62.5 lbs., involving an error of ½ of one per cent. on the side of safety. Therefore, the pressure of one square foot of surface at a depth of h feet below the surface is $62.5\,h$. It will thus be seen that the intensity of pressure increases uniformly with the depth, and furthermore it is always normal (at right angle) to the surface acted

upon, because, being a frictionless liquid, the pressure is equal in all directions. To find the total pressure, then, on any plane surface one foot wide, whether inclined or not, it is necessary only to find the intensity of pressure per square foot at its "middle point" and multiply this result by the length of the plane. Moreover, as will be shown in the next article, under "Center of Pressure," the total pressure, instead of being assumed as a distributed force, may be assumed to act at the center of pressure, i.e., at the center of gravity of pressure on any "rigid" surface, thereby greatly simplifying the calculations.

The following are common examples of total pressure, the heavy line in each figure representing the edge of the plane, *one foot wide*, acted upon:

(H and h are in feet; pressure is in lbs. against the plane ab).

Fig. 3.

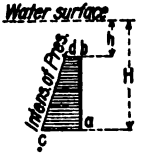

Fig. 4.

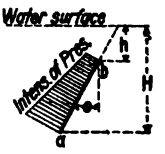

Fig. 5.

(1). Vertical plane ab, just touching surface of water. (Fig. 3.)

$$\text{Total pressure} = \frac{62.5H}{2} \cdot H = \frac{62.5H^2}{2} \text{ lbs.}$$

If the surface of the plane makes an angle θ with the vertical, multiply the above result by secant θ.

(2). Vertical plane ab, submerged below the surface. (Fig. 4.)

$$\text{Total pressure} = 62.5\left(\frac{H+h}{2}\right)(H-h) = 62.5\left(\frac{H^2-h^2}{2}\right) \text{ lbs.}$$

(3). Inclined plane ab, submerged below the surface. (Fig. 5.)

$$\text{Total pressure} = 62.5\left(\frac{H^2-h^2}{2}\right) \sec\theta \text{ lbs.}$$

This pressure will not be horizontal, but normal to the plane ab; example (2) illustrates the horizontal component of this pressure.

The above formulas will be found useful in calculating the total pressure on any section of a dam, whether the face is vertical or sloping. The point of application of the resultant pressure will now be discussed.

Center of Pressure.—If the total pressure against any surface as ab, in the preceding illustrations, is concentrated as a single *resultant* force, this force will act *at* the *center of pressure*, and *through* the center of gravity of the distributed force. In *Example* 1, above, the resultant P of the distributed force is $\frac{62.5H^2}{2}$ lbs., acting horizontally through the center of gravity *c.g.* of the pressure triangle $a\ b\ c$, at a point p on the face of the dam, distant $\frac{2}{3}H$ below the water surface (Fig. 15). Thus, p is the center of pressure, for the head H, on the plane ab, whether vertical or inclined. Likewise, in *Example* 2, the resultant pressure on the plane ab is $P = \frac{62.5}{2}(H^2-h^2)$ lbs., and the center of pressure, p, is $\frac{2}{3}\left(H+\frac{h^2}{H+h}\right)$ feet below the water surface (Fig. 16). In *Example* 3, the same value, $\frac{2}{3}\left(H+\frac{h^2}{H+h}\right)$, holds true for the distance to the center of pressure below the surface, while the total pressure is of course greater—or $\frac{62.5}{2}(H^2-h^2)$ times sec angle of inclination with the vertical. For reference, these values are tabulated as follows:—

Fig. 6.

Fig. 7.

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F HYDROSTATIC PRESSURES ON VERTICAL AND INCLINED SURFACES.

ure e.	Angle of Plane with Vertical.	Surface of Water Above Top of Plane. h	Surface of Water Above Bottom of Plane. H	Resultant Pressure on Plane 1-Ft. Wide. P	Distance Center of Pressure Below Surface of Water.
		Ft.	Ft.	Lbs.	Ft.
l.....	....	0	H	$31.25\ H^2$	$\frac{2}{3}H$
l.....	θ	0	H	$31.25\ H^2 \sec\theta$	$\frac{2}{3}H$
rged.	....	h	H	$31.25\ (H^2-h^2)$	$\frac{2}{3}\left(H+\frac{h^2}{H+h}\right)$
l; rged.	θ	h	H	$31.25\ (H^2-h^2) \sec\theta$	$\frac{2}{3}\left(H+\frac{h^2}{H+h}\right)$

f Gravity of Trapezoid.—quired to find the center of e trapezoid whose top is T B (Fig. 8). Lay off b and t and t', $=T$. The diagthe extreme points intercenter of gravity, o. The bisects T and B. Ana- $\frac{D}{2}-\frac{D}{6}\left(\frac{B-T}{B+T}\right)$. In calcu-

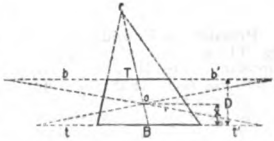

Fig. 8.

e gravity dam the section or of water pressure is frequently divided into trapezoidal ertical forces of the masonry being the weight of each trapehrough its center of gravity. The area of the trapezoid=

ce the weight of a vertical section of masonry one foot thick,

r pressure due to the corresponding pressure trapezoid, are:

, $W=mD\frac{T+B}{2}$; when m=wt. of one cubic foot of masonry.

essure, $P=wD\frac{T+B}{2}$; when w=wt. of one cubic foot of water.

essure, $P_n=wD\frac{T+B}{2}\sec\theta$; when w=wt. of one cu. ft. of water.

ustrated in Fig. 9.

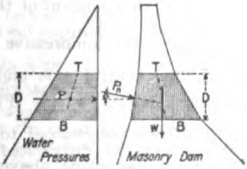

Fig. 9.

ngular Dam.—The simplest form of dam is the triangular c, Fig. 10. Let ac be any assumed base distant h below the distant H below the water surface. If w is the weight per

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cubic foot of water and m is the weight per cubic foot of masonry, we have, from the preceding:

The resultant horizontal pressure, $P = \frac{w H^2}{2}$.

The total weight of masonry, $W = \frac{m b h}{2}$.

Now, if it is desired that the resultant, R, shall intersect the base, b, at the edge of the "middle third," as shown in Fig. 10, we have that

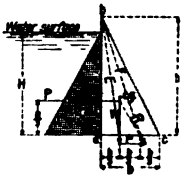

Fig. 10.

$$\frac{P}{W} = \frac{b}{3} \div \frac{H}{3} = \frac{b}{H}, \text{ whence by substitution,}$$

$$\frac{w H^2}{2} \div \frac{m b h}{2} = \frac{b}{H}; \text{ or } b = \sqrt{\frac{w H^3}{m h}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

If $H = h$, this reduces to $$b = h\sqrt{\frac{w}{m}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

If w (water) $= 62.5$, and m (masonry) $= 146$,* $b = 0.654\,h$ ..... (3)

It is desirable, sometimes, to have the resultant cut the base $\frac{2}{3}\,b$ (instead of $\frac{1}{3}\,b$) back from the toe, in which case we will have, $b = 0.7\,h$ ..... (4)

Pressure on Foundations.—If two *stiff* pencil erasers, Fig. 11, are pressed *moderately* together by equal and opposite forces W and W' applied *near* the left end, a, it will be seen that they do not remain in contact throughout their entire length, but separate at the right end, e. Moreover, it will be found that the length of contact to the right of the applied force is double the contact length to the left, and therefore $\frac{2}{3}$ of the total contact b. Similarly, we have in the case of the dam $a\,b\,c$. Fig. 10, that when the reservoir is *empty* and consequently there is no water pressure on the face ab, the resultant force is W, the weight of the masonry, acting as shown in Fig. 11. Hence, we may say that for a triangular dam with a vertical face ab the resultant weight when dam is empty will produce a pressure intensity on the base varying uniformly from zero at the lower toe, c, to a maximum ($m\ h$) at the upper heel, a, Fig. 12. The exact results given above may be deduced by mathematical analysis, which, however, will be omitted here.

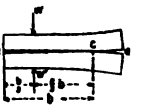

Fig. 11.

Fig. 12.

Let us now consider the forces acting on the plane $a\,c$, Fig. 10, due to the *water pressure* P. Imagine the dam to be a vertical beam of length $a\,b$ and "fixed" at the lower end, $a\,c$. Consider the depth of the beam, b, and the width (vertical with the page), unity. The acting force, P, is $\frac{w H^2}{2}$, the lever arm is $\frac{H}{3}$, and hence the bending moment about the section $a\,c$ is $\frac{w H^2}{2} \cdot \frac{H}{3} = \frac{w H^3}{6}$. The resisting moment of the beam at the section $a\,c$ is† $\frac{1}{6}\,f\,b^2$. Equating, $f = \frac{w H^3}{b^2}$ = the compressive stress at c and the tensile stress at a, considering the neutral axis midway between. If the reservoir is full of water, $H = h$ therefore $f = \frac{w h^3}{b^2}$ (Fig. 13).

Fig. 13.

By combining Figs. 12 and 13 we are enabled to obtain the vertical component of the distributed stress on the foundation or on any plane above same. In addition to this there is the horizontal

* Specific gravity $= 146 \div 62.5 = 2.336$; often assumed at $2\frac{1}{4}$.

† The resisting moment of a rectangular beam about a neutral axis passing through the center of section is $M_r = \frac{1}{6}\,f \times$ breadth of beam $\times$ (depth of beam)^2; in which f = outer fiber stress.

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shear ($=P$) which may be considered as distributed in intensity varying directly with the vertical pressure at any point. Hence the resultant intensities will be parallel with the resultant R, Fig. 10.

Combining the stress due (1) to the weight of the masonry dam, and (2) to the water pressure on the face $a\ c$, we have, for the triangular dam,

Vertical stress at a per sq. ft. $= f_a = \frac{w H^2}{b^2} - m\,h$ ..... (5)

" " " c " " " $= f_c = \quad 0 - \frac{w H^2}{b^2}$ ..... (6)

The result is tension if "plus," compression if "minus."

The following example is solved by equations (5) and (6). Wt. of water, w, is assumed at 62.5; masonry, m, 146 lbs. per cubic foot; see also Fig. 10.

Example.—What is the effect on the resultant pressure, R, Fig. 10, when the reservoir is being filled?

Solution.—(*a*) When the reservoir is empty we have from equation (5), $f_a = -146\ h$, that is, the intensity of compression per sq. ft. at a is equal to 146 × height of dam in feet. If the limiting pressure on masonry is assumed at 30,000 lbs. per sq. ft. it will be seen that the limiting height of the dam will be, $h = \frac{30\,000}{146} = 205\frac{1}{2}$ feet, when the dam is empty, and that the pressure on the foundation uniformly decreases from 30,000 lbs. per sq. ft. at a to zero at c (see Fig. 12). (*b*) If, now, the reservoir is gradually filled, the horizontal pressure P increases with H^2 and the distance to center of pressure above the base increases with $\frac{H}{3}$. The resultant pressure R will gradually swing to the right from the vertical position W and with increasing magnitude. It will be found also that the intensity of pressure at a will gradually decrease with the corresponding increase of pressure at c, while R is traveling across the middle third of the base. By the use of equations (5) and (6) the intensities of pressure f_a and f_c can readily be obtained by substituting the values of the unknown quantities. (*c*) Lastly, we will consider the reservoir as full, that is, the water is assumed to be at its flood height. In this case, H may be slightly less than, equal to, or greater than, h. The resultant R, Fig. 10, is now cutting the base at, say, the extreme right edge of the middle third, whence the compression at a is reduced to zero, while the compression at c has reached the maximum. Therefore it is evident from equation (5) that if $f_a = 0$,

$$\frac{w H^2}{b^2} = m\,h \quad \text{..... (7)}$$

from which, the value of any factor may be found by assuming values for the other factors. Thus, $b = \sqrt{\frac{w H^2}{m\,h}}$, which compare with equation (1).

If $H = h$, this reduces to equation (2), or $b = h\sqrt{\frac{w}{m}}$. Assuming water (w) at 62.5, and masonry (m) at 146, we have, $b = 0.654\ h$, equation (3). If now this value of b is substituted in equation (3) and $w = 62.5$, we have, when reservoir is full,

$$f_c = -\frac{62.5\,H^2}{(0.654\,h)^2} = -\frac{62.5\,h^2}{0.4277\,h^2} = -146\,h, \text{ or}$$

the same result which was obtained for f_a with the reservoir empty. It is to be noted, also, that the most economical type of dam approaches the triangular section with a vertical up-stream face.

General Formulas for Pressure on Foundations.—For any type of gravity dam the following formulas may be applied with a reasonable degree of accuracy, or at least sufficiently so for all practical purposes:

NOTATION.

V = total weight in lbs. of "section-foot" of masonry above plane $a\ c$.
b = width of base $a\ c$ in feet, with center of base at origin, o.

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x = distance in feet from origin o, to point of application of the resultant, R, on $a\,c$; $+x$ if downstream, $-x$ if up-stream, from o.
f_a = vertical intensity of stress in lbs. per sq. ft. at a.
f_c = vertical intensity of stress in lbs. per sq. ft. at c.
β = angle of inclination of resultant R with the vertical.
F_a = intensity of stress in lbs. per sq. ft., parallel with R, at a.
F_c = intensity of stress in lbs. per sq. ft. parallel with R, at c.

Fig. 14.

FORMULAS.

$$f_a = -\frac{W}{b}\left(1-\frac{6x}{b}\right) \quad \ldots\ldots\ldots\ldots (7)$$

$$f_c = -\frac{W}{b}\left(1+\frac{6x}{b}\right) \quad \ldots\ldots\ldots\ldots (8)$$

$$F_a = -\frac{W}{b}\left(1-\frac{6x}{b}\right)\sec\beta \quad \ldots\ldots\ldots\ldots (9)$$

$$F_c = -\frac{W}{b}\left(1+\frac{6x}{b}\right)\sec\beta \quad \ldots\ldots\ldots\ldots (10)$$

If the result is *minus*, the stress is *compression*; if *plus*, it is *tension*.

Note that f_a and f_c are vertical components of F_a and F_c; also that when $+x$ or $-x$ is equal to $\frac{b}{6}$, the resultant pressure cuts the base at the edge of the middle third; whence f_a is respectively equal to 0 or to $-\frac{2W}{b}$, and f_c is respectively equal to $-\frac{2W}{b}$ or to 0. That is to say, that when the resultant cuts the base within the limits of the middle third there will be no tension in any part of the masonry.

Table 2, next page, is based on allowable tension in the masonry, but in actual practice tension is not allowable in the masonry of a gravity dam.

Factor of Safety Against Overturning.—With the resultant pressure R cutting the base at the lower edge of the middle third, Fig. 10, when the reservoir is full and the water pressure P is maximum, it is to be noted that the factor of safety of the dam, against overturning, is 2, because if P is doubled, R will pass through the lower toe at c, whence the dam will be on the point of overturning. The nearer to the *center* of the middle third that R intersects the base, the greater is the factor of safety. If it intersects in the middle third, the factor is 2 or greater; if in the outer third, the factor is between 1 and 2; if at the outer toe, c, the factor is 1; if outside the outer toe, the factor is less than 1, and hence the dam will overturn. These *principles* apply not only to the triangular dam but to any practical type.

Shear.—The horizontal shear on any plane $a\,c$ is equal to the horizontal component of the total water pressure on a section-foot above that plane, or equal to $P\cos\theta$. If W = the total weight of the masonry resting on the joint ac, and f = the coefficient of friction, then $f\,(W+P\sin\theta)$ = the total resisting force due to friction. Hence, if the joint ac is assumed to be a horizontal plane with *no adhesion*, but simply friction, between the two surfaces in contact, we have, for equilibrium,

$$P\cos\theta \leqq f\,(W+P\sin\theta) \ldots\ldots\ldots\ldots (11)$$

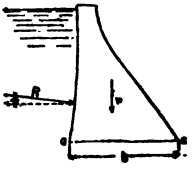

Fig. 15.

In ordinary practice, f may be assumed at $\frac{3}{4}$, hence $P\cos\theta$ must not exceed $\frac{3}{4}\,(W+P\sin\theta)$. Equation (11) may be used safely in designing because $f\,(W+P\sin\theta)$ does not represent the entire resisting force. In fact, horizontal and unbroken joints are avoided in practical construction, and hence considerable shearing resistance is introduced, thereby increasing the factor of safety against sliding, materially.

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SSURE ON FOUNDATIONS FOR VARIOUS VALUES OF x.
(See Fig. 14).

nsity of Pressure on Foundations in Lbs. per Square Foot.				Remarks.
	f_c	F_u	F_d	
$\frac{W}{b}$	$\pm\ 0$	$-\frac{2W}{b}\operatorname{Sec}\beta$	$\pm\ 0$	Resultant at up-stream edge of middle third.
$\frac{W}{b}$	$-\frac{0.1W}{b}$	$-\frac{1.9W}{b}\operatorname{Sec}\beta$	$-\frac{0.1W}{b}\operatorname{Sec}\beta$	
$\frac{W}{b}$	$-\frac{0.4W}{b}$	$-\frac{1.6W}{b}\operatorname{Sec}\beta$	$-\frac{0.4W}{b}\operatorname{Sec}\beta$	
$\frac{W}{b}$	$-\frac{0.7W}{b}$	$-\frac{1.3W}{b}\operatorname{Sec}\beta$	$-\frac{0.7W}{b}\operatorname{Sec}\beta$	
	$-\frac{W}{b}$	$-\frac{W}{b}\operatorname{Sec}\beta$	$-\frac{W}{b}\operatorname{Sec}\beta.$	Resultant at center of base or origin, o.
$\frac{7W}{b}$	$-\frac{1.3W}{b}$	$-\frac{0.7W}{b}\operatorname{Sec}\beta$	$-\frac{1.3W}{b}\operatorname{Sec}\beta$	
$\frac{4W}{b}$	$-\frac{1.6W}{b}$	$-\frac{0.4W}{b}\operatorname{Sec}\beta$	$-\frac{1.6W}{b}\operatorname{Sec}\beta$	
$\frac{25W}{b}$	$-\frac{1.75W}{b}$	$-\frac{0.25W}{b}\operatorname{Sec}\beta$	$-\frac{1.75W}{b}\operatorname{Sec}\beta$	Resultant pressure $\frac{3}{8}b$ from down-stream toe of dam.
$\frac{1W}{b}$	$-\frac{1.9W}{b}$	$-\frac{0.1W}{b}\operatorname{Sec}\beta$	$-\frac{1.9W}{b}\operatorname{Sec}\beta$	
0	$-\frac{2W}{b}$	$\pm\ 0$	$-\frac{2W}{b}\operatorname{Sec}\beta$	Resultant at down-stream edge of middle third.
$.2\frac{W}{b}$	$-\frac{2.2W}{b}$	$+\frac{0.2W}{b}\operatorname{Sec}\beta$	$-\frac{2.2W}{b}\operatorname{Sec}\beta$	These values of x give tension in up-stream face, which is not allowable in practice.
$.5\frac{W}{b}$	$-\frac{2.5W}{b}$	$+\frac{0.5W}{b}\operatorname{Sec}\beta$	$-\frac{2.5W}{b}\operatorname{Sec}\beta$	
$.8\frac{W}{b}$	$-\frac{2.8W}{b}$	$+\frac{0.8W}{b}\operatorname{Sec}\beta$	$-\frac{2.8W}{b}\operatorname{Sec}\beta$	
$.1\frac{W}{b}$	$-\frac{3.1W}{b}$	$+\frac{1.1W}{b}\operatorname{Sec}\beta$	$-\frac{3.1W}{b}\operatorname{Sec}\beta$	
$.4\frac{W}{b}$	$-\frac{3.4W}{b}$	$+\frac{1.4W}{b}\operatorname{Sec}\beta$	$-\frac{3.4W}{b}\operatorname{Sec}\beta$	
$.7\frac{W}{b}$	$-\frac{3.7W}{b}$	$+\frac{1.7W}{b}\operatorname{Sec}\beta$	$-\frac{3.7W}{b}\operatorname{Sec}\beta$	
$2.0\frac{W}{b}$	$-\frac{4W}{b}$	$+\frac{2.0W}{b}\operatorname{Sec}\beta$	$-\frac{4W}{b}\operatorname{Sec}\beta$	

Plus = tension; minus = compression. See notation, page 849.

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Author's Type of Dam.—The author submits Fig. 16 as a type which may be used, ordinarily, up to 200 ft. in height, every part to be enlarged proportionately. The dimensions shown in the figure are based on a height of *unity*. If the proposed height of dam is 50 ft., multiply each dimension by 50; if 100 ft., multiply by 100; if 200 ft., multiply by 200; etc. In approaching the upper limit of height, care must be used to see that the maximum allowable pressures per square foot on the base and foundations are not exceeded. Table 2, preceding, may be consulted with regard to the pressure on foundations. If for any proposed height, say 225 ft., the pressure on the foundations would be excessive, the type may be proportioned for a height which will not produce excessive pressure, say for 200 ft., and the side lines projected further downward, below the E' level, the necessary distance. Note that the up-stream face below the B' level is a series of chords touching an imaginary parabolic curve at their vertices, while the down-stream face is a straight line; also that above the B' level the up-stream face is vertical and the down-stream face is a parabola. The design is based on the masonry having a specific gravity of 2½, which is equivalent to 145.83×3 lbs. per cubic foot, hence the engineer is cautioned not to use this type, unmodified, for masonry of less specific gravity.

Calculations of Author's Type of Dam (Fig. 16).—The triangular type of dam, with an apex at b, Fig. 10, is never adopted in practice. The practical type is never allowed to come to a point at the top, but has a certain top width, say about one-tenth the height of the dam, giving mass to resist any forces acting at the surface such as those due to floating ice, logs, etc. This width also provides usually for foot-path and roadway, either expressly or incidentally, and for a general promenade if the dam is high and in a picturesque site. Another departure from the triangular dam is the upstream face $a\ b$, which is battered more or less instead of being vertical. With these conditions imposed the problem becomes, by comparison, a complicated one when we wish to design a type having practical lines, containing the least amount of masonry, and whose resultant lines of pressure must generally coincide with the edges of the middle third. In Fig. 10, the triangular type, line E represents the line of resultant pressure when the reservoir is empty, and line F when full. They are both straight lines and are drawn from b to the base, cutting the latter in three equal parts. Likewise they will cut any parallel plane above the base in a similar manner, and hence the triangular type is an ideal one, easy to calculate.

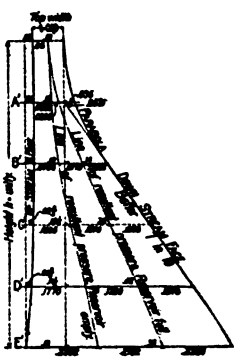

Fig. 16.—Author's Type of Dam.—Unit Dimensions.

There are several methods in use in designing dams. The "cut-and-try" method is the one here presented, and Fig. 16 is the result of the "second trial." It is practical, economical, safe, and pleasing to the eye; in fact, the outline was determined not a little by the general effect, keeping in mind certain well-known principles.

NOTATION.

h = height of dam in feet —assumed as 1;
t = width of top of dam in feet —equal to 0.1 h;
w = specific gravity of water —equal to 1;
m = specific gravity of masonry —assumed as 2½;
P = total spec. grav. water pressure on dam above any plane in question;
W = total spec. grav. weight of masonry above any plane in question;
d = depth of water above the plane in question; or depth of section considered, from top of dam downward.

CALCULATION.

Fig. 16 represents the second and final design. (The first design is not shown, but had a greater width at the A' level and contained about 7 per cent. more masonry.) It is to be noted that in the following

calculations the *horizontal* position of the *center of gravity* of the masonry W above any plane in question is desired, but not the vertical position; likewise, the *vertical* position of the *center of pressure* of the water pressure P, above any plane, is required. The vertical component of P, acting downward on the sloping up-stream face, will be neglected. This is on the side of safety, as, if considered, it would throw the line of resultant pressure for "full reservoir" inward toward the center of the middle third. Calculations will be made of the forces *above* the respective planes A', B', C', D' and E', taken in order. In other words, the structure is considered as cut successively by these planes and the problem consists in finding the point of applicant of the resultant force on each plane, (*a*) for reservoir empty, and (*b*) for reservoir full. Connecting all the (*a*) points gives the line of resultant pressure when empty, and connecting all the (*b*) points gives the line of resultant pressure when full.

A' Level.—This plane is 0.2 h below top of dam. Considering one section-foot of dam, and the specific gravity of the masonry equal $2\frac{1}{2}$, we have the acting forces as shown in Fig. 17, in which W_a and W_b act alone (above the plane) when the dam is empty, and the force P is added when the dam is full. Or, we may consider $W_a + W_b = W$ acting vertically through the center of gravity of the total section above A', cutting the base at e and being resisted, when empty, by the equal and opposite force R_e. But when the reservoir is full, the added force P will make the resultant take the direction $o\,f$, cutting the base at f, and being resisted by the equal and opposite force R_f.

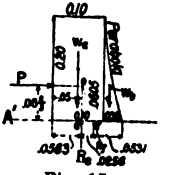

Fig. 17.

Note that the point o is the intersection of the vertical force W with the horizontal force P, and that, relatively,

$W_a = 2\frac{1}{2}\,(.10 \times .20)\,h^2$, acting vertically through center of rectangle;

$W_b = 2\frac{1}{2}\left(\frac{.035 \times .20}{3}\right)h^2$, acting vertically $\frac{3}{10} \times .035\,h$ from left face of parabolic spandril. Hence,

$W = W_a + W_b = 2\frac{1}{2}\left(\frac{.335 \times .20}{3}\right)h^2$, acting vertically $.0063^*\,h$ to right of W_a, that is, through the point o.

$P = \frac{1}{2}\,d^2 = \frac{1}{2}\,(.2\,h)^2 = .02\,h^2$, acting horizontally $.06\frac{2}{3}\,h$ above A'. Taking moments, we find the distance $e\,f = .06\frac{2}{3}\,h\,\frac{P}{W} = .0256\,h$. It is to be noted that both resultants intersect the A' base well within the limits of the middle third.

B' Level.—This plane is 0.4 h below top of dam and is calculated in the same manner as the preceding. Note that W_c includes W_a and that W_d includes W_b. Then, relatively,

$W_c = 2\frac{1}{2}\,(.10 \times .40)\,h^2$,

$W_d = 2\frac{1}{2}\left(\frac{.14 \times .40}{3}\right)h^2$,

$W = W_c + W_d = 2\frac{1}{2}\left(\frac{.44 \times .40}{3}\right)h^2$,

acting vertically through o', $.0293\,h$ to right of W_c. Hence, R_e acts $.0793\,h$ from left hand face or $.0007\,h$ outside of the middle third. For a dam 100 ft. high this amounts to only $\frac{1}{2}$ inch in a total width on the B' level of 24 ft.—an amount insignificant.

$P = \frac{1}{2}\,d^2 = \frac{1}{2}\,(.4\,h)^2 = .08\,h^2$, acting $0.13\frac{1}{3}\,h$ above B' level. Taking moments in the case of the full reservoir we find the distance $e\,f = 0.13\frac{1}{3}\,h\,\frac{P}{W} = .0779\,h$.

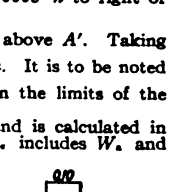

Fig. 18.

C' Level.—In Fig. 19, $W_c + W_d$ is the relative weight of the masonry above the B' level, while W_e is the relative weight of the trapezoidal section-foot between the B' and the C' levels. The vertical arrows indicating

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* $.0063 = \frac{.0605\,W_b}{W_a + W_b}$. See page 295 for determining the position of the center of gravity of two or more parallel forces.

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the weights, pass through the respective centers of gravity. The total weight W ($=W_s+W_d+W$) intersects the C' plane at e, distant .1225 h from the face of the dam; or .0053 h outside the limit of the middle third, amounting to about 6⅞ ins. on a total base of 38 ft. 4 ins. for a dam 100 ft. high. For full reservoir, we have,

$W_s = 2\frac{1}{2} \times .062\frac{1}{2}\ h^2$,
$W = 2\frac{1}{2} \times .121\ h^2$,
$P = \frac{1}{2} d^2 = \frac{1}{2} (.6\ h)^2 = .18\ h^2$, acting $0.20h$ above C' level. Taking moments in the case of the full reservoir we find the distance $ef = .2\ h \frac{P}{W} = .1275h$.

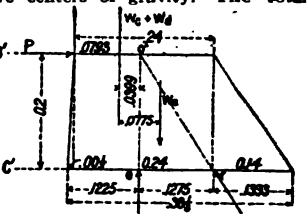

Fig. 19.

This is on the assumption that P acts horizontally. Note, however, that below the B' level the pressure is really normal to a sloping face and if so considered the resultant, under full reservoir pressure, would cut the C' plane a little to the left of f, thereby increasing the factor of safety.

D' Level.—Briefly discussed, the relative weight of the new trapezoidal section-foot, between the C' and the D' levels is

$W_t = 2\frac{1}{2} \times .091\frac{1}{3}\ h^2$,
$W = 2\frac{1}{2} \times .212\frac{1}{3}\ h^2$,
$P = \frac{1}{2}d^2 = \frac{1}{2} (.8\ h)^2 = .32\ h^2$, acting $0.26\frac{2}{3}\ h$ above D' level. Completing the calculation it is found that when the reservoir is empty the resultant cuts the D' plane .1770 h from the up-stream face, or .0008 h outside the limit of the middle third, amounting to 1 in. on a total base of 53 ft. 4 ins. for a dam 100 feet high. When the reservoir is full the resultant cuts the D plane .1720 h farther from the face, and within the middle third.

E' Level.—For the section between D' and E' levels,

$W_s = 2\frac{1}{2} \times .122\frac{1}{2}\ h^2$,
$W = 2\frac{1}{2} \times .335\ h^2$,
$P = \frac{1}{2}d^2 = \frac{1}{2}h^2$, acting $\frac{1}{3}\ h$ above E' level.

With the reservoir empty the resultant cuts the base .2382 h from the up-stream face, which is .0082 h within the limit of the middle third. When the reservoir is full the resultant cuts the E' plane .2132 h farther from the up-stream face.

For general dimensions and quantities, see Tables 3 and 4, following.

QUANTITIES IN MASONRY, ROCK-FILL AND EARTH DAMS.

Tables 4, 5 and 6, following, will be found useful in estimating the relative quantities in masonry, rock-fill and earth dams, from profiles or contour maps of any particular site. Quantities are in cubic yards per 100-ft. section of dam. For other sectional lengths the yardage will of course be proportional. Thus, for 50-ft. lengths, divide by 2; for 25-ft. lengths, divide by 4; etc.

The scientific design of masonry dams, laid in cement mortar, is subject to more accurate determination than are those of rock-fill or earth. For the two latter classes we are guided mainly by the behavior of the types that have been constructed, and by carefully studying the causes of failure of those that have not stood the test. The actual cross-section of the dam itself is only one of the necessary elements of strength. The foundations should be selected and prepared carefully to receive the superstructure or dam proper; the outlet constructed with care to prevent leakage; and the wasteway ample to provide for flood discharges.

A word in regard to the effect of *profile* on the resisting power of a dam. In our calculations we assume one section-foot of dam and one section-foot of water pressure acting against it—the resultant forces in each section-foot acting independently of any other section-foot. This theory would hold true in practice for a straight dam of constant height and of indefinite length, but for all ordinary cases, as for instance Fig. 20, where the ground line is irregular, calling for different heights for each section, it is impossible for any section to act independently of any other. Thus, under full reservoir head, the high section A will tend to deflect downstream a greater amount than will one of less height,

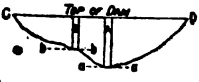

Fig. 20.—Profile of Dam-Site.

cause in the former case the deflection will extend down to the while in the latter case it will extend only as low as *bb*. Hence, tresses are set up between adjacent sections, tending to *decrease* rning moment for the higher sections, near the middle of the dam.

RAL TABLE OF DIMENSIONS AND QUANTITIES FOR MASONRY DAMS—AUTHOR'S TYPE, FIG. 16.

(See also Table 4.)

Width of Top of Dam in terms of total height, h. (2)	Width of Base of Section in terms of total height, h. (3)	Width of Base of Section in terms of depth of section, d. (4)	Area in Sq. Ft. of Section of depth d in terms of h^2. (5)	Cu. Yds. per 100 Lin. Ft. of Dam (for section depth d), in terms of h^2. (6)	Remarks. (7)
.10 h	.10035 h	5.017 d	.00200 h^2	.0074 h^2	
"	.10140 "	2.535 "	.00402 "	.0149 "	
"	.10315 "	1.719 "	.00607 "	.0225 "	
"	.10560 "	1.320 "	.00815 "	.0302 "	
"	.10875 "	1.088 "	.01029 "	.0381 "	
"	.11260 "	.938 "	.01250 "	.0463 "	
"	.11715 "	.837 "	.01480 "	.0548 "	
"	.12240 "	.765 "	.01724 "	.0639 "	
"	.12835 "	.713 "	.01970 "	.0730 "	
"	.13500 "	.675 "	.02233 "	.0827 "	
"	.14235 "	.647 "	.02511 "	.0930 "	
"	.15040 "	.627 "	.02803 "	.1038 "	
"	.15915 "	.612 "	.03113 "	.1153 "	
"	.16860 "	.602 "	.03440 "	.1274 "	The quantities in this table are expressed, generally, in terms of h, the height of the standard type, Fig. 16. If it is desired to express them in terms of t, the width of top of dam, we have only to substitute the values of h and h^2 expressed in terms of t and t^2. Thus, as $t = \frac{h}{10}$ and $t^2 = \frac{h^2}{100}$ we have, $h = 10t$ and $h^2 = 100t^2$. We may therefore use with this table the top width instead of the height h.
"	.17875 "	.596 "	.03788 "	.1403 "	
"	.18960 "	.593 "	.04156 "	.1539 "	
"	.20115 "	.592 "	.04546 "	.1684 "	
"	.21340 "	.593 "	.04961 "	.1837 "	
"	.22635 "	.596 "	.05400 "	.2000 "	
"	.24000 "	.600 "	.05867 "	.2173 "	
"	.25433 "	.606 "	.06361 "	.2356 "	
"	.26867 "	.611 "	.06884 "	.2550 "	
"	.28300 "	.615 "	.07436 "	.2754 "	
"	.29733 "	.619 "	.08016 "	.2969 "	
"	.31167 "	.623 "	.08625 "	.3194 "	
"	.32600 "	.627 "	.09263 "	.3431 "	
"	.34033 "	.630 "	.09929 "	.3677 "	
"	.35467 "	.633 "	.10624 "	.3935 "	
"	.36900 "	.636 "	.11348 "	.4203 "	
"	.38333 "	.639 "	.12100 "	.4481 "	
"	.39833 "	.642 "	.12882 "	.4771 "	
"	.41333 "	.646 "	.13693 "	.5071 "	
"	.42833 "	.649 "	.14535 "	.5383 "	
"	.44333 "	.652 "	.15407 "	.5706 "	
"	.45833 "	.655 "	.16308 "	.6040 "	
"	.47333 "	.657 "	.17240 "	.6385 "	
"	.48833 "	.660 "	.18202 "	.6741 "	
"	.50333 "	.662 "	.19193 "	.7109 "	
"	.51833 "	.665 "	.20215 "	.7487 "	
"	.53333 "	.667 "	.21267 "	.7877 "	
"	.54900 "	.670 "	.22349 "	.8277 "	
"	.56467 "	.672 "	.23463 "	.8690 "	See, also, Table 4.
"	.58033 "	.675 "	.24608 "	.9114 "	
"	.59600 "	.677 "	.25784 "	.9550 "	
"	.61167 "	.680 "	.26992 "	.9997 "	
"	.62733 "	.682 "	.28231 "	1.0456 "	
"	.64300 "	.684 "	.29501 "	1.0926 "	
"	.65867 "	.686 "	.30803 "	1.1409 "	
"	.67433 "	.688 "	.32135 "	1.1902 "	
"	.69000 "	.690 "	.33500 "	1.2407 "	

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4.—Specific Table of Quantities in Masonry Dams—Author's Type, Fig. 16.

(See also Table 3.)

Depth d of Section in Feet. d (1)	Cubic Yards of Masonry per 100 Lineal Feet of Dam, for Section of Depth d from Top (Fig. 21). Top = 20′. h = 200′. (2)	Top = 18′. h = 180′. (3)	Top = 16′. h = 160′. (4)	Top = 14′. h = 140′. (5)	Top = 12′. h = 120′. (6)	Top = 10′. h = 100′. (7)	Top = 8′. h = 80′. (8)	Top = 6′. h = 60′. (9)	Top = 4′. h = 40′. (10)	Top = 2′. h = 20′. (11)
4	296	267	237	208	178	149	119	90	61	33
8	596	536	478	419	360	302	244	187	132	87
12	899	810	723	636	549	463	379	298	224	179
16	1207	1091	976	861	748	639	529	386	348	315
20	1524	1381	1239	1099	961	827	701	588	511	496
24	1852	1683	1516	1351	1191	1038	898	782	717	
28	2193	1998	1807	1621	1442	1274	1126	1017	966	
32	2554	2330	2118	1912	1717	1539	1391	1294	1260	
36	2919	2680	2448	2227	2020	1837	1696	1613	1600	
40	3309	3051	2802	2568	2354	2173	2044	1976	1985	
44	3720	3445	3183	2939	2722	2550	2435	2384		
48	4153	3864	3591	3342	3129	2969	2869	2836		
52	4612	4311	4031	3781	3577	3431	3345	3323		
56	5096	4787	4504	4259	4067	3935	3866	3877		
60	5612	5296	5014	4778	4600	4481	4431	4467		
64	6157	5840	5563	5339	5176	5071	5041			
68	6735	6420	6153	5943	5793	5706	5696			
72	7350	7041	6786	6589	6453	6385	6398			
76	8000	7702	7461	7278	7157	7109	7146			
80	8692	8406	8178	8009	7905	7877	7941			
84	9424	9152	8938	8784	8698	8690				
88	10199	9941	9741	9601	9535	9550				
92	11016	10772	10586	10463	10417	10456				
96	11876	11645	11473	11369	11343	11409				
100	12778	12561	12404	12320	12314	12407				
104	13723	13519	13379	13315	13332					
108	14710	14520	14399	14354	14396					
112	15739	15564	15463	15438	15507					
116	16812	16653	16572	16567	16664					
120	17926	17787	17725	17743	17867					
124	19084	18965	18922	18965						
128	20286	20187	20164	20234						
132	21533	21454	21451	21549						
136	22825	22765	22785	22911						
140	24160	24121	24165	24319						
144	25541	25521	25592							
148	26966	26966	27065							
152	28434	28458	28585							
156	29948	29996	30151							
160	31507	31581	31763							
164	33110	33212								
168	34760	34889								
172	36456	36613								
176	38199	38383								
180	39988	40200								
184	41824									
188	43705									
192	45634									
196	47607									
200	49630									

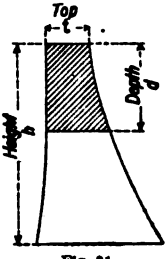

Fig. 21

Note.—For absolute quantities, use the second column only; the last nine columns are for possible sections which must be verified or modified by trial calculations.

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5.—Table of Quantities in Rock-Fill Dams.

rea of Up-tream Face, 00 Ft Long nd of epth *d*. q. Ft. (2)	Cubic Yards of Masonry per 100 Lineal Feet of Dam, for Section of Depth *d*, from Top (Fig. 22). Top= 24′. (3)	Top= 22′. (4)	Top= 20′. (5)	Top= 18′. (6)	Top= 16′. (7)	Top= 14′. (8)	Top= 12′. (9)	Top= 10′. (10)	Top= 8′. (11)
447	400	370	341	311	281	252	222	193	163
894	889	830	770	711	652	593	533	474	415
1342	1467	1378	1289	1200	1111	1022	933	844	756
1789	2133	2015	1896	1778	1659	1541	1422	1304	1185
2236	2889	2741	2593	2444	2296	2148	2000	1852	1704
2683	3733	3556	3378	3200	3022	2844	2667	2489	2311
3130	4667	4459	4252	4044	3837	3630	3422	3215	3007
3578	5689	5452	5215	4978	4741	4504	4267	4030	3793
4025	6800	6533	6267	6000	5733	5467	5200	4933	4667
4472	8000	7704	7407	7111	6815	6519	6222	5926	5630
4919	9289	8963	8637	8311	7985	7659	7333	7007	
5366	10667	10311	9956	9600	9244	8889	8533	8178	
5814	12133	11748	11363	10978	10593	10207	9822	9437	
6261	13689	13274	12859	12444	12030	11615	11200	10785	
6708	15333	14889	14444	14000	13556	13111	12667	12222	
7155	17067	16593	16119	15644	15170	14696	14222		
7602	18889	18385	17881	17378	16874	16370	15867		
8050	20800	20267	19733	19200	18667	18133	17600		
8497	22800	22237	21674	21111	20548	19985	19422		
8944	24889	24296	23704	23111	22519	21926	21333		
9391	27074	26452	25830	25207	24585	23963			
9838	29363	28711	28059	27407	26756	26104			
10286	31756	31074	30393	29711	29030	28348			
10733	34252	33541	32830	32119	31407	30696			
11180	36852	36111	35370	34630	33889	33148			
11627	39556	38785	38015	37244	36474				
12074	42363	41563	40763	39963	39163				
12522	45274	44444	43615	42785	41956				
12969	48289	47430	46570	45711	44852				
13416	51407	50519	49630	48741	47852				
13863	54630	53711	52793	51874					
14310	57956	57007	56059	55111					
14758	61385	60407	59430	58452					
15205	64919	63911	62904	61896					
15652	68556	67519	66481	65444					
16099	72296	71230	70163						
16546	76141	75044	73948						
16994	80089	78963	77837						
17441	84141	82985	81830						
17888	82896	87111	85926						
18336	92556	91341							
18783	96919	95674							
19230	101385	100111							
19677	105956	104652							
20125	110630	109296							
20572	115407								
21019	120289								
21466	125274								
21913	130363								
22361	135556								

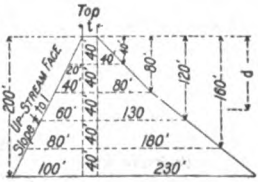

Fig. 22.

Note.—Area of up-stream face is given in order to estimate the quantity of material in the facing.

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6.—Table of Quantities in Earth Dams.

Fig. 23.

Depth *d* of Section from Top of Dam, in Feet.	Area of Up-Stream Face, 100-ft. Long and of Depth *d*. Sq. Ft.	Cubic Yards of Earth per 100 Lineal Feet of Dam, for Section of Depth *d*, from Top (Fig. 23).								
		Top = 28′	Top = 26′	Top = 24′	Top = 22′	Top = 20′	Top = 18′	Top = 16′	Top = 14′	Top = 12′
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
4	1077	563	533	504	474	444	415	385	356	326
8	2154	1422	1363	1304	1244	1185	1126	1067	1007	948
12	3231	2578	2489	2400	2311	2222	2133	2044	1956	1867
16	4308	4030	3911	3793	3674	3556	3437	3318	3200	3081
20	5385	5778	5630	5481	5333	5185	5037	4889	4741	4593
24	6462	7822	7644	7467	7289	7111	6933	6756	6578	6400
28	7539	10163	9956	9748	9541	9333	9126	8919	8711	8504
32	8616	12800	12563	12326	12089	11852	11615	11378	11141	10904
36	9693	15733	15467	15200	14933	14667	14400	14133	13867	13600
40	11770	19111	18815	18519	18222	17926	17630	17333	17037	16741
44	12847	22785	22459	22133	21807	21481	21156	20830	20504	20178
48	13924	26756	26400	26044	25689	25333	24978	24622	24267	23911
52	15001	31111	30726	30341	29956	29570	29185	28800	28415	28030
56	16078	35763	35348	34933	34519	34104	33689	33274	32859	32444
60	17155	40711	40267	39822	39378	38933	38489	38044	37600	37156
64	18233	45956	45481	45007	44533	44059	43585	43111	42637	42163
68	19310	51496	50993	50489	49985	49481	48978	48474	47970	47467
72	20387	57333	56800	56267	55733	55200	54667	54133	53600	53067
76	21464	63467	62904	62341	61778	61215	60652	60089	59526	58963
80	22541	69896	69304	68711	68119	67526	66933	66341	65748	65156
84	23618	76622	76000	75378	74756	74133	73511	72889	72267	71644
88	24695	83763	83111	82459	81807	81156	80504	79852	79200	78544
92	25772	91200	90519	89837	89156	88474	87793	87111	86430	85744
96	26849	98933	98222	97511	96800	96089	95378	94667	93956	93244
100	27926	106963	106222	105481	104741	104000	103259	102519	101778	101037
104	29003	115289	114519	113748	112978	112207	111437	110667	109896	109126
108	30080	123911	123111	122311	121511	120711	119911	119111	118311	117511
112	31157	132830	132000	131170	130341	129511	128681	127852	127022	126193
116	32234	142193	141333	140474	139615	138756	137896	137037	136178	135319
120	33311	151852	150963	150074	149185	148296	147407	146519	145630	144741
124	34388	161807	160889	159970	159052	158133	157215	156296	155378	154459
128	35465	172059	171111	170163	169215	168267	167319	166370	165422	164474
132	36542	182607	181630	180652	179674	178696	177719	176741	175763	174785
136	37619	193452	192444	191437	190430	189422	188415	187407	186400	185391

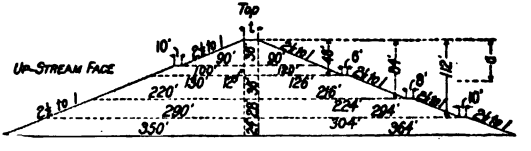

Fig. 23.

Note.—Area of up-stream face is given in order to estimate the quantity of material in the facing.

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EXCERPTS AND REFERENCES.

iyar Dam and Irrigation Works, So. India (Eng. News,).—Illustrated, with section of dam. Article contains a table principal dimensions of Periyar dam as compared with those ɔary dams over 150 ft. in total height, as follows:

ıe.	Depth of Water, Ft.	Ht. Above Base, Ft.	Top Width Ft.	Base Width Ft.	Area Profile Sq.Ft.	Top Length Ft.	Material.
N. Y.....	150.	219.	18.	200.	28400.	2240.	Stone.
ı.....	155.	176.	12.	138.5	10772.	1200.	Conc.
ce.....	164.	170.6	9.91	161.	10712.	328.	Stone.
.....	162.3	170.33	14.75	154.5	11596.	546.	Stone.
al.....	.....	170.	20.	176.	16660.	700.	Conc.
n.....	153.54	164.24	35.73	144.29	16349.	925.	Stone.
e.....	.....	161.43	.....	.....	.....	.....	Stone.
.....	137.9	156.82	16.40	126.98	6780.	.....	Stone.
ium.....	147.64	154.2	49.22	216.5	18708.	771.	Stone.

ed New Type of Masonry Dam (By Geo. L. Dillman. Trans. Vol. XLIX).

ive Cross-Sections of the High Earth Dams of the World Nov. 28, 1901).—Illustrations of 17 dams, giving sections, of core wall, etc., including: The earth portion of New Croton ft., masonry core wall, water slope 2:1); San Leandro Dam, ft., puddle wall of stiff blue clay, water slope 3:1, down-2.4:1); Turner's Embankment, Liverpool (ht. 105 ft.; clay all extending down 90 ft. into sand, earth, stones, gravel and face 3:1 with 15″ pitching); Titicus Dam, N. Y. (ht. 100 ft., wall, down-stream face 2.5:1, water face 2.4:1 with 12″ and 18″ paving.

eesman Dam and Reservoir (By C. L. Harrison and S. H. ans. A. S. C. E., Vol. LIII).—See page 131 of Transactions ve sections of the following dams: New Croton, Lake Chees-ı, Periyar, Furens, Villar, Ban, and Sweetwater dams—in the height, beginning with the highest.

n With Concrete Foundations, at Redridge, Mich. (Eng. 5, 1901).—Illustrated.

al Formula for the Length of Waterways (By J. P. Frizell. ug. 8, 1901).—For discussions, see Eng. News, Sept. 5 and

aud High Earth Dam, Cal. (By B. Bassell. Eng. News,).—Illustrated, with very complete description of method of rolling, etc.

ractice in Hydraulic-Fill Dam Construction (By J. D. ns. A. S. C. E., Vols. LVIII and LX).

Review of Dam and Reservoir Failures in the U. S. (By Eng. News, June 19, 1902).—Ten listed under "Insufficient Overtopping;" 10 under "Failures Due to Water Leaking aid Through Embankments;" 6 under "Failures of Reservoir under "Embankments and Walls Undermined When Built or Yielding Material;" 6 under "Poor Workmanship or ;" 4 under "Ice Pressure;" 8 under "Miscellaneous or Un-." In addition to the above, 48 other failures of dams and referred to, the causes of failure being unrecorded.

as a Core for Dams and Reservoir Embankments (Eng. 5, 1903).

for Hollow Reinforced-Concrete Dams (By H. W. Foster. ec. 24, 1903).—Illustrated.

ity of Surcharged Masonry Dams (By E. Sherman Gould. ar. 3, 1904).

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Rubble Concrete Dam for the Atlanta Water & Electric Power Co. (Eng. News, July 7, 1904).—Illustrated section of rollway portion of dam with graphical determinations of resultant pressures. "The advantages of rubble concrete for many kinds of masonry work, and particularly for massive structures like masonry dams, are gradually being recognized by engineers. In many cases where large yardage of masonry is required the use of rubble concrete will effect a saving in time and cost over rubble masonry work or fine concrete work. As an illustration, rubble concrete composed of 40% large stone and 60% of 1:2½:5 concrete requires 7% of the volume to be of cement, while rubble masonry composed of 65% large stone and 35% of 1:2½ mortar requires 10% of the volume to be of cement."

Investigation of Stresses in High Masonry Dams of Short Spans (By G. Y. Wisner and E. T. Wheeler. Eng. News, Aug. 10, 1905).—Relates to the proposed curved Pathfinder dam (cross-section: height 210′, top width 10′, bottom width 94′, up-stream batter 0.15, down-stream batter 0.25), with diagrams giving results of calculations.

Earth Dams with Concrete Core Walls (By Clemens Herschel. Eng. News, Sept. 7, 1905).

Computation of Height of Backwater Above Dams (Eng. News, Nov. 1, 1906; also Nov. 29, 1906, with tables).

Large Reinforced-Concrete Dam at Ellsworth, Me. (Eng. News, May 23, 1907).—Illustrated section; 64 ft. high.

Large Electrically Operated Gates for the Roosevelt Dam, Ariz. (By F. W. Hanna. Eng. News, Mar. 30, 1907).—Illustrated.

Electrically Operated Service Gates for the Shoshone and Pathfinder Dams (By F. W. Hanna. Eng. News, Jan. 2, 1908).—Illustrated.

Reinforced-Concrete Diaphrams for Earth Dams (By B. M. Hall. Eng. News, Feb. 6, 1908).—Illustrated.

Combination Dam and Bridge of Reinforced Concrete (Eng. News, April 9, 1908).—Illustrated.

The Design of Buttressed Dams of Reinforced Concrete (By R. C. Beardsley, Eng. News, April 23, 1908).—Illustrated.

Progress on the Roosevelt Dam; with Cost Data (By C. W. Smith. Eng. News, Sept. 10, 1908).

Movable Dams and Lock at the Power Plant on the Chicago Drainage Canal (Eng. News, Nov. 12, 1908).—Illustrated.

Cast-Iron Sluice Gates for the Fens Gate Chamber, Charles River Basin, Boston, Mass. (By W. H. Sears. Eng. News, Feb. 25, 1909).—Illustrated.

Investigations of the Saturization of Earth Dams (Eng. Rec., Aug. 21, 1909).—Results of experiments by Desmond Fitzgerald: 1. Clay banks are more completely saturated than well-drained banks. 2. Clay banks are more slowly saturated and part more slowly with their water of saturation. 3. In high banks it is unsafe to have nothing but clayey material, a down-stream section of well-drained material being essential.

Partial Failure Through Undermining of the Zuni Dam, New Mexico (Eng. News, Dec. 2, 1909).—Combined hydraulic earth fill, 60,120 cu. yds., up-stream section; and rock fill, 40,160 cu. yds., down-stream section. Up-stream: slope 3 : 1, rock rip-rap 18 ins. deep on gravel 12 ins. deep. Down-stream: slope 1¼ : 1. Illustrated. The spillway, south abutment and extreme south end of dam were undermined by the passage of water underneath a cap of lava rock which flanked the dam and extended beneath the spillway.

The Eastwood Multiple-Arch Dam (Eng. Rec., Jan. 15, 1910).—Abstract of article by John S. Eastwood, in the "Journal of Electricity, Power and Gas," Oct. 30, 1909. The Hume-Bennett dam consists of 12 arches, each 50-ft. span, resting on 13 buttresses, the end walls of the last buttress at each end being extended into the opposite bank as a core wall, as they are above the normal water line and have no water load. The elevation of the water line is 5,300, that of the crest of the middle six arches is 5,303, and the remainder of the crest to the ends is 5,304 ft. above mean tide level. This allows a 330-ft. crest for any freshet that may occur when the spillway flashboards were accidently left in their openings. The entire structure

rests on sound bedrock. Mr. Eastwood found that to give the required stability with the greatest economy it was desirable to build the top 16 ft. of the dam with vertical arches, and all arches up to 20 ft. high at the spring line were built vertical. All arches higher than 20 ft. are carried vertical to within 16 ft. of the top at the crown line, and then slope to the foundations at an angle of 32 deg. The arch ring thickness is increased as required for the water pressure. All of the vertical part of the arch wall is 18 in. thick; the wall increases in thickness from this point at the rate of 1 ft. to each 24 ft. vertical, or a little more than required for water load. The buttresses are all 2 ft. thick at the top and project 8 ft. from the inside spring line to the down-stream end, all corners being clipped. The batter of the down-stream end is 5 in. to 1 ft., and of the sides 1 in 24 to the base on each side. Each buttress is finished on its down-stream end with wing buttresses or counterforts. The 12 spillway openings are located in the three middle arches of the dam, the openings being 5 x 8 ft. each, to be closed to any desired height by means of flashboards. The structure was reinforced throughout when necessary by means of railroad iron scrap and old logging cable. . . . The buttress forms consist of 2 x 4 in. studding set about 20 in. apart on centers and spliced where not long enough to reach the top; a framework was first built and lined with 12-in. lumber of 10 and 12-in. widths, lightly nailed to the inside of the studs, the studs being braced to the trestle. The shapes of the counterfort forms were such that they braced themselves when once boarded up. The arch forms were built up from the bottom, using 2 x 4-in. studding and ½ x 6 in. stuff nailed on double. . . . Crushed granite was used for the coarse aggregate, the crusher-run being mixed with sand from an adjacent pit; the mix was approx. 1 : 2 : 4. The forms were not removed for at least a week after the concrete was laid, and the walls were kept wet by a night watchman and a day crew. A union of the different day's work was made by scarifying the surface of the old work and washing off with a hose; dry cement was then sprinkled over this surface and concreting begun, a few batches of concrete with excess of mortar being laid in contact, the work being carried up as nearly level as possible. The junctions of the walls were made on the center of buttresses, the reinforcement being left protruding to tie them together. . . . The water face and the parts of the down-stream face were plastered, the water face with two coats of 1 to 1½ cement plaster and selected sand and a wash of neat cement on the bases of the middle arches. A base seal of mortar was placed along the line of contact with the rock. . . . The structure contains 2,207 cu. yds. of concrete and was built in 114 days. . . . All parts of dam are in compression, max. stress 187.5 lbs. per sq. in. safety factor 16) being at bases of arch rings. Max. stress in shear, 50 lbs. per sq. in. Rates of overturning, 1 : 3.6. . . . Cost of dam, including plastering, about $21 per cu. yd. or $46,000 for the structure; cement costing a little over $5.00 per bbl., delivered.

Arched Masonry Dam at Las Vegas, N. M. (By C. W. Sherman. Eng. News, Oct. 27, 1910).—Description and illustrations. Also contains a table of 23 curved masonry dams, giving max. height, base thickness, top thickness, max. stress in arch, radius of up-stream face, top length, character of rock, date of building.

Movable Dams on the New York State Barge Canal (Eng. News, Dec. 8, 1910).—Description, with 14 illustrations, of the type of dam known as the large dam with the Boulé gates.

Illustrations of Various Types of Dams.

Description.	Eng. News.
Rubble-faced dam 47′ high	June 13, 1901.
Rock-fill dam 100′ high with steel core	Jan. 2, '02.
Aswan dam (66′ high) and reservoir on the River Nile	Aug. 14, '02.
Swam masonry dam 70′ high, Waterbury, Conn.	May 7, '03.
Water Falls masonry dams 50′ and 152′ high	June 18, '03.
Low rein.-conc. dam 11′ high (Ambursen type)	Nov. 5, '03.
Curved concrete dam 110′ high, Barossa, So. Australia	April 7, '04.
Curved Masonry dam 230′ high, Lake Cheesman, Colo.	May 12, '04.
Power dam 30′ high on the Penobscot River	Sept. 1, '04.
Develt masonry dam 260′ high, Arizona	Jan. 12, '05.
Long steel dam at Sweinfurt, Bavaria	Jan. 19, '05.
Small concrete dams—one on pile foundations	Feb. 9, '05.

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Description.	Eng. News.
Hollow rein.-conc. dam 25′ high, at Schuylerville	April 27, '05.
Debris Barrier No. 1, Yuba River, Cal.	June 15, '05.
Arched masonry dam 80′ high, Cheyenne, Wyo.	June 29, '05.
Gatun dam 270′ to bed rock (Panama Canal)	July 27, '05.
Pile foundation for movable dam	July 27, '05.
Movable dam and lock of the Rice I. & I. Assn, La.	Sept. 28, '05.
Structural steel dams (F. H. Bainbridge)	Sept. 28, '05.
Crib dam 20′ high with sheet steel piling	Nov. 3, '05.
New Croton dam, with balanced gate valve (Wegmann)	Oct. 4, '06.
The Mercedes curved masonry dam 130′ high	Nov. 1, '06.
Collapsible steel dam crest, Bear River, Utah	Oct. 3, '07.
*Hauser Lake steel dam, Missouri R., Mont.	Nov. 14, '07.
Lock Gates of the Charles River dam	July 9, '08.
Butterfly dam on Chicago Drainage Canal	July 22, '09.
Revolving segmental sluice-gates for Sterling dam	Aug. 5, '09.
Plan and cross-section of Shoshone dam	Dec. 9, '09.
Failure of concrete dam, 10 ft. high, at Danville, N. Y.	Jan. 13, '10.
Designs for rebuilding the Austin dam, Texas	Apr. 14, '10.
Curved masonry dams in New South Wales	May 19, '10.
Construction of Cataract Dam, Sidney, N. S. W.	June 23, '10.
Reinforced buttressed dam, Ottawa, Can.	June 30, '10.
Dike, mosquito extermination work, Welfleet, Mass.	Aug. 11, '10.
Design and constr. of movable dam and lock, Lockport	Oct. 6, '10.
The La Prele, hollow reinforced-conc. dam, 130 ft. high	Nov. 10, '10.
Buttressed masonry dam reinforced with steel I-beams	Nov. 24, '10.
Diamond drill borings for a dam, Clackamas R., Ore.	Dec. 22, '10.
	Eng. Rec.
Experiments with rubber models of dams, to study stresses	Mar. 6, '09.
Rein.-conc. dam, buttressed, 16′ 6″ high, 11′ 6″ base	Mar. 27, '09.
Section of the Arrowhead hydraulic fill dam	Apr. 3, '09.
Olive Bridge dam (cyclopean masonry); earth dike	Sept. 4, '09.
Diagram, principal stresses and planes in the masonry dam	Oct. 2, '09.
Cross-section of hydraulic fill dam, South Carolina	Oct. 2, '09.
Cross-section Kensico dam, Catskill water supply	Dec. 25, '09.
Cross-section rock-fill crib dam, power develop., Mont.	Mar. 12, '10.
Construction plant for the Holter dam, Montana	Oct. 29, '10.

*See Eng. News of April 30, 1908, for description of failure.

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50.—FOUNDATIONS.

ral Discussion.—Equally as important as the superstructure, is ructure or foundation. This may be defined as that part of the designed to carry and distribute safely the forces from the super- to the *foundation bed*, and *vice versa*, with resulting stability and njury or undue stress in any part. Good foundation work in both d execution is less dependent on theory than structural work in It has to draw mainly on practical experience in the particular vork at hand, being largely tempered of course with good judg- certain well-known, established principles.

e design of any structure we must know, and keep clearly in mind, ng forces," hence it is necessary in designing foundations to know: ature of the superimposed loads, (2) the nature of the foundation d (3) the nature of the reactions, at the foundation bed. The first ay be calculated with a reasonable degree of accuracy, assuming lation and foundation bed to be *relatively rigid;* the second is le of equally accurate determination; while the deduced stresses undations, and the resultant reactions at the foundation bed, are roportioned to the loads above. If now for any reason *unequal* t takes place, new stresses will be introduced which may be con- at variance with the calculated stresses in many of the parts, n may work serious injury to the structure as a whole. It there- lves upon the engineer to so proportion the foundation and founda- that not only will each part be sufficiently strong to support the *also that they will be relatively strong and rigid*, or at least sufficiently vent unequal settlement.

lation Bed.—In general, the character of material desirable for n bed will take rank in about the following order:

Solid rock (granite, limestone, sandstone);
Hardpan or cemented gravel (composed of gravel, sand, and lime or clay binder);
Gravel and sand (compact and dry, as in gravel pit);
Indurated clay (hard, dry and well drained);
Sand (dry);
Gravel;
Boulders and gravel;
Clay and sand;
Common clay;
Sand and loam;
Loam.

This classification, or any other for that matter, may be considered to be more or less elastic, depending (1) upon the amount of moisture present; (2) upon the lateral confinement of the material; (3) upon the underlying strata; (4) upon the nature of the superimposed loads.

arious classes of material for foundation bed will be discussed in ollows:

olid Rock.—Granite, when tested in small cubes exhibits a safe- so-called crushing strength of, say, 12000 lbs. per square inch; salt and trap, not less than that amount; limestone (and marble), sandstone, 5000 lbs. Some of the best specimens, however, which tested give, respectively, twice the above value for granite, or ; three times the above value for limestone, or 24000 lbs.; and s the above value for sandstone, or 15000 lbs. With these large small specimens, where in reality the blocks are ruptured by stresses," it would be problematical to assign even approximate the crushing strength of well-constructed masonry foundations n which the "crumbling resistance" would increase faster than the resisting surface. And, furthermore, it would be mere conjecture n upper limit to the resisting power of rock *in situ* and forming ation bed. A good, solid rock bed is the best that can be had, afely support any structure which man is liable to build upon it. paring the foundation bed, it should be leveled or "stepped" so ane or planes shall be normal to the acting forces. All perishable red material should be removed and the exposed surface of the

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rock should not be so smooth as to unsafely decrease resistance to sliding of the foundation above. Especially should this be looked after in the case of a submerged pier or of a dam, where the lateral pressure, due to the current of the stream, or to ice, logs, or hydrostatic pressure, would be considerable.

A bed-rock foundation is not always easily obtainable on account of its depth below the ground surface or, in marine work, below the water surface. It is therefore often more economical to choose for a foundation bed a poorer character of material not so deep, thereby saving considerable in excavation but requiring, generally, a more expensive foundation "footing" (see Figs. 1, 2 and 3). Hence it is that often after the excavation for the foundation is started the plans are changed when it has become evident that a stratum has been reached that is "good enough" for all practical purposes; or, on the other hand, when the expected stratum as revealed by the borings does not meet the requirements, and another stratum at greater depth is decided upon. (Excavation processes are described farther on.)

1.—Actual Bearing Pressures on Bed Rock.

Foundations of New Croton Dam.—Calculated pressures limited to 15 tons per sq. ft. at base of dam on rock surface, the resultant pressure being kept within the middle third of section.

Manhattan Life Building, New York City.—Pressure at base of caissons on bed rock, 10.8 tons per sq. ft.

American Surety Building.—Pressure at base of caissons, 7½ tons per sq. ft.

Gillender Building.—Pressure at base of caissons on bed rock, 12 tons per sq. ft.

(*b.*) *Hard-Pan.*—Next to solid rock there is no better material for foundation bed than cemented gravel or hard-pan. When well cemented, and in thick and extensive beds, it is capable of sustaining safely without injury and with comparatively slight settlement, a quiescent loading of 10 tons per square foot (= 138.8×8 lbs. per square inch).

(*c.*) *Gravel and Sand.*—Large, thick beds of well-compacted gravel and sand, free from wash by the action of water, can generally be counted on to sustain about 8 tons per square foot of quiet loading, or equal to 111.1×1 lbs. per square inch. For such a loading, however, the foundation bed should be at least 10 or 12 feet below the surface of the ground, and the under strata must be firm.

(*d.*) *Indurated Clay.*—By the term "indurated" we mean the hard variety, usually containing in sufficient quantities such binding materials as carbonate of lime, silicates of aluminum and magnesium, iron oxides, etc. When in the natural bed, at considerable depth, and free from the softening action of water, this material may be loaded safely to 6 tons per square foot (equivalent to 83.3×3 lbs. per square inch). If it contains a proper intermixture of gravel and sand it approaches hard-pan, and the bearing power is increased.

(*e.*) *Dry Sand.*—Under the most favorable conditions, that is, where the material is confined as in a deep trench and not allowed to spread out from the pressure above, and also where it is reasonably dry and free from wash, sand will easily support a load of 4 tons per square foot (= 55.5×5 lbs. per square inch). For large buildings, however, where unequal settlement to any considerable extent would be liable to produce cracks in the masonry walls, the maximum limit of allowable pressure should be fixed at 3 tons per square foot. 2½ tons or even 2 tons is quite common practice under ordinary conditions of wetness, but where there is no wash from running water. For the above conditions the underlying strata must of course be firm. Where the amount of settlement is of minor importance there is really no practical limit to the bearing power of sand, hence the wide range of values assumed. In some instances 10 tons per square foot has been exceeded.

2.—Actual Bearing Pressures on Sand.

Washington Monument, Washington, D. C.—Pressure at base resting on sand bed 2 ft. thick, about 11 tons per sq. ft.; and with wind pressure added, about 14 tons.

Piers of Brooklyn Suspension Bridge.—At base of piers, 44 ft. below bed of river, pressure on layer of sand 2 ft. thick resting on bed rock, 5½ tons per sq ft.

St. Paul Building, New York City.—Continuous grillage over entire area. Pressure on compact sand, 3.2 tons per sq. ft.

World Building, New York City.—Inverted arches over continuous concrete footings. Pressure on dense, fine sand, 4.7 tons per sq. ft.

Spreckles Building, San Francisco.—Continuous grillage. Pressure on dense, wet sand, 2½ tons per sq. ft.

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(*f.*) *Gravel*; (*g.*) *Boulders and Gravel.*—Some very important structures are found on these classes of material, both above and under water. If properly prepared and protected from lateral bulging and wash such a foundation may be loaded with 4 tons per square foot (=55.5×5 lbs. per square inch). If, further, it is compacted with dry or moist sand not subject to wash, the bearing power will be increased, say to 6 or 8 tons, depending upon the binding quality of the material and on local conditions; if a moderate quantity of clay is intermixed with the sand and the resultant matrix is sufficient to fill the voids of the gravel the higher value (8 tons) may be used with safety. Such a material if thoroughly hardened would form hard-pan.

3.—Actual Bearing Pressures on Gravel.

Piers of Cincinnati Suspension Bridge.—At base of piers, 12 ft. below low water, pressure on gravel bed (neglecting skin friction of piers), 4 tons per sq. ft.

(*h.*) *Clay and Sand*; (*i.*) *Common Clay.*—Ordinary soft clay or clay and sand, when wet, is subject to considerable displacement if heavily loaded; and under ordinary conditions it is best not to allow more than 1 ton per square foot (=13.8×8 lbs. per square inch) for important buildings where much settlement would be harmful. But this bearing power may be greatly increased by proper drainage or by a close (lateral) confinement of the material, so that well-drained clay or such clay containing some sand may be loaded with 2 tons per square foot. A naturally dry common clay bed of considerable thickness and extent will stand three tons with allowable settlement; and this may be increased to 4 tons for a mixture of sand and clay where the former predominates and the latter is sufficient for a binder. Hard, firm clay unmixed with sand will also stand 4 tons and if mixed as a binder with good, coarse sand the bearing power will be greatly augmented. Fine clay and sand when thoroughly saturated with water forms quicksand.

4.—Actual Pressures on Clay and Sand.

Capitol Building, Albany, N. Y.—Clay with some sand. Pressure allowed, 2 tons per sq. ft.

Congressional Library, Washington, D. C.—Yellow clay mixed with sand. Pressure allowed, 2½ tons per sq. ft.

(*j*) *Sand and Loam*; (*k.*) *Loam.*—Loam is too spongy and compressible to be relied upon, from an engineering standpoint, to support any structure, and hence the excavation is always carried through it to some firmer foundation bed beneath. It is a mixture of clay, sand, vegetable mould, and decayed animal matter. But when mixed with a large proportion of sand it is less objectionable especially when well tamped. Railway trestle posts which rest on mud sills supported by this material, are frequently subject to considerable settlement.

Practical Tests of Soils.—Where the bearing power of any particular soil is in question it may be tested by loading a vertical timber of given cross-section (the larger the better) and resting on the soil at the bottom of a pit, with a weight equal to or greater than the proposed intensity of loading. Allowance will be made, however, for the results obtained as the settlement due to loading such a limited area would be excessive.

Selection of Site for Building.—Generally, the site for the erection of a structure is determined from necessity rather than from choice. That is to say, conditions other than those of local character govern and fix the location. But there are many instances where other sites than those selected would have been chosen from pure economy of cost of foundation had direction been given to its consideration.

Examination of Soil.—The simplest and most satisfactory method of determining the character of the soil is by digging pits. This should be done generally before making estimates and letting contracts, on all important work requiring excavation in general, and including foundation work at no great depth. For the former we may cite railroad cuts, irrigation canals, and waterways in general; while for the latter, may be mentioned dam sites, and sites for bridge piers, buildings, etc. The pits need only large enough for a man to find room to excavate. Where great depth is required borings must be made.

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Borings in Soil.—For soft earth and clay, up to about 100 feet in depth, a common wood auger with levers, turned by hand, may be used. It may have a diameter of 1½ to 2½ inches and be worked on the end of a sectional iron rod easily made by any blacksmith. It is usually started by one-man power, but after being sunk a short distance two or more men may be required. During the process of boring, samples are brought up and recorded together with their distances below the surface.

For the harder strata and at greater depth, artesian well boring tools are employed. (See *Eng. News*, Vol. XXI, page 324, for illustrations.)

Estimating Loads on Foundations.—These loads include:

(1.) Forces due to the weight of the structure itself, so distributed as to *conform with the reactions*, whether vertical or inclined. This is not always an easy matter, but the principle should be kept clearly in mind.

(2.) The live loads *or that percentage of them liable to act in unison* to produce maximum stresses in any part of the foundation. The term "live loads" will here be considered to embrace those loads for which the structure was *principally* designed, as for instance people, furniture and merchandise for buildings, water pressure for dams, etc.

(3.) The wind loads (also other natural forces of an accessory character such as snow, pressure of ice, etc.), considering possible tension in the foundations as well as compression; also shearing.

(4.) Temperature stresses, due to changes in temperature affecting the length of certain principal members confined between parts of the foundation, as for instance in the two-hinged steel arch.

(5.) Impact loads due to moving machinery, as steam engines, steam hammers, dynamos, etc.

For Buildings.—There are two main classes of foundations in use for buildings, namely, continuous foundations, and independent piers. The latter are generally employed where a suitable, natural foundation bed exists only at great depth, in which case the continuous foundation would be more expensive. As more or less settlement always occurs after a building is erected, *due almost wholly to the constant pressure of the dead load*, it is best to proportion the *area* of all foundation footings to the dead load only, using a "reduced" and *uniform* working pressure for same, so that, selecting that pier which we will call the "Index" pier or part of foundation which sustains the least ratio of dead load to total load, we have,

$$\frac{\text{Reduced working pressure}}{\text{Allowable maximum pressure}} = \frac{\text{Dead load stress at footing of column}}{\text{Maximum stress at footing of column}} \quad (1)$$

Under the subject of Buildings, page 822, it will be noted that for high buildings the maximum stress at footing of column is reduced by diminishing the live load for each floor below the top one a certain percentage, as it is not likely that all floors "in column" will be loaded fully at the same time.

It may be stated here that vault shafts running up through buildings should be carried on a separate foundation from that of the main building. Engine and boiler foundations should also be independent.

Care should be taken that the line of resultant pressure at the foundation footing should come well within the middle third if the foundation bed is solid rock, and it should be practically central for any bed which is soft or springy. Eccentricity of loading may unduly increase the intensity of pressure, and also cause tipping, cracking, and perhaps rupture, of the masonry wall.

For City Building Codes, see Table 5, following page.

For Dams.—To save the reader the time and trouble of looking into the matter it will be stated that for a gravity dam with reservoir empty, it will not be necessary to consider any additional stresses in the masonry or on the foundation bed, due to wind pressure against the down-stream face. Assuming a dam 200 ft. high with a base of 69 ft., and considering a wind pressure of 50 lbs. per sq. ft. acting horizontally (with no vertical component), then under the most unfavorable conditions the intensity of stress at either toe would not exceed 9 lbs. per square inch. In calculating the resultant stress due to any pressure at the top of the dam, as for instance logs or ice, it is necessary only to consider the dam as a beam fixed at one end with length equal to the height of the dam and with depth equal to its base, and assuming the neutral axis of the section to bisect the base.

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5.—Bearing Power of Soils for Buildings.

(Extracts from various City Codes.)

[Loads are in tons of 2000 lbs. per sq. ft.]

New York (1906).—Where no test is made, different soils, excluding mud, at bottom of footings, shall be deemed safe to sustain the following loads per sq. ft.: Soft clay, 1 ton; ordinary clay and sand together, in layers, wet and springy, 2 tons; loam, clay or fine sand, firm and dry, 3 tons; very firm, coarse sand, stiff gravel or hard clay, 4 tons, or as otherwise determined by the Commissioner of Buildings.

Chicago (1907).—If the soil is a layer of pure clay at least 15 ft. thick, without admixture of any foreign substance excepting gravel, the load shall not exceed 1½ tons per sq. ft.; pure clay in layers at least 15 ft. thick, dry and thoroughly compressed, 2½ tons; dry sand, at least 15 ft. thick, and without admixture of clay, loam or other foreign substance, 2 tons; clay and sand mixed, 1½ tons.

Philadelphia (1907).—Foundations of other materials than piles shall be so proportioned that the loads upon the soil shall not exceed the limits for the different kinds of soil than herein given, to wit: Sand and loose gravel, 3½ tons per sq. ft.; dry, hard clay, 3½ tons; cemented gravel, 6 tons.

Cleveland (1907).—Good, sound, natural earth shall not be loaded to more than the following: Gravel and coarse sand well cemented, or rock or hard shale unexposed to the action of air, frost or water, 8 tons per sq. ft; dry, hard clay or fine sand, compact and well cemented, 4 tons; moderately dry clay or clean, dry sand, 2 tons; soft, wet sand, 1 ton; quicksand or alluvial soils, ½ ton; the sand underlying the City of Cleveland above the lake level, commonly called "quicksand," when drained of its ground water without puddling or disturbing the foundation may be loaded, 3 tons.

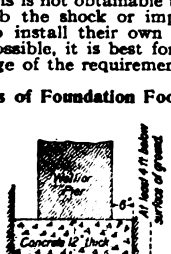
San Francisco (1910).—Soft clay, 1 ton per sq. ft.; sand and clay mixed, 2 tons; firm, dry clay, 3 tons; hard clay, 4 tons; loam or fine, dry sand, 3 tons; compact sand, 4 tons; coarse gravel, 6 tons; shale rock, 10 tons; hard rock, 20 tons.

Buffalo (1909).—In no case shall the soil under any building be loaded with a weight greater than 3½ tons per sq. ft. If the soil is composed of other materials than hard clay or gravel then the area of the foundation shall be extended as directed until the pressure is reduced to a safe limit.

District of Columbia (1906).—(Practically the same as N. Y. City Code.) Soft clay, 1 ton per sq. ft.; ordinary clay and sand together, in layers, wet, 2 tons; loam, clay, or fine sand, firm and dry, 3 tons; very firm, coarse sand, stiff gravel, or hard clay, 4 tons, or as otherwise determined by the Inspector of Buildings.

For Machines, Dynamos, etc.—The weight of the machine as a whole is not alone to be considered. A more massive foundation may be required for a light machine than for a heavy machine with the same weight of moving parts. The foundation should be on a natural, hard bed, but where this is not obtainable the foundation itself should be massive enough to absorb the shock or impact of the machine. Manufacturers usually prefer to install their own machines and prepare the foundations, and, where possible, it is best for them to do so on account of their practical knowledge of the requirements of the case.

Types of Foundation Footings:—



g. 1.—*Least dimensions* for ordinary concrete footing under brick or rubble wall or pier. Where more bearing area is required on foundation bed the offset and thickness are increased or the sides of the footing sloped, as in Fig. 2.

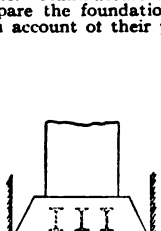


Fig. 2.—Concrete footing with or without I-beams, for heavily loaded walls, yielding foundations, or both. One or more of the beams is often imbedded in the concrete to give stiffness and absorb shock due to heavy machinery, as in power houses, etc.

Problem.—Let it be required to design a footing of I-beam construction under a steel column, carrying at bottom of footing a maximum load of 748,000 lbs., and on a soil capable of sustaining 3 tons (6000 lbs.) per square foot.

Solution.—By using formula (1), page 866, we find that the *reduced working pressure* on foundation is, say, 5200 lbs. per square foot, hence the total area of the footing should be $748{,}000 \div 5200 = 143.8$ square feet, or say a square base of length $B = 12$ ft. Now, tentatively, we may assume that $A = \frac{B}{4} = 3$ ft.; $a = \frac{B}{4} = 3$ ft.; $b = \frac{3B}{8} = 4\frac{1}{2}$ ft.; $c = \frac{B}{8} = 1\frac{1}{2}$ ft. Then for the bottom tier of beams each cantilever arm a will sustain a total upward pressure of $5200 \times a \times B = 5200 \times 3 \times 12 = 187200$ lbs., producing a bending moment of $187200 \times \frac{a}{2} = 280{,}800$ ft.-lbs. Using a fiber stress of 16000 lbs. per square inch for steel I-beams the above moment calls for either 12 9″ 21 lb. beams, 9 10″ 25 lb. beams, or 6 12″ 31 lb. beams*. The 12″ beams are the more economical, while the 10″ beams give closer and better spacing and will be adopted. Similarly, for the middle tier of beams, each cantilever arm b will sustain a total upward pressure (neglecting weight of lower tier of beams) of $5200 \times b \times B = 280800$ lbs., producing a bending moment of 631800 ft. lbs., and calling for 5 20″ 65 lb. beams. Lastly, for the upper tier, each cantilever arm c will (be assumed to) support one-quarter the total load or 187,200 lbs., producing a bending moment of 140,400 ft.-lbs., and calling for 5 10″ 25 lb. beams.

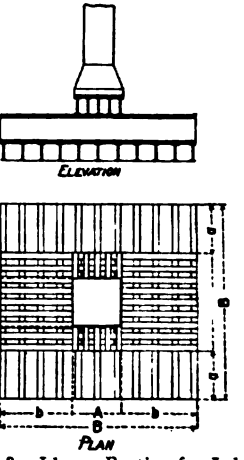


Fig. 3.—I-beam Footing for Independent (isolated) Piers. Beams to be imbedded in concrete.

Other proportions may be assumed for the areas of the tiers of beams if it is thought that economy may result.

Coffer-Dams.—A coffer-dam is a fixed enclosure built *in situ* around a proposed foundation for the purpose of shutting out the water during construction of the latter. Such a dam may be formed by building an earth embankment around the site; by constructing a water-tight casing of any material, as iron or wood; by sinking cribs; by driving sheet piling; or by a combination of these methods. In any event there will be more or less leakage, and either hand or machine pumps will have to be installed to keep the site dry.

By Earth Embankment.—A tight dam may be made with gravel and clay, but other soils may be used if there is a matrix sufficient to fill the voids. This construction is used where the water is shallow and quiet or where there is but little current. The width of dam at top may vary from 3 ft. upward, with side slopes of 2 or $2\frac{1}{2}$ to 1. For a depth greater than 5 ft. other methods are generally used. Sometimes sand in bags is employed to good advantage, especially where there is some current.

By Water-Tight Casing.—The casing may be in the form of a large wooden crib with single or double shell. If of single shell, the seams are calked tight, while if the shell is double the intervening space or chamber is packed with clay puddle, to prevent leakage. The size of the shell timbers depends on the depth of water and also on the interior bracing. For coffer-dams in ordinary bridge foundation work, 12″x12″ timbers are frequently used, dove-tailed or rather halved at the joined ends or corners. The bracing may extend entirely across the crib from shell to shell, and be replaced by shorter struts abutting against the masonry as the latter inter-

* See Tables of Properties of I-beams, pages 554 and 555.

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cepts it in being projected upward. In designing the shell, remember that the hydrostatic pressure in lbs. per sq. ft. $= 62.5\,H$, in which H is the depth in ft. below the water surface. (See Dams, page 846.) Thus for a horizontal shell timber of length, say, 6 ft. between interior bracing, and whose center is 30 ft. below the surface, we have, if d equals thickness of shell,

$$\text{Bending moment} = \frac{wl^2}{8} = \frac{62.5 \times 30 \times 6 \times 6 \times 12}{8}$$

$$\text{Resisting moment} = \tfrac{1}{6}\, f\, b\, d^2; \text{ in which } f = 1000, \text{ and } b = 12.$$

$$\text{Equating, } d^2 = \frac{62.5 \times 30 \times 6 \times 6 \times 12 \times 6}{8 \times 1000 \times 12} = 50.63; \text{ or, say } d = 8 \text{ ins.}$$

The design of such a crib is quite simple under favorable circumstances, but in a swift stream with an uneven bed, part of which may be rock, it offers many difficulties. The bottom of the crib in this case should conform closely with the rock bottom. For a silt bed the bottom of crib should be sharpened to penetrate the silt. Piles may be driven to hold the crib in place. This leads to another construction, namely, that of driving one or more rows of piles and sheathing them, thus obtaining the same result in another way. The piles should be well diagonal-braced where necessary to resist bending, remembering that where not so braced the resisting moment of the section in inch-lbs. is $= 0.0982$ diameter$^3 \times$ allowable outer fiber stress f of pile per square inch—say $f = 1000$.

By Sinking Cribs.—A large crib coffer-dam is sometimes formed by sinking several small cribs in line around the site, and planking and calking them on the outside. The cribs are made rectangular, of squared timbers framed in the ordinary manner, and containing cells or chambers, somewhat above the level of the bottom, to be weighted with stone so as to sink into the river bed. Guide piles are often driven for this work. In sinking cribs where there is current two or more clusters of piles may be driven up stream, or auxiliary cribs may be sunk to which are attached adjustable steel cables for "dropping" the cribs into position.

By Driving Sheet Piling.—This is one of the most common methods in use, both in wet ground and in shallow water; and where it can be employed effectively, is simple and cheap. Some of the principal forms of wooden sheet piling are the following:

Fig. 4.—Large Square Piles.

Fig. 5.—Plain Single Sheeting.

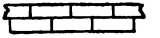

Fig. 6.—Double Sheeting.

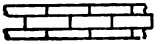

Fig. 7.—Triple Sheeting.

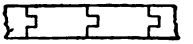

Fig. 8.—Matched Piles.

Fig. 9.—V-Matched Piles.

Fig. 10.—Tongue-and-Groove Piles.

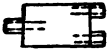

Fig. 11.—Built Matched Pile.

Fig. 12.—Built V-Matched Pile.

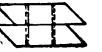

Fig. 13.—Built W-Matched Pile.

Fig. 14.

Fig. 15.

Fig. 16.

Figs. 14, 15, 16.—Details of Wakefield Sheet Piling.

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Other forms are in use but they are no improvement over the above. The lower ends of the piles are usually cut on the slant so that in driving they will crowd against those in place and close the joints (Fig. 17). They are also sharpened so as to "broom" if there is solid rock bottom. A good method to secure tight joints is to drive from both ends of the line toward the center and then drive home a good tight-fitting "key" pile. Sheet piles are usually driven between guides or horizontal waling pieces which may be supported by a row of ordinary round piles, or sawed piles, previously driven. After the sheeting is driven it is keyed tightly or wedged in the wales.

Fig. 17.

STEEL SHEET PILING comes in many forms, some of which are composed of standard rolled sections riveted together, while others are special rolled sections not requiring any riveting.

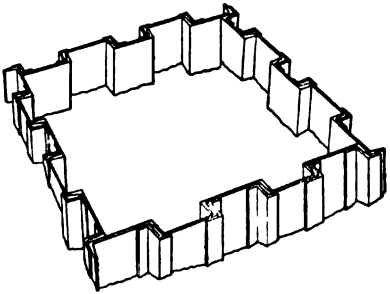
Fig. 18.

Fig. 18 illustrates an interlocking channel-bar piling caisson driven in place and ready for excavating. The piling is that of the Friestedt* type. This piling in place will weigh from 23 to 67.5 lbs. per sq. ft. Piling which will weigh when interlocked about 41 lbs. per sq. ft. will have a moment of inertia of 69.21 and a radius of gyration of 1.6.

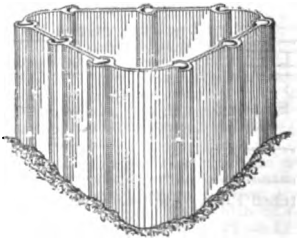
Fig. 19.

Fig. 19 illustrated the plain rolled section as manufactured by the United States Steel Piling Co. of Chicago. The actual cost of this piling depends somewhat upon the specifications. If ordered cut to lengths it can be furnished on cars at approximately $42.00 per net ton. Where corner pieces are required there is a net extra of approximately $10.00 per net ton for the corners only, and there is also a net extra of $2.00 per ton for punching each piece with pulling holes where purchasers prefer shop punching, prior to shipment. The above prices are for August, 1906.

* Manufactured by the Friestedt Interlocking Channel Bar Co., of Chicago.

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in coffer-dam construction may often be stopped by the puddle, with perhaps an intermixture of straw and gravel if the e foundation bed. A more expensive but effective method, and be applied to deep rock fissures, is by grouting. Sometimes arpaulin coated with asphalt and weighted to the bottom will fact the use of canvas can generally be recommended in various eap and convenient. The joints of the upper-work should be the seams may be asphalted or greased if desired.
e coffer-dam is constructed the water is pumped out and the excavated to a firm foundation bed.

ndations.—In soft, marshy ground it is often best to use a pile and for permanent construction the piles should be cut off at eet below the level of so-called continual saturation, or low Many of our tall office buildings, up to 12 stories or more in on piles, as do also many of our large masonry bridge piers. pile" unless specially named otherwise will be assumed to be of

ng Power of Piles.—Some engineers have claimed that piles riven with the large end downward; and this would be logical that the direct bearing of the lower end were *all* the supporting he pile received.* But it must be remembered that with the butt and the pile tapering downward it receives considerable vertical long the *sides* in addition to the largely increased "friction." to its being the most practical way there is another point in ving the small end downward, namely, that generally the soil compactness with the depth below the surface and hence tends ter lateral stiffness at the lower end where it is required. This rtant where a pile is driven to a hard stratum, through com- oft materials, thereby acting as a column subject to bending. a column may be considered as "fixed" at the lower end.† g piling so supported and driven through loose, soft material hould not exceed ½ to ⅓ the allowable load on a short column sectional area. In any case the loading should be limited by formula‡ which takes into consideration the resistance at the y (see Table 6, following page):

$$\ddagger P = \frac{2\,w\,h}{s+1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

here P = safe|| load in tons, on pile;
w = weight of hammer, in tons;
h = height of free fall of hammer, in feet;
s = penetration of pile at last blow, in inches.

practice to drive a pile ordinarily until the penetration at the ess than ½ inch, using of course a moderately heavy hammer; be taken that the pile itself is not being injured by "brooming," bottom, top, or any other part. Many piles are seriously verdriving. An experienced man can tell, generally, when a "driven" by the action of the pile and hammer, and the

rs.¶—The main essentials of a pile driver are (1) a hammer a derrick with vertical (or inclined) leads with guides for the ork in, and (3) power to operate the hammer, so that in its strike the head of the pile and ram the latter into the ground. onsider such accessory devices as the explosion of gunpowder n the head of the pile as the hammer descends, because the practical utility of such methods have not been demonstrated.

ertain conditions, such as to prevent subsequent uplifting, iven butt end downward, as when the sub-stratum is quick-sand. it is supported more or less throughout its length by the nce of the material through which it is driven unless it be etically, a pile may assume all the conditions of strength a short strut to that of a long column with round ends, as en through silt to a solid rock bed.

on's formula for drop-hammer. For steam-hammer, $P = \frac{2\,w\,h}{s+0.1}$.

safety of 6.

iption of Pile-pulling machines, see *Eng. News*, April 16, 1903.

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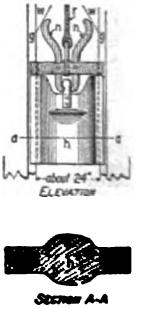
6.—Safe Bearing Power of Piles, in Tons of 2000 Lbs.
Driven by Drop Hammer.
[By Wellington's Formula (1) preceding page: $P = 2\,wh \div (s+1)$.]

Penetration at Last Blow. Inches.	Weight of Hammer in Tons, for Drop of 20 Feet.*								
	½ ton (1000 lbs.)	¾ ton (1500 lbs.)	1 ton (2000 lbs.)	1¼ tons (2500 lbs.)	1½ tons (3000 lbs.)	1¾ tons (3500 lbs.)	2 tons (4000 lbs.)	2¼ tons (4500 lbs.)	2½ tons (5000 lbs.)
⅛	17.78	26.67	35.56	44.44	53.33	62.62	71.11	80.00	88.89
¼	16.00	24.00	32.00	40.00	48.00	56.00	64.00	72.00	80.00
⅜	14.55	21.82	29.09	36.36	43.64	50.91	58.18	65.45	72.73
½	13.33	20.00	26.67	33.33	40.00	46.67	53.33	60.00	66.67
⅝	12.31	18.46	24.62	30.77	36.92	43.08	49.23	55.38	61.54
¾	11.43	17.14	22.86	28.57	34.29	40.00	45.71	51.43	57.14
⅞	10.67	16.00	21.33	26.67	32.00	37.33	42.67	48.00	53.33
1	10.00	15.00	20.00	25.00	30.00	35.00	40.00	45.00	50.00
1¼	8.89	13.33	17.78	22.22	26.67	31.11	35.56	40.00	44.44
1½	8.00	12.00	16.00	20.00	24.00	28.00	32.00	36.00	40.00
1¾	7.27	10.91	14.55	18.18	21.82	25.45	29.09	32.73	36.36
2	6.67	10.00	13.33	16.67	20.00	23.33	26.67	30.00	33.33
2¼	6.15	9.23	12.31	15.38	18.46	21.54	24.62	27.69	30.77
2½	5.71	8.57	11.43	14.29	17.14	21.00	22.86	25.71	28.57
2¾	5.33	8.00	10.67	13.33	16.00	18.67	21.33	24.00	26.67
3	5.00	7.50	10.00	12.50	15.00	17.50	20.00	22.50	25.00
3½	4.44	6.67	8.89	11.11	13.33	15.56	17.78	20.00	22.22
4	4.00	6.00	8.00	10.00	12.00	14.00	16.00	18.00	20.00
5	3.33	5.00	6.67	8.33	10.00	11.67	13.33	15.00	16.67
6	2.86	4.29	5.71	7.14	8.57	10.00	11.43	12.86	14.29

The drop-hammer.—This may consist of a heavy block of oak when some hastily improvised machine is desired, but the cast-iron ram as shown in Fig. 17 is the type quite universally employed. In the illustration the hammer *h* is engaged by the nippers *n*, and is released when the latter are drawn up against the wedge *w* fastened to the guides *g*, at the top of the derrick. The hammer may also be released at any height below the top by pulling a tripping rope attached to the nippers. The hammer line or hoisting rope *r* runs over a 10" to 18" sheave (2 sheaves are fixed at the top—one for the hammer line and the other for the pile line) and can be operated either by horse power or by hoisting engine. These hammers weigh from 1200 lbs. upward, 2000 to 2500 lbs. being a very satisfactory medium weight. Greater speed can be obtained by the use of a hoisting engine with a friction clutch or friction drum, so that the rope and hammer may be released by the engine driver at any moment. In such a case the hammer rope is fastened permanently to the hammer which is level on top, instead of being depressed as in Fig. 20. It is to be noted that the hammer should be heavier for this method as it has to "overhaul" the hammer rope in its descent. Such hammers weighing 3500 lbs. are very efficient.



Figs. 20. Drop-Hammer with Nippers.

The steam-hammer.—Fig. 21 is an illustration of an improved type of steam-hammer† with a "gravity" action. The total machine, which may

* For any other drop the bearing is proportional. Thus, for 15 ft. drop, multiply by ¾; for 25 ft. drop, multiply by 5/4; etc.

† The Warrington steam hammer as manufactured by the Vulcan Iron Works, Chicago. The first steam hammer for pile driving was applied by James Nasmyth in 1845.

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weigh as much as 5 tons, is suspended from the top and between the leads or gins of the derrick, like a common drop-hammer; but in this case it is allowed simply to *rest* on the pile to be driven. The ram *h*, whose weight is about half that of the total machine, slides vertically on four circular guides and is connected to a piston rod operated from the cylinder *c*. Steam is led into the lower part of the cylinder through a flexible tube, thus raising the ram which is then allowed to drop by its own weight. The amount of "drop" may be lessened if the cylinder is double acting.

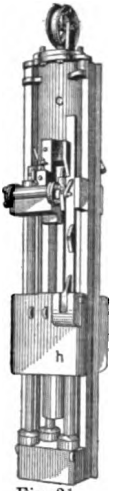

Fig. 21. Steam Pile-Driver.

The derrick.—The ordinary pile driver derrick is a simple affair consisting of two upright leaders with guides for directing the hammer (Fig. 20), and supported by a framework (for fore-and-aft and lateral bracing) resting on a platform or horizontal frame. The frame may rest on a scow, on a car, or on rollers. If on a scow, it is usually fixed; on a car, it is allowed to swing laterally around a vertical pivot; on rollers, it is given forward and lateral movement by using two sets of wooden rollers, one above the other, at right angles to each other. A *tilting* driver for driving batter piles may be constructed by allowing the leaders to swing on a horizontal pivot attached to the *A*-frame of the derrick near the top; or by pivoting the whole derrick frame on V-bolters which will allow lateral tipping. Ratchet devices may be used for tilting the leads or the derrick frame, and holding them in position while driving the piles. The following dimensions may be taken as mere "hints" for design for scow and heavy land derricks: For a 60-ft. derrick, leads 8"x 10"; guides 4"x4" sheathed with 4"x½" iron; spreads (lateral) 8"x10" for height of 55 ft. and total base of 18 ft.; ladder strings (fore and aft) 6"x12" for height of 60 ft. and base of 18 ft.; horizontal bracing 6"x8" to 8"x10" (the latter supporting platforms); diagonal bracing 4"x12"; platform frame 12"x12" and 12"x14"; caps (for supporting sheaves) 20". For a height less than 60 feet the dimensions may be proportioned out as the square root of the height; thus, for a derrick 30 ft. high multiply $\sqrt{\frac{1}{2}}$ or $\frac{7}{10}$. Less amount of bracing is of course needed for a low derrick than for a high one and where it is not over 20 or 25 ft. the diagonal bracing is not required. For a steel frame, proportion the members for equal strength and stiffness to the above. A car-derrick may be made to hinge at the foot of the ladder strings and lie flat when not in use. For a light, portable land driver is required the above proportions may be reduced.

The power.—This may be either man, horse, steam, gas, hydraulic, pneumatic, or electric. The first named is now only employed in driving sheet piling, and to a limited extent. The horse is used frequently in outlying districts where it would be expensive to ship a hoisting engine for the limited amount of work to be done. Steam is mostly employed, and usually the hoisting engine or by operating the steam hammer (see page 872). Hydraulic pressure may be used in driving piles for foundation work in submarine tunnels where a steady pressure, without shock, is absolutely necessary. For a large amount of work requiring several drivers, electric power, delivered from a central plant, has proven economical. Fig. 22 is a light, portable steam pile driver for height up to 50 feet. A diagonal bracing may be used if deemed advisable but should not be added, to increase the portable weight, unless the work is heavy and demands it. Driving is often assisted by the water jet.

Fig. 22. Light, Portable Land Driver.

The water jet.—If the soil is sandy great assistance in driving piles may be rendered by attaching the end of a small pipe to the foot of a pile and playing a stream of water into the sand as the pile descends. If the material is pure sand, driving often becomes unnecessary, the pile settling readily under the weight of the hammer. In these cases a driver is not used at all, the pile being forced down by bringing some other weight to bear

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upon it, as by block and tackle or by lever. Softer wood for piles can be used in such cases than would be required for ordinary driving.

Pile shoes.—These may be used in hard driving to prevent brooming of pile. The shoe may be of cast iron or steel, usually fitted to the point of pile after the latter has been sharpened or shaped to receive it. Cones made of sheet steel are sometimes used. Provision should be made, by lugs or straps, for spiking or bolting the shoes to the piles. Shoes should have either a point or an edge, for penetration.

Common Pile Foundations.—There is danger in driving piles too close. Instances are frequent when piles already driven have been weakened considerably by subsequent too-close driving. 2′-9″ centers is about as close as ordinary piles should be driven, and 3′-0″ is much better; but local conditions sometimes demand a minimum spacing of 2′-6″ or even 2′, especially if the piles are small.

Piles should be driven to firm foundation, and it is sometimes necessary to drive them in "tandem" or one above the other to reach a suitable supporting soil. Fig. 23 shows the dowel connection used for splicing same. Piles spliced in this manner have been driven considerably over 100 ft. in depth.

Fig. 23. Dowel Splice.

Cutting off piles.—Grades for cutting off piles are given by, say, driving a tack in the side of the pile either at the desired level or at certain established distance below it. The cut-off is made with a cross-cut saw resting on two short horizontal guide sticks nailed on opposite sides of pile, with top edges at grade. For cutting off-piles *under water* there are several methods: (1) The above method may be employed sometimes during *absolute* low water, by sawing off two or three feet below the surface, provided the cut-off is near the river bottom; (2) under the same conditions as above it may be advisable to cut off all the piles just above the surface and then nail horizontal guide strips on which to suspend a "gage" saw operated by hand and which will cut off the piles at the true level a certain distance below the guides; (3) a circular saw fixed horizontally at the lower end of a vertical shaft operated from a scow, may be used; (4) if the water is too rough or deep and greater care is required it is best to cut the piles off above water as in method number (2) and construct a level platform on which to operate the circular saw, say 4 ft. diameter, from a movable machine (a pile driver is sometimes rigged up for this purpose) mounted on rollers.

Foundations *on* piles are of many kinds and are discussed under Caissons, and what follows.

"*Dead-Men.*"—These are short piles sometimes "planted" through soft material to a firm bed and packed around the sides with gravel or sand, wet and well tamped. They should be well braced laterally.

Iron Piles.—These are cast iron, wrought iron and steel, being best (most durable) in the order named. Cast iron piles may sometimes be used to advantage in hard-driving soil where a wooden pile would broom and where the ground becomes alterably wet and dry. Steel shapes, as I-beams, may be used in wharf construction (fresh water) where considerable lateral thrust has to be resisted. They should be coated with asphalt before driving.

*Screw Piles.**—A screw pile consists of a cast iron shoe, shaped something like a cartridge, surrounded by about 1½ turns of spiral disk thread 5 feet more or less in diameter and mounted on the lower end of a vertical shaft which, when turned, screws the pile into the ground. The shaft is usually of steel and about $\frac{1}{10}$ the diameter of the screw. Screw piles are particularly adapted to soft soil, but are seldom used in this country excepting for wharf work, anchoring beacons, and light-house construction. Their use in bridge foundations has been extremely limited. The circular area of the screw presents resistance against an uplifting as well as downward force.

Disk Piles.†—A circular "disk" of cast iron fastened to the lower end of an iron shaft and sunk by the water jet (see page 873) is the usual form

* See *Eng. News*, Oct. 15, 1903.
† See Trans. A. S. C. E., Vol. VIII, page 227-37; Coney Island Pier

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e. The thickness of the cast disk may be, say, about one-third its the latter being proportioned to the required bearing area, and out 2 ft. for an ordinary shore-sand foundation. The diameter t is proportioned to the superimposed loading and may be equal ckness of the disk, more or less. Screw piles are more common piles.

Piles.—Sand piles are formed by driving ordinary piles, pulling immediately filling the hole with wet sand, well compacted in s. Such piles if well constructed will usually have a greater ower than the wooden piles withdrawn, provided, of course, the firm enough to give lateral support.

te Piles.—A concrete pile may be constructed in a similar manner d pile, just described—differing from a concrete pier in that it teral support from the soil, which the pier does not. But the of concrete pile, in place, is either (1) a concrete core enclosed or steel shell, or (2) a steel reinforcement encased in concrete the ordinary concrete-steel column). In either case the piles iven with the drop- or the steam hammer; or they may be sunk with the water jet, by a small metal tube centrally located in, nating at the point of, the pile; or the water jet may be used assist the driving. Under this heading we will include only s encased in a metal shell, and then follow with a discussion of -concrete piles, under a separate heading.

*Jet Concrete Piles.**—Fig. 24 illustrates ethod of constructing a concrete pile d of sinking same by the water jet. the telescopic shell (similar to a collapsi- drinking cup) is assembled at the sur- the pile is to be sunk; the shoe, with ozzle, is attached to the water pipe, , under pressure of 40 lbs. more or less, hrough the nozzle into the sand; as e telescopic sections sink they are filled rete, and the "spreaders" are introduced at the joints to prevent The process is simple and effective. The piles can be sunk to cable depth in sand, and they form a safe and permanent founda-

Fig. 24.

*Shell Concrete Piles.**—The finished pile consists of a metal "shell" 20 sheet iron, driven into the soil and then filled with concrete. is driven by inserting into it a tightly fitting iron "core" which onsidered as the "temporary pile" and which is driven in the ner by the drop or the steam hammer, and with a follower to protect the head of the core from After driving, the metal core is collapsed (made to ss diameter) and withdrawn, leaving the metal shell which latter is now filled with concrete. The metal sts essentially of three main parts, split longitudinally, ater "halves" being split by the third, acting as a dge between them. When the key is lowered the are spread apart as in Fig. 25. When raised, the ses ready to be withdrawn. The diameter of the e 6 ins. at the point and, say, 16 to 20 ins. at the

Fig. 25.

rced-Concrete Piles.—These may be built in various forms, as circular, (modified) triangular, (modified) tangular (for sheet piling), etc. Figs. 26, 27, 28 and mple sections.

imes four more rods are placed intermediately between those shown making 8 reinforced rods instead of 4. For rectangular section, often used—3 on each side.

d of rods or expanded metal, I-beams are sometimes used, for sheet piling where lateral stiffness is required. The I-beams pairs, connected at intervals by riveted diaphragms.

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At the point of the pile the reinforcement rods are brought together, and may be banded with wire, or welded. At the top, the rods are imbedded in concrete; but after driving, the concrete may be picked away and the rods exposed in order to bind with the new-laid concrete for foundations.

Fig. 26.

Fig. 27.

Fig. 28.

Fig. 29.

Fig. 26.—Circular, tapering pile, reinforced with $\frac{1}{4}$"° and $1\frac{1}{2}$"° steel rods, united at bottom of pile.
Fig. 27.—Cylindrical pile, reinforced with expanded metal.
Fig. 28.—(Modified) triangular pile, reinforced with 1"° steel rods, and $\frac{1}{4}$"° wire ties.
Fig. 29.—"Square" pile, reinforced with 4 – 1"° steel rods with various arrangements of $\frac{1}{4}$"° wire ties. The central hole is for water jet.

Concrete piles are made in moulds. They should be kept sufficiently wet while curing, and away from the sun. In driving, a follower should be used to protect the concrete, at the head of the pile, from injury.

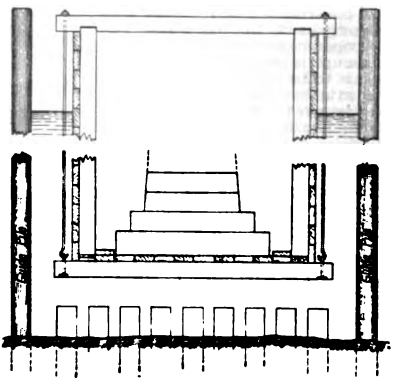

Fig. 30.

Open Caissons.—An "open" caisson is a water-tight box without a top, in which a masonry pier is built and sunk on to a prepared foundation. The sides are detachable from the bottom (see Fig. 30), and hence may be removed after the pier is sunk in place, and used in the construction of succeeding piers. Guide piles are usually driven to assist in sinking the caisson.

The following rules will be found useful in proportioning the thickness of planking on the sides of the caisson, and subject to hydrostatic pressure (taking into consideration the deflection as well as the strength): *Hard woods*: For hydrostatic head of 36 ft. make thickness of planking in inches equal to the unsupported span in feet; for $4\frac{1}{2}$ ft. head make the thickness one-half of the above; for intermediate heads make thickness directly proportional between the two. *Soft woods*: For hydrostatic head of 30 ft. make thickness of planking in inches equal to the unsupported span in feet; for $3\frac{3}{4}$ ft. head make the thickness one-half of the above; for intermediate heads make thickness directly proportional between the two.

In proportioning the uprights, note that for deep caisson work they may be braced by horizontal struts from side to side or from the sides to the built masonry pier.

For the construction of *concrete* piers in open caissons, the wooden forms are built inside and separate from the sides of the caisson—leaving a space between, all around.

For large or deep piers the floor of the caisson may be of two or three courses of 12-inch timber; and the sides may be planked with two thicknesses of planking (the inner course being laid diagonally), or they may be composed of crib timbers dovetailed or halved together and well drift-bolted. Guide piles are sometimes driven in pairs instead of in single line.

Crib Piers.—A crib pier is simply a wooden crib (see page 869) or box constructed of logs or of squared timbers framed and bolted together, and sunk to a natural or a prepared foundation bed, by filling the chambers with gravel, rock, etc. A crib differs from a caisson in that the latter is supposed to be water-tight. The crib may or may not have a bottom. If it is sunk to a natural bed the bottom is omitted (which is the usual form) and the sides are projected downward a few feet below the inner bracing and chamber floors so as to cut into the bed and get a good stable bearing. The bottom of the sides of the crib is sometimes extended by blocking, to conform with the natural bed, especially if the latter is solid rock, in which case the crib is often bolted thereto.

The crib may be of rectangular form (with vertical or inclined sides) or it may have a V-end up-stream, and perhaps down-stream also. The up-stream end should be protected with angle iron or steel rail, against floating ice and logs. In framing the sides, the timbers (say 12"x12") may rest squarely on each other or be separated a few inches; and framed into the cross timbers, say every four feet apart. Interior longitudinal timbers are also framed in, about the same distance apart, forming vertical chambers about 4-ft. square. The bottoms of these chambers are planked, to hold the filling. All the timbers should be securely drift-bolted together. If the crib is designated to support a heavy superimposed load the chambers should be made smaller by increasing the number of longitudinal and transverse timbers.

The crib is usually sunk directly on the bottom, if hard; but if soft, it is sometimes sunk on a prepared pile foundation, and the bottom protected from scour by a deposit of rip-rap. If the crib projects above low water it becomes merely a temporary structure as the timbers are exposed to rot. Crib piers are specially permanent when entirely submerged below low water mark, and when not subject to the attacks of the *teredo* (see page 360). As such, a crib may act as a sub-foundation on which may be constructed a concrete or stone masonry pier.

Pile Piers.—A pile pier is a pile sub-foundation projected upward to support the superstructure direct. It is at best a temporary affair and lacks the lateral stability of masonry. Pile piers and abutments are used largely in every new country where timber abounds, for supporting railway and highway bridge spans, on account of low first cost and rapidity of construction. As Bridge Engineer of a western railroad the writer constructed many of the bridges from end of track, as the latter was laid using little delay in the tracklaying, and utilizing the construction trains for the transportation of material to the bridge sites.

A simple form of bridge pier consists of two or more rows of piles (5 or more in a row), each row capped with a 12"x14" cap dapped ½ inch and drift-bolted with 1"x20" drift bolts. On top of these caps, cross-caps are drift-bolted to receive the pedestals of the span. The inside rows of piles may be projected up-stream in the form of a V. The rows of piles should be sway-braced with 4"x12" planking, both longitudinally and transversely; secured at the ends, to piles and caps, with screw bolts; and double spiked at intersections with piles. The outsides of piers may be sheathed all around with 3-inch or 4-inch planking, laid longitudinally with staggered joints. The nose of the pier may be driven with batter piles and sheathed with iron as a protection against ice and logs. Rip-rap may be deposited around the piles, to prevent scour; also pile clusters may be driven up-stream, as ice breakers or fenders.

Tubular Piers.—Under this heading the writer desires to bring to the attention of the reader a variety of piers which are tubular in form, com-

posed of various materials, sunk by various processes, and occupying various positions when in place. When we say "tubular in form" we mean that they are composed of a tubular "shell" which is lowered into position and filled with, say, concrete or some other equally serviceable material.

Shape.—The circular section is the one most commonly used and hence if the section is uniform throughout, the pier is cylindrical in form; if the section decreases uniformly upward it is conical. Often two cylinders of different diameters (the smaller one above) are joined by a conical frustum. They are usually placed in pairs for supporting bridge trusses, being braced to each other by a web plate, or by horizontal and diagonal bracing. The center pier of a drawbridge may be one large circular cylinder or may consist of, say, 6 or 8 separate cylinders surrounding a central one of larger diameter, all braced rigidly together. When used singly for center piers of draw spans the oval or elliptical section is sometimes preferred.

Materials.—Timber staves, say from 6″ to 12″ thick and planed with sides radial have been constructed in the circular or "barrel" form similar to the sides of a water tank or a section of stave pipe. The bottom is provided with an iron shoe if it is to be sunk into the soil. Instances are on record where a masonry shell, as of brick, has been used as a tubular pier and sunk into the ground to considerable depths. But the most commonly used materials are the metals—cast iron, wrought iron and steel. Cast-iron shells with metal 1-in. thick, more or less, and with flanges inside for bolting the sections together, present a smooth exterior surface for sinking, and are very durable. Wrought iron and steel are generally preferred. The metal is from ¼-in. to ⅜-in. thick and the riveted tubes are composed of sections from 5 to 6 ft. in length. Both butt- and lap joints are used. The cylinders are stiffened at the top with an outside circular angle-iron and the top covered, when finished, with a plate. There is no reason why a reinforced-concrete shell should not prove economical under certain favorable conditions.

Sinking in Place.—The method of placing or sinking the tubes, or their method of support after being placed, often determines the *name* of the pier. For instance, a metal cylinder encasing a cluster of piles is called a Cushing cylinder pier, "Cushing pier," or Cushing pile, named after the inventor of the system. If the same cylinder (or usually two cylinders) is set on a timber platform resting on piles, it is called a "platform pier," platform cylinder pier, or simply a cylinder pier. If it is sunk in the ground it may receive the name of cylinder pier, or "tubular pier." If sunk by the pneumatic process (compressed air) it is called a "pneumatic cylinder," pneumatic tube, or pneumatic pile. Sinking the cylinder in place, in the foundation bed, may be accomplished either by: (1) Weighting the tube with pig iron and excavating inside, whence it descends by overcoming the frictional resistance* on the sides; (2) Dredging, when in water, and loading as before; (3) Pneumatic Process, for great depth under water, excavating in a "working chamber" under compressed air, as with the pneumatic caisson.

Cushing Piers.—Fig. 31 illustrates two styles of casing and two styles of piling ordinarily used for Cushing piers. The wrought-iron or steel cylinders (say ¼″ to ⅜″ metal) are usually placed after the piles are driven and bolted together, but sometimes one of more bottom sections (5 to 6 ft. lengths each) are set up and riveted together in place and then the piles are driven inside of them. Care must be used not to bulge the sides in driving. After the cylinders are set and connected with bracing (at least at the bottom) they are filled with concrete, deposited in layers and thoroughly tamped around the piles. Soft silt, logs, boulders, etc, should be removed (previously) from the bottom and the cylinders should rest on as firm a foundation bed as practicable. The horizontal bracing between the cylinders may be composed of two channels with pin connections to angles or bent plates riveted to the cylinders. They may be connected top and bottom, with tie plates or lattice bars, or be reinforced with plates. Two 10″ channels are common for small piers. Adjustable rods, singly or in pairs, are often used for the diagonal bracing, but stiff members are preferred. Howe truss bracing

* The frictional resistance for cylinder piers may vary from about 300 lbs. per sq. ft. in mud, up to as much as say 1500 lbs. per sq. ft. in gravel.

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been used considerably in the West where timber is plentiful; that is, diagonal braces are of timber, say 10" to 12", tied with horizontal rods, and upward. The web brac- is usually sheathed on both s with 2-in. planking, projecting le beyond high and low water. of the best forms of bracing is eb plate connected with the ders by vertical angles riveted to. The web is stiffened at in- ls by horizontal angle stiffen- Much of this description, espe- that relating to the bracing, pply to Platform Piers, which ext be described. Plenty of should be used around the to prevent scour. Ends of ould be cut off at different ons.

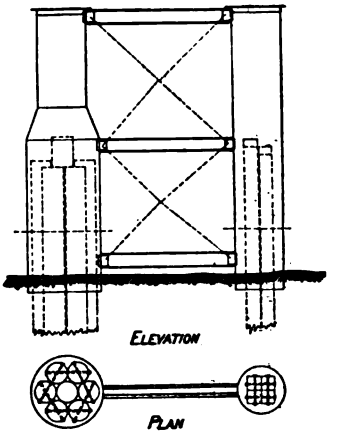

Fig. 31.

form Cylinder Piers.—Plat- ers are constructed by driv- or more rows of piles, cap- m below low water mark timbers laid longitudinally pier, and drift-bolted to ; and then laying a plat- 6" to 12" timbers trans- nd spiked to the caps. nders are then placed on orm and securely bolted rough holes in the bottom flange angles. Sometimes the piles 32), are allowed to project upward through the platform into ers, thus forming a com- form and Cushing pier. The re filled with concrete and ilar to the "Cushing piers" ibed. For heavy masonry latform consists of a "gril- nbers (say 12"x12") usually courses or more, laid at drift-bolted together and piles. On this grillage the laid.

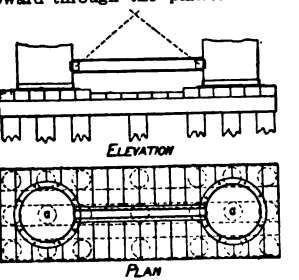

Fig. 32.

ic Cylinder Piers.—Instead g cylinder piers by the cess or on platforms, as just described, they are a firm foundation bed g the material from the ow the bottom, as they descend, and ting them. If the tubes are sunk he material may be removed often sing an orange-peel bucket for this re this can be done it will be found al. But for deep foundation work process is generally used.

Process.—In Fig. 33, *W* is the com- king chamber, connected with the air lock. When the workmen or through the lock into or out of the oors *a* and *b* are worked like the lock, because the compressed air

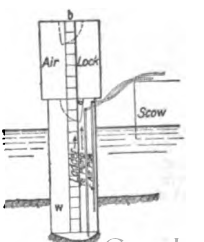

Fig. 33.

eumatic process is meant the *plenum* air process. The *vacuum* process is ed.

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in *W* is at a higher pressure than the atmospheric pressure. The compressed air, sand and water pumps are on the scow. The material excavated is raised by a windlass. When the air lock has been sunk to water level, a new section of cylinder is inserted and the air lock placed above it. Guide piles may be driven to guide the descending cylinders.

Pneumatic Foundations.—The main essentials for the prosecution of deep foundation work under water, for large bridge piers, are:

(1) The pneumatic caisson (an inverted, air-tight, "open" caisson), which forms the working chamber, and supports the masonry pier;

(2) The crib, a cob-like (sometimes solid grillage) construction of timbers above the pneumatic caisson, really forming a part of it, and on which the masonry pier is built;

(3) The coffer-dam (sometimes omitted), built on top of the crib so that masonry can be laid dry even when below the water level;

(4) The pneumatic tubes, consisting of air shafts, air locks, etc.;

(5) The machinery scow, containing boilers, air compressors, engines, and dynamos for lighting;

(6) The excavating tools, as picks, shovels, windlass for hoisting, etc.

(7) The sand lift, for forcing out the sand;

(8) The mud pump, for pumping out the mud.

Fig. 34 illustrates the first four essentials, and Fig. 33 shows the arrangement of main shaft and air lock, enlarged. Note that the air lock occupies (Fig. 34) about a central position in the shaft, high enough to be out of danger from flooding, and low enough to be economical. After the pier is sunk to bedrock the shafts, as well as the working chamber, are filled solid with concrete.

The main essentials will be discussed briefly in detail, as follows:*

The Pneumatic Caisson.—The caisson may be separate from the crib, as shown in Fig. 34, in which case the roof is supported by longitudinal and transverse trusses extending through the working chamber (but not shown in the illustration). But the more modern method is the combined crib and caisson, by which means the crib becomes a part (sometimes the whole) of the truss system, to transfer the central weight, over the chamber, on to the cutting edges of the caisson, during the process of sinking. The combined crib and caisson is typically represented in Fig. 35, which is a general plan and longitudinal section of the coffer-dam and caisson for South Pier of Brooklyn tower foundation of the new East River Bridge (Williamsburg) New York City. Fig. 36 shows a vertical section of same, transverse to bridge axis. Figs. 37 to 40 show details of caisson for North Pier, and Figs 41 to 43 show details of coffer-dam for either pier.

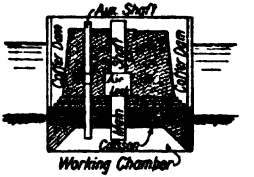

Fig. 34.—Separate Caisson.

The working chamber (Fig. 35) was 7 ft. high and divided by bulkheads. All seams were calked with two strands of oakum; the chamber was then lined with 3-in. plank, the joints (and spikes to fasten them) being treated as above, and then painted with white lead, making it air-tight. Fig. 40 shows the roof plan of caisson.

The Crib.—Where a crib is used above the caisson proper, it may be a solid grillage, or it may be divided into separate vertical compartments to be filled separately with concrete as the caisson is sunk; or the compartments may be staggered (offset) vertically or open so the concrete filling will form one monolithic mass. The accompanying illustrations (Figs. 36 to 39) show an open crib of the latter type.

* The illustrations of Williamsburg bridge foundations are adapted from the official working drawings.

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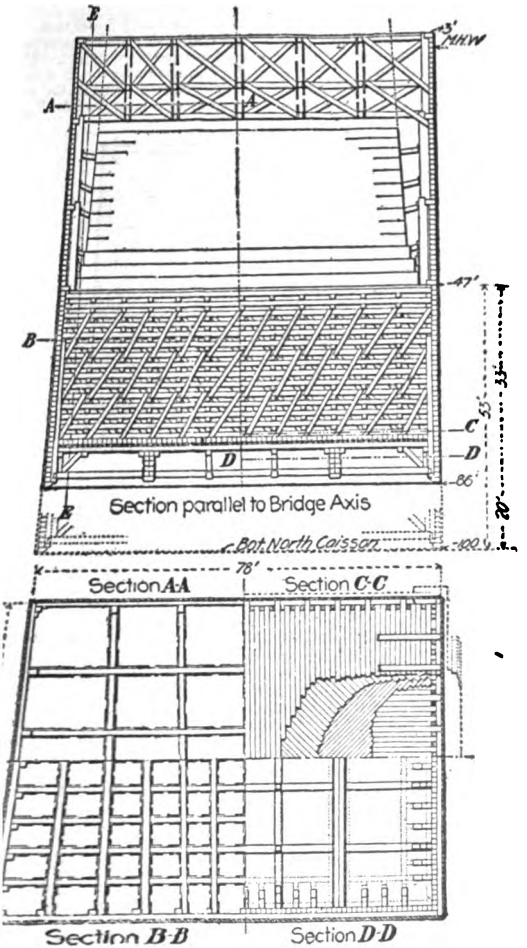

Fig. 35.—Combined Crib and Caisson.
(See pages 880 and 882.)

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The Coffer-Dam.—The coffer-dam as applied to the pneumatic foundation, is really an open caisson (see page 876) resting on top of the crib; or if the crib is omitted, it rests directly on the pneumatic caisson. But sometimes the coffer-dam itself is omitted, as when the masonry is built directly on the crib or on the pneumatic caisson, and its top kept *above high water* as the pier sinks. If this is done, no coffer-dam is needed to shut out the water. But it is not always safe, advisable, or possible (economically) to construct the masonry with the rate of progress corresponding with the sinking of the pier, and hence the coffer-dam is employed. It is usually erected in sections, one above the other. Note in Fig. 35 the substituted bracing against the masonry as the latter is built upward. Figs. 41, 42 and 43 show the bracing in detail.

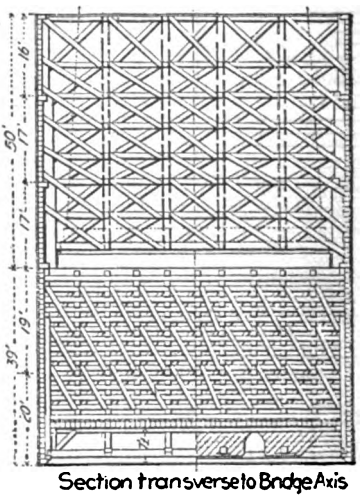

Fig. 36.—Combined Crib and Caisson. (See page 880.)

The Freezing Process.—Soft, flowing mud and quicksand (especially) are the most difficult materials to be encountered in sinking foundations and shafts, excavating for wells, or driving tunnels; and it has naturally occurred to a few inventors to devise means for freezing this material, somewhat beyond the area to be excavated (leaving frozen walls for temporary lateral support), so the actual excavation can be made by ordinary methods, as with the pick and shovel.

Any practical system consists essentially in driving a number of tubes into the ground around the proposed excavation, and a little beyond its outer limits. These are called the "freezing tubes" and may be, say, from 4" to 10" in diameter. They are closed at the lower end and should penetrate the full depth of the soft material. Inside of these tubes are fitted the "circulating pipes," about 1" to 1½" diam., with lower ends open, and extending practically to the bottom of the tubes. The circulating pipes are connected at their tops by a "circulating ring" into which the freezing liquid is pumped; descending through the pipes into the large tubes where by absorbing the heat from the surrounding soil, the latter is frozen; then emerging from the tops of the tubes into the "collector rings" and reservoir. From the reservoir it passes to the refrigerating machine, and then to the pump, thus completing the cycle. Certain modifications of this process have been introduced, but the process in any form is seldom used. The circulating liquid may be a solution of calcium chloride or brine. Ammonia machines have been used with success, with magnesium chloride as the circulating medium.

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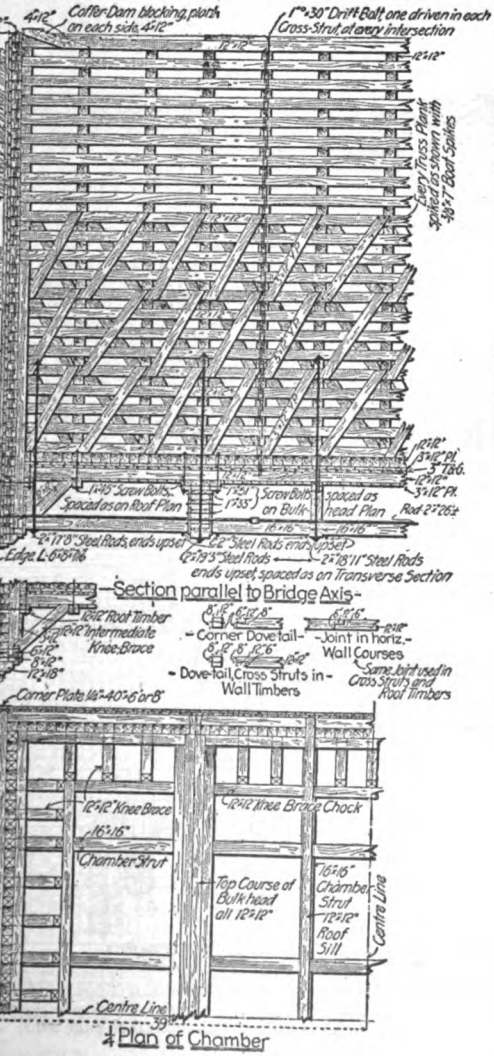

Fig. 37.—Open Crib Caisson. (See page 880.)

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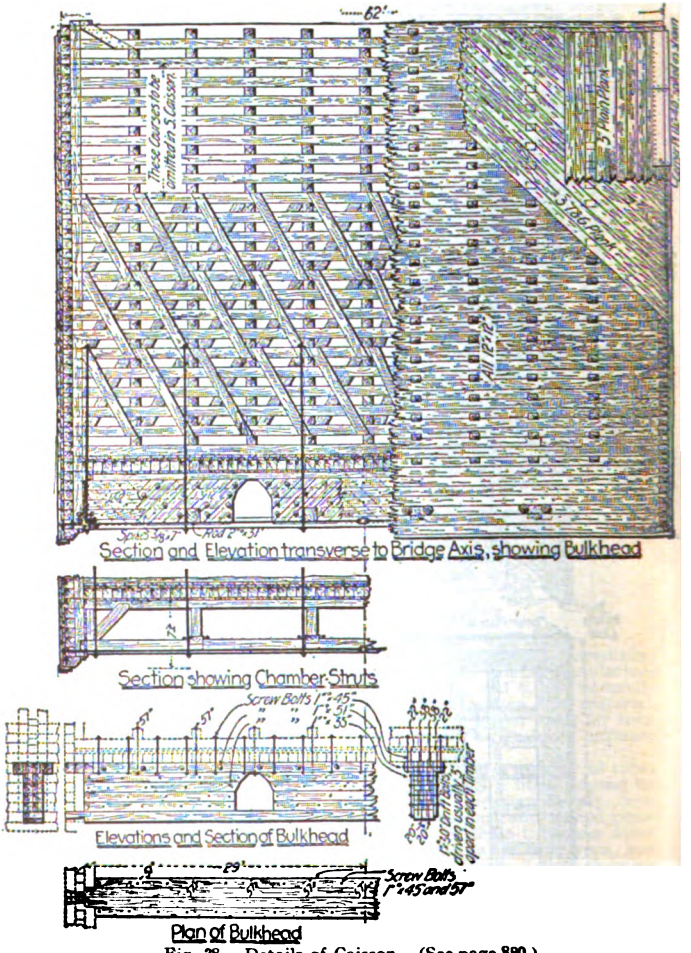

Fig. 38.—Details of Caisson. (See page 880.)

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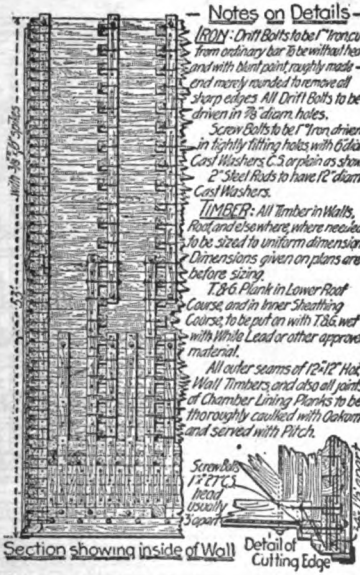

Fig. 39.—Details of Caisson. (See page 880.)

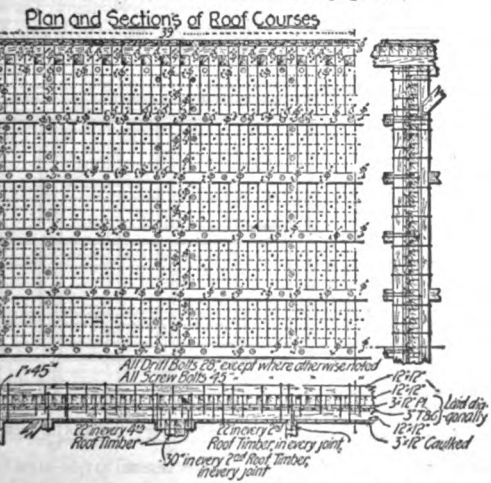

Fig. 40.—Roof Plan of Caisson. (See page 880.)

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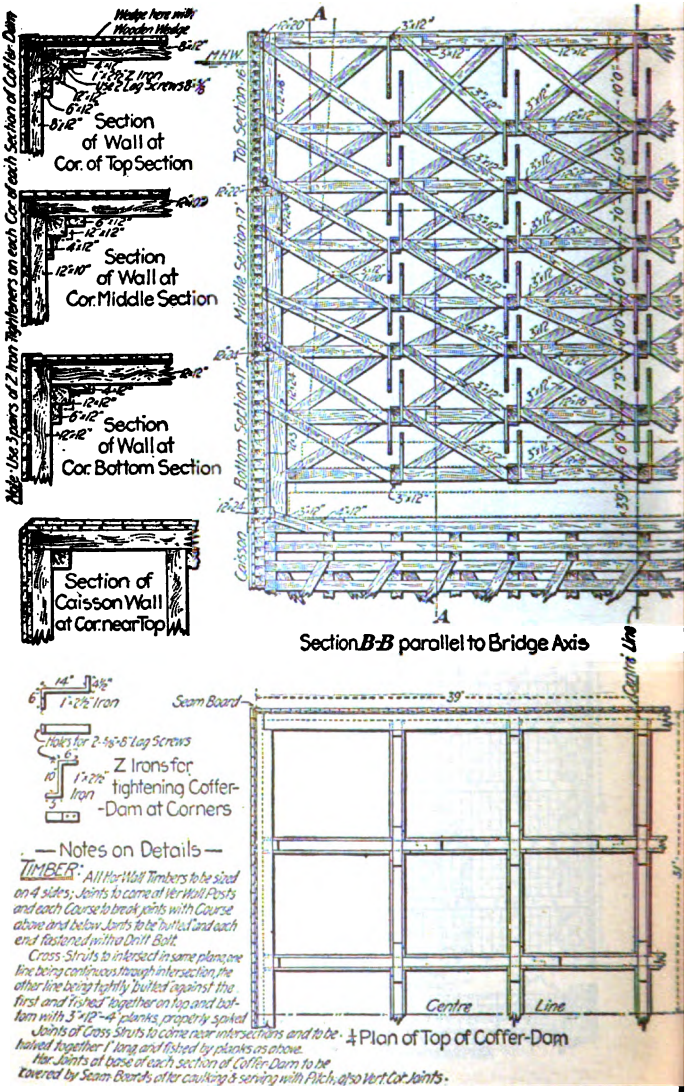

Fig. 41.—Details of Cofferdam. (See pages 880, 882.)

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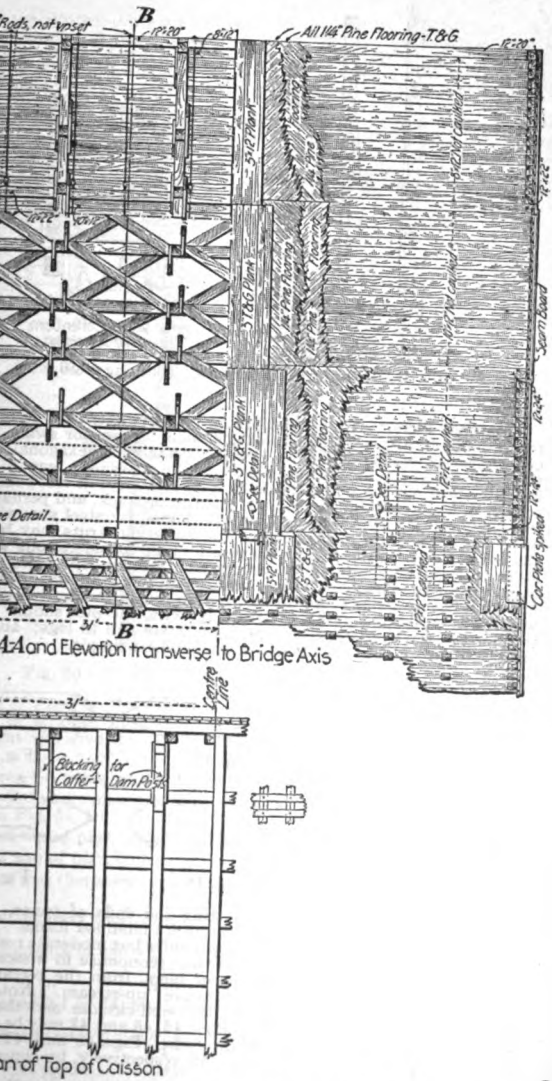
Rods, not upset
B
12"x20"
8"x12"
All 1½" Pine Flooring-T&G
12"x20"
5"x12" Plank
5"x6" Plank
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Sawed Timber
Sawed Timber
Sawed Timber

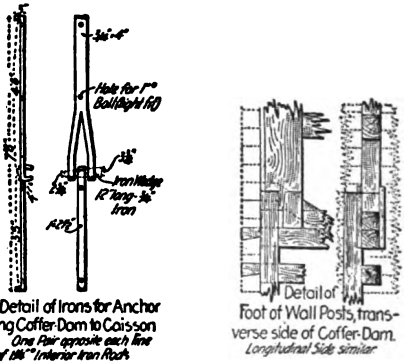

Fig. 43.—Details of Cofferdam. (See pages 880, 882.)

Masonry Piers.*—In sectional plan, masonry piers are designed largely to accommodate the superimposed loadings. Thus, for swing bridges the center pier would naturally be circular, octagonal, hexagonal or square—the circle being the most economical and the square the least. If the draw is center-bearing, the pier may be of solid masonry; if rim-bearing, it may consist of a circular shell to support the rim or track, and perhaps a central pier or core; the rim and core may be joined by steel struts, or concrete webs, radiating from the latter; or the radiating struts may be used with the central core omitted. But there are other considerations which affect, more or less, the shape of the center pier, namely, the sides of the draw openings for the passage of water craft should be straight, or in continuous line with the up-stream and down-stream arms of the draw rest; where the water way is limited and the stream is swift, the pier should be designed to offer the least resistance to the flow, to ice and to logs; and lastly, for structural reasons, the sectional plan should be simple.

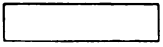

Fig. 44.

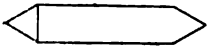

Fig. 45.

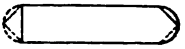

Fig. 46.

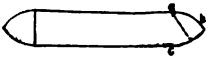

Fig. 47.

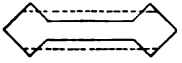

Fig. 48.

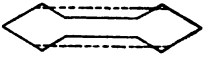

Fig. 49.

For ordinary river piers supporting the ends of spans, we naturally choose the rectangular section, or one of its modified forms. The modifications are based on such sections as will offer but moderate resistance to the flow of the stream and at the same time economize in masonry. Figs. 44 to 49 show various sectional plans of piers, from the rectangular to the double diamond. The right-hand ends are "up-stream." Note that Fig. 46 shows two types of ends, namely, the semi-circular and the 45° pointed; also that the down-stream ends of Figs. 45, 46 and 47 may be either square, or symmetrical with the up-stream ends. Figs. 48 and 49 illustrate the saving in masonry over Figs. 46 and 45, respectively, by comparing the full

* For masonry *abutments*, see Sec. 25, Masonry, pages 436 and 437.

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lines with the dotted (between the ends of the latter); and although not altogether pleasing in appearance they may be (and have been) used as concrete piers in replacing some existing cylinder piers, where the other forms above would have overloaded the pile foundation. Where concrete piers are built hollow, or cobbed (with cross walls), they may generally be reinforced with steel rods.

Summarizing in general, the rectangular type, Fig. 44, is suitable for a land pier; Fig. 46, with semicircular ends, for a land or shore pier; and Fig. 47, with circular arcs (*a*, *b* and *c* being equidistant), for a channel pier in the swiftest current. By combining 47 with 46 (semicircular ends), using the former type below high water and the latter type above, there is obtained a pier at once efficient, economical and graceful (see Fig. 50), and suitable for our deepest rivers and swiftest currents.

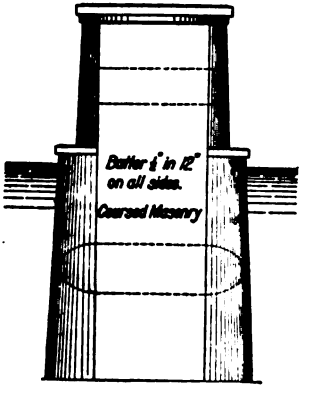

Fig. 50.—Practical Type for High, River Piers.

Contents of Piers by Prismoidal Formula.—Where the sides of the piers *battered* in straight lines the *average sectional area* multiplied by the ical height will give the cubic contents. The average sectional area is al to

$\frac{1}{6}$ (top area + 4 times middle area + bottom area).

For an ordinary masonry pier of rectangular s-section, Fig. 51, let

length of top of pier, under coping, in ft.;

width of top of pier, under coping, in ft.;

height of pier (between coping and footing), in ft.;

batter of masonry (horizontal + vertical).

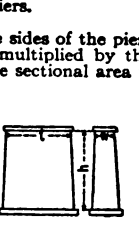

Fig. 51.

, cubic contents in feet $= \frac{h}{6}[wl + 4(w+bh)(l+bh) + (w+2bh)(l+2bh)]$ . . (1)

$= h[wl + bh(l+w) + \frac{4}{3}b^2h^2]$ . . . . . (2)

the batter is 1 in 24, we have, since $b = \frac{1}{24}$,

contents in feet $= h\left[wl + \frac{h(l+w)}{4\times 6} + \frac{h^2}{6\times 8\times 9}\right]$ . . . . . (3)

in yards = (above, divided by 3×9) . . . . . (4)

r small piers, the batter is usually 1 in 12, or $\frac{1}{4}$ in 12; for large piers, $\frac{1}{8}$.

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crete piers with metal shells (Cooper)	Nov. 6, '02.
iers for the Quebec Cantilever bridge	Jan. 29, '03.
tment and parapet wall for a skew R. R. bridge	Sept. 24, '03.
te pier with a sheet pile and canvas cofferdam	May 30, '05.
oden caisson for pivot pier	June 7, '06.
oncrete caisson for dock-wall construction	Jan. 17, '07.
idge abutment of T-section	Feb. 14, '07.
ft with steel and concrete lining, soft ground	Feb. 21, '07.
ndations in shifting ground	May 21, '08.
oncrete caissons for breakwater at Algoma, Wis.	Oct. 5, '08.
nder foundations for a R. R. drawbridge	June 10, '09.
issons (30′ x 64′), B. & O. R. R. bridge	Nov. 18, '09.
d of sinking building footings, Chicago	Nov. 18, '09.
with conc. shell and removable steel core	Dec. 16, '09.
for McKinley Bridge across Mississippi River	Jan. 6, '10.
ngs for protecting piles against teredo	Jan. 13, '10.
ide bracket, for under-water piles	June 16, '10.
oncrete under water, slope walls, Lachine canal	Aug. 11, '10.
	Eng. Rec.
orm for foundation bearing tests	July 10, '09.
nforced-concrete piles	Dec. 11, '09.
Boston terminal of the B. & A. R. R.	Dec. 25, '09.
iles for Evansville filter foundation	Feb. 19, '10.
of 300-ton column on quicksand	May 14, '10.
butments for double track R. R. bridge	July 23, '10.
onc. R. R. bridge pier, Willamette Riv., Portland	Aug. 6, '10.
sson of the Quebec bridge	Oct. 1, '10.
of St. Louis Municipal Bridge structure	Oct. 15, '10.
ndations for Municipal Building, New York	Nov. 5, '10.
crete shaft lining and working chamber	Nov. 12, '10.
d caisson for Beaver Bridge, Pittsburg	Nov. 19, '10.
ting concrete crib for a lighthouse	Nov. 19, '10.
erdam for laying 48-in. gas main, New York City	Dec. 3, '10.
method of sinking, Bankers Trust Co.'s. Bldg.	Dec. 10, '10.

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51.—WHARVES, PIERS AND DOCKS.

(References: Foundations, Sec. 50; Breakwaters, Sec. 52.)

Definitions.—A *wharf* is essentially a platform structure projecting outward from the shore, and alongside of which water-crafts may be moored for the exchange of freight or passengers. The term "quay" is applied distinctly to a wharf which skirts the shore, and runs about parallel with, and extends to no great distance beyond, the shore line. A *pier*, on the other hand, is a wharf which projects outward from the shore a considerable distance; is supported usually on piles or piers; and hence has "open waterways" beneath the platform.

A *Dock* is an artificial receiving basin for water-craft—for loading, unloading, repairing, etc. It need not necessarily be "closed." A common form of dock is the "open" basin between adjacent wharves or piers; thus, we speak of "docking" a vessel, or bringing her up alongside one of the wharves or piers. Where the rise and fall of the tide is excessive and would interfere with loading and unloading, "closed" docks may be used, as those at London and Liverpool. These are provided with gates which are opened only at full tide. It is but a step from the common "closed" dock to the Graving Dock.*

Foundations.—One of the most important features to be considered in the construction of wharves and piers is the foundation. The kind of foundation most advisable to use will depend upon: (1) the uses for which the structure is designed; (2) the nature and character of the soil and foundation bed, and depth of same; (3) the currents, tide limits, and depths of water; (4)the restrictions by the Government, State (and City) as called for by the established—

Pierhead and Bulkhead Lines.—Piers with open waterways may be constructed to the pierhead lines, while wharves of solid construction may be built only to the bulkhead lines. Where two sets of lines exist, one set established by the Government and the other set by the State, the set nearest the shore is supposed to govern. The U. S. Engineers have authority to interpret the true position of established Government harbor lines, and no encroachment is allowed beyond without the approval of the Sec'y of War. Such approval may often be obtained to meet certain exigencies in local conditions.

Construction Methods.—Common methods of *wharf* construction inside of bulkhead lines are: (a) By sinking timber cribs filled with stone, around the sides (inside) of the wharf area, and then filling in behind with earth, perhaps by *dredging* from the outside. (b) By constructing wharf walls of stone or concrete masonry, instead of sinking cribs, and filling in behind them. Care should be taken to have these walls rest on a good sub-foundation or foundation bed, that is, either natural or artificially prepared, as they are really retaining walls of the most treacherous kind: the earth backing is saturated and has a flat angle of repose, and when the wharf is loaded the overturning force may be increased greatly; moreover, the resistance of the wall itself to overturning is decreased considerably when

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* A *Graving Dock* (commonly called a Dry-dock) is an (excavated) basin into which a vessel can be floated, the gates closed, the water forced out, and the hull exposed for inspection, repairs, cleaning, painting, etc. (See Paper No. 1016, Trans. A. S. C. E., June, 1906, entitled "A New Graving Dock at Nagasaki, Japan.") A *Floating Dock* (commonly called a Floating Dry-dock) is what its name implies and need not be defined. (See Paper No. 1042, Trans. A. S. C. E., June, 1907, entitled "The Naval Floating Dock —Its Advantages, Design and Construction," by Leonard M. Cox. Mr. Cox defines the "Marine Railway" and the "Lift Dock" as additional forms of Repair Docks.)

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nersed in water. It is a good plan to back such walls with stone spalls slag, before filling. (c) By any of the methods used in the construction open piers, which will now be explained.

Piers are usually of timber construction—at least the floors—supported piling. The latter may be timber-, screw,- disk-, concrete- or cylinder-. *ber Piles* are most commonly used. They are driven in rows (some of piles are frequently driven slanting to give lateral stability), and capped a say 12"x12" timbers thoroughly drift-bolted to piles. The floor may composed of one or more thicknesses of 3" or 4" planking laid on say 14" stringers resting on the caps and drift-bolted or toe-nailed thereto. uard from 5"x8" to 10"x12" surrounds the edge of the pier. Snubbing s are driven where required near edge of pier, and allowed to project ve the floor level. Fender piles, singly or in cluster, are driven usually he corners of piers to receive the shock of vessels making landing. The of the pier are thoroughly sway-braced with 3"x10" to 4"x12" planking. re the soil is sandy, *screw piles* or *disk piles* are sometimes employed. long iron pier at Coney Island is supported on disk piles with disks 24" nd 9" thick, the wrought-iron shafts (tubes) being 8½" outside diameter. piles are calculated to support 5 tons or more per sq. ft. of disk area. are sunk with the water jet. Where timber piles are subject to the cks of the *teredo nevalis*" (see page 360) the expense of repairs is con- able, hence *concrete* piles and *cylinder* piles are often employed.

Ferry Slips and Bridge Aprons.—A ferry slip is a dock for a ferry boat; bridge or apron being an adjustable roadway, for rise and fall of tides, nd from the ferry in the slip. Probably the largest ferry boats in the d are those plying between San Francisco and Oakland, Cal. Some of lips in New York City are designed for boats 250 ft. upward in length. tremendous force with which these boats sometimes strike the slips in ng, especially during foggy weather, requires the latter to be constructed e strongest manner compatible with the requisite elasticity, and im- ements are constantly being made in their design. Local conditions be studied for each particular case, as those which fit one locality not fit another.

he throat or shore end of the slip is usually made to conform closely the shape of the ferry boat for say ¼ to ⅓ its length, and from thence es outward more or less to the mouth. Sometimes one wing only is ded beyond the middle in order to facilitate landing, under certain ions of wind and tide.

ne following illustrations were prepared by the Dept. of Docks and s, New York City, Chas. W. Staniford, Engineer-in-Chief, and were ned the writer through the courtesy of Mr. S. W. Hoag, Jr., of the eering Department.

39TH STREET FERRY, MANHATTAN:

(a) Plans of Crib with Dolphin:

g. 1.—General Plan of Outer End of West Pier with Dolphin.

gs. 2 to 7.—General Details of Crib.

(b) Plans of Ferry Dolphin:

. 8.—Part Plan of Ferry Dolphin.

s. 9 to 13.—General Detail of Ferry Dolphin.

(c) Plans of Ferry Bridge.

. 14.—Part Plan and Cross-Section of Bridge.

s. 15 to 17.—General Elevation and Details of Bridge.

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ne piling along the Seattle (Wash.) water front is often rendered by this sea-worm in two or three years' time, being almost completely ombed.

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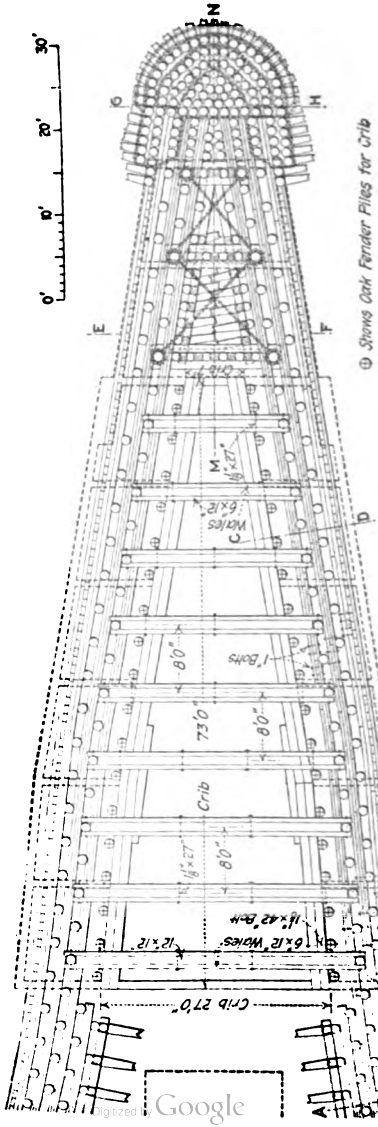

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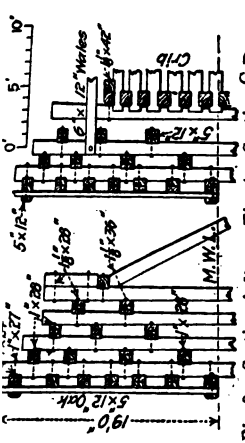

Fig. 3.—Section A-B.

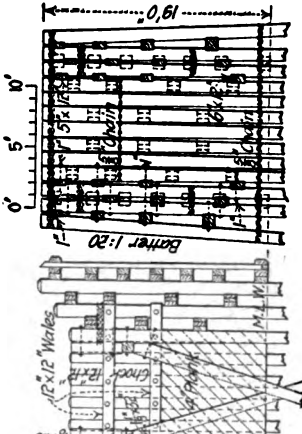

Fig. 4.—Section C-D.

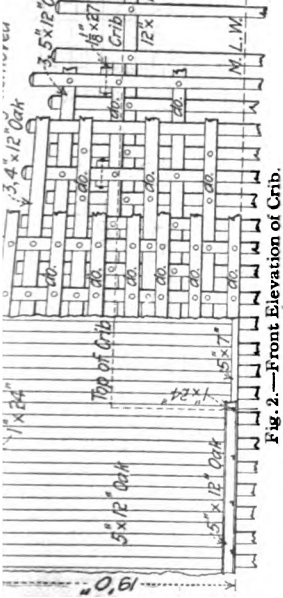

Fig. 2.—Front Elevation of Crib.

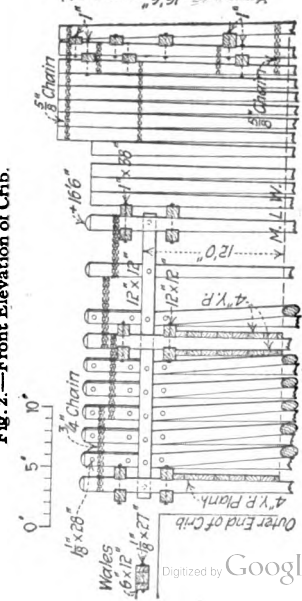

Fig. 7.—Section M-N.

Fig. 6.—Section G-H.

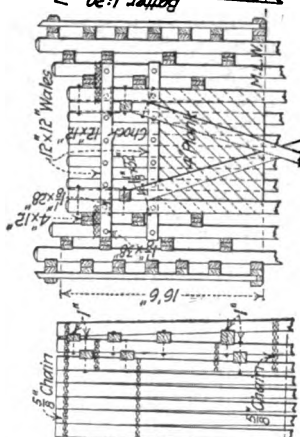

Fig. 5.—Section E-F.

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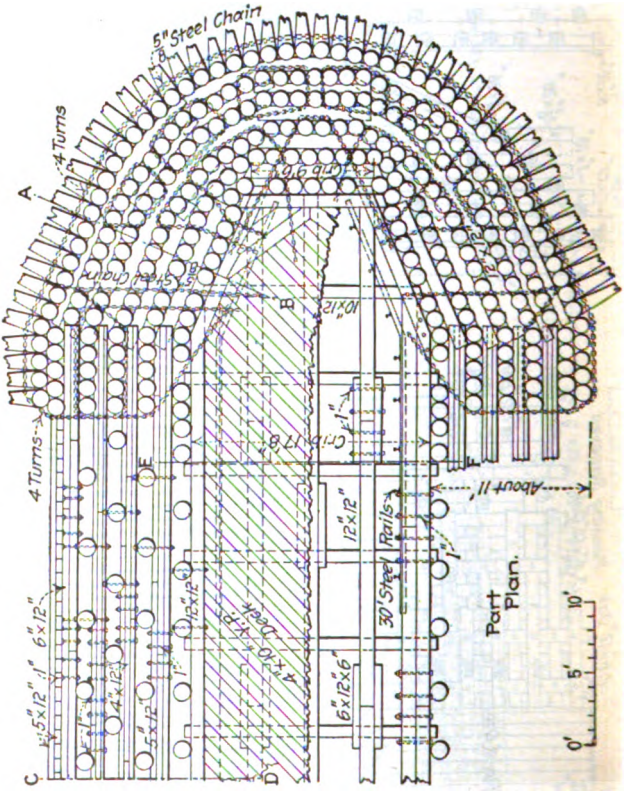

Fig. 8.—Part Plan of Ferry Dolphin.
(For Details, see Figs. 9 to 13, on following page.)

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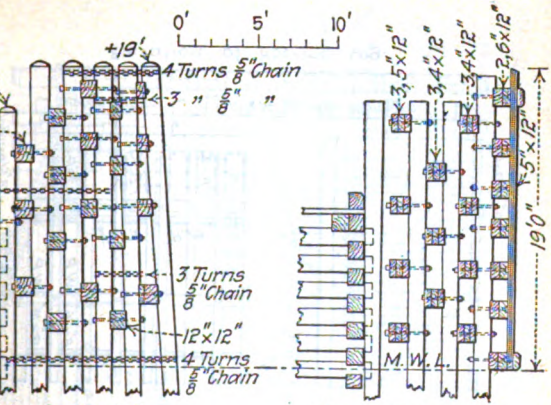


. 9.—Section A-B. Fig. 10.—Section C-D.

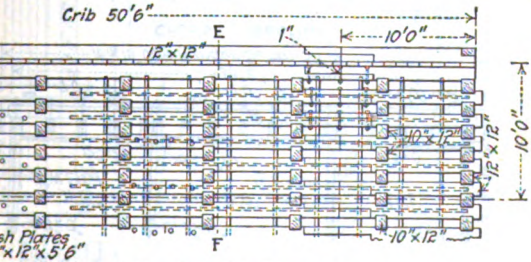
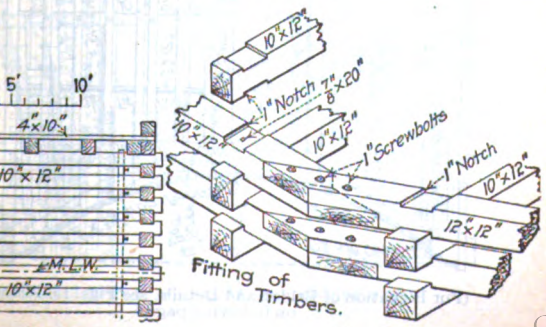


Fig. 11.—Part Side Elevation.



.—Section E-F. Fig. 13.

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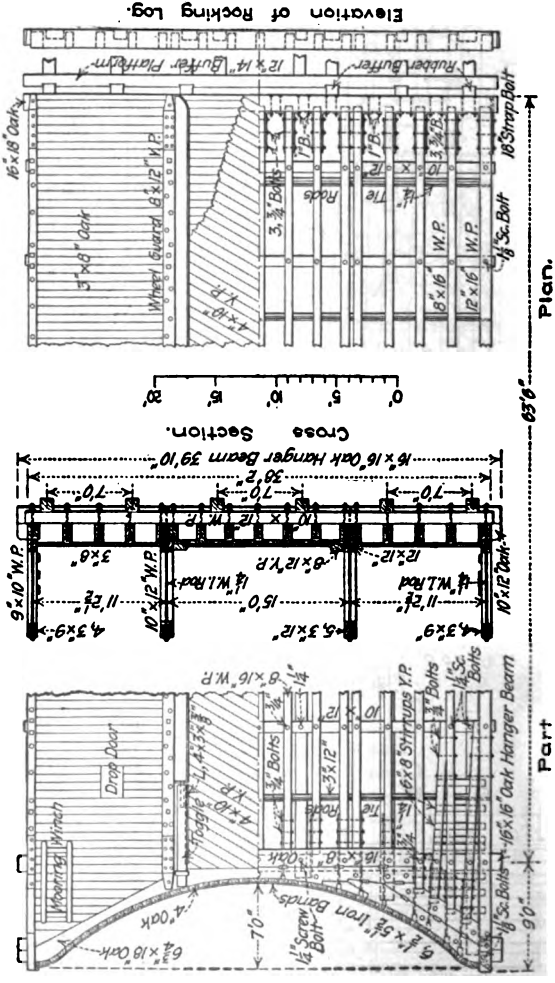

Fig. 14.—Part-Plan and Cross Section of Ferry Bridge.

(For Elevation of Bridge and Details, see Figs. 15, 16 and 17, on following page.)

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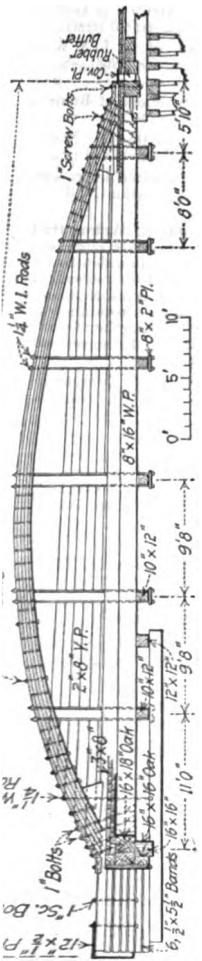

Fig. 15.—Longitudinal Section of Bridge.

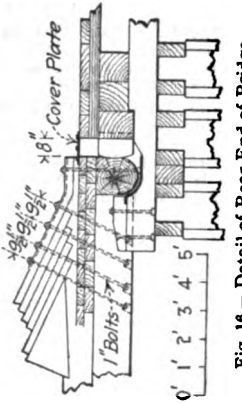

Fig. 16.—Detail of Rear End of Bridge.

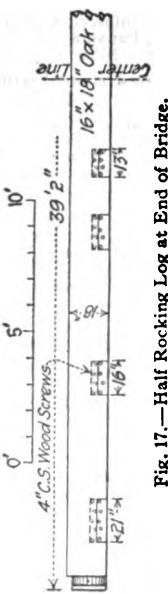

Fig. 17.—Half Rocking Log at End of Bridge.
(See Fig. 14.)

For Plan of Bridge, see Fig. 14, on preceding page.)

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EXCERPTS AND REFERENCES.

The U. S. Steel Floating Dry-Dock for Cavite, Philippine Islands (By J. S. Schultz. Eng. News, Dec. 10, 1903).—Illustrated.

Novel Steel Pier Construction at Lome, Africa ("Le Genie Civil" of July 15, 1905; Eng. News, May 24, 1906).—Illustrated structural details.

The Terminal Station and Ferry House of D., L. & W. R. R., at Hoboken (By C. C. Hurlbut. Eng. News, Sept. 20, 1906).—Sixteen illustrations.

Steamship Terminal With Concrete Pile Piers at Brunswick, Ga., A. & B. Ry. (Eng. News, Dec. 20, 1906).—Illustrated.

Dock Walls at the Port of Koenigsberg, Prussia (Eng. News, Aug. 22, 1907).—Illustrations showing methods of driving piles, and bracing.

New Piers for Transatlantic Steamships, Chelsea Improvement, N. Y. City (Eng. News, Jan. 14, 1909).—Twelve illustrations and double-page insert.

Light Reinforced-Concrete Wharf Construction, Madras Harbor, India (Eng. News, Nov. 11, 1909).—Construction comprises reinforced-concrete piles (reinforced with 1-in. rods) driven 8 ft. apart to a depth of 8 ft. below bottom and tied back to an anchor by means of 30-lb. old rails completely encased in concrete. Back of these piles are sunk reinforced-concrete slabs to a point below river bottom, the slabs acting as a retaining wall to hold back the earth. The tops of the piles are joined together by an arch construction topped by a coping of old rails. A 1 : 2 : 4 concrete was used for all the work. Illustrated.

Reinforced Concrete Wharf of the United Fruit Company, Bocas del Toro, Panama (By T. H. Barnes. Trans. A. S. C. E., Vol. LXVI., Mar., 1910).—Illustrated, with cost data.

Illustrations Useful for Reference:—

Description.	Eng. News.
Standard car-ferry transfer bridge	Dec. 19, 1901.
Large ore dock of the C. & N.-W. Ry., Escanaba, Mich.	July 30, '03.
New graving dock at Kobe, Japan	Sept. 24, '03.
Concrete dry-dock at Kiel, Germany	Dec. 3, '03.
Details of Reinforced-concrete crib-work wharf	May 26, '04.
Details of standard pile pier, Dept. Docks and Ferries, N. Y.	May 18, '05.
Elevation and plan of fireproof wharf, Tampico, Mex.	June 8, '05.
Solid pier construction in Baltimore harbor	July 19, '06.
Plan and details of reinforced-concrete pier, Atlantic City	July 26, '06.
Reinforced-concrete retaining wall and quay	April 20, '09.
Design of Concrete Naval Dry-Dock, Pearl Harbor, Hawaii	Dec. 9, '09.
Rein.-conc. piers at U. S. Naval Station, Philippine Islands	Dec. 15, '10.
	Eng. Rec.
Walls, girders, ties, anchorage, etc., Baltimore piers	May 8, '09.
Details of ferry platforms and bridges, C. R. R. of N. J.	May 22, '09.
Sewells Point coal pier, Virginia Ry.	Feb. 5, '10.
Sea-Wall bulkheads in connection with streets and buildings	Feb. 26, '10.
Cross-section of deck of rein.-conc. deck of ore dock	Nov. 12, '10.

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52.—BREAKWATERS.

General Discussion.—This subject forms one of the most important in River and Harbor Improvements. The term "breakwater" is quite distinct from the so-called "reaction (curved) breakwater," which latter belongs rather under the head of *Jetties* (page 905) and might be termed a "breakwater" jetty or "lee" jetty.

A Breakwater is an arm-like construction projected in the water and designed to form an artificial harbor for sea-going crafts. The foundation is stone, rip-rap, gravel, etc., either deposited loose or sunk in timber cribs. Where the timber cribs are used they are, for permanent construction, projected upward only to within about 2 ft. of low water, thus forming a substructure on which the superstructure is built. The superstructure* is best constructed of stone- or concrete blocks weighing from 1 to 10 or 15 tons; or of concrete deposited *en masse*. If the bottom is soft or silty, a trench should first be dredged on the line of the breakwater, removing all soft material liable to cause trouble by excessive settlement. This method will generally be more satisfactory than the one sometimes employed of using a gravel core in the breakwater and trusting permanent settlement to take place as the breakwater is built up. Although the latter method has proved successful in some instances, there are records of utter failure during the first heavy storm after completion of the work.

For an excellent discussion of breakwater construction, see Paper No. 971, Trans. A. S. C. E., June, 1904, entitled "The Breakwater at Buffalo, N. Y.," by Emile Low. See also Paper No. 15 of Transactions, Vol. IV—Part A, page 324, entitled "The Delaware, Sandy Bay and San Pedro Breakwaters," by C. H. McKinstry. Considerable valuable data on breakwater construction is contained in Vol. VIII—Part 4, Annual Report (1904) of Chief of Engineers, War Department (U. S.). The following illustrations are from these sources:

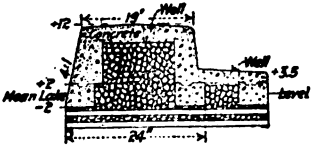

Fig. 1.—Buffalo (N. Y.) Breakwater.

Types of Breakwaters.—Fig. 1 is a plan of minimum cross-section for replacing the decayed timber superstructure of a portion of the old Buffalo breakwater, with a new superstructure of concrete and stone "shell construction." The maximum cross-section is 32 ft. on the harbor side and 2 ft. on the lake side, making a total width of 78 ft.

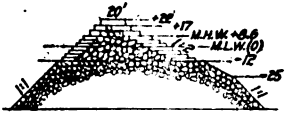

Fig. 2.—Sandy Bay (Mass.) Breakwater.

Fig. 2 is a plan of the Sandy Bay (Mass.) breakwater, showing the section adopted in 1902.

* The use of timber cribs projecting above high water is becoming more and more obsolete. But see Fig. 4.

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Fig. 3 is a cross-section of the Delaware Bay c
structed, 1897–1901. The total length of superstruct

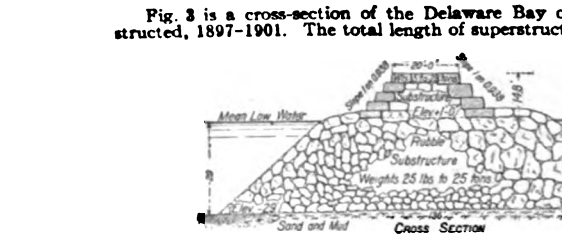

Fig. 3.—Delaware Bay Outer Break
$39.43 per lin. ft.; total length of substructure, 80
per lin. ft.

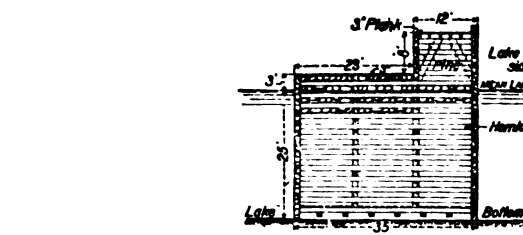

Fig. 4.—Oswego (N. Y.) Outer Brea

Fig. 4 is a cross-section of the Oswego (N. Y.) o
crib), constructed, 1884–1900.

AVERAGES PER LINEAL FOOT FOR FIG

Fig.	Materials.	Volumes Cu. Yds.
3	Cross-section above mean low water..	20.37
	Cross-section below mean low water..	108.30
	Total cross-section above sea bottom	128.67
	Rubble stone.....	120.14
	Capping stone.....	8.53
	Cost of superstructure.....	
	Cost of substructure.....	
	Total cost.....	
	(Approximate voids: rubble, 39%; capping stone, 10%.)	
4	Cross-section above mean lake level..	7.5
	Cross-section below mean lake level..	29.81
	Total cross-section above lake bottom	37.31
	Timber.....	7.44
	Stone.....	28.20
	Cost of old structure.....	
	Cost of foundation.....	
	Cost of new superstructure.....	
	Total cost 2970 ft.....	
	Total cost 570 ft.....	

Notable Breakwaters.—The following table was
the direction of Maj. Theo. A. Bingham, Corps of E

1.—STATISTICS OF NOTABLE BREAKWATERS.

Location.	Style of Breakwater.	Greatest Continuous Length. Ft.	Depth at Low water. Ft.	Total Height Ft.	Width of Base. Ft.	Width of Top. Ft.	Cost per Foot. $	Total cost of Continuous Section. $	Remarks.
Plymouth....	Rubble mound, capped with granite, laid in conc.	5100	32.5	55	380	65	1470	7,500,000	Long sea slope.
Portland.....	Rubble mound.....	6400	62.5	73	345	65	655	4,192,000	Long sea slope; does not include cost of quarrying stone; done by convict labor.
Tees.....	Rubble mound.....	12810	13.	46	221	51	00	000	Built of slag; price paid for removal of slag covered cost of construction.
Algiers.....	Rubble and concrete..... block mound.	.....	105.	.....	235	46	615	.....	Sea slope long'l; great saving of material; concrete, blocks 13 cu. yds. each.
Alexandria...	Rubble and concrete..... block mound.	9675	26.	40	100	43	1034	10,000,000	Concrete blocks, 13 cu. yds. = 20 tons each.
Marseilles....	Rubble and concrete ..... block mound, with slight superstructure.	11930	56.	80	257	110	500	6,000,000	Driveway 98.5 ft. wide, and path on top.
Cherbourg...	Rubble mound and superstructure founded at low water.	12178	50.	73	320	Mound 90 Sup'tr 30	1098	13,372,500	Greatest sheltered area, 2000 acres. Fort in center. 70 years building.
Holyhead.....	Rubble mound with superstructure founded above low water.	7860	41.	72	408	59	816	6,425,000	Driveway 40 ft. Path 13 ft. Long sea slope.
Genoa.....	Sorted rubble mound and superstructure, conc. toe.	.....	50.	84	316	M'nd 141 Sup'tr 49	640	.....	Driveway 40 ft. Path 9 ft. Storehouses in arches of superstructure.
Alderney.....	Sorted rubble mound and stone superstructure.	4680	123.	154	480	Mound 62 Sup'tr 35	1175	6,345,000	Driveway 30 ft. Path 14 ft. Greatest depth of water and area of cross-section in world, 4200 ft. from shore.
Dover.....	Upright wall stone and con.	.....	37.	75	60	48	1616	.....	Bottom of chalk, firm, was leveled off.
Buffalo.....	Rubble mound capped and crib work (conc. capping).	10000	30.	42	150	14	125	1,637,000	7250 ft. of rubble mound.
Delaware....	Rubble mound capped with stone.	8040	29.	44	136	20	217	1,746,000	Outer breakwater, on mattress foundation.
San Pedro...	Rubble mound with stone superstructure founded at low water.	8500	48.	62	180	20	285	2,901,000	Under construction. Estimated cost.
Sandy Bay...	Rubble mound with stone superstructure founded below low water.	9200	50.	72	205	20	361	6,828,000	Modeled after Delaware, not completed; cost estimated.

EXCERPTS AND REFERENCES.

The Sea Wall of La Punta, Havana (By W. M. Black. Eng. News, Nov. 14, 1901).—Illustrated: Fig. 1 shows section through steps of sea wall; Fig. 3 shows section of concrete toe with projecting stones to check run of waves. The article gives the composition of the various concretes used.

The Materials for the Concrete of the Buffalo Breakwater (By Emile Low. Eng. News, Sept. 11, 1902).—The gravel and sand were obtained from the bed of the Niagara River by means of a so-called "sand-sucker," described as follows: The vessel consists of a wooden hull 132 ft. long, 30.2 ft. beam and 7.2 ft. depth. The propelling machinery consists of a double non-condensing engine with a steam cylinder 14" dia. and 16" stroke. The boiler is allowed to carry 30 lbs. steam pressure. At the bow is located a centrifugal pump, driven by two direct-connected engines with cylinders 9" dia. and 9" stroke. The suction pipe is 12" dia., and is also a discharge pipe. Located on the deck of the scow is a large wooden box, 85' long, 24' 9" wide and 3' 10" deep, divided into two compartments by wooden bulkheads 4" thick. The capacity of the box is 325 cu. yds. The water charged with gravel and sand is pumped from the river bed (generally 12' deep) and flows into the flume, with screens of ¼" wire spaced ¼" apart in frames 11"x 24". Five tables are given showing various properties of the aggregates, and data regarding the manufactured concrete blocks.

Wave Action in Relation to Engineering Structures (By D.D. Gaillard. Professional Paper No. 31 of the Corps of Engrs., U. S. A.; Eng. News, Feb. 23, 1905).—Paper deals with: Definitions and theory; Height and Length of Waves; Reduction in Height of Waves on Passing into a Closed Harbor; Velocity of Waves; Per Cent of Wave Above Water Level; Depth in which Waves Break; Dynamometer Tests of Wave Force; Comparison of Theoretical Wave Force; etc. Tables and formulas are given.

Reinforced-Concrete Caissons for Breakwaters (By W. V. Judson. Eng. News, July 8, 1909).—Illustrated: plan of caisson and method of computing stresses. Estimated cost of Algoma breakwater, if built of stone-filled wooden cribs, placed on pile foundation and capped with a standard concrete superstructure (the cheapest permanent form) was $105.18 per lin. ft. Estimate for caisson breakwater was $103.74. Actual cost of caisson breakwater was $75.67+$2.62 per lin. ft.

Illustrations of Breakwaters, for Reference:—

Description.	Eng. News.
Plans of crib breakwater at Welland Canal entrance	May 15, 1902.
Plans of concrete breakwater at Buffalo (T. W. Symons)	May 29, '02.
Section and views of Cleveland breakwater	Mar. 23, '07.
Concrete breakwater at Harbor Beach, Mich.	Mar. 28, '07.
Sections of breakwaters in fishery harbors of Scotland; Cost	Dec. 22, '10.

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53.—JETTIES.

General Discussion.—At the mouths of navigable rivers where there is cross-current, whether the river empties into another or into the sea, sand bars are liable to form, shoaling the water and obstructing navigation The cost of dredging deep-water channels through these bars and maintaining them is sometimes enormous, hence jetties are often constructed to reduce this annual expense.

Jetties are structures designed to change the shape and velocity of the moving volume of water so that it will do the work, in part at least, of deepening the channel. The volume is contracted laterally and deepened vertically, and the velocity is increased, thereby cutting out the channel and carrying the material further on, some of it out to sea.

"Twin" jetties are formed by two jetty arms converging. One is called the "windward" jetty and the other the "lee" jetty. The windward jetty is placed on the side of the channel on which the sand-drift *predominates*; the "lee" jetty, on the other side. As twin jetties are very expensive it is customary on large projects to build one jetty first, watch the effect of scouring of channel, shifting of sand bars, etc., and then with this additional data plan the second arm. In selecting the single jetty, sometimes the windward- and sometimes the leë- jetty is chosen. The argument in favor of the latter is that the channel is practically confined from further movement "leeward," as the sand drifts from the "windward"* and is swept on or carried away by the current; whereas with the single windward jetty the channel may fluctuate in position instead of remaining permanent and deep. Each particular case requires a special solution.

The "reaction breakwater," so-called, is a single S-shaped jetty designed to increase the scour by creating a centrifugal force to the current, thereby narrowing the volume laterally and increasing the velocity, similar to the natural winding cuttings in river beds. The effect of this type of jetty has not fully been demonstrated, and will be watched with much interest.

Jetty Construction.—Most of the jetties now built are of rock fill, that is, rubble stone dumped from scows or trains. The trains are run out on temporary pile trestles constructed along the line of the jetty. The material is paid for by the yard or ton. Brush and rock are sometimes used but the use of brush has practically given way to rock alone, especially on large work, on account of the action of the teredo and liability to unequal settlement.

Small jetties and bank protections of streams are often constructed by driving two rows of piles, facing them on the inside with planking, bolting and bracing the rows together, and filling the space between with brush and rock. The brush is often tied together in bundles or *fascines* (see Fig. 1). The bottom layer is placed on the bed of the stream transversely to the direction of the jetty, and on these are placed other fascines laying longitudinally between the piles. Between the fascines and the bottom of the side planking,† boxes filled with rock are sunk between the piles; and the balance of the space above, between the planking, is filled with rock, forming a pile crib. This type is often used for the protection of bridge abutments, and frequently as real jetties in deepening the channel.

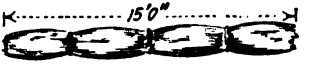

Fig. 1.—Fascine.

EXCERPTS AND REFERENCES.

The Improvement of the Entrance to Cumberland Sound, Georgia and Florida (By J. H. Bacon. Eng. News, May 12, 1903).—Shows general plans of the jetty-work improvement; table of official record of contract work; and table of sand movement at Cumberland Sound.

Complicated Reinforced-Concrete Jetty-Head, Thames Haven, England (Eng. News, April 22, 1909).—Illustrated.

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* The terms "windward" and "lee" relate to the sand drift and not to the direction of the wind, although they are frequently in the same direction.

† The side planking consists of plank spiked or bolted to the inside of piles and laid as far below low water as practicable.

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54.—EARTHWORK.*

Uncertain Cost.—In any engineering work the chance of an accurate estimate of the cost diminishes as earthwork becomes the important item; for in the purchase of materials and supplies a fairly correct estimate may be had in advance, but where the labor factor enters largely *in a direct way* great uncertainty exists. Especially is this true where the character of work to be *performed*, as in grubbing, clearing, and "earthwork," is problematical, the quality of labor uncertain, and the rate of wages unstable.

Before making an estimate the engineer should consider: (1) the kind and quality of material to be handled, by digging test pits and by boring; (2) the most approved method of doing the work; (3) the availability of good contractors, superintendents, and foremen; (4) the quality and price of labor. These will be considered in the order mentioned.

Kind and Quality of Material.—No contract should be let or work started before full information as to the character of the material has been obtained. The cost of pits and borings is merely incidental. Not only are they a practical guarantee of the correctness of the estimates but they give timely information necessary for the pruchase and proper disposition on the work of the necessary tools and machinery. The writer has seen steam shovels delivered where nothing short of dynamite could be used, and the expense of such changes and incidental delays is enormous. For trench work, test pits will generally determine in advance whether shoring plank will be required, so they can be ordered in time. One of the first things to look for is the material "hard-pan," "cemented gravel" or "glacial drift," as it is variously termed. When not classified it is a source of contention. Lying beneath the soil, unseen, it is often from 3 to 5 times as expensive to remove as is the material above, especially in trench work where it cannot be loosened economically or handled with the steam shovel. It comes between the earth and the loose rock or the sand-rock classifications and should be mentioned in specifications. Stiff clay is sometimes as hard to work as many of the hard-pans, and some idea should be had of the proportionate amount of this material. If it is large the estimate of cost of "earth" should be raised accordingly. Loam is about the easiest material to shovel by hand, while sand and gravel are more difficult. Loose and solid rock are considered under the next subject heading (Section 55). Sandrock is a partially formed sandstone. It may be in various degrees of hardness, and should receive a separate classification.

Approved Methods of Handling.—Space will permit only of the briefest mention of the methods employed, simply to recall them to the attention of the reader.

Clearing and Grubbing.—After the trees are cut down the stumps are usually blown out with giant powder (No. 2). An effective method, however, which is sometimes employed, is to snatch them out with a donkey engine. This has been done economically on railway right-of-way in the State of Washington, where the stumpage is thick. Quite a common method is to twist them out by using horses at the end of a horizontal lever chained rigidly to the stump; but only small stumps can be removed in this manner. The method of burning stumps is a slow process and not generally resorted to on engineering work. It takes many years for stumps to rot. The brush-hook is very serviceable in cutting brush, especially small willows and alders. Grubbing the roots of trees is best accomplished with the grubbing-hoe or mattock. Strange to say, estimates for clearing and grubbing are almost universally too low. The price is usually *per acre*, or sometimes per *station* on railroad work. In reservoir work for domestic water supply the grubbing must be done most carefully in order that the top soil may be removed unobstructed. Under railroad fills the

* For earthwork Tables, see Sec. 59, Railroads, page 1017, etc.

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often left standing, unless the depth of fill is small. For trench or pipe-laying, it is best that the grubbing and excavation con- together—to the same party.

ing Earth.—The team with driver, and plow with holder, are the mical outfit generally; but four or six horses, with two men at re often used on railroad work especially if the material is hard. bial "six-mule-team" requirement often denotes the upper limit earth classification. Hard-pan may be plowed generally with tfit but *not always economically* and it may be cheaper to use arse, black). For narrow trench work and for shallow bank ex- such material, the common pick is used.

g and Conveying.—For widening and deepening large railroad eam shovel stands pre-eminent. The size of the shovel (capacity dipper, or scoop) should be gaged to the depth of cut or rather to t of material which can be loaded on the cars from one position, g up" is expensive. A shallow cut would call for a pony shovel, shallow, hand shoveling might be cheaper. A cu.-yd. bucket is a or an average cut. In considering the use of any (steam) plant t on cost, the depreciation, the operating expenses, and the st be proportioned to the total yardage, hence the greater the e more expensive may be the outfit, provided of course that it tly economical while in operation. Errors are often made in oo complete a plant for the work; and also in the other direction. ial is usually loaded in trains of self-dumping cars operated by . If the delivery track to the steam shovel is on a proper grade, 2%, the cars may be operated by gravity in making-up the ns. If flat cars are used they may be unloaded cheaply with a the full length of train.

ng out long sand cuts, where the material can be hand shoveled sening, a good outfit consists of small dump-cars (side-dumping) train on a narrow gage track. There is usually more or less of ade from cut to fill, and this gravity power may be assisted by h draw the empty cars back.

heeled carts with removable slat bottoms, and two-wheeled are used in excavating cellars where the material is removed from t they have practically been crowded out by the wheel and drag ordinary railroad grading. The drag scraper is specially efficient sting from cuts and for short leads generally; the wheel scraper, for s. The Fresno scraper is popular in the West and is considered most efficient scrapers on the market. Wheelbarrows are used muddy places.

ous conveyor systems have been employed for side-waste from cuttings. The plane rubber belt and the movable-bucket types et demonstrated their practicability. The combined hoist and are generally employed in deep sewer excavations, large buckets Steel cableways are sometimes used in mining and river-bottom for dams and foundations.

Fig. 1.

avation of the Chicago Drainage Canal developed types of the ane and movable bridge conveyors unique in their magnitude. simply steel structures for conveying loaded cars from the canal over the spoil banks for dumping. The first named was a canti- econd a simple span, placed transversely to the canal and resting arallel with the canal (see Fig. 1).

Carson trenching machine is of this type. See "Cable Hoist- for general use on large work; published by The Trenton Iron

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There are many graders, trench machines and excavators on the market, many of which are greatly overrated in efficiency and capacity. Some of them work well in loose soils but are useless in the harder materials liable to be encountered. Caution should be exercised in their selection.

Under favorable conditions where there is plenty of water, excavation and fill by hydraulic method is sometimes the cheapest. The favorable conditions are a gravity supply of water obtained with little expense, the right kind of material as sand or fine gravel, sufficient grade from cut to fill, and a short conveying distance. The material is washed from the natural bank by concentrating a stream from a nozzle, or giant, and is carried in a sluice box under a constant stream of running water. The required grade of the sluice may be as great as 8 or 10 per cent for very coarse material. Earth dams constructed in this manner are plentiful in California. Part of the water front at Tacoma, Wash., has been filled by washing down the steep bank and sluicing the material into the bay. If there is no gravity supply and pumping is required the chances of the hydraulic method being the cheapest are greatly lessened.

One other method which may be touched upon in this connection is by hydraulic dredging. By the use of a rotary cutter and a suction pump the fine material from the bottom is sucked up and forced into a sheet-iron pipe leading to the shore. Filling operations along our water fronts bear witness to the efficiency of this method. Material is frequently delivered one-half mile from the dredge, and sometimes to a much greater distance.

Superintendence.—Some years ago a certain engineer known to the writer entered into a term contract as Chief Engineer with a contracting firm at a stated salary and a certain percentage of the profits. One of the first contracts secured was for the construction of headworks and a 30-mile pipe line for water works in one of our prominent western cities. The amount of the contract was over $900,000.00. The estimate was based on prevailing prices, the prices for material being guaranteed in most cases. A *profit of* 25% was added to all estimated costs, including materials furnished. The Treasurer* volunteered to go up and superintend the work, as it would be a nice outing for him. They had other contracts on hand but this was the largest, and the profit on the steel pipe material alone, delivered, was about $50,000.00. The wood-stave pipe construction was under an experienced man. The Treasurer was a fair book-keeper. When the work was about half completed the City Engineer became desperate and called at the main office. "Fully a hundred thousand dollars had been wasted on the line. Men all over the line working without sufficient tools; one gang of 12 with only one shovel between them, and no pick. Material from the trench being wasted improperly, that had to be used again for back-fill." Upon investigation it was found that these were facts; that the office force was up to its ears in book-keeping; that there was no *active* superintendent or line boss; that not one of the foremen had been furnished with a profile of the line or had been instructed where to make his spoil banks; and after the pipe had been laid, extra borrow pits had to be opened for back-fill! It proved to be an expensive "outing," and the loss fell most heavily upon the Treasurer himself who was the principal owner. Here is a case where a good superintendent could have saved the company ten times his salary.

Good foremen keep track of how much each man is doing, know what he ought to do, and whether he is doing a day's work. It is a mistake to drive men who are resting occasionally, as the one who "keeps moving" may not be doing half as much work. Poor foremen can wipe out the profits in a contract even although they may appear to be "hustlers."

Labor.—The common labor problem is a difficult one to solve. Generally, "white" labor is the most profitable but it is hard to secure, exclusively, on large work. Italian and Chinese come next in order. A method commonly employed with some contractors is to select one or two good men in each gang, secretly pay them from 25 to 50 cents more per day and let them "set the pace." As a further incentive a small bonus is sometimes offered to the other men if a certain amount of work is accomplished. In

* There was considerable rivalry between the Treasurer and the Secretary as to who should "boss" the job, which was acknowledged to have been the best one ever landed on the Pacific Coast up to that time.

he cost of earthwork the labor cost is the main item. It may n as a fixed rule that, with the same class of men the cost of ses more rapidly than the increase in wage, because under ay demanded by "good times" the men will actually do less ay; they will not work as steadily; and there will be more the working force, preventing thorough organization. Thus, if ork cost A dollars with labor at $1 per day, it might cost 2A the same labor at $1.75 per day, the extra cost, ¼A, being educe materially the contractor's profit. Hence, estimates based labor costs should be used with caution.

mbankment.—Dry earth embankment for railroad and other is universally carried up with slope of 1½ horizontal to 1 verti- ams and reservoir embankments, much flatter, as 2 to 1, 2½ to 1. The latter are built in horizontal layers and usually rolled, rmer are built largely by side and end dumping. To illustrate the best reservoir embankments are composed of fine gravel clay to form a good binder, being *puddled* by sheep or cattle ver it during construction, and sprinkled with a moderate vater. "Spike" rollers are an imitation of the above. Grooved etter than plane rollers.

ge of Earth.—Earth in place swells when loosened, but under f pressure and moisture it tends to recede to its original volume ne cases, to lesser volume than it previously occupied. The en illustrated in laying water pipe: A trench is dug, pipe is laid, k-filling is done with thorough wetting and tamping. Often, to be borrowed to fill the trench, notwithstanding that the pipe splaced a small portion of it.

ject of shrinkage has received much attention from various has provoked considerable discussion, especially during the ears. The variance in the different conclusions drawn is of mainly to the lack of definite and careful experiments and ex- data, and partly to incorrect analysis. The special problems ailway embankments are:

at quantities in "cut" (unmeasured) shall be assumed to be uivalent to the volume of the material after being placed in ill" (and measured therein for final estimate)?

at shall be the tentative cross-section of a fill so that when prac- ally final settlement has obtained, the lines of the fill or em- nkment shall be those of another cross-section, predetermined?

ssing these questions a great many factors have to be considered, ich are the following:

re Earths? Earths are mainly metallic oxides which are prac- luble in water, as alumina, glucina, yttria, thorina, etc.—the ths, as baryta, lime, magnesia, strontia, etc., being somewhat water. Chemically speaking, earths are dry, inodorous, un- substances, fusible only at extremely high temperatures.

Soils? Soils are earths, composed of the broken and weathered earth's crust, mixed with varying proportions of organic matter s, either decomposed or in a state of decomposition. Sedentary se soils which rest upon the parent rock from which they were ransported soils are those soils which have been removed and sewhere by the action of water, glaciers, or wind; if by wind, ally termed Aeolian soils.

nsity of Soil in its natural bed depends upon its composition, nd method of deposition; also upon the action of the vegetable life in the soil, in connection with the action of the percolating of frost. Thus, heavy soils, as hard clay and cemented gravel, y water or glacial action, under pressure, will be very dense; soils, as volcanic ash, deposited dry by the wind, will naturally ensity. But these light soils may become dense through the lacial action above described; while the heavy surface soils, so y in turn become light and porous through the action of the er—dissolving certain of the chemical elements and giving them lants or carrying them away, or carrying away (perhaps into soils), the small (or even large) suspended particles themselves;

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and the voids thus created being further increased by the action of frost, tending to swell the soil. Hence it is seen that soils may have different densities, even if of the same composition, when lying in their natural beds.

The Effect of Water on excavated soils is to settle them, mechanically, and to make them more compact; with the notable exception of clay, which swells when moistened and shrinks again when dried. Thus, with most soils the water will deposit the finely suspended particles of matter into the voids of the coarser material, decrease the friction, and settle it into a denser mass; while with clay the admixture of water produces a colloidal state, causing the mass to expand.

The Effect of Temperature on soil- or earth embankment is mainly the effect of drying out the water, or of producing frost action; the direct expansion or contraction due to temperature, being extremely slight, and negligible.

The Effect of Vertical Compression or Jarring on an earth embankment, is a downward tendency to settlement and a lateral tendency to expansion. This statement as to lateral expansion does not of course take into consideration any side-surface slides of the mass, or wash of slopes from rain.

The Effect of Stirring or Puddling is to make the soil or earth denser by decreasing the percentage of voids. The following experiment was made with coarse sand, which had all passed through a ¼-in. sieve (16 meshes to the inch): A box of one cu. ft. capacity was filled with the sand, then jarred to settle it, and again filled to the top. Water was then poured in, filling all the voids in the sand, the amount of water used being 0.342 cu. ft., thus measuring the voids in the sand as 34.2%. The wet sand was then thoroughly stirred, and settled in the box to 82.5% of its original volume, the voids in the stirred sand being therefore 20.2% of the final volume.

In the following discussion the term earth will be considered to include soil.

Swellage (*w*) of Earth takes place, usually, when it is first dug. This is due to the loosening of the material, thus increasing the voids. The more thoroughly it is loosened the greater will be the swellage. But it can immediately be Compressed or compacted to its original volume, or be made to Shrink below its original volume, if water and sufficient pressure are applied. The Percentage of Swellage is the percentage of increase in volume of the loose material excavated, based on the original volume of the material in situ.

Compression (*k*) of Earth takes place, more or less, in forming any embankment; that is, the loose material is compacted somewhat, even by its own weight, while being placed. A low embankment formed by shovel work, or by dumping from a cableway, or from a train supported on a trestle, especially during dry weather, would naturally show little compression during the short period of time required in its construction; while the material forming a high embankment might be compressed appreciably under the same conditions of construction, owing to the increased weight of the bank and the element of time, both of which are important factors. If the track were supported directly on the embankment itself, or the material delivered in wheelbarrows, or especially if carts or scrapers were used, the compression would be increased; while if the material were spread in thin layers and rolled with a heavy roller, the compression would be much greater. The Percentage of Compression is the percentage of reduction of volume in placing the material in the embankment, based on the volume of the loose material after being excavated.

Contraction (*c*) of Earth in embankment continues, perhaps, indefinitely; but the rate of contraction, under constant conditions, decreases with time. That is to say, under the same conditions the rate of contraction decreases as the material in the embankment becomes more dense. The rate of contraction, however, may be increased by pressure, by jarring or shaking and usually by moisture or sprinkling. Thus we have as factors tending to contract the material in the embankment: The pressure of the embankment itself, the pressure and jarring due to moving trains, and the sprinkling due to rainfall; added to this, is the wash of the slopes which, for our immediate purpose, will be included under the present heading. The Percentage of Contraction in Embankment is the percentage of decrease in volume at any stated time after the work is completed, based on the volume of the compacted materials as placed in embankment.

Shrinkage (*s*) of Earth is also measured in per cent, and is the ratio of the loss in volume of earth (measured) in embankment at any specified time.

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he original volume which the same material occupied before being l (that is, in situ).

kage may be obtained approximately by the following formula:

Notation (from preceding discussion):

ge % = ratio of increase in volume of loose material excavated, to olume of same material before being excavated;

ression % = ratio of decrease in volume of placed material in nbankment, to volume of same material loose after being ex-vated;

action % = ratio of decrease in volume of material in embankment t any specified time), to volume of same material when placed in nbankment;

kage % = ratio of decrease in volume of material in embankment t any specified time), to volume of same material before being xcavated.

Formulas:

Shrinkage $s = 1-(1+w)(1-k)(1-c)$ ..... (1)

n measured immediately embankment is completed,

$s = 1-(1+w)(1-k)$ ..... (2)

wrong conclusions arise in current discussions on shrinkage of rtly from the erroneous assumption of some writers that the is based on the volume of earth in embankment or fill, instead of iginal volume in excavation.

s 1, 2 and 3, following, give the approximate values of $(1 + w)$, $-c)$ and s, in Formula (1). They are intended merely as a vering average and extreme ranges, and in no way absolute, as ne apparent from the greatly varying conditions of construction y discussed.

PROXIMATE VALUES OF $(1+w)$ TO BE USED IN FORMULA (1).
(w = Swellage.)

Material.	Voids.	$(1+w)$.
sted rock, large masses, irregular	.47	1.80
ken rock, smaller masses, as for riprapping	.43	1.75
shed trap, granite and harder rocks, run of crusher	.41	1.70
shed limestone, sandstone and softer rocks, run of rusher	.39	1.65
rtz rock crushed fine—loose	.37	1.60
estone crushed to fine grains—loose	.35	1.55
cial drift, cemented gravel, clay-and-gravel, extra ense	*(.33)	1.50
ented gravel, hard—well loosened	*(.31)	1.45
ented gravel, muck, compact hard-pan, av	*(.29)	1.40
y-and-gravel, ordinary—well loosened	*(.26)	1.35
y-sand-gravel mixture, average	*(.23)	1.30
d-and-gravel, compact—well loosened	*(.20)	1.25
m, sandy loam, average	*(.17)	1.20
d or gravel, ordinary	*(.13)	1.15

itional voids due to loosening of material.

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2.—APPROXIMATE VALUES OF $(1-k)$ TO BE USED IN FORMULA (1).
(k = Compression.)

Class.	Material, and Method of Placing in Embankment.	$(1-k)$.
1	Blasted rock, large masses	1.00
2	Broken rock as for riprap: (a) Carelessly dumped	1.00
	(b) More carefully placed	.80
3	Crushed trap, granite and the harder rocks:	
	(a) Loosely placed	1.00
	(b) Thoroughly shaken in transportation	.93
	(c) Thoroughly rolled	.77
4	Crushed limestone, sandstone and the softer rocks:	
	(a) Loosely placed	1.00
	(b) Thoroughly shaken in transportation	.91
	(c) Thoroughly rolled	.74
5	Quartz rock crushed to sand, loose	.....
6	Limestone crushed to fine grains, loose	.....
7	Glacial drift, cemented gravel, clay-and-gravel, extremely dense:	
	(a) Cableways or wheelbarrows used, dry weather, low embankment	.80
	(b) Carts or scrapers used, some rain, medium embankment	.70
	(c) Material thoroughly sprinkled and rolled, high embankment	.60
8	Cemented gravel, or clay-and-sand, very hard, well loosened:	
	(a) Cableways or wheelbarrows used, dry weather, low embankment	.80
	(b) Carts or scrapers used, some rain, medium embankment	.71
	(c) Material thoroughly sprinkled and rolled, high embankment	.60
9	Cemented gravel, muck, and compact hard-pan, average:	
	(a) Cableways or wheelbarrows used, dry weather, low embankment	.80
	(b) Carts or scrapers used, some rain, medium embankment	.72
	(c) Material thoroughly sprinkled and rolled, high embankment	.60
10	Clay-and-gravel, ordinary, well loosened:	
	(a) Cableways or wheelbarrows used, dry weather, low embankment	.80
	(b) Carts or scrapers used, some rain, medium embankment	.73
	(c) Material thoroughly sprinkled and rolled, high embankment	.62
11	Clay-sand-gravel mixture, average:	
	(a) Cableways or wheelbarrows used, dry weather, low embankment	.80
	(b) Carts or scrapers used, some rain, medium embankment	.75
	(c) Material thoroughly sprinkled and rolled, high embankment	.65
12	Sand-and-gravel, compact:	
	(a) Cableways or wheelbarrows used, dry weather, low embankment	.85
	(b) Carts or scrapers used, some rain, medium embankment	.80
	(c) Material thoroughly sprinkled and rolled, high embankment	.67
13	Loam, sandy loam, average:	
	(a) Cableways or wheelbarrows used, dry weather, low embankment	.90
	(b) Carts or scrapers used, some rain, medium embankment	.80
	(c) Material thoroughly sprinkled and rolled, high embankment	.60
14	Sand or gravel, ordinary:	
	(a) Cableways or wheelbarrows used, dry weather, low embankment	.95
	(b) Carts or scrapers used, some rain, medium embankment	.90
	(c) Material thoroughly sprinkled and rolled, high embankment	.70

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ROXIMATE VALUES OF $(1-c)$ TO BE USED IN FORMULA (1).
(c = Contraction.)

, and Method of Placing.	One Year After Embankment is Completed.				After Final Settlement of Embankment.			
	Weather—				Weather—			
	Dry.	Wet.	Dry.	Wet.	Dry.	Wet.	Dry.	Wet.
	$(1-c)$	$(1-c)$	$*s$	$*s$	$(1-c)$	$(1-c)$	$*s$	$*s$
t, cem.-grav., etc., ex. den:								
eways, dry, low emb'km't.	.92	.84	-.10	-.01	.90	.80	-.08	.04
s, scrapers, rain, med. embk	.95	.90	.00	.05	.94	.88	.01	.08
erial rolled wet, high embk.	1.00	1.00	.10	.10	.99	.99	.11	.11
., clay-and-sand, very hard								
eways, dry, low emb'km't.	.92	.84	-.07	.03	.90	.80	- 04	.07
s, scrapers, rain, med. embk	.95	.90	.02	.07	.94	.88	.03	.09
erial rolled wet, high embk.	1.00	1.00	.13	.13	.99	.99	14	.14
., muck, hard-pan, average:								
eways, dry, low emb'km't.	.92	.85	-.03	.05	.90	.80	-.01	.10
s, scrapers, rain, med. embk	.95	.90	.04	.09	.94	.88	.05	.11
erial rolled wet, high embk.	1.00	1.00	.16	.16	.99	.99	.17	.17
grav., ord., well loosened:								
eways, dry, low emb'km't	.92	.86	.01	.07	.90	.84	.03	.09
s, scrapers, rain, med. embk	.95	.91	.06	.10	.94	.89	.07	.12
erial rolled wet, high embk.	.99	.99	.17	.17	.99	.99	.18	.18
-gravel mixture, average:								
eways, dry, low emb'km't.	.92	.87	.04	.10	.91	.84	.05	.13
s, scrapers, rain, med. embk	.96	.91	06	.11	.94	.88	.08	.14
erial rolled wet, high embk.	.99	99	.16	.16	.99	.99	.17	.17
-gravel, compact:								
eways, dry, low emb'km't.	.93	.88	.02	.07	.92	.85	.04	.10
s, scrapers, rain, med. embk	.96	.92	.04	.08	.95	.89	.05	.11
erial rolled wet, high embk.	1.00	1.00	.16	.16	.99	.99	.17	.17
ndy loam, average:								
eways, dry, low emb'km't.	.92	.84	.01	.09	.90	.80	.03	.14
s, scrapers, rain, med. embk	.95	.90	.09	.14	.94	.87	.10	.17
erial rolled wet, high embk.	.99	.98	.29	.29	.98	.97	30	.30
ravel, ordinary:								
eways, dry, low emb'km't.	.96	.92	-.05	.00	.95	.90	-.03	.02
s, scrapers, rain, med. embk	.98	.96	-.02	.00	.97	.95	-.01	.01
erial rolled wet, high embk.	1.00	.99	.19	.20	.99	.98	.20	.21

nents on the Shrinkage of Earth are scattered, and most of them
'actory because the data are not complete. In all experiments
ge the prismoidal formula should be used in the calculations,
rrors amounting to from 3 to 10 per cent would be quite usual in
and enough to destroy the value of the results obtained.
comes necessary to base an estimate on the quantity in embank-
eas the contract price was based on the quantity in excavation,
s should be made on the shrinkage of the material.† The usual
making these experiments is to dig a rectangular-shaped pit, or
then place the excavated material back in the pit or trench
itions similar to those in making the embankment. The shrink-
t will then be the difference between the original volume of the
final volume of the placed material, divided by the original
he pit.

Morris, about two-thirds of a century ago, made some measure-
large scale, on the shrinkage of earth. "The embankments were
ayers by carts and scrapers, and one winter intervened between

of s are actual shrinkage averages from original volume in
Where the minus sign appears before these values, it indicates
rinkage, or actual increase in volume by that amount.

contracts on railroad work stipulate that when material is
n embankment a certain percentage, say 10%, shall be added
ge.

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the commencement and completion of the work." The results of the measurements, involving nearly 44,000 cubic yards, are: Shrinkage of yellow clayey soil, 9.25 to 10.15 per cent; shrinkage of light sandy soil, 12.93 per cent; mean average shrinkage, 10.3 per cent. Based on these experiments some authors give the "shrinkage of earth" as 9 to 13 per cent, while others have widened the range to 8 to 15 per cent. Most railroads use about 10 per cent as an average working basis.

The Am. Ry. Eng. & M. of W. Assn. Committee Report for 1907 recommends shrinkage allowance for both height and width in new banks, 20 replies favoring this, while two favored allowance for vertical shrinkage only, and two for horizontal shrinkage only. The following shrinkage values were recommended:

	Suggested by Members.	Recommended by Committee.
Black dirt, trestle filling	7 to 30%	15%
Black dirt, raising under traffic	4 to 20%	5%
Clay, trestle filling	5 to 30%	10%
Clay, raising under traffic	2 to 20%	5%
Sand, trestle filling	3 to 15%	6%
Sand, raising under traffic	2 to 15%	5%

In building the Tabeaud Dam, 1900-1902, near Jackson, Cal., tests of the earth material used showed the following average weights per cu. ft.:

Dust dry soil (angle of repose 36°)	84.0 lbs.
Soil fully saturated, 52% moisture (angle of repose 23°)	101.7 lbs.
Natural bank soil, 19% moisture (angle of repose 44°)	116.5 lbs.
Delivered from wagons, moist and loose (showing swellage of 52%)	76.6 lbs.
Loose dirt from dam, shaken down and measure struck (swellage 45.6%)	80.0 lbs.
Material in dam, 38% gravel and grit, thoroughly sprinkled and rolled (showing shrinkage of 12.4%)	133.0 lbs.

Experiments made by Mr. D. C. Henny, on earth material for the Cold Springs Dam, Umatilla Irrigation Project, Oregon, show the following results:

*Sample.	Specific Gravity.		Percentage Voids.			†Uniformity Coefficient.	‡Effective Size—mm.
	Constituents.	Mass.	Dry. Loose.	Dry. Compact.	Wet Rammed.		
A. Surface soil	2.52	1.41	59	49	44	3.9	.023
B. Fine subsoil	2.65	1.55	54	43	45	4.0	.025
C. Gravel	2.90	1.91	42	37	34	9.2	.559
D. Volcanic ash	2.66	0.94	74	68	65	5.5	.010
E. Coarse subsoil	2.93	1.57	55	49	46	1.5	.079
Mixtures: B. C.							
75% 25%	2.64	1.75	50	39	38	3.0	.066
67% 33%	2.83	1.76	47	41	39	.....	.....
50% 50%	2.90	1.91	42	33	34	14.7	.076
33% 67%	2.87	1.95	41	35	32	20.0	.076
25% 75%	2.83	2.01	47	43	29	21.0	.086
20% 80%	2.91	2.04	47	40	30	76.7	.091
15% 85%	2.84	1.88	41	35	34	18.0	.162

*Sample A is the 12-in. surface soil in the bottom lands, being darker in color than the deeper-lying soil, but containing a sensibly greater quantity of organic matter, as roots and vegetable fibers. Sample B is taken from 1 to 4 ft. below the surface, being slightly lighter in color than A, and with less vegetable matter. Sample C is coarse sand gravel and from the steep side-hill, the mass of the material being a coarse sand or fine gravel with a considerable proportion of gravel that would be retained on a one-inch mesh; also occasional large cobbles, and a scantiness of fine sand. Sample D is volcanic ash, almost pure white in color; when in place it appears to be slightly indurated. Sample E is a coarse soil, coarser in appearance than A and B but otherwise greatly resembling them; it lies from 1 to 4 ft. below the surface. Samples A, B and E all contain a considerable proportion of volcanic ash and vegetable matter, and are fairly representative of soils in that section of the West. †Massachusetts State Board of Health standard.

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Experiments made by Mr. Emery Sudler, on soils for use in construction of earth dam for water-works reservoir, Baltimore, Md., show the following results:

Properties or Condition of Material.		Yellow to Brown Clay, from layer 2-ft. to 4-ft. thick.	Soft, rotten Serpentine Rock, below the clay.
Weight in lbs. per cubic foot when moist.....	In place.....	116.	108.
	Loose.....	78.	74.5
	Compressed (in 4-in. layer in 6-in. test pipe, at about 150 lbs. per sq. in.).....	131.6	129.3
Volume in cubic feet corresponding to one cubic foot in place..	Loose.....	1.5	1.45
	Compressed	.86	.84
Absorption by weight.....		.007	.007

This table shows that when loosened the clay swelled 50% and the rotten rock swelled 45%; after being compressed the loose clay showed a compression of 40.7% and the loose rotten rock a compression of 42.4%; while the ultimate shrinkage of the material in place when compressed, was: clay, 11.9% and rotten rock, 16.5%.

Shrinkage in Volume—Vertical Shrinkage.—If each particle in an earth embankment shrinks vertically (not laterally) how will the settlement of the top of an embankment compare with the shrinkage in volume?

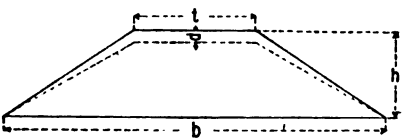

Fig. 2.

Solution.—Let the full lines, Fig. 2, represent the original fill, and the dotted lines the top and slopes after vertical shrinkage. Then will $d+h$ represent the ratio of vertical shrinkage, as well as shrinkage in volume. For

$$\frac{\text{Final volume}}{\text{Original volume}} = (h-d)\frac{(b+t)}{2} + h\frac{(b+t)}{2} = \frac{h-d}{h} = 1 - \frac{d}{h}.$$

Earthwork is always measured in excavation where possible; but it often happens that a contractor will start in on a borrow-pit before it is cross-sectioned. For this and other reasons it is sometimes necessary or advisable to measure up the embankments as a basis for payment. Considerable judgment must therefore be exercised in the question of "shrinkage."

Performance of Work.—The following items are selected and digested from the columns of *Engineering-Contracting*,* and give what may be considered fair averages of good work under the conditions named. For detailed information see original articles. References are made to the files of *Eng'r-Contr.* in the following manner; thus, *E.-C.*, (date, page).

Sewer Trench in stiff clay, wet and soft in spots, but tough digging; at West Allis (near Milwaukee), Wis.; by "Buckeye Traction Ditcher," a machine with buckets on the periphery of a large wheel, operated by steam, costing about $4600.00, and manufactured by the Van Buren, Heck & Marvin Co., of Findlay, O. Estimated performance (less than actual) 900 lin. ft. of trench 2 ft. wide by 7 ft. deep in 10 hrs. at cost of 5 cts. per cu. yd., including all items of pay roll, fuel, interest and depreciation.—*E.-C.*, Jan., 1906, p. 7.

* Published weekly by the Myron C. Clark Publishing Co., Chicago, Ill.

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New York Subway. Earth excavation in a typical section of about ½ mile. Brooklyn Extension: Excavation proper (labor 1.60, materials and plant 0.32, power 0.02, dump charges at 60 cts. per load 0.25), $2.19 per cu. yd.; bracing and sheeting (labor 0.78, materials and plant 0.37), $1.15 per cu. yd.; pumping and draining (labor 0.01, materials and plant 0.01, power 0.01), $0.03 per cu. yd.; bridges and barricades (labor 0.10, materials and plant 0.14), $0.24 per cu. yd.; backfilling (labor), $0.01 per cu. yd. Grand total, $3.62 per cu. yd.—*E.-C.*, Feb. 1906, p. 30.

Panama Canal. Cost per cu. yd. of mixed excavation (96052 c. y. hard rock, 254252 c. y. soft rock, 391340 c. y. earth), for a total of 741644 cu. yds., between July 1, 1904, and June 30, 1905: July, 65.4 c., Aug., 50.6 c., Sept., 57.3 c., Oct. 54.1 c., Nov. 50.1 c., Dec. 52.8 c., Jan. 47.8 c., Feb. 46.5 c., Mar. 43.3 c., Apr. 52.5 c., May 83.8 c., June 102.7 c. Excavators and steam shovels were used. Roughly, the above costs include up-keep, depreciation, etc., of plant.—*E.-C.*, Feb., 1906, p. 44.

Intercepting Sewers, Chicago. (1) Excavating with Ennis type derrick, equipped with a 1 cu. yd. Haywood orange-peel bucket, an 8¾" by 10" double-drum hoisting engine and 60-ft. boom. Cost, exclusive of wear and tear of machinery, 6.6 cts per cu. yd. (2) Other methods were used also.—*E.-C.*, Apr., 1906, p. 96.

R. R. Excavation with Elevating Grader. 7 examples, by D. J. Hauer. Material, average earth when dry. Machine built by the Nat'l Drill and Mfg. Co. of New York City. Wagons, drop bottom patent dump, of 3-yds. capacity. (1) R. R. cut, 400 ft. long, 45 ft. wide on top. Cost (loading 0.130, handling 0.111, dumping 0.041, water boy 0.001, foreman 0.012), 29.5 cts. per cu. yd. Av. output, 206 cu. yds. per day; lead, 400 ft. (2) R. R. cut about 1400 ft. long, 20 ft. wide and 2 ft. deep. Cost (loading 0.067, handling 0.078, dumping 0.011, water boy 0.002, foreman 0.007), 16.5 cts per cu. yd. Av. capacity, 380 cu. yds. per day; lead 1000 ft. (3) R. R. cut, 30 ft. roadbed for double track. Cost (load. 0.046, haul. 0.072, dump. 0.016, water 0.001, foreman 0.005), 14 cts. per cu. yd. Max. output at top of cut, 510 cu. yds. per day; av. output, 300 cu. yds; lead 600 ft. (4) R. R. cut for double track. Cost (load. 0.108, haul. 0.149, dump. 0.019, water 0.003, foreman 0.010), 28.9 cts. per cu. yd. Av. output, 284 cu. yds. per day; lead, 700 ft. (5) R. R. cut. Cost (l. 0.061, h. 0.077, d. 0.018, w. 0.002, f. 0.006), 16.4 cts. per cu. yd. Av. output, 417 cu. yds. per day; lead, 500 ft. (6) R. R. borrow-pit. Cost (l. 0.098, h. 0.094, d. 0.049, w. 0.003, f. 0.009), 25.3 cts. per cu. yd. Av. output, 260 cu. yds. per day; could have been increased if more wagons had been used. Lead, 500 ft. (7) R. R. borrow-pit. Cost (l. 0.153, h. 0.260, d. 0.050, w. 0.002, f. 0.015), 48 cts per cu. yd. Av. output, 167 cu. yds. per day; material hauled 1700 ft.; management inferior.—*E.-C.*, Apr., 1906, p. 102.

Steam Shovel Work on Ann Arbor R. R. in 1895. Cost figures cover loading, hauling and placing under track, but make no allowance for rental of plant, locomotive or cars, nor for depreciation of plant. Labor, $1.15. Cost per cu. yd.: Sand, 7.22 to 13.88 cts; sand (very light face), 17.24 cts.; sand (all work lowering by hand charged against this cut), 25.44 cts.; sand (light face), 13.25 cts.; quicksand, 13.98 and 15.95 cts.; gravel, 8.93 and 14.37 cts.; gravel (long haul), 19.81 cts.; clay, 9.60 to 14.01 cts.; clay (hard pan), 17.65 cts.; sand and gravel, 6.48 and 8.58 cts.; sand and gravel (very light face), 17.31 cts.; sand and clay, 10.49 cts.—*E.-C.*, May 30, 1906, p. 151.

Sewer Tunnel at Cleveland, Using Hydraulic Shield. Sewer, 13½ ft. dia., built of four rings of No. 1 shale brick laid in Portland cem. mortar. Shield, 16½ ft. dia., 4 ft. long, of ¾" metal, and weight about 16 tons; upper half provided with follower 7 ft. long, of ¾" steel, bolted to shield. Shield pushed forward by 12 hydraulic jacks, 5" dia. and 26" long. Water led to jacks by pipes with swinging joint; av. pressure about 700 lbs. per sq. in., but pump could develop 6000 lbs. Material, hard, dry quicksand, sometimes mixed with clay. Cost of tunnel per lin. ft.: 8 c. y. excav., underground labor, at 73 cts.=$5.44; 8 c. y. excav., surface labor, at 48 cts.=$3.82, 2.62 c. y. brickwork, underground labor, at $1.12=$2.95; 2.62 c. y. brickwork, surface labor, at 73 cts.=$1.91; 1100 bricks @$9 per M=$9.90; 2.1 bbls. cement (1:3 mortar), at $1.70=$3.57; 1 c. y. sand at $1=$1.00; plant, 50% of first cost, distributed over 1625 lin. ft.=$5.00; lumber=$1.05; shafts or manholes=$1.00. Total $35.64.—*E.-C.*, July 25, 1906, p. 22.

Clearing and Grubbing Land; and Blasting Stumps. Area, 9 acres; trees, 6 ins. to 3 ft. in dia., with average about 20 ins., consisting of oak, hickory, chestnut, etc.; number of trees cut was over 1100, and number of stumps blasted was 1212. Trees under 6" dia. classed as brush, and stumps were grubbed with mattocks. For blasting stumps the following were used: 1 churn drill, 1 large auger, and 1 bucket, costing in all about $80. Total cost per acre was as follows, with Italian labor at $1.25: Chopping $18.84, grubbing and clearing $15.53, making cord wood $10.14, blasting $73.73, grubbing after blasting $35.26, grinding axes $0.65, tools $9.00; total cost per acre $163.25.—*E.-C.*, Feb. 27, 1907, p. 98.

Wash Drill Borings, Deep Waterways Survey, Great Lakes to Atlantic Tide Waters, 1897–1900. The process consisted in alternately "driving casing" and "drilling" until "bottom" was reached. Where obstructions were encountered that could not be passed by drilling, they were removed by pulling the drill rod and lifting the casing 3 or 4 ft. and then firing a stick or two of dynamite at the bottom of the hole. (a) On the Tonawanda-Olcott and La Salle-Lewiston Routes, compris-

ing 404 holes bored to an aggregate depth of 9624 ft. The cost of borings (including total cost of plant) was 68.63 cts. per lin. ft., the material being sand, gravel, clay, and hardpan. (b) On the Western Division of the Oswego-Mohawk Route—from Oswego to Rome—750 holes were bored to an aggregate depth of 33711 ft., and at an average cost of 70.07 cts. per lin. ft. For the Oswego river and harbor work, the machines were mounted on small flatboats with open wells at the center; and the work on Oneida Lake was done through the ice. (c) On the Eastern Division of the Oswego-Mohawk Route there were made 290 soundings by hand with a steel rod, and 1562 actual borings, together amounting to 55521 ft., aggregate depth, at a cost of 54.19 cts. per lin. ft. The borings varied from a few feet to 190 ft. in depth. Four types of machines were used, viz.: 1 Pierce well-boring machine, 1 Sullivan wash drill, and 2 home-made affairs. The material encountered was, sand, 20706 ft.; clay, 9880 ft.; earth, 7611 ft.; sand and clay, 3176 ft.; gravel, 2815 ft.; sand and gravel, 2728 ft.; sand, clay and gravel, 1843 ft.; quicksand, 1529 ft.; clay and shale, 903 ft.; sand, loam and mud, 900 ft., clay and gravel, 760 ft.; rock, 626 ft.; mud, 417 ft.; miscellaneous, 1628 ft. (d) On the Champlain Route from Ogdensburg to Lake St. Francis there were 148 sand borings totaling 7052 ft., and 151 water borings totaling 2123 ft. The cost of the 9175 ft. of borings was 84.18 cts. per lin. ft. (e) On the Hudson River Division of the Champlain Route 57991 lin. ft. of borings cost 12.35 cts. per lin. ft. (f) On the Hudson River Survey, Hudson to Troy, the borings were made with an outfit mounted on a catamaran and on scows. Silt, clay, coarse and fine sand, gravel, and boulders were penetrated. A 2½-in. casing and "B drill rods," with X-bits were used. In all, 1385 borings were made aggregating 28965 ft. in depth and costing 25.07 cts. per lin. ft.—*E.-C.*, Mar. 27, 1907, p. 132.

Diamond Drill Borings, Deep Waterways Survey, Great Lakes to Atlantic Tide Waters, 1897–1900:

Route	(1) Tonawanda-Olcott.	(2) La Salle-Lewiston.	(3) Oswego, W'n Div'n.	(4) Champlain; N'n Div n
No. of holes	9	5	7	4
Depth in feet	1022.9	574.4	521.6	342.5
Standpipe, feet	118.4	101.0	241.7	90.7
Rock drilled, feet	904.5	473.4	279.9	251.8
Cost of boring, per lin. ft.	$2.494		$3.738	$4.366
Cost Details, per Lin. Ft.: Rental of drilling outfit	0.769		1.150	1.099
Carbon	0.423		0.139	0.490
Labor	0.542		1.016	1.024
Teamster	0.124		0.345	0.368
Teaming, extra	0.077		0.177	0.190
Superintendence	0.321		0.479	0.547
Repairs	0.042		0.101	0.072
Coal (and wood)	0.039		0.066	0.073
Lumber	0.030		.....	.....
Core boxes	0.015		0.010	0.020
Freight and Express	0.042		0.143	0.225
Traveling expenses	0.042		0.058	0.201
Sundries	0.028		0.054	0.057

—*E.-C.*, Mar. 13, 1907, p. 108.

R. R. Grading with Wheeled Scrapers. Five examples:

Material.	Ex. 1. Sandy loam.	Ex. 2. Good clay.	Ex. 3. Wet clay.	Ex. 4. Fine sand.	Ex. 5. Loam-clay-sand.	Av.	%
ead, in ft.	260.	300.	400.	500.	700.	432.	
oreman ($3.00)	$.017	$.019	$.026	$.024	$.020	$.021	6.0
crapers ($4.75)	.138	.158	.216	.222	.210	.189	51.6
owing ($9.20)	.052	.057	.080	.073	.053	.063	17.2
atching ($6.00)	.034	.037	.052	.050	.030	.040	10.9
oaders ($1.60)	.018	.020	.028	.026	.020	.022	6.0
imping (men $1.50)	.008	.016	.039	.027	.033	.025	6.8
ater boy ($1.00)	.006	.004	.009	.008	.001	.006	1.5
tal cost per cu.yd.	$.273	$.311	$.450	$.430	$.367	$.366	(100.0)

—*E.-C.*, Sept. 25, 1907, p. 184.

Trenching and Backfilling for Pipe Sewer, Centerville, Iowa. Data from which le was compiled was furnished by Mr. M. A. Hall, engineer in charge, from daily orts by inspectors, and Mr. Hall suggests *adding* 10% for possible omissions in

labor rate. There were 53 jobs in all, and these are numbered by "side and top" method. The following information is p—dia. of pipe, in ins.; t—size of trench, width by depth; d—yard for digging trench; b—cost in cents per cubic yard to 18 cts. per hour.

[Add 10% to *d*: =cost (cents) per cu. yd. for digging; *b*=same for back-filling.]

		0	1	2	3	4	5	6	7	8	9
0	p		1 15"	2 15"	3 15"	4 15"	16"	5 12"	6 12"	7 12"	8 12"
	t		3 x 6.5'	3 x 7.7'	3 x 8.9'	3 x 9'	3 x 3.5'	2 x 7.7'	2 x 8 2'	2 x 8 2'	2 x 8.6'
	d		26.75c.	14.07c.	48.77c.	44.68c.	19.23c.	34.61c.	53.27c.	32.62c.	57.92c.
	b		.....	7.27c.	5.88c.	4.24c.	7.28c.	.....	.....	4.25c.	8.30c.
10	p	9 12"	10 12"	11 12"	12 10"	10 10"	5 8"	5 8"	10 8"	5 8"	12 8"
	t	2 x 9.6'	2 x 10.7'	2 x 11'	2 x 7'	2 x 12'	22" x 6.3'	22" x 7.3'	22" x 7.3'	22" x 7.8'	22" x 8.1'
	d	43.72c.	41.34c.	50.33c.	15.70c.	52.14c.	35.53c.	40.04c.	36.92c.	25.00c.	45.07c.
	b	8.89c.	2.75c.	9.28c.	9.00c.	.....	4.67c.	5.78c.	.....	7.81c.	9.35c.
20	p	2 8"	13 8"	11 8"	10 8"	14 8"	14 8"	15 8"	16 8"	17 8"	10 8"
	t	22" x 8.1'	22" x 8.2'	22" x 8.6'	22" x 8.9'	22" x 9'	22" x 9'	22" x 9.1'	22" x 9.1'	22" x 9.2'	22" x 9.3'
	d	39.93c.	46.52c.	35.76c.	36.47c.	41.10c.	40.57c.	35.98c.	63.90c.	33.03c.	37.19c.
	b	5.29c.	10.89c.	4.15c.	4.30c.	3.39c.	4.75c.	—.....	11.90c.	12.38c.	5.13c.
30	p	10 8"	18 8"	14 8"	14 8"	19 8"	10 8"	10 8"	20 8"	10 8"	10 8"
	t	22" x 9 5'	22" x 9.5'	22" x 9.5'	22" x 9.6'	22" x 9.6'	22" x 9.7'	22" x 9.8'	22" x 9.9'	22" x 10'	22" x 10'
	d	33.08c.	31.73c.	43.50c.	45.13c.	37.68c.	37.47c.	46.49c.	45.89c.	41.18c.	33.18c.
	b	7.08c.	7.51c.	4.66c.	5.91c.	6.34c.	5.52c.	5.58c.	7.17c.	4.72c.	4.54c.
40	p	21 8"	10 8"	22 8"	14 8"	17 8"	23 8"	23 8"	10 8"	10 8"	17 8"
	t	22" x 10'	22" x 10.1'	22" x 10.1'	22" x 10.1'	22" x 10.3'	22" x 10.4'	22" x 10.4'	22" x 10.6'	22" x 10.6'	22" x 10.7'
	d	36.31c.	39.18c.	57.22c.	38.69c.	24.67c.	45.26c.	42.64c.	48.61c.	33.37c.	37.62c.
	b	4.31c.	8.04c.	5.42c.	9.59c.	5.33c.	.....	18.67c.	3.36c.	6.29c.	.....
	p	24 8"	25 8"	26 8"	26 8"						

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draulic Fill Dams. (1) Croton dam on the Muskegon River, Mich.; a feet below the junction of the Big and Little Muskegon. Sluiced em- 50 ft. long, 55 ft. high and 20 ft. wide on top; containing 104000 cu. and gravel. Maximum distance sluiced, 800 ft.; grades generally 8 to of finished embankment, 2 to 1 and 3 to 1. Two Ramsey and two rotary fire pumps were used, operated by electric power. Total cost nt, and superintendence (½ cent), 7.3 cents per cu. yd. (2) Lyons dam, valley. Embankment 150 ft. long and containing 23000 cu. yds. of avel was sluiced into place. Total cost including plant minus salvage, endence (3 cents), 36.01 cts. per cu. yd. The high cost was due to estles to carry the troughs, and from trouble with ice.—*E.-C.*, Nov. 13,

; Steam Shovel Excavation. (1) Earth. Excavation by steam shovel ansportation 19.42 cts., tracks 8.30 cts., dumps 15.48 cts., general ex- s., arbitrary to cover cost of plant 12.00 cts.; grant total 67.94 cts. per Rock. Add 25.57 cts, to above for blasting, making grand total of r cu. yd. (3) Dredging. Cost of dredging 1,213,597 cu. yds. of soft ma- measurement, by one dredge was about 10½ cts. per cu. yd.—*E.-C.*, p. 32.

ork Classification.—The object of classification is to fix a basis ing the cost of work by unit prices which will be fair for any terial encountered and for any reasonable changes in the original st of all, the engineer who draws up the specifications should y in mind the kinds of material he is classifying, not merely by h is often misleading, but by a thorough practical knowledge demonstrated to intending bidders. The writer, as contracting as been on work where test pits were dug showing material engineer who drew up the specifications could not classify him- mpression received, however, was that it would be classified as gravel," but when the work was completed it was paid for as earth" excavation, although blasting was the cheapest method g it. In fairness to the engineer it should be stated that he ved the powder bill, which was quite an item; but it did not xtra expense for that material over our unit price for "common

of the above, the writer believes that all material which cannot d accurately in specifications should be exposed to view in test nples of classification, and referred to as such, together with s of same as accurate as possible.

Classifications.—The following is from the very valuable paper Dennis, entitled "Uniformity of Requirement and Clearness of n in Agreements for the Graduation of Railroads," published in C. E., June, 1907 (p. 321).

SOLID ROCK. (Abstracts from Specifications.)

. H. & H.—All rock or stone containing 1 cu. yd. or more. All other h. *Erie.*—Rock in masses exceeding 1 cu. yd., which cannot be re- ut blasting. *P. R. R.*—[Same as Erie.] *B. & O.*—Rock in solid beds or h may best be removed by blasting. *Ches. & O.*—Rock in ledges and sses exceeding ½ cu. yd., which may best be removed by blasting. —Rock in masses which may best be removed by blasting. *Southern.*— ses of more than 1 cu. yd., which may best be removed by blasting. —Stone in solid masses or ledges. *C., B. & Q.*—Stratified rock weighing 0 lbs. per cu. ft., which can only be removed by blasting. *Chi. & Alt.*— rock and rock occurring in masses which can only be removed by blast- ust ring under hammer. *Great Nor.*—Rock in place, in removing which y to resort to drilling and blasting. *A., T. & S. F.*—Rock in solid beds its original or stratified position - - - - other material which can be hout continuous drilling and blasting, but which is as difficult - - - - - or limestone. *Ill. Cent.*—Rock in solid beds or masses in its original - - which may best be removed by blasting. Everything else classi- hon excavation. *N. P.*—All rock in masses that cannot be removed ng and blasting. *Mo. P.*—Rock in solid beds or masses, in its original ch can only be removed by continuous blasting.

LOOSE ROCK. (Abstracts from Specifications.)

ate, shale or other rock which can properly be removed by steam r bar, without blasting, although blasting may be resorted to on favor- s to facilitate the work - - - - detached masses, 3 cu. ft. to 1 cu. yd one and detached rock lying in separate and continuous masses con- ver 1 cu. yd.; also all slate or other rock that can be quarried without

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blasting, although blasting may be resorted to occasionally. *B. & O.*—Slate, coal, shale, soft friable sandstone and soapstone, detached masses 3 cu. ft. to 1 cu. yd. *Ches. & O.*—Shale, slate, ochre, which can be removed with pick and bar, and is soft and loose enough to be removed without blasting, although blasting may be resorted to occasionally. Detached masses 3 cu. ft. to 1 cu. yd. *Norf. & W.*—Shale soapstone, and other rock which can be removed by pick and bar, and is soft and loose enough to be removed without blasting, although blasting may be resorted to occasionally. Detached masses 1 cu. ft. to 1 cu. yd. *Southern.*—[Same as Norf. & W.] *"Big Four."*—Shale, coal, slate, soft sandstone, soapstone, conglomerate stratified limestone in layers less than 6 in. Detached masses 3 cu. ft. to 1 cu. yd. *C. B. & Q.*—Stratified rock which can be removed by pick and bar and weighing more than 140 lbs. per cu. ft. Detached masses 3 cu. ft. to 1 cu. yd. *Chi. & Alt.*—Stratified rock which can be removed by pick and bar - - - - and masses between 3 cu. ft. and 1 cu. yd. *Great Nor.*—Slate and other rock, and loose enough to be removed without blasting, although blasting may be resorted to occasionally. Detached masses 3 cu. ft. to 1 cu. yd. *A., T. & S. F.*—Hard shale or soapstone - - - - in original or stratified position, boulders in gravel, cemented gravel, hardpan - - - and other material requiring - - - - use of pick and bar or which cannot be plowed with 10-in. plow and 6-horse team. *Ill. Cent.*—(No loose rock. Everything but solid rock classed as common excavation.) *N. P.*—Slate, soft sandstone, or other rock than can be - - -removed without blasting. Detached rock between 1 cu. ft. and 1 cu.yd. *Mo. P.*—All rock - - - - which requires for its removal steam shovel or pick and bar, without blasting, although blasting may be resorted to at the option of the contractor. Detached masses 1 to 18 cu. ft.

EXCERPTS AND REFERENCES.

Some References to Earthwork and Especially to Shrinkage.—(1). Notes on Earthwork. By Geo. J. Specht. Tech. Soc. Pac. Coast Transactions, May, 1885. A collection and digest of data on shrinkage up to that time. (2). Shrinkage of Earthwork. By P. J. Flynn. Tech. Soc. Pac. Coast Transactions, read June 5, 1885. Refers to experiments on shrinkage, made in India, and gives tables of shrinkage. Also gives numerous references and data. (3). Shrinkage—Growth. J. C. Nagle. "A Field Manual for Railroad Engineers," 1887. (4). Shrinkage of Earthwork. W. M. Patton. "Civil Engineering." Patton says: Sand shrinks about 10%; sand and gravel, 8%; earth and loam, 10 to 12%; gravelly clay, 8 to 10%; puddled clay and soil, 20 to 25%; rock excavation produces a larger mass by from 25% in cases of small fragments and 60 or 70% when in blocks carelessly piled up. (5). Shrinkage of Macadam Under Rolling. See Eng. News of Feb. 11, 1904. (See, also, Eng. News, Jan. 14 ,1904, under Macadam Road Construction Along Charles River.

A Novel Method of Constructing High Embankments in Switzerland (Eng. News, Aug. 21, 1902).—By use of temporary suspension bridge; illustrated.

The Cost of Hydraulic Excavation for Embankments and for Placer Mining (Eng. News, Nov. 27, 1902).—Cost data on several works, also references to other data.

The Buckeye Trench Digging Machine (Eng News, Aug. 6, 1903).—Illustrated.

Discussions on Clearing and Grubbing (Eng. News, Jan., 1904.)

Time Required to Load Wagons with a Steam Shovel (By J. S. Ely. Eng. News, July 14, 1904).—Table.

A Cross-Cut Excavating Machine for Drainage Ditches (Eng. News, Sept. 7, 1905).—Illustrated.

Machine for Spreading and Leveling Material (Eng. News, Jan. 4, 1906).—Illustrated.

Cost of Steam Shovel Work by Railway Force and by Contract (By J. C. Sesser. Bulletin 81, Nov., of Am. Ry. Eng. & M. of W. Assn.; Eng. News, Jan. 17, 1907).—Work by company force, 18.7 cts. per cu. yd.; contract, 26.0 cts.; saving, 7.3 cts.

Excavating Machines on the N. Y. State Barge Canal (Eng. News, June 6, 1907).—Illustrated.

Gravel Spreader Used on the Colorado River Levee Construction (By H. T. Cory. Eng. News, July 11, 1907).—Illustrated. "Cost of work done by the machine was about one-tenth of a cent per yard of material spread. Machine cost $300. Its operation required a locomotive and four men.

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An Unloading Machine for Dumping Cars in Building Embankments on the Western Pacific Ry. (Eng. News, Aug. 22, 1907).—Illustrated; consists of a circular loop at end of top of embankment, for cars to run around and dump.

Hydraulic Construction of Large Embankments on the Chi., Mil. & Puget Sound Ry. (Eng. News, May. 27, 1909).—Twelve illustrations of construction work.

Machine for Excavating Trenches and Foundations in Frozen Ground (Eng. News, June 17, 1909.)—Used in Winnipeg, Canada "In that city the frost penetrates to a depth of 5 to 6½ ft. Previous to the introduction of this machine, all earth excavation during 5 or 6 months of the winter had been done by hand picking or by blasting the frozen earth with black powder, the former costing about $1.35 per cu. yd., and the latter about $1.25 per cu. yd. down to 93 cents per cu. yd. By the machine, the cost has been reduced to from 11 to 30 cents per cu. yd. The ("Drop-Chisel") machine is illustrated.

Excavation Methods, Fourth Avenue Subway, Brooklyn (Eng. Rec., Dec. 3, 1910).—Described and illustrated.

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55.—ROCK EXCAVATION.

Subject Divisions.—Under "Quarrying" (see page 419) are discussed the methods of excavating "dimension" and other stone for subsequent use in masonry construction. The present subject deals with the most approved methods of excavating *open rock cuts* and *trenches* for railways, highways, canals, sewers, etc. For subaqueous excavation, including blasting under water, see "Dredging," page 927, and for tunnel work see "Tunneling" page 933.

Open Rock Cuts.—The cheapest form of open cut is the side-hill cut (Fig. 1), in which *c* is the cut, *f* the fill, and *w* the waste. A common method of operation is to begin drilling with holes *a* at the bottom of the cut, using moderate blasts and working back in steps to *b*. But where the quantity in cut *c* greatly exceeds the fill *f*, it is often advisable to begin at *b*, using deep holes and large charges of explosives. By this means a more effective use of the powder can be had in not only loosening the rock but in wasting a large proportion of it at the same time. Solid rock is supposed to stand about "vertical," hence the drill holes *b* as shown. If the material is seamy and contains more or less loose rock the latter may be taken out to slope as required, but it is not necessary to take out the whole cut in the same slope.*

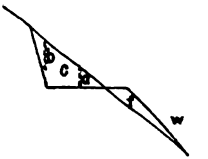

Fig. 1.

Drilling.—The most common form of rock cut is the "thorough" cut as shown in Fig. 2. The rock is excavated in benches, *a*, *b* and *c*, but not so regularly as shown in the Fig. unless channeling machines are used as was the case on the Chicago Canal. The rise or lift may be assumed as about equal to the tread, but this depends on the size of blocks that can be handled economically, the depth of economical drilling, the quality and character of stone and seams, and whether *horizontal* holes at the bottom of rise are drilled to assist in blasting. The spacing of the holes may be assumed about equal to their depth, if in single row and the rock is not too hard. For trap and granite the spacing should be closer, ordinarily. The rise may vary from a few feet up to 12 or even 20 ft., the deeper holes requiring the larger drills, say 3 to 3½ ins., and operated by machine. Churn drills up to 3 ins. in dia. (1⅛" bar) may be used for vertical holes in soft rock, where machine drills are not available. Each drill is operated by two or more men who raise it, turn it 'round a little and let it plunge back into the hole. Sometimes the drill is loaded to give more weight and hence become more effective. For two-hand hammer drilling (one man holding and turning drill, and two men striking), the hole is usually started with a 1¼ to 1½-in. bit and using 10-lb. hammers. The dia. of bit is decreased with depth of hole, to prevent binding, and the limiting depth is about 8-ft. Octagonal bars from ⅞-in. to 1-in. are used. For one-hand drilling (hammer 4½ lbs.), the drills are usually 1 in to 1¼ in. with octagonal bars ⅞-in to ⅞-in. in dia. The minimum diameter of hole (at bottom) for use of dynamite is about ⅞ in. The hand drill is often called a jumper. Rotary drills, rotating drills, or augers are names given to (hand or machine) drills which *bore* solid holes, or annular rings around solid cores. They are generally used in the softer rocks where heavy blasts are required. Such drills can, however, penetrate almost any material liable to be encountered, instances being recorded in which borings have been made through imbedded steel rails used for foundations. Core drills

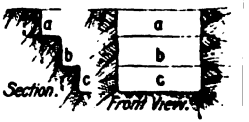

Fig. 2.

* The writer knows of an instance where a whole rock cut was taken out to slope of ½ to 1, where $\frac{2}{10}$ of it could have been vertical, or at least ¼ to 1.

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ularly designed for well-borings, both water and oil; and for borings to determine the geological formation at different depths, f which are brought up in the form of cylindrical cores. They ither steel (hardened) or diamond (black)-pointed, for effective

ning in Rock.—Trench Excavation, for water pipes, sewers, etc., pensive than open-cut excavation, but less expensive than tunnel n. The increase in cost over open-cut work is due (1) to the greater difficulty of drilling, per cubic yard removed; (2) to the limiting explosives, thereby reducing the duty of drilling, powder and to the large proportion of excavation required *outside* of "neat rench, without added compensation; (4) to the great amount of up of sides and bottom of trench preparatory to laying the pipe. discussed as follows:

wing to the large amount of drilling required, machine drills should d where the preliminary cost, reduced to rental, is warranted by yardage. Comparing cost of operation alone with that of hand saving of 40 to 60 cents per cubic yard is often effected. The ng steam drill is perhaps the most commonly used for this work, compressed air may be cheaper. Gasoline compressors are some- aper than steam compressors especially on small jobs. Electric so should be considered where electric power is available.

arger blasts may be fired if ballasted with logs, timbers or blasting e the flying rocks are liable to cause damage, as in cities. Mr. ette in his "Rock Excavation," describes a blasting mat woven hemp rope, 1½ in dia. or larger. The mat is woven like a cane- air, the first set of strands being looped over 1-in. gas pipes. It is with heavy timbers, thus effectively checking the progress of the agments of rocks. The narrow outlines of the trench, however, duty of the charge.

n estimating the cubic yards in trench excavation it is well to e trench 24 ins. wider than the external diameter of pipe or e laid, and 6 ins below grade. Bell holes have to be dug at the s and there is more or less trimming to be done along the sides m so that this seems no more than a just compensation. Engi- etimes let contracts based on the lineal feet of pipe laid irrespective f trench or quality of material to be excavated. This throws the examination of the soil, etc., upon the contractor (who usually uess), and is not recommended. If for any reason the line of the changed after such a contract has been let, complications invari- between engineer and contractor.

eference has just been made to trimming up the bottom and e trench to receive the pipe for laying. There is another item st not be overlooked, namely, lowering the grade after the trench finished. Instances are of record where contractors have been lower the grade from a few inches to a few feet at a price based iginal bid per cubic yard, whereas such work sometimes costs es the amount received, the specifications admitting of no redress. th the above is a trench mostly in earth excavation but with a unt of rock at the bottom. In such a case the lack of proper ay prove very expensive to the contractor.

o Drainage Canal.—The material was classified as glacial drift 0 cu. yds.) and solid rock (12,300,000 cu. yds). The glacial drift tough, indurated clay and cemented gravel or hardpan, inter- h boulders and capped with prairie soil. The solid rock was increasing in hardness with the depth. All the material except- p soil, required blasting, the hardpan standing vertically like . The best method of loosening the hardpan was by dynamite charged at the ends of small tunnels. Fig. 3 shows sections al in the two classified materials named.

ock section is 160 ft. wide at base, and 36 ft. deep. The drills ted mostly by compressed air at 80 to 90 lbs. pressure per sq. in., tral compressor, through 8 or 10-in. mains with 6 to 8-in. branches. ls were commonly supplied with 2-in. feed pipes 175 to 230 ft. ng into 3½ x 6½ in. cylinders. Bits of the X pattern were found best. The rock was taken out in 3 12-ft. lifts (Fig. 2); and 40%

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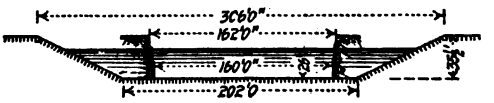
dynamite proved to be the best generally. The sticks were 1½ x 6 ins., weighing ⅔ lb., and 10 to 25 sticks were charged in a hole. The price per lb., including caps and fuse, was about 12 cents and about 1 lb. of dynamite was used per cu. yd. of rock.



Chicago Main Drainage Channel.

Fig. 3.

Channeling machines of the Sullivan and Ingersoll types were used in order to secure smooth side walls to the canal. The average machine weighed 11000 lbs., was operated on a track 30 ft. long, and struck 250 blows per min. The width of the channel cut by the bit was 2⅞ ins. at the top, decreasing ⅛ in. for each 2 ft. of depth. The speed of channeling was 130 to 200 sq. ft. per 10 hrs. on the upper lift (where the rock was softer) and about half as much on the two lower lifts. The lifts were 12 ft. each. The cost of the channeling varied from 8 to 25 (say 17) cents per sq. ft., or from 3 to 7 (say 6) cents per cu. yd. excavated.

Steam shovels of the Bucyrus type were employed to a limited extent in loading rocks on cars. The cars were operated by incline and hoist methods. But these were generally more expensive than the cantilever, cableway and derrick methods of conveyance as shown in the two following tables.

1.—Cost in Cents per Cu. Yd. (Solid).

	Channeling.	Drilling.	Explosives.	General Expense.	Pumping.	Conveying.	Pit Force.	Dump Force.	Total.
Brown Cantilever.....	3.9	4.1	8.0	3.2	1.0	3.6	14.6	0.0	38.3
Lidgerwood Cableway..	3.7	3.8	8.4	2.7	1.0	3.6	15.6	0.0	38.8
Hullett-McMyler Derrick	3.9	4.0	7.4	2.5	1.8	5.3	18.3	0.0	43.2
Hullett Conveyor.....	4.1	3.7	8.5	3.8	1.2	6.2	21.4	0.0	48.9
Car Hoist, No. 1.....	3.7	3.9	9.1	2.7	0.8	3.1	24.8	5.1	53.1
Car Hoist, No. 2.....	3.9	3.6	8.9	3.2	0.9	1.2	22.9	2.3	47.1
Car Hoist, No. 3.....	4.0	5.0	10.7	3.1	1.2	1.2	26.4	4.8	56.5

Note.—Shop repairs, drill sharpening and plant rental not included.

2.—Output in Cubic Yards by Conveyors.*

	Section.	Cu. Yds.	Cu. Yds. per 10 Hrs.	Cu. Yds. per Man in Pit per 10 Hrs.
Brown Cantilever.....	10	443,750	478	10.45
Lidgerwood Cableway.....	8	600,725	397	10.25
Hullett-McMyler Derrick....	7	180,406	217	8.52
Hullett Cantilever.....	7	109,397	235	9.91
Hullett Conveyor.....	9	178,839	335	6.85
Car Hoist, No. 1.....	8	131,674	285	6.96
Car Hoist, No. 2.....	10	60,341	269	6.98
Car Hoist, No. 3.....	9	308,531	463	6.82
Double Boom Derrick.....	14	324,880	282	8.22
St. Paul Derrick.....	14	63,700	153	8.22

* Compiled by Mr. W. G. Potter. Common labor received 15 cents per hour.

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mance of Work.—A few examples* are given to illustrate methods of various classes of rock excavation, as follows:

nitz Rock Breaker for excavating rock under water, designed by F. W. 'r of Empire Eng. Corporation,† used in excavating (about 260000 cu. k about 18 to 25 ft. under water, in improvement of Black Rock Harbor. Y. Contract price, $1.85 per cu. yd. The rock is broken by a cutter of a steel cylinder 28 ins. dia., about 25 ft. long and weighing 46000 lbs., he bottom with a hardened steel cutting point. The weight is hoisted upon the rock bottom from a height of six ft. or more.—‡*Eng. News*, 907, p. 236.

g and Blasting Soft Shale Rock Under Water, at Ashtabula Harbor, Ohio. 85 ft. long by 30 ft. wide, held in position by four spuds, one at each quipped with Ingersoll-Sargent steam drils on trucks for horizontal and operated vertically by hydraulic lifts. *Drill Boat "A"* worked days of drilling and blasting 4744 cu. yds. (place measure) during month of was 28.5 cts. per cu. yd., the average depth of holes being 6.15 ft. spaced t. apart. Of this amount, the labor cost of drilling and blasting was 6.7 ost of explosives was 6.7 cts. *Drill Boat "B"* worked both day and night st for the day shift was 23.6 cts. per cu. yd., while for the night shift it to only 20.4 cts. per cu. yd., there being less breeze during the night verage amount of explosive (45%-dynamite) used was ¾ lb. per lin. ft. d costing 15 cts. per lb. The blasted material was excavated with dipper *Eng.-Contr.*, Aug. 7, 1907, p. 83.

ating Granite in Open Cuts on the Grand Trunk Pacific R. R.—Data y Geo. C. McFarlane, eng'r and contr. Steam drills used in large cuts. rills in smaller cuts. 1" steel to make drills gaged to 1⅜", used for entire le. Gang of three men drilled per day average of 29 ft. in dark hornblende, l granite, and 18 to 19 ft. in trap and diabase rock. Rock-cuts 20 ft. wide side slopes 1:4 or as material will stand. On most open cut work it has stom to drill the blast hole on or near the center line of the cut, so as to rge of explosive at the center of the mass to be moved. A 12-ft. hole is ft. burden, while a 25-ft. hole is given from 20 to 25-ft. burden. but Mr. used for the 20-ft. roadbed two holes 10-ft. on each side of center line, not to exceed 15 ft. After drilling, the bottom of the holes are chambered he charge by springing with dynamite. In a bottom bench where a heavy red, no more than a ft. is chambered. In upper benches it is permissible 2 or 3 ft. of hole. The first springs are held down by water tamping. g, water tamping is nearly as good as sand, except when charges of over re used. With heavy charges the water tamping permits the dynamite the hole and make it ragged. Sand tamping has to be drawn after each e sand is free from clayey material it can be drawn quickly by spudding urn drill and pumping up the cuttings. Sometimes it can be blown out eam blowpipe. With water tamping, a cap and fuse are used, unless the ed, when the spring is made with electric exploders. The fuse, usually , after being split is held under water for 5 or 6 seconds to kill any fire the tape, and then dropped into the hole. Unless the drop fuse were ater, it might ignite dynamite adhering to the sides of the hole, causing e explosion. After each water-tamped spring, the hole is blown out with mped out with a sludge pump. The springing opens up the rock jointing es very closely where the burden of the shot will cleave from the solid, cessive springing charges indicate the ratio of enlargement of the pocket. s of cost are as follows: (a) Fully ⅓ of the rock cuts are sublet to gangs en at from $1.15 to $1.35 per cu. yd. They are given free cars and rails; rged $4.50 per week for board, $5.50 per day for team and driver, $11.50 60% dynamite, and $2.75 per keg for black powder. (b) Cost of block d granite boulder 10 x 12 x 9 ft., containing 36 cu. yds., was (48" drill 3 lbs. 60% dynamite $0.66, reblasting 2 fragments $1.42) $4.50 or 12½ yd. (c) Cost of block holing red granite boulder 16 x 14 at top and bottom, 15 ft. deep, containing 92 cu. yds., was (96" machine drill hole s. 60% dynamite $3.52, reblasting 2 fragments $1.45) $12.24 or 13⅓ cts. (d) Cost of blockholing black granite boulder triangular in shape, 4 x 5 x containing 1.1 cu. yds., was (7" hole 50 cts., ½ lbs. 60% dynamite 8 cts.) 2½ cts. per cu. yd.—*Eng.-Contr.*, Nov. 27, 1907, p. 301.

rence to articles giving more complete description are given at each example.

York City. ‡ See also *Eng.-Contr.*, Dec. 18, 1907, p. 343, for ns of cutter and detailed information.

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EXCERPTS AND REFERENCES.

The Well Driller for Drilling Blasting Holes (Eng. News, June 23, 1904).—On the excavation work for the Wabash R. R., in Ohio, in drilling brown sandstone the holes were put down to a depth of 24 ft. with a 3" bit, and the drill averaged two such holes per day of 10 hours. The cost of labor, fuel and water was about 12½ cents per ft. of hole drilled. In the blue standstone, which is softer, an average of 60 ft. per day was drilled. Formerly, with churn drills by hand, the cost in brown sandstone has been 38 cents per ft. of hole, the holes being 20 to 30 ft. deep; the steam drills, up to depths of 20 ft., and in sandstone no harder, reducing this cost but very little. The well-drill holes being large (3") are never "sprung" more than three times in the sandstone, whereas the steam-drill holes (1¾") must be sprung 4 or 5 times.

Methods of Subaqueous Rock Excavation, Buffalo Harbor, N. Y. (Eng. News, July 6, 1905).

Rock Excavation by Mechanical Power Instead of Explosives (Eng. News, June 25, 1908).—Editorial on the Lobnitz rock breaker and similar machines.

Illustrations of Machines, Tools, etc.

Description.	Eng. News.
Austrian drill boat with screw-operated spuds	May 26, '10.
	Eng. Rec.
Excavating submerged rock with a drill boat, N. Y., N. H. & H. R. R.	Jan. 8, '10.

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56.—DREDGING.

Discussion.—In estimating the cost of dredging in any locality epend upon the *amount of dredging* to be done, as well as the erial, method of disposal, depth and roughness of water, climatic s affecting the number of working days per year, and other ns. The cost of dredging may vary from 3 to 4 cents up to about d. for earthy material; and for solid rock where subaqueous blasting are required the total expense of removal may amount e per cubic yard.

re three methods of measuring dredged material, namely, by prior and by subsequent surveys or soundings; (2) in *scows*, or by displacement; and (3) in *fill*. The *first* method is generally y especially in shallow cuts where any error in average depth nt to considerable in percentage; and again, the excavated lmost sure to become partly filled again with current drift, nt would represent an error in yardage against the contractor. easurements are very useful as a check in many cases. The d, in scows, is usually adopted and is probably the best where ject is "dredging" and not "filling" or reclamation. In this t should not be forgotten that dredged earth will swell from cent (average say 25) and the method of measurement should the specifications and contract. Scows should be self-dumping cost of delivering or wasting will be that of towage mainly. thod of measurement is in fill where the object is reclamation. easurements are also generally kept as a check or even as the for payment. The writer was employed in an engineering connection with the filling of what is now known as a part of ark, Boston, being the trapezium bounded by Beacon St., West (now Massachusetts Ave.), the Boston & Albany R. R., and e. Much of the area was under water. It was cross-sectioned ares and soundings from rafts were taken at range and stationions, in connection with gage (tide) readings at stated intervals taking the soundings (2 or 3 men at a pole) there were usually namely, top of mud, bottom of soft mud, and "stiff mud" e filling consisted of dredged material from the Charles River, scows as long as the depth of water would permit, then the as brought in on B. & A. R. R. cars from gravel banks at niles distant. The character of the mud bottom precluded the estimating the quantity of fill accurately by the method of ng and sounding, and payments were made by scow and easurements. It was estimated, however, that the amount of *below* the top of the mud was equal to from 20 to 40% of the mud respectively. In hydraulic dredging for reclamation it to estimate "in fill" only, and it should be stated clearly in

—Any machine that will excavate material under water is ge. Perhaps the simplest form of dredge is the drag scraper back and forth over the bottom by the use of cables. There ard cables to give the bucket the correct incline for scooping ial, and one back cable to steady the bucket and haul it back. sually mounted on scows or floating hulks and may be operated electricity. The latter is seldom used however excepting edging is centralized, of great magnitude, and practically conhich case there may be a central power station on shore, feeding dredges.

are usually classified according to the excavating machine, of re four principal types, namely, (1) the dipper dredge, (2) the

uller & Whitney, Boston, Mass.

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grapple dredge, (3) the bucket elevator dredge, (4) the hydraulic dredg Each of these is particularly adapted to certain kinds of dredging and the are many modifications.

The Dipper dredge (Fig. 1) is really a long-handled dipper or shovel which is filled by pressing the scoop down into the mud while being swung radially outward. The material is dumped through the bottom of the bucket, which is movable (usually hinged). This is the best all-around type of dredge for general use. The buckets average 1 to 2 cu. yds., but may be of any size up to 6, 10 or even 15 cu. yds. capacity. They are capable of working in very soft and very hard material and are generally useful around wharves, and for channel and canal excavation. 30 ft. of water is about the limiting depth for good work.

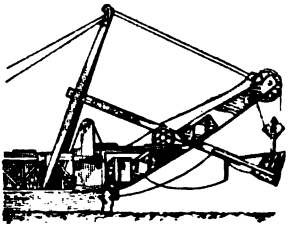

Fig. 1.
Dipper Dredge.

The Clam-shell bucket (Fig. 2) is suspended from a derrick boom on scow and is a form of grapple dredge. The bucket is lowered with jaw open, and sinks into the mud by its own weight. The jaws are then closed around the material and the bucket is raised and dumped. The capacity of a clam-shell bucket is about the same as that of a dipper, but it is especially adapted to deep dredging.

The Orange-peel bucket (Fig. 3) is another form of grapple, and may have 3 or more segments which are open during its descent and, after pressing into the mud, are closed. In addition to its regular work in dredging, it is frequently employed in excavating inside of cylinders which are being sunk for foundations.

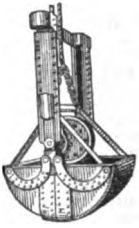

Fig. 2.
Clam-shell Bucket.

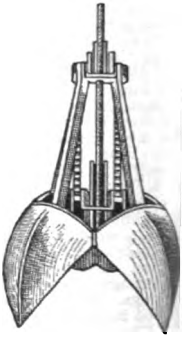

Fig. 3.
Orange-peel Bucket.

The (Bucket) Elevator Dredge or bucket ladder dredge consists of an endless ladder chain to which buckets are attached. These buckets scoop up the material from the bottom, are elevated to the top of the ladder, and the material is dumped in a chute or on a belt conveyo which deposits it as required. They are particularly adapted to "skim dredging in soft material for canal work, etc. They leave a smooth bot tom. The buckets may have a capacity of say $^1/_{10}$ to $^1/_8$ cu. yd. each, an a speed of 30 to 40 ft. per min.

For discussion of sub-aqueous electric power cable, see *Eng. News*, Ma 7, 1903, and Aug. 13, 1903.

The Hydraulic Dredge* is used in the improvement of waterways an in the reclamation of low lands. It works best in fine material whic remains suspended in a swift-moving volume of water. The princip features of the hydraulic dredge are the rotary cutter or stirrer to loose the material and keep it in suspension, and the centrifugal pump to pum the suspended material up into the delivery pipe and to its destinatio If the material is light and loose the rotary cutter is sometimes replaced b the water-jet, but ordinarily the cutter is better, and must be used to compacted materials. Sand is handled easily, but if it is sharp and hard it wears out the pumps quickly. Coarse gravel is out of the question. So clayey mud is probably the best. The delivery pipe is of sheet iron or ste (sometimes wood-stave pipe), with an average diameter of say 10 to 20 in

* For valuable data on Dredges, see *Eng. News* of July 28, 1904.

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be 2000 to 3000 ft. in length. Between the dredge and the shore orted on pontoons, either single or double.
ing "with" and "against" the current of a stream are subjects for , and depend somewhat on local conditions.

ng Under Water is required in removing ledge rock and large The following is from Gillette's "Rock Excavation," from data by C. Y. Dixon, U. S. Ass't Engr.:

mouth of the Detroit River it is usually necessary to drill and blast the fore it may be excavated. The drill boats in use (1903) are from 60 to and from 25 to 30 ft. wide, and are held in position by four "spuds," one ner. They are equipped with two or more Ingersoll steam drills supported frames having trucks to permit of the drills being moved horizontally dge of the boat. The drills are raised and lowered during the operation lic lifts. The boiler furnishes the steam for operating the drills, the l in connection with the hydraulic lifts, the forge the electric light plant machinery with which the ordinary drill boat is equipped. The holes, ns. in dia., are made at the corners of 5-ft. squares to a depth of about *he required* depth, at the rate of about 5 ft. per hour per drill. One pound namite is used per lin. ft. of drill hole. The holes are charged by inserting namite with the exploder and battery wires attached to the bottom of a the battery wires leading out through a slit in the side of the pipe. This pipe into the drill hole, the dynamite shoved down with a long ram-rod, and thdrawn, a wire spring clamped to the dynamite stick preventing its com- the hole. The wires are then attached to the battery and the dynamite During this operation the drill boat is not moved, nor does the work of the other drills cease except at the time of firing. On two occasions, he charge of dynamite came out of the hole, and was exploded directly the boat, causing it to sink almost immediately. This was due to careless- pulling the wires up taut they will indicate whether the charge is in place. supply is kept at the drill boat, stored in a small scow trailing down- a safe distance. Dipper dredges are used in excavating the material. in length from 80 to 135 ft. and in width from 30 to 40 ft., and are held by three spuds (36 ins. square), two at the bow and one at the stern. entire width of the area to be improved has been worked over by the t by cut, the derrick scow follows after to remove such loose pieces of y have been left projecting above the required depth. The derrick scows from 80 to 100 ft. long and from 20 to 25 ft. wide and they are equipped dinary hoisting engine and derrick capable of lifting from 12 to 18 tons, plete diving outfit. When lifting the boulders, the derrick scow is pinned ported on two spuds, each about 1 ft. square. The material to be moved means of an iron bar, about 30 ft. long, suspended from the side of the required depth. Any obstruction struck by this bar, as the scow is nproved area, is removed by the derrick by means of a chain, which is osition by a diver.

llowing summary of the above will be of interest. It should be noted that al at Ballard's Reef was limestone bedrock, clay hardpan (50%), and ith two ft. of loose material overlying the bedrock; at Lime Kiln Crossing al was mainly limestone bedrock, with no overlying material; at Am- Reach the material was limestone bedrock, clay and boulders, with gener- two ft. of loose material above.

DREDGING DETROIT RIVER, 1900–1903.

	Ballard's Reef.	Lime Kiln Crossing.	Amherstb'g Reach.
ove 23 ft. depth (place measure)	74,143	101,072	98,332
tal (place measure)	135,548	121,707	209,821
tal (scow measure)	153,097	168,633	.....
ed, sq. yds.	225,000	61,000	225,000
pth dredged, pay material	1 ft.	5 ft.	1.3 ft.
pth dredged, total exc	1.8 ft.	6 ft.	2.8 ft.
pth of water over material	21 ft.	18 ft.	20.07 ft.
rs, worked	7,248	3,945	9,021
rs, delayed	3,386	1,490	2,890
rs, total	10,634	5,435	12,911
nths (12-hr. days)	34	17.4	39
onth	$3,000	$3,200	$3,200
	$102,000	$55.720	$124,800
yd. (place measure), pay material	$1.38	$0.55	$1.27
yd. (place measure), total exc.	$0.75	$0.46	$0.60
per hr., working time	19.0	43.0	23.3
per hr., total time	12.8	22.4	16.2
cu. yds., per hr., soft material	150	250	200

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DRILLING.

		Ballard's Reef.	Lime Kil Crossing
Drill hours	Worked	24,442	37,746
	Delayed	982	1,278
	Total	25,424	39,024
Number of holes drilled		30,023	29,236
Number of feet drilled		191,850	240,591
Ft. per hr., actual work		7.9	6.4
Ft. per cu. yd., pay material		2.6	2.4
Ft. per cu. yd., total exc		1.4	1.9
Distance between holes		5 ft.	5 ft.
Average depth of holes		6.2 ft.	8.2 ft.
Average depth of pay material		1.0 ft.	5.0 ft.
Percentage of drilling below pay depth		84.0 %	37.5 %
No. of lbs. of 60% dynamite		110,305	222,396
Lbs. per cu. yd., pay material		0.5	2.2
Lbs. per cu. yd., total excav		0.8	1.8
Total cost of drilling		$59,235	$105,245
Cost per cu. yd., pay material		$0.80	$1.04
Cost per cu. yd., total excav		$0.44	$0.865
Cost per lin. ft. drilled		$0.31	$0.44
Cost per drill hour		$2.25	$2.69

Note.—75% dynamite often produces the best results.

DERRICK SCOWS.

	Ballard's Reef.	Lime Kiln Crossing.
Cost	at $9.70 11.2 mos. $10,865 Tug service included in dredging.	13.0 mos. at $9.70 $12,610 Tug servi included dredging
Cost per sq. yd. of area improved	$0.0475	$0.22
Cost per cu. yd. of material removed by diver	$5.73	$2.84

SUMMARY OF COST.

	Ballard's Reef.	Lime Kil Crossing
Dredging	$102,000	$55,72
Drilling	59,235	105,24
Derrick Scows	10,865	12,61
Totals	$172,100	$173,57
Cost per cu yd. of pay material	$2.32	$1.71
Cost per cu. yd. of total excavation	$1.27	$1.42

Gold Dredging.—This has become a very profitable ind of the streams of California. Mr. W. P. Hammon, who h experience in this class of work, writes the author from M under date of Sept. 12, 1906, as follows:

In handling gravel in our work we use elevator dredges entire dredge not being so efficient or economical; in fact, in most place the latter is altogether impracticable for the reason that where boulders are frequent, in the operation of a hydraulic dredge, the nest at the intake of the suction pipe thereby preventing gold from flow of water.

We have operated elevator dredges with steam and electric c from our experience consider that electric power is at least 50 pe nomical, to say nothing of the great convenience electricity has o estimate is based on power at 1½ cents per kilowatt-hour, and w ord aboard ship.

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ormance of Work.—The following examples are given mainly for e to the articles themselves, in which complete details and illustra- given; the abstracted matter being sufficiently full to present to er some idea of the nature of the article, and to bring out some s in the design and execution of this class of work.

rful Dipper Dredge "Majestic' with Edward Cable-Storage Drum. De- and built for Edward Bros., of Sault Ste. Marie, Mich., sub-contractors on ovement of the West Neebish Channel in the St. Mary's River, Mich. 116x40 ft., and 13 ft. deep, with two steel trusses 24 ft. deep extending length. Steel boom 60 ft. long, stepped into a steel casting at the bow orted by four cables 2½ ins. dia.; boom swung by two 2-in. cables on a ntable. Dipper handle 54 ft. long and weighing 15 tons; with 5-ft. dipper cavate to a depth giving 30 ft. of water; operated by horizontal engine P., capable of exerting, through the gearing, a pull of 200 tons upon the ble, and of dredging solid beds of soft limestone rock without blasting. spuds are 43 ins. square by 52 ft. long, each composed of four sticks dressed s., fitted into cast steel shoe at bottom and bolted together for 8 ft. from m with 24 bolts 1½ ins. in dia. At the top, the sticks are held together by and ⅜" x 10". For the intermediate space there is no bolting, the timbers so as to give a certain flexibility or spring. The spud foot is 10x12 ft. of timbers 16 ins. square. The dredge can be lifted about 2 ft. above its osition, by means of a 2-in cable running over the top of each spud, thus e spuds from 2 to 6 ft. into the mud. The capacity of the dredge is about cu. yds. per hour in rock alone, and about 10 times that amount when n soft material and using an 8-yd. dipper. The Jackson & Church Co., of Mich., controls the patents for the Edward cable-storage drum, which or a cable about 1000 ft. long to be used, and when this breaks, only about t. are lost, the working length being replaced by paying out sufficient cable storage part of the drum.—*Eng. News*, Feb. 7, 1907, p. 145; and Oct. 10, 73.

orting the Discharge Pipe of a Dredge. Device used in dredging the harbor Ind. The 24-in. pipe line was suspended from pontoons made of pairs of inders built of boiler plate, water-tight, and from 10 to 80 ft. in length. of cylinders is fastened together by timbers under which the discharge ung, the latter being laid with flexible joints at intervals, for the necessary aterial is delivered from 1000 to 3000 ft. from dredge.—*Eng.-Contr.*, Sept. p. 160.

aulic Dredge "Oneida," used on the New York State Barge Canal. De- Mr. Emile Low. Designed by Mr. Lindon W. Bates, Pres. the Empire g Corporation, contractors. Length over all, 97 ft.; extreme beam, 17.5 d depth to deck, 10 ft.; light draft, 5.5 ft. Can be taken through locks. on pipes, on sides, extending from centrifugal pump aft to cutter forward; , 19½ ins., No. 9 B. W. G. metal, connected forward to two Bates curved joints. Two movable suction pipes or suction ladders are built-up pipes, ng an area of 291 sq ins., at the lower ends of which are attached the ds 5 ft. 6 ins. dia. and 3 ft 8 ins. high The cutters and dredge are illus- *Eng. News*. When working, pontoons are attached to side of dredge to lity. During Oct., 1906, 38173 cu. yds. were dredged at cost of 50.5 cts. ; during Nov., 1906, (two weeks spent in repairs), 144 882 cu. yds. were t cost of 30.8 cts. per cu. yd.—*Eng. News*, Dec. 5, 1907, p. 619.

aulic Dredge "Francis T. Simmons," used in Lincoln Park Reclamation Michigan. Dredge designed by Mr. A. W. Robinson, and built by Atlantic t Co., 111 Broadway, New York City. Hull, 148 ft. long, 38 ft. wide, and s. deep. Main pump has 30-in. suction and discharge, and main engines triple expansion marine type of 1200 I. H. P There are two double- rine boilers 11 ft. 6 ins. x 18 ft. long with eight corrugated furnaces. The pe is carried by a very strong steel frame, and is fitted with a powerful digging the clay. A capacity of 3000 cu. yds. per hour in clay is said to reached.—*Eng.-Contr.*, Dec. 25, 1907, p. 356.

EXCERPTS AND REFERENCES.

ods of Measuring Dredged Material in the Channel Improve- the Delaware and Schuylkill Rivers (C. H. Ott. Proc. Engrs. Club , April number; Eng. News, July 3, 1902).—Illustrations of sound- submerged sweep used with rowboat, and sweep and derrick used er. Description and methods used.

mproved Centrifugal Dredging Pump (By M. M. Patrick. Eng. ly 9, 1903).—Illustrated.

Electrical Equipment of a Gold Dredge (By R. L. Montague. ws, July 16, 1903).—Illustrated.

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Cutters for Hydraulic Dredges Working in Hard Material (Eng. News, June 15, 1905).—Used at Alameda, Cal. Illustrated.

The Work of a Ladder Dredge and Belt Conveyor System on the Fox River, Wisconsin (By L. M. Mann. Eng. News, Oct. 25, 1906).—Illustrated.

Dredging Operations at Warroad, Lake of the Woods, Minn., by U. S. Gov't (By Emile Low. Eng. News, Nov. 29, 1906).—Cost data.

The Fruhling System of Suction Dredging (By John Reid. Eng. News, Mar. 5, 1908).—Table of comparison of this type with American hydraulic hopper dredges.

Large Elevator Dredge for Work in Boston Harbor (Eng. News, Jan. 27, 1910).—Capacity of each bucket is 1¼ cu. yds. and the bucket chain is driven ordinarily at a rate of 14 buckets per min., so that the dredging capacity with full buckets would be 1,100 cu. yds. per hour. This has been exceeded by more than 30% for shorter periods in actual use. For driving the bucket chain, a double tandem compound steeple engine is provided, with cylinders 12 x 16 ins. by 18-in. stroke. This is entirely separate from the engines for driving the vessel's propeller shaft. The ladder frame is of steel and is of sufficient length to work at a depth of 51 ft. After a year's service, the buckets with their pins and bushings are reported in good condition and the general loss of time and cost of repairs is said to compare favorably with that of a dipper dredge on the same class of work. The material encountered at different parts of the channel included hardpan, clay, stone and gravel. The ordinary yardage for a single day's work was about 8,000, and under favorable conditions as much as 10,800 cu. yds.

Working Costs of Gold Dredging in California (By Charles Janin and W. B. Winston. Mining and Scientific Press, July, 1910).—The total operating expense per cu. yd. varies from 9.23 cts., for difficult digging, down to 2.30 cts. for easy digging (fine gravel).

Illustrations.

Description.	Eng. News.
Light-draft stern-wheel suction dredge for Niger river	June 16, '10.
	Eng. Rec.
Gold dredging and rock crushing in California	July 16, '10.

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57.—TUNNELING.

itions.—In a broad sense, a *tunnel* is an artificial underground and *tunneling* treats of the methods of construction, embracing sses of excavation, timbering and lining. Either or both of the d may sometimes be omitted as when the excavation is through d rock. Tunnels are usually horizontal or nearly so; but they nclined in any direction: a vertical tunnel is called a *shaft*. For nt enlargement, a small tunnel projected ahead of the full section *heading* if in upper part of section,* and a *drift* if at bottom of A *loop heading* is a small auxiliary tunnel loop from the main In blasting, the quantity of rock which should be loosened by a harge of powder is called the *burden*. *Muck* is a term applied cavated material (including broken rock) from a tunnel.

of Tunnels.—Railway-, highway-, subway-, aqueduct-, mining-, sewer-, ventilation-, sub-aqueous-.

ds of Tunneling.—There are many methods in use—some of them national or provincial—dependent upon the class of material to ted, the machinery available, the position of the tunnel, and local nces:

cut, dry tunneling—for subways, etc.
gh, dry tunneling (including the deep subaqueous-).
solid rock, by headings or by drifts.
common earth, by driving full section at once; by driving either the upper or the lower half first; by leaving a middle core sub-pillar to be taken out last.
gh, wet tunneling—not subaqueous.
quickstand, by shield or by freezing process.
ueous tunneling—not deep.
shield—with or without compressed air.
e-bottom tunneling—by dredging, and—
nstructing coffer dams.
nking caissons.
us "tunneling." (Proposed for English Channel but not adopted.)
ibes suspended, supported, or sub-floated in water.

Cut Tunneling.—Much of our modern subway construction is art at least, by the open-cut method, if the required cutting is not . The sections to be built are opened up in convenient lengths of or more, and the street traffic is supported on wooden framework e processes of excavating and lining the tunnel. The construction at analogous to that of a big sewer, but much more intricate. of the cut may be shored up by sheet piling, say 2½ ins. thick, hind longitudinal girts braced apart by transverse struts of 10 x bers with single or double wedges at the ends (see Fig. 1).

l portals also may be constructed in open-ll as by the shield method. In Europe the ctice is to construct the portals first, while ited States they are finished after the main completed.

cuts were formerly very rare at depths an 65 ft., for any considerable distance, but the practice is quite common to run them t 80 ft. in depth.

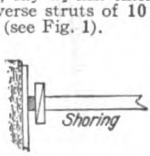

Fig. 1.—Shoring.

ling-" vs. "Drift-" Method.—The first named is the distinctive American tunneling. A wedge of rock is first blasted out on the the top of the section, by drilling two rows of holes with the drills

etimes the term "heading" is used in a broad sense as applying to ce tunnel smaller than the full section.

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converging toward each other at their points. This "center cut" is then widened laterally into a "heading" from which the bench below is easily worked by vertical drilling supplemented by horizontal drilling at the sides of the section. Sometimes a central core is left to support the centering temporarily. There are various modifications of the above. They differ essentially however from the prevailing European method of first driving a "drift" at the *bottom* of the section and working *upward*. In either method, where possible, the work progresses in benches so two or more drilling gangs may be employed at the same time. Headings and drifts are often advanced a few hundred feet beyond the main, full section—sometimes a thousand feet of more.

Drilling and Blasting.—Hand drilling, and even steam drilling, is being supplanted by drills operated with compressed air or electricity. Where the necessary water power is available hydraulic drills are used, and sometimes show great economy over any other power; at the same time each drill is supplied with a jet of water for cleaning out the hole and laying the dust in the tunnel. Compressed air has the advantage of incidentally ventilating the tunnel. Electricity becomes economical as the depth of tunnel increases from the face or portal.

Drills may be mounted on tripods, on cars, on columns (one or two drills to each column) which brace against the top and bottom of the tunnel and have a wide range of action, on bars which brace against the sides of the tunnel, etc. The size of an *air* drill is denoted by the inner diameter of the cylinder.

Dynamite of high grade, say 75% down to 60 or 50, is used in center-cut blasting in hard rock, as granite, trap, basalt, syenite, gneiss, etc., while for soft or seamy rock and for trimming up on the sides of the tunnel a lower grade of dynamite, say 40%, is better.

The "Radialaxes" channeling machine, manufactured by the Ingersoll-Rand Co., has been used to some extent, and its general efficiency will be watched with interest.

Timbering.—In hard, solid rock, timbering is not required and the finished tunnel is often left rough without lining. But in loose, seamy rocks and in the common soils, the sides and roofs have to be supported. Round timbers are generally used for timbering because they are cheaper, although sawed timbers are more easily framed and handled. In very loose material, lagging or sheeting is driven longitudinally of the tunnel behind transverse, segmental girts, or it may be driven transversely behind longitudinal girts. The girts are supported by timber props or struts, size about 12 x 12 ins. if sawed. Another method is to omit the lagging and lay the longitudinal girts close together, supported by segmental ribs of timbers. If these segmental ribs are placed close together even the girts may be omitted. Figs. 2 and 3 show simple types of rafter timbering in small tunnels in which the lower space is left open.

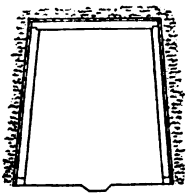

Fig. 2.

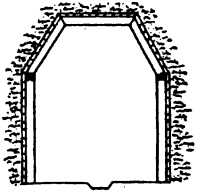

Fig. 3.

Lining.—The lining may be of timber, brick, stone masonry, concrete, etc. In subaqueous work the lining is frequently of cast iron, steel or reinforced concrete. The thickness of plain concrete or brick lining in the upper or arched section will of course depend largely upon the kind of material through which the tunnel is driven. In *loose* material of considerable depth the thickness of the concrete lining may be *assumed* at not less than 2 ft. for a single track railroad tunnel, and somewhat more for a double track tunnel. This thickness increases gradually in the lower bench walls.

middle of the section, to about 150% neat thickness at the foot-
g. 4 is a section of the *Gallitzin tunnel through the Alle-
n the Pennsylvania R. R
oundation footings are sometimes very expensive and difficult, if
averns, seamy rock or soft material If the last named is en-

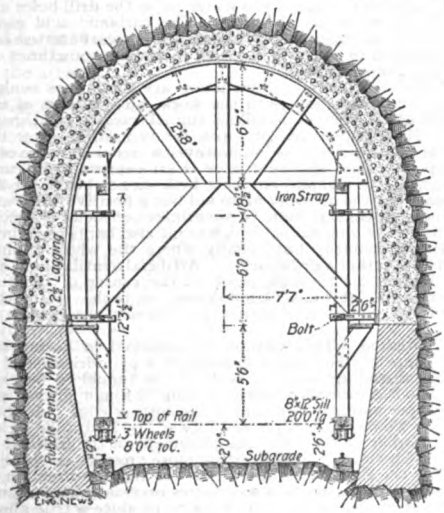

Fig. 4.

the side walls are frequently continued below the track in the
nverts or inverted arches, of concrete; and "weep holes" should
the lower part of the walls to drain off the surplus of outside
ich if allowed to accumulate might cause undue pressure on the
softening the sustained material.

nent and Grade.—The following remarks apply particularly to
unnels:
ight alinement conduces to economy in both length and sectional
nnel; and in cost of excavation, timbering and lining, per unit
of material as well as in total. It also minimizes cost of operation,
es risk of accident. Curves may be introduced when other con-
s outweigh the above, or from necessity.
s are absolutely essential in long tunnels, for drainage. As tunnels
d generally at the highest points of the lines the grade summits are
ally near their centers, so that in driving the tunnels up-grade
end they are drained naturally. If it is assured, however, that
ling heavy freight traffic will be in one direction it might be better
he whole tunnel on a down-grade in that direction, thus reducing
e nuisance (aiding ventilation) and the chance of trains being
For drainage, a grade of about 2 ft. per mile is the minimum—
mile is better.

ation.—The greatest difficulties met with in the ventilation of
railroad tunnels are during their construction. It is estimated
workman fouls about 25 cubic feet of air per minute; and a horse

Eng. News, Dec. 6, 1906.

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5 or 6 times as much. If we add to this about 10 cubic feet per man for fouling by dust and gases from explosions, we have a basis for estimating the *minimum* number of cubic feet of air which must be supplied at the heading. This must be increased when men are working in different parts of the tunnel and also when locomotives are employed to handle the cars. The purity of the atmosphere can be aided materially by the use of compressed air drills and also by the use of the water jet in the drill holes and by water sprinkling elsewhere to lay the dust. The carbonic acid gas exhaled in breathing may be augmented by "fire damp" or carburetted hydrogen gas which lies pocketed in rocks in the coal measures, sometimes encountered by railway and other tunnels.

Vertical shafts in tunnel construction are sometimes sunk to increase the number of headings and push the work, but the cost of sinking them and the subsequent cost of handling tunnel excavation through them is very great. These shafts are left open for ventilation after the tunnel is completed. It is a serious question however whether they are of much value for this purpose. The best ventilator is an express train running downgrade through the tunnel and emitting little or no (black) smoke, in which case the tunnel shaft is not only of no aid but a positive hindrance to clearing the tunnel. A central shaft is sometimes beneficial if located in about the middle of a very long tunnel which point also happens to be the summit of two ascending grades; but usually where the whole tunnel is on one grade the shaft is rather a drawback. Artificial ventilation is accomplished usually by forcing air through pipes to the center of the tunnel, or by suction at the ends (through closed doors), or both. With the advent of electricity in the operation of trains the difficulty disappears.

"Shield" Method.—This method is employed in boring subways and subaqueous tunnels. The shield consists of a cylindrical steel shell with a cutting edge pressed against the head of the tunnel to be excavated. A detachable hood is provided, when boring through gravel and sand, to replace the upper and side portions of the cutting edge. As the excavation progresses, the whole shield is pressed forward by hydraulic jacks or shoving rams; and in order to preserve true alinement in the finished tunnel it is necessary to steer the shield by exerting more pressure on some of the jacks, thereby giving the shield a certain "lead" or angular direction, either laterally or vertically, or both. It is sometimes necessary also to give the shield a constant lead in one direction in order to produce a true alinement when passing through certain classes of material. For subaqueous tunneling, where compressed air is used, the shield is a very complicated machine, and the progress of the work hinges largely on its proper design.*

"Dredging" Method.—This method was employed by McMullen & McBean,† sub-contractors, in the construction of the west half of the river portion of the Harlem River Tunnel, New York City, for the Rapid Transit Railroad. Mr. McBean describes the work as follows:

First a channel was dredged across the river bottom to within a few feet of the full depth of excavation required to build the tunnel. In this channel, foundation piles and a row of specially prepared heavy timber sheeting, along each side and across the ends, were driven and cut off to a true plane about 25 ft. below the surface of the water. This sheeting forms the sides and ends of a pneumatic working chamber. For the roof of this chamber a platform of timber 40 ins. in thickness and extending the full width and length of the tunnel section, was built and sunk and rested on the cut off sheeting which formed the sides and ends as above described. Simultaneously with pumping the water from under this roof, compressed air was forced into the chamber under a pressure corresponding to the pressure of the water above the roof. Inside this chamber the west half of the tunnel was built and then the timber roof was removed.

"Caisson" Method.—This consists in building and sinking caissons to the required level below the surface of the river bed and later connecting them together so that they form sections of the completed tunnel. See *Eng. News* of Feb. 15, 1906, and April 11, 1907, for illustrated description of

* See articles on "The Construction of the Pennsylvania R. R. Tunnels Under the Hudson River at New York City," by James Forgie, in *Eng. News* of Dec. 13, 1906, and Feb. 28, 1907.

† Mr. D. D. McBean is credited with the design of this method of tunneling.

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od, proposed by Mr. L. Chagnand, a Paris contractor, and applied
nstruction of a tunnel crossing the Seine on Line 4 of the Metro-
ilway of Paris. It may be noted here that in connecting the
ctions in place, the "Freezing" method is proposed. The first
nk measured 54 ft. wide, 41 ft. high, 118 ft. long, and weighed
s. without the cast iron lining. See also *Le Genie Civil*, Dec. 2,

mance of Work.—References are given here to recent tunnel con-
under the most approved methods; particular pains being taken
o the attention of the young engineer the *various classes of work*
ng.

f Building Subways in New York City.—Nearly four pages devoted to
sts of about ½-mile section of Brooklyn extension. Cost, including plant,
000. Some of the unit costs are: Earth excavation, $3.62 per cu. yd.;
ation, $5.71 per cu. yd.; concrete for foundations, $4.61 per cu. yd;
r arches, $7.69 per cu. yd.; steel work, about 3½ cts. per lb. in place;
ng, about $24.59 per cu. yd.—*Eng.-Contr.*, Feb., 1906, p. 29.
a Small Rock Tunnel, 9 ft. diam. and about 3000 ft. long. About 2.46
excavation, in hard trap rock, per lin. ft. of tunnel. Total cost, including
ciation, repairs, etc., $9.36 per cu. yd. or $23.02 per lin. ft. 75% dyna-
sed. Data by F. L. Pruyn.—*Eng.-Contr.*, May 16, 1906.
atskill Aqueduct of the Additional Water Supply System of New York
r three pages of descriptive matter, with illustrations of standard types
t: in dry loose earth, on embankment, for grade tunnel (timbered and
rock), and for Hunter's Brook siphon (single and double ring sections).—
, Dec. 19, 1906.
o (Lawrence Ave.) Intercepting Sewer.—Length of land portion, Lake
o Chicago River, 8220 ft., through clay. Internal dia. of lining, 16 ft.;
a. of shield, 20′-1″; brick lining in 4 rings, backed with solid timbering
Contract price, $79.50 per lin. ft. Article contains about 2½ pages of
matter relating to methods and cost, with illustrations of draw knives
—*Eng.-Contr.*, Feb. 6, 1907.
ss of Work in Lowering the Tunnels under the Chicago River.—About
of descriptive matter, illustrated. Tunnels at Van Buren, Washington
e Sts. to be lowered to meet Federal Government requirement for 26-ft.
ead of 17-ft. depth heretofore maintained.—*Eng. News*, Sept. 13, 1906,
1907.
ubways in British Cities and in Paris.—Over four pages, with numerous
s.—*Eng. News*, Mar. 14, 1907
Collapsible Steel Centering for Concrete Conduit Work.—Illustrated.
d for sewer work.—*Eng.-Contr.*, Feb. 13, 1907.
l of Relining the Hodges Pass Tunnel, Oregon Short Line Ry.—Two
illustrations.—*Eng.-Contr.*, Feb. 20, 1907.
ay Tunnel under Thames River, at London, Eng.—Four pages with
strations including plans of shield —*Eng. News*, Dec. 19, 1907.
ls of drilling and Mucking in a Rock Tunnel.—Abstract of article in May,
of *Mine and Quarry*,* by W. P. J. Dinsmore. An interesting article of
ages, illustrated, treating of the plan of work, arrangement of drill holes,
e muck, ventilation, etc.—*Eng.-Contr.*, Jan. 1, 1908.

Notable Tunnels.—The following data have been compiled from
urces:

n Tunnel.—Twin single-track R. R. tunnels Nos. 1 and 2, 55.76 ft. apart
hrough Alps between Baill, Switzerland, and Iselle, Italy, on line of Jura-
y. Length 19729 meters = 64728 ft. = 12.26 miles. Tunnel No. 1 con-
898–1905) with clean width of 14.75 to 16.4 ft., and height above rail,
sectional area, 250 sq. ft.; cost $180 per lin. ft.; drift method; thickness
, 14 to 24 inches. Tunnel No. 2 was simultaneously put through as a *head-*
ide for ventilation and drainage; to be enlarged later.
Gothard Tunnel.—(1872–82.) Double-track R. R. Between Goschenen
Switzerland. Length 48887 ft. = 9.26 miles. Clear width 26.24 ft.; clear
8 ft. Cost $246 per lin. ft.
enis Tunnel.—(1862–71.) Double track R. R. Between Fourneau, France,
nèche, Italy. Length 42157 ft. = 8 miles. Clear width 26.24 ft.; clear
ft. Cost $380 per lin. ft.
Tunnel.—(1880–84.) Double-track R. R. Austria-Switzerland. Width
eight 25 ft. Cost $208 per lin. ft.
Tunnel.—(1854–76.) Double-track; Troy and Greenfield R. R., Mass.
81 ft. = 4.75 miles. Clear width 26 ft.; clear height 21 ft. Cost about
. ft.

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Cascade Tunnel.—(1897–1900.) Single track, through Cascade Mts., on Great Northern Ry. Length 13813 ft. = 2.62 miles. Clear width 16 ft.; clear height above base of rail, 21.5 ft. Concrete lining, 2 to 3½ ft. thick, replacing temporary timber lining.

Stampede Tunnel.—(1886–88.) Single track, through Cascade Mts., on Northern Pacific R. R. Length 9850 ft. = 1.87 miles. Contract price $118 per lin. ft., without masonry lining.

Busk Tunnel.—(1890–93.) Single track, through Rocky Mts., on Colorado Midland R. R. Length 9395 ft. = 1.78 miles. Clear width 15 ft.; clear height 21 ft.

Musconetcong Tunnel.—(1872–75.) Double-track R. R., on Easton and Amboy (L. V. R. R.). Length 4879 ft.; clear width 26 ft.; clear height 21 ft. [H. S. Drinker, author of "Tunneling," was a resident engineer on this work.]

EXCERPTS AND REFERENCES.

A Proposed Ventilating System for the Park Ave. Tunnel, N. Y. City (By A. H. Cary. Eng. News, Aug. 8, 1901).—Illustrated.

Subaqueous Tunnel Siphons of the Mass. Pipe Line Gas Co. (W. W. Cummings. Jl. Assn. of Eng. Soc., June, 1901; Eng. News, Oct. 3, 1901).—Illustrated details.

Difficult Work in Repairing a Swiss Railway Tunnel (Eng. News, Sept. 25, 1902).—Shows, by illustration, method of centering for caving roof.

Freezing Process for Building the River Tunnels of the P. R. R. at N. Y. City (By Charles Sooysmith, Inventor. Eng. News, Dec. 4, 1902).—Illustrated.

Methods of Work Adopted in Constructing the Chicago Telephone Tunnels (Eng. News, Feb. 19, 1903).—Illustrated.

Tunnel at Michel Creek Loop, Crow's Nest Pass Line, Can. Pac. Ry. (By C. R. Coutlee. Eng. News, April 2, 1903).—Illustrated.

Construction of the Simplon Tunnel (Eng. News, Aug. 13, 20, 27 1903).—Complete description of the tunnel and its construction. Illustrated.

The Pennsylvania R. R. Tunnel Under the North River, N. Y. City (Eng. News, Oct. 15, 1903).—Complete detailed description of method of construction. Illustrated.

The East River Division of the Penn. R. R. Tunnel, at N. Y. City (Eng. News, Oct. 29, 1903).—Illustrated.

The Cost of Concrete Tunnel Lining and of Tunnel Excavation (By Geo. W. Lee. Eng. News, Dec. 17, 1903).—Illustrated sections with cost data.

Remarkable Progress of the Hudson River Tunnel for the N. Y. & N. J. R. R. Co. (Eng. News, Nov. 10, 1904).—Illustrations of shield.

Waterproofing the P. R. R. Tunnels at New York (Eng. News, June 29, 1905).—Specifications.

Concrete Stringers and Tracks in Mine Shafts (Eng. News, Jan. 25, 1906).—Illustrated.

The Ventilation of Tunnels (By C. S. Churchill. Trans. A. S. C. E., Vol. LVII).—Includes the ventilation of subways.

Tunnel Lining Work in the Far West (Eng. News, Dec, 6, 1906).—Descriptive and illustrated article on lining and relining; masonry and timber.

The Construction of the P. R. R. Tunnels Under the Hudson River at N. Y. City (By James Forgie. Eng. News, Dec. 13, 20, 1907).—Complete Description of the tunnel and its construction. Plans of shield. Tables VI and VII (Eng. News, Feb. 28): Comparative Statement Giving Particulars of Some of the Principal Tunnels (Railroad and Other Than Railroad) Constructed Under Waterways. Very comprehensive.

Alpine Railway Tunnel Data (Eng. News, Dec. 5, 1907).—Table, giving: Particulars of the Four Principal Alpine Railway Tunnels.

Records in Rock Tunneling (Eng. News, April 2, 1908).—Shows progress figures—greatest advance made in any one month, in feet, for a single heading. See, also, Eng. News, Nov. 19, 1908.

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Detail Tunnel Costs in Tunnel No. 7, Los Angeles Aqueduct (By hards. Eng. News, Nov. 18, 1909).—Finished cross-section of l, with 8-in. concrete lining, 8 ft. 6 ins. wide and 10 ft. 6½ ins. high Over-breakage was 17%, making 6½ cu. yds. broken material tunnel. Total cost of tunnel, timbered and ready for lining, r lin. ft. **Drilling.**—Done by No. 7 Leyner drill, water being ough the hollow steel. Drill used approximately 66 cu. ft. free n. at 83 lbs. pressure, drilling holes to 10 ft. in depth. During riod 150 holes were drilled, aggregating 1203.3 ft. in length; average ole being 8 ft. Average speed of drilling was 21.74 ft. hole per tual drilling time, or 15.84 ft. per hour if lost time is included: ns that the average 8-ft. hole was drilled in 22 mins. Average lling was 3.6 cts. per ft. of hole. **Explosives.**—There were used 1½-in. 40% gelatine; 250 lbs. 1-in. 40% gelatine; and 150 lbs. gelatine; a total of 1,050 lbs., or 11.6 lbs. per lin. ft of tunnel, or cu. yd. place measurement. Cost (del.) was $140.87. Adding (2,700 ft., $13.85) caps (306=$2.49), and tamping stick ($0.48), total cost $157.69, which is about $0.50 per cu. yd. place measure- 0.27 per cu. yd. loose. Tables of detailed costs are not reproduced

se of Steel for Mine Timbering (By R. B. Woodworth. Paper, bering in Steel," before W. Va. Coal Mining Inst., Dec. 7, 1909; , Feb. 24, 1910).—Illustrated.

ew York Tunnel Extension of the Penn. R. R. (Trans. A. S. C. E., III., Sept., 1910).—Following papers presented:—

No. 1150. General Discussion. By C. W. Raymond.
" 1151. The North River Division. By C. M. Jacobs.
" 1152. The East River Division. By Alfred Noble.
" 1153. Meadow Div. and Harrison Transfer Yards. By E. B. Temple.
" 1154. The Bergen Hill Tunnels. By F. Lavis.
" 1155. The North River Tunnels. By B. H. M. Hewett and W. L. Brown.
" 1156. The Terminal Station-West. By B. F. Cresson, Jr.
" 1157. The Site of the Terminal Station. By G. C. Clarke.
" 1158. The Cross-Town Tunnels. By J. H. Brace and F. Mason.
" 1159. The East River Tunnels. By Brace, Mason and Woodward.

ted Cross-Sections of Some Tunnels:—

Description.	Eng. News.
Musconetcong tunnel during traffic	Nov. 7, 1901.
ss-sections Albina tunnel, Switzerland	Dec. 19, '01,
sections for Colo. Springs & Cripple Cr. Dist. Ry.	May 1, '02.
nstruction, Long Island R. R.	May 22, '02.
on of Pryor Gap tunnel, Mont., B. & M. R. R.	July 3, '02.
ons of telephone tunnels in Chicago	July 17, '02.
nstruction in N. Y. City	Sept. 18, '02.
nstruction in N. Y. City	Feb. 12, '03.
f wide arch soft ground tunneling	June 4, '03.
rch subway construction in open cut	June 11, '03.
Gallitzin tunnel, P. R. R., with concrete	Sept. 24, '03.
nstruction under Harlem River, N. Y. City	Oct. 1 and 8, '03.
derground railway tunnel in soft ground	Oct. 29, '03.
nstruction in N. Y. City	Nov. 19, '03.
f wide arch rock tunnel at Genoa, Italy	Nov. 26, '03.
on of Winston tunnel, Chi. Gt. Western Ry.	Aug. 4, '04.
on of P. R. R. tunnel under Capitol Hill, Wash. D.C.	Sept. 14, '05.
ks tunnels, Salt Lake City	Dec. 14, '05.
nethod of tunnel construction by sinking caissons	Feb. 15, '06.
unnel under Detroit River for Mich. Cent. R. R.	Feb. 15, '06.
London in connection with street improvement	July 12, '06.
work of the P. R. R. under East River, N. Y. City	July 12, '06.
locks for tunnel lining, Mex. Cent. Ry.	July 26, '06.
ons of pipe subways in London	Mar. 14, '07.
supply tunnels at Chicago	Nov. 14, '07.

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Description.	Eng. News.
Cross-section of Cascade tunnel, for electric system	Nov. 18, '09.
Lining and grouting tunnels in water-bearing material	Nov. 25, '09.
Railway tunnels; cross-sec., grades, lining, drainage	June 2, '10.
Reconstruction Washington St. Tunnel under Chicago river	July 21, '10.
Typical sections of tunnels, Catskill aqueduct	Oct. 20, '10.
	Eng. Rec.
Concrete-lined tunnel, 8′ x 9′, Los Angeles aqueduct	July 3, '09.
Sections of the Washington St. tunnel, Chicago	Dec. 11, '09.
The Bergen Hill 4-track tunnel, Erie R. R.	Dec. 18, '09.
Sections of Main St. Subway, Cambridge, Mass.	Jan. 1, '10.
Relining railroad tunnel with cast-iron segments	Jan. 1, '10.
Terryville tunnel, double track, N. Y., N. H. & H. R. R.	Feb. 26, '10.
Construction of tunnel for Great Western Power Plant	July 16, '10.
Section of tramway and pipe subway in Kingsway, London	Dec. 10, '10.

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58.—SURVEYING, MAPPING AND LEVELING.

Care of Instruments.—The correct use of surveying instruments takes into consideration their lack of precision. No instrument, however carefully it may be adjusted in the shop or field, will remain permanently accurate with ordinary field use. New instruments hold their adjustments better than old ones because the parts of precision are less worn. With good care in handling, the life of an instrument can be doubled without decreasing its duty. For the Level, there is but one main object of adjustment, namely, to preserve a *horizontal plane of sight* around a vertical axis. For the plain Transit it is necessary, in addition to the above "horizontal plane" adjustments, to preserve a *vertical plane of sight* passing through the point of the suspended plumbbob. Note that when the instruments are out of adjustment these "planes" of sight are warped into "cones" of sight, with the apex of the cone at the center of the telescope; hence the adjustments consist in changing "cones" into "planes" of sight.

To Adjust the Level.*—In running a line of levels, if the back-sights and fore-sights are of equal length the results of the leveling will be accurate even if the instrument is out of adjustment, as shown in Fig. 1. Thus, the elevation of each of the turning points at *R* will be accurately determined, but the "height of instrument" in each case will be wrong because the lines of sight are not level. For general leveling, however, the instrument should be adjusted so that all sights will be level.

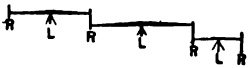

Fig. 1.

First, the cross-hairs, or line of collimation.—"Set up" the level firmly (although it need not be accurately leveled) with one diagonal pair of leveling screws *l* in line with the telescope. Open the clips *c c*, Fig. 2, so the tele-

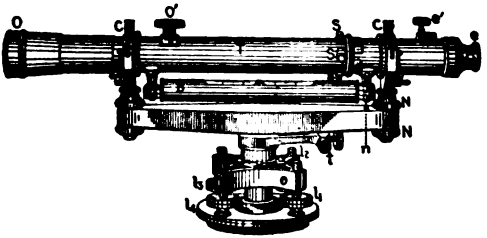

Fig. 2.—The Level.

cope will be free to revolve in the Ys. Adjust the eye-glass *e* by the milled ead *e'*, so the cross hairs appear distinctly. Sight the telescope *T* on a istant point (at the intersection of horizontal and vertical lines) about 50 to 300 feet away, as this is the usual length of sight for accurate leveling. ove the object glass *o* by the milled head *o'* so that the object is seen istinctly without "parallax" (i. e., without the cross-hairs "dancing" or ppearing to move away from a straight line of sight). With the bubble be below the telescope, as shown in the illustration, move the line of

* See also "peg-adjustment" described in the fourth adjustment of the ansit, p. 944.

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sight, by means of the leveling screws *l* and the tangent screw *t*, so the intersection of the cross-hairs will cover the distant point. Now, theoretically, if the telescope is revolved in the Ys by revolving the bubble tube completely around it, the cross-hairs are in adjustment if their intersection continues to cover the distant point. That is, the line of sight is along the central axis of the telescope and pierces the center of the object glass. When either cross-hair leaves the distant object it is out of adjustment. The cross-hairs are stretched across a circular, movable ring or diaphragm, called the *reticule*, held in position by four adjusting screws *s*. To move the horizontal cross-hair upward, loosen the top screw and tighten the bottom one. (See also Fig. 6.) The reverse operation will lower it. Similarly, the vertical cross-hair can be moved laterally by the two side screws. These

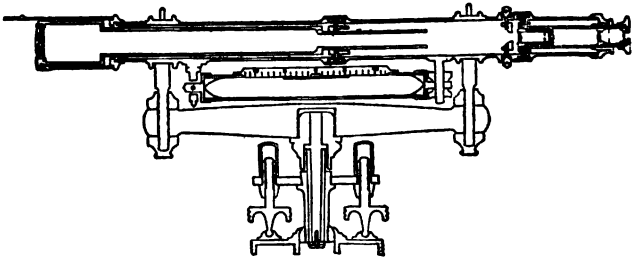

Fig. 3.—Level Telescope.

screws are turned with capstan or adjusting pins. *To adjust the horizontal cross-hair* bring it on to the distant object, with the bubble tube below the telescope; revolve the telescope half round, in the Ys, bringing the bubble tube above; correct *one-half* the *variation* from the distant object, by operating the top and bottom screws *s*. Repeat until accurately adjusted. The vertical cross-hair can be adjusted in the same manner.

Second, the bubble tube.—With the clips *c* open as before, level the instrument and lightly clamp the telescope over one set of leveling screws *l*, by means of the tangent clamp. Now level the bubble *B* accurately, gently lift the telescope out of the Ys and set it back reversed, end for end. If the bubble is still level it is in adjustment. If it moves either way from a central position it is out of adjustment. Correct *one-half* the *variation* from the central position with the leveling screws and the balance by operating the capstan nuts *n* at the right-hand end of the tube as shown in Fig. 2, thus rising one end of the tube to a level position. Repeat (after leveling the instrument again with the leveling screws *l*) until the bubble remains central in the tube upon reversing the telescope in the Ys.

Third, the Ys.—In the two previous adjustments, we have fixed the line of sight centrally through the telescope, and adjusted the bubble tube parallel with it. It now remains to adjust the Ys, in which the telescope rests, to the same level, so that when the instrument is leveled up the vertical center pin will be truly vertical, and the line of sight truly horizontal in any direction. To do this, level up the instrument carefully, and start with the telescope over one set of the leveling screws *l*. With the telescope clamped firmly in the Ys, swing half round on the vertical center pin so the telescope will rest over the same screws but reversed in direction. Correct *one-half* the *variation* of the bubble, from the central position, with the leveling screws *l*, and the balance by means of the large capstan nuts *N*, operated with large adjusting pins, and similar to the nuts *n* described in the second adjustment. (Level up and) repeat, with the same screws, until adjustment is effected. The adjustment may be repeated over the other set of screws in order to test the workmanship of the vertical center pin and bearings, but it is rarely done.

To Adjust the Transit.—Even with a transit considerably out of adjustment, leveling can be done by using equi-distant back- and fore-sights; straight lines can be run by half-revolving the alidade (commonly called

half around") between reversals of the telescope; and angles can red by the method of continuous readings (called "doubling") e "vertical" cross-hair is not perpendicular, by proper inclination escope. But the only safe and proper way is to keep the instrument ment. There are four or five principal adjustments, as follows:

the plate-bubbles.—The two bubbles *b b* are usually set at right h one another, one on a pair of standards and the other on the izontal plate. When the bubbles are adjusted and are central in the e central axis or center pin of the transit is *vertical*. To adjust

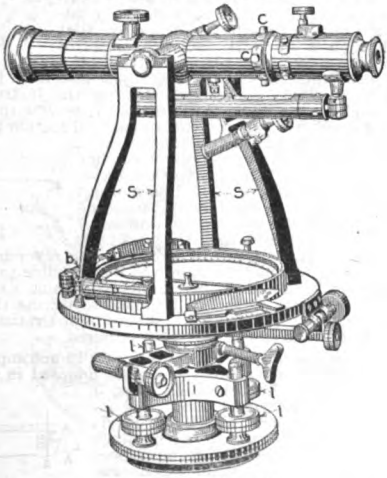

Fig. 4.—The Transit.

the bubbles, revolve the alidade so the bubble will be parallel set of diagonal leveling screws *l l*; level the instrument; revolve de 180° and correct *one-half* the *variation* of the bubble from a osition, by the leveling screws *l*, and the other half by raising ng one end of the tube, using the adjusting pin in the capstan rew or nut at one end. Repeat until the bubble remains central in when the alidade is revolved. The other bubble is adjusted

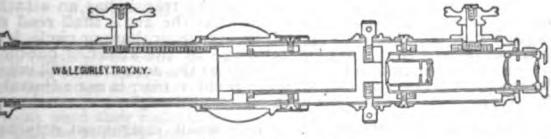

Fig. 5.—Transit Telescope.

d, the standards.—In order that the telescope will revolve about a true vertical circle, the standards *s* must support the *axis* of the in a perfectly horizontal position. In some instruments there is for raising or lowering one end of the axis in the standard, while there is no adjustment possible without returning the instrument op. To test the standards, level the instrument, sight on some t with the instrument clamped against lateral movement, revolve

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the telescope vertically downward and fix a *low* point on the line of sight. Now revolve the alidade 180°, sight again on the high point and get another low point beside the first one. A point midway between the two low points will lie in a vertical plane passing through the high point. The adjustment can then be made on the standard, using these two fixed points.

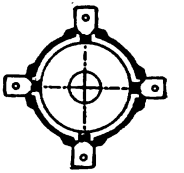

Fig. 6.

Third, the vertical hair or line of collimation.—By the two preceding adjustments we have secured a vertical axis for the alidade, and a horizontal axis for the telescope. It now remains to adjust the vertical cross-hair in the telescope so that in revolving through a vertical circle the line of sight will describe a vertical plane instead of a "cone," or in other words, so that a straight line may be produced by "reversing the instrument" (telescope). Set up the instrument on fairly level ground. Sight on a fixed point *a*, Fig. 7, reverse the telescope vertically, and set a point *b* on line in the opposite direction from *a*; revolve the alidade 180°, set the instrument again on *a*, reverse the telescope, and set a point *c* on line opposite the point *b*. Correct the vertical cross-hair by moving it laterally so the line of sight strikes *d* ($cd = \frac{1}{4}\,cb$). This is done by operating the side capstan-headed screws *c* which move (laterally) the diaphragm ring or reticule to which the cross-hairs are attached. Repeat the whole operation until, on reversing, the first and second points (*b* and *c*) coincide at *e*, on the straight line produced. If the "vertical" cross-hair is not truly perpendicular (which can be tested by sighting at a plumb-string) it should be made so during this adjustment. This is done by loosening two adjacent screws of the reticule and tapping gently against their heads in the direction required.

Fig. 7.

Fourth, the telescope bubble.—This is generally accomplished by what is called the "peg-adjustment," sometimes also adopted in the adjustment of the engineer's level. Two leveling pegs, *a* and *b*, are driven nearly level and about 250 ft. apart (Fig. 8). The instrument is set up at *A*, about a foot from the rod at *a*, and rod readings are taken at *a* and *b*, these readings being respectively A_a and A_b. Similarly, with the instrument at *B*, readings B_b and B_a are taken. Then, from the nature of the problem,

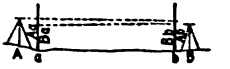

Fig. 8.

$$A_a - B_a = A_b - B_b \pm 2c \qquad (1)$$

in which c = the inclination of the line of sight from the horizontal, between the two rods. Now with the instrument at *B*, fix a point on the rod at *a*, at elevation $B_a \pm c$ above the peg; sight the telescope on this point, using the horizontal cross-hair; and bring the telescope bubble to a level in the tube by raising or lowering one end of the latter, with the adjusting screws. Equation (1), in practice, will determine whether c is + or −. Repeat the above for a new and more refined adjustment.

Fifth, the vertical arc or circle.—If the transit has an attachment for reading vertical angles, it is desirable that the angle shall read zero when the telescope is horizontal. To adjust the vertical arc or circle, level up the instrument, and then level the telescope by the attached bubble. Adjust the vernier (attached to the standards) so the zero mark will coincide with the zero of the vertical circle or arc. If the vernier is not adjustable, record the "index error" to be used with all vertical angles measured.

The Solar Attachment.—This is a small instrument attached to the upper part of the transit telescope, for determining the meridian, latitude, time, etc. It is used largely in government land surveying where the section and township-lines are supposed to run in the direction of the principal points of the compass.

Fig. 9 illustrates the solar apparatus manufactured by the Messrs. Gurley, of Troy, New York.

The circles shown represent those supposed to be drawn upon the concave surface of the heavens. When the telescope is set horizontal by its spirit level, the hour angle will be in the plane of the horizon, the polar axis will point to the zenith,

eros of the vertical arc and its vernier will coincide. If we incline ope, directed north as shown in the cut, the polar axis will descend from on of the zenith. The angle through which it moves, being laid off on the e, will be the co-latitude (90°—latitude) of the place where the instrument

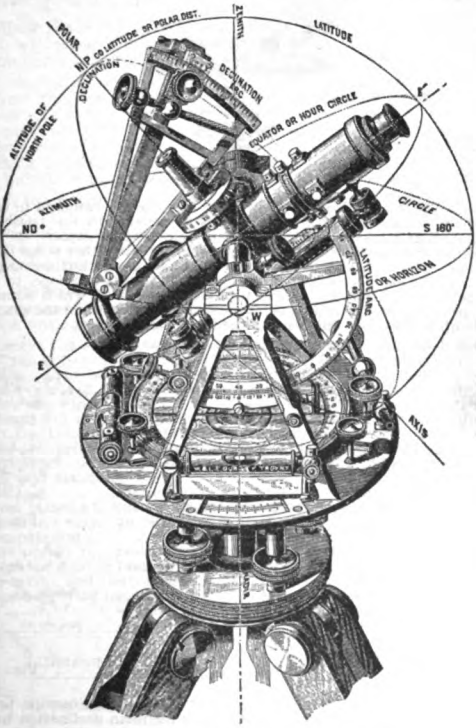

Fig. 9.—Transit with Solar Attachment.

e latitude itself being found by subtracting this angle from 90°. When the ove (north of) or below (south of) the equator, its declination, or angular bove or below, as given in the Ephemeris or Nautical Almanac, can be set he declination arc, and its image brought into position as before. In order , however, it is necessary not only that the latitude and declination are rectly upon their respective arcs, but also that the instrument is moved n until the polar axis points to the pole of the heavens, i. e., is placed in the he meridian. Thus the position of the sun's image will indicate not only le of the place, the declination of the sun for the given hour, and the time, but it will also determine the meridian, or true north and south line rough the place where the observation is made. Allowance for declina- referring to the Ephemeris, and setting off on the arc the declination of r the given day and hour, we are still able to determine its position with certainty as if it remained on the equator. When the sun's declination is m Sept. 22 to Mar. 20) the arc is turned downward, toward the plates of t; during the remainder of the year the arc is turned upward.—From *Manual*, page 71, etc.

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The Solar Instrument* manufactured by Keu (see Fig. 10), is perhaps one of the neatest and market. Attached to the transit it may be used astronomical meridian, (*B*) the latitude, and (*C*) th a vertical sighting telescope in mining engineering.

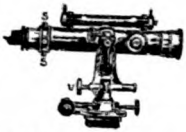

Fig. 10.—Solar Instrument.

It consists of a small telescope with prism to eyepiece standard which revolves upon a vertical axis attachable transit. This small telescope, called the solar telescope, altitude and azimuth, slow motion being imparted to it in tangent screws. The vertical axis, called the polar axis, can with the axis of the earth's rotation by inclining the transi attached, the vertical circle giving the inclination. A lev solar telescope is provided with two pointers, so placed that of them falls upon the other, the sun will be in the field of

(*A*). For determining the meridian. (*a*). Incline the transit telescope until the angle of declination† corrected for refraction, is indicated by the vertical limb or arc, depressing the telescope if the sun's declination is north, and elevating it if it is south. See Fig. 11.

(*b*). Bring the solar telescope into the vertical plane of the transit telescope, (without disturbing the position of the latter) and also to a horizontal position by means of its level. The two telescopes will now enclose an angle equal to the amount of the declination. See Fig. 12.

(*c*). Without disturbing the relative positions of the two telescopes, elevate the transit telescope (and with it the solar) until the amount of the co-latitude is indicated by the vernier of the vertical limb. See Fig. 13.

(*d*). Revolve the two telescopes together upon their respective vertical axes until the image of the sun is brought into the field of the solar telescope; when the sun is accurately bisected the transit telescope will be in the meridian and the compass needle will indicate the amount of its declination at the place of observation. It will of course considerably facilitate this last operation if, before commencing to revolve the two telescopes, the transit one is approximately pointed toward the south by means of the transit compass needle.

(*B*). For ascertaining the latitude. Direct the transit south, incline it to an amount equal to the sun's meridian d for refraction, depressing the telescope if the declination is no it is south. Now bring the solar telescope into the vertical telescope and to a perfectly horizontal position by means of it A few minutes before noon (the moment of the sun's culmin image between the two horizontal wires of the solar telescope *transit telescope* in altitude and azimuth. By means of the transit, keep the sun, as it continues to rise and travel southw relatively to the cross-hairs of the solar telescope. When it h the reading of the vertical arc of the transit, deduct from it this altitude, and the remainder is the co-latitude, which ded the latitude. The position of the two telescopes is identical Fig. 13.

(*C*). For observing the time. Having brought the two final positions for determining the meridian, that is the transit and the solar telescope bisecting the sun, revolve them both axis, without disturbing the vertical axis, until they are both

* Invented by Saegmuller, 1881.
† See American Ephemeris or Nautical Almanac.

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ed by their respective lines of sight, which can be determined by sight-
he two telescopes upon any clearly defined distant object, and taking the
of the respective readings of the transit horizontal limb, is the hour angle.
en reduced to time before or after apparent noon: One degree of arc = 4
time and 1 minute of arc = 4 seconds of time. The time obtained by
bservation is reliable to a few seconds.

djust the Solar Attachment.—Attach the instrument to a transit
adjustment, or adjust the transit as explained, page 942. In
nuller solar (Fig. 10) there are two adjustments as follows:

the polar or vertical axis.—Level the transit and also its tele-
ow revolve the solar telescope horizontally about its polar axis
whether the bubble in the small tube attached above it moves
ntral position. If so, correct *one-half* the *variation* by the screw *v*,
nd the other half by adjusting the bubble tube to a level position,
sing or lowering one end by the capstan-headed screws. Repeat,
bubble remains central when the solar telescope is revolved.

l, the horizontal hair.—The horizontal lines of sight of the transit
and the solar telescope must be parallel with each other. To
h this the reticule or diaphragm of the latter is moved upward or
l by means of upper and lower capstan-headed screws *s*, Fig. 10,
ed in the adjustment of the level.

Observation with Transit Alone.—Mr. A. W. French gives the
method* for determining the meridian† from the sun by direct
n with the transit and without the use of solar attachment:

serve the sun directly, by the aid of colored glass, and bring his image
the horizontal and vertical wires; read the vertical circle and the hori-
es. Suppose that in the first pointing the image was approaching the
bring it into the opposite quarter of the field of view, where the image
m the wires; bring the wires tangent and read as before The mean of
ngs will give the apparent altitude and the plate reading for the sun's
the transit has a full vertical circle, the telescope should be reversed
e two pointings, to eliminate all errors of adjustment. If the transit has
ical arc, no reversal can be made, and great care must be taken that the
and standards are in good adjustment and the index error accurately

d plates when pointing at any convenient mark, thus finding the angle
e sun's center and the mark.
aputation of the *P Z S* triangle enables us to find the angle between the
e north; then, addition or subtraction of the angle between the sun and
s the mark is north or south of the sun) gives the angle made by the line
mark), and the meridian.
companying form of notes and reductions needs but a few remarks.

OBSERVATION.

	Horizontal Circle Readings.		Vertical Circle Readings.	Date and Time.
	On Mark.	On Sun.		
..	240° 41′ 00″	282° 51′ 30″	30° 57′ 00″	Sept. 26, 1896, 2.30 P. M.,
b	60° 41′ 00″	102° 43′ 00″	31° 09′ 00″	standard time, 75th merid.
...	240° 41′ 00″	282° 47′ 15″	31° 03′ 00″	

Eng. News, May 20, 1897.
meridian is the great circle whose plane passes through the *zenith*
les. The *azimuth* of a body is the horizontal arc measured on the
tween the north point and the vertical circle passing through the
s, the azimuth of the sun or of a star in determining the meridian.
ation of a body is its angular distance from the equator measured
r circle; thus the declination of the sun is south in winter, north
, and zero when it crosses the equator on March 20 and Sept. 20.
e of a body is its distance from the horizon measured on a vertical
e *zenith* distance is 90° minus the altitude. "*Co*"—before declina-
ide, etc., means the complement of (90° −) these quantities.

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COMPUTATION.

Declination at Greenwich noon = 7 A. M. standard time 75th meridian - - - - - - - - - = 1° 32′ 33″ south
Hourly change = 58.5″. Change for 7½ hours = 58.5 × 7½ = 7′ 39″ south

Declination at 2 P. M - - - - - - = 1° 40′ 12″ south

Average vertical angle by observation - = 31° 03′ 00″
Correction for refraction - - - - - = 1′ 40″

True altitude - - - - - - - = 31° 01′ 20″

Latitude of Thayer School = 43° 42′ 10″
Station about 1 mile south = 1′ 00″

Latitude of station = 43° 41′ 10″

$$\text{Cos } \tfrac{1}{2} PZS = \sqrt{\frac{\sin \tfrac{1}{2} S \times \sin (\tfrac{1}{2} S - \text{co-decl.})}{\sin \text{co-alt.} \times \sin \text{co-lat.}}}$$

where S = co-decl. + co-alt + co-lat.

co-decl. = 91° 40′ 12″
co-alt. = 58° 58′ 40″
co-lat. = 46° 18′ 50″

S = 196° 57′ 42″

½ S = 98° 28′ 51″
co-decl. = 91° 40′ 12″
½ S − co-decl. = 6° 48′ 39″

	log sin	98° 28′ 51″ =	9.995225
	" "	6° 48′ 39″ =	9.074052
a. c.	" "	58° 58′ 40″ =	0.067035
a. c.	" "	46° 18′ 50″ =	0.140781
		2	19.277093

log cos ½ PZS = 9.638546
½ PZS = 64° 12′ 40″
PZS = 128° 25′ 20″
Azimuth of sun from north = 128° 25′ 20″
Angle between sun and mark = 42° 06′ 15″
Angle north—station—mark = 170° 31′ 35″

"If a single observation is made, the altitude must be changed by the semi-diameter (16′) and the horizontal angle by, not 16′, but $\frac{16'}{\text{cos of alt.}}$. The dihedral angle, whose edge is the vertical line through the instrument subtended by the semi-diameter, varies with the altitude of the sun, from 16′ for alt. = 0°, to 46′ for alt. = 67° on June 21st, in this latitude (43° 42′)."

Meridian from the North Star.—Polaris is a distant "fixed" star called the north star. It lies about in line with the two "pointers" of the "big dipper" (the two stars farthest from the "handle") and facing its open top. It is the brightest star of a cluster of three forming one end of "Urser Minor" and can be recognized easily.* The earth's axis if produced will never pierce the north star, but wabbles around it in a circle. This makes the star appear to move in a circle around the true north point (Fig. 14), and hence we consider the pole P as fixed, and the star to move around it. The

* The accompanying diagram will aid in finding Polaris at any time of the year: Hold the diagram, while facing the north, in such a position that the month, during which the observation is made, will point vertically upward. Polaris crosses the meridian on April 10th of each year at about noon.

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ance, or radial distance from pole to star varies from a little 1° to a little less than 2° angular measure, varying from day om year to year, and also with the latitude of of observation. When the star appears at E or W eastern or western elongation, respectively; when it is at its upper or lower culmination. Now if we polar distance of the star when it is at its eastern elongation, and also the latitude of the place of n, it is a very simple matter to find the azimuth r or the horizontal angle from the star to the pole, what the surveyor desires in getting the true meri-

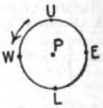

Fig. 14.

$$\text{ne of Azimuth of Star} = \frac{\text{Sine of Polar Distance}}{\text{Cos. of lat. of place of observation}} \quad \ldots (1)$$

1, below, is a table of polar distances from latitude 0, the equator, of each 3rd year, 1906–1930; and may be used in connection with formula (1) for any latitude, and for any time (year) by interpo-

-Polar Distances of Polaris for Lat. 0, the Equator.

906.0	1909.0	1912.0	1915.0	1918.0	1921.0	1924.0	1927.0	1930.0
′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
11 41	1 10 45	1 9 49	1 8 53	1 7 58	1 7 2	1 6 7	1 5 12	1 4 16
.31910	8.31341	8.30765	8.30181	8.29594	8.28999	8.28401	8.27794	8.27169
−.00569	−.00576	−.00584	−.00587	−.00595	−.00598	−.00607	−.00625	

Example of Use of Table 1.

the polar	Log sin *p. d.* for Jan. 1, 1915..... =	8.30181
p. d.) of	Add ½ diff (1915–1912) for July. 1, 1914.... =	0.00097
July 1,		
atitude N	Log sin *p. d.* for July 1, 1914..... =	8.30278
30′? Use	Log cos 38° − 30′ (latitude)..... =	9.89354
d formula		
	∴ *p. d.* = 1° − 28′ − 13″ Log sin..... =	8.40924

lar dist. at elongation of star is called the Azimuth at Elongation. titudes +25° to +72°, the direct Azimuths are given in Table 2, ng page.

bservation of Polaris for Azimuth will now be described. The ould be in adjustment and be provided with a reflector for illu- he cross-hairs. This may consist of a circular piece of bright tin diameter of the object glass, and with a hole in it about half its This annular ring may be fastened to a wire, bent around the

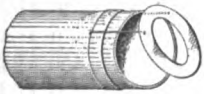

Fig. 15.

of the telescope as shown in Fig. 15. It should project outward e of about 45° and the under side may be painted white so as to light from a dark lantern (held beneath it) into the telescope. ht or illuminated target should be placed several hundred feet the transit, and *about* due north to serve in fixing a base line imuth reading to the star. The target may consist of a vertical bout $\frac{1}{16}$ of an inch in width) in a box with a light behind it. d on page 951.)

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2.—Azimuth of Polaris at Elongation Jan. 1. For Various Years and Latitudes.

Lat.	1900.0	1905.0	1906.0	1907.0	1908.0	1909.0	1910.0	Yearly Decrease.
°	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
+25	1 21.2	1 19.4	1 19.1	1 18.7	1 18.4	1 18.1	1 17.7	0 0.35
26	21.8	20.1	19 8	19.4	19.1	18.7	18.4	0.35
27	22.5	20.8	20.5	20.1	19.8	19.4	19.1	0.35
28	23.3	21.6	21.3	20.9	20.5	20.1	19.8	0.35
29	24.1	22.4	22.1	21.7	21.3	20.9	20.5	0.36
30	1 24.9	1 23.1	1 22.8	1 22.4	1 22.1	1 21.7	1 21.3	0 0.36
31	25.8	24.0	23.6	23.2	22.9	22.5	22.2	0.36
32	26.7	24.9	24.5	24.1	23.8	23.4	23.1	0.37
33	27.7	25.9	25.5	25.1	24.7	24.3	24.0	0.37
34	28.7	26.9	26.5	26.1	25.7	25.3	25.0	0.38
35	1 29.8	1 27.9	1 27.5	1 27.1	1 26.8	1 26.4	1 26.0	0 0.38
36	30.9	29.0	28.6	28.2	27.9	27.5	27.1	0.39
37	32.1	30.1	29.7	29.3	29.0	28.6	28.2	0.39
38	33.3	31.4	31.0	30.6	30.2	29.8	29.4	0.40
39	34.7	32.7	32.3	31.8	31.4	31.0	30.6	0.40
40	1 36.0	1 34.0	1 33.6	1 33.2	1 32.8	1 32.4	1 32.0	0 0.41
41	37.5	35.4	35.0	34.6	34.2	33.8	33.4	0.41
42	39.0	36.9	36.5	36.0	35.6	35.2	34.8	0.42
43	40.6	38.5	38.1	37.6	37.2	36.8	36.3	0.43
44	42.3	40.1	39.7	39.2	38.8	38.4	37.9	0.44
45	1 44.0	1 41.8	1 41.4	1 40.9	1 40.5	1 40.1	1 39.6	0 0.44
46	45.9	43.7	43.2	42.7	42.3	41.9	41.4	0.45
47	47.9	45.6	45.1	44.6	44.2	43.7	43.3	0.46
48	49.9	47.7	47.2	46.7	46.3	45.8	45.3	0.47
49	52.1	49.8	49.3	48.8	48.4	47.9	47.4	0.48
50	1 54.4	1 52.0	1 51.5	1 51.0	1 50.6	1 50.1	1 49.6	0 0.49
51	56.9	54.4	54.0	53.5	53.0	52.5	52.0	0.49
52	59.5	57.0	56.4	55.9	55.4	54.9	54.4	0.51
53	2 02.2	59.6	59.1	58.6	58.1	57.6	57.1	0.51
54	05.1	2 02.5	2 02.0	2 01.5	2 00.9	2 00.4	1 59.9	0.52
55	2 08.3	2 05.6	2 05.0	2 04.4	2 03.9	2 03.4	2 02.8	0 0.55
56	11.6	08.8	08.2	07.7	07.1	06.6	06.0	0.56
57	15.1	12.2	11.7	11.1	10.5	10.0	09.4	0.57
58	18.8	15.9	15.3	14.7	14.2	13.6	13.0	0.58
59	22.8	19.8	19.2	18.6	18.0	17.4	16.8	0.60
60	2 27.1	2 24.0	2 23.4	2 22.8	2 22.1	2 21.5	2 20.9	0 0.62
61	31.7	28.5	27.9	27.2	26.6	25.9	25.3	0.64
62	36.7	33.4	32.7	32.1	31.4	30.8	30.1	0.66
63	42.1	38.6	38.0	37.3	36.6	35.9	35.2	0.69
64	47.8	44.3	43.6	42.9	42.2	41.5	40.8	0.70
65	2 54.1	2 50.4	2 49.7	2 49.0	2 48.3	2 47.5	2 46.8	0 0.73
66	3 00.9	57.1	56.3	55.6	54.8	54.1	53.3	0.76
67	08.3	3 04.4	3 03.6	3 02.8	3 02.0	3 01.2	3 00.4	0.79
68	16.4	12.3	11.5	10.7	09.8	09.0	08.2	0.82
69	25.3	21.0	20.1	19.3	18.4	17.6	16.7	0.86
70	3 35.2	3 30.6	3 29.7	3 28.8	3 27.9	3 27.0	3 26.1	0 0.91
71	46.1	41.3	40.3	39.4	38.4	37.5	36.5	0.96
72	58.2	53.2	52.1	51.1	50.1	49.1	48.1	1.01

The azimuth is constantly growing less; hence the yearly *decrease*.

Ex.—Elongation of Polaris for Lat $N(+)38°-30'$ during June–July, 1914, or 4½ years after Jan. 1, 1910, is $1°-30'$ *minus* 4½ times $0°-0'.40=1°-28'.2$.

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(a) Observation of Polaris at Elongation.

efore the time of elongation of Polaris, bring the telescope into cutting the star with vertical wire; and with the horizontal angle ntained at zero on the vernier, follow the star by operating the e slow-motion screw. When the star ceases to move laterally it cally reached its elongation. Now measure the horizontal angle minated target, previously set (p. 949). By working rapidly, the be doubled as a check or for greater accuracy*. Better still, for any error of adjustment of the transit, it may be well to re-alidade 180° for the second reading, with the telescope reversed. of these two readings will be the correct one to use, and this be the horizontal angle between Polaris at elongation and the l base line fixed by the target. The *angle between this base line and eridian* can now be calculated by taking into consideration the f Polaris at its elongation, for the particular time and latitude, es 1 or 2, and 3†; thus, it will either be equal to the *sum* or the *diff* sured angle and the azimuth. The operator must have clearly hether the elongation is east or west† and in which direction red angle is taken, in order to lay off the *meridian* in the right A pencil sketch should be drawn showing the angles.

(b) Observation of Polaris at any Hour Angle.

ing to Fig. 14, the hour angle of Polaris (or of any star) is the time elapsed since its upper culmination, at U. The time occupied by making a complete cycle from U to U, is $23h\ 56.1\ m$, and in alf cycle, from U to L or from L to U, is assumed to be one-half $h\ 58m$. Therefore, if we know the time of upper culmination we o subtract or add $11h\ 58m$ to find the preceding or the following lower culmination. To an observer at the equator‡ (lat. 0), the hour angles of Polaris are: $0h\ 0m$ at U, $5h\ 59m$ at W, $11h\ 58.1m$ $7.1m$ at E, and $23h\ 56.1m = 0h\ 0m$ at U again. Now for any point the hour angle at western elongation will be diminished, while at ngation it will be increased; the amount of decrease or increase th the lat. of the observer. Thus, in 40° N. latitude the hour angle elongation is $5h\ 55m$, and at eastern elongation is $18h\ 01m$, out $4m$ in each case from the above as observed at the equator. be seen from the above that the azimuth of the star varies with *our angle* and the *latitude*|| of the place of observation, as well as *me* of observation.

are two kinds of *time* which must be kept clearly in mind, viz., c) Time and Astronomical Time. An astronomical day begins at

variation (deduction) of azimuth angle for time of observation to or after elongation is very small, being less than $0° - 01'$ for time. The following table shows the relation for lat. 45° N. and r 1910, the azimuth at elongation being about $1° - 40' = 100'$.

Time.	Diff. in Azimuth.	Diff. in Time.	Diff. in Asimuth.
$1m$	$0°\ 0.'001 = 0.''06$	$0h\ 25m$	$0°\ 0.'594 = 35.''6$
5	$0\ 0.\ 024 = 1.\ 4$	0 30	$0\ 0.\ 856 = 51.\ 4$
10	$0\ 0.\ 095 = 5.\ 8$	0 32. 4	0 1. = 60.
15	$0\ 0.\ 214 = 12.\ 8$	0 35	$0\ 1.\ 164 = 69.\ 8$
20	$0\ 0.\ 381 = 22.\ 9$	0 45. 9	0 2. = 120.

Table 3 we can find whether the elongation is east or west, as in the foot-note accompanying that table.

difference in time between the upper culmination and the adja-ation *diminishes* with increase in latitude; while the azimuth at *increases* with increase in latitude.

variation for longitude is very slight, and for points in the U. S. ible for ordinary surveying. Table 3 gives the local mean (astro-ime of the upper culmination of Polaris, computed for longitude 2m.) west of Greenwich. The meridian passes through the rts (near the edges) of Colorado and New Mexico, and is con-rly "central" for U. S. territory. (The eastern edge of Maine is west, the western edge of Oregon about 125° west, and the western aska about 168° west.) The error is of course greatest at the ns (at which time observation should be avoided when possible nd least at the elongations.

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noon of a civil day, and each is 24 hrs. in length. Thus, the *A. M* of a civil day corresponds to the last half of the preceding astronomical day, each lapping by 12 hrs.*

In making an observation on Polaris for azimuth, it is necessary to know: (1) the latitude of the place of observation, which may be obtained from a map; (2) the mean local time of observation, which can be calculated from the particular "standard time"† used in that locality when the longitude of the place of observation is known; (3) the horizontal measured angle from Polaris to the previously established base line, explained under (*a*), preceding, together with the mean local time of observation; (4) the hour angle of Polaris at time of observation, to be calculated from Parts I and II of Table 3; (5) the azimuth of Polaris from the hour angle, date, and latitude of place of observation, to be obtained from Table 4. (6) Find the sum or difference of the azimuth and the measured angle, explained under (*a*) preceding, and lay it off in the right direction from the established base line.

Practical Example of Use of Tables 3 and 4.

Place of observation, lat. 41° N., long. 100° W.; Mountain time, 8 30 P. M., Nov. 8, 1911; measured angle from Polaris *eastward* to base line, 2°—40′. Find the azimuth of Polaris, and the angle of base line with true meridian? Then we have—

	h	m	h	m		
Standard time of observation (merid. 105° W.)			8	30.0		
For merid. 100° W., add 5 x 4 m.				20.0		
Mean local time of obs., 1911, Nov. 8			8	50.0		
Equivalent time to Nov. 7 (add 24 h.)			32	50.0		
Astrom. time U. C. Polaris, Nov. 1‡ (Table 3, Part I)	10	48.0				
Reduction to Nov. 7 (Table 3, Part II)‖ Subtract		23.6		10 24.4		
Hour angle of Polaris at observation			22	25.6		
Subtract from				56.1		
Time argument for Table 4¶			1	30.5		
Azimuth of Polaris, at observation (lat. 41°)			0°	36′.5		R.
Measured angle eastward from Polaris to base line			2°	40′		E.
Angle of base line eastward from true meridian			3°	16′.5		R.

Therefore lay off angle 3° 16′.5 westward from base line to true meridian.

---

* Civil time *P. M.* = astronomical time with the *P. M.* omitted; thus, 5.30 P. M. = 5h 30m of the same date. Civil time *A. M.*, + 12 hrs. = astronomical time of the preceding date· thus, 5.30 *A. M.* June 2 = 17h 30m June 1. Astronomical time under 12h = civil time *P. M* of the same date· thus 5h 30m = 5.30 *P.M.* of same date. Astronomical time over 12h, with 12 hrs. deducted from it = civil time *A. M.* of the following date; thus, 17h 30m June 1 = 5.30 *A. M.* June 2.

† Standard railway time for longitude west from Greenwich: Intercolonial time, for 60° west, Eastern time, for 75° west; Central time, for 90° west; Mountain time, for 105° west; Pacific time for 120° west. 15° of longitude = 1 hr. of time (1° = 4 m.); 15′ of long. = 1 min. of time (1′ = 4 sec.); 15″ of long. = 1 sec. of time (1″ = 0.6⅔ sec.). *Ex.*—The place of observation is, say, 108° west, and the observer has Mountain time; hence the mean local time is 3 × 4 m. = 12 m. *slower* than his watch. For 102° west it would be 12 m. *faster* than his watch. *West* of standard time meridian means *deduct*; *east* means *add*.

‡ This is the nearest date preceding, in the table. Values are given in Part I for the 1st and 15th of each month; and in Part II the reduction is given for succeeding dates.

‖ Caution.—Be sure to use the same "Diff. for 1 day" in Part II as obtained from Part I, and opposite the day of the month *Subtract*; don't add.

¶ See "*Ex.*" below Table 3.

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Mean (Astronomical) Time of the Upper Culmination of , computed for Longitude 108° (7h. 12m.) west of Greenwich.

on line with any date in Part I is the hours and minutes elapsed (common watch time) since the preceding noon.]

Part I.

1901.	1902.	1903.	1904.	1905.	1906.	1907.	1908.	Diff. for 1 day.
h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	m.
39.5	6 41.0	6 42.4	6 43.9	6 41.4	6 42.8	6 44.3	6 45.7	3.95
44.2	5 45.7	5 47.1	5 48.6	5 46.1	5 47.5	5 49.0	5 50.4	3.95
37.1	4 38.6	4 40.0	4 41.5	4 39.0	4 40.4	4 41.9	4 43.3	3.95
41.9	3 43.4	3 44.8	3 46.3	3 43.8	3 45.2	3 46.7	3 48.1	3.95
46.6	2 48.1	2 49.5	2 47.0	2 48.5	2 49.9	2 51.4	2 48.9	3.94
51.5	1 53.0	1 54.4	1 51.9	1 53.4	1 54.8	1 56.3	1 53.8	3.94
44.6	0 46.1	0 47.5	0 45.0	0 46.5	0 47.9	0 49.4	0 46.8	3.94
45.6	23 47.1	23 48.5	23 46.0	23 47.5	23 48.9	23 50.4	23 47.8	3.93
42.8	22 44.3	22 45.7	22 43.2	22 44.7	22 46.1	22 47.6	22 45.1	3.93
47.8	21 49.3	21 50.7	21 48.2	21 49.7	21 51.1	21 52.6	21 50.1	3.92
41.2	20 42.7	20 44.1	20 41.6	20 43.1	20 44.5	20 46.0	20 43.5	3.92
46.4	19 47.9	19 49.3	19 46.8	19 48.3	19 49.7	19 51.2	19 48.7	3.91
43.8	18 45.3	18 46.7	18 44.2	18 45.7	18 47.1	18 48.6	18 46.1	3.91
49.0	17 50.1	17 51.9	17 49.4	17 50.9	17 52.3	17 53.8	17 51.3	3.92
42.4	16 43.9	16 45.3	16 42.8	16 44.3	16 45.7	16 47.2	16 44.7	3.92
47.6	15 49.1	15 50.5	15 48.0	15 49.5	15 50.9	15 52.4	15 49.9	3.92
41.0	14 42.5	14 43.9	14 41.4	14 42.9	14 44.3	14 45.8	14 43.3	3.92
46.1	13 47.6	13 49.0	13 46.5	13 48.0	13 49.4	13 50.9	13 48.4	3.93
43.3	12 44.8	12 46.2	12 43.7	12 45.2	12 46.6	12 48.1	12 45.6	3.93
48.3	11 49.8	11 51.2	11 48.7	11 50.2	11 51.6	11 53.1	11 50.6	3.93
41.4	10 42.9	10 44.3	10 41.8	10 43.3	10 44.7	10 46.2	10 43.7	3.93
46.4	9 47.9	9 49.3	9 46.8	9 48.3	9 49.7	9 51.2	9 48.7	3.94
43.3	8 44.8	8 46.2	8 43.7	8 45.2	8 46.6	8 48.1	8 45.6	3.94
48.1	7 49.6	7 51.0	7 48.5	7 50.0	7 51.4	7 52.9	7 50.4	3.95

Part I.—Continued.

1909.	1910.	1911.	Diff. for 1 day.
h. m.	h. m.	h. m.	m.
43.2	6 44.7	6 46.1	3.95
47.9	5 49.4	5 50.8	3.95
40.8	4 42.3	4 43.7	3.95
45.6	3 47.1	3 48.5	3.95
50.3	2 51.8	2 53.2	3.94
55.2	1 56.7	1 58.1	3.94
48.3	0 49.8	0 51.2	3.94
49.3	23 50.8	23 52.2	3.93
46.5	22 48.0	22 49.4	3.93
51.5	21 53.0	21 54.4	3.92
44.9	20 46.4	20 47.8	3.92
50.1	19 51.6	19 53.0	3 91
47.5	18 49.0	18 50.4	3.91
52.7	17 54.2	17 55.6	3.92
46.1	16 47.6	16 49.0	3.92
51.3	15 52.8	15 54.2	3.92
44.7	14 46.2	14 47.6	3.92
49.8	13 51.3	13 52.7	3.93
47.0	12 48.5	12 49.9	3.93
52.0	11 53.5	11 54.9	3.93
45.1	10 46.6	10 48.0	3.93
50.1	9 51.6	9 53.0	3.94
47.0	8 48.5	8 49.9	3.94
51.8	7 53.3	7 54.7	3.95

Part II.

Reduction of tabular times to *intermediate dates.*
Subtract the reduction when computing from a *preceding,* or add it when working from a *following* date.

Day of the month	Reduction. Arg.—"Diff. for 1 day."					No. of days elapsed.
	m. 3.91.	m. 3.92.	m. 3.93.	m. 3.94.	m. 3.95.	
	m.	m.	m.	m.	m.	
2or16	3.9	3.9	3.9	3.9	3.9	1
3or17	7.8	7.8	7.9	7.9	7.9	2
4or18	11.7	11.8	11.8	11.8	11.8	3
5or19	15.6	15.7	15.7	15.8	15.8	4
6or20	19.5	19.6	19.6	19.7	19.7	5
7or21	23.5	23.5	23.6	23.6	23.7	6
8or22	27.4	27.4	27.5	27.6	27.6	7
9or23	31.3	31.4	31.4	31.5	31.6	8
10or24	35.2	35.3	35.4	35.5	35.5	9
11or25	39.1	39.2	39.3	39.4	39.5	10
12or26	43.0	43.1	43.2	43.3	43.4	11
13or27	46.9	47.0	47.2	47.3	47.4	12
14or28	50.8	51.0	51.1	51.2	51.3	13
29	54.7	54.9	55.0	55.2	55.3	14
30	58.6	58.8	58.9	59.1	59.2	15
31	62.6	62.7	62.9	63.0	63.2	16

o find the time of upper culmination of Polaris on Nov. 8, 1911, for longi- est of Greenwich?

n.—From Part I of above table we have for Nov. 1, 1911, 10h. 48.0m., and *eduction*) for one day is 3.93 m. Now under this diff. in Part II, and in line ay of the month, 8, in the left-hand column (or in line with the number of ed since Nov. 1, namely 7, in the right-hand column) we obtain the total of 27.5m., making the astronomical time of upper culmination 10h. 48.0m. n. = 10h. 20.5m., or 10.20½ *P.M.*, local civil time. Note that in the practical

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4.—Azimuth of Polaris

[The hour angles are expressed in *mean solar* time. The occurrence of a *period* after

Star and Azimuth.

W. of N. when hour angle is *less* than $11^h\ 58^m$.

E. of N. when hour angle is *greater* than $11^h\ 58^m$.

Time argument, the star's hour angle (or $23^h\ 56^m.1$ *minus* the star's hour angle), for the year—

Polaris *above* the Pole.

To determine the true meridian, the azimuth will be laid off to the *east* when the hour angle is *less* than $11^h\ 58^m$, and to the *west* when *greater* than $11^h\ 58^m$.

Hours	1901	1902	1903	1904	1905	1906	1907	1908	1909	1910	1911	Azimuths for Latitude— 30°	32°	34°	36°	38°	40°	42°	44°	46°	48°	50°
h.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	m.	′	′	′	′	′	′	′	′	′	′	′
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	4.	4.	4.	5	5	5	5	5	5	5	5	2	2	2	2	2	2	2	2	2	2.	2.
	9.	9.	9.	9.	9.	9.	9.	9.	10	10	10	3.	3.	3.	4	4	4	4	4.	4.	4.	5
	14	14	14	14.	14.	14.	14.	14.	14.	14.	15	5.	5.	5.	5.	6	6	6	6.	6.	7	7
	19	19	19	19	19	19	19.	19.	19.	19.	19.	7	7	7.	7.	8	8	8.	8.	9	9	9.
	23.	23.	23.	24	24	24	24	24.	24.	24.	24.	9	9	9	9.	9.	10	10.	10.	11	11.	12
	28.	28.	28.	28.	29	29	29	29	29.	29.	29.	10.	11	11	11.	11.	12	12.	13	13	13.	14
	33	33	33.	33.	33.	34	34	34	34.	34.	34.	12.	12.	13	13	13.	14	14.	15	15.	16	17
	38	38	38	38.	38.	38.	39	39	39	39.	39.	14	14.	14.	15	15.	16	16.	17	18	18.	19
	42.	42.	43	43	43.	43.	44	44	44	44.	44.	16	16	16.	17	17.	18	18.	19.	20	21	21.
	47.	47.	48	48	48	48.	48.	49	49	49.	49.	17.	18	18.	19	19.	20	20.	21.	22.	23	24
	52	52.	52.	53	53	53.	53.	54	54	54.	54.	19.	20	20.	21	21.	22	22.	23.	24.	25.	26.
	57	57.	57.	58	58	58.	58.	59	59	59.	0	21.	21.	22	22.	23.	24	25	26	27	28	29
1	2	2	2.	3	3	3.	3.	4	4.	4.	5	23	23.	24	24.	25	26	27	28	29	30	31.
	7	7	7.	7.	8	8.	8.	9	9.	9.	10	25	25.	26	26.	27	28	29	30	31.	32.	34
	12	12	12.	13	13	13.	14	14	14.	15	15	27	27	27.	28.	29	30	31	32	33.	35	36.
	17	17	17.	18	18	18.	19	19.	19.	20	20.	28.	29	29.	30	31	32	33	34.	36	37.	39
	22	22	22.	23	23.	23.	24	24.	25	25.	26	30.	31	31.	32	33	34	35	36.	38	39.	41.
	27	27	27.	28	28.	29	29.	29.	30	30.	31	32	32.	33	34	35	36	37	38.	40	42	43.
	32	32.	33	33.	33.	34	34.	35	35.	36	36.	33.	34.	35	36	37	38	39.	41	42.	44	46
	37	37.	38	38.	39	39.	40	40.	41	41.	42	35.	36	37	38	39	40	41.	43	44.	46.	48.
	42.	43	43.	44	44.	45	45.	46	46.	47	47.	37	38	38.	39.	40.	42	43.	45	46	48.	50.
	47.	48	48.	49	50	50.	51	51	52	52.	53	39	39.	40.	41.	42.	44	45.	47	49	51	53
	53	53.	54	54.	55	55.	56.	57	57.	58	58.	40.	41.	42.	43.	44.	46	47.	49	51	53	55
	58.	59	59.	0	0.	1.	2	2.	3	4	4.	42.	43	44	45.	46.	48	49.	51.	53.	55.	57.
2	4	4.	5	6	6.	7	7.	8.	9	9.	10.	44	45	46	47	48.	50	51.	53.	55.	57.	60
	9	10	10.	11.	12	12.	13.	14	15	15.	16.	46	47	48	49	50.	52	53.	55.	57.	60	62.
	15	16	16.	17	18	18.	19.	20	21	21.	22.	47.	48.	49.	51	52.	54	56	57.	60	62	64.
	21	21.	22.	23	24	24.	25.	26	27	28	28.	49.	50.	51.	53	54.	56	58	60	62	64.	67
	27	27.	28.	29	30	30.	31.	32.	33	34	35	51	52	53.	55	56.	58	60	62	64.	66.	69.
	33	33.	34.	35	36	37	38	38.	39.	40.	41.	53	54	55.	56.	58.	60	62	64	66.	69	72
	39	40	40.	41.	42.	43.	44.	45	46	47	48	54.	56	57	58.	60	62	64	66	68.	71.	74.
	45	46	47	48	49	50	51	52	53	54	55	56.	57.	59	60.	62	64	66	68.	71	73.	76.
	51.	52.	53.	54.	55.	56.	57.	58.	59.	1	2	58	59.	61	62.	64	66	68	70.	73	76	79
	58.	59.	0.	1.	2.	3.	4.	6	7	8	9	60	61.	63	64.	66	68	70	72.	75	78	81.
3	5.	6.	7.	8.	10	11	12	13	14.	15.	17	61.	63	64.	66	68	70	72	74.	77.	80.	84
	12.	13.	14.	16	17	18.	19.	21	22	23.	25	63.	65	66.	68	70	72	74.	77	79.	82.	86
	19.	21	22	23.	25	26	27.	29	30.	31.	33	65	66.	68.	70	72	74	76.	79	82	85	88.
	27.	28.	30	31.	33	34.	35.	37	38.	40.	42	67	68.	70	72	74	76	78.	81	84	87	91
	35.	37	38.	39.	41	43	44.	46	47.	49.	51	69	70.	72	74	76	78	80.	83	86	89.	93.
	44	45.	47	48.	50	52	53.	55	57	59	0.	70.	72	74	75.	77.	80	82.	85.	88.	91.	95.
	53	54.	56	58	59.	1.	3.	5.	7.	9.	11.	72.	74	76	77.	79.	82	84.	87.	90.	94	98
4	2.	4.	6.	8	10	12.	14.	16.	19	21	23.	74	76	77.	79.	81.	84	86.	89.	92.	96	100.
	13	15	17.	19.	22	24	26.	29	32	34.	37.	76	77.	79.	81.	83.	86	88.	91.	95	98.	103
	24.	27	29.	32	34.	37.	40.	43.	46.	50	53.	77.	79.	81.	83.	85.	88	90.	94	97	101	105
	38	40.	43.	46.	50	53.	57.	2	6.	11	16.	79.	81.	83	85	87.	90	93	96	99.	103	107.
	54	57.	1.	5	10	16	23.	32	42.			81	83	85	87	89.	92	95	98	101.	105.	110
5	16	22	29.	40.								83	85	87	89	91.	94	97	100	103.	107.	112

Note.—From the "Ex." below Table 3, preceding page, we find that the upper culmination of Polaris occurs at 10.20$\frac{1}{4}$ *P. M.* on Nov. 8, 1914; and that the time of observation is fixed at 8.50 *P. M.*, mean local time, or 1h. 30.5m. earlier. Hence the position of Polaris at the time of observation is just to the right of *U*, Fig. 14, p. 949. Moreover, the star's hour angle at observation is 23h. 56.1m. minus 1h. 30 5m. = 22h. 25.6m., which corresponds with that given in the practical example preceding. Have graphically in mind the position of the star, and its apparent motion, and the calculations become simple.

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e or of an hour angle indicates that its value is 0ᵐ.5 greater than printed.]

R AND AZIMUTH.

en hour angle is *less* than
hour angle is *greater* than
t, the star's hour angle (or
us the star's hour angle),

POLARIS *below* THE POLE.

To determine the true meridian, the azimuth will be laid off to the *east* when the hour angle is *less* than 11ʰ 58ᵐ, and to the *west* when *greater* than 11 58ᵐ.

	1904	1905	1906	1907	1908	1909	1910	1911	Azimuths for Latitude— 30°	32°	34°	36°	38°	40°	42°	44°	46°	48°	50°
	m.								′	′	′	′	′	′	′	′	′	′	′
).	9.	m.	m.	m.	m.	m.			83	85	87	89	91.	94	97	100	103.	107.	112
3.	45	40.	34	27	18	8	m.	m.	81.	83	85	87	89.	92	95	98	101.	105.	109.
7	4	0.	56.	52.	48.	44	39	34	79.	81.	83	85	87.	90	93	96	99.	103	107
1	18.	16	13	10	7	4	0.	57	78	79.	81.	83.	85.	88	90.	93.	97	100.	104.
3.	31.	29	26.	24	21.	19	16	13	76	77.	79.	81.	83.	86	88.	91.	95	98.	102
4.	42.	40.	38.	36.	34.	32	29.	27	74.	76	77.	79.	81.	84	86.	89.	92.	96	100
5	53	51.	49.	47.	45.	43.	41.	39.	72.	74	76	77.	79.	82	84.	87.	90.	94	97.
4	2.	1	59.	57.	56	54	52.	50.	71	72.	74	76	77.	80	82.	85.	88.	91.	95
3	11.	10	8.	7	5.	4	2.	1	69	70.	72	74	75.	78	80.	83	86	89.	92.
1.	20	18.	17.	16	14.	13	11.	10	67.	68.	70.	72	74	76	78.	81	84	87	90
9.	28	27	25.	24.	23	21.	20.	19	65.	67	68.	70	72	74	76.	79	81.	84.	88
7	36	35	33.	32.	31	30	28.	27.	64	65	66.	68	70	72	74.	77	79.	82.	85.
4.	43.	42.	41.	40	39	38	36.	35.	62	63.	64.	66.	68	70	72	74.	77.	80	83
2	50.	49.	48.	47.	46.	45.	44.	43	60.	61.	63	64.	66	68	70	72.	75	78	80.
58.	57.	56.	55.	54.	53.	52.	51.	50.	58.	59.	61	62.	64	66	68	70.	73	75.	78
5.	4.	3.	2.	1.	1	0	59	58	57	58	59	60.	62.	64	66	68	70.	73	76
12	11	10.	9.	8.	7.	6.	5.	5	55	56	57.	59	60.	62	64	66	68.	71	73.
18.	17.	17	16	15	14.	13.	12.	11.	53.	54.	55.	57	58.	60	62	64	66	68.	71
24.	24	23	22.	21.	20.	20	19	18	51.	52.	53.	55	56.	58	60	62	64	66.	68.
31	30	29.	28.	28	27	26.	25.	24.	49.	50.	52	53	54.	56	58	59.	62	64	66.
37	36	35.	35	34	33.	32.	32	31	48	49	50	51	52.	54	55.	57.	59.	61.	64
43	42	41.	41	40	39.	38.	38	37.	46	47	48	49.	50.	52	53.	55.	57.	59.	61.
48.	48	47.	47	46	45.	45	44	43.	44.	45.	46.	47.	48.	50.	51.	53.	55	57	59
54.	54	53	52.	52	51.	51	50	49.	42.	43.	44.	45.	46.	48	49.	51	53	55	57
0	59.	59	58.	57.	57	56.	56	55.	41	41.	42.	43.	44.	46	47.	49	50.	52.	54.
5.	5	4.	4	3.	3	2.	2	1.	39	40	40.	41.	43	44	45.	47	48.	50	52
11	10.	10	9.	9	8.	8	7.	7	37.	38	39	40	41	42	43.	45	46.	48	49
16.	16	15.	15	14.	14	13.	13	12.	35.	36	37	38	39	40	41	42.	44	45.	47.
22	21.	21	20.	20	19.	19.	19	18.	34	34.	35	36	37	38	39	40.	42	43.	45
27.	27	26.	26	25.	25	25	24.	24	32	32.	33.	34	35	36	37	38.	39.	41	42.
32.	32	32	31.	31	30.	30	30	29.	30	31	31.	32	33	34	35	36	37.	39	40
38	37.	37	36.	36.	36	35.	35	35	28.	29	29.	30.	31	32	33	34	35.	36.	38
43	42.	42.	42	41.	41.	41	40.	40	26.	27	28	28.	29	30	31	32	33	34	35.
48	48	47.	47.	47	46.	46.	46	45.	25	25.	26	26.	27	28	29	30	31	32	33
53	53	52.	52.	52	52	51.	51	51	23	23.	24	24.	25.	26	27	27.	28.	29.	31
58.	58	58	57.	57.	57	57	56	56.	21.	22	22	22.	23	24	25	25.	26.	27.	28
3.	3	3	3	2.	2.	2	2	1.	19.	20	20.	21	21.	22	22.	23.	24	25	26
8.	8.	8	8	8	7.	7.	7	7	18	18	18.	19	19.	20	20.	21.	22	23	23.
13.	13.	13	13	12.	12.	12.	12	12	16	16.	16.	17	17.	18	18.	19	20	20.	21.
18.	18.	18	18	18	17.	17.	17.	17	14	14.	15	15	15.	16	16.	17	17.	18	19
23.	23.	23	23	23	23	22.	22.	22	12.	12.	13	13.	13.	14	14.	15	15.	16	16.
28.	28.	28.	28	28	28	27.	27.	27.	10.	11	11	11.	12	12	12.	13	13.	14	14
33.	33.	33	33	33	33	33	32.	32.	9	9	9	9.	9.	10	10.	10.	11	11.	12
38.	38.	38	38	38	38	38	38	37.	7	7	7.	7.	8	8	8	8.	.9	9	9.
43.	43.	43	43	43	43	43	43	43	5.	5.	5.	5.	6	6	6	6.	6.	7	7
48	48	48	48	48	48	48	48	48	3.	3.	3.	4	4	4	4	4	4.	4.	4.
53	53	53	53	53	53	53	53	53	2	2	2	2	2	2	2	2	2	2.	2.
58	58	58	58	58	58	58	58	58	0	0	0	0	0	0	0	0	0	0	0

—The time of lower culmination of Polaris occurs 11h. 54m. before or after the upper culmination. The time of eastern and of western elongation occurs nately 5h. 55m. before and after the time of upper culmination. Hence the longation may be obtained from the above table. See also Fig. 14, p. 949, and p. 950, for data in this connection.

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4a.—Polaris, 1912, for Meridian of Greenwich.
Civil Date and Clock Time.

Date 1912.	Upper Culmination.	Elongation, Lat. 40°.	Declination.
			° ′ +88 50 ″
Jan.	h m	h m	
1	6 47.4 P.M.	W. E. 0 46.4 A.M.	30.8
2	6 43.5	0 42.5	31.0
3	6 39.5	0 38.6	31.1
4	6 35.6	0 34.6	31.3
5	6 31.6	0 30.7	31.5
6	6 27.7	0 26.7	31.6
7	6 23.7	0 22.8	31.7
8	6 19.8	0 18.8	31.8
9	6 15.8	0 14.9	31.9
10	6 11.9	0 10.9	31.9
11	6 7.9	0 7.0	31.9
12	6 4.0	{ 0 3.0 A.M. / 11 59.1 P.M. }	32.0
13	6 0.0	11 55.1	32.0
14	5 56.1	11 51.2	32.0
15	5 52.1	11 47.2	32.1
16	5 48.2	11 43.3	32.2
17	5 44.2	11 39.3	32.2
18	5 40.3	11 35.4	32.3
19	5 36.3	11 31.4	32.3
20	5 32.4	11 27.5	32.4
21	5 28.4	11 23.5	32.4
22	5 24.5	11 19.6	32.4
23	5 20.5	11 15.6	32.3
24	5 16.6	11 11.7	32.3
25	5 12.6	11 7.7	32.2
26	5 8.7	11 3.8	32.2
27	5 4.7	10 59.8	32.1
28	5 0.8	10 55.9	32.1
29	4 56.8	10 51.9	32.0
30	4 52.9	10 48.0	32.0
31	4 48.9	10 44.0	32.0
Feb.			
1	4 45.0 P.M.	W.E. 10 40.1 P.M.	32.0
2	4 41.0	10 36.1	32.0
3	4 37.1	10 32.2	32.0
4	4 33.1	10 28.2	31.9
5	4 29.2	10 24.3	31.8
6	4 25.2	10 20.3	31.6
7	4 21.3	10 16.4	31.5
8	4 17.3	10 12.4	31.3
9	4 13.4	10 8.5	31.2
10	4 9.4	10 4.5	31.1
11	4 5.5	10 0.6	31.0
12	4 1.5	9 56.6	30.8
13	3 57.6	9 52.7	30.7
14	3 53.7	9 48.8	30.6
15	3 49.7	9 44.8	30.5
16	3 45.8	9 40.9	30.4
17	3 41.8	9 36.9	30.2
18	3 37.9	9 33.0	30.0
19	3 33.9	9 29.0	29.9
20	3 30.0	9 25.1	29.7
21	3 26.0	9 21.1	29.4
22	3 22.1	9 17.2	29.2
23	3 18.1	9 13.2	29.0
24	3 14.2	9 9.3	28.8
25	3 10.2	9 5.3	28.6
26	3 6.3	9 1.4	28.4
27	3 2.4	8 57.5	28.2
28	2 58.4	8 53.5	28.1
29	2 54.5	8 49.6	27.9
30	2 50.5	8 45.6	27.7

Date 1912.	Upper Culmination.	Elongation, Lat. 40°.	Declination.
			° ′ +88 50 ″
Mar.	h m	h m	
1	2 50.5 P.M.	W. E. 8 45.6 P.M.	27.7
2	2 46.6	8 41.7	27.5
3	2 42.6	8 37.7	27.2
4	2 38.7	8 33.8	27.0
5	2 34.7	8 29.8	26.7
6	2 30.8	8 25.9	26.4
7	2 26.9	8 22.0	26.1
8	2 22.9	8 18.0	25.8
9	2 19.0	8 14.1	25.6
10	2 15.0	8 10.1	25.3
11	2 11.1	8 6.2	25.1
12	2 7.2	8 2.3	24.9
13	2 3.2	7 58.3	24.6
14	1 59.3	7 54.4	24.4
15	1 55.3	7 50.4	24.1
16	1 51.4	7 46.5	23.9
17	1 47.5	7 42.6	23.6
18	1 43.5	7 38.6	23.3
19	1 39.6	7 34.7	22.9
20	1 35.6	7 30.7	22.6
21	1 31.7	7 26.8	22.3
22	1 27.8	7 22.9	22.0
23	1 23.8	7 18.9	21.7
24	1 19.9	7 15.0	21.4
25	1 16.0	7 11.1	21.1
26	1 12.0	7 7.1	20.9
27	1 8.1	7 3.2	20.7
28	1 4.1	6 59.2	20.4
29	1 0.2	6 55.3	20.1
30	0 56.3	6 51.4	19.8
31	0 52.3	6 47.4	19.5
Apr.			
1	0 48.4 P.M.	W. E. 6 43.5 P.M.	19.1
2	0 44.5	6 39.6	18.8
3	0 40.5	6 35.6	18.5
4	0 36.6	6 31.7	18.1
5	0 32.7	6 27.8	17.8
6	0 28.7	6 23.8	17.5
7	0 24.8	6 19.9	17.2
8	0 20.9	6 16.0	17.0
9	0 16.9	6 12.0	16.7
10	0 13.0	6 8.1	16.4
11	0 9.1	6 4.2	16.1
12	0 5.1	6 0.2	15.8
13	0 1.2 P.M.	W. E. 5 56.3 P.M.	15.5
14	11 57.3 A.M.	E. E. 6 2.2 A.M.	15.2
15	11 53.4	5 58.3	14.9
16	11 49.4	5 54.3	14.5
17	11 45.5	5 50.4	14.2
18	11 41.6	5 46.5	13.9
19	11 37.6	5 42.5	13.6
20	11 33.7	5 38.6	13.3
21	11 29.8	5 34.7	13.0
22	11 25.9	5 30.8	12.8
23	11 21.9	5 26.8	12.5
24	11 18.0	5 22.9	12.3
25	11 14.1	5 19.0	12.0
26	11 10.2	5 15.1	11.8
27	11 6.2	5 11.1	11.5
28	11 2.3	5 7.2	11.1
29	10 58.4	5 3.3	10.8
30	10 54.4	4 59.3	10.5
31	10 50.5	4 55.4	10.2

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.ARIS, 1912, FOR MERIDIAN OF GREENWICH.—Continued.
Civil Date and Clock Time.

	Elongation, Lat. 40°.	Decli-n'tion.	Date 1912.	Upper Culmination.		Elongation, Lat. 40°.	Decli-n'tion.
		° ′					° ′
		+88 50					+88 50
	h m	″		h m		h m	″
.	E. E. 4 55.4 A.M.	10.2	July 1	6 51.5 A.M.		E. E. 0 56.4 A.M.	1.6
	4 51.5	9.9	2	6 47.6		0 52.5	1.7
	4 47.6	9.7	3	6 43.7		0 48.6	1.7
	4 43.7	9.4	4	6 39.8		0 44.7	1.7
	4 39.7	9.2	5	6 35.9		0 40.8	1.7
	4 35.8	8.9	6	6 32.0		0 36.9	1.7
	4 31.9	8.7	7	6 28.0		0 32.9	1.7
	4 28.0	8.5	8	6 24.1		0 29.0	1.7
	4 24.0	8.3	9	6 20.2		0 25.1	1.7
	4 20.1	8.0	10	6 16.3		0 21.2	1.8
	4 16.2	7.8	11	6 12.4		0 17.3	1.9
	4 12.3	7.5	12	6 8.5		0 13.4	2.0
	4 8.3	7.2	13	6 4.6		0 9.5	2.1
	4 4.4	6.9	14	6 0.7		0 5.6	2.2
	4 0.5	6.7	15	5 56.7		0 1.6 A.M. / 11 57.7 P.M.	2.3
	3 56.6	6.5					
	3 52.7	6.3	16	5 52.8		11 53.8	2.5
	3 48.8	6.1	17	5 48.9		11 49.9	2.6
	3 44.8	5.9	18	5 45.0		11 46.0	2.7
	3 40.9	5.8	19	5 41.1		11 42.1	2.7
	3 37.0	5.6	20	5 37.2		11 38.2	2.8
	3 33.1	5.5	21	5 33.3		11 34.2	2.9
	3 29.2	5.3	22	5 29.3		11 30.3	3.0
	3 25.2	5.1	23	5 25.4		11 26.4	3.1
	3 21.3	4.8	24	5 21.5		11 22.5	3.2
	3 17.4	4.6	25	5 17.6		11 18.6	3.4
	3 13.5	4.4	26	5 13.7		11 14.7	3.6
	3 9.6	4.2	27	5 9.8		11 10.8	3.7
	3 5.6	4.0	28	5 5.9		11 6.8	3.9
	3 1.7	3.9	29	5 1.9		11 2.9	4.1
	2 57.8	3.7	30	4 58.0		10 59.0	4.3
			31	4 54.1		10 55.1	4.4
			Aug.				
.M.	E. E. 2 53.9 A.M.	3.6	1	4 50.2 A.M.		E. E. 10 51.2 P.M.	4.6
	2 50.0	3.5	2	4 46.3		10 47.3	4.7
	2 46.1	3.4	3	4 42.4		10 43.4	4.9
	2 42.1	3.3	4	4 38.5		10 39.4	5.0
	2 38.2	3.2	5	4 34.5		10 35.5	5.2
	2 34.3	3.0	6	4 30.6		10 31.6	5.4
	2 30.4	2.9	7	4 26.7		10 27.7	5.6
	2 26.5	2.8	8	4 22.8		10 23.8	5.8
	2 22.6	2.6	9	4 18.9		10 19.9	6.0
	2 18.6	2.5	10	4 15.0		10 16.0	6.3
	2 14.7	2.3	11	4 11.1		10 12.0	6.6
	2 10.8	2.2	12	4 7.1		10 8.1	6.8
	2 6.9	2.1	13	4 3.2		10 4.2	7.1
	2 3.0	2.1	14	3 59.3		10 0.3	7.3
	1 59.1	2.0	15	3 55.4		9 56.4	7.5
	1 55.2	2.0	16	3 51.5		9 52.5	7.7
	1 51.2	2.0	17	3 47.6		9 48.5	7.9
	1 47.3	2.0	18	3 43.6		9 44.6	8.1
	1 43.4	1.9	19	3 39.7		9 40.7	8.4
	1 39.5	1.9	20	3 35.8		9 36.8	8.6
	1 35.6	1.8	21	3 31.9		9 32.9	8.9
	1 31.7	1.7	22	3 28.0		9 29.0	9.2
	1 27.7	1.7	23	3 24.1		9 25.0	9.5
	1 23.8	1.6	24	3 20.1		9 21.1	9.8
	1 19.9	1.5	25	3 16.2		9 17.2	10.1
	1 16.0	1.5	26	3 12.3		9 13.3	10.4
	1 12.1	1.5	27	3 8.4		9 9.4	10.7
	1 8.2	1.5	28	3 4.5		9 5.4	11.0
	1 4.3	1.6	29	3 0.5		9 1.5	11.3
	1 0.4	1.6	30	2 56.6		8 57.6	11.5
	0 56.4	1.6	31	2 52.7		8 53.7	11.7
			32	2 48.8		8 49.8	12.0

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4a.—Polaris, 1912, for Meridian of Greenwich.—Concluded. Civil Date and Clock Time.

Date 1912.	Upper Culmination.	Elongation, Lat. 40°.	Declination.
			° ′
			+88 50
Sep.	h m	h m	° ′
1	2 48.8 A.M.	E. E. 8 49.8 P.M.	12.0
2	2 44.9	8 45.9	12.3
3	2 41.0	8 41.9	12.6
4	2 37.0	8 38.0	12.9
5	2 33.1	8 34.1	13.2
6	2 29.2	8 30.2	13.6
7	2 25.3	8 26.3	14.0
8	2 21.4	8 22.3	14.4
9	2 17.4	8 18.4	14.7
10	2 13.5	8 14.5	15.1
11	2 9.6	8 10.6	15.4
12	2 5.7	8 6.6	15.7
13	2 1.7	8 2.7	16.0
14	1 57.8	7 58.8	16.3
15	1 53.9	7 54.9	16.6
16	1 50.0	7 51.0	16.9
17	1 46.1	7 47.0	17.3
18	1 42.1	7 43.1	17.7
19	1 38.2	7 39.2	18.0
20	1 34.3	7 35.3	18.4
21	1 30.4	7 31.3	18:8
22	1 26.4	7 27.4	19.2
23	1 22.5	7 23.5	19.6
24	1 18.6	7 19.6	19.9
25	1 14.7	7 15.6	20.3
26	1 10.7	7 11.7	20.6
27	1 6.8	7 7.8	20.9
28	1 2.9	7 3.9	21.3
29	0 59.0	7 0.0	21.6
30	0 55.1	6 56.0	22.0
Oct.			
1	0 51.1 A.M.	E. E. 6 52.1 P.M.	22.3
2	0 47.2	6 48.2	22.7
3	0 43.3	6 44.3	23.2
4	0 39.4	6 40.3	23.6
5	0 35.4	6 36.4	24.0
6	0 31.5	6 32.5	24.4
7	0 27.6	6 28.5	24.8
8	0 23.6	6 24.6	25.2
9	0 19.7	6 20.7	25.6
10	0 15.8	6 16.7	25.9
11	0 11.8	6 12.8	26.3
12	0 7.9	6 8.9	26.6
13	0 4.0	E. E. 6 5.0 P.M.	27.0
14	0 0.1 A.M. / 11 56.2 P.M.	W.E. 5 55.2 A.M.	27.4 / 27.8
15	11 52.2	5 51.3	28.2
16	11 48.3	5 47.3	28.6
17	11 44.4	5 43.4	29.1
18	11 40.4	5 39.5	29.5
19	11 36.5	5 35.5	29.9
20	11 32.6	5 31.6	30.2
21	11 28.6	5 27.7	30.6
22	11 24.7	5 23.7	31.0
23	11 20.8	5 19.8	31.3
24	11 16.8	5 15.9	31.7
25	11 12.9	5 11.9	32.0
26	11 9.0	5 8.0	32.4
27	11 5.0	5 4.1	32.8
28	11 1.1	5 0.1	33.2
29	10 57.2	4 56.2	33.6
30	10 53.2	4 52.3	34.0
31	10 49.3	4 48.3	34.4
32	10 45.4	4 44.4	34.9

Date 1912.	Upper Culmination.	Elongation, Lat. 40°.	Declination.
			° ′
			+88 50
Nov.	h m	h m	° ′
1	10 45.4 P.M.	W. E. 4 44.4 A.M.	34.9
2	10 41.4	4 40.5	35.3
3	10 37.5	4 36.5	35.6
4	10 33.6	4 32.6	36.0
5	10 29.6	4 28.7	36.3
6	10 25.7	4 24.7	36.7
7	10 21.8	4 20.8	37.0
8	10 17.8	4 16.9	37.3
9	10 13.9	4 12.9	37.7
10	10 9.9	4 9.0	38.1
11	10 6.0	4 5.0	38.5
12	10 2.1	4 1.1	38.9
13	9 58.1	3 57.2	39.3
14	9 54.2	3 53.2	39.6
15	9 50.3	3 49.3	40.0
16	9 46.3	3 45.4	40.4
17	9 42.4	3 41.4	40.7
18	9 38.4	3 37.5	41.0
19	9 34.5	3 33.5	41.3
20	9 30.6	3 29.6	41.4
21	9 26.6	3 25.7	41.9
22	9 22.7	3 21.7	42.2
23	9 18.7	3 17.8	42.5
24	9 14.8	3 13.8	42.9
25	9 10.9	3 9.9	43.2
26	9 6.9	3 6.0	43.6
27	9 3.0	3 2.0	43.9
28	8 59.0	2 58.1	44.3
29	8 55.1	2 54.1	44.6
30	8 51.2	2 50.2	44.9
Dec.			
1	8 47.2 P.M.	W. E. 2 46.3 A.M.	45.2
2	8 43.3	2 42.3	45.5
3	8 39.3	2 38.4	45.7
4	8 35.4	2 34.4	46.0
5	8 31.4	2 30.5	46.2
6	8 27.5	2 26.5	46.5
7	8 23.5	2 22.6	46.7
8	8 19.6	2 18.6	47.0
9	8 15.7	2 14.7	47.3
10	8 11.7	2 10.8	47.6
11	8 7.8	2 6.8	47.9
12	8 3.8	2 2.9	48.2
13	7 59.9	1 58.9	48.4
14	7 55.9	1 55.0	48.7
15	7 52.0	1 51.0	48.9
16	7 48.0	1 47.1	49.1
17	7 44.1	1 43.1	49.3
18	7 40.1	1 39.2	49.5
19	7 36.2	1 35.2	49.6
20	7 32.3	1 31.3	49.8
21	7 28.3	1 27.4	50.0
22	7 24.4	1 23.4	50.2
23	7 20.4	1 19.5	50.4
24	7 16.5	1 15.5	50.7
25	7 12.5	1 11.6	50.9
26	7 8.6	1 7.6	51.1
27	7 4.6	1 3.7	51.3
28	7 0.7	0 59.7	51.5
29	6 56.7	0 55.8	51.6
30	6 52.8	0 51.8	51.7
31	6 48.8	0 47.9	51.8
32	6 44.9	0 43.9	51.9

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4b.—POLARIS, 1912, FOR VARIOUS LATITUDES.
(To accompany Tables 4a and 4c.)

de.	Corrections to the Times of Elongations for Different Latitudes.	AZIMUTHS OF POLARIS AT ELONGATION, 1912. Decl. + 88° 50′					
		0″	10″	20″	30″	40″	50″
°	W. E. m E. E.	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
25	+1.7−	1 17 14	1 17 3	1 16 52	1 16 41	1 16 30	1 16 19
26	1.6	1 17 53	1 17 42	1 17 31	1 17 20	1 17 8	1 16 57
27	1.5	1 18 34	1 18 23	1 18 11	1 18 0	1 17 49	1 17 38
28	1.4	1 19 17	1 19 6	1 18 54	1 18 43	1 18 32	1 18 20
29	1.3	1 20 2	1 19 51	1 19 39	1 19 28	1 19 16	1 19 5
30	1.2	1 20 50	1 20 38	1 20 27	1 20 15	1 20 4	1 19 52
31	1.1	1 21 40	1 21 28	1 21 17	1 21 5	1 20 53	1 20 42
32	1.0	1 22 33	1 22 21	1 22 9	1 21 57	1 21 46	1 21 34
33	0.9	1 23 28	1 23 16	1 23 4	1 22 52	1 22 40	1 22 28
34	0.7	1 24 26	1 24 14	1 24 2	1 23 50	1 23 38	1 23 26
35	0.6	1 25 27	1 25 15	1 25 3	1 24 51	1 24 39	1 24 26
36	0.5	1 26 32	1 26 19	1 26 7	1 25 55	1 25 42	1 25 30
37	0.4	1 27 39	1 27 27	1 27 14	1 27 2	1 26 49	1 26 37
38	0.2	1 28 50	1 28 37	1 28 25	1 28 12	1 27 59	1 27 47
39	+0.1−	1 30 5	1 29 52	1 29 39	1 29 26	1 29 13	1 29 0
40	0.0	1 31 23	1 31 10	1 30 57	1 30 44	1 30 31	1 30 18
41	−0.1+	1 32 45	1 32 32	1 32 19	1 32 6	1 31 52	1 31 39
42	0.3	1 34 12	1 33 58	1 33 45	1 33 32	1 33 18	1 33 5
43	0.4	1 35 43	1 35 30	1 35 16	1 35 2	1 34 48	1 34 35
44	0.6	1 37 19	1 37 5	1 36 51	1 36 37	1 36 23	1 36 9
45	0.8	1 39 0	1 38 46	1 38 32	1 38 18	1 38 4	1 37 50
46	1.0	1 40 47	1 40 32	1 40 18	1 40 3	1 39 49	1 39 35
47	1.2	1 42 39	1 42 24	1 42 10	1 41 55	1 41 40	1 41 26
48	1.4	1 44 37	1 44 22	1 44 7	1 43 52	1 43 38	1 43 23
49	1.5	1 46 42	1 46 27	1 46 12	1 45 57	1 45 42	1 45 26
50	1.7	1 48 55	1 48 39	1 48 24	1 48 8	1 47 52	1 47 37
51	1.9	1 51 15	1 50 59	1 50 43	1 50 27	1 50 11	1 49 55
52	2.1	1 53 43	1 53 26	1 53 10	1 52 54	1 52 38	1 52 22
53	2.3	1 56 20	1 56 3	1 55 47	1 55 30	1 55 13	1 54 57
54	2.6	1 59 6	1 58 49	1 58 32	1 58 15	1 57 58	1 57 41
55	2.8	2 2 3	2 1 46	2 1 29	2 1 11	2 0 54	2 0 36
56	3.0	2 5 12	2 4 54	2 4 36	2 4 18	2 4 0	2 3 42
57	3.3	2 8 33	2 8 14	2 7 56	2 7 38	2 7 19	2 7 1
58	3.6	2 12 7	2 11 48	2 11 29	2 11 10	2 10 52	2 10 33
59	3.9	2 15 56	2 15 37	2 15 17	2 14 58	2 14 39	2 14 19
60	4.2	2 20 2	2 19 42	2 19 22	2 19 2	2 18 42	2 18 22
61	4.5	2 24 25	2 24 4	2 23 44	2 23 23	2 23 3	2 22 42
62	4.9	2 29 8	2 28 47	2 28 26	2 28 4	2 27 43	2 27 22
63	5.2	2 34 14	2 33 52	2 33 30	2 33 8	2 32 46	2 32 24
64	5.6	2 39 44	2 39 21	2 38 58	2 38 35	2 38 12	2 37 50
65	6.0	2 45 41	2 45 18	2 44 54	2 44 30	2 44 6	2 43 43
66	6.4	2 52 10	2 51 45	2 51 20	2 50 56	2 50 31	2 50 7
67	6.9	2 59 13	2 58 48	2 58 22	2 57 56	2 57 31	2 57 5
68	7.5	3 6 56	3 6 30	3 6 3	3 5 36	3 5 10	3 4 43
69	8.1	3 15 25	3 14 57	3 14 29	3 14 1	3 13 34	3 13 6
70	−8.7+	3 24 46	3 24 17	3 23 48	3 23 19	3 22 49	3 22 20

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4c.—Polaris, 1912, General Tabl
(To accompany Tables 4a and 4b.)

MEAN TIME HOUR ANGLES, 1912.							AZIMUTHS OF					
Hours.	Decl. + 88° 50′						Latit					
	0″	10″	20″	30″	40″	50″	30°	32°	34°	36°	38°	40°
	m	m	m	m	m	m	′	′	′	′	′	′
0	0 0	0 0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.
	4.9	4.9	4.9	5.0	5.0	5.0	1.8	1.8	1.8	1.9	1.9	2.
	9.8	9.8	9.8	9.9	9.9	10.0	3.5	3.6	3.7	3.8	3.9	4.
	14.8	14.8	14.8	14.9	14.9	15.0	5.3	5.4	5.5	5.7	5.8	6.
	19.7	19.7	19.8	19.8	19.9	19.9	7.0	7.2	7.4	7.6	7.8	8.
	24.6	24.7	24.7	24.8	24.9	24.9	8.8	9.0	9.2	9.4	9.7	10.
	29.6	29.7	29.7	29.8	29.9	29.9	10.6	10.8	11.1	11.3	11.7	12.
	34.6	34.7	34.8	34.8	34.9	35.0	12.3	12.6	12.9	13.2	13.6	14.
	39.6	39.7	39.8	39.8	39.9	40.0	14.1	14.4	14.7	15.1	15.5	16.
	44.6	44.7	44.8	44.9	45.0	45.1	15.8	16.2	16.6	17.0	17.5	18.
	49.6	49.7	49.8	50.0	50.1	50.2	17.6	18.0	18.4	18.9	19.4	20.
	54.7	54.8	54.9	55.0	55.2	55.3	19.4	19.8	20.3	20.8	21.4	22.
	59.7	59.8	0.0	0.2	0.3	0.4	21.1	21.6	22.1	22.7	23.3	24.
1	4.8	5.0	5.1	5.3	5.5	5.6	22.9	23.4	24.0	24.6	25.3	26
	10.0	10.2	10.3	10.5	10.7	10.8	24.6	25.2	25.8	26.5	27.2	28
	15.2	15.4	15.6	15.8	15.9	16.1	26.4	27.0	27.6	28.4	29.1	30
	20.4	20.6	20.8	21.0	21.2	21.4	28.2	28.8	29.5	30.2	31.1	32
	25.7	25.9	26.1	26.4	26.6	26.8	29.9	30.6	31.3	32.1	33.0	34
	31.0	31.2	31.5	31.7	31.9	32.2	31.7	32.4	33.2	34.0	35.0	36
	36.4	36.6	36.9	37.2	37.4	37.6	33.4	34.2	35.0	35.9	36.9	38
	41.8	42.1	42.3	42.6	42.9	43.1	35.2	36.0	36.8	37.8	38.8	40
	47.4	47.7	48.0	48.2	48.5	48.8	37.0	37.8	38.7	39.7	40.8	42
	53.0	53.3	53.6	53.9	54.2	54.5	38.7	39.6	40.5	41.6	42.7	44
	58.6	58.9	59.3	59.6	59.9	0.3	40.5	41.4	42.4	43.5	44.7	46
2	4.4	4.7	5.1	5.4	5.7	6.1	42.3	43.2	44.2	45.4	46.6	48
	10.2	10.6	10.9	11.2	11.6	12.0	44.0	45.0	46.1	47.2	48.6	50
	16.2	16.6	17.0	17.4	17.7	18.1	45.8	46.8	47.9	49.1	50.5	52
	22.3	22.7	23.1	23.5	23.9	24.3	47.6	48.6	49.8	51.0	52.4	54
	28.5	28.9	29.4	29.8	30.2	30.7	49.3	50.4	51.6	52.9	54.4	56
	34.7	35.2	35.6	36.1	36.6	37.0	51.1	52.2	53.5	54.8	56.3	58
	41.1	41.6	42.1	42.6	43.0	43.5	52.8	54.0	55.3	56.7	58.3	60
	47.8	48.3	48.8	49.4	49.9	50.4	54.6	55.8	57.2	58.6	60.2	62
	54.6	55.1	55.7	56.2	56.8	57.3	56.4	57.6	59.0	60.5	62.2	64
3	1.7	2.3	2.9	3.4	4.0	4.6	58.2	59.4	60.9	62.4	64.1	66
	8.9	9.5	10.1	10.8	11.4	12.0	59.9	61.3	62.7	64.3	66.1	68
	16.4	17.1	17.7	18.4	19.0	19.7	61.7	63.1	64.6	66.2	68.0	70
	24.4	25.1	25.8	26.5	27.3	28.0	63.5	64.9	66.4	68.1	69.9	72
	32.7	33.5	34.2	35.0	35.8	36.6	65.3	66.7	68.3	70.0	71.9	74
	41.4	42.2	43.1	43.9	44.8	45.7	67.0	68.5	70.1	71.9	73.8	76
	50.6	51.5	52.4	53.3	54.3	55.2	68.8	70.3	72.0	73.8	75.8	78
4	0.3	1.3	2.3	3.3	4.3	5.4	70.6	72.1	73.8	75.7	77.7	80
	11.1	12.2	13.4	14.5	15.7	16.8	72.4	73.9	75.7	77.6	79.7	83
	23.0	24.3	25.6	26.9	28.3	29.6	74.2	75.8	77.5	79.5	81.6	84
	36.5	38.0	39.6	41.3	43.0	44.7	76.0	77.6	79.4	81.4	83.6	86
	52.8	54.8	56.9	59.1	1.3	3.6	77.8	79.4	81.3	83.3	85.5	88
5	3.1	5.6	8.2	10.9	13.7	16.6	78.7	80.4	82.2	84.3	86.5	89
	8.9	11.8	14.9	18.2	21.7	25.3	79.1	80.8	82.7	84.7	87.0	89
	15.3	18.6	22.2	26.1	30.9	36.8	79.6	81.3	83.1	85.2	87.5	90
	23.3	27.5	32.5	38.9	51.6	.....	80.0	81.7	83.6	85.7	88.0	90
	34.2	41.4	.....	.....	.....	.....	80.5	82.2	84.1	86.2	88.5	91

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4c.—Polaris, 1912, General Table.—Concluded.
(To accompany Tables 4a and 4b.)

HOUR ANGLES, 1912.					AZIMUTHS OF POLARIS, 1912.										
Decl. +88° 50′.					Latitude.										
″	20″	30″	40″	50″	30°	32°	34°	36°	38°	40°	42°	44°	46°	48°	50°
h	m	m	m	m	′	′	′	′	′	′	′	′	′	′	′
1.8	.....	.....	.....	.....	80.5	82.2	84.1	86.2	88.5	91.0	93.8	96.9	100.3	104.1	108.4
2.7	17.7	11.3	58.6	.....	80.1	81.8	83.6	85.7	88.0	90.5	93.3	96.4	99.8	103.6	107.8
1.5	28.0	24.1	19.3	13.4	79.6	81.3	83.2	85.2	87.5	90.0	92.8	95.8	99.2	103.0	107.2
1.6	35.8	32.8	29.2	25.4	79.2	80.9	82.7	84.8	87.0	89.5	92.2	95.3	98.7	102.4	106.6
1.9	42.4	39.8	36.9	33.8	78.7	80.4	82.3	84.3	86.5	89.0	91.7	94.8	98.1	101.8	106.0
5.7	53.6	51.4	49.1	46.8	77.9	79.5	81.3	83.3	85.6	88.0	90.7	93.7	97.0	100.7	104.8
2.6	11.0	9.4	7.7	6.0	76.1	77.7	79.5	81.5	83.6	86.0	88.6	91.5	94.8	98.4	102.3
4.7	25.4	24.1	22.7	21.3	74.4	76.0	77.7	79.6	81.7	84.0	86.6	89.4	92.5	96.0	99.9
1.9	37.7	36.6	35.4	34.2	72.7	74.2	75.9	77.7	79.8	82.0	84.5	87.2	90.3	93.7	97.5
1.9	48.9	47.9	46.8	45.7	70.9	72.4	74.0	75.8	77.8	80.0	82.4	85.1	88.1	91.4	95.0
1.8	58.9	57.9	57.0	56.1	69.2	70.6	72.2	73.9	75.9	78.0	80.4	83.0	85.9	89.1	92.7
4.3	8.4	7.6	6.7	5.8	67.4	68.8	70.3	72.0	73.9	76.0	78.3	80.8	83.6	86.8	90.3
1.3	17.5	16.7	15.9	15.1	65.6	67.0	68.5	70.2	72.0	74.0	76.2	78.7	81.4	84.5	87.9
4.7	26.0	25.3	24.6	23.9	63.9	65.2	66.6	68.3	70.0	72.0	74.2	76.6	79.2	82.2	85.5
1.9	34.3	33.6	33.0	32.3	62.1	63.4	64.8	66.4	68.1	70.0	72.1	74.4	77.0	79.9	83.1
2.6	42.0	41.4	40.8	40.2	60.3	61.6	63.0	64.5	66.1	68.0	70.0	72.3	74.8	77.6	80.7
4.1	49.5	49.0	48.4	47.8	58.6	59.8	61.1	62.6	64.2	66.0	68.0	70.2	72.6	75.3	78.3
7.4	56.8	56.3	55.8	55.2	56.8	58.0	59.3	60.7	62.3	64.0	65.9	68.0	70.4	73.0	75.9
1.5	4.0	3.5	3.0	2.5	55.0	56.2	57.4	58.8	60.3	62.0	63.8	65.9	68.2	70.7	73.5
1.3	10.8	10.4	9.9	9.4	53.3	54.4	55.6	56.9	58.4	60.0	61.8	63.8	66.0	68.4	71.1
1.0	17.5	17.1	16.6	16.2	51.5	52.6	53.7	55.0	56.4	58.0	59.7	61.6	63.8	66.1	68.8
1.5	24.0	23.6	23.2	22.7	49.7	50.7	51.9	53.1	54.5	56.0	57.7	59.5	61.6	63.8	66.4
1.8	30.4	30.0	29.6	29.2	47.9	48.9	50.0	51.2	52.5	54.0	55.6	57.4	59.4	61.6	64.0
7.1	36.7	36.4	36.0	35.6	46.2	47.1	48.2	49.3	50.6	52.0	53.5	55.3	57.2	59.3	61.6
1.2	42.9	42.5	42.1	41.8	44.4	45.3	46.3	47.4	48.6	50.0	51.5	53.1	55.0	57.0	59.3
1.3	48.9	48.6	48.2	47.9	42.6	43.5	44.5	45.5	46.7	48.0	49.4	51.0	52.8	54.7	56.9
1.2	54.9	54.6	54.2	53.9	40.9	41.7	42.6	43.6	44.8	46.0	47.4	48.9	50.6	52.4	54.5
1.1	0.8	0.5	0.1	59.8	39.1	39.9	40.8	41.7	42.8	44.0	45.3	46.8	48.4	50.1	52.1
5.8	6.5	6.3	6.0	5.7	37.3	38.1	38.9	39.8	40.9	42.0	43.2	44.6	46.2	47.9	49.7
2.5	12.3	12.0	11.7	11.5	35.5	36.3	37.1	38.0	38.9	40.0	41.2	42.5	44.0	45.6	47.4
3.2	17.9	17.7	17.4	17.2	33.8	34.4	35.2	36.1	37.0	38.0	39.1	40.4	41.8	43.3	45.0
3.8	23.5	23.3	23.0	22.8	32.0	32.6	33.4	34.2	35.0	36.0	37.1	38.2	39.6	41.0	42.6
9.3	29.1	28.8	28.6	28.3	30.2	30.8	31.5	32.3	33.1	34.0	35.0	36.1	37.4	38.7	40.3
4.7	34.5	34.3	34.1	33.9	28.4	29.0	29.7	30.4	31.1	32.0	32.9	34.0	35.2	36.4	37.9
0.1	39.9	39.8	39.6	39.4	26.7	27.2	27.8	28.5	29.2	30.0	30.9	31.9	33.0	34.2	35.5
5.5	45.3	45.2	45.0	44.8	24.8	25.4	25.9	26.6	27.2	28.0	28.8	29.7	30.8	31.9	33.1
0.8	50.7	50.5	50.3	50.2	23.1	23.6	24.1	24.7	25.3	26.0	26.8	27.6	28.6	29.6	30.8
5.2	56.0	55.9	55.7	55.6	21.3	21.8	22.2	22.8	23.3	24.0	24.7	25.5	26.4	27.3	28.4
1.4	1.2	1.1	1.0	0.8	19.5	19.9	20.4	20.9	21.4	22.0	22.6	23.4	24.2	25.0	26.0
5.6	6.5	6.4	6.2	6.1	17.8	18.1	18.5	19.0	19.4	20.0	20.6	21.2	22.0	22.8	23.7
1.8	11.7	11.6	11.5	11.4	16.0	16.3	16.7	17.1	17.5	18.0	18.5	19.1	19.8	20.5	21.3
7.0	16.9	16.9	16.8	16.7	14.2	14.5	14.8	15.2	15.6	16.0	16.5	17.0	17.6	18.2	18.9
2.0	22.1	22.1	22.0	21.9	12.4	12.7	13.0	13.3	13.6	14.0	14.4	14.9	15.4	15.9	16.6
7.3	27.3	27.2	27.1	27.1	10.7	10.9	11.1	11.4	11.7	12.0	12.3	12.7	13.2	13.7	14.2
2.5	32.5	32.4	32.3	32.3	8.9	9.1	9.3	9.5	9.7	10.0	10.3	10.6	11.0	11.4	11.8
7.7	37.6	37.6	37.5	37.5	7.1	7.3	7.4	7.6	7.8	8.0	8.2	8.5	8.8	9.1	9.5
2.8	42.7	42.7	42.7	42.6	5.3	5.4	5.6	5.7	5.8	6.0	6.2	6.4	6.6	6.8	7.1
7.9	47.8	47.8	47.8	47.7	3.6	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.4	4.6	4.7
3.0	53.0	53.0	52.9	52.9	1.8	1.8	1.9	1.9	1.9	2.0	2.1	2.1	2.2	2.3	2.4
8.0	58.0	58.0	58.0	58.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

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Tapes.—The best form of tape for general surveying is a 100-ft. steel ribbon with graduations to feet, tenths, and hundredths. The zero end of the tape should be *the end of the steel ribbon itself*, and it may be provided with a small handle or a large detachable handle, the latter for continuous line measurements. The 100-ft. end should also be provided with detachable handle so it can be wound in a box.

Every city surveyor should keep, in his office, a standard tape tested by the U. S. Coast and Geodetic Dept. at Washington, and certified to as correct at a certain *standard temperature*, say 60° F., and for a certain pull, say 10 to 15 lbs., when uniformly supported. This should be used as a test tape only, and never for field work. The advantage of such a standard over a fixed base (as on a pavement) is that the temperature* of the tapes need not be taken during the test. Nor is it necessary to use the spring balance, as the tapes can be brought to practically the *same* tension without it. It is necessary of course to use the same pull for the field measurements and this is one of the tricks of chaining.† The "misuse" of the thermometer in the field is often a source of "error." That is to say, the temperature of the thermometer will never indicate the temperature of the tape, exactly. Both will be affected differently by the sun's rays, surrounding air, wind and temperature of ground. The best chaining is done on cloudy days and in still air.

Temperature corrections should be *added* to a *measured length* between two *fixed points* in the field when the temperature of the tape is *above* the *standard temperature*; *subtracted* when *below*. Reverse the above when *laying out* a fixed distance, as setting one point from another.

5.—Temperature Corrections in Feet per 100 Feet.

Note.—Use signs as per table for measurements between fixed objects. Reverse signs in table when laying out fixed distances.

(From the author's "Railway Right-of-Way Surveying."‡)

Field Temperatures.	100-foot Tape Standard at following Temperatures.										
	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	
0°	−.027	−.03	−.033	−.037	−.04	−.043	−.047	−.05	−.053	−.057	0°
5°	−.023	−.027	−.03	−.033	−.037	−.04	−.043	−.047	−.05	−.053	5°
10°	−.02	−.023	−.027	−.03	−.033	−.037	−.04	−.043	−.047	−.05	10°
15°	−.017	−.02	−.023	−.027	−.03	−.033	−.037	−.04	−.043	−.047	15°
20°	−.013	−.017	−.02	−.023	−.027	−.03	−.033	−.037	−.04	−.043	20°
25°	−.01	−.013	−.017	−.02	−.023	−.027	−.03	−.033	−.037	−.04	25°
30°	−.007	−.01	−.013	−.017	−.02	−.023	−.027	−.03	−.033	−.037	30°
35°	−.003	−.007	−.01	−.013	−.017	−.02	−.023	−.027	−.03	−.033	35°
40°	.....	−.003	−.007	−.01	−.013	−.017	−.02	−.023	−.027	−.03	40°
45°	+.003	.....	−.003	−.007	−.01	−.013	−.017	−.02	−.023	−.027	45°
50°	+.007	+.003	.....	−.003	−.007	−.01	−.013	−.017	−.02	−.023	50°
55°	+.01	+.007	+.003	.....	−.003	−.007	−.01	−.013	−.017	−.02	55°
60°	+.013	+.01	+.007	+.003	.....	−.003	−.007	−.01	−.013	−.017	60°
65°	+.017	+.013	+.01	+.007	+.003	.....	−.003	−.007	−.01	−.013	65°
70°	+.02	+.017	+.013	+.01	+.007	+.003	.....	−.003	−.007	−.01	70°
75°	+.023	+.02	+.017	+.013	+.01	+.007	+.003	.....	−.003	−.007	75°
80°	+.027	+.023	+.02	+.017	+.013	+.01	+.007	+.003	.....	−.003	80°
85°	+.03	+.027	+.023	+.02	+.017	+.013	+.01	+.007	+.003	.....	85°
90°	+.033	+.03	+.027	+.023	+.02	+.017	+.013	+.01	+.007	+.003	90°
95°	+.037	+.033	+.03	+.027	+.023	+.02	+.017	+.013	+.01	+.007	95°
100°	+.04	+.037	+.033	+.03	+.027	+.023	+.02	+.017	+.013	+.01	100°

* The tapes should lie unwound in the same atmosphere for some little time before making the test.

† On very accurate city work it is often desirable to let the chainmen use the spring balance until they get accustomed to the proper tension, after which it may be discarded generally, or used only occasionally to keep them in tune. ‡ Published by McGraw-Hill Book Company, New York.

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g and the Stretch of a Tape supported at two or three points, ıitions 2 and 3, may be made to equalize each other, by apply- ɔer tension oı pull at the ends, so that the horizontal measured remain the same, and equal also to the length of the tape when hroughout as in position 1. Practical advantage is taken of the n measuring with a single- or double-sagged tape. The pull P_1, for s of course fixed when the tape is standardized, while P_2 and P_3, s 2 and 3, are obtained with the spring balance and by plumbing ed points a and b If it is desired, however, to maintain a constant for all three positions of the tape (and this is generally the

Fig. 16.

ıod) then the horizontal *distance error* must be determined and r positions 2 and 3. These "distance errors" will become the ors or "sag" errors of the tape, and are to be used in addition ıperature" corrections. Care must be used in maintaining the d tension while measuring, as a difference of 6 lbs. in pull, when uniformly supported, will affect an ordinary 100-ft. steel tape 0.01 he exact amount depends upon the size (area) of the ribbon, and s of elasticity of the steel. It can best be determined with the ape, using the spring balance and watching the amount of he apparent stretch of a sagged tape is greater than when lying differ very much in area, varying from about .0015 to .008 sq.

gineer's Chain (100 1-ft. links) is recommended for rough work inous or lava country, but it is often used where the steel (ribbon) prove more expeditious.

rveyor's or Gunter's Chain (100 7.92-in. links), 66 ft. long, is y in the West in connection with government land surveys. useful in retracing old land surveys made before the engineer's n came into use.

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6.—Table of Feet and Chains (Gunter's or Surveyor's).

(1 chain = 100 links; 1 link = 7.92 ins.)

(*a*) Chains to Feet (Exact).

Ch'ns.	Feet.	Chains.	Feet.	Chains.	Feet.	Chains.	Feet.	Chains.	Feet.
.01	.66	.21	13.86	.41	27.06	.61	40.26	.81	53.46
.02	1.32	.22	14.52	.42	27.72	.62	40.92	.82	54.12
.03	1.98	.23	15.18	.43	28.38	.63	41.58	.83	54.78
.04	2.64	.24	15.84	.44	29.04	.64	42.24	.84	55.44
.05	3.30	.25	16.50	.45	29.70	.65	42.90	.85	56.10
.06	3.96	.26	17.16	.46	30.36	.66	43.56	.86	56.76
.07	4.62	.27	17.82	.47	31.02	.67	44.22	.87	57.42
.08	5.28	.28	18.48	.48	31.68	.68	44.88	.88	58.08
.09	5.94	.29	19.14	.49	32.34	.69	45.54	.89	58.74
.10	6.60	.30	19.80	.50	33.00	.70	46.20	.90	59.40
.11	7.26	.31	20.46	.51	33.66	.71	46.86	.91	60.06
.12	7.92	.32	21.12	.52	34.32	.72	47.52	.92	60.72
.13	8.58	.33	21.78	.53	34.98	.73	48.18	.93	61.38
.14	9.24	.34	22.44	.54	35.64	.74	48.84	.94	62.04
.15	9.90	.35	23.10	.55	36.30	.75	49.50	.95	62.70
.16	10.56	.36	23.76	.56	36.96	.76	50.16	.96	63.36
.17	11.22	.37	24.42	.57	37.62	.77	50.82	.97	64.02
.18	11.88	.38	25.08	.58	38.28	.78	51.84	.98	64.68
.19	12.54	.39	25.74	.59	38.94	.79	52.14	.99	65.34
.20	13.20	.40	26.40	.60	39.60	.80	52.80	1.00	66.00

(*b*) Feet to Chains.

Feet.	Chains.	Feet.	Chains.	Feet.	Chains.	Feet.	Chains.	Feet.	Chains.
1	.015˘15	3	.045˘45	5	.075˘75	7	.106˘06	9	.136˘36
2	.030˘30	4	.060˘60	6	.090˘90	8	.121˘21	10	.151˘51

Note.—The inverted caret indicates repeating decimal; thus, 1 ft. = 0.015 15 15 15 - - - - chain.

Ex.—Reduce 18 *ch.* 45 *l.* to feet?

Solution:

.18 ch = 11.88 ft. ∴ 18 ch = 1188.00ft.
.45 ch = 29.70 ft. ∴ 45 *l* = 29.70ft.

Ans. 18 ch 45 l = 1217.70ft.
Or, mult. 18.45 by 66.

Ex.—Reduce 482.73 ft. to *ch.* and *l?*

Solution:

400ft. = 6.0606ch. .70ft. = .0106 ch.
80ft. = 1.2121ch. .03ft. = .0005 ch.
2ft. = 0.0303ch.

Ans. 7 ch. 31.41 l.
Or, divide 482.73 by 6 and 11.

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ls of Platting Angles.—There are three principal methods of ngles:

3y the Protractor. The cheapest, simplest and perhaps most y for general use is the paper protractor, a complete circle 14 ins. l graduated to ½ degrees. Steel and German silver protractors ble arms and verniers are convenient but expensive, costing from $40.00 and upward, while the large paper protractor may be or about 30 cts.

By Tangents. This consists of laying off by the use of the Table of Natural Tangents 3, p. 144.) Thus, in Fig. 17, let it be o lay off the angle $\alpha = 28° - 16'$ from the Lay off a length of Base, to any scale, ome multiple of 10, say 1000; then the length of Altitude, at to the Base, will equal 1000 mult. by tang $\alpha = 1000 \times 0.5377 =$ me scale. Draw the Hypothenuse, which now makes the required ith the Base. This method is desirable for small angles. For s, and for very long lines, the method of Chords is often preferred.

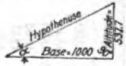

Fig. 17.

3y Chords. Assume a length of Base as some multiple of 10, say ny scale. With a radius of 1000 describe the arc $a\,b$, Fig. 18. ollowing Table of Chords to Radius 1 find orresponding to the angle α, assumed here '. This value, 0.4884, mult. by 1000, the ase, = 488.4 the desired Chord, or radius n center at a, to intersect at b. Draw Ob; is the desired angle α. Note that the se (i.e., radius of circle), mult. by twice en α does not exceed 90°.

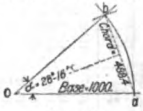

Fig. 18.

7.—Chords to Radius 1.

1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	M.
0175	.0349	.0524	.0698	.0872	.1047	.1221	.1395	.1569	.1743	0′
0180	.0355	.0529	.0704	.0878	.1053	.1227	.1401	.1575	.1749	2
0186	.0361	.0535	.0710	.0884	.1058	.1233	.1407	.1581	.1755	4
0189	.0364	.0538	.0713	.0887	.1061	.1235	.1410	.1584	.1758	5
0192	.0366	.0541	.0715	.0890	.1064	.1238	.1413	.1587	.1761	6
0198	.0372	.0547	.0721	.0896	.1070	.1244	.1418	.1592	.1766	8
0204	.0378	.0553	.0727	.0901	.1076	.1250	.1424	.1598	.1772	10
0209	.0384	.0558	.0733	.0907	.1082	.1256	.1430	.1604	.1778	12
0215	.0390	.0564	.0739	.0913	.1087	.1262	.1436	.1610	.1784	14
0218	.0393	.0567	.0742	.0916	.1090	.1265	.1439	.1613	.1787	15
0221	.0396	.0570	.0745	.0919	.1093	.1267	.1442	.1616	.1789	16
0227	.0401	.0576	.0750	.0925	.1099	.1273	.1447	.1621	.1795	18
0233	.0407	.0582	.0756	.0931	.1105	.1279	.1453	.1627	.1801	20
0239	.0413	.0588	.0762	.0936	.1111	.1285	.1459	.1633	.1807	22
0244	.0419	.0593	.0768	.0942	.1116	.1291	.1465	.1639	.1813	24
0247	.0422	.0596	.0771	.0945	.1119	.1294	.1468	.1642	.1816	25
0250	.0425	.0599	.0774	.0948	.1122	.1296	.1471	.1645	.1818	26
0256	.0430	.0605	.0779	.0954	.1128	.1302	.1476	.1650	.1824	28
0262	.0436	.0611	.0785	.0960	.1134	.1308	.1482	.1656	.1830	30
0268	.0442	.0617	.0791	.0965	.1140	.1314	.1488	.1662	.1836	32
0273	.0448	.0622	.0797	.0971	.1145	.1320	.1494	.1668	.1842	34
0276	.0451	.0625	.0800	.0974	.1148	.1323	.1497	.1671	.1845	35
0279	.0454	.0628	.0803	.0977	.1151	.1325	.1500	.1674	.1847	36
0285	.0460	.0634	.0808	.0983	.1157	.1331	.1505	.1679	.1853	38
0291	.0465	.0640	.0814	.0989	.1163	.1337	.1511	.1685	.1859	40
0297	.0471	.0646	.0820	.0994	.1169	.1343	.1517	.1691	.1865	42
0303	.0477	.0651	.0826	.1000	.1175	.1349	.1523	.1697	.1871	44
0305	.0480	.0654	.0829	.1003	.1177	.1352	.1526	.1700	.1873	45
0308	.0483	.0657	.0832	.1006	.1180	.1355	.1529	.1703	.1876	46
0314	.0489	.0663	.0838	.1012	.1186	.1360	.1534	.1708	.1882	48
0320	.0494	.0669	.0843	.1018	.1193	.1366	.1540	.1714	.1888	50
0326	.0500	.0675	.0849	.1023	.1198	.1372	.1546	.1720	.1894	52
0332	.0506	.0681	.0855	.1029	.1204	.1378	.1552	.1726	.1900	54
0334	.0509	.0683	.0858	.1032	.1206	.1381	.1555	.1729	.1902	55
0337	.0512	.0686	.0861	.1035	.1209	.1384	.1558	.1732	.1905	56
0343	.0518	.0692	.0867	.1041	.1215	.1389	.1563	.1737	.1911	58
0349	.0524	.0698	.0872	.1047	.1221	.1395	.1569	.1743	.1917	60

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7.—Chords to Radius 1.—Continued.

M.	11°	12°	13°	14°	15°	16°	17°	18°	19°	20°	21°	22°	23°	24°	25°	26°	27°	28°	29°	30°	M.
0′	.1917	.2091	.2264	.2437	.2611	.2783	.2956	.3129	.3301	.3473	.3645	.3816	.3987	.4158	.4329	.4499	.4669	.4838	.5008	.5176	0′
2	.1923	.2096	.2270	.2443	.2616	.2789	.2962	.3134	.3307	.3479	.3650	.3822	.3993	.4164	.4334	.4505	.4675	.4844	.5013	.5182	2
4	.1928	.2102	.2276	.2449	.2622	.2795	.2968	.3140	.3312	.3484	.3656	.3828	.3999	.4170	.4340	.4510	.4680	.4850	.5019	.5188	4
5	.1931	.2105	.2279	.2452	.2625	.2798	.2971	.3143	.3315	.3487	.3659	.3830	.4002	.4172	.4343	.4513	.4683	.4853	.5022	.5190	5
6	.1934	.2108	.2281	.2455	.2628	.2801	.2973	.3146	.3318	.3490	.3662	.3833	.4004	.4175	.4346	.4516	.4686	.4855	.5024	.5193	6
8	.1940	.2114	.2287	.2460	.2634	.2807	.2979	.3152	.3324	.3496	.3668	.3839	.4010	.4181	.4352	.4522	.4692	.4861	.5030	.5199	8
10	.1946	.2119	.2293	.2466	.2639	.2812	.2985	.3157	.3330	.3502	.3673	.3845	.4016	.4187	.4357	.4527	.4697	.4867	.5036	.5204	10
12	.1952	.2125	.2299	.2472	.2645	.2818	.2991	.3163	.3335	.3507	.3679	.3850	.4022	.4192	.4363	.4533	.4703	.4872	.5041	.5210	12
14	.1957	.2131	.2305	.2478	.2651	.2824	.2996	.3169	.3341	.3513	.3685	.3856	.4027	.4198	.4369	.4539	.4708	.4878	.5047	.5216	14
15	.1960	.2134	.2307	.2481	.2654	.2827	.2999	.3172	.3344	.3516	.3688	.3859	.4030	.4201	.4371	.4541	.4711	.4881	.5050	.5219	15
16	.1963	.2137	.2310	.2484	.2657	.2830	.3002	.3175	.3347	.3519	.3690	.3862	.4033	.4204	.4374	.4544	.4714	.4884	.5053	.5221	16
18	.1969	.2143	.2316	.2489	.2662	.2835	.3008	.3180	.3353	.3525	.3696	.3868	.4039	.4209	.4380	.4550	.4720	.4889	.5058	.5227	18
20	.1975	.2148	.2322	.2495	.2668	.2841	.3014	.3186	.3358	.3530	.3702	.3873	.4044	.4215	.4386	.4556	.4725	.4895	.5064	.5233	20
22	.1981	.2154	.2328	.2501	.2674	.2847	.3019	.3192	.3364	.3536	.3708	.3879	.4050	.4221	.4391	.4561	.4731	.4901	.5070	.5238	22
24	.1986	.2160	.2333	.2507	.2680	.2853	.3025	.3198	.3370	.3542	.3713	.3885	.4056	.4226	.4397	.4567	.4737	.4906	.5075	.5244	24
25	.1989	.2163	.2336	.2510	.2683	.2855	.3028	.3200	.3373	.3545	.3716	.3888	.4059	.4229	.4400	.4570	.4740	.4909	.5078	.5247	25
26	.1992	.2166	.2339	.2512	.2685	.2858	.3031	.3203	.3376	.3547	.3719	.3890	.4061	.4232	.4403	.4573	.4742	.4912	.5081	.5249	26
28	.1998	.2172	.2345	.2518	.2691	.2864	.3037	.3209	.3381	.3553	.3725	.3896	.4067	.4238	.4408	.4578	.4748	.4917	.5086	.5255	28
30	.2004	.2177	.2351	.2524	.2697	.2870	.3042	.3215	.3387	.3559	.3730	.3902	.4073	.4244	.4414	.4584	.4754	.4923	.5092	.5261	30
32	.2010	.2183	.2357	.2530	.2703	.2876	.3048	.3221	.3393	.3565	.3736	.3908	.4079	.4249	.4420	.4590	.4759	.4929	.5098	.5266	32
34	.2015	.2189	.2362	.2536	.2709	.2881	.3054	.3226	.3398	.3570	.3742	.3913	.4084	.4255	.4425	.4595	.4765	.4934	.5103	.5272	34
35	.2018	.2192	.2365	.2538	.2711	.2884	.3057	.3229	.3401	.3573	.3745	.3916	.4087	.4258	.4428	.4598	.4768	.4937	.5106	.5275	35
36	.2021	.2195	.2368	.2541	.2714	.2887	.3060	.3232	.3404	.3576	.3748	.3919	.4090	.4261	.4431	.4601	.4771	.4940	.5109	.5278	36

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4	.5356	.5524	.5691	.5859	.6025	.6191	.6357	.6522	.6687	.6851	.7015	.7178	.7341	.7503	.7664	.7825	.7986	.8145	.8304	.8463	4
5	.5359	.5527	.5694	.5861	.6028	.6194	.6360	.6525	.6690	.6854	.7018	.7181	.7344	.7506	.7667	.7828	.7988	.8148	.8307	.8466	5
6	.5362	.5530	.5697	.5864	.6031	.6197	.6363	.6528	.6693	.6857	.7020	.7184	.7346	.7508	.7670	.7831	.7991	.8151	.8310	.8468	6
8	.5367	.5535	.5703	.5870	.6036	.6202	.6368	.6533	.6698	.6862	.7026	.7189	.7352	.7514	.7675	.7836	.7996	.8156	.8315	.8473	8
10	.5373	.5541	.5708	.5875	.6042	.6208	.6374	.6539	.6704	.6868	.7031	.7195	.7357	.7519	.7681	.7841	.8002	.8161	.8320	.8479	10
12	.5378	.5546	.5714	.5881	.6047	.6214	.6379	.6544	.6709	.6873	.7037	.7200	.7362	.7524	.7686	.7847	.8007	.8167	.8326	.8484	12
14	.5384	.5552	.5719	.5886	.6053	.6219	.6385	.6550	.6715	.6879	.7042	.7205	.7368	.7530	.7691	.7852	.8012	.8172	.8331	.8489	14
15	.5387	.5555	.5722	.5889	.6056	.6222	.6387	.6553	.6717	.6881	.7045	.7208	.7371	.7533	.7694	.7855	.8015	.8175	.8334	.8492	15
16	.5390	.5557	.5725	.5892	.6058	.6225	.6390	.6555	.6720	.6884	.7048	.7211	.7373	.7535	.7697	.7857	.8018	.8177	.8336	.8495	16
18	.5395	.5563	.5730	.5897	.6064	.6230	.6396	.6561	.6725	.6890	.7053	.7216	.7379	.7541	.7702	.7863	.8023	.8183	.8341	.8500	18
20	.5401	.5569	.5736	.5903	.6070	.6236	.6401	.6566	.6731	.6895	.7059	.7222	.7384	.7546	.7707	.7868	.8028	.8188	.8347	.8505	20
22	.5406	.5574	.5742	.5909	.6075	.6241	.6407	.6572	.6736	.6901	.7064	.7227	.7390	.7551	.7713	.7873	.8034	.8193	.8352	.8510	22
24	.5412	.5580	.5747	.5914	.6081	.6247	.6412	.6577	.6742	.6906	.7069	.7232	.7395	.7557	.7718	.7879	.8039	.8198	.8357	.8516	24
25	.5415	.5583	.5750	.5917	.6083	.6249	.6415	.6580	.6745	.6909	.7072	.7235	.7398	.7559	.7721	.7881	.8042	.8201	.8360	.8518	25
26	.5418	.5585	.5753	.5920	.6086	.6252	.6418	.6583	.6747	.6911	.7075	.7238	.7400	.7562	.7723	.7884	.8044	.8204	.8363	.8521	26
28	.5423	.5591	.5758	.5925	.6092	.6258	.6423	.6588	.6753	.6917	.7080	.7243	.7406	.7568	.7729	.7890	.8050	.8209	.8368	.8526	28
30	.5429	.5597	.5764	.5931	.6097	.6263	.6429	.6594	.6758	.6922	.7086	.7249	.7411	.7573	.7734	.7895	.8055	.8214	.8373	.8531	30
32	.5434	.5602	.5769	.5936	.6103	.6269	.6434	.6599	.6764	.6928	.7091	.7254	.7417	.7578	.7740	.7900	.8060	.8220	.8378	.8537	32
34	.5440	.5608	.5775	.5942	.6108	.6274	.6440	.6605	.6769	.6933	.7097	.7260	.7422	.7584	.7745	.7906	.8066	.8225	.8384	.8542	34
35	.5443	.5611	.5778	.5945	.6111	.6277	.6443	.6608	.6772	.6936	.7099	.7262	.7425	.7586	.7748	.7908	.8068	.8228	.8386	.8545	35
36	.5446	.5613	.5781	.5947	.6114	.6280	.6445	.6610	.6775	.6939	.7102	.7265	.7427	.7589	.7750	.7911	.8071	.8230	.8389	.8547	36
38	.5451	.5619	.5786	.5953	.6119	.6285	.6451	.6616	.6780	.6944	.7108	.7270	.7433	.7595	.7756	.7916	.8076	.8326	.8394	.8552	38
40	.5457	.5625	.5792	.5959	.6125	.6291	.6456	.6621	.6786	.6950	.7113	.7276	.7438	.7600	.7761	.7922	.8082	.8241	.8400	.8558	40
42	.5462	.5630	.5797	.5964	.6130	.6296	.6462	.6627	.6791	.6955	.7118	.7281	.7443	.7605	.7766	.7927	.8087	.8246	.8405	.8563	42
44	.5468	.5636	.5803	.5970	.6136	.6302	.6467	.6632	.6797	.6961	.7124	.7287	.7449	.7611	.7772	.7932	.8092	.8251	.8410	.8568	44
45	.5471	.5638	.5806	.5972	.6139	.6305	.6470	.6635	.6799	.6963	.7127	.7289	.7452	.7613	.7774	.7935	.8095	.8254	.8413	.8571	45
46	.5474	.5641	.5808	.5975	.6142	.6307	.6473	.6638	.6802	.6966	.7129	.7292	.7454	.7616	.7777	.7938	.8098	.8257	.8415	.8573	46
48	.5479	.5647	.5814	.5981	.6147	.6313	.6478	.6643	.6808	.6971	.7135	.7298	.7460	.7621	.7782	.7943	.8103	.8262	.8421	.8579	48
50	.5485	.5652	.5820	.5986	.6153	.6318	.6484	.6649	.6813	.6977	.7140	.7303	.7465	.7627	.7788	.7948	.8108	.8267	.8426	.8584	50
52	.5490	.5658	.5825	.5992	.6158	.6324	.6489	.6654	.6819	.6982	.7146	.7308	.7471	.7632	.7793	.7954	.8113	.8273	.8431	.8589	52
54	.5496	.5664	.5831	.5997	.6164	.6330	.6495	.6660	.6824	.6988	.7151	.7314	.7476	.7638	.7799	.7959	.8119	.8278	.8437	.8594	54
55	.5499	.5666	.5834	.6000	.6166	.6332	.6498	.6662	.6827	.6990	.7154	.7316	.7479	.7640	.7801	.7962	.8121	.8281	.8439	.8597	55
56	.5502	.5669	.5836	.6003	.6169	.6335	.6500	.6665	.6829	.6993	.7156	.7319	.7481	.7643	.7804	.7964	.8124	.8283	.8442	.8600	56
58	.5507	.5675	.5842	.6009	.6175	.6341	.6506	.6671	.6835	.6999	.7162	.7325	.7487	.7648	.7909	.7970	.8129	.8289	.8447	.8605	58
60	.5513	.5680	.5847	.6014	.6180	.6346	.6511	.6676	.6840	.7004	.7167	.7330	.7492	.7654	.7815	.7975	.8135	.8294	.8452	.8610	60

7.—Chords to Radius 1.—Continued.

M.	51°	52°	53°	54°	55°	56°	57°	58°	59°	60°	61°	62°	63°	64°	65°	66°	67°	68°	69°	70°	M.
0′	.8610	.8767	.8924	.9080	.9235	.9389	.9543	.9696	.9848	1.0000	1.0151	1.0301	1.0450	1.0598	1.0746	1.0893	1.1039	1.1184	1.1328	1.1472	0′
2	.8615	.8773	.8929	.9085	.9240	.9395	.9548	.9701	.9854	1.0005	1.0156	1.0306	1.0455	1.0603	1.0751	1.0898	1.1044	1.1189	1.1333	1.1476	2
4	.8621	.8778	.8934	.9090	.9245	.9400	.9553	.9706	.9859	1.0010	1.0161	1.0311	1.0460	1.0608	1.0756	1.0903	1.1048	1.1194	1.1338	1.1481	4
5	.8623	.8780	.8937	.9093	.9248	.9402	.9556	.9709	.9861	1.0013	1.0163	1.0313	1.0462	1.0611	1.0758	1.0905	1.1051	1.1196	1.1340	1.1483	5
6	.8626	.8783	.8940	.9095	.9250	.9405	.9559	.9711	.9864	1.0015	1.0166	1.0316	1.0465	1.0613	1.0761	1.0907	1.1053	1.1198	1.1342	1.1486	6
8	.8631	.8788	.8945	.9101	.9256	.9410	.9564	.9717	.9869	1.0020	1.0171	1.0321	1.0470	1.0618	1.0766	1.0912	1.1058	1.1203	1.1347	1.1491	8
10	.8636	.8794	.8950	.9106	.9261	.9415	.9569	.9722	.9874	1.0025	1.0176	1.0326	1.0475	1.0623	1.0771	1.0917	1.1063	1.1208	1.1352	1.1495	10
12	.8642	.8799	.8955	.9111	.9266	.9420	.9574	.9727	.9879	1.0030	1.0181	1.0331	1.0480	1.0628	1.0775	1.0922	1.1068	1.1213	1.1357	1.1500	12
14	.8647	.8804	.8960	.9116	.9271	.9425	.9579	.9732	.9884	1.0035	1.0186	1.0336	1.0485	1.0633	1.0780	1.0927	1.1073	1.1218	1.1362	1.1505	14
15	.8650	.8807	.8963	.9119	.9274	.9428	.9581	.9734	.9886	1.0038	1.0188	1.0338	1.0487	1.0635	1.0783	1.0929	1.1075	1.1220	1.1364	1.1507	15
16	.8652	.8809	.8966	.9121	.9276	.9430	.9584	.9737	.9889	1.0040	1.0191	1.0341	1.0490	1.0638	1.0785	1.0932	1.1078	1.1222	1.1366	1.1510	16
18	.8657	.8814	.8971	.9126	.9281	.9436	.9589	.9742	.9894	1.0045	1.0196	1.0346	1.0495	1.0643	1.0790	1.0937	1.1082	1.1227	1.1371	1.1514	18
20	.8663	.8820	.8976	.9132	.9287	.9441	.9594	.9747	.9899	1.0050	1.0201	1.0351	1.0500	1.0648	1.0795	1.0942	1.1087	1.1232	1.1376	1.1519	20
22	.8668	.8825	.8981	.9137	.9292	.9446	.9599	.9752	.9904	1.0055	1.0206	1.0356	1.0504	1.0653	1.0800	1.0946	1.1092	1.1237	1.1381	1.1524	22
24	.8673	.8830	.8986	.9142	.9297	.9451	.9604	.9757	.9909	1.0060	1.0211	1.0361	1.0509	1.0658	1.0805	1.0951	1.1097	1.1242	1.1386	1.1529	24
25	.8676	.8833	.8989	.9145	.9299	.9454	.9607	.9760	.9912	1.0063	1.0213	1.0363	1.0512	1.0660	1.0807	1.0954	1.1099	1.1244	1.1388	1.1531	25
26	.8678	.8835	.8992	.9147	.9302	.9456	.9610	.9762	.9914	1.0065	1.0216	1.0366	1.0514	1.0662	1.0810	1.0956	1.1102	1.1246	1.1390	1.1533	26
28	.8684	.8841	.8997	.9152	.9307	.9461	.9615	.9767	.9919	1.0070	1.0221	1.0370	1.0519	1.0667	1.0815	1.0961	1.1107	1.1251	1.1395	1.1538	28
30	.8689	.8846	.9002	.9157	.9312	.9466	.9620	.9772	.9924	1.0075	1.0226	1.0375	1.0524	1.0672	1.0820	1.0966	1.1111	1.1256	1.1400	1.1543	30
32	.8694	.8851	.9007	.9163	.9317	.9472	.9625	.9778	.9929	1.0080	1.0231	1.0380	1.0529	1.0677	1.0824	1.0971	1.1116	1.1261	1.1405	1.1548	32
34	.8699	.8856	.9012	.9168	.9323	.9477	.9630	.9783	.9934	1.0086	1.0236	1.0385	1.0534	1.0682	1.0829	1.0976	1.1121	1.1266	1.1409	1.1552	34
35	.8702	.8859	.9015	.9170	.9325	.9479	.9632	.9785	.9937	1.0088	1.0238	1.0388	1.0537	1.0685	1.0832	1.0978	1.1123	1.1268	1.1412	1.1555	35
36	.8705	.8861	.9018	.9173	.9328	.9482	.9635	.9788	.9939	1.0091	1.0241	1.0390	1.0539	1.0687	1.0834	1.0980	1.1126	1.1271	1.1414	1.1557	36

	71°	72°	73°	74°	75°	76°	77°	78°	79°	80°	81°	82°	83°	84°	85°	86°	87°	88°	89°	M.
0′	1.1614	1.1756	1.1896	1.2036	1.2175	1.2313	1.2450	1.2586	1.2722	1.2856	1.2989	1.3121	1.3252	1.3383	1.3512	1.3640	1.3767	1.3893	1.4018	0′
2	1.1619	1.1760	1.1901	1.2041	1.2180	1.2318	1.2455	1.2591	1.2726	1.2860	1.2993	1.3126	1.3257	1.3387	1.3516	1.3644	1.3771	1.3897	1.4022	2
4	1.1624	1.1765	1.1906	1.2046	1.2184	1.2322	1.2459	1.2595	1.2731	1.2865	1.2998	1.3130	1.3261	1.3391	1.3520	1.3648	1.3776	1.3902	1.4026	4
5	1.1626	1.1767	1.1908	1.2048	1.2187	1.2325	1.2462	1.2598	1.2733	1.2867	1.3000	1.3132	1.3263	1.3393	1.3523	1.3651	1.3778	1.3904	1.4029	5
6	1.1628	1.1770	1.1910	1.2050	1.2189	1.2327	1.2464	1.2600	1.2735	1.2869	1.3002	1.3134	1.3265	1.3396	1.3525	1.3653	1.3780	1.3906	1.4031	6
8	1.1633	1.1775	1.1915	1.2055	1.2194	1.2332	1.2468	1.2604	1.2740	1.2874	1.3007	1.3139	1.3270	1.3400	1.3529	1.3657	1.3784	1.3910	1.4035	8
10	1.1638	1.1779	1.1920	1.2060	1.2198	1.2336	1.2473	1.2609	1.2744	1.2878	1.3011	1.3143	1.3274	1.3404	1.3533	1.3661	1.3788	1.3914	1.4039	10
12	1.1642	1.1784	1.1924	1.2064	1.2203	1.2341	1.2478	1.2614	1.2748	1.2882	1.3015	1.3147	1.3279	1.3409	1.3538	1.3665	1.3792	1.3918	1.4043	12
14	1.1647	1.1789	1.1929	1.2069	1.2208	1.2345	1.2482	1.2618	1.2753	1.2887	1.3020	1.3152	1.3283	1.3413	1.3542	1.3670	1.3797	1.3922	1.4047	14
15	1.1650	1.1791	1.1931	1.2071	1.2210	1.2348	1.2484	1.2620	1.2755	1.2889	1.3022	1.3154	1.3285	1.3415	1.3544	1.3672	1.3799	1.3925	1.4049	15
16	1.1652	1.1793	1.1934	1.2073	1.2212	1.2350	1.2487	1.2623	1.2757	1.2891	1.3024	1.3156	1.3287	1.3417	1.3546	1.3674	1.3801	1.3927	1.4051	16
18	1.1657	1.1798	1.1938	1.2078	1.2217	1.2354	1.2491	1.2627	1.2762	1.2896	1.3029	1.3161	1.3292	1.3421	1.3550	1.3678	1.3805	1.3931	1.4055	18
20	1.1661	1.1803	1.1943	1.2083	1.2221	1.2359	1.2496	1.2632	1.2766	1.2900	1.3033	1.3165	1.3296	1.3426	1.3555	1.3682	1.3809	1.3935	1.4060	20
22	1.1666	1.1807	1.1948	1.2087	1.2226	1.2364	1.2500	1.2636	1.2771	1.2905	1.3038	1.3169	1.3300	1.3430	1.3559	1.3687	1.3813	1.3939	1.4064	22
24	1.1671	1.1812	1.1952	1.2092	1.2231	1.2368	1.2505	1.2641	1.2775	1.2909	1.3042	1.3174	1.3305	1.3434	1.3563	1.3691	1.3818	1.3943	1.4068	24
25	1.1673	1.1814	1.1955	1.2094	1.2233	1.2370	1.2507	1.2643	1.2778	1.2911	1.3044	1.3176	1.3307	1.3437	1.3565	1.3693	1.3820	1.3945	1.4070	25
26	1.1676	1.1817	1.1957	1.2097	1.2235	1.2373	1.2509	1.2645	1.2780	1.2914	1.3046	1.3178	1.3309	1.3439	1.3567	1.3695	1.3822	1.3947	1.4072	26
28	1.1680	1.1821	1.1962	1.2101	1.2240	1.2377	1.2514	1.2650	1.2784	1.2918	1.3051	1.3183	1.3313	1.3443	1.3572	1.3699	1.3826	1.3952	1.4076	28
30	1.1685	1.1826	1.1966	1.2106	1.2244	1.2382	1.2518	1.2654	1.2789	1.2922	1.3055	1.3187	1.3318	1.3447	1.3576	1.3704	1.3830	1.3956	1.4080	30
32	1.1690	1.1831	1.1971	1.2111	1.2249	1.2386	1.2523	1.2659	1.2793	1.2927	1.3060	1.3191	1.3322	1.3452	1.3580	1.3708	1.3834	1.3960	1.4084	32
34	1.1694	1.1836	1.1976	1.2115	1.2254	1.2391	1.2528	1.2663	1.2798	1.2931	1.3064	1.3196	1.3326	1.3456	1.3585	1.3712	1.3839	1.3964	1.4089	34
35	1.1697	1.1838	1.1978	1.2117	1.2256	1.2394	1.2530	1.2665	1.2800	1.2934	1.3066	1.3198	1.3328	1.3458	1.5587	1.3714	1.3841	1.3966	1.4091	35
36	1.1699	1.1840	1.1980	1.2120	1.2258	1.2396	1.2532	1.2668	1.2802	1.2936	1.3068	1.3200	1.3331	1.3460	1.3589	1.3716	1.3843	1.3968	1.4093	36
38	1.1704	1.1845	1.1985	1.2124	1.2263	1.2400	1.2537	1.2672	1.2807	1.2940	1.3073	1.3204	1.3335	1.3465	1.3593	1.3721	1.3847	1.3972	1.4097	38
40	1.1709	1.1850	1.1990	1.2129	1.2267	1.2405	1.2541	1.2677	1.2811	1.2945	1.3077	1.3209	1.3339	1.3469	1.3597	1.3725	1.3851	1.3977	1.4101	40
42	1.1713	1.1854	1.1994	1.2134	1.2272	1.2409	1.2546	1.2681	1.2816	1.2949	1.3082	1.3213	1.3344	1.3473	1.3602	1.3729	1.3855	1.3981	1.4105	42
44	1.1718	1.1859	1.1999	1.2138	1.2277	1.2414	1.2550	1.2686	1.2820	1.2954	1.3086	1.3218	1.3348	1.3477	1.3606	1.3733	1.3860	1.3985	1.4109	44
45	1.1720	1.1861	1.2001	1.2141	1.2279	1.2416	1.2552	1.2688	1.2822	1.2956	1.3088	1.3220	1.3350	1.3480	1.3608	1.3735	1.3862	1.3987	1.4111	45
46	1.1723	1.1864	1.2004	1.2143	1.2281	1.2418	1.2555	1.2690	1.2825	1.2958	1.3090	1.3222	1.3352	1.3482	1.3610	1.3738	1.3864	1.3989	1.4113	46
48	1.1727	1.1868	1.2008	1.2148	1.2286	1.2423	1.2559	1.2695	1.2829	1.2962	1.3095	1.3226	1.3557	1.3486	1.3614	1.3742	1.3868	1.3993	1.4117	48
50	1.1732	1.1873	1.2013	1.2152	1.2290	1.2428	1.2564	1.2699	1.2833	1.2967	1.3099	1.3231	1.3361	1.3490	1.3619	1.3746	1.3872	1.3997	1.4122	50
52	1.1737	1.1878	1.2018	1.2157	1.2295	1.2432	1.2568	1.2704	1.2838	1.2971	1.3104	1.3235	1.3365	1.3495	1.3623	1.3750	1.3876	1.4002	1.4126	52
54	1.1742	1.1882	1.2022	1.2161	1.2299	1.2437	1.2573	1.2708	1.2842	1.2976	1.3108	1.3239	1.3370	1.3499	1.3627	1.3754	1.3881	1.4006	1.4130	54
55	1.1744	1.1885	1.2025	1.2164	1.2302	1.2439	1.2575	1.2710	1.2845	1.2978	1.3110	1.3241	1.3372	1.3501	1.3629	1.3757	1.3883	1.4008	1.4132	55
56	1.1746	1.1887	1.2027	1.2166	1.2304	1.2441	1.2577	1.2713	1.2847	1.2980	1.3112	1.3244	1.3374	1.3503	1.3631	1.3759	1.3885	1.4010	1.4134	56
58	1.1751	1.1892	1.2032	1.2171	1.2309	1.2446	1.2582	1.2717	1.2851	1.2985	1.3117	1.3248	1.3378	1.3508	1.3636	1.3763	1.3889	1.4014	1.4138	58
60	1.1756	1.1896	1.2036	1.2175	1.2313	1.2450	1.2586	1.2722	1.2856	1.2989	1.3121	1.3252	1.3383	1.3512	1.3640	1.3767	1.3893	1.4018	1.4142	60

Farm Surveying.—Let it be required to make a survey of a farm of about 150 acres, locating the roads, fences, buildings, determining the acreage, and making the map. As the farm is, say, about 30 miles from the city the surveyor decides that he will prepare to spend one day in the field, only.

The Equipment consists of 1 transit, 2 100-ft. tapes, 2 flag poles (Fig. 19), 12 pins (Fig. 20) with red flannel tied at their tops to prevent losing them, 1 axe, 1 hatchet, 1 transit plumbbob (Fig. 21), 1 plumbbob (Fig. 22) for each of the men, 1 steel frost pin (Fig. 23) if the ground is frozen, and the stakes (Fig. 24),

Fig. 19.

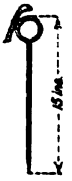

Fig. 20.

Fig. 21.

Fig. 22.

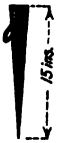

Fig. 23.

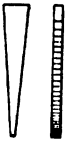

Fig. 24.

Fig. 25.

hubs (Fig. 25), tacks, etc. (The stakes may perhaps be procured on the ground, but it is often cheaper to take them from the office.) Four men, say, besides the chief of party, comprise the working outfit.

The Traverse of the farm is represented (Fig. 26) by the broken instrument-line *A B C D E A*, which closely follows the fence lines (not shown). In running around the farm with this transverse, lettered or numbered stakes are set on the instrument lines opposite all bends in the fence lines; these stakes are located by base line measurements, and from them the offset distances are measured to the fences, thus completely tying-in the farm boundary. Any building as *H* may be located by running a spur instrument line from some point on the traverse, as *B*. The traverse itself is determined by the lengths of the measured base lines, *A B*, *B C*, *C D*, etc., and by the measured angles at *A*, *B*, *C*, etc. *Inaccuracies* in measurement, both in base line distances and in angles, will usually creep into the work and hence the traverse will seldom *close*, that is, it will have to be *adjusted*. Absolute *errors*, however, can be eliminated by measuring the base lines twice and by "repeating" all angles, and these should be examined carefully before leaving the field.

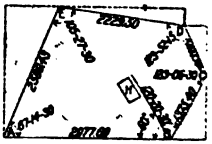

Fig. 26.

The Adjustment of the Traverse may be made in several ways, but the simplest and most practical is as follows: Let *A*, *B*, *C*, etc., be the interior angles, measured in the field, at the respective corners of the traverse (Fig. 26). The sum of these angles should equal 540° (= 180° × number of sides − 360°) if there is perfect accuracy in the field work. If the angles add up to within a few minutes of 540° this variation may be proportioned among all the angles so their sum will equal 540°. But greater weight should be given to those measured angles with the (longest and) *clearest* foresights, as *A*, especially if the angle "doubled" accurately when measured in the field; and less weight should be given to, say, *C* and *D*, especially if the angles did not "double" or "repeat" properly. Having adjusted the angles (Fig. 27) so their sum is 540°, assume one side of the traverse as a base line, say *A B*, and cut the exterior of the traverse into right angle triangles as shown by the dotted lines; calculate the angles a, c_1, c_2 and e, from the

Fig. 27.

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sted interior angles A, B, etc.; solve the sides of the triangles, using measured base lines of the traverse, in each case, as the hypothenuse onnection with the calculated angles a, c_1, etc.

The final test is now made by adding together the horizontal distances e top of the traverse, which we will call W, and comparing with the of the horizontal distance at the bottom, which we will call E; like- the sum of the right-hand vertical distances N should "equal" the left- vertical distances S. If there has been no radical error in the field , they should check fairly well; thus, Fig. 27, starting from A, we have,

(+)E	(+) N	(−) W	(−) S
-2977.69	+1187.06	− 458.60	−2387.76
- 694.70	+ 917.47	−2211.39	
	+ 283.61	−1001.88	
-3672.39	+2388.14	−3671.87	−2387.76
-3671.87	−2387.76		
0.52			

and + 0.38 = closing errors. (See Fig. 28.)

o "close" the traverse, construct a closing diagram as per Fig. 29, off the base line measurements $A\,B$, $B\,C$, etc., consecutively and sufficient accuracy, from Fig. 27. At the right-hand end, at A, lay off losing errors, 0.38, 0.52 and the hypothenuse or closing line 0.64,

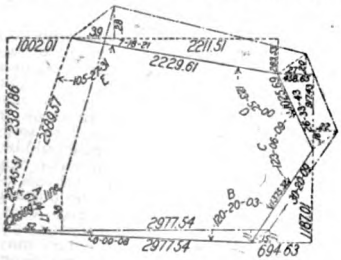

Fig. 28.—Adjusted Traverse.

lly above the base line $A\,A$, and draw rays to A, intersecting verticals from B, C, D and E. The scaled distances from the base line to ntersections will be the closing errors to be used at these points, B, nd E, in Fig. 28, the closing line at each point being parallel to the line shown dotted at A, and the errors at any point being propor- o the total distance on the base line from the initial point A, in the

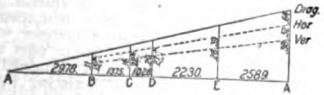

Fig. 29.—Closing Diagram.

n of the lettering. Fig. 28 need not be drawn to scale, the adjusted being shown by full lines and the unadjusted traverse (greatly ated by using an enlarged scale for the closing errors) being shown by d-dot lines. Now construct external triangles around the adjusted lines, calculate their sides (applying difference in errors obtained g. 28 to the calculated sides in Fig. 27) and their angles. From gles, calculate, by addition and subtraction, the new interior an- the adjusted traverse. All calculations should be made in regular lculation books (unruled pages), numbered and indexed. Cross- es are always desirable for field books, calculation books and office

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The Office Plan is made on heavy manila detail paper The adjusted traverse is shown in red ink. Angle points on the instrument lines are indicated by being enclosed in a small triangle; and ordinary hubs, at other points, by a small circle. The property lines are in black (India) ink, as are also the outlines of buildings. Lanes may be indicated by parallel dotted black lines, and streets by full shaded lines. Fences may be shown by fine black lines (unless they mark the property boundary) and should be lettered "*Fence*," "*Stone wall*," etc.; or they may be indicated by broken lines as for instance alternate dashes and dots. All corrected measurements and angles are given on the office plan as they appear in the field book or calculation book, and cross reference is made to each other. The lettering is slanting, and is made by single strokes of the pen, including names of streets, buildings, etc. All information should be recorded, including the acreage of the property. This is usually obtained by calculating the area of the traverse and making the necessary additions and subtractions when the boundary line of the property is respectively exterior or interior to the traverse lines. Field measurements are taken in such a way as to simplify office calculations. Both magnetic and true north are shown. The title is preferably in the lower right-hand corner and should include the name of the property (owner), location, date of survey and by whom, reference to field book, date of plan, and scale. The author has found it convenient to cut out a small 45° triangle from the lower right-hand corner of the plan so that when the plans are lying in the case drawer, the thumb can be inserted and the number of the plan sought can be seen readily. The plan numbers are adjacent to the corners so cut.

The Finished Map which is furnished to the owner of the property is on mounted white paper with rough surface. Paragon paper or its equal is recommended. The transfer is made with the steel or needle point from the office plan. All instrument lines are omitted, but the property lines are shown with distances and angles just as though the instrument lines had actually traced them and they were the real traverse. In other words, the property line traverse should contain sufficient data so it can be plotted to "close," and also be described in deed. All lines should be in India ink. The slant-block lettering, plain or fancy, is easy to make, neat, and clean. The upright Roman lettering for street names is perhaps more desirable than the block lettering, but requires greater care in proportioning and execution. The title should be in taste with the general map, neat and compact. It should always include the scale, date, and surveyor's name. It is a mistake to color a map too highly. The property boundary may be shaded with a diluted carmine, and the streets tinted with burnt sienna. Buildings may be tinted with the proper colors* to represent wood, brick, stone, etc. The north point should be neat and artistic but not coarse. It should be placed in a position to "balance" the map. The border may consist of a heavily leaded line between two finer lines; or one of the latter may be omitted. The corners are generally made as in Fig. 30, but may be curved to various patterns.

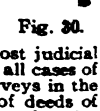

Fig. 30.

City-Lot Surveying.—The functions of the surveyor are almost judicial in character. No statutes are framed or can be framed to meet all cases of conflicting deed lines. These conflicts arise from inaccurate surveys in the past when land was cheap, and also from improper wording of deeds of conveyance. The inaccurate surveys were due in part to the use of chains which were longer or shorter than the present standard. This is the cause of much of the "surplus" and "deficiency" existing in many of our city blocks, amounting in some cases to several inches per hundred feet. Jersey City, N. J., is a notable example of surplus, some of the blocks being 4 ins. per 100 ft. too long. The sensible way is to distribute this surplus proportionately or, in other words, to *use the same length of chain by which the blocks were laid out originally*. But this cannot be held to in all cases because

* Technical or conventional colors may be purchased in liquid form (25 cts. per bottle) in the following colors: 1 cast iron, 2 wrought iron, 3 steel, 4 copper, 5 brass, 6 machinery, 7 leather, 8 light wood, 9 dark wood, 10 brick, 11 stone, 12 brown stone, 13 Prussian blue, 14 gamboge, 15 yellow ochre, 16 vermillion, 17 burnt sienna, 18 carmine. The same may be purchased in water colors (10 cts. per half pan) with the exception that Chinese white is substituted for burnt sienna (17).

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he lines have been established by mutual consent, expressed or Perhaps one or more buildings have been erected and have absorbed ıot quite all the surplus in the block. If on the other hand there ıency of total measurement, we have a more serious problem to Those having the prior deeds are apt to oppose any proportionate ɔn of the "deficiency," throwing it entirely in the last lot con- Existing buildings have much to do with the solution of these . The purpose of the surveyor is to : any benefits or losses as equally as with injustice to no one. It is well to re- hat in making out deeds of original ıce of lots, the description of each lot ɔe referred to the same street line. Fig. 31, the west line of 1st St. may ed as the initial base of all lots in block he width of lot 8 should read "25 ft. ese" to the east line of 2nd St.

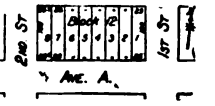

Fig. 31.

t lines in small cities should be fixed by stone monuments set on nes at the street intersections. As the cities grow in size these nts are bound to be disturbed, but they have served their purpose nience for ready use. Offsets to buildings may now be used or the nt points transferred to the manholes* which have supplanted the onuments. The point selected on the building should be such as lily be described, as *above* or *below* the *water table*; the corner of a is the best, as being definite. Where sewers, water works and ır lines have been introduced more rapidly than substantial build- ɾe been erected, it is quite customary to transfer the stone monu- rom the centers to the corners of the streets, say on 2-ft. to 10-ft. nes.

ernment Land Surveying.—The following is a digest of General ffice "Circular on Restoration of Lost and Obliterated Corners and sion of Sections," Revision of June 1, 1909.

"obliterated" corner is one where no visible evidence remains of the the original surveyor in establishing it; but it is not a "lost" corner ɔcation has been preserved beyond all question by acts of land- , and by the memory of those who knew and recollect the true situs ɔriginal monument.

SYNOPSIS OF ACTS OF CONGRESS.

Y 20, 1785. Prescribing mode of survey for the "Western Territory, said y to be divided into "townships of six miles square, by running lines due *N* and others crossing them at right angles," as near as may be. Further pro- hat the first line running *N* and *S* should be on the Ohio river, at a point due ı the western terminus of a line run as the south boundary of the State of and the first line running *E* and *W* should begin at the same point and extend h the whole territory. In these initial surveys only the exterior lines of the ıips were surveyed, but the plats were marked by subdivisions into sections square, numbered from 1 to 36, commencing with No. 1 in the southeast corner township, and running from *S* to *N* in each tier to No. 36 in the northwest of the township; mile corners were established on the township lines. The embraces what is known as the "Seven Ranges" in Ohio.

AY 18, 1796. "Territory northwest of the River Ohio, and above the mouth Kentucky River." Section 2 provided for dividing lands "by *N* and *S* lines ɔcording to the true meridian, and by others crossing them at right angles, so form townships of 6 miles square," etc. Also that "one-half of said townships, them alternately, should be subdivided into sections containing, as nearly as ɔe, 640 acres each, by running through the same each way parallel lines at the f every two miles; and by marking a corner on each of said lines at the end of mile." Also that "the sections shall be numbered, respectively, beginning No. 1 in the northeast section, and proceding west and east alternately through ɔwnship, with progressive numbers till the 36th is completed.†"

ıAY 10, 1800. Amendatory to the foregoing. "Townships west of the Musking- which are directed to be sold in quarter townships, to be subdivided into half ɔns of 320 acres each, as nearly as may be, by running parallel lines through the : from *E* to *W*, and from *S* to *N*, at a distance of one mile from each other, and

* Four marks with a cold chisel on the fixed iron rim and at quadrant ıts will determine the true center.

† This method of numbering sections is still in use.

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marking corners, at the distance of each half mile on the lines running from *E* to *W*, and at the distance of each mile on those running from *S* to *N*. And the interior lines of townships intersected by the Muskingum, and of all townships lying east of that river, which have not heretofore been actually subdivided into sections, shall also be run and marked - - -. And in all cases where the exterior lines of the township thus to be subdivided into sections or half-sections, shall exceed or shall not extend six miles, the excess or deficiency shall be specially noted, and added to or deducted from the western or northern ranges of sections or half-sections in such townships, according as the error may be in running the lines from *E* to *W* or from *S* to *N*."

JUNE 1, 1796. Act "regulating the grants of land appropriated for military services, etc., provided for dividing the "U. S. Military Tract," in Ohio, into townships 5 miles square, each to be subdivided in quarter townships containing 4000 acres.

MARCH 1, 1800. Amendatory of the foregoing act. Section 6 enacted that the Secretary of the Treasury was authorised to subdivide the quarter townships into lots of 100 acres, bounded as nearly as practicable by parallel lines 160 perches in length by 100 perches in width. [1 perch = 1 rod = 16.5 ft. = ¼ chain.] These subdivisions into lots were made upon the plats in the office of the Secretary of the Treasury and did not agree with the actual survey made later, many fractional lots being entirely crowded out. This fact may explain some of the difficulties met with in the district thus subdivided.

FEBRUARY 11, 1805. This act directs the subdivision of land into quarter sections, and provides that all corners marked in the field shall be established as the proper corners of the sections or quarter sections which they were intended to designate, and that corners of half and quarter sections not marked shall be placed as nearly as possible "equidistant from those two corners which stand on the same line." Also that "the boundary line actually run and marked [in the field] shall be established as the proper boundary lines of the sections, or subdivisions, for which they were intended, and the length of such lines as returned by either of the surveyors aforesaid shall be held and considered as the true length thereof and the boundary lines which shall not have been actually run and marked as aforesaid shall be ascertained by running straight lines from the established corners to the opposite corresponding corners, but in those portions of the fractional townships where no such opposite or corresponding corners have been or can be fixed, the said boundary lines shall be ascertained by running from the established corners due *N* and *S*, or *E* and *W* lines, as the case may be, to the water course, Indian boundary line, or other external boundary of such fractional township."

APRIL 24, 1820. This act provides for the sale of public lands in half-quarter sections, and requires that "in every case of the division of a quarter section the line for the division thereof shall run *N* and *S*, - - - and fractional sections, containing 160 acres and upward, shall in like manner, as nearly as practicable, be subdivided into half-quarter sections, under such rules and regulations as may be prescribed by the Secretary of the Treasury; but fractional sections containing less than 160 acres shall not be divided."

MAY 24, 1824. This act provides "that whenever, in the opinion of the President of the U. S., a departure from the ordinary mode of surveying land on any river, lake, bayou or water course would promote the public interest, he may direct the surveyor-general in whose district such land is situated, and where the change is intended to be made, under such rules and regulations as the President may prescribe, to cause the lands thus situated to be surveyed in tracts of two acres in width, fronting on any river, bayou, lake, or water course, and running back the depth of forty acres."

APRIL 5, 1832. This act directed the subdivision of the public lands into quarter-quarter sections; that in every case of the division of a half-quarter section the dividing line should run *E* and *W*, and that fractional sections should be subdivided, under rules and regulations prescribed by the Secretary of the Treasury. Under this provision the Secretary directed that fractional sections containing less than 160 acres, or the residuary portion of a fractional section, after the subdivision into as many quarter-quarter sections as it is susceptible of, may be subdivided into lots, each containing the quantity of a quarter-quarter section as nearly as practicable, by so laying down the line of subdivision that they shall be 20 chains wide, which distances are to be marked on the plat of subdivision, as are also the areas of the quarter-quarters and residuary fractions.

These *two last acts* provided that the corners and contents of half-quarter and quarter-quarter sections should be ascertained as nearly as possible in the manner and on the principles prescribed in the act of February 11, 1805.

GENERAL RULES FROM THE FOREGOING ACTS.

1st. Boundaries established and returned by the duly appointed Government surveyors, when approved by the surveyor general and accepted by the government, are unchangeable.

2nd. Original township, section, and quarter-section corners established by the Government surveyors must stand as the true corners which they were intended to represent, whether the corners be in place or not.

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That quarter-quarter corners not established by the Government surveyors aced on a straight line joining the section and quarter-section corners and etween them, except on the last half-mile of section lines closing on the west boundaries of the township, or on other lines between fractional

That all subdivisional lines of a section running between corners established inal survey of a township must be straight lines, running from the proper ne section line to its opposite corresponding corner in the opposite section

That in a fractional section where no opposite corresponding corner has n be established, any required subdivision line of such section must be run roper original corner in the boundary line due *E* and *W*, or *N* and *S*, as ay be, to the water course, Indian reservation, or other boundary of such th due parallelism to sectional lines.

Extinct corners of the Government surveys must be restored to their cations, whenever it is possible to do so; and hence resort should always l to the marks of the survey in the field. The locus of the missing corner t be identified on the ground by the aid of the mound, pits, line trees, es, etc., described in the original survey, as their indentification affords the s of relocating the missing corner in its original position. Next to this, convincing testimony of citizens as to the locality it originally occupied taken, if such can be obtained. In any case, whether the locus of the ked by the one means of the other, such locus should always be tested and by measurements to *known* corners.

ional cases may be noted as follows: (*a*) When new measurements are single line to determine the position thereon for a restored lost corner er corner on a section line), or when new measurements are made between rners on two lines for the purpose of fixing by their intersection the posi- estored missing corner (as a corner common to 4 sections or townships), ate measurements, using the original field notes, must be used, recognizted errors in the original chaining. (*b*) When the relocated corner cannot harmonize with the field notes in all directions it sometimes becomes the surveyor to place it according to the requirements of one line and against another line. For instance, if the line between sections 30 and 31, reported n length [2 chains short,] would draw the missing corner on range line stward out of range with the other exterior corners, the presumption rong that the range line had been run straight and the length of the secrongly reported, because experience shows that west random lines are s less important than range lines and more liable to error. (*c*) Again, rner on a standard parallel has been obliterated, it is proper to assume placed in line with other corners, and if an anomolous length of line reported ctions 3 and 4 would throw the closing corner into the northern townreyor would properly assume that the older survey of the standard line is the length of the later and minor line. The marks or corners found on closing to a standard parallel fix its *location*, but its *length* should be limited al intersection, at which point the lost closing corner may be placed. ct rule of the law that "all corners marked in the field shall be established ers which they were intended to designate," and the further rule that of lines returned by the surveyors shall be held and considered as the thereof," are found in some cases to be impossible of fulfillment in all t once, and discretion must be applied by the surveyor. (*e*) In a case of an out existing closing corner, which was set some distance out of the true dary of Missouri and Kansas, it was held by this office that a surveyor the fractional section should preserve the boundary as a straight line, not regard said closing corner as a proper corner of the adjacent fractional said corner was considered as fixing the position of the line between two ections, but that its length extended to a new corner to be set on the new ine. (*f*) The principle of the preponderance of one line over another of ance has been recognized in the rule for restoring a section corner common rnships. The new corner should be placed *on* the township line; and nts to check its position by distances to corners within the townships are nfirm it if found to agree well, but should not cause it to be placed off the l not to agree, if the general condition of the boundary supports the prehat it was properly alined.

To Restore Lost or Obliterated Corners.

restore corners on base lines and standard parallels.—By proportionate nts on the line, using original field notes and joining the nearest identified ndard corners on opposite sides of missing corner or corners. (*a*) "Standard eans standard township, section, quarter-section, and meander corners; osing corners as are used in the original survey to determine the position rd parallel, or established during the survey of the same (*b*) A lost or closing corner from which a standard parallel has been initiated or to s been directed will be reestablished in its original place by proportionate

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measurement from the corners used in the original survey to determine its position; measurements from corners on the opposite side of the parallel will not control. (*c*) A missing closing corner originally established during the survey of a standard parallel as a corner from which to project surveys *south* will be restored to its original position by considering it a standard corner and treating it accordingly. (*d*) Therefore, from the preceding, using proportionate measurements, we have: "As the original field-note distance between the selected known corners is to the new measure of said distance, so is the original field-note length of any part of the line to the required new measure thereof. (*e*) As existing original corners must not be disturbed, discrepancies between the new and the original field-note measurements of the line joining the selected original corners will not affect measurements beyond said corners. Proportionate measurements are to be used between them. (*f*) After having checked each new location by measurement to the nearest known corners, new corners will be established permanently and new bearings and measurements taken to prominent objects, and recorded for future reference.

2. *Restoration of township corners common to four townships*.—Two cases: *1st*. Where the position of the original corner has been made to depend upon measurements on two lines at right angles to each other: A line will first be run connecting the nearest identified original corners on the meridional township lines, north and south of the missing corner, and a temporary corner will be placed at the proper proportionate distance, thus determining the corner in a north and south direction only. Next, the nearest original corners on the latitudinal township lines will be connected and a point thereon determined in a similar manner, near the intersection with the meridional line just run. The intersection of these two lines will define the position for establishing the true corner. *2nd*.—Where the original corner has been located by measurements on one line only; for example, as a guide meridian: Restoration of corner is effected by proportionate measurements on said line, as previously explained.

3. *Reestablishment of corners common to two townships*.—The two nearest known corners on the township line (the same not being a base or a correction line) to be corrected as in case No. 1, by a right line, and the missing corner established by proportionate distance, and to be "checked" upon by measurements laterally to nearest known section or quarter-section corners.

4. *Reestablishment of closing corners*.—Measure from the quarter-section, section, or township corner east or west, as the case may be, to the next preceding or succeeding corner in the order of original establishment, and reestablish the missing corner by proportionate measurement.

5. *Reestablishment of interior section corners*.—Same manner as corners common to four townships. When a number of corners are missing on all sides of the one sought to be reestablished, the entire distance must be measured between the nearest existing recognized corners both *N* and *S*, and *E* and *W*, in accordance with the rule laid down, and the new corner reestablished by proportionate measurement.

6. *Reestablishment of quarter-section corners on township boundaries*.—Only one set of quarter-section corners are actually marked in the field on township lines, and they are established when the township exteriors are run. When double section corners are found, the quarter-section corners are considered generally as standing midway between the corners of their respective sections, and when required to be established or reestablished, they should generally be so placed.

7. *Reestablishment of quarter-section corners on closing section lines between fractional sections*.—Must be reestablished according to the original measurement of 40 chains from the last interior section corner, or rather that distance corrected by proportional measurement of original field notes and the new measurement on closing line.

8. *Reestablishment of interior quarter-section corners*.—The missing quarter-corner (in the later surveys) must be established equidistant between the section corners marking the line, according to the field notes of the original survey.

9. *Where double corners were originally established, one of which is standing, to reestablish the other*.—It being remembered that the corners established when the exterior township lines were run, belong to the sections in the townships north and west of those lines, the surveyor must first determine beyond a doubt to which sections the existing corner belongs. This may be done by testing the corners and distances to witness trees or other objects noted in the original field notes of the survey, and by remeasuring distances to known corners. Having determined to which township the existing corner belongs, the missing corner may be reestablished in line north or south of the existing, as the case may be, at the distance stated in the field notes of the original survey, by proportionate measurement, and tested by retracement to the opposite corresponding corner of the section to which the missing corner belongs.

SUBDIVISION OF SECTIONS.

1. *Subdivision of sections into quarter sections*.—Run straight lines from the established quarter-section corners, U. S. surveys, to the opposite corresponding corners. The point of intersection of these lines will be the legal center of the section. (*a*) Upon the lines closing on the north and west boundaries of a township, the

ection corners are established by the U. S. deputy surveyors, but in sub-such sections said quarter-corners should be so placed as to suit the calcu-f the areas of the quarter sections adjoining the township boundaries as l upon the official plat, adopting proportionate measurements where the surements of the north and west boundaries of the section differ from the neasurements.

ubdivision of fractional sections.—Where opposite corresponding corners have or cannot be fixed, the subdivision lines should be ascertained by running established corners due *N*, *S*, *E* or *W*, as the case may be, to the water course, oundary line, or other boundary of such fractional section. (*a*) The law the section lines surveyed and marked in the field by the U. S. deputy sur-be due *N* and *S* or *E* and *W* lines, but in actual experience this is not always Hence, in order to carry out the spirit of the law, it will be necessary in he subdivisional lines through fractional sections to adopt mean courses e section lines are not due lines, or to run the subdivision line parallel to *W*, or *N* boundary of the section, as conditions may require, where there osite sectional line.

ubdivision of quarter sections into quarter-quarters.—Preliminary to the sub-f quarter sections, the quarter-quarter corners will be established at points etween the section and quarter-section corners, and between quarter cor-the center of the section, except on the last half mile of the lines closing on or west boundaries of a township, where they should be placed at 20 chains, nate measurement, to the north or west of the quarter-section corner, quarter-quarter section corners having been established as directed e subdivision lines of the quarter section will be run straight between corresponding quarter-quarter section corners on the quarter-section s. The intersection of the lines thus run will determine the place for the nmon to the four quarter-quarter sections.

ubdivision of fractional quarter sections.—The subdivision lines of fractional ctions will be run from properly established quarter-quarter section corners ue *N*, *S*, *E* or *W*, to the lake, water course, or reservation which renders s fractional, or parallel to the east, south, west, or north boundary of the ction, as conditions may require. (See par. 2 (*a*).)

roportionate measurement.—By "proportionate measurement" is meant a ent having the same ratio to that recorded in the original field notes as *of chain* used in the new measurement has to the *length of chain* used in al survey, assuming that the original and new measurements have been made. For example: the length of the line from the quarter-section corner st side of sec. 2, T. 24 N., R. 14 E., Wisconsin, to the north line of the town-he United States deputy surveyor's chain, was reported as 45.40 chains, e county surveyor's measure is reported as 42.90 chains; then the distance quarter-quarter section corner should be located north of the quarter-rner would be determined as follows As 45.40 chains, the Government f the whole distance, is to 42.90 chains, the county surveyor's measure of distance, so is 20.00 chains, original measurement, to 18.90 chains by the rveyor's measure, showing that by proportionate measurement in this case r-quarter section corner should be set at 18.90 chains north of the quarter-rner, instead of 20.00 chains north of such corner, as represented on the it. In this manner the discrepancies between original and new measure-equitably distributed.

USEFUL TABLES IN PUBLIC LANDS SURVEYS.

(From the Manual of 1902.)

ystem of rectangular surveying, authorized by law May 20, 1785 67), was first employed in the survey of U. S. public lands in of Ohio.

boundary line between the States of Penn. and Ohio, known as 's line," in longitude 80° 32′ 20″ west from Greenwich, is the to which the first surveys are referred. The townships east of to R., in Ohio, are numbered from south to north, commencing 1 on the Ohio River, while the ranges are numbered from east to inning with No. 1 on the east boundary of the State, except in the ignated "U. S. Military Land," in which the townships and ranges bered, respectively, from the south and east boundaries of said tract. 1875, numbered and locally-named principal meridians and base e been established as shown by Table 8, following.

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8.—MERIDIANS AND BASE LINES OF UNITED STATES SURVEYS.

Principal Meridians.	Governing surveys (wholly or in part) in States of—	Longitude W. from Greenwich.			Runs near—	Base line.	Latitude N.			Runs near or with—
		°	′	″			°	′	″	
First	Ohio	84	48	50	State line					Findlay, Ohio.
Second	Indiana and Illinois	86	28	00	Lebanon and Indianapolis.		38	28	20	Salem, Ind.
Third	Illinois	89	10	15	Cairo		38	28	20	Centralia.
Fourth	Wisconsin and Minnesota	90	28	45	Platteville, Wis.		42	30	00	State line, Wis.
	Illinois	90	28	45	Rock Island, Ill.		40	00	30	Beardstown, Ill.
Fifth	Arkansas, Missouri, Iowa, So. Dakota, North Dakota, and Minnesota.	91	03	42	Powhatan, Ark., and Dubuque, Iowa.		34	44	00	Little Rock.
Sixth	Kansas, Nebraska, Wyoming, Colorado, and South Dakota	93	23	00	Wichita, Kans., and Columbus, Neb.		40	00	00	On State line.
Michigan	Michigan	84	22	24	Lansing	Michigan	42	26	30	Detroit.
Tallahassee	Florida	84	16	42	Tallahassee	Tallahassee	30	28	00	Tallahassee.
St. Stephens	Alabama and Mississippi	88	02	00	Mobile	St. Stephens	31	00	00	State line.
Huntsville	Alabama	86	34	45	Huntsville	Huntsville	35	00	00	North State line.
Choctaw	Mississippi	90	14	45	Jackson, Miss.	Choctaw	31	54	40	Hazelhurst.
Chickasaw	do	89	15	00	Holly Springs	Chickasaw	34	59	00	Old Tennessee boundary.
Washington	do	91	09	15	Natchez	St. Stephens	31	00	00	State line.
St. Helena	Louisiana	91	09	15	Baton Rouge	do	31	00	00	do.
Louisiana	do	92	24	15	Alexandria	do	31	00	00	do.
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	34	15	25	Socorro.

9.—AZIMUTHS OF THE SECANT, AND OFFSETS, IN FEET, TO THE PARALLEL.
Arguments: latitude in left-hand column, and distance from starting point at top of table.
(For example of use of table, see Fig. 32.)

	Azimuths and Offsets at—							Deflect'n Angle and nat. tan. to Rad. 66ft
	0 miles.	½ mile.	1 mile.	1½ miles.	2 miles.	2½ miles.	3 miles.	
30	89° 58'.5 1.93 N.	89° 58'.7 0.87 N.	89° 59'.0 0.00	89° 59'.2 0.67 S.	89° 59'.5 1.15 S.	89° 59'.7 1.44 S.	90° (E. or W.) 1.54 S.	3' 00".2 0.69 ins.
31	89° 58'.4 2.01 N.	89° 58'.6 0.91 N.	89° 58'.9 0.00	89° 59'.2 0.70 S.	89° 59'.5 1.20 S.	89° 59'.7 1.50 S.	90° (E. or W.) 1.60 S.	3' 07".4 0.72 ins.
32	89° 58'.4 2.09 N.	89° 58'.6 0.94 N.	89° 58'.9 0.00	89° 59'.2 0.73 S.	89° 59'.5 1.25 S.	89° 59'.7 1.56 S.	90° (E. or W.) 1.67 S.	3' 15".0 0.75 ins.
33	89° 58'.3 2.17 N.	89° 58'.5 0.97 N.	89° 58'.8 0.00	89° 59'.1 0.76 S.	89° 54' 4 1.30 S.	89° 59' 7 1.62 S.	90° (E. or W.) 1.73 S.	3' 22".6 0.78 ins.
34	89° 58'.2 2.25 N.	89° 58'.5 1.01 N.	89° 58'.8 0.00	89° 59'.1 0.79 S.	89° 59'.4 1.35 S.	89° 59'.7 1.69 S.	90° (E. or W.) 1.80 S.	3' 30".4 0.81 ins.
35	89° 58'.2 2.33 N.	89° 58'.5 1.05 N.	89° 58'.8 0.00	89° 59'.1 0.82 S.	89° 59'.4 1.40 S.	89° 59'.7 1.75 S.	90° (E. or W.) 1.87 S.	3' 38".4 0.84 ins.
36	89° 58'.1 2.42 N.	89° 58'.4 1.09 N.	89° 58'.7 0.00	89° 59'.0 0.85 S.	89° 59'.4 1.46 S.	89° 59' 7 1.82 S.	90° (E. or W.) 1.94 S.	3' 46".4 0.87 ins.
37	89° 58'.0 2.51 N.	89° 59'.3 1.13 N.	89° 58'.6 0.00	89° 58'.9 0.88 S.	89° 59' 3 1.51 S.	89° 59' 7 1.89 S.	90° (E. or W.) 2.01 S.	3' 55".0 0.90 ins.
38	89° 58'.0 2.61 N.	89° 58'.3 1.17 N.	89° 58'.6 0.00	89° 58' 9 0.91 S.	89° 59'.3 1.56 S.	89° 59'.7 1.95 S.	90° (E. or W.) 2.08 S.	4 03".6 0.93 ins.
39	89° 57'.9 2.70 N.	89° 58'.2 1.21 N.	89° 58'.6 0.00	89° 58' 9 0.94 S.	89° 59'.3 1.62 S.	89° 59'.7 2.02 S.	90° (E. or W.) 2.16 S.	4' 12".6 0.97 ins.
40	89° 57'.8 2.79 N.	89° 58'.1 1.25 N.	89° 58'.5 0.00	89° 58'.9 0.98 S.	89° 59'.3 1.68 S.	89° 59'.7 2.10 S.	90° (E. or W.) 2.24 S.	4'.21".6 1.00 ins.
41	89° 57'.7 2.89 N.	89° 58'.0 1.30 N.	89° 58'.4 0.00	89° 58' 8 1.02 S.	89° 59'.2 1.74 S.	89° 59'.6 2.17 S.	90° (E. or W.) 2.32 S.	4' 31".2 1.04 ins.
42	89° 57'.7 3.00 N.	89° 58'.0 1.35 N.	89° 58'.4 0.00	89° 58'.8 1.05 S.	89° 59'.2 1.80 S.	89° 59'.6 2.25 S.	90° (E. or W.) 2.40 S.	4' 40".8 1.08 ins
43	89° 57'.6 3.11 N.	89° 58'.0 1.40 N.	89° 58'.4 0.00	89° 58'.8 1.08 S.	89° 59'.2 1.86 S.	89° 59'.6 2.33 S.	90° (E. or W.) 2.48 S.	4' 50".8 1.12 ins.
44	89° 57'.5 3.22 N.	89° 57'.9 1.45 N.	89° 58'.3 0.00	89° 58'.7 1.12 S.	89° 59'.2 1.93 S.	89° 59'.6 2.41 S.	90° (E. or W.) 2.57 S.	5' 01".0 1.16 ins.
45	89° 57'.4 3.33 N.	89° 57'.8 1.50 N.	89° 58'.3 0.00	89° 58'.7 1.16 S.	89° 59'.1 2.00 S.	89° 59'.5 2.49 S.	90° (E. or W.) 2.66 S.	5' 11".8 1.20 ins.
46	89° 57'.3 3.44 N.	89° 57'.7 1.55 N.	89° 58'.2 0.00	89° 58'.6 1.21 S.	89° 59'.1 2.07 S.	89° 59'.5 2.59 S.	90° (E. or W.) 2.76 S.	5'.22".8 1.24 ins.
47	89° 57'.2 3.57 N.	89° 57'.6 1.61 N.	89° 58'.1 0.00	89° 58'.6 1.25 S.	89° 59'.1 2.14 S.	89° 59'.5 2.67 S.	90° (E. or W.) 2.86 S.	5' 34".2 1.28 ins.
48	89° 57'.1 3.70 N.	89° 57'.5 1.66 N.	89° 58'.0 0.00	89° 58'.5 1.30 S.	89° 59'.0 2.22 S.	89° 59'.5 2.78 S.	90° (E. or W.) 2.96 S.	5' 46".2 1.33 ins.
49	89° 57'.0 3.82 N.	89° 57'.5 1.72 N.	89° 58'.0 0.00	89° 58'.5 1.34 S.	89° 59'.0 2.30 S.	89° 59'.5 2.87 S.	90° (E. or W.) 3.06 S.	5' 58".6 1.38 ins.
50	89° 56'.9 3.96 N.	89° 57'.4 1.78 N.	89° 57'.9 0.00	89° 58'.4 1.39 S.	89° 59'.0 2.38 S.	89° 59'.5 2.97 S.	90° (E. or W.) 3.17 S.	6' 11".4 1.43 ins.

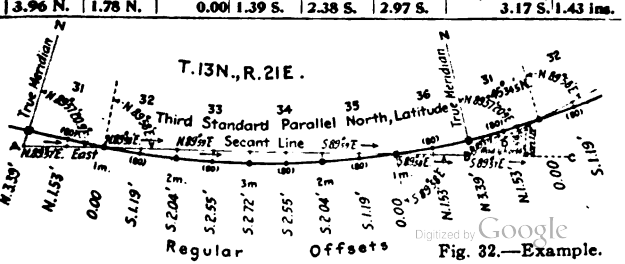

Fig. 32.—Example.

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10.—Azimuths of the Tangent to the Parallel.

The azimuth is the smaller angle the tangent makes with the true meridian and always measured from the north and towards the tangential points.

Latitude.	1 mile.	2 miles.	3 miles.	4 miles.	5 miles.	6 miles.
°	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
30	89 59 30.0	89 58 59.9	89 58 29.9	89 57 59.9	89 57 29.9	89 56 59.8
31	89 59 28.8	89 58 57.5	89 58 26.3	89 57 55.0	89 57 23.8	89 56 52.5
32	89 59 27.5	89 58 55.0	89 58 22.5	89 57 50.0	89 57 17.5	89 56 45.0
33	89 59 26.2	89 58 52.5	89 58 18.7	89 57 44.9	89 57 11.2	89 56 37.4
34	89 59 24.9	89 58 49.9	89 58 14.8	89 57 39.7	89 57 04.6	89 56 29.6
35	89 59 23.6	89 58 47.2	89 58 10.8	89 57 34.4	89 56 58.0	89 56 21.6
36	89 59 22.2	89 58 44.4	89 58 06.8	89 57 28.9	89 56 51.1	89 56 13.4
37	89 59 20.8	89 58 41.6	89 58 02.5	89 57 23.3	89 56 44.1	89 56 05.0
38	89 59 19.4	89 58 38.8	89 57 58.2	89 57 17.5	89 56 36.9	89 55 56.3
39	89 59 17.9	89 58 35.8	89 57 53.7	89 57 11.6	89 56 29.6	89 55 47.5
40	89 59 16.4	89 58 32.8	89 57 49.2	89 57 05.5	89 56 21.9	89 55 38.3
41	89 59 14.8	89 58 29.6	89 57 44.4	89 56 59.3	89 56 14.1	89 55 28.9
42	89 59 13.2	89 58 26.4	89 57 39.6	89 56 52.8	89 56 06.0	89 55 19.2
43	89 59 11.5	89 58 23.1	89 57 34.6	89 56 46.2	89 55 57.7	89 55 09.2
44	89 59 09.8	89 58 19.6	89 57 29.5	89 56 39.3	89 55 49.1	89 54 58.9
45	89 59 08.0	89 58 16.1	89 57 24.1	89 56 32.1	89 55 40.2	89 54 48.2
46	89 59 06.2	89 58 12.4	89 57 18.6	89 56 24.8	89 55 31.0	89 54 37.2
47	89 59 04.3	89 58 08.6	89 57 12.9	89 56 17.1	89 55 21.4	89 54 25.7
48	89 59 02.3	89 58 04.6	89 57 06.9	89 56 09.2	89 55 11.5	89 54 13.8
49	89 59 00.2	89 56 00.5	89 57 00.7	89 56 00.9	89 55 01.2	89 54 01.4
50	89 58 58.1	89 57 56.2	89 56 54.3	89 55 52.6	89 54 50.5	89 53 48.5

Latitude.	7 miles.	8 miles.	9 miles.	10 miles.	11 miles.	12 miles.
°	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
30	89 56 29.8	89 55 59.8	89 55 29.8	89 54 59.7	89 54 29.7	89 53 59.7
31	89 56 21.3	89 55 50.0	89 55 18.8	89 54 47.6	89 54 16.3	89 53 45.1
32	89 56 12.5	89 55 40.0	89 55 07.6	89 54 35.1	89 54 02.6	89 53 30.1
33	89 56 03.6	89 55 29.9	89 54 56.1	89 54 22.3	89 53 48.5	89 53 14.8
34	89 55 54.5	89 55 19.4	89 54 44.4	89 54 09.3	89 53 34.2	89 52 59.1
35	89 55 45.2	89 55 08.8	89 54 32.3	89 53 55.9	89 53 19.5	89 52 43.1
36	89 55 35.6	89 54 57.8	89 54 20.0	89 53 42.3	89 53 04.5	89 52 26.7
37	89 55 25.8	89 54 46.6	89 54 07.4	89 53 28.2	89 52 49.1	89 52 09.9
38	89 55 15.7	89 54 35.1	89 53 54.5	89 53 13.9	89 52 33.2	89 51 52.6
39	89 55 05.4	89 54 23.3	89 53 41.2	89 52 59.1	89 52 17.0	89 51 34.9
40	89 54 54.7	89 54 11.1	89 53 27.5	89 52 43.8	89 52 00.2	89 51 16.6
41	89 54 43.7	89 53 58.5	89 53 13.4	89 52 28.2	89 51 43.0	89 50 57.8
42	89 54 32.4	89 53 45.6	89 52 58.8	89 52 12.0	89 51 25.2	89 50 38.4
43	89 54 20.8	89 53 32.3	89 52 43.8	89 51 55.4	89 51 06.9	89 50 18.5
44	89 54 08.7	89 53 18.5	89 52 28.4	89 51 38.2	89 50 48.0	89 49 57.8
45	89 53 56.3	89 53 04.3	89 52 12.3	89 51 20.4	89 50 28.4	89 49 36.4
46	89 53 43.4	89 52 49.5	89 51 55.7	89 51 01.9	89 50 08.1	89 49 14.2
47	89 53 30.0	89 52 34.3	89 51 38.6	89 50 42.9	89 49 47.2	89 48 51.4
48	89 53 16.1	89 52 18.4	89 51 20.7	89 50 23.0	89 49 25.3	89 48 27.6
49	89 53 01.7	89 52 01.9	89 51 02.1	89 50 02.4	89 49 02.6	89 48 02.8
50	89 52 46.6	89 51 44.7	89 50 42.8	89 49 40.9	89 48 39.0	89 47 37.1

Note.—For example of use of table, see Fig. 33, next page.

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11.—Offsets, in Chains, from Tangent to Parallel.

[Chains.]

Latitude. Deg.	Miles.											
	1	2	3	4	5	6	7	8	9	10	11	12
30	0.006	0.023	0.053	0.09	0.14	0.21	0.29	0.37	0.47	0.58	0.71	0.84
31	0.006	0.024	0.055	0.10	0.15	0.22	0.30	0.39	0.49	0.60	0.74	0.88
32	0.006	0.025	0.057	0.10	0.16	0.23	0.31	0.40	0.51	0.63	0.76	0.91
33	0.007	0.026	0.059	0.10	0.16	0.24	0.32	0.42	0.53	0.65	0.79	0.95
34	0.007	0.027	0.061	0.11	0.17	0.25	0.33	0.43	0.55	0.68	0.82	0.98
35	0.007	0.028	0.064	0.11	0.18	0.25	0.35	0.45	0.57	0.70	0.86	1.02
36	0.007	0.029	0.066	0.12	0.18	0.26	0.36	0.47	0.59	0.73	0 89	1.06
37	0.008	0.031	0.068	0.12	0.19	0.27	0.37	0.48	0.61	0.75	0.91	1.10
38	0.008	0.032	0.071	0.13	0.20	0.28	0.38	0.50	0.64	0.78	0.95	1.14
39	0.008	0.033	0.074	0.13	0.20	0.29	0.40	0.52	0.66	0.81	0.99	1.18
40	0.008	0.034	0.076	0.13	0.21	0.30	0.41	0.54	0.68	0.84	1.02	1 22
41	0.009	0.035	0.079	0.14	0.22	0.32	0.43	0.56	0.70	0.87	1.06	1.26
42	0.009	0.036	0.082	0.14	0.23	0.33	0.44	0.58	0.73	0.90	1.09	1.31
43	0.009	0.038	0.085	0.15	0.24	0.34	0.46	0.60	0.75	0.93	1.14	1.35
44	0.010	0.039	0.088	0.16	0.24	0.35	0.48	0.62	0.79	0.97	1.18	1.40
45	0.010	0.040	0.091	0.16	0.25	0.36	0.49	0.64	0.81	1.00	1.22	1.45
46	0.010	0.042	0.094	0.17	0.26	0.37	0.51	0.66	0.84	1.04	1.26	1.50
47	0.011	0.044	0.097	0.17	0.27	0.39	0.53	0.68	0.87	1.07	1.31	1.56
48	0.011	0.045	0.101	0.18	0.28	0.40	0.55	0.71	0.91	1.12	1.35	1.61
49	0.012	0.046	0.104	0.19	0.29	0.42	0.57	0.74	0.93	1.16	1.40	1.67
50	0.012	0.048	0.108	0.19	0.30	0.43	0.59	0.77	0.97	1.20	1.45	1.73

Note.—For use of above table, see example below (Fig. 33). Note that in the table the offsets are in chains, and *not* in feet.

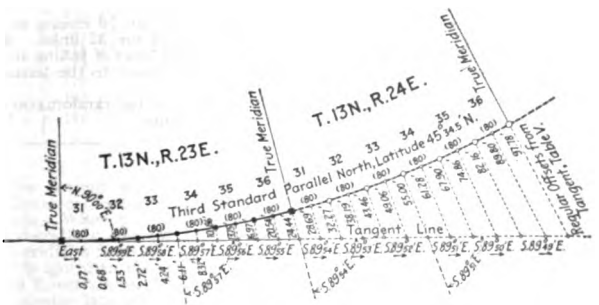

Fig. 33.—Example of use of Tables 10 and 11, for latitude 45 deg. 34.5 m. N.

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12.—Correction of Randoms.

Links and Minutes of Arc, showing departure in running 80.00 chs. at any course from 1 to 60 minutes (or difference in latitude for 90° minus angle).

Angle.	Departure	Angle.	Departure	Angle.	Departure	Angle.	Departure	Angle.	Departure	Angle.	Departure
Min.	Links.	Min.	Links.	Min.	Links.	Min.	Links.	Min.	Links.	Min.	Links.
1	2½	11	25¾	21	49	31	72½	41	95¾	51	119
2	4¾	12	28	22	51½	32	74¾	42	98	52	121½
3	7	13	30½	23	53¾	33	77	43	100½	53	123¾
4	9½	14	32¾	24	56	34	79½	44	102¾	54	126
5	11¾	15	35	25	58½	35	81¾	45	105	55	128½
6	14	16	37½	26	60¾	36	84	46	107½	56	130¾
7	16½	17	39¾	27	63	37	86½	47	109¾	57	133
8	18¾	18	42	28	65½	38	88¾	48	112	58	135½
9	21	19	44½	29	67¾	39	91	49	114½	59	137¾
10	23½	20	46¾	30	70	40	93½	50	116¾	60	140

Remarks.—Table 12, showing the departure or falling at 80 chains distance, for any number of minutes up to 60, can also be used in finding the minutes of correction of a random course corresponding to the number of links of falling. For distance less than 1 mile, the links of falling must be proportionately increased; for example, if the falling at 70 chains is 28 links, the correction of the course will be 14 minutes for 32 links. For township exteriors and other long lines, the number of links of falling must be divided by the number of miles to bring the calculation to the basis of the table.

Table 12 may be used to determine the return from the random course, also, by keeping in mind clearly just what is being done.

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ONVERGENCY OF MERIDIANS SIX MILES LONG AND SIX MILES RT, AND OTHER RELEVANT DATA, TO LATITUDE 70° NORTH.

Convergency.		Difference of longitude per range.		Difference of latitude for—	
On the parallel.	Angle.	In arc.	In time.	1 mile in arc.	1 Tp. in arc.
Links.	′ ″	′ ″	*Seconds.*		
41.9	3 0	6 0.36	24.03		
43.6	3 7	6 4.02	24.27		
45.4	3 15	6 7.93	24.53	0′.871	5′.225
47.2	3 23	6 12.00	24.80		
49.1	3 30	6 16.31	25.09		
50.9	3 38	6 20.95	25.40		
52.7	3 46	6 25.60	25.71		
54.7	3 55	6 30.59	26.04	0′.870	5′.221
56.8	4 4	6 35.81	26.39		
58.8	4 13	6 41.34	26.76		
60.9	4 22	6 47.13	27.14		
63.1	4 31	6 53.22	27.55		
65.4	4 41	6 59.62	27.97	0′.869	5′.217
67.7	4 51	7 6.27	28.42		
70.1	5 1	7 13.44	28.90		
72.6	5 12	7 20.93	29.39		
75.2	5 23	7 28.81	29.92		
77.8	5 34	7 37.10	30.47	0′.869	5′.212
80.6	5 46	7 45.79	31.05		
83.5	5 59	7 55.12	31.67		
86.4	6 12	8 4.83	32.32		
89.6	6 25	8 15.17	33.01		
92.8	6 39	8 26.13	33.74	0′.868	5′.207
96.2	6 54	8 37.75	34.52		
99.8	7 9	8 50.07	35.34		
103.5	7 25	9 3.18	36.22		
107.5	7 42	9 17.12	37.14		
111.6	8 0	9 31.97	38.13	0′.867	5′.202
116.0	8 19	9 47.83	39.19		
120.6	8 38	10 4.78	40.32		
125.5	8 59	10 22.94	41.52		
130.8	9 22	10 42.42	42.83		
136.3	9 46	11 3.38	44.22	0′.866	5′.198
142.2	10 11	11 25.97	45.73		
148.6	10 38	11 50.37	47.36		
155.0	11 8	12 16.82	49.12		
162.8	11 39	12 45.55	51.04		
170.7	12 13	13 16.88	53.12	0′.866	5′.195
179.3	12 51	13 51.15	55.41		
188.7	13 31	14 28.77	57.92		
199.1	14 15	15 10.26	60.68	0′.866	5′.193

REMARKS ON TABLE 13.

second column of Table 13 contains the convergency of two ıs six miles long and six miles apart, measured on a parallel of When the parallel of latitude passing through the south end of ridians, and forming the south boundary of the township of which dians form the meridional boundaries, is coincident with a tabular given in the first column, the required convergency will be ob- irectly from the second column (see Fig. 34); while for other than ular latitudes, it will be obtained by simple proportion (Fig. 35). d column contains the angle of convergency. (abc, Figs. 34 and 35.) the purpose of computing convergency within the boundaries of a township, said boundaries may be regarded as straight lines and the p a plane figure, generally a trapezoid; the convergency of any ular part thereof, bounded by meridional and latitudinal section ill be determined as follows: Multiply the convergency for the p, determined as above directed, by the length of the tract in miles

and decimals of a mile, divided by 6, and the product by the width of the tract divided by 6; the resulting product will be the convergency required. (See Fig. 34.)

To obtain the convergency of the meridional boundaries of any tract bounded by section lines, or other lines of legal subdivision, within a township, proceed as follows: Divide the tract into the least possible number of rectangular parts and compute the convergency for each tract; then, take the sum of the convergencies thus determined. (See example, Fig. 36). The convergency of two meridians of equal length, in the same latitude, is proportional to their distance apart; e. g., the convergency of two meridians 6 miles long, separated by 5 ranges, latitude 38°, is 56.8 lks. × 5 = 2.84 chains.

Convergency of meridians in the same latitudes, and not exceeding 24 miles in length, may be computed by an approximate proportion, which combines the advantages of convenience with an accuracy sufficient for the ordinary wants of the land surveyor; the proportion is this: *The cosines of the latitudes are to each other as the lengths of the intercepted parallels.* The following example illustrates the use of this rule:

The distance between the Principal Meridian and first range line west, in latitude 42° 39′ 07″, is 6 miles; what is the convergency of the two range lines at the Base Line, the meridional distance being 24 miles?

cos 42° 39′07″ : cos 43° :: 480.00 chs : 477.31 chs., which proportion may be worked with natural cosines, or more expeditiously by logarithms, as follows:

a. c. log cos	42° 39′ 07″	0.133427
log cos	43°	9.864127
log	480.00	2.681241
log	477.30	2.678795

The difference..... 2.70 chs. is the convergency required.

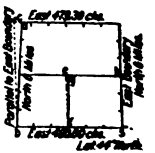

Fig. 34.

ac = Convergency on the Parallel.
abc = Angle of Convergency.

Table 13; opposite Latitude 44°, will be found 70.1 links, the convergency.

North Boundary = 480.00 − 0.70 = 479.30 chs.

For Convergency of the meridians *sh* and *fg*, we have:

70.1 × ⅔ × ⅔ = 17.15 Links, as in the text.

Required the Convergency for a Township in Lat. 38° 29′ N.

From Table 13:

Convergency in Latitude 38°, = 56.8 Links;
" " " 39°, = 58.8 "

Difference = 2.0 "

Fig. 35.

Also; 29′ = 0°.48;
then, 0°.48 × 2.0 = 0.96 links;
ac = 56.8 + 0.96 = 57.76 links, the convergency required.
North boundary = 478.86 − 0.58* = 478.28 chains.

* Taken to nearest whole link.

Tabular Convergency, is 80.6 links.

Convergency for the tract *abcdefgh*:

Conv. of A; 80.6 × ⅔ × ⅔ = 22.39 links;
" " B; 80.6 × ⅔ × ⅓ = 13.42 "
" " C; 80.6 × ⅓ × ⅓ = 8.96 "

Convergency Required = 44.77 "

Also;
Conv. for E. tract is 17.92 "
" " S.W. " " 17.91 "

Total convergency 80.60 links, for Township.

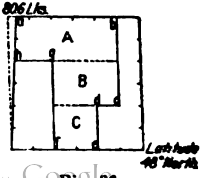

Fig. 36.

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NGTH OF A DEGREE OF LATITUDE COMPUTED TO MINUTES.

30°	31°	32°	33°	34°	35°	36°	37°	38°	Lat.
Chains	*Chains*	*Chains*	*Chains*	*Chains*	*Chains*	*Chains*	*Chains*	*Chains*	′
5509.97	5510.82	5511.67	5512.55	5513.44	5514.34	5515.25	5516.18	5517.11	0
09.99	10.83	11.69	12.56	13.45	14.35	15.27	16.19	17.13	1
10.00	10.84	11.70	12.58	13.47	14.37	15.28	16.21	17.14	2
10.01	10.86	11.72	12.59	13.48	14.38	15.30	16.22	17.16	3
10.03	10.87	11.73	12.61	13.50	14.40	15.31	16.24	17.17	4
10.04	10.89	11.75	12.62	13.51	14.42	15.33	16.25	17.19	5
10.06	10.90	11.76	12.64	13 53	14.43	15.34	16.27	17.20	6
10.07	10.91	11.78	12.65	13.54	14.45	15.36	16.28	17.22	7
10.08	10.93	11.79	12.67	13.56	14.46	15.38	16.30	17.23	8
10.10	10.94	11.81	12.68	13.57	14.48	15.39	16.32	17.25	9
10.11	10.96	11.82	12.70	13.59	14.49	15.41	16.33	17.27	10
10.13	10.97	11.83	12.71	13.60	14.51	15.42	16.35	17.28	11
10.14	10.99	11.85	12.73	13.62	14.52	15.44	16.36	17.30	12
10.15	11.00	11.86	12.74	13.63	14.54	15.45	16.38	17.31	13
10.17	11.01	11.88	12.76	13.65	14.55	15.47	16.39	17.33	14
10.18	11.03	11.89	12.77	13.66	14.57	15.48	16.41	17.34	15
10.19	11.04	11.91	12.79	13.68	14.58	15.50	16.42	17.36	16
10.21	11.06	11.92	12.80	13.69	14.60	15.51	16.44	17.38	17
10.22	11.07	11.94	12.81	13.71	14.61	15.53	16.46	17.39	18
10.24	11.09	11.95	12.83	13.72	14.63	15.54	16.47	17.41	19
10.25	11.10	11.96	12.84	13.74	14.64	15.56	16.49	17.42	20
10.26	11.11	11.98	12.86	13.75	14.66	15.57	16.50	17.44	21
10.28	11.13	11.99	12.87	13.77	14.67	15.59	16.52	17.45	22
10.29	11.14	12.01	12.89	13.78	14.69	15.61	16.53	17.47	23
10.31	11.16	12.02	12.90	13.80	14.70	15.62	16.55	17.49	24
10.32	11.17	12.04	12.92	13.81	14.72	15.64	16.56	17.50	25
10.33	11.19	12.05	12.93	13.83	14.73	15.65	16.58	17.52	26
10.35	11.20	12.07	12.95	13.84	14.75	15.67	16.60	17.53	27
10.36	11.21	12.08	12.96	13.86	14.76	15.68	16.61	17.55	28
10.38	11.23	12.10	12.98	13.87	14.78	15.70	16.63	17.56	29
10.39	11.24	12.11	12.99	13.89	14.79	15.71	16.64	17.58	30
10.41	11.26	12.12	13.01	13.90	14.81	15.73	16.66	17.60	31
10.42	11.27	12.14	13.02	13.92	14.82	15.74	16.67	17.61	32
10.44	11.29	12.15	13.04	13.93	14.84	15.76	16.69	17.63	33
10.45	11.30	12.17	13.05	13.95	14.86	15.77	16.70	17.64	34
10.46	11.31	12.18	13.07	13.96	14.87	15.79	16.72	17.66	35
10.48	11.33	12.20	13.08	13.98	14.89	15.81	16.74	17.67	36
10.49	11.34	12.21	13.10	13.99	14.90	15.82	16.75	17.69	37
10.50	11.36	12.22	13.11	14.01	14.92	15.84	16.77	17.71	38
10.52	11.37	12.24	13.13	14.02	14.93	15.85	16.78	17.72	39
10.53	11.39	12.26	13.14	14.04	14.95	15.87	16.80	17.74	40
10.55	11.40	12.27	13.16	14.05	14.96	15.88	16.81	17.75	41
10.56	11.42	12.29	13.17	14.07	14.98	15.90	16.83	17.77	42
10.57	11.43	12.30	13.18	14.08	14.99	15.91	16.84	17.78	43
10.59	11.44	12.31	13.20	14.10	15.01	15.93	16.86	17.80	44
10.60	11.46	12.33	13.21	14.11	15.02	15.94	16.88	17.82	45
10.62	11.47	12.34	13.23	14.13	15.04	15.96	16.89	18.83	46
10.63	11.49	12.36	13.24	14.14	15.05	15.98	16.91	17.85	47
10.65	11.50	12.37	13.26	14.16	15.07	15.99	16.92	17.86	48
10.66	11.52	12.39	13.27	14.17	15.08	16.01	16.94	17.88	49
10.67	11.53	12.40	13.29	14.19	15.10	16.02	16 95	17.89	50
10.69	11.54	12.42	13.30	14.20	15.11	16.04	16.97	17.91	51
10.70	11.56	12.43	13.32	14.22	15.13	16.05	16.98	17.93	52
10.72	11.57	12.45	13.33	14.23	15.15	16.07	17.00	17.94	53
10.73	11.59	12.46	13.35	14.25	15.16	16.08	17.02	17.96	54
10.74	11.60	12.48	13.36	14.26	15.18	16.10	17.03	17.97	55
10.76	11.62	12.49	13.38	14.28	15.19	16.11	17.05	17.99	56
10.77	11.63	12.51	13.39	14.29	15.21	16.13	17.06	18.00	57
10.79	11.65	12.52	13.41	14.31	15.22	16.15	17.08	18.02	58
10.80	11.66	12.53	13.42	14.32	15.24	16.16	17.09	18.04	59
5510.82	5511.67	5512.55	5513.44	5514.34	5515.25	5516.18	5517.11	5518.05	60

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14.—Length of a Degree of Latitude.—Concluded.

Lat.	39°	40°	41°	42°	43°	44°	45°	46°	47°	48°	Lat.
′	*Chains*	*Chains*	*Chains*	*Chains*	*Chains*	*Chains*	*Chains*	*Chains*	*Chains*	*Chains*	
0	5518.05	5519.00	5519.96	5520.92	5521.88	5522.85	5523.81	5524.78	5525.75	5526.72	0
1	18.07	19.02	19.97	20.93	21.90	22.86	23.83	24.80	25.77	26.73	1
2	18.08	19.03	19.99	20.95	21.91	22.88	23.85	24.82	25.78	26.75	2
3	18.10	19.05	20.00	20.96	21.93	22.89	23.86	24.83	25.80	26.76	3
4	18.11	19.06	20.02	20.98	21.94	22.91	23.88	24.85	25.82	26.78	4
5	18.13	19.08	20.04	21.00	21.96	22.93	23.90	24.86	25.83	26.80	5
6	18.15	19.10	20.05	21.01	21.98	22.94	23.91	24.88	25.85	26.81	6
7	18.16	19.11	20.07	21.03	21.99	22.96	23.93	24.90	25.86	26.83	7
8	18.18	19.13	20.08	21.04	22.01	22.98	23.94	24.91	25.88	26.84	8
9	18.19	19.14	20.10	21.06	22.02	22.99	23.96	24.93	25.90	26.86	9
10	18.21	19.16	20.12	21.08	22.04	23.01	23.98	24.94	25.91	26.88	10
11	18.22	19.18	20.13	21.09	22.06	23.02	23.99	24.96	25.93	26.89	11
12	18.24	19.19	20.15	21.11	22.07	23.04	24.01	24.98	25.94	26.91	12
13	18.26	19.21	20.16	21.12	22.09	23.06	24.02	24.99	25.96	26.92	13
14	18.27	19.22	20.18	21.14	22.11	23.07	24.04	25.01	25.98	26.94	14
15	18.29	19.24	20.20	21.16	22.12	23.09	24.06	25.03	25.99	26.96	15
16	18.30	19.25	20.21	21.17	22.14	23.10	24.07	25.04	26.01	26.97	16
17	18.32	19.27	20.23	21.19	22.15	23.12	24.09	25.06	26.02	26.99	17
18	18.34	19.29	20.24	21.20	22.17	23.14	24.11	25.07	26.04	27.00	18
19	18.35	19.30	20.26	21.22	22.19	23.15	24.12	25.09	26.06	27.02	19
20	18.37	19.32	20.28	21.24	22.20	23.17	24.14	25.11	26.07	27.04	20
21	18.38	19.33	20.29	21.25	22.22	23.19	24.15	25.12	26.09	27.05	21
22	18.40	19.35	20.31	21.27	22.23	23.20	24.17	25.14	26.10	27.07	22
23	18.41	19.37	20.32	21.29	22.25	23.22	24.19	25.15	26.12	27.09	23
24	18.43	19.38	20.34	21.30	22.27	23.23	24.20	25.17	26.14	27.10	24
25	18.45	19.40	20.36	21.32	22.28	23.25	24.22	25.19	26.15	27.12	25
26	18.46	19.41	20.37	21.33	22.30	23.27	24.23	25.20	26.17	27.13	26
27	18.48	19.43	20.39	21.35	22.31	23.28	24.25	25.22	26.19	27.15	27
28	18.49	19.45	20.40	21.36	22.33	23.30	24.27	25.23	26.20	27.17	28
29	18.51	19.46	20.42	21.38	22.35	23.31	24.28	25.25	26.22	27.18	29
30	18.53	19.48	20.44	21.40	22.36	23.33	24.30	25.27	26.23	27.20	30
31	18.54	19.49	20.45	21.41	22.38	23.35	24.32	25.28	26.25	27.21	31
32	18.56	19.51	20.47	21.43	22.40	23.36	24.33	25.30	26.27	27.23	32
33	18.57	19.53	20.48	21.45	22.41	23.38	24.35	25.32	26.28	27.25	33
34	18.59	19.54	20.50	21.46	22.43	23.40	24.36	25.33	26.30	27.26	34
35	18.60	19.56	20.52	21.48	22.44	23.41	24.38	25.35	26.31	27.28	35
36	18.62	19.57	20.53	21.49	22.46	23.43	24.40	25.36	26.33	27.29	36
37	18.64	19.59	20.55	21.51	22.48	23.44	24.41	25.38	26.35	27.31	37
38	18.65	19.60	20.56	21.53	22.49	23.46	24.43	25.40	26.36	27.33	38
39	18.67	19.62	20.58	21.54	22.51	23.48	24.44	25.41	26.38	27.34	39
40	18.68	19.64	20.60	21.56	22.52	23.49	24.46	25.43	26.39	27.36	40
41	18.70	19.65	20.61	21.57	22.54	23.51	24.48	25.44	26.41	27.37	41
42	18.72	19.67	20.63	21.59	22.56	23.52	24.49	25.46	26.43	27.39	42
43	18.73	19.68	20.64	21.61	22.57	23.54	24.51	25.48	26.44	27.41	43
44	18.75	19.70	20.66	21.62	22.59	23.56	24.52	25.49	26.46	27.42	44
45	18.76	19.72	20.68	21.64	22.60	23.57	24.54	25.51	26.47	27.44	45
46	18.78	19.73	20.69	21.65	22.62	23.59	24.56	25.52	26.49	27.45	46
47	18.79	19.75	20.71	21.67	22.64	23.60	24.57	25.54	26.51	27.47	47
48	18.81	19.76	20.72	21.69	22.65	23.62	24.59	25.56	26.52	27.49	48
49	18.83	19.78	20.74	21.70	22.67	23.64	24.61	25.57	26.54	27.50	49
50	18.84	19.80	20.76	21.72	22.69	23.65	24.62	25.59	26.56	27.53	50
51	18.86	19.81	20.77	21.74	22.70	23.67	24.64	25.61	26.57	47.53	51
52	18.87	19.83	20.79	21.75	22.72	25.69	24.65	25.62	26.59	27.56	52
53	18.89	19.84	20.80	21.77	22.73	23.70	24.67	25.64	26.60	27.57	53
54	18.91	19.86	20.82	21.78	22.75	23.72	24.69	25.65	26.62	27.58	54
55	18.92	19.88	20.84	21.80	22.77	23.73	24.70	25.67	26.64	27.60	55
56	18.94	19.89	20.85	21.82	22.78	23.75	24.72	25.69	25.65	27.61	56
57	18.95	19.91	20.87	21.83	22.80	23.77	24.73	25.70	26.67	27.63	57
58	18.97	19.92	20.88	21.85	22.81	23.78	24.75	25.72	26.68	27.65	58
59	18.98	19.94	20.90	21.86	22.83	23.80	24.77	25.73	26.70	27.66	59
60	5519.00	5519.96	5520.92	5521.88	5522.85	5523.81	5524.78	5525.75	5526.72	5527.68	60

LENGTH OF A DEGREE OF LONGITUDE COMPUTED TO MINUTES.

	30°	31°	32°	33°	34°	35°	36°	37°	38°	Lat.
s	Chains	Chains	Chains	Chains	Chains	Chains	Chains	Chains	Chains	′
7	4795.82	4747.01	4696.75	4645.06	4591.96	4537.45	4481.56	4424.29	4365.68	0
0	95.02	46.19	95.90	44.19	91.06	36.53	80.61	23.33	64.69	1
2	94.22	45.36	95.05	43.32	90.16	35.61	79.67	22.36	63.70	2
4	93.42	44.53	94.20	42.44	89.26	34.69	78.73	21.40	62.72	3
6	92.61	43.71	93.35	41.57	88.37	33.77	77.78	20.43	61.73	4
8	91.81	42.88	92.50	40.69	87.47	32.84	76.84	19.46	60.74	5
0	91.01	42.05	91.65	39.82	86.57	31.92	75.89	18.49	59.75	6
2	90.20	41.22	90.80	38.94	85.67	31.00	74.95	17.53	58.76	7
4	89.40	40.39	89.94	38.06	84.77	30.08	74.00	16.56	57.77	8
6	88.59	39.56	89.09	37.19	83.87	29.15	73.05	15.59	56.77	9
8	87.79	38.73	88.24	36.31	82.97	28.23	72.11	14.62	55.78	10
0	86.98	37.90	87.38	35.43	82.07	27.30	71.16	13.65	54.79	11
2	86.18	37.07	86.53	34.55	81.17	26.38	70.21	12.68	53.80	12
4	85.37	36.24	85.67	33.68	80.26	25.46	69.26	11.71	72.81	13
6	84.56	35.41	84.82	32.80	79.36	24.53	68.32	10.74	51.81	14
7	83.76	34.58	83.96	31.92	78.46	23.60	67.37	09.77	50.82	15
9	82.95	33.75	83.11	31.04	77.56	22.68	66.42	08.80	49.83	16
1	82.14	32.92	82.25	30.16	76.65	21.75	65.47	07.82	48.83	17
2	81.33	32.08	81.40	29.28	75.75	20.83	64.52	06.85	47.84	18
4	80.52	31.25	80.54	28.40	74.85	19.90	63.57	05.88	46.84	19
5	79.71	30.42	79.68	27.52	73.94	18.97	62.62	04.91	45.85	20
7	78.90	29.58	78.82	26.64	73.04	18.04	61.67	03.93	44.85	21
8	78.09	28.75	77.97	25.75	72.13	17.11	60.72	02.96	43.85	22
0	77.28	27.92	77.11	24.87	71.23	16.19	59.77	01.98	42.86	23
1	76.47	27.08	76.25	23.99	70.32	15.26	58.81	01.01	41.86	24
2	75.66	26.25	75.39	23.11	69.41	14.33	57.86	4400.04	40.86	25
3	74.85	25.41	74.53	22.22	68.51	13.40	56.91	4399.06	39.87	26
5	74.04	24.57	73.67	21.34	67.60	12.47	55.96	98.08	38.87	27
6	73.22	23.74	72.81	20.45	66.69	11.54	55.00	97.11	37.87	28
7	72.41	22.90	71.95	19.57	65.78	10 61	54.05	96.13	36.87	29
8	71.60	22.06	71.09	18.69	64.88	09.67	53.09	95.16	35.87	30
9	70.78	21.22	70.22	17.80	63.97	08.74	52.14	94.18	34.87	31
0	69.97	20.39	69.36	16.91	63.06	07.81	51.19	93.20	33.87	32
1	69.16	19.55	68.50	16.03	62.15	06.88	50.23	92.22	32.87	33
2	68.34	18.71	67.64	15.14	61.24	05.94	49.27	91.25	31.87	34
3	67.53	17.87	66.77	14.26	60.33	05.01	48.32	90.27	30.87	35
4	66.71	17.03	65.91	13.37	59.42	04.08	47.36	89.29	29.87	36
5	65.89	16.19	65.05	12.48	58.51	03.14	46.41	88.31	28.87	37
5	65.08	15.35	64.18	11.59	57.60	02.21	45.45	87.33	27.87	38
6	64.26	14.51	63.32	10.70	56.68	01.28	44.49	86.35	26.87	39
7	63.44	13.67	62.45	09.81	55.77	4500.34	43.53	85.37	25.86	40
8	62.52	12.82	61.59	08.93	54.86	4499.40	42.57	84.39	24.86	41
8	61.81	11.98	60.72	08.04	53.95	98.47	41.62	83.41	23.86	42
9	60.99	11.14	59.85	07.15	53.03	97.53	40.66	82.42	22.85	43
9	60.17	10.30	58.99	06.26	52.12	96.59	39.70	81.44	21.85	44
0	59.35	09.45	58.12	05.36	51.21	95.66	38.74	80.46	20.85	45
0	58.53	08.61	57.25	04.47	50.29	94.72	37.78	79.48	19.84	46
1	57.71	07.76	56.38	03.58	49.38	93.78	36.82	78.49	18.84	47
1	56.89	06.92	55.51	02.69	48.46	92.84	35.86	77.51	17.83	48
1	56.07	06.07	54.65	01.80	47.55	91.91	34.89	76.53	16.82	49
2	55.25	05.23	53.78	00.90	46.63	90.97	33.93	75.54	15.82	50
2	54.43	04.38	52.91	4600.01	45.71	90.03	32.97	74.56	14.81	51
2	53.60	03.54	52.04	4599.12	44.80	89.09	32.01	73.57	13.80	52
2	52.78	02.69	51.17	98.22	43.88	88.15	31.04	72.59	12.80	53
2	51.96	01.84	50.30	97.33	42.96	87.21	30.08	71.60	11.79	54
2	51.13	01.00	49.42	96.44	42.04	86.27	29.12	70.62	10.78	55
2	50.31	4700.15	48.55	95.54	41.13	85.32	28.15	69.63	09.77	56
2	49.49	4699.30	47.68	94.64	40.21	84.38	27.19	68.64	08.76	57
2	48.66	98.45	46.81	93.75	39.29	83.44	26.22	67.66	07.75	58
2	47.84	97.60	45.94	92.85	38.37	82.50	25.26	66.67	06.74	59
2	4747.01	4696.75	4645.06	4591.96	4537.45	4481.56	4424.29	4365.68	4305.73	60

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15.—Length of a Degree of Longitude.—Concluded.

Lat.	39°	40°	41°	42°	43°	44°	45°	46°	47°	48°	Lat.
′	Chains	Chains	Chains	Chains	Chains	Chains	Chains	Chains	Chains	Chains	′
0	4305.73	4244.47	4181.91	4118.06	4052.96	3986.62	3919.05	3850.28	3780.33	3709.22	0
1	04.72	43.44	80.85	16.99	51.87	85.50	17.91	49.12	79.15	08.03	1
2	03.71	42.41	79.80	15.91	50.77	84.38	16.78	47.97	77.98	06.83	2
3	02.70	41.37	78.75	14.84	49.67	83.27	15.64	46.81	76.80	05.63	3
4	01.69	40.34	77.69	13.76	48.58	82.15	14.50	45.65	75.63	04.44	4
5	4300.68	39.31	76.64	12.69	47.48	81.03	13.36	44.50	74.45	03.24	5
6	4299.67	38.27	75.58	11.61	46.38	79.91	12.23	43.34	73.27	02.05	6
7	98.65	37.24	74.53	10.53	45.28	78.79	11.09	42.18	72.00	3700.85	7
8	97.64	36.20	73.47	09.46	44.19	77.68	09.95	41.02	70.92	3699.65	8
9	96.63	35.17	72.41	08.38	43.09	76.56	08.81	39.86	69.74	98.46	9
10	95.61	34.13	71.36	07.30	41.99	75.44	07.67	38.70	68.56	97.26	10
11	94.60	33.10	70.30	06.22	40.89	74.32	06.53	37.54	67.38	96.06	11
12	93.59	32.06	69.24	05.14	39.79	73.20	05.39	36.38	66.20	94.86	12
13	92.57	31.02	68.18	04.07	38.69	72.08	04.25	35.22	65.02	93.66	13
14	91.56	29.99	67.12	02.99	37.59	70.96	03.11	34.06	63.84	92.46	14
15	90.54	28.95	66.07	01.91	36.49	69.84	01.97	32.90	62.66	91.26	15
16	89.52	27.91	65.01	4100.83	35.39	68.72	3900.83	31.74	61.48	90.60	16
17	88.51	26.87	63.95	4099.75	34.29	67.59	3899.69	30.58	60.30	88.86	17
18	87.49	25.84	62.89	98.67	33.19	66.47	98.54	29.42	59.12	87.66	18
19	86.48	24.80	61.83	97.58	32.09	65.35	97.40	28.26	57.94	86.46	19
20	85.46	23.76	60.77	96.50	30.98	64.23	96.26	27.09	56.76	85.26	20
21	84.44	22.72	59.71	95.42	29.88	63.11	95.12	25.93	55.57	84.06	21
22	83.42	21.68	58.65	94.34	28.78	61.98	93.97	24.77	54.39	82.86	22
23	82.40	20.64	57.58	93.26	27.67	60.86	92.83	23.60	53.21	81.66	23
24	81.39	19.60	56.52	92.17	26.57	59.73	91.68	22.44	52.02	80.46	24
25	80.37	18.56	55.46	91.09	25.47	58.61	90.54	21.28	50.84	79.25	25
26	79.35	17.52	54.40	90.01	24.36	57.49	89.40	20.11	49.66	78.05	26
27	78.33	16.48	53.44	88.92	23.26	56.36	88.25	18.95	48.47	76.85	27
28	77.31	15.43	52.27	87.84	22.15	55.24	87.11	17.78	47.29	75.64	28
29	76.29	14.39	51.21	86.75	21.05	54.11	85.96	16.62	46.10	74.44	29
30	75.27	13.35	50.14	85.67	19.94	52.98	84.81	15.45	44.92	73.24	30
31	74.24	12.31	49.08	84.58	18.84	51.86	83.67	14.29	43.73	72.03	31
32	73.22	11.26	48.02	83.50	17.73	50.73	82.52	13.12	42.55	70.83	32
33	72.20	10.22	46.95	82.41	16.62	49.60	81.37	11.95	41.30	69.62	33
34	71.18	09.18	45.89	81.33	15.52	48.48	80.23	10.79	40.18	68.42	34
35	70.16	08.13	44.82	80.24	14.41	47.35	79.08	09.62	38.99	67.21	35
36	69.13	07.09	43.75	79.15	13.30	46.22	77.93	08.45	37.80	66.01	36
37	68.11	06.04	42.69	78.07	12.19	45.09	76.78	07.28	36.62	64.80	37
38	67.09	05.00	41.62	76.98	11.09	43.96	75.63	06.11	35.43	63.59	38
39	66.06	03.95	40.55	75.89	09.98	42.83	74.48	04.95	34.24	62.39	39
40	65.04	02.90	39.49	74.80	08.87	41.71	73.34	03.78	33.05	61.18	40
41	64.01	01.86	38.42	73.71	07.76	40.58	72.19	02.61	31.86	59.97	41
42	62.99	4200.81	37.35	72.62	06.65	39.45	71.04	01.44	30.67	58.76	42
43	61.96	4199.76	36.28	71.53	05.54	38.32	69.89	3900.27	29.48	57.56	43
44	60.93	98.72	35.21	70.44	04.43	37.18	68.74	3799.10	28.30	56.35	44
45	59.91	97.67	34.14	69.35	03.32	36.05	67.58	97.93	27.11	55.14	45
46	58.88	96.62	33.08	68.26	02.21	34.92	66.43	96.76	25.92	53.93	46
47	57.85	95.57	32.01	67.17	4001.10	33.79	65.28	95.59	24.73	52.72	47
48	56.83	94.52	30.93	66.08	3999.98	32.66	64.13	94.41	23.53	51.51	48
49	55.80	93.47	29.86	64.99	98.87	31.53	62.98	93.24	22.34	50.30	49
50	54.77	92.42	28.79	63.90	97.76	30.39	61.82	92.07	21.15	49.09	50
51	53.74	91.37	27.72	62.81	96.65	29.26	60.67	90.90	19.96	47.88	51
52	52.71	90.32	26.65	61.71	95.53	28.13	59.52	89.72	18.77	46.67	52
53	51.68	89.27	25.58	60.62	94.42	26.99	58.36	88.55	17.58	45.46	53
54	50.66	88.22	24.51	59.53	93.31	25.86	57.21	87.38	16.38	44.25	54
55	49.63	87.17	23.43	58.43	92.19	24.73	56.06	86.20	15.19	43.03	55
56	48.59	86.12	22.36	57.34	91.08	23.59	54.90	85.03	14.00	41.82	56
57	47.56	85.07	21.29	56.25	89.96	22.46	53.75	83.86	12.80	40.61	57
58	46.53	84.02	20.21	55.15	88.85	21.32	52.59	82.68	11.61	39.40	58
59	45.50	82.96	19.14	54.06	87.73	20.19	51.44	81.51	10.41	38.18	59
60	4344.47	4181.91	4118.06	4052.96	3986.62	3919.05	3850.28	3780.33	3709.22	3636.97	60

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Stadia.—Transits may be ordered with stadia, usually without pense. The stadia is a ring or diaphragm inserted between the eye- d the center of the telescope, and across which are stretched two al wires, called "stadia wires," so spaced that they will subtend a on a leveling rod equal to 0.01* the distance of the rod from a point P (where the visual rays cross), a few inches (=the focal) in front of the objective. The distance from the center of the ent (intersection of plumb-line with telescope) to the point P for rument is called the "constant" for that instrument.† This con- its coordinate values if the line of sight is not horizontal, must be the values reduced from stadia readings.

Stadia Reduction Table, following, was calculated by Mr. Arthur for use in the Pennsylvania Geological Survey, and may be used g the horizontal distance and the elevation of an object located from sit by stadia.

Example of Use of Table.

me "constant" for instrument = 1.00;
ia reading on rod = 4.26 ft. (rod always vertical);
instrument is set up over a hub whose elevation is known, and to a point on rod whose reading, from the middle cross-wire, is height of instrument above the hub; and in this position the stadia and the vertical angle are taken.)
ical angle = 12° − 19′. Then from Table, by interpolation:
al dist. of object from inst. = 95.45 × 4.26 + 0.98 = 407.60 ft.
dist. of object from hub = 20.84 × 4.26 + 0.22 = 89.00 ft.

Form of Field Notes. (Left-hand Page.)
(Right-hand page is for notes and sketches.)

.) me	Hor. Ang.‡ nt at Sta 84 318° − 30′	Vert. Ang. (Elev. = 287. 12° − 19′	Cor. Dist. 5). Zero (ho 407.6	Diff. in Elev. r. ang.) on 89.0	Elev. 83. 376.5

y ratio may be used when ordering; but 0.01 is usual and con- The diaphragm may be ordered with the stadia wires either "fixed" stable."
ch instrument has its particular "constant," whose value is fur- y the maker, and may vary from 0.75 to 1.25 ft. The constant is the distance from center of instrument, or axis, to the objective, instrument is focussed on a distant object, *plus* the focal distance f, ce from the plane of the cross-wires to the objective. Hence, the for any instrument may readily be found by focussing to some bject and measuring from the objective to the cross-wires and to r of the axis, and adding these measurements together. This sum distance from center of axis to the point P, noted above.
asured right-hand from back-sight [Sta. 83] up to 360°.

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16.—STADIA REDUCTION TABLE.

For Reduction of Stadia Readings to Horizontal Distances and Elevations.

Minutes	0°		1°		2°		3°		4°		5°		6°		7°		8°		9°		10°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	100.00	0.00	99.97	1.74	99.88	3.49	99.73	5.23	99.51	6.96	99.24	8.68	98.91	10.40	98.51	12.10	98.06	13.78	97.55	15.45	96.98	17.10
2	100.00	0.06	99.97	1.80	99.87	3.55	99.72	5.28	99.51	7.02	99.23	8.74	98.90	10.45	98.50	12.15	98.05	13.84	97.53	15.51	96.96	17.16
4	100.00	0.12	99.97	1.86	99.87	3.60	99.71	5.34	99.50	7.07	99.22	8.80	98.88	10.51	98.48	12.21	98.03	13.89	97.52	15.56	96.94	17.21
6	100.00	0.17	99.96	1.92	99.87	3.66	99.71	5.40	99.49	7.13	99 21	8.85	98.87	10.57	98.47	12.26	98.01	13.95	97.50	15.62	96.92	17.26
8	100.00	0.23	99.96	1.98	99.86	3.72	99.70	5.46	99.48	7.19	99.20	8.91	98.86	10.62	98.46	12.32	98.00	14.01	97.48	15.67	96.90	17.32
10	100.00	0.29	99.96	2.04	99.86	3.78	99.69	5.52	99.47	7.25	99.19	8.97	98.85	10.68	98.44	12.38	97.98	14.06	97.46	15.73	96.88	17.37
12	100.00	0.35	99.96	2.09	99.85	3.84	99.69	5.57	99.46	7.30	99.18	9.03	98.83	10.74	98.43	12.43	97.97	14.12	97.44	15.78	96.86	17.43
14	100.00	0.41	99.95	2.15	99.85	3.90	99.68	5.63	99.46	7.36	99.17	9.08	98.82	10.79	98.41	12.49	97.95	14.17	97.43	15.84	96.84	17.48
16	100.00	0.47	99.95	2.21	99.84	3.95	99.68	5.69	99.45	7.42	99.16	9.14	98.81	10.85	98.40	12.55	97.93	14.23	97.41	15.89	96.82	17.54
18	100.00	0.52	99.95	2.27	99.84	4.01	99.67	5.75	99.44	7.48	99.15	9.20	98.80	10.91	98.39	12.60	97.92	14.28	97.39	15.95	96.80	17.59
20	100.00	0.58	99.95	2.33	99.83	4.07	99.66	5.80	99.43	7.53	99.14	9.25	98.78	10.96	98.37	12.66	97.90	14.34	97.37	16.00	96.78	17.65
22	100.00	0.64	99.94	2.38	99.83	4.13	99.66	5.86	99.42	7.59	99.13	9.31	98.77	11.02	98.36	12.72	97.88	14.40	97.35	16.06	96.76	17.70
24	100.00	0.70	99.94	2.44	99.82	4.18	99.65	5.92	99.41	7.65	99.11	9.37	98.76	11.08	98.34	12.77	97.87	14.45	97.33	16.11	96.74	17.76
26	99.99	0.76	99.94	2.50	99.82	4.24	99.64	5.98	99.40	7.71	99.10	9.43	98.74	11.13	98.33	12.83	97.85	14.51	97.31	16.17	96.72	17.81
28	99.99	0.81	99.93	2.56	99.81	4.30	99.63	6.04	99.39	7.76	99.09	9.48	98.73	11.19	98.31	12.88	97.83	14.56	97.29	16.22	96.70	17.86
30	99.99	0.87	99.93	2.62	99.81	4.36	99.63	6.09	99.38	7.82	99.08	9.54	98.72	11.25	98.29	12.94	97.82	14.62	97.28	16.28	96.68	17.92
32	99.99	0.93	99.93	2.67	99.80	4.42	99.62	6.15	99.38	7.88	99.07	9.60	98.71	11.30	98.28	13.00	97.80	14.67	97.26	16.33	96.66	17.97
34	99.99	0.99	99.92	2.73	99.80	4.48	99.62	6.21	99.37	7.94	99.06	9.65	98.69	11.36	98.27	13.05	97.78	14.73	97.24	16.39	96.64	18.03
36	99.99	1.05	99.92	2.79	99.79	4.53	99.61	6.27	99.36	7.99	99.05	9.71	98.68	11.42	98.25	13.11	97.76	14.79	97.22	16.44	96.62	18.08
38	99.99	1.11	99.92	2.85	99.79	4.59	99.60	6.33	99.35	8.05	99.04	9.77	98.67	11.47	98.24	13.17	97.75	14.84	97.20	16.50	96.60	18.14
40	99.99	1.16	99.92	2.91	99.78	4.65	99.59	6.38	99.34	8.11	99.03	9.83	98.65	11.53	98.22	13.22	97.73	14.90	97.18	16.55	96.57	18.19
42	99.99	1.22	99.91	2.97	99.78	4.71	99.59	6.44	99.33	8.17	99.01	9.86	98.64	11.59	98.20	13.28	97.71	14.95	97.16	16.61	96.55	18.24
44	99.99	1.28	99.91	3.02	99.77	4.76	99.58	6.50	99.32	8.23	99.00	9.92	98.62	11.64	98.19	13.33	97.68	15.01	97.14	16.66	96.53	18.30

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0	96.36	18.73	95.68	20.34	94.94	21.92	94.15	23.47	93.30	25.00	92.40	26.50	91.45	27.96	90.45	29.39	89.40	30.78	88.30	32.14				
2	96.34	18.78	95.65	20.39	94.91	21.97	94.12	23.52	93.27	25.05	92.37	26.55	91.42	28.01	90.42	29.44	89.36	30.83	88.26	32.18				
4	96.32	18.84	95.63	20.44	94.89	22.02	94.09	23.58	93.24	25.10	92.34	26.59	91.39	28.06	90.38	29.48	89.33	30.87	88.23	32.23				
6	96.29	18.89	95.61	20.50	94.86	22.08	94.07	23.63	93.21	25.15	92.31	26.64	91.35	28.10	90.35	29.53	89.29	30.92	88.19	32.27				
8	96.27	18.95	95.58	20.55	94.84	22.13	94.04	23.68	93.18	25.20	92.28	26.69	91.32	28.15	90.31	29.58	89.26	30.97	88.15	32.32				
10	96.25	19.00	95.56	20.60	94.81	22.18	94.01	23.73	93.16	25.25	92.25	26.74	91.29	28.20	90.28	29.62	89.22	31.01	88.11	32.36				
12	96.23	19.05	95.53	20.66	94.79	22.23	93.98	23.78	93.13	25.30	92.22	26.79	91.26	28.25	90.24	29.67	89.18	31.06	88.08	32.41				
14	96.21	19.11	95.51	20.71	94.76	22.28	93.95	23.83	93.10	25.35	92.19	26.84	91.22	28.30	90.21	29.72	89.15	31.10	88.04	32.45				
16	96.18	19.16	95.49	20.76	94.73	22.34	93.93	23.88	93.07	25.40	92.15	26.89	91.19	28.34	90.18	29.76	89.11	31.15	88.00	32.49				
18	96.16	19.21	95.46	20.81	94.71	22.39	93.90	23.93	93.04	25.45	92.12	26.94	91.16	28.39	90.14	29.81	89.08	31.19	87.96	32.54				
20	96.14	19.27	95.44	20.87	94.68	22.44	93.87	23.99	93.01	25.50	92.09	26.99	91.12	28.44	90.11	29.86	89.04	31.24	87.93	32.58				
22	96.12	19.32	95.41	20.92	94.66	22.49	93.84	24.04	92.98	25.55	92.06	27.04	91.09	28.49	90.07	29.90	89.00	31.28	87.89	32.63				
24	96.09	19.38	95.39	20.97	94.63	22.54	93.81	24.09	92.95	25.60	92.03	27.09	91.06	28.54	90.04	29.95	88.96	31.33	87.85	32.67				
26	96.07	19.43	95.36	21.03	94.60	22.60	93.79	24.14	92.92	25.65	92.00	27.13	91.02	28.58	90.00	30.00	88.93	31.38	87.81	32.72				
28	96.05	19.48	95.34	21.08	94.58	22.65	93.76	24.19	92.89	25.70	91.97	27.18	90.99	28.63	89.97	30.04	88.89	31.42	87.77	32.76				
30	96.03	19.54	95.32	21.13	94.55	22.70	93.73	24.24	92.86	25.75	91.93	27.23	90.96	28.68	89.93	30.09	88.86	31.47	87.74	32.80				
32	96.00	19.59	95.29	21.18	94.52	22.75	93.70	24.29	92.83	25.80	91.90	27.28	90.92	28.73	89.90	30.14	88.82	31.51	87.70	32.85				
34	95.98	19.64	95.27	21.24	94.50	22.80	93.67	24.34	92.80	25.85	91.87	27.33	90.89	28.77	89.86	30.19	88.78	31.56	87.66	32.89				
36	95.96	19.70	95.24	21.29	94.47	22.85	93.65	24.39	92.77	25.90	91.84	27.38	90.86	28.82	89.83	30.23	88.75	31.60	87.62	32.93				
38	95.93	19.75	95.22	21.34	94.44	22.91	93.62	24.44	92.74	25.95	91.81	27.43	90.82	28.87	89.79	30.28	88.71	31.65	87.58	32.98				
40	95.91	19.80	95.19	21.39	94.42	22.96	93.59	24.49	92.71	26.00	91.77	27.48	90.79	28.92	89.76	30.32	88.67	31.69	87.54	33.02				
42	95.89	19.86	95.17	21.45	94.39	23.01	93.56	24.55	92.68	25.05	91.74	27.52	90.76	28.96	89.72	30.37	88.64	31.74	87.51	33.07				
44	95.86	19.91	95.14	21.50	94.36	23.06	93.53	24.60	92.65	26.10	91.71	27.57	90.72	29.01	89.69	30.41	88.60	31.78	87.47	33.11				
46	95.84	19.96	95.12	21.55	94.34	23.11	93.50	24.65	92.62	26.15	91.68	27.62	90.69	29.06	89.65	30.46	88.56	31.83	87.43	33.15				
48	95.82	20.02	95.09	21.60	94.31	23.16	93.47	24.70	92.59	26.20	91.65	27.67	90.66	29.11	89.61	30.51	88.53	31.87	87.39	33.20				
50	95.79	20.07	95.07	21.66	94.28	23.22	93.45	24.75	92.56	26.25	91.61	27.72	90.62	29.15	89.58	30.55	88.49	31.92	87.35	33.24				
52	95.77	20.12	95.04	21.71	94.26	23.27	93.42	24.80	92.53	26.30	91.58	27.77	90.59	29.20	89.54	30.60	88.45	31.96	87.31	33.28				
54	95.75	20.18	95.02	21.76	94.23	23.32	93.39	24.85	92.49	26.35	91.55	27.81	90.55	29.25	89.51	30.65	88.41	32.01	87.27	33.33				
56	95.72	20.23	94.99	21.81	94.20	23.37	93.36	24.90	92.46	26.40	91.52	27.86	90.52	29.30	89.47	30.69	88.38	32.05	87.24	33.37				
58	95.70	20.28	94.97	21.87	94.17	23.42	93.33	24.95	92.43	26.45	91.48	27.91	90.48	29.34	89.44	30.74	88.34	32.09	87.20	33.41				
60	95.68	20.34	94.94	21.92	94.15	23.47	93.30	25.00	92.40	26.50	91.45	27.96	90.45	29.39	89.40	30.78	88.30	32.14	87.16	33.46				
C = 0.75	0.73	0.15	0.73	0.16	0.73	0.17	0.73	0.19	0.72	0.20	0.72	0.21	0.72	0.23	0.71	0.24	0.71	0.25	0.70	0.26				
= 1.00	0.98	0.20	0.98	0.22	0.97	0.23	0.97	0.25	0.96	0.27	0.96	0.28	0.95	0.30	0.95	0.32	0.94	0.33	0.94	0.35				
1.25	1.22	0.25	1.22	0.27	1.21	0.29	1.21	0.31	1.20	0.34	1.20	0.36	1.19	0.38	1.19	0.40	1.18	0.42	1.17	0.44				

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16.—Stadia Reduction Table.—Concluded.

Minutes	21°		22°		23°		24°		25°		26°		27°		28°		29°		30°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	87.16	33.46	85.97	34.73	84.73	35.97	83.46	37.16	82.14	38.30	80.78	39.40	79.39	40.45	77.96	41.45	76.50	42.40	75.00	43.30
2	87.12	33.50	85.93	34.77	84.69	36.01	83.41	37.20	82.09	38.34	80.74	39.44	79.34	40.49	77.91	41.48	76.45	42.43	74.95	43.33
4	87.08	33.54	85.89	34.82	84.65	36.05	83.37	37.23	82.05	38.38	80.69	39.47	79.30	40.52	77.86	41.52	76.40	42.46	74.90	43.36
6	87.04	33.59	85.85	34.86	84.61	36.09	83.33	37.27	82.01	38.41	80.65	39.51	79.25	40.55	77.81	41.55	76.35	42.49	74.85	43.39
8	87.00	33.63	85.80	34.90	84.57	36.13	83.28	37.31	81.96	38.45	80.60	39.54	79.20	40.59	77.77	41.58	76.30	42.53	74.80	43.42
10	86.96	33.67	85.76	34.94	84.52	36.17	83.24	37.35	81.92	38.49	80.55	39.58	79.15	40.62	77.72	41.61	76.25	42.56	74.75	43.45
12	86.92	33.72	85.72	34.98	84.48	36.21	83.20	37.39	81.87	38.53	80.51	39.61	79.11	40.66	77.67	41.65	76.20	42.59	74.70	43.47
14	86.88	33.76	85.68	35.02	84.44	36.25	83.15	37.43	81.83	38.56	80.46	39.65	79.06	40.69	77.62	41.68	76.15	42.62	74.65	43.50
16	86.84	33.80	85.64	35.07	84.40	36.29	83.11	37.47	81.78	38.60	80.41	39.69	79.01	40.72	77.57	41.71	76.10	42.65	74.60	43.53
18	86.80	33.84	85.60	35.11	84.35	36.33	83.07	37.51	81.74	38.64	80.37	39.72	78.96	40.76	77.52	41.74	76.05	42.68	74.55	43.56
20	86.77	33.89	85.56	35.15	84.31	36.37	83.02	37.54	81.69	38.67	80.32	39.76	78.92	40.79	77.48	41.77	76.00	42.71	74.49	43.59
22	86.73	33.93	85.52	35.19	84.27	36.41	82.98	37.58	81.65	38.71	80.28	39.79	78.87	40.82	77.42	41.81	75.95	42.74	74.44	43.62
24	86.69	33.97	85.48	35.23	84.23	36.45	82.93	37.62	81.60	38.75	80.23	39.83	78.82	40.86	77.38	41.84	75.90	42.77	74.39	43.65
26	86.65	34.01	85.44	35.27	84.18	36.49	82.89	37.66	81.56	38.78	80.18	39.86	78.77	40.89	77.33	41.87	75.85	42.80	74.34	43.67
28	86.61	34.06	85.40	35.31	84.14	36.53	82.85	37.70	81.51	38.82	80.14	39.90	78.73	40.92	77.28	41.90	75.80	42.83	74.29	43.70
30	86.57	34.10	85.36	35.36	84.10	36.57	82.80	37.74	81.47	38.86	80.09	39.93	78.68	40.96	77.23	41.93	75.75	42.86	74.24	43.73
32	86.53	34.14	85.31	35.40	84.06	36.61	82.76	37.77	81.42	38.89	80.04	39.97	78.63	40.99	77.18	41.97	75.70	42.89	74.19	43.76
34	86.49	34.18	85.27	35.44	84.01	36.65	82.72	37.81	81.38	38.93	80.00	40.00	78.58	41.02	77.13	42.00	75.65	42.92	74.14	43.79
36	86.45	34.23	85.23	35.48	83.97	36.69	82.67	37.85	81.33	38.97	79.95	40.04	78.54	41.06	77.09	42.03	75.60	42.95	74.09	43.82
38	86.41	34.27	85.19	35.52	83.93	36.73	82.63	37.89	81.28	39.00	79.90	40.07	78.49	41.09	77.04	42.06	75.55	42.98	74.04	43.84
40	86.37	34.31	85.15	35.56	83.89	36.77	82.58	37.93	81.24	39.04	79.86	40.11	78.44	41.12	76.99	42.09	75.50	43.01	73.99	43.87
42	86.33	34.35	85.11	35.60	83.84	36.80	82.54	37.96	81.19	39.08	79.81	40.14	78.39	41.16	76.94	42.12	75.45	43.04	73.93	43.90
44	86.29	34.40	85.07	35.64	83.80	36.84	82.49	38.00	81.15	39.11	79.76	40.18	78.34	41.19	76.89	42.15	75.40	43.07	73.88	43.93
46	86.25	34.44	85.03	35.68	83.76	36.88	82.45	38.04	81.10	39.15	79.72	40.21	78.30	41.22	76.84	42.19	75.35	43.10	73.83	43.95
48	86.21	34.48	84.98	35.72	83.72	36.92	82.41	38.06	81.06	39.18	79.67	40.24	78.25	41.26	76.79	42.22	75.30	43.13	73.78	43.98
50	86.17	34.52	84.94	35.76	83.67	36.96	82.36	38.11	81.01	39.22	79.62	40.28	78.20	41.29	76.74	42.25	75.25	43.16	73.73	44.01
52	86.12	34.57	84.90	35.80	83.63	37.00	82.32	38.15	80.97	39.26	79.58	40.31	78.15	41.32	76.69	42.28	75.20	43.18	73.68	44.04
54	86.08	34.61	84.86	35.85	83.59	37.04	82.27	38.19	80.92	39.29	79.53	40.35	78.10	41.35	76.64	42.31	75.15	43.21	73.63	44.07
56	86.05	34.65	84.82	35.89	83.54	37.08	82.23	38.22	80.87	39.33	79.48	40.38	78.06	41.39	76.59	42.34	75.10	43.24	73.58	44.09
58	86.01	34.69	84.77	35.93	83.50	37.12	82.18	38.26	80.83	39.36	79.44	40.42	78.01	41.42	76.55	42.37	75.05	43.27	73.53	44.12
60	85.97	34.73	84.73	35.97	83.46	37.16	82.14	38.30	80.78	39.40	79.39	40.45	77.96	41.45	76.50	42.40	75.00	43.30	73.47	44.15
0.75	0.70	0.27	0.69	0.29	0.69	0.30	0.68	0.31	0.68	0.32	0.67	0.33	0.66	0.35	0.66	0.36	0.65	0.37	0.65	0.38
1.50	0.99	0.37	0.98	0.38	0.93	0.40	0.91	0.41	0.90	0.43	0.89	0.45	0.89	0.46	0.88	0.48	0.87	0.49	0.86	0.51
2.25	1.21	0.46	1.20	0.48	1.18	0.50	1.14	0.52	1.13	0.54	1.12	0.56	1.11	0.58	1.10	0.60	1.09	0.62	1.08	0.64

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Leveling Correction for Earth's Curvature and Refraction.—A *level* "surface" around the earth is a spheroid. Now this spheroid is of the exact size and shape of the "mean" earth when every point of its surface is at zero elevation, corresponding generally with mean sea level. This is the datum spheroidal "plane" used in leveling. Any "plane" *above* said datum, i. e., at a higher elevation, if extended around the globe, will form a spheroidal level surface whose *perpendicular distance* from the datum plane will be constant at points of equal latitude; but will decrease gradually from the equator toward the poles. Hence, a meridian line of levels if run at a considerable height above the datum plane would be subject to correction; but the error is generally so small compared with other errors that it is disregarded in practical leveling.

Ordinary sources of error in leveling are eliminated by taking equal back-sights and fore-sights and by using other precautions. Those due to curvature of the earth and refraction of light rays passing through the atmosphere must be *corrected*, where a single sight is taken on a distant object from a fixed position of the level. Let t (Fig. 37) be the point of sight of the telescope and let it be required to find the elevation of the point p upon which rests a leveling rod hp. Let the point l on the rod be at the same elevation as t; then lp, the desired height, is the height of instrument above p, and the radius of the curve tl is the radius of the earth. If no atmosphere were present to cause refraction, the line of sight would be the horizontal line th, but on account of refraction the real line of sight takes the curve tr whose radius is about 7 times the radius of the earth; hence $hl = 7$ times hr, or $rl = {}^6/_7 hl$ (nearly). Now as tr and tl are short curves of very large radii we may consider them as parabolas and intersecting the leveling rod at angles of 90°. If D represents the horizontal distance from instrument to rod, we have, from the nature of the problem, that hr, rl and hl are each proportional to D^2. Moreover, while the correction for curvature *adds* to the apparent elevation of the distant object, the correction for refraction *subtracts* from this by about $\frac{1}{7}$ part. The combined (difference) correction for curvature and refraction is *additive*, as per the following table.

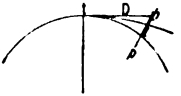

Fig. 37.

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17.—Correction for Earth's Curvature, and Refraction.

(Add "Curvature and Refraction" to apparent Elevation of object.)

Distance. Feet.	Curvature and Refraction.	Distance. Miles.	Correction in Feet for— Curvature.	Refraction.	Curvature and Refraction.	Distance. Miles.	Correction in Feet for — Curvature.	Refraction.	Curvature and Refraction.
300	.002	1	0.7	0.1	0.6	34	771.0	108.0	663.3
400	.003	2	2.7	0.4	2.3	35	817.4	114.4	703.0
500	.005	3	6.0	0.8	5.2				
600	.007	4	10.7	1.5	9.2	36	864.8	121.1	743.7
700	.010	5	16.7	2.3	14.4	37	913.5	127.9	785.6
						38	963.5	134.9	828.6
800	.013	6	24.0	3.4	20.6	39	1014.9	142.1	872.8
900	.017	7	32.7	4.6	28.1	40	1067.6	149.5	918.1
1000	.020	8	42.7	6.0	36.7				
1100	.025	9	54.0	7.6	46.4	41	1121.7	157.0	964.7
1200	.030	10	66.7	9.3	57.4	42	1177.0	164.8	1012.2
						43	1233.7	172.7	1061.0
1300	.035	11	80.7	11.3	69.4	44	1291.8	180.8	1111.0
1400	.040	12	96.1	13.4	82.7	45	1351.2	189.2	1162.0
1500	.046	13	112.8	15.8	97.0				
1600	.052	14	130.8	18.3	112.5	46	1411.9	197.7	1412.2
1700	.059	15	150.1	21.0	129.1	47	1474.0	206.3	1267.7
						48	1537.3	215.2	1322.1
1800	.066	16	170.8	23.9	146.9	49	1602.0	224.3	1377.7
1900	.074	17	192.8	27.0	165.8	50	1668.1	233.5	1434.6
2000	.082	18	216.2	30.3	185.9				
2200	.099	19	240.9	33.7	207.2	51	1735.5	243.0	1492.5
2400	.118	20	266.9	37.4	229.5	52	1804.2	252.6	1551.6
						53	1874.3	262.4	1611.9
2600	.139	21	294.3	41.2	253.1	54	1945.7	272.4	1673.3
2800	.161	22	322.9	45.2	277.7	55	2018.4	282.6	1735.8
3000	.184	23	353.0	49.4	303.6				
3200	.210	24	384.3	53.8	330.5	56	2092.5	292.9	1799.6
3400	.237	25	417.0	58.4	358.6	57	2167.9	303.5	1864.4
						58	2244.6	314.2	1930.4
3600	.266	26	451.1	63.1	388.0	59	2322.7	325.2	1997.5
3800	.296	27	486.4	68.1	418.3	60	2402.1	336.3	2065.8
4000	.328	28	523.1	73.2	449.9				
4200	.362	29	561.2	78.6	482.6	61	2482.8	347.6	2135.2
4400	.397	30	600.5	84.1	516.4	62	2564.9	359.1	2205.8
						63	2648.3	370.8	2277.5
4600	.434	31	641.2	89.8	551.4	64	2733.0	382.6	2350.4
4800	.472	32	683.3	95.7	587.6	65	2819.1	394.7	2424.4
5000	.512	33	726.6	101.7	624.9	66	2906.5	406.9	2499.6

Ex.—The rod reading on an object distant 3200 ft. from the level is 5.00 ft.; and the height of instrument (*H. I.*) is 300.00 ft. Find the elevation of the object.

Ans.—The *apparent* elevation is 295.00 ft.; and the *true* elevation is 295.21 ft.

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18.—Allowable Errors in Leveling.

Distance, Miles.	Distance, Feet.	Allowable Error.*	Distance, Miles.	Distance, Feet.	Allowable Error.*
.1	528	.005	.0189	100	.0022
.125	660	.006	.04	200	.003
.25	1320	.008	.06	300	.004
.333	1760	.009	.08	400	.004
.5	2640	.011	.09	500	.005
.625	3300	.013	.11	600	.005
.75	3960	.014	.13	700	.006
1	5280	.016	.15	800	.006
1.25	6600	.018	.17	900	.007
1.5	7920	.020	.19	1000	.007
1.75	9240	.021	.23	1200	.008
2	10560	.023	.27	1400	.008
2.5	13200	.025	.30	1600	.009
3	15840	.028	.38	2000	.010
4	21120	.032	.47	2500	.011
5	26400	.036	.57	3000	.012
6	31680	.039	.66	3500	.013
8	42240	.045	.76	4000	.014
10	52800	.051	.85	4500	.015
12	63360	.055	.95	5000	.016
15	79200	.062	1.14	6000	.017
20	105600	.072	1.33	7000	.018
25	132000	.080	1.52	8000	.020
30	158400	.088	1.70	9000	.021
40	211200	.101	1.89	10000	.022
50	264000	.113	2.08	11000	.023
60	316800	.124	2.27	12000	.024
70	369600	.134	2.46	13000	.025
80	422400	.143	2.65	14000	.026
90	475200	.152	2.84	15000	.027
100	528000	.160	3.03	16000	.028
125	660000	.179	3.22	17000	.029
150	792000	.196	3.41	18000	.030
175	924000	.212	3.79	20000	.031
200	1056000	.226	4.17	22000	.033
250	1320000	.253	4.73	25000	.035
300	1584000	.277	5.68	30000	.038
350	1848000	.299	6.63	35000	.041
400	2112000	.320	7.58	40000	.044
450	2376000	.339	8.52	45000	.047
500	2640000	.358	9.47	50000	.049

* Error (in feet) leveling "up and back" must not exceed

$.016\sqrt{\text{distance in miles (one way)}}$.

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SUMMARY OF COSTS.

	Detail topography.		Topography for 5-ft. contours.	
	Deens Bridge.	The Hemlocks site.	Lower End of basin.	West Branch.
Area, in acres	213	51	1310	200
Number of Shots	2392	1010	3215	781
Length of traverse, in feet	17990	8600	52350	10520
Number of traverses	5	.....	.....	3
Number of courses	27	25	62	16
Days work, Sat. = full day	7½	8	16	3
Total cost, excl. transportation	$171.50	$150.40	$434.40	$77.80
Total cost, incl. transportation	$181.10	$190.40	$514.40	$107.80
Acres surveyed per day	29	6	82	67
Shots per acre	11	20	2.4	3.9
Feet of traverse, per acre	84	168	40	52
Cost per acre, excl. transportation	$0.81	$2.94	$0.33	$0.39
Cost per acre, incl. transportation	$0.85	$3.74	$0.39	$0.54

Illustrations.

Description.	Eng. Rec.
Concrete boundary monuments, with costs	Jan. 1, '10.

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59.—RAILROADS.

A.—GENERAL DISCUSSION.

Existing Mileage, etc.—There are (1907) about 212,500 miles of steam ilroads in operation in the United States, or one mile to each 400 inhabi-ints. The total assets of these roads are estimated at $15,500,000,000 or 73,000 per mile or $180 per inhabitant; the gross annual earnings, $2,000,-00,000 or $9400 per mile or $23 per inhabitant; and the annual operating cpenses, $1,340,000,000 or $6300 per mile or $16 per inhabitant. There re about 7½ miles of line per 100 sq. miles of land, equivalent to 13.8 sq. iles per mile of line.

Both the earning capacity and operating expenses of a road are depen-ent to a greater or less extent upon its location: the former upon the neral route selected, and the latter upon the detailed location adopted y the engineer. This last factor embraces the two important subjects of radients and Curvature, and the problems which confront the locating ngineer are: What shall be their maximum values on a particular line on a particular Division? and what price shall be paid for their *reduction*, er cent of grade and per degree of curvature? These questions, especially e former, also appear constantly during the *operation* of the road, in the rdinary *improvements* of the line.

Economic Principles.—Private capital very seldom seeks investment nless it "promises" to be a paying one *from the start*, and railroad building fers no exception to this rule. In the design and construction of works of ublic utility from public funds, provision is made for future increase of popula-on, on the grounds of economy. For instance, the estimated population 0 years hence is generally assumed as a basis for determining the capac-ies of reservoirs and sizes of distributing mains for domestic water supply ystems installed in eities and towns. The reason for this is that they are orks of quite permanent character and cannot be "enlarged" without reat expense. Hence we have an illustration of a plant over-efficient for number of years after construction, but economically so because the st of gradually increasing the capacity of the plant would have exceeded e interest on the surplus capital invested in the beginning. On the ther hand, let us take the case of a growing industry such as a manufac-ring plant. Provision is here made (wisely) for future expansion of usiness. The site is selected where additional land may be purchased hen required. Additional buildings are erected also when needed. There but little surplus or idle capital invested in the plant dependent on the rowth of the business, and furthermore, when additions are made, ma-inery of the latest pattern is installed. Thus the plant is efficient and conomical at all times.

Railroad Projection cannot be governed wholly by either of the above xtreme cases. The first would require too great an outlay at the start hich would not be warranted by the early receipts of the road, and the cond would make subsequent improvement too expensive. The New ork West Shore and Buffalo Railroad is an example of a brand new road ade to order for future generations, while the Pennsylvania System is uilt up of short lines which served at one time local interests. The first amed now caters to second-class traffic, while the second has grown strong y virtue of its natural location and expansion. On the other hand there re roads which have been too much localized. They can never become through" lines unless they reduce their *gradients* and *curvature*, and the through freight" traffic is usually the best source of revenue. Exceptions ay be made, however, to some small roads like the Columbia Southern nd the Central of New Jersey, where the prevailing freight traffic (coal, in e latter case) is in one direction—that of the descending grades.

Gradients, in a line, may be established either (1) from necessity, as en the two terminal points to be joined together are at different eleva-

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tions above sea level; (2) to reduce distance, as in passing over a long range of hills instead of around it; (3) to reduce the cost of construction incident to deep cuttings, high embankments and long tunnels.

The "ruling" grade on a location is the maximum grade allowable on any part of the line, and is sometimes called the "limiting" grade. For mountainous sections it is generally fixed at about 2%, more or less, and arbitrarily adhered to* by the locating engineers. Sometimes, however, the ruling grade is changed after location is begun. If a low mountain pass is discovered it may be decreased; if certain unforeseen difficulties are encountered in the topography of the country it may be increased. It is always best to have the *heavy* grades bunched together continuously if possible and not scattered throughout the line. By this arrangement they may be taken from the class of "limiting" grade, for single engine trains, and placed in the class of "pusher" grade, where assistant engines or pushers can be operated economically at one point. In this way the limiting grade proper, on the line, may be said to be lowered, which is decidedly advantageous to the reduced operating expenses of the road.

The Traction Force of a locomotive is the train resistance which it can overcome. It cannot exceed the "adhesion" of the driving wheels to the rails; the "adhesion" should not exceed the "cylinder" power of the engine; the "cylinder" power should not exceed the "boiler" power. We will assume then, that the engine is properly designed: that the cylinder power is a little in excess of the adhesion, and that the boiler power is just sufficient to cause slipping of drivers, when using sand. Then *tractive force equals adhesion*. The adhesion or tractive force may be assumed, for our present purpose, at ¼† *the total weight on the drivers*, that is, this force can be exerted horizontally in moving the train.

The train resistance *on a level track* comprises rolling friction proper, journal friction, reduced effect of traction due to curvature, air resistance, etc. We will assume it to be about 8 *lbs.* ‡ *per short ton* or $^1/_{250}$ the total weight of train—engine, tender and cars. Hence, using the same unit of weight throughout,

$$\text{Traction force} = \frac{\text{Total wt. on drivers}}{4} = \frac{\text{Total wt. of train}}{250} = \text{Train resistance} \quad (1)$$

$$\text{Total wt. of train} = 62.5 \times \text{total wt. on engine drivers} \ldots\ldots\ldots\ldots (2)$$

$$\text{Gross car loads} = 62.5 \times \text{total wt. on drivers} - \text{wt. of engine and tender} \ldots (3)$$

$$\text{Net car loads} = 62.5 \times \text{total wt. on drivers} - \text{wt. of engine, tender and empty cars} \ldots\ldots (4)$$

The effect of an *ascending grade* on train resistance is calculated easily. The level-grade tractive force is simply increased by the *total weight of train* × *the rate* (or %) *of the grade*||. Hence, for any ordinary grade and using the same unit of weight throughout, we have, from (1),

$$\text{Tractive force} = \frac{\text{Total wt. on drivers}}{4} = \text{Total wt. of train } (\tfrac{1}{250} + \text{rate of grade}) \ldots\ldots (5)$$

$$\text{Total wt. of train} = \frac{\text{Total wt. on drivers}}{.016 + 4 \times \text{rate of grade}} \ldots\ldots (6)$$

$$\text{Max. rate (or \%) of grade} = \frac{\text{Total wt. on drivers}}{4 \times \text{total wt. of train}} - .004 \ldots\ldots (7)$$

---

* Compensation amounting to .04 (or .05) % of grade per degree of curve is introduced in order to equalize the tractive resistance.

† May vary from ⅛ to ½; recent experiments give 0.23 to 0.235.

‡ May vary from 4 to 10; 7 to 8 lbs. is usually assumed.

|| This is a slight approximation. The rate (or %) of grade is equal to $\frac{v}{h}$ or *tangent* α (Fig. 1), the angle of inclination which grade line makes with the horizontal, whereas the multiplier *should be* $\frac{v}{l}$ or sine α. For such slight inclinations as railroad grades the error is inappreciable. It is on the side of safety.

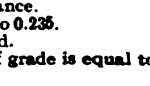

Fig. 1.

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oblem 1.—An engine with tender weighs 138 tons. The weight on vers is 80 tons.

nd total weight of train for a level haul?
nd total weight of train for haul up a 1% grade?
nd the ruling grade for a total train weight of $833\frac{1}{3}$ tons?

ution.—For (*a*) use (2), (*b*) use (6), (*c*) use (7); then,

otal weight of train $= 62.5 \times 80 = 5000$ tons.

otal weight of train $= \dfrac{80}{.016 + .04} = 1429$ tons.

uling (maximum) grade $= \dfrac{80}{3333} - .004 = .02$, or 2% grade.

e.—As the engine and tender weigh 138 tons this amount must be ed from the total weight of train to get the gross-car-loads. To find freight the weight of empty cars must also be deducted. The of cattle-, box- and coal cars is about $\frac{4}{10}$* their normal *capacity*; - and platform cars, $\frac{1}{3}$. Hence if the above train of 833 tons is a rain the net freight would $= \frac{5}{7}(833 - 138) = 496 +$ tons, or about $\frac{6}{10}$ l weight of the train for a 2% grade. If we take into consideration ght of the caboose and also that the cars are not all loaded to their capacity, the ratio of net freight is still further reduced. Moreover, o of weight on the drivers, approximately,—

l weight of engine and tender, is.....	1 : 1.725;
l weight of train.....	1 : 10.417;
ht of train minus engine and tender.....	1 : 8.692;
freight (cattle) only, not including wt. of cars.....	1 : 6.200.

following table is calculated from equation (6) and gives the total of train (including engine and tender) which may be hauled up grades, assuming the weight on the drivers as *unity*.

at is, weight empty $= \frac{3}{7}$ weight loaded.

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1.—Ratio of Total Weight of Train, T, to Weight on Drivers, D, for Various Grades.

(Weight on Drivers, D, Assumed as Unity.)

Calculated from Formula 6, preceding.

Rate of Grade.		Total Wt. of Train. $\frac{T}{D}$	Rate of Grade.		Total Wt. of Train. $\frac{T}{D}$	Rate of Grade.		Total Wt. of Train. $\frac{T}{D}$
Per 100.	Ft. per Mile.		Per 100.	Ft. per Mile.		Per 100.	Ft. per Mile.	
Level	Level	62.50	1.00	52.800	17.86	2.00	105.600	10.42
0.02	1.056	59.52	1.02	53.856	17.61	2.02	106.656	10.33
0.04	2.112	56.82	1.04	54.912	17.36	2.04	107.712	10.25
0.06	3.168	54.35	1.06	55.968	17.12	2.06	108.768	10.16
0.08	4.224	52.08	1.08	57.024	16.89	2.08	109.824	10.08
0.10	5.280	50.00	1.10	58.080	16.67	2.10	110.880	10.00
0.12	6.336	48.08	1.12	59.136	16.45	2.12	111.936	9.92
0.14	7.392	46.30	1.14	60.192	16.23	2.14	112.992	9.84
0.16	8.448	44.64	1.16	61.248	16.03	2.16	114.048	9.77
0.18	9.504	43.10	1.18	62.304	15.82	2.18	115.104	9.69
0.20	10.560	41.67	1.20	63.360	15.63	2.20	116.160	9.62
0.22	11.616	40.32	1.22	64.416	15.43	2.22	117.216	9.54
0.24	12.672	39.06	1.24	65.472	15.24	2.24	118.272	9.47
0.26	13.728	37.88	1.26	66.528	15.06	2.26	119.328	9.40
0.28	14.784	36.76	1.28	67.578	14.88	2.28	120.384	9.33
0.30	15.840	35.71	1.30	68.640	14.71	2.30	121.440	9.26
0.32	16.896	34.72	1.32	69.696	14.53	2.32	122.496	9.19
0.34	17.952	33.78	1.34	70.752	14.37	2.34	123.552	9.12
0.36	19.008	32.89	1.36	71.808	14.20	2.36	124.608	9.06
0.38	20.064	32.05	1.38	72.864	14.04	2.38	125.664	8.99
0.40	21.120	31.25	1.40	73.920	13.89	2.40	126.720	8.93
0.42	22.176	30.49	1.42	74.976	13.74	2.42	127.776	8.87
0.44	23.232	29.76	1.44	76.032	13.59	2.44	128.832	8.80
0.46	24.288	29.07	1.46	77.088	13.44	2.46	129.888	8.74
0.48	25.344	28.41	1.48	78.144	13.30	2.48	130.944	8.68
0.50	26.400	27.78	1.50	79.200	13.16	2.50	132.000	8.62
0.52	27.456	27.18	1.52	80.256	13.02	2.52	133.056	8.56
0.54	28.512	26.60	1.54	81.312	12.89	2.54	134.112	8.50
0.56	29.568	26.04	1.56	82.368	12.76	2.56	135.168	8.45
0.58	30.624	25.51	1.58	83.424	12.63	2.58	136.224	8.39
0.60	31.680	25.00	1.60	84.480	12.50	2.60	137.280	8.33
0.62	32.736	24.51	1.62	85.536	12.38	2.62	138.336	8.28
0.64	33.792	24.04	1.64	86.592	12.25	2.64	139.392	8.22
0.66	34.848	23.58	1.66	87.648	12.14	2.66	140.448	8.17
0.68	35.904	23.15	1.68	88.704	12.02	2.68	141.504	8.12
0.70	36.960	22.73	1.70	89.760	11.90	2.70	142.560	8.06
0.72	38 016	22.32	1.72	90.816	11.79	2.72	143.616	8.01
0.74	39.072	21.93	1.74	91.872	11 68	2.74	144.672	7.96
0.76	40.128	21.55	1.76	92.928	11.57	2.76	145.728	7.91
0.78	41.184	21.19	1.78	93.984	11.47	2.78	146.784	7.86
0.80	42.240	20.83	1.80	95.040	11.36	2.80	147.840	7.81
0.82	43.296	20.49	1.82	96.096	11.26	2.82	148.896	7.76
0.84	44.352	20.16	1.84	97.152	11.16	2 84	149.952	7.71
0.86	45.408	19 84	1.86	98.208	11.06	2.86	151.008	7.67
0 88	46.464	19.53	1.88	99.264	10.96	2.88	152.064	7.62
0.90	47.520	19.23	1.90	100.320	10.87	2.90	153.120	7.58
0.92	48.576	18.94	1.92	101.376	10.78	2.92	154.176	7.53
0.94	49.632	18.66	1 94	102.432	10.68	2.94	155.232	7.49
0.96	50.688	18.38	1.96	103.488	10.59	2.96	156.288	7.44
0.98	51.744	18.12	1.98	104.544	10.50	2.98	157.344	7.40
1.00	52.800	17.86	2.00	105.600	10.42	3 00	158.400	7.35

Note.—To find the gross weight of train *behind the tender*: Multiply the weight of the driving wheels by the figures under "Total wt. of Train," for the particular grade, and deduct weight of engine and tender.

For the net freight, multiply this result by $\frac{2}{3}$, approximately; but see page 993.

The Allowable Expense for Grade Reduction will now be considered. A few hints only can be given and these based on data of very general character. Let Fig. 2 represent a modern freight train, in which

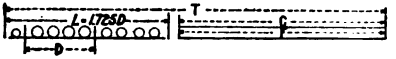

Fig. 2.

D = weight on engine drivers (consolidation type);
L = weight of locomotive and tender = 1.725 D;
T = weight of train $\left(\text{multiplying values } \frac{T}{D} \text{ in preceding Table by } D\right)$;
C = weight of loaded cars, which (see Table 1) = $\left(\frac{T}{D} - 1.725\right) D$;
F = weight of net freight hauled, which = $\frac{8}{7}\left(\frac{T}{D} - 1.725\right) D$, approx.

We will assume that trains are made up and hauled over a Division of 100 miles, without regard to the nature of the traffic on the balance of the road; that there are 1,000,000 tons of freight annually, hauled by 138-ton engines with 80 tons on drivers; that the cost per train mile is $1.00; and that no pusher engines are used.

Fig. 3.

Ques.—What will be the ruling grade* on the Division, provided a saving of $200,000 can be effected in cost of construction for each 0.1% grade above a level grade; the interest value of money being at the rate of 5 per cent?

Ans.—We can calculate readily the cost of hauling this freight by determining, for various grades, from the preceding discussion: (1) The net freight $F = \frac{8}{7}\left(\frac{T}{D} - 1.725\right) D$, hauled per train; (2) The number of trains per year required to haul the 1,000,000 tons; (3) The total cost of haul, at $1.00 per train mile. Column (4), in the following table, gives the increased cost of the annual haul for any grade over that for a grade 0.1 per cent less. Column (5) shows the annual interest on the $200,000 at 5 per cent, which equates nearly with $9910 in column (4), opposite a 2% grade, which is therefore the required ruling grade. Any other rate of interest than 5 per cent would equate differently and give a different ruling grade. Of course other considerations naturally affect the problem to a greater or less extent.

Remarks.—Problems of this character are, in their nature, extremely concrete, and therefore cannot be truly represented by merely abstract formulas, which serve only as guides. For instance, the probable increase in future traffic (either immediate or remote) should be taken into account. If the future traffic is almost certain to be immediately and largely increased, the grade reduction should be proportionately great. Another fact to be borne in mind is, that the cost of grade reduction after track is laid is much greater than before, and especially so when under heavy traffic. But, as a compensating effect, the road is better able to stand this extra expense at such a time, both as to available cash and traffic economy. The "Lake Shore" road has expended hundreds of thousands of dollars in reducing grades by a small fraction of one per cent for a distance of a few miles, and while this work was going on the writer counted on one Sunday, *27 sections* of a "single freight train."

* Of course the grade must be assumed to be long so the momentum of the train cannot be counted on as affecting the problem.

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2.—Cost of Haul on Various Grades.
And determination of Ruling Grade.
(See preceding discussion.)

Grade per 100.	Net Freight per Train. Tons. (1)	No. of Trains per Year Required. (2)	Total Cost of Hauling 1,000,000 Tons at $1 per Train Mile (3)	Increased Cost of Haul over that for grade 0.1 lower. (4)	Interest on $200,000 at 5%. (5)	Remarks.
Level	3472.9	287.9	$28,790	0		
0.1	2758.6	362.5	36,250	$7,460		
0.2	2282.6	438.1	43,810	7,560		
0.3	1942.0	514.9	51,490	7,680		
0.4	1687.1	592.7	59,270	7,780		
0.5	1488.9	671.7	67,170	7,900		Direct loss if grade is reduced below 2%; but justified, possibly, if based on future increased traffic, higher cost per train mile, or lower cost for grade reduction or interest.
0.6	1330.0	751.8	75,190	8,020		
0.7	1200.3	833.1	83,310	8,120		
0.8	1091.7	916.0	91,600	8,290		
0.9	1000.3	999.7	99,970	8,370		
1.0	922.0	1084.6	108,460	8,490		
1.1	854.0	1171.0	117,100	8,640		
1.2	794.6	1258.5	125,850	8,750		
1.3	742.0	1347.7	134,770	8,920		
1.4	695.1	1438.6	143,860	9,090		
1.5	653.4	1530.4	153,040	9,180		
1.6	615.7	1624.1	162,410	9,370		
1.7	581.4	1719.9	171,990	9,580		
1.8	550.6	1816.3	181,630	9,640		
1.9	522.6	1913.6	191,360	9,730		
2.0	496.9	2012.7	201,270	9,910	$10,000	Ruling Grade.
2.1	472.9	2114.8	211,480	10,210		
2.2	451.7	2218.0	221,800	10,320		
2.3	430.6	2322.5	232,250	10,450		
2.4	411.7	2428.9	242,890	10,640		
2.5	394.0	2538.1	253,810	10,920		Direct saving is effected by cutting down all grades above 2%.
2.6	377.4	2649.5	264,950	11,140		
2.7	362.0	2762.4	276,240	11,290		
2.8	347.7	2875.9	287,590	11,350		
2.9	334.4	2991.5	299,150	11,560		
3.0	321.4	3111.1	311,110	11,960		

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Curvature, with increased Distance,* in a line may arise from four primaty considerations, namely, (1) *to increase the revenue* of the road by passing through towns not on an air line between terminal points; (2) *to reduce cost of construction,* as by avoiding deep cuts, high fills, long tunnels, expensive bridge crossings, etc.; (3) *to reduce cost of operation,* as by avoiding steep grades; and (4) *to reduce cost of maintenance,* as by choosing a line with a permanent roadbed, cheaply maintained, instead of a "structural" line involving much expense in repairs and renewals.

(1). In a new country, thinly settled and without competing roads, any departure from an economically located "air" line, to tap a lateral region, is justifiable when the "richness" of that region is about proportional to the additional cost of reaching it, assuming that the cost per mile of such changed line as well as its future growth of business will be, say, proportional to that of the whole line. If there is any question on this latter point it may better be tapped by a spur. The population of a region is in no wise the only safe criterion on which to base probable business. A cattle-raising country sparsely populated is very deceptive in this respect.

(2). We have just considered a case (1) where increased curvature (and distance) on a line would be justified by an increase in revenue about proportional to the *increased cost* of the line. Let *A B C*, Fig. 4, be such a line giving increased revenue over the direct line *A a C*. We will consider now whether some change in location from *A a C* would be justified by *reduced cost of construction,* of such amount that the annual interest on same would be equal to the increased net revenue received in case (1)? In the first case the line *A B C* is considered *more expensive* to construct than the line *A a C*, while in the present case the longer line is *cheaper*. As a matter of fact there is no relation between the two cases, although at first glance there appears to be. In the former case we are increasing our business say proportionately to our investment by running the line through *B*. In the latter case we are not willing to increase the length of our line to pass through *B*, but rather through some point *B'* (where there is no business) so that *the annual increase in the operating expenses on the line A B'C over that of A a C shall not exceed the interest on the decreased cost of construction.* The *actual* cost of the line between *A* and *C* is another matter. This should be fixed within certain limits, however, having due regard to the amount and quality of traffic between *O-O'*, the terminal centers-of-gravity of haul; and also to the quality of the improvements made on those parts of the line *A O* and *C O'*. If the bulk of traffic is "through" traffic, *O* and *O'* may be considered practically to be at the terminal points of the line.

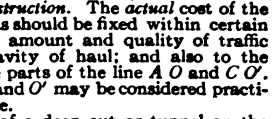

Fig. 4.

(3). Let us now suppose that instead of a deep cut or tunnel on the line *A a C* (Fig. 4) as in the preceding case (2), we are confronted with a long ridge which will have to be surmounted with heavy grades if the "air" line is to be maintained. Here we may resort to the expedient of selecting some route as *A b C*, carrying with it reduced grades and increased curvature and distance. In fixing the new route we now apply the opposite rule to that of case (2): The line should pass through some point *b* so that *the annual decrease in the operating expenses on the line A b C over that of A a C should exceed the interest on the increased cost of construction.*

(4). The cost of maintenance should generally be considered as a part of the "operating" expenses in the two preceding cases, (2) and (3). But the term may have a greater significance apart from such association. Two lines having equally objectionable grades may be constructed, (*a*) following the natural contour of the country, and (*b*) shortening the distance and cutting out curvature by the use of bridges and trestles. The difference in cost of maintenance and renewals in favor of (*a*) may outweigh all other considerations against it. The increased curvature would have to be considerable to become a serious matter, as the cost of maintaining curved track is but slightly more than that for maintaining straight track. The life of ties in curved track is shortened from 3 to 5% annually per degree of curvature. The excess cost per train mile is inappreciable for a slightly increased length of line (not at all proportional to the length), and it is also but slightly affected by the introduction of moderately flat curves. Steep grades are especially to be avoided.

* *Curvature* is used, generally, in this discussion, in its broadest sense as including distance.

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Location of the Line.—This comprises three main operations, as follows:

B. Reconnoissance, or general field inspection.
C. Preliminary Survey, with instruments.
D. Location, or final determination of the line.

Topographical maps of many sections of the country may be had from the Government and from the several States. Those of the geological surveys are especially valuable in fixing the general route of the line in the reconnoissance and in the subsequent detailed surveys.

B.—THE RECONNOISSANCE.

This is the Field Examination necessary in fixing the "critical" points on the line, prior to the preliminary survey, in order to reduce the expense of the latter. It picks out the low mountain passes, the tunnel locations, and the favorable river- and other crossings. If well conducted it may dictate also, within close limits, the ruling grade and maximum curvature. The principal instruments used are the aneroid barometer, thermometer, pedometer (if on foot), cyclometer (if by wheel), odometer (if by wagon). These three latter are used for measuring the distance traveled. A hand level will be found useful, also a pocket compass, if detailed information is necessary in any particular locality. The thermometer is used in connection with the aneroid barometer. The transit with stadia is often used advantageously at this early stage. (See Stadia Reduction Table, page 984.)

The Aneroid Barometer is useful in determining the altitude of any point above sea level, or the relative difference in altitude between two or more points. It consists of a small, circular, air-tight box, in vacuo, with one side sensitive to the pressure of the atmosphere outside. The heavier the atmospheric pressure (the lower the altitude) the more it is pressed inward, and this movement is multiplied and transmitted to a recording index, like the hand of a clock, which denotes the pressure in inches (on the inner circle) corresponding to the mercurial column of an ordinary barometer. It is to be noted that outside the mercurial scale there is also a direct reading scale, to hundreds of feet, giving the direct altitude *approximately*. All aneroids now sold by the best makers are "compensated" for change or difference in temperature (see Fig. 5); but this does not mean necessarily that they are absolutely exact, but merely that they are nearly so and may be used "singly" with some degree of accuracy. When, however, extreme accuracy is required in getting the difference in elevation between two stations—one called the upper station and the other the lower station—two compensated aneroids are used, one at each station. Readings are taken at the *same time* and—

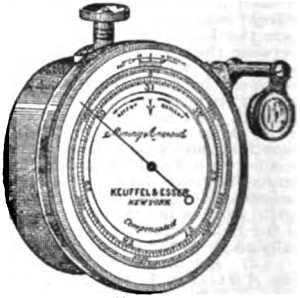

Fig. 5.

$$\text{The difference in elevation} = (H-h)\left(\frac{900+T+t}{1000}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which H = reading in feet on elevation scale at upper station;
h = reading in feet on elevation scale at lower station;
T = temperature in degrees Fahrenheit at upper station;
t = temperature in degrees Fahrenheit at lower station.

If the temperature at both stations is 50°F. it is seen that $H-h$ represents the true difference in elevation, and there is no correction for temperature. Of course the aneroids will have to be compared by taking observations together at some station and the difference or index error noted for subsequent observations. The author has seen it stated by some of our prominent writers that the smaller aneroids of 1¾ to 2¼ inches diameter give as accurate results as the larger ones. The author's observations are to the contrary and are confirmed by the following from Messrs. Keuffel & Esser, New York City: "All our aneroids are compensated and their readings do not require any further corrections than the one referred to [formula (1)

preceding]. It is our judgment that the small pocket aneroids are less accurate than the larger ones; not only are the dials of the larger instruments more closely graduated and permit of finer reading, but the larger instruments are also more sensitive on account of the larger size of their vacuum boxes. Furthermore, the instrumental error arising from the elastic reaction of the counter spring and rocker is greatly reduced."

The Mercurial Barometer is seldom used in railroad reconnoissance, having been supplanted by the aneroid, previously described. The aneroid is much more convenient to carry; there is no correction for latitude nor for variation in gravity due to altitude above the earth. For extremely accurate work, however, in establishing absolute elevations above sea level, the mercurial barometer is used. The following Tables are from Appendix 10, Report U. S. Coast and Geodetic Survey for 1881.

3.—Barometric Elevations for Temperature 50° F.

(For use with Mercurial Barometer.)

Note.—For temperatures other than 50°, see Correction Table, No. 4.

[Elevation in Feet above sea level.]

Height of Barom. Ins.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Rem.
11	27336	27090	26846	26604	26364	26126	25890	25656	25424	25194	*Caution.*—Remember to *subtract* the Difference.
12	24966	24740	24516	24294	24073	23854	23637	23421	23207	22995	
13	22785	22576	22368	22162	21958	21757	21557	21358	21160	20962	
14	20765	20570	20377	20186	19997	19809	19623	19437	19252	19068	
15	18886	18705	18525	18346	18168	17992	17817	17643	17470	17298	
16	17127	16958	16789	16621	16454	16288	16124	15961	15798	15636	
17	15476	15316	15157	14999	14842	14686	14531	14377	14223	14070	
18	13918	13767	13617	13468	13319	13172	13025	12879	12733	12589	
19	12445	12302	12160	12018	11877	11737	11598	11459	11321	11184	
20	11047	10911	10776	10642	10508	10375	10242	10110	9979	9848	
21	9718	9589	9460	9332	9204	9077	8951	8825	8700	8575	
22	8451	8327	8204	8082	7960	7838	7717	7597	7477	7358	
23	7239	7121	7004	6887	6770	6654	6538	6423	6308	6194	
24	6080	5967	5854	5741	5629	5518	5407	5296	5186	5077	
25	4968	4859	4751	4643	4535	4428	4321	4215	4109	4004	
26	3899	3794	3690	3586	3483	3380	3277	3175	3073	2972	
27	2871	2770	2670	2570	2470	2371	2272	2173	2075	1977	
28	1880	1783	1686	1589	1493	1397	1302	1207	1112	1018	
29	924	830	736	643	550	458	366	274	182	91	
30	000	—91	—181	—271	—361	—451	—540	—629	—717	—805	

Ex. 1.—The mean temp. of two stations whose diff. of elev. is desired is 50° F. The barom. reading at upper station is 24.62 ins.; and at lower station, 28.165. Find the difference in elevation of the two stations.

Solution.—Using proportional differences—

Elev. upper station (24.62 ins.) = 5385 ft.
Elev. lower station (28.165 ins.) = 1720 ft.

Ans.—Diff. in elevation.... = 3665 ft.

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4.—Barometric Correction Table for Temperature.

(To be used in connection with Table 3, preceding.)

Note.—Mult. value obtained from Table 3, by the Coefficients in this table.

[Coefficients.]

Mean Temp.	Coef.	Diff. per Deg.	Mean Temp.	Coef.	Diff. per Deg.	Mean Temp.	Coef.	Diff. per Deg.
0°	.8975		30°	.9620		60°	1.0262	
		.00219			.00214			.00210
10°	.9194		40°	.9834		70°	1.0472	
		.00214			.00215			.00205
20°	.9408		50°	1.0049		80°	1.0677	
		.00212			.00213			.00202
30°	.9620		60°	1.0262		90°	1.0879	

Ex. 2.—Now for any other mean temp. of the two stations than 50° F., say 65° F., we find the coef. for mean temp. in Table 4, and mult. this into the result obtained from Table 3 for 50° F. Thus, for 65° F., the coef. is $1.0262+.00210\times5=1.0367$. Solving *Ex.* 1, for a mean temp. of 65° F., we have $3665\times1.0367=3800$ ft. = diff. in elev. of the two stations.

C.—THE PRELIMINARY SURVEY.

The Organization for the preliminary survey, as ordinarily conducted, comprises full parties for transit-, level-, and topographic work. This survey consists in developing a broken line on the ground, that can be used later as a base in projecting the final location. The preliminary line should be nearly identical, practically, with the final location so that the latter will be "fully covered" by the topography. In cases where it is evident that any part of the location will be radically different from the preliminary line as run, a new preliminary line should be run immediately covering that portion, and the old line "abandoned" and marked so in the note books. Equated stationing should be used instead of introducing the terms "long station" or "short station."

The Locating Engineer will select, usually, some critical point where the line has to come, in both position and elevation, for a starting point of the survey. Where a mountain range has to be crossed it is customary to begin at the summit and work down, but this does not always hold. If there is no pass low enough, and a tunnel is imperative, it is often necessary to make a quick topographical survey "along" the main range, running transverse lines from the main summit line at points where it is thought the length of tunnel will be the least, at the desired elevation. This will not necessarily be at the "pass" and may be a considerable distance from it. (In connection with tunnel location, the geological formation should be studied and also the possibility of one or more shafts to facilitate construction.)

The Transitman should be careful to hold closely to any grade line which may have been decided upon from the reconnoissance data. For this purpose the transit should be provided with a Gradienter Screw by which the telescope may be inclined to the required grade. It consists of a clamp and slow-motion screw, so that one complete revolution of the latter raises or lowers the line of sight of the telescope 1 foot vertically in a horizontal distance of 100 feet. The edge of the head is divided into 100 parts for minute readings, and the number of complete turns of the screw are indicated by a graduated bar. Stakes are set every 100-ft., or station, and hubs at every transit point. In a true preliminary line no curves are run, but it is sometimes convenient to fit in a curve around a hillside to facilitate the work of topography and later location. The accuracy with which preliminary lines are run depends somewhat upon the circumstances of the case. The policy of the Southern Pacific R. R. Co. is to run very accurate preliminary surveys so that the subsequent location can be calculated to a nicety in the office. Magnetic bearings should be taken at every position of the transit, as a check on the angles.

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5.—Grades in Feet per 100-Ft. and Feet per Mile.
Part I. [Feet per Mile. (*Exact.*)]

.00	.01	.02	.03	.04	.05	.06	.07	.08	.09	P. P.
—	.528	1.056	1.584	2.112	2.640	3.168	3.696	4.224	4.752	
5.280	5.808	6.336	6.864	7.392	7.920	8.448	8.976	9.504	10.032	
10.560	11.088	11.616	12.144	12.672	13.200	13.728	14.256	14.784	15.312	
15.840	16.368	16.896	17.424	17.952	18.480	19.008	19.536	20.064	20.592	
21.120	21.648	22.176	22.704	23.232	23.760	24.288	24.816	25.344	25.872	
26.400	26.928	27.456	27.984	28.512	29.040	29.568	30.096	30.624	31.152	
31.680	32.208	32.736	33.264	33.792	34.320	34.848	35.376	35.904	36.432	
36.960	37.488	38.016	38.544	39.072	39.600	40.128	40.656	41.184	41.712	
42.240	42.768	43.296	43.824	44.352	44.880	45.408	45.936	46.464	46.992	
47.520	48.048	48.576	49.104	49.632	50.160	50.688	51.216	51.744	52.272	
52.800	53.328	53.856	54.384	54.912	55.440	55.968	56.496	57.024	57.552	
58.080	58.608	59.136	59.664	60.192	60.720	61.248	61.776	62.304	62.832	
63.360	63.888	64.416	64.944	65.472	66.000	66.528	67.056	67.584	68.112	
68.640	69.168	69.696	70.224	70.752	71.280	71.808	72.336	72.864	73.392	
73.920	74.448	74.976	75.504	76.032	76.560	77.088	77.616	78.144	78.672	
79.200	79.728	80.256	80.784	81.312	81.840	82.368	82.896	83.424	83.952	.528
84.480	85.008	85.536	86.064	86.592	87.120	87.648	88.176	88.704	89.232	1 .053
89.760	90.288	90.816	91.344	91.872	92.400	92.928	93.456	93.984	94.512	2 .106
95.040	95.568	96.096	96.624	97.152	97.680	98.208	98.736	99.264	99.792	3 .158
100.320	100.848	101.376	101.904	102.432	102.960	103.488	104.016	104.544	105.072	4 .211
105.600	106.128	106.656	107.184	107.712	108.240	108.768	109.296	109.824	110.352	5 .264
110.880	111.408	111.936	112.464	112.992	113.520	114.048	114.576	115.104	115.632	6 .317
116.160	116.688	117.216	117.744	118.272	118.800	119.328	119.856	120.384	120.912	7 .370
121.440	121.968	122.496	123.024	123.552	124.080	124.608	125.136	125.664	126.192	8 .422
126.720	127.248	127.776	128.304	128.832	129.360	129.888	130.416	130.944	131.472	9 .475
132.000	132.528	133.056	133.584	134.112	134.640	135.168	135.696	136.224	136.752	
137.280	137.808	138.336	138.864	139.392	139.920	140.448	140.976	141.504	142.032	
142.560	143.088	143.616	144.144	144.672	145.200	145.728	146.256	146.784	147.312	
147.840	148.368	148.896	149.424	149.952	150.480	151.008	151.536	152.064	152.592	
153.120	153.648	154.176	154.704	155.232	155.760	156.288	156.816	157.344	157.872	
158.400	158.928	159.456	159.984	160.512	161.040	161.568	162.096	162.624	163.152	
163.680	164.208	164.736	165.264	165.792	166.320	166.848	167.376	167.904	168.432	
168.960	169.488	170.016	170.544	171.072	171.600	172.128	172.656	173.184	173.712	
174.240	174.768	175.296	175.824	176.352	176.880	177.408	177.936	178.464	178.992	
179.520	180.048	180.576	181.104	181.632	182.160	182.688	183.216	183.744	184.272	
184.800	185.328	185.856	186.384	186.912	187.440	187.968	188.496	189.024	189.552	
190.080	190.608	191.136	191.664	192.192	192.720	193.248	193.776	194.304	194.832	
195.360	195.888	196.416	196.944	197.472	198.000	198.528	199.056	199.584	200.112	
200.640	201.168	201.696	202.224	202.752	203.280	203.808	204.336	204.864	205.392	
205.920	206.448	206.976	207.504	208.032	208.560	209.088	209.616	210.144	210.672	

1.—Rate of grade = 1.937 ft. ft. Then ft. per mile = 1.937 ÷ 0.37 = 102.274.

Ex. 2.—By inverse operation, the rate of grade may be obtained when ft. per mile is given.

Part II. [Feet per Mile. (*Exact.*)]

0	.1	.2	.3	.4	.5	.6	.7	.8	.9	P. P.
211.20	216.48	221.76	227.04	232.32	237.60	242.88	248.16	253.44	258.72	5.28
										1 .528
264.00	269.28	274.56	279.84	285.12	290.40	295.68	300.96	306.24	311.52	2 1.056
316.80	322.08	327.36	332.64	337.92	343.20	348.48	353.76	359.04	364.32	3 1.584
369.60	374.88	380.16	385.44	390.72	396.00	401.28	406.65	411.84	417.12	4 2.112
422.40	427.68	432.96	438.24	443.52	448.80	454.08	459.36	464.64	469.92	5 2.640
475.20	480.48	485.76	491.04	496.32	501.60	506.88	512.16	517.44	522.72	6 3.168
										7 3.696
528.00	533.28	538.56	543.84	549.12	554.40	559.68	564.96	570.24	575.52	8 4 224
										9 4.752

3.—By direct operation: grade = 4.25 per 100 ft. per mile = 224.40.

Ex. 4.—By inverse operation: Ft. per mile = 250. Then rate of grade per 100 ft. = 4.735.

ber of 100-Ft. Stations Corresponding to Distance in Miles.
(Use above Table, No. 5, with revised headings.)

1.—By direct operation: es = 118.8 stations; 22.5 188 stations.

Ex. b.—By inverse operation: 500.00 stations = 9.4 + .06 ˇ96 miles; 5000 sta. = 94.6 ˇ96 miles.

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7.—Grade Angles Corresp'd'g to Rates of Grade in Feet per 100-Ft.

Part I. [Grade Angle.]

Ft. per 100 ft.	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09	P. P.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	°
.0	— — —	21	41	1 02	1 23	1 43	2 04	2 24	2 45	3 06	
.1	3 26	3 47	4 08	4 28	4 49	5 09	5 30	5 51	6 11	6 32	
.2	6 53	7 13	7 34	7 54	8 15	8 36	8 56	9 17	9 38	9 58	
.3	10 19	10 39	11 00	11 21	11 41	12 02	12 23	12 43	13 04	13 24	
.4	13 45	14 06	14 26	14 47	15 08	15 28	15 49	16 09	16 30	16 51	
.5	17 11	17 32	17 53	18 13	18 34	18 54	19 15	19 36	19 56	20 17	
.6	20 38	20 58	21 19	21 39	22 00	22 21	22 41	23 02	23 23	23 43	
.7	24 04	24 24	24 45	25 06	25 26	25 47	26 08	26 28	26 49	27 09	
.8	27 30	27 51	28 11	28 32	28 53	29 13	29 34	29 54	30 15	30 36	
.9	30 57	31 17	31 38	31 58	32 19	32 39	33 00	33 21	33 41	34 02	20
1.0	34 23	34 43	35 04	35 24	35 45	36 05	36 26	36 47	37 08	37 28	1 2
1.1	37 49	38 09	38 30	38 51	39 11	39 32	39 53	40 13	40 34	40 54	2 4
1.2	41 15	41 35	41 56	42 17	42 38	42 58	43 19	43 39	44 00	44 21	3 6
1.3	44 41	45 02	45 23	45 43	46 04	46 24	46 45	47 06	47 26	47 47	4 8
1.4	48 08	48 28	48 49	49 09	49 30	49 51	50 11	50 32	50 52	51 13	5 10
1.5	51 34	51 54	52 15	52 36	52 56	53 17	53 37	53 58	54 19	54 39	6 12
1.6	55 00	55 21	55 41	56 02	56 22	56 43	57 04	57 24	57 45	58 06	7 14
1.7	58 26	58 47	59 07	59 28	59 49	1 00 09	1 00 30	1 00 51	1 01 11	1 01 32	8 16
1.8	1 01 52	1 02 13	1 02 34	1 02 54	1 03 15	1 03 35	1 03 56	1 04 17	1 04 37	1 04 58	9 18
1.9	1 05 19	1 05 39	1 06 00	1 06 20	1 06 41	1 07 02	1 07 22	1 07 43	1 08 04	1 08 24	
2.0	1 08 45	1 09 05	1 09 26	1 09 47	1 10 07	1 10 28	1 10 48	1 11 09	1 11 30	1 11 50	
2.1	1 12 11	1 12 32	1 12 52	1 13 13	1 13 33	1 13 54	1 14 15	1 14 35	1 14 56	1 15 16	21
2.2	1 15 37	1 15 58	1 16 18	1 16 39	1 17 00	1 17 20	1 17 41	1 18 01	1 18 22	1 18 43	1 2
2.3	1 19 03	1 19 24	1 19 44	1 20 05	1 20 26	1 20 46	1 21 07	1 21 28	1 21 48	1 22 09	2 4
2.4	1 22 29	1 22 50	1 23 11	1 23 31	1 23 52	1 24 12	1 24 33	1 24 54	1 25 14	1 25 35	3 6
2.5	1 25 56	1 26 16	1 26 37	1 26 57	1 27 18	1 27 39	1 27 59	1 28 20	1 28 40	1 29 01	4 8
2.6	1 29 22	1 29 42	1 30 03	1 30 24	1 30 44	1 31 05	1 31 25	1 31 46	1 32 07	1 32 27	5 11
2.7	1 32 48	1 33 08	1 33 29	1 33 50	1 34 10	1 34 31	1 34 51	1 35 12	1 35 33	1 35 53	6 13
2.8	1 36 14	1 36 35	1 36 55	1 37 16	1 37 36	1 37 57	1 38 18	1 38 38	1 38 59	1 39 19	7 15
2.9	1 39 40	1 40 01	1 40 21	1 40 42	1 41 02	1 41 23	1 41 44	1 42 04	1 42 25	1 42 45	8 17
3.0	1 43 06	1 43 27	1 43 47	1 44 08	1 44 29	1 44 49	1 45 10	1 45 30	1 45 51	1 46 12	9 19
3.1	1 46 32	1 46 53	1 47 13	1 47 34	1 47 55	1 48 15	1 48 36	1 48 56	1 49 17	1 49 38	
3.2	1 49 58	1 50 19	1 50 39	1 51 00	1 51 21	1 51 41	1 52 02	1 52 22	1 52 43	1 53 04	
3.3	1 53 24	1 53 45	1 54 05	1 54 26	1 54 47	1 55 07	1 55 28	1 55 48	1 56 09	1 56 30	
3.4	1 56 50	1 57 11	1 57 32	1 57 52	1 58 13	1 58 33	1 58 54	1 59 15	1 59 35	1 59 56	
3.5	2 00 16	2 00 37	2 00 58	2 01 18	2 01 39	2 01 59	2 02 20	2 02 41	2 03 01	2 03 22	
3.6	2 03 42	2 04 03	2 04 24	2 04 44	2 05 05	2 05 25	2 05 46	2 06 07	2 06 27	2 06 48	
3.7	2 07 08	2 07 29	2 07 50	2 08 10	2 08 31	2 08 51	2 09 12	2 09 33	2 09 53	2 10 14	
3.8	2 10 34	2 10 55	2 11 16	2 11 36	2 11 57	2 12 17	2 12 38	2 12 58	2 13 19	2 13 40	
3.9	2 14 00	2 14 21	2 14 41	2 15 02	2 15 23	2 15 43	2 16 04	2 16 24	2 16 45	2 17 06	

Ex. 1.—Rate of grade = 1.937 ft. per 100 ft. Then grade angle = 1°06′20″ + 15″ = 1°06′35″.

Ex. 2.—By inverse operation, the rate of grade may be obtained when grade angle is given.

Part II. [Grade Angle.]

Ft. per 100 ft.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	P. P.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	°
4	2 17 26	2 20 52	2 24 18	2 27 44	2 31 10	2 34 36	2 38 01	2 41 27	2 44 53	2 48 19	3 25
5	2 51 45	2 55 10	2 58 36	3 02 09	3 05 27	3 08 53	3 12 19	3 15 44	3 19 10	3 22 36	10 21
6	3 26 01	3 29 27	3 32 52	3 36 18	3 39 43	3 43 08	3 46 34	3 49 59	3 53 24	3 56 50	20 41
7	4 00 15	4 03 40	4 07 06	4 10 31	4 13 56	4 17 21	4 20 46	4 24 11	4 27 36	4 31 01	30 62
8	4 34 26	4 37 51	4 41 16	4 44 41	4 48 06	4 51 30	4 54 55	4 58 20	5 01 45	5 05 09	40 82
9	5 08 34	5 11 59	5 15 23	5 18 48	5 22 12	5 25 37	5 29 01	5 32 25	5 35 50	5 39 14	50 103
10	5 42 38	5 46 02	5 49 26	5 52 50	5 56 15	5 59 39	6 03 03	6 06 27	6 09 51	6 13 14	60 124
											70 144
											80 165
											90 185

Ex. 3.—By direct operation: Rate of grade = 4.25 per 100 ft. Then grade angle = 2° 26′ 01″.

Ex. 4.—By inverse operation: Grade angle = 2° 42′ 39″. Then rate of grade per 100 ft. = 4.735.

Notes.

Grades may be run with the transit by the use of the gradienter attachment. See also Table No. 8, following.

8.—Grades in Feet per Mile Reduced to Feet per 100-Ft.

[Grade in Feet per 100 Ft.]

Ft. per Mile.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	P. P.
0—.	——	.01894	.03788	.05682	.07576	.09470	.11364	.13258	.15152	.17045	
1—.	.18939	.20833	22727	.24621	.26515	.28409	.30303	.32197	.34091	.35985	1894
2—.	.37879	.39773	.41667	.43561	.45455	.47348	.49242	.51136	.53030	.54924	1 189
3—.	.56818	.58712	.60606	.62500	.64394	.66288	.68182	.70076	.71970	.73864	2 379
4—.	.75758	.77652	.79545	.81439	.83333	.85227	.87121	.89015	.90909	.92803	3 568
											4 758
5—.	.94697	.96591	.98485	1.00379	1.02273	1.04167	1.06061	1.07955	1.09848	1.11742	5 947
6—.	1.13636	1.15530	1.17424	1.19318	1.21212	1.23106	1.25000	1.26894	1.28788	1.30682	6 1136
7—.	1.32576	1.34470	1.36364	1.38258	1.40152	1.42045	1.43939	1.45833	1.47727	1.49621	7 1326
8—.	1.51515	1.53409	1.55303	1.57197	1.59091	1.60985	1.62879	1.64773	1.66667	1.68561	8 1515
9—.	1.70455	1.72348	1.74242	1.76136	1.78030	1.79924	1.81818	1.83712	1.85606	1.87500	9 1705
10—.	1.89394	1.91288	1.93182	1.95076	1.96970	1.98864	2.00758	2.02652	2.04545	2.06439	
11—.	2.08333	2.10227	2.12121	2.14015	2.15909	2.17803	2.19697	2.21591	2.23485	2.25379	1893
12—.	2.27273	2.29167	2.31061	2.32955	2.34848	2.36742	2.38636	2.40530	2.42424	2.44318	1 189
13—.	2.46212	2.48106	2.50000	2.51894	2.53788	2.55682	2.57576	2.59470	2.61364	2.63258	2 379
14—.	2.65152	2.67045	2.68939	2.70833	2.72727	2.74621	2.76515	2.78409	2.80303	2.82197	3 568
											4 757
15—.	2.84091	2.85985	2 87879	2.89773	2.91667	2.93561	2.95455	2.97348	2.99242	3.01136	5 947
16—.	2.030303	0.04924	3.06818	3.08712	3.10606	3.12500	3.14394	3.16288	3.18182	3.20076	6 1136
17—.	3 21970	23864	3.25758	3.27652	3.29545	3.31439	3.33333	3.35227	3.37121	3.39015	7 1325
18—.	2 40909	3.42803	3.44697	3.46591	3.48485	3.50379	3.52273	3.54167	3.56061	3.57955	8 1514
19—.	3 59848	3.61742	3.63636	3.65530	3.67424	3.69318	3.71212	3.73106	3.75000	3.76894	9 1704
20—.	3.78788	3.80682	3.82576	3.84470	3.86364	3.88258	3.90152	3.92045	3.93939	3.95833	

Ex. 1.—The grade of a road is 95.3 ft. per mile = (1.79924 + .00568) ft. per 100 ft.

9.—Grade Angles Corresponding to Grades in Feet per Mile.

[Grade Angle.]

Ft. per Mile.	0.	1.	2	3.	4.	5.	6.	7.	8.	9.	P. P.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	
0—.	— —	39	1 18	1 57	2 36	3 15	3 54	4 33	5 13	5 52	
1—.	6 31	7 10	7 49	8 28	9 07	9 46	10 25	11 04	11 43	12 22	39″
2—.	13 01	13 40	14 19	14 58	15 38	16 17	16 56	17 35	18 14	18 53	1 4
3—.	19 32	20 11	20 50	21 29	22 08	22 47	23 26	24 05	24 44	25 24	2 8
4—.	26 03	26 42	27 21	28 00	28 39	29 18	29 57	30 36	31 15	31 54	3 12
											4 16
5—.	32 33	33 12	33 51	34 30	35 09	35 49	36 28	37 07	37 46	38 25	5 20
6—.	39 04	39 43	40 22	41 01	41 40	42 19	42 58	43 37	44 16	44 55	6 23
7—.	45 34	46 12	46 53	47 32	48 11	48 50	49 29	50 08	50 47	51 26	7 27
8—.	52 05	52 44	53 23	54 02	54 41	55 20	55 59	56 38	57 17	57 57	8 31
9—.	58 36	59 15	59 54	1 00 33	1 01 12	1 01 51	1 02 30	1 03 09	1 03 48	1 04 27	9 35
10—.	1 05 06	1 05 45	1 06 24	1 07 03	1 07 42	1 08 21	1 09 00	1 09 39	1 10 18	1 10 58	
11—.	1 11 37	1 12 16	1 12 55	1 13 34	1 14 13	1 14 52	1 15 31	1 16 10	1 16 49	1 17 28	40″
12—.	1 18 07	1 18 46	1 19 25	1 20 04	1 20 43	1 21 22	1 22 01	1 22 40	1 23 19	1 23 58	1 4
13—.	1 24 38	1 25 17	1 25 56	1 26 35	1 27 14	1 27 53	1 28 32	1 29 11	1 29 50	1 30 29	2 8
14—.	1 31 08	1 31 47	1 32 26	1 33 05	1 33 44	1 34 23	1 35 02	1 35 41	1 36 20	1 36 59	3 12
											4 16
15—.	1 37 38	1 38 17	1 38 56	1 39 35	1 40 14	1 40 53	1 41 32	1 42 11	1 42 50	1 43 30	5 20
16—.	1 44 09	1 44 48	1 45 27	1 46 06	1 46 45	1 47 24	1 48 03	1 48 42	1 49 21	1 50 00	6 24
17—.	1 50 39	1 51 18	1 51 57	1 52 36	1 53 15	1 53 54	1 54 33	1 55 12	1 55 51	1 56 30	7 28
18—.	1 57 09	1 57 48	1 58 27	1 59 06	1 59 45	2 00 24	2 01 03	2 01 42	2 02 21	2 03 00	8 32
19—.	2 03 39	2 04 18	2 04 57	2 05 36	2 06 15	2 06 54	2 07 33	2 0 12	2 08 51	2 09 30	9 36
20—.	2 10 09	2 10 48	2 11 27	2 12 06	2 12 45	2 13 24	2 14 03	2 14 42	2 15 21	2 16 00	

Ex. 1.—The grade of a road is 95.3 ft. per mile; hence, grade angle = 1° 01′ 51″ + 12″.

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The Levelman starts usually from an established bench mark (*B. M.*), frequently at an assumed elevation, as near as possible to the correct elevation above sea level. He follows closely behind the transitman, taking elevations on the ground at every station and also at intermediate points where the profile demands. He should keep the transit at the correct elevation at every transit point and sometimes give levels ahead of the transit. He establishes temporary bench marks on every transit hub, and permanent bench marks, say, every half mile. They should be described accurately in the note book by exact stationing on the line and by distance to right or left of same. Check levels should be run every 10 miles, more or less. If work is slack, the levelman can often aid the topographer over certain stretches by taking side levels. Two rodmen can often be used to great advantage. The combined target- and self-reading rod is the best—the former for turning points, and the latter for ordinary ground elevations.

The Topographer gets the ground elevations at stations on the center line from the levelman, either the night before or during the day, at intervals. His duties are to take such notes that contour lines can be platted accurately on the maps. It is perhaps needless to say that in some cases topography should be taken extremely accurate, and especially so if the location lines are determined primarily in the office. Perhaps the most common method of taking topography is with the hand level, for elevation, and by pacing, for distance. Often the (cloth) tape is used. There are two methods of keeping the notes: (1) To give the relative elevation at right angle to the center line at each station, and the distance out, to each break in the ground, as $\frac{+4.3}{20}$; and (2) to "sketch in" the contour lines directly in the field, giving the distances out from center line. The clinometer is very useful in moderately sloping country. The transitman may often help the topographer on very steep hillsides by taking vertical slope angles at right angle to the line.

The Mapping consists in platting the instrument line from the transit notes, and the topography or contour lines from the topography notes. The former may be platted with a protractor or by tangent- or chord deflection,* from each previous tangent line; or a base (say north and south) may be established on the map from which each tangent line is laid off by calculated angle. Sometimes the lines are platted from calculated latitudes and departures of the angle points or points of intersection (*P. I.'s*) of the tangents. This latter method has the advantage of accuracy. The magnetic bearings are a check on the calculated bearings of each tangent and errors of reading angles in the field, both in amount and direction.

D.—THE LOCATION SURVEY

The Location is the Objective, or the desired end sought; the other surveys are simply means to this end. The reconnoissance may sometimes be reduced to a mere inspection of the country in a most casual manner. The preliminary survey may often consist in running a few compass lines to determine the general route, or even these may be omitted. But the location survey consists in the final establishment on the ground of the line as it is to be built: running in the tangents and joining them with the proper curves. Of course there are generally minor changes in the line during its construction, but these subsequent changes might be considered part of the location proper.

The Profile and Grades are subjects for constant study; the latter even after the road is constructed and in operation. The grade line (= base of cut or top of fill, and is called sub-grade during construction) is usually adjusted to the profile by the use of a fine thread stretched over the latter in various positions, studying at the same time the "equalization of cuts and fills." By this phrase we do not mean necessarily that the quantities in the cuts should equal those in the fills, although this might hold true for certain saw-toothed profiles or profiles with short haul from cut to fill; and especially where the material in the cuts, instead of borrowed material, would naturally be used in the fills. If the material in cuts is rock, and borrowed material may be had more cheaply for the fills, the grade line would naturally be raised to reduce the cost of cutting. Generally, it may be stated that the grade line is so adjusted that the quantities in cut are

* See Table of Chords, page 959.

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t less than those in fill. In this connection see *earthwork tables*,* 0, etc; and also notes on *shrinkage*, page 909.

oad Curves.—Curves are introduced in both the grade and aline- roads to give easy change of direction to moving trains. The *kind* used for either purpose is chosen by reason of its convenience in ut on the ground. Hence we use, generally, parabolic vertical or connecting grade lines, and circular horizontal curves for con- angents.

cal Parabolic Curves are easily calculated and laid out. The principle arabola is that the *offset* from a *fixed base* is proportional to the *the distance* in a *uniform direction* from a *fixed point*. In Fig. 6,

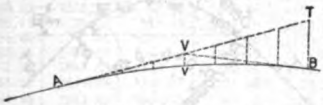

Fig. 6.—Parabolic Vertical Curve.

the vertex or summit of two grade lines passing through A and B, ely; and let it be required to put in a parabolic curve AvB of any ay 800 feet Then A and B, the limiting points of the curve, will ions either side of the vertex. Calculate the elevations of these two Produce AV to T and calculate the maximum offset TB. Then

quare of distance in *stations* from A to any other station is equal to

cal offset at that station from the line AVT to the parabolic curve.

$v = \frac{TB}{8\times 8}\times 4\times 4 = \frac{TB}{4}$. The level and rod are used for vertical off-

vations), and the chain for horizontal distances. Valley curves are n a similar manner to summit curves.

zontal Circular Curves may be simple, compound or reversed. To y be added the infinitely compounded curves—the parabola, and parabola; and also the many forms of spiral and easement curves e approximations to the latter.

le Curves are arcs of circles, and are denoted either by the radius rcle or by the "degree of curve," generally the latter. The degree may be defined as the angle subtended at the center of the circle 00-ft. station" on the arc, whether the chord lengths are 100-ft. flat curves as of 5° or less), 50-ft. (used for sharp curves from 5° or 25-ft. (used for curves above 10° and for turnouts and switches). is seen that the degree of curve is comparable with the radius only *length of chord* by which the curve is laid out, is given; hence,

t. chords, the sine of	$\frac{1}{2}$	the degree of curve	= 50 ft. ÷	radius in ft.
t. " " "	$\frac{1}{4}$	" " "	= 25 ft. ÷	" "
t. " " "	$\frac{1}{8}$	" " "	= 12½ft. ÷	" "

ich the degree of curve may be obtained by multiplying the corres- angles by 2, 4 and 8, respectively. Throughout the following n 100-ft. chords will be assumed unless otherwise noted.

y out a simple curve of D degrees from a point, P. C. (called point or beginning of curve), situated on the tangent line T_1, Fig. p the transit at the *P. C.*, and turn off, *by addition*, from the produced, the successive angles $\frac{D}{2}$ for each 100-ft. station. The

se tables may be copied on profile paper in the form of cubic yards ft. station and arranged so that the quantities can be read off by matching the zero of the table to the grade line of the profile, ing the quantity opposite the ground line. The tables are to feet , or to feet and half feet, and the heights are to the same vertical the profile. *Caution*: Note that the quantities must be increased e ground slope is not level.

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angle $\frac{D}{2}$ is called the deflection angle per station and is always equal to ½ the central angle per station; or ½ the degree of curve, D. From this it will be seen that the *total* deflection angle to any point on the curve, from a tangent to the curve at instrument point, is equal to ½ the *total* central

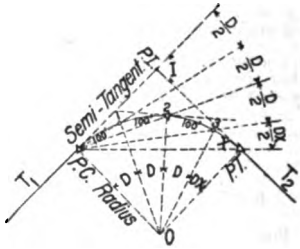

Fig. 7.—Circular Horizontal Curve.

angle subtended by the two points. Thus, the deflection angle to point 2 is $2\times\frac{D}{2}$; to point 3, is $3\times\frac{D}{2}$; and to the $P.\ T.$, is $(3+x)\times\frac{D}{2}$, in which x is any *decimal part* of a 100-ft. station. If, in Fig. 7, the degree of curve $D=30°-30'$, and $x=60$ ft., we have that the total deflection to the end of curve or point of tangent $P.\ T.=(3+x)\times\frac{D}{2}=3.6\times\frac{3°-30'}{2}=6°-18'$; and that the total central angle $=12°36'$. Moreover, the central angle subtended by the *total length of curve*, from the $P.\ C.$ to the $P.\ T.$, is equal to the *angle of intersection* I at the point of intersection $P.\ I.$ of the adjoining tangents T_1 and T_2 produced. The portion of a tangent produced which lies between the $P.\ I.$ and the $P.\ C.$ or $P.\ T.$ is variously termed the "semi-tangent," "tangent distance" or "vertex distance." The length of a curve is the distance between the $P.\ C.$ and $P.\ T.$, measured in chord lengths, and not the true length of the arc. Thus, in Fig. 7, if the last chord x is $\frac{6}{10}$ of a 100-ft. station, the total length of curve is 360 feet. If the degree of curve is $3°-30'$, the radius $=50\div\sin\frac{3°-30'}{2}=1637.28$ ft.; and if the angle of intersection I is $12°-36'$, the semi-tangent $=$ radius $\times\tan\frac{12°-36'}{2}=180.76$ ft. The last is simply the solution of a right-angle triangle with hypothenuse joining o and $P.\ I.$, and with angle at the base (at o) equal to $\frac{I}{2}$. The "external" is the shortest distance from the $P.\ I.$ to the curve, and is measured along the hypothenuse just mentioned. Clearly, it is the length of the hypothenuse minus the radius, and in the present instance is equal to radius $\times$ exsec* $\frac{12°-36'}{2}=9.95$ ft. In fitting a curve in between two adjoining tangents of a preliminary survey, the external distance is often of primary importance in arbitrarily fixing the position (and degree) of the curve. When the degree of the curve is determined the semi-tangents are calculated and laid off from the $P.\ I.$, and the curve run in as explained above.

* Exsecant = secant − 1.

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10.—Radii of English Curves, in Feet.* (Chords 100 Feet.)
Note.—See Foot-note for using this table for Metric curves.
[Radii in Feet.]

Minutes.	Degree of Curve.							
	0°	1°	2°	3°	4°	5°	6°	7°
0′	Infinite	5729.65	2864.93	1910.08	1432.69	1146.28	955.366	819.020
1	343775.	5635.72	2841.26	1899.53	1426.74	1142.47	953.722	817.077
2	171887.	5544.83	2817.97	1889.09	1420.85	1138.69	950.093	815.144
3	114592.	5456.82	2795.06	1878.77	1415.01	1134.94	947.478	813.238
4	85943.7	5371.56	2772.53	1868.56	1409.21	1131.21	944.877	811.303
5	68754.9	5288.92	2750.35	1858.47	1403.46	1127.50	942.291	809.397
6	57295.8	5208.79	2728.52	1848.48	1397.76	1123.82	939.719	807.499
7	49110.7	5131.06	2707.04	1838.59	1392.10	1120.16	937.161	805.611
8	42971.8	5055.59	2685.89	1828.82	1386.49	1116.52	934.616	803.731
9	38197.2	4982.33	2665.06	1819.14	1380.92	1112.91	932.086	801.860
10	34377.5	4911.15	2644.58	1809.57	1375.40	1109.33	929.569	799.997
11	31252.3	4841.98	2624.39	1800.10	1369.92	1105.76	927.066	798.144
12	28647.8	4774.74	2604.51	1790.73	1364.49	1102.22	924.576	796.299
13	26444.2	4709.33	2584.93	1781.45	1359.10	1098.70	922.100	794.462
14	24555.4	4645.69	2565.65	1772.27	1353.75	1095.20	919.637	792.634
15	22918.3	4583.75	2546.64	1763.18	1348.45	1091.73	917.187	790.814
16	21485.9	4523.44	2527.92	1754.19	1343.15	1088.28	914.750	789.003
17	20222.1	4464.70	2509.47	1745.28	1337.65	1084.85	912.326	787.210
18	19098.6	4407.46	2491.29	1736.48	1332.77	1081.44	909.915	785.405
19	18093.4	4351.67	2473.37	1727.75	1327.63	1078.05	907.517	783.618
20	17188.8	4297.28	2455.70	1719.12	1322.53	1074.68	905.131	781.840
21	16370.2	4244.23	2438.29	1710.56	1317.46	1071.34	902.758	780.069
22	15626.1	4192.47	2421.12	1702.10	1312.43	1068.01	900.397	778.307
23	14946.7	4141.96	2404.19	1693.72	1307.45	1064.71	898.048	776.552
24	14323.6	4092.66	2387.50	1685.42	1302.50	1061.43	895.712	774.806
25	13751.0	4044.51	2371.04	1677.20	1297.58	1058.16	893.388	773.067
26	13222.1	3997.49	2354.80	1669.06	1292.71	1054.92	891.076	771.336
27	12732.4	3951.54	2338.78	1661.00	1287.87	1051.70	888.776	769.613
28	12277.7	3906.64	2322.98	1653.01	1283.07	1048.48	886.488	767.897
29	11854.3	3862.74	2307.39	1645.11	1278.30	1045.31	884.211	766.190
30	11459.2	3819.83	2292.01	1637.28	1273.57	1042.14	881.946	764.489
31	11089.6	3777.85	2276.84	1629.52	1268.87	1039.00	879.693	762.797
32	10743.0	3736.79	2261.86	1621.84	1264.21	1035.87	877.451	761.112
33	10417.5	3696.61	2247.08	1614.22	1259.58	1032.76	875.221	759.434
34	10111.1	3657.29	2232.49	1606.68	1254.98	1029.67	873.002	757.764
35	9822.18	3618.80	2218.09	1599.21	1250.42	1026.60	870.795	756.101
36	9549.34	3581.10	2203.87	1591.81	1245.89	1023.55	868.598	754.445
37	9291.29	3544.19	2189.84	1584.48	1241.40	1020.51	866.412	752.796
38	9046.75	3508.02	2175.98	1577.21	1236.94	1017.49	864.238	751.155
39	8814.78	3472.59	2162.30	1570.01	1232.51	1014.50	862.075	749.521
40	8594.42	3437.87	2148.79	1562.88	1228.11	1011.51	859.922	747.894
41	8384.80	3403.83	2135.44	1555.81	1223.74	1008.55	857.780	746.274
42	8185.16	3370.46	2122.26	1548.80	1219.40	1005.60	855.648	744.661
43	7994.81	3337.74	2109.24	1541.86	1215.30	1002.67	853.527	743.055
44	7813.11	3305.65	2096.39	1534.98	1210.82	999.762	851.417	741.456
45	7639.49	3274.17	2083.68	1528.16	1206.57	996.867	849.317	739.864
46	7473.42	3243.29	2071.13	1521.40	1202.36	993.988	847.228	738.279
47	7314.41	3212.98	2058.73	1514.70	1198.17	991.126	845.148	736.701
48	7162.03	3183.23	2046.48	1508.06	1194.01	988.280	843.080	735.129
49	7015.87	3154.03	2034.37	1501.48	1189.88	985.451	841.021	733.564
50	6875.55	3125.36	2022.41	1494.95	1185.78	982.638	838.972	732.005
51	6740.74	3097.20	2010.59	1488.48	1181.71	979.840	836.933	730.454
52	6611.12	3069.55	1998.90	1482.07	1177.66	977.060	834.904	728.909
53	6486.38	3042.39	1987.35	1475.71	1173.65	974.294	832.885	727.370
54	6366.26	3015.71	1975.93	1469.41	1169.66	971.544	830.876	725.838
55	6250.51	2989.48	1964.64	1463.16	1165.70	968.810	828.876	724.312
56	6138.90	2963.71	1953.48	1456.96	1161.76	966.091	826.886	722.793
57	6031.20	2938.39	1942.44	1450.81	1157.85	963.387	824.905	721.280
58	5927.22	2913.49	1931.53	1444.72	1153.97	960.698	822.934	719.774
59	5826.76	2889.01	1920.75	1438.68	1150.11	958.025	820.973	718.273
60	5729.65	2864.93	1910.08	1432.69	1146.28	955 366	819.020	716.779

* Table 10, above, may be used for Metric curves: Radii of curves in meters = values in above table mult. by $\frac{1}{100}$ chord in meters. Ex.—For chord = 20 meters, and degree of curve = 3° 30′: Radius = $\frac{1637.28}{5}$ meters.

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10.—Radii of English Curves, in Feet.—Concluded.

(Chords 100 Ft.)

Note.—See Foot-note preceding page, for use of this table for Metric curves.

[Radii in Feet.]

Min.	Degree of Curve. 8°	9°	Min.	Degree of Curve. 10°	12°	14°	16°	18°	Degree of Curve.	
0′	716.779	637.275	0′	573.686	478.339	410.275	359.265	319.623	20° 0′	287.939
1	715.291	636.099	2	571.784	477.018	409.306	358.523	319.037	10	285.563
2	713.810	634.928	4	569.896	475.705	408.341	357.784	318.453	20	283.267
3	712.335	633.761	6	568.020	474.400	407.380	357.048	317.871	30	280.985
4	710.865	632.599	8	566.156	473.102	406.424	356.315	317.292	40	278.746
5	709.402	631.440	10	564.305	471.810	405.473	355.585	316.715	50	276.541
6	707.945	630.286	12	562.466	470.526	404.526	354.859	316.139	21° 0′	274.370
7	706.493	629.136	14	560.638	469.249	403.583	354.135	315.566	10	272.234
8	705.048	627.991	16	558.823	467.978	402.645	353.414	314.993	20	270.132
9	703.609	626.849	18	557.019	466.715	401.712	352.696	314.426	30	268.062
10	702.175	625.712	20	555.227	465.459	400.782	351.981	313.860	40	266.024
11	700.748	624.579	22	553.447	464.209	399.857	351.269	313.295	50	264.018
12	699.326	623.450	24	551.678	462.966	398.937	350.560	312.732	22° 0′	262.042
13	697.910	622.325	26	549.920	461.729	398.020	349.854	312.172	10	260.096
14	696.499	621.203	28	548.174	460.500	397.108	349.150	311.613	20	258.180
15	695.095	620.087	30	546.438	459.276	396.200	348.450	311.056	30	256.292
16	693.696	618.974	32	544.714	458.060	395.296	347.752	310.502	40	254.431
17	692.302	617.865	34	543.001	456.850	394.396	347.057	309.949	50	252.599
18	690.914	616.760	36	541.298	455.646	393.501	346.365	309.399	23° 0′	250.793
19	689.532	615.660	38	539.606	454.449	392.609	345.676	308.850	10	249.013
20	688.156	614.563	40	537.924	453.259	391.722	344.990	308.303	20	247.258
21	686.785	613.470	42	536.253	452.073	390.838	344.306	307.759	30	245.529
22	685.419	612.380	44	534.593	450.894	389.959	343.625	307.216	40	243.825
23	684.059	611.295	46	532.943	449.722	389.084	342.947	306.675	50	242.144
24	682.704	610.214	48	531.303	448.556	388.212	342.274	306.136	24° 0′	240.487
25	681.354	609.136	50	529.673	447.395	387.345	341.598	305.599	10	238.853
26	680.010	608.062	52	528.053	446.241	386.481	340.928	305.064	20	237.241
27	678.671	606.992	54	526.443	445.093	385.621	340.260	304.531	30	235.652
28	677.338	605.926	56	524.843	443.951	384.765	339.595	304.000	40	234.084
29	676.008	604.864	58	523.252	442.814	383.913	338.933	303.470	50	232.537
30	674.686	603.805		11°	13°	15°	17°	19°	25° 0′	231.011
31	673.369	602.750	0′	521.671	441.684	383.065	338.273	302.943	10	229.506
32	672.056	601.698	2	520.100	440.559	382.220	337.616	302.417	20	228.020
33	670.748	600.651	4	518.539	439.440	381.380	336.962	301.893	30	226.555
34	669.446	599.607	6	516.986	438.326	380.543	336.310	301.371	40	225.108
35	668.148	598.567	8	515.443	437.219	379.709	335.660	300.851	50	223.680
36	666.856	597.530	10	513.909	436.117	378.880	335.013	300.333	26° 0′	222.271
37	665.568	596.497	12	512.385	435.020	378.054	334.369	299.816	10	220.879
38	664.286	595.467	14	510.869	433.929	377.231	333.727	299.302	20	219.506
39	663.008	594.441	16	509.363	432.844	376.412	333.088	298.789	30	218.150
40	661.736	593.419	18	507.865	431.764	375.597	332.451	298.278	40	216.811
41	660.468	592.400	20	506.376	430.690	374.786	331.816	297.768	50	215.489
42	659.205	591.384	22	504.896	429.620	373.977	331.184	297.260	27° 0′	214.183
43	657.947	590.372	24	503.425	428.557	373.173	330.555	296.755	10	212.893
44	656.694	589.364	26	501.962	427.498	372.372	329.928	296.250	20	211.620
45	655.446	588.359	28	500.507	426.445	371.574	329.303	295.748	30	210.362
46	654.202	587.357	30	499.061	425.396	370.780	328.689	295.247	40	209.119
47	652.963	586.359	32	497.624	424.354	369.989	328.061	294.748	50	207.891
48	651.729	585.364	34	496.195	423.316	369.202	327.443	294.251	28° 0′	206.678
49	650.499	584.373	36	494.774	422.283	368.418	326.828	293.756	10	205.480
50	649.274	583.385	38	493.361	421.256	367.637	326.215	293.262	20	204.296
51	648.054	582.400	40	491.956	420.233	366.859	325.604	292.770	30	203.125
52	646.838	581.419	42	490.559	419.215	366.085	324.996	292.279	40	201.969
53	645.627	580.441	44	489.171	418.203	365.315	324.390	291.790	50	200.826
54	644.420	579.466	46	487.790	417.195	364.547	323.786	291.303	29° 0′	199.696
55	643.218	578.494	48	486.417	416.192	363.783	323.184	290.818	10	198.580
56	642.021	577.526	50	485.051	415.194	363.022	322.585	290.334	20	197.476
57	640.828	576.561	52	483.694	414.201	362.264	321.989	289.851	30	196.385
58	639.639	575.599	54	482.344	413.212	361.510	321.394	289.371	40	195.306
59	638.455	574.641	56	481.001	412.229	360.758	320.801	288.892	50	194.240
60	637.275	573.686	58	479.666	411.250	360.010	320.211	288.414	30° 0′	193.185
			60	478.339	410.275	359.265	319.623	287.939		

11.—*Semi-Tangents and Externals to a 1° English Curve, in Feet.†

(Chords 100 Feet.)

Note.—See Foot-note for using this table for Metric curves.

Intersection Angle.	External.	Semi-Tangent.			Intersection Angle.	External.	Semi-Tangent.		
		Distance.	Diff. for 0° 10′	Correct'n per Deg. of curve			Distance.	Diff. for 0° 10′	Correct'n per Deg. of curve
I	E	S-T	$\frac{d}{6}$	Add	I	E	S-T	$\frac{d}{6}$	Add
0°	.00	0.00	8.333	.000	50°	592.32	2671.8	10.183	.034
1	.22	50.00	8.335	.001	51	618.39	2732.9	10.267	.035
2	.87	100.01	8.338	.001	52	645.17	2794.5	10.367	.036
3	1.96	150.04	8.340	.002	53	672.66	2856.7	10.450	.036
4	3.49	200.08	8.347	.003	54	700.89	2919.4	10.550	.037
5	5.46	250.16	8.353	.004	55	729.85	2982.7	10.633	.038
6	7.86	300.28	8.360	.005	57	759.58	3046.5	10.733	.039
7	10.71	350.44	8.370	.005	58	790.08	3110.9	10.850	.040
8	13.99	400.66	8.378	.006	59	821.37	3176.0	10.950	.040
9	17.72	450.93	8.392	.006	60	853.46	3241.7	11.050	.041
10	21.89	501.28	8.403	.007	61	886.38	3308.0	11.167	.042
11	26.50	551.70	8.418	.008	62	920.14	3375.0	11.283	.043
12	31.56	602.21	8.433	.008	63	954.75	3442.7	11.400	.044
13	37.07	652.81	8.450	.009	64	990.24	3511.1	11.533	.044
14	43.03	703.51	8.468	.010	65	1026.6	3580.3	11.650	.045
15	49.44	754.32	8.488	.010	66	1063.9	3650.2	11.783	.046
16	56.31	805.25	8.508	.010	67	1102.2	3720.9	11.917	.047
17	63.63	856.30	8.532	.011	68	1141.4	3792.4	12.050	.048
18	71.42	907.49	8.553	.012	69	1181.6	3864.7	12.200	.049
19	79.67	958.81	8.582	.013	70	1222.7	3937.9	12.333	.050
20	88.39	1010.3	8.600	.014	71	1265.0	4011.9	12.500	.051
21	97.58	1061.9	8.633	.014	72	1308.2	4086.9	12.650	.052
22	107.24	1113.7	8.667	.015	73	1352.6	4162.8	12.817	.053
23	117.38	1165.7	8.700	.016	74	1398.0	4239.7	12.983	.054
24	128.00	1217.9	8.717	.017	75	1444.6	4317.6	13.150	.055
25	139.11	1270.2	8.767	.017	76	1492.4	4396.5	13.333	.056
26	150.71	1322.8	8.800	.018	77	1541.4	4476.5	13.517	.057
27	162.81	1375.6	8.833	.019	78	1591.6	4557.6	13.700	.058
28	175.41	1428.6	8.867	.019	79	1643.0	4639.8	13.900	.059
29	188 51	1481.8	8.917	.020	80	1695.8	4723.2	14.083	.060
30	202.12	1535.3	8.950	.021	81	1749.9	4807.7	14.317	.061
31	216.25	1589.0	9.000	.021	82	1805.3	4893.6	14.517	.062
32	230.90	1643.0	9.033	.022	83	1862.2	4980.7	14.750	.063
33	246.08	1697.2	9.083	.023	84	1920.5	5069.2	14.967	.064
34	261.80	1751.7	9.150	.024	85	1980.4	5159.0	15.217	.065
35	278.05	1806.6	9.183	.024	86	2041.7	5250.3	15.450	.066
36	294.86	1861.7	9.233	.025	87	2104.7	5343.0	15.700	.067
37	312.22	1917.1	9.300	.026	88	2169.2	5437.2	15.983	.068
38	330.15	1972.9	9.350	.026	89	2235.5	5533.1	16.233	.070
39	348.64	2029.0	9.400	.027	90	2303.5	5630.5	16.533	.071
40	367.72	2085.4	9.467	.028	91	2373.3	5729.7	16.800	.072
41	387.38	2142.2	9.533	.028	92	2444.9	5830.5	17.117	.073
42	407.64	2199.4	9.600	.029	93	2518.5	5933.2	17.433	.075
43	428.50	2257.0	9.650	.030	94	2584.0	6037.8	17.750	.076
44	449.98	2314.9	9.733	.031	95	2671.6	6144.3	18.083	.078
45	472.08	2373.3	9.800	.031	96	2751.3	6252.8	18.433	.079
46	494.82	2432.1	9.867	.032	97	2833.2	6363.4	18.800	.080
47	518.20	2491.3	9.950	.033	98	2917.3	6476.2	19.167	.082
48	542.23	2551.0	10.033	.034	99	3003.8	6591.2	19.567	.083
49	566.94	2611.2	10.100	.034	100	3092.7	6708.6	19.950	.085
50	592.32	2671.8				3184.1	6828.3		.086

Examples of Use of Table.

Note.—Where *externals* are required accurately, *add* the following correction to result obtained from table:

(a) For $I = 0°$ to $50°$, Cor. $= +.000003\,I^2\,D$.

(b) For $I = 100°$, Cor. $= +.00004\,I^2\,D$.

In which I = intersection angle, and D = degree of curve, both in degrees.

Ex. 1.—To find the external E to 3° curve for $I = 20°\,13'$:

Solution:

For 1° cv. $E = 88.39$ for $I = 20°$
$+ 1.99$ for 13 diff.
$- 90.38$ for $I = 20°\,13'$
do. Dividing by 3, for 3° curve,
For 3° cv. $E = 30.13$ for $I = 20°\,13'$
Correction $+ .00$ ft'm (a) above
$E = 30.13$ Ans.

Ex. 2.—To find the semi-tang. S-T to a 4° curve, for $I = 16°\,25'$:

Solution:

For 1° cv. S-$T = 805.25$ for $I = 16°$
$+ 21.27$ for 25′ diff.
826.52 for $I = 16°\,25'$
do. Dividing by 4, for 4° curve,
For 4° cv. S-$T = 206.63$ for $I = 16°\,25'$
Correction $.04\ (.010 \times 4)$
S-$T = 206.67$ Ans.

These results, to hundredths, are closer than will ordinarily be measured in the field. Note that the corrections are very small and can generally be disregarded when I and D are small.

* Semi-tangents and externals are (almost exactly) inversely proportional to the degree of curve D, for the same intersection angle I.

† Table 11, above, may be used for Metric curves: Semi-tangents and externals in meters = values in above table mult. by $\frac{1}{100}$ chord in meters. Ex.—For chord = 20 meters, and intersection angle = 12°; then semi-tang = $\frac{2.21}{5}$ meters, and external = $\frac{31.56}{5}$ meters, for a 1° curve.

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12.—Minutes and Seconds Reduced to Decimals of a Degree or Hour.*

(Either Angular Measure or Time Measure.)
Note.—See Foot-note regarding use of this table.
[Decimals of a Degree or Hour.]

Add for seconds.	P. P.
	.00139
1	.00028
2	.00056
3	.00083
4	.00111

Min.	Seconds. 0"	5"	10"	15"	20"	25"	30"	35"	40"	45"	50"	55"
0'	.00000	.00139	.00278	.00417	.00556	.00694	.00833	.00972	.01111	.01250	.01389	.01528
1	.01667	.01806	.01944	.02083	.02222	.02361	.02500	.02639	.02778	.02917	.03056	.03194
2	.03333	.03472	.03611	.03750	.03889	.04028	.04167	.04306	.04444	.04583	.04722	.04861
3	.05000	.05139	.05278	.05417	.05556	.05694	.05833	.05972	.06111	.06250	.06389	.06528
4	.06667	.06806	.06944	.07083	.07222	.07361	.07500	.07639	.07778	.07917	.08056	.08194
5	.08333	.08472	.08611	.08750	.08889	.09028	.09167	.09306	.09444	.09583	.09722	.09861
6	.10000	.10139	.10278	.10417	.10556	.10694	.10833	.10972	.11111	.11250	.11389	.11528
7	.11667	.11806	.11944	.12083	.12222	.12361	.12500	.12639	.12778	.12917	.13056	.13194
8	.13333	.13472	.13611	.13750	.13889	.14028	.14167	.14306	.14444	.14583	.14722	.14861
9	.15000	.15139	.15278	.15417	.15556	.15694	.15833	.15972	.16111	.16250	.16389	.16528
10	.16667	.16806	.16944	.17083	.17222	.17361	.17500	.17639	.17778	.17917	.18056	.18194
11	.18333	.18472	.18611	.18750	.18889	.19028	.19167	.19306	.19444	.19583	.19722	.19861
12	.20000	.20139	.20278	.20417	.20556	.20694	.20833	.20972	.21111	.21250	.21389	.21528
13	.21667	.21806	.21944	.22083	.22222	.22361	.22500	.22639	.22778	.22917	.23056	.23194
14	.23333	.23472	.23611	.23750	.23889	.24028	.24167	.24306	.24444	.24583	.24722	.24861
15	.25000	.25139	.25278	.25417	.25556	.25694	.25833	.25972	.26111	.26250	.26389	.26528
16	.26667	.26806	.26944	.27083	.27222	.27361	.27500	.27639	.27778	.27917	.28056	.28194
17	.28333	.28472	.28611	.28750	.28889	.29028	.29167	.29306	.29444	.29583	.29722	.29861
18	.30000	.30139	.30278	.30417	.30556	.30694	.30833	.30972	.31111	.31250	.31389	.31528
19	.31667	.31806	.31944	.32083	.32222	.32361	.32500	.32639	.32778	.32917	.33056	.33194
20	.33333	.33472	.33611	.33750	.33889	.34028	.34167	.34306	.34444	.34583	.34722	.34861
21	.35000	.35139	.35278	.35417	.35556	.35694	.35833	.35972	.36111	.36250	.36389	.36528
22	.36667	.36806	.36944	.37083	.37222	.37361	.37500	.37639	.37778	.37917	.38056	.38194
23	.38333	.38472	.38611	.38750	.38889	.39028	.39167	.39306	.39444	.39583	.39722	.39861
24	.40000	.40139	.40278	.40417	.40556	.40694	.40833	.40972	.41111	.41250	.41389	.41528
25	.41667	.41806	.41944	.42083	.42222	.42361	.42500	.42639	.42778	.42917	.43056	.43194
26	.43333	.43472	.43611	.43750	.43889	.44028	.44167	.44306	.44444	.44583	.44722	.44861
27	.45000	.45139	.45278	.45417	.45556	.45694	.45833	.45972	.46111	.46250	.46389	.46528
28	.46667	.46806	.46944	.47083	.47222	.47361	.47500	.47639	.47778	.47917	.48056	.48194
29	.48333	.48472	.48611	.48750	.48889	.49028	.49167	.49306	.49444	.49583	.49722	.49861
30	.50000	.50139	.50278	.50417	.50556	.50694	.50833	.50972	.51111	.51250	.51389	.51528
31	.51667	.51806	.51944	.52083	.52222	.52361	.52500	.52639	.52778	.52917	.53056	.53194
32	.53333	.53472	.53611	.53750	.53889	.54028	.54167	.54306	.54444	.54583	.54722	.54861
33	.55000	.55139	.55278	.55417	.55556	.55694	.55833	.55972	.56111	.56250	.56389	.56528
34	.56667	.56806	.56944	.57083	.57222	.57361	.57500	.57639	.57778	.57917	.58056	.58194
35	.58333	.58472	.58611	.58750	.58889	.59028	.59167	.59306	.59444	.59583	.59722	.59861
36	.60000	.60139	.60278	.60417	.60556	.60694	.60833	.60972	.61111	.61250	.61389	.61528
37	.61667	.61806	.61944	.62083	.62222	.62361	.62500	.62639	.62778	.62917	.63056	.63194
38	.63333	.63472	.63611	.63750	.63889	.64028	.64167	.64306	.64444	.64583	.64722	.64861
39	.65000	.65139	.65278	.65417	.65556	.65694	.65833	.65972	.66111	.66250	.66389	.66528
40	.66667	.66806	.66944	.67083	.67222	.67361	.67500	.67639	.67778	.67917	.68056	.68194
41	.68333	.68472	.68611	.68750	.68889	.69028	.69167	.69306	.69444	.69583	.69722	.69861
42	.70000	.70139	.70278	.70417	.70556	.70694	.70833	.70972	.71111	.71250	.71389	.71528
43	.71667	.71806	.71944	.72083	.72222	.72361	.72500	.72639	.72778	.72917	.73056	.73194
44	.73333	.73472	.73611	.73750	.73889	.74028	.74167	.74306	.74444	.74583	.74722	.74861
45	.75000	.75139	.75278	.75417	.75556	.75694	.75833	.75972	.76111	.76250	.76389	.76528
46	.76667	.76806	.76944	.77083	.77222	.77361	.77500	.77639	.77778	.77917	.78056	.78194
47	.78333	.78472	.78611	.78750	.78889	.79028	.79167	.79306	.79444	.79583	.79722	.79861
48	.80000	.80139	.80278	.80417	.80556	.80694	.80833	.80972	.81111	.81250	.81389	.81528
49	.81667	.81806	.81944	.82083	.82222	.82361	.82500	.82639	.82778	.82917	.83056	.83194
50	.83333	.83472	.83611	.83750	.83889	.84028	.84167	.84306	.84444	.84583	.84722	.84861
51	.85000	.85139	.85278	.85417	.85556	.85694	.85833	.85972	.86111	.86250	.86389	.86528
52	.86667	.86806	.86944	.87083	.87222	.87361	.87500	.87639	.87778	.87917	.88056	.88194
53	.88333	.88472	.88611	.88750	.88889	.89028	.89167	.89306	.89444	.89583	.89722	.89861
54	.90000	.90139	.90278	.90417	.90556	.90694	.90833	.90972	.91111	.91250	.91389	.91528
55	.91667	.91806	.91944	.92083	.92222	.92361	.92500	.92639	.92778	.92917	.93056	.93194
56	.93333	.93472	.93611	.93750	.93889	.94028	.94167	.94306	.94444	.94583	.94722	.94861
57	.95000	.95139	.95278	.95417	.95556	.95694	.95833	.95972	.96111	.96250	.96389	.96528
58	.96667	.96806	.96944	.97083	.97222	.97361	.97500	.97639	.97778	.97917	.98056	.98194
59	.98333	.98472	.98611	.98750	.98889	.99028	.99167	.99306	.99444	.99583	.99722	.99861

*Ex. in Angular measure: 20° 35′ 17″ = 20° 35′ 15″ + 2″ = 20.58806 deg.
Ex. in Time measure: [illegible]

The Various Problems in Simple Curves may all be solved without the use of formulas. It is necessary only to draw a sketch of the conditions of the problem for any particular case, set down the *known* data governing the case, and solve trigonometrically for the *unknown*. The position of the center of the circle or circular arc is the *key* to all solutions. A few hints will be given to illustrate:

To move a curve (shown in full lines, Fig. 8) joining two tangents (T_1 and T_2), so it will end (dotted lines) in a parallel tangent (T_3): The distance moved ($=m$) = the perpendicular distance between the tangents (d) ÷ sine of intersection angle (I).

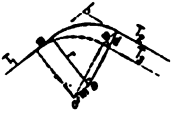

Fig. 8.

To change the radius r (full) to r' (dotted) so that curve ending in tangent T_2 shall end in parallel tangent T_3 (Fig. 9): The end of the new curve lies on the long chord as shown. Then $n = d \div \sin \frac{I}{2}$, and $r - r' = \frac{n}{2} \div \sin\frac{I}{2}$; therefore $r' = r - \left(d \div 2\sin^2\frac{I}{2}\right)$, in which d = perpendicular distance between T_2 and T_3. Or, $r' = r - d \div (1 - \cos I) = r - d \div \text{vers } I$. The reverse holds true when radius is increased.

Fig. 9.

To join a curve (c) and a point (P) by a tangent (T_2): The tangent T_1 and the curve c fit the contour line around the hill H, and it is proposed to throw off a tangent T_2, from some point on the curve, so it will pass through the point P, ahead. One practical method of doing this in the field is to assume the position of the *P. T.*, turn the angle for the tangent ahead, measure the distance and offset to P, and from this data calculate the true position of the *P. T.*, and "move ahead" or "back up" accordingly. If, however, the position of the point P has been located, the position of the *P. T.* can be fixed accurately by calculation. Reduce the location of P (no matter how located) to the distance d from the center of the circle at o, and calculate the angle α. The distance r is the radius of the curve. This gives a calculated tie from the *P. C.* to P. Now

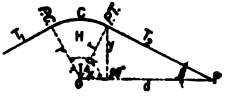

Fig. 10.

$$x^2 + y^2 = r^2 \quad \text{.....(1)}$$

and

$$\text{Tan } \beta = \frac{x}{y} = \frac{y}{d - x}; \text{ or } y^2 + x^2 = dx \quad \text{.....(2)}$$

Equating (1) and (2).

$$dx = r^2, \text{ or } x = \frac{r^2}{d} \quad \text{.....(3)}$$

From which y, $\tan \beta \left(= \frac{x}{y}\right)$, $I\ (= \alpha - 90° + \beta)$, etc., can be found.

Compound Curves are made up of two or more curves of different radii, directly joining each other, and curving in the same direction. They are not difficult of analysis. They key to the solution of any problem lies in putting the data in the right shape and working from the centers of the curves. Fig. 11 needs no explanation other than that the *P. C. C.* is the point of compound curve, and the total angle of intersection between tangents T_1 and T_2 is the sum of the separate intersection angles I and i. Note also that the length of tangent to the *P. C. C.* is equal to the *S-T* of the large curve + the *s-t* of the small curve; hence the total vertex distances, v' and v, from the *P. C.* and *P. T.*, respectively, bear a relation to the rest of the data. The following table illustrates a few simple problems and solutions which may be had from Fig. 11. The known quantities are indicated by letters; and the answers required, by blank spaces in each line. The formula for solution in each case is at the right. Problems with other data may be reduced to these forms before solving.

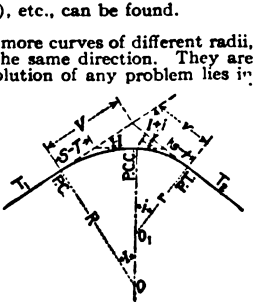

Fig. 11.

13.—Solutions of Compound Curve Problems.

(See Fig. 11).

Note.—Capital letters refer to the curve of larger radius; small letters to curve of smaller radius.

No.	Angles of Intersection.			Radii.		Vertex Distanc's		Solution.
	$I+i$	I	i	R	r	V	v	(Descriptions refer to Fig. 11.)
1	.....	I	i	R	r	....	....	Solve for S-T, s-t, small triangle, V, v. $(I+i)$.
2	$I+i$	I or i		R	....	V	....	$R-r=[R \text{ vers } (I+i)-V \sin (I+i)]+\text{vers } i$; and $v=R \sin (I+i)-(R-r) \sin i-V \cos (I+i)$.
3	$I+i$	....	....	R	r	V	....	Vers $i=[R \text{ vers } (I+i)-V \sin (I+i)]+(R-r)$; and $v=R \sin (I+i)-(R-r) \sin i-V \cos (I+i)$.
4	$I+i$	....	....	R	....	V	v	Tan $\frac{1}{2}I=[R \text{ vers } (I+i)-V \sin (I+i)]+[R \sin (I+i)-V \cos (I+i)-v]$; and $R-r=[R \sin (I+i)-V \cos (I+i)-v]+\sin i$.
5	$I+i$	I or i		.....	r	....	v	$R-r=[v \sin (I+i)-r \text{ vers } (I+i)]+\text{vers } I$; and $V=(R-r) \sin I+r \sin (I+i)-v \cos (I+i)$.
6	$I+i$	....	....	R	r	....	v	Vers $I=[v \sin (I+i)-r \text{ vers } (I+i)]+(R-r)$; and $V=(R-r) \sin I+r \sin (I+i)-v \cos (I+i)$.
7	$I+i$	....	....	.....	r	V	v	Tan $\frac{1}{2}I=[v \sin (I+i)-r \text{ vers } (I+i)]+[V+v \cos (I+i)-r \sin (I+i)$; and $R-r=[V+v \cos (I+i)-r \sin (I+i)]+\sin I$.

Note.—For Nos. 1, 2 and 5, either I or i may be given.

Reversed Curves should be avoided, especially for main line traffic. This may be done by "separating" the two simple curves on either side of the point of reversed curve (*P. R. C.*) and joining them by a short

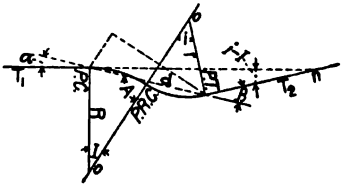

Fig. 12.

tangent. The curves may be separated by moving them back from the *P. C.* and *P. T.*, or by making them "sharper." To find the relation existing between the known and unknown data in Fig. 12:

Let d = the length of the line joining the *P. C.* with the *P. T.*;

α = the angle which the line d makes with T_1;

β = the angle which the line d makes with T_2.

Then $\sin A = \frac{1}{2}(\cos \alpha + \cos \beta)$ from which the angle A, is obtained:

$I = \alpha + 90° - A$;

$i = \beta + 90° - A = I - \alpha + \beta$;

and $R = (d \sin A) + (\sin I + \sin i)$ .....(1)

In Fig. 13 the distance d is measured between points on two tangents T_1 and T_2; and the angles I and i, which the line d makes with these tangents, are known. It is required to fit in a reverse curve of common radius R:

$$R = \frac{d}{\tan \frac{1}{2} I + \tan \frac{1}{2} i}.$$

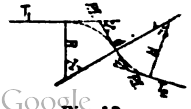

Fig. 13.

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The Cubic Parabola is the principal form of parabolic curve used, and is fundamental of most so-called spiral- or easement curves. The equation of the cubic parabola is $y = nx^3$, in which x is the abscissa, and y the ordinate to any point p, Fig. 14. The constant n is a decimal, and may have any value: small for a flat curve, and large for a sharp curve. If the cubic parabola is laid off by deflection angles from the point o on the tangent T, then the deflection angle* to point 1 is d; to point 2, $4d$; to point 3, $9d$; to point 4, $16d$; to point 5, $25d$; etc. That is, while the ordinate y to any point p is proportional to x^3, the deflection angle is proportionate to x^2. It is to be noted also that x is measured along the tangent produced and not along the curve. The effect of this is, of course, to gradually increase the lengths of the "stations" from point o: as 4–5 > 3–4 > 2–3 > 1–2 > 0–1. For this reason the cubic parabola has never been popular, but has given way to the spiral curve. Both of them are very flat at the ends and grow rapidly sharper toward the center of the curve.

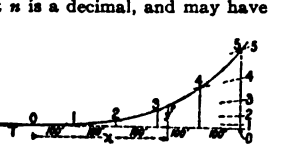

Fig. 14.

The Spiral Curve is a modification of the cubic parabola. It is based on *chords of equal length*, with the curve *compounded* at the end of each chord. The chords may be of any length from 10 ft. to 50 ft., while 30 ft. is quite usual. The degree of curve of the first arc, subtended by the chord 0–1, is the common difference for the degree of curve of successive arcs. Thus, if the central angle of the curve 0–1 is A, then 0–1 will incline at an angle of $\frac{1}{2}A$ with the tangent T; 1–2, at an angle of $\frac{1}{2}A \times 4 = 2A$; 2–3, $\frac{1}{2}A \times 9 = 4\frac{1}{2}A$; 3–4, $\frac{1}{2}A \times 16 = 8A$; 4–5, $\frac{1}{2}A \times 25 = 12\frac{1}{2}A$; etc. This can be proved by making the central angles A, $2A$, $3A$, $4A$, $5A$, respectively. The offsets from tangent T to points 1, 2, 3, etc. are calculated successively, knowing the length of chords and their angles of inclination with T; as are also the horizontal distances between the offset lines, as x_1, x_2, etc. Then, the tangent of the deflection angle from o to any point on the curve is $\frac{y}{x}$ †.

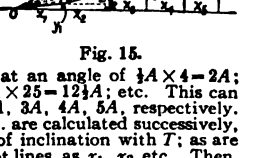

Fig. 15.

Simple Easement Curves are sometimes approximated from simple curves, as follows: Run out the regular simple curve from the tangent T as shown dotted. Lay off the required curve *inside*, by measuring the offset distance d from the outer curve. Lay off the point $P.S.$ of the spiral starting from the tangent, and also the end of the spiral at the $P.C.C.$ where it joins the new curve, *equidistant* from the point c, which *bisects* the offset distance d at the $P.C.$ The offset distance may average from 2 to 4 ft.; and the distance from $P.S.$ to $P.C.$ may vary from 50 ft. to 100 ft. The writer can recommend these curves for fast-train service. Note that the inner curve may be "run in" directly with the instrument, the outer curve being omitted.

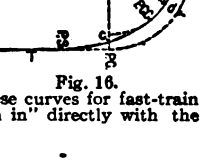

Fig. 16.

E.—RIGHT OF WAY.

Filing of Location.—After the final location is adopted it is filed with the Secretary of State for the particular State through which the line is projected. The following is a typical description of location: Beginning at a stake on the shore of Huron Bay, near high-water mark and westerly about 325 feet from the westerly corner of the old stone fort in the town of Stamford, county of Huntoon, and State of Ohio; thence north 84° 30′ west, four thousand three hundred ten (4310) feet to a stake in the county road in front of Judge Pritchard's farm-house; thence by a curve to the

* Approximately; but almost exact for flat curves. To be strictly exact, the "*natural tangent* of the deflection angle" should be substituted for "deflection angle."

† The "Railroad Spiral," by W. H. Searles, contains tables for laying out any spiral in the field, with instrument at any point on the spiral.

right, with a radius of nineteen hundred ten (1910) feet, a distance of three hundred fifty (350) feet to a stake; thence by a tangent to said curve North 74° West, 12652 feet to a stake - - - - etc.

Purchase and Condemnation.—The Real-Estate Agent of the road is provided with maps of the located line showing land lines, owners' names, and widths of right-of-way desired through the various parcels of land. The usual width is 100 feet, but this should be exceeded in the case of heavy cuts and fills. The following table, No. 14, will be useful for reference in this connection. Where land cannot be purchased at a reasonable figure it may be condemned and the condemnation price so fixed is called an "award."

Oftentimes the award may include a parcel of land of considerable size, not required strictly for right-of-way purposes. In such cases it is a great mistake for the R. R. Co. to dispose of any of this land without first considering whether it is liable to be needed for a passing siding, or for a spur track to some prospective manufacturing plant. The writer can instance many cases in which supposed surplus property has been disposed of at a low figure and repurchased for an amount four or five times as great.

14.—Table for Finding Width of Right-of-Way for Cuts and Fills.

Note.—Total width between slope stakes = width of base of roadway + sum of horizontal distances for slopes (from table).

[Horizontal Distance in Feet for One Slope.]

Height in Ft.	Side Slope.										
	⅛ to 1.	¼ to 1.	⅓ to 1.	½ to 1.	1 to 1.	1¼ to 1.	1½ to 1.	1¾ to 1.	2 to 1.	2½ to 1.	2½ to 1.
4	.8	1	2	3	4	5	6	7	8	9	10
8	1.6	2	4	6	8	10	12	14	16	18	20
12	2.4	3	6	9	12	15	18	21	24	27	30
16	3.2	4	8	12	16	20	24	28	32	36	40
20	4.0	5	10	15	20	25	30	35	40	45	50
24	4.8	6	12	18	24	30	36	42	48	54	60
28	5.6	7	14	21	28	35	42	49	56	63	70
32	6.4	8	16	24	32	40	48	56	64	72	80
36	7.2	9	18	27	36	45	54	63	72	81	90
40	8.0	10	20	30	40	50	60	70	80	90	100
44	8.8	11	22	33	44	55	66	77	88	99	110
48	9.6	12	24	36	48	60	72	84	96	108	120
52	10.4	13	26	39	52	65	78	91	104	117	130
56	11.2	14	28	42	56	70	84	98	112	126	140
60	12.0	15	30	45	60	75	90	105	120	135	150
64	12.8	16	32	48	64	80	96	112	128	144	160
68	13.6	17	34	51	68	85	102	119	136	153	170
72	14.4	18	36	54	72	90	108	126	144	162	180
76	15.2	19	38	57	76	95	114	133	152	171	190
80	16.0	20	40	60	80	100	120	140	160	180	200
84	16.8	21	42	63	84	105	126	147	168	189	210
88	17.6	22	44	66	88	110	132	154	176	198	220
92	18.4	23	46	69	92	115	138	161	184	207	230
96	19.2	24	48	72	96	120	144	168	192	216	240
100	20.0	25	50	75	100	125	150	175	200	225	250
104	20.8	26	52	78	104	130	156	182	208	234	260
108	21.6	27	54	81	108	135	162	189	216	243	270
112	22.4	28	56	84	112	140	168	196	224	252	280
116	23.2	29	58	87	116	145	174	203	232	261	290
120	24.0	30	60	90	120	150	180	210	240	270	300

Ex.—Width of base of roadway = 28 ft.; height. 48 ft. (ground slope about level;) side slopes, 1½ to 1. Then width between slope stakes = 28 + 60 + 60 = 148 ft. (Retaining walls are often used to narrow the required right-of-way.)

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15.—Acres per 100-Ft. Station for Various Widths in Feet.

Note.—Supply space occupied by dash in left-hand column by the unit ire at top of columns.

[Acres per 100-Ft. Station.]

in Ft.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	P. P
	—	.00230	.00459	.00689	.00918	.01148	.01377	.01607	.01837	.02066	For
	.02296	.02525	.02755	.02984	.03214	.03444	.03673	.03903	.04132	.04362	Tenths
	.04591	.04821	.05051	.05280	.05510	.05739	.05969	.06198	.06428	.06657	.00230
	.06887	.07117	.07346	.07576	.07805	.08035	.08264	.08494	.08724	.08953	1 .00023
	.09183	.09412	.09642	.09871	.10101	.10331	.10560	.10790	.11019	.11249	2 .00046
											3 .00069
	.11478	.11708	.11938	.12167	.12397	.12626	.12856	.13085	.13315	.13545	4 .00092
	.13774	.14004	.14233	.14463	.14692	.14922	.15152	.15381	.15611	.15840	5 .00115
	.16070	.16299	.16529	.16758	.16988	.17218	.17447	.17677	.17906	.18136	6 .00138
	.18365	.18595	.18825	.19054	.19284	.19513	.19743	.19972	20202	.20432	7 .00161
	.20661	.20891	.21120	.21350	.21579	.21809	.22039	.22268	.22498	.22727	8 .00184
											9 .00207
	.22957	.23186	.23416	.23646	.23875	.24105	.24334	.24564	.24793	.25023	

Ex.—Width = 90.75 ft. Then we have 0.20661 + 0.00161 + 0.00011 = 0833 acre per 100-ft. station.

16.—Acres per Mile for Various Widths in Feet.

(See also Table 17, following.)

Note.—Supply space occupied by dash in left-hand column by the unit ire at top of column.

[Acres per Mile.]

in Ft.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	P. P
	—	.121	.242	.364	.485	.606	.727	.848	.970	1.091	For
	1.212	1.333	1.455	1.576	1.697	1.818	1.939	2.061	2.182	2.303	Tenths
	2.424	2.545	2.667	2.788	2.909	3.030	3.152	3.273	3.394	3.515	.121
	3.636	3.758	3.879	4.000	4.121	4.242	4.364	4.485	4.606	4.727	1 .012
	4.848	4.970	5.091	5.212	5.333	5.455	5.576	5.697	5.818	5.939	2 .024
											3 .036
	6.061	6.182	6.303	6.424	6.545	6.667	6.788	6.909	7.030	7.152	4 .048
	7.273	7.394	7.515	7.636	7.758	7.879	8.000	8.121	8.242	8.364	5 .061
	8.485	8.606	8.727	8.848	8.970	9.091	9.212	9.333	9.455	9.576	6 .073
	9.697	9.818	9.939	10.061	10.182	10.303	10.424	10.545	10.667	10.788	7 .085
	10.909	11.030	11.152	11.273	11.394	11.515	11.636	11.758	11.879	12.000	8 .097
											9 .109
	12.121	12.242	12.364	12.485	12.606	12.727	12.848	12.970	13.091	13.212	

Ex.—Width = 90.75 ft. Then we have 10.909 + 0.085 + 0.006 = 11.000 es per mile.

17.—Acres for 1 Foot in Width and for Various Distances in Miles.

(Use above Table No. 16, changing the headings as below.)

Note.—For strip 100 ft. wide, mult. values in table by 100.

[Acres for 1 Foot in Width.]

is- ice. es.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	P. P.
			Body of Table same as Table 16.								

Ex.—Strip 1 ft. wide and 90.75 miles long. Then we have 10.9094 085 + 0.006 = 11.000 acres. For strip 100 ft. wide we have 1100 acres.

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F.—CONSTRUCTION.

Earthwork Calculations.—The preliminary and location survey maps and profiles should give all the information necessary to estimate the cost of constructing the line. Borings should be made or test-pits dug occasionally in order to classify the material in excavation. Several methods are used in making preliminary estimates of earthwork, and the three following are worthy of note:

(1).—The cross-section at each station, and at intermediate points if necessary, may be plotted on cross-section paper from the profile and

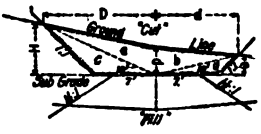

Fig. 17.

topography notes, as shown in Fig. 17. The distances D, d, H, h (and p) are then scaled and used in the following formula for area of cross-section in "cut," and similarly in "fill:"

$$\text{Area in sq. ft.} = \tfrac{1}{2}\,[p(D+d)+10\,(H+h)] \qquad (1)$$

where 10 ft. is ½ the width of roadbed in excavation. For quantities in fill, 10 would be replaced by 7, if the roadbed is to be 14 ft. wide. The cubical contents in feet of the solid figure of which the above cross-section is considered as the "average area" is found by multiplying this area by ½ the sum of the distances to the adjacent cross-sections on either side. The *summation* of the contents in cubic feet is reduced to cubic yards, as the cost price is in that denomination. (Use planimeter if preferred.)

(2).—If the topography has been taken with a clinometer and the ground line is straight, as in Fig. 18, instead of broken as shown in Fig. 17, there are three methods in use for determining areas:

Fig. 18.

(a).—Plat the ground line G by means of the profile height h and the angle of inclination α; scale G and H. Then, for the etched portion or cut: Area $= \frac{1}{2}GH - C$; in which C is the area of triangle below roadbed base, to V. In the figure, the constant C is 100 sq. ft. It varies with the side slopes and width of roadbed. (Use planimeter if preferred.)

(b).—Prepare tables of areas for various heights h, and various ground slopes α; and for the standard roadbed and side slopes. For loose rock in excavation the side slope may be say ½ : 1; in embankment, 1 : 1. Earth 1 : 1 for cut, and 1½ : 1 for fill. The calculation of the table may be based on the following formula:

$$\text{Area required, cut or fill} = \tfrac{1}{2}(h+v)^2 \cos^2\alpha[\cot(\beta+\alpha)+\cot(\beta-\alpha)] - C \qquad (2)$$

In which Area required = area of the etched portion in Fig. 18,

h = center height of cut or fill,
v = vertex distance below roadbed,
α = angle of ground slope with the horizontal,
β = angle of side slopes with the horizontal,
C = area of triangle (ht. $= v$) below roadbed.

When $\beta = 45°$ the formula is somewhat simplified. It is to be noted that $(h+v)$ may be assumed as unity for various ground and side slopes, and afterward expanded by squares, finally deducting the value of the constant area C. The ground slope must not intercept the roadbed.

(c).—Tables may be prepared giving *correction areas* for slopes, to be *added* to tables for "level sections" (see Case 3), instead of tables of actual areas as previously described. From Fig. 18 it will be seen that sloping ground always gives a *plus* correction, as the small triangle below the elevated side of the ground line G is greater than the one above the depressed side. The correction tables may be for areas of cross-section, or

for cubic yards or cubic feet per station of 100 ft., 50 ft., etc. See Table 22, page 1060; also Table 27, page 1039.

(3).—Tables of "level sections" may be used in preliminary estimates where the ground is fairly "level;" or in connection with the previously described correction tables, where the transverse ground line is sloping. These tables may be calculated from the following formula, modified from formula (2), page 1016, by making $\alpha = 0$, and using the previous notation:

$$A = \text{Area required} = (h+v)^2 \cot \beta - C \ldots\ldots\ldots\ldots (3)$$

Or, giving C ($= v^2 \cot \beta$) its value in terms of v and β,

$$A = (h^2 + 2\,h\,v) \cot \beta \ldots\ldots\ldots\ldots (3)$$

When the side slopes are 1 : 1, $\beta = 45°$, and

$$A = (h+v)^2 - C = h^2 + 2\,h\,v \ldots\ldots\ldots\ldots (4)$$

When the side slopes are 1½ : 1;

$$A = \tfrac{2}{3}\,(h+v)^2 - C = \tfrac{2}{3}\,(h^2 + 2\,h\,v); \ldots\ldots\ldots\ldots (5)$$

and similarly for any other side slope.

These tables may be copied on profile paper in the form of cubic yards per 100-ft. station, and arranged so that the quantities can be read off directly by matching the zero of the table to the grade line of the profile, and reading the quantity opposite the ground line. The profile tables are usually to feet in height; or to feet and half feet.

List of Earthwork Tables, Following.

(And also tables relevant thereto.)

Table No.	Description.	Page.
18.	Multiplication table, up to 60×60	1018-1019
18a.	Multiplication table, up to 89×35	1020
19.	End areas reduced to cu. yds. per station (Equiv. 1-10)	1021
20.	End areas (1-27000) reduced to cu. yds. per station	1022-1027
21.	Method of calculating earthwork tables	1028-1029
22.	Formulas for calculating ground-slope quantities	1030-1033
23.	Level sections; Heights, 0-60 ft.; Roadway, 14 ft.; Slopes, 1½ to 1	1034
24.	" " " 60-120 " " 14 " " 1½ to 1	1035
25.	" " " 0-60 " " 16 " " 1 to 1	1036
26.	" " " 0-60 " " 16 " " 1½ to 1	1037
27.	" " " 0-60 " " 18 " " 1 to 1	1038
	with corrections for ground slopes	1039
28.	Level sections; Heights, 60-100 ft.; Roadway, 18 ft.; Slopes, 1 to 1	1040
	with corrections for ground-slopes	1041
29.	Formulas for extending tables of level sections to any side slopes	1042
30.	Factors for extending tables of level sections to other widths and slopes	1042
	Level sections; Heights, 0-60 ft.; Roadway, 18 ft.; Slopes, 1½ to 1	1043
	" " " 0-60 " " 20 " " ¼ to 1	1044
	" " " 0-60 " " 20 " " ½ to 1	1045
	" " " 0-60 " " 20 " " 1 to 1	1046
	" " " 0-60 " " 22 " " 1 to 1	1047
	" " " 0-60 " " 24 " " 1½ to 1	1048
	" " " 0-60 " " 26 " " 1½ to 1	1049
	" " " 0-60 " " 28 " " 1 to 1	1050
	" " " 0-60 " " 28 " " 1½ to 1	1051
	" " " 0-60 " " 30 " " 1 to 1	1052
	" " " 60-100 " " 14-30 " " 1 to 1	1053
	" " " 60-100 " " 14-30 " " 1½ to 1	1054
	Prismoidal correction table	1057-1058

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18.—Multiplication Table up to 60×60, for Finding Areas of Cross Sections

Note.—The Table is divided into Parts I and II. The *greater* factor denotes which Part to use and will be found as a *heading* or *footing*. *Caution.*—In finding the areas of triangles, by using the base and altitude as factors, *remember to divide the product by* 2.

Part II. Greater Factor, **31** to **60**.

Lesser Factor.	60	59	58	57	56	55	54	53	52	51	50	49	48	47	46	45	44	43	42	41	40	39	38	37	36	35	34	33	32	31
1	60	59	58	57	56	55	54	53	52	51	50	49	48	47	46	45	44	43	42	41	40	39	38	37	36	35	34	33	32	31
2	120	118	116	114	112	110	108	106	104	102	100	98	96	94	92	90	88	86	84	82	80	78	76	74	72	70	68	66	64	62
3	180	177	174	171	168	165	162	159	156	153	150	147	144	141	138	135	132	129	126	123	120	117	114	111	108	105	102	99	96	93
4	240	236	232	228	224	220	216	212	208	204	200	196	192	188	184	180	176	172	168	164	160	156	152	148	144	140	136	132	128	124
5	300	295	290	285	280	275	270	265	260	255	250	245	240	235	230	225	220	215	210	205	200	195	190	185	180	175	170	165	160	155
6	360	354	348	342	336	330	324	318	312	306	300	294	288	282	276	270	264	258	252	246	240	234	228	222	216	210	204	198	192	186
7	420	413	406	399	392	385	378	371	364	357	350	343	336	329	322	315	308	301	294	287	280	273	266	259	252	245	238	231	224	217
8	480	472	464	456	448	440	432	424	416	408	400	392	384	376	368	360	352	344	336	328	320	312	304	296	288	280	272	264	256	248
9	540	531	522	513	504	495	486	477	468	459	450	441	432	423	414	405	396	387	378	369	360	351	342	333	324	315	306	297	288	279
10	600	590	580	570	560	550	540	530	520	510	500	490	480	470	460	450	440	430	420	410	400	390	380	370	360	350	340	330	320	310
11	660	649	638	627	616	605	594	583	572	561	550	539	528	517	506	495	484	473	462	451	440	429	418	407	396	385	374	363	352	341
12	720	708	696	684	672	660	648	636	624	612	600	588	576	564	552	540	528	516	504	492	480	468	456	444	432	420	408	396	384	372
13	780	767	754	741	728	715	702	689	676	663	650	637	624	611	598	585	572	559	546	533	520	507	494	481	468	455	442	429	416	403
14	840	826	812	798	784	770	756	742	728	714	700	686	672	658	644	630	616	602	588	574	560	546	532	518	504	490	476	462	448	434
15	900	885	870	855	840	825	810	795	780	765	750	735	720	705	690	675	660	645	630	615	600	585	570	555	540	525	510	495	480	465
16	960	944	928	912	896	880	864	848	832	816	800	784	768	752	736	720	704	688	672	656	640	624	608	592	576	560	544	528	512	496
17	1020	1003	986	969	952	935	918	901	884	867	850	833	816	799	782	765	748	731	714	697	680	663	646	629	612	595	578	561	544	527
18	1080	1062	1044	1026	1008	990	972	954	936	918	900	882	864	846	828	810	792	774	756	738	720	702	684	666	648	630	612	594	576	558
19	1140	1121	1102	1083	1064	1045	1026	0007	988	969	950	931	912	893	874	855	836	817	798	779	760	741	722	703	684	665	646	627	608	589
20	1200	1180	1160	1140	1120	1100	1080	1060	1040	1020	1000	980	960	940	920	900	880	860	840	820	800	780	760	740	720	700	680	660	640	620
21	1260	1239	1218	1197	1176	1155	1134	1113	1092	1071	1050	1029	1008	987	966	945	924	903	882	861	840	819	798	777	756	735	714	693	672	651
22	1320	1298	1276	1254	1232	1210	1188	1166	1144	1122	1100	1078	1056	1034	1012	990	968	946	924	902	880	858	836	814	792	770	748	726	704	682
23	1380	1357	1334	1311	1288	1265	1242	1219	1196	1173	1150	1127	1104	1081	1058	1035	1012	989	966	943	920	897	874	851	828	805	782	759	736	713
24	1440	1416	1392	1368	1344	1320	1296	1272	1248	1224	1200	1176	1152	1128	1104	1080	1056	1032	1008	984	960	936	912	888	864	840	816	792	768	744
25	1500	1475	1450	1425	1400	1375	1350	1325	1300	1275	1250	1225	1200	1175	1150	1125	1100	1075	1050	1025	1000	975	950	925	900	875	850	825	800	775
26	1560	1534	1508	1482	1456	1430	1404	1378	1352	1326	1300	1274	1248	1222	1196	1170	1144	1118	1092	1066	1040	1014	988	962	936	910	884	858	832	806
27	1620	1593	1566	1539	1512	1485	1458	1431	1404	1377	1350	1323	1296	1269	1242	1215	1188	1161	1134	1107	1080	1053	1026	999	972	945	918	891	864	837
28	1680	1652	1624	1596	1568	1540	1512	1484	1456	1428	1400	1372	1344	1316	1288	1260	1232	1204	1176	1148	1120	1092	1064	1036	1008	980	952	924	896	868
29	1740	1711	1682	1653	1624	1595	1566	1537	1508	1479	1450	1421	1392	1363	1334	1305	1276	1247	1218	1189	1160	1131	1102	1073	1044	1015	986	957	928	899
30	1800	1770	1740	1710	1680	1650	1620	1590	1560	1530	1500	1470	1440	1410	1380	1350	1320	1290	1260	1230	1200	1170	1140	1110	1080	1050	1020	990	960	930
31	1860	1829	1798	1767	1736	1705	1674	1643	1612	1581	1550	1519	1488	1457	1426	1395	1364	1333	1302	1271	1240	1209	1178	1147	1116	1085	1054	1023	992	961
32	1920	1888	1856	1824	1792	1760	1728	1696	1664	1632	1600	1568	1536	1504	1472	1440	1408	1376	1344	1312	1280	1248	1216	1184	1152	1120	1088	1056	1024	
33	1980	1947	1914	1881	1848	1815	1782	1749	1716	1683	1650	1617	1584	1551	1518	1485	1452	1419	1386	1353	1320	1287	1254	1221	1188	1155	1122	1089		

37	2220	2183	2146	2109	2072	2035	1998	1961	1924	1887	1850	1813	1776	1739	1702	1665	1628	1591	1554	1517	1480	1443	1406	1369						841	870	29
38	2280	2242	2204	2166	2128	2090	2052	2014	1976	1938	1900	1862	1824	1786	1748	1710	1672	1634	1596	1558	1520	1482	1444					784	812	840	28
39	2340	2301	2262	2223	2184	2145	2106	2067	2028	1989	1950	1911	1872	1833	1794	1755	1716	1677	1638	1599	1560	1521			676	702	728	756	783	810	27
40	2400	2360	2320	2280	2240	2200	2160	2120	2080	2040	2000	1960	1920	1880	1840	1800	1760	1720	1680	1640	1600			625	650	675	700	725	750	775	26
41	2460	2419	2378	2337	2296	2255	2214	2173	2132	2091	2050	2009	1968	1927	1886	1845	1804	1763	1722	1681			576	600	624	648	672	696	720	744	25
42	2520	2478	2436	2394	2352	2310	2268	2226	2184	2142	2100	2058	2016	1974	1932	1890	1848	1806	1764			529	552	575	598	621	644	667	690	713	24
43	2580	2537	2494	2451	2408	2365	2322	2279	2236	2193	2150	2107	2064	2021	1978	1935	1892	1849		484	506	528	550	572	594	616	638	660	682	704	23
44	2640	2596	2552	2508	2464	2420	2376	2332	2288	2244	2200	2156	2112	2068	2024	1980	1936		441	462	483	504	525	546	567	588	609	630	651	672	21
45	2700	2655	2610	2565	2520	2475	2430	2385	2340	2295	2250	2205	2160	2115	2070	2025		400	420	440	460	480	500	520	540	560	580	600	620	640	20
46	2760	2714	2668	2622	2576	2530	2484	2438	2392	2346	2300	2254	2208	2162	2116		361	380	399	418	437	456	475	494	513	532	551	570	589	608	19
47	2820	2773	2726	2679	2632	2585	2538	2491	2444	2397	2350	2303	2256	2209		324	342	360	378	396	414	432	450	468	486	504	522	540	558	576	18
48	2880	2832	2784	2736	2688	2640	2592	2544	2496	2448	2400	2352	2304		289	306	323	340	357	374	391	408	425	442	459	476	493	510	527	544	17
49	2940	2891	2842	2793	2744	2695	2646	2597	2548	2499	2450	2401		256	272	288	304	320	336	352	368	384	400	416	432	448	464	480	496	512	16
50	3000	2950	2900	2850	2800	2750	2700	2650	2600	2550	2500		225	240	255	270	285	300	315	330	345	360	375	390	405	420	435	450	465	480	15
51	3060	3009	2958	2907	2856	2805	2754	2703	2652	2601		196	210	224	238	252	266	280	294	308	322	336	350	364	378	392	406	420	434	448	14
52	3120	3068	3016	2964	2912	2860	2808	2756	2704		169	182	195	208	221	234	247	260	273	286	299	312	325	338	351	364	377	390	403	416	13
53	3180	3127	3074	3021	2968	2915	2862	2809		144	156	168	180	192	204	216	228	240	252	264	276	288	300	312	324	336	348	360	372	384	12
54	3240	3186	3132	3078	3024	2970	2916		121	132	143	154	165	176	187	198	209	220	231	242	253	264	275	286	297	308	319	330	341	352	11
55	3300	3245	3190	3135	3080	3025		100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250	260	270	280	290	300	310	320	10
56	3360	3304	3248	3192	3136		81	90	99	108	117	126	135	144	153	162	171	180	189	198	207	216	225	234	243	252	261	270	279	288	9
57	3420	3363	3306	3249		64	72	80	88	96	104	112	120	128	136	144	152	160	168	176	184	192	200	208	216	224	232	240	248	256	8
58	3480	3422	3364		49	56	63	70	77	84	91	98	105	112	119	126	133	140	147	154	161	168	175	182	189	196	203	210	217	224	7
59	3540	3481		36	42	48	54	60	66	72	78	84	90	96	102	108	114	120	126	132	138	144	150	156	162	168	174	180	186	192	6
60	3600		25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130	135	140	145	150	155	160	5
		16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96	100	104	108	112	116	120	124	128	4
		9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75	78	81	84	87	90	93	3
	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	62	2
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	1
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30		Lesser Factor.

Part I. Greater Factor, **1** to **30.**

Ex's.—Multiply—59 by 14.

Solutions:—

59
14
II, 826
Ans.

59.4 by 14.8

59.4
14.8
II, 826.32 I.
59 × .8 = 47.2 II.
14 × .4 = 5.6 I.
879.12 *Ans.*

594 by 148. Divide each factor by 10; solve as per method to left; and multiply result by 100.

Ans.—87912.

Ex's.—Multiply—37 by 23.

Solutions:—

37.9 by 23.5

37.9
23.5
II, 851.45 I.
37 × .5 = 18.5 II.
23 × .9 = 20.7 I.
II, 851
Ans.
890.65 *Ans.*

Note that the product of the whole numbers is placed to the left of decimal point, and the product of the decimals to the right. Then cross-multiplications.

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18a.—Multiplication Table from 61 to 89, by 2 to 35, for Finding Areas of Cross-Sections.

Lesser Factor.	Greater Factor, 61 to 89.																													Lesser Factor.
	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	
2	122	124	126	128	130	132	134	136	138	140	142	144	146	148	150	152	154	156	158	160	162	164	166	168	170	172	174	176	178	2
3	183	186	189	192	195	198	201	204	207	210	213	216	219	222	225	228	231	234	237	240	243	246	249	252	255	258	261	264	267	3
4	244	248	252	256	260	264	268	272	276	280	284	288	292	296	300	304	308	312	316	320	324	328	332	336	340	344	348	352	356	4
5	0305	0310	0315	0320	0325	0330	0335	0340	0345	0350	0355	0360	0365	0370	0375	0380	0385	0390	0395	0400	0405	0410	0415	0420	0425	0430	0435	4440	0445	5
6	366	372	378	384	390	396	402	408	414	420	426	432	438	444	450	456	462	468	474	480	486	492	498	504	510	516	522	528	534	6
7	427	434	441	448	455	462	469	476	483	490	497	504	511	518	525	532	539	546	553	560	567	574	581	588	595	602	609	616	623	7
8	488	496	504	512	520	528	536	544	552	560	568	576	584	592	600	608	616	624	632	640	648	656	664	672	680	688	696	704	712	8
9	549	558	567	576	585	594	603	612	621	630	639	648	657	666	675	684	693	702	711	720	729	738	747	756	765	774	783	792	801	9
10	0610	0620	0630	0640	0650	0660	0670	0680	0690	0700	0710	0720	0730	0740	0750	0760	0770	0780	0790	0800	0810	0820	0830	0840	0850	0860	0870	0880	0890	10
11	671	682	693	704	715	726	737	748	759	770	781	792	803	814	825	836	847	858	869	880	891	902	913	0924	0935	0946	0957	0968	0979	11
12	732	744	756	768	780	792	804	816	828	840	852	864	876	888	900	912	0924	0936	0948	0960	0972	0984	0996	1008	1020	1032	1044	1056	1068	12
13	793	806	819	832	845	858	871	884	897	910	923	0936	0949	0962	0975	0988	1001	1014	1027	1040	1053	1066	1079	092	1105	118	131	144	157	13
14	854	868	882	896	910	924	0938	0952	0966	0980	0994	1008	1022	1036	1050	1064	078	092	106	1120	134	148	162	176	190	204	218	232	246	14
15	0915	0930	0945	0960	0975	0990	1005	1020	1035	1050	1065	1080	1095	1110	1125	1140	1155	1170	1185	1200	1215	1230	1245	1260	1275	1290	1305	1320	1335	15
16	0976	0992	1008	1024	1040	1056	072	088	104	120	136	152	168	184	200	216	232	248	264	280	296	312	328	344	360	376	392	408	424	16
17	1037	1054	071	088	105	122	139	156	173	190	207	224	241	258	275	292	309	326	343	360	377	394	411	428	445	462	479	496	513	17
18	098	116	134	152	170	188	206	224	242	260	278	296	314	332	350	368	386	404	422	440	458	476	494	512	530	548	566	584	602	18
19	159	178	197	216	235	254	273	292	311	330	349	368	387	406	425	444	463	482	501	520	539	558	577	596	615	634	653	672	691	19
20	1220	1240	1260	1280	1300	1320	1340	1360	1380	1400	1420	1440	1460	1480	1500	1520	1540	1560	1580	1600	1620	1640	1660	1680	1700	1720	1740	1760	1780	20
21	281	302	323	344	365	386	407	428	449	470	491	512	533	554	575	596	617	638	659	680	701	722	743	764	785	806	827	848	869	21
22	342	364	386	408	430	452	474	496	518	540	562	584	606	628	650	672	694	716	738	760	782	804	826	848	870	892	1914	1936	1958	22
23	403	426	449	472	495	518	541	564	587	610	633	656	679	702	725	748	771	794	817	840	863	886	909	1932	1955	1978	2001	2024	2047	23
24	464	488	512	536	560	584	608	632	656	680	704	728	752	776	800	824	848	872	896	1920	1944	1968	1992	2016	2040	2064	088	112	136	24
25	1525	1550	1575	1600	1625	1650	1675	1700	1725	1750	1775	1800	1825	1850	1875	1900	1925	1950	1975	2000	2025	2050	2075	2100	2125	2150	2175	2200	2225	25
26	586	612	638	664	690	716	742	768	794	820	846	872	898	924	1950	1976	2002	2028	2054	080	106	132	158	184	210	236	262	288	314	26
27	647	674	701	728	755	782	809	836	863	890	917	1944	1971	1998	2025	2552	079	106	133	160	187	214	241	268	295	322	349	376	403	27
28	708	736	764	792	820	848	876	904	1932	1960	1988	2016	2044	2072	100	128	156	184	212	240	268	296	324	352	380	408	436	464	492	28
29	769	798	827	856	885	914	1943	1972	2001	2030	2059	088	117	146	175	204	233	262	291	320	349	378	407	436	465	494	523	552	581	29
30	1830	1860	1890	1920	1950	1980	2010	2040	2070	2100	2130	2160	2190	2220	2250	2280	2310	2340	2370	2400	2430	2460	2490	2520	2550	2580	2610	2640	2670	30
31	891	922	1953	1984	2015	2046	077	108	139	170	201	232	263	294	325	356	387	418	449	480	511	542	573	604	635	666	697	728	759	31
32	1952	1984	2016	2048	080	112	144	176	208	240	272	304	336	368	400	432	464	496	528	560	592	624	656	688	720	752	784	816	848	32
33	2013	2047	079	112	145	178	211	244	277	310	343	376	409	442	475	508	541	574	607	640	673	706	739	772	805	838	871	904	2937	33
34	2074	2108	142	176	210	244	278	312	346	380	414	448	482	516	550	584	618	652	686	720	754	788	822	856	890	2924	2958	2992	3026	34
35	2135	2170	2205	2240	2275	2310	2345	2380	2415	2450	2485	2520	2555	2590	2625	2660	2695	2730	2765	2800	2835	2870	2905	2940	2975	3010	3045	3080	3115	35

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19.—Unit and Decimal Areas of Cross-Section Reduced to Cu. Yds. per 100-Ft. Station.

(See Table 20, following, for general use.)

Note.—The values in the table are cu. yds. per 100-ft. station corresponding to the unit area in the first column when used *in the proper decimal place or denomination as per headings of columns.*

[Cu. Yds. per 100-Ft. Station.]

Heading Multipliers.	Mil.	H	T	Thous.	h	t	Units	Decimals		Heading Multipliers.
	(7) O ,	(6) O	(5) O	(4) O ,	(3) O	(2) O	(1) O .	(.1) O	(.2) O	
1	3 703 704	370 370	37 037	3 703.7	370.4	37.04	3.70	0.37	.04	1
2	7 407 407	740 741	74 074	7 407.4	740.7	74.07	7.41	0.74	.07	2
3	11 111 111	1 111 111	111 111	11 111.	1 111.1	111.11	11.11	1.11	.11	3
	(7)	(6)	(5)	(4)	(3)	(2)	(1)	(.1)	(.2)	
4	14 814 815	1 481 481	148 148	14 815.	1 481.5	148.15	14.81	1.48	.15	4
5	18 518 519	1 851 852	185 185	18 519.	1 851.9	185.19	18.52	1.85	.19	5
6	22 222 222	2 222 222	222 222	22 222.	2 222.2	222.22	22.22	2.22	.22	6
	(7)	(6)	(5)	(4)	(3)	(2)	(1)	(.1)	(.2)	
7	25 925 926	2 592 593	259 259	25 926.	2 592.6	259.26	25.93	2.59	.26	7
8	29 629 630	2 962 963	296 296	29 630.	2 963.0	296 30	29.63	2.96	.30	8
9	33 333 333	3 333 333	333 333	33 333.	3 333.3	333.33	33.33	3.33	.33	9

Ex.—The average sectional area of one station (100 ft.) of the Culebra cut was 40600 sq. ft. Find the cu. yds. in the station?

Solution.—From above Table—

4 (5) = 148 148
6 (3) = 2 222

150 370 cu. yds. Ans.

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20.—Cubic Yards in 100-Ft. Station, for—

(Column headings are Units of numbers in first column.)

[Cu. Yds., from formula $\frac{100A}{3\times 9}$.]

Area. Sq. Ft.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
-.	0.0	3.7	7.4	11.1	14.8	18.5	22.2	25.9	29.6	33.3
1-.	37.0	40.7	44.4	48.1	51.9	55.6	59.3	63.0	66.7	70.4
2-.	74.1	77.8	81.5	85.2	88.9	92.6	96.3	100.0	103.7	107.4
3-.	111.1	114.8	118.5	122.2	125.9	129.6	133.3	137.0	140.7	144.4
4-.	148.1	151.9	155.6	159.3	163.0	166.7	170.4	174.1	177.8	181.5
5-.	185.2	188.9	192.6	196.3	200.0	203.7	207.4	211.1	214.8	218.5
6-.	222.2	225.9	229.6	233.3	237.0	240.7	244.4	248.1	251.9	255.6
7-.	259.3	263.0	266.7	270.4	274.1	277.8	281.5	285.2	288.9	292.6
8-.	296.3	300.0	303.7	307.4	311.1	314.8	318.5	322.2	325.9	329.6
9-.	333.3	337.0	340.7	344.4	348.1	351.9	355.6	359.3	363.0	366.7
10-.	370.4	374.1	377.8	381.5	385.2	388.9	392.6	396.3	400.0	403.7
11-.	407.4	411.1	414.8	418.5	422.2	425.9	429.6	433.3	437.0	440.7
12-.	444.4	448.1	451.9	455.6	459.3	463.0	466.7	470.4	474.1	477.8
13-.	481.5	485.2	488.9	492.6	496.3	500.0	503.7	507.4	511.1	514.8
14-.	518.5	522.2	525.9	529.6	533.3	537.0	540.7	544.4	548.1	551.9
15-.	555.6	559.3	563.0	566.7	570.4	574.1	577.8	581.5	585.2	588.9
16-.	592.6	596.3	600.0	603.7	607.4	611.1	614.8	618.5	622.2	625.9
17-.	629.6	633.3	637.0	640.7	644.4	648.1	651.9	655.6	659.3	663.0
18-.	666.7	670.4	674.1	677.8	681.5	685.2	688.9	692.6	696.3	700.0
19-.	703.7	707.4	711.1	714.8	718.5	722.2	725.9	729.6	733.3	737.0
20-.	740.7	744.4	748.1	751.9	755.6	759.3	763.0	766.7	770.4	774.1
21-.	777.8	781.5	785.2	788.9	792.6	796.3	800.0	803.7	807.4	811.1
22-.	814.8	818.5	822.2	825.9	829.6	833.3	837.0	840.7	844.4	848.1
23-.	851.9	855.6	859.3	863.0	866.7	870.4	874.1	877.8	881.5	885.2
24-.	888.9	892.6	896.3	900.0	903.7	907.4	911.1	914.8	918.5	922.2
25-.	925.9	929.6	933.3	937.0	940.7	944.4	948.1	951.9	955.6	959.3
26-.	963.0	966.7	970.4	974.1	977.8	981.5	985.2	988.9	992.6	996.3
27-.	1000.0	1003.7	1007.4	1011.1	1014.8	1018.5	1022.2	1025.9	1029.6	1033.3
28-.	1037.0	1040.7	1044.4	1048.1	1051.9	1055.6	1059.3	1063.0	1066.7	1070.4
29-.	1074.1	1077.8	1081.5	1085.2	1088.9	1092.6	1096.3	1100.0	1103.7	1107.4
30-.	1111.1	1114.8	1118.5	1122.2	1125.9	1129.6	1133.3	1137.0	1140.7	1144.4
31-.	1148.1	1151.9	1155.6	1159.3	1163.0	1166.7	1170.4	1174.1	1177.8	1181.5
32-.	1185.2	1188.9	1192.6	1196.3	1200.0	1203.7	1207.4	1211.1	1214.8	1218.5
33-.	1222.2	1225.9	1229.6	1233.3	1237.0	1240.7	1244.4	1248.1	1251.9	1255.6
34-.	1259.3	1263.0	1266.7	1270.4	1274.1	1277.8	1281.5	1285.2	1288.9	1292.6
35-.	1296.3	1300.0	1303.7	1307.4	1311.1	1314.8	1318.5	1322.2	1325.9	1329.6
36-.	1333.3	1337.0	1340.7	1344.4	1348.1	1351.9	1355.6	1359.3	1363.0	1366.7
37-.	1370.4	1374.1	1377.8	1381.5	1385.2	1388.9	1392.6	1396.3	1400.0	1403.7
38-.	1407.4	1411.1	1414.8	1418.5	1422.2	1425.9	1429.6	1433.3	1437.0	1440.7
39-.	1444.4	1448.1	1451.9	1455.6	1459.3	1463.0	1466.7	1470.4	1474.1	1477.8
40-.	1481.5	1485.2	1488.9	1492.6	1496.3	1500.0	1503.7	1507.4	1511.1	1514.8
41-.	1518.5	1522.2	1525.9	1529.6	1533.3	1537.0	1540.7	1544.4	1548.1	1551.9
42-.	1555.6	1559.3	1563.0	1566.7	1570.4	1574.1	1577.8	1581.5	1585.2	1588.9
43-.	1592.6	1596.3	1600.0	1603.7	1607.4	1611.1	1614.8	1618.5	1622.2	1625.9
44-.	1629.6	1633.3	1637.0	1640.7	1644.4	1648.1	1651.9	1655.6	1659.3	1663.0
45-.	1666.7	1670.4	1674.1	1677.8	1681.5	1685.2	1688.9	1692.6	1696.3	1700.0
46-.	1703.7	1707.4	1711.1	1714.7	1718.5	1722.2	1725.9	1729.6	1733.3	1737.0
47-.	1740.7	1744.4	1748.1	1751.9	1755.6	1759.3	1763.0	1766.7	1770.4	1774.1
48-.	1777.8	1781.5	1785.2	1788.9	1792.6	1796.3	1800.0	1803.7	1807.4	1811.1
49-.	1814.8	1818.5	1822.2	1825.9	1829.6	1833.3	1837.0	1840.7	1844.4	1848.1
50-.	1851.9	1855.6	1859.3	1863.0	1866.7	1870.4	1874.1	1877.8	1881.5	1885.2

Supply the space occupied by the dash (-), by figure at top of column.

P. P.

	3.7	
1	.4	
2	.7	
3	1.1	
4	1.5	
5	1.9	
6	2.2	
7	2.6	
8	3.0	
9	3.3	
	3.8	
1	.4	
2	.8	
3	1.1	
4	1.5	
5	1.9	
6	2.3	
7	2.7	
8	3.0	
9	3.4	

Ex.—Average area of cross-section of station is 321.4 sq. ft. Find yardage for station?

321. = 1188.9

(P. P. col.) .4 = 1.5

Ans. 1190.4 cu. yds.

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—GIVEN AREAS IN SQ. FT. OF CROSS-SECTION.
ve decimal point to right or left for both Areas and Cu. Yds.)
[Cu. Yds., from formula $\frac{100A}{3\times9}$.]

).	1.	2.	3.	4.	5.	6.	7.	8.	9.	P. P.
51.9	1855.6	1859.3	1863.0	1866.7	1870.4	1874.1	1877.8	1881.5	1885.2	
88.9	1892.6	1896.3	1900.0	1903.7	1907.4	1911.1	1914.8	1918.5	1922.2	
25.9	1929.6	1933.3	1937.0	1940.7	1944.4	1948.1	1951.9	1955.6	1959.3	
63.0	1966.7	1970.4	1974.1	1977.8	1981.5	1985.2	1988.9	1992.6	1996.3	
00.0	2003.7	2007.4	2011.1	2014.8	2018.5	2022.2	2025.9	2029.6	2033.3	
37.0	2040.7	2044.4	2048.1	2051.9	2055.6	2059.3	2063.0	2066.7	2070.4	
74.1	2077.8	2081.5	2085.2	2088.9	2092.6	2096.3	2100.0	2103.7	2107.4	
11.1	2114.8	2118.5	2122.2	2125.9	2129.6	2133.3	2137.0	2140.7	2144.4	
48.1	2151.9	2155.6	2159.3	2163.0	2166.7	2170.4	2174.1	2177.8	2181.5	
85.2	2188.9	2192.6	2196.3	2200.0	2203.7	2207.4	2211.1	2214.8	2218.5	
22.2	2225.9	2229.6	2233.3	2237.0	2240.7	2244.4	2248.1	2251.9	2255.6	
59.3	2263.0	2266.7	2270.4	2274.1	2277.8	2281.5	2285.2	2288.9	2292.6	
96.3	2300.0	2303.7	2307.4	2311.1	2314.8	2318.5	2322.2	2325.9	2329.6	
33.3	2337.0	2340.7	2344.4	2348.1	2351.9	2355.6	2359.3	2363.0	2366.7	
70.4	2374.1	2377.8	2381.5	2385.2	2388.9	2392.6	2396.3	2400.0	2403.7	
07.4	2411.1	2414.8	2418.5	2422.2	2425.9	2429.6	2433.3	2437.0	2440.7	
44.4	2448.1	2451.9	2455.6	2459.3	2463.0	2466.7	2470.4	2474.1	2477.8	3.7
81.5	2485.2	2488.9	2492.6	2496.3	2500.0	2503.7	2507.4	2511.1	2514.8	1 .4
18.5	2522.2	2525.9	2529.6	2533.3	2537.0	2540.7	2544.4	2548.1	2551.9	2 .7
55.6	2559.3	2563.0	2566.7	2570.4	2574.1	2577.8	2581.5	2585.2	2588.9	3 1.1
										4 1.5
92.6	2596.3	2600.0	2603.7	2607.4	2611.1	2614.8	2618.5	2622.2	2625.9	5 1.9
29.6	2633.3	2637.0	2640.7	2644.4	2648.1	2651.9	2655.6	2659.3	2663.0	6 2.2
66.7	2670.4	2674.1	2677.8	2681.5	2685.2	2688.9	2692.6	2696.3	2700.0	7 2.6
03.7	2707.4	2711.1	2714.8	2718.5	2722.2	2725.9	2729.6	2733.3	2737.0	8 3.0
40.7	2744.4	2748.1	2751.9	2755.6	2759.3	2763.0	2766.7	2770.4	2774.1	9 3.3
77.8	2781.5	2785.2	2788.9	2792.6	2796.3	2800.0	2803.7	2807.4	2811.1	
14.8	2818.5	2822.2	2825.9	2829.6	2833.3	2837.0	2840.7	2844.4	2848.1	3.8
51.9	2855.6	2859.3	2863.0	2866.7	2870.4	2874.1	2877.8	2881.5	2885.2	1 .4
88.9	2892.6	2896.3	2900.0	2903.7	2907.4	2911.1	2914.8	2918.5	2922.2	2 .8
25.9	2929.6	2933.3	2937.0	2940.7	2944.4	2948.1	2951.9	2955.6	2959.3	3 1.1
										4 1.5
63.0	2966.7	2970.4	2974.1	2977.8	2981.5	2985.2	2988.9	2992.6	2996.3	5 1.9
00.0	3003.7	3007.4	3011.1	3014.8	3018.5	3022.2	3025.9	3029.6	3033.3	6 2.3
37.0	3040.7	3044.4	3048.1	3051.9	3055.6	3059.3	3063.0	3066.7	3070.4	7 2.7
74.1	3077.8	3081.5	3085.2	3088.9	3092.6	3096.3	3100.0	3103.7	3107.4	8 3.0
11.1	3114.8	3118.5	3122.2	3125.9	3129.6	3133.3	3137.0	3140.7	3144.4	9 3.4
48.1	3151.9	3155.6	3159.3	3163.0	3166.7	3170.4	3174.1	3177.8	3181.5	
85.2	3188.9	3192.6	3196.3	3200.0	3203.7	3207.4	3211.1	3214.8	3218.5	
22.2	3225.9	3229.6	3233.3	3237.0	3240.7	3244.4	3248.1	3251.9	3255.6	
59.3	3263.0	3266.7	3270.4	3274.1	3277.8	3281.5	3285.2	3288.9	3292.6	
96.3	3300.0	3303.7	3307.4	3311.1	3314.8	3318.5	3322.2	3325.9	3329.6	
33.3	3337.0	3340.7	3344.4	3348.1	3351.9	3355.6	3359.3	3363.0	3366.7	
70.4	3374.1	3377 8	3381.5	3385.2	3388.9	3392.6	3396.3	3400.0	3403.7	
07.4	3411.1	3414.8	3418.5	3422.2	3425.9	3429.6	3433.3	3437.0	3440.7	
44.4	3448.1	3451.9	3455.6	3459.3	3463.0	3466.7	3470.4	3474.1	3477.8	
81.5	3485.2	3488.9	3492.6	3496.3	3500.0	3503.7	3507.4	3511.1	3514.8	
18.5	3522.2	3525.9	3529.6	3533.3	3537.0	3540.7	3544.4	3548.1	3551.9	
55.6	3559.3	3563.0	3566.7	3570.4	3574.1	3577.8	3581.5	3585.2	3588.9	
92.6	3596.3	3600.0	3603.7	3607.4	3611.1	3614.8	3618.5	3622.2	3625.9	
29.6	3633.3	3637.0	3640.7	3644.4	3648.1	3651.9	3655.6	3659.3	3663.0	
66.7	3670.4	3674.1	3677.8	3681.5	3685.2	3688.9	3692.6	3696.3	3700.0	
03.7	3707.4	3711.1	3714.8	3718.5	3722.2	3725.9	3729.6	3733.3	3737.0	

Average area of cross- Yardage for 321.4 sq. ft. = 1190.4 cu. yds.
3214 sq. ft. Find yard- " " 3214 " " = 11904 " "
ation? Ans.

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20.—Cubic Yards in 100-Ft. Stations, for—
(Column headings are Units of numbers in first column.)

[Cu. Yds., from formula $\frac{100A}{3\times9}$.]

Supply the space occupied by the dash (-), by figure at top of column.

Area. Sq. Ft.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	P. P.
100-.	3703.7	3707.4	3711.1	3714.8	3718.5	3722.2	3725.9	3729.6	3733.3	3737.0	
101-.	3740.7	3744.4	3748.1	3751.9	3755.6	3759.3	3763.0	3766.7	3770.4	3774.1	
102-.	3777.8	3781.5	3785.2	3788.9	3792.6	3796.3	3800.0	3803.7	3807.4	3811.1	
103-.	3814.8	3818.5	3822.2	3825.9	3829.6	3833.3	3837.0	3840.7	3844.4	3848.1	
104-.	3851.9	3855.6	3859.3	3863.0	3866.7	3870.4	3874.1	3877.8	3881.5	3885.2	
105-.	3888.9	3892.6	3896.3	3900.0	3903.7	3907.4	3911.1	3914.8	3918.5	3922.2	
106-.	3925.9	3929.6	3933.3	3937.0	3940.7	3944.4	3948.1	3951.9	3955.6	3959.3	
107-.	3963.0	3966.7	3970.4	3974.1	3977.8	3981.5	3985.2	3988.9	3992.6	3996.3	
108-.	4000.0	4003.7	4007.4	4011.1	4014.8	4018.5	4022.2	4025.9	4029.6	4033.3	
109-.	4037.0	4040.7	4044.4	4048.1	4051.9	4055.6	4059.3	4063.0	4066.7	4070.4	
110-.	4074.1	4077.8	4081.5	4085.2	4088.9	4092.6	4096.3	4100.0	4103.7	4107.4	
111-.	4111.1	4114.8	4118.5	4122.2	4125.9	4129.6	4133.3	4137.0	4140.7	4144.4	
112-.	4148.1	4151.9	4155.6	4159.3	4163.0	4166.7	4170.4	4174.1	4177.8	4181.5	
113-.	4185.2	4188.9	4192.6	4196.3	4200.0	4203.7	4207.4	4211.1	4214.8	4218.5	
114-.	4222.2	4225.9	4229.6	4233.3	4237.0	4240.7	4244.4	4248.1	4251.9	4255.6	
115-.	4259.3	4263.0	4266.7	4270.4	4274.1	4277.8	4281.5	4285.2	4288.9	4292.6	
116-.	4296.3	4300.0	4303.7	4307.4	4311.1	4314.8	4318.5	4322.2	4325.9	4329.6	3 7
117-.	4333.3	4337.0	4340.7	4344.4	4348.1	4351.9	4355.6	4359.3	4363.0	4366.7	1 .4
118-.	4370.4	4374.1	4377.8	4381.5	4385.2	4388.9	4392.6	4396.3	4400.0	4403.7	2 .7
119-.	4407.4	4411.1	4414.8	4418.5	4422.2	4425.9	4429.6	4433.3	4437.0	4440.7	3 1.1
											4 1.5
120-.	4444.4	4448.1	4451.9	4455.6	4459.3	4463.0	4466.7	4470.4	4474.1	4477.8	5 1.9
121-.	4481.5	4485.2	4488.9	4492.6	4496.3	4500.0	4503.7	4507.4	4511.1	4514.8	6 2.2
122-.	4518.5	4522.2	4525.9	4529.6	4533.3	4537.0	4540.7	4544.4	4548.1	4551.9	7 2.6
123-.	4555.6	4559.3	4563.0	4566.7	4570.4	4574.1	4577.8	4581.5	4585.2	4588.9	8 3.0
124-.	4592.6	4596.3	4600.0	4603.7	4607.4	4611.1	4614.8	4618.5	4622.2	4625.9	9 3.3
125-.	4629.6	4633.3	4637.0	4640.7	4644.4	4648.1	4651.9	4655.6	4659.3	4663.0	
126-.	4666.7	4670.4	4674.1	4677.8	4681.5	4685.2	4688.9	4692.6	4696.3	4700.0	3 8
127-.	4703.7	4707.4	4711.1	4714.8	4718.5	4722.2	4725.9	4729.6	4733.3	4737.0	1 .4
128-.	4740.7	4744.4	4748.1	4751.9	4755.6	4759.3	4763.0	4766.7	4770.4	4774.1	2 .8
129-.	4777.8	4781.5	4785.2	4788.9	4792.6	4796.3	4800.0	4803.7	4807.4	4811.1	3 1.1
											4 1.5
130-.	4814.8	4818.5	4822.2	4825.9	4829.6	4833.3	4837.0	4840.7	4844.4	4848.1	5 1.9
131-.	4851.9	4855.6	4859.3	4863.0	4866.7	4870.4	4874.1	4877.8	4881.5	4885.2	6 2.3
132-.	4888.9	4892.6	4896.3	4900.0	4903.7	4907.4	4911.1	4914.8	4918.5	4922.2	7 2.7
133-.	4925.9	4929.6	4933.3	4937.0	4940.7	4944.4	4948.1	4951.9	4955.6	4959.3	8 3.0
134-.	4963.0	4966.7	4970.4	4974.1	4977.8	4981.5	4985.2	4988.9	4992.6	4996.3	9 3.4
135-.	5000.0	5003.7	5007.4	5011.1	5014.8	5018.5	5022.2	5025.9	5029.6	5033.3	
136-.	5037.0	5040.7	5044.4	5048.1	5051.9	5055.6	5059.3	5063.0	5066.7	5070.4	
137-.	5074.1	5077.8	5081.5	5085.2	5088.9	5092.6	5096.3	5100.0	5103.7	5107.4	
138-.	5111.1	5114.8	5118.5	5122.2	5125.9	5129.6	5133.3	5137.0	5140.7	5144.4	
139-.	5148.1	5151.9	5155.6	5159.3	5163.0	5166.7	5170.4	5174.1	5177.8	5181.5	
140-.	5185.2	5188.9	5192.6	5196.3	5200.0	5203.7	5207.4	5211.1	5214.8	5218.5	
141-.	5222.2	5225.9	5229.6	5233.3	5237.0	5240.7	5244.4	5248.1	5251.9	5255.6	
142-.	5259.3	5263.0	5266.7	5270.4	5274.1	5277.8	5281.5	5285.2	5288.9	5292.6	
143-.	5296.3	5300.0	5303.7	5307.4	5311.1	5314.8	5318.5	5322.2	5325.9	5329.6	
144-.	5333.3	5337.0	5340.7	5344.4	5348.1	5351.9	5355.6	5359.3	5363.0	5366.7	
145-.	5370.4	5374.1	5377.8	5381.5	5385.2	5388.9	5392.6	5396.3	5400.0	5403.7	
146-.	5407.4	5411.1	5414.8	5418.5	5422.2	5425.9	5429.6	5433.3	5437.0	5440.7	
147-.	5444.4	5448.1	5451.9	5455.6	5459.3	5463.0	5466.7	5470.4	5474.1	5477.8	
148-.	5481.5	5485.2	5488.9	5492.6	5496.3	5500.0	5503.7	5507.4	5511.1	5514.8	
149-.	5518.5	5522.2	5525.9	5529.6	5533.3	5537.0	5540.7	5544.4	5548.1	5551.9	
150-.	5555.6	5559.3	5563.0	5566.7	5570.4	5574.1	5577.8	5581.5	5585.2	5588.9	

Ex.—Average area of cross-section is 1295.3 sq. ft. Find yardage for station?

1295. = 4796.3
(P. P. Col.) .3 = 1.1

Ans. 4797.4 cu. yds.

-GIVEN AREAS IN SQ. FT. OF CROSS-SECTION.—Continued.
e decimal points to right or left for both Areas and Cu. Yds.)

[Cu. Yds., from formula $\frac{100A}{3\times9}$.]

0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	P. P.
5555.6	5559.3	5563.0	5566.7	5570.4	5574.1	5577.8	5581.5	5585.2	5588.9	
5592.6	5596.3	5600.0	5603.7	5607.4	5611.1	5614.8	5618.5	5622.2	5625.9	
5629.6	5633.3	5637.0	5640.7	5644.4	5648.1	5651.9	5655.6	5659.3	5663.0	
5666.7	5670.4	5674.1	5677.8	5681.5	5685.2	5688.9	5692.6	5696.3	5700.0	
5703.7	5707.4	5711.1	5714.8	5718.5	5722.2	5725.9	5729.6	5733.3	5737.0	
5740.7	5744.4	5748.1	5751.9	5755.6	5759.3	5763.0	5766.7	5770.4	5774.1	
5777.8	5781.5	5785.2	5788.9	5792.6	5796.3	5800.0	5803.7	5807.4	5811.1	
5814.8	5818.5	5822.2	5825.9	5829.6	5833.3	5837.0	5840.7	5844.4	5848.1	
5851.9	5855.6	5859.3	5863.0	5866.7	5870.4	5874.1	5877.8	5881.5	5885.2	
5888.9	5892.6	5896.3	5900.0	5903.7	5907.4	5911.1	5914.8	5918.5	5922.2	
5925.9	5929.6	5933.3	5937.0	5940.7	5944.4	5948.1	5951.9	5955.6	5959.3	
5963.0	5966.7	5970.4	5974.1	5977.8	5981.5	5985.2	5988.9	5992.6	5996.3	
6000.0	6003.7	6007.4	6011.1	6014.8	6018.5	6022.2	6025.9	6029.6	6033.3	
6037.0	6040.7	6044.4	6048.1	6051.9	6055.6	6059.3	6063.0	6066.7	6070.4	
6074.1	6077.8	6081.5	6085.2	6088.9	6092.6	6096.3	6100.0	6103.7	6107.4	
6111.1	6114.8	6118.5	6122.2	6125.9	6129.6	6133.3	6137.0	6140.7	6144.4	
6148.1	6151.9	6155.6	6159.3	6163.0	6166.7	6170.4	6174.1	6177.8	6181.5	3.7
6185.2	6188.9	6192.6	8196.3	6200.0	6203.7	6207.4	6211.1	6214.8	6218.5	1 .4
6222.2	6225.9	6229.6	6233.3	6237.0	6240.7	6244.4	6248.1	6251.9	6255.6	2 .7
6259.3	6263.0	6266.7	6270.4	6274.1	6277.8	6281.5	6285.2	6288.9	6292.6	3 1.1
										4 1.5
6296.3	6300.0	6303.7	6307.4	6311.1	6314.8	6318.5	6322.2	6325.9	6329.6	5 1.9
6333.3	6337.0	6340.7	6344.4	6348.1	6351.9	6355.6	6359.3	6363.0	6366.7	6 2.2
6370.4	6371.4	6377.8	6381.5	6385.2	6388.9	6392.6	6396.3	6400.0	6403.7	7 2.6
6407.4	6411.1	6414.8	6418.5	6422.2	6425.9	6429.6	6433.3	6437.0	6440.7	8 3.0
6444.4	6448.1	6451.9	6455.6	6459.3	6463.0	6466.7	6470.4	6474.1	6477.8	9 3.3
6481.5	6485.2	6488.9	6492.6	6496.3	6500.0	6503.7	6507.4	6511.1	6514.8	
6518.5	6522.2	6525.9	6529.6	6533.3	6537.0	6540.7	6544.4	6548.1	6551.9	3.8
6555.6	6559.3	6563.0	6566.7	6570.4	6574.1	6577.8	6581.5	6585.2	6588.9	1 .4
6592.6	6596.3	6600.0	6603.7	6607.4	6611.1	6614.8	6618.5	6622.2	6625.9	2 .8
6629.6	6633.3	6637.0	6640.7	6644.4	6648.1	6651.9	6655.6	6659.3	6663.0	3 1.1
										4 1.5
6666.7	6670.4	6674.1	6677.8	6681.5	6685.2	6688.9	6692.6	6696.3	6700.0	5 1.9
6703.7	6707.4	6711.1	6714.8	6718.5	6722.2	6725.9	6729.6	6733.3	6737.0	6 2.3
6740.7	6744.4	6748.1	6751.9	6755.6	6759.3	6763.0	6766.7	6770.4	6774.1	7 2.7
6777.8	6781.5	6785.2	6788.9	6792.6	6796.3	6800.0	6803.7	6807.4	6811.1	8 3.0
6814.8	6818.5	6822.2	6825.9	6829.6	6833.3	6837.0	6840.7	6844.4	6848.1	9 3.4
6851.9	6855.6	6859.3	6863.0	6866.7	6870.4	6874.1	6877.8	6881.5	6885.2	
6888.9	6892.6	6896.3	6900.0	6903.7	6907.4	6911.1	6914.8	6918.5	6922.2	
6925.9	6929.6	6933.3	6937.0	6940.7	6944.4	6948.1	6951.9	6955.6	6959.3	
6963.0	6966.7	6970.4	6974.1	6977.8	6981.5	6985.2	6988.9	6992.6	6996.3	
7000.0	7003.7	7007.4	7011.1	7014.8	7018.5	7022.2	7025.9	7029.6	7033.3	
7037.0	7040.7	7044.4	7048.1	7051.9	7055.6	7059.3	7063.0	7066.7	7070.4	
7074.1	7077.8	7081.5	7085.2	7088.9	7092.6	7096.3	7100.0	7103.7	7107.4	
7111.1	7114.8	7118.5	7122.2	7125.9	7129.6	7133.3	7137.0	7140.7	7144.4	
7148.1	7151.9	7155.6	7159.3	7163.0	7166.7	7170.4	7174.1	7177.8	7181.5	
7185.2	7188.9	7192.6	7196.3	7200.0	7203.7	7207.4	7211.1	7214.8	7218.5	
7222.2	7225.9	7229.6	7233.3	7237.0	7240.7	7244.4	7248.1	7251.9	7255.6	
7259.3	7263.0	7266.7	7270.4	7274.1	7277 8	7281.5	7285.2	7288.9	7292.6	
7296.3	7300.0	7303.7	7307.4	7311.1	7314.8	7318 5	7322.2	7325.9	7329.6	
7333.3	7337.0	7340.7	7344.4	7348.1	7351.9	7355.6	7359.3	7363.0	7366.7	
7370.4	7371.4	7377.8	7381.5	7385.2	7388.9	7392.6	7396.3	7400.0	7403.7	
7407.4	7411.1	7414.8	7418.5	7422.2	7425.9	7429.6	7433.3	7437.0	7440.7	

—Average area of cross- is 1845.8 sq. ft. Find for station?

(P. P. Col.)

1845. =6833.3
.8= 3.0
Ans. 6836.3 cu. yds.

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20.—Cubic Yards in 100-Ft. Station, for—

(Column headings are Units of numbers in first column.)

[Cu. Yds., from formula $\frac{100A}{3\times9}$.]

Area. Sq. Ft.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	P. P.
200-.	7407.4	7411.1	7414.8	7418.5	7422.2	7425.9	7429.6	7433.3	7437.0	7440.7	
201-.	7444.4	7448.1	7451.9	7455.6	7459.3	7463.0	7466.7	7470.4	7474.1	7477.8	
202-.	7481.5	7485.2	7488.9	7492.6	7496.3	7500.0	7503.7	7507.4	7511.1	7514.8	
203-.	7518.5	7522.2	7525.9	7529.6	7533.3	7537.0	7540.7	7544.4	7548.1	7551.9	
204-.	7555.6	7559.3	7563.0	7566.7	7570.4	7574.1	7577.8	7581.5	7585.2	7588.9	
205-.	7592.6	7596.3	7600.0	7603.7	7607.4	7611.1	7614.8	7618.5	7622.2	7525.9	
206-.	7629.6	7633.3	7637.0	7640.7	7644.4	7648.1	7651.9	7655.6	7659.3	7663.0	
207-.	7666.7	7670.4	7674.1	7677.8	7681.5	7685.2	7688.9	7692.6	7696.3	7700.0	
208-.	7703.7	7707.4	7711.1	7714.8	7718.5	7722.2	7725.9	7729.6	7733.3	7737.0	
209-.	7740.7	7744.4	7748.1	7751.9	7755.6	7759.3	7763.0	7766.7	7770.4	7774.1	
210-.	7777.8	7781.5	7785.2	7788.9	7792.6	7796.3	7800.0	7803.7	7807.4	7811.1	
211-.	7814.8	7818.5	7822.2	7825.9	7829.6	7833.3	7837.0	7840.7	7844.4	7848.1	
212-.	7851.9	7855.6	7859.3	7863.0	7866.7	7870.4	7874.1	7877.8	7881.5	7885.2	
213-.	7888.9	7892.6	7896.3	7900.0	7903.7	7907.4	7911.1	7914.8	7918.5	7922.2	
214-.	7925.9	7929.6	7933.3	7937.0	7940.7	7944.4	7948.1	7951.9	7955.6	7959.3	
215-.	7963.0	7966.7	7970.4	7974.1	7977.8	7981.5	7985.2	7988.9	7992.6	7996.3	
216-.	8000.0	8003.7	8007.4	8011.1	8014.8	8018.5	8022.2	8025.9	8029.6	8033.3	3.7
217-.	8037.0	8040.7	8044.4	8048.1	8051.9	8055.6	8059.3	8063.0	8066.7	8070.4	1 .4
218-.	8074.1	8077.8	8081.5	8085.2	8088.9	8092.6	8096.3	8100.0	8103.7	8107.4	2 .7
219-.	8111.1	8114.8	8118.5	8122.2	8125.9	8129.6	8133.3	8137.0	8140.7	8144.4	3 1.1
											4 1.5
220-.	8148.1	8151.9	8155.6	8159.3	8163.0	8166.7	8170.4	8174.1	8177.8	8181.5	5 1.9
221-.	8185.2	8188.9	8192.6	8196.3	8200.0	8203.7	8207.4	8211.1	8214.8	8218.5	6 2.2
222-.	8222.2	8225.9	8229.6	8233.3	8237.0	8240.7	8244.4	8248.1	8251.9	8255.6	7 2.6
223-.	8259.3	8263.0	8266.7	8270.4	8274.1	8277.8	8281.5	8285.2	8288.9	8292.6	8 3.0
224-.	8296.3	8300.0	8303.7	8307.4	8311.1	8314.8	8318.5	8322.2	8325.9	8329.6	9 3.3
225-.	8333.3	8337.0	8340.7	8344.4	8348.1	8351.9	8355.6	8359.3	8363.0	8366.7	
226-.	8370.4	8371.4	8377.8	8381.5	8385.2	8388.9	8392.6	8396.3	8400.0	8403.7	3.8
227-.	8407.4	8411.1	8414.8	8418.5	8422.2	8425.9	8429.6	8433.3	8437.0	8440.7	1 .4
228-.	8444.4	8448.1	8451.9	8455.6	8459.3	8463.0	8466.7	8470.4	8474.1	8477.8	2 .8
229-.	8481.5	8485.2	8488.9	8492.6	8496.3	8500.0	8503.7	8507.4	8511.1	8514.8	3 1.1
											4 1.5
230-.	8518.5	8522.2	8525.9	8529.6	8533.3	8537.0	8540.7	8544.4	8548.1	8551.9	5 1.9
231-.	8555.6	8559.3	8563.0	8566.7	8570.4	8574.1	8577.8	8581.5	8585.2	8588.9	6 2.3
232-.	8592.6	8596.3	8600.0	8603.7	8607.4	8611.1	8614.8	8618.5	8622.2	8625.9	7 2.7
233-.	8629.6	8633.3	8637.0	8640.7	8644.4	8648.1	8651.9	8655.6	8659.3	8663.0	8 3.0
234-.	8666.7	8670.4	8674.1	8677.8	8681.5	8685.2	8688.9	8692.6	8696.3	8700.0	9 3.4
235-.	8703.7	8707.4	8711.1	8714.8	8718.5	8722.2	8725.9	8729.6	8733.3	8737.0	
236-.	8740.7	8744.4	8748.1	8751.9	8755.6	8759.3	8763.0	8766.7	8770.4	8774.1	
237-.	8777.8	8781.5	8785.2	8788.9	8792.6	8796.3	8800.0	8803.7	8807.4	8811.1	
238-.	8814.8	8818.5	8822.2	8825.9	8829.6	8833.3	8837.0	8840.7	8844.4	8848.1	
239-.	8851.9	8855.6	8859.3	8863.0	8866.7	8870.4	8874.1	8877.8	8881.5	8885.2	
240-.	8888.9	8892.6	8896.3	8900.0	8903.7	8907.4	8911.1	8914.8	8918.5	8922.2	
241-.	8925.9	8929.6	8933.3	8937.0	8940.7	8944.4	8948.1	8951.9	8955.6	8959.3	
242-.	8963.0	8966.7	8970.4	8974.1	8977.8	8981.5	8985.2	8988.9	8992.6	8996.3	
243-.	9000.0	9003.7	9007.4	9011.1	9014.8	9018.5	9022.2	9025.9	9029.6	9033.3	
244-.	9037.0	9040.7	9044.4	9048.1	9051.9	9055.6	9059.3	9063.0	9066.7	9070.4	
245-.	9074.1	9077.8	9181.5	9085.2	9088.9	9092.6	9096.3	9100.0	9103.7	9107.4	
246-.	9111.1	9114.8	9118.5	9122.2	9125.9	9129.6	9133.3	9137.0	9140.7	9144.4	
247-.	9148.1	9151.9	9155.6	9159.3	9163.0	9166.7	9170.4	9174.1	9177.8	9181.5	
248-.	9185.2	9188.9	9192.6	9196.3	9200.0	9203.7	9207.4	9211.1	9214.8	9218.5	
249-.	9222.2	9225.9	9229.6	9233.3	9237.0	9240.7	9244.4	9248.1	9251.9	9255.6	
250-.	9259.3	9263.0	9266.7	9270.4	9274.1	9277.8	9281.5	9285.2	9288.9	9292.6	

Supply the space occupied by the dash (-), by figure at top of column.

Ex.—Average area of cross-section is 2436.2 sq. ft. Find yardage for station?

2436. = 9022.2
(P. P. Col.) .2 = .7
Ans. 9023.9 cu. yds

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—GIVEN AREAS IN SQ. FT. OF CROSS-SECTION.—Concluded.
ove decimal point to right or left for both Areas and Cu. Yds.)

[Cu. Yds., from formula $\frac{100A}{3\times9}$.]

0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	P. P.
9259.3	9263.0	9266.7	9270.4	9274.1	9277.8	9281.5	9285.2	9288.9	9292.6	
9296.3	9300.0	9303.7	9307.4	9311.1	9314.8	9318.5	9322.2	9325.9	9329.6	3.7
9333.3	9337.0	9340.7	9344.4	9348.1	9351.9	9355.6	9359.3	9363.0	9366.7	1 .4
9370.4	9371.4	9377.8	9381.5	9385.2	9388.9	9392.6	9396.3	9400.0	9403.7	2 .7
9407.4	9411.1	9414.8	9418.5	9422.2	9425.9	9429.6	9433.3	9437.0	9440.7	3 1.1
										4 1.5
9444.4	9448.1	9451.9	9455.6	9459.3	9463.0	9466.7	9470.4	9474.1	9477.8	5 1.9
9481.5	9485.2	9488.9	9492.6	9496.3	9500.0	9503.7	9507.4	9511.1	9514.8	6 2.2
9518.5	9522.2	9525.9	9529.6	9533.3	9537.0	9540.7	9544.4	9548.1	9551.9	7 2.6
9555.6	9559.3	9563.0	9566.7	9570.4	9574.1	9577.8	9581.5	9585.2	9588.9	8 3.0
9592.6	9596.3	9600.0	9603.7	9607.4	9611.1	9614.8	9618.5	9622.2	9625.9	9 3.3
9629.6	9633.3	9637.0	9640.7	9644.4	9648.1	9651.9	9655.6	9659.3	9663.0	
9666.7	9670.4	9674.1	9677.8	9681.5	9685.2	9688.9	9692.6	9696.3	9700.0	3.8
9703.7	9707.4	9711.1	9714.8	9718.5	9722.2	9725.9	9729.6	9733.3	9737.0	1 .4
9740.7	9744.4	9748.1	9751.9	9755.6	9759.3	9763.0	9766.7	9770.4	9774.1	2 .8
9777.8	9781.5	9785.2	9788.9	9792.6	9796.3	9800.0	9803.7	9807.4	9811.1	3 1.1
										4 1.5
9814.8	9818.5	9822.2	9825.9	9829.6	9833.3	9837.0	9840.7	9844.4	9848.1	5 1.9
9851.9	9855.6	9859.3	9863.0	9866.7	9870.4	9874.1	9877.8	9881.5	9885.2	6 2.3
9888.9	9892.6	9896.3	9900.0	9903.7	9907.4	9911.1	9914.8	9918.5	9922.2	7 2.7
9925.9	9929.6	9933.3	9937.0	9940.7	9944.4	9948.1	9951.9	9955.6	9959.3	8 3.0
9963.0	9966.7	9970.4	9974.1	9977.8	9981.5	9985.2	9988.9	9992.6	9996.3	9 3.4

—Average area of cross-
is 2583.7 sq. ft. Find
for station?

2583. =9566.7
(P. P. Col.) .7= 2.6
Ans. 9569.3 cu. yds.

Remarks on above Table 20.

e 20 gives the number of cubic yards in 100-ft. station corresponding in sq. ft. of cross-section up to 2700 sq. ft., equivalent to 10000 cu. 100-ft. station for the area named. The main table advances by as, the units being at tops of columns; the Prop. Parts columns ed for interpolation for tenths of sq. ft. Thus, for an average area section of 2622 sq. ft. the yardage per station equals 9711.1 cu. yds. ea is increased by 0.4 sq. ft. the yardage would be increased by 1.5 giving 9712.6 cu. yds for an area of 2622.4 sq. ft. By manipulating nal point to the right of left simultaneously for both areas and cu. range of the table may be extended. Thus, for an area of 26224 e have 97126 cu. yds. For areas up to 270 sq. ft. it will be found nt to move the decimal point one place to the left, considering the headings as tenths of sq. ft. Thus, for an area of 262.2 sq. ft. we 1.1 cu. yds., obtained by moving the decimal point for the yardage e to left also. For areas above 270 and up to 2700 sq. ft., the decimal n the table should be preserved.

e average area of cross-section is for less than a 100-ft. station, say ., the yardage would of course be proportional, i. e., ½ for 50 ft., ft., etc.

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21.—Method of Calculating Earthwork Tables for Level Sections.

(From a page of the writer's calculation of Table 26, page 1037.)

1st.—Select foolscap paper, horizontally ruled

2nd.—Draw vertical lines, in pencil, to represent the decimal points of numbers in columns 1, 2, 3, 4, 5, etc.; thus insuring the figures being "in column" and saving considerable time in making decimal points.

3rd.—Let w = width of roadway, in ft. ($w = 16$ in the present case); s = the side slopes of excavation or embankment (thus, in the present case, using slopes of $1\frac{1}{2}$ to 1, $s = 1.5$); h = height of level cutting, in ft.; and Q = quantity in cu. yds. per 100-ft station for a level cut or fill ground line.

Then $Q = (2sh + 2w)\frac{h}{2} \cdot \frac{100}{3 \times 9}$; or, $= (2sh + 2w)h \cdot \frac{100}{6 \times 9}$; or, $= (sh + w)h \cdot \frac{100}{3 \times 9}$.

Sometimes one and sometimes another of the above equations will be found most convenient, depending upon s and h.

4th.—Calculate Q for $h = 0.1$ as for line c, column 1*:

$$Q = (2 \times 1.5 \times 0.1 + 2 \times 16)0.1 \times \frac{100}{6 \times 9} = 5.981 \text{ cu. yds.} \quad \text{Similarly,}$$

for $h = 0.2$, $Q = (2 \times 1.5 \times 0.2 + 2 \times 16)0.1 \times \frac{100}{3 \times 9} = 12.074$ cu. yds.

∴Diff. for 0.1 ft. from $h = 0.1$ to $h = 0.2$ ft. = 6.093 cu. yds. (See opp. line 1.) This number, 6.093, forms a primary base for calculating all the Differences (see numbers in *italics*) in the following table; and these Differences increase by a constant increment which will next be explained.

5th.—Find the successive Differences, lines 1, 3, 5, 7, etc. (in Italics), by calculating the constant incremental increase i for the particular slope s (in the present case $s = 1.5$) and for the successive increase in height d (in the present case $d = 0.1$ ft.); and adding this increment successively, using the first Difference, 6.093, as a base. Now the value of i in cu. yds. for any slope $= \frac{2s}{3 \times 9}$ cu. yds.; therefore for slope of $1\frac{1}{2}$ to 1, as in the present case.

$i = \frac{3}{9} = 0.111\vee1$. Hence we have for Differences in column 1: $6.093 + 0.111\vee1 = 6.204$ (line 3); $6.204 + 0.111\vee1 = 6.315$ (line 5); $6.315 + 0.111\vee1 = 6.426$ (line 7); etc. Now glancing down the column we notice a similarity of the three groups of numbers, the number (Difference) in each group being a unit larger than the corresponding one in the group above. Hence it is essential that these Differences be arranged in groups, each successive group of Differences being set down from the preceding group by mental calculation. Note in this connection also that the Differences in successive columns are increased by 3.000 from those in the same line in the preceding column. Thus, 6.093, 9.093, 12.093, etc., indefinitely. Now, as to the number of Differences in each group, before repetition occurs: In the present case this number is 9 because $i = \frac{3}{9}$; for a slope of 1 to 1, $i = \frac{3}{27}$, therefore if 27 Differences are arranged in each column they will increase by 2.000 in line horizontally, when the slope is 1 to 1. Hence, the number of Differences in each column should be arranged for the particular slope. For a slope of $1\frac{1}{2}$ to 1, as in the present case, any number of groups of 9 will be convenient.

6th.—By successive addition, the quantities for each successive height h may now be obtained. Thus, for $h = 0.3$ ft., $Q = 18.278$ cu. yds.; for $h = 1$ ft., $Q = 64.815$ cu. yds.; for $h = 9.7$ ft., $Q = 1097.538$ cu. yds., etc.

Remarks.—The value of S at foot of each column is the sum of the Differences (in *italics*) in that column. Only one column need be added (these values of S are simply for checking the regular additions in each column) as the successive sums will increase by, in the present case, $27 \times 3 = 81$. Note method of checking each column by lines a, b and c.

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* See table on opposite page.

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The following is a tabular arrangement, explained p. 1028, for calculating the quantities Q in cu. yds. in Table 26, with ground line level, roadway $w = 16$ft., slopes $s = 1\frac{1}{2}$ to 1. The numbers in *italics*, opposite the *odd line* Nos., are Differences for successive tenths of feet in height, while the quantities in cu. yds., corresponding to the height in feet., are given opposite the *even line* Nos.

[Ex.—For height of 3.2 ft. there are 246.519 cu. yds. per station.]

No.	(Check) Height. Feet.	Col. 1.	Height. Feet.	Col. 2.	Height. Feet.	Col. 3.	Height. Feet.	Col. 4.	Height. Feet.	Col 5.	Height. Feet.	Col. 6
				203 500 / 5 981		284 500 / 209 481		365 500 / 493 981		446 500 / 859 481		527 500 / 1305 981
	.1 =	5 981		209 481		493 981		859 481		1305 981		1833 481
1		*6 093*		*9 093*		*12 093*		*15 093*		*18 093*		*21 093*
2	.2 =	12 074		218 574		506 074		874 574	11. =	1324 074		1854 574
3		*6 204*		*9 204*		*12 204*		*15 204*		*18 204*		*21 204*
4		18 278	3. =	227 778		518 278		889 778		1342 278		1875 778
5		*6 315*		*9 315*		*12 315*		*15 315*		*18 315*		*21 315*
6		24 593		237 093		530 593	8.5 =	905 093		1360 593		1897 093
7		*6 426*		*9 426*		*12 426*		*15 426*		*18 426*		*21 426*
8	.5 =	31 019		246 519		543 019		920 519		1379 019	14. =	1918 519
9		*6 538*		*9 538*		*12 538*		*15 538*		*18 538*		*21 538*
0		37 557		256 057	6. =	555 557		936 057		1397 557		1940 057
1		*6 648*		*9 648*		*12 648*		*15 648*		*18 648*		*21 648*
2		44 205		265 705		568 205		951 705	11.5 =	1416 205		1961 705
3		*6 759*		*9 759*		*12 759*		*15 759*		*18 759*		*21 759*
4		50 964	3.5 =	275 464		580 964		967 464		1434 964		1983 464
5		*6 870*		*9 870*		*12 870*		*15 870*		*18 870*		*21 870*
6		57 834		285 334		593 834	9. =	983 334		1453 834		2005 334
7		*6 981*		*9 981*		*12 981*		*15 981*		*18 981*		*21 981*
8	1. =	64 815		295 315		606 815		999 315		1472 815	14.5 =	2027 315
9		*7 093*		*10 093*		*13 093*		*16 093*		*19 093*		*22 093*
0		71 908		305 408	6.5 =	619 908		1015 408		1491 908		2049 408
1		*7 204*		*10 204*		*13 204*		*16 204*		*19 204*		*22 204*
2		79 112		315 612		633 112		1031 612	12. =	1511 112		2071 612
3		*7 315*		*10 315*		*13 315*		*16 315*		*19 315*		*22 315*
4		86 427	4. =	325 927		646 427		1047 927		1530 427		2093 927
5		*7 426*		*10 426*		*13 426*		*16 426*		*17 426*		*22 426*
6		93 853		336 353		659 853	9.5 =	1064 353		1549 853		2116 353
7		*7 537*		*10 537*		*13 537*		*16 537*		*19 537*		*22 537*
8	1.5 =	101 390		346 890		673 390		1080 890		1569 390	15. =	2138 890
9		*7 648*		*10 648*		*13 648*		*16 648*		*19 648*		*22 648*
0		109 038		357 538	7. =	687 038		1097 538		1589 038		2161 538
1		*7 759*		*10 759*		*13 759*		*16 759*		*19 759*		*22 759*
2		116 797		368 297		700 797		1114 297	12.5 =	1608 797		2184 297
3		*7 870*		*10 870*		*13 870*		*16 870*		*19 870*		*22 870*
4		124 667	4.5 =	379 167		714 667		1131 167		1628 667		2207 167
5		*7 981*		*10 981*		*13 981*		*16 981*		*19 981*		*22 981*
6		132 648		390 148		728 648	10. =	1148 148		1648 648		2230 148
7		*8 093*		*11 093*		*14 093*		*17 093*		*20 093*		*23 093*
8	2. =	140 741		401 241		742 741		1165 241		1668 741	15.5 =	2253 241
9		*8 204*		*11 204*		*14 204*		*17 204*		*20 204*		*23 204*
0		148 945		412 445	7.5 =	756 945		1182 445		1688 945		2276 445
1		*8 315*		*11 315*		*14 315*		*17 315*		*20 315*		*23 315*
2		157 260		423 760		771 260		1199 760	13. =	1709 260		2299 760
3		*8 426*		*11 426*		*14 426*		*17 426*		*20 426*		*23 426*
4		165 686	5. =	435 186		785 686		1217 186		1729 686		2323 186
5		*8 537*		*11 537*		*14 537*		*17 537*		*20 537*		*23 537*
6		174 223		446 723		800 223	10.5 =	1234 723		1750 223		2346 723
7		*8 648*		*11 648*		*14 648*		*17 648*		*20 648*		*23 648*
8	2.5 =	182 871		458 371		814 871		1252 371		1770 871	16. =	2370 371
9		*8 759*		*11 759*		*14 759*		*17 759*		*20 759*		*23 759*
0		191 630		470 130	8. =	829 630		1270 130		1791 630		2394 130
1		*8 870*		*11 870*		*14 870*		*17 870*		*20 870*		*23 870*
2		200 500		482 000		844 500		1288 000	13.5 =	1812 500		2418 000
3		*8 981*		*11 981*		*14 981*		*17 981*		*20 981*		*23 981*
4		209 481	5.5 =	493 981		859 481		1305 981		1833 481		2441 981
	(Ch'k.)	203 500 + 81 =		284 500 + 81 =		365 500 + 81 =		446 500 + 81 =		527 500 + 81 =		608 500

22.—Formulas for Ground-Slope Quantities.

To be added to, or subtracted from, quantities for level sections.
(For application, see Tables 27 and 28.)

Note.—x = hor. dist. (ft.) from cen. line to point where ground slope meets limit of section.
y = vert. dist. (ft.) " sub-grade " " " " " " " " "

Excav. or Emb. Slope.	*Tan β.	Ground Slope. Case.	x.	y.	Area in Sq. Ft. A On One Side of Center.	Quan. in Cu. Yds. Q For a 100-ft. Sta'n On One Side of Center.
General	General	U	$\frac{h+\frac{1}{2}w\tan\beta}{\tan\beta-\tan\alpha}$	$h+x\tan\alpha$	$\frac{(h+\frac{1}{2}w\tan\beta)^2\tan\alpha}{2\tan\beta(\tan\beta-\tan\alpha)}$	$\frac{100A}{3\times 9}$ (See Table 20)
"	"	D_1	$\frac{h+\frac{1}{2}w\tan\beta}{\tan\beta+\tan\alpha}$	$h-x\tan\alpha$	$\frac{(h+\frac{1}{2}w\tan\beta)^2\tan\alpha}{2\tan\beta(\tan\beta+\tan\alpha)}$	$\frac{100A}{3\times 9}$ "
"	"	D_2	$h\cot\alpha$	0	$\frac{h}{2}[h(\cot\beta-\cot\alpha)+w]$	$\frac{100A}{3\times 9}$ "
¼ : 1	4	U	$\frac{h+2w}{4-\tan\alpha}$	$\frac{4h+2w\tan\alpha}{4-\tan\alpha}$	$\frac{(h+2w)^2\tan\alpha}{32-8\tan\alpha}$	$\frac{100A}{3\times 9}$ "
"	"	D_1	$\frac{h+2w}{4+\tan\alpha}$	$\frac{4h-2w\tan\alpha}{4+\tan\alpha}$	$\frac{(h+2w)^2\tan\alpha}{32+8\tan\alpha}$	$\frac{100A}{3\times 9}$ "
"	"	D_2	$h\cot\alpha$	0	$\frac{h}{2}[h(\frac{1}{4}-\cot\alpha)+w]$	$\frac{100A}{3\times 9}$ "

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$\frac{3}{4}:1$	$1\frac{1}{3}$	U	$\frac{h+\frac{3}{4}w}{1\frac{1}{3}-\tan\alpha}$	$\frac{1\frac{1}{3}h+\frac{3}{4}w\tan\alpha}{1\frac{1}{3}-\tan\alpha}$	$\frac{(h+\frac{3}{4}w)^2\tan\alpha}{3\frac{1}{9}-2\frac{2}{3}\tan\alpha}$	$\frac{100A}{3\times 9}$	"
"	"	D_1	$\frac{h+\frac{3}{4}w}{1\frac{1}{3}+\tan\alpha}$	$\frac{1\frac{1}{3}h-\frac{3}{4}w\tan\alpha}{1\frac{1}{3}+\tan\alpha}$	$\frac{(h+\frac{3}{4}w)^2\tan\alpha}{3\frac{1}{9}+2\frac{2}{3}\tan\alpha}$	$\frac{100A}{3\times 9}$	"
"	"	D_2	$h\cot\alpha$	0	$\frac{h}{2}[h(\frac{4}{3}-\cot\alpha)+w]$	$\frac{100A}{3\times 9}$	"
$1:1$	1	U	$\frac{h+\frac{1}{2}w}{1-\tan\alpha}$	$\frac{h+\frac{1}{2}w\tan\alpha}{1-\tan\alpha}$	$\frac{(h+\frac{1}{2}w)^2\tan\alpha}{2-2\tan\alpha}$	$\frac{100A}{3\times 9}$	"
"	"	D_1	$\frac{h+\frac{1}{2}w}{1+\tan\alpha}$	$\frac{h-\frac{1}{2}w\tan\alpha}{1+\tan\alpha}$	$\frac{(h+\frac{1}{2}w)^2\tan\alpha}{2+2\tan\alpha}$	$\frac{100A}{3\times 9}$	"
"	"	D_2	$h\cot\alpha$	0	$\frac{h}{2}[h(1-\cot\alpha)+w]$	$\frac{100A}{3\times 9}$	"
$1\frac{1}{4}:1$	0.8	U	$\frac{h+0.4w}{0.8-\tan\alpha}$	$\frac{0.8h+0.4w\tan\alpha}{0.8-\tan\alpha}$	$\frac{(h+0.4w)^2\tan\alpha}{1.28-1.6\tan\alpha}$	$\frac{100A}{3\times 9}$	"
"	"	D_1	$\frac{h+0.4w}{0.8+\tan\alpha}$	$\frac{0.8h-0.4w\tan\alpha}{0.8+\tan\alpha}$	$\frac{(h+0.4w)^2\tan\alpha}{1.28+1.6\tan\alpha}$	$\frac{100A}{3\times 9}$	"
"	"	D_2	$h\cot\alpha$	0	$\frac{h}{2}[h(1\frac{1}{2}-\cot\alpha)+w]$	$\frac{100A}{3\times 9}$	"

* Note that $\tan\beta$ = the reciprocal of $\cot\beta$; and that $\cot\beta$ = the ordinary ratio of "excav. or emb. slope" expressed as a fraction or decimal. Thus, for slope of $1\frac{1}{2}$ to $1 = 1\frac{1}{2} : 1 = 1\frac{1}{2} + 1$, $\cot\beta = 1\frac{1}{2} = \frac{3}{2} = 1.5$, and $\tan\beta$ = its reciprocal $= \frac{2}{3} = 0.66\dot{6}$, etc.

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22.—FORMULAS FOR GROUND-SLOPE QUANTITIES.—Concluded.

To be added to, or subtracted from, quantities for level sections.
(For application, see Tables 27 and 28.)

Note.—x = hor. dist. (ft.) from cen. line to point where ground slope meets limit of section.
y = vert. dist. (ft.) " sub-grade " " " " " " " " "

Excav. or Emb. Slope.	*Tan β.	Ground Slope. Case.	x.	y.	Area in Sq. Ft. A On One Side of Center.	Quan. in Cu. Yds. Q For a 100-ft. Sta'n On One Side of Center.
1½ : 1	⅔	U	$\frac{h+\frac{1}{3}w}{\frac{2}{3}-\tan\alpha}$	$\frac{\frac{2}{3}h+\frac{1}{3}w\tan\alpha}{\frac{2}{3}-\tan\alpha}$	$\frac{(h+\frac{1}{3}w)^2\tan\alpha}{\frac{8}{9}-\frac{4}{3}\tan\alpha}$	$\frac{100A}{3\times 9}$ (See Table 20)
"	"	D_1	$\frac{h+\frac{1}{3}w}{\frac{2}{3}+\tan\alpha}$	$\frac{\frac{2}{3}h-\frac{1}{3}w\tan\alpha}{\frac{2}{3}+\tan\alpha}$	$\frac{(h+\frac{1}{3}w)^2\tan\alpha}{\frac{8}{9}+\frac{4}{3}\tan\alpha}$	$\frac{100A}{3\times 9}$ "
"	"	D_2	$h\cot\alpha$	0	$\frac{h}{2}[h(1\frac{1}{2}-\cot\alpha)+w]$	$\frac{100A}{3\times 9}$ "
1¾ : 1	4/7	U	$\frac{h+\frac{2}{7}w}{\frac{4}{7}-\tan\alpha}$	$\frac{\frac{4}{7}h+\frac{2}{7}w\tan\alpha}{\frac{4}{7}-\tan\alpha}$	$\frac{(h+\frac{2}{7}w)^2\tan\alpha}{\frac{32}{49}-\frac{8}{7}\tan\alpha}$	$\frac{100A}{3\times 9}$ "
"	"	D_1	$\frac{h+\frac{2}{7}w}{\frac{4}{7}+\tan\alpha}$	$\frac{\frac{4}{7}h-\frac{2}{7}w\tan\alpha}{\frac{4}{7}+\tan\alpha}$	$\frac{(h+\frac{2}{7}w)^2\tan\alpha}{\frac{32}{49}+\frac{8}{7}\tan\alpha}$	$\frac{100A}{3\times 9}$ "
"	"	D_2	$h\cot\alpha$	0	$\frac{h}{2}[h(1\frac{3}{4}-\cot\alpha)+w]$	$\frac{100A}{3\times 9}$ "
2 : 1	0.5	U	$\frac{h+\frac{1}{4}w}{0.5-\tan\alpha}$	$\frac{\frac{1}{2}h+\frac{1}{4}w\tan\alpha}{0.5-\tan\alpha}$	$\frac{(h+\frac{1}{4}w)^2\tan\alpha}{0.5-\tan\alpha}$	$\frac{100A}{3\times 9}$ "
"	"	D_1	$\frac{h+\frac{1}{4}w}{0.5+\tan\alpha}$	$\frac{\frac{1}{2}h-\frac{1}{4}w\tan\alpha}{0.5+\tan\alpha}$	$\frac{(h+\frac{1}{4}w)^2\tan\alpha}{0.5+\tan\alpha}$	$\frac{100A}{3\times 9}$ "
"	"	D^2	$h\cot\alpha$	0	$\frac{h}{2}[h(2-\cot\alpha)+w]$	$\frac{100A}{3\times 9}$ "

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"	"	D_2	$h \cot \alpha$	0	$\frac{h}{2}[h(1\frac{1}{2}-\cot \alpha)+14]$	$\frac{100A}{3\times 9}$	"

† Example 2.—Roadway (w), 18 ft.; excav. slopes, 1 : 1.

1 : 1	1	U	$\frac{h+9}{1-\tan \alpha}$	$\frac{h+9\tan \alpha}{1-\tan \alpha}$	$\frac{(h+9)^2 \tan \alpha}{2-2\tan \alpha}$	$\frac{100A}{3\times 9}$	"
"	"	D_1	$\frac{h+9}{1+\tan \alpha}$	$\frac{h-9\tan \alpha}{1+\tan \alpha}$	$\frac{(h+9)^2 \tan \alpha}{2+2\tan \alpha}$	$\frac{100A}{3\times 9}$	"
"	"	D_2	$h \cot \alpha$	0	$\frac{h}{2}[h(1-\cot \alpha)+18]$	$\frac{100A}{3\times 9}$	"

* Note that $\tan \beta$ = the reciprocal of $\cot \beta$; and that $\cot \beta$ = the ordinary ratio of "excav. or emb. slope" expressed as a fraction or decimal. Thus, for slope of $1\frac{1}{2}$ to $1 = 1\frac{1}{2} : 1 = 1\frac{1}{2} \div 1$, $\cot \beta = 1\frac{1}{2} = \frac{3}{2} = 1.5$, and $\tan \beta$ = its reciprocal $= \frac{2}{3} = 0.66^{\vee}6$, etc.

† Ground-slope corrections in Tables 27 and 28 were calculated from the formulas under Example 2, above.

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23.—Level Sections (Earthwork); Height., 0–60 Ft. Base of Roadway, 14 Ft. Side Slopes, 1½ to 1.

Note.—The last two columns enable us to use any other base than 14 ft.: Ex.—Given height, 34.5 ft.; roadway 12 ft. Then we have, 8401.4÷(251.85÷3.70)=8145.8 cu. yds. (For Ht. >60 ft., see Tables 24, 41.)

[Cu. Yds. per 100-Ft. Station.]

Ht. Ft.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Width of 2 Ft. Cu. Yds
0	.....	5.2	10.6	16.1	21.6	27.3	33.1	39.0	45.0	51.2	.....
1	57.4	63.8	70.2	76.8	83.5	90.3	97.2	104.2	111.3	118.6	7.41
2	125.9	133.4	141.0	148.6	156.4	164.4	172.4	180.5	188.7	197.1	14.81
3	205.6	214.1	222.8	231.6	240.5	249.5	258.7	267.9	277.3	286.7	22.22
4	296.3	306.0	315.8	325.7	335.7	345.8	356.1	366.4	376.9	387.5	29.63
5	398.1	408.9	419.9	430.9	442.0	453.2	464.6	476.1	487.6	499.3	37.04
6	511.1	523.0	535.0	547.2	559.4	571.8	584.2	596.8	609.5	622.3	44.44
7	635.2	648.2	661.3	674.6	687.9	701.4	714.9	728.6	742.4	756.3	51.85
8	770.3	784.5	798.7	813.1	827.5	842.1	856.8	871.6	886.5	901.5	59.26
9	916.7	931.9	947.3	962.7	978.3	994.0	1009.8	1025.7	1041.7	1057.8	66.67
10	1074.1	1090.4	1106.9	1123.5	1140.1	1156.9	1173.9	1190.9	1208.0	1225.2	74.07
11	1242.6	1260.1	1277.6	1295.3	1313.1	1331.0	1349.0	1367.2	1385.4	1403.8	81.48
12	1422.2	1440.8	1459.5	1478.3	1497.2	1516.2	1535.3	1554.6	1573.9	1593.4	88.89
13	1613.0	1632.6	1652.4	1672.4	1692.4	1712.5	1732.7	1753.1	1773.6	1794.1	96.30
14	1814.8	1835.6	1856.5	1877.5	1898.7	1919.9	1941.3	1962.7	1984.3	2006.0	103.70
15	2027.8	2049.7	2071.7	2093.8	2076.1	2138.4	2160.9	2183.5	2206.1	2228.9	111.11
16	2251.8	2274.9	2298.0	2321.2	2344.6	2368.1	2391.6	2415.3	2439.1	2463.0	118.52
17	2487.0	2511.2	2535.4	2559.8	2584.2	2608.8	2633.5	2658.3	2683.2	2708.2	125.93
18	2733.3	2758.6	2783.9	2809.4	2835.0	2860.6	2886.4	2912.4	2938.4	2964.5	133.33
19	2990.7	3017.1	3043.6	3070.1	3096.8	3123.6	3150.5	3177.5	3204.7	3231.9	140.74
20	3259.3	3286.7	3314.3	3342.0	3369.8	3397.7	3425.7	3453.8	3482.1	3510.4	148.15
21	3538.9	3567.5	3596.1	3624.9	3653.8	3682.9	3712.0	3741.2	3770.6	3800.0	155.56
22	3829.6	3859.3	3889.1	3919.0	3949.0	3979.1	4009.4	4039.7	4070.2	4100.8	162.96
23	4131.5	4162.3	4193.2	4224.2	4255.3	4286.6	4317.9	4349.4	4381.0	4412.6	170.37
24	4444.4	4476.4	4508.4	4540.5	4572.7	4605.1	4637.6	4670.1	4702.8	4735.6	177.78
25	4768.5	4801.5	4834.7	4867.9	4901.3	4934.7	4968.3	5002.0	5035.8	5069.7	185.19
26	5103.7	5137.8	5172.1	5206.4	5240.9	5275.5	5310.1	5344.9	5379.8	5414.9	192.59
27	5450.0	5485.2	5520.6	5556.1	5591.6	5627.3	5663.1	5699.0	5735.0	5771.2	200.00
28	5807.4	5843.8	5880.2	5916.8	5953.5	5990.3	6027.2	6064.2	6101.3	6138.6	207.41
29	6175.9	6213.4	6251.0	6288.6	6326.4	6364.4	6402.4	6440.5	6478.7	6517.1	214.81
30	6555.6	6594.1	6632.8	6671.6	6710.5	6749.5	6788.7	6827.9	6867.3	6906.7	222.22
31	6946.3	6986.0	7025.8	7065.7	7105.7	7145.8	7186.1	7226.4	7266.9	7307.5	229.63
32	7348.1	7388.9	7429.9	7470.9	7512.0	7553.2	7594.6	7636.1	7677.6	7719.3	237.04
33	7761.1	7803.0	7845.0	7887.2	7929.4	7971.8	8014.2	8056.8	8099.5	8142.3	244.44
34	8185.2	8228.2	8271.3	8314.6	8357.9	8401.4	8445.0	8488.6	8532.4	8576.3	251.85
35	8620.4	8664.5	8708.7	8753.1	8797.6	8842.1	8886.8	8931.6	8976.5	9021.5	259.26
36	9066.7	9111.9	9157.3	9202.7	9248.3	9294.0	9339.8	9385.7	9431.7	9477.8	266.67
37	9524.1	9570.4	9616.9	9663.5	9710.1	9756.9	9803.9	9850.9	9898.0	9945.2	274.07
38	9992.6	10040	10088	10135	10183	10231	10279	10327	10375	10424	281.48
39	10472	10521	10569	10618	10667	10716	10765	10815	10864	10913	288.89
40	10963	11013	11062	11112	11162	11212	11263	11313	11364	11414	296.30
41	11465	11516	11567	11618	11669	11720	11771	11823	11874	11926	303.70
42	11978	12029	12081	12134	12186	12238	12291	12343	12396	12449	311.11
43	12502	12555	12608	12661	12715	12768	12822	12875	12929	12983	318.52
44	13037	13091	13145	13200	13254	13309	13363	13418	13473	13528	325.93
45	13583	13639	13694	13749	13805	13861	13916	13972	14028	14084	333.33
46	14141	14197	14254	14310	14367	14424	14480	14537	14595	14652	340.74
47	14709	14767	14824	14882	14940	14998	15056	15114	15172	15230	348.15
48	15289	15347	15406	15465	15524	15583	15642	15701	15761	15820	355.56
49	15880	15939	15999	16059	16119	16179	16239	16300	16360	16421	362.96
50	16481	16542	16603	16664	16725	16787	16848	16909	16971	17033	370.37
51	17094	17156	17218	17280	17343	17405	17467	17530	17593	17656	377.78
52	17719	17782	17845	17908	17971	18035	18098	18162	18226	18290	385.19
53	18354	18418	18482	18546	18611	18675	18740	18805	18870	18935	392.59
54	19000	19065	19131	19196	19262	19327	19393	19459	19525	19591	400.00
55	19657	19724	19790	19857	19923	19990	20057	20124	20191	20259	407.41
56	20326	20393	20461	20529	20596	20664	20732	20800	20869	20937	414.81
57	21005	21074	21143	21212	21280	21349	21419	21488	21557	21627	422.22
58	21696	21766	21836	21906	21976	22046	22116	22186	22257	22327	429.63
59	22398	22469	22540	22611	22682	22753	22825	22896	22968	23039	437.04
60	23111	23183	23255	23327	23399	23472	23544	23617	23689	23762	444.44

Note that Base, Slope, and Cu. Yds. in this table may all be multiplied by the same factor; thus, using factor of 1½ for height of 49.7 ft., we have, 24450 cu. yds. for base of 21 ft. and slopes 2¼ to 1.

Add for Tenths of Feet in Height.

	P. P.
	7.41
1	.74
2	1.48
3	2.22
4	2.96
5	3.70
6	4.44
7	5.19
8	5.93
9	6.67

Use with preceding column only.

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LEVEL SECTIONS (Earthwork); HEIGHT, 60–120 FT.
USE OF ROADWAY, 14 FT. SIDE SLOPES, 1½ TO 1.

he last two columns enable us to use any other base than 14 ft.: on height, 94.5 ft.; roadway 15 ft. Then we have, 54513+ 70) = 54863 cu. yds. (See also Table 41.)

[Cu. Yds. per 100-Ft. Station.]

.1	.2	.3	.4	.5	.6	.7	.8	.9	Width of 2 Ft. Cu. Yds	
23183	23255	23327	23399	23472	23544	23617	23689	23762	444.44	Add for Tenths of Feet in Height
23908	23981	24055	24128	24201	24275	24349	24422	24496	451.85	
24645	24719	24793	24868	24942	25017	25092	25167	25242	459.26	
25392	25467	25543	25618	25694	25770	25846	25922	25998	466.67	
26150	26227	26303	26380	26457	26534	26611	26688	26765	474.07	
26920	26998	27075	27153	27231	27309	27387	27465	27544	481.48	
27701	27779	27858	27937	28016	28095	28175	28254	28333	488.89	
28493	28572	28652	28732	28812	28893	28973	29054	29134	496.30	
29296	29377	29458	29539	29620	29701	29783	29864	29946	503.70	
30110	30192	30274	30356	30438	30521	30603	30686	30769	511.11	
30935	31018	31101	31185	31268	31352	31435	31519	31603	518.52	
31771	31855	31940	32024	32109	32193	32278	32363	32448	525.93	
32619	32704	32789	32875	32961	33046	33132	33218	33305	533.33	
33477	33564	33650	33737	33824	33911	33998	34085	34172	540.74	
34347	34434	34522	34610	34698	34786	34874	34962	35050	548.15	
35227	35316	35405	35494	35583	35672	35761	35851	35940	555.56	
36119	36209	36299	36389	36479	36569	36660	36750	36841	562.96	
37022	37113	37204	37295	37387	37478	37569	37661	37753	570.37	
37936	38028	38121	38213	38305	38398	38490	38583	38676	577.78	
38862	38955	39048	39141	39235	39328	39422	39516	39610	585.19	
39798	39892	39986	40081	40175	40270	40365	40460	40555	592.59	
40745	40841	40936	41032	41127	41223	41319	41415	41511	600.00	
41704	41800	41897	41993	42090	42187	42284	42381	42479	607.41	
42673	42771	42869	42966	43064	43162	43260	43359	43457	614.81	
43654	43753	43852	43951	44050	44149	44248	44347	44447	622.22	
44646	44746	44846	44946	45046	45146	45246	45347	45447	629.63	P. P.
45649	45750	45851	45952	46053	46155	46256	46358	46459	637.04	7.41
46663	46765	46867	46969	47072	47174	47277	47379	47482	644.44	1 .74
47688	47791	47895	47998	48101	48205	48309	48412	48516	651.85	2 1.48
48725	48829	48933	49038	49142	49247	49352	49457	49562	659.26	3 2.22
49772	49877	49983	50088	50194	50300	50406	50512	50618	666.67	4 2.96
50830	50937	51044	51150	51257	51364	51471	51578	51685	674.07	5 3.70
51900	52008	52115	52223	52331	52439	52547	52655	52764	681.48	6 4.44
52981	53089	53198	53307	53416	53525	53635	53744	53853	688.89	7 5.19
54073	54182	54292	54402	54513	54623	54733	54844	54954	696.30	8 5.93
55176	55287	55398	55509	55620	55731	55843	55954	56066	703.70	9 6.67
56290	56402	56514	56626	56738	56851	56963	57076	57189	711.11	Use with preceding column only.
57415	57528	57641	57755	57868	57982	58095	58209	58323	718.52	
58551	58665	58780	58894	59009	59123	59238	59353	59468	725.93	
59699	59814	59929	60045	60161	60276	60392	60508	60624	733.33	
60857	60974	61090	61207	61324	61441	61558	61675	61792	740.74	
62027	62144	62262	62380	62498	62616	62734	62852	62970	748.15	
63207	63326	63445	63564	63683	63802	63921	64041	64160	755.56	
64399	64519	64639	64759	64879	64999	65120	65240	65361	762.96	
65602	65723	65844	65965	66087	66208	66329	66451	66573	770.37	
66816	66938	67060	67183	67305	67428	67550	67673	67796	777.78	
68042	68165	68288	68411	68535	68658	68782	68906	69030	785.19	
69278	69402	69526	69651	69775	69900	70025	70150	70275	792.59	
70525	70651	70776	70902	71027	71153	71279	71405	71531	800.00	
71784	71910	72037	72163	72290	72417	72544	72671	72799	807.41	
73053	73181	73309	73436	73564	73692	73821	73949	74077	814.81	
74334	74463	74592	74721	74850	74979	75108	75237	75367	822.22	
75616	75756	75886	76016	76146	76276	76406	76537	76667	829.63	
76929	77060	77191	77322	77453	77585	77716	77848	77979	837.04	
78243	78375	78507	78639	78772	78904	79037	79169	79302	844.44	
79568	79701	79835	79968	80101	80235	80369	80502	80636	851.85	
80895	81039	81173	81308	81442	81577	81712	81847	81982	859.26	
82252	82387	82523	82658	82794	82930	83066	83202	83338	866.67	
83610	83747	83883	84020	84157	84294	84431	84568	84705	874.07	
84970	85118	85255	85393	85531	85669	85807	85945	86084	881.48	
86331	86499	86638	86777	86916	87055	87195	87334	87437	888.89	

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25.—Level Sections (Earthwork); Height, 0–60 Ft.

Base of Roadway, 16 Ft. Side Slopes, 1 to 1.

Note.—The last two columns enable us to use any other base than 14 ft.: Ex.—Given height, 20.3 ft.; roadway 14 ft. Then we have, 2729.2 − (148.15 + 2.22) = 2578.8 cu. yds. (For Ht. >60 ft., see Tables 28, 40.)

[Cu. Yds. per 100-Ft. Station.]

Ht. Ft.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Width of 2 Ft. Cu. Yds
0	.....	6.0	12.0	18.1	24.3	30.6	36.9	43.3	49.8	56.3	.....
1	63.0	69.7	76.4	83.3	90.2	97.2	104.3	111.4	118.7	126.0	7.41
2	133.3	140.8	148.3	155.9	163.6	171.3	179.1	187.0	195.0	203.0	14.81
3	211.1	219.3	227.6	235.9	244.3	252.8	261.3	270.0	278.7	287.4	22.22
4	296.3	305.2	314.2	323.3	332.4	341.7	351.0	360.3	369.8	379.3	29.63
5	388.9	398.6	408.3	418.1	428.0	438.0	448.0	458.1	468.3	478.6	37.04
6	488.9	499.3	509.8	520.3	531.0	541.7	552.4	563.3	574.2	585.2	44.44
7	596.3	607.4	618.7	630.0	641.3	652.8	664.3	675.9	687.6	699.3	51.85
8	711.1	723.0	735.0	747.0	759.1	771.3	783.6	795.9	808.3	820.8	59.26
9	833.3	846.0	858.7	871.4	884.3	897.2	910.2	923.3	936.4	949.7	66.67
10	963.0	976.3	898.8	1003.3	1016.9	1030.6	1044.3	1058.1	1072.0	1086.0	74.07
11	1100.0	1114.1	1128.3	1142.6	1156.9	1171.3	1185.8	1200.3	1215.0	1229.7	81.48
12	1244.4	1259.3	1274.2	1289.2	1304.3	1319.4	1334.7	1350.0	1365.3	1380.8	88.89
13	1396.3	1411.9	1427.6	1443.3	1459.1	1475.0	1491.0	1507.0	1523.1	1539.3	96.30
14	1555.6	1571.9	1588.3	1604.8	1621.3	1638.0	1654.7	1671.4	1688.3	1705.2	103.70
15	1722.2	1739.3	1756.4	1773.7	1791.0	1808.3	1825.8	1843.3	1860.9	1878.6	111.11
16	1896.3	1914.1	1932.0	1950.0	1968.0	1986.1	2004.3	2022.6	2040.9	2059.3	118.52
17	2077.8	2096.3	2115.0	2133.7	2152.4	2171.3	2190.2	2209.2	2228.3	2247.4	125.93
18	2266.7	2286.0	2305.3	2324.8	2344.3	2363.9	2383.6	2403.3	2423.1	2443.0	133.33
19	2463.0	2483.0	2503.1	2523.3	2543.6	2563.9	2584.3	2604.8	2625.3	2646.0	140.74
20	2666.7	2687.4	2708.3	2729.2	2750.2	2771.3	2792.4	2813.7	2835.0	2856.3	148.15
21	2877.8	2899.3	2920.9	2942.6	2964.3	2986.1	3008.0	3030.0	3052.0	3074.1	156.56
22	3096.3	3118.6	3140.9	3163.3	3185.8	3208.3	3231.0	3253.7	3276.4	3299.3	162.96
23	3322.2	3345.2	3368.3	3391.4	3414.7	3438.0	3461.3	3484.8	3508.3	3531.9	170.37
24	3555.6	3579.3	3603.1	3627.0	3651.0	3675.0	3699.1	3723.3	3747.6	3771.9	177.78
25	3796.3	3820.8	3845.3	3870.0	3894.7	3919.4	3944.3	3969.2	3994.2	4019.3	185.19
26	4044.4	4069.7	4095.0	4120.3	4145.8	4171.3	4196.9	4222.6	4248.3	4274.1	192.59
27	4300.0	4326.0	4352.0	4378.1	4404.3	4430.6	4456.9	4483.3	4509.8	4536.3	200.00
28	4563.0	4589.7	4616.4	4643.3	4670.2	4697.2	4724.3	4751.4	4778.7	4806.0	207.41
29	4833.3	4860.8	4888.3	4915.9	4943.6	4971.3	4999.1	5027.0	5055.0	5083.0	214.81
30	5111.1	5139.3	5167.6	5195.9	5224.3	5252.8	5281.3	5310.0	5338.7	5367.4	222.22
31	5396.3	5425.2	5454.2	5483.3	5512.4	5541.7	5571.0	5600.3	5629.8	5659.3	229.63
32	5688.9	5718.6	5748.3	5778.1	5808.0	5838.0	5868.0	5898.1	5928.3	5958.6	237.04
33	5988.9	6019.3	6049.8	6080.3	6111.0	6141.7	6172.4	6203.3	6234.2	6265.2	244.44
34	6296.3	6327.4	6358.7	6390.0	6421.3	6452.8	6484.3	6515.9	6547.6	6579.3	251.85
35	6611.1	6643.0	6675.0	6707.0	6738.1	6771.3	6803.6	6835.9	6868.3	6900.8	259.26
36	6933.3	6966.0	6998.7	7031.4	7064.3	7097.2	7130.2	7163.3	7196.4	7229.7	266.67
37	7263.0	7296.3	7329.8	7363.3	7396.9	7430.6	7464.3	7498.1	7532.0	7566.0	274.07
38	7600.0	7634.1	7668.3	7702.6	7736.9	7771.3	7805.8	7840.3	7875.0	7909.7	281.48
39	7944.4	7979.3	8014.2	8049.2	8084.3	8119.4	8154.7	8190.0	8225.3	8260.8	288.89
40	8296.3	8331.9	8367.6	8403.3	8439.1	8475.0	8511.0	8547.0	8583.1	8619.3	296.30
41	8655.6	8691.9	8728.3	8764.8	8801.3	8838.0	8874.7	8911.4	8948.3	8985.2	303.70
42	9022.2	9059.3	9096.4	9133.7	9171.0	9208.3	9245.8	9283.3	9320.9	9358.6	311.11
43	9396.3	9434.1	9472.0	9510.0	9548.0	9586.1	9624.3	9662.6	9700.9	9739.3	318.52
44	9777.8	9816.3	9855.0	9893.7	9932.4	9971.3	10010	10049	10088	10127	325.93
45	10167	10206	10245	10285	10324	10364	10404	10443	10483	10523	333.33
46	10563	10603	10643	10683	10724	10764	10804	10845	10885	10926	340.74
47	10967	11007	11048	11089	11130	11171	11212	11254	11295	11336	348.15
48	11378	11419	11461	11503	11544	11586	11628	11670	11712	11754	355.56
49	11796	11839	11881	11923	11966	12008	12051	12094	12136	12179	362.96
50	12222	12265	12308	12351	12395	12438	12481	12525	12568	12612	370.37
51	12656	12699	12743	12787	12831	12875	12919	12963	13008	13052	377.78
52	13096	13141	13185	13230	13275	13319	13364	13409	13454	13499	385.19
53	13544	13590	13635	13680	13726	13771	13817	13863	13908	13954	392.59
54	14000	14046	14092	14138	14184	14231	14277	14323	14370	14416	400.00
55	14463	14510	14556	14603	14650	14697	14744	14791	14839	14886	407.41
56	14933	14981	15028	15076	15124	15171	15219	15267	15315	15363	414.81
57	15411	15459	15508	15556	15604	15653	15701	15750	15799	15847	422.22
58	15896	15945	15994	16043	16092	16142	16191	16240	16290	16339	429.63
59	16389	16439	16488	16538	16588	16638	16688	16738	16788	16839	437.04
60	16889	16939	16990	17040	17091	17142	17192	17243	17294	17345	444.44

Note that Base, Slope, and Cu. Yds. in this table may all be multiplied by the same factor; thus, using factor of ¾ for height of 27.2 ft., we have, 3264 cu. yds. for base of 12 ft. and slopes ¾ to 1.

Add for Tenths of Feet in Height.

	P. P.
1	.74
2	1.48
3	2.22
4	2.96
5	3.70
6	4.44
7	5.19
8	5.93
9	6.67

Use with preceding column only.

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—LEVEL SECTIONS (Earthwork); HEIGHT, 0–60 FT.
ASE OF ROADWAY, 16 FT. SIDE SLOPES, 1½ TO 1.
ie last two columns enable us to use any other base than 16 ft.:
en height, 39.7 ft.; roadway 14 ft. Then we have, 11109—
) = 10815 cu. yds. (For Ht. >60 ft., see Tables 24, 41.)

[Cu. Yds. per 100-Ft. Station.]

.1	.2	.3	.4	.5	.6	.7	.8	.9	Width of 2 Ft. Cu. Yds	
6.0	12.1	18.3	24.6	31.0	37.6	44.2	51.0	57.8	.....	Add for Tenths of Feet in Height.
71.9	79.1	86.4	93.9	101.4	109.0	116.8	124.7	132.6	7.41	
148.9	157.3	165.7	174.2	182.9	191.6	200.5	209.5	218.6	14.81	
237.1	246.5	256.1	265.7	275.5	285.3	295.3	305.4	315.6	22.22	
336.4	346.9	357.5	368.3	379.2	390.1	401.2	412.4	423.8	29.63	
446.7	458.4	470.1	482.0	494.0	506.1	518.3	530.6	543.0	37.04	
568.2	581.0	593.8	606.8	619.9	633.1	646.4	659.9	673.4	44.44	
700.8	714.7	728.6	742.7	756.9	771.3	785.7	800.2	814.9	51.85	
844.5	859.5	874.6	889.8	905.1	920.5	936.1	951.7	967.5	59.26	
999.3	1015.4	1031.6	1047.9	1064.4	1080.9	1097.5	1114.3	1131.2	66.67	
165.2	1182.4	1199.8	1217.2	1234.7	1252.4	1270.1	1288.0	1306.0	74.07	
342.3	1360.6	1379.0	1397.6	1416.2	1435.0	1453.8	1472.8	1491.9	81.48	
530.4	1549.9	1569.4	1589.0	1608.8	1628.7	1648.6	1668.7	1688.9	88.89	
729.7	1750.2	1770.9	1791.6	1812.5	1833.5	1854.6	1875.8	1897.1	96.30	
940.1	1961.7	1983.5	2005.3	2027.3	2049.4	2071.6	2093.9	2116.4	103.70	
161.5	2184.3	2207.2	2230.1	2253.2	2276.4	2299.8	2323.2	2346.7	111.11	
394.1	2418.0	2442.0	2466.1	2490.3	2514.6	2539.0	2563.6	2588.2	118.52	
637.8	2662.8	2687.9	2713.1	2738.4	2763.9	2789.4	2815.0	2840.8	125.93	
892.6	2918.7	2944.9	2971.3	2997.7	3024.2	3050.9	3077.6	3104.5	133.33	
158.6	3185.8	3213.1	3240.5	3268.1	3295.7	3323.5	3351.3	3379.3	140.74	
435.6	3463.9	3492.4	3520.9	3549.5	3578.3	3607.2	3636.1	3665.2	148.15	
723.8	3753.2	3782.7	3812.4	3842.1	3872.0	3902.0	3932.1	3962.3	156.56	
023.0	4053.6	4084.2	4115.0	4145.8	4176.8	4207.9	4239.1	4270.4	162.96	
333.4	4365.0	4396.8	4428.7	4460.6	4492.7	4524.9	4557.3	4589.7	170.37	
654.9	4687.6	4720.5	4753.5	4786.6	4819.8	4853.1	4886.5	4920.1	177.78	
987.5	5021.3	5055.3	5089.4	5123.6	5157.9	5192.4	5226.9	5261.5	185.19	P. P.
1311.2	5366.1	5401.2	5436.4	5471.8	5507.2	5542.7	5578.4	5614.1	192.59	7.41
1646.0	5722.1	5758.3	5794.6	5831.0	5867.6	5904.2	5941.0	5977.8	200.00	1 .74
1991.9	6089.1	6126.4	6163.9	6201.4	6239.0	6276.8	6314.7	6352.6	207.41	2 1.48
2348.9	6467.3	6505.7	6544.2	6582.9	6621.6	6660.5	6699.5	6738.6	214.81	3 2.22
2717.1	6856.5	6896.1	6935.7	6975.5	7015.3	7055.3	7095.4	7135.6	222.22	4 2.96
3096.4	7256.9	7297.5	7338.3	7379.2	7420.1	7461.2	7502.4	7543.8	229.63	5 3.70
3486.7	7668.4	7710.1	7752.0	7794.0	7836.1	7878.3	7920.6	7963.0	237.04	6 4.44
3888.2	8091.0	8133.8	8176.8	8219.9	8263.1	8306.4	8349.9	8393.4	244.44	7 5.19
4300.8	8524.7	8568.6	8612.7	8656.9	8701.3	8745.7	8790.2	8834.9	251.85	8 5.93
4724.5	8969.5	9014.6	9059.8	9105.1	9150.5	9196.1	9241.7	9287.5	259.26	9 6.67
5159.3	9425.4	9471.6	9517.9	9564.4	9610.9	9657.5	9704.3	9751.2	266.67	
5605.2	9892.4	9939.8	9987.2	10035	10082	10130	10178	10226	274.07	
6062	10371	10419	10468	10516	10565	10614	10663	10712	281.48	
6520	10860	10909	10959	11009	11059	11109	11159	11209	288.89	
6980	11360	11411	11462	11513	11563	11615	11666	11717	296.30	
7440	11872	11923	11975	12027	12079	12132	12184	12236	303.70	
7902	12394	12447	12500	12553	12606	12660	12713	12767	311.11	
8364	12928	12982	13036	13090	13145	13199	13254	13308	318.52	
8828	13473	13528	13583	13638	13694	13749	13805	13861	325.93	
9293	14029	14085	14141	14198	14254	14311	14368	14425	333.33	
9759	14596	14653	14711	14768	14826	14883	14941	14999	340.74	
10225	15174	15232	15291	15350	15408	15467	15526	15585	348.15	
10691	15763	15823	15882	15942	16002	16062	16122	16182	355.56	
11158	16364	16424	16485	16546	16607	16668	16729	16790	362.96	
11626	16975	17037	17099	17161	17223	17285	17347	17410	370.37	
12095	17598	17661	17723	17787	17850	17913	17977	18040	377.78	
12565	18231	18295	18359	18424	18488	18552	18617	18682	385.19	
13035	18876	18941	19006	19072	19137	19203	19268	19334	392.59	
13506	19532	19598	19665	19731	19798	19864	19931	19998	400.00	Use with preceding column only.
13978	20199	20266	20334	20401	20469	20537	20605	20673	407.41	
14451	20877	20946	21014	21083	21152	22221	21289	21359	414.81	
14925	21567	21636	21706	21775	21845	21915	21985	22056	422.22	
15400	22267	22338	22408	22479	22550	22621	22692	22764	429.63	
15876	22978	23050	23122	23194	23266	23338	23411	23483	437.04	
16353	23701	23774	23847	23920	23993	24066	24140	24213	444.44	

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27.—Level Sections (Earthwork); Height, 0–60 Ft. Also—Base of Roadway, 18 Ft. Side Slopes, 1 to 1.

Note.—The last two columns enable us to use any other base than 18 ft.: Ex.—Given height, 14.8 ft.; roadway 19 ft. Then we have, 1797.9+ ½(103.70+5.93) = 1852.7 cu. yds. (For Ht. >60 ft., see Tables 28, 40.)

[Cu. Yds. per 100-Ft. Station.]

Ht. Ft.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Width of 2 Ft. Cu. Yds
0	.....	6.7	13.5	20.3	27.3	34.3	41.3	48.5	55.7	63.0	.....
1	70.4	77.8	85.3	92.9	100.6	108.3	116.1	124.0	132.0	140.0	7.41
2	148.1	156.3	164.6	172.9	181.3	189.8	198.4	207.0	215.7	224.5	14.81
3	233.3	242.3	251.3	260.3	269.5	278.7	288.0	297.4	306.8	316.3	22.22
4	325.9	335.6	345.3	355.1	365.0	375.0	385.0	395.1	405.3	415.6	29.63
5	425.9	436.3	446.8	457.4	468.0	478.7	489.5	500.3	511.3	522.3	37.04
6	533.3	544.5	555.7	567.0	578.4	589.8	601.3	612.9	624.6	636.3	44.44
7	648.1	660.0	672.0	684.0	696.1	708.3	720.6	732.9	745.3	757.8	51.85
8	770.4	783.0	795.7	808.5	821.3	834.3	847.3	860.3	873.5	886.7	59.26
9	900.0	913.4	926.8	940.3	953.9	967.6	981.3	995.1	1009.0	1023.0	66.67
10	1037.0	1051.1	1065.3	1079.6	1093.9	1108.3	1122.8	1137.4	1152.0	1166.7	74.07
11	1181.5	1196.3	1211.3	1226.3	1241.3	1256.5	1271.7	1287.0	1302.4	1317.8	81.48
12	1333.3	1348.9	1364.6	1380.3	1396.1	1412.0	1428.0	1444.0	1460.1	1476.3	88.89
13	1492.6	1508.9	1525.3	1541.8	1558.4	1575.0	1591.7	1608.5	1625.3	1642.3	96.30
14	1659.3	1676.3	1693.5	1710.7	1728.0	1745.4	1762.8	1780.3	1797.9	1815.6	103.70
15	1833.3	1851.1	1869.0	1887.0	1905.0	1923.1	1941.3	1959.6	1977.9	1996.3	111.11
16	2014.8	2033.4	2052.0	2070.7	2089.5	2108.3	2127.3	2146.3	2165.3	2184.5	118.52
17	2203.7	2223.0	2242.4	2261.8	2281.3	2300.9	2320.6	2340.3	2360.1	2380.0	125.93
18	2400.0	2420.0	2440.0	2460.2	2480.5	2500.8	2521.2	2541.7	2562	2582.9	133.33
19	2603.6	2624.4	2645.2	2666.2	2687.2	2708.2	2729.5	2750.7	2772.0	2793.4	140.74
20	2814.8	2836.3	2857.9	2879.6	2901.3	2923	2945.0	2967.0	2989.0	3011.1	148.15
21	3033.3	3055.6	3077.9	3100.3	3122.8	3145.4	3168.0	3190.7	3213.5	3236.3	155.56
22	3259.3	3282.3	3305.3	3328.5	3351.7	3375.0	3398.4	3421.8	3445.3	3468.9	162.96
23	3492.6	3516.3	3540.1	3564.0	3588.0	3612.0	3636.1	3660.3	3684.6	3708.9	170.37
24	3733.3	3757.8	3782.4	3807.0	3831.7	3856.5	3881.3	3906.3	3931.3	3956.3	177.78
25	3981.5	4006.7	4032.0	4057.4	4082.8	4108.3	4133.9	4159.6	4185.3	4211.1	185.19
26	4237.0	4263.0	4289.0	4315.1	4341.3	4367.6	4393.9	4420.3	4446.8	4473.4	192.59
27	4500.0	4526.7	4553.5	4580.3	4607.3	4634.3	4661.3	4688.5	4715.7	4743.0	200.00
28	4770.4	4797.8	4825.3	4852.9	4880.6	4908.3	4936.1	4964.0	4992.0	5020.0	207.41
29	5048.1	5076.3	5104.6	5132.9	5161.3	5189.8	5218.4	5247.0	5275.7	5304.5	214.81
30	5333.3	5362.3	5391.3	5420.3	5449.5	5478.7	5508.0	5537.4	5566.8	5596.3	222.22
31	5625.9	5655.6	5685.3	5615.1	5645.0	5675.0	5605.0	5635.1	5665.3	5695.6	229.63
32	5725.9	5956.3	5986.8	6017.4	6048.0	6078.7	6109.5	6140.3	6171.3	6202.3	237.04
33	6233.3	6264.5	6295.7	6327.0	6358.4	6389.8	6421.3	6452.9	6484.6	6516.3	244.44
34	6548.1	6580.0	6612.0	6644.0	6676.1	6708.3	6740	6772.9	6805.3	6837.8	251.85
35	6870.4	6903.0	6935.7	6968.5	7001.3	7034.3	7067.3	7100.3	7133.5	7166.7	259.26
36	7200.0	7233.4	7266.8	7300.3	7333.9	7367.6	7401.3	7435.1	7469.0	7503.0	266.67
37	7537.0	7571.1	7605.3	7639.6	7673.9	7708.3	7742.8	7777.4	7812.0	7846.7	274.07
38	7881.5	7916.3	7951.3	7986.3	8021.3	8056.5	8091.7	8127.0	8162.4	8197.8	281.48
39	8233.3	8268.9	8304.6	8340.3	8376.1	8412.0	8448.0	8484.0	8520.1	8556.3	288.89
40	8592.6	8628.9	8665.3	8701.8	8738.4	8775.0	8811.7	8848.5	8885.3	8922.3	296.30
41	8959.3	8996.3	9033.5	9070.7	9108.0	9145.4	9182.8	9220.3	9257.9	9295.6	303.70
42	9333.3	9371.1	9409.0	9447.0	9485.0	9523.1	9561.3	9599.6	9637.9	9676.3	311.11
43	9714.8	9753.4	9792.0	9830.7	9869.5	9908.3	9947.3	9986.3	10025	10064	318.52
44	10104	10143	10182	10222	10261	10301	10341	10380	10420	10460	325.93
45	10500	10540	10580	10620	10661	10701	10741	10782	10822	10863	333.33
46	10904	10944	10985	11026	11067	11108	11149	11191	11232	11273	340.74
47	11315	11356	11398	11440	11481	11523	11565	11607	11649	11691	348.15
48	11733	11776	11818	11860	11903	11945	11988	12031	12073	12116	355.56
49	12159	12202	12245	12288	12332	12375	12418	12462	12505	12549	362.96
50	12593	12636	12680	12724	12768	12812	12856	12900	12945	12989	370.37
51	13033	13078	13122	13167	13212	13256	13301	13346	13391	13436	377.78
52	13481	13527	13572	13617	13663	13708	13754	13800	13845	13891	385.19
53	13937	13983	14029	14075	14121	14168	14214	14260	14307	14353	392.59
54	14400	14447	14493	14540	14587	14634	14681	14728	14776	14823	400.00
55	14870	14918	14965	15013	15061	15108	15156	15204	15252	15300	407.41
56	15348	15396	15445	15493	15541	15590	15638	15687	15736	15784	414.81
57	15833	15882	15931	15980	16029	16079	16128	17177	16227	16276	422.22
58	16326	16376	16425	16475	16525	16575	16625	17675	16725	16776	429.63
59	16826	16876	16927	16977	17028	17079	17129	17180	17231	17282	437.04
60	17333	17384	17436	17487	17538	17590	17641	17693	17745	17796	444.44

Note that Base, Slope, and Cu. Yds. in this table may all be multiplied by the same factor; thus, using factor of 1½ for height of 47.3 ft., we have, 17160 cu. yds. for base of 27 ft. and slopes 1½ to 1.

Add for Tenths of Feet in Height.

	P. P.
1	7.41
	.74
2	1.48
3	2.22
4	2.96
5	3.70
6	4.44
7	5.19
8	5.93
9	6.67

Use with preceding column only.

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—CORRECTIONS FOR GROUND SLOPES NOT LEVEL.

BASE OF ROADWAY, 18 FT. SIDE SLOPES, 1 TO 1.

(See Explanation and Formulas in Table 22.)

Note.—α = angle of ground slope (*G. S.*) to right of left of center line; "*G. S.* = 1:10," "*G. S.* = 2:10," etc., means tan α. Up or (+) slopes from center indicate additive corrections, and down or (−) slopes subtract- from quantities in table on opposite page for level sections.

litive (+) and subtractive (−) Corrections in Cu. Yds. per 100-Ft. Sta.]

Center Ht. Ft.	G. S. = 1:10. $\alpha = 5°.7$		G. S. = 2:10. $\alpha = 11°.3$		G. S. = 3:10. $\alpha = 16°.7$		G. S. = 4:10. $\alpha = 21°.8$		G. S. = 5:10. $\alpha = 26°.6$	
	+ Up	− Down	+ Up	− Down	+ Up	− Down	+ Up	− Down	+ Up	− Down
1	21	17	46	26	79	29	123	31	185	31
2	35	20	56	37	96	49	149	56	224	59
3	30	24	67	44	114	62	178	75	267	83
4	34	28	78	52	134	72	208	89	313	104
5	40	33	91	60	156	84	242	104	363	121
6	46	38	104	69	179	96	278	119	417	139
7	53	43	119	79	203	109	316	135	474	158
8	59	49	134	89	229	124	357	153	535	178
9	67	55	150	100	257	138	400	171	600	200
10	74	61	167	111	287	154	446	191	669	223
	82	67	185	123	317	171	494	212	741	247
	91	73	204	136	350	188	544	233	817	272
	100	81	224	149	384	207	597	256	896	299
	109	89	245	163	420	226	653	280	980	327
	119	97	267	178	457	246	711	305	1067	356
	129	105	289	193	496	267	771	331	1157	386
	139	114	313	209	537	289	834	358	1252	417
	150	123	338	225	579	312	900	386	1350	450
	161	132	363	242	622	335	968	415	1452	484
	173	142	389	260	667	359	1038	445	1557	519
	185	152	417	278	714	385	1111	476	1667	556
	198	162	445	297	763	411	1186	508	1780	593
	311	172	474	316	813	438	1264	542	1896	632
	224	183	504	336	864	465	1344	576	2017	672
	238	195	535	357	917	494	1427	612	2141	714
	252	206	567	378	971	524	1512	648	2269	756
	267	218	600	400	1029	554	1600	686	2400	800
	282	230	634	423	1087	585	1690	724	2535	845
	297	243	669	446	1146	617	1783	764	2674	891
	313	256	704	469	1207	650	1878	805	2817	939
	329	269	741	494	1270	684	1975	847	2963	988
	346	283	778	519	1334	718	2075	889	3113	1038
	363	297	817	544	1400	754	2178	933	3267	1089
	380	311	856	571	1467	790	2283	978	3424	1141
	398	326	896	598	1537	827	2390	1024	3585	1195
	417	341	938	625	1607	865	2500	1071	3750	1250
	435	356	980	653	1679	904	2612	1120	3919	1306
	455	372	1023	682	1753	944	2727	1169	4091	1364
	475	388	1067	711	1829	985	2844	1219	4267	1422
	494	404	1112	741	1906	1026	2964	1270	4446	1482
	514	421	1157	772	1984	1068	3086	1323	4630	1543
	535	438	1204	803	2064	1112	3211	1376	4817	1606
	556	455	1252	835	2146	1156	3338	1431	5007	1669
	578	473	1300	867	2229	1200	3468	1486	5201	1734
	600	491	1350	900	2314	1246	3600	1543	5400	1800
	622	509	1400	934	2401	1293	3734	1601	5603	1867
	645	528	1452	968	2489	1340	3871	1659	5807	1936
	668	547	1504	1003	2579	1388	4011	1719	6017	2006
	692	566	1557	1038	2670	1438	4153	1780	6230	2077
	716	586	1612	1074	2763	1488	4297	1842	6446	2149
	741	606	1667	1111	2857	1538	4444	1905	6667	2222
	766	626	1723	1148	2953	1590	2593	1969	6891	2297
	792	647	1780	1186	3051	1643	4746	2034	7119	2373
	818	668	1838	1225	3150	1696	4900	2100	7350	2450
	845	690	1896	1264	3251	1750	5057	2167	7585	2528
	872	711	1956	1304	3353	1806	5216	2235	7824	2608
	900	733	2017	1344	3457	1862	5378	2305	8067	2689
	928	756	2078	1385	3563	1918	5542	2375	8313	2771
	957	778	2141	1427	3670	1976	5708	2447	8563	2854
	986	802	2204	1469	3779	2035	5878	2519	8817	2939

28.—Level Sections (Earthwork); Height, 60-100 Ft. Also—Base of Roadway, 18 Ft. Side Slopes, 1 to 1.

Note.—The last two columns enable us to use any other base than 18 ft.:
Ex.—Given height, 88.5 ft.; roadway 17 ft. Then we have, 34968 − ½(651.85 + 3.70) = 34580 cu. yds. (See also Table 40.)

[Cu. Yds. per 100-Ft. Station.]

Ht. Ft.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Width of 2 Ft. Cu. Yds	Add for Tenths of Feet in Height.	
60	17333	17384	17436	17487	17538	17590	17641	17693	17745	17796	444.44		
61	17848	17900	17952	18004	18056	18108	18161	18213	18265	18318	451.85		
62	18370	18423	18476	18528	18581	18634	18687	18740	18793	18847	459.26		
63	18900	18953	19007	19060	19114	19168	19221	19275	19329	19383	466.67		
64	19437	19491	19545	19600	19654	19708	19763	19817	19872	19927	474.07		
65	19981	20036	20091	20146	20201	20256	20312	20367	20422	20478	481.48		
66	20533	20589	20645	20700	20756	20812	20868	20924	20980	21036	488.89		
67	21093	21149	21205	21262	21318	21375	21432	21488	21545	21602	496.30		
68	21659	21716	21773	21831	21888	21945	22003	22060	22118	22176	503.70		
69	22233	22291	22349	22407	22465	22523	22581	22640	22697	22756	511.11		
70	22815	22873	22932	22991	23049	23108	23167	23226	23285	23344	518.52		
71	23404	23463	23522	23582	23641	23701	23761	23820	23880	23940	525.93		
72	24000	24060	24120	24180	24241	24301	24361	24422	24482	24543	533.33		
73	24604	24664	24725	24786	24847	24908	24969	25031	25092	25153	540.74		
74	25215	25276	25338	25400	25461	25523	25585	25647	25709	25771	548.15		
75	25833	25896	25958	26020	26083	26145	26208	26271	26333	26396	555.56	P. P.	
76	26459	26522	26585	26648	26712	26775	26838	26902	26965	27029	562.96	7.41	
77	27093	27156	27220	27284	27348	27412	27476	27540	27605	27669	570.37	1	.74
78	27733	27798	27862	27927	27992	28056	28121	28186	28251	28316	577.78	2	1.48
79	28381	28447	28512	28577	28643	28708	28774	28840	28905	28971	585.19	3	2.22
80	29037	29103	29169	29235	29301	29368	29434	29500	29567	29633	592.59	4	2.96
81	29700	29767	29833	29900	29967	30034	30101	30168	30236	30303	600.00	5	3.70
82	30370	30438	30505	30573	30641	30708	30776	30844	30912	30980	607.41	6	4.44
83	31048	31116	31185	31253	31321	31390	31458	31527	31596	31664	614.81	7	5.19
84	31733	31802	31871	31940	32010	32079	32148	32217	32287	32356	622.22	8	5.93
85	32426	32496	32565	32635	32705	32775	32845	32915	32985	33056	629.63	9	6.67
86	33126	33196	33267	33337	33408	33479	33549	33620	33691	33762	637.04		
87	33833	33904	33976	34047	34118	34190	34261	34333	34405	34476	644.44		
88	34548	34620	34692	34764	34836	34908	34981	35053	35125	35198	651.85		
89	35270	35343	35416	35488	35561	35634	35707	35780	35853	35927	659.26		
90	36000	36073	36147	36220	36294	36368	36441	36515	36589	36663	666.67		
91	36737	36811	36885	36960	37034	37108	37183	37257	37332	37407	674.07		
92	37481	37556	37631	35006	37781	37856	37932	38007	38082	38158	681.48		
93	38233	38309	38385	38460	38536	38612	38688	38764	38840	38916	688.89		
94	38993	39069	39145	39222	39298	39375	39452	39528	39605	39682	696.30		
95	39759	39836	39913	39991	40068	40145	40223	40300	40378	40456	703.70		
96	40533	40611	40689	40767	40845	40923	41001	41080	41158	41236	711.11		
97	41315	41393	41472	41551	41629	41708	41787	41866	41945	42024	718.52		
98	42104	42183	42262	42342	42421	42501	42581	42660	42740	42820	725.93		
99	42900	42980	43060	43140	43221	43301	43381	43462	43542	43623	733.33		
100	43704	43784	43865	43946	44027	44108	44189	44271	44352	44433	740.74		

Use with preceding column only.

Note that Base, Slope, and Cu. yds. in above table may all be multiplied by the same factor; thus, using factor of ⅔ for height of 66.3 ft., we have 13800 cu. yds. for base of 12 ft. and slopes ⅔ to 1.

Examples of Use of Table 28.

Ex. 1.—Find the number of cu. yds. in a 100-ft. station: roadway 18 ft., excav. slopes 1 to 1, ground slope 3 in 10 straight across, and center height 63 ft?

Solution:

Level cutting.....	18900	cu. yds.
Up slope, add.....	4114	" "
	23014	" "
Down slope, sub..	2215	" "
Ans.....	20799	" "

Ex. 2.—Same, but slope "up" 4 in 10 to left of center, and "down" 1 in 10 to right?

Solution:

	+ 18900	cu. yds.
	+ 6400	" "
	− 873	" "
Ans.....	+ 24427	" "

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—CORRECTIONS FOR GROUND SLOPES NOT LEVEL.
BASE OF ROADWAY, 18 FT. SIDE SLOPES, 1 TO 1.
(See Explanation and Formulas in Table 22.)

.—α = angle of ground slope (*G. S.*) to right or left of center line; *S.* = 1:10," "*G. S.* = 2:10," etc., means tan α. Up or (+) slopes center indicate additive corrections, and down or (−) slopes sub- from quantities in table on opposite page for level sections.

e (+) and subtractive (−) Corrections in Cu. Yds. per 100-Ft. Sta.]

G. S. = 1:10. α = 5°.7		G. S. = 2:10. α = 11°.3		G. S. = 3:10. α = 16°.7		G. S. = 4:10. α = 21°.8		G. S. = 5:10. α = 26°.6	
+ Up	− Down	+ Up	− Down	+ Up	− Down	+ Up	− Down	+ Up	− Down
980	802	2204	1469	3779	2035	5878	2519	8817	2939
1008	825	2269	1512	3889	2094	6049	2593	9074	3025
1037	848	2334	1556	4001	2154	6223	2667	9335	3112
1067	873	2400	1600	4114	2215	6400	2743	9600	3200
1097	897	2467	1645	4229	2277	6579	2820	9869	3290
1127	922	2535	1690	4346	2340	6760	2897	10141	3380
1157	947	2604	1736	4464	2404	6944	2976	10417	3472
1188	972	2674	1783	4584	2468	7131	3056	10696	3565
1220	998	2745	1830	4706	2534	7320	3137	10980	3660
1252	1024	2817	1878	4829	2600	7511	3219	11267	3756
1284	1051	2889	1926	4953	2667	7705	3302	11557	3852
1317	1077	2963	1975	5079	2735	7901	3386	11852	3951
1350	1105	3038	2025	5207	2804	8100	3471	12150	4050
1384	1133	3113	2075	5337	2874	8301	3558	12452	4151
1417	1160	3189	2126	5467	2944	8505	3645	12757	4252
1452	1188	3267	2178	5600	3015	8711	3733	13067	4356
1487	1216	3345	2230	5734	3088	8920	3823	13380	4460
1522	1245	3424	2283	5870	3161	9131	3913	13696	4565
1557	1274	3504	2336	6007	3235	9344	4005	14017	4672
1593	1304	3585	2390	6146	3309	9560	4097	14341	4780
1630	1334	3667	2445	6287	3385	9779	4191	14669	4890
1667	1364	3750	2500	6429	3462	10000	4286	15000	5000
1704	1394	3834	2556	6572	3539	10223	4381	15335	5112
1742	1425	3919	2613	6717	3617	10449	4478	15674	5225
1780	1456	4004	2669	6863	3696	10678	4576	16017	5339
1818	1488	4091	2727	7013	3776	10909	4675	16363	5454
1857	1519	4178	2785	7163	3857	11142	4775	16713	5571
1896	1552	4267	2844	7314	3938	11378	4876	17067	5689
1936	1584	4356	2904	7467	4021	11616	4978	17424	5808
1976	1617	4446	2964	7622	4104	11857	5081	17785	5928
2017	1650	4538	3025	7779	4188	12100	5186	18150	6050
2059	1684	4630	3086	7937	4274	12346	5291	18519	6173
2099	1717	4723	3148	8096	4359	12594	5397	18891	6297
2141	1752	4817	3211	8257	4446	12844	5505	19267	6422
2183	1786	4912	3274	8420	4534	13098	5613	19647	6549
2226	1821	5007	3338	8584	4623	13353	5723	20030	6677
2269	1856	5104	3403	8750	4712	13611	5833	20417	6806
2312	1892	5202	3468	8917	4802	13872	5945	20807	6936
2356	1927	5300	3534	9087	4893	14135	6058	21203	7067
2400	1964	5400	3600	9257	4985	14400	6171	21600	7200
2445	2000	5500	3667	9429	5077	14668	6286	22002	7334

—*G. S.* = + 2 in 10 to left of center, and − 3 in 10 to right; center 30 ft. Then add 3667 cu. yds. to, and subtract 3385 cu. yds. from, . yds. obtained from table of level sections on opposite page.

Interpolations for Table 28.

—When center height is in feet and tenths, interpolate for tenths from table.

—When the ground slope is intermediate between those slopes to head the columns, direct interpolation will be close enough for purposes for preliminary estimates, etc. Otherwise, use the exact in Table 22.

—When the roadway, w, is greater or less than 18 ft., multiply table by $\left(\frac{h+\frac{1}{2}w}{h+9}\right)^2$.

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29.—Formulas for Extending Tables of Level Sections to any Side Slopes.

Side Slopes	For Use with Tables with Slopes 1½ to 1. + means *add* to / − means *sub.* from the quantities in table.	For Use with Tables with Slopes 1 to 1. + means *add* to / − means *sub.* from the quantities in table.
¼ to 1	$-\frac{100h^2}{3\times9}\cdot\frac{10}{8}$ c.y. $=-\frac{100}{3\times9}[1\frac{1}{4}h^2]$* c.y.	$-\frac{100h^2}{4\times9}$ c. y. $=-\frac{100}{3\times9}[\frac{1}{4}h^2]$*
½ to 1	$-\frac{100h^2}{3\times9}$ " $=-\frac{100}{3\times9}[\ h^2\ ]$ "	$-\frac{100h^2}{6\times9}$ " $=-\frac{100}{3\times9}[\frac{1}{2}h^2]$ "
¾ to 1	$-\frac{100h^2}{4\times9}$ " $=-\frac{100}{3\times9}[\frac{3}{4}h^2]$ "	$-\frac{100h^2}{12\times9}$ " $=-\frac{100}{3\times9}[\frac{1}{4}h^2]$ "
1 to 1	$-\frac{100h^2}{6\times9}$ " $=-\frac{100}{3\times9}[\frac{1}{2}h^2]$ "	.....
1¼ to 1	$-\frac{100h^2}{12\times9}$ " $=-\frac{100}{3\times9}[\frac{1}{4}h^2]$ "	$+\frac{100h^2}{12\times9}$ c. y. $=+\frac{100}{3\times9}[\frac{1}{4}h^2]$ "
1½ to 1	.....	$+\frac{100h^2}{6\times9}$ " $=+\frac{100}{3\times9}[\frac{1}{2}h^2]$ "
1¾ to 1	$+\frac{100h^2}{12\times9}$ c.y. $=+\frac{100}{3\times9}[\frac{1}{4}h^2]$ "	$+\frac{100h^2}{4\times9}$ " $=+\frac{100}{3\times9}[\frac{3}{4}h^2]$ "
2 to 1	$+\frac{100h^2}{6\times9}$ " $=+\frac{100}{3\times9}[\frac{1}{2}h^2]$ "	$+\frac{100h^2}{3\times9}$ " $=+\frac{100}{3\times9}[\ h^2\ ]$ "

* *Note.*—Quantities in brackets [ ] are + or − *areas* in sq. ft. due to change in cross-section by change in side slope, and correspond with areas [*A*] in Table 20. Hence, the cu. yds. to be added or subtracted may be obtained from [*A*] by the use of Table 20.

29a.—Factors (*F*) for Extending Tables (*T*) of Level Sections to Given Side Slopes (First Column) and Widths of Roadway (*R*).

(Tables 24, 28, 40 and 41 are not included here.)

Side Slopes.	*R*	*T*	*F*	*R*	*T*	*F*	*R*	*T*	*F*	*R*	*T*	*F*	*R*	*T*	*F*	*R*	*T*	*F*
1/5 to 1	4	33	1/5	6	39	1/5	8	32	0.4	16	31	0.8						
¼ to 1	3	30	1/6	4	25	¼	5	33	¼	7	37	¼	10	32	½	20	31	1
½ to 1	6	30	⅓	8	25	½	9	27	½	10	33	½	11	34	½	14	37	½
	15	39	½	20	32	1	40	31	2									
¾ to 1	7	23	½	8	26	½	9	30	½	12	35	½	13	36	½	14	37	½
	15	33	¾	21	37	¾	30	22	1½	60	31	3						
1 to 1	12	30	⅔	16	25	1	18	27	1	20	33	1	22	34	1	28	37	1
	30	39	1	40	32	2	80	31	4									
1¼ to 1	15	30	5/6	20	25	1¼	25	33	1¼									
1½ to 1	14	23	1	16	26	1	18	30	1	24	35	1	26	36	1	27	27	1½
	28	38	1	30	33	1½	33	34	1½	45	39	1½	60	32	3	120	31	6
1¾ to 1	21	30	1 1/6	28	25	1¾	35	33	1¾									
2 to 1	24	30	1 1/3	32	25	2	36	27	2	40	33	2	44	34	2	60	38	2
2¼ to 1	21	23	1½	24	26	1½	27	30	1½	36	25	2¼	45	33	2¼			
2½ to 1	30	30	1⅔	40	25	2½	45	27	2½	50	33	2½	55	34	2½			
2¾ to 1	33	30	1 5/6	44	25	2¾	55	33	2¾									
3 to 1	28	23	2	32	26	3	35	30	2	48	35	2	55	27	3	60	33	3
	66	34	3															

Note.— *R*=width of bottom of canal or railway cut, or top of embankment. *R* may be increased or decreased by consulting the last two columns of the tables referred to.

Ex.—To find the number of cu yds. in a 100-ft. station for a canal 100 ft. wide at bottom and with side slopes 1½ to 1: Consult Table 21 and multiply the cu. yds. corresponding to any given height by the factor 6.

30.—Level Sections (Earthwork); Height, 0–60 Ft.
Base of Roadway, 18 Ft. Side Slopes, 1½ to 1.

Note.—The last two columns enable us to use any other base than 18 ft.: Ex.—Given height, 46.4 ft.; roadway 20 ft. Then we have, 15054+ 0.74+2.96) = 15398 cu. yds. (For Ht. >60 ft., see Tables 28, 40.)

[Cu. Yds. per 100-Ft. Station.]

Ht. Ft.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Width of 2 Ft. Cu. Yds	P. P. 7.41	
0	.....	6.7	13.6	20.5	27.6	34.7	42.0	49.4	56.9	64.5	.....		Add for Tenths of Feet in Height.
1	72.2	80.1	88.0	96.1	104.2	112.5	120.9	129.4	138.0	146.7	7.41		
2	155.5	164.5	173.5	182.7	191.9	201.3	210.8	220.4	230.1	240.0	14.81		
3	249.9	260.0	270.1	280.4	290.8	301.3	311.9	322.6	333.4	344.5	22.22		
4	355.5	366.7	378.0	389.4	400.9	412.5	424.2	436.0	448.0	460.0	29.63		
5	472.2	484.5	496.9	509.4	522.0	534.7	547.6	560.5	573.6	586.7	37.04		
6	600.0	613.4	626.9	640.5	654.2	668.1	682.0	696.1	710.2	724.5	44.44		
7	738.9	753.4	768.0	782.7	797.6	812.5	827.6	842.7	858.0	873.4	51.85		
8	888.9	904.5	920.2	936.1	952.0	968.1	984.2	1000.5	1016.9	1033.4	59.26		
9	1050.0	1066.7	1083.6	1100.5	1117.6	1134.7	1152.0	1169.4	1186.9	1204.5	66.67		
10	1222.2	1240.1	1258.0	1276.1	1294.2	1312.5	1330.9	1349.4	1368.0	1386.7	74.07		
11	1405.6	1424.5	1443.6	1462.7	1482.0	1501.4	1520.9	1540.5	1560.2	1580.1	81.48		
12	1600.0	1620.1	1640.2	1660.5	1680.9	1701.4	1722.0	1742.7	1763.6	1784.5	88.89		
13	1805.6	1826.7	1848.0	1869.4	1890.9	1912.5	1934.2	1956.1	1978.0	2000.1	96.30		
14	2022.2	2044.5	2066.9	2089.4	2112.0	2134.7	2157.6	2180.5	2203.6	2226.7	103.70		
15	2250.0	2273.4	2296.9	2320.5	2344.2	2368.1	2392.0	2416.1	2440.2	2465.5	111.11		
16	2488.9	2513.4	2538.0	2562.7	2587.6	2612.5	2637.6	2662.7	2688.0	2713.4	118.52		
17	2738.9	2764.5	2790.2	2816.1	2842.0	2868.1	2894.2	2920.5	2946.9	2973.4	125.93		
18	3000.0	3026.7	3053.6	3080.5	3107.6	3134.7	3162.0	3189.4	3216.9	3244.5	133.33		
19	3272.2	3300.1	3328.0	3356.1	3384.2	3412.5	3440.9	3469.4	3498.0	3526.7	140.74		
20	3555.6	3584.5	3613.6	3642.7	3672.0	3701.4	3730.9	3760.5	3790.2	3820.1	148.15		
21	3850.0	3880.1	3910.2	3940.5	3970.9	4001.4	4032.0	4062.7	4093.6	4124.5	155.56		
22	4155.6	4186.7	4218.0	4249.4	4280.9	4312.5	4344.2	4376.1	4408.0	4440.1	162.96		
23	4472.2	4504.5	4536.9	4569.4	4602.0	4634.7	4667.6	4700.5	4733.6	4766.7	170.37		
24	4800.0	4833.4	4866.9	4900.5	4934.2	4968.1	5002.0	5036.1	5070.2	5104.5	177.78		
25	5138.9	5173.4	5208.0	5242.7	5277.6	5312.5	5347.6	5382.7	5418.0	5453.4	185.19		
26	5488.9	5524.5	5560.2	5596.1	5632.0	5668.1	5704.2	5740.5	5776.9	5813.4	192.59		
27	5850.0	5886.7	5923.6	5960.5	5997.6	6034.7	6072.0	6109.4	6146.9	6184.5	200.00	1	.74
28	6222.2	6260.1	6298.0	6336.1	6374.2	6412.5	6450.9	6489.4	6528.0	6566.7	207.41	2	1.48
29	6605.6	6644.5	6683.6	6722.7	6762.0	6801.4	6840.9	6880.5	6920.2	6960.1	214.81	3	2.22
30	7000.0	7040.1	7080.2	7120.5	7160.9	7201.4	7242.0	7282.7	7323.6	7364.5	222.22	4	2.96
31	7405.6	7446.7	7488.0	7529.4	7570.9	7612.5	7654.2	7696.1	7738.0	7780.1	229.63	5	3.70
32	7822.2	7864.5	7906.9	7949.4	7992.0	8034.7	8077.6	8120.5	8163.6	8206.7	237.04	6	4.44
33	8250.0	8293.4	8336.9	8380.5	8424.2	8468.1	8512.0	8556.1	8600.2	8644.5	244.44	7	5.19
34	8688.9	8733.4	8778.0	8822.7	8867.6	8912.5	8957.6	9002.7	9048.0	9093.4	251.85	8	5.93
35	9138.9	9184.5	9230.2	9276.1	9322.0	9368.1	9414.2	9460.5	9506.9	9553.4	259.26	9	6.67
36	9600.0	9646.7	9693.6	9740.5	9787.6	9834.7	9882.0	9929.4	9976.9	10025	266.67		Use with preceding column only.
37	10072	10120	10168	10216	10264	10313	10361	10409	10458	10507	274.07		
38	10556	10605	10654	10703	10752	10801	10851	10901	10950	11000	281.48		
39	11050	11100	11150	11200	11251	11301	11352	11403	11454	11505	288.89		
40	11556	11607	11658	11709	11761	11813	11864	11916	11968	12020	296.30		
41	12072	12125	12177	12229	12282	12335	12388	12441	12494	12547	303.70		
42	12600	12653	12707	12761	12814	12868	12922	12976	13030	13085	311.11		
43	13139	13193	13248	13303	13358	13413	13468	13523	13578	13633	318.52		
44	13689	13745	13800	13856	13912	13968	14024	14081	14137	14193	325.93		
45	14250	14307	14364	14421	14478	14535	14592	14649	14707	14765	333.33		
46	14822	14880	14938	14996	15054	15113	15171	15229	15288	15347	340.74		
47	15406	15465	15524	15583	15642	15701	15761	15821	15880	15940	348 15		
48	16000	16060	16120	16181	16241	16301	16362	16423	16484	16545	355.56		
49	16606	16667	16728	16789	16851	16913	16974	17036	17098	17160	362.96		
50	17222	17285	17347	17409	17472	17535	17598	17661	17724	17787	370.37		
51	17850	17913	17977	18041	18104	18168	18232	18296	18360	18425	377.78		
52	18489	18553	18618	18683	18748	18813	18878	18943	19008	19073	385.19		
53	19139	19205	19270	19336	19402	19468	19534	19601	19667	19733	392.59		
54	19800	19867	19934	20000	20068	20135	20202	20269	20337	20405	400.00		
55	20472	20540	20608	20676	20744	20813	20881	20949	21018	21087	407.41		
56	21156	21225	21294	21363	21432	21501	21571	21641	21710	21780	414.81		
57	21850	21920	21990	22061	22131	22201	22272	22343	22414	22485	422.22		
58	22556	22627	22698	22769	22841	22913	22984	23056	23128	23200	429.63		
59	23272	23345	23417	23489	23562	23635	23708	23781	23854	23927	437.04		
60	24000	24073	24147	24221	24294	24368	24442	24516	24590	24665	444.44		

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31.—Level Sections (Earthwork); Height, 0-60 Ft. Base of Roadway, 20 Ft. Side Slopes, ¼ to 1.

Note.—The last two columns enable us to use any other base than 20 ft.:
Ex.—Given height, 43.9 ft.; roadway 18 ft. Then we have, 5036.3 − (318.52 + 6.67) = 4711.1 cu. yds.

[Cu. Yds. per 100-Ft. Station]

Ht. Ft.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Width of 2 Ft. Cu. Yds
0	.....	7.4	14.9	22.3	29.8	37.3	44.8	52.3	59.9	67.4	.....
1	75.0	82.6	90.2	97.9	105.5	113.2	120.9	128.6	136.3	144.1	7.41
2	151.8	159.6	167.4	175.3	183.1	191.0	198.8	206.7	214.7	222.6	14.81
3	230.6	238.5	246.5	254.5	262.6	270.6	278.7	286.8	294.9	303.0	22.22
4	311.1	319.3	327.4	335.6	343.9	352.1	360.3	368.6	376.9	385.2	29.63
5	393.5	401.9	410.2	418.6	427.0	435.4	443.9	452.3	460.8	469.3	37.04
6	477.8	486.3	494.9	503.4	512.0	520.6	529.2	537.9	546.5	555.2	44.44
7	563.9	572.6	581.3	590.1	598.9	607.6	616.4	625.3	634.1	643.0	51.85
8	651.8	660.7	669.7	678.6	687.6	696.5	705.5	714.5	723.6	732.6	59.26
9	741.7	750.8	759.9	769.0	778.1	787.3	796.4	805.6	814.9	824.1	66.67
10	833.3	842.6	851.9	861.2	870.5	879.9	889.2	898.6	908.0	917.4	74.07
11	926.9	936.3	945.8	955.3	964.8	974.3	983.9	993.4	1003.0	1012.6	81.48
12	1022.2	1031.9	1041.5	1051.2	1060.9	1070.6	1080.3	1090.1	1099.9	1109.6	88.89
13	1119.4	1129.3	1139.1	1149.0	1158.9	1168.8	1178.7	1188.6	1198.6	1208.5	96.30
14	1218.5	1228.5	1238.6	1248.6	1258.7	1268.8	1278.9	1289.0	1299.1	1309.3	103.70
15	1319.4	1329.6	1339.9	1350.1	1360.3	1370.6	1380.9	1391.2	1401.5	1411.9	111.11
16	1422.2	1432.6	1443.0	1453.4	1463.9	1474.3	1484.8	1495.3	1505.8	1516.3	118.52
17	1526.9	1537.4	1548.0	1558.6	1569.2	1579.9	1590.5	1601.2	1611.9	1622.6	125.93
18	1633.3	1644.1	1654.9	1665.6	1676.4	1687.3	1698.1	1709.0	1719.9	1730.8	133.33
19	1741.7	1752.6	1763.6	1774.5	1785.5	1796.5	1807.6	1818.6	1829.7	1840.8	140.74
20	1851.9	1863.0	1874.1	1885.3	1896.4	1907.6	1918.9	1930.1	1941.3	1952.6	148.15
21	1963.9	1975.2	1986.5	1997.9	2009.2	2020.6	2032.0	2043.4	2054.9	2066.3	156.56
22	2077.8	2089.3	2100.8	2112.3	2123.9	2135.4	2147.0	2158.6	2170.2	2181.9	162.96
23	2193.5	2205.2	2216.9	2228.6	2240.3	2252.1	2263.9	2275.6	2287.4	2299.3	170.37
24	2311.1	2323.0	2334.9	2346.8	2358.7	2370.6	2382.6	2394.5	2406.5	2418.5	177.78
25	2430.6	2442.6	2454.7	2466.8	2478.9	2491.0	2503.1	2515.3	2527.4	2539.6	185.19
26	2551.9	2564.1	2576.3	2588.6	2600.9	2613.2	2625.5	2637.9	2650.2	2662.6	192.59
27	2675.0	2687.4	2699.9	2712.3	2724.8	2737.3	2749.8	2762.3	2774.9	2787.4	200.00
28	2800.0	2812.6	2825.2	2837.9	2850.5	2863.2	2875.9	2888.6	2901.3	2914.1	207.41
29	2926.9	2939.6	2952.4	2965.3	2978.1	2991.0	3003.9	3016.8	3029.7	3042.6	214.81
30	3055.6	3068.5	3081.5	3094.5	3107.6	3120.6	3133.7	3146.8	3159.9	3173.0	222.22
31	3186.1	3199.3	3212.4	3225.6	3238.9	3252.1	3265.3	3278.6	3291.9	3305.2	229.63
32	3318.5	3331.9	3345.2	3358.6	3372.0	3385.4	3398.9	3412.3	3425.8	3439.3	237.04
33	3452.8	3466.3	3479.9	3493.4	3507.0	3520.6	3534.2	3547.9	3561.5	3575.2	244.44
34	3588.9	3602.6	3616.3	3630.1	3643.9	3657.6	3671.4	3685.3	3699.1	3713.0	251.85
35	3726.9	3740.8	3754.7	3768.6	3782.6	3796.5	3810.5	3824.5	3838.6	3852.6	259.26
36	3866.7	3880.8	3894.9	3809.0	3923.1	3937.3	3951.4	3965.6	3979.9	3994.1	266.67
37	4008.3	4022.6	4036.9	4051.2	4065.5	4079.9	4094.2	4108.6	4123.0	4137.4	274.07
38	4151.9	4166.3	4180.8	4195.3	4209.8	4224.3	4238.9	4253.4	4268.0	4282.6	281.48
39	4297.2	4311.9	4326.5	4341.2	4355.9	4370.6	4385.3	4400.1	4414.9	4429.6	288.89
40	4444.4	4459.3	4474.1	4489.0	4503.9	4518.8	4533.7	4548.6	4563.6	4578.5	296.30
41	4593.5	4608.5	4623.6	4638.6	4653.7	4668.8	4683.9	4699.0	4714.1	4729.3	303.70
42	4744.4	4759.6	4774.9	4790.1	4805.3	4820.6	4835.9	4851.2	4866.5	4881.9	311.11
43	4897.2	4912.6	4928.0	4943.4	4958.9	4974.3	4989.8	5005.3	5020.8	5036.3	318.52
44	5051.9	5067.4	5083.0	5098.6	5114.2	5129.9	5145.5	5161.2	5176.9	5192.6	325.93
45	5208.3	5224.1	5239.9	5255.6	5271.4	5287.3	5303.1	5319.0	5334.9	5350.8	333.33
46	5366.7	5382.6	5398.6	5414.5	5430.5	5446.5	5462.6	5478.6	5494.7	5510.8	340.74
47	5526.9	5543.0	5559.1	5575.3	5591.4	5607.6	5623.9	5640.1	5656.3	5672.6	348.15
48	5688.9	5705.2	5721.5	5737.9	5754.2	5770.6	5787.0	5803.4	5819.9	5836.3	355.56
49	5852.8	5869.3	5885.8	5902.3	5918.9	5935.4	5952.0	5968.6	5985.2	6001.9	362.96
50	6018.5	6035.2	6051.9	6068.6	6085.3	6102.1	6118.9	6135.6	6152.4	6169.3	370.37
51	6186.1	6203.0	6219.9	6236.8	6253.7	6270.6	6287.6	6304.5	6321.5	6338.5	377.78
52	6355.6	6372.6	6389.7	6406.8	6423.9	6441.0	6458.1	6475.3	6492.4	6509.6	385.19
53	6526.9	6544.1	6561.3	6578.6	6595.9	6613.2	6630.5	6647.9	6665.2	6682.6	392.59
54	6700.0	6717.4	6734.9	6752.3	6769.8	6787.3	6804.8	6822.3	6839.9	6857.4	400.00
55	6875.0	6892.6	6910.2	6927.9	6945.5	6963.2	6980.9	6998.6	7016.3	7034.1	407.41
56	7051.9	7069.6	7087.4	7105.3	7123.1	7141.0	7158.9	7176.8	7194.7	7212.6	414.81
57	7230.6	7248.5	7266.5	7284.5	7302.6	7320.6	7338.7	7356.8	7374.9	7393.0	422.22
58	7411.1	7429.3	7447.4	7465.6	7483.9	7502.1	7520.3	7538.6	7556.9	7575.2	429.63
59	7593.5	7611.9	7630.2	7648.6	7667.0	7685.4	7703.9	7722.3	7740.8	7759.3	437.04
60	7777.8	7796.3	7814.9	7833.4	7852.0	7870.6	7789.2	7807.9	7826.5	7845.2	444.44

Note that Base, Slope, and Cu. Yds. in this table may all be multiplied by the same factor; thus, using factor of 0.8 for height of 28 ft., we have, 2240 cu. yds. for base of 16 ft. and slopes $^1/_5$ to 1. (Slope $^1/_4$ to 1 is frequently used in solid rock excavation.)

Add for Tenths of Feet in Height.

	P. P.
1	.74
2	1.48
3	2.22
4	2.96
5	3.70
6	4.44
7	5.19
8	5.93
9	6.67

Use with preceding column only.

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32.—Level Sections (Earthwork); Height, 0–60 Ft. Base of Roadway, 20 Ft. Side Slopes ½ to 1.

Note.—The last two columns enable us to use any other base than 20 ft.: Ex.—Given height, 18.1 ft.; roadway 22 ft. Then we have 1947.4+ .33+0.74) = 2081.5 cu. yds.

[Cu. Yds. per 100-Ft. Station.]

ft.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Width of 2 Ft. Cu. Yds
0	.....	7.4	14.9	22.4	29.9	37.5	45.1	52.8	60.4	68.2	.....
1	75.9	83.7	91.6	99.4	107.3	115.3	123.3	131.3	139.3	147.4	7.41
2	155.6	163.7	171.9	180.2	188.4	196.8	205.1	213.5	221.9	230.4	14.81
3	238.9	247.4	256.0	264.6	273.3	281.9	290.7	299.4	308.2	317.1	22.22
4	325.9	334.8	343.8	352.8	361.8	370.8	379.9	389.1	398.2	407.4	29.63
5	416.7	425.9	435.3	444.6	454.0	463.4	472.9	482.4	491.9	501.5	37.04
6	511.1	520.8	530.4	540.2	549.9	559.7	569.6	579.4	589.3	599.3	44.44
7	609.3	619.3	629.3	639.4	649.6	659.7	669.9	680.2	690.4	700.8	51.85
8	711.1	721.5	731.9	742.4	752.9	763.4	774.0	784.6	795.3	805.9	59.26
9	816.7	827.4	838.2	849.1	859.9	870.8	881.8	892.8	903.8	914 8	66.67
0	925.9	937.1	948.2	959.4	970.7	981.9	993.3	1004.6	1016.0	1027.4	74.07
1	1038.9	1050.4	1061.9	1073.5	1085.1	1096.8	1108.4	1120.2	1131.9	1143.7	81.48
2	1155.6	1167.4	1179.3	1191.3	1203.3	1215.3	1227.3	1239.4	1251.6	1263.7	88.89
3	1275.9	1288.2	1300.4	1312.8	1325.1	1337.5	1349.9	1362.4	1374.9	1387.4	96.30
4	1400.0	1412.6	1425.3	1437.9	1450.7	1463.4	1476.2	1489.1	1501.9	1514.8	103.70
5	1527.8	1540.8	1553.8	1566.8	1579.9	1593.1	1606.2	1619.4	1632.7	1645.9	111.11
6	1659.3	1672.6	1686.0	1699.4	1712.9	1726.4	1739.9	1753.5	1767.1	1780.8	118.52
7	1794.4	1808.2	1821.9	1835.7	1849.6	1863.4	1877.3	1891.3	1905.3	1919.3	125.93
8	1933.3	1947.4	1961.6	1975.7	1989.9	2004.2	2018.4	2032.8	2047.1	2061.5	133.33
9	2075.9	2090.4	2104.9	2119.4	2134.0	2148.6	2163.3	2177.9	2192.7	2207.4	140.74
0	2222.2	2237.1	2251.9	2266.8	2281.8	2296.8	2311.8	2326.8	2341.9	2357.1	148.15
1	2372.2	2387.4	2402.7	2417.9	2433.3	2448.6	2464.0	2479.4	2494.9	2510.4	156.56
2	2525.9	2541.5	2557.1	2572.8	2588.4	2604.2	2619.9	2635.7	2651.6	2667.4	162.96
3	2683.3	2699.3	2715.3	2731.3	2747.3	2763.4	2779.6	2795.7	2811.9	2828.2	170.37
4	2844.4	2860.8	2877.1	2893.5	2909.9	2926.4	2942.9	2959.4	2976.0	2992.6	177.78
5	3009.3	3025.9	3042.7	3059.4	3076.2	3093.1	3109.9	3126.8	3143.8	3160.8	185.19
6	3177.8	3194.8	3211.9	3229.1	3246.2	3263.4	3280.7	3297.9	3315.3	3332.6	192.59
7	3350.0	3367.4	3384.9	3402.4	3419.9	3437.5	3455.1	3472.8	3490.4	3508.2	200.00
8	3525.9	3543.7	3561.6	3579.4	3597.3	3615.3	3633.3	3651.3	3669.3	3687.4	207.41
9	3705.6	3723.7	3741.9	3760.2	3778.4	3796.8	3815.1	3833.5	3851.9	3870.4	214.81
0	3888.9	3907.4	3926.0	3944.6	3963.3	3981.9	4000.7	4019.4	4038.2	4057.1	222.22
1	4075.9	4094.8	4113.8	4132.8	4151.8	4170.8	4189.9	4209.1	4228.2	4247.4	229.63
2	4266.7	4285.9	4305.3	4324.6	4344.0	4363.4	4382.9	4402.4	4421.9	4441.5	237.04
3	4461.1	4480.8	4500.4	4520.2	4539.9	4559.7	4579.6	4599.4	4619.3	4639.3	244.44
4	4659.3	4679.3	4699.3	4719.4	4739.6	4759.7	4779.9	4800.2	4820.4	4840.8	251.85
5	4861.1	4881.5	4901.9	4922.4	4942.9	4963.4	4984.0	5004.6	5025.3	5045.9	259.26
6	5066.7	5087.4	5108.2	5129.1	5149.9	5170.8	5191.8	5212.8	5233.8	5254.8	266.67
7	5275.9	5297.1	5318.2	5339.4	5360.7	5381.9	5403.3	5424.6	5446.0	5467.4	274.07
8	5488.9	5510.4	5531.9	5553.5	5575.1	5596.8	5618.4	5640.2	5661.9	5683.7	281.48
9	5705.6	5727.4	5749.3	5771.3	5793.3	5815.3	5837.3	5859.4	5881.6	5903.7	288.89
0	5925.9	5948.2	5970.4	5992.8	6015.1	6037.5	6059.9	6082.4	6104.9	6127.4	296.30
1	6150.0	6172.6	6195.3	6217.9	6240.7	6263.4	6286.2	6309.1	6331.9	6354.8	303.70
2	6377.8	6400.8	6423.8	6446.8	6469.9	6493.1	6516.2	6539.4	6562.7	6585.9	311.11
3	6609.3	6632.6	6656.0	6679.4	6702.9	6726.4	6749.9	6773.5	6797.1	6820.8	318.52
4	6844.4	6868.2	6891.9	6915.7	6939.6	6963.4	6987.3	7011.3	7035.3	7059.3	325.93
5	7083.3	7107.4	7131.6	7155.7	7179.9	7204.2	7228.4	7252.8	7277.1	7301.5	333.33
6	7325.9	7350.4	7374.9	7399.4	7424.0	7448.6	7473.3	7497.9	7522.7	7547.4	340.74
7	7572.2	7597.1	7621.9	7646.8	7671.8	7696.8	7721.8	7746.8	7771.9	7797.1	348.15
8	7822.2	7847.4	7872.7	7897.9	7923.3	7948.6	7974.0	7999.4	8024.9	8050.4	355.56
9	8075.9	8101.5	8127.1	8152.8	8178.4	8204.2	8229.9	8255.7	8281.6	8307.4	362.96
0	8333.3	8359.3	8385.3	8411.3	8437.3	8463.4	8489.6	8515.7	8541.9	8568.2	370.37
1	8594.4	8620.8	8647.1	8673.5	8699.9	8726.4	8752.9	8779.4	8806.0	8832.6	377.78
2	8859.3	8885.9	8912.7	8939.4	8966.2	8993.1	9019.9	9046.8	9073.8	9100.8	385.19
3	9127.8	9154.8	9181.9	9209.1	9236.2	9263.4	9290.7	9317.9	9345.3	9372.6	392.59
4	9400.0	9427.4	9554.9	9482.4	9509.9	9537.5	9565.1	9592.8	9620.4	9648.2	400.00
5	9675.9	9703.7	9731.6	9759.4	9787.3	9815.3	9843.3	9871.3	9899.3	9827.4	407.41
6	9955.6	9983.7	10012	10040	10068	10097	10125	10154	10182	10210	414.81
7	10239	10267	10296	10325	10353	10382	10411	10439	10468	10497	422.22
8	10526	10555	10584	10613	10642	10671	10700	10729	10758	10787	429.63
9	10817	10846	10875	10905	10934	10963	10993	11022	11052	11082	437.04
0	11111	11141	11170	11200	11230	11260	11290	11319	11349	11379	444.44

Add for Tenths of Feet in Height.

P.	P. 7.41
1	.74
2	1.48
3	2.22
4	2.96
5	3.70
6	4.44
7	5.19
8	5.93
9	6.67

Use with preceding column only.

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33.—Level Sections (Earthwork); Height, 0–60 Ft. Base of Roadway, 20 Ft. Side Slopes 1 to 1.

Note.—The last two columns enable us to use any other base than 20 ft.: Ex.—Given height, 54.7 ft.; roadway 21 ft. Then we have, 15134+ ½ (400.00+5.19) = 15337 cu. yds. (For Ht. >60 ft., see Tables 28, 40.)

[Cu. Yds. per 100-Ft. Station.]

Ht. Ft.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Width of 2 Ft. Cu. Yds
0	.....	7.4	15.0	22.6	30.2	38.0	45.8	53.7	61.6	69.7	.....
1	77.8	86.0	94.2	102.6	111.0	119.4	128.0	136.6	145.3	154.1	7.41
2	163.0	171.9	180.9	190.0	199.1	208.3	217.6	227.0	236.4	246.0	14.81
3	255.6	265.2	275.0	284.8	294.7	304.6	314.7	324.8	335.0	345.2	22.22
4	355.6	366.0	376.4	387.0	397.6	408.3	419.1	430.0	440.9	451.9	29.63
5	463.0	474.1	485.3	496.6	508.0	519.4	531.0	542.6	554.2	566.0	37.04
6	577.8	589.7	601.6	613.7	625.8	638.0	650.2	662.6	675.0	687.4	44.44
7	700.0	712.6	725.3	738.1	751.0	763.9	776.9	790.0	803.1	816.3	51.85
8	829.6	843.0	856.4	870.0	883.6	897.2	911.0	924.8	938.7	952.6	59.26
9	966.7	980.8	995.0	1009.2	1023.6	1038.0	1052.4	1067.0	1081.6	1096.3	66.67
10	1111.1	1126.0	1140.9	1155.9	1171.0	1186.1	1201.3	1216.6	1232.0	1247.4	74.07
11	1263.0	1278.6	1294.2	1310.0	1325.8	1341.7	1357.6	1373.7	1389.8	1406.0	81.48
12	1422.2	1438.6	1455.0	1471.4	1488.0	1504.6	1521.3	1538.1	1555.0	1571.9	88.89
13	1588.9	1606.0	1623.1	1640.3	1657.6	1675.0	1692.4	1710.0	1727.6	1745.2	96.30
14	1763.0	1780.8	1798.7	1816.6	1834.7	1852.8	1871.0	1889.2	1907.6	1926.0	103.70
15	1944.4	1963.0	1981.6	2000.3	2019.1	2038.0	2056.9	2075.9	2095.0	2114.1	111.11
16	2133.3	2152.6	2172.0	2191.4	2211.0	2230.6	2250.2	2270.0	2289.8	2309.7	118.52
17	2329.6	2349.7	2369.8	2390.0	2410.2	2430.6	2451.0	2471.4	2492.0	2512.6	125.93
18	2533.3	2554.1	2575.0	2595.9	2616.9	2638.0	2659.1	2680.3	2701.6	2723.0	133.33
19	2744.4	2766.0	2787.6	2809.2	2831.0	2852.8	2874.7	2896.6	2918.7	2940.8	140.74
20	2963.0	2985.2	3007.6	3030.0	3052.4	3075.0	3197.6	3120.3	3143.1	3166.0	148.15
21	3188.9	3211.9	3235.0	3258.1	3281.3	3304.6	3328.0	3351.4	3375.0	3398.6	155.56
22	3422.2	3446.0	3469.8	3493.7	3517.6	3541.7	3565.8	3590.0	3614.2	3638.6	162.96
23	3663.0	3687.4	3712.0	3736.6	3761.3	3786.1	3811.0	3835.9	3860.9	3886.0	170.37
24	3911.1	3936.3	3961.6	3987.0	4012.4	4038.0	4063.6	4089.2	4115.0	4140.8	177.78
25	4166.7	4192.6	4218.7	4244.8	4271.0	4297.2	4323.6	4350.0	4376.4	4403.0	185.19
26	4429.6	4456.3	4483.1	4510.0	4536.9	4563.9	4591.0	4618.1	4645.3	4672.6	192.59
27	4700.0	4727.4	4755.0	4782.6	4810.2	4838.0	4865.8	4893.7	4921.6	4949.7	200.00
28	4977.8	5006.0	5034.2	5062.6	5091.0	5119.4	5148.0	5176.6	5205.3	5234.1	207.41
29	5263.0	5291.9	5320.9	5350.0	5379.1	5408.3	5437.6	5467.0	5496.4	5526.0	214.81
30	5555.6	5585.2	5615.0	5644.8	5674.7	5704.6	5734.7	5764.8	5795.0	5825.2	222.22
31	5855.6	5886.0	5916.4	5947.0	5977.6	6008.3	6039.1	6070.0	6100.9	6131.9	229.63
32	6163.0	6194.1	6225.3	6256.6	6288.0	6319.4	6351.0	6382.6	6414.2	6446.0	237.04
33	6477.8	6509.7	6541.6	6573.7	6605.8	6638.0	6670.2	6702.6	6735.0	6767.4	244.44
34	6800.0	6832.6	6865.3	6898.1	6931.0	6963.9	6996.9	7030.0	7063.1	7096.3	251.85
35	7129.6	7163.0	7196.4	7230.0	7263.6	7297.2	7331.0	7364.8	7398.7	7432.6	259.26
36	7466.7	7500.8	7535.0	7569.2	7603.6	7638.0	7672.4	7707.0	7741.6	7776.3	266.67
37	7811.1	7846.0	7880.9	7915.9	7951.0	7986.1	8021.3	8056.6	8092.0	8127.4	274.07
38	8163.0	8198.6	8234.2	8270.0	8305.8	8341.7	8377.6	8413.7	8449.8	8486.0	281.48
39	8522.2	8558.6	8595.0	8631.4	8668.0	8704.6	8741.3	8778.1	8815.0	8851.9	288.89
40	8888.9	8926.0	8963.1	9000.3	9037.6	9075.0	9112.4	9150.0	9187.6	9225.2	296.30
41	9263.0	9300.8	9338.7	9376.6	9414.7	9452.8	9491.0	9529.2	9567.6	9606.0	303.70
42	9644.4	9683.0	9721.6	9760.3	9799.1	9838.0	9876.9	9915.9	9955.0	9994.1	311.11
43	10033	10073	10112	10151	10191	10231	10270	10310	10350	10390	318.52
44	10430	10470	10510	10550	10590	10631	10671	10711	10752	10793	325.93
45	10833	10874	10915	10956	10997	11038	11079	11120	11162	11203	333.33
46	11244	11286	11328	11369	11411	11453	11495	11537	11579	11621	340.74
47	11663	11705	11748	11790	11832	11875	11918	11960	12003	12046	348.15
48	12089	12132	12175	12218	12261	12305	12348	12391	12435	12479	355.56
49	12522	12566	12610	12654	12698	12742	12786	12830	12874	12919	362.96
50	12963	13007	13052	13097	13141	13186	13231	13276	13321	13366	370.37
51	13411	13456	13502	13547	13592	13638	13684	13729	13775	13821	377.78
52	13867	13913	13959	14005	14051	14097	14144	14190	14236	14283	385.19
53	14330	14376	14423	14470	14517	14564	14611	14658	14705	14753	392.59
54	14800	14847	14895	14943	14990	15038	15086	15134	15182	15230	400.00
55	15278	15326	15374	15423	15471	15519	15568	15617	15665	15714	407.41
56	15763	15812	15861	15910	15959	16008	16058	16107	16156	16206	414.81
57	16256	16305	16355	16405	16455	16505	16555	16605	16655	16705	422.22
58	16756	16806	16856	16907	16958	17008	17059	17110	17161	17212	429.63
59	17263	17314	17365	17417	17468	17519	17571	17623	17674	17726	437.04
60	17778	17830	17882	17934	17986	18038	18090	18143	18195	18247	444.44

Note that Base, Slope, and Cu. Yds. in this table may all be multiplied by the same factor; thus, using factor of 1¼ for height of 34 ft., we have, 8500 cu. yds. for base of 25 ft. and slopes 1¼ to 1.

Add for Tenths of Feet in Height.

	P. P.
1	.74
2	1.48
3	2.22
4	2.96
5	3.70
6	4.44
7	5.19
8	5.93
9	6.67

Use with preceding column only.

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34.—Level Sections (Earthwork); Height, 0–60 Ft.

Base of Roadway, 22 Ft. Side Slopes, 1 to 1.

Note.—The last two columns enable us to use any other base than 22 ft.: Ex.—Given height, 17.2 ft.; roadway 24 ft. Then we have, 2497.2+ (.93+1.48)=2624.6 cu. yds. (For Ht. >60 ft., see Tables 28, 40.)

[Cu. Yds. per 100-Ft. Station.]

Ht. Ft.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Width of 2 Ft. Cu. Yds	
0	.....	8.2	16.4	24.8	33.2	41.7	50.2	58.9	67.6	76.3	.....	Add for Tenths of Feet in Height.
1	85.2	94.1	103.1	112.2	121.3	130.6	139.9	149.2	158.7	168.2	7.41	
2	177.8	187.4	197.2	207.0	216.9	226.9	236.9	247.0	257.2	267.4	14.81	
3	277.8	288.2	298.7	309.2	319.9	330.6	341.3	352.2	363.1	374.1	22.22	
4	385.2	396.3	407.6	418.9	430.2	441.7	453.2	464.8	476.4	488.2	29.63	
5	500.0	511.9	523.9	535.9	548.0	560.2	572.4	584.8	597.2	609.7	37.04	
6	622.2	634.9	647.6	660.3	673.2	686.1	699.1	712.2	725.3	738.6	44.44	
7	751.9	765.2	778.7	792.2	805.8	819.4	833.2	847.0	860.9	874.9	51.85	
8	888.9	903.0	917.2	931.4	945.8	960.2	974.7	989.2	1003.9	1018.6	59.26	
9	1033.3	1048.2	1063.1	1078.1	1093.2	1108.3	1123.6	1138.9	1154.2	1169.7	66.67	
10	1185.2	1200.8	1216.4	1232.2	1248.0	1263.9	1279.9	1295.9	1312.0	1328.2	74.07	
11	1344.4	1360.8	1377.2	1393.7	1410.2	1426.9	1443.6	1460.3	1477.2	1494.1	81.48	
12	1511.1	1528.2	1545.3	1562.6	1579.9	1597.2	1614.7	1632.2	1649.8	1667.4	88.89	
13	1685.2	1703.0	1720.9	1738.9	1756.9	1775.0	1793.2	1811.4	1829.8	1848.2	96.30	
14	1866.7	1885.2	1903.9	1922.6	1941.3	1960.2	1979.1	1998.1	2017.2	2036.3	103.70	
15	2055.6	2074.9	2094.2	2113.7	2133.2	2152.8	2172.4	2192.2	2212.0	2231.9	111.11	
16	2251.9	2271.9	2292.0	2312.2	2332.4	2352.8	2373.2	2393.7	2414.2	2434.9	118.52	
17	2455.6	2476.3	2497.2	2518.1	2539.1	2560.2	2581.3	2602.6	2623.9	2645.2	125.93	
18	2666.7	2688.2	2709.8	2731.4	2753.2	2775.0	2796.9	2818.9	2840.9	2863.0	133.33	
19	2885.2	2907.4	2929.8	2952.2	2974.7	2997.2	3019.9	3042.6	3065.3	3088.2	140.74	
20	3111.1	3134.1	3157.2	3180.3	3203.6	3226.9	3250.2	3273.7	3297.2	3320.8	148.15	
21	3344.4	3368.2	3392.0	3415.9	3439.9	3463.9	3488.0	3512.2	3536.4	3560.8	155.56	
22	3585.2	3609.7	3634.2	3658.9	3683.6	3708.3	3733.2	3758.1	3783.1	3808.2	162.96	
23	3833.3	3858.6	3883.9	3909.2	3934.7	3960.2	3985.8	4011.4	4037.2	4063.0	170.37	
24	4088.9	4114.9	4140.9	4167.0	4193.2	4219.4	4245.8	4272.2	4298.7	4325.2	177.78	P. P.
25	4351.9	4378.6	4406.3	4432.2	4459.1	4486.1	4513.2	4540.3	4567.6	4594.9	185.19	7.41
26	4622.2	4649.7	4677.2	4704.8	4732.4	4760.2	4788.0	4815.9	4843.9	4871.9	192.59	
27	4900.0	4928.2	4956.4	4984.8	5013.2	5041.7	5070.2	5098.9	5127.6	5156.3	200.00	1 .74
28	5185.2	5214.1	5243.1	5272.2	5301.3	5330.6	5359.9	5389.2	5418.7	5448.2	207.41	2 1.48
29	5477.8	5507.4	5537.2	5567.0	5596.9	5626.9	5656.9	5687.0	5717.2	5747.4	214.81	3 2.22
30	5777.8	5808.2	5838.7	5869.2	5899.9	5930.6	5961.3	5992.2	6023.1	6054.1	222.22	4 2.96
31	6085.2	6116.3	6147.6	6178.9	6210.2	6241.7	6273.2	6304.8	6336.4	6368.2	229.63	5 3.70
32	6400.0	6431.9	6463.9	6495.9	6528.0	6560.2	6592.4	6624.8	6657.2	6689.7	237.04	6 4.44
33	6722.2	6754.9	6787.6	6820.3	6853.2	6886.1	6919.1	6952.2	6985.3	7018.6	244.44	7 5.19
34	7051.9	7085.2	7118.7	7152.2	7185.8	7219.4	7253.2	7287.0	7320.9	7354.9	251.85	8 5.93
35	7388.9	7423.0	7457.2	7491.4	7525.8	7560.2	7594.7	7629.2	7663.9	7698.6	259.26	9 6.67
36	7733.3	7768.2	7803.1	7838.1	7873.2	7908.3	7943.6	7978.9	8014.2	8049.7	266.67	
37	8085.2	8120.8	8156.4	8192.2	8228.0	8263.9	8299.9	8335.9	8372.0	8408.2	274.07	
38	8444.4	8480.8	8517.2	8553.7	8590.2	8626.9	8663.6	8700.3	8737.2	8774.1	281.48	
39	8811.1	8848.2	8885.3	8922.6	8959.9	8997.2	9034.7	9072.2	9109.8	9147.4	288.89	
40	9185.2	9223.0	9260.9	9298.9	9336.9	9375.0	9413.2	9451.4	9489.8	9528.2	296.30	
41	9566.7	9605.2	9643.9	9682.6	9721.3	9760.2	9799.1	9838.1	9877.2	9916.3	303.70	
42	9955.6	9994.9	10034	10074	10113	10153	10192	10232	10272	10312	311.11	Use with preceding column only.
43	10352	10392	10432	10472	10512	10553	10593	10634	10674	10715	318.52	
44	10756	10796	10837	10878	10919	10960	11001	11043	11084	11125	325.93	
45	11167	11208	11250	11291	11333	11375	11417	11459	11501	11543	333.33	
46	11585	11627	11670	11712	11755	11797	11840	11883	11925	11968	340.74	
47	12011	12054	12097	12140	12184	12227	12270	12314	12357	12401	348.15	
48	12444	12488	12532	12576	12620	12664	12708	12752	12796	12841	355.56	
49	12885	12930	12974	13019	13064	13108	13153	13198	13243	13288	362.96	
50	13333	13379	13424	13469	13515	13560	13606	13651	13697	13743	370.37	
51	13789	13835	13881	13927	13973	14019	14066	14112	14159	14205	377.78	
52	14252	14299	14345	14392	14439	14486	14533	14580	14628	14675	385.19	
53	14722	14770	14817	14865	14912	14960	15008	15056	15104	15152	392.59	
54	15200	15248	15296	15345	15393	15442	15490	15539	15588	15636	400.00	
55	15685	15734	15783	15832	15881	15931	15980	16029	16079	16128	407.41	
56	16178	16227	16277	16327	16377	16427	16477	16527	16577	16627	414.81	
57	16678	16728	16779	16829	16880	16931	16981	17032	17083	17134	422.22	
58	17185	17236	17288	17339	17390	17442	17493	17545	17596	17648	429.63	
59	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	

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35.—Level Sections (Earthwork); Height, 0-60 Ft.

Base of Roadway, 24 Ft. Side Slopes, 1½ to 1.

Note.—The last two columns enable us to use any other base than 24 ft.: Ex.—Given height, 43.1 ft.; roadway 22 ft. Then we have, 14151 − (318.52 + 0.74) = 13832 cu. yds. (For Ht. >60 ft., see Tables 24, 41.)

[Cu. Yds. per 100-Ft. Station.]

Ht. Ft.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Width of 2 Ft. Cu. Yds
0	.....	8.9	18.0	27.2	36.4	45.8	55.3	64.9	74.7	84.5	.....
1	94.4	104.5	114.7	124.9	135.3	145.8	156.4	167.2	178.0	188.9	7.41
2	200.0	211.2	222.4	233.8	245.3	256.9	268.6	280.5	292.4	304.4	14.81
3	316.6	238.9	341.2	353.7	366.3	379.0	391.9	404.8	417.8	431.0	22.22
4	444.4	457.8	471.3	484.9	498.6	512.4	526.4	540.4	554.6	568.8	29.63
5	583.3	597.8	612.4	627.1	642.0	656.9	671.9	687.1	702.3	717.7	37.04
6	733.3	748.9	764.7	780.5	796.4	812.5	828.7	844.9	861.3	877.8	44.44
7	894.4	911.2	928.0	944.9	962.0	979.2	996.4	1013.8	1031.3	1048.9	51.85
8	1066.7	1084.5	1102.4	1120.5	1138.7	1156.9	1175.3	1193.8	1212.4	1231.2	59.26
9	1250.0	1268.9	1288.0	1307.2	1326.4	1345.8	1365.3	1384.9	1404.7	1424.5	66.67
10	1444.4	1464.5	1484.7	1504.9	1525.3	1545.8	1566.4	1587.2	1608.0	1628.9	74.07
11	1650.0	1671.2	1692.4	1713.8	1735.3	1756.9	1778.7	1800.5	1822.4	1844.5	81.48
12	1866.7	1888.9	1911.3	1933.8	1956.4	1979.2	2002.0	2024.9	2048.0	2071.2	88.89
13	2094.4	2117.8	2141.3	2164.9	2188.7	2212.5	2236.4	2260.5	2284.7	2308.9	96.30
14	2333.3	2357.8	2382.4	2407.2	2432.0	2456.9	2482.0	2507.2	2532.4	2557.8	103.70
15	2583.3	2608.9	2634.7	2660.5	2686.4	2712.5	2738.7	2764.9	2791.3	2817.8	111.11
16	2844.4	2871.2	2898.0	2924.9	2952.0	2979.2	3006.4	3033.8	3061.3	3088.9	118.52
17	3116.7	3144.5	3172.4	3200.5	3228.7	3256.9	3285.3	3313.8	3342.4	3371.2	125.93
18	3400.0	3428.9	3458.0	3487.2	3516.4	3545.8	3575.3	3604.9	3634.7	3664.5	133.33
19	3694.5	3724.5	3754.7	3784.9	3815.3	3845.8	3876.4	3907.2	3938.0	3968.9	140.74
20	4000.0	4031.2	4062.4	4093.8	4125.3	4156.9	4188.7	4220.5	4252.4	4284.5	148.15
21	4316.7	4348.9	4381.3	4413.8	4446.4	4479.2	4512.0	4544.9	4578.0	4611.2	155.56
22	4644.4	4677.8	4711.3	4744.9	4778.7	4812.5	4846.4	4880.5	4914.7	4948.9	162.96
23	4983.3	5017.8	5052.4	5087.2	5122.0	5156.9	5192.0	5227.2	5262.4	5297.8	170.37
24	5333.3	5368.9	5404.7	5440.5	5476.4	5512.5	5548.7	5584.9	5621.3	5657.8	177.78
25	5694.4	5731.2	5768.0	5804.9	5842.0	5879.2	5916.4	5953.8	5991.3	6028.9	185.19
26	6066.7	6104.5	6142.4	6180.5	6218.7	6256.9	6295.3	6333.8	6372.4	6411.2	192.59
27	6450.0	6488.9	6528.0	6567.2	6606.4	6645.8	6685.3	6724.9	6764.7	6804.5	200.00
28	6844.4	6884.5	6924.7	6964.9	7005.3	7045.8	7086.4	7127.2	7168.0	7208.9	207.41
29	7250.0	7291.2	7332.4	7373.8	7415.3	7456.9	7498.7	7540.5	7582.4	7624.5	214.81
30	7666.7	7708.9	7751.3	7793.8	7836.4	7879.2	7922.0	7964.9	8008.0	8051.2	222.22
31	8094.4	8137.8	8181.3	8224.9	8268.7	8312.5	8356.4	8400.5	8444.7	8488.9	229.63
32	8533.3	8577.8	8622.4	8667.2	8712.0	8756.9	8802.0	8847.2	8892.4	8937.8	237.04
33	8983.3	9028.9	9074.7	9120.5	9166.4	9212.5	9258.7	9304.9	9351.3	9397.8	244.44
34	9444.4	9491.2	9538.0	9584.9	9632.0	9679.2	9726.4	9773.8	9821.3	9868.9	251.85
35	9916.7	9964.5	10012	10061	10109	10157	10205	10254	10302	10351	259.26
36	10400	10449	10498	10547	10596	10646	10695	10745	10795	10845	266.67
37	10894	10945	10995	11045	11095	11146	11196	11247	11298	11349	274.07
38	11400	11451	11502	11554	11605	11657	11709	11761	11812	11865	281.48
39	11917	11969	12021	12074	12126	12179	12232	12285	12338	12391	288.89
40	12444	12498	12551	12605	12659	12713	12766	12821	12875	12929	296.30
41	12983	13038	13092	13147	13202	13257	13312	13367	13422	13478	303.70
42	13533	13589	13645	13701	13756	13813	13869	13925	13981	14038	311.11
43	14094	14151	14208	14265	14322	14379	14436	14494	14551	14609	318.52
44	14667	14725	14782	14840	14899	14957	15015	15074	15132	15191	325.93
45	15250	15309	15368	15427	15486	15546	15605	15665	15725	15785	333.33
46	15844	15905	15965	16025	16085	16146	16206	16267	16328	16389	340.74
47	16450	16511	16572	16634	16695	16757	16819	16881	16942	17005	348.15
48	17067	17129	17191	17254	17316	17379	17442	17505	17568	17631	355.56
49	17694	17758	17821	17885	17949	18013	18076	18141	18205	18269	362.96
50	18333	18398	18462	18527	18592	18657	18722	18787	18852	18918	370.37
51	18983	19049	19115	19181	19246	19313	19379	19445	19511	19578	377.78
52	19644	19711	19778	19845	19912	19979	20046	20114	20181	20249	385.19
53	20317	20385	20452	20521	20589	20657	20725	20794	20862	20931	392.59
54	21000	21069	21138	21207	21276	21346	21415	21485	21555	21625	400.00
55	21694	21765	21835	21905	21975	22046	22116	22187	22258	22329	407.41
56	22400	22471	22542	22614	22685	22757	22829	22901	22972	23045	414.81
57	23117	23189	23261	23334	23406	23479	23552	23625	23698	23771	422.22
58	23844	23918	23991	24065	24139	24213	24286	24361	24435	24509	429.63
59	24583	24658	24732	24807	24882	24957	25032	25107	25182	25258	437.04
60	25333	25409	25485	25561	25636	25713	25789	25865	25941	26018	444.44

Note that Base, Slope, and Cu. Yds. in this table may all be multiplied by the same factor; thus, using factor of 5/6 for height of 38 ft., we have, 9500 cu. yds. for base of 20 ft. and slopes 1¼ to 1.

Add for Tenths of Feet in Height.

P.	P.
1	7.41
2	14.81
3	22.22
4	29.63
5	37.04
6	44.44
7	51.85
8	59.26
9	66.67

Use with preceding column only.

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36.—Level Sections (Earthwork); Height, 0–60 Ft.
Base of Roadway, 26 Ft. Side Slopes 1½ to 1.

Note.—The last two columns enable us to use any other base than 26 ft.:
Ex.—Given height, 22.2 ft.; roadway 27 ft. Then we have, 4875.8+
(2.96+1.48) = 4958.0 cu. yds. (For Ht. >60 ft., see Tables 24, 41.)

[Cu. Yds. per 100-Ft. Station.]

Ht. Ft.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Width of 2 Ft. Cu. Yds	
0	.....	9.7	19.5	29.4	39.4	49.5	59.8	70.1	80.6	91.2	.....	Add for Tenths of Feet in Height.
1	101.9	112.6	123.6	134.6	145.7	156.9	168.3	179.8	191.3	203.0	7.41	
2	214.8	226.7	238.7	250.9	263.1	275.5	287.9	300.5	313.2	326.0	14.81	
3	338.9	351.9	365.0	378.3	391.6	405.1	418.7	432.4	446.1	460.1	22.22	
4	474.1	488.2	502.4	516.8	531.3	545.8	560.5	575.3	590.2	605.2	29.63	
5	620.4	635.6	651.0	666.4	682.0	697.7	713.5	729.4	745.4	761.5	37.04	
6	777.8	794.1	810.6	827.2	843.9	860.6	877.6	894.6	911.7	928.9	44.44	
7	946.3	963.8	981.3	999.0	1016.8	1034.7	1052.7	1070.9	1089.1	1107.5	51.85	
8	1125.9	1144.5	1163.2	1182.0	1200.9	1219.9	1239.0	1258.3	1277.6	1297.1	59.26	
9	1316.7	1336.4	1356.1	1376.1	1396.1	1416.2	1436.4	1456.8	1477.3	1497.8	66.67	
10	1518.5	1539.3	1560.2	1581.2	1602.4	1623.6	1645.0	1666.4	1688.0	1709.7	74.07	
11	1731.5	1753.4	1775.4	1797.5	1819.8	1842.1	1864.6	1887.2	1909.9	1932.6	81.48	
12	1955.6	1978.6	2001.7	2024.9	2048.3	2071.8	2095.3	2119.0	2142.8	2166.7	88.89	
13	2190.7	2214.9	2239.1	2263.5	2287.9	2312.5	2337.2	2362.0	2386.9	2411.9	96.30	
14	2437.0	2462.3	2487.6	2513.1	2538.7	2564.4	2590.1	2616.1	2642.1	2668.2	103.70	
15	2694.4	2720.8	2747.3	2773.8	2800.5	2827.3	2854.2	2881.2	2908.4	2935.6	111.11	
16	2963.0	2990.4	3018.0	3045.7	3073.5	3101.4	3129.4	3157.5	3185.8	3214.1	118.52	
17	3242.6	3271.2	3299.9	3328.6	3357.6	3386.6	3415.7	3444.9	3474.3	3503.8	125.93	
18	3533.3	3563.0	3592.8	3622.7	3652.7	3682.9	3713.1	3743.5	3773.9	3804.5	133.33	
19	3835.2	3866.0	3896.9	3927.9	3959.0	3990.3	4021.6	4053.1	4084.7	4116.4	140.74	
20	4148.1	4180.1	4212.1	4244.2	4276.4	4308.8	4341.3	4373.8	4406.5	4439.3	145.15	
21	4472.2	4505.2	4538.4	4571.6	4605.0	4638.4	4672.0	4705.7	4739.5	4773.4	155.56	
22	4807.4	4841.5	4875.8	4910.1	4944.6	4979.2	5013.9	5048.6	5083.6	5118.6	162.96	
23	5153.7	5188.9	5224.3	5259.8	5295.3	5331.0	5366.8	5402.7	5438.7	5474.9	170.37	
24	5511.1	5547.5	5583.9	5620.5	5657.2	5694.0	5730.9	5767.9	5805.0	5842.3	177.78	P. P.
25	5879.6	5917.1	5954.7	5992.4	6030.1	6068.1	6106.1	6144.2	6182.4	6220.8	185.19	7.41
26	6259.3	6297.8	6336.5	6375.3	6414.2	6453.2	6492.4	6531.6	6571.0	6610.4	192.59	
27	6650.0	6689.7	6729.5	6769.4	6809.4	6849.5	6889.8	6930.1	6970.6	7011.2	200.00	1 .74
28	7051.9	7092.6	7133.6	7174.6	7215.7	7256.9	7298.3	7339.8	7381.3	7423.0	207.41	2 1.48
29	7464.8	7506.7	7548.7	7590.9	7633.1	7675.5	7717.9	7760.5	7803.2	7846.0	214.81	3 2.22
30	7888.9	7931.9	7975.0	8018.3	8061.6	8105.1	8148.7	8192.4	8236.1	8280.1	222.22	4 2.96
31	8324.1	8368.2	8412.4	8456.8	8501.3	8545.8	8590.5	8635.3	8680.2	8725.2	229.63	5 3.70
32	8770.4	8815.6	8861.0	8906.4	8952.0	8997.7	9043.5	9089.4	9135.4	9181.5	237.04	6 4.44
33	9227.8	9274.1	9320.6	9367.2	9413.9	9460.6	9507.6	9554.6	9601.7	9648.9	244.44	7 5.19
34	9696.3	9743.8	9791.3	9839.0	9886.8	9934.7	9982.7	10031	10079	10127	251.85	8 5.93
35	10176	10225	10273	10322	10371	10420	10469	10518	10568	10617	259.26	9 6.67
36	10667	10716	10766	10816	10866	10916	10966	11017	11067	11118	266.67	Use with preceding column only.
37	11169	11219	11270	11321	11372	11424	11475	11526	11578	11630	274.07	
38	11681	11733	11785	11838	11890	11942	11995	12047	12100	12153	281.48	
39	12206	12259	12312	12365	12418	12472	12525	12579	12633	12687	288.89	
40	12741	12795	12849	12903	12958	13013	13067	13122	13177	13232	296.30	
41	13287	13342	13398	13453	13509	13564	13620	13676	13732	13788	303.70	
42	13844	13901	13957	14014	14071	14127	14184	14241	14298	14356	311.11	
43	14413	14470	14528	14586	14643	14701	14759	14818	14876	14934	318.52	
44	14993	15051	15110	15169	15228	15287	15346	15405	15464	15524	325.93	
45	15583	15643	15703	15763	15823	15883	15943	16003	16064	16125	333.33	
46	16185	16246	16307	16368	16429	16490	16552	16613	16675	16736	340.74	
47	16798	16860	16922	16984	17046	17109	17171	17234	17297	17359	348.15	
48	17422	17485	17548	17612	17675	17738	17802	17866	17929	17993	355.56	
49	18057	18122	18186	18250	18315	18379	18444	18519	18574	18639	362.96	
50	18704	18769	18834	18900	18965	19031	19097	19163	19229	19295	370.37	
51	19361	19427	19494	19561	19627	19694	19761	19828	19895	19962	377.78	
52	20030	20097	20165	20232	20300	20368	20436	20504	20572	20641	385.19	
53	20709	20778	20847	20915	20984	21053	21122	21192	21261	21330	392.59	
54	21400	21470	21539	21609	21679	21750	21820	21890	21961	22031	400.00	
55	22102	22173	22244	22315	22386	22457	22528	22600	22671	22743	407.41	
56	22815	22887	22959	23031	23103	23175	23248	23321	23393	23466	414.81	
57	23539	23612	23685	23758	23832	23905	23979	24052	24126	24200	422.22	
58	24274	24348	24422	24497	24571	24646	24721	24795	24870	24945	429.63	
59	25020	25096	25171	25246	25322	25398	25473	25549	25625	25702	437.04	
60	25778	25854	25931	26007	26084	26161	26238	26315	26392	26469	444.44	

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37.—Level Sections (Earthwork); Height, 0–60 Ft. Base of Roadway, 28 Ft. Side Slopes, 1 to 1.

Note.—The last two columns enable us to use any other base than 28 ft.: Ex.—Given height, 57.5 ft.; roadway 26 ft. Then we have, 18208 – (422.22 + 3.70) = 17782 cu. yds. (For Ht. >60 ft., see Tables 28, 40.)

[Cu. Yds. per 100-Ft. Station.]

Note that Base, Slope, and Cu. Yds. in this table may all be multiplied by the same factor; thus, using factor of ½ for height of 45.1 ft., we have, 6105 cu. yds. for base of 14 ft. and slopes ½ to 1.

Ht. Ft.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Width of 2 Ft. Cu. Yds
0	.....	10.4	20.9	31.4	42.1	52.8	63.6	74.4	85.3	96.3	.....
1	107.4	118.6	129.8	141.1	152.4	163.9	175.4	187.0	198.7	210.4	7.41
2	222.2	234.1	246.1	258.1	270.2	282.4	294.7	307.0	319.4	331.9	14.81
3	344.4	357.1	369.8	382.6	395.4	408.3	421.3	434.4	447.6	460.8	22.22
4	474.1	487.4	500.9	514.4	528.0	541.7	555.4	569.2	583.1	597.1	29.63
5	611.1	625.2	639.4	653.7	668.0	682.4	696.9	711.4	726.1	740.8	37.04
6	755.6	770.4	785.4	800.4	815.5	830.6	845.8	861.1	876.5	891.9	44.44
7	907.5	923.0	938.7	954.5	970.3	986.1	1002.1	1018.1	1034.2	1050.4	51.85
8	1066.7	1083.0	1099.4	1115.9	1132.4	1149.1	1165.8	1182.6	1199.4	1216 3	59.26
9	1233.3	1250.4	1267.6	1284.8	1302.1	1319.4	1336.9	1354.4	1372.0	1389.7	66.67
10	1407.4	1425.2	1443.1	1461.1	1479.1	1497.2	1515.4	1533.7	1552.0	1570.4	74.07
11	1588.9	1607.4	1626.1	1644.8	1663.6	1682.4	1701.3	1720.3	1739.4	1758.6	81.48
12	1777.8	1797.1	1816.4	1835.9	1855.4	1875.0	1894 7	1914.4	1934.2	1954.1	88.89
13	1974.1	1994.1	2014.2	2034.4	2054.7	2075.0	2095.4	2115.9	2136.4	2157.1	96.30
14	2177.8	2198.6	2219.4	2240.3	2261.3	2282.4	2303.6	2324.8	2346.1	2367.4	103.70
15	2388.9	2410.4	2432.0	2453.7	2475.4	2497.2	2519.1	2541.1	2563.1	2585.2	111.11
16	2607.4	2629.7	2652.0	2674.4	2696.9	2719.4	2742.1	2764.8	2787.6	2810.4	118.52
17	2833.3	2856.3	2879.4	2902.6	2925.8	2949.1	2972.4	2995.9	3019.4	3043.0	125.93
18	3066.7	3090.4	3114.2	3138.1	3162.1	3186.1	3210.2	3234.4	3258.7	3283.0	133.33
19	3307.4	3331.9	3356.4	3381.1	3405.8	3430.6	3455.4	3480.3	3505.3	3530.4	140.74
20	3555.6	3580.8	3606.1	3631.4	3656.9	3682.4	3708.0	3733.7	3759.4	3785.2	148.15
21	3811.1	3837.1	3863.1	3889.2	3915.4	3941.7	3968.0	3994.4	4020.9	4047.4	155.56
22	4074.1	4100.8	4127.6	4154.4	4181.3	4208.3	4235.4	4262.6	4289.8	4317.1	162.96
23	4344.4	4371.9	4399.4	4427.0	4454.7	4482.4	4510.2	4538.1	4566.1	4594.1	170.37
24	4622.2	4650.4	4678.7	4707.0	4735.4	4763.9	4792.4	4821.1	4849.8	4878.6	177.78
25	4907.4	4936.3	4965.3	4994.4	5023.6	5052.8	5082.1	5111.4	5140.9	5170.4	185.19
26	5200.0	5229.7	5259.4	5289.2	5319.1	5349.1	5379.1	5409.2	5439.4	5469.7	192.59
27	5500.0	5530.4	5560.9	5591.4	5622.1	5652.8	5683.6	5714.4	5745.3	5776.3	200.00
28	5807.4	5838.6	5869.8	5901.1	5932.4	5963.9	5995.4	6127.0	6158.7	6190.4	207.41
29	6122.2	6154.1	6186.1	6218.1	6250.2	6282.4	6314.7	6347.0	6379.4	6411.9	214.81
30	6444.4	6477.1	6509.8	6542.6	6575.4	6608.3	6641.3	6674.4	6707.6	6740.8	222.22
31	6774.1	6807.4	6840.9	6874.4	6908.0	6941.7	6975.4	7009.2	7043.1	7077.1	229.63
32	7111.1	7145.2	7179.4	7213.7	7248.0	7282.4	7316.9	7351.4	7386.1	7420.8	237.04
33	7455.6	7490.4	7525.3	7560.3	7595.4	7630.6	7665.8	7701.1	7736.4	7771.9	244.44
34	7807.4	7843.0	7878.7	7914.4	7950.2	7986.1	8022.1	8058.1	8094.2	8130.4	251.85
35	8166.7	8203.0	8239.4	8275.9	8212.4	8349.1	8385.8	8423.6	8459.4	8496.3	259.26
36	8533.3	8570.4	8607.6	8644.8	8682.1	8719.4	8756.9	8794.4	8832.0	8869.7	266.67
37	8907.4	8945.2	8983.1	9021.1	9059.1	9097.2	9135.4	9173.7	9212.0	9250.4	274.07
38	9288.9	9327.4	9366.1	9404.8	9443.6	9482.4	9521.3	9560.3	9599.4	9638.6	281.48
39	9677.8	9717.1	9756.4	9795.9	9835.4	9875.0	9914.7	9954.4	9994.2	10034	288.89
40	10074	10114	10154	10194	10235	10275	10315	10356	10396	10437	296.30
41	10478	10519	10559	10600	10641	10682	10724	10765	10806	10847	303.70
42	10889	10930	10972	11014	11055	11097	11139	11181	11223	11265	311.11
43	11307	11350	11392	11434	11477	11519	11562	11605	11648	11690	318.52
44	11733	11776	11819	11863	11906	11949	11992	12036	12079	12123	325.93
45	12167	12210	12254	12298	12342	12386	12430	12474	12519	12563	333.33
46	12607	12652	12696	12741	12786	12831	12875	12920	12965	13010	340.74
47	13056	13101	13146	13191	13237	13282	13328	13374	13419	13465	348.15
48	13511	13587	13603	13649	13695	13742	13788	13834	13881	13927	355.56
49	13974	14021	14068	14114	14161	14208	14255	14303	14350	14397	362.96
50	14444	14492	14539	14587	14635	14682	14730	14778	14826	14874	370.37
51	14922	14970	15019	15067	15115	15164	15212	15261	15310	15359	377.78
52	15407	15456	15505	15554	15604	15653	15702	15751	15801	15850	385.19
53	15900	15950	15999	16049	16099	16149	16199	16249	16299	16350	392.59
54	16400	16450	16501	16551	16602	16653	16704	16754	16805	16856	400.00
55	16907	16959	17010	17061	17112	17164	17215	17267	17319	17370	407.41
56	17422	17474	17526	17578	17630	17682	17735	17787	17839	17892	414.81
57	17944	17997	18050	18103	18155	18208	18261	18314	18368	18421	422.22
58	18474	18527	18581	18634	18688	18742	18795	18849	18903	18957	429.63
59	19011	19065	19119	19174	19228	19282	19337	19391	19446	19501	437.04
60	19556	19610	19665	19720	19775	19831	19886	19941	19996	20052	444.44

Add for Tenths of Feet in Height.

	P. P.
	7.41
1	.74
2	1.48
3	2.22
4	2.96
5	3.70
6	4.44
7	5.18
8	5.93
9	6.67

Use with preceding column only.

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38.—Level Sections (Earthwork); Height., 0–60 Ft.

Base of Roadway, 28 Ft. Side Slopes, 1½ to 1.

ote.—The last two columns enable us to use any other base than 28 ft.:
c.—Given height, 33.6 ft.; roadway 30 ft. Then we have, 9756.4+
4+4.44)=10005.3 cu. yds. (For Ht. >60 ft., see Tables 24, 41.)

[Cu. Yds. per 100-Ft. Station.]

	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Width of 2 Ft. Cu. Yds	
)	.....	10.4	21.0	31.6	42.4	53.2	64.2	75.3	86.5	97.9	.....	Add for Tenths of Feet in Height.
1	109.3	120 8	132.5	144.3	156.1	168.1	180.2	192.4	204.8	217.2	7.41	
2	229.6	242.3	255.0	267.9	280.9	294.0	307.2	320.5	334.0	347.5	14.81	
3	361.2	374.9	388.8	402.8	416.9	431.1	445.4	459.9	474.4	489.1	22.22	
4	503.7	518.6	533.6	548.6	563.9	579.3	594.7	610.2	625.8	641.6	29.63	
5	657.5	673.4	689.5	705.7	722.1	738.5	755.0	771.7	788.4	805.3	37.04	
6	822.2	839.3	856.5	873.8	891.2	908.8	926.4	944.2	962.0	980.0	44.44	
7	998.1	1016.4	1034.7	1053.1	1071.6	1090.3	1109.0	1127.9	1146.9	1166.0	51.85	
8	1185.2	1204.5	1223.9	1243.5	1263.1	1282.9	1302.7	1322.7	1342.8	1363.0	59.26	
9	1383.3	1403.8	1424.3	1444.9	1465.7	1486.6	1507.6	1528.6	1549.8	1571.2	66.67	
10	1592.6	1614.1	1635.8	1657.5	1679.4	1701.4	1723.5	1745.7	1768.0	1790.4	74.07	
11	1813.0	1835.6	1858.4	1881.2	1904.2	1927.3	1950.5	1973.8	1997.3	2020.8	81.48	
12	2044.4	2068.2	2092.1	2116.1	2040 1	2164.3	2188.7	2213.1	2237.6	2262.3	88.89	
13	2287.0	2311.9	2336.9	2362.0	2387.2	2412.5	2437.9	2463.5	2489.1	2514.9	96.30	
14	2540.7	2566.7	2592.8	2619.0	2645.3	2671.8	2698.3	2724.9	2751.7	2778.6	103.70	
15	2805.6	2832.6	2859.9	2887.2	2914.6	2942.1	2969.8	2997.5	3025.4	3053.4	111.11	
16	3081.5	3109.7	3138.0	3166.4	3195.0	3223.6	3252.4	3281.2	3310.2	3339.3	118.52	
17	3368.5	3397.8	3427.3	3456.8	3486.4	3516.2	3546.1	3576.1	3606.1	3636.4	125.93	
18	3666.7	3697.1	3727.6	3758.3	3789.0	3819.9	3850.9	3882.0	3913.2	3944.5	133.33	
19	3975.9	4007.5	4039.1	4070.9	4102.7	4134.7	4166.8	4199.0	4231.3	4263.8	140.74	
20	4296.2	4328.9	4361.7	4394.6	4427.6	4460.6	4493.9	4527.2	4560.6	4594.1	148.15	
21	4627.8	4661.5	4695.4	4729.4	4763.5	4797.7	4832.0	4866.4	4901.0	4935.6	155.56	
22	4970.4	5005.2	5040.2	5075.3	5110.5	5145.8	5181.3	5216.8	5252.4	5288.2	162.96	
23	5324.1	5360.1	5396.1	5432.4	5468.7	5505.1	5541.6	5578.3	5615.0	5651.9	170.37	
24	5688.9	5726.0	5763.2	5800.5	5837.9	5875.5	5913.1	5950.9	5988.7	6026.7	177.78	P. P.
25	6064.8	6103.0	6141.3	6179.8	6218.3	6256.9	6295.7	6334.6	6373.6	6412.6	185.19	7.41
26	6451.9	6491.2	6530.6	6570.1	6609.8	6649.5	6689.4	6729.4	6769.5	6809.7	192.59	
27	6850.0	6890.4	6931.0	6971.6	7012.4	7053.2	7094.2	7135.3	7176.5	7217.8	200.00	1 .74
28	7259.3	7300.8	7342.4	7384.2	7426.1	7468.1	7510.1	7552.4	7594.7	7637.1	207.41	2 1.48
29	7679.6	7722.3	7765.0	7807.9	7850.9	7894.0	7937.2	7980.5	8023.9	8067.5	214.81	3 2.22
30	8111.1	8154.9	8198.7	8242.7	8286.8	8331.0	8375.3	8419.8	8464.3	8508.9	222.22	4 2.96
31	8553.7	8598.6	8643.6	8688.6	8733.9	8779.2	8824.6	8870.1	8915.8	8961.5	229.63	5 3.70
32	9007.4	9053.4	9099.5	9145.7	9192.0	9238.4	9285.0	9331.6	9378.4	9425.2	237.04	6 4.44
33	9473.2	9519.3	9566.5	9613.8	9661.3	9708.8	9756.4	9804.2	9852.1	9900.1	244.44	7 5.19
34	9948.1	9996.4	10045	10093	10142	10190	10239	10288	10337	10386	251.85	8 5.93
35	10435	10484	10534	10583	10633	10683	10732	10782	10832	10882	259.26	9 6.67
36	10933	10983	11034	11084	11135	11186	11237	11288	11339	11391	266.67	Use with preceding column only.
37	11443	11494	11546	11598	11649	11701	11753	11806	11858	11910	274.07	
38	11963	12016	12068	12121	12174	12227	12281	12334	12387	12441	281.48	
39	12494	12548	12602	12656	12710	12764	12819	12873	12928	12982	288.89	
40	13037	13092	13147	13202	13257	13312	13368	13423	13479	13535	296.30	
41	13591	13647	13703	13759	13815	13872	13928	13985	14042	14099	303.70	
42	14156	14213	14270	14327	14385	14442	14500	14558	14615	14673	311.11	
43	14731	14790	14848	14906	14965	15024	15082	15141	15200	15259	318.52	
44	15318	15378	15437	15497	15556	15616	15676	15736	15796	15856	325.93	
45	15917	15977	16038	16098	16159	16220	16281	16342	16403	16465	333.33	
46	16526	16587	16649	16711	16773	16835	16897	16959	17021	17084	340.74	
47	17146	17209	17272	17335	17398	17461	17524	17587	17651	17714	348.15	
48	17778	17842	17905	17969	18033	18098	18162	18226	18291	18356	355.56	
49	18420	18485	18550	18615	18680	18746	18811	18877	18942	19008	362.96	
50	19074	19140	19206	19272	19339	19405	19472	19538	19605	19672	370.37	
51	19739	19806	19873	19940	20008	20075	20143	20211	20279	20347	377.78	
52	20415	20483	20551	20620	20688	20757	20826	20894	20963	21032	385.19	
53	21102	21171	21241	21310	21380	21450	21519	21589	21659	21730	392.59	
54	21800	21870	21941	22012	22082	22153	22224	22295	22366	22438	400.00	
55	22509	22581	22653	22724	22796	22868	22940	23012	23085	23157	407.41	
56	23230	23302	23375	23448	23521	23594	23667	23741	23814	23888	414.81	
57	23961	24035	24109	24183	24257	24331	24405	24480	24554	24629	422.22	
58	24704	24779	24854	24929	25004	25079	25155	25230	25306	25381	429.63	
59	25457	25533	25609	25686	25762	25838	25915	25992	26068	26145	437.04	
60	26222	26299	26376	26454	26531	26609	26686	26764	26842	26920	444.44	

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39.—Level Sections (Earthwork); Height, 0–60 Ft. Base of Roadway, 30 Ft. Side Slopes, 1 to 1.

Note.—The last two columns enable us to use any other base than 30 ft.: Ex.—Given height, 41.1 ft.; roadway 32 ft. Then we have, 10823+ (303.70+0.74)=11127 cu. yds. (For Ht. >60 ft., see Tables 28, 40.)

[Cu. Yds. per 100-Ft. Station.]

Ht. Ft.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	Width of 2 Ft. Cu. Yds	P.	P.
0	.....	11.1	22.4	33.7	45.0	56.5	68.0	79.6	91.3	103.0	.....		
1	114.8	126.7	138.7	150.7	162.8	175.0	187.3	199.6	212.0	224.5	7.41		
2	237.0	249.7	262.4	275.1	288.0	300.9	313.9	327.0	340.1	353.4	14.81		
3	366.7	380.0	393.5	407.0	420.6	434.3	448.0	461.8	475.7	489.7	22.22		
4	503.7	517.8	532.0	546.3	560.6	575.0	589.5	604.0	618.7	633.4	29.63		
5	648.1	663.0	677.9	692.9	708.0	723.1	738.4	753.7	769.0	784.5	37.04		
6	800.0	815.6	831.3	847.0	862.8	878.7	894.7	910.7	926.8	943.0	44.44		
7	959.3	975.6	992.0	1008.5	1025.0	1041.7	1058.4	1075.1	1092.0	1108.9	51.85		
8	1125.9	1143.0	1160.1	1177.4	1194.7	1212.0	1229.5	1247.0	1264.6	1282.3	59.26		
9	1300.0	1317.8	1335.7	1353.7	1371.7	1389.8	1408.0	1426.3	1444.6	1463.0	66.67		
10	1481.5	1500.0	1518.7	1537.4	1556.1	1575.0	1593.9	1612.9	1632.0	1651.1	74.07		
11	1670.4	1689.7	1709.0	1728.5	1748.0	1767.6	1787.3	1807.0	1826.8	1846.7	81.48		
12	1866.7	1886.7	1906.8	1927.0	1947.3	1967.6	1988.0	2008.5	2029.0	2049.7	88.89		
13	2070.4	2091.1	2112.0	2132.9	2153.9	2175.0	2196.1	2217.4	2238.7	2260.0	96.30		
14	2281.5	2303.0	2324.6	2346.3	2368.0	2389.8	2411.7	2433.7	2455.7	2477.8	103.70		
15	2500.0	2522.3	2544.6	2567.0	2589.5	2612.0	2634.7	2657.4	2680.1	2703.0	111.11		
16	2725.9	2748.9	2772.0	2795.1	2818.4	2841.7	2865.0	2888.5	2912.0	2935.6	118.52		
17	2959.3	2983.0	3006.8	3030.7	3054.7	3078.7	3102.8	3127.0	3151.3	3175.6	125.93		
18	3200.0	3224.5	3249.0	3273.7	3298.4	3323.1	3348.0	3372.9	3397.9	3423.0	133.33		
19	3448.1	3473.4	3498.7	3524.0	3549.5	3575.0	3600.6	3626.3	3652.0	3677.8	140.74		
20	3703.7	3729.7	3755.7	3781.8	3808.0	3834.3	3860.6	3887.0	3913.5	3940.0	148.15		
21	3966.7	3993.4	4020.1	4047.0	4073.9	4100.9	4128.0	4155.1	4182.4	4209.7	156.56		
22	4237.0	4264.5	4292.0	4319.6	4347.3	4375.0	4402.8	4430.7	4458.7	4486.7	162.96		
23	4514.8	4543.0	4571.3	4599.6	4628.0	4656.5	4685.0	4713.7	4742.4	4771.1	170.37		
24	4800.0	4828.9	4857.9	4887.0	4916.1	4945.4	4974.7	5004.0	5033.5	5063.0	177.78		
25	5092.6	5122.3	5152.0	5181.8	5211.7	5241.7	5271.7	5301.8	5332.0	5362.3	185.19		
26	5392.6	5423.0	5453.5	5484.0	5514.7	5545.4	5576.1	5607.0	5637.9	5668.9	192.59	7.41	
27	5700.0	5731.1	5762.4	5793.7	5825.0	5856.5	5888.0	5919.6	5951.3	5983.0	200.00	1	.74
28	6014.8	6046.7	6078.7	6110.7	6142.8	6175.0	6207.3	6239.6	6272.0	6304.5	207.41	2	1.48
29	6337.0	6369.7	6402.4	6435.1	6468.0	6500.9	6533.9	6567.0	6600.1	6633.4	214.81	3	2.22
30	6666.7	6700.0	6733.5	6767.0	6800.6	6834.3	6868.0	6901.8	6935.7	6969.7	222.22	4	2.96
31	7003.7	7037.8	7072.0	7106.3	7140.6	7175.0	7209.5	7244.0	7278.7	7313.4	229.63	5	3.70
32	7348.1	7383.0	7417.9	7452.9	7488.0	7523.1	7558.4	7593.7	7629.0	7664.5	237.04	6	4.44
33	7700.0	7735.6	7771.3	7807.0	7842.8	7878.7	7914.7	7950.7	7986.8	8023.0	244.44	7	5.19
34	8059.3	8095.6	8132.0	8168.5	8205.0	8241.7	8278.4	8315.1	8352.0	8388.9	251.85	8	5.93
35	8425.9	8463.0	8500.1	8537.4	8574.7	8612.0	8649.5	8687.0	8724.6	8762.3	259.26	9	6.67
36	8800.0	8837.8	8875.7	8913.7	8951.7	8989.8	9028.0	9066.3	9104.6	9143.0	266.67		
37	9181.5	9220.0	9258.7	9297.4	9336.1	9375.0	9413.9	9452.9	9492.0	9531.1	274.07		
38	9570.4	9609.7	9649.0	9688.5	9728.0	9767.6	9807.3	9847.0	9886.8	9926.7	281.48		
39	9966.7	10007	10047	10087	10127	10168	10208	10248	10289	10330	288.89		
40	10370	10411	10452	10493	10534	10575	10616	10657	10699	10740	296.30		
41	10781	10823	10865	10906	10948	10990	11032	11074	11116	11158	303.70		
42	11200	11242	11285	11327	11369	11412	11455	11497	11540	11583	311.11		
43	11626	11669	11712	11755	11798	11842	11885	11928	11972	12016	318.52		
44	12059	12103	12147	12191	12235	12279	12323	12367	12411	12456	325.93		
45	12500	12544	12589	12634	12678	12723	12768	12813	12858	12903	333.33		
46	12948	12993	13039	13084	13129	13175	13221	13266	13312	13358	340.74		
47	13404	13450	13496	13542	13588	13634	13681	13727	13773	13820	348.15		
48	13867	13913	13960	14007	14054	14101	14148	14195	14242	14290	355.56		
49	14337	14384	14432	14480	14527	14575	14623	14671	14719	14767	362.96		
50	14815	14863	14911	14960	15008	15056	15105	15154	15202	15251	370.37		
51	15300	15349	15398	15447	15496	15545	15595	15644	15693	15743	377.78		
52	15793	15842	15892	15942	15992	16042	16092	16142	16192	16242	385.19		
53	16293	16343	16393	16444	16495	16545	16596	16647	16698	16749	392.59		
54	16800	16851	16902	16954	17005	17056	17108	17160	17211	17263	400.00		
55	17315	17367	17419	17471	17523	17575	17627	17680	17732	17784	407.41		
56	17837	17890	17942	17995	18048	18101	18154	18207	18260	18313	414.81		
57	18367	18420	18473	18527	18581	18634	18688	18742	18796	18850	422.22		
58	18904	18958	19012	19066	19121	19175	19229	19284	19339	19393	429.63		
59	19448	19503	19558	19613	19668	19723	19778	19834	19889	19944	437.04		
60	20000	20056	20111	20167	20223	20279	20335	20391	20447	20503	444.44		

Note that Base, Slopes, and Cu. Yds. in this table may all be multiplied by the same factor; thus, using factor of ½ for height of 36 ft., we have, 4400 cu. yds. for base of 15 ft. and slopes ½ to 1.

Add for Tenths of Feet in Height.

Use with preceding column only.

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40.—Level Sections (Earthwork); Height, 60–100 Ft.

Bases of Roadway, 14–30 Ft. Side Slopes, 1 to 1.

Note.—The last two columns enable us to use any other base than given: Ex.—Given height. 71.5 ft.: roadway 32 ft. Then we have, 26879 + 530 = 27409 cu. yds. (See also Table 28.)

[Cu. Yds. per 100-Ft. Station.]

Ht. Ft.	Width of Roadway in Feet.									Width of 2 Ft. Cu. Yds	Width of 10 Ft. Cu. Yds
	14	16	18	20	22	24	26	28	30		
60	16444	16889	17333	17778	18222	18667	19111	19556	20000	444.44	2222.2
.5	16694	17142	17590	18038	18486	18934	19382	19831	20279	448.15	2240.7
61	16944	17396	17848	18300	18752	19204	19656	20107	20559	451.85	2259.3
.5	17197	17653	18108	18564	19019	19475	19931	20386	20842	455.56	2277.8
62	17452	17911	18370	18830	19289	19748	20207	20667	21126	459.26	2296.3
.5	17708	18171	18634	19097	19560	20023	20486	20949	21412	462.96	2314.8
63	17967	18433	18900	19367	19833	20300	20767	21233	21700	466.67	2333.3
.5	18227	18697	19168	19638	20108	20579	21049	21519	21990	470.37	2351.9
64	18489	18963	19437	19911	20385	20859	21333	21807	22281	474.07	2370.4
.5	18753	19230	19708	20186	20664	21142	21619	22097	22575	477.78	2388.9
65	19019	19500	19981	20463	20944	21426	21907	22389	22870	481.48	2407.4
.5	19286	19771	20257	20742	21227	21712	22197	22682	23168	485.19	2425.9
66	19556	20044	20533	21022	21511	22000	22489	22978	23467	488.89	2444.4
.5	19827	20319	20812	21305	21797	22290	22782	23275	23768	492.59	2463.0
67	20100	20596	21093	21589	22085	22582	23078	23574	24070	496.30	2481.5
.5	20375	20875	21375	21875	22375	22875	23375	23875	24375	500.00	2500.0
68	20652	21156	21659	22163	22667	23170	23674	24178	24681	503.70	2518.5
.5	20931	21438	21945	22453	22960	23468	23975	24482	24990	507.41	2537.0
69	21211	21722	22233	22744	23256	23767	24278	24789	25300	511.11	2555.6
.5	21494	22008	22523	23038	23553	24068	24582	25097	25612	514.81	2574.1
70	21778	22296	22815	23333	23852	24370	24889	25407	25926	518.52	2592.6
.5	22064	22586	23108	23631	24153	24675	25197	25719	26242	522.22	2611.1
71	22352	22878	23404	23930	24456	24982	25507	26033	26559	525.93	2629.6
.5	22642	23171	23701	24231	24760	25290	25819	26349	26879	529.63	2648.1
72	22933	23467	24000	24533	25067	25600	26133	26667	27200	533.33	2666.7
.5	23227	23764	24301	24838	25375	25912	26449	26986	27523	537.04	2685.2
73	23522	24063	24604	25144	25685	26226	26767	27307	27848	540.74	2703.7
.5	23819	24364	24908	25453	25997	26542	27086	27631	28175	544.44	2722.2
74	24119	24667	25215	25763	26311	26859	27407	27956	28504	548.15	2740.7
.5	24419	24971	25523	26075	26627	27179	27731	28282	28834	551.85	2759.3
75	24722	25278	25833	26389	26944	27500	28056	28611	29167	555.56	2777.8
.5	25027	25586	26145	26705	27264	27823	28382	28942	29501	559.26	2796.3
76	25333	25896	26459	27022	27585	28148	28711	29274	29837	562.96	2814.8
.5	25642	26208	26775	27342	27908	28475	29042	29608	30175	566.67	2833.3
77	25952	26522	27093	27663	28233	28804	29374	29944	30515	570.37	2851.9
.5	26264	26838	27412	27986	28560	29134	29708	30282	30856	574.07	2870.4
78	26578	27156	27733	28311	28889	29467	30044	30622	31200	577.78	2888.9
.5	26894	27475	28056	28638	29219	29801	30382	30964	31545	581.48	2907.4
79	27211	27796	28381	28967	29552	30137	30722	31307	31893	585.19	2925.9
.5	27531	28119	28708	29297	29886	30475	31064	31653	32242	588.89	2944.4
80	27852	28444	29037	29630	30222	30815	31407	32000	32593	592.59	2963.0
81	28500	29100	29700	30300	30900	31500	32100	32700	33300	600.00	3000.0
82	29156	29763	30370	30978	31585	32193	32800	33407	34015	607.41	3037.0
83	29819	30433	31048	31663	32278	32893	33507	34122	34737	614.81	3074.1
84	30489	31111	31733	32356	32978	33600	34222	34844	35467	622.22	3111.1
85	31167	31796	32426	33056	33685	34315	34944	35574	36204	629.63	3148.1
86	31852	32489	33126	33763	34400	35037	35674	36311	36948	637.04	3185.2
87	32544	33189	33833	34478	35122	35767	36411	37056	37700	644.44	3222.2
88	33244	33896	34548	35200	35852	36504	37156	37807	38459	651.85	3259.3
89	33952	34611	35270	35930	36589	37248	37907	38567	39226	659.26	3296.3
90	34667	35333	36000	36667	37333	38000	38667	39333	40000	666.67	3333.3
91	35389	36063	36737	37411	38085	38759	39433	40107	40781	674.07	3370.4
92	36119	36800	37481	38163	38844	39526	40207	40889	41570	681.48	3407.4
93	36856	37544	38233	38922	39611	40300	40989	41678	42367	688.89	3444.4
94	37600	38296	38993	39689	40385	41082	41778	42474	43170	696.30	3481.5
95	38352	39056	39758	40462	41166	41869	42573	43277	43981	703.70	3518.5
96	39111	39822	40533	41244	41956	42667	43378	44089	44800	711.11	3555.6
97	39878	40596	41315	42033	42752	43470	44189	44907	45626	718.52	3592.6
98	40652	41378	42104	42830	43556	44282	45007	45733	46459	725.93	3629.6
99	41433	42167	42900	43633	44367	45100	45833	46567	47300	733.33	3666.7
100	42222	42963	43704	44444	45185	45926	46667	47407	48148	740.74	3703.7

41.—Level Sections (Earthwork); Height, 60–100 Ft.

Bases of Roadway, 14–30 Ft. Side Slopes, 1½ to 1.

Note.—The last two columns enable us to use any other base than given: Ex.—Given height, 68.5 ft.; roadway 32 ft. Then we have, 33679 + 507 = 34186 cu. yds. (See also Table 24.)

[Cu. Yds. per 100-Ft. Station.]

Ht. Ft.	Width of Roadway in Feet.									Width of 2 Ft. Cu. Yds	Width of 10 Ft. Cu. Yds
	14	16	18	20	22	24	26	28	30		
60	23111	23556	24000	24444	24889	25333	25778	26222	26667	444.44	2222.2
.5	23472	23920	24368	24816	25264	25713	26161	26609	27057	448.15	2240.7
61	23835	24287	24739	25191	25643	26094	26546	26998	27450	451.85	2259.3
.5	24201	24657	25113	25568	26024	26479	26935	27390	27846	455.56	2277.8
62	24570	25030	25489	25948	26407	26867	27326	27785	28244	459.26	2296.3
.5	24942	25405	25868	26331	26794	27257	27720	28183	28646	462.96	2314.8
63	25317	25783	26250	26717	27183	27650	28117	28583	29050	466.67	2333.3
.5	25694	26164	26635	27105	27575	28046	28516	28987	29457	470.37	2351.9
64	26074	26548	27022	27496	27970	28444	28918	29393	29867	474.07	2370.4
.5	26457	26935	27413	27890	28368	28846	29324	29801	30279	477.78	2388.9
65	26843	27324	27806	28287	28769	29250	29731	30213	30694	481.48	2407.4
.5	27231	27716	28201	28687	29172	29657	30142	30627	31113	485.19	2425.9
66	27622	28111	28600	29089	29578	30067	30556	31044	31533	488.89	2444.4
.5	28016	28509	29001	29494	29987	30479	30972	31464	31957	492.59	2463.0
67	28413	28909	29406	29902	30398	30894	31391	31887	32383	496.30	2481.5
.5	28813	29313	29813	30313	30813	31313	31813	32313	32813	500.00	2500.0
68	29215	29719	30222	30726	31230	31733	32237	32741	33244	503.70	2518.5
.5	29620	30127	30635	31142	31650	32157	32664	33172	33679	507.41	2537.0
69	30028	30539	31050	31561	32072	32583	33094	33606	34117	511.11	2555.6
.5	30438	30953	31468	31983	32498	33012	33527	34042	34557	514.81	2574.1
70	30852	31370	31889	32407	32926	33444	33963	34481	35000	518.52	2592.6
.5	31268	31790	32313	32835	33357	33879	34401	34924	35446	522.22	2611.1
71	31687	32213	32739	33265	33791	34317	34843	35369	35894	525.93	2629.6
.5	32109	32638	33168	33698	34227	34757	35287	35816	36346	529.63	2648.1
72	32533	33067	33600	34133	34667	35200	35733	36267	36800	533.33	2666.7
.5	32961	33498	34035	34572	35109	35646	36183	36720	37257	537.04	2685.2
73	33391	33930	34471	35012	35553	36093	36634	37175	37717	540.74	2703.7
.5	33824	34368	34912	35457	36001	36546	37090	37635	38179	544.44	2722.2
74	34259	34807	35356	35904	36452	37000	37548	38096	38644	548.15	2740.7
.5	34698	35250	35801	36353	36905	37457	38009	38561	39113	551.85	2759.3
75	35139	35694	36250	36806	37361	37917	38472	39028	39583	555.56	2777.8
.5	35583	36142	36701	37261	37820	38379	38938	39498	40057	559.26	2796.3
76	36030	36593	37156	37719	38281	38844	39407	39970	40533	562.96	2814.8
.5	36479	37046	37613	38179	38746	39313	39879	40446	41013	566.67	2833.3
77	36931	37502	38072	38643	39213	39783	40354	40924	41494	570.37	2851.9
.5	37387	37961	38535	39109	39683	40257	40831	41405	41979	574.07	2870.4
78	37844	38422	39000	39578	40156	40733	41311	41889	42467	577.78	2888.9
.5	38305	38887	39468	40050	40631	41212	41794	42375	42957	581.48	2907.4
79	38769	39354	39939	40524	41109	41694	42280	42865	43450	585.19	2925.9
.5	39235	39824	40412	41001	41590	42179	42768	43357	43946	588.89	2944.4
80	39704	40296	40889	41481	42074	42667	43259	43852	44444	592.59	2963.0
81	40650	41250	41850	42450	43050	43650	44250	44850	45450	600.00	3000.0
82	41607	42215	42822	43430	44037	44644	45252	45859	46467	607.41	3037.0
83	42576	43191	43806	44420	45035	45650	46265	46880	47494	614.81	3074.1
84	43556	44178	44800	45422	46044	46667	47289	47911	48533	622.22	3111.1
85	44546	45176	45806	46435	47065	47694	48324	48954	49583	629.63	3148.1
86	45548	46185	46822	47459	48096	48733	49370	50007	50644	637.04	3185.2
87	46561	47206	47850	48494	49139	49783	50428	51072	51717	644.44	3222.2
88	47585	48237	48889	49541	50193	50844	51496	52148	52800	651.85	3259.3
89	48620	49280	49939	50598	51257	51917	52576	53235	53894	659.26	3296.3
90	49667	50333	51000	51667	52333	53000	53667	54333	55000	666.67	3333.3
91	50724	51398	52072	52746	53420	54094	54768	55443	56117	674.07	3370.4
92	51793	52474	53156	53837	54519	55200	55881	56563	57244	681.48	3407.4
93	52872	53561	54250	54939	55628	56317	57006	57694	58383	688.89	3444.4
94	53963	54659	55356	56052	56748	57444	58141	58837	59533	696.30	3481.5
95	55065	55769	56472	57176	57880	58583	59287	59991	60695	703.70	3518.5
96	56178	56889	57600	58311	59022	59733	60444	61156	61867	711.11	3555.6
97	57302	58020	58739	59457	60176	60894	61613	62331	63050	718.52	3592.6
98	58437	59163	59889	60615	61341	62067	62793	63519	64245	726.93	3629.6
99	59583	60317	61050	61783	62517	63250	63983	64717	65450	733.33	3666.7
100	60741	61481	62222	62963	63704	64444	65185	65926	66667	740.74	3703.7

Note that Base, Slope, and Cu. Yds. in this table may all be multiplied by the same factor. See side-notes to preceding tables.

Slope-Staking is one of the first operations after the location has been ıdopted and filed with the proper authorities to secure condemnation rights. t consists of setting slope-stakes at oints where the side slopes of cut and fill ıtersect the ground line; and embraces lso, in its widest sense, intermediate :ross-sectioning" as shown in Fig. 19. he illustration shows the "grade rod" ethod, which is considered the best. ıe "grade rod" is the difference in ele- tion between the height of instru- ınt and the top of fill or bottom of cut, for the station at which : slope stakes are to be set; and is used directly with ground-rod readings. us, for the left-hand slope stake, $10.2+2.3=12.5=$distance *below* top of and is marked -12.5. The "distance out" (from center line stake) responding to -12.5 is $12.5\times 1\frac{1}{2}+7.0=25.8$. Hence $\frac{-12.5}{25.8}$ is the posi- of the slope stake. Sometimes two or three trials have to be made to :t the point which will give the proper relation between elevation, and ance from center.

Fig. 19.

The field notes are kept as shown at the bottom of sketch, Fig. 19; are on the right-hand page of the note book, under *Left - Center - Right*. he left-hand page are *Station*, *Grade Elevation*, $+S.$, *H. I.*, $-S.$, *B. M.* and *Rod*, if the complete records of bench marks and turning points are * Sometimes, however, the records of the turnings are kept on loose s and thrown away and the balance of the above notes, including the section notes, are all on the left-hand page, with an added column *round Elevation*; the right-hand page being reserved for office ations of quantities in excavation (including solid rock, loose nd earth) and embankment. The first named method is the best as is a complete record of all field operations. The office copy may be form last mentioned; and it is best to copy the field notes every day ce record.

thwork Computation from cross-section notes as in Fig. 19, is ›y cutting the figure up into triangles, rectangles and trapezoids, n be calculated directly from the field notes. Thus, the area of the ınd half of the figure $=\frac{16.0}{2}\times(26.2-7.0)+7.0\times\frac{16.2+16.4}{2}$. In this te that the intermediate cross-section is taken 7.0 ft. out from one-half width of roadbed, a wise thing to do where practicable d an intermediate elevation is necessary) as it simplifies office on. On the left-hand side this was not done and we have for area ılf of figure: $11.0\times\left(\frac{15.4+16.4}{2}\right)+14.8\times\left(\frac{15.4+12.5}{2}\right)-12.5\times$ $\left.\frac{7.0}{}\right)$. The deduction is for the triangle outside the slope at T, as :luded in the previous quantities. Fig. 17, page 1016, presents the orm of sketch for computation and this will obtain when only the : or fill is given in addition to the slope-stake notes. Thus, area $D+d)$, and area $c+d=\frac{10}{2}(H+h)$. ction is called a "three-level"

rismoidal Formula and **Pris- rection Formula** are used for *e* determination of the *vol-* rismoids, where the method as" will not suffice. The fol- ıssion is based on the "three- ›n, Fig. 20.

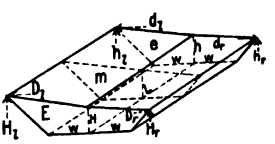

Fig. 20.

ı = the number of the station, as $1095+50$; *Grade Elevation* = n of sub-grade, i. e., bottom of cut or top of fill; $+S$ = back- rod reading on the bench mark (*B. M.*) to determine the height ıt (*H. I.*); $-S$ = fore-sight, or the rod reading to determine the turning point (*T. P.*) or *B. M.*, from the *H. I.*

Let E = area of the end section E,
e = " " " " e,
m = " " middle " m,
L = perp dist between E and e,
H, H_r, H_l = respective elevations above roadbed at E,
h, h_r, h_l = " " " " " e,
D_r, D_l = respective distances out from center at E,
$D = D_r + D_l$,
d_r, d_l = respective distances out from center at e,
$d = d_r + d_l$,
W = one-half width of roadbed at E,
w = " " " " e.
S_e = solidity or volume by end areas,
V_p = volume by prismoidal formula,
C_p = prismoidal correction volume $= \pm(S_e - V_p)$.

Then, $E = \frac{H}{2}(D_r + D_l) + \frac{W}{2}(H_r + H_l)$,

$$e = \frac{h}{2}(d_r + d_l) + \frac{w}{2}(h_r + h_l), \text{ and since } W = w,$$

$$m = \frac{H+h}{8}(D+d) + \frac{W}{4}(H_r + H_l + h_r + h_l).$$

By end areas in cu. ft.,

$$S = \frac{L}{2}(E+e) = \frac{L}{4}[HD + hd + W(H_r + H_l + h_r + h_l)] \quad \ldots\ldots (1)$$

By prismoidal formula, in. cu. ft.,

$$V_p = \frac{L}{6}(E + 4m + e) = \frac{L}{12}[HD + hd + 3W(H_r + H_l + h_r + h_l) + (H+h)(D+d)] \quad \ldots\ldots (2)$$

By prismoidal correction, in cu. ft.,

$$C_p = (S_e - V_p) = \frac{L}{12}(HD + hd - hD - Hd) = \frac{L}{12}(H-h)(D-d) \quad \ldots\ldots (3)$$

By prismoidal correction, in cu. yds. for 100-ft. station,

$$C_p = \frac{100}{12 \times 27}(H-h)(D-d) = \frac{1}{3.24}(H-h)(D-d)$$
$$= 0.308642(H-h)(D-d) \quad \ldots\ldots (4)$$

The correction C_p is to be *subtracted* if $(H-h)(D-d)$ is *positive* (usual).
The correcton C_p is to be *added* if $(H-h)(D-d)$ is *negative* (rare).

The Prismoidal Correction table on pages 1057-8 is made up from the following equivalents, which may be used direct, if desired:

$(H-h)(D-d)$. (Feet.)	Pris. Cor. C_p for 100-Ft. Sta. (Cubic Yards.)	$(H-h)(D-d)$. (Feet.)	Pris. Cor. C_p for 100-Ft. Sta. (Cubic Yards.)
1	.308 642	6	1.851 852
2	.617 284	7	2.160 494
3	.925 926	8	2.469 136
4	1.234 568	9	2.777 778
5	1.543 210	10	3.086 420

Example.—The following cross-sections were taken at stations 1 and 2, roadbed 20 ft. wide, and side slopes 1 on 1:

	LEFT.	CENTER.	RIGHT.	REMARKS.
Sta. 2.	$\frac{+6.4}{16.4}$	+4.2	$\frac{+3.1}{13.1}$.	$h = 4.2$; $d = 16.4 + 13.1 = 29.5$
Sta. 1.	$\frac{+8.7}{18.7}$	+5.5	$\frac{+4.3}{14.3}$.	$H = 5.5$; $D = 18.7 + 14.3 = 33.0$

Find the quantity of earth to be removed from Station 1 – 2?
Solution.—By end areas, S_e ..... = 491.11 Cu. Yds.
Prismoidal correction for $(H-h)(D-d)$, + 4.55 = 1.40 "
Therefore, by prismoidal formula, quantity..... = 489.71 " Ans.

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42.—Prismoidal Corrections* $\left[=\frac{100}{12\times 27}(H-h)\,(D-d)\right]$.

[Cu. Yds. per 100-Ft. Station.]

h)	Tenths.									
d)	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
).	.....	.0309	.0617	.0926	.1235	.1543	.1852	.2160	.2469	.2778
).	.3086	.3395	.3704	.4012	.4321	.4629	.4938	.5247	.5556	.5864
).	.6173	.6481	.6790	.7099	.7407	.7716	.8024	.8333	.8642	.8950
).	.9259	.9568	.9877	1.019	1.049	1.080	1.111	1.142	1.173	1.204
.	1.235	1.265	1.296	1.327	1.358	1.389	1.420	1.451	1.481	1.512
.	1.543	1.574	1.605	1.636	1.666	1.697	1.728	1.759	1.790	1.821
.	1.852	1.883	1.914	1.944	1.975	2.006	2.037	2.068	2.099	2.130
.	2.160	2.191	2.222	2.253	2.284	2.315	2.346	2.377	2.408	2.438
.	2.469	2.500	2.531	2.562	2.593	2.624	2.654	2.685	2.716	2.747
.	2.778	2.809	2.840	2.870	2.901	2.932	2.963	2.994	3.025	3.056
.	3.086	3.117	3.148	3.179	3.210	3.241	3.272	3.303	3.333	3.364
.	3.395	3.426	3.457	3.488	3.518	3.549	3.580	3.611	3.642	3.673
.	3.704	3.735	3.765	3.796	3.827	3.858	3.889	3.920	3.951	3.981
.	4.012	4.043	4.074	4.105	4.136	4.167	4.197	4.228	4.259	4.290
.	4.321	4.352	4.383	4.413	4.444	4.475	4.516	4.537	4.568	4 599
.	4.629	4.660	4.691	4.722	4.753	4.784	4.815	4.846	4.877	4.907
.	4.938	4.969	5.000	5.031	5.062	5.093	5.123	5.154	5.185	5.216
.	5.247	5.278	5.308	5.339	5.370	5.401	5.432	5.463	5.494	5.524
.	5.556	5.586	5.617	5.648	5.679	5.710	5.740	5.772	5.802	5.833
.	5.864	5.895	5.926	5.957	5.988	6.019	6.049	6.080	6.111	6.142
.	6.173	6.204	6.235	6.265	6.296	6.327	6.358	6.389	6.420	6.451
.	6.481	6.512	6.543	6.574	6.605	6.636	6.667	6.698	6.728	6.759
.	6.790	6.821	6.852	6.883	6.914	6.944	6.975	7.006	7.037	7.068
.	7.099	7.130	7.160	7.191	7.222	7.253	7.284	7.315	7.346	7.377
.	7.407	7.438	7.469	7.500	7.531	7.562	7.593	7.623	7.654	7.685
.	7.716	7.747	7.778	7.809	7.840	7.870	7.901	7 932	7.965	7.994
.	8.024	8.056	8.086	8.117	8.148	8.179	8.210	8.241	8.274	8.302
.	8.333	8.364	8.395	8.426	8.457	8.488	8.519	8.549	8.582	8.611
.	8.642	8.673	8.704	8.735	8.765	8.796	8.827	8.858	8.891	8.920
.	8.950	8.981	9.012	9.043	9.074	9.105	9.136	9.167	9.200	9.228
.	9.259	9.290	9.321	9.352	9.383	9.414	9.444	9.475	9.508	9.537
.	9.568	9.599	9.630	9.660	9.691	9.722	9.753	9.784	9.817	9.846
.	9.877	9.907	9.938	9.969	10.00	10.03	10.06	10.09	10.12	10.15
.	10.19	10.22	10.25	10.28	10.31	10 34	10.37	10.40	10.43	10.46
.	10.49	10.52	10.56	10.59	10.62	10.65	10.68	10.71	10.74	10.77
.	10.80	10.83	10.86	10.90	10.93	10.96	10.99	11.02	11 05	11.08
.	11.11	11.14	11.17	11.20	11.23	11.27	11.30	11.33	11.36	11.39
.	11.42	11.45	11.48	11.51	11.54	11.57	11.60	11.64	11.67	11.70
.	11.73	11.76	11.79	11.82	11.85	11.88	11.91	11.94	11.98	12.01
.	12.04	12.07	12.10	12.13	12.16	12.19	12.22	12.25	12.28	12.31
.	12.35	12.38	12.41	12.44	12.47	12.50	12.53	12.56	12.59	12.62
.	12.65	12.69	12.72	12.75	12.78	12.81	12.84	12.87	12.90	12.93
.	12.96	12.99	13.02	13.06	13.09	13.12	13.15	13.18	13.21	13.24
.	13.27	13.30	13.33	13.36	13.40	13.43	13.46	13.49	13.52	13.55
.	13.58	13.61	13.64	13.67	13.70	13.73	13.77	13.80	13.83	13.86
.	13.89	13.92	13.95	13.98	14.01	14.04	14.07	14.10	14.14	14.17
.	14.20	14.23	14.26	14.29	14.32	14.35	14.38	14.41	14.44	14.48
.	14.51	14.54	14.57	14.60	14.63	14.66	14.69	14.72	14.75	14.78
.	14.81	14.85	14.88	14.91	14.94	14.97	15.00	15.03	15.06	15.09
.	15.12	15.15	15.19	15.22	15.25	15.28	15.31	15.34	15.37	15.40
.	15.43	15.46	15.49	15.52	15.56	15.59	15.62	15.65	15.68	15.71

orrection to be *subtracted* when $(H-h)\,(D-d)$ is *positive*. (Usual.)
" " *added* " " " " *negative*. (Rare.)

42.—Prismoidal Corrections* $[=\frac{100}{12\times 27}(H-h)\ (D-d)]$.—Concluded.

[Cu. Yds. per 100-Ft. Station.]

$(H-h)\times(D-d)$	Tenths.									
	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
50.	15.43	15.46	15.49	15.52	15.56	15.59	15.62	15.65	15.68	15.71
51.	15.74	15.77	15.80	15.83	15.86	15.90	15.93	15.96	15.99	16.02
52.	16.05	16.08	16.11	16.14	16.17	16.20	16.23	16.27	16.30	16.33
53.	16.36	16.39	16.42	16.45	16.48	16.51	16.54	16.57	16.60	16.64
54.	16.67	16.70	16.73	16.76	16.79	16.82	16.85	16.88	16.91	16.94
55.	16.98	17.01	17.04	17.07	17.10	17.13	17.16	17.19	17.22	17.25
56.	17.28	17.31	17.35	17.38	17.41	17.44	17.47	17.50	17.53	17.56
57.	17.59	17.62	17.65	17.69	17.72	17.75	17.78	17.81	17.84	17.87
58.	17.90	17.93	17.96	17.99	18.02	18.06	18.09	18.12	18.15	18.18
59.	18.21	18.24	18.27	18.30	18.33	18.36	18.40	18.43	18.46	18.49
60.	18.52	18.55	18.58	18.61	18.64	18.67	18.70	18.73	18.77	18.80
61.	18.83	18.86	18.89	18.92	18.95	18.98	19.01	19.04	19.07	19.11
62.	19.14	19.17	19.20	19.23	19.26	19.29	19.32	19.35	19.38	19.41
63.	19.44	19.48	19.51	19.54	19.57	19.60	19.63	19.66	19.69	19.72
64.	19.75	19.78	19.81	19.85	19.88	19.91	19.94	19.97	20.00	20.03
65.	20.06	20.09	20.12	20.15	20.19	20.22	20.25	20.28	20.31	20.34
66.	20.37	20.40	20.43	20.46	20.49	20.52	20.56	20.59	20.62	20.65
67.	20.68	20.71	20.74	20.77	20.80	20.83	20.86	20.90	20.93	20.96
68.	20.99	21.02	21.05	21.08	21.11	21.14	21.17	21.20	21.23	21.27
69.	21.30	21.33	21.36	21.39	21.42	21.45	21.48	21.51	21.54	21.57
70.	21.61	21.64	21.67	21.70	21.73	21.76	21.79	21.82	21.85	21.88
71.	21.91	21.94	21.98	22.01	22.04	22.07	22.10	22.13	22.16	22.19
72.	22.22	22.25	22.28	22.31	22.35	22.38	22.41	22.44	22.47	22.50
73.	22.53	22.56	22.59	22.62	22.65	22.69	22.72	22.75	22.78	22.81
74.	22.84	22.87	22.90	22.95	22.96	22.99	23.02	23.06	23.09	23.12
75.	23.15	23.18	23.21	23.24	23.27	23.30	23.33	23.36	23.40	23.43
76.	23.46	23.49	23.52	23.55	23.58	23.61	23.64	23.67	23.70	23.73
77.	23.77	23.80	23.83	23.86	23.89	23.92	23.95	23.98	24.01	24.04
78.	24.07	24.10	24.14	24.17	24.20	24.23	24.26	24.29	24.32	24.35
79.	24.38	24.41	24.44	24.48	24.51	24.54	24.57	24.60	24.63	24.66
80.	24.69	24.72	24.75	24.78	24.81	24.85	24.88	24.91	24.94	24.97
81.	25.00	25.03	25.06	25.09	25.12	25.15	25.19	25.22	25.25	25.28
82.	25.31	25.34	25.37	25.40	25.43	25.46	25.49	25.53	25.56	25.59
83.	25.62	25.65	25.68	25.71	25.74	25.77	25.80	25.83	25.86	25.90
84.	25.93	25.96	25.99	26.02	26.05	26.08	26.11	26.14	26.17	26.20
85.	26.23	26.27	26.30	26.33	26.36	26.39	26.42	26.45	26.48	26.51
86.	26.54	26.57	26.60	26.64	26.67	26.70	26.73	26.76	26.79	26.82
87.	26.85	26.88	26.91	26.94	26.98	27.01	27.04	27.07	27.10	27.13
88	27.16	27.19	27.22	27.25	27.28	27.31	27.35	27.38	27.41	27.44
89.	27.47	27.50	27.53	27.56	27.59	27.62	27.65	27.69	27.72	27.75
90.	27.78	27.81	27.84	27.87	27.90	27.93	27.96	27.99	28.02	28.06
91.	28.09	28.12	28.15	28.18	28.21	28.24	28.27	28.30	28.33	28.36
92.	28.40	28.43	28.46	28.49	28.52	28.55	28.58	28.61	28.64	28.67
93.	28.70	28.73	28.77	28.80	28.83	28.86	28.89	28.92	28.95	28.98
94.	29.01	29.04	29.07	29.10	29.14	29.17	29.20	29.23	29.26	29.29
95.	29.32	29.35	29.38	29.41	29.44	29.48	29.51	29.54	29.57	29.60
96.	29.63	29.66	29.69	29.72	29.75	29.78	29.81	29.85	29.88	29.91
97.	29.94	29.97	30.00	30.03	30.06	30.09	30.12	30.15	30.19	30.22
98.	30.25	30.28	30.31	30.34	30.37	30.40	30.43	30.46	30.49	30.53
99.	30.56	30.59	30.62	30.65	30.68	30.71	30.74	30.77	30.80	30.83
100.	30.86	30.90	30.93	30.96	30.99	31.02	31.05	30.08	31.11	31.14

Note.—H, h, D and d are in feet.

* Correction to be *subtracted* when $(H-h)\ (D-d)$ is *positive*. (Usual.)
" " *added* " " " " *negative*. (Rare.)

Correction for Curvature, in earthwork computation, is very often neglected. Let A, Fig. 21, be the total area of the cross-section at any station on a curve; d_A the horizontal distance from the center of the section to the center of gravity of A*; R the radius of the curve. Then the correction for curvature may be embodied by using a new area, $A_c = A\left(1 \pm \frac{d_A}{R}\right)$, and maintaining the distance between stations as measured on the center line; $\frac{d_A}{R}$ to be added if R_A, the radius to the center of gravity of A, is greater than R, and subtracted if $R_A < R$. This is based on the theory that the volume of a solid of revolution is equal to the area revolved multiplied by the length of the path traced by its center of gravity. From this, we have, Volume for one station $= A \times 100\left(1 \pm \frac{d_A}{R}\right)$; but this is clearly equal to $100 A_c = 100A\left(1 \pm \frac{d_A}{R}\right)$. In one case the length of station is maintained while the area of the section is considered to be increased or decreased; in the other case the reverse is assumed.

Fig. 21.

"Haul" is a term applied to the average "lead" or horizontal distance between the *centers of gravity* of the same material "in place" and "in fill." In Fig. 22, let $G.L.$ be the grade line, H the haul, $F.H.$ the free haul, and $O.H.$ the overhaul or paid haul. Then, overhaul = haul − free haul. The free haul may be 500 ft. more or less, according to the specifications and contract. The centers of gravity are determined after the division lines d are fixed, from the estimated quantities, in much the same way as the center of gravity of A, Fig. 21, was determined. "Shrinkage" is a refinement which may be considered if the quantities are large. "Waste" and "borrow" will affect haul, and notes should be made (on the profile) of the manner in which all material on the work has been handled.

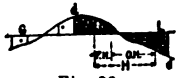

Fig. 22.

Roadbed.—The standard roadbed cross-sections should be adopted before the grade line is established, in order to equalize cuts and fills, where

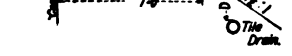

Fig. 23.

necessary, during construction. Each road has its own standard or rather standards, for the cross-section varies with the amount of traffic, height of embankment, whether on tangent or curve, main or branch line, etc. Fig. 23 shows about the minimum width of roadway that should be considered, namely, 14 ft. and 18 ft. at sub-grade for embankment and excavation, respectively. These should be increased rather than diminished, especially if on the main line, with high embankment, on a long curve, under heavy traffic. The width of roadway in embankment on existing roads varies from 12 ft. (minimum for cheap roads) to 20 ft. Allowance must be made for shrinkage of embankment as the width of roadbed *at sub-grade* will narrow three times the amount of vertical shrinkage, when side slopes are 1½ to 1. For double track lines the roadbed for single track is increased by the distance between track centers, say 13 ft. The size of ditches in cuts will depend on the length of cut, amount of drainage, and slope of ditch. Drain tile should be placed below the frost line.

* $A = a_1 + a_2 + a_3 + a_4$; and $d_A = [a_2(d_1 - d_2) + a_3(d_1 + d_3) + a_4(d_1 + d_4)]A - d_1$.

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Rails and Fastenings.—Rails are rolled usually in 30-ft. lengths, although 33-ft. rails have lately been introduced on main-line track. Shorter lengths, varying by two feet, or even by one foot, are furnished for preserving "opposite" or "broken" joints in track laid on curves, and for switch "leads," etc. Rails are designated by the "weight per yard," as 60-lb., 70-lb., 80-lb., etc. For instance, a 30-ft. 80-lb rail will weigh 800 lbs. In order to decrease the number of existing standard rail sections and the consequent expense in rolling, as well as to improve the type and facilitate quick delivery from the mills, the Am. Soc. of C. E., in 1893, adopted a standard type called the "American Society Standard," shown in Fig. 24 and in Table 43, following.

Splices for rail joints have developed from the primitive "chair" which simply gave *vertical support* to the rail joint placed directly over the tie, to the "fish plate" (invented in the early 40's) which gave *vertical stiffness* to the joint (see Fig. 27, below), and later to the "angle bar" which gives *lateral* as well as *vertical stiffness* to the joint. Many improved forms of angle bars have appeared on the market from time to time, but only those of uniform cross-section, that can be rolled continuously, have become standard. Figs. 24, 26 and 28 show standard types.

Bolts for fastening the splice bars to the rail are often provided with lock-nuts to prevent the bolts from working loose, or at least from working loose too rapidly. One of the simplest and perhaps most common forms is a steel ring of square or hexagonal section cut beveled with sharp corners and forming a complete spiral curve of one turn. This is placed on the bolt and when the nut is screwed on, the spiral form of the lock-nut is compressed to a nearly circular form, and the sharp points pressing against the steel on either side prevent the nut and bolt from loosening. Weights and dimensions of splice bolts are given in Table 43.

TABLE 43, giving standard dimensions of rails, splice plates and bolts, will be found on the two following pages. The accompanying illustrations, Figs. 24, 26 and 27, are also a part of this table, as per references given therein. Fig. 24 is the Am. Soc. C. E. Standard rail section (see pages 1061-2). Figs. 26 and 27 are Penn. Steel Co. standards (see page 1062). Figs. 25 and 28, with tables of dimensions, will be found on pages 1061 and 1062, respectively.

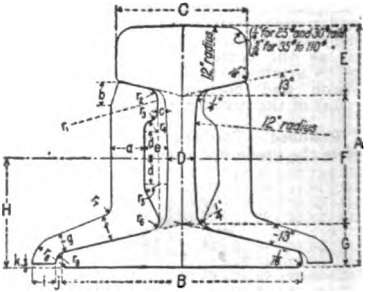

Fig. 24.

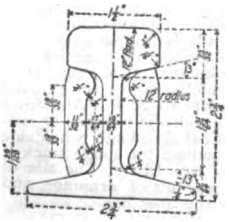

Fig. 27.

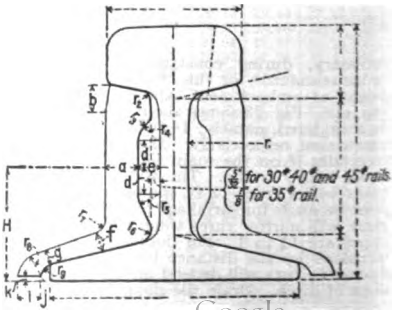

Fig. 26.

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Adapted from Penn. Steel Co. (P. S. C.).

Part I.—Rails and Bolts. (Holes in rails punched 1/16" larger in dia. than bolt.)

Note.—Dimensions of Rails in table (see Fig. 24) are Am. Soc. C. E. standards. Bolts to match splice plates.

Weight of Rails				P. S. C. Section Number	Dimensions of Rails, in Millimeters				Dimensions of Rails, in Inches.								Size of Bolts, in Inches		Wt. of One Bolt and Nut, Lbs.
		Single Track																	
Lbs. per Yard	Kgs. per Meter	Gross Tons per Mile	Tonnes per Kilo-meter		A	C	D	F	A	B	C	D	E	F	G	H	Dia.	Lgth.	
25	12.4	39.29	24.80	264	69.9	38.1	7.5	37.7	2¾	2¾	1 1/16	1/32	3/32	1 21/32	3/4	1 3/16	3/8	2¼	.355
30	14.9	47.14	29.76	269	76.2	41.3	8.3	40.9	3	3	1 5/8	3/32	3/32	1 13/16	17/32	1 3/16	3/8	2½	.373
35	17.4	55.00	34.72	265	82.6	44.5	9.1	44.1	3¼	3¼	1¾	3/32	1/8	1 13/16	9/32	1 3/16	3/8	2½	.373
40	19.8	62.86	39.68	138	88.9	47.6	9.53	47.2	3½	3½	1⅞	3/32	1/4	1 13/16	5/8	1 7/16	5/8	3	.524
45	22.3	70.71	44.64	266	93.7	50.8	10.7	50.0	3 11/16	3 11/16	2	3/32	5/16	1 15/16	3/4	1 3/16	5/8	3	.524
50	24.8	78.57	49.61	129	98.4	54.0	11.1	52.4	3⅞	3⅞	2 1/8	1/16	1/8	2 1/16	11/16	1 5/16	3/4	3¼	.757
55	27.3	86.43	54.57	130	103.2	57.2	11.9	62.8	4 1/16	4 1/16	2¼	1/16	3/16	2 3/16	13/16	1 5/16	3/4	3½	.789
60	29.8	94.29	59.53	244	108.0	60.3	12.3	57.5	4¼	4¼	2⅜	3/32	1/4	2 3/16	13/16	1 5/16	3/4	3¾	.821
65	32.2	102.14	64.49	236	112.7	61.1	12.7	60.3	4 1/16	4 1/16	2½	1/8	1/4	2 3/16	13/16	1 5/16	3/4	3¾	.821
70	34.7	110.00	69.45	237	117.5	61.9	13.1	62.7	4⅝	4⅝	2 7/16	3/32	1/4	2 3/16	13/16	2 3/16	3/4	4	.853
75	37.2	117.86	74.41	214	122.2	62.7	13.5	64.7	4 11/16	4 11/16	2 9/16	3/32	1/4	2 3/16	13/16	2 1/8	3/4	4	.853
80	39.7	125.71	79.37	251	127.0	63.5	13.9	66.7	5	5	2½	3/32	5/16	1 15/16	7/8	2 3/16	7/8	4¼	1.197
85	42.2	133.57	84.33	235	131.8	65.1	14.3	69.9	5 3/16	5 3/16	2 9/16	1/16	3/16	2 3/4	7/8	2 1/4	7/8	4¼	1.197
90	44.6	141.43	89.29	245	136.5	66.7	14.3	72.6	5⅜	5⅜	2⅝	1/16	3/16	2 13/16	7/8	2 3/16	7/8	4½	1.240
95	47.1	149.29	94.25	267	141.3	68.3	14.3	75.8	5 9/16	5 9/16	2 11/16	1/16	3/16	2 13/16	7/8	2 11/16	7/8	4¾	1.283
100	49.6	157.14	99.21	247	146.1	69.9	14.3	78.2	5¾	5¾	2¾	1/16	3/16	3 1/16	7/8	2½	1	4¾	1.672
110	54.6	172.86	109.13	268	155.6	73.0	14.7	84.9	6 1/8	6 1/8	2⅞	3/32	3/16	3 3/16	1	2 13/16	1	5	1.727

Fig. 25.—Spacing of Holes.

For Fig. 24, section of rail and fastenings, see opposite page.

Small rails manufactured by P. S. C. are (Fig. 24):

12-lb. rails; A = 2" B = 2" C = 1 1/16" D = 3/16" E = 13/32" F = 1 3/16" H = 29/32"

16-lb. rails; A = 2 11/16" B = 2 11/16" C = 1 1/16" D = 1/8" E = 13/32" F = 1 3/16" H = 1"

20-lb. rails; A = 2½" B = 2½" C = 1 1/16" D = 1/4" E = 13/32" F = 1 1/16" H = 1 3/16"

Spacing of holes (Fig. 25):

For Rails 12 to 45 lbs. per ft.: A = 2" B = 4"

" " 50 lbs. and over: A = 3 11/16" B = 5"

" " 50 lbs. and over: A = 1 1/16" B = 5" C = 6"

43.—Standard Dimensions of Rails and Fastenings.—Concluded.

Adapted from Penn. Steel Co. (P. S. C.).

Part II.—Splice Plates. Note.—Splice Plates in table are adapted to Am. Soc. C. E. Standard Rails.

Wt. of Rail per Yard, Lbs.	Weight of Splice Bar: Lbs. per Ft.	Weight of Splice Bar: Kgs. per Meter	Thickness of S-B in Millimeters.	Dimensions of Splice Bar, in Inches. a	b	c	d	e	f	g	H	i	j	k	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	Reference to Figure
25	2.0	2.97	8.7	11/16				11/4			1 3/16						1/4	1/4	1/4	1/4				Fig. 27
30	3.5	5.20	10.3	13/16	1/8			3/2	3/8	7/32	1 1/32	1/8	1/8	1/8			1/4	1/4	1/8	1/4	1/8	1/8	1/8	26
35	3.9	5.80	11.1	7/8	3/16		11/32	3/2	3/8	7/32	1 1/32	1/8	1/8	1/8		1/8	1/8	1/4	1/4	1/4	1/8	1/8	1/8	26
40	4.7	6.99	11.9	15/16	3/16		11/32	7/8	7/8	7/32	1 1/32	1/8	1/8	1/8		1/8	1/8	1/4	1/4	1/8	1/8	3/8	1/8	26
45	5.5	8.18	12.7	1 1/32	1/4		11/32	31/32	31/32	1/4	1 1/4	1/8	1/8	1/8		1/8	1/8	1/4	1/4	1/8	1/8	1/8	1/8	26
50	6.0	8.93	13.5	1 1/16	3/16	1/8	11/32	7/8	7/8	3/8	1 3/8	3/16	3/16	1/8	12	1/8	1/8	1/4	1/4	1/8	1/8	1/8	1/8	24
55	7.2	10.70	15.1	1 1/8	1/4	1/8	11/32	15/16	15/16	7/16	1 1/2	3/16	3/16	1/8	12	1/4	1/4	1/4	1/4	1/8	1/8	1/8	1/8	24
60	8.0	11.90	15.9	1 3/16	1/8	3/16	3/8	15/16	15/16	1/2	1 5/8	3/16	1/8	3/16	12	1/4	1/4	1/4	3/8	1/4	1/4	1/2	1/8	24
65	8.8	13.10	16.7	1 1/4	1/4	3/16	3/8	1 1/16	1/8	1/8	1 3/4	1/8	1/8	3/16	12	1/4	1/4	1/4		1/4	1/4	1/2	1/8	24
70	9.5	14.14	17.5	1 1/8	1/2	1/4	3/8	1 1/8	1/8	1/2	2 1/4	1/8	1/8	1/4	12	1/4	1/4	1/4	1/4	1/4	1/4	1/8	1/8	24
75	10.4	15.47	18.3	1 3/16	1/4	1/8	1/2	1 1/8	1/8	1/2	2 1/8	1/8	1/4	1/8	12	1/4	1/4	1/4	1/4	1/4	1/4	1/2	1/8	24
80	11.3	16.80	19.1	5/4	1/4	1/4	1/2	1 1/2	1/2	5/8	2 1/4	1/8	1/4	1/8	12	1/4	1/4	1/4	1/4	1/4	1/4	1/2	1/8	24
85	12.0	17.86	19.8	1 3/16	1/4	1/8	1/2	1 7/16	1/8	5/8	2 1/4	1/2	1/4	1/8	12	1/4	1/4	1/4	1/4	1/8	1/8	1/8	1/8	24
90	12.8	19.05	20.6	1 1/4	5/8	1/8	1/2	1 7/16	1/8	1/8	2 1/4	1/2	1/4	1/8	12	1/4	1/4	1/4	1/4	1/8	1/8	1/8	1/8	24
95	16.24	24.17	21.4	1 7/16	3/4	1/8	1/2	1 1/2	5/8	5/8	2 3/8	1/2	1/4	1/8	12	1/4	1/4	1/4	1/4	5/8	5/8	5/8	1/8	24
100	15.0	22.32	22.2	7/8	1/4	1/8	1/2	1 1/8	7/8	5/8	2 1/2	1/2	1/4	1/8	12	1/4	1/4	1/4	1/4	7/8	7/8	7/8	1/8	24
110	18.6	27.68	23.8	15/16	3/4	1/8	1/8	1/8	11/16	3/4	2 11/16	1/8	1/4	1/8	12	1/4	1/4	1/4	1/4	1/8	1/8	11/16	1/8	24

For Figs. 24, 26 and 27, see page 1060.

(Fig. 28.)

Wt. of Rails, Lbs.	L'gth	Bolts	D	E	F	G	H	I	J	K	L
12 to 45	16"	4		4"	4 1/2"	4"		‡	‡	‡	3/4"
50 and over	24"	4		6"	6"	5"		1 1/2"	1 7/8"	5 1/2"	3/4"
50 and over	30"	6	6"	6"	4"	5"	6"	6"	20"	4"	3/4"

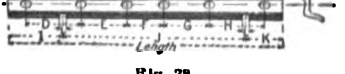

Fig. 28.

For Single Track, the number of 30-ft. rails required per mile of track $= \frac{2 \times 5280}{30} = 352$; of 33-ft. rails, 320; of 50-ft. rails, 211.2. The number of *short* ons (of 2000 lbs.) per mile of single track = 1.76 × wt. in lbs. of single rail er yd.; thus, there are required 176 short tons of 100-lb. rails per mile of ngle track. The number of *long tons* (of 2240 lbs.) per mile of single track = $\frac{11}{10}$ × wt. in lbs. of single rail per yard.; thus, there are required 157$\frac{1}{7}$ long ons of 100-lb. rails per mile of single track. For 90-lb. rails, mult. 176 and 57$\frac{1}{7}$, respectively, by $\frac{9}{10}$; for 80-lb. rails, mult. by $\frac{8}{10}$; etc.

44.—Weight of Rails per Yard Reduced to Tons per Mile of Single Track.

(Short tons at 2000 lbs.; long tons at 2240 lbs.)

Wt. per Yard. Lbs.	Short Tons per Mile.	Long Tons per Mile.	Wt. per Yard. Lbs.	Short Tons per Mile.	Long Tons per Mile.	Wt. per Yard. Lbs.	Short Tons per Mile.	Long Tons per Mile.
8	14.08	$12\frac{4}{7}$	56	98.56	88	76	133.76	$119\frac{2}{7}$
12	21.12	$18\frac{6}{7}$	57	100.32	$89\frac{4}{7}$	78	137.28	$122\frac{2}{7}$
16	28.16	$25\frac{1}{7}$	60	105.60	$94\frac{2}{7}$	80	140.80	$125\frac{5}{7}$
25	44.00	$39\frac{2}{7}$	62	109.12	$97\frac{3}{7}$	85	149.60	$133\frac{4}{7}$
30	52.80	$47\frac{1}{7}$	64	112.64	$100\frac{4}{7}$	90	158.40	$141\frac{3}{7}$
35	61.60	55	65	114.40	$102\frac{1}{7}$	95	167.20	$149\frac{2}{7}$
40	70.40	$62\frac{6}{7}$	68	119.68	$106\frac{6}{7}$	100	176.00	$157\frac{1}{7}$
45	79.20	$70\frac{5}{7}$	70	123.20	110	105	184.80	165
50	88.00	$78\frac{4}{7}$	72	126.72	$113\frac{1}{7}$	110	193.60	$172\frac{6}{7}$
52	91.52	$81\frac{5}{7}$	75	132.00	$117\frac{6}{7}$	120	211.20	$188\frac{4}{7}$

Note.—Values in above table are exact. Fractions of long tons may be iced to decimals of long tons and to pound equivalents as follows:).14286 l. t. = 320 lbs.; $\frac{2}{7}$ = 0.28571 l. t. = 640 lbs.; $\frac{3}{7}$ = 0.42857 l. t. = 960 $\frac{4}{7}$ = 0.57143 l. t. = 1280 lbs.; $\frac{5}{7}$ = 0.71429 l. t. = 1600 lbs.; $\frac{6}{7}$ = 0.85714 l. t. =) lbs.

Middle Ordinates for "bending" (curving) rails, to be laid on curves, be obtained from the following formulas:

$$= \frac{L^2}{8R} \text{ (practically exact); or, } M' = \frac{.001L^2D}{9 \times 5} \text{ (nearly exact)} \ldots\ldots\ldots (1)$$

$$= \frac{3L^2}{2R} \text{ (practically exact); or, } M'' = \frac{.004L^2D}{3 \times 5} \text{ (nearly exact)} \ldots\ldots\ldots (2)$$

ich M' = middle ordinate to the curved rail, *in feet*;
M'' = middle ordinate to the curved rail, *in inches*;
L = length of rail in feet;
R = radius of curve in feet;
D = degree of curve in degrees and decimals.

ote that in the above formulas either the radius or the degree of curve ɔe used, both being exact for the flat curves, say up to 4° or 5°. For a rail and 10° curve, the "radius" formula gives $M' = 0.196$ (exact), the "degree of curve" formula gives $M' = 0.200$ (2% large); same r 20° curve gives $M' = 0.391$ and $M' = 0.400$, respectively, instead of the correct value. For shorter rails the errors decrease, hence it ı that either formula may be used for all practical purposes.

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45.—Middle Ordinates in Inches for Curving Rails.

(Degrees of curve and radii are for 100-ft. chords.)

For Ordinates in Feet, see Table 46.

Note.—Ordinates are practically proportional to the square of length of rail. Thus, for 60-ft. rail, mult. value for 30, by 4; for 45-ft. rail, mult value for 30, by 2¼; for 8-ft. rail, divide value for 16, by 4; etc.

[Middle Ordinates, in Inches.]

Degree of Curve.	Radius. Feet.	Length of Rail, or Arc, in Feet.													
		100	50	33	30	28	26	24	22	20	18	16	14	12	10
0.5	11459.2	1 5/16	3/16	3/64	1/4	3/32	3/16	1/8	3/64	3/64	3/32	1/16	1/64	1/64	1/64
1.	5729.7	2 7/16	5/16	5/64	1/2	3/16	5/64	1/16	1/64	3/32	3/64	1/32	1/64	3/128	1/64
1.5	3819.8	3 5/8	3/8	7/64	3/4	9/16	7/64	3/32	1/16	9/64	1/16	3/64	1/32	1/64	1/64
2.	2864.9	5 1/16	1 1/16	9/64	15/16	3/8	9/64	3/16	3/32	3/32	5/64	1/32	3/128	1/64	1/64
2.5	2292.0	6 3/4	1 3/8	3/16	1 1/8	3/4	3/16	1/8	3/64	3/64	3/32	3/64	1/64	3/128	1/128
3.	1910.1	7 15/16	1 15/16	3/8	1 7/16	15/16	3/8	3/16	3/32	9/64	3/16	3/32	1/64	1/64	1/64
3.5	1637.3	9 1/8	2 1/8	1	1 5/8	1 1/8	1/2	1/4	3/16	3/16	3/32	1/16	1/64	1/64	1/64
4.	1432.7	10 5/16	2 9/16	1 1/64	1 7/8	1 5/16	9/16	3/16	3/16	3/16	3/32	1/16	1/64	1/64	1/64
4.5	1273.6	11 3/8	2 7/8	1 3/16	1 15/16	1 3/8	5/8	3/16	3/16	3/16	3/32	1/16	1/64	1/64	1/64
5.	1146.3	13 1/64	3 3/32	1 3/64	1 3/16	1 3/16	3/4	3/16	3/16	3/16	3/32	3/64	1/64	1/64	1/64
5.5	1042.1	14 3/8	3 1/8	1 3/16	1 1/2	1 1/4	3/4	3/16	3/16	3/16	3/32	3/64	1/64	1/64	1/64
6.	955.4	15 11/16	3 9/16	1 1/8	1 5/8	1 7/16	1 1/16	3/16	3/16	3/16	3/32	3/64	1/64	1/64	1/64
6.5	881.9	17	4 1/4	1 3/16	1 3/4	1 1/2	1 1/8	3/16	3/16	3/16	3/32	3/64	1/64	1/64	1/64
7.	819.0	18 1/2	4 5/8	1 5/8	1 3/4	1 11/16	1 1/4	1 1/16	3/16	3/16	3/32	3/64	1/64	1/64	1/64
7.5	764.5	17 3/8	4 3/4	2 3/64	1 7/8	1 3/8	1 3/8	1 1/8	3/16	3/16	3/32	3/64	1/64	1/64	1/64
8.	716.8	20 1/8	5 1/8	2 9/32	1 7/8	1 11/16	1 3/8	1 1/8	1 1/64	3/16	3/32	3/64	1/64	1/64	1/64
8.5	674.7	22 7/16	5 5/8	2 3/4	2	1 13/16	1 3/8	1 1/8	1 1/64	3/16	3/32	3/64	1/64	1/64	1/64
9.	637.3	23 3/4	5 15/16	2 15/16	2 3/16	1 15/16	1 3/8	1 1/8	1 1/64	3/16	3/32	3/64	1/64	1/64	1/64
9.5	603.8	24 13/16	6 1/8	3 1/8	2 1/8	1 15/16	1 1/16	1 1/8	1 1/64	1	3/32	3/64	1/64	1/64	1/64
10.	573.7	26 3/4	6 5/8	3 5/16	2 1/4	2 3/64	1 5/16	1 1/8	1 1/64	1 3/64	3/32	3/64	1/64	1/64	1/64
11.	521.7	28 13/16	7 1/16	3/4	2 1/8	2 1/4	1 3/16	1 1/8	1 1/64	1 5/64	3/32	3/64	1/64	1/64	1/64
12.	478.3	31 5/16	7 3/16	3 3/8	2 1/8	2 3/8	1 3/8	1 1/8	1 1/64	1 5/64	3/32	3/64	1/64	1/64	1/64
13.	441.7	33 11/16	8 3/16	3 3/8	3 3/64	2 5/16	1 3/8	1 1/8	1 1/64	1 5/64	3/32	3/64	1/64	1/64	1/64
14.	410.3	36 3/16	9 1/16	3 9/16	3 1/16	2 9/16	2 3/16	1 5/16	1 1/64	1 5/64	3/32	3/64	1/64	1/64	1/64
15.	383.1	39 3/16	9 13/16	4 1/64	3 3/16	3 1/64	2 1/16	2 1/4	1 5/64	1 7/64	1 1/64	1 3/64	3/64	3/64	3/64
16.	359.3	41 11/16	10 7/16	4 3/8	3 1/4	3 9/32	2 13/16	2 1/8	2 1/64	1 11/64	1 13/64	1 3/64	3/64	3/64	3/64
17.	338.3	44 1/2	11 1/8	4 3/4	4	3 5/16	3 3/32	2 1/16	2 3/64	1 3/16	1 5/64	1 1/64	3/64	3/64	3/64
18.	319.6	46 13/16	11 7/16	5 7/64	4 7/32	3 13/16	3 1/16	2 1/8	2 1/64	1 1/4	1 5/64	1 1/64	3/64	3/64	3/64
19.	302.9	49 1/16	12 3/8	5 1/8	4 3/8	3 13/16	3 3/16	2 1/8	2 1/64	1 1/4	1 5/64	1 1/64	3/64	3/64	3/64
20.	287.9	51 3/4	13 1/64	5 3/8	4 3/4	4 3/32	3 3/16	3	2 3/64	2 3/32	1 11/64	1 3/64	1 3/64	3/64	3/64
21.	274.4	54 3/16	13 5/8	5 11/16	4 5/8	4 9/32	3 5/16	3 1/8	2 11/64	2 3/32	1 13/64	1 7/64	1 3/64	3/64	3/64
22.	262.0	57 1/16	14 3/8	6 1/8	5 1/64	4 3/8	3 7/16	3 1/16	2 11/64	2 1/16	1 15/64	1 3/64	1 3/64	3/64	3/64
23.	250.8	59 3/4	14 15/16	6 3/16	5 1/32	4 1/2	3 7/16	3 1/16	2 11/64	2 1/16	1 15/64	1 3/64	1 3/64	3/64	3/64
24.	240.5	62 9/16	15 5/16	6 3/16	5 1/8	4 9/16	3 1/2	3 1/16	3 1/64	2 1/8	2 1/64	1 1/64	1 1/64	3/64	3/64
25.	231.0	64 15/16	16 3/32	7 1/16	5 3/8	5 3/32	4 3/16	3 1/16	3 1/64	2 3/16	2 1/64	1 1/64	1 1/64	3/64	3/64
26.	222.3	67 1/2	16 7/8	7 1/16	6 3/64	5 1/32	4 3/16	3 7/16	3 3/64	2 3/16	2 1/64	1 1/64	1 1/64	1 3/64	3/64
27.	214.2	69 3/32	17 3/16	7 3/8	6 9/32	5 3/16	4 1/4	4 1/64	3 1/32	2 3/16	2 3/64	1 1/64	1 1/64	1 3/64	3/64
28.	206.7	72 1/16	18 1/8	7 13/16	6 3/16	5 1/16	4 1/8	4 1/64	3 1/32	2 3/16	2 1/64	1 1/64	1 1/64	1 3/64	3/64
29.	199.7	74 3/16	18 9/16	8 1/16	6 1/4	5 3/16	5 1/32	4 1/64	3 1/32	3	2 1/64	1 1/64	1 1/64	1 3/64	3/64
30.	193.2	77 1/2	19 3/8	8 3/16	6 3/8	6 3/64	5 1/4	4 1/16	3 1/64	3 3/64	2 3/64	1 1/64	1 1/64	1 3/64	3/64

Note.—For reduction of inches and fractions to decimals of a foot, see Table 10, page 223. No particular refinement is necessary in curving rails: ordinates to the nearest ¼" are close enough, usually.

The quarter ordinates are practically three-fourths the middle ordinate.

46.—Middle Ordinates in Feet for Curving Rails.

(Degrees of curve and radii are for 100-ft. chords.)

For Ordinates in Inches, see Table 45.

Note.—Ordinates are practically proportional to the square of length of Thus, for 60-ft. rail, mult. value for 30, by 4; for 45-ft. rail, mult. ie for 30, by 2¼; for 8-ft. rail, divide value for 16, by 4, etc.

[Middle Ordinates, in Feet.]

Radius. Feet.	Length of Rail, or Arc, in Feet.													
	100	50	33	30	28	26	24	22	20	18	16	14	12	10
11459.2	.109	.027	.012	.010	.008	.006	.005	.004	.004	.003	.002	.002	.001	.001
5729.7	.217	.054	.024	.020	.016	.013	.011	.009	.008	006	.005	.004	.003	.002
3819.8	.327	.082	.036	.029	.026	.021	.018	.016	.013	.010	.008	.006	.004	.003
2864.9	.436	.109	.047	.038	.034	.029	.025	.021	.017	.014	.011	.008	.006	.004
2292.0	.546	.136	.059	.049	.043	.037	.031	.027	.022	.018	.014	.010	.007	.005
1910.1	.655	.164	.071	.058	.051	.044	.037	.031	.026	.022	.017	.012	.009	.006
1637.3	.763	.191	.083	.070	.061	.052	.043	.037	.031	.025	.020	.015	.011	.008
1432.7	.872	.218	.095	.079	.069	.060	.050	.042	.035	.029	.023	.018	.013	.009
1273.6	.982	.245	.107	.088	.077	.067	.056	047	.039	.032	.026	.020	.015	.010
1146.3	1.090	.273	.119	.099	.086	.074	.063	.053	.044	.035	.029	.022	.016	.011
1042.1	1.199	.300	.131	108	.094	.082	.070	.059	.048	.039	.032	.024	.018	.012
955.4	1.308	.327	.142	.117	.102	.088	.076	.064	.052	.042	.034	.026	.019	.013
881.9	1.417	.354	.154	.128	112	.097	.082	.069	.057	.046	.037	.028	.021	.014
819.0	1.525	.381	.166	.137	.120	.104	.088	.074	.061	.049	.039	.030	.022	.015
764.5	1.634	.408	.178	.146	.127	.111	.094	.079	.065	.053	.042	.032	.024	.016
716.8	1.743	.436	.190	.158	.137	.119	.100	.085	.070	.056	.045	.034	.025	.017
674.7	1.851	.463	.202	166	.145	.126	.106	.090	.074	.060	.048	.036	.027	.018
637.3	1.961	.490	.214	.175	.153	.133	.112	.095	.078	.063	.050	.038	.029	.019
603.8	2.069	517	.225	.187	.163	.141	.119	.101	.083	.067	.054	.042	.031	.021
573.7	3.178	.545	.237	.196	.171	.148	.125	.106	.087	.071	.057	.045	.032	.022
521.7	2.394	.598	.261	.216	.188	.163	.139	.117	.096	.078	.063	.049	.036	.024
478.3	2.611	.653	.284	236	.206	.179	.151	.128	.105	.085	.069	.053	.039	.026
441.7	2.828	.707	.308	.254	.222	.192	.163	.138	.113	.092	.075	.057	.042	.028
410.2	3.043	.761	.332	.275	.239	.207	.175	.148	.122	.099	.080	061	.045	.030
383.1	3.258	.816	.356	.295	.257	.223	.188	.159	.131	.106	.085	.065	.049	.033
359.3	3.474	.870	.379	.313	.273	.236	.200	.170	.139	.113	.091	.070	.053	.035
338.3	3.688	.924	.403	.333	.290	.252	.213	180	.148	.120	.096	.074	.055	.037
319.6	3.903	.978	.426	.351	.306	.265	225	.190	.156	.127	.102	.078	.058	.039
302.9	4.117	1.031	.450	.371	.324	.280	.238	.201	.165	.134	.108	.082	.061	.044
287.9	4.330	1.085	.473	392	.341	.296	.250	.212	.174	.141	.114	.087	.066	.041
274.4	4.543	1.138	.496	.410	.357	.309	.262	222	.182	.148	.120	.091	.069	.046
262.0	4.756	1.192	.520	.430	.375	.325	.275	.233	.191	.155	.126	.096	.072	.048
250.8	4.968	1.245	.543	.450	.390	.338	.287	.243	.199	.162	.131	.100	.075	.050
240.5	5.178	1.298	566	469	.408	.354	.299	.253	.208	.169	.137	.104	.078	.052
231.0	5.390	1.351	.589	486	.424	.367	.311	.263	.216	.176	.142	.108	.081	.054
222.2	5.600	1.404	.612	.506	.441	.382	.323	.274	.225	.183	.148	.112	.084	.056
214.2	5.810	1.457	.635	.524	.457	.396	.335	.284	.233	.190	.153	.116	.087	.058
206.7	6.019	1.510	.658	545	.475	.411	348	.294	.242	.197	.158	.120	.090	.060
199.7	6.226	1.563	.681	.564	491	.424	.361	.303	.250	.203	.163	.124	.093	.062
193.2	6.434	1.615	.704	.582	.507	.438	.373	.313	.259	.210	.168	.128	.096	.064

Note.—For reduction of decimals of a foot to inches and fractions, see le 10, page 223. No particular refinement is necessary in curving rails: nates to the nearest ¼" are close enough, usually.
The quarter ordinates are practically three-fourths the middle ordinate.

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47.—Chord Lengths of Curved Rails.

(Degrees of curve and radii are for 100-ft. chords.)

[Chord Lengths, in Feet.]

Degree of Curve.	Radius. Feet.	Length of Rail, or Arc, in Feet.								
		100	50	33	30	26	22	18	14	10
2	2864.9	99.995	49.999	33.000	30.000	26.000	22.000	18.000	14.000	10.000
4	1432.7	99.980	49.997	32.999	30.000	26.000	22.000	18.000	14.000	10.000
6	955.4	99.954	49.994	32.998	29.999	26.000	22.000	18.000	14.000	10.000
8	716.8	99.919	49.990	32.997	29.998	25.999	22.000	18.000	14.000	10.000
10	573.7	99.871	49.983	32.996	29.997	25.998	21.999	18.000	14.000	10.000
12	478.3	99.818	49.977	32.994	29.996	25.997	21.998	17.999	14.000	10.000
14	410.3	99.753	49.969	32.991	29.993	25.996	21.997	17.999	13.999	10.000
16	359.3	99.677	49.960	32.988	29.991	25.994	21.997	17.999	13.999	10.000
18	319.6	99.593	49.949	32.985	29.989	25.993	21.996	17.998	13.999	10.000
20	287.9	99.498	49.937	32.982	29.986	25.991	21.995	17.997	13.999	9.999
22	262.0	99.394	49.924	32.978	29.984	25.989	21.994	17.996	13.998	9.999
24	240.5	99.281	49.910	32.974	29.981	25.987	21.993	17.996	13.998	9.999
26	222.3	99.157	49.894	32.970	29.978	25.985	21.992	17.995	13.998	9.999
28	206.7	99.027	49.878	32.965	29.974	25.983	21.990	17.994	13.997	9.999
30	193.2	98.887	49.860	32.960	29.970	25.980	21.988	17.993	13.997	9.999

Note.—For reduction of decimals of a foot to inches and fractions, see Table 10, page 223.

To Find the Degree of Curvature of Laid Track.

(See Formulas 1 and 2, and Notation, page 1063.)

On maintenance work it is often necessary to find the degree of curvature of laid track, on a curve which has been more or less shifted and deranged by the trackmen; and then to run in a regular or a spiral curve which will best fit the existing track: so as to require the least possible shifting of the latter.

From the approximate formulas (1) and (2), page 1063, it will be seen that there are direct ratios between the degree of curve (D), the curved length of rail (L), and the middle ordinate (M). If D is in degrees, and L and M in feet, we have, by transposition,

$$D=\frac{45000\,M}{L^2} \text{ (approximate)} \quad \ldots\ldots (3)$$

For a 100-ft. length of rail, $L=100$, hence,

$$D=4.5\,M \text{ (approximate)} \quad \ldots\ldots (4)$$

$$=4.58\,M \text{ (nearly exact)} \quad \ldots\ldots (5)$$

For a 30-ft. length of rail, $L=30$, hence,

$$D=50\,M \text{ (approximate)} \quad \ldots\ldots (6)$$

$$=51\,M \text{ (nearly exact)} \quad \ldots\ldots (7)$$

For a 21′ 3″ length of rail,

D = the middle ordinate in hundredths of a foot. ..... (8)

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.—Middle Ordinates for 20- and 30-Ft. Chords and 30-Ft. Arcs. Also, Chord Lengths of 30-Ft. Arcs.

e.—Radii are given in feet and tenths; other dimensions in feet (′) and inches (″).

Rad. Ft.	Middle Ordinates for 20′ Ch.	Middle Ordinates for 30′ Ch.	Middle Ordinates for 30′ Arc	Ch. of 30′ Arc.	Rad Ft.	Middle Ordinates for 20′ Ch.	Middle Ordinates for 30′ Ch.	Middle Ordinates for 30′ Arc	Ch. of 30′ Arc.
Infinite	0′0″	0′ 0″	0′ 0″	30′ 0″	90	0′ 6 11/16″	1′ 3 1/16″	1′ 2 13/16″	29′ 10 5/16″
5729.7	0′ 1/8″	0′ 1/4″		30′ 0″	89	0′ 6 3/4″	1′ 3 1/4″	1′ 3 1/8″	29′ 10 7/16″
2864.9	0′ 3/16″	0′ 3/8″		30′ 0″	88	0′ 6 13/16″	1′ 3 7/16″	1′ 3 5/16″	29′ 10 1/2″
1910.1	0′ 1/4″	0′ 1/2″		30′ 0″	87	0′ 6 7/8″	1′ 3 5/8″	1′ 3 1/2″	29′ 10 9/16″
1432.7	0′ 5/16″	0′ 5/8″		30′ 0″	86	0′ 7″	1′ 3 13/16″	1′ 3 5/8″	29′ 10 5/8″
1146.3	0′ 3/8″	0′ 1 3/16″		30′ 0″	85	0′ 7 1/16″	1′ 4″	1′ 3 13/16″	29′ 10 3/4″
955.4	0′ 7/16″	0′ 1 3/8″		30′ 0″	84	0′ 7 3/16″	1′ 4 3/16″	1′ 4″	29′ 10 13/16″
819.0	0′ 1/2″	0′ 1 9/16″		30′ 0″	83	0′ 7 1/4″	1′ 4 3/8″	1′ 4 1/8″	29′ 10 15/16″
716.8	0′ 13/16″	0′ 1 7/8″		30′ 0″	82	0′ 7 5/16″	1′ 4 9/16″	1′ 4 1/4″	29′ 10″
637.3	0′ 15/16″	0′ 2 1/16″		30′ 0″	81	0′ 7 7/16″	1′ 4 11/16″	1′ 4 3/8″	29′ 9 15/16″
573.7	0′ 1 1/16″	0′ 2 5/16″		29′ 11 13/16″	80	0′ 7 1/2″	1′ 5″	1′ 4 13/16″	29′ 9 7/8″
521.7	0′ 1 1/8″	0′ 2 9/16″		29′ 11 13/16″	79	0′ 7 5/8″	1′ 5 1/4″	1′ 5 1/16″	29′ 9 3/4″
478.3	0′ 1 3/16″	0′ 2 11/16″		29′ 11 13/16″	78	0′ 7 3/4″	1′ 5 7/16″	1′ 5 1/4″	29′ 9 11/16″
441.7	0′ 1 1/4″	0′ 3 1/16″		29′ 11 13/16″	77	0′ 7 13/16″	1′ 5 11/16″	1′ 5 9/16″	29′ 9 1/2″
410.3	0′ 1 5/16″	0′ 3 1/4″		29′ 11 13/16″	76	0′ 7 7/8″	1′ 5 15/16″	1′ 5 13/16″	29′ 9 5/16″
383.1	0′ 1 3/8″	0′ 3 1/2″		29′ 11 7/8″	75	0′ 8 1/16″	1′ 6 3/16″	1′ 5 15/16″	29′ 9 3/8″
359.3	0′ 1 7/16″	0′ 3 3/4″		29′ 11 7/8″	74	0′ 8 1/8″	1′ 6 7/16″	1′ 6 3/16″	29′ 9 1/16″
338.3	0′ 1 1/2″	0′ 4″		29′ 11 7/8″	73	0′ 8 1/4″	1′ 6 11/16″	1′ 6 7/16″	29′ 9″
319.6	0′ 1 9/16″	0′ 4 1/4″		29′ 11 7/8″	72	0′ 8 3/8″	1′ 6 15/16″	1′ 6 11/16″	29′ 8 13/16″
302.9	0′ 2″	1′ 4 1/2″		29′ 11 7/8″	71	0′ 8 7/16″	1′ 7 3/16″	1′ 6 15/16″	29′ 8 5/16″
287.9	0′ 2 1/16″	0′ 4 11/16″		29′ 11 13/16″	70	0′ 8 1/2″	1′ 7 7/16″	1′ 7 3/16″	29′ 8 1/4″
274.4	0′ 2 1/8″	0′ 4 15/16″		29′ 11 13/16″	69	0′ 8 5/8″	1′ 7 11/16″	1′ 7 7/16″	29′ 8 1/16″
262.0	0′ 2 3/16″	0′ 5 1/4″		29′ 11 13/16″	68	0′ 8 3/4″	1′ 8 1/4″	1′ 7 11/16″	29′ 7 15/16″
250.8	0′ 2 1/4″	0′ 5 9/16″		29′ 11 13/16″	67	0′ 9″	1′ 8 9/16″	1′ 8 1/4″	29′ 9″
240.5	0′ 2 5/16″	0′ 5 13/16″		29′ 11 3/4″	66	0′ 9 1/16″	1′ 8 13/16″	1′ 8 9/16″	29′ 8 7/8″
231.0	0′ 2 3/8″	0′ 6 1/16″		29′ 11 3/4″	65	0′ 9 3/16″	1′ 9″	1′ 8 13/16″	29′ 8 13/16″
225	0′ 2 13/16″	0′ 6″		29′ 11 3/4″	64	0′ 9 7/16″	1′ 9 3/8″	1′ 9″	29′ 8 11/16″
220	0′ 2 7/8″	0′ 6 1/8″		29′ 11 11/16″	63	0′ 9 9/16″	1′ 9 7/16″	1′ 9 5/16″	29′ 8 5/8″
215	0′ 2 15/16″	0′ 6 1/4″		29′ 11 11/16″	62	0′ 9 3/4″	1′ 10 1/8″	1′ 9 11/16″	29′ 8 1/2″
210	0′ 3″	0′ 6 5/16″		29′ 11 11/16″	61	0′ 9 7/8″	1′ 10 1/2″	1′ 10″	29′ 8 3/8″
205	0′ 3 1/8″	0′ 6 9/16″		29′ 11 11/16″	60	0′ 10 1/16″	1′ 10 7/8″	1′ 10 3/8″	29′ 8 1/4″
200	0′ 3 3/16″	0′ 6 3/4″		29′ 11 11/16″	59	0′ 10 1/4″	1′ 11 1/4″	1′ 10 3/4″	29′ 8 1/8″
195	0′ 3 5/16″	0′ 6 15/16″		29′ 11 11/16″	58	0′ 10 5/16″	1′ 11 11/16″	1′ 11 1/8″	29′ 8″
190	0′ 3 3/8″	0′ 7 1/8″		29′ 11 5/8″	57	0′ 10 5/8″	2′ 0 3/8″	1′ 11 15/16″	29′ 7 7/8″
185	0′ 3 1/2″	0′ 7 5/16″		29′ 11 5/8″	56	0′ 10 13/16″	2′ 0 15/16″	1′ 11 13/16″	29′ 7 11/16″
180	0′ 3 9/16″	0′ 7 1/2″		29′ 11 5/8″	55	0′ 11″	2′ 1″	2′ 0 3/8″	29′ 7 5/8″
175	0′ 3 5/8″	0′ 7 11/16″		29′ 11 5/8″	54	0′ 11 3/16″	2′ 1 5/8″	2′ 0 13/16″	29′ 7 3/8″
170	0′ 3 3/4″	0′ 7 15/16″		29′ 11 5/8″	53	0′ 11 5/16″	2′ 2″	2′ 1 5/8″	29′ 7 3/8″
165	0′ 3 7/8″	0′ 8 3/8″		29′ 11 3/8″	52	0′ 11 1/2″	2′ 2 1/8″	2′ 1 13/16″	29′ 7″
160	0′ 3 15/16″	0′ 8 7/16″		29′ 11 3/8″	51	0′ 11 7/8″	2′ 3 1/16″	2′ 2 1/8″	29′ 6 13/16″
155	0′ 3 7/8″	0′ 8 15/16″		29′ 11 3/8″	50	1′ 0 1/8″	2′ 3 5/16″	2′ 2 13/16″	29′ 6 5/8″
150	0′ 4″	0′ 9″	0′ 9″	29′ 11 1/4″	49	1′ 0 1/4″	2′ 4 1/8″	2′ 3 5/16″	29′ 6 3/8″
145	0′ 4 1/8″	0′ 9 3/8″	0′ 9 5/16″	29′ 11 1/4″	48	1′ 0 3/8″	2′ 4 7/8″	2′ 4 1/8″	29′ 6 1/8″
140	0′ 4 3/16″	0′ 9 11/16″	0′ 9 9/16″	29′ 11 1/4″	47	1′ 0 13/16″	2′ 5 5/8″	2′ 4 7/8″	29′ 5 13/16″
135	0′ 4 5/16″	0′ 10 1/16″	0′ 10″	29′ 11 1/4″	46	1′ 1 1/16″	2′ 6 3/8″	2′ 5 5/8″	29′ 5 5/8″
130	0′ 4 1/2″	0′ 10 5/16″	0′ 10 3/8″	29′ 11 3/16″	45	1′ 1 3/8″	2′ 6 15/16″	2′ 5 13/16″	29′ 5 3/8″
125	0′ 4 13/16″	0′ 10 15/16″	0′ 10 13/16″	29′ 11 3/16″	44	1′ 1 13/16″	2′ 7 7/8″	2′ 6 13/16″	29′ 5 1/8″
120	0′ 5″	0′ 11 3/16″	0′ 11 1/8″	29′ 11 1/16″	43	1′ 2 1/8″	2′ 8 1/16″	2′ 7 1/8″	29′ 4 11/16″
115	0′ 5 1/8″	0′ 11 11/16″	0′ 11 1/8″	29′ 11″	42	1′ 2 1/4″	2′ 9 1/8″	2′ 7 13/16″	29′ 4 3/8″
110	0′ 5 5/16″	1′ 0 5/16″	1′ 0 1/8″	29′ 10 7/8″	41	1′ 2 1/2″	2′ 10 1/8″	2′ 8 9/16″	29′ 4″
105	0′ 5 1/2″	0′ 0 13/16″	1′ 0 1/8″	29′ 10 3/4″	40	1′ 3″	2′ 11″	2′ 9 1/8″	29′ 3 3/8″
100	0′ 6″	1′ 1 5/16″	1′ 1″	29′ 10 5/8″	39	1′ 3 1/8″	3′ 0″	2′ 10 1/8″	29′ 3 1/8″
99	0′ 6 1/16″	1′ 1 3/8″	1′ 1 1/8″	29′ 10 5/8″	38	1′ 4 1/16″	3′ 1″	2′ 11 1/8″	29′ 2 1/8″
98	0′ 6 1/8″	1′ 1 7/8″	1′ 1 1/4″	29′ 10 5/8″	37	1′ 4 1/2″	3′ 2 1/8″	3′ 0″	29′ 2″
97	0′ 6 3/16″	1′ 2″	1′ 1 1/4″	29′ 10 1/16″	36	1′ 5″	3′ 3 1/8″	3′ 0 13/16″	29′ 1 13/16″
96	0′ 6 1/4″	1′ 2 1/8″	1′ 2 1/16″	29′ 10 1/16″	35	1′ 5 1/8″	3′ 4 1/2″	3′ 2″	29′ 1 1/16″
95	0′ 6 5/16″	1′ 2 1/4″	1′ 2 1/16″	29′ 10 1/2″	34	1′ 6 1/8″	3′ 5 7/8″	3′ 3 1/8″	29′ 0 7/8″
94	0′ 6 3/8″	1′ 2 5/16″	1′ 2 5/16″	29′ 10 1/2″	33	1′ 6 3/4″	3′ 7 1/8″	3′ 4 3/8″	28′ 11 1/2″
93	0′ 6 1/2″	1′ 2 3/8″	1′ 2 3/8″	29′ 10 7/16″	32	1′ 7″	3′ 8 13/16″	3′ 5 7/8″	28′ 10 1/2″
92	0′ 6 5/8″	1′ 2 5/8″	1′ 2 5/8″	29′ 10 7/16″	31	1′ 7 1/8″	3′ 10 1/8″	3′ 6 11/16″	28′ 10 1/8″

Ordinates for 30-ft. arcs, down to 150-ft. radius, are practically the same as for corresponding 30-ft. chords.

Track Spikes, or railroad spikes as they are commonly called, are square in cross-section, with a flared hook head and wedge point. Spikes of the same size vary considerably in weight, but the following table shows an average, and assumes 10560 spikes per mile of single track.

49.—TRACK SPIKES.

Size Meas. Under Head. Inches.	Average Wt. of 100 Spikes. Pounds.	Average Number per Keg of 200 Pounds	Quantity of Spikes per Mile of Single Track. Ties 2 feet c. to c. 4 spikes per tie.		Rails Used. Weight per Yard. Pounds.
			Pounds.	Kegs.	
5½ x ⅝	66.67	300	7040	35.20	75 to 100
5½ x 9/16	53.33	375	5630	28.16	45 " 75
5 x 9/16	50.00	400	5280	26.40	40 " 56
5 x ½	44.44	450	4690	23.47	35 " 40
4½ x ½	37.74	530	3990	19.93	30 " 35
4 x ½	33.33	600	3520	17.60	25 " 35
4½ x 7/16	29.41	680	3110	15.53	20 " 30
4 x 7/16	27.78	720	2930	14.67	20 " 30
3½ x 7/16	22.22	900	2350	11.73	16 " 25
4 x ⅜	20.00	1000	2110	10.56	16 " 25
3½ x ⅜	16.81	1190	1770	8.87	16 " 20
3 x ⅜	16.13	1240	1700	8.52	16 " 20
2½ x ⅜	14.93	1340	1580	7.88	8 " 16

Note.—There are 4 spikes per tie. The above table assumes the ties to be spaced 24" centers. For other spacing allow as follows:

For main line, 16 ties to 30-ft. rail, *increase* values in cols. 4 and 5 by 1/16.
" " " 17 " 33-ft. " " " " " " 1/32.
" " " 18 " 33-ft. " " " " " " 1/16.
" sidings, 14 " 30-ft. " *decrease* " " " " 1/16.

Rail Joints are almost universally "square." That is, the rails are cut off square as they come from the rolls. Such a joint is shown at *S* in Fig. 29.

Fig. 29.—Square, Miter and Lap Joints.

In the same Fig. the "miter" joint is shown at *M*, and the "lap" joint at *L*. The miter- or beveled joint is made by sawing off the rails on a bevel of, say, 45° to 60° or 65°. The advantage claimed was that *longer rails* could be used with *reduced effect* of "hammering" on rail ends by trains, as the open joints caused by temperature contraction of rail would be robbed of their ill effects if beveled. Steel rails will vary about .0008 of their length under a change of temperature of 120° F.; hence 30-ft. rails if laid with closed joints at +100°F. will have between ¼ and 5/16 in. joints when the temperature falls to 20° below zero. The miter joint has never come into general use. It was formerly used quite extensively on the Lehigh Valley R. R., and may now be seen there, but it has generally been replaced with the square rail joint. The lap- or scarf joint, *L*, claims the advantage of the miter joint above mentioned; in addition, it is free from danger of any projecting point catching a wheel flange, especially if the track is curved. If the lap is long the joint is stiffened vertically, a decided advantage. It is seldom if ever employed in the United States.

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"Shims" are pieces of wood or iron inserted at rail joints when track is laid, in order to give the proper spacing of joint. The thickness of the shim depends upon the temperature. A good rule for thickness of shim is the following:

$$S = .00128\,L\,(T - t)$$

in which S = thickness of shim, *in 16ths of an inch*;
L = length of rail, in feet;
T = "hottest" temp. to be expected in that locality, in degs. F.
t = prevailing temp. when track is laid, in degs. F.

The Suspended Joint (i. e., where the joint comes between two supporting ties) has practically superseded the "supported" joint which rests directly on a single tie with the usual spacing. But better still, by far, is the use of three ties closely spaced at each joint with angle- or splice bars long enough (say 3½ to 4 ft.) to get direct support from the two outside ties. Such an arrangement may properly be called a 3-tie "supported" joint.

Alternate, Staggered or Broken Joints are usually preferred on main line to the "opposite" or "even" joints. The advantages of "broken" joints are: (1) The intensity of shock due to passing trains is reduced about 50% although the number of shocks is doubled; (2) the track can be kept in better surface and line on tangents; (3) on curves, one line of rails stiffens the other at the joints and aids in preserving uniform curvature even if the rails were not curved properly prior to laying. The advantage of "opposite" joints is purely one of cost in tracklaying, and hence for second-class yards, or for slow train service generally, they may do.

Cross Ties.—Ordinary track ties are usually 8 or 8½ ft. long; bridge ties for single track, 12 ft.; and switch ties, up to 15 ft. or more in length.

Tables 50 and 51, respectively, give the number of cubic feet and feet board measure in ties of various dimensions.

Tables 52 and 53 are bills of switch ties for No. 6 and No. 8 frog, respectively. To find the bill of material for any other frog: Draw the switch to scale, and scale off the lengths of ties. (For turnouts and switches, see page 1075.)

50.—Cubic Feet in Wooden Ties of Various Dimensions.

[Cubic Feet.]

Length. Ins.	Sectional Dimension, in Inches.									
	6 x 8	6 x 9	6 x 10	7 x 8	7 x 9	7 x 10	8 x 10	8 x 12	9 x 10	9 x 12
1	.028	.031	.035	.032	.036	.042	.046	.056	.052	.063
2	.056	.063	.069	.065	.073	.081	.093	.111	.104	.125
3	.083	.094	.104	.097	.109	.122	.139	.167	.156	.188
6	.167	.188	.208	.194	.219	.243	.278	.333	.313	.375
9	.250	.281	.313	.292	.328	.365	.417	.500	.469	.563
	.333	.375	.417	.389	.438	.486	.556	.667	.625	.750
	.667	.750	.833	.778	.875	.972	1.111	1.333	1.250	1.500
	1.000	1.125	1.250	1.167	1.313	1.458	1.667	2.000	1.875	2.250
	2.000	2.250	2.500	2.333	2.625	2.917	3.333	4.000	3.750	4.500
	2.667	3.000	3.333	3.111	3.500	3.889	4.444	5.333	5.000	6.000
6	2.833	3.188	3.542	3.306	3.719	4.132	4.722	5.667	5.313	6.375
	3.000	3.375	3.750	3.500	3.938	4.375	5.000	6.000	5.625	6.750
	3.333	3.750	4.167	3.889	4.375	4.861	5.556	6.667	6.250	7.500
	3.667	4.125	4.583	4.278	4.813	5.347	6.111	7.333	6.875	8.250
	4.000	4.500	5.000	4.667	5.250	5.833	6.667	8.000	7.500	9.000
	4.333	4.875	5.417	5.056	5.688	6.319	7.222	8.667	8.125	9.750
	4.667	5.250	5.833	5.444	6.125	6.806	7.778	9.333	8.750	10.500
	5.000	5.625	6.250	5.833	6.563	7.292	8.333	10.000	9.375	11.250
	5.333	6.000	6.667	6.222	7.000	7.778	8.889	10.667	10.000	12.000
	5.667	6.375	7.083	6.611	7.438	8.264	9.444	11.333	10.625	12.750
	6.000	6.750	7.500	7.000	7.875	8.750	10.000	12.000	11.250	13.500

Ex.—A tie 7"x9"x10'6" will contain (4.375+.219=)4.594 cu. ft.; and such ties at 48 lbs. per cu. ft. will weigh 220,500 lbs.

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51.—Feet Board Measure (B. M.) in Wooden Ties of Various Dimensions.

(See also Table 4, Section 20.)

[Ft. B. M.]

Lgth. Ft.	Lgth. Ins.	Sectional Dimension, in Inches. 6 x 8	6 x 9	6 x 10	7 x 8	7 x 9	7 x 10	8 x 10	8 x 12	9 x 10	9 x 12
	1	.33	.38	.42	.39	.44	.49	.56	.67	.63	.75
	2	.67	.75	.83	.78	.88	.97	1.11	1.33	1.25	1.50
	3	1.00	1.13	1.25	1.17	1.31	1.46	1.67	2.00	1.88	2.25
	6	2.00	2.25	2.50	2.33	2.63	2.92	3.33	4.00	3.75	4.50
	9	3.00	3.38	3.75	3.50	3.94	4.38	5.00	6.00	5.63	6.75
1		4.00	4.50	5.00	4.67	5.25	5.83	6.67	8.00	7.50	9.00
2		8.00	9.00	10.00	9.33	10.50	11.67	13.33	16.00	15.00	18.00
3		12.00	13.50	15.00	14.00	15.75	17.50	20.00	24.00	22.50	27.00
6		24.00	27.00	30.00	28.00	31.50	35.00	40.00	48.00	45.00	54.00
8		32.00	36.00	40.00	37.33	42.00	46.67	53.33	64.00	60.00	72.00
8	6	34.00	38.25	42.50	39.67	44.63	49.58	56.67	68.00	63.75	76.50
9		36.00	40.50	45.00	42.00	47.25	52.50	60.00	72.00	67.50	81.00
10		40.00	45.00	50.00	46.67	52.50	58.33	66.67	80.00	75.00	90.00
11		44.00	49.50	55.00	51.33	57.75	64.17	73 33	88.00	82.50	99.00
12		48.00	54.00	60.00	56.00	63.00	70.00	80.00	96.00	90.00	108.00
13		52.00	58.50	65.00	60.67	68.25	75.83	86.67	104.00	97.50	117.00
14		56.00	63.00	70.00	65.33	73.50	81.67	93.33	112.00	105.00	126.00
15		60.00	67.50	75.00	70.00	78.75	87.50	100.00	120.00	112.50	135.00
16		64.00	72.00	80.00	74.67	84.00	93.33	106.67	128.00	120.00	144.00
17		68.00	76.50	85.00	79.33	89.25	99.17	113.33	136.00	127.50	153.00
18		72.00	81.00	90.00	84.00	94.50	105.00	120.00	144.00	135.00	162.00

Ex.—A tie 8"x10"x12'3" will contain (80.000+1.667=) 81.667 ft. B. M.; and 1000 such ties will contain 81667 ft. B. M.

52.—Bill of Switch Ties for No. 6 Frog.

(8"x10" is a good size; lengths are given to nearest 3".)

Num. Req.	Length	Num. Req.	Length	Num. Req.	Length	Num. Req.	Length	Num. Req.	Length	Num. Req.	Length
2	8' 3"	1	9' 3"	1	10' 3"	1	11' 3"	1	12' 6"	1	13' 6"
6	8' 6"	2	9' 6"	1	10' 6"	1	11' 6"	1	12' 9"	3	14' 0"
2	8' 9"	2	9' 9"	1	10' 9"	1	11' 9"	1	13' 0"	1	14' 3"
3	9' 0"	1	10' 0"	1	11' 0"	1	12' 0"	1	13' 3"		

Total lin. ft. in above bill, using exact lengths = 380 lin. ft.
Total lin. ft. in above bill, using 1-ft. lengths = 393 lin. ft.

53.—Bill of Switch Ties for No. 8 Frog.

(8"x10" is a good size; lengths are given to nearest 3".)

Num. Req.	Lgth.	Num. Req.	Lgth.	Num. Req.	Lgth.	Num. Req.	Lgth.	Num. Req.	Lgth.	Num. Req.	Lgth.	Num. Req.	Lgth.
2	8' 0"	3	9' 0"	2	10' 0"	1	11' 0"	1	12' 0"	1	13' 0"	1	14' 3"
3	8' 3"	2	9' 3"	1	10' 3"	1	11' 3"	2	12' 3"	1	13' 3"	2	14' 6"
3	8' 6"	2	9' 6"	2	10' 6"	2	11' 6"	1	12' 6"	2	13' 6"		
3	8' 9"	1	9' 9"	1	10' 9"	1	11' 9"	1	12' 9"	3	14' 0"		

Total lin. ft. in above bill, using exact lengths = 484 lin. ft.
Total lin. ft. in above bill, using 1-ft. lengths = 505 lin. ft.

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"Winter cut" ties are the best, and hewed ties are better than sawed. Planing improves sawed ties in shedding water and preventing decay. Creosoting lengthens the life of ties, and the creosote oil in ties so preserved has a beneficial effect on spikes in preventing rust.

Wooden ties are the only kind used to any great extent in the United States at the present time. The old stone tie has been abandoned. When the wooden tie is supplanted it will probably be by the steel tie, the steel-concrete tie, or the steel-paper tie; and then only gradually and on roads of the first class.

Oak ties are the best, and white oak is the best variety as it holds the spikes better. The other varieties frequently used are bur oak, post oak, chestnut oak and red oak. Together they comprise at least one-half of the total number of ties in use. The average life of the best oak tie is about eight years, varying inversely with the humidity of the atmosphere, and depending upon the amount of traffic, position in the track (whether on curves or tangent), etc. Chestnut ties will last about as long as oak but do not hold a spike as well. Cedar ties are the longest lived, but they do not wear well under heavy traffic unless tie-plates are used, in which case they will last about twice as long as oak. Red cedar is not now readily obtainable, white cedar being much more plentiful. Next to oak, the various varieties of pine furnish most of the ties now being used: Yellow pine, in the South and East; loblolly pine in the Southwest; California mountain pine, Oregon pine (Douglas fir or Washington pine), on the Pacific Coast; Michigan pine in the Northwest; etc. In the New England States, hemlock and spruce are much used, although inferior to the Southern yellow pine. Black walnut is used in the middle West, and redwood in California. The life of redwood is about up to that of cedar if tie-plates are used, the wood being very soft.

Tie Plates are used on wooden ties, particularly those of soft wood, to prevent the rails from cutting into them; otherwise the life of such ties would be measured by the amount of heavy traffic passing over them, rather than by their resistance to the action of the weather, etc. The life of soft ties, then, may be increased: (1) by creosoting, to resist the action of the weather, as already explained; (2) by the use of tie plates, to resist erosion.

The usual construction of tie plates consists simply of flat, plain or bowed plates say 6"x 8", with bottom lugs or flanges say ½" deep, which are driven into the tie. The plate is provided with 2, 3 or 4 square holes gaged to the rail flange, for spiking. In the earlier patterns there were lugs above the plate on one or both sides of the rail flange, but in more recent designs these are omitted. Figs. 30 to 33 show types of the Servis, Walhauper, Fox and Diamond tie plates.

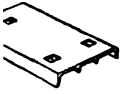


Fig. 30.

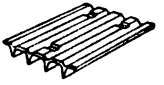


Fig. 31.

Fig. 32.

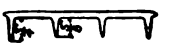


Fig. 33.

The thickness of metal may vary from $\frac{1}{16}$" to ⅜". The cost of tie plates is very nominal, varying from 5 to 15 cents each. They are used generally on certain points instead of universally, as at rail joints, where there is much hammering; on heavy grades where sand is much used; on expensive ties and switch ties; on curves; and in places generally where the renewal of ties would be unusually expensive, as at stations and crossings, and in tunnels.

Rail Braces are designed to resist the outward lateral thrust of the rail of passing trains; hence they are placed on the outside of rail, pressed against it, and spiked solidly to the ties. The most primitive form of brace, and one which may be seen in almost any yard, at switches, is

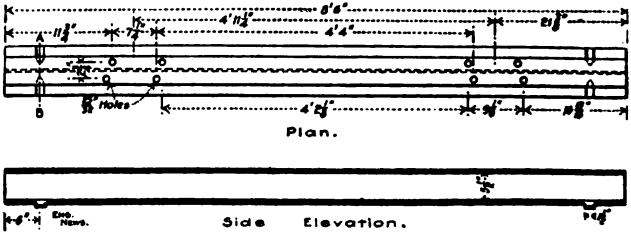
Stone ties were tried in the early days on the Boston & Worcester R. R. (part of the B. & A. R. R. system) but were abandoned on account of cost and also because they did not furnish a sufficiently elastic base.

a bent fish-plate with one end pressing against the web of the rail, just under the head, and the other end spiked to the tie. Many patterns are made of cast iron or cast steel. Fig. 34 shows the Alkins brace of forged steel; and Fig. 35 shows the Edwards brace, being a combination rail brace and tie plate. Rail braces are used on curves and at switches, and in general where double spiking is insufficient.

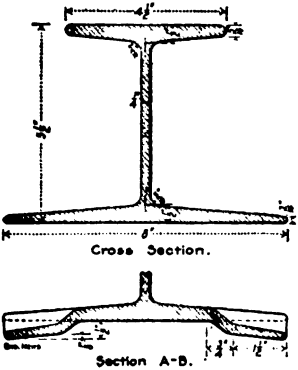
Fig. 34.

Fig. 35.

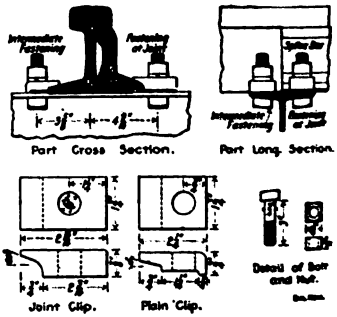
Steel Ties are much used in Europe and somewhat in the United States, but are at present avoided here on account of the expense as compared with wooden ties. Those in Europe are generally of the trough type. Figs. 36, 37 and 38 show a special I-beam section, after the original design of Mr. C. Buhrer, roadmaster of the L. S. & M. S. Ry., and manufactured by the Carnegie Steel Co., for the Bessemer & Lake Erie Ry. The tie is 8′ 6″ long and weighs 19.36 lbs. per ft. Section A - B of Figs. 37 shows the depression lug 6″ from ends of tie to prevent



Figs. 36.



Figs. 37.



Figs. 38.

lateral movement in the ballast. Figs. 38 show details of rail fastenings, including those at joint where angle splice bars are used.*

Concrete-Steel Ties may be said to be in the experimental stage. Those interested in these types may find designs of the †Campbell tie and the ‡Percival tie in *Eng. News*, Oct. 5, 1905, page 349.

* For full description, see *Eng. News*, Aug. 24, 1905, page 202.
† Mr. R. B. Campbell, Gen. Man., the Elgin, Joliet & Eastern Ry.
‡ Mr. H. E. Percival, Galveston, Tex.

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Ballast may be broken stone, gravel, cinders, sand or dirt. The first ıamed is by far the best and should be of about the size that will pass hrough a 2½ in. ring. Most roads using a great amount of broken stone have heir own quarries and rock-crushing plants instead of breaking the stone ›y hand. Portable stone crushers are also used. The advantages of broken tone and gravel over the finer materials are: (1) good drainage, (2) firm ›earing and solid ballast packing for the ties; (3) absence of frost, (4) lack f retention of moisture to rot the ties, (5) freedom from dust, unpleasant ɔ passengers and injurious to the wearing parts of the rolling stock.

For estimating the amount of ballast per mile of track, a sketch should e drawn of the ballast cross-section desired (see Fig. 23, page 1059) and om this deduct the cubic contents of the ties from Table 50, page 1069.

hus from the figure, we have,

ı. ft., gross, of ballast per lin. ft. of single track..... = 9.5
educt for tie (6″x 8″ – 8′ spaced 24″ centers) by table = 1.333

ı. ft., net, of ballast per lin. ft..... = 8.167

ı. yds. net of ballast per mile $= 8.167 \times \frac{5280}{27} = 1597.$

ence 1600 cu. yds. of ballast per mile of single track is the very least ıt can be assumed. This is for a depth of 12 ins. and a top width of 8 ft. nerally the ballast is much deeper and wider, say 18″ below top of tie, d 10 to 12 ft. wide.

Gage of Track and Wheels.—The *minimum* "Standard" Gage* of track the United States and Canada (also in England and most European ıntries) is 4′ 8½″. On roads where this minimum gage is used for straight ck, the gage is widened for track on curves, say about $\frac{3}{64}$″ per each ree of curvature, as per the following table.

54.—Increase in Gage for Various Degrees of Curvature.
(Based on about $\frac{3}{64}$″ per degree of curve.)

. of ve.	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°
. in e.	$\frac{1}{16}$″	$\frac{3}{32}$″	$\frac{1}{8}$″	$\frac{3}{16}$″	$\frac{1}{4}$″	$\frac{1}{4}$″	$\frac{5}{16}$″	$\frac{3}{8}$″	$\frac{7}{16}$″	$\frac{1}{2}$″	$\frac{1}{2}$″	$\frac{9}{16}$″	$\frac{5}{8}$″	$\frac{5}{8}$″

)n some roads in the U. S., mostly in the South, the "Standard" Gage 9″. This is true also of some main-line freight tracks on the P. R. R. m. In such cases the gage is seldom widened for ordinary curves. ɔ is also another "Standard" Gage employed by a very few roads, 4′ 8¾″, which may be considered to be a compromise between the two above mentioned.

ig. 39 shows the Master Car Builders' (M. C. B.) standard wheel (and) gage, which has become universally standard. Note that the side

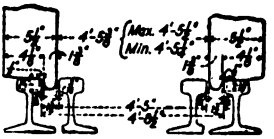

Fig. 39.

the wheels is ⅝″ for a 4′ 8½″ gage, and that for a 4′ 9″ gage it would : to ⅞″.

age of track is the distance between "inside heads," or "gage sides,"

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The following data regarding gages will be found useful in connection with the calculation of turnouts, switches, crossovers, crossings, etc.

55.—VARIOUS GAGES AND HALF GAGES OF TRACK, IN FEET AND METERS WITH LOGARITHMIC VALUES.

Gage g. Ft.	Ins.	Gage g. Ft.	Log	Gage g. Meters.	Log.	$\frac{g}{2}$. Ft.	Ins.	$\frac{g}{2}$. Ft.	Log.	$\frac{g}{2}$. Meters.	Log.
2	0	2.	.3010300	.6096	9.7850458	1	0	1.	.0000000	.3048	9.4840158
2	3	2.25	.3521825	.6858	9.8361983	1	1½	1.125	.0511525	.3429	9.5351683
2	6	2.5	.3979400	.7620	9.8819558	1	3	1.25	.0969100	.3810	9.5809258
2	9	2.75	.4393327	.8382	9.9233485	1	4½	1.375	.1383027	.4191	9.6223185
3	0	3.	.4771213	.9144	9.9611371	1	6	1.5	.1760913	.4572	9.6601071
3	3	3.25	.5118834	.9906	9.9958992	1	7½	1.625	.2108534	.4953	9.6948692
3	6	3.5	.5440680	1.0668	0.0280838	1	9	1.75	.2430380	.5334	9.7270538
3	9	3.75	.5740313	1.1430	0.0580471	1	10½	1.875	.2730013	.5715	9.7570171
4	0	4.	.6020600	1.2192	0.0860758	2	0	2.	.3010300	.6096	9.7850458
4	3	4.25	.6283889	1.2954	0.1124047	2	1½	2.125	.3273589	.6477	9.8113747
4	6	4.5	.6532125	1.3716	0.1372283	2	3	2.25	.3521825	.6858	9.8361983
4	8½	4.7083	.6728672	1.4351	0.1568830	2	4½	2.3542	.3718372	.7176	9.8558530
4	8¾	4.7292	.6747847	1.4415	0.1588005	2	4¾	2.3646	.3737547	.7207	9.8577705
4	9	4.75	.6766936	1.4478	0.1607094	2	4½	2.375	.3756636	.7239	9.8596794
5	0	5.	.6989700	1.5240	0.1829858	2	6	2.5	.3979400	.7620	9.8819558
5	3	5.25	.7201593	1.6002	0.2041751	2	7½	2.625	.4191293	.8001	9.9031451
5	6	5.5	.7403627	1.6764	0.2243785	2	9	2.75	.4393327	.8382	9.9233485
5	9	5.75	.7596678	1.7526	0.2436836	2	10½	2.875	.4586378	.8763	9.9426536
6	0	6.	.7781513	1.8288	0.2621671	3	0	3.	.4771213	.9144	9.9611371

Various Track Gages are used in different countries as follows: In the United States, Canada, England and most European countries, the standard gage is 4′ 8½″ = 1.435 meters. This is true of Austria, Switzerland, Germany, France, Hungary, Italy, Sweden, and Balkan. In Russia, the standard gage is 1.524 meters with the exception of the Warschau-Wien and Warschau-Bromberg which have a standard of 1.435 meters. In Spain, the standard is 1.676 meters, and this gage also prevails mostly in the East Indies, Argentine and Chile. It corresponds to the old English gage of 5′ 6″. The gage of the Great Western R. R., in England, was changed from 7 ft. (in connection with 4′ 8½″) wholly to standard gage in 1890. In Ireland, the standard gage is 5′ 3″. Narrow gages are used in many countries. In Norway, Cape Colonies, South Australia, Japan and Java, a gage of 3′ 6″ is used largely. In Brazil, Algiers, Greece and Corsica, 1 meter is common. Gages of 1 meter, 0.75*m.* and 0.60*m.* are used to some extent in Germany for narrow gage extensions, etc. Many roads in Switzerland are of 1m. gage. The gage of the Festiniog-Bahn, in Wales, is only 0.591 meter.

The "Best" Standard Gage, for universal inter-traffic, has gradually sifted down to that of about 4′ 8½″. Wide gages of 5 to 6 ft., and narrow gages of 3 to 3½ ft., have been changed to the above standard almost universally throughout North America, and the chances are that the same standardization will prevail ultimately in South America also. Pertinent to this question, the writer has lately been of the opinion that, with the enormous locomotives now being built, and with the limited head room, a wider gage, say 5′ 3″ to 5′ 6″ would have been a better standard to adopt than the present one. From the standpoint of the locomotive manufacturer, the following letter, under date of September 8, 1906, from the Baldwin Locomotive Works, Burnham, Williams & Co., of Philadelphia, in reply to a query from the writer, is interesting:

Your favor of September 4th was duly received.

The capacity of any railway, as an instrument for transportation of goods, is proportionate to its gage. In fact, the practicable weight of locomotives and the practicable capacity of cars, increase directly in proportion to increased width of gage. In designing some of the heaviest locomotives now required for freight traffic, it would be a great comfort if the gage were wider than 4′ 8½″. As a broad proposition, however, we do not think that even the congestion of traffic upon the principal trunk lines has yet reached a point limited by the size of locomotives, nor that wider gage than 4′ 8½″ has generally become necessary.

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Turnouts and Switches.—A turnout (for switching trains from one track to another) consists essentially of a switch, a frog, and the connecting "lead" rails.

Switches may be classed under three main heads, namely, the stub-switch, the split-switch, and the Wharton switch. Where there is a double turnout from the same track, the diverting switch is called a three-throw switch. Fig. 40 shows a *double* "slip" switch, very useful in switching-yards

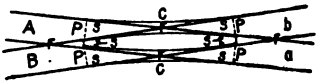

Fig. 40.

or economizing room, and particularly adapted to sharp crossings, with nterchangeable traffic. For instance, a train may pass from either track n one side of the crossing to either track on the other, by operating the witches *s*. In the Fig., the switches are set for "crossing traffic" on tracks *a* and *Bb*. It is to be noted that the outside curved rails *c* are continuous ıroughout. The letters *F* denote position of frogs; and *P*, the points of vitches. A *single* slip switch has but one curved rail *c* with its correspond-g gage rail, instead of two as shown in Fig. 40.

Frogs are devices for allowing the flanges of wheels to pass unobstructed ong one rail crossing another rail. There are three distinct classes, namely, iff or rigid frogs, spring or spring rail frogs, and movable-point frogs.

Stiff frogs may be made up of rail sections, or of solid steel castings. g. 41 illustrates the shape of a stiff rail frog, the heads of rails only being

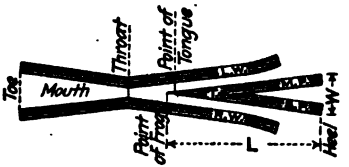

Fig. 41.

m. *LW* and *RW* are left- and right-wings respectively; and *MP* and are main- and side-points. Note that the "point of frog" is at the section of the outside lines of *MP* and *SP* and a little beyond the blunt of tongue. All frogs are designated by numbers. Thus, if we let *L* l the length from point of frog to heel, and *W* equal the width at heel,

$$\text{'frog number'}\ n = \frac{L}{W}.^{*} \qquad (1)$$

range from numbers 4 to 24, but the usual limits are 6 and 12, while 9 are perhaps the most common. We will show further on how the *umber* determines practically the radius or degree of the turnout and it will be seen also what bearing the *kind of switch* and *length of* ave on the problem, for any particular *gage of track*.

lid cast *manganese* steel frogs will greatly outwear the *ordinary* rail ay about 9 to 1, while they cost about 4 or 5 times as much. They e had "one-sided" or "double sided" depending on whether the heavy is mainly on one track, or is "balanced." There is a saving in cost of 25% in favor of the former.

ing Rail Frogs are usually placed on the main line because there is no shock in riding over them, as there is over the rigid frogs. Fig. P. & R. Railway standard, showing the right wing *RW* pressed against the tongue by the spring, thus forming practically a con-

s the ratio $L \div W$ is constant for any given frog, its number may determined by measuring the distance L', from the theoretical point to a point where $W' =$ say 4"; then $n = L' \div 4$.

56.—Properties of Frog Angles ϕ — With Logarithmic Values.

Note.—Logarithmic values are exact for the given frog numbers; the frog angles are to the nearest second. (Angles to the nearest minute are close enough, usually.)

For values of cosec ϕ and cot ϕ, see Table 64.

Part I.—Properties of Frog Angles, ϕ.

Frog No. n.	Frog Angle ϕ.	Nat Sin ϕ	Log Sin ϕ	Nat Cos ϕ	Log Cos ϕ	Nat Tan ϕ	Log Tan ϕ	Nat Sec ϕ	Log Sec ϕ.
4	14°15′00″	.246154	9.3912067	.969231	9.9864273	.253968	9.4047794	1.03175	Log sec ϕ = colog cos ϕ = 1 (or 11) − log cos ϕ. Thus, log sec 14° 15′ 00″ = 0.013572h.
4½	12 40 49	.219512	9.3414586	.975610	9.9892761	.225000	9.3521826	1.02800	
5	11 25 16	.198020	9.2967086	.980198	9.9913138	.202020	9.3053948	1.02020	
5½	10 23 20	.180328	9.2560628	.983607	9.9928314	.183333	9.2632415	1.01667	
6	9 31 38	.165517	9.2188433	.986207	9.9939680	.167832	9.2248753	1.01399	
6½	8 47 51	.152941	9.1845244	.988235	9.9948604	.154762	9.1896440	1.01190	
7	8 10 16	.142132	9.1526919	.989848	9.9955684	.143590	9.1571235	1.01026	
7½	7 37 41	.132743	9.1230128	.991150	9.9961396	.133929	9.1268732	1.00893	
8	7 09 10	.124514	9.0952169	.992218	9.9966071	.125490	9.0986099	1.00784	
8½	6 43 59	.117241	9.0690810	.993103	9.9969944	.118056	9.0720865	1.00694	
9	6 21 35	.110769	9.0444192	.993846	9.9973192	.111455	9.0471000	1.00619	
9½	6 01 32	.104972	9.0210750	.994475	9.9975939	.105556	9.0234811	1.00556	
10	5 43 29	.099751	8.9989157	.995012	9.9978285	.100251	9.0010871	1.00501	
10½	5 27 09	.095023	8.9778270	.995475	9.9980304	.095455	8.9797966	1.00455	
11	5 12 18	.090722	8.9577109	.995876	9.9982054	.091097	8.9595855	1.00414	
11½	4 58 45	.086792	8.9384821	.996226	9.9983580	.087121	8.9401239	1.00379	
12	4 46 19	.083189	8.9200655	.996534	9.9984920	.083478	8.9215724	1.00348	
12½	4 34 52	.079872	8.9023957	.996805	9.9986103	.080128	8.9037854	1.00321	
13	4 24 19	.076809	8.8854147	.997046	9.9987151	.077037	8.8866996	1.00296	
14	4 05 27	.071338	8.8533184	.997452	9.9988921	.071520	8.8544263	1.00253	
15	3 49 06	.066593	8.8234264	.997780	9.9990349	.066741	8.8243915	1.00222	
16	3 34 47	.062439	8.7954561	.998049	9.9991517	.062561	8.7963043	1.00196	
17	3 22 10	.058773	8.7691755	.998271	9.9992486	.058874	8.7699369	1.00173	
18	3 10 56	.055513	8.7443926	.998458	9.9993298	.055598	8.7450628	1.00154	
19	3 00 54	.052595	8.7209457	.998616	9.9993985	.052668	8.7215473	1.00139	
20	2 51 51	.049969	8.6986987	.998751	9.9994571	.050031	8.6992415	1.00125	
21	2 43 40	.047592	8.6775346	.998867	9.9995076	.047646	8.6780270	1.00113	
22	2 36 14	.045431	8.6573530	.998967	9.9995513	.045478	8.6578017	1.00103	
23	2 29 27	.043458	8.6380670	.999055	9.9995895	.043499	8.6384776	1.00095	
24	2 23 13	.041649	8.6196003	.999132	9.9996230	.041685	8.6199773	1.00087	

$$\text{Vers } \phi = \sin \phi . \tan \frac{\phi}{2} = 2 \sin^2 \frac{\phi}{2}.$$

tinuous main line rail. When a train takes the *SP* rail, the *RW* is crowded open by the wheel flange, but springs back after the train has passed Spring-rail frogs are "rights" and "lefts" and hence care must be used in ordering them. Other types are the Vaughan, Wood, Ajax, Eureka, etc. *Double* spring rail frogs are seldom used.

Fig. 42.—Spring Rail Frog.

Movable-Point Frogs are shown in Fig. 43, as manufactured by Wm. Wharton, Jr., & Co., Philadelphia. Note that this type might well be used in Fig. 40 at the central frogs *F*. The cut is self-explanatory. The movable points also may be of manganese steel if required.

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ND PROPERTIES OF ½ FROG ANGLES $\frac{\phi}{2}$.

WITH LOGARITHMIC VALUES.

Note.—Logarithmic values are exact for the given frog numbers; the angles are to the nearest second. (Angles to the nearest minute close enough, usually.)

Part II.—Properties of ½ Frog Angles, $\frac{\phi}{2}$.

$\frac{\phi}{2}$.	Nat Sin $\frac{\phi}{2}$	Log Sin $\frac{\phi}{2}$	Nat Cos $\frac{\phi}{2}$	Log Cos $\frac{\phi}{2}$	Nat Tan $\frac{\phi}{2}$	Log Tan $\frac{\phi}{2}$	Nat Sec $\frac{\phi}{2}$	Log Sec $\frac{\phi}{2}$.
7°07′ 30″	.124035	9.0935433	.992278	9.9966333	.125	9.0969100	1.00778	Log sec $\frac{\phi}{2}$ = colog cos $\frac{\phi}{2}$ = 1 (or 11) − log cos $\frac{\phi}{2}$. Thus, log sec 7° 07′ 30″ = 0.0033667.
6 20 25	.110432	9.0430931	.993884	9.9973356	.111111	9.0457575	1.00615	
5 42 38	.099504	8.9978393	.995037	9.9978393	.1	9.0000000	1.00499	
5 11 40	.090536	8.9568201	.995893	9.9982128	.090909	8.9586073	1.00412	
4 45 49	.083045	8.9193160	.996546	9.9984973	.083333	8.9208188	1.00347	
4 23 55	.076696	8.8847755	.997055	9.9987189	.076923	8.8860566	1.00295	
4 05 08	.071247	8.8527670	.997459	9.9988950	.071429	8.8538720	1.00255	
3 48 51	.066519	8.8229457	.997785	9.9990370	.066667	8.8239087	1.00222	
3 34 35	.062378	8.7950334	.998053	9.9991534	.0625	8.7958800	1.00195	
3 21 59	.058722	8.7688011	.998274	9.9992499	.058824	8.7695511	1.00173	
3 10 47	.055470	8.7440584	.998460	9.9993308	.055556	8.7447275	1.00154	
3 00 46	.052559	8.7206457	.998618	9.9993993	.052632	8.7212464	1.00138	
2 51 45	.049938	8.6984278	.998752	9.9994578	.05	8.6989700	1.00125	
2 43 35	.047565	8.6772889	.998868	9.9995081	.047619	8.6777807	1.00113	
2 36 09	.045408	8.6571292	.998969	9.9995518	.045455	8.6575773	1.00103	
2 29 22	.043437	8.6378622	.999056	9.9995899	.043478	8.6382722	1.00094	
2 23 09	.041631	8.6194122	.999133	9.9996234	.041667	8.6197888	1.00087	
2 17 26	.039968	8.6017129	.999201	9.9996528	.04	8.6020600	1.00080	
2 12 09	.038433	8.5847057	.999261	9.9996791	.038462	8.5850267	1.00074	
2 02 43	.035692	8.5525652	.999363	9.9997232	.035714	8.5528420	1.00064	
1 54 33	.033315	8.5226375	.999445	9.9997589	.033333	8.5228787	1.00056	
1 47 24	.031235	8.4946380	.999512	9.9997880	.03125	8.4948500	1.00049	
1 41 05	.029399	8.4683333	.999568	9.9998122	.029412	8.4685211	1.00043	
1 35 28	.027767	8.4435301	.999614	9.9998325	.027778	8.4436975	1.00039	
1 30 27	.026307	8.4200661	.999654	9.9998497	.026316	8.4202164	1.00035	
1 25 56	.024992	8.3978043	.999688	9.9998643	.025	8.3979400	1.00031	
1 21 50	.023803	8.3766276	.999717	9.9998769	.023810	8.3767507	1.00028	
1 18 07	.022721	8.3564351	.999742	9.9998878	.022727	8.3565473	1.00026	
1 14 43	.021734	8.3371396	.999764	9.9998974	.021739	8.3372422	1.00024	
1 11 37	.020829	8.3186645	.999783	9.9999058	.020833	8.3187588	1.00022	

Vers $\frac{\phi}{2} = \sin\frac{\phi}{2}.\tan\frac{\phi}{4} = 2\sin^2\frac{\phi}{4}$.

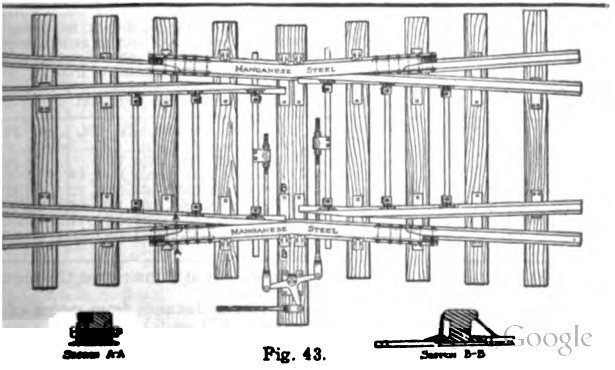

Fig. 43.

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Crossing Frogs are usully rigid, and made up of rail sections with perhaps cast steel frog junctions. There is a form of movable frog consisting of short pieces of rail on miniature turntables that can be turned in any desired direction, thus making a continuous rail of either track.

Stub Switches.—The stub switch is the cheapest and most primitive form. Its use is now confined to second-class yards and spur connections with sidings, having disappeared entirely from main yards. It should never be used for main line connection. Fig. 44 illustrates the essential features. The head blocks *H B* are at the junction of the switch rails *S* with the main lead (rail) *M L* and turnout lead (rail) *TL*, which "lead" to the toe of frog; also with the continuous main line rail *M* and turnout rail *T*. The "throw" of the switch is clearly the distance between the *gage* sides of rails *M* and *TL* or *ML* and *T*, at the head blocks, and is usually 5, 5½ (or 6) ins.* The switch rails are tied together with a front rod and three or more back rods. The length of switch rails is governed by the frog number or by the degree of turnout curve. The following table was calculated by the author in 1887 while he was Resident Engineer of a western road and was used by the foremen in laying out all switches, including the terminal yards at Toledo. The sharpest frog was No. 9, and the table was used for both stub and split switches; but for frog numbers higher than 9 the table does not apply strictly to the latter. The virtue of the table lies in the offset distances between the gage sides of *ML* and *TL* rails, Fig. 44, as given in the last seven columns but one. These offsets are measured at points 10-, 20-, 30-ft., etc. from theoretical point of frog (Fig. 41). By this means the position of the *TL* rail is fixed quickly, and from it the *T* rail is gaged. The efficiency of the work per gang was increased from two switches in three days to one switch per day, in broken stone ballast.

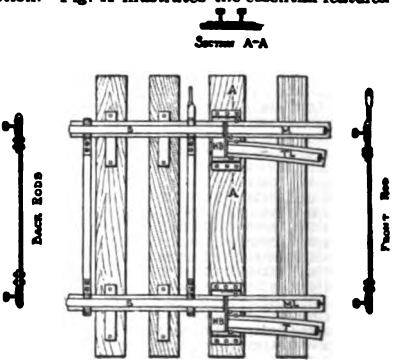

Fig. 44.

57.—Table for Laying out Switches. Gage 4′ 8½″. Throw 5″.

Frog No.	Frog Angle.	Turnout Curve.		Theoretical Lead.	Stub Switch. †		Split Switch. †		Offset distances in ft. from gage side of main lead to gage side of turnout lead at following distances from point of frog.							Frog No.
		Degree.	Radius.		Rail	Lead	Rail	Lead	10ft.	20ft.	30ft.	40ft.	50ft.	60ft.	70ft.	
n	° ′	° ′	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	Ft.	n
6	9 32	16 58	339.0	56.5	16.9	39.6	15	54.6	1.51	2.73	3.65					6
7	8 10	12 26	461.4	65.9	19.6	46.3	15	61.3	1.31	2.30	3.13	3.74				7
8	7 09	9 31	602.7	75.3	22.5	52.9	15	67.9	1.16	2.16	2.99	3.65	4.14			8
9	6 22	7 31	762.8	84.8	25.3	59.5	15	74.5	1.04	1.96	2.74	3.39	3.92			9
10	5 43	6 05	941.7	94.2	28.0	66.1	15	81.1	0.95	1.78	2.52	3.15	3.66	4.08		10
11	5 12	5 02	1139.4	103.6	30.8	72.7	15	87.7	0.86	1.64	2.32	2.93	3.44	3.87	4.21	11
12	4 46	4 14	1356.0	113.0	33.6	79.4	15	94.4	0.80	1.52	2.17	2.76	3.26	3.68	4.03	12

* Distance between rail heads should be about 3 ins.; and the throw is equal to this distance plus width of railhead.

† "Rail" means switch rail; "lead" means distance from point of frog to toe of stub switch (*H. B.*), or to point of split switch.

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Main Frog No.	Main Frog Angle.	Turn-out Radius.	Degree of Turn-out Curve.	Middle Ordinate, Outer Rail.		Quarter Ordinate, Outer Rail.		Theoretical Lead.	For 5" Throw.			For 5½" Throw.			Middle or Crotch Frog.			Theoretical Lead.	H.B. to P.F.		Dist. Back from Main Frog on C. L. Main Track.
				Straight Main Track.	Curved Main Track. Ch'ng per Deg. of cv.	Straight Main Track.	Curved Main Track. Ch'ng per Deg. of cv.		Central Angle.	Theoretical Switch Length.	H. B. to P. F.	Central Angle.	Theoretical Switch Length.	H. B. to P. F.	Frog No.	Frog Angle.			For 5" Throw.	For 5½" Throw.	
n	ϕ	R		m		q		l	$\varkappa$	s	$l-s$	$\varkappa$	s	$l-s$	n'	ϕ'		l'	$l'-s$	$l'-s$	*
	° ′ ″	Ft.	° ′ ″	Ins.	Ins.	Ins.	Ins.	Ft.	° ′ ″	Ft.	Ft.	° ′ ″	Ft.	Ft.		° ′ ″	Ft.	Ft.	Ft.	Ft.	Ft.
4	14 15 00	150.667	38 45 48	14 3/16	3/8	10 5/8	3/32	37.667	4 13 45	11.29	26.38	4 26 09	11.84	25.83	2.82	20 07 36	26.74	15.45	14.90	10.93
4½	12 40 49	190.688	30 24 09	14 3/16	3/8	10 5/8	3/32	42.375	3 45 55	12.68	29.70	3 56 56	13.29	29.08	3.17	17 54 52	30.05	17.37	16.76	12.33
5	11 25 16	235.417	24 31 29	14 3/16	3/8	10 5/8	3/32	47.083	3 23 33	14.07	33.01	3 33 29	14.76	32.33	3.53	16 06 19	33.38	19.31	18.62	13.71
5½	10 23 20	284.854	20 13 08	14 3/16	3/8	10 5/8	3/32	51.792	3 05 12	15.47	36.33	3 14 14	16.22	35.57	3.88	14 40 55	36.70	21.23	20.48	15.09
6	9 31 38	339.000	16 57 48	14 3/16	3/8	10 5/8	5/8	56.500	2 49 52	16.86	39.64	2 58 10	17.68	38.82	4.23	13 27 57	40.02	23.16	22.34	16.48
6½	8 47 51	397.854	14 26 22	14 3/16	3/8	10 5/8	3/32	61.208	2 36 53	18.26	42.95	2 44 32	19.06	42.15	4.59	12 26 07	43.35	25.09	24.29	17.86
7	8 10 16	461.417	12 26 30	14 3/16	1⅛	10 5/8	3/32	65.917	2 25 44	19.65	46.26	2 32 51	20.61	45.30	4.94	11 33 04	46.67	27.02	26.06	19.23
7½	7 37 41	529.688	10 49 59	14 1/8	1 1/16	10 11/32	3/32	70.625	2 16 04	21.05	49.57	2 22 42	22.08	48.55	5.30	10 47 02	50.00	28.95	27.92	20.63
8	7 09 10	602.667	9 31 05	14 1/8	1 1/16	10 11/32	1⅛	75.333	2 07 36	22.46	52.88	2 13 50	23.55	51.78	5.65	10 06 44	53.32	30.86	29.77	22.01
8½	6 43 59	680.354	8 25 45	14 1/8	1 1/16	10 11/32	1¼	80.042	2 00 07	23.85	56.19	2 05 58	25.01	55.03	6.01	9 31 08	56.65	32.80	31.64	23.40
9	6 21 35	762.750	7 31 02	14 1/8	1⅛	10 11/32	1 3/16	84.750	1 53 28	25.25	59.50	1 59 00	26.48	58.27	6.36	8 59 30	59.97	34.72	33.49	24.78
9½	6 01 32	849.854	6 44 45	14 1/8	2 3/16	10 11/32	1 7/16	89.458	1 47 30	26.65	62.81	1 52 44	27.94	61.52	6.71	8 31 10	63.30	36.65	35.36	26.16
10	5 43 29	941.667	6 05 15	14 1/8	2 3/16	10 11/32	1¾	94.167	1 42 08	28.04	66.13	1 47 08	29.42	64.75	7.07	8 05 40	66.63	38.59	37.21	27.54
10½	5 27 09	1038.19	5 31 16	14 1/8	2 3/16	10 11/32	1 13/16	98.875	1 37 18	29.45	69.43	1 42 02	30.88	68.00	7.42	7 42 36	69.96	40.51	39.08	28.92
11	5 12 18	1139.42	5 01 48	14 1/8	2 3/16	10 11/32	2 3/32	103.583	1 32 52	30.84	72.74	1 37 24	32.34	71.24	7.77	7 21 36	73.28	42.44	40.94	30.30

REMARKS.—"Shortened" leads are generally used in practice as giving better line to the turnout curve, where high frog numbers are used. Where shortened leads are adopted, they are fixed to suit rail lengths without cutting.

* This last column gives the distance between points of frogs, measured along center line of main track.

58.—THEORETICAL (OR STUB) SWITCHES. GAGE, 4' 8½".—Concluded.
(See Figs. 44, 45 and 46, pages 1078, 1082 and 1083.)

n Main Frog No.	Main Frog Angle.	Turn-out Radius.	Degree of Turn-out Curve.	Middle Ordinate, Outer Rail. Straight Main Track.	Middle Ordinate, Outer Rail. Curved Main Track. Ch'ng per Deg. of cv.	Quarter Ordinate, Outer Rail. Straight Main Track.	Quarter Ordinate, Outer Rail. Curved Main Track. Ch'ng per Deg. of cv.	Theoretical Lead.	For 5" Throw. Central Angle.	For 5" Throw. Theoretical Switch Length.	For 5" Throw. H. B. to P. F.	For 5½" Throw. Central Angle.	For 5½" Throw. Theoretical Switch Length.	For 5½" Throw. H. B. to P. F.	Middle or Crotch Frog. Frog No.	Middle or Crotch Frog. Frog Angle.	Middle or Crotch Frog. Theoretical Lead.	Middle or Crotch Frog. H. B. to P. F. For 5" Throw.	Middle or Crotch Frog. H. B. to P. F. For 5½" Throw.	Dist. Back from Main Frog on C. L. Main Track.
	ϕ	R		m		q		l	κ	s	$l-s$	κ	s	$l-s$	n'	ϕ'	l'	$l'-s$	$l'-s$	$\bullet$
	° ′ ″	Ft.	° ′ ″	Ins.	Ins.	Ins.	Ins.	Ft.	° ′ ″	Ft.	Ft.	° ′ ″	Ft.	Ft.		° ′ ″	Ft.	Ft.	Ft.	Ft.
11½	4 58 45	1245.35	4 36 07	14½	3 1/16	10 11/16	2 3/16	108.292	1 28 51	32.24	76.05	1 33 11	33.82	74.48	8.13	7 02 24	76.61	44.37	42.79	31.69
12	4 46 19	1356.00	4 13 35	14 1/8	3 1/16	10 11/16	2 1/32	113.000	1 25 09	33.64	79.36	1 29 18	35.28	77.72	8.48	6 44 52	79.94	46.30	44.66	33.06
12½	4 34 52	1471.35	3 53 41	14 1/8	3 1/32	10 11/16	2 1/16	117.708	1 21 45	35.04	82.67	1 25 44	36.75	80.96	8.84	6 28 40	83.26	48.22	46.51	34.45
13	4 24 19	1591.42	3 36 03	14 1/8	3 1/16	10 11/16	2 1/16	122.417	1 18 37	36.44	85.97	1 22 27	38.22	84.20	9.19	6 13 44	86.59	50.15	48.37	35.83
14	4 05 27	1845.67	3 06 17	14 1/8	4 1/16	10 11/16	3 1/16	131.833	1 13 00	39.24	92.59	1 16 34	41.16	90.68	9.90	5 47 04	93.25	54.01	52.09	38.59
15	3 49 06	2118.75	2 42 16	14 1/8	5 1/16	10 11/16	3 11/16	141.250	1 08 08	42.04	99.21	1 11 28	44.09	97.16	10.60	5 23 58	99.91	57.87	55.82	41.34
16	3 34 47	2410.67	2 22 37	14 1/8	5 11/16	10 11/16	4 1/16	150.667	1 03 52	44.82	105.84	1 07 00	47.03	103.64	11.31	5 03 44	106.56	61.73	59.53	44.11
17	3 22 10	2721.42	2 06 20	14 1/8	6 11/16	10 11/16	4 9/16	160.083	1 00 08	47.64	112.44	1 03 04	49.97	110.12	12.01	4 45 52	113.22	65.56	63.25	46.86
18	3 10 56	3051.00	1 52 41	14 1/8	7 1/16	10 11/16	5 1/16	169.500	0 56 48	50.45	119.05	0 59 34	52.90	116.60	12.73	4 30 00	119.87	69.42	66.97	49.63
19	3 00 54	3399.42	1 41 08	14 1/8	8 1/16	10 11/16	5 11/16	178.917	0 53 48	53.24	125.68	0 56 20	55.84	123.08	13.43	4 15 48	126.53	73.29	70.69	52.39
20	2 51 51	3766.67	1 31 16	14 1/8	9 1/16	10 11/16	6 5/16	188.333	0 51 07	56.04	132.29	0 53 37	58.78	129.55	14.14	4 03 00	133.18	77.14	74.40	55.15
21	2 43 40	4152.75	1 22 47	14 1/8	10 1/16	10 11/16	7 1/16	197.750	0 48 41	58.84	138.91	0 51 04	61.72	136.03	14.84	3 51 28	139.86	81.02	78.14	57.89
22	2 36 14	4557.67	1 15 26	14 1/8	11 1/16	10 11/16	8 1/16	207.167	0 46 28	61.63	145.53	0 48 44	64.64	142.53	15.55	3 40 56	146.50	84.87	81.86	60.67
23	2 29 27	4981.42	1 09 01	14 1/8	12 1/16	10 11/16	9 1/16	216.583	0 44 27	64.44	152.14	0 46 38	67.60	148.98	16.26	3 31 20	153.16	88.72	85.50	63.42
24	2 23 13	5424.00	1 03 23	14 1/8	13 1/16	10 11/16	10 1/16	226.000	0 42 36	67.24	158.76	0 44 41	70.53	155.47	16.97	3 22 32	159.82	92.58	89.29	66.18

REMARKS. "Shortened" leads are generally used in practice as giving better line to the turnout curve, when high-flow [illegible] used. Where shortened leads are adopted they are [illegible] will [illegible] without a curving.

With Logarithms of R.

(See Fig. 45, page 1082.)

Part I.—R and g in Feet. **Part II.—Corresponding R and g in Meters.**

Frog No. n.	g=3′ 6″		g=4′ 8½″		g=5′ 3″		g=5′ 6″		g=1.067 m.		g=1.435 m.		g=1.6 m.		g=1.676 m.	
	Rad. R.	Log. R.	Rad. R.	Log. R.	Rad. R.	Log. R.	Rad. R.	Log. R.	Rad. R.	Log. R.	Rad. R.	Log. R.	Rad. R.	Log. R.	Rad. R.	Log. R.
4	112.000	2.0492180	150.667	2.180172	168.000	2.2253093	176.000	2.2455127	34.138	1.5332338	45.923	1.6620330	51.207	1.7093251	53.645	1.7295285
4½	141.750	2.1515231	190 688	2.2803222	212.625	2.3276143	222.750	2.3478177	43.205	1.6355388	58.122	1.7643380	64.808	1.8116301	67.894	1.8318335
5	175.000	2.2430380	235.417	2.3718372	262.500	2.4191293	275.000	2.4393327	53.340	1.7270538	71.755	1.8558530	80.010	1.9031451	83.820	1.9233485
5½	211.750	2.3258234	284.854	2.4546226	317.625	2.5019147	332.750	2.5221181	64.542	1.8098392	86.824	1.9386384	96.812	1.9859305	101.422	2.0061339
6	252.000	2.4014005	339.000	2.5301997	378.000	2.5774918	396.000	2.5976952	76.810	1.8854163	103.327	2.0142155	115.215	2.0615076	120.701	2.0817110
6½	295.750	2.4709248	397.854	2.5997239	443.625	2.6470160	464.750	2.6672194	90.145	1.9549405	121.266	2.0837397	135.217	2.1310318	141.656	2.1512352
7	343.000	2.5352941	461.417	2.6640933	514.500	2.7113854	539.000	2.7315888	104.547	2.0193099	140.640	2.1481091	156.820	2.1954012	164.288	2.2156046
7½	393.750	2.5952206	529.688	2.7240197	590.625	2.7713118	618.750	2.7915152	120.015	2.0792363	161.449	2.2080355	180.023	2.2553276	188.595	2.2755310
8	448.000	2.6512780	602.667	2.7800772	672.000	2.8273693	704.000	2.8475727	136.551	2.1352938	183.693	2.2640930	204.826	2.3113851	214.580	2.3315885
8½	505.750	2.7039359	680.354	2.8327350	758.625	2.8800271	794.750	2.9002305	154.153	2.1879516	207.372	2.3167508	231.229	2.3640429	242.240	2.3842463
9	567.000	2.7535831	762.750	2.8823822	850.500	2.9296743	891.000	2.9498777	172.822	2.2375988	232.487	2.3663980	259.233	2.4136901	271.577	2.4338935
9½	631.750	2.8005453	849.854	2.9293444	947.625	2.9766365	992.750	2.9968399	192.588	2.2845610	259.036	2.4133602	288.837	2.4606523	302.591	2.4808557
10	700.000	2.8450980	941.667	2.9738972	1050.00	3.0211893	1100.00	3.0413927	213.360	2.3291138	287.021	2.4579130	320.041	2.5052051	335.281	2.5254085
10½	771.750	2.8874766	1038.19	3.0162758	1157.63	3.0635679	1212.75	3.0837713	235.230	2.3714924	316.440	2.5002916	352.845	2.5475837	369.647	2.5677871
11	847.000	2.9278834	1139.42	3.0566826	1270.50	3.1039747	1331.00	3.1241781	258.166	2.4118992	347.295	2.5406984	387.249	2.5879905	405.690	2.6081939
11½	925.750	2.9664937	1245.35	3.0952929	1388.63	3.1425850	1454.75	3.1627884	282.169	2.4505095	379.585	2.5793087	423.254	2.6266008	443.409	2.6468042
12	1008.00	3.0034605	1356.00	3.1322597	1512.00	3.1795518	1584.00	3.1997552	307.239	2.4874763	413.310	2.6162755	460.859	2.6635676	482.804	2.6837710
12½	1093.75	3.0389180	1471.35	3.1677172	1640.63	3.2150093	1718.75	3.2352127	333.376	2.5229338	448.470	2.6517330	500.063	2.6990251	523.876	2.7192285
13	1183.00	3.0729847	1591.42	3.2017839	1774.50	3.2490760	1859.00	3.2692794	360.579	2.5570005	485.065	2.6857997	540.869	2.7330918	566.624	2.7532952
14	1372.00	3.1373541	1845.67	3.2661533	2058.00	3.3134454	2156.00	3.3336488	418.186	2.6213699	562.560	2.7501691	627.280	2.7974612	657.150	2.8176646
15	1575.00	3.1972806	2118.75	3.3260797	2362.50	3.3733718	2475.00	3.3935752	480.061	2.6812963	645.796	2.8100955	720.091	2.8573876	754.381	2.8775910
16	1792.00	3.2533380	2410.67	3.3821372	2688.00	3.4294293	2816.00	3.4496327	546.203	2.7373538	734.773	2.8661530	819.304	2.9134451	858.319	2.9336485
17	2023.00	3.3059959	2721.42	3.4347950	3034.50	3.4820871	3179.00	3.5022905	616.611	2.7900116	829.489	2.9188108	924.917	2.9661029	968.961	2.9863063
18	2268.00	3.3556431	3051.00	3.4844422	3402.00	3.5317343	3564.00	3.5519377	691.288	2.8396588	929.947	2.9684580	1036.93	3.0157501	1086.31	3.0359535
19	2527.00	3.4026052	3399.42	3.5314044	3790.50	3.5786965	3971.00	3.5988999	770.231	2.8866210	1036.14	3.0154202	1155.35	3.0627123	1210.36	3.0829157
20	2800.00	3.4471580	3766.67	3.5759572	4200.00	3.6232493	4400.00	3.6434527	853.442	2.9311738	1148.08	3.0599730	1280.16	3.1072651	1341.12	3.1274685
21	3087.00	3.4895366	4152.75	3.6183358	4630.50	3.6656279	4851.00	3.6858313	940.919	2.9735524	1265.76	3.1023516	1411.38	3.1496437	1478.59	3.1698471

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60.—THEORETICAL SWITCHES FOR ANY GAGE g. Note.—All dimensions are in units in which g is taken.

Part I.—Properties of Outer Rail. Part II.—Properties of Center Line.

Frog No. n.	Frog Angle ϕ.	$R' = g\left[\frac{1}{\text{vers}\,\phi} - g\,[2n^2+\frac{1}{2}]\right]$ = g times	Log $\left[\frac{1}{\text{vers}\,\phi}\right]$	$l = g\left[\cot\frac{\phi}{2} - g\,[2n]\right]$ = g times	$d = g\left[\text{cosec}\frac{\phi}{2} - g\sqrt{(2n)^2+1}\right]$ = g times	Log $\left[\text{cosec}\frac{\phi}{2}\right]$	$m = \frac{\text{vers}\,\frac{\phi}{2}}{\text{vers}\,\phi} - g$ = g times	Log $\left[\frac{\text{vers}\,\frac{\phi}{2}}{\text{vers}\,\phi}\right]$
4	14°15′00″	32.5	1.5118834	8.	8.0623	0.9064567	.25097	9.3996202
4½	12 40 49	41.	1.6127839	9.	9.0554	0.9569069	.25077	9.3992703
5	11 25 16	50.5	1.7032914	10.	10.0499	1.0021607	.25062	9.3990190
5½	10 23 20	61.	1.7853298	11.	11.0454	1.0431799	.25051	9.3988326
6	9 31 38	72.5	1.8603380	12.	12.0416	1.0806840	.25043	9.3986908
6½	8 47 51	85.	1.9294189	13.	13.0384	1.1152245	.25037	9.3985801
7	8 10 16	98.5	1.9934362	14.	14.0357	1.1472330	.25032	9.3984924
7½	7 37 41	113.	2.0530784	15.	15.0333	1.1770543	.25028	9.3984210
8	7 09 10	128.5	2.1089031	16.	16.0312	1.2049666	.25024	9.3983629
8½	6 43 59	145.	2.1613680	17.	17.0294	1.2311989	.25022	9.3983150
9	6 21 35	162.5	2.2108534	18.	18.0278	1.2559416	.25019	9.3982746
9½	6 01 32	181.	2.2576786	19.	19.0263	1.2793543	.25017	9.3982402
10	5 43 29	200.5	2.3021144	20.	20.0250	1.3015722	.25016	9.3982112
10½	5 27 09	221.	2.3443923	21.	21.0238	1.3227111	.25014	9.3981859
11	5 12 18	242.5	2.3847117	22.	22.0227	1.3428708	.25013	9.3981640
11½	4 58 45	265.	2.4232459	23.	23.0217	1.3621378	.25012	9.3981451
12	4 46 19	288.5	2.4601458	24.	24.0208	1.3805878	.25011	9.3981284
12½	4 34 52	313.	2.4955443	25.	25.0200	1.3982871	.25010	9.3981137
13	4 24 19	338.5	2.5295587	26.	26.0192	1.4152943	.25009	9.3981005
14	4 05 27	392.5	2.5938397	28.	28.0179	1.4474248	.25008	9.3980785
15	3 49 06	450.5	2.6536948	30.	30.0167	1.4773625	.25007	9.3980604
16	3 34 47	512.5	2.7096939	32.	32.0156	1.5053630	.25006	9.3980459
17	3 22 10	578.5	2.7623034	34.	34.0147	1.5316667	.25005	9.3980338
18	3 10 56	648.5	2.8119100	36.	36.0139	1.5564699	.25005	9.3980238
19	3 00 54	722.5	2.8588379	38.	38.0132	1.5799339	.25004	9.3980151
20	2 51 51	800.5	2.9032613	40.	40.0125	1.6021957	.25004	9.3980079
21	2 43 40	882.5	2.9457147	42.	42.0119	1.6232724	.25003	9.3980017
22	2 36 14	968.5	2.9860996	44.	44.0114	1.6431649	.25003	9.3979964
23	2 29 27	1058.5	2.0246005	46.	46.0109	1.6620684	.25003	9.3979917
24	2 24 18	1152.5	3.0615100	48.	48.0104	1.6801350	.25003	9.3979878

Frog No. n.	$R = R' - \frac{g}{2}\,[2n^2]$ = g times	Log. $[2n^2]$	Remarks.
4	32.	1.5051500	Values are exact.
4½	40.5	1.6074550	
5	50.	1.6989700	
5½	60.5	1.7817554	
6	72.	1.8573325	
6½	84.5	1.9268567	
7	98.	1.9912261	
7½	112.5	2.0511525	
8	128.	2.1072100	
8½	144.5	2.1598678	
9	162.	2.2095150	
9½	180.5	2.2564772	
10	200.	2.3010300	
10½	220.5	2.3434086	
11	242.	2.3838154	
11½	264.5	2.4224257	
12	288.	2.4593925	
12½	312.5	2.4948500	
13	338.	2.5289167	
14	392.	2.5932861	
15	450.	2.6532125	
16	512.	2.7092700	
17	578.	2.7619278	
18	648.	2.8115750	
19	722.	2.8585372	
20	800.	2.9030900	
21	882.	2.9454466	
22	968.	2.9857754	
23	1058.	3.0244957	
24	1152.	3.0615800	

Values are exact.

l = g times these values. Values are exact.

d = g times these values.

m = g times these values.

R = g times these values. Values are exact.

Note simply the Radius R may be obtained from Log. g and the frog number n, with equal simplicity, R' and l, in Part I.

Fig. 45.

Turnout Curves for Stub Switches are simple curves. In staking them out on the ground it is necessary only to set two stakes, one marked *P. F.*,

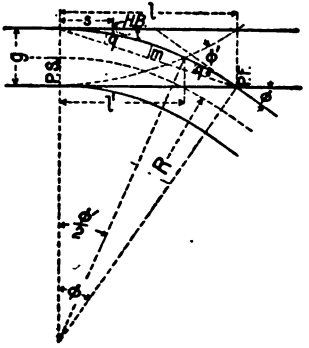

Fig. 46.

opposite the point of frog, and the other marked *P. S.*, opposite the point of switch. They should be set on the frog side of the track to indicate the direction of the turnout. The *P. F.* is usually located about half a frog length from a rail joint to save one cut of rail. No refinement is necessary in the calculation of turnouts, and the following formulas may be used where the main track is straight, as in Fig. 46:

Let ϕ = frog angle = central angle of turnout curve;
n = frog number (see Fig. 41);
l = theoretical lead to point of main frog;
l' = theoretical lead to point of crotch frog;
g = gage of track;
R = radius of turnout curve;
s = switch length, *P. S.* to *H. B.*;
m = middle ordinate of turnout curve;
q = quarter ordinates of turnout curve.
t = throw of switch.

All distances in feet.

Then, $\tan \frac{\phi}{2} = \frac{g}{l}$;

$$\cot \frac{\phi}{2} = \frac{l}{g} = 2n;$$

$$n = \tfrac{1}{2} \cot \frac{\phi}{2} = \frac{l}{2g} = \sqrt{R + 2g};$$

$$R = g + \text{vers } \phi - \frac{g}{2} = 2gn^2;$$

$$= l + \sin \phi - \frac{g}{2};$$

$$l = 2gn = g \cot \frac{\phi}{2} - \left(R + \frac{g}{2}\right) \sin \phi;$$

$$s = 2n\sqrt{gt} = \sqrt{2Rt};$$

$$t = s^2 + 2R;$$

$$m = g + 4 \text{ (nearly)};$$

$$q = \tfrac{3}{4}g + 4 = \tfrac{3}{16}g \text{ (nearly)}.$$

All the above formulas may be used with perfect safety for field work. Other formulas may be deduced from these by transposing or equating.

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Double Turnouts (=three-throw turnouts) require two main-line frogs and one crotch frog. If the two turnouts are opposite and of equal radii, then, Fig. 46, ϕ', the angle of the crotch frog, is equal to double the central angle $\frac{\phi'}{2}$. But $\left(R+\frac{g}{2}\right)$ vers $\frac{\phi'}{2}=\frac{g}{2}$, hence vers $\frac{\phi'}{2}=\frac{g}{2}\div\left(R+\frac{g}{2}\right)$, from which ϕ' is obtained.

Further, if n' = the number of the crotch frog,
and l' = the crotch frog (point) distance from the *P. S.*, we have,

$$R=4\,n'^2g \text{ (nearly)}=2\,n^2g;$$

$$n'=n\div\sqrt{2} \text{ (nearly)}=0.7071\,n \text{ (nearly)};$$

$$l'=0.707\,l \text{ (nearly)}.$$

Hence n' and l' are respectively equal to n and l multiplied by 0.707 (nearly). See Table 58.

Curved Main Track.—The above remarks apply to turnouts from *straight* main-line track, in connection with the stub switch. It is to be remembered that nearly all the formulas used in practical track work are approximate and close enough to the exact values, which may be obtained by trigonometric calculation. We will show now how the foregoing formulas may be applied to turnouts from *curved* main-line track. In order to illustrate we will consider Fig. 46 to be warped so the main line is curved instead of straight:

Let $T°$ = degree of curve of turnout from straight track;
$t°$ = degree of curve of turnout from curved main line;
$M°$ = degree of curve of main line after curving.
Then $t°=T°+M°$ when main line is curved *toward* turnout;
$t°=T°-M°$ when main line is curved *away from* turnout.

For instance, a No. 9 frog calls for a 7° 30′ curve from a straight track; it calls for 10° 30′ curve from the *inside* of a 3° main-line curve; and for a 4° 30′ curve from the *outside* of a 3° main-line curve. Hence, when the two curves are in the same direction we have to find their difference, and when in *opposite* directions we have to find their sum, to get the degree of curve $t°$ in Table 58, preceding, and the desired frog number, n, corresponding thereto.

Split Switches.—In our consideration of the stub switch, which is really for second-class track work, we have assumed all frogs in the turnout curves to be curved, whereas they are straight, and should be so considered in first-class track work where the split switch is used, although formerly this refinement was not considered necessary. This introduces a short tangent in the turnout at the point of frog. Moreover, split switch points are straight and it is now customary to take this fact into consideration in calculating the turnout *curve*, which really extends only from the heel of switch (*P. C.*) to the toe of frog (*P. T.*), joining the two short tangents above mentioned, instead of from point of switch to point of frog as assumed in Table 57, preceding.

The following is the practice of the Weir Frog Co., of Cincinnati, Ohio:

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1.—TURNOUTS FOR SPLIT SWITCHES AND SPRING FROGS. GAGE 4′ 8½″.
(Curve is Tangent to Switch Angle at Heel of Switch and to Frog Angle at Toe of Frog.)

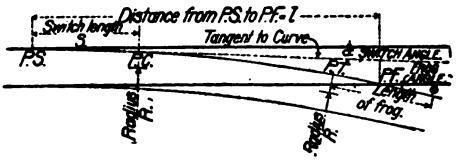

Fig. 47.

(a).—Switch and Frog Variable.

Frog No.	Frog Angle. ϕ.	Radius of Curve. R.	Degree of Curve.	Switch Angle. α.	Switch Length. s.	Distance from Point of Switch to Actual Point of Frog.	Lgth of Frog.	Length of Main Point of Frog.
1	14° 15′	125.868′	46° 49′	3° 20′ 29″	7′ 6″	33′ 9″	6′ 0″	3′ 9″
1½	12° 41′	164.569′	35° 22′	"	"	36′ 7″	6′ 0″	3′ 9″
2	11° 25′	202.054′	28° 39′	2° 30′ 27″	10′ 0″	43′ 7¾″	6′ 0″	3′ 10″
2½	10° 23′	244.318′	23° 37′	"	"	46′ 3¼″	6′ 6″	3′ 10½″
3	9° 32′	289.453′	19° 53′	1° 40′ 16″	15′ 0″	57′ 2½″	6′ 8″	4′ 3″
3½	8° 48′	343.249′	16° 45′	"	"	60′ 3″	7′ 0″	4′ 6″
4	8° 10′	403.942′	14° 13′	"	"	63′ 4″	7′ 0″	4′ 6½″
4½	7° 38′	468.794′	12° 15′	"	"	66′ 3½″	7′ 6″	5′ 0″
5	7° 09′	535.773′	10° 42′	"	"	69′ 0½″	8′ 0″	5′ 3″
6	6° 22′	678.440′	8° 27′	"	"	74′ 2″	10′ 0″	6′ 4½″
7	5° 44′	855.803′	6° 42′	"	"	79′ 5¼″	11′ 0″	7′ 1½″
8	5° 12′	1048.987′	5° 28′	"	"	84′ 0⅝″	12′ 0″	7′ 8″
9	4° 46′	1259.507′	4° 33′	"	"	88′ 6¼″	14′ 0″	8′ 9″

(b).—15 Ft. Switches.

5	9° 32′	231.21	24° 56′	1° 40′ 16″	15′ 0″	53′ 9″	15′ 0″	8′ 0″
6	8° 10′	335.50	17° 08′	"	"	60′ 1″	"	"
7	7° 09′	461.08	12° 27′	"	"	64′ 1¾″	"	"
8	6° 22′	609.62	9° 25′	"	"	71′ 10¾″	"	"
9	5° 44′	783.02	7° 19′	"	"	77′ 4¾″	"	"

(c).—18 Ft. Switches.

5	9° 32′	228.97	25° 15′	1° 23′ 34″	18′ 0″	57′ 6¾″	15′ 0″	8′ 0″
6	8° 10′	331.04	17° 22′	"	"	64′ 2½″	"	"
7	7° 09′	453.04	12° 40′	"	"	70′ 7″	"	"
8	6° 22′	596.05	9° 37′	"	"	76′ 8½″	"	"
9	5° 44′	761.28	7° 32′	"	"	82′ 7″	"	"

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62.—Three-Throw Turnouts. Split Switches.
Gage 4′ 8½″; Throws 5″.

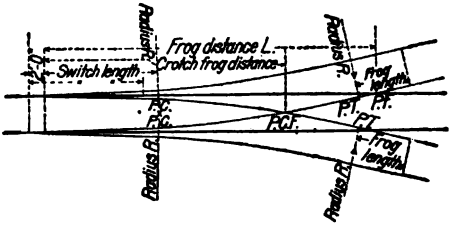

Fig. 48.

Main Line Frog No.	Crotch Frog Number.	Crotch Frog Angle.	Distance from Point of Switch to Actual Point of Crotch Frog.	Length of Crotch Frog.	Length of Actual Main Point of Crotch Frog.
4	2.76	20° 34′	29′ 9⅛″	5′ 0″	3′ 3″
4½	3.12	18° 12′	31′ 8″	5′ 0″	3′ 3″
5	3.51	16° 14′	37′ 5¾″	5′ 6″	3′ 6″
5½	3.83	14° 53′	39′ 2″	5′ 6″	3′ 6″
6	4.21	13° 34′	41′ 0⅞″	6′ 0″	3′ 9″
6½	4.56	12° 32′	42′ 9¼″	6′ 0″	3′ 9″
7	4.91	11° 37′	44′ 5⅝″	6′ 0″	3′ 10″
7½	5.26	10° 52′	46′ 0¼″	6′ 6″	3′ 10½″
8	5.58	10° 14′	47′ 5⅝″	6′ 6″	3′ 10½″
9	6.20	9° 14′	50′ 1⅛″	7′ 0″	4′ 6″
10	6.84	8° 21′	52′ 8⅝″	7′ 0″	4′ 6½″
11	7.57	7° 33′	57′ 8⅛″	8′ 0″	5′ 3″
12	8.17	7° 00′	60′ 0¾″	9′ 0″	5′ 11″

Split switches are made either stiff or with springs.

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The following is the practice of the Buda Manufacturing Co.:

68.—Turnouts from Straight Track. Split Switches.

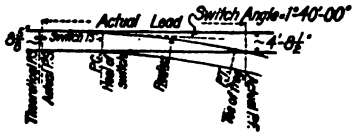

Fig. 49.

(a).—Properties of Turnouts.

Frog Number.	Frog Angle.	Radius of Curve.	Degree of Curve.	Lead.	Length of Curve.	Mid. Ord. of Curve.
4	14° 15′	121.841′	48° 27′	43′ 9½″	26′ 9⅛″	8 11/16″
5	11° 25′	193.991′	29° 52′	50′ 4 11/16″	33′ 0⅛″	8 7/16″
6	9° 32′	283.525′	20° 19′	56′ 8 ″	38′ 11⅛″	8 ″
7	8° 10′	393.603′	14° 36′	62′ 10⅝″	44′ 7 11/16″	7 15/16″
8	7° 10′	516.219′	11° 8′	68′ 2¼″	49′ 6⅝″	7½″
9	6° 22′	667.734′	8° 36′	73′ 8½″	54′ 10 11/16″	6¾″
10	5° 44′	841.083′	6° 49′	78′ 11 7/16″	59′ 8⅜″	6⅜″

Frogs.

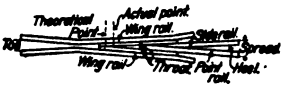

Fig. 50.

(b).—Properties of Frogs.

Frog Angle.	Length.	Actual		Theoretical		Spread.	
		Point to Heel.	Point to Toe.	Point to Heel.	Point to Toe.	Heel.	Toe.
14° 15′	6′ 0″	3′ 7 ″	2′ 5 ″	3′ 9″	2′ 3″	11¼″	6¾″
11° 25′	7′ 0″	4′ 3½″	2′ 8½″	4′ 6″	2′ 6″	10 11/16″	6 ″
9° 32′	8′ 0″	5′ 0 ″	3′ 0 ″	5′ 3″	2′ 9″	10½″	5½″
8° 10′	9′ 0″	5′ 8½″	3′ 3½″	6′ 0″	3′ 0″	10 1/16″	5⅛″
7° 10′	10′ 0″	6′ 2 ″	3′ 10 ″	6′ 6″	3′ 6″	9¾″	5¼″
6° 22′	11′ 0″	6′ 10½″	4′ 1½″	7′ 3″	3′ 9″	9⅝″	5 ″
5° 44′	12′ 0″	7′ 7 ″	4′ 5 ″	8′ 0″	4′ 0″	9⅝″	4 13/16″

Formulas for Split Switches.

(a).—Assuming Frog to be *curved*.

ion:

g = gage;
s = length of switch rail;
t = throw of switch;
x = switch angle;
β = frog angle;
R = radius of turnout curve;

$R' = R + \frac{g}{2}$; l = lead.

in feet.

Fig. 51.

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Formulas (Approximate):

$$\text{Sin } \alpha = \frac{t}{s};$$

$$l = s + \frac{g-t}{\tan \frac{1}{2}(\phi + \alpha)};$$

$$R' = \frac{g-t}{\cos \alpha - \cos \phi}.$$

(b).—Assuming Frog to be *Straight.*

Notation:

f = straight length of frog to *P. F.*, in feet.
Balance of notation as on preceding page.

Formulas (Approximate):

$$\text{Sin } \alpha = \frac{t}{s};$$

$$l = s + \frac{g-t-f \cos \phi}{\tan \frac{1}{2}(\phi + \alpha)} + f \cos \phi;$$

$$R' = \frac{g-t-f \sin \phi}{\tan \frac{1}{2}(\phi + \alpha)}.$$

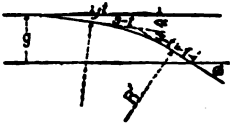

Fig. 52.

Note.—Switches can be planned graphically very easily by drawing them to scale and scaling the dimensions. After the switch length and switch angle are drawn for one rail, the frog, with angle ϕ and length f, may be "slid" long the other "rail" until the semi-tangents, s-t, are equal.

Wharton Switch.—The virtue of the Wharton switch lies in the fact that no frog is necessary for a turnout curve from the main line, for which it is especially designed, and that *the main line rails are therefore unbroken.* Fig. 53 is a general plan of the switch as patented April 2, 1901, showing it in position for main-line traffic. The switch rails are on a grade, rising from the points, and when thrown over and set for the siding the wheels of the train *ride up* on them, clearing the flanges from the main track. The trip rail is pivoted so that when the switch is set for the siding, the end *G* of guard, nearest the point of switch, swings outward and hugs the main rail. Hence, a train coming heel on, on the main line, crowds the trip rail inward and throws the switch automatically, for the main line.

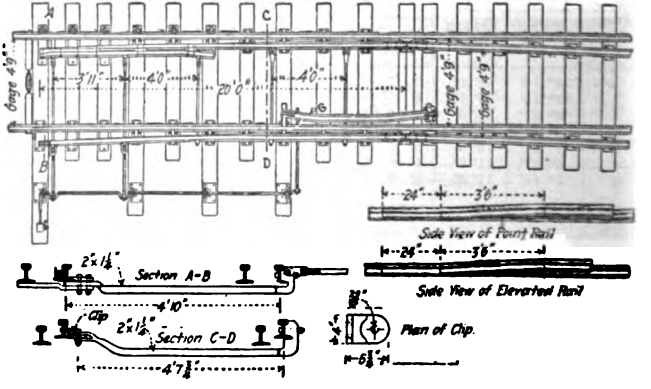

Fig. 53.—Wharton Switch. (See also Fig. 43.)

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64.—Ladder Tracks. Spacing of Frogs.

(Any Gage.)

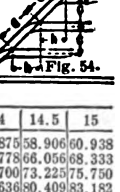

Fig. 54.

Part I.—Diagonal (Direct) Distances *d* between Frog Points. [*d* in Feet.]

Frog No.	Distances Between Track Centers, *c*, in Feet.											
	General*	10	10.5	11	11.5	12	12.5	13	13.5	14	14.5	15
4	4.06250 c	40.625	42.656	44.688	46.719	48.750	50.781	52.813	54.844	56.875	58.906	60.938
4½	4.55555 c	45.556	47.833	50.111	52.389	54.667	56.944	59.222	61.500	63.778	66.056	68.333
5	5.05000 c	50.500	53.025	55.550	58.075	60.600	63.125	65.650	68.175	70.700	73.225	75.750
5½	5.54546 c	55.455	58.227	61.000	63.773	66.546	69.318	72.091	74.864	77.636	80.409	83.182
6	6.04167 c	60.417	63.438	66.458	69.479	72.500	75.521	78.542	81.563	84.583	87.604	90.625
6½	6.53846 c	65.385	68.654	71.923	75.192	78.462	81.731	85.000	88.269	91.538	94.808	98.077
7	7.03571 c	70.357	73.875	77.393	80.911	84.429	87.946	91.464	94.982	98.500	102.02	105.54
7½	7.53333 c	75.333	79.100	82.867	86.633	90.400	94.167	97.933	101.70	105.47	109.23	113.00
8	8.03125 c	80.318	84.328	88.344	92.359	96.375	100.39	104.41	108.42	112.44	116.45	120.47
8½	8.52941 c	85.294	89.559	93.824	98.088	102.35	106.62	110.88	115.15	119.41	123.68	127.94
9	9.02778 c	90.278	94.792	99.306	103.82	108.33	112.85	117.36	121.88	126.39	130.90	135.42
9½	9.52632 c	95.263	100.03	104.79	109.55	114.32	119.08	123.84	128.61	133.37	138.13	142.89
10	10.0250 c	100.25	105.26	110.28	115.29	120.30	125.31	130.33	135.34	140.35	145.36	150.38
10½	10.5238 c	105.24	110.50	115.76	121.02	126.29	131.55	136.81	142.07	147.33	152.60	157.86
11	11.0227 c	110.23	115.74	121.25	126.76	132.27	137.78	143.30	148.81	154.32	159.83	165.34
11½	11.5217 c	115.22	120.98	126.74	132.50	138.26	144.02	149.78	155.54	161.30	167.06	172.83
12	12.0208 c	120.21	126.22	132.23	138.24	144.25	150.26	156.27	162.28	168.29	174.30	180.31
12½	12.5200 c	125.20	131.46	137.72	143.98	150.24	156.50	162.76	169.02	175.28	181.54	187.80
13	13.0192 c	130.19	136.70	143.21	149.72	156.23	162.74	169.25	175.76	182.27	188.78	195.29
14	14.0179 c	140.18	147.19	154.20	161.21	168.21	175.22	182.23	189.24	196.25	203.26	210.27
15	15.0167 c	150.17	157.68	165.18	172.69	180.20	187.71	195.22	202.73	210.23	217.74	225.25
16	16.0156 c	160.16	168.16	176.17	184.18	192.19	200.20	208.20	216.21	224.22	232.23	240.23
17	17.0147 c	170.15	178.65	187.16	195.67	204.18	212.68	221.19	229.70	238.21	246.71	255.22
18	18.0139 c	180.14	189.15	198.15	207.16	216.17	225.17	234.18	243.19	252.19	261.20	270.21
19	19.0132 c	190.13	199.64	209.15	218.65	228.16	237.67	247.17	256.68	266.18	275.69	285.20
20	20.0125 c	200.13	210.13	220.14	230.14	240.15	250.16	260.16	270.17	280.18	290.18	300.19
21	21.0119 c	210.12	220.62	231.13	241.64	252.14	262.65	273.15	283.66	294.17	304.67	315.18
22	22.0114 c	220.11	231.12	242.13	253.13	264.14	275.14	286.15	297.15	308.16	319.17	330.17
23	23.0109 c	230.11	241.61	253.12	264.63	276.13	287.64	299.14	310.65	322.15	333.66	345.16
24	24.0104 c	240.10	252.11	264.11	276.12	288.12	300.13	312.14	324.14	336.15	348.15	360.16

Part II.—Horizontal Distances *h* between Frog Points. [*h* in Feet.]

Frog No.	Distances Between Track Centers, *c*, in Feet.											
	General†	10	10.5	11	11.5	12	12.5	13	13.5	14	14.5	15
4	3.93750 c	39.375	41.344	43.313	45.281	47.250	49.219	51.188	53.156	55.125	57.094	59.063
4½	4.44444 c	44.444	46.667	48.889	51.111	53.333	55.556	57.778	60.000	62.222	64.444	66.667
5	4.95000 c	49.500	51.975	54.450	56.925	59.400	61.875	64.350	66.825	69.300	71.775	74.250
5½	5.45455 c	54.545	57.273	60.000	62.727	65.455	68.182	70.909	73.637	76.364	79.091	81.818
6	5.95833 c	59.583	62.562	65.542	68.521	71.500	74.479	77.458	80.438	83.417	86.396	89.375
6½	6.46154 c	64.615	67.846	71.077	74.308	77.539	80.769	84.000	87.231	90.462	93.692	96.923
7	6.96428 c	69.643	73.125	76.607	80.089	83.571	87.054	90.536	94.018	97.500	100.98	104.46
7½	7.46667 c	74.667	78.400	82.133	85.867	89.600	93.333	97.067	100.80	104.53	108.27	112.00
8	7.96875 c	79.688	83.672	87.656	91.641	95.625	99.610	103.59	107.58	111.56	115.55	119.53
8½	8.47059 c	84.706	88.941	93.177	97.412	101.65	105.88	110.12	114.35	118.59	112.82	127.06
9	8.97222 c	89.722	94.208	98.694	103.18	107.67	112.15	116.64	121.12	125.61	130.10	134.58
9½	9.47368 c	94.737	99.474	104.21	108.95	113.68	118.42	123.16	127.89	132.63	137.37	142.11
10	9.97500 c	99.750	104.74	109.73	114.71	119.70	124.69	129.68	134.66	139.65	144.64	149.63
10½	10.4762 c	104.76	110.00	115.24	120.48	125.71	130.95	136.19	141.43	146.67	151.91	157.14
11	10.9773 c	109.77	115.26	120.75	126.24	131.73	137.22	142.70	148.19	153.68	159.17	164.66
11½	11.4783 c	114.78	120.52	126.26	132.00	137.74	143.48	149.22	154.96	160.70	166.44	172.17
12	11.9792 c	119.79	125.78	131.77	137.76	143.75	149.74	155.73	161.72	167.71	173.70	179.69
12½	12.4800 c	124.80	131.04	137.28	143.52	149.76	156.00	162.24	168.48	174.72	180.96	187.20
13	12.9808 c	129.81	136.30	142.79	149.28	155.77	162.26	168.75	175.24	181.73	188.22	194.71
13	13.9821 c	139.82	146.81	153.80	160.79	167.79	174.78	181.77	188.76	195.75	202.74	209.73
14	14.9833 c	149.83	157.32	164.82	172.31	179.80	187.29	194.78	202.27	209.77	217.26	224.75
15	15.9844 c	159.84	167.84	175.83	183.82	191.81	199.81	207.80	215.79	223.78	231.77	239.77
16	16.9853 c	169.85	178.35	186.84	195.33	203.82	212.32	220.81	229.30	237.79	246.29	254.78
17	17.9861 c	179.86	188.85	197.85	206.84	215.83	224.83	233.82	242.81	251.81	260.80	269.79
18	18.9868 c	189.87	199.36	208.85	218.35	227.84	237.34	246.83	256.32	265.82	275.31	284.80
19	19.9875 c	199.88	209.87	219.86	229.86	239.85	249.84	259.84	269.83	279.83	289.82	299.81
20	20.9881 c	209.88	220.38	230.87	241.36	251.86	262.35	272.85	283.34	293.83	304.33	314.82
21	21.9886 c	219.89	230.88	241.87	252.87	263.86	274.86	285.85	296.85	307.84	318.83	329.83
22	22.9891 c	229.89	241.39	252.88	264.37	275.87	287.36	298.86	310.35	321.85	333.34	344.84
23	23.9896 c	239.90	251.89	263.89	275.88	287.88	299.87	311.86	323.86	335.85	347.85	359.84

65.—Crossovers. Spacing of Frogs.

Gage = 4′ 8½″.

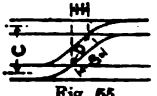

Fig. 65.

Part I.—Lengths *S* (Feet) of Straight Track between Frog Points.

Frog No.	Distances Between Track Centers, *c*, in Feet.												
	General*	10	10.5	11	11.5	12	12.5	13	13.5	14	14.5	15
4	d- 37.667	2.958	4.990	7.021	9.052	11.084	13.116	15.147	17.178	19.209	21.241	23.272
4½	d- 42.375	3.181	5.458	7.736	10.014	12.292	14.569	16.847	19.125	21.403	23.681	25.958
5	d- 47.083	3.417	5.942	8.467	10.992	13.517	16.042	18.567	21.092	23.617	26.142	28.667
5½	d- 51.792	3.663	6.435	9.208	11.981	14.754	17.526	20.299	23.072	25.844	28.617	31.390
6	d- 56.500	3.917	6.938	9.958	12.979	16.000	19.021	22.042	25.063	28.083	31.104	34.125
6½	d- 61.208	4.176	7.446	10.715	13.984	17.253	20.522	23.792	27.061	30.330	33.599	36.869
7	d- 65.917	4.440	7.958	11.476	14.994	18.512	22.030	25.548	29.065	32.583	36.101	39.619
7½	d- 70.625	4.708	8.475	12.242	16.008	19.775	23.542	27.308	31.075	34.842	38.608	42.375
8	d- 75.333	4.979	8.994	13.010	17.026	21.042	25.057	29.073	33.089	37.104	41.120	45.136
8½	d- 80.042	5.252	9.517	13.782	18.046	22.311	26.576	30.840	35.105	39.370	43.634	47.899
9	d- 84.750	5.528	10.042	14.556	19.070	23.584	28.097	32.611	37.125	41.639	46.153	50.667
9½	d- 89.458	5.805	10.568	15.332	20.095	24.858	29.621	34.384	39.147	43.911	48.674	53.437
10	d- 94.167	6.083	11.096	16.108	21.121	26.133	31.146	36.158	41.171	46.183	51.196	56.208
10½	d- 98.875	6.363	11.625	16.887	22.149	27.411	32.673	37.934	43.196	48.458	53.720	58.982
11	d-103.583	6.644	12.155	17.667	23.178	28.689	34.201	39.712	45.223	50.735	56.246	61.757
11½	d-108.292	6.925	12.686	18.447	24.208	29.969	35.729	41.490	47.251	53.012	58.773	64.534
12	d-113.000	7.208	13.218	19.229	25.239	31.250	37.260	43.270	49.281	55.291	61.302	67.312
12½	d-117.708	7.492	13.752	20.012	26.272	32.532	38.792	45.052	51.312	57.572	63.832	70.092
13	d-122.417	7.775	14.285	20.795	27.304	33.814	40.324	46.833	53.343	59.852	66.362	72.871
14	d-131.833	8.346	15.355	22.364	29.373	36.382	43.391	50.400	57.409	64.418	71.426	78.436
15	d-141.250	8.917	16.425	23.934	31.442	38.950	46.459	53.967	61.475	68.984	76.492	84.000
16	d-150.667	9.490	17.497	25.505	33.513	41.521	49.528	57.536	65.544	73.552	81.560	89.567
17	d-160.083	10.064	18.571	27.078	35.586	44.093	52.600	61.108	69.615	78.123	86.630	95.137
18	d-169.500	10.639	19.646	28.653	37.660	46.667	55.674	64.681	73.688	82.695	91.702	100.71
19	d-178.917	11.215	20.722	30.229	39.735	49.242	58.748	68.255	77.762	87.268	96.775	106.28
20	d-188.333	11.792	21.798	31.804	41.810	51.817	61.823	71.829	81.835	91.842	101.85	111.85
21	d-197.750	12.369	22.875	33.381	43.887	54.393	64.899	75.405	85.911	96.417	106.92	117.43
22	d-207.167	12.947	23.953	34.959	45.964	56.970	67.976	78.982	89.987	100.99	112.00	123.00
23	d-216.583	13.526	25.031	36.537	48.042	59.548	71.053	82.558	94.064	105.57	117.08	128.58
24	d-226.000	14.104	26.109	38.114	50.120	62.125	74.130	86.135	98.140	110.15	122.15	134.16

Part II.—Horizontal Distances *H* (Feet) between Frog Points.

Frog No.	Distances Between Track Centers, *c*, in Feet.												
	General†	10	10.5	11	11.5	12	12.5	13	13.5	14	14.5	15
4	h- 37.667	1.708	3.677	5.646	7.615	9.583	11.552	13.521	15.490	17.458	19.427	21.396
4½	h- 42.375	2.069	4.292	6.514	8.736	10.958	13.181	15.403	17.625	19.847	22.069	24.292
5	h- 47.083	2.417	4.892	7.367	9.842	12.317	14.792	17.267	19.742	22.217	24.692	27.167
5½	h- 51.792	2.754	5.481	8.208	10.935	13.663	16.390	19.117	21.845	24.572	27.299	30.026
6	h- 56.500	3.083	6.062	9.042	12.021	15.000	17.979	20.958	23.938	26.917	29.896	32.875
6½	h- 61.208	3.407	6.638	9.869	13.100	16.331	19.561	22.792	26.023	29.254	32.484	35.715
7	h- 65.917	3.726	7.208	10.690	14.172	17.654	21.137	24.619	28.101	31.583	35.065	38.547
7½	h- 70.625	4.042	7.775	11.508	15.242	18.975	22.708	26.442	30.175	33.968	37.642	41.375
8	h- 75.333	4.354	8.339	12.323	16.308	20.292	24.276	28.261	32.245	36.230	40.214	44.199
8½	h- 80.042	4.664	8.899	13.135	17.370	21.605	25.840	30.076	34.311	38.546	42.782	47.017
9	h- 84.750	4.972	9.458	13.944	18.431	22.917	27.403	31.889	36.375	40.861	45.347	49.833
9½	h- 89.458	5.279	10.016	14.752	19.489	24.226	28.964	33.700	38.437	43.174	47.910	52.647
10	h- 94.167	5.583	10.571	15.558	20.545	25.533	30.520	35.508	40.495	45.483	50.470	55.458
10½	h- 98.875	5.887	11.125	16.363	21.601	26.839	32.077	37.316	42.554	47.792	53.030	58.268
11	h-103.583	6.190	11.679	17.167	22.656	28.144	33.632	39.121	44.610	50.098	55.587	61.076
11½	h-108.292	6.491	12.230	17.969	23.708	29.447	35.186	40.926	46.665	52.404	58.143	63.882
12	h-113.000	6.792	12.782	18.771	24.761	30.750	36.740	42.730	48.719	54.709	60.698	66.688
12½	h-117.708	7.092	13.332	19.572	25.812	32.052	38.292	44.532	50.772	57.012	63.252	69.492
13	h-122.417	7.391	13.882	20.372	26.863	33.353	39.843	46.334	52.824	59.315	65.805	72.295
14	h-131.833	7.988	14.979	21.970	28.961	35.952	42.943	49.934	56.925	63.916	70.907	77.898
15	h-141.250	8.583	16.075	23.566	31.058	38.550	46.042	53.533	61.025	68.517	76.008	83.500
16	h-150.667	9.177	17.170	25.162	33.154	41.146	49.138	57.131	65.123	73.115	81.107	89.099
17	h-160.083	9.770	18.262	26.755	35.248	43.740	52.233	60.726	69.219	77.711	86.204	94.697
18	h-169.500	10.361	19.354	28.347	37.340	46.333	55.326	64.319	73.312	82.305	91.298	100.29
19	h-178.917	10.951	20.445	29.938	39.431	48.925	58.418	67.912	77.405	86.899	96.392	105.89
20	h-188.333	11.542	21.535	31.529	41.523	51.517	61.510	71.504	81.498	91.492	101.49	111.48
21	h-197.750	12.131	22.625	33.119	43.613	54.107	64.601	75.095	85.589	96.083	106.58	117.07
22	h-207.167	12.719	23.714	34.708	45.702	56.696	67.691	78.685	89.679	100.67	111.67	122.66
23	h-216.583	13.308	24.802	36.297	47.791	59.286	70.780	82.275	93.770	105.26	116.76	128.25
24	h-226.000	13.896	25.891	37.886	49.880	61.875	73.870	85.865	97.860	109.85	121.85	133.84

65.—Crossovers. Spacing of Frogs.—Concluded.

Gage = 4′ 8½″.

Part III.—Direct Distances *D* (Feet) between Frog Points.

Frog No.	Distances Between Track Centers, c, in Feet.											
	General.	10	10.5	11	11.5	12	12.5	13	13.5	14	14.5	15
4	$D=\sqrt{S^2+g^2}=\sqrt{H^2+(c-g)^2}$ (Fig. 55.)	5.56	6.86	8.45	10.20	12.04	13.93	15.86	17.81	19.78	21.75	23.74
4½		5.68	7.21	9.06	11.06	13.16	15.31	17.49	19.70	21.91	24.14	26.38
5		5.82	7.58	9.69	11.96	14.31	16.72	19.15	21.61	24.08	26.56	29.05
5½		5.96	7.97	10.34	12.87	15.49	18.15	20.84	23.55	26.27	29.00	31.74
6		6.12	8.38	11.01	13.81	16.68	19.59	22.54	25.50	28.47	31.46	34.45
6½		6.29	8.81	11.70	14.75	17.88	21.05	24.25	27.47	30.69	33.93	37.17
7		6.47	9.25	12.40	15.72	19.10	22.53	25.98	29.44	32.92	36.41	39.90
7½		6.66	9.69	13.12	16.69	20.33	24.01	27.71	31.43	35.16	38.89	42.64
8		6.85	10.15	13.84	17.66	21.56	25.50	29.45	33.42	37.40	41.39	45.38
8½		7.05	10.62	14.56	18.65	22.80	26.99	31.20	35.42	39.65	43.89	48.13
9		7.26	11.09	15.30	19.64	24.05	28.49	32.95	37.42	41.90	46.39	50.88
9½		7.47	11.57	16.04	20.64	25.30	29.99	34.70	39.43	44.16	48.90	53.64
10		7.69	12.05	16.78	21.64	26.55	31.50	36.46	41.44	46.42	51.41	56.41
10½		7.92	12.54	17.53	22.64	27.81	33.01	38.23	43.45	48.69	53.93	59.17
11		8.14	13.04	18.28	23.65	29.07	34.52	39.99	45.47	50.95	56.44	61.94
11½		8.37	13.53	19.04	24.66	30.34	36.04	41.76	47.49	53.22	58.96	64.71
12		8.61	14.03	19.80	25.67	31.60	37.56	43.53	49.51	55.49	61.48	67.48
12½		8.85	14.54	20.56	26.69	32.87	39.08	45.30	51.53	57.76	64.01	70.25
13		9.09	15.04	21.32	27.71	34.14	40.60	47.07	53.55	60.04	66.53	73.02
14		9.58	16.06	22.85	29.75	36.69	43.65	50.62	57.60	64.59	71.58	78.58
15		10.08	17.09	24.39	31.79	39.23	46.70	54.17	61.66	69.15	76.64	84.13
16		10.59	18.12	25.94	33.84	41.79	49.75	57.75	65.71	73.70	81.70	89.69
17		11.11	19.16	27.48	35.88	44.34	52.81	61.29	69.77	78.26	86.76	95.25
18		11.64	20.20	29.04	37.95	46.90	55.87	64.85	73.84	82.83	91.82	100.82
19		12.16	21.25	30.59	40.01	49.47	58.94	68.42	77.90	87.39	96.89	106.38
20		12.70	22.30	32.15	42.07	52.03	62.00	71.98	81.97	91.96	101.96	111.95
21		13.23	23.35	33.71	44.14	54.60	65.07	75.55	86.04	96.53	107.02	117.52
22		13.78	24.41	35.27	46.20	57.17	68.14	79.12	90.11	101.10	112.10	123.09
23		14.32	25.47	36.84	48.27	59.73	71.21	82.70	94.18	105.67	117.17	128.67
24		14.87	26.53	38.40	50.34	62.30	74.28	86.26	98.25	110.25	122.24	134.24

EXCERPTS AND REFERENCES.

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New Steel Rail Specifications of P. R. R. (Eng. News, April 16, 1908).

New Rail Sections and Specifications of the Am. Ry. Assn. (Eng. ws, May 14, 1908).—Two types illustrated: A and B, each rolled at 60-, , 80-, 90-, and 100-lb. Tables of properties.

Standards of Track Construction on American Railways (Eng. News, ne 4, 1908).—Four large tables comprising tabulated data from 59 rail- ds: Table 1.—Standard Practice as Applied to Rails and Rail Joints **ail**; weight per yard; length; type of section; lightest in main track. **l Joints**; type; square or broken; number of bolts; size of bolts; spac- of bolts; nutlock; nuts. **Splice bars**; length; weight per pair. Special patented joints used). Table 2.—Standard Practice as to Ties and Plates **es**; wood, and where obtained; average life in main track; cost; re- **d** for wear or decay mainly; size, length, thickness, width; number 33-ft. rail; spacing at joints; preservative process; number of treated n use. **Tie-plates**; make; size; weight; where used). Table 3.— **dard Practice as to Frogs and Switches (Frogs**; standard pattern; **g or rigid**; numbers, for main track; numbers, for yards; flangeway **uard rail**, ins. **Switches**; standard pattern; length of switch rail. **llaneous**). Table 4.—Standard Practice as to Spikes, Ballast and **Protection (Spikes**; style; size; screw spikes used. **Ballast**; kind

used; size of stone; depth under tie. **Guard rails for curves; on what curves; flangeway, ins.; are tie rods or bars used to hold gage on sharp curves?)**

Readjustment of Curves and Tangents in Maintenance-of-Way Work (By W. H. Wilms. Eng. News, Sept. 17, 1908).—Illustrated.

Some Special Designs of Rails and Tie-Plates (Eng. News, Oct. 15, 1908).—Illustrated sections of 85-lb. rails for Western Pac. Ry. (5½" x 5½") and Great Nor. Ry. (5" x 5"); also plans of four types of metal tie-plates in use on different railways.

Cast Manganese-Steel Rails on Curves, Boston Elev. Ry. (By H. M. Steward. Eng. News, Oct. 22, 1908).—Table showing comparative life of rails of ordinary Bessemer, high carbon Bessemer, nickel-steel, manganese-steel and open-hearth.

A Study of Rail Pressures and Stresses in Track Produced by Different Types of Locomotives on Curves (By E. E. Stetson. Bulletin No. 104, Oct., 1908, Am. Ry. Assn.; Eng. News, Nov. 26, 1908).—Extensive table; and analytic discussion of pressures and stresses.

A Wedge Rail Fastening for Steel Ties (Eng. News, Dec. 24, 1908).—Illustrated. It requires no bolts or clamps; the width of gage can be varied as required by wear; neatness of gage can be obtained, even if the rail section is not exact; gives greater resistance to shearing, as compared with bolts; the fastening can be insulated; and it is impossible for derailed wheels to destroy the fastening.

New Rail Specifications for the P. R. R. (Eng. News, Jan. 14, 1909).

A 1200-Volt Direct-Current Electric Railway (Eng. News, Jan. 21, 1909).—The Pittsburg, Harmony, Butler & New Castle Ry. Table of horse powers.

The Disadvantages of Concrete Foundations for Railway Crossings (By R. P. Black. Eng. News, April 29, 1909).—(1) The concrete gives too rigid a bearing under frogs, causing an anvil blow at frogs which soon wears down the points, especially at flat crossings, on account of the greater distance between points. (2) This anvil or solid blow is hard on bolts, especially when high speed is maintained over frogs. (3) In a high speed track concrete cracks away from timbers on account of the excessive jar. (4) It is hard to maintain a good surface on the track, as the concrete footing does not permit of raising. (5) The rigid bearing is hard on equipment. (6) The concrete foundation for crossing is a failure. The only case where it can be used to good advantage is where traffic is light, speed low and angle of crossing 90° more or less.

Earthwork Calculations for Side Hill Work (By R. S. Beard. Eng. News, June 10, 1909.—Diagrams, formulas and tables.

A Railway Transfer Table Without a Pit (By H. V. Miller. Eng. News, July 15, 1909).—Illustrated.

Standard Specifications for Structural Steel for Buildings (Proc. A. S. T. M., Vol. IX., 1909).—Adopted Aug. 16, 1909.

Standard Specifications for Bessemer and for Open-Hearth Steel Rails (Proc. A. S. T. M., Vol. IX., 1909).—Adopted by Aug. 16, 1909.

Taper Curves Used on Southern Pacific R. R. (Oct. 28, 1909).—Tables, formulas and illustrations.

Special Type of Track Construction for Tunnels and Subways (Eng. News, Aug. 19, 1909).—Illustrated, with tables of cost.

Tie-Plates and Braces for Guard Rails on Sharp Curves; Natal Gov't Rys. (Eng. News, Nov. 11, 1909).—Illustrated description.

Street Railway Track Construction and Paving (Report of Comm. on Way, Am. St. and Interurban Ry. Eng'g Assn., Oct. 4 to 8, 1909; Eng. News, Nov. 11, 1909).—**Use of T-Rails in Paved Streets:**—Recommendations: (1) For track construction where type of pavement will permit, as in macadam or other shallow pavement, the T-rail weighing not less than

lbs. per yd. adopted as "recommended practice" by the Am. Ry. Eng. and M. of W. Assn., 1908. (2) For heavy service in connection with deep block pavements, a T-rail 7 ins. high with a 6-in. base, 17/32-in. web and a head of 2¼ ins. wide and 1-11/16 ins. deep, weighing about 100 lbs. per yd., as illustrated (not reproduced here). (3) For light service, in connection with deep block pavement, a T-rail 7 ins. high, 6-in. base, 7/16-in web, 2½ by 1-9/32 in. head, and weighing 80 lbs. per yd., as illustrated (not reproduced here). (4) For heavy service in connection with deep block pavement on streets where traffic is confined to the railway strip, etc. (cities of large class), the half-grooved (or "Trilby") section recommended in 1907. **Other Subjects Discussed**:—Track in paved streets; Opinions of City Engineers, etc., as to St. Ry. track construction; Widening gages on curves; Cost and life of steel ties; Wear of gage lines of rails on tangents; Creosoted wood-block pavement; Spacing of ties; Setting of concrete foundation; Life of rail joints on paved streets; Efficiency of electrically brazed and soldered rail bonds; Discussions on rails, ties, foundations, paving.

Train Resistance by Various Formulas (Eng. Rec., June 4, 1910).—Table of train resistance in pounds per ton by various formulas: (1) Am. Ry. Eng. & M. W. Assn., Bulletin 114, p. 4; (2) Ditto, Bulletin 84, p. 100; (3) Am. Loco. Co., Bulletin 1001, p. 3; (4) Am. Ry. Eng. & M. W. Assn., Bulletin 120, p. 26. Experiments indicate that train resistance increases with the speed.

Elementary Theory of the Gyroscope in the Brennan Monorail Car (By E. V. Huntington. Eng. News, July 21, 1910).—Discussion of: (a) Steady precession; (b) Accelerated precession; (c) Application to the monorail car; (d) Efficiency of the Brennan apparatus; (e) Proportions of the car; (f) Proofs of theorems. Illustrated.

The Track and Line Construction of Electric Railways (Eng. News, Oct. 27, 1910).—Tabular results of inquiries as to general lines of practice on 18 interurban railways and 17 street railways in various parts of the country. The tables are: (I) Length, track arrangement and ballasting; (II) Grades and curves; (III) Rails and rail joints; (IV) Ties and tie-plates; (V) Frogs, switches and switch-stands; (VI) Pole equipment; (VII) Line and car equipment, signals, etc. These tables are accompanied by a general discussion of prevailing practices.

Mountain Rack Railways, and the Jungfrau Ry. in Switzerland (By E. . Corthell. Eng. News, Oct. 27, 1910).—Includes a table of 57 railways in Europe, Asia and Australia, and North and South America, giving date of building, gage of track, length, grades of adhesion and rack, kind of traction, minimum curve on rack, number and weight of locomotives, and train weight.

Track Construction on the Chicago Street Railways (Eng. News, Nov. 1910).—"The complete reconstruction of the street railway tracks in Chicago was one of the requirements of the new system of municipal regulation and the agreement between the city and the companies, which went into effect about three years ago, and by which the city exercises a strong control over the construction, equipment, operation and financial affairs of the street railways." The supervision of this work is in the hands of a Board of Supervising Engineers, composed of Bion J. Arnold, George Weston, Harvey B. Fleming, John Z. Murphy and A. L. Drum. This article gives illustrated descriptions of:—(1) Types of track construction (grooved girder rails on steel ties embedded in concrete; same with wooden ties; improved girder rails on wooden ties on broken stone ballast; T-girder rails with brick paving; tram-head girder rails on wooden ties in macadam). (2) Track material (rails, with table; rail joints; ties; tie-plates; spikes and fish bars; switches and frogs). (3) Foundation and paving (broken stone; concrete; paving; track spacing; track grade and street grade; curves at street intersections). (4) Methods of construction. (5) Track deflection.

The Electric Bolt Lock as Applied to Interlocking (Report of Committee on Power Interlocking, presented at annual meeting of Railway Signal Assn., Oct. 11-13, 1910; Eng. News, Nov. 3, 1910).—The principal merit is its simplicity.

Gravity Freight Classification Yard for the P. R. R. at Northumberland, (By W. A. MacCart. Eng. News, Nov. 17, 1910).—Illustrated description.

Important Illustrations for Reference.

Description.	Eng. News.
Railway ditching machine for small cuts and fills	Jan. 20, '10.
Ties: rein.-conc., steel-and-wood, steel-and-concrete	Feb. 17, '10
Track construction with 114-lb. rails, Belgium	Apr 14, '10.
120-ft. turntable for Malet locomotives, A. T. & S. F. Ry.	June 23, '10.
American electric locomotives (tables)	Aug. 4, '10.
The design of the electric locomotive	Aug. 4, '10.
How to run transition curves without tables (C. P. Howard)	Oct. 13, '10.
Concrete and timber snow sheds on Gt. Nor. Ry.	Dec. 15, '10.
A deflection recorder for track switches, N. Y., O. & W. Ry.	Dec. 22, '10.
	Eng. Rec.
Street railway tract construction in Charlotte, N. C.	Apr. 24, '09.
Curves showing relation of train resistance to velocity	July 31, '09.
Track details on German railroads; anti-creeper	Jan. 29, '10.
Cross-sections of single and double track roadbed	Feb. 19, '10
Elevated car storage yard, Interborough Rap. Tr. Co., N. Y.	Oct. 15, '10.

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60.—HIGHWAYS.

A.—TRACTION.

Power of a Horse.—It is generally estimated that an average horse weighing 1200 lbs. can exert a force of 100 lbs. for a day of 10 hours at the rate of $2\frac{1}{2}$ miles per hr., on a fairly level grade; and for a short haul he can exert a force of $2\frac{1}{2}$ times the above, or 250 lbs. A constant force of 100 lbs. at $2\frac{1}{2}$ miles (13200 ft.) per hr. is equal to just two-thirds of a horsepower (H. P.), and for 10 hrs. it is equal to $6\frac{2}{3}$ horsepower-hours, or 13 200 000 ft.-lbs. of work.

Effect of Road Surfaces on Traction.—The tractive force required to move one ton of 2000 lbs. on various kinds of level roads is approximately as follows:

	F	W	$F:W$	$= A$
Earth road.....	100 lbs. per ton of	2000 lbs.	= 1:20	= .05
Macadam road.....	40 " "	" "	= 1:50	= .02
Granite blocks.....	30 " "	" "	= 1:66⅔	= .015
Brick pavement....	25 " "	" "	= 1:80	= .0125
Asphalt pavement..	20 " "	" "	= 1:100	= .01
Steel rails..... From	8 " "	" "	= 1:250	= .004
	7½ " "	" "	= 1:266⅔	= .00375
To...	7 " "	" "	= 1:285⅐	= .0035

Experiments in Iowa showed the following tractive resistances: Brick pavement, 25.4 to 58 lbs. per ton; asphalt pavement, 23.3 to 67.8 lbs. per ton. (See page 1142.)

Effect of Grades on Traction.—In the above case, if we let F = the tractive force, and W = the load, then will $\frac{F}{W} = A$, the tangent* of the angle of repose, or grade of the road at which the load would just begin to slide or descend of its own weight. Hence, if F lbs. are required to move a load W on a level, it is clearly evident that $2F$ lbs. would be required to haul it up a grade $G = A$, $3F$ lbs. up a grade $G = 2A$, etc., approximately. This proposition is often erroneously neglected, the usual formula given being, $F = WG$, in which F = tractive force, W = load, and G = grade. The correct formulas are:

$$F = W\,(A + G) \text{ in ascending} \qquad (1)$$

$$F = W\,(A - G) \text{ in descending} \qquad (2)$$

Hence

$$W = \frac{F}{A + G} \text{ in ascending} \qquad (3)$$

Records of actual tests appear in various works stating that a horse which can pull 1000 lbs. on a level road, can pull only 900 lbs. up a 1% grade, 810 lbs. up a 2% grade, 750 lbs. up a $2\frac{1}{2}$% grade, 720 lbs. up a $2\frac{1}{2}$% grade, 640 lbs. up a $3\frac{1}{2}$% grade, 540 lbs. up a 4% grade, 500 lbs. up a $4\frac{1}{2}$% grade, 400 lbs. up a 5% grade, and 250 lbs. up a 10% grade.

Problem.—On an extremely bad earth road it requires a constant force of 100 lbs. to pull 1000 lbs. on a level. What would be the maximum allowable grade, assuming that F could be increased to 250 lbs. while ascending grade?

Solution.—Transposing equation (3), above, we have, since $A = 100 \div 1000$,

Maximum grade, $G = F \div W - A$

$$= \frac{250}{1000} - \frac{100}{1000}$$

$$= 0.15 = 15\%. \quad \text{Ans.}$$

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* Approximately; see Sec. 59, Railroads, page 992.

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B.—ROADS AND STREETS.

Definitions.—A Street is a public way in a city or town, and consists generally of a roadway and two sidewalks. A Road consists essentially of a roadway or public thoroughfare through a county district, and with or without sidewalks. City streets are usually paved, while roads, and roadways of streets in small towns, are merely surfaced. Roads and streets may be classed according to the kind of surface or pavement, the selection of which will depend upon the kind and amount of traffic, grades, cleanliness desired, material available, climate, allowable first cost, cost of maintenance, etc.

Dirt Roads, sometimes dignified by the name Earth Roads, are the pioneers in any new country. Dirt from the sides is simply thrown up into the center, forming a sort of crown for lateral shedding of rain water. In the Middle West, dirt roads are constructed very rapidly and cheaply by plowing one or two furrows on either side, and using scrapers in casting this material up for the crown of the road. Where extensive road-making is contemplated, it is well to figure on regular road-making machines.

Corduroy Roads are probably among the first in any new country, and in thinly populated sections generally, to supplant the common dirt road in low, marshy, wet ground. The typical corduroy road consists of round sticks of wood a few inches in diameter, laid transversely across the road. These are sometimes supplemented with half-round sticks, or slabs from saw-mills. At best they are but makeshifts, and give way sooner or later to plank- or other construction of surface of a smoother character and calling for less tractive power in hauling. Corduroy roads are often improved by crowning them with gravel, using sticks or poles as a foundation.

Plank Roads are usually the first form of improvement in timbered sections where there is much rainfall. On many of our old maps, in various sections of the country, we may see the "Old Plank Road" shown in dotted lines. The typical plank streets now being constructed in our small towns of the Pacific Northwest are composed of planking 3" to 4" thick, laid transversely, and spiked (or not) to longitudinal wooden stringers of size say 4" by 10", more or less, spaced about 4-ft. centers. The sidewalk planking is usually about 2" thick, laid with a slope of about ¼" per ft. toward the curb. A simple gutter is shown in Fig. 1. For the roadway, the planking is sometimes laid level from gutter to gutter, and sometimes crowned in the form of a parabola, the quarter points being ¼ the height of the middle. The method sometimes adopted of laying the middle third of the roadway level, and the sides sloping, is particularly objectionable because of the two continuous longitudinal joints formed along the edges of the level portion. If the street is on a steep grade, it is generally best to have the longitudinal stringers under the planking "broken" and not "continuous," in order to prevent excessive wash of the soil. Two nails are used at each intersection of plank with stringer. Barbed wire nails are most commonly preferred. Sometimes the planks, if very heavy, are simply laid on the stringers without spiking, but this method is objectionable. The stringers should be imbedded firmly in the ground.

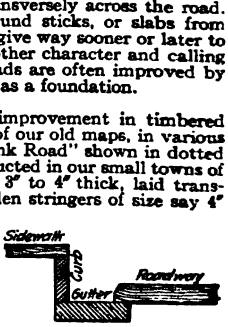

Fig. 1.

Gravel Roads are excellent when properly made. Angular, pit gravel is the best; smooth, sea-polished gravel will never bind properly unless a binder is added, and that increases the expense. After the gravel has been screened, ½" to 1½", it is spread on the ground in 3" or 4" layers and thoroughly compacted, after sprinkling, with a steam road roller weighing from 3 to 10 tons. A small quantity of clay added to the gravel acts as a binder and it is still further improved if mixed with crushed gravel or small broken limestone. Sand should not be used.

Gravel Walks are constructed practically in the same manner. The main thing to look out for in gravel construction is good drainage, as the clay binder is easily softened when saturated. Tile or box under-drains should be laid to carry off the surplus water, and their proper use greatly decreases the cost of repairs. Coal tar is sometimes used with gravel in making foot-walks, but the result is usually unsatisfactory.

Broken-Stone Pavement has undergone rapid improvement since the advent of the rock crusher and the steam road-roller. The original Telford and Macadam methods, about 1825, have been supplanted by more modern methods of construction. In providing for a good pavement, under-drainage should be provided when required; a sub-grade should be prepared by removing all perishable matter and the top soil; the under-soil should be compacted and a foundation of gravel prepared when necessary, if the best results are expected. The broken-stone should not be larger than 2" for the softer rock or 1½" for the harder, and if clean it should not (generally) be screened, as the stone dust and chips make good binders. A soft stone foundation and a hard stone surface are the best. Sand and gravel may be employed to fill the voids, but very little if any clay or loam should be used. Crushed granite should never be used. Trap or basalt is best for the surface, and limestone for the bottom. The material is spread in layers of about 4" to 4½", sprinkled sufficiently, and compacted with steam road rollers weighing 5 to 10 tons. It will roll to about $\frac{1}{10}$ the spread thickness. The thickness of broken stone pavement is usually from 6" to 10", although 1" is quite common. It is to be noted that a thin pavement laid on a good gravel foundation is often superior to a much thicker pavement laid directly on the soil. The rolling should begin at the sides and work toward the crown of the road.

Hydraulic-Cement Pavement consists of a 3" to 6" concrete base underlying a wearing surface composed of one part hydraulic cement to two parts finely crushed rock, 1½" or more in thickness. The concrete base may rest on a gravel or cinder bed. The finished surface may be flushed with a 1 to 1 sand and cement mixture.

Cement Sidewalks usually consist of a 1" to 1½" wearing surface composed of a 1 to 1 sand and cement mixture, overlaying a 3" to 4" concrete base resting on cinders. The wearing surface is often flushed with a pure, nearly pure, cement mortar.

Wood-Block Pavements have given good results when properly constructed, but the expense of preparing good foundations necessary to keep the blocks in even surface is considerable. The best foundation is a layer of concrete, say 4" or more in thickness, and this is the practice in Europe where this kind of pavement has reached its highest perfection. It is also becoming standard in many of the principal cities of the United States. Experiments have been made in New York City and elsewhere in this country with varying degrees of success, but it can be stated that, generally, the trend is toward some more permanent form of construction. This is the case also in many of our western cities, even those of the Northwest where timber is very plentiful but where the wood block is being supplanted by asphalt, brick and Belgian block pavements. In section, the blocks may be of various shapes, round, square, rectangular, etc., but the rectangular section is perhaps the most common. The blocks should be laid with the fibers or grain vertical and with close joints. In the cheaper construction, in some parts of the West, they are laid on a bed of sand resting on a gravel foundation, or on planking of single or double thickness, and with joints packed with sand. In the better construction, they are laid on a bed of tar spread on a concrete foundation, with joints smeared with tar, asphalt or cement, and expansion joints provided at intervals and along the curb. Many engineers prefer to lay the blocks so the course of joints be diagonal to direction of traffic. If laid square they are more easily opened by the corks of the horses' shoes. A common objection to wooden block pavement is made on the ground of cleanliness or sanitation. The blocks should be treated with some preservative, as creosote, but this is not always done. Blocks so treated expand much less after being laid, and provision need be made therefor; but if laid untreated they will swell by the absorption of moisture, and careful provision must be made for expansion, at frequent intervals. The writer has seen "blisters" raised on the surface because of lack of provision in this respect. The harder woods, oak, are not generally used in this country, as they wear too slippery.

Cobblestone Pavement consists of cobblestones from 4" to 6" in longest diameter, set vertically, with the fattest end up, in a bed of gravel, thoroughly rammed, and with joints filled with gravel. This pavement is but a makeshift for roadways, and is being supplanted by Belgian blocks or other pavement of better quality. It is frequently used for gutters.

Belgian Block Pavement consists of hard trap or basalt blocks of stone laid in parallel courses and rammed into a bed of sand, with sand joints. The blocks are usually about 7 ins. high and 5 or 6 ins. square.

Granite Block Pavement is supplanting the Belgian blocks as the latter did the cobblestones, and it is now considered the best pavement for heavy traffic. The blocks may be, say 7″ deep, 3″ wide and 10″ long, laid with continuous parallel joints or courses at right angle to the direction of traffic, on a concrete foundation not less than 4″ thick; 6″ to 8″ is better for unusually heavy traffic. They are laid directly on a thin layer of sand, well rammed to a firm bed, and the joints filled with bituminous cement.

Brick Pavement has become quite popular in recent years in certain sections of the country where a moderately durable pavement is required for all-around traffic at not too great expense. The brick should be uniformly hard-burned and tough, and subjected to rigid inspection before being laid. They should be laid on edge in parallel courses (with broken joints) at right angle with the direction of traffic, on Portland cement concrete 4″ or more in thickness, with a cushion layer of sand, and with small joints so the sand will work up into the joints during rolling. The bricks may be of the ordinary size, or vitrified *blocks* may be used. At street intersections, the courses may be diagonal with either street. There is no advantage in laying bricks herring-bone fashion as in Fig. 2, except perhaps to please the eye, as in sidewalk fancies. Straight, transverse courses give a better foothold for the horses. Bituminous cement, or, better, Portland cement grout should be used in filling the joints after the bricks are laid. Sand is far inferior and should never be used.

Fig. 2.

Asphalt Pavement takes first rank for combined general serviceability, low tractive resistance, cleanliness and hygienic properties. It consists, in general, of about a 4″ base of hydraulic cement concrete or bituminous concrete, for a foundation. On this base is laid a ½″ (finished) cushion coat or binder which contains about 3% more asphalt cement than the surface coat. The surface or wearing coat is laid with a finished thickness of about 2″. It is composed of asphaltic cement (85 parts pure asphalt and 15 parts heavy petroleum oil) 15%, sand and stone dust 80%, and crushed carbonate of lime 5%, more or less; the proportions often being varied. A bituminous base is composed of broken stone coated or mixed with coal-tar cement. It usually calls for a slightly thicker cushion coat than the above, say 1″ to 1½″, and also a 1½″ wearing coat.

Asphalt Paving Blocks are made in a variety of shapes, and laid on sand, gravel, or concrete foundation.

Bituminous-Rock Pavement is made from bituminous sandstone or limestone, of which extensive quarries are worked in California, Kentucky, and elsewhere. The quarried rock is broken up, melted, and rolled while hot. A proper amount of asphalt is added if necessary to give the required proportions. The product is commonly called rock asphalt. This is heated to about 200° F. and spread in a finished layer, after rolling, of about 2½″ in thickness.

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C.—PAVEMENT SPECIFICATIONS.

ALLEGHENY COUNTY (PA.) ROAD SPECIFICATIONS.

(Geo. T. Barnsley, Chf. Rd. Engr.)

Work by Contractor.—Do clearing, grubbing, leveling, grading, surfac-ing; make excavations, embankments, ditches, drains, gutters; construct masonry, stonework; build fences and protection railings required. In fact, complete road and structures. **Excavation.**—Straight classification, and in-cludes trees and clearing. Embankments rolled in layers not over 12" thick. Earthwork measured and paid for by cu. yd. in excav. When required, top soil to be removed from road surface and deposited as directed. May require clay and spongy material to be removed to any depth and replaced with gravel or coarse stone. Where possible, embankment slopes to be covered with 3" of surface loam. No work on covered drains, paved gutters or foundations to be paid for in excavation. Ordinarily, no allowance for excavation beyond lines of cross-section. **Clearing.**—Trees, stumps, bushes, roots, etc., to be removed; and no perishable matter allowed under em-bankments. **Drainage.**—Where required, a trench 12" wide at bottom and 15" wide at top to be excavated at least 30" below sub-grade; at least 3" of gravel to be placed in the bottom, and on this lay salt-glazed vitrified drain-pipe as directed, with bell and spigot joints, laid with open joints, ordi-narily; then fill to 6" above pipe with gravel between 1" and ¼" screening; then fill to top of trench with stone between 3" and 2" screening. Open ditches paid for as excavation; covered drains, by the lin. ft., price to include all expenses of trenching, furnishing and laying pipe, refilling, etc. **Dry Rubble.**—For small culverts, ordinarily, and for retaining walls. Spalls used only where needed for leveling or pinning; large stones for foundation courses, and for heads and faces of culverts. Covering-stones of culverts to be not less than 12" thick, laid close together, and cracks closed with pin-ners. All walls to be of coursed rubble, laid with best bed down; breaking joints at least 1 ft., and to have no pinners on the face; joints not over ¼". Wind walls of culverts, and all retaining walls, to be capped with roughly-dabbled coping stones 16" thick, at least 24" wide, and as long as possible. This masonry paid for by cu. yd., actual measurement. **Fencing.**—Posts to be straight locust, not less than 6" dia., with knots hewn down to face; spaced 8 ft. apart and set 3 ft. in ground, with 3½ ft. above surface. Top 4" sq. and notched into top of post so all surfaces will incline 45°; in addition to spiking, it shall be held with a 1/16" by 1½" iron strap 30" long, securely nailed. Side rail, 2" by 6", notched into inside of posts and spiked. Paid for by lin. ft. in place. **Shaping Sub-grades.**—Before the foundation is laid, the roadbed shall be shaped and rolled with a steam roller of at least 10 tons, all resulting depressions filled, and the surface again rolled. **Foundations.**—Upon the sub-grade so prepared, a foundation to be laid according to method a, b, or c: (a) For clay or wet soil, a Telford founda-tion, with stones 6" to 8" deep, not over 4" wide on top, and 6" to 16" long; set by hand with the broadest edges down and longest side across the road, on the sub-grade. Not over 10% to be less than 7" deep. Stones to fit joints; projecting points to be broken off with hammer; wedging stones to be driven until foundation is to grade and 8" thick. Foundation to be rolled with not less than 10-ton roller. (b) For soil of medium resistance, spread evenly to finished depth of 8" broken stone between 3½" and 2½" ring dia.; roll this with roller of not less than 10 tons until the whole mass is firmly imbedded into the earth sub-way and the top is 4" below finished grade. (c) Where nature of soil will permit, spread broken stone 6" finished depth, and roll as in b. **Surfacing.**—Upon the foundation prepared according to a, b or c, spread two layers of broken, close-grained, hard rock, granite, ligonier or limestone, free from dirt or dust, and broken fairly uniform or regular cubes, and comparatively free from flakes or splinters; crushing strength of not less than 20000 lbs. per sq. in. Con-tractor required to furnish certified copies of railroad weights for macadam material shipped and placed, or cu. yds. of same when furnished by local inspector. The first layer to consist of 2" to 2½" broken stone and to be 2" thick when consolidated; rolled with at least a 10-ton roller, all depressions filled with stone of same quality, and again rolled to finished surface 2" below finished grade. The second layer to consist of 2" in consolidated thickness of ¾" to 1½" broken stone to which may be added a proportionate amount of ¼" to ¾" screenings, free from dust; the whole surface then to be

rolled to finished grade. If ordered, dust from the crusher shall be laid on as a binding course, just sufficient to bond the top and make the surface smooth, and in no case thicker than ½"; then sprinkled and the whole rolled until mud flushes to the surface, and until roller causes no wave in surface. Engineer may vary macadam surfacing from 6" for heavy traffic, to 4" for light traffic; and increase width of macadam from 14 ft. to 22 ft., or decrease it to 10 ft. **Mile Stones.**—Cut granite, 10" sq. at top and 12" sq. at bottom; 5½ ft. long, 3 ft. above ground.

BOSTON (MASS.) PAVEMENT SPECIFICATIONS.

(William Jackson, City Engineer.)

Granite Block Pavement—Brick Sidewalk.

Covers, etc.—City to reset and repair catch-basin and manhole covers and other structures to be left in street. **Preparing Site.**—Ground to be brought to proper sub-grades; bottom of excavation for sidewalks and edgestones to be rammed and rolled, and bottom for roadway to be watered thoroughly, made solid and of even surface, with heavy steam road roller, places not accessible to be tamped with hand rammers; unsuitable bottoms to be excavated and refilled. **Edgestones.**—(a) New edgestones, including circulars and corners, to be of Quincy, Cape Ann, or other equally good granite, all of same color, cut in lengths not less than 6 ft., free from bunches and depressions, and have horizontal beds; ends to entire depth to be square with top, and set with mortar joints not over ¾"; to be out of wind; hammered surfaces to be full to line; to be 7" wide on top and 20" deep; to be hammered on top, fine pointed 3" down on the back, squared with the top, and fine pointed 10" down on the face; remainder of face to be straight split; face to be cut square with top. (b) Excavation to be 18" wide, and its bottom to be at the sub-grade of 24" below top of finished edgestone. (c) Upon this bottom, the foundation to consist of clean coarse gravel 4" thick when rammed; then more gravel to be spread, the edgestone laid thereon, with closed joints, spaces under stone thoroughly filled with gravel and tamped firmly to grade. (d) Excavation on each side of edgestone then filled to the sub-grade of roadway and sidewalk, respectively, with clean gravel, laid in 4" layers, each rammed and tamped under and around the edgestone, and joints carefully pointed, top, front and back, with mortar, of equal parts Natural hydraulic cement and clean, sharp sand. (e) Good clean gravel then to be laid, without ramming, against and up to top of edgestone on sidewalk side; after which the roadway paving is to be laid and rammed, care being taken not to disturb the grade of stone; and thereafter all gravel and other material on the sidewalk side to be excavated to depth of 12" below top of edgestone, and replaced by gravel in 4" layers, rammed, to sub-grade of sidewalk. **Roadway.**—(Either gravel base or concrete base.) **Gravel Base.**—Sub-grade to be 12" below finished surface of roadway; and on this lay gravel base, consisting of coarse-screened paving gravel, not larger than ¾", thoroughly rammed into a solid layer, 4" thick when completed. **Concrete Base.**—Sub-grade to be 16" below finished surface of roadway; and on this lay concrete base, consisting of 1 part Portland cement, 3 parts screened coarse sharp sand, and 7 parts broken stone, not larger than 2½" and very few smaller than ¼", and evenly graded between these sizes. (b) Templates and other forms used to hold concrete in place, to be set true to lines and grades, and secured firmly. (c) Cement and sand, just before concrete is to be used, to be mixed dry, then add only enough water to make a paste, thoroughly worked with hoes or other tools; broken stone is then wet, after which the materials are handled rapidly to the end; paste is spread evenly over pile of stones, on platform, and the whole turned over at least twice, thoroughly mixed, put in place at once, and thoroughly rammed so that interstices between stones are filled with mortar and water flushes to surface, made true and parallel with finished roadway. (d) After ramming, concrete allowed to set; defects remedied with good concrete. (e) When Portland cement is specified it is to be Dykerhoff, Star Stettin, Alsen, Alpha, Lehigh, Vulcanite or Atlas; when Natural cement is specified it is to be Natural hydraulic cement, equal to best Rosendale; other brands can only be substituted when approved. No cement will be tested in cars, or in course of transportation, or on the street. **Granite Block Paving.**—(a) Standard granite blocks, 3½" to 4" wide, 7½" to 8" deep, and 9" to 14" long (average not less than 11½"); edges to be sharp and straight, right angle both horizontally and vertically, faces to be straight-split, free from bunches

and depressions exceeding ½", and carefully piled. (b) Upon the gravel or concrete base, spread a layer of clean, coarse-screened bedding sand, on which lay the blocks in courses of uniform depth, at right angle with street line (ordinarily), with close joints, the longitudinal joints broken by lap of at least 2", sufficient sand being used to bring blocks to grade, after thorough ramming; then covered, and covering raked and swept until joints are filled, blocks then thoroughly rammed to unyielding bed, with surface parallel with grade and crown required; then again covered and raked and swept as before; blocks again rammed until solid and secure at grade and crown of finished roadway; no ramming done within 15 ft. of paving being laid; one rammer to each paver. (c) If blocks are laid with Gravel Joints, cover blocks (after being rammed) with clean, coarse-screened sand, dried by artificial heat if necessary, and rake and sweep until joints are filled; entire area covered with 1" layer. (d) If blocks are laid with Pitch Joints, lay as above, but the covering is to be washed pebbles, equal to best Long Island white pebbles, ¾" to 1½", thoroughly heated and raked or swept, filling joints to within 1" of surface; joints then filled with paving cement of proper consistency, flush to grade and crown of finished roadway; the cement left upon top of blocks to be covered with dry sand, sufficient to absorb the cement if required. The paving cement to be obtained by the direct distillation of coal tar, and kept at a temperature of 300° F. while being used. **Flagging Crosswalks.**—(a) Granite flagging stone, each exactly 2 ft. wide, not less than 4 ft. long, of same thickness as the others, not less than 6" nor more than 7", of best grade and quality, uniform color, top rough pointed, and ends jointed and square-cut to full depth of stone. (b) Upon the gravel or concrete base, spread a layer of clean, coarse-screened bedding sand, in which lay the flagging crosswalks; to be rammed and tamped to a solid and unyielding bed, sufficient sand being used to bring surface of flagging to grade and crown of finished roadway, after ramming. (c) If crosswalks are to be laid with Gravel Joints, fill joints with sand as for block paving with gravel joints. (d) If crosswalks are to be laid with Pitch Joints, fill joints with paving cement as for block paving with pitch joints. **Brick Sidewalks.**—(a) Bricks to be burned hard entirely through, straight-edged, of compact texture, regular and uniform in shape and size; bricks which after being thoroughly dried and then immersed in water for 24 hrs. absorb more than 16% of their volume, may be rejected; any edge of a brick sidewalk not against a curb or buildings is to be supported by a continuous spruce plank, 2" by 8", held by 2" by 4" spruce stakes driven in the ground. (b) Excavation for sidewalk is to have its bottom brought to subgrade 8" below finished surface of walk, and on this lay foundation consisting of coarse-screened paving gravel, not larger than ¾", rolled and rammed so as to be 4" thick when completed. (c) On this foundation, spread a layer at least 2" thick of clean, sharp sand, parallel with finished grade of walk; on this, the bricks are to be laid on their widest side, in courses of uniform width and depth, at right angle with street, or in herring-bone fashion, with close joints, all longitudinal joints broken by at least 2"; the bricks then to be covered with clean, fine, dry sand, using screen of 20 meshes to an inch; and upon the bricks, a plank, covering several courses, is to be placed and rammed carefully with a heavy hammer to a firm bed, with surface to proper grade; then spread fine sand over surface and sweep or rake so as to fill joints.

Wood Block Pavement—Brick Sidewalk.

Covers, etc.—(Same as for granite block pavement.) **Preparing Site.**—(Same as for granite block pavement.) **Edgestones.**—(Same as for granite block pavement.) **Roadway.**—(Either gravel base or concrete base.) **Gravel Base.**—(Same as for granite block pavement.) **Concrete Base.**—(Same as for granite block pavement, except that sub-grade is to be 10½" below finished surface of roadway for wood block pavement.) **Flagging Crosswalks.**—(Same as for granite block pavement.) **Wood Block Pavement.**—(a) Southern long-leaf yellow pine, not less than 90% of heart, texture permitting satisfactory treatment; sticks inspected at works before being sawed into blocks. (b) Blocks to be of sound timber, free from bark, loose or rotten knots, or other defects which would be detrimental to life of block or interfere with paving; no second growth timber allowed. (c) Blocks to be well made, rectangular and of uniform size; depth (parallel to fiber) 4"; length not less than 8"; width not less than 4"; depth and width to be exact. (d) The method of Treatment to conform to the best and most advanced knowledge of the art, the purpose being to allow contractors to manufacture block by

following any preferred detail and by use of any process which may properly be adapted to secure the results demanded, namely, that all parts of each block shall be thoroughly impregnated with the preservative (an antiseptic and water-proofing oil), not less than 20 lbs. per cu. ft. of wood; the block not to split or warp, and to have a specific gravity greater than that of water. (e) The preservative to have a specific gravity not less than 1.12 at 68° F. When distilled in a retort, with the thermometer suspended not less than 1" above the oil, it is to lose not more than 35% up to 315° C. and not more than 50% up to 370° C. Oil to be free from adulteration or foreign material. (f) After treatment the blocks are to show such waterproof qualities that, after being dried in an oven at a temperature of 100° for a period of 24 hours, weighed, and then immersed in clear water for a period of 24 hours and weighed, the gain in weight is not to be greater than 3%. (g) Material and blocks may be rejected if not satisfactory. (h) Upon the surface of the concrete foundation is to be spread a bed of cement mortar ½" thick, the surface to be composed of slow-setting Portland cement and clean, sharp sand, free from pebbles over ¼" diameter, 1 part cement to 4 parts sand; this mortar top to be thoroughly rammed into place with concrete rammers until all unevenness in the concrete is taken up, and is then to be "struck" to a true surface parallel to top of finished pavement. (i) On this mortar surface, lay the blocks with the grain vertical and at such an angle with the curb as may be directed; to be laid in parallel courses with tight joints, firmly imbedded in the mortar bed so as to form a true and even surface. (j) Joints then to be filled with cement grout, 2 parts clean sand and 1 part Portland cement, mixed to a liquid form, and the surface of the block slushed with same and the joints swept until completely filled; expansion joints, filled with a paving cement of proper consistency, to be made next the edgestones. Surface then covered with ½" of screened sand. (k) Where grade of street is more than 3%, the blocks are to be not less than 8" nor more than 10" long, and the upper edge of each block is to be cut away for a width of ¼", and a depth of 1", to provide a transverse groove between each course of blocks when laid in place; or such other construction is to be used as will provide an equally good foothold for horses. **Brick Sidewalks.**—(Same as granite block pavement.)

Asphalt Pavement.

Bituminous Concrete Binder.—(a) On the concrete base, covered with Trinidad asphalt, lay the binder. (b) In making, use 15 gallons Trinidad asphaltic cement and 1 cu. yd. of crushed stone, not over ¾", heated and thoroughly mixed. (c) In using, this binder, while hot and plastic, is to be evenly spread and thoroughly rolled until the roller ceases to make any impression, the compressed binder to be at least 1½" thick, on which an asphalt wearing surface is to be laid. **Trinidad Wearing Surface.**—For Trinidad asphalt pavement, the wearing surface is to be composed of (1) Trinidad Lake asphalt, specially refined and brought to a uniform standard of purity and gravity; (2) heavy petroleum oil, freed from all impurities and brought to a specific gravity of 18° to 22° Baumé, and a fire test of 250° F.; (3) sand entirely free from clay or other objectionable material, of such size that none of it will pass through a No. 80 screen, and all through a No. 10 screen; (4) powdered carbonate of lime of such degree of fineness that 15% by weight will pass through a No. 100 screen, and all through a No. 26 screen. (b) In making, 100 parts of the asphalt and 15 to 20 parts of the petroleum oil are to be made into an asphaltic cement, which is to have fire test of 250° F., and a temperature of 60° F. is to have a specific gravity of 1.19; this cement and the sand are to be kept heated separately to about 300° F., and the carbonate of lime is to be kept cold; 70 to 83% of the sand so heated and 5 to 15% of the lime while cold are to be thoroughly mixed together, but the lime may be reduced or omitted if the sand is satisfactory in quality and quantity; to this mixture, while so heated, is to be added 12 to 15% of asphaltic cement at a temperature of about 300° F., kept at that temperature, and thoroughly mixed in a suitable apparatus. (c) In using, spread the above mixture, at about 250° F., evenly on the concrete binder by hot iron rakes, to produce a uniform surface; then compress with tamping irons and hand rollers and sweep a small amount of dry hydraulic cement over it; then thoroughly compact by steam roller, not less than 5 tons, until roller fails to make any impression on surface, the finished wearing surface not to be less than 1½" thick. **Sicilian Wearing Surface.**—(a) For Sicilian asphalt pavement, the wearing surface is

to be composed of (1) natural bituminous limestone rock, mined by the "United Limmer & Verwohle Rock Asphalt Co., Limited," at Ragusa, Sicily; (2) limestone rock mixed by said company at Vorwohle, Germany. (b) In making, 3 or 4 parts of the Ragusa rock are to be thoroughly mixed with 1 part of the Verwohle rock, and this mixture to be crushed, pulverized to a powder and passed through a fine sieve, nothing being added or taken from the powder. (c) In using, spread the above mixture, at about 160° F., evenly upon the concrete base, and compress evenly by heated hand rollers and rammers, smooth by heated smoothers and roll for 2 or 3 days with a heavy iron roller until it ceases to make any impression, the finished wearing surface not to be less than 2" thick. **Coal Tar Painting.**—The wearing surface, to a width of 24" from curb, is to be painted with coal tar distillate and ironed with hot smoothing irons, to make a continuous layer without holes.

Bitulithic Pavement.

Sub-Grade.—6" below finished surface of roadway. **Crushed Stone Foundation.**—Upon the sub-grade, lay a foundation consisting of a layer of hard crushed stones to a depth of 6", and compress with heavy steam road-roller. Upon these stones, spread a thin layer of Warren's No. 1 Puritan brand bituminous semi-liquid cement, to be flexible and to unite freely with the cold stones. Upon this cement, spread a heavy coating consisting of 1 gallon of Warren's No. 24 Puritan brand hard bituminous cement to each sq. yd. of surface, the wearing surface immediately spread thereon, and the stones firmly bound together and with the wearing surface by this coating. **Wearing Surface.**—Hard crushed trap rock to be heated in a rotary mechanical dryer to a temperature of about 250° F. This material then to be elevated, passed through a rotary screen having 6 sections, each with a different sized opening, the largest $1\frac{1}{4}$" and the smallest $\frac{1}{10}$" diameter, the materials to be separated by these sections into 6 lots, each lot consisting of the materials passing through one of the sections of the screen into a separate compartment or bin. The materials in each lot are then weighed separately and mixed with the materials of each of the other lots into batches, in the proportions which shall have been determined by laboratory tests to give the best results, that is, the most dense mixture of mineral aggregate having inherent stability; and if the fine crushed rock does not provide the best proportion of fine-grained particles there must be supplied not more than 15% of hydraulic cement, pulverized stone or very fine sand. Each batch is then passed into a "Twin Pug" or other approved form of mixer, and then mixed with a sufficient quantity of Warren's No. 21 Puritan brand bituminous waterproof cement to thoroughly coat all the materials and fill all voids; the cement when used is to be at a heat between 200° and 250° F., and the amount used with each batch is to be accurately weighed and used in such proportions as shall have been determined by laboratory tests to give the best results, the mixing to continue until mixture is a uniform bituminous concrete, that will when cold have as closely as practicable the solidity and density of solid stone. This concrete is immediately after mixing to be hauled to the street, spread on the No. 24 cement, and compressed with a steam road-roller to finished thickness of 2". **Surface Finish.**—On the wearing surface, a thin coating of Warren's quick-drying bituminous flush-coat composition is to be so spread over the surface that any unevenness or honeycombing in the concrete is filled. A thin layer of stone chips is then to be rolled into the surface so it will be gritty and not slippery. **In General.**—Each layer of the work to be kept as free as possible from dirt so that the layers will unite. Bituminous cement used shall be free from water, petroleum oil, water gas or process tars, and all light oil, naphthalin and other crystalline matter susceptible to atmospheric influences removed by refining.

Macadam Roadway—Crushed-Stone Sidewalk.

Granite Block Paving.—To be used for gutters and brows for crosswalks; blocks to be $3\frac{1}{2}$" to $4\frac{1}{2}$" wide, 7" to 8" deep, and 7" to 12" long (average not less than 10"). Sub-grade 12" below finished surface; on this, the gravel base 6" thick when completed. On this foundation, spread a layer of bedding sand, and in this lay the blocks, in courses of uniform width and depth at right angle with street line (ordinarily), with close joints, longitudinal joints varying at least 2", enough sand used to bring blocks to grade; usual spreading of gravel on surface, sweeping and ramming. **Brick Block Paving.**—To be used for gutters and crosswalks; blocks to be re-pressed, hard, tough, com-

pletely vitrified, rectangular in shape, uniform in size and color, homogeneous in texture, and substantially 3" wide, 4" deep, 9" long. Rattler Test for abrasion to be as specified by Natl. Brick Manfrs'. Ass'n*; average loss in weight not to exceed 20%, and no one brick to lose more than 24%. Absorption Test, using 5 bricks previously subjected to rattler test, or bricks broken in half; bricks to be dried 48 hrs. at 230° to 250° F., then weighed and immersed in water 48 hrs., wiped and weighed, increase in weight not to exceed 4%. Sub-grade for brick blocks to be 10" below finished surface; on this, lay 4" layer of paving gravel. On this foundation, spread a layer of bedding sand, and in this lay the bricks, in courses of uniform width and depth at right angle with street line (ordinarily), with close joints, longitudinal joints lapping at least 2", enough sand used to bring blocks to grade, usual spreading of gravel on surface, sweeping and ramming. **Macadam Surface.**—Sub-grade 6" below finished surface; on this lay the macadam surface, made as follows: Hard, durable, broken stones, either of the best quality of broken trap or Roxbury conglomerate, or of acceptable field stone, 2½" to 1" diameter screen, free from round or other ill-shaped or improper stones, to be spread over whole surface of base, and thoroughly rolled and packed with 15-ton steam road-roller, until surface is ½" below finished roadway; spaces between stones then to be filled with fine screenings or binding gravel applied in at least 3 layers, each layer thoroughly worked in by wetting and rolling aforesaid before the next layer is applied, and during the operation the surface to be brought, with the broken stone, to a finished grade. **Crushed-Stone Sidewalk.**—Sub-grade 5" below finished surface of walk; on this, spread 2½" to 1" broken stone, making a 3" layer after rolling and ramming. On this, a layer of No. 2 crushed stone to be spread and thoroughly rolled to 2" thick. On this, spread a layer of fine screenings, trimmed, watered and rolled with steam, horse or hand rollers so as to make a hard, compact sidewalk at required grade. Edges to be supported by spruce plank if required.

THE PROPER CONSTRUCTION OF BRICK STREET PAVEMENTS.

(Will P. Blair†.)

Sub-Grade.—Must be drained, graded, compacted and parallel with grade of finished street; not essentially different than required for other pavements. A depression here and there, a spot of loose earth, a lack of thorough compaction, or a wet condition due to improper drainage will be followed by disaster to the street as a whole. **Foundation.**—(a) Impossible to define the proportions of cement, sand, broken stone or gravel that shall compose the mix, because of the varied qualities of these materials; but in order to secure maximum strength they must be mixed dry in the first instance, and then thoroughly mixed after the water is applied. (b) Either in the machine or hand mixing, an intelligent supervision is worth while at all times. The value of the concrete is often reduced at least 50% by carelessness, by ignorance or indifference, by application of too much or too little water, by lack of proper proportion of some one or another of the other ingredients composing the foundation, resulting in 1, 2, or 3 sq. yds. of the concrete foundation being of no more value than merely loose piles of stone or gravel. (c) The concrete surface as it is put in place must have a uniform surface with grade of finished street, and the surface must be smooth. This cannot be accomplished by the eye; the grade stakes should be set at no greater distance apart than 4 or 5 ft. If any stone used in the concrete exceeds 2" in largest diameter it will be next to impossible to accomplish the condition desired. Sufficient water should be used in the mixing so that one man can smooth the top with an ordinary dirt shovel—never should it be so stiff as to call into use a rammer. When we say "smooth surface" we mean that a greater variation than 1" shall not be allowed. **Sand Cushion.**—Must be 2" thick; if less, it will not afford a sufficient relief from the vibration created by the impact of travel; if more, it cannot be sufficiently compacted to afford a support to the load coming upon the brick street, and prevent cracking and crushing of the joints of the cement filler which is required in finishing the street. Thus, this cushion must be of such a thickness that will afford relief from the impact and weight, slight though it be, yet sufficiently unyielding to furnish the support for the load it must bear. **Expansion Cushion.**—This must be provided, after the sand cushion is spread, by

* See page 507.

† Secretary National Paving Block Manufacturers' Association.

icing next the curb a board of sufficient width to extend above the height the brick; and in order that it may be drawn readily, it is advisable that vedge be dropped at intervals of 3 or 4 ft. behind this board and extending ove it from 3" to 4"; the wedges to be ½" thick at top; the thickness of ard varying with width of street, providing sufficient thickness ranging from to 1½"*. **Laying.**—The brick should be placed in the street with the best ge up. This is a rule universally required of brick construction in masonry rk. In order that this shall be done, the brick should be delivered to the rson who drops them into the street with the face placed to suit the hand eration of such person, called the brick layer. The brick should not be d in place in close contact with one another. Such practice will result in brick being chipped, and it will be impossible to put in the cement filler perly in the interstices. **Inspection.**—After the bricks are placed, they uld be inspected before being rolled so that as few bricks as possible will disturbed after the rolling. **Rolling.**—The roller should be a light one, m 4 to 5 tons; one that is easily handled, and can move rapidly upon the face of the brick. The rolling should proceed from each side along the b, working toward the center of the street; then cross-rolling at angles 45°; again rolling longitudinally and cross-rolling as before, continuing s process until the bricks are thoroughly compacted into the sand, so that grade of the pavement shall be as intended and the inequalities of the hion ironed out by the sand being pushed up in the interstices of the ck, a condition always found in the case of properly rolled streets by an ven amount pressed upwards in the interstices running from ½" to ¾" and sibly 1" in some cases. (The use of the horse roller and the 8 to 10-ton am roller should be prohibited). **Wetting.**—After rolling, the bad bricks uld be replaced with good ones, the street swept clean, and then sprinkled, the use of a nozzle either upon a sprinkling can or a hose which will mit but the finest spray of water to come upon the street. **Cement Filler.**— sand to be clean, sharp and dry; the mixing, not over ½ bu. of sand and ie amount of Portland cement, to be placed in box and mixed dry until is is of even shade; water then added, forming mix. like thick cream. st be kept in constant motion from time of mixing until floated into ts. Mix. to be removed from the box to the street surface with a scoop vel; box to be 3½ to 4 ft. long, 27" to 30" wide and 14" deep, with one ier low, and 8" to 10" above pavement. The mix., from the moment it hes the bricks, shall be thoroughly swept into the joints. Two boxes e provided where street is under 20 ft. wide; over 20 ft., 3 boxes. This k of filling should be carried forward in line until an advance of 15 to 20 ls has been made, when the same force shall be turned back and cover same space in like manner, except that the proportions for the *second shall be ⅔ Portland cement and ⅓ sand.* To avoid possibility of thickening ny point, there should be a man with a sprinkling can, the head per- ted with small holes, sprinkling gently the surface ahead of the sweepers. in ½ to ¾ hour after the second coat is applied, and grout between joints fully subsided, and initial set is taking place, the whole surface is to be tly sprinkled and all surplus mixture left on the tops of the bricks t into the joints, bringing them up flush and full. Then, after sufficient for evaporation has taken place, a ½" layer of sand shall be spread over whole surface, and if under a hot summer sun, the sand should be kled occasionally for a few days.

CINCINNATI (OHIO) PAVEMENT SPECIFICATIONS.

Boulder Pavement.

ub-Grade.—Brought to even surface, parallel with grade proposed for ment, making necessary excavation and embankment. Soft or spongy , etc., to be removed, and space filled with broken stone, rammed or l. Sub-grade surface to be compacted by rolling with steam roller ing not less than 250 lbs. per lin. in. of roller; portions not accessible rammed. Finished sub-grade to be 14" below surface of pavement. **dation.**—Upon the sub-grade thus prepared the entire surface of the vay between the gutters will be spread evenly with a layer of sound, hill limestone, broken into fragments as nearly regular as practicable, ver 2½" dia.; the layer to be of such thickness that when thoroughly

* This cushion to be composed of pitch or asphaltum composition, ⅔ the allotted space, the remaining top third to be filled with sand.

compacted its surface shall be 6" above true surface of sub-grade, using roller above described. Where additional material is required, after rolling, to bring surface to proper grade, the rolled surface must be loosened to depth of 2" to receive the new material, and afterward rerolled. **Gravel Layer.**—On the broken-stone foundation, spread a layer of gravel, loose, and of sufficient depth in which to pave the boulders. The gravel must be clean and free from animal or vegetable matter or refuse; it must not contain more than 15% of clay or loam, nor pebbles exceeding 1" longest diameter. **Paving (General).**—In paving, the foundation work shall be kept laid to proper grade, rolled or rammed into proper slope or shape at least 100 ft. ahead of paving; be laid in sections of not less than 100 ft. in length, entirely free from gravel, rubbish, etc., and thoroughly swept, ready for inspection. **Boulders.**—The boulders will be laid down between the gutter-flagging; to be of good shape free from flaws or breaks, of hard, imperishable substance, no sandstone or limestone boulders to be used, no stone to measure less than 4" nor more than 6" in longest diameter; the stones to be carefully assorted and so placed that the largest shall be next the gutter-flagging and gradually diminish in size to the center. **Laying.**—No stone when set in an upright position, to show a horizontal diameter of less than 3" or more than 6" in any direction, and they must be set firmly on the foundation in a perfectly upright position, with small ends down, and as closely and compactly together as possible; none to be laid flat or on side edge. When boulders have been set for a distance of 60 ft., the first 50 ft. must be lightly rammed, after which a covering of gravel, sufficient only to fill the interstices, will be spread over the surface and thoroughly broomed in, when the whole will be thoroughly rammed with not less than 40-lb. rammers. When two sections (aggregating 110 ft.) have been treated, the first 100 ft. will be again covered with gravel, broomed, rammed, and ready for inspection. As soon as each section of 100 ft. is accepted, a final covering of 2" of gravel will be spread over the entire surface.

DETROIT (MICH.) PAVEMENT SPECIFICATIONS.

General.

Old Pavement and curbing to be measured to contractor as excavation; all old material and rubbish, including surface dirt, to be removed, city reserving curbing, etc. for new pavement, same to be delivered free by contractor to nearest city yard or for distance of 1 mile, if required. Contractor to use care in removal of old cushion sand to prevent mixture with other material. **Grading.**—After excavation to sub-grade, should there be places in street which are not firm, the earth must be taken out and the space refilled with crushed stone and rolled. Before laying the concrete foundation and after the curbstone has been set, the sub-grade shall be rolled with 7-ton roller furnished and operated by City, at cost to contractor of ½c. per sq. yd. After this rolling, high places shall be brought to sub-grade and depressions filled with concrete at expense of contractor. Sub-grade to be properly planked before teaming is allowed. **Curb Trench.**—Trench to be excavated on each side of roadway to depth sufficient to set curb on concrete base 6" deep and of such width as to allow concrete backing to the curb of 4" thickness; bottom to be smoothly trimmed parallel to curb grade. **Stone Curbing.**—Old curbing taken up shall be rejointed, edges rounded, retopped, refaced and reset wherever directed, as per specifications for jointing, facing and setting new curb. New curb to be of best quality of granite, Medina, North River Blue, Elyria, Berea, or other curb as may be bid upon and ordered. The stone shall be 4" thick (ordinarily), at least 3 ft. long, and 18" deep; upper corner next to roadway to be rounded with radius of 1½". Top and face of above-named stone curb to be dressed to what is known as 4-cut work, true and even, and the softer curb to be crandall dressed, true and smooth, all with close joints at the ends of at least 7" below top of curb, and a joint of not to exceed ¼" for the remaining 18" depth of curb; stones to have straight and even face on gutter side to depth of 7" below top. Back of curb to be dressed 3" down from top. Top to be dressed to a straight line, and to ¼" bevel in 5", and to uniform thickness or 4" (ordinarily). Stones to be taken out of wind, set with close joints to street line and grade on concrete bed 6" deep, and to full width of trench, and backed up with concrete to within 4" of top of curb; the remaining 4" behind curb to be filled with suitable earth well compacted. The crushed stone for concrete to be ½" to 1". **Concrete Curbing.**—Concrete for cement curb, plain or rein-

forced with metal, shall consist of not more than 4 parts of 1" to ½" broken stone or slag, 2 parts sharp sand, and 1 part Portland cement; curbing to be of approved construction and finish. **Foundation.**—When the roadbed has been prepared, it shall be covered with a layer of concrete not less than 6" thick, and rammed until the surplus cement mortar appears on the surface, which shall be smooth and parallel to roadbed. No teaming allowed until set and covered with plank; defects to be repaired before work proceeds. **Concrete.**—Broken stone, 2" to ½", may be from boulders, granite, syenite, slag, or hard limestone; it must be clean, screened if necessary to free it from dirt or stone refuse, and wetted before being placed on mixing boards. The concrete shall consist of 1 part natural cement, 2 parts sand, and 4 parts stone or slag; or 1 part Portland cement, 3 parts sand, and 6 parts stone or slag; depending upon which cement is specified.

Brick Pavement on Concrete Foundation.

Cushion.—Coat of clean, sharp, bank, lake or river sand, well screened, to be spread over concrete foundation to depth of 1½" when compacted. **Brick Paving.**—Shall consist of best quality of sound, hard, burned paving brick, or cement brick, made especially for street paving purposes, and to stand all reasonable tests as to durability and fitness, to which paving material is usually subjected. Bricks to be round or bevel-edged, straight, free from cracks and other defects, of uniform size, and of approved quality, equal to approved sample in office. **Handling and Piling Brick.**—Brick to be handled with brick tongs, carefully, to avoid breakage or chipping, and piled on street in rectangular piles, with uniform tiers or courses to aid counting. **Manner of Laying.**—Upon the cushion, the pavement to be laid with a single layer of brick on edge, end to end, in right angle or diagonal courses across the street, as may be directed, except at street intersections and along street railway tracks where the courses are to be placed at such angles as may be designated. Bricks to be set in straight courses, with body of bricks close together, sides and ends touching, and breaking joints at least 2" with the bricks in adjoining courses; to be set perpendicular to grade of street, and to height of from ¼" to 1", or as may be directed, above the true grade and crown of street when finished, to allow for settlement and pounding and rolling. Whole bricks to be used, except in starting a course or in making a closure, when not less than half bricks may be used in breaking joints, tight and close at ends. **Rolling and Tamping.**—The paving when laid, and before filling of the joints and top dressing is put on, shall be rolled three or more times lengthwise of street, with not less than 7-ton roller, furnished and operated by City at ½c. per sq. yd. Parts which cannot be rolled shall be rammed. **Tar Filling.**—Whenever tar filler is specified, the joints to be filled to the bottom with paving cement obtained from the direct distillation of coal tar, and shall be residuum thereof, such as is ordinarily numbered 5 and 6 at the manufactory, or any other approved composition; quality and temperature to be approved. Extra material and care to be used at gutters, catch-basins, etc., to prevent leakage of water to sub-roadway. **Grout Filling.**—A joint, ½" in width, next to and parallel to the curbstone, to be filled to top with a composition of coal tar cement, mixed with at least 10% of refined asphalt, and the whole mixed with sufficient still wax to prevent softening or brittleness in hot or cold weather. On streets with car tracks, three rows of brick to be laid along outside of rails in form of stretchers, with broken joints, and all joints filled with above composition. Balance of pavement to be filled with grout, composed of 1 part Portland cement and 1 part sand. The grout to be prepared in small quantities, stirred while being applied to pavement, and swept into joints with proper brooms; no settlings or residue to be used. Filling to be done in two or more applications of grout; the first ½" in depth from the bottom to be filled with grout somewhat thinner than required for the remainder; balance with a thicker grout, and if necessary refilled; brick to be presently wet. Teaming and traffic prohibited for about one week. **Top Dressing.**—Surface of paving to be covered with ½" top dressing of sand. **Joining Stone.**—At intersections with paved streets and alleys having a cement surface, the pavement shall be finished up to a Medina stone over 4" thick, not less than 16" deep, and not less than 30" long, to be set on concrete bed 6" deep, 8" wide and backed with 4" of concrete to within top. Stones to be dressed on top, pointed down on both sides to bottom of surfacing material, having good joint for depth of 3" from top, and to grade.

Sheet Asphalt Pavement on Concrete Foundation.

Binder.—Upon the concrete bed a binder course to be laid, composed of clean, broken stone, varying in size from fine to coarse, all to pass a 1½" ring in its larger dimensions. Stone after being heated shall not contain less than 5% nor more than 15% of material passing a No. 10 screen. Stone to be heated not higher than 350° F. in suitable appliances; then thoroughly mixed by machinery with asphaltic cement, such as is acceptable for surface cement; penetration, 60 to 90, at 77° F., City standard, at such temperatures and in such proportions that the resulting binder will have life and gloss without an excess of cement. Should it appear dull, from overheating or lack of cement, it will be rejected. While hot, it will be hauled upon the work and spread upon the base, so that when compacted it will be at least 1½" thick, and immediately rammed and rolled until it is cold. **Wearing Surface.**—Upon the binder course will be laid the wearing surface, or pavement proper, the binding material of which must be a cement prepared from asphalt, refined until free from water and volatile oils. This surface to be composed of asphaltic cement, clean, sharp-grained sand and fine absorbent mineral dust. The Asphaltic Cement must be prepared from refined asphalt of one of the following brands: Trinidad, Bermudez, Obispo, or any other equally as good. The refined asphalt shall be softened into a proper asphaltic cement by the addition of a suitable flux. The flux must be either a residuum from eastern petroleum oil, Texas petroleum oil, or a maltha from which the light oils and water have been removed by distillation. The asphaltic cement to be satisfactory, practically free from water, and within the range of 40 and 70 penetration (amount of penetration to be fixed by Department of Public Works), at 77° F., City standard. The Sand to be hard grained and moderately sharp. On sifting, it should have at least 15% caught on a 40 mesh to the inch screen; 25% pass an 80 mesh, 10% of which must pass a 100 mesh screen. If the sand used does not contain the desired fine material, mineral dust can be added to make up the deficiency, and in any case at least 5% of such mineral dust shall be used. The Mineral Dust shall be fine, absorbent inorganic dust, not acted upon by water, the whole of which shall pass a 30 mesh screen, and at least 75% pass a 100 mesh screen. The Asphalt Paving Mixture to be composed of above materials mixed in proportions by weight, depending upon their character and the street traffic and character of asphalt, and will be determined by the inspector; but the per cent bitumen in any mixture, soluble in carbon di-sulphide, shall not exceed the limits 9 to 13 per cent. Proportions of mixture must not be varied from those specified. The sand, or the mixture of sand and stone dust, and the asphaltic cement, shall be heated separately to about 300° F. The dust, if limestone, will be mixed while cold with hot sand, in the required proportions, and then mixed with the asphaltic cement at the required temperature and in the proper proportion, in a suitable apparatus, so as to effect a thoroughly homogeneous mixture. The mixture thus prepared will be brought to the street in carts, at a temperature of not less than 230° nor more than 350° F., depending on the asphalt in use; canvas covers to be used if the temperature of the air is less than 60° F. It is then to be spread to a thickness of at least 3" by means of hot rakes, to a uniform grade, so that when compressed it will have a finished thickness of at least 2". The surface to be compressed by rolling, after which a small amount of hydraulic cement will be swept over it, and it will then be thoroughly compressed by a steam roller weighing not less than 175 lbs. to the inch run, the rolling being continued for not less than 5 hours for each 1000 yds. of surface. Contractor to furnish a 10-year guarantee. **Retaining Stone.**—(Same as for brick pavement.)

Cedar Block Pavement on Concrete Foundation.

Cushion.—(Same as for brick pavement.) **Cedar Blocks** to be 4" long, of best quality of sound, selected, live timber, stripped of all bark and free from traces of rot or indications of decay, and not less than 4½" nor more than 9" in diameter and so selected in size as to make a close-jointed pavement. **Filling.**—Spaces between blocks to be filled with screened gravel or crushed granite or boulders of size varying from ½" to 1" diameter and free from dust, sand, loam or thin stone, screened, when necessary, through a wire screen set at an angle of 60°, with meshes of not less than 8" lengthwise by ½" in width; tamped with iron tamping bars as required, and then the surface well rolled by City roller at cost to contractor of ½c. per sq. yd. After rolling

spaces between gravel or stone filling of the blocks to be completely filled from bottom to top with paving cement obtained from the direct distillation of coal tar, and shall be the residuum thereof, such as is ordinarily numbered 5 or 6. Quality and temperature to be approved. Extra care at gutters, catch-basins, etc. **Top Dressing.**—(Same as for brick pavement.) **Retaining Plank.**—Where cedar pavement is laid, the pavement at intersections of unpaved streets and alleys to be finished up to a piece of timber 3″ thick and 12″ deep, set on 6″ of concrete, and extending across the proposed width of roadway of such street or alley. For all other kinds of pavement a stone header will be used, similar to, and set as stated for retaining stone. **Retaining Stone.**—(Same as for brick pavement.)

EASTON (PA.) PAVEMENT SPECIFICATIONS.

(John McNeal, City Engineer.)

Macadam and Telford Roads.

Work.—Done by City force; not by contract. **Grading.**—Completed rade to have slope of from ½″ to 1″ per ft. from center to sides, according to ercentage of grade of street. **Roadbed.**—Must be rolled firm with steam oad roller; depressions formed by rolling to be filled and rolled again to nished sub-grade. **Macadam Foundation.**—On the sub-grade, place 3″ to ″ crushed stone, spread evenly, and roll with road roller until none of the tones move under the roller; all material to be added dry, but water added head of the roller. This course to be 5″ thick. **Telford Foundation.**—On the ib-grade, place the bottom course composed of stones 8″ to 12″ long, 3″ to ″ wide, and 5″ deep, vertically by hand on their broadest edges and pointed : the top; stones to be laid in lengthwise courses across the street, and all terstices filled with broken stone, wedged with a hammer; projecting oints to be broken off to surface grade. This course to be thoroughly rolled ıtil stones do not rock under the roller. Clay may be used as a binder on is course if directed. **Second Course.**—(Same for either macadam or tel-rd.) On the foundation, lay a 3″ course of crushed stone, 1½″ to 2″, and ll until firm and solid, water being applied ahead of the roller. **Binder.**—ıe binder for the bottom and second course shall be limestone screenings, plying water ahead of the roller if necessary. **Surface.**—On the second ırse, a coat of 50% of ¾″ stone and 50% of screenings, properly mixed, l about 1″ thickness, shall be applied dry and rolled once before wetting, n alternate watering and rolling until finally completed, when the sur-e must be uniform to shape and grade. (The several courses of material st be of required depth after rolling, allowance for compression being at ıt one-half.) **Rolling.**—Each course to be rolled with the utmost thor-hness, the roller starting from the sides and working toward the center.

Concrete Curbs, Gutters and Sidewalks.

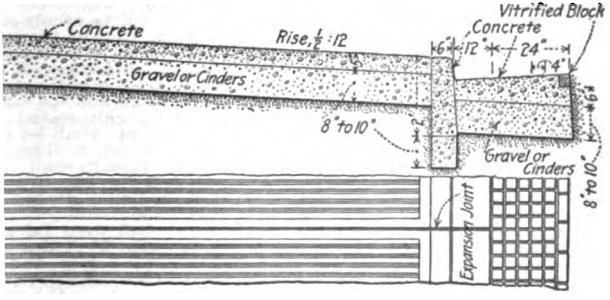

Fig. 3.—Concrete Curb, Gutter and Sidewalk.

ter sidewalk is excavated and shaped to proper depth and grade, the t curb, gutter and sidewalk shall be constructed in place, upon a bed vel or cinders 8″ to 10″ deep, well consolidated by ramming to an even

surface, and moistened before the concrete is placed thereon. The curb, gutter and sidewalk to be composed of concrete formed by mixing dry, 1 part Portland cement, 2 parts coarse, clean sand and 4 parts clean screened limestone or trap rock, crushed to pass through a 1½" mesh screen, to which shall be added sufficient water to form a concrete that when placed in the templets and thoroughly rammed, free mortar will appear on the surface. The ramming of the concrete in the forms shall be done with the proper tamping bars and other tools to insure a compact mass with full square corners. All exposed surfaces to be covered with a finished coat 1" thick, composed of 1 part cement, 1 part clean, fine hard stone screenings, and 1 part clean, coarse sand, applied before concrete has hardened. Top facing of curb, gutter and sidewalk to be thoroughly troweled to insure perfect contact; when sufficiently hard it shall be troweled and floated to a smooth true surface. Concrete curb to be 6" thick at top, 8" at base, and 24" deep, exclusive of foundation. Concrete gutter to be 6" deep, 3 ft. wide on all streets more than 20 ft. wide, and 2 ft. wide on narrower streets; surface of outside edge to be grooved with 4" squares for width of 2 ft. on the 3-ft. gutters, and 1 ft. on 2-ft. gutters, to prevent slipping of horses. Concrete sidewalk to be at least 5" deep. For the entire depth of curb, gutter and sidewalk, joints to be made with tar paper or by means of removable plates to form expansion joints or planes of weakness; joints to be not more than 10 ft. apart.

REINFORCED CONCRETE FOUNDATIONS.

See article entitled "Reinforced Concrete Foundations over Excavations on Paved Streets," in Trans. A. S. C. E., Vol. LX, p. 217 (1908).

EL PASO (TEX.) PAVEMENT SPECIFICATIONS.

(F. H. Todd, City Engineer.)

Petrolithic Pavement.

Preparing Roadway.—The roadway shall be so excavated or filled that after thorough rolling, or tamping with hand rammers at such points as cannot be well done with roller, its surface shall be 2" below and approximately parallel with surface of finished street. Soft and boggy places not affording a firm foundation shall be dug out, refilled and thoroughly tamped with good sound earth, cinders, gravel, slag, stone or concrete, as may be directed; the contractor to be paid for this excavation the same price as other excavation, if the soft and boggy place was not caused by him, but if caused by him, then at his own proper cost and expense. The entire roadway shall then be ploughed to a depth of not less than 6" nor more than 9", and *thoroughly* pulverized by cultivating, harrowing, or such other method as will accomplish the result. **Foundation.**—On this thoroughly pulverized roadway, including the intersections of all streets and alleys up to the property lines of the street being improved, the roadway shall be evenly coated with liquid asphalt, ½ gal. per sq. yd. of surface; it shall then be thoroughly cultivated to a depth of 6" until the liquid asphalt which has been applied is thoroughly mixed with the soil; then a second application of liquid asphalt, ½ gal. per sq. yd., shall be made and the area again well and thoroughly cultivated to a depth of 6", until the liquid asphalt and the material comprising the surface of the street are well and thoroughly mixed; then the third application of liquid asphalt, ½ gal. per sq. yd., shall be evenly spread over the entire roadway and the area for a third time well and thoroughly cultivated in such a manner that the liquid asphalt shall become thoroughly mixed with the street surface to a depth of 6". The street shall be thoroughly watered after each application of the liquid asphalt. The surface of the street shall then be brought to a grade approximately parallel to grade of finished street. **Tamping.**—The street shall then be tamped with a petrolithic rolling tamper until it is solid to within 2" of the surface. When the tamping of the road with the rolling tamper is begun, the rolling tamper shall be immediately followed by a cultivator set so as not to disturb the sub-base already tamped; said cultivator being reset as the tamping progresses, so as to cultivate to shallower depths. The cultivator shall be used continually during the tamping, the purpose being to prevent a too rapid solidification whereby the roadway would be solidified without being compacted from the bottom up. Upon this base prepared as above specified shall be applied liquid asphaltum, ½ gal. per sq. yd. of street surface. Wearing

Surface.—On the foundation, shall be placed a layer of hard, durable, crushed stone, 2" to 1" screen, spread evenly and to such thickness that after it has been thoroughly sprinkled, and rolled with a roller weighing not less than 10 tons, its surface shall be parallel to and ¾" below surface of finished street. Liquid asphaltum shall then be applied, ½ gal. per sq. yd. of surface; then a layer of crushed rock, 1½" to ¼" screen, shall be spread evenly to depth of ¾"; then the surface shall be thoroughly watered and rolled, followed by a coating of liquid asphaltum applied at rate of ½ gal. per sq. yd. then a light coating of rock screenings ¼" and under in size, and in sufficient quantity to absorb all the surface liquid asphaltum and produce a uniform surface. The pavement shall then be watered and rolled until it becomes hard, smooth, true to grade and cross-section, free from all hollows and other irregularities, until but slight movement takes place under the action of the roller. In streets or avenues having a street-railway track, the street shall be excavated to a point 6" below the ties and 6" beyond the end of the ties, and this space filled with broken stone or slag to the upper surface of the ties; this filling to be so well tamped with stones of assorted sizes, from 3" in greatest- to ¼" in least dimension, that but slight movement will take place under the ties during the passage of the electric car The space between the rails shall then be filled in the same manner as is provided for wearing surface on other portions of the street. It will be necessary, however, to provide a roller so formed as to make the flange-ways similar in form to that approved by the City Council, Jan. 9, 1908. With this roller, the surface of the street between the rails must be so compressed that but slight movement takes place under a 10-ton road roller. That portion of the roadway outside of the rails, shall be treated in a manner similar to the wearing surface of the remaining portion of the street. In case rock is encountered in excavating, it will be necessary to remove that to a depth of at least 3" below finished surface of street. **Liquid Asphaltum.**—The liquid asphaltum used shall contain not less than 75% of asphaltum at 80° penetration when tested at a temperature of 77° F. The specific gravity shall not be lower than 10° or higher than 11° Baumé, at a temperature of 60° F., and shall not contain more than 2% of water and sediment. In all cases the liquid asphaltum shall be applied at a temperature between 200° and 250° F. It shall be applied to the roadway by an approved form of sprinkler, such as will give a uniform distribution over the entire surface of the roadway **Concrete.**—The cement must be equal in quality to the best American Portland cement, and opportunity shall be provided to test it for 30 days before it is used in the work, in order to prove its strength and soundness. The Sand, for mortar, must be fine, sharp, and have grains of different sizes so proportioned as to make a dense aggregate. The Concrete, for pavement foundation or street railway foundation, must be made from 1 part by measure of Portland cement, parts sand, and 7 parts clean, sound, hard, broken rock, or clean gravel, of various sizes from 2½" in greatest- to ¼" in least dimension, and these sizes so proportioned as to give the greatest density. Measuring Boxes shall be provided, if required. After the cement, sand and rock or gravel have been thoroughly mixed dry, either by hand or machinery, it shall be wet in such manner as not to wash away the cement, and at the same time the mixing shall be done so that the wet particles of cement and sand are thoroughly incorporated as mortar, and each stone covered with mortar, the whole mass being in it *just enough* water to make the concrete, when in place, slightly "spongy." From the time water is first applied to the batch until the concrete is thoroughly rammed in place, the work must proceed rapidly and the ramming must be so well done that no voids remain in the concrete, and the mortar flushes to the surface. Before the concrete sets, but after it is rammed in place, enough hard, clean, broken stone, 2½" to 1½", to about cover ½ of the surface shall be evenly spread on it. These stones shall then be rammed through into the concrete to hold them firmly and yet leave it rough enough to securely hold the petrolithic wearing surface. The Concrete for shall be composed of 1 part Portland cement, 2½ parts sand, and 5 clean, hard, durable broken stone, ¼" to 2" in greatest dimension and in proportions as to give the densest mixture, and placed in the same manner as specified for concrete foundation. After the concrete has been thoroughly rammed in the gutter, and while it is yet soft, 1½" of mortar composed of 1 part Portland cement and 2 parts sand and as much fine crushed stone as will make the densest aggregate, shall be placed on top and well rammed, after which the surface shall be finished with the proper tools to conform accurately to the given lines and grades. Proper forms are provided for all concrete work. Expansion Joints of roofing paper

and bitumen, or of bitumen alone, must be provided and placed at such pe
and in such manner in the concrete as may be required, but not n
together than 20 ft. All concrete work shall be covered with earth and
after it has set and shall be kept wet for at least 1 week and shall be prote
from injury for at least 10 days after laying, to allow it to set prop
Marginal Curb.—Whenever a paved street is joined to an unpaved o
marginal curb of hard, durable stone at least 15" deep, 5" thick, and
long and with top surface broken to a straight line, must be provided
set, if required. This curb shall be set in 6" of the same class of concret
specified for foundation, and backed up with the same to within 6" of
top. The upper surface must conform to the cross-section of the st
General.—The price bid per sq. yd. for complete petrolithic pavement
include the excavation or filling required to bring the street to its establi
grade and surface, and further, must include the furnishing, placing
manipulation of all material necessary for the construction of this paveme
including all labor and necessary implements. No wearing surface shall
laid when the temperature of the air is below 40° F., and preferably,
asphaltum for the foundation and surface shall be applied during war
dry weather.

LOS ANGELES (CAL.) PAVEMENT SPECIFICATIONS.

(Homer Hamlin, City Engineer.)

Graveled Streets.

(Oiled.)

Sub-Grade.—For the roadway, shall be 4" below surface of finished wo
unless otherwise indicated or directed. **Grading.**—Shall include all fil.in
excavation, shaping and trimming required to bring surface of street
grade and cross-section. Mud and other soft material, to a depth of 2 f
shall be taken out and the space filled with good earth or gravel. All fill
to be with good sound earth; the embankment to be carried up of full wid
in horizontal layers, not over 1 ft. thick, the teams to travel as evenly
possible over the whole surface of each layer, both going and coming. Aft
the street has been brought to the required grade and cross-section, the surfa
shall be thoroughly moistened and rolled with a roller weighing not less th
250 lbs. to the inch width of tire, until it is unyielding. Depressions made
the rolling shall be leveled up with good earth and again rolled. Such portio
of the street as cannot be reached by the roller, and all places excavated bel
grade and refilled, and all pipe trenches and other places that cannot
properly compacted by the roller, shall be tamped solid, and in case of w
weather or soft or muddy ground, making the use of the roller unsafe
impracticable, the rolling shall not be undertaken until the ground has b
come sufficiently dry. The sub-grade shall then be tested for grade, cro
section and condition. **Surfacing Roadway.**—Upon the sub-grade, sprea
layer of good gravel, to have a thickness of 4" (ordinarily) after rollin
The surface of this layer for a depth of 1" is to be raked free from all sto
larger than 1" in greatest dimension. If no gutters are provided, the
larger stones shall be raked to the curb and distributed over a strip 2 ft.
width next to the curb; if gutters are provided, the stones are to be d
tributed on a strip 2 ft. wide next to the gutter. This layer of gravel is to
uniformly spread on the roadway, and well moistened; then well ramm
for at least 1 ft. from the gutters, should these be paved; or if not pave
then 1 ft. from the curb. The remaining portion of the roadway shall th
be rolled with a roller weighing not less than 250 lbs. to the inch width
tire. The rolling of roadway shall commence at the rammed portion.
depressions must be promptly filled, moistened, and again rolled. T
sprinkling and rolling must continue until the surface is uniformly ha
compact, and in such condition that it will not yield or cut up under t
wheels of a heavily loaded wagon. **Oiling.**—Oil shall then be distribut
evenly over the entire surface of roadway, 1 gal. per sq. yd. Coarse, sha
sand shall then be sprinkled over the entire surface of roadway until no fr
oil can be seen. After a lapse of not less than 12 hrs., oil shall again be d
tributed over entire surface, ½ gal. per sq. yd. Entire surface of roadw
shall again be sprinkled with coarse, sharp sand until the oil is complete
absorbed, and then rolled with a roller weighing not less than 250 lbs. to t
inch width of tire until the surface is unyielding. In all cases, sufficient sa
shall be used to prevent the oil material from picking up. Total amount
oil used shall not be less than 1½ gals. per sq. yd. of street surface. In proc

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of rolling, care must be taken not to soil the curbs or walks. After the oiling has begun, it shall be carried on diligently and continually to its completion. Sand used in covering the oil must be distributed in piles along the sides of the street before the oil is applied, and must be spread quickly and in sufficient quantity to prevent the oiled surface from picking up. Oil shall not be applied to the surface of a street while in a wet condition. During and immediately after rolling, the surface of the street shall be gone over with brooms or rakes and all irregularities removed.

Oil.—(a) The oil used shall be a natural oil treated to remove water or sediment, or one from which the volatile material has been removed by distillation. It must not have been injured by over-heating, and it must not be obtained by adding solid asphalt to lighter oils, or by cutting asphalt with distillates. (b) Temperature. All oil must be delivered at the point required for sprinkling, at a temperature not less than 150° F. (c) Measurement. In determining the quantity of oil delivered, the correction for expansion by heat shall be as follows: 60° F. shall be considered normal temperature; subtract 0.0004 of measured volume for each °F. above 60° F., as a correction for expansion by heat. (d) Volatility. The oil shall not contain more than 8% of matter volatile when said oil is heated slowly to 220° F. and maintained at that temperature for 15 minutes. (e) Asphalt. After being freed from water and sediment, the oil shall contain not less than 70% of asphalt, having a temperature of 77° F., a penetration of 80°, District of Columbia standard. The percentage of asphalt shall be determined by heating a weighed amount of said oil in an evaporating oven to a temperature of 400° F. until it has reached the proper consistency, when the weight of the residue shall be determined and the per cent calculated. (f) Water and Sediment. Deduction will be made for water and sediment in exact proportion to the percentage of water and sediment found therein, which must not exceed 2%. (g) Tank wagons. All tank wagons used for the delivery of this oil must first be submitted to the Department of Oil Inspection, which will gauge and stamp into the steel heads of said tanks the capacity in gallons, which shall be the official rating. (h) All oil used shall be tested by the Department of Oil Inspection.

Surfacing Sidewalk Areas.—In cases where the plans provide for cement sidewalks on portions of the street to be improved, and do not provide for such walks over its entire length, then there shall be constructed at the accepted places gravel walks 2" deep and of a width and location corresponding to those for the cement walk provided for. In cases where the plans do not provide for cement walks on the street to be improved, then gravel sidewalks 2" deep and 5 ft. wide shall be constructed, except, however, in cases where the total width of the sidewalk area is less than 5 ft., in which event the total area of the sidewalk is to be graveled. In the construction of the graveled walks, the same quality of material as that used in the roadbed may be employed. It shall be raked free from large stones, sprinkled and rolled until firm.

Bituminized Brick Gutters.

Sand Cushion.—Upon the concrete base (the surface of which is 6" below finished grade, and thoroughly watered for at least 48 hours, before receiving the tion coat, and swept free from all dirt and rubbish) shall be spread a layer of sand 2" deep. The sand need not necessarily be sharp, but it must be screened, dry and free from more than 3% of loamy matter. It shall be spread by the aid of a templet and made to conform smoothly to the true shape of the gutter. There shall be no disturbance of the surface of the cushion coat previous to laying the bituminized brick thereon. **Laying Bituminized Brick.**—Upon the cushion coat shall be laid the bituminized brick, vertically on edge, and in close contact with each other. Brick in finishing rows must be laid so as to break joints at least 2". No bats or parts of bricks shall be used except for the purpose of closure or for breaking joints in starting courses. After the bricks are laid they shall be thoroughly rejected and all warped, spalled and chipped brick removed and replaced by perfect ones. The edge of the gutter next the curb shall then be carefully tamped by hand and the whole gutter shall then be rolled until all joints are thoroughly bedded and the tops lie in a smooth surface conforming to the grade and cross-section of gutter. Asphalt of a composition hereinafter described, and heated to a temperature of 300° F., shall then be poured into the joints until they are full and remain full to height of top of brick. Surplus asphalt shall then be removed, before it has become stiff, from the surface of

the gutter, and fine sand shall be swept over the top until all stickiness is removed. **Bituminized Brick.**—Shall be obtained by subjecting ordinary brick of the quality specified hereinafter to a bath of asphalt, also of the quality hereinafter described, and heated to a temperature of from 300° to 325° F., until at least 80% of the cross-section of the brick shall have become saturated with asphalt. They shall be free from warps, cracks, chips or other flaws, and the surfaces shall be free from superfluous asphalt and in condition to lay closely together. All bituminized brick will be subject to the following **Abrasion Test.**—This test shall be made in a foundry rattler whose inside diameter is 28" and inside length is 20". Such a number of whole, dry brick that their total volume shall equal, as nearly as possible, 8% of the cubic contents of the rattler, shall be placed therein. There shall then be added an abrasive charge of 300 lbs. of cast iron blocks as follows: 10 blocks about 2½" square and 4½" long, with edges rounded to about ¼" radius and weighing 7½ lbs. each, and 225 lbs. of cubical blocks about 1½" on a side and with square corners and edges. The rattler shall be revolved 1800 times at a speed of from 28 to 30 rev. per min. The loss by abrasion during such test shall not exceed 20% of the original weight of the brick. **Ordinary Brick.**—Shall be whole, sound brick with smooth, rectangular surfaces and straight edges and must give a clear ringing sound when struck together. They shall be uniform in quality, free from laminations, and shall run in size from 8" to 8½" long, 4" wide and from 2" to 2½" thick and burned to a medium degree of hardness. All brick shall be culled or sorted by the contractor before being treated to an asphalt bath, and will be subject to the following test: Three or more bricks shall be broken across, thoroughly dried, weighed, then immersed in water for 24 hours and weighed again. The absorption shall be determined by the difference between the two weights, and it shall not exceed 15% nor be less than 12% of the dry weight of the brick; otherwise the brick from which the tested samples were selected, shall be rejected. **Asphalt.**—This must be prepared from California products. It shall be a mixture of refined liquid asphalt with a refined solid asphalt or be an oil asphalt, and must be free from admixture with any residues obtained by the artificial distillation of coal, coal tar or paraffine oil. The asphalt must be homogeneous and its consistency at the time of its use in the bath must fall within the limits of 60° and 80° penetration by the District of Columbia standard. It must be adhesive and ductile and also slightly elastic at a temperature of 32° F. When 20 grams are heated to a temperature of 300° F. for 5 consecutive hours in an uncovered cylindrical dish 3½ cm. high by 5½ cm. in diameter, it must not lose more than 1% in weight, and its penetration must not be reduced, as a result of such heating, more than 50%. It must, when ready for use, contain at least 90% of bitumen soluble in carbon disulphide. It shall be soluble in cold carbon tetrachloride to the extent of at least 97%. Not less than 70% shall be soluble in 86° naphtha. It shall not contain more than 15% of fixed carbon on ignition. When the asphalt is prepared by mixing a solid oil asphalt with a liquid asphalt, the solid oil asphalt shall be prepared by distilling the crude oil until the asphaltic residuum has a penetration not less than 50° by the District of Columbia standard, and shall not be prepared by mixing or fluxing a more solid asphalt with a liquid or softer asphalt. The refined liquid asphalt used in softening a solid asphalt must be a stiff residuum of petroleum oil with an asphalt base. It must be free from water and from light oils volatile at less than 250° F. When 20 grams are heated to a temperature of 300° F. for 5 consecutive hours in an uncovered cylindrical dish 3½ cm. high by 5½ cm. in diameter, it must not lose more than 5% in weight. It must contain not less than 99% of bitumen soluble in carbon disulphide.

MARYLAND STATE HIGHWAY SPECIFICATIONS.

(Maryland Geological Survey.)

Macadam Construction.

Class A, B and C.—Thickness after rolling, to be as follows:

Class A. 1st course, 3"; .....2nd, 3"; .....3rd, as described later.
Class B. " 5", in 2 layers; " 3"; ..... " "
Class C. " 5", gravel; ..... " 3", stone; " "

Roadbed.—Natural earth bed prepared and rolled until firm and hard; a small amount of clay to be added if sandy or other soil will not compact readily under roller. In Cuts and Fills, roadbed is to be graded 24 ft. wide

Roadbed prepared for broken stone surface to be 14 ft. wide* and rolled firm and hard; depressions filled with earth and rerolled. Old earth roadbed, where there is no change in grade, is to be shaped to proper cross-section, elevations and depressions removed, and surface rolled hard and smooth. The portion of the roadbed prepared for the broken stone, is to be below the sides by an amount equal to thickness of 1st course of stone, to prevent spreading at sides. Roadbed to have cross-slope of ¼" to 1 ft. **First Course.**—Sound broken stone, 3" to 2", known as "No. 1" size. If approved it may be gravel, 3" to 1", with not more than 25% less than 1". No layer of crushed stone to be spread thicker than 6" before being thoroughly rolled. Broken stone or gravel for 1st course to be rolled with steam roller weighing not less than 10 tons, until compacted firm and smooth; sprinkling with water or lightly spreading with sand if needed; rolling to begin at sides and work toward center. Unevenness or depressions to be remedied. **Shoulders.**—After 1st course is made, construct shoulders along each side for width of at least 5 ft.; against these shoulders, spread broken stone for second course; the shoulders with the 14 ft. of broken stone will make a total width of 24 ft., to be cross-sloped ¼" to 1 ft. **Second Course.**—Same width as first course. Broken stone 1" to 2", known as No."2" size. Unless otherwise specified the stone for this course shall be trap rock with a "coefficient of wear" as determined by tests made at the laboratory of the Highway Division of the Maryland Geological Survey, of not less than 15, or limestone with a "coefficient of wear" of not less than 10. The broken stone to be spread upon the 1st course, to a uniform thickness, and rolled with not less than a 10-ton roller, sprinkling with water or lightly spreading with sand or other material if necessary, until surface is hard and smooth; cross-slope of surface, ¼" to 1 ft. Unevenness and depressions to be remedied. **Third Course.**—Trap rock screenings, from 1" to dust; other material may be used if approved; limestone screenings to be used with a limestone 2nd course. Upon the 2nd course, and in quantity just enough to cover it, the screenings are to be spread dry, then sprinkled with a sprinkling cart, and rolled with not less than a 10-ton roller, beginning at the sides. If after rolling the screenings, the No. 2 stone appears at the surface, use additional screenings. Rolling and watering to continue until the water flushes to the surface; the rolling to extend over whole width or road and shoulders. Unevenness and depressions to be remedied.

NATIONAL ASSOCIATION OF CEMENT USERS.

(Philadelphia, Pa.)

Portland Cement Sidewalk Specifications.

(Adopted January, 1908.)

Materials.—(1) Cement shall meet requirements of specification for Portland cement of the A. S. T. M., and adopted by this association (Spec. No. 1), January, 1906. (2) Sand shall pass a No. 4 screen and be free from foreign matter, except loam and clay up to 5% when not occurring as a coating on the sand grains. Not more than 40% shall be retained on a No. 10 sieve; or 5% pass a No. 10 and be retained on a No. 20; or 35% pass a No. 20 and be retained on a No. 30; or 35% pass a No. 30 and be retained on a No. 40; or 5% pass a No. 40 and be retained on a No. 50. Not more than 20% shall pass a No. 50 sieve; or 70% pass a No. 10 and be retained on a No. 40; or 5% pass a No. 20 and be retained on a No. 50 sieve. (3) Stone shall be crushed from clean, sound, hard, durable rock, be screened dry through a mesh, and be retained on a ¼" mesh. (4) Screenings from the crushed stone, if they meet the requirements for sand, may be used as sand if approved. Gravel shall be clean, hard, and vary in size from ¼" to ¾" screening; unscreened gravel shall be clean, hard, and contain no particles larger than ¾", proportions of fine and coarse to be determined and corrected to agree with requirements for concrete. (6) Water to be clean, free from oil, sulphuric acid and strong alkalies. **Forms.**—(7) Lumber, free from warp, not less than 1½" thick; all mortar and dirt to be removed from forms previously used. (8) *Setting.*—The forms shall be well staked to the established lines and grades, and their upper edges shall conform with finished

* Mr. E. F. Ruggles, First Assistant Engineer, writes the author as follows: "As a rule, our roads are built with a width of 12 ft. for the macadam; though we have built, where the travel requires it, a good deal of 14-ft. macadam."

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grade of sidewalk, which shall have sufficient rise from curb to provide proper drainage; this rise not to exceed ¼″ per ft., except where such rise shall parallel length to walk. (9) Cross Forms, at each block division, shall be put in the full width of walk and at right angle to side forms. (10) Expansion Joints. A metal parting strip ½″ thick shall take the place of the cross-forms at least once in every 50 lin. ft. of sidewalk. When sidewalk has become hard, this parting strip shall be removed and joint filled with suitable material prior to opening the walk to traffic. Similar joints shall be provided where new sidewalks abut curbing or other artificial stone sidewalk. (11) Wetting. All forms to be thoroughly wetted before any material is deposited against them. **Size and Thickness of Blocks.**—(12) In Business Districts, blocks shall be so divided that no dimension shall be greater than 6 ft.; thickness of sidewalk shall correspond directly with the greatest dimension of the walks as follows: 6″ thick for block 6 by 6 ft.; 5½″ for block 5 by 5 ft.; 5″ for block 4½ by 4½ ft.; 4″ for block 4 by 4 ft. (13) In Residence Districts, thickness of sidewalk shall be as follows: 6″ thick for block 6 by 6 ft.; 5″ for block 5 by 5 ft.; 4″ for block 4 by 4 ft.; 3″ for block 3 by 3 ft.; it being permissible to lay sidewalks with a thickness at the edges 25% less than at center. (14) Minimum Thickness of walk to be 3″ in any case. **Sub-Base.**—(15) Preparation. Sub-base to be thoroughly rammed, and all soft spots removed and replaced by suitable hard material. (16) Fills. When a fill exceeding 1 ft. thick is required, it shall be thoroughly compacted by flooding and tamping in layers not over 6″ thick, and shall have a slope of not less than 1 to 1½. The top of all fills shall extend at least 12″ beyond the sidewalk. (17) Wetting. While compacting, the sub-base shall be thoroughly wetted and shall be maintained in that condition until the concrete is deposited. **Base.**—(18) Proportions. The concrete for the base shall be so proportioned that the cement shall overfill the (19) Voids* in the sand by at least 5%, and the mortar shall overfill the voids in the stone by at least 10%. The proportions shall not exceed 1 part cement to 8 parts of the other materials. When the voids are not determined, the concrete shall be: 1 part cement, 3 parts sand or screenings, and 5 parts stone or gravel. A sack of cement (94 lbs.) shall be considered to have a volume of 1 cu. ft. (20a) Hand Mixing. Spread sand evenly on level water-tight platform; spread cement upon sand; mix thoroughly dry to uniform color; add water in a spray, and turn mass until homogeneous mortar of even consistency is obtained; to this mortar, add the required amount of stone or gravel previously drenched, and mix the whole until the aggregate is thoroughly coated with mortar. When unscreened gravel is used, the cement and gravel shall be thoroughly mixed dry until no streaks of cement are visible; water shall be added with a spray in sufficient quantity to render when thoroughly mixed, a concrete equal to that specified above. Water may be added during the process of mixing, but the concrete shall be turned at least once immediately after its addition. (20b) Mechanical Mixing. Machine mixing will be acceptable when a concrete equal in quality to that specified above is obtained; mixing to be thorough. (21) Retempering will not be permitted. (22) Depositing. The concrete shall be deposited within 1 hour after being mixed, and shall be transferred to the forms in water-tight wheelbarrows; the barrows not to be filled so full as to allow mortar to slop out, and shall not be run over freshly laid concrete. The concrete to be spread evenly and tamped until water flushes to the top. (23) Separation of Blocks shall be done with a tool not over 6″ wide and ¼″ thick, and to insure

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* To determine voids, fill a vessel with sand and let net weight of sand equal B. Fill same vessel with water and let net weight of water equal A.

$$\text{Per cent voids} = \frac{A \times 2.65 - B}{A \times 2.65} \times 100.$$

This formula may also be used in determining voids in crushed stone and screenings by substituting for 2.65 the specific gravity of the stone.

The following is a more simple method of determining voids in coarse aggregate: Fill a vessel with the aggregate and let net weight equal B. Add water slowly until it just appears on the surface, and weigh. Let net weight equal A. Fill same vessel with water and let net weight equal C.

$$\text{Per cent voids} = \frac{A - B}{C} \times 100.$$

Use a vessel of not less than one-half (½) cubic foot capacity. The larger the vessel, the more accurate the results.

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complete separation the groove should be cut through into the sub-base. Fill the groove with dry sand before the top coat is spread, and the top coat should be cut through to the sand after floating and troweling and a jointer run in the groove; then again draw a trowel through the groove, so as to insure a complete separation of the block. (24) Protection. Workmen not permitted to walk on freshly laid concrete, and where sand or dust collects on the base it shall be carefully removed before the wearing surface is applied. **Wearing Surface.**—(25) Thickness. ¾". (26) Mixing. The mortar to be mixed in the same manner as the mortar for the base, but using 1 part cement to 2 parts of sand or screenings, and it shall be of such consistency as will not require tamping, but will be readily floated with a straight-edge. (27) Depositing. Spread mortar on the base within 30 minutes after mixing, and in no case shall more than 50 minutes elapse between the time that the concrete for the base is mixed and the time that the wearing course is floated. Float a thin coat of mortar on the base before spreading the wearing surface. (28) Marking. After being worked to an approximately true surface, the block markings shall be made directly over the joints in the base with a tool which shall cut clear through to the base and completely separate the wearing courses of adjacent blocks. (29) Edges. All surface edges of blocks to be rounded to radius of not less than ¼". (30) Troweling. When partially set, the surface shall be troweled smooth. (31) Roughening wearing surface. On grades exceeding 5%, the surface shall be roughened, by using a grooving tool, toothed roller, brush, wooden float or other suitable tool, or by working coarse sand or screenings into the surface. (32) Color. If color is desired, only mineral colors shall be used, which shall be incorporated with the entire working surface. **Single Coat Work.**—(23) Proportions. Single coat work shall be composed of 1 part cement, 2 parts sand, 4 parts gravel or crushed stone, and the blocks separated as provided for in the specifications for two-coat work. (34) Finishing. The concrete shall be thoroughly compacted by tamping and evenly struck off and smoothed to the top of mold. Then, with a suitably grooved tool the coarser particles of the concrete tamped to the necessary depth so as to finish the same as two-coat work. **Protection and Grading.**—(35) Protection. When completed, the sidewalk shall be kept moist and protected from traffic and the elements for at least 3 days; the forms to be removed with great care, and when removed, earth shall be banked against the edges of the walk. (36) Grading, after the walks are ready for use, should be on the curb side of the sidewalk, 1½" lower than the sidewalk, and not less than ¼" per ft. fall toward the curb or gutter. On the property side of the walk, the ground should be graded back at least 2 ft. and not lower than the walk; this will insure the frost throwing the walk alike on both sides.

MANHATTAN (N. Y. CITY) BOROUGH PAVEMENT SPECIFICATIONS.

Granite Block Pavement.

Blocks.—Shall be of a durable, sound and uniform quality of granite; " to 12" long, 3½" to 4½" wide, and 7" to 8" deep; same quality as to hardness, color and grain. No outcrop, soft, brittle or laminated stone accepted. Blocks to be rectangular on top and sides, uniform in thickness, to lay closely, and with fair and free surfaces, free from bunches. Other dimensions of blocks may be used for special construction. Stone from each quarry shall be piled and laid separately in different sections of the work; no mixing of stones from different quarries. **Sand Cushion.**—On the concrete foundation (previously prepared 6" thick) place a layer of clean, course, dry sand to such a depth (not less than 1½") as may be necessary to bring the surface of pavement when thoroughly rammed, to the proper grade. **Laying.**—On this sand bed, and to grade and crown specified, lay the blocks at right angle to line of street, or at such angle as may be directed; each course to be straight and regular, with the end joints by lap of at least 3"; stones of different width not to be laid in the same course, except on curves; joints to be close, except where gravel filling is used the joints between courses shall not exceed ¼". After the blocks are laid, they shall be covered with clean, hard and dry gravel (previously heated and dried), to be brushed in until the joints are filled therewith to within 3" of the top; the gravel to be washed white quartz, free from sand or dirt, and ⅛" to ⅜" mesh screenings. **Ramming.**—Blocks must then be rammed and ramming repeated until they are brought to an unyielding bearing with a uniform surface, true to even grade and crown; no ramming to be done within 20 ft. of face of work being

laid. **Joints.**—After ramming, the pavement cement heated to 300° F. shall then be poured into the joints until same are full and remain full to top of gravel. Hot gravel shall then be poured along the joints flush with top of blocks; and paving cement again poured in joints, filling all voids. **Paving Cement.**—Shall be composed of 20 parts of refined asphalt and 3 parts of residuum oil, mixed with 100 parts of coal-tar pitch such as is ordinarily numbered 4 at the manufactory, the proportions to be determined by weight.

Wood Block Pavement.

Foundation.—6" thick, including 5½" of concrete proper and ½" of mortar top surface, ordinarily. **Blocks.**—(a) Either of southern long-leaf yellow pine, southern black gum, Norway pine or tamarack, not less than 90% of heart; texture permitting satisfactory treatment; inspection at works, in the stick, before being sawed into blocks. (b) All blocks shall be of sound timber, free from bark, loose or rotten knots, or other defects detrimental to life of blocks or to laying; no second-growth timber allowed. (c) Blocks shall be well made, rectangular and of uniform dimensions: Depth (parallel to fiber) 3½", length 6" to 10", width 3" to 4"; in any one contract, blocks to be of same timber, and depth and width shall not vary more than ⅛". (d) Blocks to be treated with an antiseptic and waterproof mixture, not more than 75% per cent of which shall be creosote or heavy oil of coal tar, and at least 25% of which shall be resin; all parts of each block to be thoroughly treated, injecting not less than 20 lbs. per cu. ft. (e) Treated pine blocks shall weigh as much as water; treated gum blocks, at least 59 lbs. per cu. ft.; any other wood, at least 20 lbs. per cu. ft. more than its recognized weight untreated. Blocks cut from the several classes of timber will require different treatment, hence the exact methods of applying the mixture will not be specified, but must conform in every respect to the best and most advanced knowledge of the art. (f) The creosote oil at 68° F. shall have a specific grav. of not less than 1.12; when distilled in a retort with the thermometer suspended not less than 1" above the oil, it shall lose not more than 35% up to 315° C., and not more than 50% up to 370° C. Oil to be free from adulteration or foreign material. (g) The resin to be solid resin obtained from pine; and reduced to a fine dust by grinding and then incorporated with the hot creosote oil in a suitable mixing tank until the proper proportions are secured. (h) After treatment, blocks not to gain more than 3½% in weight after being oven-dried at 100° for 24 hours and then immersed in water 24 hours. **Analysis of Treated Block.**—Fine turnings from the block shall be placed in an extraction apparatus and the oil completely extracted therefrom with ether or carbon bisulphide; the oil then placed in a still and distilled; the portion up to 120° C., consisting of the solvent, is to be collected apart; the oil then distilled up to 370° C. The oil thus obtained must conform in all respects to the requirements of (h), above. **Mortar Bed.**—On concrete foundation, spread ½" layer of mortar composed of 1 part Portland cement to 4 parts clean, sharp sand, free from pebbles over ¼" diameter; the mortar top to be "struck" 3½" below and parallel to top of finished pavement. The mortar bed to be laid as follows: On surface of concrete foundation, before mortar bed is laid, set strips of wood 4" wide by ½" thick, or strips of steel 4" by ⅛", and of convenient length; these strips to be set parallel and about 8 to 10 ft. apart, running from curb to curb, and imbedded in mortar so that top surface shall be 3½" below grade of finished pavement; the space between two strips having been filled with mortar, a true and even top surface shall be struck by using an iron-shod straight-edge on the strips as a guide, the strips to be removed and the places filled with mortar as the blocks are laid. **Laying.**—On this mortar surface the blocks are laid, with the grain vertical, in parallel courses, at angles as directed, tight joints as possible, each block being firmly imbedded in the mortar bed so as to form a true and even surface. Expansion joints, ½", shall be used along each curb, and across the street every 100 ft. The joints shall then be filled with cement grout (2 parts sand and 1 part Portland cement, mixed to a perfectly liquid form) and the surface of the blocks shall be slushed with same and joints swept until completely filled; surface then covered with ½" of screened sand. **Grooved Blocks.**—Where grade exceeds 3%, the blocks shall be between 6" and 10" long, the upper edge of each block to be cut away for a width of ¼" and depth of 1", so as to provide transverse grooves between each course (as a foothold for horses); or other equally good construction. Blocks to be laid (in usual manner), 4" lap, whole blocks used, and covered with sand when laid.

RICHMOND (N. Y. CITY) BOROUGH PAVEMENT SPECIFICATIONS.

(Louis L. Tribus, Commissioner.)

Iron Slag Block Pavement.

Blocks.—Iron slag blocks, 8″ to 9″ long, 3½″ wide, 3″ deep; shall be hard, durable and perfect; upper edges to be chamfered. On grades of 6% or over, the joints between blocks are to be left open to receive hot, clean gravel, with paving cement; for grades below 6%, the joints will be laid close without gravel but filled with paving cement. **Sand Cushion.**—On the foundation (concrete) place about a 2″ layer of clean, dry sand to bring surface of pavement, when rolled, to proper grade; sand to be screened and free from stones and rubbish; cushion to be brought to required form and crown by means of template resting on curbs and drawn forward a few feet, ahead of laying. **Laying.**—Blocks laid on edge at right angle to curb line, except at street intersections where they shall be laid as required. End joints to be broken by lap of half length of block. At every 4th course, or as often as directed, blocks shall be closed up by hammering and the course straightened. End joints to be closed by means of crowbar applied at ends next the curbs before the closures are made. Whole blocks used except in beginning or closing a course, or as directed. **Rolling.**—As soon as street block has been laid, the pavement shall be swept clean and rolled with 5-ton roller until all blocks are thoroughly imbedded in the sand cushion; all depressed surfaces to be relaid; pavement then rerolled until finished surface is smooth and even, to required grade and crown. Broken or chipped blocks are to be replaced. **Expansion Joints.**—Before laying the blocks, laths ½″ thick shall be placed next each curb. The spaces thus formed shall be filled with hot paving cement composed as follows: **Paving Cement.**—Shall be a bituminous material either natural or artificial, free from coal tar or any products of coal tar distillation. It shall be waterproof, free from water or decomposition products, remain ductile and pliable at all climatic temperatures to which it may be subjected in actual use, and shall not run in the joints in the hottest temperature in summer, nor become hard and brittle through the action of frost. It shall conform to the following requirements: 99% or over by weight shall be soluble in carbon bisulphide; specific gravity 106° F. to be not less than 1; 100 grams of this cement not to lose more than 10% weight when maintained at a uniform temperature of 400° F. for hours in a cylindrical vessel 3½″ diameter and 1″ high; amount of fixed carbon not to be more than 12% and it shall show a flashing point, open oil tester, of more than 510° F., and shall not contain more than 2½% of paraffin scale; if obtained by mix of bituminous materials, it shall be homogeneous, free from water and light oils, obtained by agitation with hot air at temperature of not more than 400° until all the mass is blended completely, and shall be free from granular accumulations. Penetration test: 32° F. with No. 2 needle and 100 grams weight for 5 seconds, shall not be less than 1 millimeter; at 115° F., No. 2 needle, 50 grams, not less than 8 and more than 15 mm.; ½ gram of the material when made into a ball shall melt and drip through an aperture 1 mm. in diameter at less than 220° F. Paving cement shall be heated on the work to a temperature between 400° and 450° F., in quantities to allow this temperature to be maintained in kettles during progress of pouring; none with temperature below 400° to be used. It shall then be put in a conical can and poured in the interstices of blocks till the filler is flush with top of blocks; repeating filling if necessary. All joints between blocks shall be filled with this hot paving cement, pouring from center to sides, but no flushing of the pavement will be permitted. Where girder rails are used, the space between the web of rail and joining blocks shall be filled with mortar, composed of Portland cement 1 to 4. **Underdraining.**—Pipes shall be vitrified, salt-glazed stoneware jointed with proper collars, pipes to be 4″ inside diameter and 12″ to 24″

Vitrified Brick Pavement.

Blocks.—The carriageway to be paved with best quality of repressed paving blocks made of shale or clay, and with ½″ lugs on the sides; uniform in size, color and quality and of same make; blocks to be from 2½″ wide, 7″ to 10″ long, and 4″ to 4½″ deep, exclusive of projection. Selected (34 blocks of average quality from each 50000 or less) to be subject to the following tests: Bricks to be free from lime and magnesia in lumps of pebbles and shall show no signs of cracking or spawling on removal in water 96 hours; when subjected to tests for abrasion, loss in

weight not to be more than 20%; shall have a specific gravity of not less than 2.3; shall not absorb more than 3% of water when dried at 212° F. for 48 hours and afterward immersed for 48 hours in water (test to be made on blocks which have been subjected to abrasion test); for transverse test, they shall show a modulus of rupture of not less than 2000 lbs. per sq. in. when tested on edge as laid in the pavement, the modulus to be computed by the formula $R = 3lw + 2bd^2$, in which R is the modulus of rupture, l the length between supports ($=6$), b and d breadth and depth, all in inches, and w the load, in lbs., producing rupture. **Sand Cushion, Laying, Rolling, Expansion Joints, Paving Cement, Underdraining.**—(Same as for iron slag block pavement.)

Asphalt Block Pavement.

Blocks.—The asphalt blocks shall be 4¾″ to 5¼″ wide, 11¾″ to 12¼″ long, and 2¾″ to 3¼″ deep; and composed of 6 to 8 parts asphaltic cement, 86 to 82 parts crushed trap rock, and 8 to 10 parts inorganic stone dust. The Asphaltic Cement shall be composed of refined and natural asphalt, or asphaltum, fluxed with liquid petroleum residuum or refined maltha or liquid asphalt; no residuum of petroleum other than that contained in the flux, to be used; the refined asphalt and flux shall be mixed in proper and approved proportions; the bituminous flux to be free from impurities and brought to a specific gravity of from 18° to 22° Beaume, and a first test not less than 300° F., and shall contain no appreciable amount of light oils, or matter volatile under 250° F.; the distillate of the petroleum oil, if used at 400° F. for 30 hours, shall not exceed 10%. The Crushed Trap Rock shall not exceed ½″; the size to be nearly cubical as possible, and graded from maximum size to dust, so as to give the mineral aggregate with a minimum percentage of voids. The Inorganic Dust shall be pulverized stone, free from loam, clay or other earthy material; no weathered rock or dust from same to be used. Blocks when laid shall have a specific gravity of not less than 2.45, and when dried for 1 day at a temperature of 150° F., and then immersed in water 7 days, they shall not absorb more than 1% water. **Laying.**—On the concrete surface, after same has been swept and wetted, spread a layer of cement mortar (composed of 1 part slow-setting Portland cement and 4 parts clean sharp sand, free from gravel over ¼″ diameter) to such thickness that when struck to a surface 3″ below and parallel to the grade of completed pavement its depth shall be nowhere less than ½″; the spreading and surfacing to be as follows: On the surface of the concrete, set strips of wood 4″ wide by ¾″ thick, or strips of steel 4″ by ¼″, and of greatest convenient length; these strips to be set parallel and about 8 or 10 ft. apart, running from curb to curb, and imbedded in mortar throughout their length so top surface shall be 3″ below and parallel to grade of finished pavement; the space between two strips having been filled with mortar, an even surface shall be struck by using an ironshod straight-edge on the strips as a guide, and as soon as the bed has been struck, the strip which would interfere with laying the block shall be removed and its place filled with mortar with a trowel. On this mortar surface, the blocks are to be laid, in courses at right angle to line of street (ordinarily), each course to be of uniform width and depth, laid to proper crown and grade, with close joints, and end joints broken by a lap of at least 4″. The surface shall present no greater variation than ¼″ between adjoining blocks. Blocks fractured or broken shall be replaced. **Sand Joints and Covering.**—When laid, the blocks shall be covered with clean, fine sand,

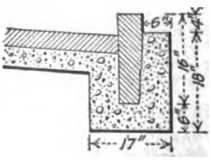

Fig 4.—Curb on Concrete Foundation.

entirely free from loam or earthy matter, perfectly dry and screened through a sieve having not less than 20 meshes per lin. in.; the sand to be swept and brushed into the joints and left on the surface until such time when the pavement shall be swept clean for final inspection, and defects remedied.

RICHMOND (IND.) STREET CROWNING.

(H. L. Weber, City Engineer.)

Crowning is parabolic: $h_1 = \frac{H}{9}$; $h_2 = 4h_1 = \frac{4}{9} H$.

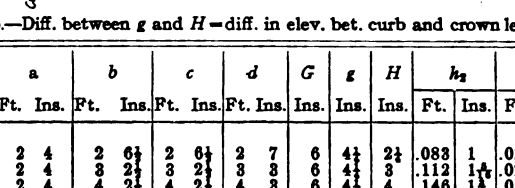

Fig. 5.—Diff. between g and H = diff. in elev. bet. curb and crown levels.

Width of Street. Ft.	a Ft.	a Ins.	b Ft.	b Ins.	c Ft.	c Ins.	d Ft.	d Ins.	G Ins.	g Ins.	H Ins.	h_2 Ft.	h_2 Ins.	h_1 Ft.	h_1 Ins.
20	2	4	2	6½	2	6½	2	7	6	4½	2½	.083	1	.021	¼
24	2	4	3	2½	3	2½	3	3	6	4½	3	.112	1 1/16	.026	1/16
30	2	4	4	2½	4	2½	4	3	6	4½	4	.146	1½	.036	1/16
30	2	4	4	2½	4	2½	4	3	4	2½	2½	.083	1	.021	¼
36	2	4	5	2½	5	2½	5	3	6	4½	5	.188	2¼	.047	1/16
40	2	4	5	10½	5	10½	5	11	6	4½	6	.224	2½	.052	⅛
*45	(2	4)	(5	2)	7	6	7	6	6	(4½)	(6)	.288	3 1/16	.073	⅛
†45	(2	4)	(5	2)	7	6	7	6	6	(4½)	(4½)	.222	2⅝	.056	3/16
‡45	(2	3)	(6	9)	6	9	6	9	6	(4½)	(4½)	.157	1⅞	.039	⅛
48	2	4	7	2½	7	2½	7	3	6	4½	7	.260	3⅛	.062	¼

* Crowning is calculated for $H = 7\frac{1}{2}''$ at curb line; and level of crown is $1\frac{1}{2}''$ above level of curb.

† Half width of street to be used on side next to stone curb. Crowning s calculated for $H = 6''$ at curb line; and crown is level with curb.

‡ Half width of street to be used on side next to cement curb; and :rown is level with curb.

SYRACUSE (N. Y.) PAVEMENT SPECIFICATIONS.

(H. C. Allen, City Engineer.)

General.

Excavation.—Includes earth, rock or other material necessary to be :moved from work, to depth required to reach sub-grade, and work con- :cted with adjusting street intersections and grading slopes back of curb ne. Excavations below sub-grade shall be made up with cement concrete. ırplus material shall be deposited within an average distance of 2000 ft., or : disposed of by contractor. **Embankment.**—Shall start from a well-prepared ıse, mellowed or stepped on sloping ground, and be carried up in horizontal yers not over 4" thick, each layer carefully rammed or rolled, and well ıtered. **Rolling and Ramming.**—After the sub-grade has been brought to .es prescribed, it shall be rolled with steam roller of not less than 5 tons, til surface is firm and compact. Portions inaccessible to roller shall be nmed, and all depressions, defects, etc., made by the contractor, and ain rolled and rammed.

Vitrified Brick Pavement.

Concrete Foundation.—Upon the sub-grade thus prepared, lay a bed of rtland cement concrete 6" deep, of 1 part cement, 3 parts sand, 6 parts ken stone. Use best quality of Portland cement; sand to be clean, coarse l sharp, free from foreign matter; broken stone to consist of hard durable ne, varying in size from 2½" to ¾" in diameter, and free from dust or dirt. **shion.**—Upon the concrete foundation, thoroughly set and dry, carefully ead ½" layer of clean, coarse, sharp sand, or fine screened gravel, free from n, dirt or vegetable matter; leaving surface true and even and parallel grade. **Paving Bricks.**—Shall be made and burned especially for street

paving purposes, and shall stand all reasonable tests as to durability and fitness, to which paving material is usually subjected; the material to be burned in down-draught kilns or furnaces. Bricks to be square and straight, with sharp or slightly beveled edges, free from cracks or other defects, and of uniform size and pattern, and approved quality, equal to approved samples; specific gravity not less than 2. Size may be: length 7½" to 8½", width 2¼" to 2½", depth 3½" to 4". The absorption of water by any one brick to be not greater than 3%; average absorption of all bricks tested not to exceed 2% of their dry weight; tests to be made on either abraded or broken bricks by drying them for 12 hours in an oven and then soaking them 12 hours in water. At least three bricks shall be used for this test. The bricks shall also be tested in the standard rattler* and by the method of the National Brick Manf. Assn. and Am. Soc. of Municipal Improvements. **Sample Bricks.**—Three or more bricks of the kind or quality to be used on the paving shall be furnished with each proposal; the bricks to be labeled with bidder's and maker's names and addresses. **Manner of Laying.**—Upon the cushion, the pavement to be constructed with a single layer of bricks, laid on edge, end to end, in courses at right angles with the curb line, except at street intersections, where courses are to be placed at such angles as directed. Bricks to be set in courses across the street, which must be kept true and parallel, with the body of the bricks close together, sides and ends touching, and breaking joints at least 3" with bricks in adjoining courses; they are to be set perpendicular to grade of street, and to a height of from ¼" to ⅜" (ordinarily) above the true grade and crown of street when finished, to provide for settlement in pounding. Whole bricks to be used except in starting and closing courses at curbs, catch-basins and street structures, when not less than half bricks may be used in breaking joints, which shall be tight and close at ends. **Ramming and Tamping.**—The paving when laid, and either before or after the filling of the joints and top dressing is put on, as may be directed, shall be thoroughly rammed not less than three times with a paver's rammer of 90-lb. weight; the blows of the rammer must not be made directly on the bricks, but upon a 2" plank not less than 10 ft. long and 12" wide, which will be laid upon the surface of the pavement, which must conform to true grade and crown of street. **Grout,** composed of equal parts Portland cement and sand, with proper amount of water, to be poured upon the pavement and swept to and fro until every joint is filled flush with the surface of the pavement, and continued until joints are entirely filled. Wet sand then to be spread over the entire pavement ½" thick, and kept wet until ordered removed.

Sandstone Block Pavement.

Concrete Foundation.—(Same as for vitrified brick pavement.) **Cushion.**—Upon the concrete foundation, thoroughly set hard, carefully spread 2" layer of clean, coarse sand, free from loam, dirt or vegetable matter; leaving surface true and even and parallel to grade. **Sandstone Blocks.**—Shall be of best quality of Medina or Potsdam sandstone, not less than 3" nor more than 5" thick, not less than 6" nor more than 7" deep, and from 7" to 12" long; must be sufficiently dressed to present rectangular faces with straight edges on top, bottom and sides, and all blocks whose faces vary more than ⅛" from the rectangular shape will be rejected. The sides and ends of the blocks must be so dressed that they will make joints not to exceed ¼" in width. If necessary to obtain a satisfactory surface, the top surface of the blocks to be cut or "axed" off smooth, sides and ends to receive similar treatment when necessary to secure the ¼" joint. The stones to be set tight together, perpendicular to grade, so as to break joints at least 2", in uniform rows across the street at right angles to line of curb, except at street intersections and other places as may be directed. When laid, the blocks to be carefully rammed as may be directed, with a paver's rammer, no iron being allowed on its lower surface to come in contact with the pavement, which is to be surfaced by using a long straight-edge; shall conform to established grade and crown. **Grout Filling.**—Pour into joints a Portland cement grout composed of 2 parts clean, sharp sand to 1 part Portland cement, of approved quality, together with enough water to make proper grout, which will be poured upon the pavement and swept to and fro until every joint is filled flush with surface of pavement, operation to be continued until joints are entirely filled. Wet sand then to be spread over entire pavement 1" thick, and kept wet until ordered removed.

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* See page 507.

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Asphalt Sheet Pavement.

Upon the concrete foundation, thoroughly set and hard, shall be laid the wearing surface, which is divided into two classes of standard grade sheet asphalt: Refined Asphalt and Rock Asphalt. It is intended to admit the use of any asphalt of reputation which can be made into a suitable paving mixture.

Refined Asphalt Sheet Pavement will consist of (1) refined asphalt (an asphalt after it has been freed wholly or in part from water or organic matter by being heated); (2) a hydro-carbon flux or softening agent; (3) sand and fine powder of carbonate ot lime. **Asphaltic Flux.**—Properties required are: It shall contain no material volatile under 300° F.; shall be chemically stable and not lose its fluidity by molecular change; shall dissolve the asphalt and not simply form a mechanical mixture with it; must not volatilize more than 5% when heated for 7 hours at 325° F. The flash points shall be taken in a N. Y. State closed oil tested. **Asphaltic Cement.**—The refined asphalt is to be combined with the flux to produce asphaltic cement, to be tested as follows: Must not flash below 350° F.; must contain no water; must not show more than 4% loss by weight on being heated at 325° F; and must not show a loss of more than 8½% in weight on being heated 7 hours at 350° F. It must not contain more than 4½% of carbonaceous matter insoluble in carbon bi-sulphide, and must not show more than 15% of fixed carbon, and must not contain more than 3% of paraffine scale. The test for consistency of penetration of the asphaltic cement shall be the distance expressed in $^1/_{100}$ of a centimeter that a No. 2 needle will penetrate into it at 25° C. (77° F.) under a weight of 100 grams in 5 seconds of time, the needle to penetrate direct without friction. **Sand.**—Sand used for body of wearing surface shall be clean and sharp and composed of grains not easily crushed. Shall be graded in size of grains to reduce voids to a minimum, to secure which, a quantity of powdered carbonate of lime, from 5 to 15%, shall be added. The sand grains to be graded in size about as follows: Retained on 10 mesh per lineal inch, 3%; on 20 mesh, 5%; 40 mesh 25%; 60 mesh 25%; 80 mesh 12%; 100 mesh 18%; passed 100 mesh, 12%; total 100%. **Mixing.**—The wearing surface of graded aggregate and sufficient asphaltic cement to fill the voids when laid shall contain no trace of coal-tar, water, appreciable amount of light oils, no matter volatile at a temperature of 250° F. It shall yield, when extracted with bisulphide of carbon and after evaporation of the solvent, not less than 9½% nor more than 13% of bitumen, and at least 68% of the extracted bitumen shall be soluble in petroleum naphtha. The sand and asphaltic cement to be heated separately to about 300° F. The pulverized carbonate of lime, while cold, will be mixed with the hot sand in the required proportion and then mixed with the asphaltic cement at the required temperature, and in proper proportion, in a suitable apparatus, so as to effect a thoroughly homogeneous mixture. Sand boxes and tar and asphalt gauges will be weighed up daily. **Laying.**—Pavement mixture thus prepared will be laid on the foundation (same as for vitrified brick pavement) in two coats: The first, or cushion coat, will contain 2 to 4% more asphaltic cement than is required for the surface mixture, and laid so as to give a thickness of ½" after being consolidated by rollers. The second, or surface coat, prepared as required, to be laid on the cushion coat; it will be brought to the ground in parts at a temperature not less than 250° nor more than 300° F., and if the temperature of the air is less than 50° the contractor must provide canvas covers for use in transit; the mixture to be spread with hot iron rakes to form grade, and to have thickness of 2" after ultimate compression. The surface to be compressed by a hand roller, after which a small amount of hydraulic cement will be swept over it, then thoroughly compressed with a heavy steam roller, continued as long as it makes an impression on the surface, at least 5 hours for each 1000 sq. yds. of surface. **Gutters.**—Strip of cement 12" wide next to curb, to be coated with hot pure asphalt, and finished with hot smoothing irons, price to be included in price for pavement.

Rock Asphalt Pavement.—Shall consist of one or more natural bituminous sandstones or bituminous sandstone rocks. If necessary, they are to be mixed together or a quantity of natural asphalt added to secure proper proportion of bitumen, between 9 and 10%. A bituminous limestone shall be coarse-grained, as nearly as possible a pure carbonate thoroughly and evenly impregnated with asphalt, with no more impurities than the standard German asphalt of Limmer or Vorwohle. **Laying Pavement.**—(1) The lumps of rock to be crushed and pulverized and the powder passed through a fine

sieve. (2) This powder to be heated in a suitable apparatus to a temperature of about 200° F., and brought to the ground at such temperature, in carts, and spread on the concrete foundation (same as for vitrified brick pavement) previously prepared. (3) Then skillfully compressed by heated hand rollers and rammers until it shall have required thickness of 2½". (4) Surface then made even by heated smoothers. (5) Finally, after completion, to be rolled with a heavy roller at least 5 hours for each 1000 sq. yds., a small amount of hydraulic cement to be swept over the surface during rolling.

Creosoted Wood Block Pavement.

Concrete Foundation.—(Same as for vitrified brick pavement.) **Mortar Bed.**—On the foundation, spread a ½" bed of Portland cement mortar, 1 part cement and 2 parts sand, surfaced true and smooth and parallel to finished grade and cross-section. **Blocks.**—Long-leaf yellow pine, 50% heart, treated as described below; blocks to be of sound timber, free from bark, sap wood, loose or rotten knots, etc.; no second-growth timber to be allowed. Blocks not less than 3" wide, 5" to 9" long and 4" deep, uniform in depth and thickness; to be treated throughout with an antiseptic and water-proofing mixture as follows. **Treatment of Blocks.**—Mixture shall contain 50% of dead oil of tar, known as creosote oil; its specific gravity shall not be less than 1.12 at 68° F., it shall lose not more than 40% when distilled in a flask or retort for 30 minutes up to 600° F. The distillate to contain 4% tar acids and at least 12% naphthalin. Specific gravity of residue obtained by distilling the mixture up to 600° F. must be at least 1.15. After treatment, specific gravity of block shall be greater than that of water; and shall show such waterproof qualities that, after being dried in an oven at a temperature of 100° for 24 hrs., weighed and then immersed in water for 24 hours, the gain in weight not to be greater than 3%. Blocks to be treated as follows: Blocks to be placed in an air-tight cylinder, and when doors are closed the dry heat is to be raised to 215° F. without pressure, for 1 hour, to get rid of moisture. Then heat to be increased, pressure to be applied and both are to be raised gradually to avoid injury to fiber, for 2 hours, until heat has reached about 285° and pressure about 90 lbs., and both are to be held there for 1 hour. The heat is then to be shut off and the tanks allowed to cool gradually for 1 hour; then heat reduced to 250° and pressure to about 40 lbs. Pressure is then blown off and heat still further reduced. Vacuum is then applied until about 26" is raised, and while under vacuum the creosote mixture (which shall contain no tar, petroleum, or petroleum residue) is to be run into the cylinders at a temperature of 175° to 200°, and hydraulic pressure is to be applied, reaching 200 lbs. per sq. in., and kept at this point until 20 lbs. of the mixture per cu. ft. has been absorbed. The liquid is then run off, and the wood placed in another cylinder and milk of lime at a temperature of about 150° is run in and hydraulic pressure of about 200 lbs. applied for from ½ to 1 hour. The antiseptic and waterproof mixture shall not contain more than 2% water at any time. The quantity specified for each tar acid, naphthalin and residue is the minimum for each. **Laying Blocks.**—Blocks to be set immediately upon the cement mortar bed, before it has set, and driven together as closely as possible. Pavement to be constructed with a single layer of the blocks laid on edge, with grain vertical, end to end in courses at right angle with the curb line except at street intersections where the courses are to be placed as directed. Blocks to be set in courses across the street; must be kept true and parallel, sides and ends touching, and breaking joints at least 3" with blocks in adjoining courses. Only whole blocks to be used except in starting and closing courses. **Expansion Joints** of bituminous cement to be placed at curb lines and across the street at intervals of 50 ft. Gutter joints to be 1" and cross joints ½". To make these, a plank shall be inserted and the blocks laid against it; the plank then removed, and the crack thus left filled with bituminous cement which shall have a temperature of at least 300° F. Pavement then to be rolled with a hand roller until tops of blocks are even. **Bituminous Cement** used shall not flow at 120° F., and shall not become brittle at 0° F.; shall be proof against street liquids, and pliable rather than rigid. **Filling Joints.**—After pavement is rolled, bituminous cement heated to at least 300° F. shall be poured along, filling each crack, and only when blocks are dry.

Bitulithic Pavement.

Foundation.—Bituminous, or concrete. **Bituminous Foundation.**—On the sub-foundation, crushed hard limestone which will pass a 3½" ring, to be spread to a depth of 6", and compressed with heavy steam road roller. On

this, after rolling, spread a heavy coating of Puritan brand bituminous cement, to make foundation unite with bitulithic wearing surface: One gallon of the cement to each sq. yd. of foundation. **Concrete Foundation.**—(Same as for vitrified brick pavement.) **Wearing Surface.**—On the foundation, lay the wearing surface, composed of carefully selected, sound, hard, crushed trap or syenite rock, mixed with bitumen, as follows: After heating stone in rotary mechanical mixer to temperature of about 250° F., it is elevated and passed through a rotary screen, having sections with various size openings; difference in width of openings in successive sections not to exceed ¼" in sections with openings less than ½", and not to exceed ½" in sections with openings more than ½". The several sizes of stone thus separated by the screen sections shall pass into a bin with sections corresponding to screen sections. From these, the stone is drawn into a weigh box, resting on a scale having seven beams; the stone to be weighed, using the proportions previously determined by laboratory tests to give the best results; that is, the most dense mixture of mineral aggregate, and one having inherent stability. If the crushed stone in the wearing surface does not provide the best proportions of fine-grained particles, such deficiency must be supplied by the use of not to exceed 25% hydraulic cement, pulverized stone, or very fine sand. From the weigh-box each batch of mineral aggregate, of different sizes weighed as above, shall pass into a "twin pug," or other approved form of mixer. In this mixer shall be added a sufficient quantity of Puritan brand bituminous waterproof cement, to coat all the particles of stone and to fill all voids in the mixture. Before mixing, the bituminous cement shall be heated to between 200° and 250° F., and the amount in each batch shall be weighed and used in such proportions as have been determined. Mixing to continue until the combination is a uniform bituminous concrete. In this condition it is to be hauled to the street, and spread on the prepared foundation to such a depth that, after compression with a steam road roller, it shall have a thickness of 2". The proportioning shall be such that the compressed mixture shall have the density of solid stone, as nearly as practicable. **Surface Finish.**—After rolling the wearing surface, spread over it, while it is still warm, a thin coating of quick-drying bituminous flush coat composition, by means of a suitable flush coat spreading machine provided with a flexible spreading and and adjustable device for regulating the quantity and uniformity of the composition. On grades of over 4% a mineral flush coat may be used in place of the liquid flush coat. While the flush coat is still warm, spread over it, in at least two coats, fine particles of hot crushed stone, in sufficient quantity to cover the surface of the pavement; these stone chips to be spread by means of a suitable stone spreading machine, so designed as to provide a storage receptacle of at least 5 cu. ft. capacity and to rapidly and uniformly spread over the surface of the pavement properly. The hot stone chips to immediately be rolled into the surface until it has become cool. **Patents.**—Agreement of Warren Bros. Co., on file with City Engineer, to license all contractors desiring to bid for the work to lay bitulithic pavement in accordance with its patents.

TORONTO (ONT.) PAVEMENT SPECIFICATIONS.

(C. H. Rust, City Engineer.)

A.—Grading.

Excavation.—Levels and cross-sections may be varied to conform to sills of buildings, grades of intersecting streets, lanes, carriage ways, etc. **Trenches, etc.**—Trenches or excavations that have been made for or in connection with sewers, private drains, gas or water pipes, telephone or electric wires, pipes or conduits, street or other railway works, or any other lawful purpose, and which are not thoroughly settled, shall be opened out and re-filled with gravel, well pounded, in layers of not over 4"; no extra allowance to be made to contractor. **Defective Places.**—Soft, boggy, wet, muddy or defective places must be wholly removed and filled with gravel, as in above clause, with no extra allowance. **Boulders, Trees, Etc.**—Boulders, stones, rocks, stumps, trees, posts, etc., to be removed when directed, without extra pay. **Excavations below Grade.**—Where excavation is made below proper level of sub-grade, it shall be made up with concrete where foundation is concrete, or with gravel in all other cases, without extra pay. **Rolling, Etc.**—Sub-grade to be rolled with steam roller weighing not less than ... tons; to be omitted when engineer shall consent in writing. Portions inaccessible to roller shall be rammed. Settlements, etc., to be repaired, and again rolled or rammed. Engineer

reserves privilege of testing sub-grade with City Roller. **Slopes.**—In cuttings, excavation to be made for a sufficient distance above and behind curbing, to form a slope of 2 horizontal to 1 vertical. **Oil Macadam.**—Where a macadam or broken stone roadway has to be excavated, the old material must be picked out and screened separately, and the stone delivered by contractor where directed, within 1 mile, without pay; extra haul at ½c. per cu. yd. per 100 ft.

B.—Cedar Block Pavement.

Character of Pavement.—When gravel; sand or broken stone is used for foundation, it must be watered, rolled, rammed and consolidated, until quite hard and compact, using a 12½-ton roller, unless a lighter one is allowed. **Blocks.**—Shall be of first-growth, sound white cedar, stripped of bark. No pin hole more than ½" diameter, and not more than 3 pin holes allowed on wearing surface of block. Blocks from 5" to 11" in diameter, and shall not show more than ½" of sap wood at any part of outside edge. **Laying.**—After blocks are laid, they shall be rammed with a pounder of 80 lbs. or more, 12" diameter and flat on bottom. The whole surface then rolled with a roller weighing at least 5 tons. When pavement has been brought to surface, it shall (unless other filling is called for) be covered with a sufficient quantity of gravel to fill all joints, being worked in with suitable brooms. Surplus material, if any, then to be swept off, and pavement to be rammed as before. To be repeated if necessary. Finally, pavement to be covered with ⅝" layer of good clean gravel. **Board Bed.**—Shall be laid upon the foundation bed if required. Shall be pine, 9" to 12" wide, perfectly sound, etc.; and thoroughly swabbed on both sides and ends with approved composition, or dipped in same, or other preservative process. Where double boards are used, the lengthwise boards shall be 12 and 16 ft. in length, respectively, laid so as to break joint. **Concrete Bed.**—When a concrete bed is required it shall be laid as specified under "C.—Concrete," below. After the bed has set hard, spread a layer of clean, sharp sand so that it will be ½" thick after the blocks are laid upon it and rammed. **Grouting.**—When grout filling is required between the blocking it shall be composed of 1 part Portland cem., 3 parts coarse, sharp, clean sand, thoroughly mixed and flooded, and swept into all joints; repeating same until joints are full and flush with surface. Surface then to be covered with at least ⅝" layer of approved gravel. **Tar Filling.**—When tar composition is required for filling, it shall be composed of 1 part coal-tar to 2 parts pitch, boiled and freed from moisture, and applied at a temperature of 250 to 275° F., filling all joints. A paving pitch, when approved, may be used instead, and applied similarly, or as directed. Clean gravel, ½" to ¾", dried and heated, must be used with above composition; the gravel to be swept into the interstices hot, and the composition applied at once, completely filling joints and flushing to surface. **Composition and Gravel.**—With board foundation, use not less than 2 gallons (imperial) of composition per sq. yd. of pavement; without board foundation, use not less than 3 gallons, and as much more as needed to fill flush to top of pavement. After ramming, pavement to be swept clean, and covered with hot composition, upon which shall immediately be spread heated gravel or stone chippings, ¼" to ½", at least ½" deep.

C.—Concrete.

Proportions.—Upon the sub-grade, lay a bed of concrete, ... ins. thick, composed of 1 part best Portland cement, 3 parts clean, sharp sand, 7 parts broken stone or furnace slag; the proportions may be varied to 1 part cement and 10 parts of the sand, broken stone and slag. Gravel may be required instead of broken stone.

D.—Asphalt Pavement.

Kinds of Asphalt.—Use best quality of following asphalt: Trinidad, Bermudez, Venezuela, Natural Rock, California, or other equally as good, and approved. No coal-tar, or any other product thereof, or any other inferior products, will be accepted. **Cushion Coat.**—When light asphalt is specified no binder or cushion coat will be required. When heavy asphalt is specified a binder course 1" thick will be laid directly on the concrete foundation, but it will not be required where Natural Rock asphalt is used. This underlayer to be rolled and consolidated, while fresh and hot, with a steam roller weighing not less than 5 tons, to a finished thickness of 1". No binder asphalt to be laid during wet weather. Should Rock asphalt be used it should be at least 2" thick. **Surface Coat.**—Shall be 2" thick after ultimate compression.

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E.—Brick Pavement.

Foundation.—A base of Portland cement concrete 4" or 6" in depth. **Cushion.**—Where no thickness is shown, the depth of sand will be at least 1½". **Paving Bricks.**—Must not be less than 2½" by 8½" by 4". **Paving Blocks.**—Must not be less than 3½" by 8½" by 4". **Tests** shall be made for absorption and abrasion. **Laying.**—(Usual manner.) **Ramming and Rolling.**—First with 80-lb. rammer, then with 5-ton roller. **Grout Filling.**—Composed of equal parts best Portland cement and sand, mixed; water added, stirred and immediately used, filling joints to no more than half their depth. Then thicker grout, 2 cement to 1 sand, for balance of joint. **Sand Coating.**—Not less than ½".

F.—Macadam Roadway.

Foundation Course.—Upon the sub-grade, lay a course of stone, 5" in thickness; stones to be laid by hand, largest sides down, and in line at right angle to the curb, and breaking joint as much as practicable. The upper surface of stones not to be less than 6" nor more than 9" in width, nor less than 12" nor more than 15" in length. The stones to be set close together and bound by wedging in small stones and filling the interstices with stone chipping so as to form a compact bed. Stones projecting above the surface must be broken off, great care being used not to loosen the foundation. No wedging is to be done within 20 ft. of the work being laid. Next, spread evenly over this course, clean gravel to fill all interstices, and then roll and re-roll until thoroughly consolidated. **Intermediate Course.**—Upon the foundation, place a layer of broken stone, as granite, trap, or hard limestone, of approved quality. Not more than 5% of the stone shall be less than 1¼" and no particle of stone shall be more than 3" in breadth. After this course has been evenly spread over the street (and raked if necessary) so as to present a uniform surface, a layer of coarse, clean sand is to be spread upon it, sufficient to fill voids, and the surface then watered (if necessary) and rolled, more sand and stone being applied where and as required, and the rolling and watering continued until an even, hard and uniform surface is obtained, after which the sand is swept up and removed from the surface. **Top Course.**—Consists of broken stone, ... ins. in depth, as uniform in size as possible, no particle to be more than 2½", and not more than 5% to be less than 1¼", in length or breadth. The surface shall be raked evenly, watered, rolled, repaired, brought fully up to grade, and then re-rolled until firm, compact and true. A sufficient layer of good, coarse, clean sand is then spread over the surface, rolled, and flooded with water, to carry the sand into all interstices; more sand to be added as rolling and watering progresses, so as to have finished thickness of ¼" to ½".

K.—Concrete Sidewalk.

Foundation.—After the street has been graded, a foundation shall be laid, composed of coarse gravel or suitable soft coal cinders, to a depth of 4", after being consolidated by pounding or rolling with a suitable and approved roller, weighing at least 1 ton, and the whole brought to an even surface. Whilst pounding, a small quantity of water may be used through a sprinkler, as directed. **Templates.**—When required, the contractor must furnish wooden templates, cut to exact form and slope of the walk, for use by the

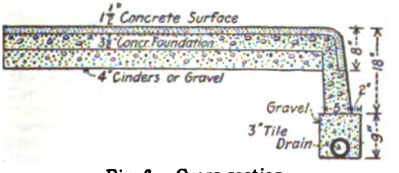

Fig. 6.—Cross-section.

inspectors. **Concrete Base.**—Upon this foundation, a layer of concrete, 3½" thick, shall be laid, composed of 1 part Portland cement (of approved quality), 2 parts of clean, sharp, coarse sand, and 5 parts of approved furnace

slag, broken stone or screened gravel, thoroughly free from stone over 2″ diameter, and free from clay, loam, dirt or other impurities. The concrete thus made shall be rammed with iron rammers into one solid mass, and until it has a straight and even surface. **Divisions.**—Every 6 ft. a cut shall be made completely through the concrete before it is set, with an iron for that purpose, not less than ¼″ in width. The opening shall then be filled in with clean, sharp sand. A clear space not less than ½″ must be left between back of curbing and abutting ends of sidewalks to allow room for expansion, excepting where the walk and curb are combined. **Heavy Surface.**—On the concrete base, before it has had time to set, lay the wearing surface, 1½″ thick, composed of 1 part Portland cement, 1 part clean, sharp, coarse sand, and 3 parts crushed granite or quartzite. **Light Surface.**—On the concrete base, before it has had time to set, lay the wearing surface 1″ thick, composed of 1 part Portland cement, 1 part clean, sharp, coarse sand, and 3 parts of screened pea gravel, crushed granite, quartzite or suitable hard limestone. (See Fig. 6.)

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D.—CARE OF ROAD SURFACES.

DUST PREVENTIVES.*

CLASSIFICATION.

Two classes: 1st, water, salt solutions, certain light oils and tars, and oil and tar emulsions; 2nd, the heavier oils, tars, semi-solid and solid materials. Salt solutions are valuable because the dissolved salt has a considerable affinity for water and keeps the road moist long after a surface treated with water alone would have become dry. The light oils and tars, and oil and tar emulsions, leave upon the road surface a comparatively small amount of true binding base after the volatile products have evaporated. The heavy oils and tars contain a greater amount of binding base, hence more lasting. The semi-solid and solid preparations usually contain a still greater amount of binder, and also other materials of a solid nature, such as rocks, sand or clay. With some few exceptions, all the true binders are bitumens, either natural or artificial.

TARS, THEIR MANUFACTURE AND PROPERTIES.

Coal Tars.—Coal is by far the most important source of tar. (a) Tar from Coke Ovens is made as follows: The coal is charged into long narrow chambers or retorts of about 4 or 5 tons capacity and heated by means of flues set in the retort walls; volatile matter held in the coal passes out through an opening in the top and is conducted through a series of washers and scrubbers, as in gas manufacture, to remove the tar and ammonia; the purified gas is then allowed to pass into a holder from which it is drawn as needed for burning under the retorts. (b) Tar from Gas Plants is unavoidable in the manufacture of illuminating gas; the bituminous coal is placed in fire-clay retorts about 8 ft. long, 15" high and 18" wide; 6 or 8 retorts set together in a furnace and forming a "bench;" a number of these benches built together is called a "stack;" the retorts are heated by means of a coke fire or by generator gas. The tar which collects in the hydraulic main, the condensers, and the tar towers, is run into large wells where it is allowed to settle; the accompanying ammoniacal liquor, being lighter than the tar, rises and is drawn off; the crude oil tar which remains is a black acid fluid with peculiar odor, and with specific gravity from 1.1 to 1.2. It represents about 5% of weight of coal. The true tarry products are artificial bitumens; the natural bitumens being found in various mineral oils and asphalts. The nature and value of tar vary with the coal used, and with the temperature and other conditions under which it is produced. Free carbon, having no binding value, will prove detrimental. (c) Refined Coal Tar is obtained by fractional distillation for the separation of certain constituents used in the arts; the residue left in the still is known as coal-tar pitch, and is a thick viscous material while hot. It represents the true binding base of the tar, and if the tar is produced at comparatively low temperature the residue is composed mainly of bitumens. After cooling for a few hours it is run out of the still and is graded as soft, medium or hard, according to its condition when cold. The dead oils, or heavier distillation products, and of less value than the other volatile distillates, are often run back into the still before the pitch is drawn off, in which case the pitch is liquid when cold. In preparing a tar for dust prevention, most of the valuable products are removed by fractionation, and the least valuable—as some of the carbolic and all the dead oils—are run back into the pitch until it reaches about the consistency of a heavy crude tar, sometimes adding dead oils from previous distillation, if necessary. These oils give life to the tar, and if percentage of pitch is not reduced too low the mixture has certain advantages over the crude tar, and it is comparatively free from naphthalin and anthracine and contains none of the volatile oils and ammoniacal liquor found in the latter. (d) Dehydrated Tar (crude) is sometimes prepared for dust prevention, the water being to remove all water, ammonium compounds, and some of the light oils. The absence of water makes it easier to handle when applied hot, and probably allows of a better absorption of the tar by the road surface. Water in tar hastens disintegration of the heavy binding materials; the ammoniacal liquor may saponify some of the oily products, mix with the water and wash out. Dehydrated tar may be prepared by boiling the crude material in open kettles until its boiling point lies between 105° and 110° C.

* Digest of Bulletin No. 34, Office of Public Roads, U. S. Dept. of Agric., Prepared by Hubbard, Assistant Chemist.

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Water-Gas Tar has been used to some extent as a dust layer and road preservative. It is first obtained by admitting steam into a chamber called a generator which contains coke heated to incandescence; the water vapor reacts with this coke to form certain products and the mixture of gases is led into another chamber called the carburetor, where it meets a spray of hot oil, which is thus volatilized and carried to another chamber known as the superheater, where most of the hydrocarbons combine to form a permanent gas. The gas thus produced is washed with water and passed through extractors and scrubbers in much the same manner as ordinary coal gas, in order to remove the tarry products. The product is entirely different from ordinary coal-tar and contains a relatively small amount of heavy bitumens; the base is more or less thin, and of poorer binding quality than that of good coal tar; it may be used to advantage in certain instances, being cheap and easily handled. It compares quite favorably with the lighter oils, and oil and tar emulsions.

Composition of Tars.—The following Table shows some of the properties of crude coal tar, refined coal tar and water-gas tar, previously described. The notes to the table refer to the condition of distillates and residues when cold:

SPECIFIC GRAVITY AND COMPOSITION OF TAR PRODUCTS.

Kind of Tar.	Specific Gravity.	Ammoniacal Water.	Total Light Oils to 170°C.	Total Dead Oils 170°-270°C.	Residue (by difference).
		Per cent.	*Per cent.*	*Per cent.*	*Per cent.*
Water-gas tar....	1.041	2.4	*a*21.6	*b*52.0	*c*24.0
Crude coal tar....	1.210	2.0	*d*17.2	*e*26.0	*f*54.8
Refined coal tar..	1.177	0.0	*b*12.8	*g*47.6	*f*39.6

a Distillate mostly liquid.
b Distillate all liquid.
c Pitch very brittle.
d Distillate mostly solid.
e Distillate one-half solid.
f Pitch hard and brittle.
g Distillate one-third solid.

THE APPLICATION OF TARS.

Application to finished road surfaces.—The primitive method in this country is: Road surface first thoroughly swept to remove all dust; hot tar then spread on and thoroughly broomed in; road then, if possible, closed to traffic 12 hours to allow tar to soak in; at end of that time, or sooner, a coat of clean sand or stone chips applied to absorb excess of tar, and surface then rolled several times to bring it to proper condition quickly. The tar is heated in an open kettle preferably mounted on wheels and fitted with a portable fire-box. It is usually brought to its boiling point—about 190° F. (If temperature of crude tar is raised above 190° F. when heated it is very likely to foam up, boil over and catch fire.)—before being spread upon the road, although a lower temperature is sometimes sufficient; and if the kettle is of the above type the tar may be run out upon the road by means of a hose, the kettle being kept just in advance of the work; two kettles will allow continuous working, one being charged and heated while the other is in use: kettles to hold easily 9 barrels or about 450 gallons. Application of tar by mechanical means is also being used, notably in England.

Use of tar in road construction.—When a New Road is under construction, or an old road being resurfaced, the road should first be shaped and consolidated as well as possible without the use of water. The voids should be filled well with clean, fine stone chips free from dust, but an excessive amount of rolling should be avoided because if the roller is used too freely the larger stones will become rounded and covered with dust, thus preventing the tar from adhering properly. Hot tar may be applied to all the courses if desired, but sometimes only the upper course is so treated. After the tar has been applied, a dressing of fine material is spread on and the whole road well rolled. The tar may be spread by hand, but it is most economical to use a tar spreader: the spraying apparatus is mounted on wheels and is so arranged that the tar is forced from the tank in which it is heated, into an air receiver under a pressure of 150 to 350 lbs. per sq. in.; the necessary power

for pumping the air and the liquid into the receiver being obtained by means of chain drive from the road wheel. From the receiver the tar is distributed upon the road by means of specially designed spraying nozzles.

Amount and cost of materials.—According to conditions and methods of application, a surface-treated road will require from 0.35 to 0.70 gallons of tar per sq. yd. when applied by hand; and as small as 0.21 gallon has been used with good results when applied by machine, for first treatment. With either method the application of tar must be repeated from time to time, though less is required at each successive application. If tar is applied as road is built, as much as 1.5 gallons per sq. yd. are often consumed if spread by hand; but by means of devices like the pneumatic tar sprayers, it is claimed that the road stones to a depth of 3½" may be well covered with about 0.6 gallon per sq. yd. Crude coal tar can ordinarily be purchased from gas or coke companies at from 3 to 5 cts. per gallon; refined tars run from 6 to 12 cts. and even higher. The cost of treatment in France, by machine, will average about 3 cts., and by hand 5 cts.; in this country, where it is generally applied by hand, the cost ranges from about 6 to 12 cts. or more per sq. yd.

Oils, Their Classification and Properties.

Oil fields.—There are seven distinct oil fields in the United States: 1st the Appalachian (including New York, Penn., W. Va., southeastern Ohio, and parts of Ky. and Tenn.) produces oils known as eastern oils or paraffin petroleums, and which are therefore of use only as temporary binders in dust suppression; 2nd the Ohio-Indiana field produces oils much like those of the Appalachian and are also classed as paraffin oils; 3rd the Colorado field, similar to above; 4th the Wyoming field with oils varying from the lighter oils, to the heavy asphaltic oils which are found principally in California; 5th the California field produces oils of the most varied character, consisting mainly of more or less dense asphaltic hydrocarbons, none of the components being of the paraffin series, the percentage of asphaltic residue usually high and of good binding character, the oils being considered the best for use as permanent binders; 6th the Texas field contains oils of a mixed character, with some paraffin as well as a greater or less amount of asphaltic residue, some having been used successfully as dust preventives, with others unfit for this purpose; 7th the Kansas field (including Oklahoma) produces oils quite similar to those from Texas. The same is true of Louisiana. In general, the eastern oils are of the paraffin type and useless as permanent binders; the western oils are of asphaltic character and of great value as permanent binders; while the southern oils are of a mixed character, their value as dust preventives lying in the relative amount of asphalt base contained.

Refining.—Although crude oil is used to a great extent in the West as a dust preventive, it is often customary in the East to partially distill oils containing asphaltic residues before using them, thus recovering many of the more valuable constituents and producing residual oils having a much better binding quality because they contain a larger percentage of asphalt base. Crude petroleum is an oily liquid, of unpleasant odor, with specific gravity from 0.73 to 0.97, according to locality from which it is derived; color from greenish brown to nearly black, often reddish brown or orange when viewed by transmitted light; sometimes fluorescent. The crude petroleum is refined by means of fractional distillation, somewhat similar to that for crude coal tar. The most valuable products are the kerosene, or burning oils, and the method called "cracking" is employed to increase their yield; this consists in modifying the fire, during process of distillation, so that only the bottom of the still is intensely heated, while top and sides, being exposed to the air, become somewhat cooled; thus the heavy oil vapors are condensed within the still itself, and upon dropping back into the residuum, which is much hotter than their boiling point, break up into lighter oils with lower boiling points, with a separation at the same time of free carbon or coke, which is deposited in the residuum. The paraffin petroleum residuums contain a large amount of paraffin hydrocarbons and paraffin scale or crude paraffin, and are unsuitable for road surface treatment. The base held by the California petroleums is composed of bitumens resembling asphalt; the residuum contains no paraffin and, if cracking has not been employed in its preparation, carries but little free carbon; both the crude oil and the residuums, if properly prepared, are excellent binders and give the best results of any oils which have been used as dust preventives. The semi-asphaltic oils, as from Texas, carry an asphaltic base, but also a considerable amount of paraffin hydrocarbons and

1% or more of paraffin scale; are somewhat inferior to the Califorina products but often give good results.

Comparisons of crude oils and residuums.—The two following Tables show some results obtained from an examination of various crude and refined petroleums in the New York Testing Laboratory.

RESULTS OF TESTS OF CRUDE PETROLEUMS.

Kind of Oil.	Spec. Grav.	Flashing Point.	Volatil'y at 110°C. 7 hours.	Volatil'y at 160°C. 7 hours.	Volatil'y at 205°C. 7 hours.	Residue.
		°C.	*Per cent.*	*Per cent.*	*Per cent.*	*Per cent.*
Pennsylvania, paraffin.	0.801	(*a*)	47.3	58.0	68.0	*b*32.0
Texas, semi-asphaltic..	.904	43	20.0	27.0	49.0	*c*51.0
California, asphaltic...	.939	26	.....	.....	*d*42.7	*e*57.3

a Ordinary temperature.
b Soft.
c Quick flow.
d Volatility at 200°, 7 hours.
e Soft maltha; sticky.

RESULTS OF TESTS OF PETROLEUM RESIDUUMS.

Kind of Oil.	Spec. Grav.	Flashing Point.	Volatil'y at 200°C. 7 hours.	Residue.	Solid Paraffin.	Fixed Carbon.
		°C.	*Per cent.*	*Per cent.*	*Per cent.*	*Per cent.*
Pennsylvania, paraffin.	0.920	186	14.2	*a*85.8	11.0	3.0
Texas, semi-asphaltic..	.974	214	6.2	*a*93.8	1.7	3.5
California, asphaltic...	1.006	191	17.3	*a*82.7	0.0	6.0

a Soft.

THE APPLICATION OF THE HEAVIER OILS.

To macadam surfaces.—Holes and inequalities should be repaired; not necessary to remove all dust as in case of tar, but sticks, leaves, etc., should be removed; crude oil either hot or cold, according to its viscosity and ability to penetrate the road surface; much cheaper applied cold. A covering of sharp sand or ½" stone screenings should be applied after the oil has been allowed to penetrate as much as possible, in order to take up all excess. and the surface well compacted by rolling, additional sand or screenings being thrown on wherever the oil shows a tendency to force its way to the surface and produce a sticky condition. Sometimes 2 or 3 courses of oil and screenings are applied.

During construction of macadam road.—The greatest success has been in California where the heaviest asphaltic oils are found; and the residuums obtained from the partial distillation of these oils have, so far, given the best results. The treatment is essentially the same as with tar, above described. The macadam is built in the usual manner and each course thoroughly rolled until the whole road is consolidated. A road constructed in this manner will require from ¾ to 1½ gallons of oil per sq. yd.

To gravel roads.—A gravel road is oiled in much the same way whether it is an old road or one under construction, as only the upper course is treated in either case. Good drainage is very essential. The oil may be applied either hot or cold, according to its viscosity, by any method previously described. Where the treated surface is loose and contains a considerable amount of clay, the oil may be worked into the upper course by raking, insuring an equal distribution. After application of oil, the road should be rolled until properly compacted, adding fresh material as the oil works to the surface. Condensed specifications from biennial report (1906) by Commissioner of Department of Highways of California are: Sub-grade to be thoroughly rolled; then 2 layers of gravel, bottom layer 5" and top layer 3" after being rolled; 1st layer containing not larger than 2½" stone; gravel to

be evenly spread, well moistened, rammed 1 ft. from gutter or curb, and remaining portion rolled; depressions filled, moistened, and again rolled to unyielding surface; on this surface place the top layer of gravel, no stones over 1" diameter, and compacted in same manner. Oil should then be evenly distributed over the entire surface, ½ gallon per sq. yd., and covered with clean, sharp sand until no oil can be seen; after 12 hours, another application of oil and sand in same manner, and rolled to unyielding surface. Use crude oil applied at temperature between 150° and 190° F.

To earth roads.—(a) The oil is simply sprinkled on the road, laying the dust and incidentally hardening the surface. Alkali soils disintegrate the oil and destroy its binding qualities. A sandy loam is the most suitable for treatment, usually giving good results when properly treated with an oil of good binding quality. Clay is probably the worst of all, as it does not absorb the oil well and exhibits a tendency to ball up and give trouble; sand should therefore be added to the clayey surface. Special attention should be paid to drainage, the roadbed to be dry when the oil is applied. (b) Another method: The road is first plowed to deptn of 6" and properly crowned; all clods and lumps broken up by means of a harrow, and roadway well sprinkled with water; a specially constructed rolling tamper is then used by which the lower portion of the loose earth is compacted to depth of about 2", except in cases where sub-grade is unusually firm. After the lower portion is made firm, a heavy asphaltic oil is applied, 1½ gallons per sq. yd., and a cultivator passed over the road until the oil and earth are thoroughly mixed. The tamper is then used again, and the road is further compacted until only 1½" of loose material remain on top. Oil is again applied, and surface rolled with the tamper until firm, and finally it is ironed down with an ordinary roller, additional applications of earth being made wherever necessary to take up any excess of oil. A road constructed in this manner will require from 2½ to 3 gallons of oil per sq. yd. It is hard and dustless and resembles asphalt. California oils are the best. Texas and Kentucky oils cost from 4 to 8 cts. per gallon. The residuums and special preparations vary from 2 to 12 cts.

SPECIFICATIONS FOR COAL TARS.

I. When used as temporary binder it is usually employed in form of an emulsion; specifications difficult.

II. When used as a semi-permanent binder it is necessary that sufficient binding base be present to last through dusty season; and for economy, material to be sufficiently fluid to apply cold; hence—

1. Coal tar to be formed at low temperature, such as produced from by-product coke ovens or by gas plants.

2. A crude tar may be employed because of cheapness.

3. If not sufficiently fluid to apply cold, enough water-gas tar may be added to bring it to proper consistency, but proportion of latter should not exceed 50% of mix.

III. When used as a permanent binder for surface application, either a crude or a refined product may be employed, preferably the latter.

1. If a crude tar, it should have the following properties:

(a) Same as II, 1.

(b) Its specific gravity to be between 1.15 and 1.19.

(c) It should be free from water-gas tar and oil tar.

2. If a refined tar, the following might be specified:

(a) Same as II, 1.

(b) Its specific gravity to be between 1.17 and 1.20.

(c) No water nor ammoniacal liquor to be present; boiling point above 110° C. (230° F.).

(d) Upon distillation, at least 40% by volume of pitch should remain after all oils have been driven off below 270° C. (518°F.).

IV. When used as a permanent binder in road construction it is necessary that more binding base be present than is usually found in the crude

product. A refined product should therefore be employed; either prepared from a crude tar by the contractor or a special preparation purchased.

1. In the former case, a mixture may be prepared as follows from a crude tar, which meets the specifications set forth in III, 1:

(a) The tar should be heated until boiling point is raised to at least 110° C. (230° F.).

(b) One-tenth part or more of good soft pitch should then be dissolved in the tar while hot; the quantity added should be sufficient to produce a pitch residue of at least 50% by volume after all oils have been driven off under 270° C. (518° F.).

2. A refined tar for this purpose should meet the requirements as suggested for III, 2, except that the pitch remaining after volatile oils under 270° C. have been driven off should amount to at least 50% by volume.

In some cases, especially where climate is warm throughout the year, a tar considerably thicker in pitch may be preferred.

SPECIFICATIONS FOR MINERAL OILS.

(a) The oil shall have a specific gravity of not less than 0.95. (b) Its flash point shall not be lower than 300° F. (c) It shall be free from water as determined by the gasoline test. (d) When heated to 400° F. its loss in weight should not be over 35%. The character of the residue should be smooth and nearly solid when cold, but not so hard that it may not be easily dented with the finger, and when soft it should pull to a long, thin thread. (e) The oil shall be soluble in carbon disulphide to the extent of 98%, and in 88° naphtha to at least 88%.

EXPERIMENTS WITH DUST PREVENTIVES.

Note.—Following is from circular No. 89, Office of Public Roads, U. S. Dept. of Agric., issued April 20, 1908. During the past few years a number of preparations for laying and preventing dust have appeared on the market in competition with crude materials, such as coal tar and petroleum, and it was therefore decided by the Office of Public Roads to carry on a series of experiments during the summer of 1907 with a view to determining, if possible, the relative value of these preparations and crude products and their adaptability to different conditions. Experiments were conducted at Wayland, Mass., Washington, D. C., and Bowling Green, Ky. Following are results:

Experiments at Wayland, Mass.—The water-gas tar was obtained from a local gas company at $1.50 per barrel of 50 gallons, delivered. The crude coal tar, in 50-gal. bbls. at $2 per bbl., del. It had been produced at a low temperature and contained a good pitch base. The special coal-tar product was supplied free in 50-gal. bbls., the Office paying the freight from Boston to Wayland. It contained no water, was free from the extremely volatile oils present in the crude tar, and held a good pitch base. The other properties are shown in comparison with the water-gas and coal tars in preceding Table. Labor cost per 8-hour day was as follows: Common labor, $1.50 to $1.85; single teams, $3; double teams, $5; foreman, $3; steam roller, $12. The cost of repairs per sq. yd. of road surface was from 2.6 to 3.3 cts. When gravel was used it was obtained from pits near the road, costing the commission $1.08 per cu. yd. Clean trap screenings, 1½", or pea stone, however, was used whenever it could be obtained and was furnished at $1.10 per ton by a rock-crushing plant located about 4 miles from nearest section of work. Thirteen experiments were made as summarized in the two following Tables.

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COST DATA OF TAR EXPERIMENTS.

Experiment No.	Material Supplied.	Cost of repairs per square yard.		Cost of application per square yard.			Cost of removing waste material per square yard.	Total cost per square yard.	Total cost of entire area.
		Labor.	Material.	Labor.	Material.	Surfacing.			
1	Water-gas tar.....	$0.022	$0.010	$0.049	$0.027	$0.008	$0.013	$0.129	$220.74
2	"	.023	.010	.049	.011	.008	.013	.114	19.03
3	"	.....	.....	.006	.009	.....	.....	.015	7.30
4	"	.....	.....	.013	.008	.....	.012	.033	6.60
5	Coal tar.....	.018	.008	.061	.024	.008	.010	.129	32.46
6	"	.018	.008	.056	.028	.009	.010	.129	129.14
7	"	.....	.....	.044	.017	.014	.013	.088	52.82
8	Water-gas and coal-	.018	.008	.058	.033	.010	.010	.137	57.49
9	" ..... tars	.....	.....	.030	.015	.007	.010	.062	58.26
10	"	.....	.....	.037	.017	.008	.010	.072	119.89
11	Special tar mixture.....	.....	.....	.046	.026	.017	.013	.102	27.27
12	Special tar preparation	.023	.010	.148	.090	.033	.012	.316	31.64
13	"	.....	.....	.058	.040	.019	.010	.127	103.45
	Total.....	72.28	36.00	382.07	203.38	82.26	90.10	.....	866.09

MISCELLANEOUS DATA OF TAR EXPERIMENTS.

Experiment No.	Material Applied.	Surface Application.	Number of gals. of material applied per sq. yd.	Surfacing material, cubic yds. per sq. yd.	Waste material removed, cubic yds. per sq. yd.	Area covered, square yards.
1	Water-gas tar.....	Gravel....	0.90	0.0079	0.009	1,700
2	"	"	.38	.0080	.009	167
3	"	None.....	.30	.....	.....	500
4	"	"	.25	.....	.009	200
5	Coal tar.....	Gravel....	.60	.0080	.009	250
6	"	"	.70	.0080	.009	1,000
7	"	"	.42	.0133	.009	600
8	Water-gas and coal tars....	"	.90	.0096	.009	417
9	"	Pea stone.	.43	.0086	.009	933
10	"	Gravel....	.48	.0072	.009	1,667
11	Special tar mixture.....	Pea stone.	.42	.0202	.009	267
12	Special tar preparation.....	"	1.50	.0400	.009	100
13	"	"	.67	.0226	.009	817
	Total.....	.....	.....	.....	.....	11,118

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Experiments at Washington, D. C.—With calcium chloride, as a dust preventive; tested on portion of macadam driveway in Agricultural Dept. grounds, Washington, D. C. The roadway is built of trap rock, held in position by a soft limestone binder; the screenings of the binder pulverized rapidly under traffic, forming a light dust continually raised by passing vehicles, and carried away by the wind; the road was thus becoming stripped of its binding material. In preparing for treatment, all dust and dirt were scraped from surface of roadway; a solution was prepared by mixing 300 lbs. of commercial calcium chloride (granular, containing 75% calcium chloride and 25% moisture) with 300 gallons of water in an ordinary street sprinkler, agitating the liquid thoroughly before applying. It was then applied from one sprinkling head, and the sprinkler passed slowly back and forth over the road to facilitate the complete absorption of the solution; each application consisting of 600 gallons over an area of 1582 sq. yds., or 0.38 gallon per sq. yd. The first application was made July 13, 1907, followed by a similar one July 15, to increase the efficacy of the treatment. The effect was marked. The texture of the road surface was completely changed: before treatment, raveling was excessive in spots and the whole surface seemed loosely knit together; after treatment of July 15, this condition changed and the road surface became smooth, compact and resilient. The third treatment, August 3rd, was given because certain points exposed to the most severe wear were showing signs of raveling; the results of this treatment, and of successive ones, were most satisfactory and not unlike those attending the first two treatments. The calcium chloride is charged at rate of $16 per ton, f. o. b. cars at Baltimore; freight 13 cts. per 100 lbs. Specific gravity of solutions, 1.053 to 1.060. Following is Table of cost of applying:

COST OF APPLYING CALCIUM CHLORIDE.

Item.	Cost.
600 pounds calcium chloride, at $18.60 per ton	$5.58
3 men for 1½ hours, at 15 cents per hour	.675
1-horse sprinkling wagon for 1½ hours, at 35 cents per hour	.525
Total cost of 1,582 square yards	6.78
Cost per square yard at this rate	.0043
Total cost of five applications	33.90
Cost per square yard of five applications	.0235

Experiments at Bowling Green, Ky.—Materials used were Kentucky rock asphalt tested for its fitness as a binder in macadam construction, crude Kentucky oil, and special preparation of residuum oils, the last two of which were used as dust preventives.

(a) *Rock Asphalt Experiment.*—The rock asphalt used is a natural product formed in the Chester group of subcarboniferous rocks over a course extending through Breckenridge, Grayson, Edmonson, Logan, and Warren counties, marking the edge of the coal fields lying in the western part of Kentucky. It is a fine-grained sandstone impregnated with mineral pitch or bitumen, the latter averaging from 6 to 8%, with a maximum of 12%. After quarrying and crushing, 2" size, it is further passed between steel rollers, the finished product being a mass of individual grains of sand, each thoroughly coated with a film of mineral pitch, adhering to surrounding grains and packing very firmly if subjected to pressure; with a rich dark brown color with a slight luster which gradually disappears as the bitumen hardens and dries. If chilled when compacted, a lump becomes very hard and tough; if warmed in the hand, the bitumen becomes soft and semi-fluid and the individual grains of sand fall from the mass of their own weight. The test was made on Cemetery pike, a main thoroughfare. The form of construction originally adopted was a 20-ft. Telford road. When the surface had been worn away exposing the foundation, it was repaired and brought to grade with a sharp gravel containing about 20% sand and clay, the layer being about 8" thick when compacted. Previous to the experiment, it was loosened to a depth of 4" by means of a spiked roller and a heavy harrow, and shoveled out by hand. The sub-grade was then made to conform to crown of roadway, planned to be 4½" in 9 ft., or an average of ½" per ft. After rolling the sub-grade the wearing course of stone was laid; it consisted of crushed limestone,

1" to 1½", spread in a 4" layer, rolled once to turn down the sharp edges of the stone and form a smooth, even surface. The rock asphalt was then thrown on with shovels and spread to a depth of 1½", care being taken to break all lumps and to work all the asphalt rock possible into the interstices of the stone without disturbing the latter. As the work progressed, the roller was kept moving back and forth parallel to axis of roadway, working from outer edge to crown as in ordinary macadam construction. The inadvisability of working the material when chilled and damp was apparent, for the portion of the road laid at a temperature of 65° F. failed to become hard and firm for several hours after subsequent applications had compacted satisfactorily. The cost of the work is shown in the following Table. Labor ranged from $1.20 to $1.25, and teams cost $3 per day of 10 hours; roller was loaned, cost of operation being $2.50 per day for engineer plus cost of fuel; stone was delivered on roadway at $1.20 per cu. yd. and was spread 4" thick uncompacted, making cost per sq. yd. delivered 13 cts; cost of asphalt charged at market price of $5 per ton f. o. b. cars at Bowling Green; it was spread about 1½" thick, or at rate of 24.5 sq. yds. per ton.

COST DATA OF ROCK ASPHALT EXPERIMENT.

Item.	Cost per square yard.	Total cost.	Percentage of total.
	Cents.	*Dollars.*	*Per cent.*
haping sub-grade	5.66	43.60	11.8
tone on work	13.71	105.60	28.8
preading stone	.78	5.97	1.7
olling stone	.09	.67	.3
sphalt on work	23.77	183.10	50.0
preading asphalt	1.44	11.06½	3.1
olling asphalt	2.18	16.78½	4.3
Total	47.63	366.79	100.0

(b) *Oils.*—In connection with work on rock asphalt, experiments were ade to determine the comparative value of a residuum oil preparation and ude oil, as dust preventives. The oil preparation is a patented mixture of siduum oils, combined with a view to obtaining such proportions of asaltic and lighter oils as shall be best fitted for immediate dust laying and rmanent improvement of the roadway. The tests were made on Cemetary ke beyond the point where the rock asphalt work ended. The general ndition of the gravel roadway was unsatisfactory for the purposes of such test; the cross section of the roadway was quite flat, the crown having en completely worn down, so that lateral drainage was defective; also ere were extensive pockets of loose material, characteristic of roads made gravel containing a large percentage of sand and clay. Heavy rains had len for several days preceding the application of the oil, and although the d surface was quite dry there was a large amount of moisture in the road-l, thus prevening the rapid absorption of the oil by the gravel before the poration of the lighter oils took place. The oils were applied over a lth of 12 ft. of the roadway by means of an oil sprinkler adapted to the form spreading of heavy liquids. A tank load of the oil preparation was ead over an area of 841 sq. yds., or an average of 0.903 gallon per sq. yd.; temperature of the oil 87° F. due to its exposure to the sun; it was heavy l was absorbed very slowly by the gravel, about 4 days being required to se it to become thoroughly incorporated into the surface of the road; gravel becoming very compact and showing few traces of wheel marks ept at points where repairs had been made, in which cases the cementing cess took place more slowly. In the case of the crude oil, 5 tank wagons, total of 3712 gallons, were applied to 4416 yards of surface, making an rage of 0.84 gallon per sq. yd. An experiment was also made with crude on macadam road; there was about ¼" of dust on roadway consisting ely of powdered limestone; the roadway was given an application of gallons over an area of 1452 sq. yds., or at rate of 0.52 gallon per sq. yd. following Table gives a statement of the cost of repairs, materials, and lication of the oils.

COST DATA OF OIL EXPERIMENTS.

Experiment.	Item.	Cost per square yard.	Total cost.
		Cents.	*Dollars.*
Special oil preparation	Repairs and ditching	0.57	4.79
	Oil	.13	110.00
	Application	.47	3.93
	Total cost	1.17	118.72
Crude oil on gravel road	Repairs and ditching	.33	14.38
	Oil	4.46	196.94
	Application	.16	7.28
	Total cost	4.95	218.60
Crude oil on macadam road	Oil	2.75	40.00
	Application	.11	1.64
	Total cost	2.86	41.64

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EXCERPTS AND REFERENCES.

The Asphalt and Bituminous Rock Deposits of the U. S. (By G. H. Eldridge; U. S. Geol. Surv.; Eng. News, June 5, 1902).—

TABLE I.—CLASSIFICATION OF NATURAL HYDROCARBONS.

Gaseous			Marsh gas. Natural gas.
Bituminous	Fluid		Naphtha. Petroleum.
	Viscous (malthite)		Maltha. Mineral tar. Brea. Chapapote.
	Elastic		Elaterite (mineral caoutchouc). Wurtzilite.
	Solid	Asphaltic	Albertite. Impsonite. Grahamite. Nigrite. Uintaite (gilsonite).
		Coal	Lignite. Bituminous coal. Semi-bituminous coal. Anthracite coal.
esinous			Succinite (amber). Copalite. Amberite, etc.
ereous			Ozocerite. Hatchetite, etc.
rystalline			Fichtelite. Hartite, etc.

TABLE II.—GROUPING OF NATURAL AND ARTIFICIAL BITUMINOUS COMPOUNDS.

tural	Mixed with limestone (asphaltic limestone)	Seyssel, Val de Travers, Lobsan, Illinois, Utah, etc.
	Mixed with silica and sand (asphaltic)	California, Kentucky, Utah, etc.
	Mixed with earthy matter (asphaltic)	Trinidad, Cuba, California, Utah.
	Bituminous schists	Canada, California, Kentucky, Virginia, etc.
	Fluid	Thick oils from distilled petroleum, "residuum."
ficial	Viscous	Gas tar. Pitch.
	Solid	Refined Trinidad asphaltic earth. Mastic of asphaltite. Gritted asphaltic mastic. Paving compounds.

he Adjustment of Macadam Road Design to Various Subgrade Soils ort of Mass. Highway Commission; Eng. News, Sept. 4, 1902).—Deals Sand and Gravel, Clay, and Sandy Loam.

aving a Country Road With Brick (By Sam. Houston. Paper, Soc. C. E. & Surv., Jan. 20, 1902; Eng. News, Sept. 25, 1902).—Illus- l details in cross-section. Specifications and discussion under the ring headings: Underdrains; Foundation of Broken Vitrified Pipe; ied Clay Curbing; Crown of Pavement and Curve of Summer Road; Pavement. Cost data is given showing that 5479 ft. of road cost 7.72; the total width of road between ditches is 26 ft., with 10. ft. of pavement proper; etc.

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An Experimental Steel Trackway in N. Y. City (Eng. News, Dec. 4, 1902).—Illustrated cross-section of trackway, and details of rail section used in Murray St. Rails are special channel section, 40 ft. long, and weigh 25 lbs. per lin. ft. Approximate cost of trackway, laid, complete, $7,500 per mile.

The Design of a Bituminous Macadam Road for Salem Co., N. J. (Eng. News, Sept. 24, 1903).—Illustrated cross-section of 30-ft. road.

Data on Roads and Pavements in Iowa (By A. Marston. Report, Iowa Eng. Soc.; Eng. News, Feb. 9, 1905).—Table of traction tests on brick and asphalt pavements in various stages of condition. Traction resistance: Brick pavement, 25.4 to 58 lbs. per ton. Asphalt pavement, 23.3 to 67.8 lbs. per ton.

Experience With Various Pavements on Streets With Heavy Grades (By C. G. Anderson. Paper, Ill. Soc. of Engrs. and Surv., Jan., 1907; Eng. News, Mar. 14, 1907).—Illustrated cross-sections showing different styles of paving employed.

Bituminized Dirt Roads at Santa Monica, Cal. (Eng. News, May 16, 1907).—Illustration of latest type of road-tamping roller.

Street Railway Track and Paving at Fort Wayne, Ind. (Eng. News June 20, 1907).—Illustrations of track and paved streets, nose block for street railway track, and reinforced concrete poles.

Notes on Tar Macadam (By C. F. Wike, England; Eng. News, Aug. 8, 1907).—Initial cost of tar macadam roads in England is about 55 to 60 cents per sq. yd., exclusive of foundation; and the annual charge (including initial cost) for a period of 14 years has averaged 8 cents per sq. yd.—down to 5 cents.

The Use of T-Rails for Street Railway Tracks in Cities (By C. G. Reel, Paper, Am. St. & Int. Ry. Assn., Oct., 1907; Eng. News, Oct. 31, 1907).—Illustrations: Standard T-rail construction in Milwaukee; latest construction in Kingston, N. Y., showing special brick outside of rail.

Cost of Brick Pavements and Cement Mortar Curbs at Centerville, Iowa (By M. G. Hall. Eng. News, April 2, 1908).—Tables of costs.

Cost of Oiling Roads in N. Y. State (Eng. News, May 14, 1908).—Table showing costs of experimental road oiling, for macadam road, sand road, and gravel road.

Concrete Paving for Streets (Eng. News, Aug. 20, 1908).—Costs, and illustrated sections of Streets. Specifications.

Specifications and Notes on Macadam Road Construction (By A. N. Johnson. Paper, West. Soc. of Engrs., Oct., 1908; Eng. News, Nov. 5, 1908).

The Use of Asphaltic Flux for Coating Macadam Roads at Palo Alto, Cal. (By J. F. Byxbee. Eng. News, May. 13, 1909).—Specifications and description of method. Total cost for material and labor, 5¼ cents per sq. yd.

An "Accelerated Test" of Road Wear by Automobile Traffic in Germany (Eng. News, July 8, 1909).—Illustrated.

Method of Keeping Data Relating to Street Lines and Grades, in Brookline, Mass. (By H. A. Varney. Eng. News, July 8, 1909).—Illustrated sample pages of data sheets.

Sampittic Surfacing (By W. W. Crosby. Trans. A. S. C. E., Vol. LXIV., Sept. 1909).—Specifications.

Inverted Macadam Road Construction (Eng. Rec., Jan. 8, 1910).—Inverted macadam road construction has been adopted in a number of cases by Mr. A. N. Johnson, highway engineer of Illinois, as a means of meeting conditions which obtain in many parts of the State. The specifications proposed to cover this method of construction and the reasons for placing the fine material on the bottom and the large pieces of stone at the top were described in a paper Mr. Johnson presented before the Western Soc. of Engrs. This paper was printed in the Eng. Rec. of Nov. 7, 1908.

Vitrified Clay Curbing for Streets and Roads (Eng. News, Jan. 13, 1910).—Illustrated. The blocks are hollow and form continuous drains, so that the usual line of broken stone for drainage of the roadbed is not required where this form of curb is used. Has been used for eight years on the country

road between Toledo and Calumet, O., and is said to be in excellent condition, showing practically no wear.

Sidewalk Practice in Chicago (By N. E. Murray. Paper Ill. Soc. Engrs. and Survrs., Jan. 26, 28, 1910; Eng. News, Feb. 17, 1910).—The average cost of cement walks laid in Chicago from 1901 to 1908, inclusive, based on the total cost ($8,918,278) divided by the total mileage (1862) was $4,787 per mile, or 15.11 cents per sq. ft. This price was for walks complete, including filling, which in many instances was from 2 ft. to 6 ft. in depth and is not a fair average cost for the ordinary cement walk. Average Chicago prices for labor and material, give following average cost of material delivered on the work: Cinders, 50 cts. per cu. yd.; cement, $1.20 per bbl. (3.8 cu. ft.); sand, $1.75 per cu. yd.; gravel, $1.50 per cu. yd. An ordinary concrete sidewalk gang in Chicago is usually composed of six men paid as follows (for 8 hours): 1 finisher at 65 cts., $5.20; 1 helper at 47½ cts., $3.80; 4 laborers at 37½ cts., $12; total, $21. Assuming that this gang of six men can construct 600 square feet of walk per day (a fair assumption, borne out by experience), we have, as total cost, 13.51 cents per sq. ft.; thus:—

Cinders (20% shrinkage), 20.83 cu. yds. at 50 cts.....		$10.42
Base, 4½ ins. (1 : 2½ : 5):		
Cement, 9.77 bbls. at $1.20.....	$11.72	
Sand, 3.47 cu. yds. at $1.75.....	6.07	
Gravel, 6.85 cu. yds. at $1.50.....	10.28	28.07
Wearing coat, ¾-in. (2 : 3):		
Cement, 5.56 bbls. at $1.20.....	6.67	
Sand, 1.17 cu. yds. at $1.75.....	2.04	8.71
Water, 1 mill per sq. ft.....		.60
Labor, 1 gang one day.....		21.00
Use of tools, waste of material, etc., at 2%.....		1.37
Supt. and office expenses, at 5%.....		3.51
Profit at 10%.....		7.36
Total 600 sq. ft. at 13.51 cts. per sq. ft.....		$81.04

Paving Practice in Chicago (By P. E. Green. Trans. A. S. C. E., Vol. XVI., Mar 1910).—**Crown of roadway:** Chicago ordinance calls for arc of circle, but the parabola is mostly used in construction; the formula is $y=\frac{x^2+a^2}{b}$, in which b=depth of gutter below grade of center or roadway, a=half roadway, x=hor. dist. from center of roadway, and y=vert. dist. below the grade. Above formula applies to roadways up to 100 ft. and to pavements having a rigid wearing surface; for wider roadways, and with macadam surfaces, the curve should approach a straight line from center of roadway to gutters. The Rosewater (Omaha) formula for height of crown $H=W(100-4P)\div 5000$, in which H=height of crown, W=width of roadway, and P=percentage of grade; this formula is best adapted to park roads or boulevards, or for streets having stiff grades where little crown is necessary. For residence streets a good formula is $H=0.02W$. Illustrations of girder rail and pavement.

Two Years' Experience in Dust Suppression on New Jersey Roads (By James Owen. Paper before State Sanitary Assn., of N. J.; Eng. Rec., Dec. 1910).—Mr. Owen records some of his failures in road construction, and draws certain conclusions as the results of his experience: "In construction, use no penetration of heavy oil, but construct the road as of ordinary macadam and provide after complete consolidation of surface a coating of 45% using about one-half gallon per sq. yd., and in the maintenance contract the year provide for two applications in that period. With this practice entire satisfaction has been realized." It is apparent that an ordinary macadam surface, even when oiled, is not satisfactory, but satisfaction has been obtained with various patent pavements as Amiesite, Filbertine, Warrenite and what is known as road asphalt. These all use the broken stone with a plastic mixture injected, in most cases asphalt. **Cost.**—Before the automobile era, the cost of maintaining a good surface upon the Essex County roads was 3 cts. per sq. yd. per annum. This was increased in later years to about 5 cts., and including the oiling, amounts to 6 cts. The cement pavements alluded to cost from 80 cts. to $1.20 per sq. yd. It is obvious that those pavements must last 15 to 20 years to be on the same monetary basis as ordinary repairs.

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Tests of Various Road Surfacing Materials by the Ohio State Highway Department (Bulletin No. 12, O. S. H. D.; Eng. News, Nov. 10, 1910).—The following table shows the wear on Nelson Avenue experimental road, one year after its construction:

Section.	8 ft. East.	4 ft. East.	Center line.	4 ft. West.	8 ft. West.
1. Glutin	.07 ft.	.07 ft.	.00 ft.	.00 ft.	.00 ft.
2. Standard Asphalt	.02 ft.	.07 ft.	.06 ft.	.08 ft.	.09 ft.
3. Pioneer Asphalt	.07 ft.	.07 ft.	.05 ft.	.03 ft.	.03 ft.
4. Tarvia "X"	.02 ft.	.04 ft.	.04 ft.	.05 ft.	.03 ft.
5. Tarvia "B"	.08 ft.	.04 ft.	.03 ft.	.00 ft.	.00 ft.
6. Indian Asphalt	.01 ft.	.03 ft.	.01 ft.	.03 ft.	.02 ft.
7. Ugite	.04 ft.	.05 ft.	.05 ft.	.03 ft.	.05 ft.
8. Fairfield Asphalt	.04 ft.	.04 ft.	.04 ft.	.05 ft.	.00 ft.
9. Asphaltoilene	.05 ft.	.04 ft.	.04 ft.	.05 ft.	.03 ft.
10. Rock Asphalt	.00 ft.	.00 ft.	.02 ft.	.03 ft.	.04 ft.
11. Carbo-Via	.03 ft.	.09 ft.	.07 ft.	.08 ft.	.07 ft.
12. Concrete Macadam	.04 ft.	.05 ft.	.05 ft.	.01 ft.	.00 ft.
13. Taroid	.02 ft.	.01 ft.	.03 ft.	.00 ft.	.00 ft.
14. Petrolithic	.03 ft.	.01 ft.	.06 ft.	.00 ft.	.00 ft.
15. Limestone Concrete	.04 ft.	.06 ft.	.02 ft.	.00 ft.	.00 ft.
16. Gravel Concrete	.00 ft.	.00 ft.	.00 ft.	.00 ft.	.01 ft.
17. Water-bound Macadam	.01 ft.	.03 ft.	.03 ft.	.01 ft.	.01 ft.

Illustrations and Specifications.

Description.	Eng. News.
Specifications for bituminous concrete paving	Mar. 17, '10.
Specifications for sheet asphalt pavement	Mar. 17, '10.
Specifications for concrete sidewalk, curbs, street pavement	Mar. 17, '10.
Suggestions for street pavement crowns, with formulas	May 5, '10.
General plans for location of street conduits and pipes, Seattle	May 12, '10.
Formulas for street crowns—curbs at different elevations	June 30, '10.
	Eng. Rec.
Combined concrete and gutter in Salt Lake City	Oct. 15, '10.
Typical sections of covered conduits (10′ x 20′) at Jones Falls	Dec. 3 '10.

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61.—HYDROSTATICS.

Hydrostatics, in its broadest sense, treats of the conditions of equilibrium and pressure of *fluids** at rest. As the term "fluid" comprehends both *liquids* and gases our discussion will be confined to the former, and especially to *water*. To the engineer, Water may be considered as practically *frictionless* and *incompressible*. In other words, we assume it to be a perfect fluid possessing no statical friction; and not subject to increased density under pressure, to any perceptible degree.† From this it follows (1) that the intensity of pressure (in lbs. per sq. in., or lbs. per sq. ft.) at any given point in the liquid is equal in all directions; (2) that, neglecting the weight of the atmosphere above, the intensity of pressure is directly proportional to the depth below the surface; (3) that the intensity of pressure is directly proportional to the density (=mass of a unit of volume) or to the weight, of a unit volume of the liquid; (4) that the pressure is always normal to any plane, pressed surface; (5) that the pressure on a curved surface may be resolved into one or more resultant pressures each acting normal to a tangent plane projected on the curved portion.

The reader is referred to the subject of Dams, pages 845, etc., for many of the elementary principles of hydrostatics which bear particularly on that subject, and they will not be repeated here.

Atmospheric Pressure may be neglected in most hydrostatic calculations because its effects are usually balanced. For instance, the effect of the atmospheric pressure on the up-stream water surface at a dam exerts so much additional force tending to overturn the structure, but it must be remembered that an equal and opposite pressure is exerted against the down-stream face of the dam, tending to preserve equilibrium, and hence the effect is neutralized‡. The height of the atmosphere above sea level is variously estimated at from 40 to 200 miles. Whatever the height may be it is certain that, beginning with the maximum density at sea level, it becomes exceedingly rarefied above an elevation of 30 to 40 miles. A column of air at sea level will balance a column of water 34 ft. in height or a column of mercury 2½ ft. (30 ins.) in height, each column exerting a pressure of 14.7 lbs. per sq. in. This is based on a cubic ft. of water weighing about 62.5 lbs., and the specific gravity of mercury at 13.6.

Air, dry, at atmospheric pressure, and at a temperature of 56° F., weighs exactly $\frac{1}{12}$ of a pound per cubic foot. (See Table of weight of air, page 463.)

Water is 773 times as heavy as the denser air at 0° C. A column of water 1 sq. in. in section and 1 ft. high weighs about 0.434 lb.; or, in other words, 1 ft. "head" corresponds to a "pressure" of about 0.434 lb. per sq. in. Hence a pressure of 1 lb. per sq. in. corresponds to a head of about 2.304 ft. These values should be committed to memory.|| Table No. 1, following, when used decimally will give corresponding pressures in lbs. per sq. in. for any given heads in ft., and *vice versa*.

---

* "*Incompressible*" fluids, only, are included in the modern acceptation of the term *hydrostatics*.

† Under one atmosphere (14.7 lbs. per sq. in.) fresh water is compressed to 0.99995 its original volume, amounting to an increased density of 0.0032 per cu. ft.; salt water to 0.999956 its original volume. Sea water one mile in depth below the surface, equal to 156 additional atmospheres or 2292 lbs. per sq. in., is compressed only to 0.99995566 its original volume; hence the additional 156 atmospheres increases its density only about ⅘ of 1% more than does one atmosphere.

‡ This is only partly true, as a partial vacuum is frequently formed at the down-stream face when the water is *flowing over* the crest of the dam.

|| The refinement sometimes employed by using about 62.424 to 62.428 lbs. as the weight of a cu. ft. of water at its maximum density, is unnecessary in ordinary engineering problems. The value 62.5, used above, involves an error of only 0.5 lb. pressure per sq. in. for a 1000-ft. head, equivalent to about $^1/_{10}$ of 1 per cent on the side of safety. But see Tables 3, 4 and 5, following.

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Hydrostatic Pressure.—There are two *Pressure Units* in general use in the United States, as follows:

(a) "Lbs. per sq. in." corresponding to head in ft., as given in Tables 1, 3 and 4, is used in the design of water pipes, tanks, sewers, etc.

(b) "Lbs. per sq. ft." corresponding to head in ft., is used in the design of dams. See Tables 2 and 5.

Let P = total pressure in lbs. on any submerged plane surface;
p' = pressure in lbs. per sq. ft.;
p'' = pressure in lbs. per sq. in.;
h = height of column of water or "head," in ft.;
a' = area of submerged surface acted upon, in ft.;
a'' = area of submerged surface acted upon, in ins.;
w' = wt. of a column of water 1 ft high and 1 ft. sq , in lbs. = 62.5 lbs.;
w'' = wt. of a column of water 1 ft. high and 1 in. sq., in lbs. = .434 lb.

Then, neglecting atmospheric pressure (14.7 lbs. per sq in), we have,

$$P = w'\,a'\,h = 62.5\,a'\,h \quad \ldots\ldots (1)$$

$$= w''\,a''\,h = .434\,a''\,h \quad \ldots\ldots (2)$$

$$p' = w'\,h = 62.5\,h \quad \ldots\ldots (3)$$

$$p'' = w''\,h = .434\,h \quad \ldots\ldots (4)$$

$$h = \frac{p'}{w'} = .016\,p' \quad \ldots\ldots (5)$$

$$= \frac{p''}{w''} = 2.304\,p'' \quad \ldots\ldots (6)$$

1.—Head and Pressure Equivalents.—Lb. per Sq. In.
(Water assumed at 62.5 lbs. per cu. ft.)

Head. Feet.	Pressure. Lbs. per Sq. In.	Pressure. Lbs. per Sq. In.	Head. Feet.
1	0.434	1	2.304
2	0.868	2	4.608
3	1.302	3	6 912
4	1.736	4	9.216
5	2.170	5	11.520
6	2.604	6	13.824
7	3.038	7	16.128
8	3.472	8	18.432
9	3.906	9	20.736
10	4.340	10	23.040

Example.—What pressure in lbs. per sq. in. corresponds to a head of 104.8 ft.?

Solution.—(From above table.)

100 = 43.4
4 = 1.736
.8 = .347

Ans. 45.483 lbs.

2.—Head and Pressure Equivalents.—Lbs. per Sq. Ft.
(Water assumed at 62.5 lbs. per cu. ft.)

Head. Feet.	Pressure. Lbs. per Sq. Ft.	Pressure. Lbs. per Sq. Ft.	Head. Feet.
1	62.5	1	.016
2	125.	2	.032
3	187.5	3	048
4	250.	4	.064
5	312.5	5	.080
6	375.	6	.096
7	437.5	7	.112
8	500.	8	.128
9	562.5	9	.144
10	625.	10	.160

Example.—Pressure corresponding to 82.4 ft. head = 5000 + 125 + 25 = 5150 lbs. per sq. ft. Or, = 62.5 × 82.4 (= 80 × 82.4), by formula.

Example.—Head corresponding to pressure of 1250 lbs. per sq. ft. = 16 + 3.2 + .8 = 20 ft.
Or, = .016 × 1250, by formula.

3.—Head of Water, in Feet Corresponding to Given Pressures in Lbs. Per Sq In.

Tenths of Lbs.	P. P. 2.3068
1	.231
2	.461
3	.692
4	.923
5	1.153
6	1.384
7	1.615
8	1.845
9	2.076

Note.—Weight of water assumed at 62.424 lbs. per cu. ft.
imal point may be moved simultaneously to right or left for Pressure and Head.
[Head of Water, in Feet.]

Units.

0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
.....	2.307	4.614	6.920	9.227	11.534	13.841	16.148	18.454	20.761
23.068	25.375	27.682	29.988	32.295	34.602	36.909	39.216	41.522	43.829
46.136	48.443	50.750	53.056	55.363	57.670	59.977	62.284	64.590	66.897
69.204	71.511	73.818	76.124	78.431	80.738	83.045	85.352	87.658	89.965
92.272	94.579	96.886	99 192	101.499	103.806	106.113	108.420	110.726	113.033
115.340	117.647	119.954	122.260	124.567	126.874	129.181	131.488	133.794	136.101
138.408	140.715	143.022	145.328	147.635	149.942	152.249	154.556	156.862	159.169
161.476	163.783	166.090	168.396	170.703	173.010	175.317	177.624	179.930	182.237
184.544	186.851	189.158	191.464	193.771	196.078	198.385	200.692	202.998	205.305
207.612	209.919	212.226	214.532	216.839	219.146	221.453	223.760	226.066	228.373
230.680	232.987	235.294	237.600	239.907	242.214	244.521	246.828	249.134	251.441
253.748	256.055	258.362	260.668	262.975	265.282	267.589	269.896	272.202	274.509
276.816	279.123	281.430	283.736	286.043	288.350	290.657	292.964	295.270	297.577
299.884	302.191	304.498	306.804	309.111	311.418	313.725	316.032	318.338	320.645
322.952	325.259	327.566	329.872	332.179	334.486	336.793	339.100	341.406	343.713
346.020	348.327	350.634	352.940	355.247	357.554	359.861	362.168	364.474	366.781
369.088	371.395	373.702	376.008	378.315	380.622	382.929	385.236	387.542	389.849
392.156	394.464	396.770	399.076	401.383	403.690	405.997	408.304	410.610	412.917
415.224	417.531	419.838	422.144	424.451	426.758	429.065	431.372	433.678	435.985
438.292	440.599	442.906	445.212	447.519	449.826	452.133	454.440	456.746	459.053
461.360	463.667	465.974	468.280	470.587	472 894	475.201	477.508	479.814	482.121
484.428	486.735	489.042	491.348	493.655	495 962	498.269	500.576	502.882	505.189
507.496	509.803	512.110	514.416	516.723	519.030	521.337	523.644	525.950	528.257
530.564	532.871	535.178	537.484	539.791	542.098	544.405	546.712	549.018	551.325
553.632	555.939	558.246	560.552	562.859	565.166	567.473	569.780	572.086	574.393
576.700	579.007	581.314	583.620	585.927	588.234	590.541	592.848	595.154	597.461
599.768	602.075	604.382	606.688	608.995	611.302	613.609	615.916	618.222	620.529
622.836	625.143	627.450	629.756	632.063	634.370	636.677	638.984	641 290	643.597
645.904	648.211	650.518	652.824	655.131	657.438	659.745	662.052	664 358	666.665
668.972	671.279	673.586	675 892	678.199	680.506	682 813	685.120	687.426	689.733
692.040	694.347	696.654	698.960	701.267	703.574	705 881	708.188	710.494	712.801
715.108	717.415	719.722	722.028	724.335	726.642	728 949	731.256	733.562	735.869
738.176	740.483	742.790	745.096	747 403	749.710	752 017	754.324	756.630	758.937
761.244	763.551	765.858	768.164	770 471	772.778	775 085	777.392	779.698	782.005
784.312	786.619	788.926	791.232	793.539	795.846	798.153	800.460	802.766	805.073
807.380	809.687	811.994	814.300	816.607	818.914	821.221	823.528	825.834	828.141
830.448	832.755	835.062	837.368	839.675	841.982	844.289	846.596	848.902	851.209
853.516	855.823	858.130	860.436	862.743	865.050	867.357	869.664	871.970	874.277
876.584	878.891	881.198	883.504	885.811	888.118	890.425	892.732	895.038	897.345
899.652	901.959	904.266	906.572	908.879	911.186	913.493	915.800	918.106	920.413
922.720	925.027	927.334	929.640	931.947	934.254	936.561	938.868	941.174	943.481
945.788	948.095	950.402	952.708	955.015	957 322	959.629	961.936	964.242	966.549
968.856	971.163	973.470	975 776	978.083	980.390	982 697	985.004	987.310	989.617
991.924	994.231	996.538	998 844	1001.151	1003.458	1005 765	1008.072	1010.378	1012.685
1014.992	1017.299	1019.606	1021 912	1024.219	1026.526	1028.833	1031.140	1038.446	1035.753
1038.060	1040.367	1042.674	1044.980	1047.287	1049.594	1051.901	1054.208	1056.514	1058.821
1061.128	1063.435	1065.742	1068.048	1070.355	1072.662	1074.969	1077.276	1079.582	1081.889
1084.196	1086.503	1088.810	1091.116	1093.423	1095.730	1098.037	1100.344	1102.650	1104.957
1107.264	1109.571	1111.878	1114.184	1116.491	1118.798	1121.105	1123.412	1125.718	1128.025
1130.332	1132.639	1134.946	1137.252	1139.559	1141.866	1144.173	1146.480	1148.786	1151.093
1153.400	1155.707	1158.014	1160.320	1162.627	1164.934	1167.241	1169.548	1171.854	1174.161
1176.468	1178.775	1181.082	1183.388	1185.695	1188.002	1190.309	1192.616	1194.922	1197.229
1199.536	1201.843	1204.150	1206.456	1208.763	1211.070	1213.377	1215.684	1217.990	1220.297
1222.604	1224.911	1227.218	1229.524	1231.831	1234.138	1236.445	1238.752	1241.058	1243.365
1245.672	1247.979	1250.286	1252.592	1254.899	1257.206	1259.513	1261.820	1264.126	1266.433
1268.740	1271.047	1273.354	1275.660	1277.967	1280.274	1282.581	1284.888	1287.194	1289.501
1291.808	1294.115	1296.422	1298.728	1301.035	1303.342	1305.649	1307 956	1310 262	1312.569
1314.876	1317.183	1319.490	1321.796	1324.103	1326.410	1328.717	1331 024	1333.330	1335.637
1337.944	1340.251	1342.558	1344.864	1347.171	1349.478	1351.785	1354.092	1356.398	1358.705
1361.012	1363.319	1365.626	1367.932	1370.239	1372.546	1374.853	1377.160	1379.466	1381.773
1384.080	1386.387	1388.694	1391.000	1393.307	1395.614	1397.921	1400.228	1402.534	1404.841

Example.—A pressure of 45.3 lbs. per sq. in. is equivalent to (moving the imal point one place to left in 453 and 1044.98) 104.5 ft. head.

4.—Pressures in Lbs. per Sq. In. Corresponding to Given Heads of Water, in Feet.

Note.—Weight of water assumed at 62.424 lbs. per cu. ft.

Decimal point may be moved simultaneously to right or left for Head and Pressure.

[Pressure in Lbs. per Sq. In.]

Tenths of Feet.	P. P.
	.4335
1	.0434
2	.0867
3	.1301
4	.1734
5	.2168
6	.2601
7	.3035
8	.3468
9	.3902

Head. Feet.	Units. 0.	1.	2.	3.	4.	5.	6.	7.	8.	9
-.	.....	.4335	.8670	1.3005	1.7340	2.1675	2.6010	3.0345	3.4680	3.9015
1-.	4.3350	4.7685	5.2020	5.6355	6.0690	6.5025	6.9360	7.3695	7.8030	8.2365
2-.	8.6700	9.1035	9.5370	9.9705	10.4040	10.8375	11.2710	11.7045	12.1380	12.5715
3-.	13.0050	13.4385	13.8720	14.3055	14.7390	15.1725	15.6060	16.0395	16.4730	16.9065
4-.	17.3400	17.7735	18.2070	18.6405	19.0740	19.5075	19.9410	20.3745	20.8080	21.2415
5-.	21.6750	22.1085	22.5420	22.9755	23.4090	23.8425	24.2760	24.7095	25.1430	25.5765
6-.	26.0100	26.4435	26.8770	27.3105	27.7440	28.1775	28.6110	29.0445	29.4780	29.9115
7-.	30.3450	30.7785	31.2120	31.6455	32.0790	32.5125	32.9460	33.3795	33.8130	34.2465
8-.	34.6800	35.1135	35.5470	35.9805	36.4140	36.8475	37.2810	37.7145	38.1480	38.5815
9-.	39.0150	39.4485	39.8820	40.3155	40.7490	41.1825	41.6160	42.0495	42.4830	42.9165
10-.	43.3500	43.7835	44.2170	44.6505	45.0840	45.5175	45.9510	46.3845	46.8180	47.2515
11-.	47.6850	48.1185	48.5520	48.9855	49.4190	49.8525	50.2860	50.7195	51.1530	51.5865
12-.	52.0200	52.4535	52.8870	53.3205	53.7540	54.1875	54.6210	55.0545	55.4880	55.9215
13-.	56.3550	56.7885	57.2220	57.6555	58.0890	58.5225	58.9560	59.3895	59.8230	60.2565
14-.	60.6900	61.1235	61.5570	61.9905	62.4240	62.8575	63.2910	63.7245	64.1580	64.5915
15-.	65.0250	65.4585	65.8920	66.3255	66.7590	67.1925	67.6260	68.0595	68.4930	68.9265
16-.	69.3600	69.7935	70.2270	70.6605	71.0940	71.5275	71.9610	72.3945	72.8280	73.2615
17-.	73.6950	74.1285	74.5620	74.9955	75.4290	75.8625	76.2960	76.7295	77.1630	77.5965
18-.	78.0300	78.4635	78.8970	79.3305	79.7640	80.1975	80.6310	81.0645	81.4980	81.9315
19-.	82.3650	82.7985	83.2320	83.6655	84.0990	84.5325	84.9660	85.3995	85.8330	86.2665
20-.	86.7000	87.1335	87.5670	88.0005	88.4340	88.8675	89.3010	89.7345	90.1680	90.6015
21-.	91.0350	91.4685	91.9020	92.3355	92.7690	93.2025	93.6360	94.0695	94.5030	94.9365
22-.	95.3700	95.8035	96.2370	96.6705	97.1040	97.5375	97.9710	98.4045	98.8380	99.2715
23-.	99.7050	100.1385	100.5720	101.0055	101.4390	101.8725	102.3060	102.7395	103.1730	103.6065
24-.	104.0400	104.4735	104.9070	105.3405	105.7740	106.2075	106.6410	107.0745	107.5080	107.9415
25-.	108.3750	108.8085	109.2420	109.6755	110.1090	110.5425	110.9760	111.4095	111.8430	112.2765
26-.	112.7100	113.1435	113.5770	114.0105	114.4440	114.8775	115.3110	115.7445	116.1780	116.6115
27-.	117.0450	117.4785	117.9120	118.3455	118.7790	119.2125	119.6460	120.0795	120.5130	120.9465
28-.	121.3800	121.8135	122.2470	122.6805	123.1140	123.5475	123.9810	124.4145	124.8480	125.2815
29-.	125.7150	126.1485	126.5820	127.0155	127.4490	127.8825	128.3160	128.7495	129.1830	129.6165
30-.	130.0500	130.4835	130.9170	131.3505	131.7840	132.2175	132.6510	133.0845	133.5180	133.9515
31-.	134.3850	134.8185	135.2520	135.6855	136.1190	136.5525	136.9860	137.4195	137.8530	138.2865
32-.	138.7200	139.1535	139.5870	140.0205	140.4540	140.8875	141.3210	141.7545	142.1880	142.6215
33-.	143.0550	143.4885	143.9220	144.3555	144.7890	145.2225	145.6560	146.0895	146.5230	146.9565
34-.	147.3900	147.8235	148.2570	148.6905	149.1240	149.5575	149.9910	150.4245	150.8580	151.2915
35-.	151.7250	152.1585	152.5920	153.0255	153.4590	153.8925	154.3260	154.7595	155.1930	155.6265
36-.	156.0600	156.4935	156.9270	157.3605	157.7940	158.2275	158.6610	159.0945	159.5280	159.9615
37-.	160.3950	160.8285	161.2620	161.6955	162.1290	162.5625	162.9960	163.4295	163.8630	164.2965
38-.	164.7300	165.1635	165.5970	166.0305	166.4640	166.8975	167.3310	167.7645	168.1980	168.6315
39-.	169.0650	169.4985	169.9320	170.3655	170.7990	171.2325	171.6660	172.0995	172.5330	172.9665
40-.	173.4000	173.8335	174.2670	174.7005	175.1340	175.5675	176.0010	176.4345	176.8680	177.3015
41-.	177.7350	178.1685	178.6020	179.0355	179.4690	179.9025	180.3360	180.7695	181.2030	181.6365
42-.	182.0700	182.5035	182.9370	183.3705	183.8040	184.2375	184.6710	185.1045	185.5380	185.9715
43-.	186.4050	186.8385	187.2720	187.7055	188.1390	188.5725	189.0060	189.4395	189.8730	190.3065
44-.	190.7400	191.1735	191.6070	192.0405	192.4740	192.9075	193.3410	193.7745	194.2080	194.6415
45-.	195.0750	195.5085	195.9420	196.3755	196.8090	197.2425	197.6760	198.1095	198.5430	198.9765
46-.	199.4100	199.8435	200.2770	200.7105	201.1440	201.5775	202.0110	202.4445	202.8780	203.3115
47-.	203.7450	204.1785	204.6120	205.0455	205.4790	205.9125	206.3460	206.7795	207.2130	207.6465
48-.	208.0800	208.5135	208.9470	209.3805	209.8140	210.2475	210.6810	211.1145	211.5480	211.9815
49-.	212.4150	212.8485	213.2820	213.7155	214.1490	214.5825	215.0160	215.4495	215.8830	216.3165
50-.	216.7500	217.1835	217.6170	218.0505	218.4840	218.9175	219.3510	219.7845	220.2180	220.6515
51-.	221.0850	221.5185	221.9520	222.3855	222.8190	223.2525	223.6860	224.1195	224.5530	224.9865
52-.	225.4200	225.8535	226.2870	226.7205	227.1540	227.5875	228.0210	228.4545	228.8880	229.3215
53-.	229.7550	230.1885	230.6220	231.0555	231.4890	231.9225	232.3560	232.7895	233.2230	233.6565
54-.	234.0900	234.5235	234.9570	235.3905	235.8240	236.2575	236.6910	237.1245	237.5580	237.9915
55-.	238.4250	238.8585	239.2920	239.7255	240.1590	240.5925	241.0260	241.4595	241.8930	242.3265
56-.	242.7600	243.1935	243.6270	244.0605	244.4940	244.9275	245.3610	245.7945	246.2280	246.6615
57-.	247.0950	247.5285	247.9620	248.3955	248.8290	249.2625	249.6960	250.1295	250.5630	250.9965
58-.	251.4300	251.8635	252.2970	252.7305	253.1640	253.5975	254.0310	254.4645	254.8980	255.3315
59-.	255.7650	256.1985	256.6320	257.0655	257.4990	257.9325	258.3660	258.7995	259.2330	259.6665
60-.	260.1000	260.5335	260.9670	261.4005	261.8340	262.2675	262.7010	263.1345	263.5680	264.0015

Space occupied by dash, to be supplied with unit figure at top of column.

Example.—At a depth of 362.4 ft. below the surface, the water pressure is 156.9270+.1734=157.1004 lbs. per sq. in.

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5.—Pressures in Lbs. per Sq. Ft.
Corresponding to
Given Heads of Water, in Feet.

Note.—Weight of water assumed at 62.424 lbs. per cu. ft.
Decimal point may be moved simultaneously to right or left for Head and Pressure.
[Pressure in Lbs. per Sq. Ft.]

Tenths of Feet.	P. P. 62.424
1	6.24
2	12.48
3	18.73
4	24.97
5	31.21
6	37.45
7	43.70
8	49.94
9	56.18

Head. Feet.	Units.									
	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
-.	.....	62.42	124.85	187.27	249.70	312.12	374.54	436.97	499.39	561.82
1-.	624.24	686.66	749.09	811.51	873.94	936.36	998.78	1061.21	1123.63	1186.06
2-.	1248.48	1310.90	1373.33	1435.75	1498.18	1560.60	1623.02	1685.45	1747.87	1810.30
3-.	1872.72	1935.14	1997.57	2059.99	2122.42	2184.84	2247.26	2309.69	2372.11	2434.54
4-.	2496.96	2559.38	2621.81	2684.23	2746.66	2809.08	2871.50	2933.93	2996.35	3058.78
5-.	3121.20	3183.62	3246.05	3308.47	3370.90	3433.32	3495.74	3558.17	3620.59	3683.02
6-.	3745.44	3807.86	3870.29	3932.71	3995.14	4057.56	4119.98	4182.41	4244.83	4307.26
7-.	4369.68	4432.10	4494.53	4556.95	4619.38	4681.80	4744.22	4806.65	4869.07	4931.50
8-.	4993.92	5056.34	5118.77	5181.19	5243.62	5306.04	5368.46	5430.89	5493.31	5555.74
9-.	5618.16	5680.58	5743.01	5805.43	5867.86	5930.28	5992.70	6055.13	6117.55	6179.98
10-.	6242.40	6304.82	6367.25	6429.67	6492.10	6554.52	6616.94	6679.37	6741.79	6804.22
11-.	6866.64	6929.06	6991.49	7053.91	7116.34	7178.76	7241.18	7303.61	7366.03	7428.46
12-.	7490.88	7553.30	7615.73	7678.15	7740.58	7803.00	7865.42	7927.85	7990.27	8052.70
13-.	8115.12	8177.54	8239.97	8302.39	8364.82	8427.24	8489.66	8552.09	8614.51	8676.94
14-.	8739.36	8801.78	8864.21	8926.63	8989.06	9051.48	9113.90	9176.33	9238.75	9301.18
15-.	9363.60	9426.02	9488.45	9550.87	9613.30	9675.72	9738.14	9800.57	9862.99	9925.42
16-.	9987.84	10050.26	10112.69	10175.11	10237.54	10299.96	10362.38	10424.81	10487.23	10549.66
17-.	10612.08	10674.50	10736.93	10799.35	10861.78	10924.20	10986.62	11049.05	11111.47	11173.90
18-.	11236.32	11298.74	11361.17	11423.59	11486.02	11548.44	11610.86	11673.29	11735.71	11798.14
19-.	11860.56	11922.98	11985.41	12047.83	12110.26	12172.68	12235.10	12297.53	12359.95	12422.38
20-.	12484.80	12547.22	12609.65	12672.07	12734.50	12796.92	12859.34	12921.77	12984.19	13046.62
21-.	13109.04	13171.46	13233.89	13296.31	13358.74	13421.16	13483.58	13546.01	13608.43	13670.86
22-.	13733.28	13795.70	13858.13	13920.55	13982.98	14045.40	14107.82	14170.25	14232.67	14295.10
23-.	14357.52	14419.94	14482.37	14544.79	14607.22	14669.64	14732.06	14794.49	14856.91	14919.34
24-.	14981.76	15044.18	15106.61	15169.03	15231.46	15293.88	15356.30	15418.73	15481.15	15543.58
25-.	15606.00	15668.42	15730.85	15793.27	15855.70	15918.12	15980.54	16042.97	16105.39	16167.82
26-.	16230.24	16292.66	16355.09	16417.51	16479.94	16542.36	16604.78	16667.21	16729.63	16792.06
27-.	16854.48	16916.90	16979.33	17041.75	17104.18	17166.60	17229.02	17291.45	17353.87	17416.30
28-.	17478.72	17541.14	17603.57	17665.99	17728.42	17790.84	17853.26	17915.69	17978.11	18040.54
29-.	18102.96	18165.38	18227.81	18290.23	18352.66	18415.08	18477.50	18539.93	18602.35	18664.78
30-.	18727.20	18789.62	18852.05	18914.47	18976.90	19039.32	19101.74	19164.17	19226.59	19289.02
31-.	19351.44	19413.86	19476.29	19538.71	19601.14	19663.56	19725.98	19788.41	19850.83	19913.26
32-.	19975.68	20038.10	20100.53	20162.95	20225.38	20287.80	20350.22	20412.65	20475.07	20537.50
33-.	20599.92	20662.34	20724.77	20787.19	20849.62	20912.04	20974.46	21036.89	21099.31	21161.74
34-.	21224.16	21286.58	21349.01	21411.43	21473.86	21536.28	21598.70	21661.13	21723.55	21785.98
35-.	21848.40	21910.82	21973.25	22035.67	22098.10	22160.52	22222.94	22285.37	22347.79	22410.22
36-.	22472.64	22535.06	22597.49	22659.91	22722.34	22784.76	22847.18	22909.61	22972.03	23034.46
37-.	23096.88	23159.30	23221.73	23284.15	23346.58	23409.00	23471.42	23533.85	23596.27	23658.70
38-.	23721.12	23783.54	23845.97	23908.39	23970.82	24033.24	24095.66	24158.09	24220.51	24282.94
39-.	24345.36	24407.78	24470.21	24532.63	24595.06	24657.48	24719.90	24782.33	24844.75	24907.18
40-.	24969.60	25032.02	25094.45	25156.87	25219.30	25281.72	25344.14	25406.57	25468.99	25531.42
1-.	25593.84	25656.26	25718.69	25781.11	25843.54	25905.96	25968.38	26030.81	26093.23	26155.66
2-.	26218.08	26280.50	26342.93	26405.35	26467.78	26530.20	26592.62	26655.05	26717.47	26779.90
3-.	26842.32	26904.74	26967.17	27029.59	27092.02	27154.44	27216.86	27279.29	27341.71	27404.14
4-.	27466.56	27528.98	27591.41	27653.83	27716.26	27778.68	27841.10	27903.53	27965.95	28028.38
5-.	28090.80	28153.22	28215.65	28278.07	28340.50	28402.92	28465.34	28527.77	28590.19	28652.62
6-.	28715.04	28777.46	28839.89	28902.31	28964.74	29027.16	29089.58	29152.01	29214.43	29276.86
7-.	29339.28	29401.70	29464.13	29526.55	29588.98	29651.40	29713.82	29776.25	29838.67	29901.10
8-.	29963.52	30025.94	30088.37	30150.79	30213.22	30275.64	30338.06	30400.49	30462.91	30525.34
9-.	30587.76	30650.18	30712.61	30775.03	30837.46	30899.88	30962.30	31024.73	31087.15	31149.58
40-.	31212.00	31274.42	31336.85	31399.27	31461.70	31524.12	31586.54	31648.97	31711.39	31773.82
1-.	31836.24	31898.66	31961.09	32023.51	32085.94	32148.36	32210.78	32273.21	32335.63	32398.06
2-.	32460.48	32522.90	32585.33	32647.75	32710.18	32772.60	32835.02	32897.45	32959.87	33022.30
3-.	33084.72	33147.14	33209.57	33271.99	33334.42	33396.84	33459.26	33521.69	33584.11	33646.54
4-.	33708.96	33771.38	33833.81	33896.23	33958.66	34021.08	34083.50	34145.93	34208.35	34270.78
5-.	34333.20	34395.62	34458.05	34520.47	34582.90	34645.32	34707.74	34770.17	34832.59	34895.02
6-.	34957.44	35019.86	35082.29	35144.71	35207.14	35269.56	35331.98	35394.41	35456.83	35519.26
7-.	35581.68	35644.10	35706.53	35768.95	35831.38	35893.80	35956.22	36018.65	36081.07	36143.50
8-.	36205.92	36268.34	36330.77	36393.19	36455.62	36518.04	36580.46	36642.89	36705.31	36767.74
9-.	36830.16	36892.58	36955.01	37017.43	37079.86	37142.28	37204.70	37267.13	37329.55	37391.98
50-.	37454.40	37516.82	37579.25	37641.67	37704.10	37766.52	37828.94	37891.37	37953.79	38016.22

At a depth of 344 ft. below the surface, the pressure is 21473.86 lbs. per sq. ft.
At a depth of 34.4 ft. below the surface, the pressure is 2147.386 lbs. per sq. ft.

The Center of Pressure on any submerged plane surface is the point of resultant pressure on that surface. We have explained, under Dams, page 846, the position of the center of pressure on rectangular surfaces. We will now present a general formula for finding the center of pressure on *any* plane surface, and whether vertical or inclined.

Let $t\,b$, Fig. 1, represent the edge of a submerged plane figure of any shape (as a rectangle, circle, triangle, etc.);
α, the angle which this plane makes with the water surface;
D_x, the inclined distance (ft.) from A to the center of gravity, or to the horizontal neutral axis, of the figure $t\,b$;
D_0, the inclined distance to the center of pressure;
h_x, the head in feet on the center of gravity;
h_0, the head in feet on the center of pressure;
a', the area in sq. ft. of the surface $t\,b$. Also—

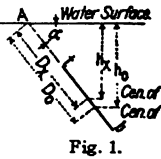

Fig. 1.

Let I_x = the moment of inertia of the figure $t\,b$ about a horizontal neutral axis passing through its center of gravity;
I_A = the moment of inertia of the figure $t\,b$ about a horizontal axis passing through A,
$= I_x + a' D_x^2$;
S = the statical moment about A,
$= a' D_x$;

$$\text{Then } D_0 = \frac{I_A}{S} = \frac{I_x + a' D_x^2}{a' D_x} \quad \ldots\ldots (7)$$

$$\text{And } h_0 = D_0 \sin \alpha = \frac{I_x + a' D_x^2}{a' D_x} \sin \alpha \quad \ldots\ldots (8)$$

Having obtained h_0 we can easily find the total pressure P from formulas (1) and (8); thus, making $h = h_0$,

$$P = 62.5\, a'\, h = 62.5\, a'\, h_0 = 62.5 \left(\frac{I_x + a' D_x^2}{D_x}\right) \sin \alpha \quad \ldots\ldots (9)$$

If the submerged figure $t\,b$ is rigid, the pressure P may be considered as a resultant pressure acting normal to the surface $t\,b$ at the center of pressure distant h_0 ft. vertically (or D_0 ft. inclined) below the surface of the water. Moreover, if the figure $t\,b$ is vertical, $\alpha = 90°$ and $\sin \alpha = 1$ in equations (8) and (9); hence, $h_0 = D_0$.

The practical application of equations (8) and (9) consists in substituting the proper values for I_x, a', D_x and $\sin \alpha$ in the second members of the equations. The angle α, the distance $A\,t$, and the shape and dimensions of the figure $t\,b$ will usually be given; then find I_x and $D_x - A\,t$, from the tables in Section 29, page 524, etc.

Problem.—Let $t\,b$ represent the section of a triangular plane of altitude $t\,b = 12$ ft. and horizontal base at bottom $b = 8$ ft. Let it be submerged so that $A\,t = 10$ ft., and $\alpha = 60°$. Find the head h_0 on the center of pressure? Find the total pressure P?

Solution.—From Table 6, next page, we deduce $I_x = 8 \times \frac{(12)^3}{36} = 384$, and $D_x = 10 + \frac{2}{3} \times 12 = 18$. Area of triangle $a' = 12 \times 4 = 48$; $\sin \alpha = 0.866$. Then, (8), $h_0 = \frac{384 + 48 \times 324}{48 \times 18} \times 0.866 = 18\frac{2}{3} \times 0.866 = 15.973$ ft.; and $P = 62.5 \times 48 \times 15.973 = 47{,}919$ lbs. Ans.

Any other shaped figure may be treated in the same manner. The center of pressure will always be found *below* the center of gravity of the figure.

If the figure $t\,b$ is a rectangle, in a vertical plane, and with the top t at A, just touching the surface of the water, then $D_0 = h_0 = \frac{2}{3}\, t\,b$ (Fig. 1).

The position of the center of pressure is essential in designing large hydraulic gates, valves, etc.

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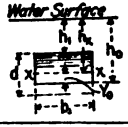
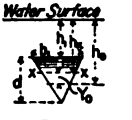
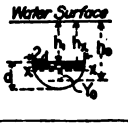
6.—CENTERS OF PRESSURE, h_0, ON VERTICAL ORIFICES [illegible]

(a) General Cases—Surface of water *above* tops of orifices.

Properties.	General. (1)	RECTANGLE. (2)	TRIANGLE. (3)	CIRCLE. (4)	SEMICIRCLE. (5)
					
I_x	About neutral axis $x-x$.	$\frac{b_1 d^3}{12}$	$\frac{b_1 d^3}{36}$	$\frac{\pi d^4}{64}$	$\left(\frac{\pi}{8}-\frac{8}{9\pi}\right)d^4$
a'	= Area.	$b_1 d$	$\frac{b_1 d}{2}$	$\frac{\pi d^2}{4}$	$\frac{\pi d^2}{2}$
h_x	= distance to neutral axis.	$h_1+\frac{d}{2}$	$h_1+\frac{d}{3}$	$h_1+\frac{d}{2}$	$h_1+\frac{4d}{3\pi}$
I_A	About axis at water surface.	$a'\left(h_1^2+h_1 d+\frac{d^2}{3}\right)$	$\frac{b_1 d^3}{12}+b_1 d\, h_1\left(\frac{h_1}{2}+\frac{d}{3}\right)$	$\frac{5\pi d^4}{64}+\frac{\pi d^2 h_1}{4}(h_1+d)$	$\pi d^2\left(\frac{d^2}{4}+h_1^2\right)+\frac{4d^3 h_1}{3}$
h_0	$=\frac{I_x+a'h_x^2}{a'h_x}$.	$I_A+\left[b_1 d\left(h_1+\frac{d}{2}\right)\right]$	$I_A+\left[\frac{b_1 d}{2}\left(h_1+\frac{d}{3}\right)\right]$	$I_A+\left[\frac{\pi d^2}{4}\left(h_1+\frac{d}{2}\right)\right]$	$I_A+\left[\frac{\pi d^2}{2}\left(h_1+\frac{4d}{3\pi}\right)\right]$
y_0	= neutral axis to center of pressure.	$h_0-\left(h_1+\frac{d}{2}\right)$	$h_0-\left(h_1+\frac{d}{3}\right)$	$h_0-\left(h_1+\frac{d}{2}\right)$	$h_0-\left(h_1+\frac{4d}{3\pi}\right)$

And when the tops of the orifices touch the surface of the water, h_1 becomes zero and we have, in each case, the following values:

(b) Special Cases—Surface of water *at* tops of orifices.

h_x	= distance to neutral axis.	$\frac{d}{2}$	$\frac{d}{3}$	$\frac{d}{2}$	$\frac{4d}{3\pi}=0.424\,d$
I_A	$=I_x+a'h_x^2$.	$\frac{b_1 d^3}{3}$	$\frac{b_1 d^3}{12}$	$\frac{5\pi d^4}{64}$	$\frac{\pi d^4}{8}$
h_0	$=\frac{I_A}{a'h_x}$.	$\frac{2d}{3}$	$\frac{d}{2}$	$\frac{5d}{8}$	$\frac{3\pi d}{16}=0.589\,d$
y_0	$=h_0-h_x$.	$\frac{d}{6}$	$\frac{d}{6}$	$\frac{d}{8}$	$0.165\,d$

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Pressure in Pipes, Tanks, etc.—In Fig. 2, let h be the total head in ft., at any level, on each diameter d of sections A, B and C. A is a water pipe leading from the water tank B, which in turn is connected by the tube t with the tank C through the vertical tube T. Then it follows that the water surfaces in B and T must remain at the same level. From formulas (3) and (4) we have for either section A, B or C, at a depth h below the surface:

Pressure in lbs. per sq. ft. $= 62.5\ h$;
Pressure in lbs. per sq. in. $= .434\ h$.

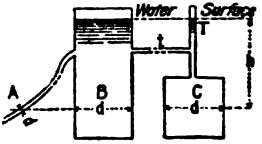

Fig. 2.

If d' = the diameter in feet, and d'' = the diameter in inches, then A, B and C will each have to resist a bursting pressure of $62.5\ h\ d'$ lbs. per lin. ft., or $.434\ h\ d''$ lbs. per lin. in. of pipe or tank; and *each side* would have to resist *one-half* that pressure.

The combination of the tube T inserted in the tank C (with t omitted) illustrates what is called the "hydrostatic paradox." The *unit* pressure at d due to the head h remains the same no matter what the diameter of T. Clearly, the diameter of T does not affect the unit pressure; the height h does. Thus, a heavy cask, as C, may be made to burst if even a small tube, as T, is filled with water so as to give the required bursting head.

Flotation.—The weight of a substance is proportional to its density; and the relative density of a substance, referred to water, is called its specific gravity.* Hence, of two substances of equal weight, that one having the least volume has the greater specific gravity. As the volume of a body is affected by heat, it follows that the specific gravity of substances decreases with a rise in temperature, the formation of ice being a phenomenal exception to this law. Pure water at 4° C. its maximum density, is assumed to have a specific gravity of 1; and all other substances, at 0° C. are referred to that standard. A porous substance will increase in density by absorbing water; thus, water-soaked logs is an instance of this kind.

Any solid with specific gravity greater than unity will sink; if less than unity it will float. Any floating body, whether solid or not, will displace a volume of water whose weight is equal to the weight of the body; if the body sinks, the weight of the volume of water displaced will be less than that of the body.

The Depth of Flotation depends upon the specific gravity of the body, if solid; upon the *average* specific gravity of the volume, if hollow; and upon

Fig. 3.

the surface form of the body. Assuming the average specific gravity of volume to be less than unity, let d = depth of flotation,

s = average specific gravity of volume of body;

then for any rod, bar, tube, cylinder, etc., of uniform cross-section and of length l, *floating vertically*,

$$d = s\,l \qquad (10)$$

For the same, if *lying horizontal*, d can be obtained by fixing the water surface on the end section at such elevation (Fig. 3) that

$$s = \frac{\text{shaded area below water surface}}{\text{total area of the section}} \qquad (11)$$

Buoyancy.—Let Figs. 4, 5 and 6 represent oval sections, and Fig. 7 a circular section, of any body floating in water, with the surface at S; then—

Stable equilibrium is represented by Fig. 4,
Unstable equilibrium, by Fig. 6;
Neutral equilibrium, by Fig. 7.

* For discussion of Specific Gravity, see page 460, etc.

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The position in Fig. 5 can obtain only when some *outside force* is applied, as will be seen from the following discussion: The center of buoyancy B in each figure below is the center of gravity of the displaced water; and G is the

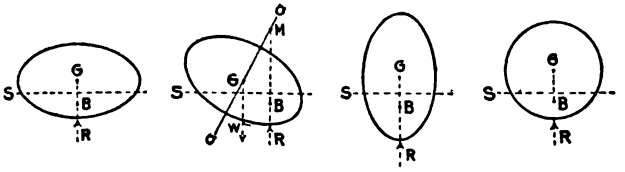

Fig. 4. Fig. 5. Fig. 6. Fig. 7.

center of gravity of the body. Then, as long as G and B are in the same vertical line there is some kind of equilibrium, as stable, unstable, or neutral, because the resultant downward weight of the body, acting in a vertical line passing through its center of gravity G, must be equal and opposite to the resultant upward pressure R of the water below the body. Furthermore, the upward resultant pressure R will not be changed in position, direction or amount if we imagine the body to be removed from the "depression" in the water and that depression refilled with the displaced water; for equilibrium would be maintained by the equal weight of the displaced water, its resultant passing vertically through its center of gravity B. Hence if R, acting vertically, is equal and opposite to the resultant weight of the body and also to the displaced water, acting through their respective centers of gravity G and B, then it follows that G and B are in the same vertical line when the body is in equilibrium.

The equilibrium of a floating body may be tested by noting the position of the "metacenter" M when the body is slightly disturbed in any direction from a position of rest. (M is the intersection of the "equilibrium axis" $a-a$ with a vertical through B.):

(a) When M rises above G it indicates that the body was in stable equilibrium.

(b) When M falls below G it indicates that the body was in unstable equilibrium.

(c) When M coincides with G it indicates neutral equilibrium.

In the above, Fig. 5 shows that Fig. 4 is in stable equilibrium. In the same way it can be proved that Fig. 6 is unstable, and Fig. 7 neutral.

The following rules apply not only to floating bodies but to supported bodies in general.

1) A body is in stable equilibrium when a slight change tends to raise its center of gravity;

2) A body is in unstable equilibrium when a slight change tends to lower its center of gravity;

3) A body is in neutral equilibrium when a slight change neither raises nor lowers its center of gravity.

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62.—HYDRAULICS.

Hydraulics embraces the application of the principles of both hydrostatics and hydrokinetics;* for a fluid at rest, as treated by hydrostatics, is but the lower limit of a condition of motion. It therefore treats of the laws governing the pressure, flow and energy of water (and other liquids) with the accompanying phenomena. These laws, however, are but imperfectly understood and, like all other branches of engineering, hydraulics is not an exact science. *Theoretical* hydraulics assumes no loss of energy during the flow, a condition which can never obtain in practice; but its great value, in the investigation of any problem, lies in fixing the upper limit of efficiency which can ever be expected. *Practical* hydraulics is founded on theoretical hydraulics, but takes into consideration the losses of energy during the flow. These losses are deduced from experiments.

Theory of Flow.—Under *hydrostatics* we have discussed the relation between the static head h and the pressure p of still water. In hydraulic computations it is convenient to reduce all pressures, velocities and losses of every description to equivalent heads and losses of head. Moreover, the unit of pressure used is, in English units, the "lb. per sq. in." Thus, referring particularly to a pipe line, we have:

p = pressure in lbs. per sq. in. = $0.434\ h_p$;
H = hydrostatic head, or simply static head = $2.304\ p$;
h_e = entry head, or loss of head at entry;
h = velocity head, or head due to velocity at given section;
h_a = "velocity of approach" head, or *gain* in head a given section due to velocity of approach *above the given section*;
h_f = friction head, or loss of head due to friction;
h_s = "suction" head, or head due to suction, acting in the direction of the flow;
h_c = "curvature" head, or loss of head due to curves or bends;
h_p = pressure head, or head due to the *resultant* pressure at a given section (i. e., piezometer head);
h_x = "expansion head," or loss of head due to expansion of section;
h_k = "contraction head," or loss of head due to contraction of section;
h_l = total loss of head above a given section.

Then, using the same sub-notation for the velocity, we have, by theory: V = theoretic velocity due to H, or $V^2 = 2gH$; $v_a^2 = 2g\ h_a$;

$v^2 = 2g\ h$; $\quad v_a^2 = 2g\ h_a$; $\quad v_f^2 = 2g\ h_f$; $\quad v_s^2 = 2g\ h_s$;
$v_e^2 = 2g\ h_e$; $\quad v_p^2 = 2g\ h_p$; $\quad v_x^2 = 2g\ h_x$; $\quad v_k^2 = 2g\ h_k$;

$v_l^2 = 2g\ h_l$; in which the gravity acceleration $g = 32.16$, and $\sqrt{2g}$ = say 8.02.

Considering the gains and losses in the pipe line, the following relations exist:

$$\underbrace{h_p = H}_{\text{Pres. Stat.}} + \underbrace{h_a + h_s}_{\text{Gains: Appr. Suct.}} - \underbrace{h}_{\text{Veloc.}} - \underbrace{(h_e + h_f + h_c + h_x + h_k)}_{\text{Losses: Entry, Fric., Curv., Expan., Contr.}} \quad \ldots (1)$$

$$\frac{v_p^2}{2g} = \frac{V^2}{2g} + \frac{v_a^2}{2g} + \frac{v_s^2}{2g} - \frac{v^2}{2g} - \left(\frac{v_e^2}{2g} + \frac{v_l^2}{2g} + \frac{v_c^2}{2g} + \frac{v_x^2}{2g} + \frac{v_k^2}{2g}\right) \quad \ldots (2)$$

If we neglect the velocity of approach and the suction head, and let h represent the losses in head (mostly friction), equations (1) and (2) reduce to

$$h_p = H - h - h_l \quad \ldots (3)$$

$$\frac{v_p^2}{2g} = \frac{V^2}{2g} - \frac{v^2}{2g} - \frac{v_l^2}{2g} \quad \ldots (4)$$

Combining (3) and (4), we have the general equation (5), following.

* The term hydrodynamics was formerly defined as the science which treats of the *motion* of liquids (now included under hydrokinetics), but it now has a broader acceptation: The science which treats of the laws of *force* as applied to fluids. Hence it comprises hydrostatics and hydrokinetics.

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locity and Discharge.—The velocity of discharge v at any section e obtained from equation (1), by substituting $\frac{v^2}{2g}$ for h, and trans- as follows:

$$v = \sqrt{2g\,(H - h_p - h_l)} \qquad (5)$$

ressure head h_p equals zero for any given section, and we also assume s h_l, then

$$v = \sqrt{2g\,H} = 8.02\sqrt{H} \qquad (6)$$

following is a table of theoretic velocities for various heads, calcu- om equation (6). See, also, table on page 283.

THEORETIC VELOCITIES v FOR VARIOUS HEADS h. (EQUATION 6.)

Veloc. Ft. per Sec.	Head. Ft.	Veloc. Ft. per Sec.	Head. Ft.	Veloc. Ft. per Sec.	Head. Ft.	Veloc. Ft. per Sec.	Head. Ft.	Veloc. Ft. per Sec.	Head. Ft.	Veloc. Ft. per Sec.
.57	.37	4 88	.93	7.73	3.1	14.1	18.5	34.5	68	66.1
.80	.38	4.94	.94	7.78	3.2	14.3	19.0	35.0	69	66.6
.98	.39	5.01	.95	7.82	3.3	14.6	.5	35.4	70	67.1
1.13	.40	5.07	.96	7.86	3.4	14.8	20.0	35.9	71	67.6
1.27	.41	5.14	.97	7.90	3.5	15.0	.5	36.3	72	68.0
1.39	.42	5.20	.98	7.94	3.6	15.2	21.0	36.8	73	68.5
1.50	.43	5.26	.99	7.98	3.7	15.4	.5	37.2	74	69.0
1.60	.44	5.32	1.00	8.02	3.8	15.6	22.0	37.6	75	69.5
1.70	.45	5.38	1.02	8.10	3.9	15.8	.5	38.0	76	69.9
1.79	.46	5.44	1.04	8.18	4.0	16.0	23.0	38.5	77	70.4
1.88	.47	5.50	1.06	8.26	.2	16.4	.5	38.9	78	70.8
1.96	.48	5 56	1.08	8.33	.4	16.8	24.0	39.3	79	71.3
2.04	.49	5.61	1.10	8.41	.6	17.2	.5	39.7	80	71.7
2.12	.50	5.67	1.12	8.49	.8	17.6	25	40.1	81	72.2
2.19	.51	5.73	1.14	8.56	5.0	17.9	26	40.9	82	72.6
2.27	.52	5.78	1.16	8.64	.2	18.3	27	41.7	83	73.1
2 34	.53	5.84	1.18	8.71	.4	18.6	28	42.4	84	73.5
2.41	.54	5.89	1.20	8.79	.6	19.0	29	43.2	85	73.9
2.47	.55	5.95	1.22	8.86	.8	19.3	30	43.9	86	74.4
2.54	.56	6.00	1.24	8.93	6 0	19.6	31	44.7	87	74.8
2.60	.57	6.06	1.26	9.00	.2	20.0	32	45.4	88	75.2
2.66	.58	6.11	1.28	9.07	.4	20.3	33	46.1	89	75.7
2.72	.59	6.16	1.30	9.14	.6	20.6	34	46.8	90	76.1
2.78	.60	6.21	1.32	9.21	.8	20.9	35	47.4	91	76.5
2.84	.61	6.26	1.34	9.28	7.0	21.2	36	48.1	92	76.9
2.89	.62	6.31	1.36	9.35	.2	21.5	37	48 8	93	77.3
2.95	.63	6.37	1.38	9.42	.4	21.8	38	49.4	94	77.8
3.00	.64	6.42	1 40	9.49	.6	22.1	39	50.1	95	78.2
3.05	.65	6.47	1.42	9.56	.8	22.4	40	50.7	96	78.6
3.11	.66	6.52	1.44	9.62	8.0	22.7	41	51.4	97	79.0
3.16	.67	6.56	1.46	9.69	.2	23.0	42	52.0	98	79.4
3.21	.68	6.61	1.48	9.76	.4	23.3	43	52.6	99	79.8
3.26	.69	6.66	1.50	9.82	.6	23.5	44	53.2	100	80.2
3.31	.70	6.71	1.52	9.89	.8	23.8	45	53.8	110	84.1
3.35	.71	6.76	1.54	9.95	9.0	24.1	46	54.4	120	87.9
3.40	.72	6.80	1.56	10.02	.2	24.3	47	55.0	130	91.4
3.45	.73	6.85	1.58	10.08	.4	24.6	48	55.6	140	94.9
3.50	.74	6.90	1.60	10.14	.6	24.8	49	56.1	150	98.2
3.54	.75	6.95	1.65	10.30	.8	25.1	50	56.7	175	106.1
3.59	.76	6.99	1.70	10.46	10.0	25.4	51	57.3	200	113.4
3.68	.77	7.04	1.75	10.61	.5	26.0	52	57.8	225	120.3
3.76	.78	7.08	1.80	10.76	11.0	26.6	53	58.4	250	126.8
3.85	.79	7.13	1.85	10.91	.5	27.2	54	58.9	275	133.0
3.93	.80	7.17	1.90	11.05	12.0	27.8	55	59.5	300	138.9
4.01	.81	7.22	1.95	11.20	.5	28.4	56	60.0	325	144.6
4.09	.82	7.26	2.00	11.34	13.0	28.9	57	60.6	350	150.0
4.17	.83	7.31	2.10	11.62	.5	29.5	58	61.1	375	155.3
4.24	.84	7.35	2.20	11.90	14.0	30.0	59	61.6	400	160.4
4.32	.85	7.39	2 30	12.16	.5	30.5	60	62.1	450	170.1
4.39	.86	7 44	2.40	12.43	15.0	31.1	61	62.6	500	179.3
4.47	.87	7.48	2 50	12 68	.5	31.6	62	63.1	550	188.1
4.54	.88	7.53	2.60	12.93	16.0	32.1	63	63.7	600	196.4
4.61	.89	7.57	2 70	13.18	.5	32.6	64	64.2	700	212.2
4.68	.90	7.61	2.80	13.42	17.0	33.1	65	64.7	800	226.8
4.74	.91	7 65	2.90	13.66	.5	33.5	66	65.2	900	240.6
4.81	.92	7.69	3.00	13.89	18.0	34.0	67	65.6	1000	253.6

The theoretic velocity, therefore, is the same as that which would be acquired by the water (or any other body) falling freely in vacuo through the height H. Also, from equation (6), we have

$$H = .01555\, v^2 \qquad (7)$$

The Discharge through a pipe, when the velocity is known, is obtained from the following simple formula:

$$q = a\, v \qquad (8)$$

In which q = discharge in cu. ft. per *sec.;
a = area of cross-section of flowing water, in sq. ft.;
v = mean velocity of flow, in ft. per sec.

Then, from (8) and (5) we have, for any practical case,

$$q = 8.02\, a \sqrt{H - h \quad - h_1} \qquad (9)$$

which takes into consideration all the losses of head. If there are no losses in head, and no pressure head at the section considered, then we have for the theoretical discharge, since, $h_p = 0$, $h_1 = 0$,

$$q = 8.02\, a \sqrt{H} \qquad (10)$$

in which $8.02 \sqrt{H}$ is the theoretic velocity (equation 6) whose values are given in Table 1, preceding.

†Table 2, following, gives the areas of pipes in square feet for various diameters in feet and inches:

Problem 1.—What is the least diameter of pipe that could possibly be used for discharging 300 cubic ft. per sec. under a 14-ft. head?

Solution.—Neglecting friction and other losses we have from Table 1, page 1155, that the theoretic velocity is 30 ft. per second. Without the use of any formula‡ we know that the area of pipe required $= \frac{300}{30} = 10$ sq. ft.; and, from Table 2, page 1157, the corresponding diameter is 3 ft. 6¼ ins. Hence, we know that the diameter would have to be larger than 3 ft. 6¼ ins. to take care of the friction- and other losses.

In practice, after a pipe has been properly designed to meet the conditions of the problem and take care of all losses of head, it is customary to increase the diameter of the pipe somewhat: (1) to provide for future increased demands on the supply, and (2) to anticipate the roughening of the inner surface of the pipes from rust or vegetable growth. Sewer pipes are increased usually about 2 ins. in dia.; water mains, about the same; and small pipes, 10 to 50% in area.

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* To find cubic feet per minute,	multiply q by	60
" " " hour,	" "	3,600
" " " 24 hours,	" "	86,400
" " " 30 days,	" "	2,592,000
" " " 365 days,	" "	31,536,000

To reduce cubic feet to gallons, multiply by $\frac{1728}{231} = \frac{576}{7 \times 11} = 7.48052$.

† See, also, tables of circles, pages 230-235.

‡ Or, from equation (5), $q = a\, v$, we have $a = \frac{q}{v} = \frac{300}{30}$.

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2.—Areas of Pipes in Sq. Ft. for Diameters in Feet and Inches.

Diameter. Ft.	Ins.	Fraction of an Inch. 0/8	1/8	1/4	3/8	1/2	5/8	3/4	7/8
0	0	0	.000085	.00034	.00077	.00137	.00213	.00307	.00418
	1	.00545	.00690	.00852	.0103	.0123	.0144	.0167	.0192
	2	.0218	.0246	.0276	.0308	.0341	.0376	.0412	.0451
	3	.0491	.0533	.0576	.0621	.0668	.0717	.0767	.0819
	4	.0873	.0928	.0985	.1044	.1104	.1167	.1231	.1296
	5	.1364	.1433	.1503	.1575	.1650	.1726	.1803	.1883
	6	.1964	.2046	.2131	.2217	.2304	.2394	.2485	.2578
	7	.2673	.2769	.2867	.2967	.3068	.3171	.3276	.3382
	8	.3491	.3601	.3712	.3826	.3941	.4056	.4176	.4296
	9	.4418	.4541	.4667	.4794	.4922	.5053	.5185	.5319
	10	.5454	.5591	.5730	.5871	.6013	.6157	.6303	.6450
	11	.6600	.6750	.6903	.7057	.7213	.7371	.7530	.7691
1	0	.7854	.8019	.8185	.8352	.8522	.8693	.8866	.9041
	1	.9218	.9396	.9575	.9757	.9940	1.013	1.031	1.050
	2	1.069	1.088	1.108	1.127	1.147	1.167	1.187	1.207
	3	1.227	1.247	1.268	1.289	1.310	1.331	1.353	1.374
	4	1.396	1.418	1.440	1.462	1.485	1.507	1.530	1.553
	5	1.576	1.599	1.623	1.646	1.670	1.694	1.718	1.742
	6	1.767	1.792	1.817	1.842	1.867	1.892	1.917	1.943
	7	1.969	1.995	2.021	2.047	2.074	2.100	2.127	2.154
	8	2.182	2.209	2.237	2.264	2.292	2.320	2.348	2.376
	9	2.405	2.434	2.463	2.492	2.521	2.550	2.580	2.610
	10	2.640	2.670	2.700	2.730	2.761	2.792	2.823	2.854
	11	2.885	2.916	2.948	2.980	3.012	3.044	3.076	3.109
2	0	3.142	3.174	3.207	3.240	3.274	3.307	3.341	3.375
	1	3.409	3.443	3.477	3.512	3.547	3.581	3.616	3.651
	2	3.687	3.722	3.758	3.794	3.830	3.866	3.903	3.939
	3	3.976	4.013	4.050	4.087	4.125	4.162	4.200	4.238
	4	4.276	4.314	4.353	4.391	4.430	4.469	4.508	4.547
	5	4.587	4.626	4.666	4.706	4.746	4.786	4.827	4.868
	6	4.909	4.950	4.991	5.032	5.074	5.115	5.157	5.199
	7	5.241	5.283	5.326	5.369	5.412	5.455	5.498	5.541
	8	5.585	5.629	5.673	5.717	5.761	5.805	5.850	5.895
	9	5.940	5.985	6.030	6.075	6.121	6.167	6.213	6.259
	10	6.305	6.351	6.398	6.445	6.492	6.539	6.586	6.633
	11	6.681	6.729	6.777	6.825	6.874	6.922	6.971	7.020
3	0	7.069	7.118	7.167	7.216	7.266	7.316	7.366	7.416
	1	7.467	7.517	7.568	7.619	7.670	7.721	7.773	7.824
	2	7.876	7.928	7.980	8.032	8.084	8.137	8.190	8.243
	3	8.296	8.349	8.403	8.456	8.510	8.564	8.618	8.672
	4	8.727	8.781	8.836	8.891	8.946	9.001	9.057	9.112
	5	9.168	9.224	9.280	9.336	9.393	9.450	9.507	9.564
	6	9.621	9.678	9.736	9.794	9.852	9.910	9.968	10.026
	7	10.085	10.144	10.203	10.262	10.321	10.380	10.440	10.499
	8	10.559	10.619	10.680	10.740	10.801	10.862	10.923	10.984
	9	11.045	11.106	11.168	11.229	11.291	11.353	11.416	11.478
	10	11.541	11.604	11.667	11.730	11.793	11.856	11.920	11.984
	11	12.048	12.112	12.177	12.241	12.306	12.371	12.436	12.501
4	0	12.566	12.632	12.698	12.764	12.830	12.896	12.962	13.028
	1	13.095	13.162	13.229	13.296	13.364	13.431	13.499	13.567
	2	13.635	13.703	13.772	13.840	13.909	13.978	14.047	14.116
	3	14.186	14.256	14.326	14.396	14.466	14.536	14.606	14.677
	4	14.748	14.819	14.890	14.962	15.033	15.105	15.176	15.249
	5	15.321	15.393	15.465	15.538	15.611	15.684	15.758	15.831
	6	15.904	15.978	16.052	16.126	16.200	16.274	16.349	16.424
	7	16.499	16.574	16.649	16.724	16.800	16.876	16.952	17.028
	8	17.104	17.181	17.257	17.334	17.411	17.488	17.565	17.643
	9	17.721	17.799	17.876	17.954	18.033	18.111	18.190	18.269
	10	18.348	18.427	18.506	18.586	18.665	18.745	18.826	18.906
	11	18.986	19.067	19.147	19.228	19.309	19.390	19.472	19.553

Note.—Areas of pipes are proportional to the squares of their diameters.

Example.—The area of pipe 2′ 6″ in dia. = 4.909 sq. ft. Then, for a pipe 0″ dia., $a = 4.909 \times 4$; for 7′ 6″ dia., $a = 4.909 \times 9$; for 10′ 0″ dia., $a = 4.909 \times 16$; etc. See, also, Tables 13-15, pages 230-235.

Velocity inversely proportional to a and to d^2.—When a pipe of variable cross-section is discharging a constant, full volume of water, as per Fig. 1, we have,

$$q = a\,v = a_1\,v_1 = a_2\,v_2 = a_3\,v_3, \text{ etc} \ldots\ldots\ldots\ldots (11)$$

But $a = \frac{\pi d^2}{4}$; $a_1 = \frac{\pi d_1^2}{4}$; $a_2 = \frac{\pi d_2^2}{4}$; etc. Then $q = \frac{\pi d^2}{4} v = \frac{\pi d_1^2}{4} v_1 = \frac{\pi d_2^2}{4} v_2$, etc. . (12)

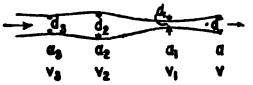

Fig. 1.

Whence $\quad v = \frac{q}{a}; \quad v_1 = \frac{q}{a_1}; \quad v_2 = \frac{q}{a_2}$; etc $\ldots\ldots\ldots\ldots (13)$

Or, $\quad v = \frac{4q}{\pi d^2}; \quad v_1 = \frac{4q}{\pi d_1^2}; \quad v_2 = \frac{4q}{\pi d_2^2}$; etc $\ldots\ldots\ldots\ldots (14)$

In which q = discharge in cubic feet per second;
d, d_1, d_2, d_3 = diameter of pipe in feet at different sections;
a, a_1, a_2, a_3 = area of pipe in sq. ft. at sections d, d_1, d_2, d_3;
v, v_1, v_2, v_3 = velocity of flow in ft. per sec. at sections a, a_1, a_2, a_3;

$$\pi = 3.1416; \qquad \frac{\pi}{4} = 0.7854.$$

It is to be noted that the above formulas represent the practical relations which may exist in any pipe line regardless of friction and loss of head. These formulas may be transposed in various ways. If it is desired to substitute the "head" of water in place of the velocity we must be careful to use the velocity head h and not the static head H. Thus, from (6), (13) and (14) we have

$$q = \frac{\pi d^2}{4} \times 8.02\sqrt{h} = 6.298908\, d^2 \sqrt{h} \ldots\ldots\ldots\ldots (15)$$

whence $\quad d = 2\sqrt{\frac{q}{8.02\,\pi\sqrt{h}}} = 0.3984\frac{\sqrt{q}}{\sqrt[4]{h}} \ldots\ldots\ldots\ldots (16)$

and if all friction and other losses are neglected, $h = H$,

and $\quad d = 2\sqrt{\frac{q}{8.02\,\pi\sqrt{H}}} = 0.3984\frac{\sqrt{q}}{\sqrt[4]{H}} \ldots\ldots\ldots\ldots (17)$

Hence it is seen that the diameter of the pipe is directly proportional to the square root of the discharge, and inversely proportional to the fourth root of the "head." Solving Problem 1 by equation (14), we obtain

$$d = 0.3984\frac{\sqrt{300}}{\sqrt[4]{14}} = 3.57 \text{ ft.} = 3 \text{ ft. } 6\tfrac{7}{8} \text{ ins.}$$

We will now take up the question of "losses" of energy, head and pressure, which occur during flow.

Losses During Flow.—In the following discussion reference is made to Fig. 2, showing water discharging from the upper reservoir U. R., through the pipe line $o\,d$, into the lower reservoir L. R.

o = orifice, intake or inlet end of pipe, at which a gate is placed;
d = discharge end, or outlet of pipe, at which a gate is placed;
$t\,V\,T$ = Venturi water meter, inserted in the pipe line for measuring the discharge;
V = Venturi itself, or the contracted section of the meter;
t or c = contracted section of the pipe line;
T or e = expanded section of the pipe line;
b = bend in the pipe line.

During flow, loss of head will occur at all these points; and throughout the whole line, due (1) to the friction of the water along the sides of the pipe, and (2) to the lateral or radial forces* set up by the impingement of

* Usually termed *viscosity*.

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the water particles against each other in oblique directions to the flow. But (1) and (2) are usually grouped together under the symbol h_f = loss of head due to friction (see page 1160).

There are two other losses of head which have to be considered, namely, (1) loss due to "dropping" head (decrease of H) in the upper reservoir, and (2) loss due to "rising" head, above d, in the lower reservoir (decrease of H_3); the static head H at d, is H_2+h' when the water surface in L. R. is below d; and H_3 when above d.

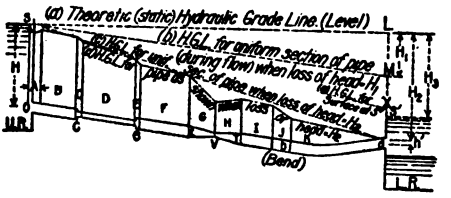

Fig. 2.—Pipe Line Between Reservoirs.

Explanation of Figure 2.—A *hydraulic grade line* is a line of "no pressure." That is, if a vertical tube or piezometer is inserted in the top of a pipe line at any point it will be found that the water will rise in the piezometer tube to the hydraulic grade line and thus indicate the pressure head h_p at that point of the line. Hence, a H. G. L. is very useful in the study of any pipe line as it gives directly the pressure head at any point. The above Figure shows five H. G. L.'s, (*a*), (*b*), (*c*), (*d*) and (*e*), which will be explained:

(*a*).—This can obtain only when a gate is closed in the pipe line as at d, or some other point. As soon as the gate is opened, water will begin to discharge at d and the end of the H. G. L. at L will drop.

(*b*).—This condition will obtain with the gate at d, *partly open*, when the H. G. L. would drop to M and the pressure head h_p at d would equal $H_2+h'-H_1$ if the surface of water in L. R. is below d; or H_3-H_1 if surface of water is above d. The total loss of head is H_1-h, the latter being the velocity head. Note that the H. G. L. $S\,M$ is straight only when the loss of head is uniform or proportional to the length of the line. The discharge at d will issue with a velocity $*v=8.02\sqrt{h}$, h forming a fractional part of H_1.

(*c*).—This is the usual condition for a pipe line of *uniform cross-section* discharging freely, that is, all gates open, and the water-level below d in L. R. Note that there is no pressure head *above the top of the pipe* at d because the efflux does not quite fill the discharge end; but the actual *mean pressure head* at d is shown in Fig. 3, by h', the depth below the free surface, and the velocity of discharge at outlet is $v=8.02\sqrt{h'}$ + *velocity of approach.*

Fig. 3.

(*d*).—The hydraulic grade line is here shown as a very broken or irregular line, and marks the elevation at which the water would rise in the piezometer tube if inserted in the pipe line at any point. The pipe is as shown in Fig. 2, and *not* uniform in section as was assumed with (*b*) and (*c*). A sharp drop (to the right) in the H. G. L. indicates a greater *rate* of loss of head, per lin. ft. of pipe, than one of flatter slope. Thus, the grade drops sharply, by comparison, at c and e, points of abrupt change in diameter; at c, the converging tube leading to the venturi V; and at the bend b. The curved form of the H. G. L. at section A is due to a loss of head at the orifice o forming a downward (first) part of curve; while the upward (second) part of the curve is due to suction by vacuum just beyond the orifice. A similar suction occurs just beyond the venturi V causing an upward H. G. L. at section H. Note that the H. G. L. at section D is sharper than at sections E, F, I and K because the area of the pipe is less, therefore the velocity, friction and rate of loss of head are greater. The conditions at the outlet d

* Treated as a weir without end contraction; see Weirs, page 1177, etc.

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are the same as described in (*c*). This case (*d*) represents a usual occurrence in practice, and the losses are described in detail in the following pages. It may be remarked here, however, that the pressure head h, at any point in the pipe line is the vertical distance from the H. G. L. to the center of pressure of the water section in the pipe. The center of pressure is usually assumed at the center of a circular pipe, although in reality it is a little below the center.

(*e*).—Here, it is assumed that the water in the lower reservoir has risen *above* the discharge end d of the pipe, to the elevation of s'. The theoretic head is therefore H_3. Now we know that the total loss of head cannot exceed the theoretic head H_3, therefore the H. G. L. must slope to some point x, *above* or *at* s'. Moreover, we know that x must be above s', otherwise there would be no pressure head ($x\ s'$) to cause a flow into L. R. We can reasonably assume $x\ s'$ to be less than h' (say about half the diameter of the pipe) and to diminish as s' rises in elevation, so that when $H_3 = o$, $xs' = o$ also.

Total Head is usually sub-divided into *entry head, velocity head* and *friction head*. That is, the force exerted by the pressure due to the total head, as in a reservoir, is expended partly (1) in overcoming the resistance at the entrance of the pipe; (2) in producing velocity of flow or discharge; (3) in producing friction (and viscosity). There is a fourth resistance which has to be overcome, namely, *curvature head**, but this is usually included in an ordinary pipe line, under *friction head*. The combined entry and velocity heads seldom exceed one foot even when the entrance is sharp-edged, in which case they are about equal. The entry head reduces with increased length of pipe line, becoming inappreciable when the length exceeds about 1000 diameters. Even for short pipe lines, less than 1000 diameters, the entry head almost entirely disappears with the use of the bell-shaped or flaring entrance. It is thus seen that the with the main considerations in any ordinary pipe line are velocity head and friction head. Moreover, one is a function of the other so that, for a given smoothness of wetted surface and a given diameter of pipe, they both increase with the hydraulic slope. The relations between velocity head (or rather velocity due to the velocity head), friction head, smoothness of wetted surface, and hydraulic slope, constitute the laws of flow and when thoroughly known, may be incorporated in a working formula.

Loss of Head due to Friction.—In long pipe lines this is really the only loss that is practically considered, or at least other incidental losses are included in "friction" losses. The "dropping" head as in emptying a reservoir may be reduced to "mean average" head or, with more exactness to several mean average heads at successive elevations of water surface. The loss of head at entrance becomes almost a negligible quantity; moreover, the orifice is usually a converging tube, thereby reducing the loss to a minimum. Similarly, converging and diverging "reducers" are inserted in a line joining pipes of different diameters, thereby greatly reducing the loss of head which would obtain if the change were abrupt as at *c* and *e*, Fig. 2. Where such changes of section occur frequently, as in lap-jointed riveted steel pipe (Fig. 4), it is customary to use the smaller diameter d in computing the flow The losses due to contraction of sections at c, and the expansion at e, together with the loss effect due to rivet heads are considerable, and it is a question whether a *uniform* pipe of diameter d would not show the greater capacity. The losses due to bends are seldom computed unless the curvature is considerable. Short curves, moderately sharp, are the best.

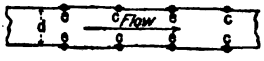

Fig 4.—Riveted Pipe.

Friction losses are deduced from experiments on pipe lines actually constructed, and from laboratory experiments. The former point to a safe

* The loss of head due to curvature of pipe, as short curves, bends, etc., is but imperfectly understood. Theoretical formulas, as those of Weisbach, are not reliable. Roughly speaking, the loss of head in a short 90° bend may be assumed at 3 to 5 times what it would be in the same length of straight pipe. For a good discussion of this subject see Paper No 911 in Trans. Am. Soc. C. E., for April, 1902, by Gardner S. Williams, Clarence W. Hubbell and George H. Fenkell.

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precedent for *similar* designs, while the latter lead to the establishing of general laws of flow, in a *relative* sense. It would hardly be safe, however, to design large pipe lines based on formulas derived from laboratory experiments alone. Many physicists have attempted to formulate laws of flow and of friction based almost wholly on theory without considering the viscosity of the flowing liquid. It is perhaps needless to say that all such formulas, while interesting, are practically useless to the engineer, and they are positively dangerous to the young student. The class of formulas which the hydraulic engineer is using to-day, and will continue to use, are founded on experimentation. Experiments reveal to us that—

(*a*) Friction F of water in pipes is proportional to some *variable power* of the velocity v, that is, $F \propto v^x$. In pipes of commercial size the values of x have been found to range from about $x = 1.75$ to $x = 2$, depending on the diameter of pipe and roughness of wetted surface, or perimeter.

(*b*) Friction is proportional to the length of the pipe, so far as at present known.

(*c*) Friction is less in curves of short radius, down to $2\frac{1}{2}$ diameters, than in curves of long radius, for the same total angle of deflection. (See Trans. Am. Soc. C. E., Vol. XLVII, pages 183, 191.)

(*d*) Friction is inversely proportional to some power of the diameter of the pipe or the hydraulic radius r of the conduit. The index of the power may be assumed as 1.2, within small limits of error.

(*e*) Friction increases rapidly with the roughness of the wetted perimeter. A newly laid conduit with a smooth surface can often deteriorate so that the velocity of discharge will decrease from 2 to 5 per cent per annum for a number of years.

Author's Hydraulic Formula.—The following general formula is based on the most reliable existing experimental data, and is applicable to the flow of water in pipes, sewers, flumes, etc.

Notation:

a = area of cross-section of water in pipe, flume, etc., in sq. ft.;
v = velocity of discharge in ft. per second $\left(v = \frac{q}{a}\right)$;
q = discharge in cubic ft. per second $(q = a\,v)$;
H = loss of head in friction, etc., in ft. per 1000 ft. $(H = 1000\,s)$;
s = hydraulic slope or sine of angle of slope $(s = .001\,H)$;
p = wetted perimeter of cross-section of pipe, etc., in ft.;
*r = hydraulic (mean) radius in ft. $= \frac{a}{p}$. $\left(\text{For circular pipe } r = \frac{d}{4}\right)$;
d = diameter of circular pipe in ft. $(d = 4r)$;
c = coefficient of smoothness of wetted surface; also alinement;
x = index of the power of the velocity, proportional to friction;
y = index of the power of d or $4r$, inversely proportional to friction.

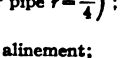

Fig. 5. Fig. 6.

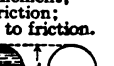

Fig. 7.

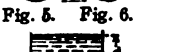

Fig. 8.

* As the velocity of discharge v increases with the mean radius r, it is desirable to design the section of the conduit so that r will be a maximum for the area a, other things equal. The circular section is ideal in this respect, and the value of r is equal to $\frac{d}{4}$ whether the circular pipe is full, as in Fig. 5, or only half full, as in Fig. 6. When not full, but more than half full, $r > \frac{d}{4}$: $r = 0.3043$ (its maximum value) when the depth of water in the circular pipe is $0.81d$. When the pipe is less than half full, $r < \frac{d}{4}$. Fig. 7 shows the section of a flume, in which r has its maximum value when $b_1 = 2d_1$. Practical considerations however usually call for a less proportionate depth. Fig. 8 is a section of a trapezoidal canal in which r is a maximum when $b_2 = 2(x - y)$, or $b_2 = 2d_2$ (cosec θ − cot θ). In practice, canals and ditches are usually designed with such slopes as will probably be maintained without wash during the flow of water.

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Formulas:

	Hyd. Radius, r.	Diameter, d.	
General:	$H=\frac{1}{c}\cdot\frac{x^2\,v^x}{(4r)^y}$	$H=\frac{1}{c}\cdot\frac{x^2\,v^x}{d^y}$	(1)

But x generally varies from 1.75 to 2.00 — somewhat nearer the former value; and y varies usually from 1.15 to 1.25. Assuming average values of x and y, we have, $x=1.82$; $x^2=3.31+$; $\frac{1}{x}=0.55$; $\frac{1}{x^2}=3.0+$; $y=1.2$.

Hence, $$H=\frac{1}{c}\cdot\frac{v^{1.82}}{3(4r)^{1.2}} \qquad H=\frac{1}{c}\cdot\frac{v^{1.82}}{3d^{1.2}} \qquad (2)$$

Equation (2) is the author's formula for loss of head in feet per 1000 feet. The various values of c, v, r, d, and H, raised to the proper powers, are given in the subjoined Tables 3, 4, 5, 6 and 7. Other forms of (2) which are useful in practice are the following:

Hydraulic slope, $$s=.001\,H \qquad (3)$$

Velocity, $$v=(3c)^{0.55}\,H^{0.55}\,(4r)^{0.66}=(3c)^{0.55}\,H^{0.55}\,d^{0.66} \qquad (4)$$

Diameter, $$d=\frac{1}{(3c)^{0.55}}\cdot\frac{1}{H^{0.55}}\cdot v^{1.82+} \qquad (5)$$

Hydraulic radius, $$r=\frac{d}{4} \qquad (6)$$

*Coefficient, $$c=\frac{v^{1.82}}{3H(4r)^{1.2}}=\frac{v^{1.82}}{3Hd^{1.2}} \qquad (7)$$

Note that H = hydraulic slope mult. by 1000.

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* Formula (7) is used in determining the coefficient c of smoothness and alinement in pipe lines actually built and tested. The values given in Table 3 are close averages from actual experiments.

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3.—VALUES OF COEFFICIENT c OF SMOOTHNESS AND ALINEMENT IN AUTHOR'S FORMULA, PRECEDING.

Class. (1)	Surface. (2)	Description of Material. (3)	Age. (4)	Alinement. (5)	$\frac{1}{c}$ (6)	c (7)	$c^{0.55}$ (8)	$(3c)^{0.55}$ (9)	$\frac{1}{(3c)^{0.55}}$ (10)
A	Extremely Smooth.	Smooth metals brass; lead Well planed timber, pipe Smooth cast iron, pipe	New. " "	Straight. " "	1.00	1.00	1.00	1.83	0.40
B	Very Smooth.	Metals; brass; lead Well planed timber, pipe Cast iron pipe Smooth steel pipe	5 yrs. " 3 yrs. New.	Few curves. " " Straight.	1.10	.91	.95	1.74	0.43
C	Fair.	Wood-stave pipe (ordinary) Cast iron pipe Smooth steel pipe	7 yrs. 5 yrs. New	Str. and curved. " Few curves.	1.20	.83	.90	1.66	0.47
D	Not Clean.	Cast iron pipe (ordinary) Riveted steel pipe Neat cement Vitrified brick	10 yrs. 4 yrs. New. New.	Str. and curved. " " "	1.50	.67	.80	1.46	0.56
E	Unclean.	Cast iron pipe Riveted steel pipe (ordinary) Brick; sewers	20 yrs. 12 yrs. 10 yrs.	Str. and curved. " "	1.90	.53	.70	1.29	0.68
F	Rough.	Cast iron pipe Riveted steel pipe Rough brickwork	30 yrs. 20 yrs.	Str. and curved. "	2.50	.40	.60	1.10	0.86
G	Very Rough.	Badly tuberculated pipe Unclean sewers	....	....	3.50	.29	.50	0.92	1.14

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4.—Values of $v^{1.82}$ for Various Values of v In Author's Formula, preceding.

Velocity v	Tenths.									
	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	0.000	0.015	0.053	0.112	0.189	0.283	0.395	0.523	0.666	0.826
1	1.00	1.19	1.39	1 61	1.84	2.09	2.35	2.63	2.91	3.22
2	3.53	3.86	4.20	4.55	4.92	5.30	5.69	6.10	6.51	6.94
3	7.39	7.84	8.31	8.78	9.27	9.78	10.29	10.82	11.36	11.91
4	12.47	13.04	13.62	14.22	14.83	15.45	16.08	16.72	17.37	18.04
5	18.71	19.40	20.10	20.81	21.53	22.26	23.00	23.75	24.52	25.29
6	26.08	26.87	27.68	28.50	29.33	30.16	31.01	31.87	32.75	33.63
7	34.52	35.42	36.34	37.26	38.19	39.14	40.09	41.06	42.03	43.02
8	44.02	45.03	46.04	47.07	48.10	49.15	50.21	51.28	52.35	53.44
9	54.54	55.65	56.77	57.89	59.03	60.18	61.34	62.51	63.69	64.87
10	66.07	67.28	68.50	69.72	70.96	72.21	73.46	74.73	76.01	77.29

Ex.—$3.1^{1.82} = 7.84$; $6.3^{1.82} = 28.50$; $9^{1.82} = 54.54$.

5.—Values of $*v^{1.50+}$ for Various Values of v In Author's Formula, preceding.

Velocity v	Tenths.									
	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	0.000	0.030	0.087	0.161	0.249	0.350	0.461	0.582	0.713	0.852
1	1.00	1.16	1.32	1.49	1.67	1.85	2.04	2.24	2.44	2.65
2	2.86	3.08	3.31	3.54	3.77	4.01	4.26	4.51	4.77	5.03
3	5.29	5.56	5.84	6.12	6.40	6.69	6.98	7.27	7 57	7.88
4	8.19	8.50	8.82	9.14	9.46	9.79	10.12	10.46	10.80	11.14
5	11.48	11.83	12.19	12.55	12.91	13.27	13.64	14.01	14.38	14.76
6	15.14	15.53	15.92	16.31	16.70	17.10	17.50	17.90	18.31	18.72
7	19.13	19.55	19.97	20.39	20.81	21.24	21.67	22.11	22.54	22.98
8	23.43	23.87	24.32	24.77	25.22	25.68	26.14	26.60	27.07	27.54
9	28.01	28.48	28.96	29.43	29.92	30.40	30.89	31.38	31.87	32.36
10	32.86	33.36	33.87	34.37	34.87	35.38	35.90	36.41	36.93	37.45

* $1.50+ = 1.82 + 1.2$.

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6.—VALUES OF $(4r)^{0.66}$, $d^{0.66}$, AND $\frac{1}{d^{1.2}}$, FOR VARIOUS VALUES OF d IN AUTHOR'S FORMULA, PRECEDING.

Equiv. Dia. in Ins.	d $(=4r)$ Ft.	$d^{0.66}=(4r)^{0.66}$	$\frac{1}{d^{1.2}}$	Equiv. Dia. in Ins.	d $(=4r)$ Ft.	$d^{0.66}=(4r)^{0.66}$	$\frac{1}{d^{1.2}}$	d $(=4r)$ Ft.	$d^{0.66}=(4r)^{0.66}$	$\frac{1}{d^{1.2}}$
.25	.0208	.0777	104.11		1.45	1.278	.640	10	4.571	.0631
.375	.0312	.1015	64.00	18	1.50	1.307	.615	11	4.868	.0563
.50	.0417	.1228	45.32		1.60	1.364	.569	12	5.155	.0507
.625	.0521	.1422	34.67	20	1.667	1.401	.542	13	5.435	.0461
.75	.0625	.1604	27.86		1.70	1.419	.529	14	5.708	.0421
.875	.0729	.1776	23.15		1.80	1.474	.494	15	5.973	.0388
1.00	.0833	.1940	19.73		1.90	1.528	.463	16	6.233	.0359
1.25	.1042	.2248	15.09	24	2.00	1.580	.435	17	6.488	.0334
1.375	.1146	.2393	13.46		2.10	1.632	.411	18	6.737	.0312
1.50	.1250	.2535	12.13		2.20	1.683	.388	19	6.982	.0292
1.75	.1458	.2806	10.08		2.30	1.733	.368	20	7.222	.02746
2.00	.1667	.3065	8.586		2.40	1.782	.350	21	7.459	.02591
2.25	.1875	.3305	7.454	30	2.50	1.831	.333	22	7.691	.02450
2.50	.2083	.3551	6.569		2.60	1.879	.3177	23	7.920	.02322
2.75	.2292	.3782	5.859		2.70	1.926	.3036	24	8.146	.02207
3.	.2500	.4005	5.278		2.80	1.973	.2907	25	8.368	.02101
3.5	.2917	.4434	4.387		2.90	2.019	.2787	26	8.588	.02007
4.	.3333	.4843	3.737	36	3.00	2.065	.2676	27	8.804	.01916
5.	.4167	.5611	2.859		3.20	2.155	.2476	28	9.018	.01834
6.	.5000	.6329	2.297		3.40	2.243	.2303	29	9.230	.01758
7.	.5833	.7007	1.909	42	3.50	2.286	.2224	30	9.438	.01688
8.	.6667	.7652	1.627		3.75	2.393	.2047	35	10.45	.01403
9.	.7500	.8271	1.412	48	4.0	2.497	.1895	40	11.41	.01195
10.	.8333	.8866	1.245	54	4.5	2.698	.1645	45	12.33	.01038
11.	.9167	.9442	1.110	60	5.0	2.893	.1450	50	13.22	.00915
12.	1.0000	1.000	1.000	66	5.5	3.081	.1293	55	14.08	.00816
13.	1.083	1.054	.908	72	6.0	3.263	.1165	60	14.91	.00735
	1.150	1.097	.846	78	6.5	3.440	.1058	65	15.72	.00668
14.	1.167	1.107	.831	84	7.0	3.612	.0968	70	16.51	.00611
	1.200	1.128	.803	90	7.5	3.780	.0891	75	17.28	.00562
15.	1.250	1.159	.765	96	8.0	3.945	.0825	80	18.03	.00520
	1.300	1.189	.730	102	8.5	4.106	.0770	85	18.77	.00484
16.	1.333	1.209	.708	108	9.0	4.264	.0716	90	19.49	.00452
	1.350	1.219	.698	114	9.5	4.419	.0674	95	20.20	.00423
	1.400	1.249	.668	120	10.0	4.571	.0631	100	20.89	.00398

EX.—For a pipe 18" dia., $d=1.5$, $r=1.5\div4$, $d^{0.66}=1.307$, $1\div d^{1.2}=0.615$.

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7.—Values of $H^{0.55}$ and $\frac{1}{H^{0.55}}$ for Various Values of H In Author's Formula, preceding.

$\frac{H}{(=1000s)}$ Ft.	$H^{0.55}$	$\frac{1}{H^{0.55}}$	$\frac{H}{(=1000s)}$ Ft.	$H^{0.55}$	$\frac{1}{H^{0.55}}$	$\frac{H}{(=1000s)}$ Ft.	$H^{0.55}$	$\frac{1}{H^{0.55}}$	$\frac{H}{(=1000s)}$ Ft.	$H^{0.55}$	$\frac{1}{H^{0.55}}$
.000	.000	∞	.10	.282	6.81	1.0	1.000	1.000	10	3.55	.147
.002	.033	177.5	.11	.297	6.29	1.1	1.054	0.924	11	3.74	.136
.004	.048	99.6	.12	.312	5.88	1.2	1.105	.859	12	3.92	.126
.006	.060	71.0	.13	.326	5.48	1.3	1.155	.804	13	4.10	.118
.008	.070	55.9	.14	.339	5.15	1.4	1.203	.755	14	4.27	.111
.010	.079	46.4	.15	.352	4.86	1.5	1.250	.713	15	4.44	.105
.012	.088	39.9	.16	.365	4.60	1.6	1.295	.676	16	4.59	.0992
.014	.096	35.1	.17	.377	4.38	1.7	1.339	.643	17	4.76	.0943
.016	.103	31.4	.18	.389	4.17	1.8	1.382	.613	18	4.90	.0899
.018	.110	28.4	.19	.401	3.99	1.9	1.423	.586	19	5.05	.0860
.020	.116	26.05	.20	.413	3.82	2.0	1.464	.561	20	5.19	.0824
.022	.123	24.06	.22	.435	3.53	2.2	1.543	.518	22	5.48	.0760
.024	.129	22.38	.24	.456	3.28	2.4	1.619	.482	24	5.74	.0708
.026	.134	20.93	.26	.477	3.07	2.6	1.691	.451	26	6.00	.0662
.028	.140	19.68	.28	.497	2.89	2.8	1.762	.424	28	6.25	.0622
.030	.145	18.58	.30	.516	2.73	3.0	1.830	.400	30	6.49	.0588
.032	.151	17.62	.32	.534	2.58	3.2	1.896	.379	32	6.73	.0557
.034	.156	16.74	.34	.552	2.46	3.4	1.960	.361	34	6.96	.0529
.036	.161	15.96	.36	.570	2.34	3.6	2.023	.344	36	7.18	.0505
.038	.166	15.26	.38	.587	2.24	3.8	2.084	.329	38	7.39	.0483
.040	.170	14.62	.40	.604	2.15	4.0	2.144	.315	40	7.61	.0463
.042	.175	14.04	.42	.621	2.06	4.2	2.202	.302	42	7.81	.0444
.044	.179	13.50	.44	.637	1.98	4.4	2.259	.291	44	8.02	.0427
.046	.184	13.01	.46	.652	1.91	4.6	2.315	.280	46	8.21	.0412
.048	.188	12.56	.48	.668	1.84	4.8	2.370	.271	48	8.41	.0398
.050	.193	12.14	.50	.683	1.78	5.0	2.42	.262	50	8.60	.0385
.055	.203	11.21	.55	.720	1.65	5.5	2.55	.242	55	9.06	.0355
.060	.213	10.43	.60	.755	1.53	6.0	2.68	.225	60	9.51	.0330
.065	.222	9.76	.65	.789	1.43	6.5	2.80	.210	65	9.93	.0308
.070	.232	9.17	.70	.822	1.35	7.0	2.92	.198	70	10.35	.0290
.075	.241	8.66	.75	.854	1.27	7.5	3.03	.187	75	10.75	.0274
.080	.249	8.20	.80	.885	1.20	8.0	3.14	.177	80	11.14	.0259
.085	.258	7.80	.85	.914	1.15	8.5	3.25	.168	85	11.51	.0247
.090	.266	7.44	.90	.944	1.09	9.0	3.35	.160	90	11.88	.0235
.095	.274	7.11	.95	.972	1.04	9.5	3.45	.153	95	12.24	.0225
.100	.282	6.81	1.00	1.000	1.00	10.0	3.55	.147	100	12.59	.0215

Ex.—For hydraulic slope $s = .004$, $H = 4.0$, $H^{0.55} = 2.144$, and $\frac{1}{H^{0.55}} = .315$.

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Problems in Use of Author's Hydraulic Formula.

(See page 1161, etc.)

Problem 1.—What loss of head per 1000 ft. would occur in an ordinary iron pipe line 20 ins. in diameter, if the velocity is 3 ft. per second?

Solution.—Use formula (2); then $\frac{1}{c}$, from Table 3, $=1.50$; $v^{1.92}$, from Table 4, $=7.39$; $\frac{1}{d^{1.3}}$, from Table 6, $=0.542$; hence, $H=1.50\times7.39\times\frac{3}{2}\times$ $.2=2.00$ ft. per 1000 ft. *Ans.*

Problem 2.—What velocity of discharge would be expected in an un-n, egg-shaped, brick sewer, whose hydraulic radius is 0.75, and hydraulic e 0.00125?

Solution.—Use formula (4); then from the Tables, $(3c)^{0.55}=1.29$; $5=1.25^{0.55}=1.13$; $(4r)^{0.66}=d^{0.66}=3^{0.66}=2.065$; hence, $v=1.29\times1.13\times2.07$ .02 ft. per sec. *Ans.*

Problem 3.—What diameter of riveted-steel pipe would be required to harge 2.4 cu. ft. per second, after it has been in use about 12 years, if hydraulic slope is 0.01225?

Solution.—Use formula (4) by trial method—assuming d, obtaining v, then q $(=av=2.4)$. *First*, assume pipe to be 12 ins. in diameter; then 1, $v=1.29\times3.97\times1=5.12$; $q=av=0.7854\times5.04=4.02$; hence, as 4.02 .4, a 12-in. pipe is too large. *Second*, assume pipe to be 10 ins. in diameter; n $d=0.833$; $v=1.29\times3.97\times0.887=4.54$; $q=av=0.5454\times4.54=2.48$, required discharge being 2.4; hence a 10-in. pipe is required. *Ans.*

Problem 4.—A conduit whose hydraulic radius is 0.875, and whose lraulic slope is .001375 has a velocity of discharge of 4.5 ft. per second. at is the coefficient c of smoothness and alinement of the conduit?

Solution.—Use formula (7), in which $v=4.5$, $H=1.375$, and $4r=3.5$; n by use of accompanying Tables, $c=\frac{15.45\times0.2224}{3\times1.375}=0.83$, which places nder Class C in Table 3.

Chezy's Hydraulic Formula assumes the velocity to be proportional to square root of both the hydraulic radius and the hydraulic slope; or

$$v=c\sqrt{rs} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

which c is a coefficient to be determined by experiment. (See Kutter's mula, following.)

Kutter's Formula, so-called, is really the Chezy formula (preceding), but th values of the coefficient c supplied so as to give it a general application. ese values of c were deduced from experiments made by Ganguillet and utter, and were primarily intended to apply to the flow of water in streams d canals. They have, however, received general application, to a greater less extent, in the design of water mains, sewers, flumes, ditches, etc. e value of the coefficient in English* measure is as follows:

$$c=\frac{\frac{1.811}{n}+41.65+\frac{.00281}{s}}{1+\frac{n}{\sqrt{r}}\left(41.65+\frac{.00281}{s}\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

which r and s are the mean radius and the slope, respectively, as explained

* In metric measure,

$$c=\frac{\frac{1}{n}+23+\frac{.00155}{s}}{1+\frac{n}{\sqrt{r}}\left(23+\frac{.00155}{s}\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

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on page 1161; and n is a coefficient depending upon the roughness of the wetted surface of the conduit, sewer, canal, or stream. Thus,

$n = .009$ for well-planned timber, perfectly aligned.*

$= .010$ for neat (pure) cement; glazed or enameled surfaces generally; smooth cast iron or iron pipes; planed timber.*

$= .011$ for cement with one-third sand, in good condition; well-jointed pipes of iron†, cement, and terra cotta.

$= .012$ for unplaned timber in good alinement, as flumes.

$= .013$ for ashlar and brickwork, well laid; ordinary metal; earthen-, cement-, stoneware- and terra cotta pipe not well pointed nor in first-class order; cement plaster and planed timber in second-class condition; generally, the materials for $n = .010$ when imperfect in quality or condition.

$= .015$ for unclean surfaces in pipes and sewers; second-class or rough brickwork; stonework, well dressed; iron-, stoneware- and terra cotta pipes with imperfect joints and in bad condition.

$= .017$ for rubble masonry in good order; brickwork, stoneware and ashlar in poor condition; generally, the materials for $n = .013$ when imperfect in quality or condition; tuberculated iron pipes

$= .020$ for canals in very firm gravel, and carefully trimmed; inferior rubble in cement; coarse, dry rubble.

$= .0225$ for coarse, dry rubble in bad condition.

$= .025$ for canals and rivers free from stones and weeds, and in good order.

$= .030$ for canals and rivers having some stones and weeds.

$= .035$ for canals and rivers in bad order, with great quantities of stones and weeds.

$= .040$ for rivers in extremely bad condition.

These values of n are to be inserted in equation (2) for finding the value of c used in equation (1). The correct use of Kutter's (or Chezy's) formula depends upon the proper selection of the values of n for the roughness of the wetted surface. The values $n = .010$ to $n = .016$ will cover all conditions for good water mains—the lower values for the smooth pipes, and the higher values for the pipes with rough surfaces. (See, also, page 1188.)

In order to simplify the calculations of the coefficients c in Kutter's formula, equations (1) and (2), the following reductions of form of equation (2) are given for various values of roughness n. It remains only to substitute the proper values of—

r, the hydraulic radius of the pipe, sewer or canal, in ft.;

d, the diameter of the circular pipe, in ft.;

s, the average slope of the pipe, or rate of fall of the hydraulic grade line.

	Using Hyd. Rad r.		Using Diameter d.	
For $n = .009$,	$c = \dfrac{\sqrt{r}\,(242.872\,s + .00281)}{s(\sqrt{r} + .37485) + .0000253}$	$= \dfrac{\sqrt{d}\,(242.872\,s + .00281)}{s(\sqrt{d} + .7497) + .0000506}$		(4)
For $n = .010$,	$c = \dfrac{\sqrt{r}\,(222.75\,s + .00281)}{s(\sqrt{r} + .4165) + .0000281}$	$= \dfrac{\sqrt{d}\,(222.75\,s + .00281)}{s(\sqrt{d} + .833) + .0000562}$		(5)
For $n = .0105$,	$c = \dfrac{\sqrt{r}\,(214.13\,s + .00281)}{s(\sqrt{r} + .437325) + .000295}$	$= \dfrac{\sqrt{d}\,(214.13\,s + .00281)}{s(\sqrt{d} + .87465) + .000059}$		(6)
For $n = .011$,	$c = \dfrac{\sqrt{r}\,(206.29\,s + .00281)}{s(\sqrt{r} + .45815) + .0000309}$	$= \dfrac{\sqrt{d}\,(206.29\,s + .00281)}{s(\sqrt{d} + .9163) + .0000618}$		(7)

* The values $n = .010$ and $n = .0105$ are often used for wood-stave pipe well laid, with dressed timber.

† The values $n = .014$ and $n = .016$ are often used for *lap-jointed* steel-riveted pipe. For foul and tuberculated iron pipe, use $n = .016+$.

	Using Hyd. Rad r.		Using Diameter d.	
For $n=.012$,	$c=\dfrac{\sqrt{r}\,(192.57\,s+.00281)}{s(\sqrt{r}+.4998)+.0000337}$	$=$	$\dfrac{\sqrt{d}\,(192.57\,s+.00281)}{s(\sqrt{d}+.9996)+.0000674}$	(8)
For $n=.013$,	$c=\dfrac{\sqrt{r}\,(180.96\,s+.00281)}{s(\sqrt{r}+.54145)+.0000365}$	$=$	$\dfrac{\sqrt{d}\,(180.96\,s+.00281)}{s(\sqrt{d}+1.0829)+.0000730}$	(9)
For $n=.014$,	$c=\dfrac{\sqrt{r}\,(171.01\,s+.00281)}{s(\sqrt{r}+.5831)+.0000393}$	$=$	$\dfrac{\sqrt{d}\,(171.01\,s+.00281)}{s(\sqrt{d}+1.1662)+.0000786}$	(10)
For $n=.015$,	$c=\dfrac{\sqrt{r}\,(162.38\,s+.00281)}{s(\sqrt{r}+.62475)+.0000422}$	$=$	$\dfrac{\sqrt{d}\,(162.38\,s+.00281)}{s(\sqrt{d}+1.2495)+.0000844}$	(11)
For $n=.017$,	$c=\dfrac{\sqrt{r}\,(148.18\,s+.00281)}{s(\sqrt{r}+.70805)+.0000478}$	$=$	$\dfrac{\sqrt{d}\,(148.18\,s+.00281)}{s(\sqrt{d}+1.4161)+.0000956}$	(12)
For $n=.020$,	$c=\dfrac{\sqrt{r}\,(132.20\,s+.00281)}{s(\sqrt{r}+.833)+.0000562}$	$=$	$\dfrac{\sqrt{d}\,(132.20\,s+.00281)}{s(\sqrt{d}+1.666)+.0001124}$	(13)
For $n=.0225$,	$c=\dfrac{\sqrt{r}\,(122.14\,s+.00281)}{s(\sqrt{r}+.93713)+.0000632}$	$=$	$\dfrac{\sqrt{d}\,(122.14\,s+.00281)}{s(\sqrt{d}+1.87426)+.0001264}$	(14)
For $n=.025$,	$c=\dfrac{\sqrt{r}\,(114.09\,s+.00281)}{s(\sqrt{r}+1.04125)+.0000703}$	$=$	$\dfrac{\sqrt{d}\,(114.09\,s+.00281)}{s(\sqrt{d}+2.0825)+.0001406}$	(15)
For $n=.030$,	$c=\dfrac{\sqrt{r}\,(102.02\,s+.00281)}{s(\sqrt{r}+1.2495)+.0000843}$	$=$	$\dfrac{\sqrt{d}\,(102.02\,s+.00281)}{s(\sqrt{d}+2.499)+.0001686}$	(16)
For $n=.035$,	$c=\dfrac{\sqrt{r}\,(93.39\,s+.00281)}{s(\sqrt{r}+1.45775)+.0000984}$	$=$	$\dfrac{\sqrt{d}\,(93.39\,s+.00281)}{s(\sqrt{d}+2.9155)+.0001968}$	(17)
For $n=.040$,	$c=\dfrac{\sqrt{r}\,(86.925\,s+.00281)}{s(\sqrt{r}+1.666)+.0001124}$	$=$	$\dfrac{\sqrt{d}\,(86.925\,s+.00281)}{s(\sqrt{d}+3.332)+.0002248}$	(18)

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Coefficients *c* in Kutter's formula, for various values of roughness *n*, mean radius *r*, and slope *s*, are given in Table 8, following. Intermediate values of *c*, for values of *n*, *r* and *s* not given in the Table, may be obtained by interpolation, by simple proportion. In cases where the slope *s* is greater than $s = .01$, section 7, the values of *c* as deduced from section 7 may be considered sufficiently accurate—a little too large for values of *r* greater than 3.28 (1 meter), and a little too small for values of *r* less than 3.28. Note that the value of *c* is independent of the slope *s* in all cases where the mean radius $r = 3.28$ (1 meter); and for this reason the "one meter" line is shown in Italics in each of the 7 sections. *Above* the one-meter line the values of *c* *increase* with the slope *s*; *below*, they *decrease*.

For explanation of the use of Kutter's formula and Table 8, see Practical Examples following the table.

8.—Coefficients *c* in Kutter's Formula, $v = c\sqrt{rs}$, English Measure for various values of *s*, *r* and *n*.

Hyd. Rad. *r* Ft.	Coefficients *n* of Roughness.														
	.009	.010	.0105	.011	.012	.013	.014	.015	.017	.020	.0225	.025	.030	.035	.040
	(1) For Slope s = .000025 = 1 in 40,000 = .132 ft. per mile.														
.1	66.0	57.1	53.4	50.1	44.6	40.0	36.3	33.1	28.1	22.8	19.6	17.2	13.7	11.4	9.7
.2	86.6	75.4	70.7	66.6	59.4	53.6	48.7	44.5	38.0	31.0	26.8	23.6	18.9	15.8	13.5
.3	100.6	87.9	82.6	77.9	69.7	63.0	57.4	52.6	45.1	36.9	32.0	28.2	22.7	19.0	16.3
.4	111.3	97.5	91.8	84.9	76.2	68.9	62.9	59.0	50.7	41.7	36.2	31.9	25.8	21.6	18.6
.6	127.3	112.1	105.7	100.0	90.1	81.9	74.9	68.9	59.5	49.1	42.9	37.9	30.8	25.8	22.3
.8	139.3	123.1	116.3	110.1	99.5	90.6	83.1	76.6	66.3	55.0	48.1	42.7	34.8	29.3	25.3
1	148.8	131.9	124.8	118.3	107.1	97.7	89.8	82.9	72.0	59.9	52.5	46.7	38.1	32.2	27.8
1.5	166.6	148.4	140.7	133.7	121.6	111.4	102.7	95.1	83.0	69.6	61.2	54.6	44.2	38.1	33.0
2	179.4	160.4	152.3	145.0	132.2	121.5	112.3	104.3	91.3	77.0	68.0	60.8	52.0	42.7	37.2
3	196.0	177.4	168.9	161.1	147.6	136.1	126.3	117.7	103.7	88.0	78.2	70.2	58.5	50.0	43.7
3.28	*201.2*	*181.1*	*172.5*	*164.6*	*151.0*	*139.3*	*129.4*	*120.7*	*106.5*	*90.6*	*80.5*	*72.4*	*60.4*	*51.7*	*45.3*
4	209.8	189.3	180.5	172.5	158.5	146.6	136.4	127.4	112.8	96.3	85.8	77.4	64.8	55.7	48.8
6	226.8	205.7	196.7	188.4	173.8	161.4	150.8	141.3	125.9	108.3	97.1	88.0	74.3	64.3	56.7
8	238.4	217.0	207.7	199.3	184.5	171.8	160.9	151.2	135.3	117.1	105.4	95.9	81.4	70.8	62.7
10	247.0	225.4	216.0	207.5	192.5	179.7	168.5	158.7	142.5	123.9	111.9	102.1	87.1	76.1	67.6
12	253.7	232.0	222.6	214.0	198.9	185.9	174.5	164.8	148.4	129.5	117.3	107.2	91.9	80.5	71.7
16	262.3	242.0	232.5	223.9	208.6	195.5	184.2	174.1	157.4	138.2	125.7	115.4	99.5	87.6	78.5
20	271.2	249.3	239.8	231.1	215.8	202.7	191.2	181.1	164.3	144.8	132.1	121.7	105.4	93.3	83.8
30	283.5	261.6	252	243.4	228.0	214.9	203.4	193.2	176.3	156.5	143.7	133.0	116.3	103.7	93.8
50	297.0	275.2	265.7	257.1	241.8	228.6	217.2	207.1	190.1	170.4	157.4	146.6	129.7	116.7	106.5
75	306.2	284.5	275.1	266.5	251.3	238.3	226.9	216.9	200.1	180.4	167.5	156.7	139.8	126.8	116.5
100	312.0	290.4	281.1	272.5	257.4	244.4	233.2	223.2	206.5	187.0	174.2	163.5	146.6	133.7	123.3
	(2) For Slope s = .00005 = 1 in 20,000 = .264 ft. per mile.														
.1	79.0	68.1	63.6	59.6	52.8	47.2	42.6	38.7	32.6	26.2	22.4	19.5	15.4	12.6	10.7
.2	100.7	87.5	82.0	77.1	68.6	61.7	55.9	51.0	43.3	35.0	30.1	26.3	20.9	17.3	14.7
.3	114.7	100.1	94.0	88.5	79.2	71.4	64.9	59.4	50.6	41.2	35.5	30.5	24.9	20.6	17.6
.4	125.0	109.5	103.0	97.2	87.1	78.8	71.8	65.8	56.3	46.0	39.8	35.0	27.5	22.8	19.5
.6	139.9	123.3	116.2	109.9	98.9	89.8	82.1	75.5	64.9	53.4	46.4	40.9	33.0	27.6	23.6
.8	150.7	133.2	125	119.1	107.6	97.9	89.8	82.7	71.5	59.1	51.5	45.6	37.0	31.0	26.6
1	159.0	141.0	133.3	126.4	114.4	104.4	95.9	88.5	76.7	63.7	55.7	49.4	40.2	33.8	29.1
1.5	174.0	155.1	147.0	139.7	127.0	116.4	107.3	99.4	86.6	72.5	63.7	56.8	46.6	39.4	34.1
2	184.3	164.9	156.6	149.1	135.9	124.9	115.5	107.2	93.9	79.0	69.8	62.4	51.4	43.7	38.0
3	198.3	178.3	169.7	161.9	148.3	136.8	126.9	118.3	104.2	88.5	78.5	70.6	58.7	50.2	43.9
3.28	*201.2*	*181.1*	*172.5*	*164.6*	*151.0*	*139.3*	*129.4*	*120.7*	*106.5*	*90.6*	*80.5*	*72.4*	*60.4*	*51.7*	*45.3*
4	207.6	187.3	178.6	170.7	156.8	145.0	134.9	126.0	111.6	95.2	84.9	76.6	64.1	55.0	48.4
6	220.0	199.3	190.5	182.4	168.2	156.1	145.8	136.6	121.7	104.7	93.9	85.2	72.0	62.4	55.1
8	228.1	207.3	198.3	190.2	175.8	163.6	153.1	143.9	128.7	111.4	100.3	91.3	77.6	67.7	60.0
10	233.9	213.0	204.1	195.9	181.4	169.1	158.6	149.3	133.9	116.4	105.1	96.0	82.1	71.8	64.0
12	238.4	217.5	208.5	200.3	185.8	173.5	162.9	153.5	138.1	120.4	109.0	99.8	85.6	75.2	67.2
16	245.1	224.1	215.1	206.8	192.3	180.0	169.3	159.9	144.3	126.5	115.0	105.7	91.3	80.6	72.3
20	249.9	228.9	219.8	211.6	197.1	184.7	174.0	164.5	149.0	131.1	119.5	110.1	95.5	84.7	76.3
30	257.6	236.7	227.6	219.4	204.9	192.5	181.8	172.4	156.8	138.8	127.2	117.7	103.0	92.0	83.5
50	266.0	245.0	236.0	227.8	213.4	201.0	190.4	181.0	165.4	147.6	136.0	126.5	111.8	100.8	92.1
75	271.5	250.6	241.7	233.5	219.1	206.8	196.2	186.9	171.4	153.7	142.2	132.8	118.1	107.2	98.6
100	274.9	254.1	245.1	237.0	222.6	210.4	199.9	190.6	175.2	157.6	146.2	136.8	122.3	111.4	102.6

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8.—Coefficients *c* in Kutter's Formula, English Measure—Cont'd.

Hyd. Rad. *r* Ft.	Coefficients *n* of Roughness.														
	.009	.010	.0105	.011	.012	.013	.014	.015	.017	.020	.0225	.025	.030	.035	.040
	(3) For Slope s = .0001 = 1 in 10,000 = .528 ft. per mile.														
.1	90.8	78.2	73.1	68.4	60.5	54.1	48.7	44.2	37.1	29.6	25.2	21.8	17.1	13.9	11.7
.2	112.7	98.0	91.8	86.3	76.9	69.1	62.6	57.0	48.3	38.9	33.3	29.0	22.9	18.8	15.9
.3	126.2	110.3	102.8	97.6	87.3	78.7	71.6	65.4	55.7	45.2	38.9	34.0	27.0	22.3	18.9
.4	135.2	119.3	112.3	105.9	95.0	85.9	78.3	71.7	60.8	50.0	43.2	37.8	30.2	25.0	21.3
.6	149.7	132.0	124.5	117.8	106.1	96.3	88.1	81.0	69.6	57.2	49.7	43.8	35.2	29.3	25.0
.8	159.2	140.9	133.2	126.2	114.0	103.8	95.2	87.8	75.8	62.6	54.5	48.2	39.0	32.6	27.9
1	166.5	147.8	139.8	132.6	120.1	109.7	100.8	93.1	80.6	66.9	58.5	51.8	42.1	35.3	30.4
1.5	179.1	159.9	151.6	144.1	131.1	120.1	110.8	102.7	89.6	74.9	65.9	58.7	48.0	40.6	35.1
2	187.7	168.0	159.6	152.0	138.6	127.4	117.8	109.5	95.9	80.7	71.2	63.7	52.5	44.6	38.7
3	198.9	178.8	170.2	162.4	148.8	137.2	127.3	118.7	104.6	88.8	78.8	70.8	58.9	50.4	44.1
3.28	*201.2*	*181.1*	*172.5*	*164.6*	*151.0*	*139.3*	*129.4*	*120.7*	*106.5*	*90.6*	*80.5*	*72.4*	*60.4*	*51.7*	*45.3*
4	206.2	186.0	177.3	169.4	155.6	143.9	133.8	125.0	110.7	94.4	84.2	76.0	63.6	54.7	48.0
6	215.7	195.3	186.5	178.5	164.5	152.6	142.4	133.5	118.8	102.1	91.6	83.1	70.2	60.8	53.8
8	221.7	201.2	192.4	184.4	170.3	158.3	148.0	139.0	124.2	107.4	96.6	88.0	74.8	65.2	57.9
10	226.1	205.5	196.7	188.6	174.5	162.5	152.1	143.1	128.2	111.2	100.4	91.6	78.3	68.6	61.1
12	229.4	208.8	200.0	191.9	177.7	165.7	155.3	146.3	131.3	114.3	103.4	94.6	81.1	71.3	63.7
16	234.2	213.6	204.7	196.7	182.5	170.4	160.1	151.0	136.0	118.8	107.9	99.0	85.4	75.4	67.8
20	237.6	217.0	208.1	200.1	185.9	173.8	163.4	154.4	138.4	122.2	111.2	102.3	88.6	78.0	70.8
30	243.1	222.5	213.7	205.6	191.4	179.4	169.0	159.9	144.9	127.8	116.8	107.8	94.2	84.0	76.2
50	248.9	228.3	219.5	211.4	197.3	185.3	175.0	165.9	151.0	133.9	123.0	114.1	100.4	90.3	82.5
75	252.7	232.2	223.3	215.3	201.2	189.3	178.9	169.9	155.0	138.1	127.2	118.4	104.8	94.8	87.0
100	255.0	234.5	225.7	217.7	203.6	191.7	181.4	172.4	157.6	140.7	129.9	121.1	107.6	97.6	89.9
	(4) For Slope s = .0002 = 1 in 5,000 = 1.056 ft. per mile.														
.1	99.4	85.8	80.1	75.0	66.4	59.3	53.4	48.4	40.6	32.3	27.4	23.7	18.5	15.0	12.6
.2	121.1	105.5	98.9	93.0	82.8	74.5	67.5	61.5	52.0	41.9	35.8	31.1	24.5	20.0	16.9
.3	134.1	117.5	110.4	104.0	93.1	84.0	76.4	69.8	59.4	48.2	41.4	36.2	28.7	23.6	19.9
.4	143.3	125.5	118.6	111.9	100.5	90.9	82.5	76.0	65.0	53.0	45.4	40.0	31.9	26.3	22.3
.6	156.0	137.7	130.0	123.0	110.9	100.8	92.2	84.9	73.0	60.0	52.0	45.8	36.8	30.5	26.0
.8	164.6	145.9	138.0	130.8	118.3	107.8	98.9	91.2	78.8	65.1	56.7	50.1	40.5	33.8	28.9
1	171.1	152.1	144.0	136.6	123.9	113.1	104.0	96.1	83.3	69.2	60.4	53.6	43.5	36.4	31.3
1.5	182.3	162.8	154.4	146.9	133.9	122.6	113.1	104.9	91.5	76.6	67.3	60.0	49.1	41.5	35.8
2	189.7	169.9	161.4	153.7	140.3	129.0	121.6	110.9	97.0	81.8	72.2	64.6	53.2	45.2	39.2
3	199.2	179.2	170.6	162.8	149.1	137.5	127.6	119.0	104.9	89.0	79.0	71.0	59.1	50.5	44.2
3.28	*201.2*	*181.1*	*172.5*	*164.6*	*151.0*	*139.3*	*129.4*	*120.7*	*106.5*	*90.6*	*80.5*	*72.4*	*60.4*	*51.7*	*45.3*
4	205.4	185.2	176.6	168.7	154.9	143.2	133.1	124.4	110.1	93.9	83.7	75.5	63.2	54.4	47.8
6	213.3	192.9	184.2	176.3	162.3	150.5	140.4	131.5	117.0	100.5	90.1	81.7	69.0	59.7	52.9
8	218.2	197.8	189.1	181.1	167.1	155.2	145.1	136.2	121.5	104.9	94.4	85.9	73.0	63.6	56.5
10	221.8	201.3	192.6	184.6	170.6	158.7	148.4	139.5	124.8	108.1	97.5	89.0	75.9	66.5	59.2
12	224.4	204.0	195.2	187.2	173.2	161.3	151.1	142.1	127.4	110.7	100.0	91.4	78.3	68.7	61.5
16	228.3	207.9	199.1	191.1	177.0	165.1	154.9	145.9	131.2	114.4	103.7	95.0	81.9	72.2	64.9
20	231.0	210.6	201.8	193.8	179.8	167.8	157.6	148.7	133.9	117.2	106.4	97.7	84.5	74.8	67.4
30	235.4	214.9	206.2	198.2	184.2	172.2	162.0	153.1	138.3	121.5	110.8	101.7	88.9	79.2	71.8
50	239.9	219.5	210.7	202.8	188.8	176.9	166.7	157.8	143.1	126.3	115.7	107.1	93.9	84.2	76.8
75	242.9	222.5	213.7	205.8	191.8	180.0	169.8	160.9	146.2	129.6	119.0	110.4	97.3	87.7	80.3
100	244.7	224.3	215.6	207.6	193.7	181.8	171.7	162.8	148.2	131.1	121.0	112.5	99.5	89.9	82.6
	(5) For Slope s = .0004 = 1 in 2,500 = 2.112 ft. per mile.														
.1	104.8	90.5	84.5	79.2	70.1	62.6	56.4	51.2	42.9	34.1	28.9	25.0	19.4	15.7	13.1
.2	126.2	110.0	103.2	97.1	86.6	77.8	70.5	64.3	54.4	43.8	37.6	32.5	25.6	20.9	17.5
.3	138.8	121.7	114.4	107.9	96.6	87.2	79.3	72.6	61.8	50.1	43.1	37.6	29.7	24.4	20.6
.4	147.6	129.8	122.3	115.5	101.7	92.0	85.6	78.4	67.2	54.8	47.3	41.4	33.0	27.2	22.9
.6	159.6	141.1	133.2	126.1	113.8	103.5	94.7	87.2	75.0	61.7	53.5	47.1	37.8	31.4	26.7
.8	167.7	148.8	140.7	133.4	120.7	110.1	101.1	93.3	80.6	66.7	58.1	51.3	41.4	34.6	29.6
1	173.8	154.5	146.4	138.9	126.0	115.1	105.9	97.9	84.9	70.5	61.7	54.6	44.3	37.1	31.9
1.5	184.1	164.4	156.0	148.4	135.1	122.8	114.4	106.1	92.6	77.6	68.2	60.7	49.7	42.0	36.3
2	190.8	170.9	162.4	155.1	141.3	129.9	120.1	111.7	97.9	82.5	72.8	65.1	53.6	45.2	39.5
3	199.5	179.4	170.8	162.9	149.3	137.7	127.8	119.2	105.0	89.1	79.1	71.1	59.2	50.6	44.2
3.28	*201.2*	*181.1*	*172.5*	*164.6*	*151.0*	*139.3*	*129.4*	*120.7*	*106.5*	*90.6*	*80.5*	*72.4*	*60.4*	*51.7*	*45.3*
4	205.0	184.8	176.1	168.3	154.5	142.8	132.8	124.1	109.8	93.6	83.5	75.3	63.0	54.2	47.6
6	212.0	191.7	183.0	175.1	161.2	149.4	139.3	130.5	116.0	99.6	89.3	80.9	68.3	59.2	52.3
8	216.4	196.0	187.3	179.4	165.4	153.6	143.5	134.6	120.1	103.6	93.1	84.7	71.9	62.7	55.6
10	219.5	199.1	190.4	182.4	168.5	156.6	146.5	137.6	123.0	106.5	95.9	87.5	74.6	65.3	58.1
12	221.8	201.5	192.7	184.8	170.8	159.0	148.8	139.9	125.3	108.7	98.1	89.6	76.7	67.3	60.1
16	225.2	204.8	196.1	188.1	174.2	162.3	152.1	143.3	128.6	112.0	101.4	92.9	79.9	70.4	63.2
20	227.6	207.2	198.5	190.5	176.5	164.7	154.5	145.6	131.0	114.3	103.8	95.2	82.2	72.7	65.5
30	231.4	211.0	202.3	194.3	180.4	168.5	158.3	149.5	134.8	118.2	107.6	99.1	86.1	76.6	69.3
50	235.3	215.0	206.2	198.3	184.4	172.5	162.4	153.5	138.9	122.4	111.8	103.3	90.4	80.9	73.7
75	237.9	217.5	208.8	200.9	187.0	175.2	165.0	156.2	141.7	125.2	114.7	106.2	93.3	83.9	76.7
100	239.4	219.1	210.4	202.5	188.6	176.8	166.7	157.9	143.3	126.9	116.4	108.0	95.2	85.8	78.6

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8.—Coefficients c in Kutter's Formula, English Measure.—Concl'd.

Hyd. Rad. r Ft.	Coefficients n of Roughness.															
	.009	.010	.0105	.011	.012	.013	.014	.015	.017	.020	.0225	.025	.030	.035	.040
	(6) For Slope s = .001 = 1 in 1,000 = 5.28 ft. per mile.														
.1	108.4	93.8	87.6	82.1	72.7	65.0	58.6	53.1	44.5	35.4	30.0	25.9	20.1	16.2	13.5
.2	129.7	113.1	106.1	99.9	89.1	80.2	72.7	66.3	56.1	45.2	38.6	33.5	26.3	21.5	18.0
.3	142.0	124.5	117.1	110.5	99.0	89.4	81.4	74.5	63.4	51.5	44.2	38.6	30.5	25.0	21.1
.4	150.5	132.5	124.8	117.9	106.0	96.0	86.8	80.4	68.8	56.1	48.4	42.4	33.7	27.8	23.5
.6	162.0	143.3	135.4	128.2	115.7	105.2	96.4	88.8	76.4	62.9	54.5	48.0	38.5	32.0	27.2
.8	169.7	150.7	142.5	135.2	122.4	111.6	102.5	94.6	81.8	67.7	59.0	52.2	42.1	35.1	30.0
1	175.5	156.1	147.9	140.4	127.4	116.5	107.1	99.1	86.0	71.5	62.5	55.4	44.9	37.6	32.3
1.5	185.2	165.5	157.1	149.4	136.1	124.9	115.3	106.9	93.4	78.2	68.8	61.3	50.2	42.4	36.6
2	191.5	171.6	163.1	155.4	141.9	130.5	120.7	112.3	98.4	82.9	73.2	65.5	53.9	45.8	39.7
3	199.6	179.5	170.9	163.1	149.4	137.8	127.9	119.3	105.1	89.2	79.2	71.2	59.2	50.7	44.3
3.28	*201.2*	*181.1*	*172.5*	*164.6*	*151.0*	*139.3*	*129.4*	*120.7*	*106.5*	*90.6*	*80.5*	*72.4*	*60.4*	*51.7*	45.3
4	204.7	184.5	175.9	168.0	154.2	142.6	132.6	123.9	109.6	93.5	83.3	75.1	62.9	54.1	47.5
6	211.2	190.9	182.2	174.3	160.4	148.7	138.6	129.8	115.4	99.1	88.7	80.4	67.9	58.8	52.0
8	215.2	194.9	186.2	178.3	164.4	152.6	142.5	133.7	119.1	102.7	92.3	83.9	71.2	62.1	55.1
10	218.1	197 8	189.0	181.1	167.2	155.4	145.2	136.4	121.9	105.4	94.9	86.5	73.7	64.5	57.4
12	220.2	199.9	191.2	183.2	169.3	157.5	147.3	138.5	123.9	107.4	97.0	88.5	75.7	66.4	59.3
16	223.3	203.0	194.3	186.3	172.4	160.6	150.4	141.6	127.0	110.5	100.0	91.5	78.6	69.3	62.1
20	225.5	205.2	196.4	188.5	174.6	162.7	152.6	143.8	129.2	112.6	102.1	93.6	80.7	71.4	64.2
30	229.0	208.6	199.9	192.0	178.0	166.2	156.1	147.3	132.7	116.1	105.7	97.2	84.3	74.9	67.7
50	232.5	212.2	203.5	195.6	181.7	169.9	159.8	151.0	136.4	119.9	109.5	101.0	88.2	78.8	71.7
75	234.8	214.5	205.8	197.9	184.0	172.3	162.2	153.4	138.9	122.4	112.0	103.6	90.8	81.5	74.4
100	236.2	216.0	207.3	199.4	185.5	173.7	163.6	154.9	140.4	124.0	113.6	105.2	92.5	83.2	76.2
	(7) For Slope s = .01 = 1 in 100 = 52.8 ft. per mile.														
.1	110.9	95.1	89.6	84.0	74.4	66.5	60.0	54.4	45.6	36.3	30.7	26.5	20.6	16.6	13.8
.2	131.9	115.1	108.0	101.7	90.7	81.7	74.1	67.6	57.2	46.1	39.4	34.2	26.8	21.9	18.4
.3	144.0	126.3	118.9	112.1	100.5	90.1	82.7	75.7	64.5	52.3	45.0	39.3	31.0	25.5	21.5
.4	152.3	134.1	126.4	119.5	107.4	97.3	88.0	81.6	69.8	57.0	49.1	43.0	34.2	28.2	23.8
.6	163.5	144.7	136.7	129.5	116.9	106.4	97.4	89.8	77.3	63.6	55.2	48.6	39.0	32.4	27.6
.8	171.0	151.8	143.7	136.3	123.4	112.6	103.4	95.5	82.6	68.4	59.6	52.7	42 5	35.5	30.3
1	176.5	157.1	148.9	141.4	128.3	117.3	107.9	99.9	86.7	72.1	63.0	55.8	45.3	38.0	32.6
1.5	185.9	166.1	157.7	150.1	136.7	125.4	115.8	107.5	93.8	78.6	69.2	61.6	50.5	42.2	36.8
2	191.9	172.0	163.5	155.8	142.2	130.8	121.0	112.6	98.7	83.2	73.4	65.7	54.1	46.0	39.9
3	199.7	179 6	171.0	163.1	149.4	137.9	127.9	119.3	105.2	89.3	79.3	71.2	59.3	50.7	44.3
3.28	*201.2*	*181.1*	*172.5*	*164.6*	*151.0*	*139.3*	*129.4*	*120.7*	*106.5*	*90.6*	*80.5*	*72.4*	*60.4*	*51.7*	45.3
4	204.6	184.4	175.7	167.9	154.1	142.4	132.4	123.7	109.5	93.3	83.2	75.0	62.8	54.0	47.4
6	210.7	190.4	181 7	173.8	160.0	148.2	138.2	129.4	115.0	98.7	88.4	80.1	67.6	58.6	51.8
8	214.5	194.2	185.5	177.6	163.7	152.0	141.8	133.1	118.6	102.2	91.8	83.4	70.8	61.7	54.7
10	217.2	196.9	188.2	180.3	166.4	154.6	144.5	135.7	121.2	104.7	94.3	85.9	73.2	64.0	57.0
12	219.3	198.9	190.2	182.3	168.4	156.6	146.5	137.7	123.1	106.2	96.2	87.8	75.0	65.8	58.8
16	222.2	201.9	193.2	185.2	171.3	159.5	149.4	140.6	126 0	109.5	99.1	90.6	77.8	68.5	61.5
20	224.2	203.9	195.2	187.3	173.3	161.5	151.4	142.6	128.1	111.6	101.1	92.7	79.8	70.5	63.4
30	227.5	207.2	198.5	190 5	176.6	164.8	154.7	145.9	131.4	114.9	104.4	96.0	83.2	73.9	66.8
50	230.8	210.5	201.8	193.9	180.0	168.3	158.2	149.4	134.9	118.4	108.0	99.6	86.4	77.6	70.5
75	233.0	212.7	204.0	196.1	182.3	170.5	160.4	151.6	137.2	120.8	110.4	102.0	89.3	80.1	73.1
100	234.3	214.1	205.4	197.5	183.6	171.9	161.8	153.0	138.6	122.2	111.9	103.5	90.9	81.7	74.7

Practical Examples in Use of Kutter's Formula and Table 8.

(1) Given, the average or mean cross-section of a stream; to find the mean radius r?

Calculate or scale the area a in sq. ft. of the section of flowing water; also, the wetted perimeter p in ft., or transverse length of (sectional) contact of flowing water with the stream bed. Then, $r = \frac{a}{p}$.

(2) Given, a stream, canal, or pipe line, etc., of average uniform section; to find the hydraulic slope s?

s = the average fall in ft. of the stream or canal, per foot of length; or the fall in ft. of any hydraulic grade line, per ft. of length.

(3) Given, the character of a stream, canal, or pipe line; to find the roughness n?

Consult the tabular values, page 1168.

(4) Given, the co-efficient c, mean radius r, and slope s; to find the velocity v?

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From Chezy's formula, $v = c\sqrt{rs}$. Note that the roughness n is not considered directly because c and n are functions of each other; but as c is given, n can be obtained if desired. (The discharge in cu. ft. per sec. $= q = a\,v$, in which $a =$ the area of cross-section of the discharging volume, in sq. ft.)

(5) Given, roughness $n = .014$, mean radius $r = 18$, slope $s = .0003$; to find velocity v?

First, find c from sections 4 and 5 of Table 8: $c = \frac{1}{4}(154.9 + 157.6 + 152.1 + 154.5) = 154.8$. Then $v = c\sqrt{rs} = 154.8 \times 4.243 \times .01732 = 11.38$ ft. per sec.

(6) Given, velocity $v = 1.2$ ft. per sec., mean radius $r = 1.5$, slope $s = .0004$; to find n?

First find c, $= \dfrac{v}{\sqrt{rs}} = \dfrac{1.2}{1.2247 \times .02} = 49.0$. Then from section 5 of preceding Table, on line with $r = 1.5$, we find that a coefficient c of 49.7 corresponds to $n = .030$; $\therefore$ use $n = .030$.

(7) Given, roughness $n = .017$, slope $s = .001$, velocity $v = 2.2$ ft. per sec.; to find the mean radius r?

Use "cut and try" method: assume r in the equation, $r = \dfrac{1}{s} \cdot \dfrac{v^2}{c^2} = 1000\,\dfrac{v^2}{c^2}$. 1st, let $r = 1$, then from section 6 of Table 8, $\dfrac{1}{s} \cdot \dfrac{v^2}{c^2} = 1000 \times \dfrac{(2.2)^2}{(86)^2} = 0.65$; 2nd, let $r = 0.8$, then $\dfrac{1}{s} \cdot \dfrac{v^2}{c^2} = 1000 \times \dfrac{(2.2)^2}{(81.8)^2} = 0.72$; 3rd, let $r = 0.75$, then $\dfrac{1}{s} \cdot \dfrac{v^2}{c^2} = 1000 \times \dfrac{(2.2)^2}{(80.45)^2} = 0.75$. $\therefore r = 0.75$. Checking, $v = c\sqrt{rs} = 80.45 \times 0.866 \times 0.03162 = 2.2$, the given velocity.

(8) Given, mean radius $r = 6$, roughness $n = .020$, and required velocity $v = 3$; to find the slope s?

Use "cut and try" method: assume s in the equation $s = \dfrac{1}{r} \cdot \dfrac{v^2}{c^2} = \dfrac{1}{6} \cdot \dfrac{v^2}{c^2}$. 1st, let $s = .0001$, then from section 6 of Table 8, $\dfrac{1}{r} \cdot \dfrac{v^2}{c^2} = \dfrac{1}{6} \cdot \dfrac{9}{(102.1)^2} = .000144$; 2nd, let $s = .00015$, then $\dfrac{1}{r} \cdot \dfrac{v^2}{c^2} = \dfrac{1}{6} \cdot \dfrac{9}{(101.3)^2} = .00015$. $\therefore s = .00015$. Checking, $v = c\sqrt{rs} = 101.3 \times 2.45 \times .01225 = 3.04$. If greater accuracy is required, assume a new slope slightly less than $s = .00015$ and try again.

The Venturi Meter.—This is an apparatus for determining the rate of discharge of water through pipes, by inserting the meter in the pipe line

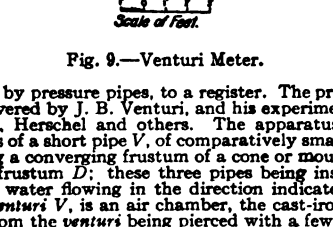

Fig. 9.—Venturi Meter.

connecting it, by pressure pipes, to a register. The principle on which it based was discovered by J. B. Venturi, and his experiments were followed Bossut, Castel, Herschel and others. The apparatus is illustrated in 9, and consists of a short pipe V, of comparatively small diameter, called *venturi*, joining a converging frustum of a cone or mouthpiece M, with a erging conical frustum D; these three pipes being inserted in the pipe PP, with the water flowing in the direction indicated by the arrows. rounding the *venturi* V, is an air chamber, the cast-iron shell separating air chamber from the *venturi* being pierced with a few small holes, care- drilled, leaving sharp, square edges on the inside face of the *venturi*.

Venturi discovered that when water flows through such a converging mouthpiece and a narrow *throat*, V, and then expands into a diverging tube or bell, D, there is a decided *decrease* of water pressure against the inside of the conduit from the beginning of the mouthpiece to the *venturi*, at which point it is a minimum, and then a decided *increase* in pressure from the *venturi* to the end of the diverging tube D. He correctly interpreted the cause of this reduction of pressure at V as due to a partial vacuum at o, just beyond the *venturi*, caused by the jet of water expanding from the throat along the tube D. In fact, not only is the pressure at the *venturi* greatly reduced, as in Fig. 11, but with high velocity of discharge the pressure changes, sometimes, from positive to negative, actually causing a *suction* in the air chamber at the *venturi*.

Mr. Herschel has clearly demonstrated that the differences in pressure at the beginning of the mouthpiece and at the *venturi* increases with the velocity of flow, and has established, by numerous experiments, a relation or relations between the two so that, knowing the difference in pressures at the two points, for a given meter, the discharge can be determined.

Fig. 10. Manometer.

The Meter Register, indicated in Fig. 9, automatically records the discharge directly, in gallons, cubic feet or pounds—per second, minute, hour or day. The register is controlled by the pressures, or difference of pressures, in the pressure pipes leading from the meter.

The Manometer (Fig. 10) is a cheap device and not automatic as a constant recorder. It is a portable instrument that can be attached to any meter tubes on the pipe line and the rate of flow determined by reading the gauge, which has been graduated to "cubic feet per second," or "gallons per day," etc., based on difference of pressure in the connecting pressure tubes.

Piezometer tubes form a very simple device for measuring the pressure and flow in pipe lines and *venturi* meters. These are glass and iron tubes inserted at the required points along the line by "tapping" the pipe so that water will rise in the piezometers to the hydraulic gradient, or line of no pressure. Let PM, PV and PD be glass piezometers inserted at points M, V, and D, Figs. 11 and 12. Each figure shows the hydraulic grade line (1)

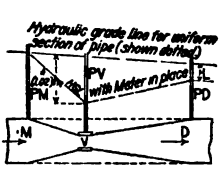

Fig. 11.
Pressure at the *venturi*.

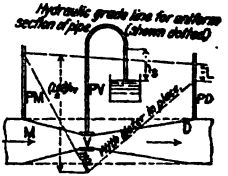

Fig. 12.
Suction at the *venturi*.

for a uniform section of pipe, and (2) with the *venturi* meter inserted in the pipe line. If there is no meter in the pipe line, that is, no contraction of area, the H. G. L. will be a straight line on a falling grade in the direction of the flow, but may be considered as "level" for such a small distance as the length of the meter would occupy, as far as the present discussion is concerned; in other words, the loss of head for this distance would be practically zero. If now the meter is inserted in the pipe line, the H. G. L. will drop with a descending grade from M to V, and with an ascending grade from V to D; but it will be noticed that it does not rise to its former level at D, there being a loss of head L in the flow through the meter from M to D. As the areas of the pipe at the piezometer PM and PD are equal, the velocities at these points are equal, hence the loss of pressure head L represents a total loss of head between those points, no part of it being compensated for by an increase in velocity. On the other hand, the loss of pressure head between M and V, represented by the difference in water levels in the piezometers PM and PV, is nearly all compensated for by the increased velocity of flow

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at V over that at M, or in other words by the increased velocity head h_v, Fig. 11. Experiments on standard meters show that this velocity head represents about 98 per cent of the lost pressure head shown by the piezometers, the other 2 per cent being a total loss.

Where there is negative pressure or suction at the *venturi* (Fig. 12) the piezometer tube is bent in the form of a siphon and the amount of negative pressure head is measured by the height h_s of the column of water in the tube. This distance is laid off *below* the center of the *venturi*, as fixing the lowest point of depression in the hydraulic grade line. The velocity head is determined by the total change of pressure head as in the preceding description, taking into account of course the small percentage of actual loss.

Fig. 13 shows the standard dimensions of the *venturi* meter in terms of the length of the *venturi* V, assumed as unity; and it is claimed that, based

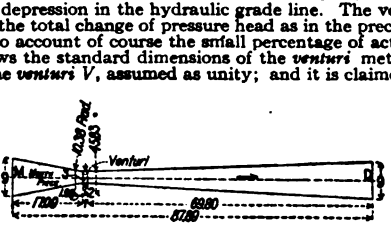

Fig. 13.—Standard Proportions of Venturi Meter.

on these proportions, for such a meter, with the diameter of the pipe ranging from one to nine feet, and with a velocity through the pipe ranging from one-half to six feet per second, the total loss of head in the mouthpiece will be about 2 per cent of the difference in pressure at these points. That is to say, the coefficient of velocity at the *venturi* closely approximates 98 per cent. The standard meter is proportioned with one main point in view, namely, to produce a marked increase in velocity at the *venturi*, with as little total loss as possible in the whole meter. In principle, it shows that if a pipe line is contracted at any point, the pressure head is reduced at that point, being partly converted into velocity head; and conversely, if it is expanded there is a *tendency* toward reconversion of velocity head into pressure head.

Leaving out the velocity of approach, the makers use the following formula for discharge:

$$q = c\,a\sqrt{(1.02 \pm)h_v} \qquad (1)$$

In which q = discharge in cu. ft. per sec., or any other units;
c = a constant or coefficient of velocity, determined by experiment;
a = area of cross-section of *venturi* V, in sq. ft.;
$(1.02 \pm)h_v$ = difference in height of piezometers PM and PV, in ft.;
h_v = increase in velocity head at V over that at M.

Note that $(1.02 \pm)h_v$ is used for simplicity of illustration in keeping clearly in mind the relative value of the difference of velocity heads h_v. It is to be noted also that the *actual* velocity at the throat or *venturi*, V_1, $=\sqrt{2\,g\,h}$ + the velocity of approach, and not simply $\sqrt{2\,g\,h}$.

Orifices, Tubes, Nozzles and Jets.—The *theoretic* discharge from an orifice is obtained by integrating or taking the summation of the discharge of each infinitesimal layer of water of transverse length x, thickness dy, and head y between the proper limiting values of the head. Thus, in computing Table 9, following, we have—

For rectangular orifice, submerged, $$q = \int_{y=h}^{y=H} x\,dy\sqrt{2\,g\,y} \qquad (2)$$

which $x = b$; hence, theoretically,

$$q = \tfrac{2}{3}\,b\,\sqrt{2\,g}\,(H^{\frac{3}{2}} - h^{\frac{3}{2}}) \qquad (3)$$

For rectangular *weirs*, the upper head $h = 0$, $H = d$, $bd = a$; $\therefore q = \tfrac{2}{3}a\,\sqrt{2\,g\,d}$.

Other forms of orifices are calculated in the same manner. The *actual* discharge and velocity are closely approximated by using *coefficients* of area of contraction, velocity and discharge. Let—

c_a = coefficient of contraction of jet;
c_v = coefficient of velocity of jet;
c_q = coefficient of discharge.

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Then, Actual area of issuing jet at its smallest section $= c_a a$; ..... (6)

Actual veloc. of issuing jet at its smallest section $= c_v v = 8.02\, c_v \sqrt{h_0}$; (5)

Actual discharge of issuing jet $= c_q q = 8.02\, c_a a\, c_v \sqrt{h_0}$; ..... (6)

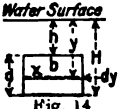
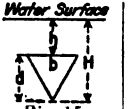
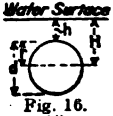
Coefficient of discharge $= c_q = c_a c_v$ ..... (7)

The values of c_a, c_v and c_q for the various shapes and conditions of orifices, nozzles, jets, etc., are determined by experiment. The coefficient of contraction of area, c_a, for small standard orifices may be assumed at about 0.63; the coefficient of velocity, c_v, at about 0.98; and the coefficient of discharge $c_q = c_a c_v = 0.63 \times 0.98 = 0.62$. These values of the coefficients substituted in equations (4), (5) and (6) will give, approximately, the results which may be expected in practice.

Table 9, following, shows the discharge, q, from rectangular, triangular and circular orifices:

9.—TABLE OF DISCHARGES FROM ORIFICES.

(a) Orifices Submerged.

	Function. (1)	Water Surface  Fig. 14. (2)	Water Surface  Fig. 15. (3)	Water Surface  Fig. 16. (4)
a	Area.....	bd	$\frac{bd}{2}$	$\frac{\pi d^2}{4}$
q	Discharge.	$c_q \frac{2}{3} b \sqrt{2g}(H^{\frac{3}{2}} - h^{\frac{3}{2}})$	Not important *	$c_q \pi r^2 \sqrt{2gH}\left(1 - \frac{r^2}{32H^2} - \frac{5r^4}{1024H^4} - \frac{105r^6}{65536H^6} \cdots\right)$

And when the tops of the orifices touch the surface of the water, h becomes zero and we have, in each case, the following values:

(b) Water Surface at Tops of Orifices.

q	Discharge.	$c_q \frac{2}{3} a \sqrt{2g\, d}$	$c_q \frac{4}{15} b\, d \sqrt{2g\, d}$	$c_q \pi r^2 \sqrt{2g\, r}\,(0.96)$

$$* \; q = c_q \frac{b}{d}\sqrt{2g}\left(\frac{H^{\frac{1}{2}}\, d^2}{2} - \frac{H^{-\frac{1}{2}}\, d^3}{6} - \frac{H^{-\frac{3}{2}}\, d^4}{32} - \frac{H^{-\frac{5}{2}}\, d^5}{80} \cdots\cdots\right).$$

COMPARISON OF ORIFICES AND TUBES.

The *standard orifice* (Fig. 17) has sharp, vertical edges. From data above $c_a = 0.625$; $c_v = 0.98$; $c_q = 0.6125$. The discharge can be increased by inserting the standard tube.

The *standard tube* (Fig. 18) is made just long enough so the expanding

Fig. 17.

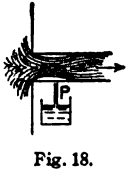


Fig. 18.

Fig. 19.

Fig. 20.

Fig. 21.

jet completely fills its extremity. Around the contracted portion of the jet there is a partial vacuum, which may be tested by tapping the tube and inserting a glass piezometer, P, with its lower end immersed in a basin of water. The water will rise in the piezometer, indicating suction. The vacuum is produced by the particles of water or spray of the jet forcing the

air out of the tube. The effect is to increase the velocity and discharge, over that of the standard orifice. The values of c_v and c_c are about 0.81 or 0.82, average.

The *conical nozzle* (Fig. 19) is designed to increase the energy of the jet. The angle of divergence of the sides of the cone is about 13° or 14° for maximum discharge, when $c_v = 0.95$, nearly. Sometimes the outer end of the cone is provided with a short straight tube. The coefficient of velocity increases with the angle of the cone.

The *Venturi meter* is simply a compound tube (Fig. 20) and is explained on page 1173, etc.

The *flaring or bell-shaped mouthpiece* (Fig. 21) decreases the loss of head at entrance, usually called Entry Head (see page 1160) and is recommended for use at the intake end of pipe lines.

Weirs.—The weir is simply a water meter. Next to the Venturi meter (see page 1173) the standard weir is probably the most accurate device for measuring the flow of water. It is practically adapted to gaging the flow of small streams, creeks, canals, ditches, and the discharge from pipe lines and sewers.

The standard weir is a rectangular, vertical opening through which the surface water is allowed to discharge. The edges of the opening should be chisel-edged, the "crest" perfectly level, and the sides perfectly plumb. Fig. 22 represents a section of a standard weir with the water flowing over the crest. Note that the surface of the discharging volume describes a curve from a point of tangency T, distant D from the crest of the dam, the drop at the crest being s. Fig. 23 is a plan of the weir showing the length of crest l, and the side contractions e and e' of the flow, for the "end contractions" E and E'. The standard weir is a weir with end contractions; the weir without end contractions is called the "suppressed" weir, because the end contractions are suppressed.

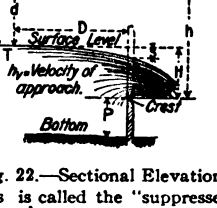


Fig. 22.—Sectional Elevation.

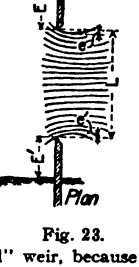


Fig. 23.

The *theoretic* discharge through a rectangular weir, neglecting velocity of approach, is obtained from the equations on pages 1175 and 1176, reducing to—

$$q = \tfrac{2}{3} L \sqrt{2g}\, H^{\frac{3}{2}} \text{ (no velocity of approach)} \quad \ldots\ldots (8)$$

in which q = discharge in cu. ft. per second;
L = length of weir, in feet (Fig. 23);
H = "surface level" head on crest, in feet (Fig. 22).

If it is desired to include velocity of approach,* v_a $(= \sqrt{2g\, h_a})$, this value must be inserted in equation (2), page 1175, before integrating; thus,

$$q = \int_{y=h}^{y=H} x\, dy \sqrt{2g\, y + v_a^2} \quad \ldots\ldots (9)$$

whence $q = \tfrac{2}{3} L \sqrt{2g}[(H+h_a)^{\frac{3}{2}} - (h+h_a)^{\frac{3}{2}}]$ (including velocity of approach) .(10)
in which h_a = velocity head of approach; and $h = 0$.

It is to be noted that these formulas, (8) and (10), are purely theoretical, and that little or no importance can be attached to them as they stand. But like all theoretic expressions, each forms a skeleton or groundwork upon which to build the more or less "emperical" formulas which are used in practice. The practical formulas are deduced from the above by supplying the proper coefficients, which are determined by experiments.

---

*The velocity of approach should be measured at the point T, Fig. 22. It should comprise generally the average velocity for some depth below the surface and not the "surface" velocity, which is often used; the latter may be close enough, however, in many instances. The surface velocity may be obtained by the surface-float method (Fig. 29); the average velocity, by the integrating float rod (page 1183), or by the current meter (page 1185), or by the Pitot tube (page 1183).

(a) Francis' Weir Formulas.

General formula applicable to any sharp-crested, surface weir:

$$q = 3.33\,(L - 0.1\,n\,H)\,[(H + h_v)^{\frac{3}{2}} - h_v^{\frac{3}{2}}] \quad \ldots\ldots (11)$$

in which q = discharge in cu. ft. per second;
L = length of weir in feet;
n = number of end contractions, as 0, 1 or 2;
H = "surface level" head on crest, in feet;
h_v = velocity head in feet $= \dfrac{\text{velocity of approach } v_o}{2g}$.

Formula (11) may be simplified to meet the following special cases:
With velocity of approach—

Contracted { Both ends: $q = 3.33\,(L - 0.2H)\,[(H + h_v)^{\frac{3}{2}} - h_v^{\frac{3}{2}}]$ ..... (12)
Contracted { One end: $q = 3.33\,(L - 0.1H)\,[(H + h_v)^{\frac{3}{2}} - h_v^{\frac{3}{2}}]$ ..... (13)

Suppressed: $q = 3.33\,L\,[(H + h_v)^{\frac{3}{2}} - h_v^{\frac{3}{2}}]$ ..... (14)

Without velocity of approach (i. e., $h_0 = 0$)—

Contracted { Both ends: $q = 3.33\,(L - 0.2\,H)\,H^{\frac{3}{2}}$ ..... (15)
Contracted { One end: $q = 3.33\,(L - 0.1\,H)\,H^{\frac{3}{2}}$ ..... (16)

Suppressed: $q = 3.33\,H^{\frac{3}{2}}$ ..... (17)

The above formulas are very extensively used in the United States. They were deduced by Mr. J. B. Francis, in 1854, from very elaborate experiments which he conducted at Lowell, Mass., on large weirs about 10 feet long. The heads ranged usually from 0.4 to 1.6 feet. The formulas are simple, logical and fairly reliable within the range of the experiments. The heads H were measured 6 ft back from the crest (Fig. 22).

(b) Bazin's Weir Formula.

General formula applicable to sharp-crested, suppressed, surface weirs is

$$^{*}q = \left(0.405 + \frac{0.00984}{H}\right)\left[1 + 0.55\left(\frac{H}{H+p}\right)^2\right] L\,H\sqrt{2g\,H} \quad \ldots\ldots (18)$$

in which p = height of crest above the bottom (Fig. 22) and the other notation as per Francis' formulas, preceding. This formula (18) takes into consideration the velocity of approach in the height of crest p, hence the former does not require any special treatment when p is known. It should be remarked that the formula was deduced from very careful experiments with standard weirs from 0.656 ft. (0.2 meter) to 6.56 ft. (2 meters) in length and with heads ranging from 2.50 to 1.00 ft., the lower heads for the longer weirs. Care should be used in extending the use of the formula beyond the above range for length L and head H. It should be noted that Bazin measured the head H at points distant $D = 16.4$ ft. back from the crest. (See Fig. 22.)

The above general formula (18) can be simplified for practical use by letting

$$m = \left(0.405 + \frac{0.00984}{H}\right)\left[1 + 0.55\left(\frac{H}{H+p}\right)^2\right]$$

hence $$q = m\,L\,H\sqrt{2g\,H} = m\sqrt{2g}\,L\,H\sqrt{H} \quad \ldots\ldots (19)$$

Either of these two forms (19) may be used in connection with the following table which gives the values of m (and of $m\sqrt{2g}$) for given values of p and H.

The second form, $q = (m\sqrt{2g})\,L\,H\sqrt{H}$ will probably be found more convenient as the table gives average values of $(m\sqrt{2g})$ for each five successive values of H, and intermediate values may be interpolated. If extreme accuracy is required the first form is preferred, remembering that $\sqrt{2g\,H} = 8.02\sqrt{H}$. Note that the last column in Table 10 gives the limiting value of m for $p = \infty$, whence $m = \left(0.405 + \dfrac{0.00984}{H}\right)$, the first part of the coefficient, as the latter part reduces to unity. Hence this value of m may be used when there is no velocity of approach, that is, when p is very large.

* The standard weir provides for free admission of air under the falling sheet of water.

10.—VALUES OF THE COEFFICIENT m IN THE FORMULA $q = m\,L\,H\,\sqrt{2gH}$, FOR A SHARP-CRESTED WEIR WITHOUT LATERAL CONTRACTION, THE AIR BEING ADMITTED FREELY BENEATH THE OVERFLOWING SHEET OF NAPPE.

Observed Head = H in Feet.	Values of the coefficient m corresponding to the height p of the weir above the bottom of the channel.									Limiting Value of m for $p = \infty$.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Value of p, in Feet.	0.656	0.984	1.312	1.640	1.968	2.624	3.280	4.920	6.560	∞
0.164	0.458	0.453	0.451	0.450	0.449	0.449	0.449	0.448	0.448	0.4481
0.197	0.456	0.450	0.447	0.445	0.445	0.444	0.443	0.443	0.443	0.4427
0.230	0.455	0.448	0.445	0.443	0.442	0.441	0.440	0.440	0.439	0.4391
0.262	0.456	0.447	0.443	0.441	0.440	0.438	0.438	0.437	0.437	0.4363
0.295	0.457	0.447	0.442	0.440	0.438	0.436	0.436	0.435	0.434	0.4340
Means	0.456	0.449	0.446	0.444	0.443	0.442	0.441	0.441	0.440	0.4400
$m\sqrt{2g}$	3.66	3.60	3.58	3.56	3.55	3.54	3.54	3.53	3.53	3.53
0.328	0.459	0.447	0.442	0.439	0.437	0.435	0.434	0.433	0.433	0.4322
0.394	0.462	0.448	0.442	0.438	0.436	0.433	0.432	0.430	0.430	0.4291
0.459	0.466	0.450	0.443	0.438	0.435	0.432	0.430	0.428	0.428	0.4267
0.525	0.471	0.453	0.444	0.438	0.435	0.431	0.429	0.427	0.426	0.4246
0.591	0.475	0.456	0.445	0.439	0.435	0.431	0.428	0.426	0.425	0.4229
Means	0.467	0.451	0.443	0.438	0.436	0.432	0.431	0.429	0.428	0.4271
$m\sqrt{2g}$	3.74	3.62	3.56	3.52	3.50	3.47	3.46	3.44	3.44	3.43
0.656	0.480	0.459	0.447	0.440	0.436	0.431	0.428	0.425	0.423	0.4215
0.722	0.484	0.462	0.449	0.442	0.437	0.431	0.428	0.424	0.423	0.4203
0.787	0.488	0.465	0.452	0.444	0.438	0.432	0.428	0.424	0.422	0.4194
0.853	0.492	0.468	0.455	0.446	0.440	0.432	0.429	0.424	0.422	0.4187
0.919	0.496	0.472	0.457	0.448	0.441	0.433	0.429	0.424	0.422	0.4181
Means	0.488	0.465	0.452	0.444	0.438	0.432	0.428	0.424	0.422	0.4196
$m\sqrt{2g}$	3.92	3.73	3.63	3.56	3.52	3.46	3.44	3.40	3.39	3.37
0.984	0.500	0.475	0.460	0.450	0.443	0.434	0.430	0.424	0.421	0.4147
1.050	0.500	0.478	0.462	0.452	0.444	0.436	0.430	0.424	0.421	0.4168
1.116	0.500	0.481	0.464	0.454	0.446	0.437	0.431	0.424	0.421	0.4162
1.181	0.500	0.483	0.467	0.456	0.448	0.438	0.432	0.424	0.421	0.4156
1.247	0.500	0.486	0.469	0.458	0.449	0.439	0.432	0.424	0.421	0.4150
Means	0.500	0.481	0.464	0.454	0.446	0.437	0.431	0.424	0.421	0.4162
$m\sqrt{2g}$	4.01	3.86	3.73	3.64	3.58	3.50	3.46	3.40	3.38	3.34
1.312	0.500	0.489	0.472	0.459	0.451	0.440	0.433	0.424	0.421	0.4144
1.378	0.500	0.491	0.474	0.461	0.452	0.441	0.434	0.425	0.421	0.4139
1.444	0.500	0.494	0.476	0.463	0.454	0.442	0.435	0.425	0.421	0.4134
1.509	0.500	0.496	0.478	0.465	0.456	0.443	0.435	0.425	0.421	0.4128
1.575	0.500	0.496	0.480	0.467	0.457	0.444	0.436	0.425	0.421	0.4122
Means	0.500	0.493	0.476	0.463	0.454	0.442	0.435	0.425	0.421	0.4133
$m\sqrt{2g}$	4.01	3.96	3.82	3.72	3.64	3.55	3.49	3.41	3.38	3.32
1.640	0.500	0.496	0.482	0.468	0.459	0.445	0.437	0.426	0.421	0.4118
1.706	0.500	0.496	0.483	0.470	0.460	0.446	0.438	0.426	0.421	0.4112
1.772	0.500	0.496	0.485	0.472	0.461	0.447	0.438	0.426	0.421	0.4107
1.837	0.500	0.496	0.487	0.473	0.463	0.448	0.439	0.427	0.421	0.4101
1.903	0.500	0.496	0.489	0.475	0.464	0.449	0.440	0.427	0.421	0.4096
1.969	0.500	0.496	0.490	0.476	0.466	0.451	0.441	0.427	0.421	0.4092
Means	0.500	0.496	0.486	0.472	0.462	0.448	0.436	0.427	0.421	0.4104
$m\sqrt{2g}$	4.01	3.98	3.90	3.79	3.71	3.60	3.50	3.42	3.38	3.29

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Problem.—What is the discharge through a weir whose crest is 7 ft. long and 2.5 ft. above the bottom; the measured head, H, being 0.75 ft.?

Solution.—The value of m in the preceding table for $p = 2.5$ and $H = 0.75$, is 0.433; then from first equation (19), $q = 0.433 \times 8.02 \times 7 \times 0.75 \times 0.866 = 15.8$ cu. ft. per second. *Ans.*

(c) Fteley and Stearns' Weir Formulas.

General formula for sharp-crested, suppressed, surface weirs, as

$$\left.\begin{aligned} q &= 0.4125\, L\, H \sqrt{2g\, H} + 0.007\, L \\ &= 3.31\, L\, H \sqrt{H} + 0.007\, L \end{aligned}\right\} \text{No velocity of approach} \ldots \quad (20)$$

and, if there is velocity of approach,

$$q = 3.31\, L\, (H + 1.5\, h_v) \sqrt{H + 1.5\, h_v} + 0.007\, L \left\{\begin{array}{l}\text{Including velocity of} \\ \text{approach.}\end{array}\right\} . \quad (21)$$

Use notation for Francis' formula, page 1178. The above formulas were deduced from experiments which they made in Boston, Mass., 1877–9, with weirs 5 to 19 ft. long and under heads ranging from 0.065 up to 1.6 feet. The heads H were measured 6 ft. back from the crest.

(d) *Parmley's Weir Formula.

General formula for sharp-crested, surface weirs, either contracted or suppressed, taking into account velocity of approach, if any:

$$q = C' K [L - 0.1\, n\, H]\, H^{\frac{3}{2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (22)$$

in which q = discharge in cu. ft. per second;
L = length of weir, in ft.;
n = number of end contractions, as 0, 1 or 2;
H = "surface level" head on crest, in ft.;
C' = constant for given value of H, from Table 11;
K = constant for given value of $\frac{a}{A}$, from Table 12;
$a = (L - 0.1\, n\, H)\, H$ = contracted area of discharge;
A = area of water section in channel of approach.

11.—Values of Coefficient C' for Given Values of H. (Equation 22.)

H	C'	H	C'	H	C'	H	C'	H	C'
Ft.		Ft.		Ft.		Ft.		Ft.	
0.10	3.580	0.40	3.385	0.70	3.351	1.00	3.334	1.60	3.301
0.15	3.520	0.45	3.376	0.75	3.349	1.10	3.329	1.70	3.296
0.20	3.478	0.50	3.368	0.80	3.346	1.20	3.324	1.80	3.290
0.25	3.444	0.55	3.362	0.85	3.343	1.30	3.319	1.90	3.285
0.30	3.420	0.60	3.358	0.90	3.340	1.40	3.313	2.00	3.280
0.35	3.400	0.65	3.354	0.95	3.337	1.50	3.307	.....	.....

12.—Values of Coefficient K for Given Values of H. (Equation 22.)

$\frac{a}{A}$	K	$\frac{a}{A}$	K	$\frac{a}{A}$	K	$\frac{a}{A}$	K	$\frac{a}{A}$	K
0.01	1.0001	0.13	1.0093	0.25	1.0344	0.37	1.0753	0.49	1.1321
0.02	1.0002	0.14	1.0108	0.26	1.0372	0.38	1.0794	0.50	1.1375
0.03	1.0005	0.15	1.0124	0.27	1.0401	0.39	1.0837	0.51	1.1431
0.04	1.0009	0.16	1.0141	0.28	1.1431	0.40	1.0880	0.52	1.1487
0.05	1.0014	0.17	1.0159	0.29	1.0463	0.41	1.0925	0.53	1.1545
0.06	1.0020	0.18	1.0178	0.30	1.0495	0.42	1.0970	0.54	1.1604
0.07	1.0027	0.19	1.0198	0.31	1.0529	0.43	1.1017	0.55	1.1664
0.08	1.0035	0.20	1.0220	0.32	1.0563	0.44	1.1065	0.56	1.1725
0.09	1.0044	0.21	1.0243	0.33	1.0599	0.45	1.1114	0.57	1.1787
0.10	1.0055	0.22	1.0266	0.34	1.0636	0.46	1.1164	0.58	1.1850
0.11	1.0066	0.23	1.0291	0.35	1.0674	0.47	1.1215	0.59	1.1915
0.12	1.0079	0.24	1.0317	0.36	1.0713	0.48	1.1267	0.60	1.1980

* Discussion by W. C. Parmley, Trans. Am. Soc. C. E., Vol. XLIV, p. 351.

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Parmley's formula (22) was not deduced from any particular set of experiments, but is based on the experiments and formulas of Bazin, Francis, and Fteley and Stearns. The result obtained is a formula comprehensive in character and giving *average* values. Bazin's experiments were made very carefully under ideal conditions in a long, smooth canal lined with cement, while those of Francis and Fteley and Stearns were conducted under conditions more nearly approaching those to be found in practice. The roughness of the stream or canal must necessarily produce an appreciable effect on the flow.

Triangular and trapezoidal weirs have been proposed in place of the standard, rectangular section, on account of the very slight variation of the coefficient of discharge for different heads H. But they have not yet come into practical use.

The Submerged Weir, so-called because the water level below the weir rises higher than the crest, is shown in Fig. 24.

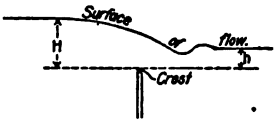

Fig. 24.—Submerged Weir.

(*a*) Fteley and Stearns' formula* for submerged weirs, without end contractions is,

$$q = m\,L\left(H + \frac{h}{2}\right)(H - h)^{\frac{1}{2}} \qquad (23)$$

in which q = discharge in cu. ft. per second;
H = up-stream head on crest, in feet;
h = down stream head on crest, in feet;
L = length of weir, in feet;

m = coefficient for given values of $\frac{h}{H}$, as in the following Table.

13.—Values of Coefficient m for Given Values of $\frac{h}{H}$.

(Equation 23.)

$\frac{h}{H}$	m	$\frac{h}{H}$	m	$\frac{h}{H}$	m	$\frac{h}{H}$	m	$\frac{h}{H}$	m
0.00	3.23	0.12	3.35	0.30	3.21	0.60	3.09	0.90	3.19
0.04	3.35	0.16	3.32	0.40	3.15	0.70	3.09	1.00	3.33
0.08	3.37	0.20	3.28	0.50	3.11	0.80	3.12	.....	.....

(*b*) Herschel's formula† for submerged weirs, based on experiments ıde by Francis and by Fteley and Stearns, is as follows:

$$q = 3.33\,L\,(c\,H)^{\frac{3}{2}} \qquad (24)$$

which c = coefficient for given values of $\frac{h}{H}$ in the following Table, the ance of the notation same as for the Fteley and Stearns' formula, preced-

* Trans. Am. Soc. C. E., Vol. XII, page 103.
† Discussion by Clemens Herschel, Trans. Am. Soc. C. E., Vol. XIV. e 194.

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14.—COEFFICIENTS c FOR GIVEN VALUES OF $\frac{h}{H}$. (Fig. 24.)

(Equation 24.)

$\frac{h}{H}$	c	$\frac{h}{H}$	c	$\frac{h}{H}$	c	$\frac{h}{H}$	c	$\frac{h}{H}$	c
0.00	1.000	0.18	0.989	0.38	0.935	0.58	0.856	0.78	0.722
.01	1.004	.20	0.985	.40	0.929	.60	0.846	.80	0.703
.02	1.006	.22	0.980	.42	0.922	.62	0.836	.82	0.681
.04	1.007	.24	0.975	.44	0.915	.64	0.824	.84	0.659
.06	1.007	.26	0.970	.46	0.908	.66	0.813	.86	0.634
.08	1.006	.28	0.964	.48	0.900	.68	0.799	.88	0.606
.10	1.005	.30	0.959	.50	0 892	.70	0.787	.90	0.574
.12	1.002	.32	0.953	.52	0.884	.72	0.771	.92	0.54
.14	0.998	.34	0.947	.54	0.875	.74	0.755	.96	0.45
.16	0.994	.36	0.941	.56	0.866	.76	0 738	1.00	0.000

Hydraulic Measurements. — Measurements of flowing water may be made by several methods, depending upon the circumstances and requirements of the particular case:

(1) By tank measurement;
(2) By venturi meter;
(3) By weir measurement;
(4) By pitot-tube meter;
(5) By floats;
(6) By current meters.

These are discussed in the order named.

Tank measurement.—This method is the most exact and can be used where the discharge is small. The discharge from a pressure-pipe line leading to a high service reservoir may be measured by using the latter as a tank and shutting off the outlet while it is being filled. The capacity of the reservoir must be determined accurately, or it may be calibrated to gauge readings above the bottom.

Venturi meter.—This is described fully on page 1173, etc. This meter is particularly adapted to measuring the discharge in pipes, and can be had in sizes up to 6 ft. or more in diameter. Results can be obtained usually within 2 or 3 per cent of the actual discharge. In ordering, it is necessary to state the diameter of the pipe and the average velocity of discharge so that the cones and *venturi* may be proportioned correctly, for accurate results. The velocity through the *venturi* must be accelerated sufficiently to cause a marked decrease in pressure, and this is done by giving the diameter of the *venturi* the proper ratio to that of the main pipe. It should be large for high velocities and small for low velocities. A register should accompany each meter.

Weir measurement.—Weir formulas are discussed on page 1177, etc. In constructing a standard weir it is necessary to have the crest perfectly level and the sides vertical. The inner edges of the opening should be sharp and chisel-edged or square cornered. If square cornered the boards or partition should be thin or they may be beveled on the downstream side, thus, . The crest is often formed of a thin sheet of metal fastened on the inside of the wooden partition. Special care must be taken to insure free access of air under the falling sheet of water below (down stream from) the crest, otherwise a partial vacuum will form there, draw the sheet inward toward the weir, and affect the discharge. In measuring the head H on the crest, it is necessary to take the elevation of the water-surface at a sufficient distance D (Fig. 22) above the weir to reach the still-water level, or practically so. Francis used $D = 6$ ft., while

Fig. 25.

Bazin used $D = 16.4$ ft. Hence the distance D may depend somewhat on the formula to be used (see pages 1177 and 1178). There are three methods in use for determining the head H. One is by setting a reading gage in the stream, using the elevation of crest as the datum plane. The second method consists in suspending a plumbbob on the end of a steel tape supported from some point E, Fig. 22, at a known elevation h above the crest, and measuring the vertical distance d to the water surface. Then the required head $H = h - d$. In order to get the measurement d accurately, the plumb-bob may be allowed to swing gently, and raised until it just ceases to cause a ripple on the water surface. The third method is by the Hook Gage, shown in Fig. 25. The point of the hook at the bottom of the rod (the hook may be attached to a leveling rod, reading to thousandths of a ft.) is raised until it pierces the "skin" of the surface, raising a slight pimple. The hook is then lowered until the pimple "just" disappears. The elevation is read by the vernier, which should be set at zero when the end of the hook is at the elevation of the crest of the weir.

Pitot tube meter.—The Pitot tube, in its primitive conception, consists essentially of a bent tube (of glass) inserted in a current of water, Fig. 26, with the open end of the lower arm squarely facing the current. Then, theoretically, will the water rise in the tube to a height $h_v = \frac{v^2}{2g}$, in which v = velocity *at that particular point* in ft. per second;

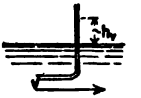

Fig. 26.

and h_v = velocity head in ft. due to v. Hence, by measuring the height h_v of the column of water in the tube, the velocity $v (= \sqrt{2g\,h_v})$ is obtained.

For measuring the velocity of flow under pressure, as in pipes, the Pitot tube, complete, comprises two pipes, crudely shown in Fig. 27. One of these pipes terminates as in Fig. 26, and records the velocity head h_v + the pressure head h_p. The other terminates as a piezometer tube in an orifice at right angle to the direction of flow, and records the pressure head h_p alone. It is clearly evident, then, that the difference in elevation of the water levels in the tubes is the velocity head h_v and that, theoretically, the velocity $v = \sqrt{2g\,h_v}$ as above.

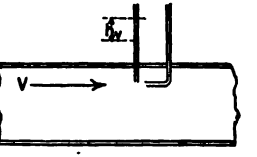

Fig. 27.

Fig. 28 shows one of the forms of Pitot tubes used in the Detroit experiments,* to receive the velocity and pressure heads. For accurate reading, the tops of both of the tubes (Fig. 27) are connected by pipes to a differential gauge consisting of two parallel glass tubes with a sliding scale between. The difference in elevation is reduced to h_v as above, or to any other equivalent expression. Mercurial or oil gauges may be used. (See Trans. Am. Soc. C. E., Vol. XLVII, pp. 72–3).

Floats.—These are classed as *surface floats*, *sub-surface floats* and *rod floats*.

Surface floats (Fig. 29) are the least accurate. Prof. Dwight Porter of the Mass. Inst. Tech. says: "These are of little value when run alone, since they are easily affected by wind, and the relation between surface and mean velocity is uncertain. As a rough approximation, the mean velocity in a vertical may be assumed as from 0.9 to 1.0 times the surface velocity in a vertical, and the mean velocity for the entire cross-section of the stream as about 0.8 times the maximum surface velocity."

Sub-surface floats consist of small cylindrical boxes, say 8 or 9 ins. in diameter, and weighted at the bottom section with a small cube of lead. These are suspended from small surface floats to any depth at which it is required to measure the velocity.

Rod floats are the most exact. They are particularly adapted to measuring the flow in canals or in streams where the depth of water is fairly constant. The length of rod should be equal to nearly the depth of the water, should project slightly above the surface and reach nearly to the bottom. They are used frequently in depths exceeding even 30 feet. The advantage of the road float is that the *mean* velocity in the vertical is obtained directly

* See Trans. Am. Soc. C. E., Vol. XLVII, p. 12.

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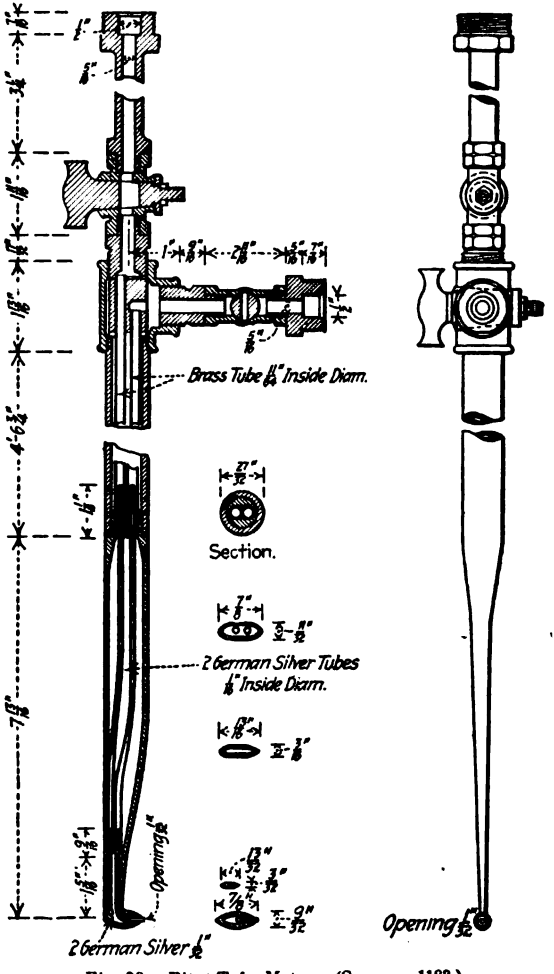

Fig. 28.—Pitot Tube Meter. (See page 1183.)

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for any vertical section of the stream or canal in which it is allowed to float at a "timed" rate between two cross-sections of the stream at a given distance apart. The floats may be made of long, tin cylinders about 2 to 2½ ins. in diameter, loaded at the bottom with lead, accurately weighted or adjusted with pebbles, and closed at the top with cork. In measuring the discharge in a canal, the cross-section of the latter is divided into vertical strips of given area, the mean velocity for each of these strips is determined by the rod float, which velocity multiplied by its area gives the discharge for that strip. The total discharge divided by the total area of cross-section gives the mean velocity. The rods are floated through the middle of the strips at the dotted lines shown in Fig. 30, which represents a section of the canal.

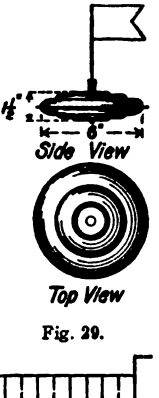

Fig. 29.

Current meters.—Current meters may be expected to give results with error ranging from 3 to 10 per cent. The greatest accuracy will be obtained in canals with a moderately high velocity of flow; the least accuracy in turbulent streams with cross currents, also when the velocity of the water is very slight, or when it contains suspended matter, as in sewers.

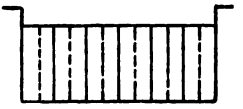

Fig. 30.

There are two principal types of current meters, namely, the cup-wheel meter and the propeller-wheel meter. Fig. 31 illustrates the former type and Fig. 32 the latter. The turns of the wheel which indicate the velocity of the water are registered by an electric meter similar to that shown in Fig. 33. All meters should be tested or rated before being used, as they are sometimes subject to fluctuations which may seriously affect the result. The meter is rated by moving it through still water at given speeds, plotting the results on cross-section paper, with "revolutions per minute" as abscissa, and "velocity in feet per second" as ordinates. The curve drawn through the plotted points is the "rating line" for the meter for *immediate* use.

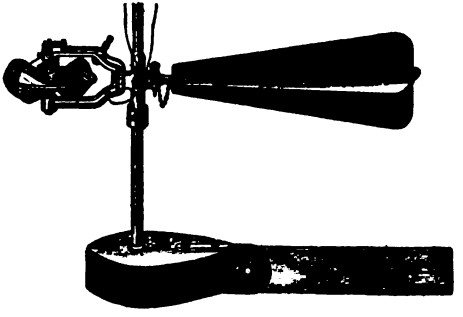

Fig. 31.—Price Meter.

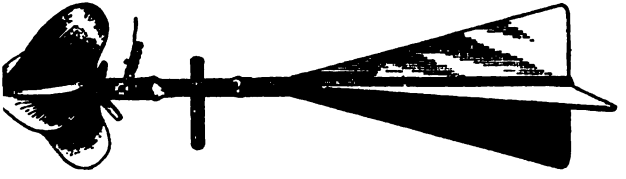

Fig. 32.—Haskell Meter.

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The current meter may be used in two ways, namely, by the "
method and by the vertical "integration" method. The former is prefe
because more accurate, and consists in holding the meter in a given po
or point of the cross-section of the stream, that is, the center of a given
for a definite length of time and noting the register of the meter for
interval. The velocity per second and the discharge for that area ca
obtained, and likewise for the other areas. The total discharge of the st
divided by the total area of cross-section gives the mean velocity, i.e.
integration method is quicker but somewhat less accurate unless very ca

Fig. 33.—Meter Register.

fully done. It consists in lowering the meter from the surface of the water to the bottom and then back again to the surface through the center of each given vertical strip as shown by the dotted lines, Fig. 30. The movement must be slow and constant and the reading of the Register must be for the actual time consumed, beginning and ending with the caps or vanes of the meter well under the surface. The result deduced is the velocity and discharge for that particular vertical strip. The other strips are treated in the same manner, and the resulting mean velocity obtained.

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EXCERPTS AND REFERENCES.

Instructions for Installing Weirs, Measuring-Flumes and Water Registers (By C. T. Johnston, and Elwood Mead, Paper, U. S. Dept. of Agriculture, and Bulletin 86, Irrigation Investigations; Eng. News, Aug. 29, 1901).—Illustrations: Fig. 1, arrangement of Cippoletti weir; Fig. 2, measuring flume and register; Figs. 3 and 4, arrangement of pulleys to magnify the record of water registers; Figs. 3a and 4a, arrangement of pulleys to reduce record of water registers; Fig. 5, the Mead stream heights or water register; Fig. 6, the Friez stream heights or water register; Fig. 7, the Leitz stream heights or water register; Fig. 8, the standard stream heights or water register. Descriptions and discussions.

Measurement of the Flow of Water in the Sudbury and Cochituate Aqueducts (By W. W. Patch. Eng. News, June 12, 1902).—Illustration of current meter apparatus; also diagrams.

Current Meter and Weir Discharge Comparisons (By E. C. Murphy. Trans. A. S. C. E., Vol. XLVII).

The Effect of Long Lengths of Hose on Fire Streams (By S. A. Charles. Paper, Am. W. W. Assn., June 12, 1902; Eng. News, July 3, 1902).—Diagram showing heights, volumes and velocities of fire streams for various lengths of hose, size of nozzle and pounds of hydrant or steamer and nozzle pressure—from experiments by S. A. Charles, combined with those of J. R. Freeman. See discussions in Eng. News of July 17 and 31, and Dec. 3, 1902.

The Flow of Water in Wood Pipes (By Theron A. Noble. Trans. A. S. C. E., Vol. XLIX).

Methods of Measuring the Flow of Streams (By J. C. Hoyt. Eng. News, Jan. 14, 1904).—Interesting diagram showing vertical velocity curves for Susquehanna River; also table of velocity determinations. See, also, Eng. News, of Aug. 4, 1904.

Notes on the Computation of Stream Gagings (By O. V. P. Stout. Paper, Sec. of Eng. and Mech., 12th Natl. Irrig. Cong., Nov. 15 to 18, 1904; Eng. News, Dec. 8, 1904).—Formulas and diagrams.

The Hydraulic Plant of the Puget Sound Power Company (By E. H. Warner. Trans. A. S. C. E., Vol. LV).—Table 1, Make-up of steel pipe; Table 3, Experiments on the hydraulics of the timber flume.

Depth of Thread of Mean Velocity in Rivers (By F. W. Hanna. Eng. News, Jan. 11, 1906).—In the measurement of the flow of rivers it is often assumed that the thread of mean velocity lies at approximately 0.6 of the total depth from the surface, varying for extreme cases between the limits of 0.5 and 0.7. There seems to be a general impression that this assumption is not capable of theoretical demonstration. Mr. Hanna proceeds to demonstrate "rationally" that these values are correct, from the assumption that the vertical velocity curve is sensibly a parabola.

An Experiment to Determine "N" in Kutter's Formula (By C. W. Babb. Eng. News, Feb. 1, 1906).—In connection with the work of the Reclamation Service of the St. Mary project in Montana, the values of "n" n earth canal were found to be: .027, .024, .023, .023, .021; average .0236, or practically .025.

Some Experiments on the Frictionless Orifice (By Horace Judd and R. S. King. Paper, Am. Assn. for the Advan. of Science, July, 1906; Eng. News, Sept. 27, 1906).—Diagrams and tables.

Additional Information on the Durability of Wooden Stave Pipe By A. L. Adams. Trans. A. S. C. E., Vol. LVIII).—Refers to the 7½ miles f wood-stave pipe laid for the Astoria city water works ten years previously. As a result of examination of the pipe line, the following conclusions are rawn: (1) Staves which are constantly subject to water pressure from within and are buried in the ground, may be very short-lived. (2) The magnitude of the water pressure, beyond a moderate head, has but little r no influence in preserving the timber. (3) The pipe laid above ground as not deteriorated to any considerable extent, nor has the pipe laid in the trenches leading from the distributing reservoir. (4) Where buried, its durability has depended upon the soil conditions and the depth of backfill. (5) When the depth of backfill has exceeded 2 ft. above the pipe, and the material has been free from vegetable matter, and has been of a fine

and impervious character, much less deterioration has taken place. (6) Whenever the staves have been in contact with loamy earth or earth containing vegetable matter, or wherever they have been covered with porous material, or to a depth less than 2 ft., rapid decay has resulted. (7) Decayed staves have been found all around the pipe. (8) Sound staves have been frequently found contiguous to badly decayed staves. (9) The character of the grain, whether slash or grain edge, has not influenced the durability. (10) The bruising of the staves during the process of erecting seems to have been one of the chief agencies in hastening decay. (11) Decay has not been confined to the outside of the pipe. (12) The pipe has not usually shown leakage as long as sound wood has remained in excess of ¼" in thickness. (13) The malleable cast band fastenings have been found to be in good condition. (14) The bands, $\frac{1}{16}$" in size, have been considerably corroded save where secured by the nut, but all have been used again by placing the nut in its original position. (15) "The pipe in the 2nd and 3rd Sections, 2½ miles, is nearly all buried in fine-grained sand, and will last perhaps 10 years more by giving it a general repairing, say 5 years hence, but the greater part of the 1st and 4th Sections will have to be replaced in about 4 years.—Superintendent." For Mr. Adam's original paper, see Trans. A. S. C. E., Vol. XXXVI; see also Trans., Vol. XLI.

The Kinetic Energy of Flowing Water (By L. F. Harza. Eng. News, Mar. 7, 1907).—Formulas for the energy in rivers, streams, pipes, etc.

Observations to Determine Values of "C" and "N" as Used in Kutter's Formula (By J. B. Lippincott).—Eight experiments described: *Exper. No. 1.*—Tunnel 446 ft. long, floor grade 0.00096, rectangular section 4½' wide x 4' deep with semi-circular arch and finished with a 1:3 cement mortar plaster. *Exper. No. 2.*—Tunnel 318 ft. long, floor grade 0.00095, section and finish same as No. 1. *Exper. No. 3.*—Canal, 800-ft. length, trapezoidal section with slopes 1:1 and bottom width 11.5 ft., concrete lined with 1:3 mortar plaster, bottom filled up with 1.5 to 2.5 ft. of fine sand. *Exper. No. 4.*—Canal, 600-ft. length, trapezoidal section with side slopes 2 on 1 and bottom width 8 ft., concrete lined with 1:3 mortar spread roughly, partially cleaned before measurements were made. *Exper. Nos. 5 and 6.*—Conduit, 700-ft. length, concrete lined with 1:3 cement mortar in smooth condition, grade of floor same as water surface. *Exper. No. 7.*—Canal, 1000-ft. tangent, lining of concrete without plaster, bottom free from sand and gravel, sides and bottom covered with thin coat of moss. *Exper. No. 8.*—Conduit, 1000-ft. length, lining of concrete tamped in behind boards and not plastered, several inches of sand and gravel on bottom, some moss and grass on sides.

TABULAR RESULTS OF EXPERIMENTS:

Experiment. No.	Discharge. Q.	Area Water Section. a.	Mean Velocity. v.	Hyd. Mean Radius. r.	Wetted Perim. p.	Hydraul. Grade. s.	Coefficient. c.	Coef. of Roughness. n.
1	70.60	14.08	5.01	1.31	10.71	0.00126	123.3	0.0128
2	74.64	15.73	4.74	1.37	11.50	0.00082	141.6	0.0113
3	60.52	30.92	1.96	1.49	20.74	0.00092	52.9	0.0284
4	8.50	6.96	1.22	0.703	9.89	0.00063	58.0	0.0218
5	13.55	5.00	2.71	0.817	6.12	0.00051	132.6	0.0111
6	14.51	5.16	2.81	0.830	6.22	0.00051	136.7	0.0108
7	15.34	6.75	2.27	0.982	6.875	0.0007	86.7	0.0166
8	26.79	10.21	2.62	0.817	12.50	0.00106	89.2	0.0157

The use of the following coefficients in Kutter's formula was suggested by the board of consulting engineers, consisting of John R. Freeman, Frederic P. Stearns and Jas. D. Schuyler: (1) For open masonry conduits of cement or smoothly plastered masonry, $n = 0.013$; (2) for concrete-lined tunnels, or covered masonry conduits, $n = 0.014$; (3) for steel pipe with rivet heads and seams projecting on the interior, $n = 0.016$; (4) for earth canals with bottom left by dredging, $n = 0.0275$.—(*Eng. News*, June 6, 1907.)

The Effect of Changes in Canal Cross-Sections Upon the Rate of Flow (By F. W. Hanna. Eng. News, June 6, 1907).—In the construction

of a canal it is usually necessary to provide for carrying its waters through culverts, flumes, siphons and other important changes of cross-section of waterway; and in order to compute properly its capacity, some estimate of the effects of such changes on the flow must be made. The mathematical discussion in this article deals with this problem.

A Solution of the Problem of Determining the Economic Size of Pipe for High-Pressure Water-Power Installation (By A. L. Adams. Trans. A. S. C. E., Vol. LIX).—Rule: "That pipe fulfills the requirements of greatest economy wherein the value of the energy annually lost in frictional resistance equals four-tenths (0.4) of the annual cost of the pipe line." Discussed by formulas.

The Flow of Water Through Submerged Tubes; Results of Experiments at the University of Wisconsin (By C. B. Stewart. Bulletin, Univ. of Wisconsin. Eng. News, Jan. 9, 1908).—Very extensive article with illustrations and diagrams.

A Logarithmic Diagram for the Flow of Water in Open Channels (By G. T. Prince. Eng. News, Feb. 6, 1908).

Coefficient of Discharge through Circular Orifices (Eng. News, July 9, 1908).—Tables of coefficients; also seven conclusions stated by the writer, Mr. H. J. Bilton.

Bazin's Hydraulic Formula ("Annales des Ponts et Chaussees" for the fourth quarter of 1897; Eng. News, Aug. 13, 1908).—Given in metric measure, as follows:—$v = 87\sqrt{rs} \div (1 + m \div \sqrt{s})$, the value of m varying with the surface of the channel as follows: (1) Very smooth (cement, planed wood, etc.), $m = 0.06$; (2) smooth (plank, brick, cut stone, etc.), $m = 0.16$; (3) rough masonry, $m = 0.46$; (3a) mixed, or intermediate, very regular earth excavation, paved slopes, etc., $m = 0.85$; (4) ordinary earth channels, $m = 1.30$; (5) earth channels in bad condition, $m = 1.75$.

A diagram for Bazin's formula for flow in open channels, prepared by O. von Voigtlander, is published in Eng. News of April 15, 1909.

A Collection of Formulas for Water-Pressure and Moments in Submerged Beams (By D. N. Showalter. Eng. News, May 27, 1909).—Formulas, illustrations and tables.

Ratings of a Pitot Tube (By E. C. Murphy. Eng. News, Aug. 12, 1909.)—Illustrated. Rating curve diagrams.

Huge (625 Cu. Ft. per Sec.) Venturi Meters, India (Eng. News, Nov. 25, 1909).—Illustrated.

The Use and Care of the Current Meter, as Practised by the U. S. Geol. Survey (By J. C. Hoyt. Trans. A. S. C. E., Vol. LXVI., Mar., 1910).—Illustrations and descriptions of various types of meters and recording devices, rating of meters, and methods of making hydraulic measurements.

Friction of Air in Small Pipes (By E. G. Harris. Univ. of Missouri Bulletin; Eng. Rec., Dec. 3 1910).—Experiments to determine the value of the coefficient c in the formula:

$$f = c\frac{lV^2}{rd^5}$$

n which f = loss of pressure in lbs. per sq. in.; l = length of pipe in ft.; V = he cu. ft. of free air passing per second; d = diam. of pipe in ins.; r = ratio f compression to atmospheres; c = the experimental coefficient. The esult of the experiments indicated that for pipes of ½" to 12" in diameter he value of $c = 0.076 - 0.00188d$, so that the above formula would reduce to:

$$f = (0.076 - 0.00188d)\frac{lV^2}{rd^5}.$$

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63.—WATER SUPPLY.

Source and Distribution.—Water supply is derived primarily from rain, snow, hail, sleet and dew, generally termed *rainfall* or precipitation. The amount of rainfall in any locality is the depth of the precipitation in inches, when melted. In some localities dew forms no inconsiderable proportion of the total. The "dew-point" is that temperature at which the air begins to deposit (more) moisture (than it takes up). It is not a fixed temperature.

The Distribution of Rainfall and Source of Water Supply may be grouped as follows: Rainfall—

- Run-off
  - Surface Water
    - Streams.
    - Reservoirs
      - Natural (lakes).
      - Artificial.
- Evaporation.
- Seepage
  - Ground Water
    - Vegetation.
    - Saturated soil.
  - Underground Water
    - Galleries.
    - Springs.
    - Wells
      - Shallow.
      - Deep driven
        - Artesian.*
        - Pumping.

Use in Various Branches of Engineering.—The consideration of water supply is pertinent to the sections which follow, namely,

64. Water Works.....page 1202.
65. Sanitation.....page 1295.
66. Irrigation.....page 1313.
67. Waterways.....page 1320.
68. Water Power.....page 1332.
70. Electric Power and Lighting.....page 1379.

Rainfall.—Engineers are concerned mainly with (1) the *average* monthly precipitation, (2) the monthly rainfall for the *driest years*, and (3) the *maximum* rates of rainfall which may be expected, in any given locality. The first two are for the consideration of Supply; the last, for Discharge.

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*Mr. Myron L. Fuller, Geologist in Charge of Underground Water Supplies, U. S. Geol. Surv. (Water Supply & Irrigation Paper No. 160), states that there is much looseness in the use of the word "artesian," and great variation of its use in different parts of the country. After a careful canvass of the leading geologists of the country engaged in hydraulic studies, he has proposed the following definitions of terms:

Artesian Principle.—The hydrostatic principle in virtue of which water confined in the materials of the earth's crust tends to rise to the level of the water surface at the highest point from which water is transmitted. Gas as an agent in causing the water to rise is expressly excluded from this definition.

Artesian Pressure.—The pressure exhibited by water confined in the earth's crust at a level lower than its static head.

Artesian Water.—That portion of the underground water which is under artesian pressure and will rise if encountered by a well or other passage affording an outlet.

Artesian System.—Any combination of geologic structures, such as basins, planes, joints, faults, etc., in which waters are confined under artesian pressure.

Artesian Basin.—A basin of porous bedded rock in which, as a result of the synclinal structure, the water is confined under artesian pressure.

Artesian Slope.—A monoclinal slope of bedded rocks in which water is confined beneath relatively impervious covers owing to the obstruction to its downward passage by the pinching out of the porous beds, by their change from a pervious to an impervious character, by internal friction, or by dykes or other obstructions.

Artesian Area.—An area underlain by water under artesian pressure.

Artesian Well.—Any well in which the water rises under artesian pressure when encountered.

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1.—Average Monthly Precipitation in the U. S., in Inches.

From Time of Establishment of Station to End of 1904.

Locality.	*	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	No. of Yrs.
Alabama.														
Birmingham	(50.94)	4.23	5.63	5.95	5.13	2.93	4.17	5.74	4.81	2.10	2.42	3.04	4.79	9
Mobile	(61.37)	4.77	5.24	7.21	4.36	4.10	5.98	6.70	6.99	4.85	3.16	3.51	4.50	34
Montgomery	(50.85)	5.29	5.02	6.26	4.53	3.84	4.34	4.34	4.46	2.70	2.27	3.27	4.53	32
Arizona.														
Flagstaff	(21.32)	2.06	3.28	1.81	1.63	1.21	0.55	1.79	3.30	2.09	1.39	1.52	0.69	7
Phoenix	(4.47)	0.72	0.35	0.28	0.20	0.03	0.13	1.21	0.99	1.13	0.32	0.68	0.43	10
Yuma	(2.84)	0.43	0.42	0.27	0.07	0.03	T.	0.13	0.32	0.17	0.23	0.28	0.49	26
Arkansas.														
Fort Smith	(41.27)	2.42	2.89	3.68	3.97	4.94	4.08	3.65	3.49	3.13	2.87	3.26	2.84	23
Little Rock	(49 68)	4.93	4.44	4.98	4.29	5.11	3.97	4.02	3.57	3.34	2.40	4.54	4.09	26
California.														
Eureka	(47.67)	7.62	7.61	7.06	4.17	2.60	1.06	0.13	0.15	1.26	2.83	5.97	7.21	18
Fresno	(8.78)	1.57	1.21	1.56	0.70	0.50	0.12	T.	0.01	0.28	0.53	0.85	1.45	17
Independence	(3.66)	0.92	0.63	0.42	0.14	0.11	0.06	0.04	0.09	0.14	0.33	0.45	0.33	8
Los Angeles	(15.36)	2.64	2.85	2.87	1.15	0.49	0.09	0.01	0.03	0.08	0.81	1.35	2.99	28
Mount Tamalpais	(30.25)	4.75	5.40	5.93	2.00	0.83	0.11	0.02	0.01	0.78	2.58	4.93	2.91	6
Point Reyes Light	(25.32)	4.90	3.32	3.69	1.68	1.27	0.20	0.10	0.10	1.06	2.19	3.47	3.34	13
Red Bluff	(25.80)	4.72	3.48	3.15	2.08	1.33	0.51	0.02	0.04	0.77	1.48	2.96	5.26	27
Sacramento	(20.20)	3.94	2.80	2.86	2.13	1.02	0.14	T.	0.01	0.33	1.08	2.10	3.79	28
San Diego	(9.72)	1.88	2.04	1.52	0.67	0.38	0.05	0.04	0.10	0.05	0.39	0.73	1.87	33
San Francisco	(22.39)	4.75	3.31	3.23	1.80	0.41	0.19	0.02	0.01	0.44	1.32	2.70	4.21	34
San Luis Obispo	(19.66)	4.80	3.35	3.48	1.56	0.40	0.10	T.	0.04	0.39	1.63	1.87	2.04	10
Colorado.														
Denver	(13.88)	0.48	0.50	0.91	1.98	2.58	1.49	1.65	1.36	0.86	0.90	0.53	0.64	32
Grand Junction	(7.61)	0.48	0.56	0.70	0.72	0.72	0.37	0.51	1.05	0.84	0.83	0.49	0.34	13
Peuble	(11.84)	0.35	0.54	0.69	1.22	1.92	1.40	2.07	1.61	0.48	0.73	0.31	0.52	17
Connecticut.														
New Haven	(47 06)	3.96	3.98	4.48	3.56	3.68	2.88	4.92	4.92	3.67	3.84	3.64	3.53	32
District of Columbia														
Washington	(42.89)	3.38	3.57	4.04	3.21	3.80	4.04	4.52	3.93	3.59	3.04	2.71	3.06	34
Florida.														
Jacksonville	(53.21)	3.12	3.36	3.41	2.82	3.94	5.51	6.19	6.06	8.16	5.21	2.45	2.98	33
Jupiter	(59.19)	3.81	2.86	3.20	2.59	4.94	6.60	5.02	5.17	9.66	10.08	2.85	2.41	17
Key West	(37.57)	2.05	1.64	1.27	1.21	2.77	4.14	3.64	4.72	6.91	5.31	2.25	1.66	34
Pensacola	(56.33)	3.97	4.53	5.53	3.07	2.84	5.10	6.85	7.99	4.61	3.65	4.08	4.11	25
Tampa	(53.99)	3.06	3.46	2.76	1.91	2.62	9.23	8.00	8.51	7.84	2.98	1.87	1.75	15
Georgia.														
Atlanta	(49.12)	5.20	4.02	5.94	3.69	3.26	4.03	4.86	4.52	3.55	2.26	3.44	4.35	26
Augusta	(48.10)	4.12	4.47	4.91	3.48	3.22	4.82	5.05	5.64	3.68	2.40	2.96	3.35	34
Macon	(46.28)	3.61	4.57	5.69	3.70	3.09	4.13	4.69	4.73	2.87	2.00	3.02	4.18	25
Savannah	(50.62)	3.12	3.30	3.66	3.15	2.81	6.04	6.06	7.77	5.67	3.57	2.44	3.03	34
Idaho.														
Boise	(12.11)	1.50	1.61	1.70	1.23	0.99	0.47	0.22	0.20	0.44	1.70	1.02	1.03	6
Lewiston	(13.46)	0.86	1.28	1.70	0.82	1.47	1.18	1.01	0.57	0.89	1.05	1.28	1.35	4
Pocatello	(11.82)	0.89	1.92	2.20	1.51	1.15	0.42	0.23	0.37	0.32	0.92	0.75	1.14	5
Illinois.														
Cairo	(41.52)	3.69	3.54	3.95	3.63	3.88	4.40	3.47	2.62	2.52	2.60	3.88	3.34	34
Chicago	(33.37)	2.05	2.30	2.59	2.74	3.45	3.63	3.62	2.88	2.94	2.55	2.53	2.09	34
Springfield	(38.67)	2.45	3.14	3.09	3.63	4.71	4.55	3.10	2.46	3.22	2.71	3.01	2.60	25
Indiana.														
Evansville	(40.77)	3.13	2.47	5.32	2.95	3.39	4.59	4.03	2.31	2.67	2.77	3.81	3.33	10
Indianapolis	(42.00)	2.86	3.24	3.97	3.46	4.03	4.35	4.17	3.18	3.35	2.74	3.62	3.03	34
Iowa.														
Charles City	(30.73)	0.96	1.07	1.84	2.51	4.02	5.16	3.96	2.83	3.21	2.39	1.39	1.39	23
Davenport	(32.82)	1.65	1.57	2.25	2.71	4.34	4.00	3.65	3.64	3.21	2.38	1.80	1.62	34
Des Moines	(32.41)	1.19	1.08	1.53	3.00	4.78	4.88	3.83	3.57	2.99	2.75	1.45	1.36	26
Dubuque	(34.62)	1.50	1.45	2.23	2.92	4.38	4.56	4.58	2.98	3.91	2.59	1.78	1.74	31
Keokuk	(35.57)	1.80	1.61	2.39	3.29	4.32	4.37	4.26	3.15	4.07	2.62	1.91	1.78	34
Sioux City	(25.41)	0.51	0.54	1.17	2.78	4.04	3.92	3.67	3.05	2.42	1.77	0.74	0.80	16

*** Figures enclosed in parentheses are totals in inches for the 12 months, and indicate average yearly precipitation for the period of years stated in the last column.**

Note.—"T" means trace—too small to measure.

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1.—Average Monthly Precipitation in the U. S., in Inches.—Continued.

Locality. *	Jan.	Feb.	Mar.	Apr.	May.	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	No. of Yrs.
Kansas.													
Concordia (27.35)	0.68	0.78	1.44	2.40	4.78	4.77	3.68	2.90	2.41	2.19	0.82	0.50	20
Dodge (18.78)	0.46	0.70	0.85	1.84	3.19	2.72	2.47	2.52	1.48	1.46	0.50	0.59	30
Topeka (34.04)	0.98	1.46	2.09	2.74	5.28	4.76	4.90	4.46	3.37	1.99	1.08	0.93	18
Wichita (30.31)	0.78	1.21	2.06	2.82	4.97	4.91	3.41	3.09	2.87	2.40	0.90	0.89	17
Kentucky.													
Lexington (41.64)	3.87	3.37	4.77	3.23	3.54	4.11	3.92	3.71	2.44	2.03	3.42	3.23	17
Louisville (43.95)	3.86	3.92	4.35	3.97	3.60	4.24	3.73	3.55	2.66	2.49	3.88	3.70	33
Louisiana.													
New Orleans (57.09)	4.53	4.62	5.14	4.98	4.01	6.19	6.36	5.68	4.63	2.96	3.74	4.25	34
Shreveport (45.72)	4.44	3.80	4.48	4.60	3.87	3.83	3.46	2.14	3.42	3.10	4.12	4.46	33
Maine.													
Eastport (42.98)	3.20	3.63	4.54	2.99	3.65	3.37	3.48	3.19	3.05	4.00	4.05	3.83	32
Portland (42.89)	3.71	3.91	3.97	3.14	3.66	3.23	3.34	3.54	3.22	3.79	3.76	3.62	33
Maryland.													
Baltimore (43.13)	3.19	3.63	3.96	3.24	3.58	3.77	4.66	4.14	3.91	3.01	2.96	3.08	34
Massachusetts.													
Boston (43.64)	3.84	3.50	4.27	3.46	3.45	3.02	3.47	4.06	3.19	3.96	4.16	3.26	33
Nantucket (37.33)	3.25	2.99	3.95	2.88	2.67	2.17	3.42	2.86	2.61	3.66	3.45	3.42	18
Michigan.													
Alpena (33.38)	2.23	1.86	1.95	2.19	3.28	3.61	3.12	3.39	3.54	3.51	2.50	2.20	32
Detroit (32.13)	1.91	2.31	2.46	2.22	3.30	3.83	3.49	2.69	2.63	2.38	2.56	2.35	34
Escanaba (31.61)	1.49	1.44	1.88	2.03	3.48	3.64	3.36	3.65	3.62	3.24	2.10	1.68	32
Grand Haven (33.70)	2.64	2.15	2.39	2.47	3.27	3.38	2.83	2.62	3.61	3.02	2.82	2.50	34
Houghton (34.24)	2.82	1.61	2.94	2.18	2.93	2.38	3.48	2.20	4.20	3.16	3.16	3.18	4
Marquette (32.43)	2.01	1.73	1.97	2.11	3.21	3.43	3.16	2.84	3.63	3.16	2.74	2.44	34
Port Huron (30.88)	1.92	2.22	2.54	2.03	3.25	3.27	2.70	2.67	2.73	2.65	2.70	2.20	31
Sault Ste. Marie (32.33)	2.37	1.57	1.79	2.05	3.08	2.91	3.00	3.05	3.68	3.27	3.10	2.46	17
Minnesota.													
Duluth (29.77)	0.98	1.04	1.56	2.13	3.45	4.39	3.75	3.43	3.55	2.73	1.50	1.26	34
Minneapolis (28.69)	0.65	0.80	1.77	2.46	3.34	3.75	4.22	3.80	3.17	2.75	0.98	1.00	14
Moorhead (24.68)	0.70	0.81	1.12	2.49	2.60	4.08	3.89	2.93	2.30	2.07	0.90	0.79	24
St. Paul (28.36)	0.90	0.88	1.61	2.40	3.28	4.35	3.51	3.44	3.26	2.40	1.21	1.12	34
Mississippi.													
Meridian (51.84)	4.89	6.16	5.36	4.02	3.94	4.87	4.92	5.12	3.01	1.90	3.04	4.61	17
Vicksburg (53.35)	5.49	4.76	6.22	5.13	4.43	4.43	4.46	3.33	3.33	2.44	4.22	5.11	33
Missouri.													
Columbia (38.42)	2.14	2.16	3.12	4.08	4.96	4.85	4.24	3.34	3.53	2.09	2.10	1.81	15
Hannibal (34.49)	2.13	1.83	2.79	3.18	5.35	3.67	4.17	3.07	3.36	1.48	1.91	1.55	13
Kansas City (37.29)	1.25	1.60	2.70	3.28	5.47	4.45	5.01	4.52	3.59	2.37	1.72	1.33	17
St. Louis (37.04)	2.22	2.87	3.46	3.42	4.30	4.62	3.57	2.44	2.85	2.19	2.84	2.26	34
Springfield (43.00)	2.45	2.47	3.93	3.82	5.73	5.09	4.18	3.91	3.37	2.77	2.68	2.60	17
Montana.													
Havre (14.18)	0.76	0.51	0.55	0.97	2.06	2.90	2.08	1.32	1.11	0.64	0.74	0.54	25
Helena (13.06)	1.06	0.69	0.84	1.12	1.98	2.08	1.16	0.64	1.14	0.77	0.78	0.80	25
Kalispell (15.50)	0.98	1.16	0.89	0.80	2.39	1.65	1.24	1.10	1.70	0.67	1.65	1.27	6
Miles City (12.28)	0.41	0.48	1.34	1.09	2.06	2.20	1.29	0.93	0.83	0.76	0.57	0.32	13
Nebraska.													
Lincoln (27.63)	0.67	0.87	1.21	2.67	4.59	4.36	4.13	3.39	2.14	2.07	0.77	0.76	20
North Platte (18.07)	0.42	0.44	0.76	2.10	2.80	3.26	2.69	2.37	1.37	1.00	0.39	0.47	30
Omaha (30.80)	0.62	0.72	1.42	3.03	4.44	5.18	4.53	3.56	2.90	2.47	0.99	0.94	34
Valentine (19.42)	0.57	0.67	1.37	2.46	2.24	3.38	3.15	2.53	1.10	0.87	0.59	0.49	16
Nevada.													
Carson City (10.75)	1.97	1.72	1.43	0.57	0.75	0.25	0.11	0.22	0.27	0.46	1.21	1.79	25
Winnemucca (8.36)	1.05	0.92	0.93	0.85	1.03	0.60	0.18	0.17	0.33	0.56	0.67	1.07	26
New Hampshire.													
Concord (40.30)	3.33	3.21	3.52	2.91	3.14	3.25	3.95	3.68	3.21	3.46	3.30	3.34	34
New Jersey.													
Atlantic City (41.11)	3.44	3.33	3.64	3.12	2.93	2.95	3.74	4.40	3.08	3.52	3.20	3.76	29
New Mexico.													
Santa Fe (14.19)	0.58	0.74	0.71	0.75	1.15	1.04	2.70	2.43	1.64	1.05	0.68	0.72	31

* Figures enclosed in parentheses are totals in inches for the 12 months, and indicate average yearly precipitation for the period of years stated in the last column.

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1.—AVERAGE MONTHLY PRECIPITATION IN THE U. S., IN INCHES.—Continued.

Locality. *	Jan.	Feb.	Mar.	Apr.	May.	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	No. of Yrs.
New York.													
Albany.....(36.84)	2.60	2.57	2.83	2.40	3.00	3.77	3.89	3.98	3.27	3.06	2.78	2.69	31
Binghamton.....(31.26)	1.72	2.00	2.93	2.11	2.69	3.24	3.11	3.35	2.82	2.98	2.04	2.27	8
Buffalo.....(32.64)	3.21	2.99	2.70	2.40	3.14	3.52	3.42	3.07	3.15	3.33	3.34	3.37	34
New York.....(44.65)	3.80	3.84	4.05	3.31	3.16	3.27	4.46	4.59	3.50	3.71	3.57	3.39	34
Oswego.....(36.06)	3.14	2.64	2.96	2.18	2.77	3.31	3.33	2.65	2.84	3.23	3.39	3.62	34
Rochester.....(34.47)	3.24	2.85	3.07	2.43	3.02	3.05	3.16	2.90	2.37	2.77	2.72	2.89	34
North Carolina.													
Asheville.....(42.45)	3.04	3.57	4.14	3.08	3.62	4.08	4.84	4.63	2.95	2.61	2.76	3.13	28
Charlotte.....(49.24)	4.23	4.54	4.71	3.45	3.84	4.55	5.31	5.25	3.27	3.37	2.95	3.77	27
Hatteras.....(61.17)	5.11	4.50	5.35	4.25	4.16	4.09	6.28	6.10	5.56	6.03	4.62	5.12	30
Raleigh.....(49.74)	3.56	4.41	4.23	3.42	5.01	4.73	6.12	5.80	3.40	3.72	2.42	2.92	18
Wilmington.....(50.97)	3.62	3.35	3.59	2.72	3.93	5.55	6.71	6.96	5.32	3.78	2.39	3.05	34
North Dakota.													
Bismarck.....(17.67)	0.53	0.64	1.00	2.22	2.58	2.58	2.57	2.12	1.04	1.10	0.63	0.66	30
Williston.....(14.96)	0.59	0.50	0.70	1.26	2.13	3.50	2.08	1.28	0.85	0.86	0.59	0.62	26
Ohio.													
Cincinnati.....(38.13)	3.28	3.42	3.72	2.88	3.41	3 94	3.38	3.34	2.35	2.21	3.18	3.02	34
Cleveland.....(34.90)	2.48	2.73	2.69	2.27	3.24	3.64	3.63	3.01	3.34	2.60	2.75	2.52	34
Columbus.....(36.98)	3.03	3.25	3.28	2.62	3.83	3.55	3.59	3.06	2.54	2.33	3.20	2.70	27
Sandusky.....(34.14)	2.14	2.58	2.68	2.44	3.30	3.90	3.67	3.27	2.69	2.37	2.78	2.32	28
Toledo.....(30.81)	1.95	2.08	2.21	2.22	3.37	3.42	3.25	2.73	2.44	2.24	2.63	2.27	34
Oklahoma.													
Oklahoma.....(31.44)	1.36	0.95	2.19	2.48	2.24	3.19	3.30	2.75	2.69	1.94	2.52	1.83	14
Oregon.													
Baker City.....(13.27)	1.31	1.49	1.34	0.97	1.73	1.16	0.44	0.44	0.77	0.99	1.12	1.51	15
Portland.....(45.48)	6.70	6.00	5.19	3.16	2.35	1.70	0.58	0.64	1.76	3.56	6.48	7.36	33
Roseburg.....(35.65)	5.86	4.98	3.90	2.61	2.04	1.15	0.37	0.35	1.12	2.70	4.43	6.14	27
Pennsylvania.													
Erie.....(39.45)	3.05	3.10	2.87	2.71	3.62	3.75	3.13	3.17	3.57	3.70	3.71	3.07	32
Harrisburg.....(37.56)	2.78	2.88	3.18	2.44	4.10	3.45	3.91	4.11	2.77	2.87	2.46	2.61	17
Philadelphia.....(40.54)	3.28	3.39	3.40	2.89	3.16	3.18	4.19	4.50	3.41	3.01	3.18	2.95	34
Pittsburg.....(36.59)	2.86	2.79	3.07	2.91	3.35	3.73	4.59	3.12	2.46	2.32	2.59	2.80	34
Scranton.....(41.62)	2.32	2.63	3.22	2.64	2.58	4.92	4.89	5.22	3.30	4.07	1.75	4.08	4
Rhode Island.													
Block Island.....(44.81)	4.02	4.32	4.43	3.76	3.71	2.75	3.21	3.50	3.08	4.14	4.06	3.83	24
Narragansett.....(47.98)	4.62	4.47	4.84	3.97	4.48	2.58	3.19	3.99	3.66	4.23	4.04	3.91	22
South Carolina.													
Charleston.....(52.57)	3.32	3.37	3.68	3.10	3.52	5.38	7.39	7.31	5.48	3.99	2.92	3.11	34
Columbia.....(47.16)	3.41	4.75	4.13	2.90	3.48	4.36	5.74	6.64	3.74	2.55	2.44	3.02	18
South Dakota.													
Huron.....(20.44)	0.49	0.48	0.98	2.73	2.66	3.67	2.80	2.60	1.60	1.29	0.55	0.59	24
Pierre.....(16.24)	0.48	0.49	1.32	2.11	1.68	3.18	2.39	1.87	1.02	0.67	0.48	0.55	14
Rapid City.....(16.49)	0.36	0.55	1.24	2.20	2.76	3.59	1.91	1.41	0.91	0.66	0.41	0.49	17
Yankton.....(25.85)	0.50	0.73	1.24	3.08	3.84	3.98	3.76	3.16	2.64	1.38	0.66	0.88	31
Tennessee.													
Chattanooga.....(50.89)	5.63	5.17	6.31	4.48	3.76	4.11	3.65	3.84	3.31	2.68	3.62	4.33	26
Knoxville.....(49.68)	5.05	5.15	5.61	4.66	3.77	4.15	4.09	4.08	2.92	2.52	3.61	4.07	34
Memphis.....(50.81)	5.40	4.98	5.67	4.95	4.29	4.47	3.32	3.39	2.99	2.63	4.37	4.35	34
Nashville.....(48.30)	4.80	4.68	5.29	4.51	3.50	4.25	4.40	3.39	3.63	2.28	3.77	3.80	34
Texas.													
Abilene.....(24.19)	0.89	1.18	1.34	2.24	3.68	3.07	2.01	2.00	3.24	2	13	1.18	1.23	19
Amarillo.....(21.03)	0.56	0.89	0.39	1.58	3.58	3.14	2.54	2.72	2.38	1.64	0.75	0.86	13
Corpus Christi.....(27.03)	2.22	2.11	1.80	1.72	2.72	2.61	1.76	2.33	4.14	1.99	2.23	1.40	18
El Paso.....(9.48)	0.50	0.36	0.33	0.21	0.37	0.55	2.15	1.65	1.40	1.05	0.42	0.49	26
Fort Worth.....(28.79)	1.87	1.29	2.83	2.50	4.24	3.54	2.72	1.80	2.43	2.72	1.65	1.20	10
Galveston.....(47.51)	3.62	3.02	3.09	3.11	3.32	4.57	4.02	5.08	5.72	4.19	4.01	3.76	34
Palestine.....(43.96)	4.15	3.47	3.81	4.00	4.94	4.07	2.98	2.36	3.35	3.48	3.73	3.62	23
San Antonio.....(27.01)	1.87	1.89	1.60	2.77	2.97	2.80	2.34	2.86	3.07	1.52	1.70	1.62	20
Taylor.....(34.53)	0.79	3.03	1.47	2.25	4.19	2.65	8.96	1.29	3.49	2.88	2.36	1.17	3

*** Figures enclosed in parentheses are totals in inches for the 12 months, and indicate average yearly precipitation for the period of years stated in the last column.**

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1.—Average Monthly Precipitation in the U. S., in Inches.—Concluded.

Locality. *	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	No. of Yrs.
Utah.													
Modena.....(7.92)	0.27	1.45	0.58	0.35	0.74	0.16	0.54	1.55	1.08	0.74	0.30	0.16	4
Salt Lake City.....(15.94)	1.33	1.40	1.99	2.13	1.97	0.73	0.52	0.74	0.80	1.50	1.40	1.43	31
Vermont.													
Northfield.....(33.03)	2.65	2.31	2.99	2.16	2.67	3.12	2.58	3.91	2.83	2.34	2.75	2.72	18
Virginia.													
Lynchburg.....(43.59)	3.79	3.71	3.90	3.18	3.97	3.81	4.07	4.14	3.75	3.33	2.82	3.12	34
Norfolk.....(50.25)	3.38	3.80	4.62	3.90	4.26	4.21	5.81	5.87	4.19	3.89	2.87	3.45	34
Richmond.....(43.28)	2.67	3.79	4.22	3.56	3.09	3.63	4.87	4.67	3.61	3.19	2.52	3.46	7
Wytheville.....(39.72)	2.44	3.54	3.78	2.92	3.83	4.64	3.78	4.41	3.00	2.60	1.99	2.79	11
Washington.													
Port Crescent.....(44.97)	6.39	5.64	4.61	2.53	1.95	1.45	0.57	0.83	1.50	3.29	8.91	7.30	6
Seattle.....(37.15)	4.33	4.03	3.31	2.97	2.26	1.60	0.80	0.50	2.12	2.96	6.31	5.96	14
Spokane.....(18.13)	2.34	1.99	1.44	1.29	1.40	1.48	0.69	0.50	0.99	1.34	2.27	2.40	24
Tacoma.....(47.34)	6.31	5.87	4.31	3.48	2.10	1.96	0.73	0.68	2.29	2.76	9.49	7.36	8
Tatoosh Island.....(92.79)	12.92	9.37	8.43	6.93	4.31	4.04	2.08	2.41	6.56	8.55	12.17	15.02	17
Walla Walla.....(17.61)	2.17	1.62	1.86	1.78	1.66	1.12	0.43	0.41	0.94	1.49	2.07	2.06	19
West Virginia.													
Elkins.....(42.21)	3.53	3.48	4.23	3.20	4.20	5.72	4.72	3.16	2.57	1.80	2.26	3.34	6
Parkersburg.....(39.32)	3.27	3.60	3.78	2.99	3.34	4.74	4.65	3.38	2.74	2.23	2.86	2.74	17
Wisconsin													
Green Bay.....(30.97)	1.68	1.70	2.32	2.49	3.47	3.46	3.46	2.63	3.37	2.63	1.90	1.86	18
La Crosse.....(30.82)	1.11	1.05	1.58	2.38	3.49	4.41	4.14	3.30	4.05	2.52	1.48	1.31	32
Milwaukee.....(30.88)	2.02	1.86	2.67	2.63	3.32	3.44	3.11	2.83	2.97	2.21	1.94	1.88	34
Madison.....(31.82)	1.66	1.56	2.18	2.38	3.40	4.19	4.13	3.10	3.24	2.48	1.76	1.74	26
Wyoming.													
Cheyenne.....(12.31)	0.37	0.40	0.70	1.39	2.25	1.53	1.79	1.47	0.99	0.67	0.37	0.38	34
Lander.....(13.34)	0.37	0.68	1.59	2.45	2.30	1.30	0.86	0.53	0.94	1.03	0.52	0.77	14
Yellowstone Park..(19.27)	2.40	1.92	2.30	1.23	1.94	1.65	1.23	1.07	0.99	1.09	1.59	1.86	18

* Figures enclosed in parentheses are totals in inches for the 12 months, and indicate average yearly precipitation for the period of years stated in the last column.

In connection with the preceding table, the following Table, No. 2, shows in respective order the percentages of rainfall to average rainfall: *First*, for the driest year; *Second*, for the two driest years; *Third*, for the three driest years; closing with 1896.

Ex.—For Pittsburg, the average rainfall, from Table 1, is 36.59 ins. Then, using the percentages in Table 2, following, we have: *First*, for the driest year, $36.59 \times 0.70 = 25.61$ ins.; *Second*, for the two driest years, $36.59 \times 0.78 = 28.54$ ins.; *Third*, for the three driest years, $36.59 \times 0.85 = 31.10$ ins.

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2.—*Percentage of Rainfall to Average Rainfall.

(To be used in connection with preceding table. See Example, p. 1194.)

North Atlantic: Boston, 60, 70, 80; New Haven, 74, 78, 82; New York, 62, 77, 80; Philadelphia, 70, 75, 80; Washington, 69, 71, 74.

South Atlantic: Wilmington, 75, 80, 81; Charleston, 48, 55, 62; Augusta, 81, 88, 87; Jacksonville, 74, 77, 83; Key West, 54, 61, 73.

Gulf and Lower Mississippi: Shreveport, 67, 75, 75; Montgomery, 76, 80, 83; Mobile, 68, 75, 78; New Orleans, 64, 75, 77; Galveston, 50, 65, 72; Nashville, 67, 73, 83; Vicksburg, 70, 74, 83.

Ohio Valley: Pittsburg, 70, 78, 85; Cincinnati, 60, 72, 71; Indianapolis, 59, 76, 82; Louisville, 74, 81, 85; Cairo, 62, 75, 81.

Lake Region: Marquette, 69, 75, 81; Detroit, 65, 72, 79; Cleveland, 71, 74, 81; Duluth, 65, 81, 88.

Upper Mississippi Valley: St. Louis, 55, 65, 75; Davenport, 56, 68, 73; Chicago, 66, 80, 86; Milwaukee, 66, 74, 73; Madison, 39, 58, 68; LaCrosse, 57, 78, 79; St. Paul, 53, 54, 75.

The Plains: Omaha, 57, 63, 70; Dodge City, 51, 58, 73; North Platte, 56, 67, 72; Denver, 59, 71, 77; Cheyenne, 39, 62, 75.

The Plateau: Yuma, 25, 50, 46; Phoenix, 52, 88, 90; Tucson, 44, 79, 80; Santa Fe, 53, 63, 66; Carson City, 57, 63, 72; Salt Lake City, 55, 64, 74; Spokane, 73, 84, 84; Walla Walla, 46, 81, 86.

Pacific Coast: Astoria (average rainfall 77 ins.), 64, 68, 77; Portland, 67, 76, 79; Red Bluff, 52, 64, 58; Sacramento, 42, 67, 84; San Francisco, 51, 73, 78; Los Angeles, 33, 48, 59; Fresno, 61, 65, 74; San Diego, 30, 54, 61.

High Intensities of Rainfall.—Flood discharges in rivers, creeks, sewers, etc., naturally occur at times of high intensities of rainfall, and hence it is important to know, for particular localities, the maximum rates of downpour which have been recorded, and which are liable to take place in the future. Considerable confusion has arisen as to the meaning of the various terms "intensity of rainfall," "maximum rate of rainfall," etc.: for even a single shower has various intensities of downpour, while the recorded "intensity" often comprises the ratio of the total downpour to the total time, omitting, perhaps, the critical data most desired for the design of the sewer, namely, a higher intensity of downpour for a shorter period of time, during the shower. In order to clarify this subject, the following notation is used:

Let d = the depth of rainfall in ins., for any given time T in hrs.;

T = time in hours corresponding to depth of precipitation d in ins.;

r = *average* rate of downpour in inches per hour during the *whole shower* $= \frac{d}{T}$, designating the length of time by a sub-numeral in the denomination of hours, thus—

$r_4 = \frac{d}{T} =$ say $\frac{4}{4}$ = average rate of downpour during the whole 4 hours of the shower;

i = *maximum* average rate of downpour in inches per hour during a *protracted period of highest intensity* $= \frac{d}{T}$, designating the length of time by a sub-numeral in the denomination of hours, thus—

$i_{1.5} = \frac{d}{T} =$ say $\frac{3}{1.5}$ = maximum average rate for 1.5 hours;

I = *maximum maximorum* rate of downpour in inches per hour during the *short time or greatest intensity* $= \frac{d}{T}$, designating the length of time by a sub-numeral in the denomination of hours, thus—

$I_{0.25} = \frac{d}{T} =$ say $\frac{.75}{.25}$ = highest intensity, for ¼ hour.

* Abstract from Table No. 6, "Public Water Supplies" by Turneaure and Russell: John Wiley & Sons, 1901. Although not strictly applicable to preceding table closing with 1904, these percentages may be used with fairly good results. Published by permission.

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Thus, from the above, say for any given shower, we have the complete data: $r_4 = 1$, $i_{1.5} = 2$, $I_{0.25} = 3$. This means that it rained 4 ins. in 4 hours., or at the average rate of 1 in. per hour; 2 ins. per hour for 1.5 hours of greatest downpour; and 0.75 in. in 15 minutes, or at the rate of 3 ins. per hour for the short period of greatest intensity. In current literature it is often problematical whether r, i or I is intended as the rate of downpour.

Formulas for Maximum Intensity of Downpour, i.

Notation.

i = average rate of downpour in ins. per hour for the time t in mins.;
t = time or duration of downpour in minutes.

Formulas.

A. N. Talbot's "Maximum formula for Eastern U. S*.

$$i = \frac{360}{t+30} \quad \text{(1)}$$

A. N. Talbot's "Ordinary" formula for Eastern U. S*.

$$i = \frac{105}{t+15} \quad \text{(2)}$$

E. Kushling's formula for New York City and vicinity:

$$i = \frac{120}{t+20} \quad \text{(3)}$$

†C. W. Sherman's formula for Eastern U. S., when $t < 180$:

$$i = \frac{420}{t+30} \quad \text{(4)}$$

But when $t > 180$ the value of i is too small.

†C. W. Sherman s "Maximum" formula for Boston:

$$i = \frac{38.64}{t^{0.687}} \quad \text{(5)}$$

†C. W. Sherman's "Ordinary" formula for Boston:

$$i = \frac{25.12}{t^{0.687}} \quad \text{(6)}$$

E. S. Dorr's "Ordinary" formula for Boston:

$$i = \frac{150}{t+30} \quad \text{(7)}$$

Standard Rain Gage.—For a complete description of the measurement of precipitation, see Weather Bureau Paper "W. B., No. 285" U. S. Dept. of Agriculture, by C. F. Marvin, 1903, from which the following cut (standard 8-in. gage) is reproduced. A is the receiver; B the overflow attachment; C the measuring tube; D the measuring stick. The diameter of the receiving tube a is just 8 ins. ($8^2 = 64$) and of the measuring tube C is 2.53 ins. ($2.53^2 = 6.4$); hence the rainfall is magnified in the latter 10 times, which facilitates accurate measurement. The measuring stick D is a strip of straight-grained cedar 0.08 in. thick, 0 5 in wide, and 24 ins. long. When the tube C is full it overflows at d into B, and is later poured into C and measured. If the precipitation is snow or hail it is melted before measuring. Snow will occupy from 7 to 34 times its depth melted.

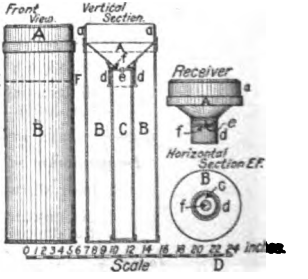

Fig. 1.—Rain Gage.

* Territory east of the Rocky Mountains.
† See Trans A. S. C. E., Vol. LIV, pp. 178 and 212.

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3.—MAXIMUM RATES OF RAINFALL BY PRECEDING FORMULAS.
For periods of time ranging from 5 minutes to 3 hours.
[Rates of downpour in inches per hour for time t.]

Formula.		Duration or time of Downpour.				
		$t=5m.$	$t=10m.$	$t=30m.$	$t=60m.$	$t=180m.$
(1) $i=\frac{360}{t+30}$.	Talbot's maximum; for Eastern U. S.	8.57	7.50	5.00	3.33	1.43
(2) $i=\frac{105}{t+15}$.	Talbot's ordinary; for Eastern U. S.	5.25	4.20	2.33	1.40	0.54
(3) $i=\frac{120}{t+20}$.	Kuehling's; for N. Y. City and vicinity.	4.80	4.00	2.40	1.50	0.60
(4) $i=\frac{420}{t+30}$.	Shermans; for Eastern U. S., when $t<180$.	12.00	10.50	7.00	4.67	2.00
(5) $i=\frac{38.64}{t^{0.657}}$.	Sherman's maximum; for Boston.	12.79	7.94	3.73	2.32	1.09
(6) $i=\frac{25.12}{t^{0.657}}$.	Sherman's ordinary; for Boston.	8.31	5.16	2.43	1.51	0.71
(7) $i=\frac{150}{t+30}$.	Dorr's ordinary; for Boston.	4.29	3.75	2.50	1.67	0.71

Note.—To obtain the total depth of rainfall for the duration of time t given: Multiply the rates or intensities i given in the table, by $\frac{t}{60}$ ($=T$, the time in hours); thus, from formula (3) the total rainfalls which it is possible to expect in 5-, 10-, 30-, 60- and 180 minutes of downpour are, respectively, $4.80\times\frac{5}{60}=0.4$ in., $4.00\times\frac{10}{60}=0.67$ in., $2.40\times\frac{30}{60}=1.20$ in., $1.50\times\frac{60}{60}=1.50$ in., and $0.60\times\frac{180}{60}=1.80$ in. In calculating the maximum rate of discharge from a catchment area into a stream or sewer, it is necessary to estimate the time required for the rain to reach such outlet, say 30 minutes, more or less, for sewers, and a longer time for streams, generally. See *Run-off*, below.

Run-off.—The term "run-off" is rather indefinite, but in general it may be defined as the surface discharge from a catchment area or basin. Each stream has its catchment area. The discharge from a stream may be estimated in three ways:

1st. By Kutter's formula (p. 1167), with sectional area, mean radius and hydraulic slope known;
2nd. By actually gaging the stream (see page 1182, etc.);
3rd. By estimating the percentage of run-off to total rainfall.

All three methods should be used if possible, for checking.

Formulas for run-off are founded on actual stream gagings, but must be used with care for the following reasons:

(*a*). Each stream, strictly spreaking, should have its own run-off formula (or formulas—see *b*, *c* and *d*).

(*b*). The run-off increases with the amount and *intensity* of rainfall.

(*c*). It is greater in the spring, when the ground is frozen and the snows are melting, than during the summer.

(*d*). It may vary considerably at different points in the same stream, the variation being due almost wholly to seepage discharge or influx from springs.

(*e*). The run-off varies generally with the compactness of the soil and sub-soil, being very slight in sand.

(*f*). It is influenced more or less by the slope of the catchment basin (contrary to some authorities).

In addition to the above, it is to be noted that the *time* of run-off affects largely any water proposition. If the run-off is immediate the waters usually

have to be stored unless the supply is very much in excess of the demand. Storage may obtain in several ways, namely, (1) by high altitude of the catchment basin, delaying the melting of the snow; (2) by forest growth in the catchment basin, decreasing evaporation; (3) by natural lakes in the catchment area; (4) by artificial storage reservoirs; (5) by sub-soil storage.

Sub-surface flow accompanies every stream and may be considered part of the run-off, under certain circumstances, if readily available. It is not included in ordinary stream measurements, but may be detected in "dry" streams by digging test pits. It may be brought to the surface and become available by the construction of tight coffer-dams extending to bed rock, or by the construction of ordinary dams with tight foundation. Many creeks apparently "dry" in summer may thus be made to yield a considerable discharge.

Formulas for Run-off.—The following notation is for the subjoined formulas:

a = area of catchment basin, in acres;
A = area of catchment basin, in square miles;
d = precipitation on catchment per month, in inches;
D = total precipitation for any period, in inches;
c = coefficient of discharge:
q = discharge in cu. ft. per second per square mile of catchment for precipitation d.
Q = total discharge in cu. ft. *per acre* of catchment for precipitation D.

Then for total run-off per acre of catchment,

$$Q = c \cdot \frac{D}{12} \cdot 43560\, a = 3630\, c\, D\, a \qquad (1)$$

in which c may vary from 0 to 100; almost always between 0.05 and 0.75; generally between 0.20 and 0.60; rough average 0.40. (But see page 1197.) The value of c in formula (1) can usually be derived from the coefficient for some other catchment, near the locality, for which the run-off has been gauged. Then, by transposition, $c = \frac{Q}{3630\, D\, a}$. The accuracy of this method is enhanced more or less if the two catchments are very dissimilar in character of soil, geology, topography, area, etc. It is far superior however to any blind assumption of percentage based on averages in distant localities.

The monthly discharge may be obtained by substituting d for D in formula (1).

Dicken's formula assumes the *maximum discharge* q_{max} in cubic feet per second to be proportional to $\sqrt{A}$; thus,

$$q_{max} = c' \sqrt{A} \qquad (2)$$

in which c' is a coefficient depending upon rainfall, character of soil, topography, geology, etc. The value of c' in formula (2) may often be obtained from similar catchments in the same locality, in about the same manner as described for c in formula (1). When c' is known this formula is useful in approximating the flood discharge from a catchment for the design of wasteways for dams.

The writer has seen it stated repeatedly that run-off is independent of the degree of slope of the catchment area. But his experience and investigations are to the contrary. Both the declivity of the ground surface and its geological formation have considerable effect on the time and amount of run-off, especially if measured near the headwaters of the effluent stream.

Run-off formulas must be used with great caution and only in connection with gagings of streams, unless the merest approximation is desired. Even then the gagings should cover a long period, by months, for a number of years, as the precipitation varies from year to year in both amount and intensity, greatly affecting the run-off. In general, the records of 20 years are desirable if the demands approach closely the minimum annual supply. The *minimum* as well as the average discharge should be sought. Much valuable data relating to discharge of streams may be obtained from the Water Supply Papers issued by the U. S. Geological Survey, Dept. of the Interior.

For a good discussion of an additive method of computing run-off to sewers, see article in *Eng. News* of March 11, 1909, from Paper by Carl H. Nordell.

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Evaporation.—Evaporation is a physical change from a solid or liquid to a vaporous or gaseous state. It is primarily due to heat. All substances emit vapors to a greater or less extent, depending largely upon the temperature and also upon many other conditions, including humidity.

Evaporation from Ice and Snow takes place at *all temperatures*. The mean evaporation from the surface of ice may be assumed at about 0.04 in., and from snow at about 0.02 in., per day of 24 hours.

Evaporation from Water surface goes on continually day and night, at all temperatures. But when the temperature is below the dew-point the additional moisture from condensation may more than offset the loss due to evaporation. The rate of evaporation increases with—

(1) Temperature of water surface;
(2) Dryness of air at water surface;
(3) Velocity of the wind at water surface.

The maximum evaporation from surface of running water exposed to hot sun and dry, hot wind in the tropics may reach 0.50 inch per day of 24 hours. But in the southern part of the United States it will rarely if ever exceed 0.40 inch per day. The average rate of evaporation from water surface of reservoirs during the summer months may be assumed safely not to exceed 0.35 inch per day in the south and southwest, and 0.18 inch in the northern states, unless the reservoirs are unusually exposed or shallow. The average rate per day for the year may be assumed at about one-half these figures, that is, 0.18 and 0.09 inches respectively. Running water evaporates more rapidly than still water; and water in a shallow pan, exposed to the sun, will evaporate more rapidly than in a reservoir.

In a catchment basin the evaporation from the land surface continues only for a definite period after each shower until the land dries, unless there is vegetation in which case it continues at a greater or less rate. Evaporation from a meadow of luxuriant grass has been found to be two or three times as rapid as from a still water surface.

4.—MONTHLY EVAPORATION IN INCHES IN THE UNITED STATES FROM WATER SURFACES (APPROXIMATE).

Locality.	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Annual.
North Atlantic Coast	1.0	1.3	1.7	2.5	2.5	3.4	3.4	3.4	2.9	2.5	2.0	1.4	28
Middle Atlantic Coast	1.7	1.8	2.3	3.8	3.8	5.0	4.8	4.5	3.7	3.5	3.0	2.1	40
South Atlantic Coast	2.5	2.6	3.3	4.3	4.1	4.8	4.4	4.3	4.1	3.7	3.3	2.6	44
Gulf Coast	2.3	2.7	3.7	4.5	4.5	4.5	4.9	5.0	5.0	4.6	3.9	2.4	48
Ohio Valley	1.8	2.2	2.8	5.1	4.7	5.3	6.0	6.0	5.5	4.2	3.4	2.0	49
Great Lakes Region	0.7	1.0	1.2	2.3	3.1	4.1	4.8	4.7	3.3	2.7	1.9	1.2	31
Upper Mississippi Valley	0.6	1.0	1.7	3.2	3.5	4.5	6.2	5.4	3.8	3.1	2.1	0.9	36
Middle Mississippi Valley	1.2	1.6	2.5	5.3	4.6	4.7	6.4	7.0	5.2	4.3	3.5	1.7	48
Missouri Valley	0.8	1.3	1.5	4.0	3.8	4.6	6.2	4.4	4.0	3.5	2.7	1.2	38
Northern Rocky Mt. Slope	1.1	2.3	1.8	5.0	5.0	6.0	7.2	6.1	5.4	3.6	2.9	1.6	48
Middle Rocky Mt. Slope	2.0	2.6	2.8	5.1	4.6	6.3	6.7	6.3	5.2	4.4	3.8	2.2	52
Southern Rocky Mt. Slope	3.2	3.6	4.4	6.0	7.5	8.3	8.8	8.9	5.3	4.7	4.2	3.1	68
Northern Plateau	1.1	2.4	3.8	5.6	6.5	6.0	9.2	8.0	5.6	3.7	2.2	1.9	56
Middle Plateau	1.4	2.5	3.6	6.3	6.5	8.1	9.2	9.2	7.4	5.5	3.9	2.4	66
Southern Plateau	3.4	4.0	5.2	7.8	9.2	11.7	9.8	9.4	7.1	6.7	5.2	3.5	83
North Pacific Coast	1.1	1.2	2.2	2.5	3.4	3.0	3.2	3.0	2.6	2.0	1.6	1.2	27
Middle Pacific Coast	2.5	3.5	4.3	4.5	4.7	5.4	6.4	6.3	6.6	7.6	4.2	3.0	59
South Pacific Coast	3.3	2.5	2.8	3.9	4.1	4.5	5.4	5.7	4.5	5.0	3.3	3.0	48

Note that the above table shows *increasing* evaporation toward the *South* for any section where conditions other than temperature are about equal. Evaporation is *less* in regions adjacent to *large bodies of water* (as the Atlantic, Pacific and Great Lakes regions) for the same latitude, because there is greater humidity in the atmosphere. Conversely, it is to be noted that the great dry plateau region in the West furnishes the greatest evaporation. Evaporation on the North Pacific Coast is low because of the almost incessant rains for six months of the year. But south of Oregon the Pacific Coast shows greater evaporation than the Atlantic Coast of the same latitude, because of dryness of the atmosphere and higher mean temperature.

Seepage.—The seepage in a catchment area is equal to the total rainfall minus the run-off and evaporation. When the ground is frozen the seepage is practically naught, while in deep, sandy soil it may equal nearly the total precipitation. It is usual to estimate the seepage and evaporation together as the *total loss* in a catchment, reservoir, stream or canal. If the latter has locks or gates, leakage is also included. The loss due to seepage and evaporation combined, for a canal in an earthen bed, will vary generally from 1½ to 2½ inches per day, depending upon size of canal, character of soil, etc. The rate of seepage often decreases with the age of the canal, as the fine particles pack into the soil bed and decrease the voids. Experiments with models have shown that the ratio of annual seepage to annual evaporation for total rainfall (with no run-off), in temperate climates is, for earth bed, about 1 : 2.4; and for sandy bed, about 5 : 1. These figures, however, cannot apply to reservoirs and canals where the process of evaporation is continuous, which was not the case with the experiments. Roughly speaking, the comparative rates of seepage in various soils is about as follows, assuming that for sandy soil to be unity: Minute gravel, 100; coarse sand, 10; fine sand, 2; sandy soil, 1; sandy clay, 0.5; clay, 0.25 to 0.00.

Mr. Elwood Mead states* that the loss from seepage and evaporation as determined from a large number of measurements made by the U. S. Dept. of Agriculture, was 2.47 per cent per mile in 1900, and 1.45 per cent per mile in 1901—grouped as follow:

Capacity of Canals.	Loss per mile. Per cent.
(*a*). Canals carrying 100 cubic ft. per second or more	0.98
(*b*). Canals carrying between 50 and 100 cu. ft. per second	2.67
(*c*). Canals carrying between 25 and 50 cu. ft. per second	5.22
(*d*). Canals carrying less than 25 cubic feet per second	7.48

It is interesting to note here that the above table may be generalized by the following law: The total loss per mile for canals carrying from 25 to 200 cubic feet per second was, in 1900 and 1901, from 1.9 to 2.0 cubic feet per second.

From evaporation alone it is probable that the loss rarely exceeds 1 per cent per mile in irrigation canals, when the velocity is 2½ to 3 feet per second. Reservoir losses from the same cause may vary from 2 to 7½ feet in depth, annually; generally from 3 to 5 ft.; rough average, 4 ft. (see Table 4, preceding page).

* "Irrigation Institutions," pp. 123-4.

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EXCERPTS AND REFERENCES.

Relation of Rainfall to Runoff in California (By J. B. Lippincott and S. G. Bennett. Eng. News, June 5, 1902).—The following basins are discussed: Sacramento River Basin, San Matea Creek, Salt Springs Valley Watershed, Stanislaus River Basin, Tuolumne River Basin, San Joaquin and Kings River Basin, Mojave River, Cuyamaca Reservoir Watershed, Sweetwater Reservoir Basins. A diagram shows the annual and mean runoff from the above watersheds. A table gives the number of "acre-ft. per sq. mile" for "depth of runoff in ins." for depths advancing by hundredths of an inch up to one inch. The table is based on 1" depth of runoff = 53⅓ acre-ft. per square mile = 640 ÷ 12.

Formulas for Computing the Cost of Impure Water Supplies (Eng. News, Nov. 15, 1906).

Works for the Purification of the Water Supply of Washington, D. C. (By Allen Hazen and E. D. Hardy. Trans. A. S. C. E., Vol. LVII).

Railways and Water Pollution, With Special Reference to the Water Supply of Seattle (Eng. News, Dec. 27, 1906).

Report of the Proposed 226-Mile Aqueduct for the Water Supply of Los Angeles, Cal. (Eng. News, Jan. 24, 1907).—Board composed of J. R. Freeman, F. P. Stearns, J. D. Schuyler. Total estimated cost, $24,485,000.

Length of Time Required to Determine Rainfall or Stream Flow Within a Given Percentage of Error (By J. C. Stevens. Eng. News, Sept. 17, 1908). —Diagram.

A New Water Supply for the City of Vancouver, B. C. (By H. M. Burwell. Eng. News, April 1, 1909).—Cost data on wood-stave pipe and steel pipe.

The Purification of Ground Waters Containing Iron and Manganese (By R. S. Weston. Trans. A. S. C. E., Vol. LXIV).

Illustrations of Water Supply Works:—

Description.	Eng. News.
Cross-section of concrete water supply conduit, Los Angeles	Aug. 27, 1903.
The California or "stovepipe" method of well construction	Nov. 12, '03.
Brick and cast-iron infiltration conduits, Columbus, O.	Feb. 11, '04.
Water-filter plant at Danville, Ill.; 11 illustrations	Aug. 28, '04.
Settling tank, and sinking shoe for pump well, Ithaca	April 20, '05.
Plans and details of sedimentation basin, Charlestown, W. Va.	June 7, '05.
Cross-sections of new conduit for Vienna water supply	Oct. 11, '06.
Cross-section of reinforced-concrete pressure conduit that fail'd	Oct. 29, '08.
Aqueduct, 7 x 40 ft.; steel frame, rein.-conc. lining	Feb. 24, '10.
	Eng. Rec.
Proposed inclined wells at shore of Lake Superior	June 19, '09.
Catskill Aqueduct, cut-and-cover construction	Jan. 8, '10.
Cross-section of 5½-ft. rein.-conc. pressure pipe line	Feb. 19, '10.
Details of anchorage for 42-in. pipe line	Feb. 19, '10.
Section (14' 2" dia.) Moodna conc. pressure tunnel	June 4, '10.
Rein.-conc. siphon for 65-ft. head, Los Angeles Aqueduct	July 9, '10.
7-ft. dia. rein.-conc. conduit under 12 to 16-ft. head	July 23, '10.

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64.—WATER WORKS.

This subject is treated under five main headings, as follows:

A.—Consumption of Water.....Page 1202.
B.—Purification of Water.....Page 1204.
C.—Reservoirs.....Page 1205.
D.—Conduits.....Page 1207.
E.—Distributing System.....Page 1280.

A.—CONSUMPTION OF WATER.

The "consumption" of water in cities and towns is measured in gallons per capita per day, of population supplied,* and includes the total amount of water "used" and *wasted*. The amount of water actually needed for domestic supply could undoubtedly be restricted to 20 gallons per capita per day, but there is a great deal of careless and wanton waste, as well as use for fire protection, manufacturing purposes, etc. Roughly spreaking, it is customary to estimate the probable consumption at 50, 60 or 80 gallons for small cities and towns about to be supplied, and based on a population 20 years in the future. For large cities, see Tables 1 and 2 following.

Water meters are most frequently used in connection with manufacturing plants; and where water is not plentiful they are useful in restricting undue waste in domestic supply, otherwise they should be omitted, as a free use of water promotes cleanliness.

1.—Population in Thousands of Persons in Various Cities of the U. S.

(To accompany Table 2, following.)

Year.	Boston. Cochituate Works. (1)	Boston. Mystic Works. (2)	Chicago. (3)	Philadelphia. (4)	Brooklyn. (5)	St. Louis. (6)	Cincinnati. (7)	Cleveland. (8)	Detroit. (9)	Milwaukee. (10)	Louisville. (11)	Providence. (12)	Lowell. (13)	Fall River. (14)	Cambridge. (15)	Lynn. (16)	New Bedford. (17)	Salem, Mass. (18)
1850	140	.....	30	121	97	78	115	17	21	20	43	42	33	12	15	14	16	20
1860	178	.....	112	566	267	161	161	43	46	45	68	51	37	14	26	19	22	22
1870	225	87	299	674	396	311	216	93	80	71	101	69	41	27	40	28	21	24
1880	306	108	503	847	567	351	255	160	116	116	124	105	59	49	53	38	27	28
1890	410	118	1209	1047	839	496	305	262	206	204	112	141	78	75	70	56	42	31
1900	.....	.....	2008	1294	1065	595	332	397	289	285	163	187	95	108	92	74	63	36
1905	†732	‡596	2060	1438	1313	714	358	462	359	343	178	214	95	107	97	83	75	38
1906	.....	.....	.....	.....	.....	750	.....	.....	384	.....	.....	.....	.....	.....	.....	85	.....	.....

* A source of error in gathering statistics on consumption of water is to include the total population of a town as being served from a certain supply, whereas a portion of the population, sometimes large, may be served from some other supply, often from wells.

† Boston, Somerville, Chelsea and Everett.

‡ Metropolitan Water Works (Boston only).

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2.—*Daily Average Consumption of Water in Gallons per Capita per Day in Various Cities of the U. S.

(See Population Statics, Table 1, preceding.)

Year	Boston. Cochituate Works. (1)	Boston. Mystic Works. (2)	Chicago. (3)	Philadelphia. (4)	Brooklyn. (5)	St. Louis. (6)	Cincinnati. (7)	Cleveland. (8)	Detroit. (9)	Milwaukee. (10)	Louisville. (11)	Providence. (12)	Lowell. (13)	Fall River. (14)	Cambridge. (15)	Lynn. (16)	New Bedford. (17)	Salem, Mass. (18)
'50	42	.....	....	....	....	....	20	....	....	....	....	....	....	....	....	....	....	....
'55	64	.....	....	....	....	....	25	....	44	....	....	....	....	....	....	....	....	....
'60	97	.....	43	36	....	....	30	14	52	....	....	....	....	....	....	....	....	....
'65	66	27	42	50	29	....	29	22	55	....	18	....	....	....	....	....	....	....
'70	66	44	73	55	47	....	48	31	64	....	23	....	....	....	44	....	....	31
1	60	56	72	55	47	....	53	36	76	....	21	....	....	....	43	....	....	38
2	63	70	74	54	53	45	54	40	88	....	22	....	....	....	38	....	....	45
3	72	77	88	56	59	51	50	43	98	....	23	....	....	....	48	41	....	55
4	72	73	96	58	54	55	55	45	97	....	24	....	....	12	50	42	....	51
'75	69	86	100	69	59	61	60	44	120	29	24	....	24	18	57	40	....	....
6	71	80	103	....	57	62	68	49	111	45	27	....	29	25	51	32	....	59
7	72	75	....	58	59	66	64	56	111	69	29	24	30	26	53	32	....	58
8	80	76	123	54	58	67	66	51	110	85	30	26	32	25	45	32	....	53
9	87	88	....	66	60	72	68	63	125	96	33	30	35	27	47	34	....	55
'80	87	87	112	68	54	72	76	65	130	108	42	34	38	28	46	32	....	55
1	94	80	....	71	56	76	87	77	145	109	56	35	40	30	46	32	86	59
2	95	73	110	76	58	76	69	68	132	114	47	33	43	36	45	37	82	63
3	97	74	....	76	58	75	66	76	146	108	52	37	46	31	47	36	78	61
4	73	65	114	74	61	63	74	83	159	101	56	34	48	26	46	39	72	60
'85	73	68	116	72	64	67	64	93	176	105	62	37	56	26	51	42	85	62
6	74	72	118	80	65	73	74	91	176	110	65	37	59	27	56	44	86	64
7	80	72	120	89	65	73	88	96	197	117	64	37	62	25	60	48	85	77
8	87	75	119	100	67	74	99	95	204	105	62	40	69	28	63	45	88	71
9	81	69	123	110	67	73	99	99	172	116	67	41	62	27	62	43	89	68
'90	83	71	126	132	68	78	111	106	161	109	100	48	69	29	62	45	98	69
1	90	75	132	140	70	77	138	111	155	112	98	50	74	31	60	49	92	87
2	96	79	135	143	79	83	123	118	154	101	103	55	73	28	66	53	88	81
3	107	86	157	150	86	83	124	130	165	108	104	60	79	27	75	54	99	66
4	101	89	152	159	85	....	129	113	156	108	97	64	75	27	69	61	85	67
'95	104	83	158	162	84	84	135	137	163	101	115	57	81	36	72	68	84	62
6	117	88	157	168	86	86	110	129	141	96	101	56	82	36	76	68	89	66
7	118	88	153	185	90	85	108	136	134	89	102	52	78	36	76	70	95	64
8	Boston, Somerville, Chelsea and Everett.	Met. W. W. (Boston only.)	147	196	84	85	95	138	143	85	100	54	78	32	86	70	102	65
9			156	199	85	....	98	153	159	85	100	53	81	35	87	79	107	70
'00			161	221	86	106	115	169	162	83	92	54	83	35	79	65	101	75
1			192	211	83	....	121	169	163	80	102	55	74	34	81	62	91	78
2			194	232	83	111	125	168	162	81	98	58	57	40	85	62	91	83
3			196	237	84	123	130	142	157	80	103	65	55	38	90	63	96	86
4	140	149	203	234	88	131	137	139	179	84	101	67	53	36	90	64	96	85
'05	140	151	200	227	91	101	124	131	168	86	103	68	58	41	93	59	95	88
6	.....	.....	....	....	....	....	....	....	160	....	....	....	....	....	....	....	....	....

* Data in Tables 1 and 2 up to the year 1890 are from Paper No. 758 of rans. A. S. C. E., Vol. XXXIV, entitled "Consumption and Waste of Vater" by Dexter Brackett. From 1890, the data have been gathered by ie writer, from official sources.

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B.—PURIFICATION OF WATER.

The purification of water for domestic use has received considerable attention in the United States during the past 15 or 20 years, and in Europe for a much longer period. Speaking generally, there are three main steps in the purification of surface water, from streams or reservoirs, namely, (1) screening; (2) sedimentation; (3) filtration. The last named is often omitted.*

Screening.—At the approach to the intake end of the pipe there should be placed a grating composed of vertical bars of wood, iron or steel to screen out the leaves and prevent the drift generally from entering the pipe. These bars are placed edgewise to the direction of flow and are stayed laterally by small rods and fillers.

Sedimentation.—The screened water is next allowed to pass into a settling basin of greater or less capacity to deposit, while at rest, the sedimentary matter which it held in suspension while in an agitated state. Turbid waters, those suspending sand, silt and clay, should necessarily thus be treated. If the particles of clay are minute, that is, as fine or finer than bacteria, it becomes necessary to add a coagulant to the water before it enters the basin. Various compounds of alum, lime and iron are used as coagulants. Perhaps sulphate of alumina is the most common. If a fair amount of coagulant is used, the time required for sedimentation will be about 24 hours, depending upon various conditions. It is to be noted that sedimentation with coagulation may be efficient in removing not only the fine particles of suspended matter above referred to, but a large percentage of bacteria as well.

Settling basins may be single or multiple and provided for continuous or intermittent flow.

"Slow" Sand Filtration.—This is essentially the so-called English system, having been employed almost exclusively in England since the early part of the Nineteenth Century. The filter beds consist of one or more water-tight reservoirs each with an area of 10,000 to 80,000 square feet. On the bottom of each reservoir is laid a system of drains for carrying off the filtered water. The materials composing the beds proper are broken stone, gravel and sand, placed in successive layers and graduated to size, with the coarser material at the bottom. The top bed of sand usually varies from 24 to 60 inches in thickness and should be composed of fine grains of uniform size.

The rate of filtration usually averages from 6 to 9 feet of water column per day of 24 hours with good results. Care must be used to see that the drains carry off the filtered water freely. The water may be delivered to the filter beds by gravity or by pumping. If it contains much sediment it should first be passed through the settling basin as previously described under Sedimentation. Two systems are employed—the continuous and the intermittent.

The result of sand filtration when highly efficient is to remove from the water its impurities and render it potable. These impurities are injurious bacteria, vegetable organic compounds (sometimes coloring the water highly) and animal pollution, as sewage. The last named is the most easily removed. A good sand filter usually removes from 98.5 to 99.5 per cent of bacteria. To eliminate vegetable organic coloring effectively, coagulants are employed—compounds of lime, alum, iron, etc.

During filtration a thin film forms on the surface of the sand and acts as a fine strainer, collecting most of the bacteria and other impurities. Although the process is (purely?) mechanical in theory yet it is a fact that water is often rendered purer *chemically* by having passed through the filter. The oxygen of the air (and of the water) is the great purifier.

"Rapid" Sand Filtration.—This system is distinctly American and is generally termed "mechanical" filtration. Mechanical filters are patented

*Another process, the Copper Sulphate treatment, is being used considerably where algae is present in the water. Many articles descriptive of copper sulphate as an algaecide, for reservoir treatment, etc., have appeared in current technical literature during the past ten years; also, of sulphate of alumina, hypochlorite of lime, etc.

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devices for increasing the rate of filtration, by agitating the sand mechanically or by compressed air, and by the use of coagulants. They have generally given satisfaction and bid fair under many conditions to compete with ordinary sand filters in beds. The first cost of mechanical filters is less than for sand filters, but the cost of operation is greater.

C.—RESERVOIRS.

Reservoirs may be classed as follows:

(*a*) Storage Reservoirs, comprising large natural basins with dam (see Dams, page 844), and wasteway constructed at outlet.

(*b*) Distributing Reservoirs (artificial) constructed in earth by excavation and embankment, with paved inner slopes:
High-service reservoirs;
Low-service reservoirs.

(*c*) Stand-Pipes or water towers of steel, reinforced-concrete or wood.

(*a*) **Storage Reservoirs*** of greater or less extent are commonly to be had on most running streams, generally near the source. At the lower end of the natural basin and at a contracted point of the out-flowing stream, as in a canon, the dam is constructed of sufficient height to guarantee a storage supply sufficient for the dry or summer months. If the supply is for domestic use the storage will be estimated in millions of gallons; if for irrigation it will be estimated in acre ft. (one acre-foot = 43,560 cubic feet); if for water power it will be estimated in cubic feet or millions of cubic feet.

(*b*) **Distributing Reservoir** sites should be selected with great care. A side-hill location should be examined very carefully for any indications of sliding land, a serious condition for reservoir construction. A slide usually reveals a wet or marshy spot at its upper end. The water collects there, filters to an inclined sub-stratum (generally clay) and by moistening or lubricating it the coefficient of friction is reduced to such an extent that sliding takes place. If, unfortunately, it is discovered that a reservoir has been constructed on such a slide, the hill-side above and around the reservoir should be sub-drained thoroughly at the sliding plane to keep it as dry as possible.† This sub-drainage may be accomplished by driving small tunnels, with branches, and laying drain tile to carry away the water.

Distributing Reservoirs situated immediately at the townsite are convenient and economical. Incidentally they serve as secondary settling basins, and hence provision should be made for "blowing" them out whenever the bottoms accumulate much sediment. The waste pipe leads from a depression in the bottom of the reservoir, and small water pipes are laid for occasional washing and cleansing. Economically, distributing reservoirs serve to lower the static head on the distributing system of pipes through the town, by receiving the supply direct, through the pressure-pipe line, from the storage reservoir or headworks. By this means the pressure on the system may often be reduced several hundred feet. If, however, the storage reservoir is not at too high an altitude the pressure-pipe line may be connected directly with the mains and allowed to discharge into them, but usually only in case of fire when high pressure is needed. The high-service and low service reservoirs are primarily designed to supply the high and the low sections of the town, respectively, each connected with its own distributing system.

It is to be noted that considerable power for lighting and pumping can often be developed at the distributing reservoirs by utilizing the available head from the source above. By this means water may be pumped to a second-pipe for supplying an isloated section of the town at considerable elevation and distance.

The outlet pipe from a reservoir should be so arranged that water may be taken at different elevations above the bottom of the reservoir, in order to avoid suspended matter and sediment.

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* See Paper entitled "Lake Cheesman Dam and Reservoir" by Messrs. Morrison and Woodard, in Trans. Am. Soc. C. E., Vol. LIII (Dec., 1904) page 89, for a very complete description of a typical storage reservoir, with construction of dam, wasteway, outlet, etc.

† See Paper entitled "A Phenomenal Land Slide" by D. D. Clarke, in Trans. Am. Soc. C. E., Vol. LIII (Dec., 1904), page 322.

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Reservoir Linings.—The bottom may be lined with 6 inches or more of concrete laid with expansion joints; on this, from $\frac{3}{4}$ to $\frac{1}{2}$ inch of cement mortar; next, a coat of liquid asphalt; and finally, a harder coat of asphalt. The side slopes may be lined with 6 inches or more of concrete laid with expansion joints; a coat of asphalt; a layer of brick dipped in hot asphalt and laid flat; and then a finishing coat of asphalt, filling up all the joints. Instead of the brick lining, a coating of asphalt or asphalt cement is sometimes employed. Expansion joints are spaced from 10 to 20 feet, depending upon the climate.

It is generally advisable to discharge the water into the reservoir through an aerating fountain.

(*c*) **Stand-Pipes** are simply upright riveted steel pipes with the lower end capped, resting on a foundation usually of concrete, and thoroughly anchored. The plates, joints and rivets will have to resist various complicated stresses if the stand-pipe is in a freezing climate:

(*a*) For static pressure, $t = \frac{0.434\,h\,r}{e\,f}$ [See notation below.].....(1)

(*b*) If the water at surface of tank is frozen and pumps are acting to force water into the tank there is an *added pressure* which would have to be reduced to equivalent head.

(*c*) Or, if the water subsides from the ice there is a *minus pressure* produced by the resulting vacuum, which may tend to collapse the tank.

(*d*) The wind pressure is a very considerable item which has to be reckoned within the design of large stand-pipes. The resisting moment M_R'' in inch-lbs., for a cylindrical section of stand-pipe, due to bending, is

$$M_R'' = \frac{fI}{y} = f\pi r^2\,t\,e \quad \text{.....(2)}$$

where f = allowable fiber stress in lbs. per sq. in.;
I = moment of inertia of cross-section, in ins.;
$y = r$ = radius of pipe or tank, in ins.;
$\pi = 3.1416$;
t = thickness of metal shell, in ins.;
e = efficiency of riveted joint, say 0.50 to 0.80.

But the bending moment M_B' in ft.-lbs., is

$$M_B' = \frac{6P}{10} \cdot \frac{d\,h^2}{2} \quad \text{.....(3)}$$

where P = wind pressure in lbs. per sq. ft for *flat* surfaces;
$\frac{6P}{10}$ = wind pressure in lbs. per sq. ft for cylindrical diameters;
d = diameter of tank, in feet;
h = depth of section considered, in feet, below top of tank.

Equating the resisting moment (2) with the bending moment (3), multiplying the latter by 12 to reduce to inch-lbs., we have

$$f\,\pi\,r^2\,t\,e = 3.6\,P\,d\,h^2 \quad \text{.....(4)}$$

whence

$$f = \frac{3.6\,P\,d\,h^2}{\pi\,r^2\,t\,e} = \frac{4.6\,P\,h^2}{d\,t\,e} \quad \text{.....(5)}$$

and if P is taken at 50 lbs. per sq. ft., we have

$$f = \frac{230\,h^2}{d\,t\,e} \quad \text{.....(6)}$$

and

$$t = \frac{230\,h^2}{d\,f\,e} \quad \text{.....(7)}$$

(*e*) In order to provide for possible corrosion, the calculated thickness of shell may be increased by say $\frac{1}{16}$ in., as with riveted steel water pipe. It should be noted that the thickness of shell at top of tank should not be less than $\frac{1}{4}$ inch.

A common method employed in the design of stand-pipes is to calculate for static pressure (*a*), wind pressure (*d*), and allow for corrosion (*e*). If soft steel is used, with an ultimate strength of 60,000 lbs., f may be taken at

15,000 lbs. where no freezing is expected. But otherwise this should be reduced to 12,000 or 10,000 lbs., depending upon the climate. The tank should be well stiffened against collapse.

An Elevated Tank is a tank supported at an elevation, usually on tower posts, hence the name water-tower. The whole structure is sometimes enclosed in a circular masonry shell.

Stand-pipes and elevated tanks are more economical in certain localities than are ordinary reservoirs. They are useful in furnishing a supply to districts where the population is small or isolated; or to sections above the altitude of the main reservoirs. They are often used in towns equipped with the direct pumping system, to provide safe pressure in case of fire; and are connected with the main line, as close to the pumping station as possible.

D.—CONDUITS.

After the water has been purified to a greater or less degree at the Headworks by *sedimentation* and perhaps by *filtration*, it is delivered by the *conduit* to the high- or low-service reservoirs or to stand-pipes, situated as near the town as proper altitude and selection of site will permit.

Conduits (or aqueducts) may be classified as follows:

I. Open conduits, or those which follow the hydraulic grade line.
- (*a*) Canals.
- (*b*) Flumes.

II. Closed conduits which follow relatively the hydraulic grade line.
- (*a*) Bored wooden pipe, not banded.
- (*b*) Salt glazed vitrified pipe.
- (*c*) Masonry aqueducts, including cement pipe.

III. Pressure pipe lines.
- (*a*) Bored wooden pipe, banded.
- (*b*) Wood stave pipe (banded).
- (*c*) Cast iron pipe.
- (*d*) Wrought iron pipe.
- (*e*) Steel pipe.
- (*f*) Attachments.
- (*g*) Specials.

Ia.—CANALS.

Where the topography will admit, considerable saving of cost may often be made by employing open channel construction from the headgate to the settling basin instead of the more expensive pipe-line construction usually adopted. But the sanitation must also be considered, as well as the possibility of freezing.

Ib.—FLUMES.

Wooden flumes may be substituted for canals (above) for small towns where a cheap construction is desired, as in the West. Flumes may be V-shaped, rectangular, or semi-circular (see Irrigation, page 1317). It is to be noted that no *open* conduit construction is to be allowed below the point where the water is purified. Steel flumes are usually semicircular.

IIa.—BORED WOODEN PIPE.

Bored Wooden Pipe has been used to a small extent in the West in certain localities where timber is cheap. Its use, however, has been confined mainly to the smaller sizes, say 6 inches or less in internal diameter. Joints are made by using wooden or cast iron hubs to couple the connecting ends. When not banded they are seldom subjected to very much static pressure. The writer has seen specimens of bored pipe 5 inches in diameter which had been removed after being in service about 40 years.

IIb.—SALT GLAZED PIPE.

Salt Glazed Vitrified Pipe is specially adapted to economic construction on small work where the topography of the ground will admit of the pipe line being laid fairly close to the hydraulic grade line, and where the static pressure is inconsiderable. It has been used successfully up to 20 inches in diameter.*

* See paper read before Am. W. W. Ass'n at Cleveland, O., April 7, 1888, by Mr. Stephen E. Babcock.

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IIc.—MASONRY AQUEDUCTS.

Masonry Aqueducts are particularly economical in conveying large supplies of water under little or no pressure. Various shapes of section are used, as circular, egg shape, rectangular with arch, and tunnel shape. The first and last named are typical of the New Croton aqueduct, the circular section being 14 feet in diameter. The ordinary section is shaped like a horse-shoe resting on an inverted arch. Rubble masonry, concrete and brick are variously employed, the first named being confined to the side walls, and the last named to the lining. The brick lining is laid generally in one, two or three layers. Concrete may replace the rubble and brick to a greater or less extent. Reinforced concrete is used for high pressures.

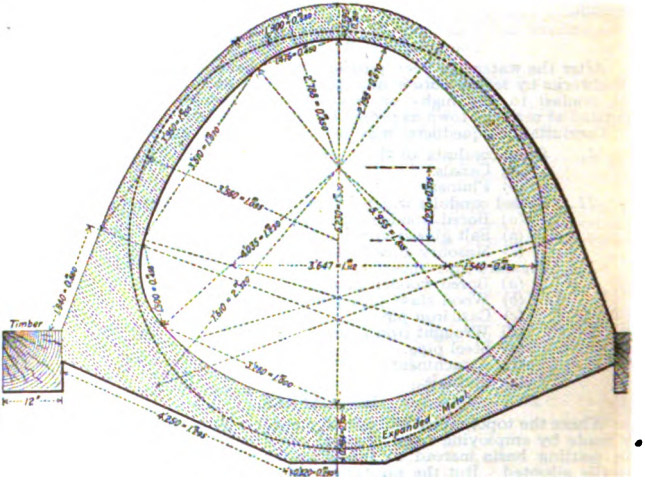

Fig. 1.—Reinforced Concrete Aqueduct.

Fig. 1 is a section of reinforced concrete aqueduct for the City of Mexico J. D. Schuyler, consulting engineer (see *Eng. News*, April 19, 1906).

IIIa.—BORED WOODEN PIPE, BANDED.

Bored Wooden Pipe (IIa) if Banded may be used under pressure, but it should be avoided generally for better construction. The cheaper form of banding is by spiral wound wiring.

IIIb.—WOOD STAVE PIPE.

Wood Stave Pipe has had a variable reputation and has been the subject of much discussion among engineers. The writer has laid a great

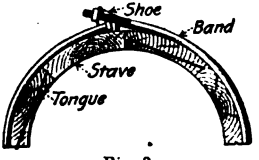

Fig. 2.

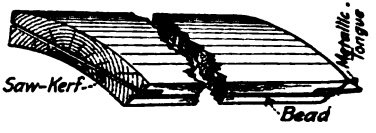

Fig. 2a.

many miles of this pipe and can recommend it for cheapness of first cost and for carrying capacity. But it yet remains to be demonstrated to what

tent it will compare in economy with other kinds of pipe when its lasting alities are better known.

Fig. 2 shows a half-section of stave pipe with staves, tongues, band ith head, nut and washer) and shoe. Fig. 2a is a view of an ordinary .ve showing the saw-kerf at one end, and the metallic tongue erted at the other. On the nearest edge is shown a beaded projection t by the planer on the finished stave to insure water-tightness, but as this ıd is subject to injury it is often omitted. Note that the tongue projects newhat beyond either edge of the stave and is embedded in adjoining ves when the pipe is cinched tightly. The metallic tongues are usually, t not always, galvanized before being placed in the pipe. Fig. 3 is a plan

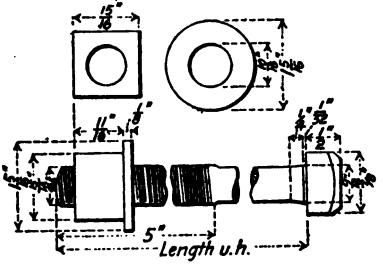

Fig. 3.

ǂ-inch band (upset) with head, nut and washer. Note that the upset is y long to allow for proper cinching, and that the length is "under head." nds should be protected from rust before the pipe is back-filled. A ımon method in the West is to dip them in hot asphalt after bending. e bending and dipping plants may be erected on the ground where the e is laid if the work is of considerable magnitude; otherwise they should ve the mill ready for laying. Fig. 4 illustrates the ordinary malleable ı shoe or coupling for the bands, and explains itself.

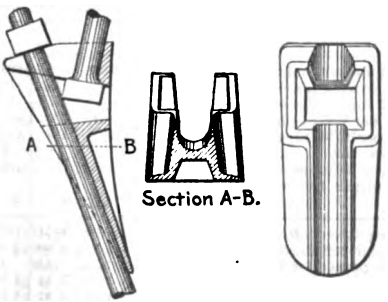

Fig. 4.

Table 3, following, was prepared by the writer some years ago, and be found useful to those who desire to make estimates on wood stave construction.

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3.—Wood Stave Pipe and Details—

Item						
	Internal diameter of pipe, ins.	8"	10"	12"	14"	16"
1	*Thickness t of staves	1	$1\frac{1}{16}$	$1\frac{1}{8}$	$1\frac{3}{16}$	$1\frac{1}{4}$
2	Finished width W, ins.	$3\frac{7}{16}$	$3\frac{7}{16}$	$3\frac{11}{16}$	$3\frac{3}{8}$	$5\frac{1}{16}$
3	Partial depth N, ins.	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{2}$
4	Partial depth O, ins.	$\frac{11}{16}$	1	$1\frac{1}{16}$	$1\frac{5}{16}$	$1\frac{5}{16}$
5	Finished width w, ins.	$2\frac{3}{8}$	$2\frac{13}{16}$	$3\frac{1}{16}$	$3\frac{1}{8}$	$4\frac{1}{8}$
6	Size of rough stave, ins.	$1\frac{1}{2}$x4	$1\frac{1}{2}$x4	$1\frac{1}{2}$x4	$1\frac{1}{2}$x4	2x6
7	No. of staves to the circle	9	11	12	14	11
8	Ft. B. M. of rough lumber per 100′ of pipe	450	550	600	700	1100
9	Size of tongue—width and B. W. G.	1"x$\frac{5}{16}$	1"x$\frac{5}{16}$	1"x$\frac{5}{16}$	1"x$\frac{5}{16}$	1"x$\frac{5}{16}$
10	No. of tongues per 100′ of pipe (average)	45	55	60	70	55
11	Wt. of tongues per 100′ of pipe, lbs.	3	4	4	5	6
	Size of band, ins.	$\frac{5}{16}$"x$\frac{7}{16}$" oval.				
12	Length of band (under head), ft. and ins.	2–11$\frac{1}{2}$	3–6$\frac{1}{2}$	4–1$\frac{1}{2}$	4–7$\frac{1}{2}$	5–2$\frac{1}{2}$
13	Wt. of band and upset, nut and head, lbs.	1.19	1.40	1.60	1.80	1.99
14	Value of s in lbs. for band section, factor of 4.	1611				
15	Allowable s in lbs. to avoid crushing staves	1035	1255	1475	1611	1611
16	Closest band spacing practicable, ins.	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$
17	Band spacing for 100′ head, ins.	$4\frac{1}{16}$+	$4\frac{3}{8}$+	$4\frac{1}{16}$−	$4\frac{1}{2}$−	$3\frac{3}{8}$+
18	No. of bands per 100′ of pipe for 100′ head	277	274	271	284	318
19	Wt. of bands per 100′ of pipe for 100′ hd., lbs.	330	384	434	511	633
20	Wt. of one shoe, lbs.	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{5}{8}$
21	Wt. of shoes per 100′ of pipe for 100′hd., lbs.	139	137	136	142	239
22	Wt. of washers per 100′ of pipe for 100′ hd., lbs.	21	21	21	22	24
	Size of band, ins.	$\frac{3}{4}$" round.				
23	Length of band (under head), ft. and ins.	2–11$\frac{1}{2}$	3–6$\frac{1}{2}$	4–1$\frac{1}{2}$	4–7$\frac{1}{2}$	5–2$\frac{1}{2}$
24	Wt. of band and upset, nut and head, lbs.	1.23	1.44	1.65	1.86	2.06
25	Value of s in lbs. for band section, factor of 4.	1656				
26	Allowable s in lbs. to avoid crushing staves	700	849	998	1146	1291
27	Closest band spacing practicable, ins.	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$
28	Band spacing for 100′ head, ins.	$2\frac{13}{16}$	3	3	3	3
29	No. of bands per 100′ of pipe for 100′ head	409	405	401	399	396
30	Wt. of bands per 100′ of pipe for 100′ hd., lbs.	503	583	662	742	816
31	Wt. of one shoe, lbs.	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{5}{8}$
32	Wt. of shoes per 100′ of pipe for 100′ hd., lbs.	205	203	200	200	297
33	Wt. of washers per 100′ of pipe for 100′ hd., lbs.	31	31	31	31	30
34	Area of pipe, sq. ft. = cu. ft. per ft. of length	.34907	.54542	.78540	1.0690	1.3963
35	Gallons of water per lin. ft. of pipe	2.611	4.080	5.875	7.997	10.44
36	Velocity v in ft. per sec. for n = .0105; s = .0001	.355	.420	.490	.550	.612
37	" " " " " s = .001	1.30	1.54	1.77	1.98	2.19
38	" " " " " s = .01	4.20	4.97	5.70	6.37	6.99
39	Discharge in million gals. per 24 hrs.; s = .0001	.08	.15	.25	.38	.55
40	" " " " " s = .001	.29	.54	.90	1.37	1.98
41	" " " " " s = .01	.95	1.76	2.88	4.40	6.30

*Finished thickness t of staves, in inches.

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— For Pipe Diameters 8 to 38 Inches.*

22″	24″	26″	28″	30″	32″	34″	36″	38″	Item
1⅜	1⅜	1⅜	1 7/16	1½	1½	1½	1 9/16	1 9/16	1
5½	5 9/16	5⅝	5⅝	5¾	5½	5½	5 9/16	5⅝	2
9/16	9/16	¾	¾	¾	9/16	9/16	9/16	9/16	3
1 11/32	1 11/32	1 11/32	1 13/32	1½—	1½—	1½—	1 9/16	1 9/16—	4
4½	5	5½	5½	5½	5	5 1/16	5 1/16+	5 1/16	5
2x6	2x6	2x6	3x6	2x6	2x6	2x6	2x6	2x6	6
14	15	16	17	18	20	21	22	23	7
1400	1500	1600	1700	1800	2000	2100	2200	2300	8
4 1½″×⅞″14	1½″×⅞″14	1½″×⅞″14	1½″×⅞″14	1½″×⅞″14	1½″×⅞″14	1½″×⅞″14	1½″×⅞″14	1½″×⅞″14	9
70	75	80	85	90	100	105	110	115	10
12	13	14	15	19	20	21	22	23	11
⅜″ round.				7/16″ round.					
6–10¼	7–4½	7–10⅝	8–5½	9–1¼	9–7⅝	10–1⅞	10–8½	11–2⅞	12
2.68	2.88	3.07	3.28	4.84	5.11	5.38	5.64	5.93	13
1656				2254					14
1656	1656	1656	1656	2254	2254	2254	2254	2254	15
1½	1½	1½	1½	1 9/16	1 9/16	1 9/16	1 9/16	1 9/16	16
2 1/16+	2 3/16	2 7/16	2⅝	3	2 27/32	2 1/16	2 7/16	2 7/16	17
411	442	473	508	399	423	445	470	493	18
1101	1273	1452	1656	1931	2162	2394	2651	2923	19
¾	¾	¾	¾	1	1	1	1	1	20
308	332	355	381	399	423	445	470	493	21
31	34	36	39	31	32	34	36	38	22
7/16″ round.				½″ round.					
6–11⅝	7–5⅝	8–0	8–6⅝	9–1½	9–7¾	10–2	10–8¾	11–3	23
3.74	4.01	4.28	4.56	6.39	6.74	7.09	7.46	7.81	24
2254				2944					25
1893	2049	2199	2254	2722	2888	2944	2944	2944	26
1 9/16	1 9/16	1 9/16	1 9/16	1⅝	1⅝	1⅝	1⅝	1⅝	27
3 1/16	3⅜	3⅝	3 7/32	3¾	3⅝	3 17/32	3 11/32	3 9/16	28
359	358	357	373	330	329	341	359	381	29
1343	1436	1528	1701	2109	2217	2418	2678	2976	30
1	1	1	1	1½	1½	1½	1½	1½	31
359	358	357	373	413	411	427	449	476	32
28	27	27	29	33	33	34	36	38	33
7 3.6398	3.1416	3.6870	4.2761	4.9087	5.5851	6.3050	7.0686	7.8758	34
19.75	23.50	27.58	31.99	36.72	41.78	47.16	52.88	58.92	35
.785	.850	.913	.947	.996	1.05	1.09	1.13	1.18	36
2.74	2.93	3.07	3.26	3.40	3.56	3.73	3.86	4.00	37
8.90	9.34	9.96	10.39	10.83	11.35	11.87	12.30	12.73	38
1 34	1.72	2.18	2.62	3.16	3.79	4.43	5.16	6.02	39
4.67	5.95	7.32	9.00	10.8	12.8	15.2	17.6	20.4	40
15.0	19.0	23.8	28.7	34.3	40.9	48.3	56.5	64.8	41

he first five items are dimensions of finished staves (see Fig.).

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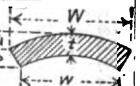
3.—Wood Stave Pipe and Details (Concluded)—



Item	Internal diameter of pipe, ins	40"	42"	44"	46"	48"	50"
1	*Thickness t of staves	$1\frac{3}{16}$	$1\frac{1}{8}$	$1\frac{1}{8}$	$1\frac{1}{8}$	$1\frac{11}{16}$	$1\frac{11}{16}$
2	Finished width W, ins	$5\frac{5}{8}$	$5\frac{11}{16}$	$5\frac{11}{16}$	$5\frac{11}{16}$	$5\frac{3}{4}$	$5\frac{3}{4}$
3	Partial depth N, ins	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{3}{16}-$	$\frac{3}{16}-$	$\frac{3}{16}-$
4	Partial depth O, ins	$1\frac{7}{16}-$	$1\frac{3}{8}-$	$1\frac{3}{8}-$	$1\frac{3}{8}-$	$1\frac{11}{16}-$	$1\frac{11}{16}-$
5	Finished width w, ins	$5\frac{1}{4}-$	$5\frac{1}{4}$	$5\frac{3}{8}-$	$5\frac{3}{16}+$	$5\frac{3}{8}$	$5\frac{3}{8}+$
6	Size of rough stave, ins	2x6	2x6	2x6	2x6	2x6	2x6
7	No. of staves to the circle	24	25	26	27	28	29
8	Ft. B. M. of rough lumber per 100' of pipe	2400	2500	2600	2700	2800	2900
9	Size of tongue—width and B. W. G.	$1\frac{1}{2}$"x#12	$1\frac{1}{2}$"x#12	$1\frac{1}{2}$"x#12	$1\frac{1}{2}$"x#12	$1\frac{1}{2}$"x#12	$1\frac{1}{2}$"x#12
10	No. of tongues per 100' of pipe (average)	120	125	130	135	140	145
11	Wt. of tongues per 100' of pipe, lbs.	32	33	35	36	38	46
	Size of band, ins	$\frac{1}{2}$" round.					
12	Length of band (under head), ft. and ins.	11–$9\frac{1}{4}$	12–4	12–$10\frac{1}{4}$	13–$4\frac{1}{2}$	13–$11\frac{3}{4}$	14–$5\frac{3}{4}$
13	Wt. of band and upset, nut and head, lbs.	8.16	8.53	8.88	9.23	9.59	9.95
14	Value of s in lbs. for band section, factor of 4	2944					
15	Allowable s in lbs. to avoid crushing staves.	2944	2944	2944	2944	2944	2944
16	Closest band spacing practicable, ins	$1\frac{3}{8}$	$1\frac{3}{8}$	$1\frac{3}{8}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$
17	Band spacing for 100' head, ins	$3\frac{1}{32}$	$2\frac{29}{32}$	$2\frac{15}{16}$	$2\frac{25}{32}$	$2\frac{9}{16}$	$2\frac{11}{16}$
18	No. of bands per 100' of pipe for 100' head	395	413	432	450	470	487
19	Wt. of bands per 100' of pipe for 100' hd., lbs.	3223	3523	3836	4154	4507	4846
20	Wt. of one shoe, lbs. (x2=2 shoes per band)	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$
21	Wt. of shoes per 100' of pipe for 100' hd., lbs.	494	516	540	562	588	609
22	Wt. of washers per 100' of pipe for 100' hd., lbs.	39	41	43	45	47	49
	Size of band, ins	$\frac{5}{8}$" round.					
23	Length of band (under head), ft. and ins.	11–$9\frac{3}{8}$	12–$4\frac{1}{4}$	12–$10\frac{1}{2}$	13–$4\frac{7}{8}$	13–$11\frac{1}{2}$	14–$5\frac{1}{2}$
24	Wt. of band and upset, nut and head, lbs.	12.96	13.54	14.08	14.63	15.21	15.75
25	Value of s in lbs. for band section, factor of 4	4602					
26	Allowable s in lbs. to avoid crushing staves.	4312	4525	4602	4602	4602	4602
27	Closest band spacing practicable, ins	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{5}{8}$	$1\frac{5}{8}$	$1\frac{5}{8}$
28	Band spacing for 100' head, ins	$4\frac{7}{16}$	$4\frac{7}{16}$	$4\frac{3}{32}$	$4\frac{3}{16}$	4	$3\frac{15}{32}$
29	No. of bands per 100' of pipe for 100' head	270	270	277	288	300	313
30	Wt. of bands per 100' of pipe for 100' hd., lbs.	3499	3656	3900	4213	4563	4914
31	Wt. of one shoe, lbs. (x2=2 shoes per band)	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$
32	Wt. of shoes per 100' of pipe for 100' hd., lbs.	405	405	416	432	450	468
33	Wt. of washers per 100' of pipe for 100' hd., lbs.	32	32	33	34	35	36
34	Area of pipe, sq. ft. = cu. ft. per ft. of length	8.7267	9.6211	10.559	11.541	12.566	13.63
35	Gallons of water per lin. ft. of pipe	65.28	71.97	78.99	86.33	94.00	102.0
36	Velocity v in ft. per sec. for n=.0105; s=.0001	1.23	1.27	1.31	1.36	1.40	1.44
37	" " " " " s=.0101	4.16	4.29	4.42	4.55	4.68	4.81
38	" " " " " s=.01	13.24	13.66	14.08	14.49	14.90	15.30
39	Discharge in million gals. per 24 hrs. s=.0001	6.95	7.88	8.93	10.1	11.4	12.7
40	" " " " " " s=.001	23.5	26.6	30.1	33.9	38.0	42.3
41	" " " " " " s=.01	74.7	84.2	96.0	108.0	121.0	134.8

* Finished thickness t of staves, in inches.

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— For Pipe Diameters 40 to 72 Inches.*

54″	56″	58″	60″	62″	64″	66″	68″	70″	72″	Item
2 3/8	2 3/8	2 1/2	2 1/2	2 1/2	2 1/2	2 9/16	2 9/16	2 9/16	2 5/8	1
7 5/8—	7 9/16	7 5/8	7 3/4—.	7 3/4+	7 3/4—	7 11/16	7 3/4	7 3/4—	7 9/16+	2
1/2	1/2—	1/2—	1/2 —	1/2—	9/16+	9/16+	9/16+	9/16+	9/16—	3
2 1/2—	2 1/2—	2 1/2—	2 1/2—	2 1/2—	2 1/2—	2 9/16—	2 9/16—	2 9/16—	2 5/8—	4
7+	7+	6 3/4+	7 1/2	7 9/16	7 9/16	7 5/8	7 5/8—	7 11/16+	7 11/16—	5
2 1/2x8	2 1/2x8	2 1/2x8	3x8	3x8	3x8	3x8	3x8	3x8	3x8	6
24	25	26	26	27	28	29	30	31	32	7
4000	4167	4333	5200	5400	5600	5800	6000	6200	6400	8
2 1/4″x5/8 10	1 3/4″x5/8 10	1 3/4″x5/8 10	1 3/4″x 5/8 10	1 3/4″ x 5/8 10	1 3/4″ x 5/8 10	1 3/4″ x 5/8 10	1 3/4″ x 5/8 10	2″x5/8 10	2″x5/8 10	9
120	125	130	130	135	140	145	150	155	160	10
61	63	64	67	70	73	75	76	95	97	11
ound.	5/8″ round.					3/4″ round.				
15–8 1/2	16–3 3/8	16–9 3/8	17–11 3/8	18–5 1/2	19–0	19–7 1/8	20–1 5/8	20–7 1/2	21–2 5/8	12
10.79	17.63	18.17	20.02	20.57	21.12	31.31	32.09	32.89	33.72	13
2944	4602					6627				14
2944	4602	4602	4602	4602	4602	6627	6627	6627	6627	15
1 1/2	1 5/8	1 5/8	1 3/4	1 3/4	1 3/4	2	2	2	2	16
2 3/4	3 13/32	3 3/32	3 1/2	3 9/16	2 31/32	4 1/8	4 1/16	3 15/32	3 13/32	17
534	353	364	382	393	405	290	297	305	314	18
5762	6223	6614	7648	8084	8554	9080	9531	10031	10588	19
1 1/2	1 1/2	1 1/2	1 3/4x2	1 3/4 x2	1 3/4x2	2x2	2x2	2x2	2x2	20
668	530	546	1337	1375	1417	1160	1188	1220	1413	21
53	41	42	45	46	47	47	48	49	51	22
ound.	3/4″ round.					7/8″ round.				
15–9 1/2	16–3 3/4	16–10	17–11 5/8	18–6 1/2	19–0 1/2	19–7 5/8	20–1 7/8	20–8 1/2	21–2 1/2	23
17.09	25.44	26.22	28.90	29.68	30.61	42.69	43.76	44.82	45.95	24
602	6627					9020				25
4602	6627	6627	6627	6627	6627	9020	9020	9020	9020	26
1 5/8	1 3/4	1 3/4	1 3/4	1 3/4	1 3/4	2 1/4	2 1/4	2 1/4	2 1/4	27
3 1/2	4 13/32	4 3/4	4 19/32	4 5/8	4 9/32	5 5/8	5 1/2	5 5/8	5 9/16	28
342	245	253	265	273	281	213	217	224	231	29
5845	6283	6634	7659	8103	8601	9093	9496	10040	10614	30
1 1/2	1 3/4	1 3/4	2x2	2x2	2x2	2 1/2x2	2 1/2x2	2 1/2x2	2 1/2x2	31
513	429	443	1060	1092	1124	958	976	1008	1040	32
40	40	41	43	44	45	53	54	56	58	33
8 15.904	17.104	18.348	19.635	20.966	22.340	23.758	25.220	26.725	28.274	34
119.0	127.9	137 3	146.9	156.8	167.1	177.7	188.7	199.9	211.5	35
1.52	1 56	1.59	1.64	1.67	1.71	1.75	1.79	1.83	1.85	36
5.03	5.16	5.28	5.41	5.50	5.63	5.75	5.87	5.96	6.08	37
16.02	16.42	16.82	17.22	17.50	17.90	18.29	18.57	18.96	19.35	38
15.6	17.2	18.9	20.8	22.5	24.7	26.8	29.2	31.2	33.8	39
51.7	57.0	62.6	68.5	74.0	81.2	88.0	95.5	103.0	111.0	40
165.0	181.0	199.0	218.0	235.0	259.0	280.0	300.0	325.0	350.0	41

The first five items are dimensions of finished staves (see Fig.).

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NOTES ON TABLE 3, PRECEDING.

Thickness of staves given is the maximum. Some engineers limit the thickness to 2 ins. even for the large sizes of pipe.

Staves can be planed readily by grinding the blades of the sticker to the proper shapes.

Sizes of rough staves are merchantable. The lumber must be clear and seasoned without checking, and *before planing*, otherwise the subsequent shrinkage will greatly affect the finished diameter of pipe.

The metal tongues should be galvanized smoothly and evenly to prevent leaky pipes; but are sometimes left plain.

Length of band may be obtained from following formulas:

For angle shoes, $L_1 = \pi\,(D + 2t + \frac{d}{2}) + d + \text{upset length} - \frac{1}{2}$.

For double shoes, $L_2 = \pi\,(D + 2t + \frac{d}{2}) + 2(d + \text{upset length} - \frac{1}{2})$.

in which L_1 = length of single band under head, in ins.
L_2 = total length of double band under head, in ins.,
π = 3.1416,
D = internal diameter of pipe in ins.,
t = thickness of stave in ins.,
d = diameter of band in ins.,

Care must be used in ordering bands. It is better to have them a little long than short. Wood stave pipe is apt to form up with a diameter from ⅛" to ¼" too large, which must be taken into consideration when ordering bands.

Bands are proportioned to not exceed a hydrostatic-pressure stress of 15,000 lbs. per sq. in. Small bands, especially on pipes of small diameter, are apt to crush the wood, hence a lower stress than 15,000 is assumed to meet this contingency, in necessary cases, as shown by the preceding table.

4.—DISCHARGE IN MILLION GALLONS PER 24 HOURS THROUGH WOOD STAVE PIPE. NEW, CLEAN AND IN FIRST-CLASS CONDITION.

(Value of roughness n = .010)

[Million Gallons per 24 Hours.]

Grade or Slope s.	Diameter of Pipe, in Inches.								
	12	16	20	24	30	36	48	60	72
.0001	0.26	0.60	1.09	1.78	3.33	5.48	11.9	21.8	35.6
.00015	0.30	0.74	1.38	2.27	4.16	6.85	14.9	27.0	43.8
.0003	0.50	1.11	2.05	3.35	6.12	9.96	21.6	38.9	63.0
.0005	0.67	1.47	2.69	4.40	8.03	13.10	28.2	50.9	82.6
.0008	0.85	1.86	3.40	5.58	10.20	16.50	35.6	64.6	104.0
.001	0.95	2.07	3.81	6.25	11.40	18.50	39.8	72.2	117.0
.003	1.68	3.65	6.68	10.90	19.90	33.40	69.5	126.0	202.0
.005	2.17	4.74	8.70	14.20	25.70	43.00	90.2	162.0	263.0
.008	2.75	6.00	11.00	18.00	32.50	53.10	114.0	206.0	332.0
.010	3.07	6.71	12.30	20.10	36.30	59.30	127.0	230.0	371.0
.016	3.88	8.50	15.50	25.40	46.00	75.10	161.0	291.0	469.0
.020	4.34	9.49	17.40	28.40	51.40	83.90	180.0	325.0	534.0

Note.—For designing ordinary wood stave pipe lines it is safer to use a coefficient of roughness n = .0105 (Table 3) rather than .010 as above. Compare discharges in Table 4 with those in Table 3.

IIIc.—CAST IRON PIPE.

Cast Iron Pipe is more durable than wood-stave-, wrought-iron-, or steel pipe, but its first cost is greater for the ordinary sizes required in a pressure-pipe line or in a distributing system. It especially commends itself for use in large cities where the increased first cost can be borne easily; also for certain portions of any pipe line demanding fairly permanent construction, that is, where cost of renewal would be excessive; and generally where "specials" are required. Before deciding on the kind of pipe to

use, chemical analyses should be made of the soil and the water, and the question of electrolysis also should be considered. Cast iron pipe should be dipped in hot coal tar or asphalt or a mixture of the two before being laid in the ground.

Formulas for Designing Cast Iron Pipe are numerous, but they all agree in taking into consideration (1) the static pressure, (2) water ram, and (3) liability to breakage from rough handling before and during the laying.

NOTATION.

t = thickness of pipe shell, in inches;
d = inside diameter of pipe, in inches;
h = pressure head in feet ($=2.304\,p$);
p = pressure of water, in lbs. per square inch ($=0.434h$);
p' = allowance for water ram, in lbs. per square inch;
r = internal radius of pipe, in ins.;
s = allowable tensile stress, in lbs. per square inch;

Based on a factor of safety of 5, s is assumed at 3,200 to 3,600 lbs.

For static pressure alone, we have,

$$t=\frac{pd}{2s}=\frac{0.217\,h\,d}{s}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Practical working formulas are as follows:

Formula used by Metropolitan Water-works, of Boston:—

$$t=\frac{(p+p')d}{2s}+0.25=\frac{(0.434\,h+p')d}{6600}+0.25\ldots\ldots(2)$$

assuming $s=3300$. Allowances for water ram are—

$p'=120$ lbs. (277 ft. hd.) for $d=$ 3 to 10 ins.;
$=110$ " (254 " ") " $d=12$ or 14 ins.;
$=100$ " (231 " ") " $d=16$ or 18 ins.;
$=90$ " (208 " ") " $d=20$ ins.;
$=85$ " (196 " ") " $d=24$ ins.;
$=80$ " (185 " ") " $d=30$ ins.;
$=75$ " (173 " ") " $d=36$ ins.;
$=70$ " (162 " ") " $d=42$ to 60 ins.

Formula recommended by R. D. Wood & Co., Philadelphia:—

$$t=\frac{(p+100)d}{7200}+0.333\left(1-\frac{d}{100}\right)\ldots\ldots\ldots\ldots\ldots(3)$$

Here, $s=3600$ (or $2s=7200$); p' is assumed at the constant value 100; and the added thickness to allow for rough handling is represented by the last term, a variable dependent on the diameter. But formula (3) easily reduces to

$$t=\frac{(p+76)d}{7200}+0.333\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

which is readily comparable with (2).

Kinds of Pipe Joints.—Cast iron pipe 3 ins. or more in diameter comes in a standard laying length of 12 feet. There are four principal types of joints, namely, (1) bell and spigot; (2) turned, page 1235; (3) flanged, page 1235; and (4) flexible joint, page 1238. These are discussed as follows.

(1) **Bell and Spigot Joint Pipe** (see p. 1220) is the most common, being almost universally employed when cast iron pipe is to be laid in trenches and back-filled. In laying the pipe it is customary to begin with some "special" as a gate or tee, etc., thrusting the spigot end of each pipe into the bell of the piece previously placed, "bell holes" having been dug in the trench where each joint is to come. When blocked or tamped to proper line and grade the joints are caulked at the inner or spigot end with oakum, jute or hemp, and then the balance of the joint is run with hot lead, and caulked tightly. In order to confine the molten lead so it will fill the joint flush with the end of the bell, a gasket is clamped around the entering pipe tightly against the bell of the other, leaving an opening at the top for pouring. Improvised gaskets may be made wholly out of clay, but manufactured gaskets composed of layers of rubber and hemp cloth, and backed by steel springs are universally employed. The ends are clamped nearly together, a pouring hole is made with clay, and the metal poured. (See page 1219 for notes on pipe laying; page 1280 for lead-melting furnace.)

5.—Cast Iron Bell and Spigot Pipe and Joints. Including Lead and Hemp per Joint.
(The McNeal Pipe and Foundry Co.)

Inside Diam. of Pipe in Inches.	Head, 100 Feet; Pressure, 43 Lbs.			Head, 200 Feet; Pressure, 86 Lbs.			Head, 300 Feet; Pressure, 130 Lbs.			Head, 400 Feet; Pressure, 173 Lbs.			Lead per Joint.	Hemp per Joint.	Inside Diam. of Pipe in Inches.
	Thickness. Ins.	Weight, Lbs. per Foot.	Weight, Lbs. per Length.	Thickness. Ins.	Weight, Lbs. per Foot.	Weight, Lbs. per Length.	Thickness. Ins.	Weight, Lbs. per Foot.	Weight, Lbs. per Length.	Thickness. Ins.	Weight, Lbs. per Foot.	Weight, Lbs. per Length.	Lbs.	Lbs.	
3	.38	13.9	167	.42	15.4	185	.45	16.7	200	.45	16.7	200	4.4	.18	3
4	.40	19.2	230	.42	20.3	243	.45	21.7	260	.47	22.1	265	5.5	.21	4
5	.42	24.6	295	.45	26.3	315	.48	28.2	338	.51	29.6	355	6.8	.27	5
6	.43	30.4	364	.47	32.8	393	.51	35.5	426	.54	37.1	445	8.0	.31	6
8	.47	42.8	513	.51	47.3	567	.56	52.0	624	.61	55.4	665	11.5	.44	8
10	.50	57.1	685	.56	63.8	765	.62	71.0	852	.68	76.7	920	14.5	.53	10
12	.53	72.5	870	.60	82.1	985	.68	92.5	1110	.75	100.8	1210	18.0	.61	12
14	.56	89.5	1074	.65	102.4	1229	.73	116.6	1399	.82	128.3	1540	21.5	.81	14
16	.60	107.8	1293	.69	124.7	1496	.79	143.6	1723	.89	158.3	1900	24.0	.94	16
18	.63	127.7	1532	.74	149.	1788	.85	172.1	2065	.96	191.7	2300	27.0	1.00	18
20	.66	149.	1788	.78	175.3	2104	.91	203.7	2444	1.03	228.3	2740	31.5	1.25	20
24	.75	200.6	2407	.87	233.6	2803	1.02	274.9	3299	1.16	306.7	3680	37.0	1.50	24
30	.87	290.2	3482	1.01	335.6	4027	1.19	398.6	4783	1.37	451.7	5420	51.0	2.06	30
36	.98	391.6	4699	1.14	455.	5460	1.36	545.3	6543	1.58	624.2	7490	75.0	3.00	36
40	1.09	483.9	5807	1.23	543.8	6525	1.48	654.8	7858	1.72	754.2	9050	85.0	3.37	40
42	1.10	512.3	6147	1.28	591.7	7100	1.54	713.6	8563	1.79	824.2	9890	90.0	3.62	42
48	1.25	665.2	7982	1.41	745.5	8946	1.71	904.8	10857	1.99	1045.8	12550	110.0	4.37	48
60	1.40	916.7	11000	1.68	1105.0	13260	2.05	1336.7	16040	2.41	1580.8	18970	150.0	6.25	60

All weights approximate only. Above sizes cast in lengths to lay 12′—440 lengths per mile. 1½″ and 2″ pipe in shorter lengths.

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(From R. D. Wood & Co. Prepared by G. A. Ellis and A. H. Howland.)

U. S. Gals. Discharged Per Min.	U. S. Gallons Discharged per Twenty-four Hours.	4-Inch Pipe.			6-Inch Pipe.			8-Inch Pipe.			10-Inch Pipe.			12-Inch Pipe.			14-Inch Pipe.		
		Veloc. Ft. per Sec.	Fric. Head. Feet.	Fric. Head. Lbs.	Veloc. Ft. per Sec.	Fric. Head. Feet.	Fric. Head. Lbs.	Veloc. Ft. per Sec.	Fric. Head. Feet.	Fric. Head. Lbs.	Veloc. Ft. per Sec.	Fric. Head. Feet.	Fric. Head. Lbs.	Veloc. Ft. per Sec.	Fric. Head. Feet.	Fric. Head. Lbs.	Veloc. Ft. per Sec.	Fric. Head Feet.	Fric. Head Lbs.
25	36,000	.64	.59	.26	.28	.11	.05	.16	.04	.02	.10	.02	.01	.07	.01	.....	.....	.....	.....
50	72,000	1.28	2.01	.87	.57	.32	.14	.32	.10	.04	.20	.04	.02	.14	.02	.01	.10	.01	.....
100	144,000	2.55	7.36	3.19	1.13	1.08	.47	.64	.29	.13	.41	.11	.05	.28	.05	.02	.21	.03	.01
150	216,000	3.83	16.05	6.95	1.70	2.28	.99	.96	.60	.26	.61	.22	.10	.43	.10	.04	.31	.05	.02
200	288,000	5.11	28.09	12.17	2.27	3.92	1.70	1.28	1.01	.44	.82	.36	.16	.57	.16	.07	.42	.08	.04
250	360,000	6.37	43.47	18.83	2.84	6.00	2.60	1.60	1.52	.66	1.02	.54	.23	.71	.24	.10	.52	.12	.05
300	432,000	7.66	62.20	26.94	3.40	8.52	3.69	1.91	2.13	.92	1.23	.75	.32	.85	.32	.14	.63	.16	.07
350	504,000	8.94	84.26	36.50	3.97	11.48	4.97	2.23	2.85	1.24	1.43	.99	.43	.99	.43	.18	.73	.21	.09
400	576,000	10.21	109.68	47.50	4.54	14.89	6.45	2.55	3.68	1.59	1.63	1.27	.55	1.13	.54	.23	.83	.27	.12
450	648,000	11.49	138.43	59.96	5.11	18.73	8.11	2.87	4.61	2.00	1.83	1.58	.69	1.28	.67	.29	.94	.33	.14
500	720,000	12.77	170.53	73.87	5.67	23.01	9.97	3.19	5.64	2.44	2.04	1.93	.84	1.42	.81	.35	1.04	.40	.17
600	864,000	15.32	244.76	106.02	6.81	32.89	14.25	3.83	8.03	3.48	2.45	2.72	1.18	1.70	1.14	.49	1.25	.55	.24
700	1,008,000	17.87	332.36	143.98	7.94	44.54	19.08	4.47	10.83	4.69	2.86	3.66	1.58	1.98	1.52	.66	1.46	.73	.32
800	1,152,000	.....	.....	.....	9.08	57.95	25.10	5.09	14.05	6.08	3.27	4.73	2.05	2.27	1.96	.85	1.67	.94	.41
900	1,296,000	.....	.....	.....	10.21	73.12	31.67	5.74	17.68	7.69	3.68	5.93	2.57	2.55	2.45	1.06	1.88	1.17	.51
1,000	1,440,000	.....	.....	.....	11.35	90.05	38.99	6.38	21.74	9.41	4.08	7.28	3.15	2.84	3.00	1.30	2.08	1.43	.62
1,200	1,728,000	.....	.....	.....	13.61	129.20	55.96	7.66	31.10	13.47	4.90	10.38	4.50	3.40	4.26	1.85	2.50	2.02	.88
1,400	2,016,000	.....	.....	.....	15.88	175.38	75.97	8.94	42.13	18.25	5.72	14.02	6.07	3.97	5.74	2.49	2.91	2.72	1.18
1,600	2,304,000	.....	.....	.....	18.15	228.62	99.03	10.21	54.84	23.75	6.53	18.22	7.89	4.54	7.44	3.22	3.33	3.51	1.52
1,800	2,592,000	.....	.....	.....	20.42	288.90	125.14	11.47	69.22	29.98	7.35	22.96	9.95	5.11	9.36	4.06	3.75	4.41	1.91
2,000	2,880,000	.....	.....	.....	22.69	356.22	154.30	12.77	85.27	36.93	8.17	28.25	12.34	5.67	11.50	5.00	4.17	5.41	2.34
2,500	3,600,000	.....	.....	.....	.....	.....	.....	15.96	132.70	57.49	10.21	43.87	19.00	7.09	17.82	7.72	5.21	8.35	3.62
3,000	4,320,000	.....	.....	.....	.....	.....	.....	.....	.....	.....	12.25	62.92	27.25	8.51	25.51	11.05	6.25	11.92	5.17
3,500	5,040,000	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	9.93	34.58	14.98	7.29	16.14	6.99
4,000	5,760,000	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	8.34	21.00	9.10
4,500	6,480,000	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	9.38	26.49	11.47

This table of the frictional heads or loss in head due to friction in pipes of various diameters and 1000 ft. long, at given

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Gals. Discharged per Min.	Gals. Discharged per Twenty-four Hours.	[illegible]-inch Pipe.			[illegible]-inch Pipe.			20-inch Pipe.			24-inch Pipe.			30-inch Pipe.			36-inch Pipe.		
		Veloc. Ft. per Sec.	Fric. Head. Feet.	Fric. Head. Lbs.	Veloc. Ft. per Sec.	Fric. Head. Feet.	Fric. Head. Lbs.	Veloc. Ft. per Sec.	Fric. Head. Feet.	Fric. Head. Lbs.	Veloc. Ft. per Sec.	Fric. Head. Feet.	Fric. Head. Lbs.	Veloc. Ft. per Sec.	Fric. Head. Feet.	Fric. Head. Lbs.	Veloc. Ft. per Sec.	Fric. Head. Feet.	Fric. Head. Lbs.
500	720,000	.80	.22	.09	.63	.13	.06	.51	.08	.04	.35	.04	.02	.23	.01	.00	.16	.01	.00
1,000	1,440,000	1.60	.76	.34	1.26	.44	.19	1.02	.27	.12	.71	.12	.05	.45	.04	.02	.32	.02	.01
1,500	2,160,000	2.39	1.63	.71	1.89	.93	.40	1.53	.56	.24	1.06	.24	.10	.68	.09	.04	.47	.04	.02
2,000	2,880,000	3.19	2.82	1.22	2.52	1.60	.69	2.04	.96	.42	1.42	.41	.18	.91	.15	.06	.63	.06	.03
2,500	3,600,000	3.99	4.34	1.88	3.15	2.45	1.06	2.55	1.47	.64	1.77	.62	.27	1.13	.22	.09	.79	.09	.04
3,000	4,320,000	4.79	6.19	2.68	3.78	3.48	1.51	3.06	2.09	.90	2.13	.87	.38	1.36	.30	.13	.95	.13	.06
3,500	5,040,000	5.59	8.37	3.63	4.41	4.70	2.03	3.57	2.81	1.22	2.48	1.16	.50	1.59	.40	.17	1.10	.17	.07
4,000	5,760,000	6.38	10.87	4.71	5.04	6.09	2.64	4.08	3.64	1.58	2.84	1.50	.65	1.82	.52	.22	1.26	.22	.09
4,500	6,480,000	7.18	13.70	5.93	5.67	7.67	3.32	4.59	4.58	1.98	3.19	1.88	.82	2.04	.64	.28	1.42	.27	.12
5,000	7,200,000	7.98	16.85	7.30	6.30	9.43	4.08	5.11	5.62	2.43	3.55	2.31	1.00	2.27	.78	.34	1.58	.33	.14
5,500	7,920,000	8.78	20.33	8.71	6.93	11.38	4.92	5.62	6.77	2.93	3.90	2.77	1.20	2.50	.94	.41	1.73	.39	.17
6,000	8,640,000	.....	.....	.....	7.57	13.49	5.84	6.13	8.03	3.48	4.26	3.28	1.42	2.72	1.11	.48	1.89	.46	.20
7,000	10,080,000	.....	.....	.....	.....	.....	.....	7.15	10.86	4.71	4.96	4.43	1.92	3.18	1.49	.65	2.21	.62	.27
8,000	11,520,000	.....	.....	.....	.....	.....	.....	.....	.....	.....	5.67	5.75	2.49	3.63	1.93	.84	2.52	.80	.35
9,000	12,960,000	.....	.....	.....	.....	.....	.....	.....	.....	.....	6.38	7.25	3.14	4.08	2.43	1.05	2.84	1.00	.43
10,000	14,400,000	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	4.54	2.98	1.29	3.15	1.23	.53
11,000	15,840,000	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	5.00	3.59	1.55	3.46	1.47	.64
12,000	17,280,000	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	5.44	4.25	1.84	3.78	1.74	.75
13,000	18,720,000	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	5.90	4.97	2.15	4.09	2.03	.88
14,000	20,160,000	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	6.36	5.75	2.49	4.41	2.35	1.02
15,000	21,600,000	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	6.80	6.58	2.85	4.73	2.69	1.17
16,000	23,040,000	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	5.05	3.04	1.32
17,000	24,480,000	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	5.36	3.43	1.49
18,000	25,920,000	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	5.68	3.83	1.66
20,000	28,800,000	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	6.30	4.71	2.04

This table (6) may be used in several ways: First, to determine the maximum discharging capacity of any size of pipe under a given head; second, to find the diameter of pipe for a given discharge and given head; third, to ascertain the frictional resistance or loss of pressure for a given rate of discharge; fourth, to find the quantity of water flowing in any pipe, from the reduction of pressure.

See examples on following page.

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Examples in use of Table 6, preceding.

(1) *The maximum discharging capacity*, for instance, of 8500 ft. of 6″ straight cast iron pipe, under a head of 195 ft., may be found as follows: First ascertain the frictional head for 1000 ft., thus: 195 ft. ÷ 8.5 = 22.94 as the fric. head in ft. per 1000 ft. By referring to the table, 1000 ft. of 6″ pipe under 23.01 ft. head will discharge 500 galls. per min.

(2) *To find the diam. of pipe for a given discharge.*—To deliver, say, 4,250,000 galls. per 24 hrs., the dist. being 20,000 ft. and the head 130 ft. The fric. head per 1000 ft. is 130 ÷ 20 = 6.5 ft., and the table shows that under 6.19 ft. head per 1000 ft. a 16″ pipe will discharge 4,320,000 galls.

(3) *To ascertain the pressure at any point* in a line of water main, given its diam., rate of disch. and static head against which the water is forced. Assume that 500 galls. of water are to be forced per minute, through 5000 ft. of 8″ cast iron pipe, laid on an incline to a height of 75 ft.; what would be the varying pressure at each 1000 ft. from the pumps? The table shows a fric. head of 5.64 ft. per 1000 ft. of 8″ pipe, discharging 500 galls. per min. If the dist. is 5000 ft., the pressure required to overcome fric. is 5 × 5.64 = 28.20 ft. head, and the total resistance head at pumps is 28.2 + 75 ft. = 103.2 ft., and for every 1000 ft. from the pumps, this head is diminished 5.64 ft. + the vert. ascent; that is, at 3000 ft. from the pumps the resistance head is 61.92 ft. Resistance head in ft. ÷ 2.3 = pres. in lbs. per sq. in.

(4) *The volume of water flowing* in a pipe may be found by ascertaining the loss of pressure or fric. resistance per 1000 ft. of pipe. To this end, if two accurate gages, entirely similar in all respects, and placed 1000 ft. apart on a line of level 12″ pipe, show a loss of 0.5 lb., the nearest figures indicate a flow of 600 galls. per min.; if the loss is 2½ lbs., a flow of 1400 galls. per min., etc. Due allowance should be made for diff. in elev. of points of observation.

Pipe-Laying Notes.

The following Notes on Pipe-Laying relate to the re-construction of the distributing system at The Dalles, Oregon, in the winter of 1898–9, and are taken from the writer's note book:

1 yarner can yarn 1000 l. f. of 6″ to 10″ C. I. pipe per day.
2 caulkers can caulk 1000 l. f. of 6″ to 10″ C. I. pipe per day.
Gang laid 700 l. f. 8″ pipe on 4th Street one day = 1¼c. per l. f.
Gang took up 2400 l. f. 8″ pipe for $60.00, at rate of 2½c. per l. f.
Cost of cutting 4″, 6″, 8″, 10″, 12″ pipe = 6c, 9c, 12c, 15c, 18c.
Cheaper to place all pipe over 10″ with derrick unless ground is sandy and soft; handle 8″ and 10″ pipe with rope at each end.
4″ and 6″ pipe can be rolled in trench if soft; otherwise by hand.
4″ pipe @ 20# = 240 lbs.; two men can carry on shoulders.
6″ pipe @ 32# = 384 lbs.; three men can carry with two bars (1 behind.)
8″ pipe @ 45# = 540 lbs.; four men can carry with two bars.
10″ pipe @ 65# = 780 lbs.; six men can carry with three bars.
12″ pipe @ 75# = 900 lbs.; eight men can carry with four bars.
12″ pipe @ 85# = 1020 lbs.; nine men can carry with five bars (1 behind).
Trucked 15–4″ (3600#), 9–6″ (3456#), 7–8″ (3780#), 4–10″ (3120#), 3–12″ (2700 to 3060#) pipe per load, two horses, two men to unload.
Required ½ cord wood for melting 1000# lead (for 1000 ft. 8″ pipe).
Required ½ cord wood for melting 250 ft. of laid, 8″ pipe for taking up. Bell-holes were dug around the joints and fires kindled.

Weights and Dimensions of Cast Iron Pipe and Specials.—The following ables are mainly from a pamphlet issued by the Engineering Department of he Metropolitan Water Board of Boston. The weight of castings is based n one cubic inch of cast iron weighing 0.2604 lb. These tables are selected ecause they are based on methods and details quite generally approved ot only in New England but elsewhere. Various foundries furnish details iffering more or less from these and from each other, and hence the sub-ined data of weights, etc., must necessarily be approximate. Five classes pipe are recognized as standards: Class A for heads up to 115 ft. (50 lbs.); ass B for heads up to 150 ft. (65 lbs.); class C for heads up to 200 ft. (87 lbs); ass D for heads up to 250 ft. (109 lbs.); and class E for heads up to 300 ft. 30 lbs. per sq. inch). (See formula (2), page 1215.)

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7.—Straight Cast-Iron Pipes with Bell and Spigot.*

(Met. Water Works.)

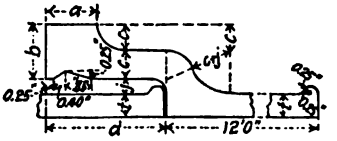

Fig. 7.

Diameter—Inches.	Class.	Dimensions in Inches.						Weight in Pounds.	
		a	b	c	d	t	j	Per Length.	Per Ft. of Straight Pipe.
4	D	1.50	1.30	0.65	3.00	0.40	0.40	230	17.3
4	E	"	"	"	"	0.45	"	255	19.7
6	D	"	1.40	0.70	"	0.46	"	380	29.2
6	E	"	"	"	"	0.50	"	415	31.9
8	D	"	1.50	0.75	3.50	0.52	"	565	43.5
8	E	"	"	"	"	0.55	"	600	46.2
10	D	"	"	"	"	0.60	"	800	62.4
10	E	"	"	"	"	0.63	"	840	65.7
12	B	"	1.60	0.80	"	0.57	"	910	70.3
12	C	"	"	"	"	0.61	"	970	75.5
12	D	"	"	"	"	0.65	"	1030	80.7
12	E	"	"	"	"	0.69	"	1095	86.0
14	B	"	1.70	0.85	"	0.61	"	1130	87.5
14	C	"	"	"	"	0.65	"	1200	93.5
14	D	"	"	"	"	0.70	"	1290	101.0
14	E	"	"	"	"	0.75	"	1380	108.6
16	B	1.75	1.80	0.90	4.00	0.65	0.50	1380	106.2
16	C	"	"	"	"	0.70	"	1485	114.8
16	D	"	"	"	"	0.75	"	1590	123.3
16	E	"	"	"	"	0.81	"	1715	133.7
20	B	"	2.00	1.00	"	0.73	"	1930	148.6
20	C	"	"	"	"	0.79	"	2080	161.2
20	D	"	"	"	"	0.85	"	2235	174.0
20	E	"	"	"	"	0.92	"	2415	188.9
24	B	2.00	2.10	1.05	"	0.80	"	2525	194.8
24	C	"	"	"	"	0.87	"	2740	212.4
24	D	"	"	"	"	0.95	"	2965	232.7
24	E	"	"	"	"	1.03	"	3220	253.1
30	B	"	2.30	1.15	4.50	0.92	"	3625	279.3
30	C	"	"	"	"	1.00	"	3930	304.3
30	D	"	2.50	1.25	"	1.10	"	4335	335.8
30	E	"	"	"	"	1.20	"	4720	367.5
36	A	"	"	"	"	0.93	"	4400	337.1
36	B	"	"	"	"	1.03	"	4850	374.4
36	C	"	"	"	"	1.13	"	5300	411.9
36	D	"	2.80	1.40	"	1.25	"	5900	457.1
36	E	"	"	"	"	1.36	"	6400	498.9
42	A	"	"	"	5.00	1.01	"	5610	424.4
42	B	"	"	"	"	1.14	"	6200	482.8
42	C	"	"	"	"	1.27	"	6975	539.4
43	D	"	3 20	1.60	"	1.40	"	7750	596.5
48	A	"	3.00	1.50	"	1.15	"	7270	554.9
48	B	"	"	"	"	1.25	"	7870	604.3
48	C	"	"	"	"	1.40	"	8700	678.9
48	D	2.25	3.50	1.75	"	1.55	"	9820	753.9
54	A	2.25	3.10	1.55	5.50	1.23	"	8770	664.9
54	B	"	"	"	"	1.35	"	9570	733.5
54	C	"	3.90	1.95	"	1.53	"	11030	834.0
60	A	"	3.20	1.60	"	1.35	"	10630	813.0
60	B	"	"	"	"	1.50	"	11750	905.6
60	C	"	4.20	2.10	"	1.70	"	13980	1029.7

* See also Tables 26, 27, 28 and 29, following.

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SPECIAL CASTINGS ARE GROUPED AS FOLLOWS:

I.—16 ins. and smaller, only Class E.
II.—20 to 24 ins., inclusive, Classes C and E.
III.—48, 54 and 60 ins., Classes B, C and E.

{Thickness of metal (except for sleeves and caps) same as for same classes of straight pipe, Table 7.

8.—BELLS OF SPECIAL CASTINGS.
(Met. W. W.)

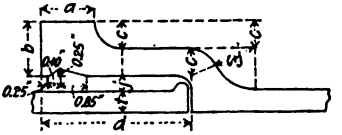

Fig. 8.

Nominal Diameter—Inches.	Class.	Dimensions in Inches.				
		a	b	c	d	j
3	E	1.50	1.20	0.60	4.00	0.40
4	"	"	1.30	0.65	"	"
6	"	"	1.40	0.70	"	"
8	"	"	1.50	0.75	"	"
10	"	"	"	"	4.50	"
12	"	"	1.60	0.80	"	"
14	"	"	1.70	0.85	"	"
16	"	1.75	1.80	0.90	5.00	0.50
20	C	"	2.00	1.00	"	"
20	E	"	"	"	"	"
24	C	2.00	2.10	1.05	"	"
24	E	"	"	"	"	"
30	C	"	2.30	1.15	"	"
30	E	"	2.50	1.25	"	"
36	C	"	"	"	"	"
36	E	"	2.80	1.40	"	"
42	C	"	"	"	"	"
42	E	"	3.20	1.60	"	"
48	B	"	3.00	1.50	"	"
48	C	"	"	"	"	"
48	E	2.25	3.50	1.75	"	"
54	B	"	3.10	1.55	"	"
60	B	"	3.20	1.60	"	"

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9.—Straight Cast-Iron Pipes, Bell and Spigot.

(Met. W. W.)

Standard, Maximum and Minimum Weights per Length and per Inch; Weight of Lead Joints and Gaskets. All Weights in Lbs.

Diam. in Ins.	Class.	St'd Wt. Per length of 12 Feet.	St'd Wt. Per Inch of Str. Pipe.	Variation Allowed. %	Max. Wt. Per length of 12 Feet.	Max. Wt. Per Inch of Str. Pipe.	Min. Wt. Per length of 12 Feet.	Min. Wt. Per Inch of Str. Pipe.	Wt. of Lead per Joint. With Gasket	Wt. of Lead per Joint. Solid Lead	Weight of Jute Gasket per Joint.
4	D	230	1.4	4	239	1.5	221	1.4	7	9.25	0.10
4	E	255	1.6	"	265	1.7	245	1.6	"	"	"
6	D	380	2.4	"	395	2.5	365	2.3	9.75	12.75	0.15
6	E	415	2.7	"	432	2.8	398	2.6	"	"	"
8	D	565	3.6	"	588	3.8	542	3.5	12.5	18.75	0.25
8	E	600	3.9	"	624	4.0	576	3.7	"	"	"
10	D	800	5.2	"	832	5.4	768	5.0	15.25	23 25	0.30
10	E	840	5.5	"	874	5.7	806	5.3	"	"	"
12	B	910	5.9	"	946	6.1	874	5.6	17.75	26.75	0.35
12	C	970	6.3	"	1009	6.5	931	6.0	"	"	"
12	D	1030	6.7	"	1071	7.0	989	6.5	18	27.25	"
12	E	1095	7.2	"	1139	7.5	1051	6.9	"	"	"
14	B	1130	7.3	"	1175	7.6	1085	7.0	20.5	31	0.40
14	C	1200	7.8	"	1248	8.1	1152	7.5	"	"	"
14	D	1290	8.4	"	1342	8.8	1238	8.1	20.75	31.5	"
14	E	1380	9.1	"	1435	9.4	1325	8.7	"	"	"
16	B	1380	8.9	"	1435	9.2	1325	8.5	31	50	0.65
16	C	1485	9.6	"	1544	10.0	1426	9.2	31	50 25	"
16	D	1590	10.3	"	1654	10.7	1526	9.9	31.25	50.5	"
16	E	1715	11.1	"	1784	11.6	1646	10.7	31.5	51	"
20	B	1930	12.4	"	2007	12.9	1853	11.9	38	61	0.80
20	C	2080	13.4	"	2163	14.0	1997	12.9	38 25	61.5	"
20	D	2235	14.5	"	2324	15.1	2146	13.9	38.5	62	"
20	E	2415	15.7	"	25.2	16.4	2318	15.1	39	63.5	"
24	B	2525	16.2	3½	26.3	16.8	2437	15.7	45	73	0.95
24	C	2740	17.7	"	2836	18.3	2644	17.1	45.25	73.5	"
24	D	2985	19.4	"	3089	20.1	2881	18.7	45.5	74	"
24	E	3230	21.1	"	33.3	21.8	3117	20.4	46	74.5	"
30	B	3625	23.3	3	3734	24.0	3516	22.6	56	100.5	1.55
30	C	3930	25.4	"	4048	26.1	3812	24.6	56.25	101	"
30	D	4335	28.0	"	4465	28.8	4205	27.1	56.5	101.5	"
30	E	4720	30.6	"	4862	31.5	4578	29.7	57	102	"
36	A	4400	28.1	"	4532	28.9	4268	27.2	66.5	120	1.95
36	B	4850	31.2	"	4995	32.1	4705	30.2	67	120.5	"
36	C	5300	34.3	"	5459	35.4	5141	33 3	67.5	121	"
36	D	5900	38.1	"	6077	39.2	5723	36.9	68	122	"
36	E	6400	41.6	"	6592	42.8	6208	40.3	68.5	123 5	"
42	A	5610	35.5	"	5778	36.6	5442	34.5	77.5	153	2.60
42	B	6290	40.2	"	6479	41.4	6101	39.0	77.5	154	"
42	C	6975	45.0	"	7184	46.3	6766	43.6	78	155	"
42	D	7750	49.7	"	7983	51.2	7518	48.2	78.5	156	"
48	A	7270	46.2	"	7488	47.6	7052	44.9	88	175	3.00
48	B	7870	50.4	"	8106	51.8	7634	48.8	88.5	176	"
48	C	8760	56.6	"	9023	58.3	8497	54.9	89	177	"
48	D	9820	62.8	"	10115	64.7	9525	61.0	89.5	178	"
54	A	8770	55.6	"	9033	57.2	8507	53.9	99	214.5	3.95
54	B	9570	61.1	"	9857	63.0	9283	59.3	99.5	215	"
54	C	11030	69.5	"	11361	71.6	10699	67.4	100	215.5	"
60	A	10630	67.8	"	10949	69.8	10311	65.7	110	238	4.40
60	B	11750	75.5	"	12103	77.7	11396	73.2	110.5	239	"
60	C	13590	85.8	"	13998	88.4	13182	83.2	111	241	"

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10.—Properties of Bells of Straight Pipes and of Special Castings.

(Met. W. W.)

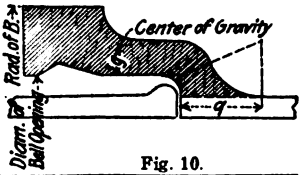

Fig. 10.

Bells of Straight Pipes.							Bells of Special Castings.					
Class.	Area (Shaded). Sq. Ins.	g	q	Diam. of Bell Openings. Ins.	Rad. of Bell. Feet.	Wt. Lbs.	Area (Shaded). Sq. Ins.	g	q	Diam. of Bell Openings. Ins.	Wt. Lbs.	Class.
E	.....	.....	.....	.....	.....	.....	3.90	0.40	1.50	4.7	15	E
D	3.61	0.46	1.55	5.60		19						
E	"	"	"	5.70	0.35	20	4.26	0.44	1.55	5.7	23	E
D	3.92	0.49	1.65	7.72		31						
E	"	"	"	7.80	0.44	31	4.62	0.47	1.65	7.8	36	E
D	4.65	0.51	1.75	9.84	0.54	41						
E	"	"	"	9.90	"	42	5.03	0.50	1.75	9.9	45	E
D	"	"	"	12.00	0.63	50						
E	"	"	"	12.06	"	50	5.40	0.49	1.75	12.1	58	E
B	5.02	0.55	1.85	13.94	0.71	62						
C	"	"	"	14.02	0.72	62						
D	"	"	"	14.10	"	62						
E	"	"	"	14.18	"	63	5.82	0.53	1.85	14.2	73	E
B	5.40	0.59	1.90	16.02	0.80	76						
C	"	"	"	16.10	0.81	76						
D	"	"	"	16.20	0.82	77						
E	"	"	"	16.30	"	77	6.25	0.57	1.90	16.3	89	E
B	6.59	0.61	2.10	18.30	0.91	105						
C	"	"	"	18.40	0.92	106						
D	"	"	"	18.50	"	106						
E	"	"	"	18.62	0.93	107	7.49	0.59	2.10	18.6	121	E
B	7.43	0.67	2.30	22.46	1.10	145						
C	"	"	"	22.58	1.11	145	8.43	0.65	2.30	22.6	165	C
D	"	"	"	22.70	"	146						
E	"	"	"	22.84	1.12	147	8.43	0.65	2.30	22.8	167	E
B	8.15	0.74	2.40	26.60	1.28	187						
C	"	"	"	26.74	1.29	188	9.20	0.72	2.40	26.6	212	C
D	"	"	"	26.90	1.30	189						
E	"	"	"	27.06	"	190	9.20	0.72	2.40	26.9	214	E
B	9.65	0.79	2.55	32.84	1.56	272						
C	"	"	"	33.00	1.57	273	10.23	0.78	2.55	32.9	289	C
D	10.63	0.85	2.75	33.20	1.59	304						
E	"	"	"	33.40	1.60	305	11.26	0.84	2.75	33.2	321	E
A	10.60	0.87	"	38.86	1.83	352						
B	"	"	"	39.03	1.84	354						
C	"	"	"	39.26	"	356	11.23	0.86	2.75	39.1	376	C
D	12.09	0.95	3.00	39.50	1.88	410						
E	"	"	"	39.72	1.89	412	12.79	0.94	3.00	39.5	433	E
A	12.80	"	"	45.02	2.11	491						
B	"	"	"	45.28	2.12	494						
C	"	"	"	45.54	2.13	497	12.80	0.95	3.00	45.3	494	C
D	15.01	1.07	3.35	45.80	2.18	589						
E	"	"	"	46.06		592	15.01	1.07	3.35	45.8	589	E
A	13.93	1.02	3.15	51.30	2.39	608						
B	"	"	"	51.50	2.40	610	13.93	1.02	3.15	51.5	610	B
C	"	"	"	51.80	2.41	614	"	"	"	51.8	614	C
D	17.13	1.23	3.60	52.10	2.46	765						
E	"	"	"	52.40		769	17.13	1.23	3.60	52.4	769	E
A	15.62	1.07	3.25	57.46	2.65	762						
B	"	"	"	57.70	2.66	765	15.62	1.07	3.25	57.7	765	B
C	20.45	1.34	3.95	58.06	2.74	1016						
A	16.20	1.13	3.35	63.70	2.92	874						
B	"	"	"	64.00	2.93	878	16.20	1.13	3.35	64.0	878	B
C	22.39	1.43	4.20	64.40	3.03	1232						

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11.—Properties of ¼ Curves.*

(Met. W. W.)

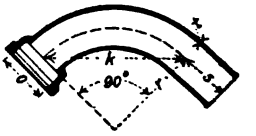

Fig. 11.

Diameter. Ins.	Dimensions in Inches.					Class.	Weight. Lbs.
	o	t	r	k	s		
4	5.7	0.45	16	22.6	8	E	80
6	7.8	0.50	"	"	"	"	125
8	9.9	0.55	"	"	10	"	185
10	12.1	0.63	"	"	12	"	265
12	14.2	0.69	"	"	"	"	340
14	16.3	0.75	18	25.5	"	"	455
16	18.6	0.81	24	34.0	"	"	680
20	22.6	0.79	"	"	"	C	835
20	22.8	0.92	"	"	"	E	950
24	26.6	0.87	30	42.4	"	C	1260
24	26.9	1.03	"	"	"	E	1465

* See also Table 31, following.

11A.—Properties of ⅛ Curves.† (Met. W. W.)

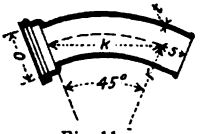

Fig. 11a.

11B.—Properties of $\frac{1}{16}$ Curves.† (Met. W. W.)

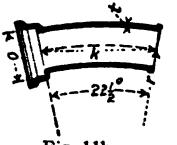

Fig. 11b.

Note.—Dimensions are in inches; weights, in lbs.

Diameter.	Class.	o	t	⅛ Curves.				$\frac{1}{16}$ Curves.		
				r	k	s	Weight	r	k	Weight
4	E	5.7	0.45	24	18.4	4	60	48	18.7	85
6	"	7.8	0.50	"	"	"	100	"	"	85
8	"	9.9	0.55	"	"	"	135	"	"	120
10	"	12.1	0.63	"	"	"	185	"	"	165
12	"	14.2	0.69	"	"	"	240	"	"	210
14	"	16.3	0.75	36	27.6		345	72	28.1	345
16	"	18.6	0.81	"	"		440	"		440
20	C	22.6	0.79	48	36.7		675	96	37.5	675
20	E	22.8	0.92	"	"		760	"	"	760
24	C	26.6	0.87	60	45.9		1050	120	46.8	1050
24	E	26.9	1.03	"	"		1215	"	"	1215
30	C	32.9	1.00	"	"		1490	"	"	1490
30	E	33.2	1.20	"	"		1780	"	"	1780
36	C	39.1	1.18	90	68.9		2810	180	70.2	2810
36	E	39.5	1.36	"	"		3400	"	"	3400
42	C	45.3	1.27	"	"		3700	"	"	3700
42	E	45.8	1.53	"	"		4470	"	"	4470
48	B	51.5	1.25	"	"		4200	"	"	4200
48	C	51.8	1.40	"	"		4650	"	"	4650
48	E	52.4	1.70	"	"		5700	"	"	5700
54	B	57.7	1.35	"	"		5100	"	"	5100
60	B	64.0	1.50	"	"		6250	"	"	6250

† See also Table 32, following.

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12.—Properties of Branches—L's, T's and Crosses.*
(Met. W. W.)

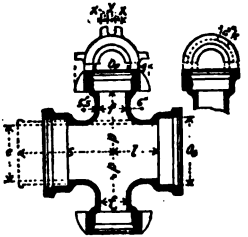

Fig. 12.

Dimensions in Inches.										Class.	Weights, in Lbs.			
											Three Way Branch.		Four Way Branch.	
e	f	l	p	s	o_s	o_r	x	y	g		2 Bells	3 Bells	3 Bells	4 Bells
4	4	11	11	23	5.70	5.70				E	120	125	155	160
6	4	11	12	"	7.80	"				"	160	165	200	205
"	6	12	"	24	"	7.80				"	190	195	245	250
8	4	11	13	23	9.90	5.70				E	215	210	250	245
"	6	12	"	24	"	7.80				"	245	240	300	295
"	8	13	"	25	"	9.90				"	270	265	345	340
10	4	11	14	23	12.1	5.70				E	285	275	320	310
"	6	12	"	24	"	7.80				"	315	305	370	360
"	8	13	"	25	"	9.90				"	345	335	415	405
"	10	14	"	26	"	12.1				"	380	370	480	470
12	4	11	15	23	14.2	5.7				E	335	340	395	380
"	6	12	"	24	"	7.8				"	390	375	440	425
"	8	13	"	25	"	9.9				"	420	405	490	475
"	10	14	"	26	"	12.1				"	460	445	550	535
"	12	15	"	27	"	14.2	1.25	1.62	2.50	"	525	510	665	650
14	4	11	16	23	16.3	5.7				E	435	415	475	455
"	6	12	"	24	"	7.8				"	475	455	525	505
"	8	13	"	25	"	9.9				"	510	490	575	555
"	10	14	"	26	"	12.1				"	550	530	640	620
"	12	15	"	27	"	14.2	1.25	1.62	2.50	"	615	595	755	735
"	14	16	"	28	"	16.3	"	"	"	"	665	645	835	815
16	4	11	17	23	18.6	5.7				E	545	530	580	565
"	6	12	"	24	"	7.8				"	580	565	630	615
"	8	13	"	25	"	9.9				"	620	605	685	670
"	10	14	"	26	"	12.1				"	665	650	750	735
"	12	15	"	27	"	14.2	1.25	1.62	2.50	"	730	715	865	850
"	14	16	"	28	"	16.3	"	"	"	"	780	765	945	930
"	16	17	"	29	"	18.6	"	"	"	"	870	855	1100	1085
20	6	12	19	24	22.6	7.8				C	710	710	760	760
"	"	"	"	"	22.8	"				E	795	770	845	820
"	8	13	"	25	22.6	9.9				C	750	750	820	820
"	"	"	"	"	22.8	"				E	840	815	905	880
"	10	14	19	26	22.6	12.1				C	800	800	890	890
"	"	"	"	"	22.8	"				E	890	865	975	950
"	12	15	"	27	22.6	14.2	1.25	1.62	2.50	C	870	870	1005	1005
"	"	"	"	"	22.8	"	"	"	"	E	965	940	1095	1070
"	14	16	"	28	22.6	16.3	"	"	"	C	925	925	1090	1090
"	"	"	"	"	22.8	"	"	"	"	E	1025	1000	1180	1155
"	16	17	"	29	22.6	18.6	"	"	"	C	1000	1000	1205	1205
"	"	"	"	"	22.8	"	"	"	"	E	1130	1095	1335	1310
"	20	19	"	31	22.6	22.6	"	"	"	C	1125	1125	1400	1400
"	"	"	"	"	22.8	22.8	"	"	"	E	1260	1235	1555	1530

* See also Table 35, following.

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12.—Properties of Branches—L's, T's and Crosses.—Continued.

Dimensions in Inches.											Weights, in Lbs.			
											3 Way Branch		4 Way Branch	
e	f	l	p	s	O_e	O_r	x	y	g	Class	2 Bells	3 Bells	3 Bells	4 Bells
24	6	12	21	24	26.6	7.8				C	915	910	980	955
"	"	"	"	"	26.9	"				E	1035	995	1080	1040
"	8	13	"	25	26.6	9.9				C	960	955	1025	1020
"	"	"	"	"	26.9	"				E	1090	1050	1150	1110
"	10	14	"	26	26.6	12.1				C	1020	1015	1105	1100
"	"	"	"	"	26.9	"				E	1150	1110	1230	1190
"	12	15	"	27	26.6	14.2	1.25	1.62	2.50	C	1090	1090	1230	1225
"	"	"	"	"	26.9	"	"	"	"	E	1240	1200	1360	1320
"	14	16	"	28	26.6	16.3	"	"	"	C	1160	1155	1315	1310
"	"	"	"	"	26.9	"	"	"	"	E	1300	1260	1450	1410
"	16	17	"	29	26.6	18.6	"		"	C	1255	1250	1475	1470
"	"	"	"	"	26.9	"	"	"	"	E	1400	1360	1610	1570
24	20	19	21	31	26.6	22.6	1.25	1.62	2.50	C	1375	1370	1640	1635
"	"	"	"	"	26.9	22.8	"	"	"	E	1560	1520	1830	1790
"	24	21	"	33	26.6	26.6	"	"	"	C	1520	1515	1860	1855
"	"	"	"	"	26.9	26.9	"	"	"	E	1730	1690	2080	2040
30	12	15	24	27	32.9	14.2	1.25	1.62	2.50	C	1490	1470	1620	1600
"	"	"	"	"	33.2	"	"	"	"	E	1740	1690	1860	1810
"	14	16	"	28	32.9	16.3	"	"	"	C	1570	1550	1720	1700
"	"	"	"	"	33.2	"	"	"	"	E	1820	1770	1960	1910
"	16	17	"	29	32.9	18.6	"	"	"	C	1680	1660	1880	1860
"	"	"	"	"	33.2	"	"	"	"	E	1940	1890	2130	2080
"	20	19	"	34	32.9	22.6	"	"	"	C	1900	1810	2140	2050
"	"	"	"	"	33.2	22.8	"	"	"	E	2210	2070	2470	2330
"	24	21	"	36	32.9	26.6	"	"	"	C	2060	1970	2370	2280
"	"	"	"	"	33.2	26.9	"	"	"	E	2410	2270	2730	2590
"	30	24	"	41	32.9	32.9	1.50	2.00	3.00	C	2410	2270	2870	2730
"	"	"	"	"	33.2	33.2	"	"	"	E	2840	2640	3320	3120
36	12	15	27	27	39.1	14.2	1.25	1.62	2.50	C	1960	1920	2070	2030
"	"	"	"	"	39.5	"	"	"	"	E	2310	2250	2410	2350
"	14	16	"	28	39.1	16.3	"	"	"	C	2050	2010	2190	2150
"	"	"	"	"	39.5	"	"	"	"	E	2410	2350	2540	2480
"	16	17	"	29	39.1	18.6	"	"	"	C	2170	2130	2360	2320
"	"	"	"	"	39.5	"	"	"	"	E	2550	2490	2720	2660
"	20	19	"	34	39.1	22.6	"	"	"	C	2450	2310	2670	2530
"	"	"	"	"	39.5	22.8	"	"	"	E	2890	2710	3120	2940
"	24	21	"	36	39.1	26.6	"	"	"	C	2640	2500	2928	2790
"	"	"	"	"	39.5	26.9	"	"	"	E	3110	2930	3490	3320
"	30	24	"	41	39.1	32.9	1.50	2.00	3.00	C	3040	2830	3450	3240
"	"	"	"	"	39.5	33.2	"	"	"	E	3610	3340	4030	3880
"	36	27	"	44	39.1	39.1	"	"	"	C	3390	3180	3940	3730
"	"	"	"	"	39.5	39.5	"	"	"	E	4050	3770	4680	4400
42	12	15	30	27	45.3	14.2	1.25	1.62	2.50	C	2520	2470	2630	2580
"	"	"	"	"	45.8	"	"	"	"	E	3010	2950	3110	3050
"	14	16	"	28	45.3	16.3	"	"	"	C	2630	2580	2760	2710
"	"	"	"	"	45.8	"	"	"	"	E	3140	3080	3250	3190
"	16	17	"	29	45.3	18.6	"	"	"	C	2780	2730	2960	2910
"	"	"	"	"	45.8	"	"	"	"	E	3290	3230	3460	3400
"	20	19	"	34	45.3	22.6	"	"	"	C	3120	2940	3330	3180
"	"	"	"	"	45.8	22.8	"	"	"	E	3720	3490	3930	3700
"	24	21	"	36	45.3	26.6	"	"	"	C	3350	3170	3600	3420
"	"	"	"	"	45.8	26.9	"	"	"	E	3990	3760	4240	4010
"	30	24	"	41	45.3	32.9	1.50	2.00	3.00	C	3830	3550	4180	3910
"	"	"	"	"	45.8	33.3	"	"	"	E	4570	4230	4980	4640
"	36	27	"	44	45.3	39.1	"	"	"	C	4210	3940	4690	4420
"	"	"	"	"	45.8	39.5	"	"	"	E	5040	4700	5590	5250
"	42	30	"	47	45.3	45.3	"	"	"	C	4700	4420	5410	5130
"	"	"	"	"	45.8	45.8	"	"	"	E	5650	5310	6460	6120
48	16	17	33	29	51.5	18.6	1.25	1.62	2.50	B	3140	3140	3320	3320
"	"	"	"	"	51.8	"	"	"	"	C	3420	3350	3600	3530
"	"	"	"	"	52.4	"	"	"	"	E	4150	4090	4300	4240
"	20	19	"	34	51.5	22.6	"	"	"	B	3510	3360	3710	3560
"	"	"	"	"	51.8	"	"	"	"	C	3840	3660	4040	3860
"	"	"	"	"	52.4	22.8	"	"	"	E	4670	4460	4900	4690

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12.—Properties of Branches—L's, T's and Crosses.—Concluded.

Dimensions in Inches.										Class.	Weights, in Lbs.			
											3 Way Branch		4 Way Branch	
e	f	l	p	s	O_s	Or	x	y	g		2 Bells	3 Bells	3 Bells	4 Bells
48	24	21	33	36	51.5	26.6	1.25	1.62	2.50	B	3750	3600	4000	3850
"	"	"	"	"	51.8	"	"	"	"	C	4110	3870	4350	4110
"	"	"	"	"	52.4	26.9	"	"	"	E	4980	4710	5210	4940
"	30	24	"	41	51.5	32.9	1.50	2.00	3.00	B	4250	4000	4600	4350
"	"	"	"	"	51.8	"	"	"	"	C	4650	4300	5000	4650
"	"	"	"	"	52 4	33.2	"	"	"	E	5660	5250	6010	5600
"	36	27	"	44	51 5	39.1	"	"	"	B	4650	4400	5080	4830
"	"	"	"	"	51 8	"	"	"	"	C	5100	4750	5510	5160
"	"	"	"	"	52.4	39.5	"	"	"	E	6200	5790	6660	6250
"	42	30	"	48	51.5	45.3	"	"	"	B	5200	4900	5800	5500
"	"	"	"	"	51.8	"	"	"	"	C	5680	5280	6300	5900
"	"	"	"	"	52.4	45.8	"	"	"	E	6900	6420	7600	7120
"	48	33	"	50	51.5	51.5	"	"	"	B	5620	5370	6400	6150
"	"	"	"	"	51.8	51.8	"	"	"	C	6150	5800	6950	6600
"	"	"	"	"	52.4	52.4	"	"	"	E	7520	7110	8490	8080

13.—Properties of Y Branches.*
(R. D. Wood & Co.)

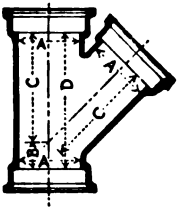

Fig. 13.

Dimensions in Inches.				Approximate Weight in Pounds.
A	B	C	D	
3	3	9	12	80
4	3	10	13	105
5	3	12	15	140
6	3.5	13.5	17	180
8	4	16	20	250
10	4 5	18.5	23	360
12	5	21	26	495
14	5.5	24.5	30	700
16	6.5	27.5	34	905
18	7	30	37	1090
20	7.5	32.5	40	1310
24	8.5	37.5	46	1920

* See also Table 37, following.

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14.—Properties of Y Branches.*
(Met. W. W.)

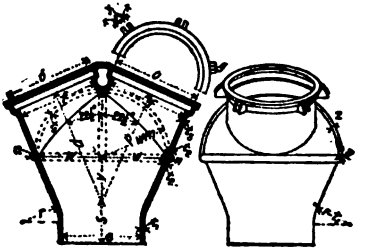

Fig. 14.

Dimensions in Inches.																Class.	Weight in Lbs.
e	f	s	p	v	w	n	r	o	t_1	t_2	t_3	z	x	y	g		
20	20	18	34	13.5	16.4	1.25	30	22.6	0.79	1.15	0.79	1.25	1.25	1.62	2.50	C	1700
"	"	"	"	"	"	1.45	"	22.8	0.92	1.35	0.92	"	"	"	"	E	1940
24	"	12	"	"	"	1.25	"	22.6	0.87	1.15	0.79	"	"	"	"	C	1690
"	"	"	"	"	"	1.45	"	22.8	1.03	1.35	0.92	"	"	"	"	E	1910
"	24	18	38	15.25	19.3	1.35	"	26.6	0.87	1.25	0.87	"	"	"	"	C	2320
"	"	"	"	"	"	1.60	"	26.9	1.03	1.50	1.03	"	"	"	"	E	2640
30	"	12	"	"	"	1.35	"	26.6	1.00	1.25	0.87	"	"	"	"	C	2300
"	"	"	"	"	"	1.60	"	26.9	1.20	1.50	1.03	"	"	"	"	E	2630
"	30	18	48	18.0	23.7	1.55	"	32.9	1.00	1.45	1.00	"	1.50	2.00	3.00	C	3740
"	"	"	"	"	"	1.90	"	33.2	1.20	1.75	1.20	"	"	"	"	E	4490
36	"	10	"	"	"	1.55	"	32.9	1.13	1.45	1.00	"	"	"	"	C	3830
"	"	"	"	"	"	1.90	"	33.2	1.36	1.75	1.20	"	"	"	"	E	4310
"	36	18	54	21.0	28.2	1.80	24	39.1	1.13	1.65	1.13	1.50	"	"	"	C	5540
"	"	"	"	"	"	2.15	"	39.5	1.36	2.00	1.36	"	"	"	"	E	6430
42	30	6	48	18.0	23.7	1.55	30	32.9	1.27	1.45	1.00	1.25	"	"	"	C	3630
"	"	"	"	"	"	1.90	"	33.2	1.53	1.75	1.20	"	"	"	"	E	4290
"	36	10	54	21.0	28.2	1.80	24	39.1	1.27	1.65	1.13	1.50	"	"	"	C	5340
"	"	"	"	"	"	2.15	"	39.5	1.53	2.00	1.36	"	"	"	"	E	6240
"	42	18	60	25.0	33.1	1.95	"	43.5	1.27	1.80	1.27	"	"	"	"	C	7500
"	"	"	"	"	"	2.45	"	45.8	1.53	2.25	1.53	"	"	"	"	E	9130
48	36	2	54	21.0	28.2	1.55	"	39.1	1.25	1.45	1.03	"	"	"	"	B	4540
"	"	"	"	"	"	1.80	"	"	1.40	1.65	1.13	"	"	"	"	C	5640
"	"	"	"	"	"	2.15	"	39.5	1.70	2.00	1.36	"	"	"	"	E	5680
"	42	10	60	25.0	33.1	1.75	"	45.3	1.25	1.60	1.14	"	"	"	"	B	6590
"	"	"	"	"	"	1.95	"	"	1.40	1.80	1.27	"	"	"	"	C	7230
"	"	"	"	"	"	2.45	"	45.8	1.70	2.25	1.53	"	"	"	"	E	8780
"	48	18	68.5	28.0	37.6	1.95	"	51.5	1.25	1.80	1.25	"	"	"	"	B	9370
"	"	"	"	"	"	2.20	"	51.8	1.40	2.05	1.40	"	"	"	"	C	10400
"	"	"	"	"	"	2.75	"	52.4	1.70	2.55	1.70	"	"	"	"	E	12750
60	"	6	"	"	"	1.95	"	51.5	1.50	1.80	1.25	"	"	"	"	B	8900

* See also Table 36, following.

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15.—PROPERTIES OF HYDRANT BRANCHES.
(R. D. Wood & Co.)

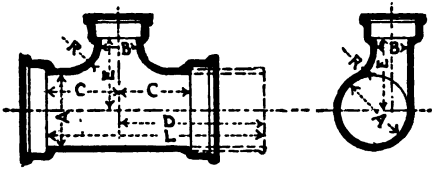

Fig. 15.

Dimensions in Inches.							Approximate Weight.
A	B	C	D	E	L	R	Lbs.
8	6	12	24	8	36	5	240
10	6	12	24	11	36	5	315
12	6	12	24	12	36	5	385
14	6	12	24	14	36	5	490
16	6	12	24	15	36	5	580
18	6	12	24	16	36	5	670
20	6	12	24	17	36	5	770
24	6	12	24	19	36	5	1000

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16.—Properties of Blow-off Branches.*
(Met. W. W.)

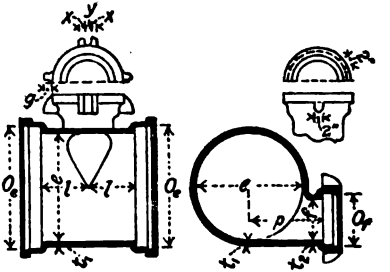

Fig. 16.

Dimensions in Inches.											Class.	Weight in Lbs.
e	f	l	p	O_e	O_f	t_1	t_2	x	y	g		
8	4	10	7	9.9	5.7	0.55	0.45	.....	.....	.....	E	195
10	"	"	8	12.1	"	0.63	"				"	255
12	"	"	10	14.2	"	0.69	"				"	320
14	"	"	11	16.3	"	0.75	"				"	390
16	"	"	12	18.6	"	0.81	"				"	500
20	6	12	14	22.6	7.8	0.79	0.50	.....	.....	.....	C	700
"	"	"	"	22.8	"	0.92	"				E	765
24	"	"	16	26.6	"	0.87	"				C	910
"	"	"	"	26.9	"	1.03	"				E	1000
30	12	13	20.5	32.9	14.2	1.00	0.69	1.25	1.62	2.50	C	1380
"	"	"	"	33.2	"	1.20	"	"	"	"	E	1560
36	"	"	23.5	39.1	"	1.13	"	"	"	"	C	1800
"	"	"	"	39.5	"	1.36	"	"	"	"	E	2100
42	"	15	26.5	45.3	"	1.27	"	"	"	"	C	2510
"	"	"	"	45.8	"	1.53	"	"	"	"	E	2900
48	12	17	29.5	51.5	"	1.25	0.69	1.25	1.62	2.50	B	3120
"	"	"	"	51.8	"	1.40	"	"	"	"	C	3340
"	"	"	"	52.4	"	1.70	"	"	"	"	E	4000
"	16	"	"	51.5	18.6	1.25	0.81	"	"	"	B	3180
"	"	"	"	51.8	"	1.40	"	"	"	"	C	3400
"	"	"	"	52.4	"	1.70	"	"	"	"	E	4150

* See also Table 38, following.

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17.—Properties of Blow-off Branches* with Manholes.
(Met. W. W.)

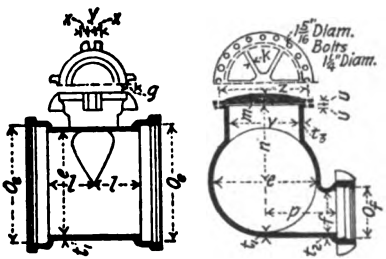

Fig. 17.

Dimensions in Inches.																						Class.	Weight. Lbs.
																				Bolts.			
s	f	l	p	o_s	o_r	t_1	t_2	x	v	g	v	n	t_3	m	u	k	s	No.	Dia		
10	12	17	20.5	32.9	14.2	1.00	0.69	1.25	1.62	2.50	20	21	1.25	1.75	1.7	1.0	25.5	20	1¼	C	2030
"	"	"	"	33.2	"	1.20	"	"	"	"	"	"	"	"	"	"	"	"	"	E	2250
16	"	"	23.5	39.1	"	1.13	"	"	"	"	"	24	"	"	"	"	"	"	"	C	2500
"	"	"	"	39.5	"	1.36	"	"	"	"	"	"	"	"	"	"	"	"	"	E	2840
12	"	"	26.5	45.3	"	1.27	"	"	"	"	"	27	"	"	"	"	"	"	"	C	3110
"	"	"	"	45.8	"	1.53	"	"	"	"	"	"	"	"	"	"	"	"	"	E	3590
18	"	"	29.5	51.5	"	1.25	"	"	"	"	"	30	"	"	"	"	"	"	"	B	3520
"	"	"	"	51.8	"	1.40	"	"	"	"	"	"	"	"	"	"	"	"	"	C	3740
"	"	"	"	52.4	"	1.70	"	"	"	"	"	"	"	"	"	"	"	"	"	E	4460
"	16	"	"	51.5	18.6	1.25	0.81	"	"	"	"	"	"	"	"	"	"	"	"	B	3600
"	"	"	"	51.8	"	1.40	"	"	"	"	"	"	"	"	"	"	"	"	"	C	3803
"	"	"	"	52.4	"	1.70	"	"	"	"	"	"	"	"	"	"	"	"	"	E	4510

* See also Table 39, following.

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18.—Properties of Manhole Pipes.*
(Met. W. W.)

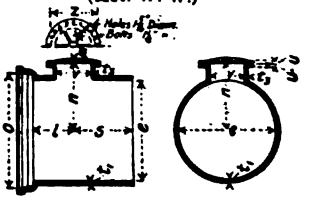

Fig. 18.

Dimensions in Inches.																
e	o	v	l	s	n	t_1	t_2	m	k	u	z	Bolts. No.	Bolts. Dia.	Class.	Weight. Lbs.
30	32.9	20	17	29	21	1.00	1.25	1.75	1.0	1.7	25.5	20	1¼	C	[illegible]
"	33.2	"	"	"	"	1.20	"	"	"	"	"	"	"	E	[illegible]
36	39.1	"	"	"	24	1.13	"	"	"	"	"	"	"	C	[illegible]
"	39.5	"	"	"	"	1.36	"	"	"	"	"	"	"	E	[illegible]
42	45.3	"	"	"	27	1.27	"	"	"	"	"	"	"	C	[illegible]
"	45.8	"	"	"	"	1.53	"	"	"	"	"	"	"	E	[illegible]
48	51.5	"	"	"	30	1.25	"	"	"	"	"	"	"	B	[illegible]
"	51.8	"	"	"	"	1.40	"	"	"	"	"	"	"	C	[illegible]
"	52.4	"	"	"	"	1.70	"	"	"	"	"	"	"	E	[illegible]
60	64.0	"	18	"	37	1.50	"	"	"	"	"	"	"	B	[illegible]

* See also Table 40, following.

19.—Properties of Sleeves.†
(R. D. Wood & Co.)

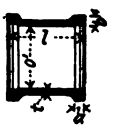

Fig. 19.

Note.—Dimensions are in inches; weights in lbs.

Diam. of Pipe.	Class.	Dimensions in Inches. a	b	l	o'	t	Weight	Diam. of Pipe.	Class.	Dimensions in Inches. a	b	l	o'	t	Weight
3	E	1.50	1.20	10	4.8	0.60	35	36	C	2.00	2.50	15	39.7	1.25	[illegible]
4	"	"	1.30	"	5.8	0.65	45	"	E	"	"	"	"	1.50	[illegible]
6	"	"	1.40	"	8.0	0.70	67	42	C	"	2.80	"	45.5	1.40	[illegible]
8	"	"	1.50	12	10.1	0.75	100	"	"	"	"	20	"	"	[illegible]
10	"	"	"	"	12.2	"	120	"	E	"	"	15	46.0	1.65	[illegible]
12	"	"	1.60	14	14.3	0.80	170	"	"	"	"	20	"	"	[illegible]
14	"	"	1.70	15	16.4	0.85	220	48	B	"	3.00	15	51.7	1.30	[illegible]
16	"	1.75	1.80	"	18.7	0.90	275	"	"	"	"	20	"	"	[illegible]
20	C	"	2.00	"	23.0	1.00	370	"	C	"	"	15	52.0	1.30	[illegible]
"	E	"	"	"	"	1.05	385	"	"	"	"	20	"	"	[illegible]
24	C	2.00	2.10	"	27.0	"	470	"	E	"	"	15	52.6	1.85	[illegible]
"	E	"	"	"	"	1.15	495	"	"	"	"	20	"	"	[illegible]
30	C	"	2.30	"	33.4	"	630	60	B	2.25	3.20	15	64.2	1.60	[illegible]
"	E	"	"	"	"	1.30	680	"	"	"	"	20	"	"	[illegible]

† See also Table 43, following.

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20.—Properties of Reducers—Type 1.*
(R. D. Wood & Co.)

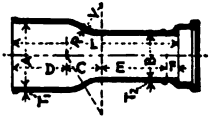

Fig. 20.

Dimensions in Inches.			Approximate Weight in Pounds.
A	B	L	No. 1.
4	3	20	55
5	4	21	65
6	4	23	75
8	6	23	110
10	6	30	155
10	8	28.6	180
12	6	32	185
12	8	30.6	210
12	10	28.8	240

* See also Table 41, following.

21.—Properties of Reducers—Type 2.†
(Met. W. W.)

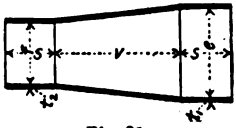

Fig. 21.

Dimensions in Inches.						Class.	Weight in Lbs.	Dimensions in Inches.						Class.	Weight in Lbs.
e	f	v	s	t_1	t_2			e	f	v	s	t_1	t_2		
14	10	20	8	0.75	0.63	E	260	36	30	32	8	1.13	1.00	C	1440
16	"	"	"	0.81	"	"	300	"	"	"	"	1.36	1.20	E	1740
"	12	"	"	"	0.69	"	330	42	"	"	"	1.27	1.00	C	1690
20	"	26	"	0.79	"	C	435	"	"	"	"	1.53	1.20	E	2040
"	"	"	"	0.92	"	E	480	"	36	"	"	1.27	1.13	C	1920
"	16	"	"	0.79	0.70	C	485	"	"	"	"	1.53	1.36	E	2320
"	"	"	"	0.92	0.81	E	565	"	"	66	"	1.27	1.13	C	3280
24	"	"	"	0.87	"	C	610	"	"	"	"	1.53	1.36	E	3970
"	"	"	"	1.03	"	E	675	48	"	32	"	1.25	1.03	B	1970
"	20	"	"	0.87	0.79	C	655	"	"	"	"	1.40	1.13	C	2200
"	"	"	"	1.03	0.92	E	775	"	"	"	"	1.70	1.36	E	2660
30	"	"	"	1.00	0.79	C	810	"	42	"	"	1.25	1.14	B	2200
"	"	"	"	1.20	0.92	E	970	"	"	"	"	1.40	1.27	C	2460
"	24	"	"	1.00	0.87	C	905	"	"	"	"	1.70	1.53	E	3000
"	"	"	"	1.20	1.03	E	1090	"	36	132	"	1.25	1.03	B	6000
36	"	32	"	1.13	0.87	C	1240	"	"	"	"	1.40	1.13	C	6700
"	"	"	"	1.36	1.03	E	1490	"	"	"	"	1.70	1.36	E	8130

† See also Table 42, following.

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22.—Properties of Caps.*
(Met. W. W.)

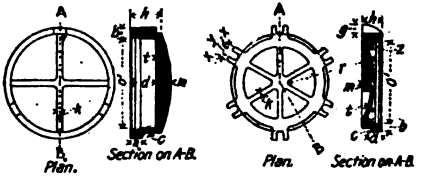

Fig. 22.

Note.—Dimensions are in inches; weights in lbs.

Dia. Pipe	Class.	Dimensions in Inches.															Weight
		a	b	c	d	o'	h	t	m	k	x	r	g	x	y		
4	E	1.50	1.3	0.65	3.0	5.8	3.60	0.60									25
6	"	"	1.4	0.70	"	8.0	3.65	0.65									40
8	"	"	1.5	0.75	3.5	10.1	4.25	0.75									60
10	"	"	"	"	"	12.2	"	"	1.50	0.75							88
12	"	"	1.6	0.80	"	14.3	"	"	1.75	"							105
14	"	"	1.7	0.85	"	16.4	4.40	0.90	1.90	"							140
16	"	1.75	1.8	0.90	4.0	18.7	5.00	1.00	2.00	"							240
20	"	"	2.0	1.00	"	23.0	5.25	"	2.50	1.00	1.75	22.35	2.50	1.25	1.62		350
24	"	2.00	2.1	1.05	4.5	27.0	6.00	1.05	3.00	"	1.80	28.65	"	"	"		490
30	"	"	2.3	1.15	"	33.4	"	1.15	2.85	"	2.00	44.60	3.00	1.50	2.00		690
36	"	"	2.5	1.25	"	39.7	"	1.25	4.00	1.25	"	44.55	"	"	"		970
42	C	"	2.8	1.40	5.0	45.5	7.00	1.50	"	1.50	"	63.25	"	"	"		1510
"	E	"	"	"	"	46.0	"	1.60	3.90	"	2.25	"	"	"	"		1570
48	B	"	3.0	1.50	5.25	51.7	8.00	1.75	4.00	"	3.00	90.0	"	"	"		2120
"	C	"	"	"	"	52.0	"	1.90	3.85	"	3.15	"	"	"	"		2300
"	E	"	"	"	"	52.6	"	2.00	3.75	"	3.45	"	"	"	"		2280

* See also Table 44, following.

23.—Properties of Plugs.†
(R. D. Wood & Co.)

Fig. 23.

Note.—Dimensions are in inches.

Diameter of Pipe.	A	D	Approximate Weight in Pounds.
3"	3.8"	5.5"	4
4	4.9	5.5	6
5	6.0	5.5	8
6	7.0	5.5	10
8	8.1	5.5	13
10	11.2	6.0	28
12	13.2	6.0	38

† See also Table 45, following.

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24.—Properties of Offsets.*
(R. D. Wood & Co.)

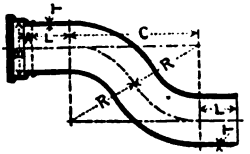

Fig. 24.

Pipe Diam. Ins.	Dimensions in Inches.						Approx. Weight in Pounds.
	R	A	B	C	L	T	
3	8	2	4.6	13.86	10	0.40	65
3	14	2	4.6	24.25	10	0.40	75
4	8	2	5.8	13.86	10	0.45	90
4	14	2	5.8	24.25	10	0.45	110
5	8	2	6.8	13.86	10	0.48	115
5	14	2	6.8	24.25	10	0.48	140
6	8	2	7.8	13.86	10	0.50	140
6	14	2	7.8	24.25	10	0.50	175
8	10	2	9.9	17.32	10	0.55	215
8	15	2	9.9	25.98	10	0.55	255
10	12	2	12.0	20.78	10	0.60	310
10	18	2	12.0	31.18	10	0.60	375
12	14	2	14.1	24.25	10	0.65	430
12	20	2	14.1	34.61	10	0.65	510

* See also Table 34, following.

(2) **Turned and Bored Joint Pipe** (see page 1215) is rarely used in the United States. During extremely cold weather the joints are liable to pull apart. If there is no danger from this cause, this kind of pipe will perhaps be economical if the cost of boring the bell end and turning the spigot end does not exceed the cost of lead joint for the ordinary bell and spigot pipe.

(3) **Flanged Joint Pipe** (see page 682) or simply flange pipe is used for special connections and open work, generally at the ends of pipe lines, as in pump-houses, gate chambers, etc. The pipes come in 12-ft. lengths, and the flanges are bolted together, with rubber or other packing between. The following represents the practice of R. D. Wood & Co., of Philadelphia, adopting the standard of National Ass'n of Master Steam and Hot Water Fitters and the Am. Soc. of Mechanical Engineers, in flange diameters and drilling:

Note.—Flexible Joint Pipe is described on page 1238.

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25.—Properties of Standard Flange Pipe. Part I.—Pressures 25 and 50 Lbs.

Notes.—5", 7", 9" and 15" are odd sizes, made to order only. 4" to 24" usually in stock, regularly made in 12' lengths, faced 1/16" short to allow for gasket. Bolts and gaskets furnished only on order. The flanges are finished approximately 1½ times the thickness of pipe + ⅛". Drilling is 1/16" larger for ½" bolts; and ⅛" larger for ⅝" bolts and above, than corresponding bolt diameters, and all drilling is in fours or multiples thereof.

Pressures given are for water; when pipes are required for steam, the pressure should not exceed ⅔ of that given in table.

Dimensions are in inches; weights in lbs. Weights are approximate only.

Inside Diam. of Pipe.	Outside Diam. of Flange.	Diameter of Bolt Circle.	Number of Bolts in Flange.	Head, 60 Feet: Pressure, 25 Lbs.						Head, 115 Feet: Pressure, 50 Lbs.						Inside Diam. of Pipe.
				Thickness of Pipe.	Weight per Foot	Weight per Length	Weight of Single Flange	Diam. of Bolts.	Weight of Bolts and Nuts per Joint.	Thickness of Pipe.	Weight per Foot	Weight per Length	Weight of Single Flange	Diam. of Bolts.	Weight of Bolts and Nuts per Joint.	
3"	7½"	6"	4	0.39"	13.0	168	6.1	½"	1.1	0.41"	13.7	177	6.3	½"	1.1	3"
4	9	7½	4	0.40	17.3	225	8.6	⅝	1.8	0.43	18.7	242	9.0	⅝	1.8	4
5	10	8½	8	0.42	22.3	288	10.2	⅝	3.8	0.46	24.7	318	10.8	⅝	3.8	5
6	11	9½	8	0.44	27.8	357	11.8	⅝	3.9	0.48	30.5	391	12.5	⅝	4.0	6
7	12½	10¾	8	0.46	33.7	435	15.5	⅝	3.9	0.52	38.3	494	17.0	⅝	4.0	7
8	13½	11¾	8	0.47	39.0	503	17.4	⅝	3.9	0.53	44.3	559	18.9	⅝	4.0	8
9	15	13¼	12	0.49	45.6	592	22.2	⅝	6.1	0.55	51.5	666	24.2	⅝	6.1	9
10	16	14¼	12	0.51	52.5	679	24.6	¾	9.4	0.58	60.2	776	26.7	¾	9.8	10
12	19	17	12	0.54	66.4	869	36.3	¾	9.6	0.63	78.0	1017	40.5	¾	10.0	12
14	21	18¾	12	0.58	82.9	1081	42.9	⅞	15.4	0.66	94.9	1233	47.0	⅞	15.9	14
15	22½	20	16	0.60	91.8	1199	48.8	⅞	20.8	0.70	107.7	1401	54.3	⅞	21.5	15
16	23½	21¼	16	0.61	99.2	1302	55.2	⅞	20.8	0.72	118.0	1538	61.2	⅞	22.2	16
18	25	22¾	16	0.65	118.8	1540	57.0	1	31.4	0.77	141.7	1828	63.8	1	32.7	18
20	27½	25	20	0.67	135.8	1769	69.7	1	39.2	0.82	167.4	2168	79.7	1	41.4	20
22	29½	27¼	20	0.72	160.4	2083	79.3	1	40.3	0.87	195.0	2520	89.9	1	42.5	22
24	32	29½	20	0.75	182.0	2376	90.8	1	40.3	0.92	224.7	2915	109.5	1	42.5	24
30	38¾	36	28	0.86	260.2	3409	143.1	1¼	78.4	1.07	325.9	4242	166.0	1¼	82.3	30
36	45¾	42¾	32	0.96	347.8	4562	209.3	1¼	91.8	1.21	441.8	5785	244.9	1¼	96.6	36
40	50½	47½	32	1.03	414.2	5522	275.8	1½	134.5	1.31	530.4	7014	324.6	1½	134.1	40
42	52½	49½	36	1.07	451.7	6011	295.4	1½	141.5	1.36	578.0	7628	345.9	1½	153.7	42
48	60½	56½	36	1.19	569.8	7602	358.4	1½	203.4	1.51	732.8	9710	408.1	1½	220.0	48

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(See Notes on preceding page.)

Pressures given are for water; when pipes are required for steam the pressure should not exceed ⅔ of that given in table.

Dimensions are in inches; weights in lbs. Weights are approximate only.

Inside Diam. of Pipe.	Outside Diam. of Flange.	Diameter of Bolt Circle.	Number of Bolts in Flange.	Head, 230 Feet: Pressure, 100 Lbs.						Head, 350 Feet: Pressure, 150 Lbs.						Inside Diam. of Pipe.
				Thickness of Pipe.	Weight per Foot	Weight per Length	Weight of Single Flange	Diam. of Bolts.	Weight of Bolts and Nuts per Joint.	Thickness of Pipe.	Weight per Foot	Weight per Length	Weight of Single Flange	Diam. of Bolts.	Weight of Bolts and Nuts per Joint.	
3"	7½"	6"	4	0.43"	14.5	187	6.5	⅝"	1.9	0.45"	15.2	196	6.7	⅝"	1.9	3"
4	9	7½	4	0.46	20.1	260	9.5	¾	3.0	0.49	21.6	279	9.9	¾	3.1	4
5	10	8½	8	0.49	26.4	340	11.4	¾	6.2	0.53	28.7	368	12.0	¾	6.4	5
6	11	9½	8	0.52	33.3	426	13.2	¾	6.4	0.56	36.0	460	13.9	¾	6.5	6
7	12½	10¾	8	0.56	41.5	534	17.9	¾	6.5	0.60	44.7	574	18.7	¾	6.6	7
8	13½	11¾	8	0.59	49.7	637	20.3	¾	6.5	0.64	54.2	693	21.4	¾	6.7	8
9	15	13¼	12	0.62	58.5	754	25.9	¾	9.9	0.68	64.5	829	27.5	¾	10.1	9
10	16	14¼	12	0.65	67.9	873	29.0	⅞	15.8	0.71	74.5	955	30.6	⅞	16.1	10
12	19	17	12	0.71	88.5	1150	44.1	⅞	16.1	0.79	99.0	1283	47.4	⅞	16.6	12
14	21	18¾	12	0.77	111.5	1443	52.4	1	24.8	0.86	125.3	1616	56.0	1	25.5	14
15	22½	20	16	0.80	123.9	1606	59.4	1	33.1	0.90	140.3	1812	64.1	1	34.4	15
16	23½	21¼	16	0.84	138.7	1800	67.7	1	33.6	0.95	157.9	2042	73.6	1	34.9	16
18	25	22¾	16	0.90	166.7	2141	70.2	1⅛	45.0	1.02	190.2	2435	75.5	1⅛	47.2	18
20	27½	25	20	0.96	197.3	2544	88.1	1⅛	57.6	1.09	225.0	2890	95.2	1⅛	59.0	20
22	29½	27¼	20	1.02	230.2	2960	99.0	1¼	75.2	1.18	268.1	3431	107.0	1¼	78.8	22
24	32	29½	20	1.08	265.5	3428	121.0	1¼	77.0	1.25	309.4	3976	131.7	1¼	80.6	24
30	38¾	36	28	1.28	392.5	5078	184.2	1⅜	131.6	1.48	456.7	5877	198.7	1⅜	140.0	30
36	45¾	42¾	32	1.46	536.1	6980	273.3	1⅜	156.8	1.71	632.0	8177	296.7	1⅜	166.4	36
40	50¾	47½	32	1.59	648.2	8509	365.2	1½	214.4	1.87	767.5	10004	396.9	1½	232.0	40
42	52¾	49½	36	1.65	705.9	9242	385.4	1½	245.7	1.94	835.5	10866	419.8	1½	265.5	42
48	59½	56	44	1.84	898.9	11811	512.1	1⅝	311.3	2.17	1067.1	13912	553.4	1⅝	335.5	48

(4) **Flexible Joint Pipe** (page 1215) is particularly advantageous and economical when it is necessary to carry the pipe line across a stream of any considerable size. If the stream is navigable and vessels are liable to anchor, the bed of the stream must be dredged on the line of the pipe, and a line of guide piles is often driven for this purpose. These piles also serve later in laying the pipe in the dredged trench. The process of laying the pipe is comparatively simple. Beginning at one shore, connection is made with the line laid up to that point (in a gradually desending deep trench toward the stream). An improvised scow with a run-way or incline is well suited to the purpose. The pipes are laid successively on the run-way, the joints are leaded, and the scow pulled ahead, allowing the pipe to gradually settle to the bottom of the trench without straining the joints excessively.

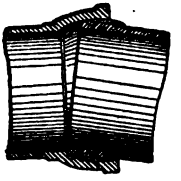

Fig. 25.

In pouring the lead for any joints the two adjacent pipes must be in a straight line. Sometimes trestle work is employed, in shallow water, and the pipes are lowered into place by block and tackle. The method of assembling the pipe on the bank, and then dragging the whole connected line into the stream is very liable to strain the joints and even pull them apart, as sometimes happens.

Flexible joint pipe is manufactured in pipe length, including pipe and joint, with bell-end machined inside; or the cast knuckle joints with flange ends or bell and spigot ends may be ordered separately for connecting with ordinary pipe.

Flexible joints are used not only with cast iron pipe but with wrought iron and steel pipe as well.*

* For design of flexible joint in connection with 52-in. riveted steel pipe, see Trans. Am. Soc. C. E., Vol. XXXIV, p. 23.

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AMERICAN WATER WORKS ASSOCIATION

*STANDARD SPECIFICATIONS FOR CAST IRON PIPE AND SPECIAL CASTINGS

WITH TABLES OF DIMENSIONS, THICKNESSES AND WEIGHTS.

Adopted May 12, 1908.

STANDARD SPECIFICATIONS.

DESCRIPTION OF PIPES.

Section 1. The pipes shall be made with hub and spigot joints, and shall conform accurately to the dimensions given in Tables 26 and 27. They shall be straight and shall be true circles in section, with their inner and outer surfaces concentric, and shall be of the specified dimensions in outside diameter. They shall be at least 12 feet in length, exclusive of socket.

Pipes with thickness and weight intermediate between the classes in Table 27 shall be made of the same outside diameter as the next heavier class. Pipes with thickness and weight less than shown by Table 27 shall be made of the same outside diameter as the Class A pipe; and pipes with thickness and weight more than shown by Table 27 shall be made of the same outside diameter as the Class D pipe.

All pipes having the same outside diameter shall have the same inside diameter at both ends. The inner diameter of the lighter pipes of each standard outside diameter shall be gradually increased for a distance of about 6 inches from each end of the pipe so as to obtain the required standard thickness and weight for each size and class of pipe.

For pipes of each size from 4-inch to 24-inch inclusive, there shall be two standards of outside diameter, and for pipes from 30-inch to 60-inch inclusive, there shall be four standards of outside diameter, as shown by Table 26.

For pipes 4-inch to 12-inch inclusive, one class of special castings shall be furnished, made from Class D pattern. Those having spigot ends shall have outside diameters of spigot ends midway between the two standards of outside diameter as shown by Table 26, and shall be tapered back for a distance of 6 inches.

For pipes from 14-inch to 24-inch inclusive, two classes of special castings shall be furnished; Class B special castings with Classes A and B pipes, and Class D special castings with Classes C and D pipes; the former shall have cast on them the letters "AB" and the latter "CD." For pipes 30-inch to 60-inch inclusive, four classes of special castings shall be furnished, one for each class of pipe, and shall have cast on them the letter of the class to which they belong.

ALLOWABLE VARIATION IN DIAMETER OF PIPES AND SOCKETS.

Section 2. Special care shall be taken to have the sockets of the required size. The sockets and spigots will be tested by circular gages, and no pipe will be received which is defective in joint room from any cause. The diameters of the sockets and the outside diameters of the spigot ends of the pipes shall not vary from the standard dimensions by more than 0.06 of an inch for pipes 16 inches or less in diameter; 0.08 of an inch for 18-inch, 20-inch and 24-inch pipes, 0.10 of an inch for 30-inch, 36-inch and 42-inch pipes; 0.12 of an inch for 48-inch, and 0.15 of an inch for 54-inch and 60-inch pipes.

ALLOWABLE VARIATION IN THICKNESS.

Section 3. For pipes whose standard thickness is less than 1 inch, the thickness of metal in the body of the pipe shall not be more than 0.08 of an inch less than the standard thickness, and for pipes whose standard thickness is 1 inch or more, the variation shall not exceed 0.10 of an inch, except

* Mr. J. M. Diven, Sec'y Am. W. W. Ass'n, writes the author as follows, under date of Sept. 3, 1908: "The specifications are not absolutely perfect yet, nor has there been any agreement between the various associations as to a universal standard. This is something greatly to be desired, and which I trust will be brought about in the next two or three years."

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that for spaces not exceeding 8 inches in length in any direction, variations from the standard thickness of 0.02 of an inch in excess of the allowance above given shall be permitted.

For special castings of standard patterns a variation of 50 per cent greater than allowed for straight pipes shall be permitted.

Defective Spigots May be Cut.

Section 4. Defective spigot ends on pipes 12 inches or more in diameter may be cut off in a lathe and a half-round wrought-iron band shrunk into a groove cut in the end of the pipe. Not more than 12 per cent of the total number of accepted pipes of each size shall be cut and banded, and no pipe shall be banded which is less than 11 feet in length, exclusive of the socket.

In case the length of pipe differs from 12 feet, the standard weight of the pipe given in Table 27 shall be modified in accordance therewith.

Special Castings.

Section 5. All special castings shall be made in accordance with the cuts and dimensions given in the tables forming a part of these specifications.

The diameters of the sockets and the external diameters of the spigot ends of the special castings shall not vary from the standard dimensions by more than 0.12 of an inch for castings 16 inches or less in diameter; 0.15 of an inch for 18-inch, 20-inch and 24-inch; 0.20 of an inch for 30-inch, 36-inch and 42-inch, and 0.24 of an inch for 48-inch, 54-inch and 60-inch. These variations apply only to special castings made from standard patterns.

The flanges on all manhole castings and manhole covers shall be faced true and smooth, and drilled to receive bolts of the sizes given in the tables. The manufacturer shall furnish and deliver all bolts for bolting-on the manhole covers, the bolts to be of the sizes shown on plans and made of the best quality of mild steel, with hexagonal heads and nuts and sound well-fitting threads.

Marking.

Section 6. Every pipe and special casting shall have distinctly cast upon it the initials of the maker's name. When cast especially to order, each pipe larger than 4-inch may also have cast upon it figures showing the year in which it was cast and a number signifying the order in point of time in which it was cast, the figures denoting the year being above and the number below, thus:

1908	1908	1908
1	2	3

etc., also any initials, not exceeding four, which may be required by the purchaser. The letters and figures shall be cast on the outside and shall not be less than 2 inches in length and ½ of an inch in relief for pipes 8 inches in diameter and larger. For smaller sizes of pipes the letters may be 1 inch in length. The weight and the class letter shall be painted conspicuously in white on the inside of each pipe and special casting after the coating has become hard.

Allowable Percentage of Variation in Weight.

Section 7. No pipes shall be accepted the weight of which shall be less than the standard weight by more than 5 per cent for pipes 16 inches or less in diameter, and 4 per cent for pipes more than 16 inches in diameter, and no excess above the standard weight of more than the given percentage for the several sizes shall be paid for. The total weight to be paid for shall not exceed for each size and class of pipe received the sum of the standard weights of the same number of pieces of the given size and class by more than 2 per cent.

No special casting shall be accepted the weight of which shall be less than the standard weight by more than 10 per cent for pipes 12 inches or less in diameter, and 8 per cent for larger sizes except that curves, Y-pieces and breeches pipe may be 12 per cent below the standard weight, and no excess above the standard weight of more than the above percentages for the several sizes will be paid for. These variations apply only to castings made from the standard patterns.

Quality of Iron.

Section 8. All pipes and special castings shall be made of cast iron of good quality, and of such character as shall make the metal of the castings

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strong, tough and of even grain, and soft enough to satisfactorily admit of drilling and cutting. The metal shall be made without any admixture of cinder iron or other inferior metal, and shall be remelted in a cupola or air furnace.

The contractor shall have the right to make and break three bars from each heat or run of metal, and the test shall be based upon the average results of the three bars. Should the dimensions of the three bars differ from those given below, a proper allowance therefor shall be made in the results of the tests.

Tests of Material.

**Section* 9. Specimen bars of the metal used, each being twenty-six inches long by two inches wide and one inch thick, shall be made without charge as often as the engineer may direct, and in default of definite instructions the contractor shall make and test at least one bar from each heat or run of metal. The bars when placed flatwise upon supports twenty-four inches apart, and loaded in the center, shall support a load of 2,000 pounds, and show a deflection of not less than 0.30 of an inch before breaking; or if preferred, tensile bars shall be made which will show a breaking point of not less than 20,000 pounds per square inch.

Casting of Pipe.

Section 10. The straight pipes shall be cast in dry sand molds in a vertical position. Pipes 16 inches or less in diameter shall be cast with the hub end up or down, as specified in the proposals. Pipes 18 inches or more in diameter shall be cast with the hub end down.

The pipes shall not be stripped or taken from the pit while showing color of heat, but shall be left in the flasks for a sufficient length of time to prevent unequal contraction by subsequent exposure.

Quality of Castings.

Section 11. The pipes and special castings shall be smooth, free from scales, lumps, blisters, sand holes and defects of every nature which unfit them for the use for which they are intended. No plugging or filling will be allowed.

Cleaning and Inspection.

Section 12. All pipes and special castings shall be thoroughly cleaned and subjected to a careful hammer inspection. No casting shall be coated unless entirely clean and free from rust, and approved in these respects by the engineer immediately before being dipped.

Coating.

Section 13 Every pipe and special casting shall be coated inside and out with coal-tar pitch varnish. The varnish shall be made from coal tar. To this material sufficient oil shall be added to make a smooth coating, tough and tenacious when cold, and not brittle nor with any tendency to scale off.

Each casting shall be heated to a temperature of 300 degrees Fahrenheit immediately before it is dipped, and shall not possess less than this temperature at the time it is put in the vat. The ovens in which the pipes are heated shall be so arranged that all portions of the pipe shall be heated to an even temperature. Each casting shall remain in the bath at least five minutes.

The varnish shall be heated to a temperature of 300 degrees Fahrenheit (or less if the engineer shall so order), and shall be maintained at this temperature during the time the casting is immersed.

Fresh pitch and oil shall be added when necessary to keep the mixture at the proper consistency and the vat shall be emptied of its contents and refilled with fresh pitch when deemed necessary by the engineer. After being coated the pipe shall be carefully drained of the surplus varnish. Any pipe or special casting that is to be recoated shall first be thoroughly scraped and cleaned.

Hydrostatic Test.

Section 14 When the coating has become hard, the straight pipes shall be subjected to a proof by hydrostatic pressure, and, if required by the engineer, they shall also be subjected to a hammer test under this pressure.

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* Pipe may be made under higher metal tests when desired. Stock pipe may be made under metal tests as low as 1,800 pounds.

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The pressures to which the different sizes and classes of pipes shall be subjected are as follows:

	20-Inch Diameter and Larger. Pounds per Square Inch.	Less than 20-Inch Diameter. Pounds per Square inch.
Class A Pipe	150	300
Class B Pipe	200	300
Class C Pipe	250	300
Class D Pipe	300	300

Weighing.

Section 15. The pipes and special castings shall be weighed for payment under the supervision of the engineer after the application of the coal-tar pitch varnish. If desired by the engineer, the pipes and special castings shall be weighed after the delivery, and the weights so ascertained shall be used in the final settlement, provided such weighing is done by a legalized weighmaster. Bids shall be submitted and a final settlement made upon the basis of a ton of 2,000 pounds.

Contractor to Furnish Men and Materials.

Section 16. The contractor shall provide all tools, testing machines, materials and men necessary for the required testing, inspection and weighing at the foundry, of the pipe and special castings; and should the purchaser have no inspector at the works, the contractor shall, if required by the engineer, furnish a sworn statement that all of the tests have been made as specified, this statement to contain the results of the tests upon the test bars.

Power of Engineer to Inspect.

Section 17. The engineer shall be at liberty at all times to inspect the material at the foundry, and the molding, casting and coating of the pipes and special castings. The forms, sizes, uniformity and condition of all pipes and other castings herein refered to shall be subject to his inspection and approval, and he may reject, without proving, any pipe or other casting which is not in conformity with the specifications or drawings

Inspector to Report.

Section 18. The inspector at the foundry shall report daily to the foundry office all pipes and special castings rejected, with the causes for rejection.

Castings to be Delivered Sound and Perfect.

Section 19. All the pipes and other castings must be delivered in all respects sound and conformable to these specifications. The inspection shall not relieve the contractor of any of his obligations in this respect, and any defective pipes or other castings which may have passed the engineer at the works or elsewhere shall be at all times liable to rejection when discovered, until the final completion and adjustment of the contract; provided, however, that the contractor shall not be held liable for pipes or special castings found to be cracked after they have been accepted at the agreed point of delivery. Care shall be taken in handling the pipes not to injure the coating, and no pipes or other material of any kind shall be placed in the pipes during transportation or at any time after they have received the coating.

Definition of the Word "Engineer."

Section 20. Wherever the word "engineer" is used herein it shall be understood to refer to the engineer or inspector acting for the purchaser and to his properly authorized agents, limited by the particular duties intrusted to them.

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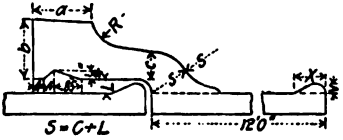
26.—Dimensions of Cast Iron Pipe.
(A. W. W. A.)
Classes A, B, C, D.



X = ¾″ on 3″ to 6″ inclusive.
V = 11/16″ on 3″ to 6″ "
X = 1″ on 8″ to 84″ "
V = ¼″ on 8″ to 84″ "

Fig. 26.

Nominal Diam Inches.	Classes.	Actual Outside Diam. Inches.	Diam. of Sockets.		Depth of S'kets.		A	B	C
			Pipe. Inches.	Special Cast'gs. Inches.	Pipe. Inches.	Special Cast'gs. Inches			
4	A	4.80	5.60	5.70	3.50	4.00	1.5	1.30	.65
4	B-C-D	5.00	5.80	5.70	3.50	4.00	1.5	1.30	.65
6	A	6.90	7.70	7.80	3.50	4.00	1.5	1.40	.70
6	B-C-D	7.10	7.90	7.80	3.50	4.00	1.5	1.40	.70
8	A-B	9.05	9.85	10.00	4.00	4.00	1.5	1.50	.75
8	C-D	9.30	10.10	10.00	4.00	4.00	1.5	1.50	.75
10	A-B	11.10	11.90	12.10	4.00	4.00	1.5	1.50	.75
10	C-D	11.40	12.20	12.10	4.00	4.00	1.5	1.60	.80
12	A-B	13.20	14.00	14.20	4.00	4.00	1.5	1.60	.80
12	C-D	13.50	14.30	14.20	4.00	4.00	1.5	1.70	.85
14	A-B	15.30	16.10	16.10	4.00	4.00	1.5	1.70	.85
14	C-D	15.65	16.45	16.45	4.00	4.00	1.5	1.80	.90
16	A-B	17.40	18.40	18.40	4.00	4.00	1.75	1.80	.90
16	C-D	17.80	18.80	18.80	4.00	4.00	1.75	1.90	1.00
18	A-B	19.50	20.50	20.50	4.00	4.00	1.75	1.90	.95
18	C-D	19.92	20.92	20.92	4.00	4.00	1.75	2.10	1.05
20	A-B	21.60	22.60	22.60	4.00	4.00	1.75	2.00	1.00
20	C-D	22.06	23.06	23.06	4.00	4.00	1.75	2.30	1.15
24	A-B	25.80	26.80	26.80	4.00	4.00	2.00	2.10	1.05
24	C-D	26.32	27.32	27.32	4.00	4.00	2.00	2.50	1.25
30	A	31.74	32.74	32.74	4.50	4.50	2.00	2.30	1.15
30	B	32.00	33.00	33.00	4.50	4.50	2.00	2.30	1.15
30	C	32.40	33.40	33.40	4.50	4.50	2.00	2.60	1.32
30	D	32.74	33.74	33.74	4.50	4.50	2.00	3.00	1.50
36	A	37.96	38.96	38.96	4.50	4.50	2.00	2.50	1.25
36	B	38.30	39.30	39.30	4.50	4.50	2.00	2.80	1.40
36	C	38.70	39.70	39.70	4.50	4.50	2.00	3.10	1.60
36	D	39.16	40.16	40.16	4.50	4.50	2.00	3.40	1.80
42	A	44.20	45.20	45.20	5.00	5.00	2.00	2.80	1.40
42	B	44.50	45.50	45.50	5.00	5.00	2.00	3.00	1.50
42	C	45.10	46.10	46.10	5.00	5.00	2.00	3.40	1.75
42	D	45.58	46.58	46.58	5.00	5.00	2.00	3.80	1.95
48	A	50.50	51.50	51.50	5.00	5.00	2.00	3.00	1.50
48	B	50.80	51.80	51.80	5.00	5.00	2.00	3.30	1.65
48	C	51.40	52.40	52.40	5.00	5.00	2.00	3.80	1.95
48	D	51.98	52.98	52.98	5.00	5.00	2.00	4.20	2.20
54	A	56.66	57.66	57.66	5.50	5.50	2.25	3.20	1.60
54	B	57.10	58.10	58.10	5.50	5.50	2.25	3.60	1.80
54	C	57.80	58.80	58.80	5.50	5.50	2.25	4.00	2.15
54	D	58.40	59.40	59.40	5.50	5.50	2.25	4.40	2.45
60	A	62.80	63.80	63.80	5.50	5.50	2.25	3.40	1.70
60	B	63.40	64.40	64.40	5.50	5.50	2.25	3.70	1.90
60	C	64.20	65.20	65.20	5.50	5.50	2.25	4.20	2.25
60	D	64.82	65.82	65.82	5.50	5.50	2.25	4.70	2.60
72	A	75.34	76.34	76.34	5.50	5.50	2.25	3.80	1.87
72	B	76.00	77.00	77.00	5.50	5.50	2.25	4.20	2.20
72	C	76.88	77.88	77.88	5.50	5.50	2.25	4.60	2.64
84	A	87.54	88.54	88.54	5.50	5.50	2.50	4.10	2.10
84	B	88.54	89.54	89.54	5.50	5.50	2.50	4.50	2.60

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27.—Thickness and Weights of Cast Iron Pipe.

(A. W. W. A.)

Classes A, B, C, D.

Note.—Dimensions are in inches; weights, in lbs.

Nominal Inside Diameter. Inches.	CLASS A 100-Feet Head. 43 Pounds Pressure.			CLASS B 200-Feet Head. 86 Pounds Pressure.			CLASS C 300-Feet Head. 130 Pounds Pressure.			CLASS D 400-Feet Head. 173 Pounds Pressure.			Nominal Inside Diameter. Inches.
	Thickness, Inches.	Weight per Foot.	Weight per Length.	Thickness, Inches.	Weight per Foot.	Weight per Length.	Thickness, Inches.	Weight per Foot.	Weight per Length.	Thickness, Inches.	Weight per Foot.	Weight per Length.	
4	0.42	20.0	240	0.45	21.7	260	0.48	23.3	280	0.52	25.0	300	4
6	0.44	30.8	370	0.48	33.3	400	0.51	35.8	430	0.55	38.3	460	6
8	0.46	42.9	515	0.51	47.5	570	0.56	52.1	625	0.60	55.8	670	8
10	0.50	57.1	685	0.57	63.8	765	0.62	70.8	850	0.68	76.7	920	10
12	0.54	72.5	870	0.62	82.1	985	0.68	91.7	1100	0.75	100.0	1200	12
14	0.57	89.6	1075	0.66	102.5	1230	0.74	116.7	1400	0.82	129.2	1550	14
16	0.60	108.3	1300	0.70	125.0	1500	0.80	143.8	1725	0.89	158.3	1900	16
18	0.64	129.2	1550	0.75	150.0	1800	0.87	175.0	2100	0.96	191.7	2300	18
20	0.67	150.0	1800	0.80	175.0	2100	0.92	208.3	2500	1.03	229.2	2750	20
24	0.76	204.2	2450	0.89	233.3	2800	1.04	279.2	3350	1.16	306.7	3680	24
30	0.88	291.7	3500	1.03	333.3	4000	1.20	400.0	4800	1.37	450.0	5400	30
36	0.99	391.7	4700	1.15	454.2	5450	1.36	545.8	6550	1.58	625.0	7500	36
42	1.10	512.5	6150	1.28	591.7	7100	1.54	716.7	8600	1.78	825.0	9900	42
48	1.26	666.7	8000	1.42	750.0	9000	1.71	908.3	10900	1.96	1050.0	12600	48
54	1.35	800.0	9600	1.55	933.3	11200	1.90	1141.7	13700	2.23	1341.7	16100	54
60	1.39	916.7	11000	1.67	1104.2	13250	2.00	1341.7	16100	2.38	1583.3	19000	60
72	1.62	1283.4	15400	1.95	1545.8	18550	2.39	1904.2	22850	....	.....	.....	72
84	1.72	1633.4	19600	2.22	2104.2	25250	....	.....	.....	....	.....	.....	84

The above weights are per length to lay 12 feet, including standard sockets; proportionate allowance to be made for any variation.

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28.—Dimensions of Pipe for High Pressure Service.

(A. W. W. A.)

Classes E, F, G, H.

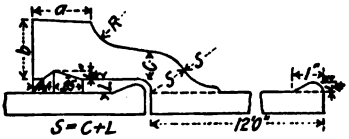

Fig. 28.

Note.—Dimensions are in inches.

Nominal Diam. Inches	Classes.	Actual Outside Diam. Inches	Diam. of Sockets. Pipe and Specials.	Depth of Sockets. Pipe and Specials.	A	B	C	R	Nominal Diam. Inches
6	E-F	7.22	8.02	4.00	1.50	1.75	0.75	1.10	6
6	G-H	7.38	8.18	4.00	1.50	1.85	0.85	1.10	6
8	E-F	9.42	10.22	4.00	1.50	1.85	0.85	1.10	8
8	G-H	9.60	10.40	4.00	1.50	1.95	0.95	1.10	8
10	E-F	11.60	12.40	4.50	1.65	1.95	0.95	1.10	10
10	G-H	11 84	12.64	4 50	1 75	2.05	1.05	1.10	10
12	E-F	13.78	14 58	4.50	1.75	2.05	1.05	1.10	12
12	G-H	14.08	14 88	4.50	1.75	2.20	1.20	1.10	12
14	E-F	15.98	16 78	4 50	2.00	2.15	1.15	1 10	14
14	G-H	16.32	17.12	4.50	2.00	2.35	1 35	1 10	14
16	E-F	18.16	18.96	4.50	2.00	2 30	1.25	1 15	16
16	G-H	18 54	19 34	4.50	2.00	2.55	1 45	1.15	16
18	E-F	20 34	21.14	4 50	2.25	2.45	1.40	1.15	18
18	G-H	20 78	21 58	4.50	2.25	2.75	1.65	1.15	18
20	E-F	22.54	23.34	4.50	2.25	2.55	1.50	1.15	20
20	G-H	23.02	23.82	4.50	2 25	2.85	1.75	1.20	20
24	E-F	26.90	27.90	5.00	2 25	2.85	1.70	1.20	24
30	E	33.10	34.10	5 00	2.25	3.25	1 80	1.50	30
30	F	33.46	34 46	5 00	2.25	3.50	2.00	1.55	30
36	E	39.60	40 60	5.00	2.25	3 70	2 05	1.70	36
36	F	40.04	41.04	5 00	2.25	4.00	2.30	1.80	36

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29.—Thickness and Weights of Cast Iron Pipe For Fire Lines and Other High Pressure Service.

(A. W. W. A.)

Classes E, F, G, H.

Note.—Dimensions are in inches, weights, in lbs.

Nominal Inside Diameter, Inches.	CLASS E 500-Feet Head. 217 Pounds Pressure.			CLASS F 600-Feet Head. 260 Pounds Pressure.			CLASS G 700-Feet Head. 304 Pounds Pressure.			CLASS H 800-Feet Head. 347 Pounds Pressure.			Nominal Inside Diameter Inches.
	Thickness, Inches.	Weight per Foot.	Weight per Length.	Thickness, Inches.	Weight per Foot.	Weight per Length.	Thickness, Inches.	Weight per Foot.	Weight per Length.	Thickness, Inches.	Weight per Foot.	Weight per Length.	
6	0.58	41.7	500	0.61	43.3	520	0.65	47.1	565	0.69	49.6	595	6
8	0.66	61.7	740	0.71	65.7	790	0.75	70.8	850	0.80	75.0	900	8
10	0.74	86.3	1035	0.80	92.1	1105	0.86	100.9	1210	0.92	106.7	1280	10
12	0.82	113.8	1365	0.89	122.1	1465	0.97	135.4	1625	1.04	143.8	1725	12
14	0.90	145.0	1740	0.99	157.5	1890	1.07	174.2	2090	1.16	186.7	2240	14
16	0.98	179.6	2155	1.08	195.4	2345	1.18	219.2	2620	1.27	232.5	2790	16
18	1.07	220.4	2645	1.17	238.4	2860	1.28	267.1	3205	1.39	286.7	3440	18
20	1.15	263.0	3155	1.27	286.3	3435	1.39	320.8	3850	1.51	344.6	4135	20
24	1.31	359.6	4315	1.45	392.9	4715	....	.....	....	....	.....	....	24
30	1.55	521.7	6260	1.73	585.4	7025	....	.....	....	....	.....	....	30
36	1.80	725.0	8700	2.02	820.0	9840	...	.....	....	....	.....	...	36

The above weights are per length to lay 12 feet, including standard sockets; proportionate allowance to be made for any variation.

For High Pressure Pipe from 6 inches to 24 inches inclusive, one class of special castings shall be furnished for Classes E and F pipe, and one class of special castings for Classes G and H pipe. For 30-inch and 36-inch pipe, one class of special castings shall be furnished for each class of pipe.

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30.—LUGS.

Number and Weights of Lugs on Outlets of Different Sizes.
(A. W. W. A).

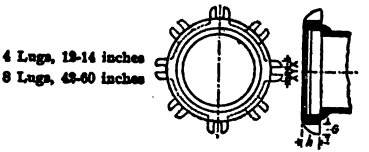

Fig. 30a.

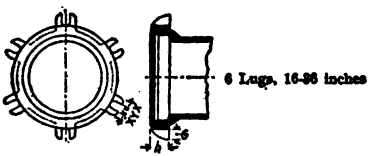

Fig. 30b.

Nominal Diameter Outlet, Inches.	Number of Pairs of Lugs.	Approximate Weight Lugs on One Bell, Pounds.	Nominal Diameter Outlet, Inches.	Number of Pairs of Lugs.	Approximate Weight Lugs on One Bell, Pounds.
12	4	32	30	6	80
14	4	32	36	6	80
16	6	56	42	8	111
18	6	56	48	8	114
20	6	56	54	8	134
24	6	56	60	8	137

Two pairs of lugs are placed on the vertical axis of each bell, the others at equal distances around circumference. H = depth of bell on all sizes.

$G = 2.50$ inches, $X = 1.25$ inches, $Y = 1.63$ inches, for 12 to 24 inches inclusive.

$G = 3.00$ inches, $X = 1.50$ inches, $Y = 2.00$ inches, for 30 to 60 inches inclusive.

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31, 32.—Properties of Curves, Bell and Spigot, ¼, ⅛, $\frac{1}{16}$. (A. W. W. A.)

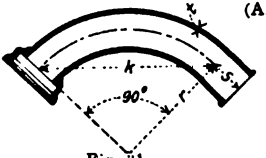

Fig. 31.

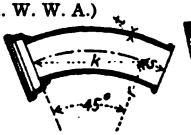

Fig. 32a.

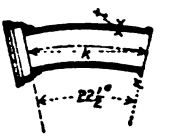

Fig. 32b.

Table 31. Table 32.

¼ Curves.						⅛ Curves.						$\frac{1}{16}$ Curves.		
Nom'al Dia. In.	Class.	Dimensions, Inches. t	r	k	Approx Weight Pounds	Nom'al Dia. In.	Class.	Dimensions, Inches. t	r	k	Approx Weight Pounds	Dimensions, Inches. r	k	Approx Weight Pounds
4	D	0.52	16	22.60	82	4	D	0.52	24	18.40	66	48	18.70	66
						6	D	0.55	24	18.40	105	48	18.70	105
6	D	0.55	16	22.60	130	8	D	0.60	24	18.40	150	48	18.70	150
						10	D	0.68	24	18.40	202	48	18.70	202
8	D	0.60	16	22.60	200	12	D	0.75	24	18.40	265	48	18.70	265
10	D	0.68	16	22.60	278	14	B	0.66	36	27.60	359	72	28.10	312
						14	D	0.82	36	27.60	442	72	28.10	382
12	D	0.75	16	22.60	366	16	B	0.70	36	27.60	445	72	28.10	388
						16	D	0.89	36	27.60	558	72	28.10	494
14	B	0.66	18	25.50	406	18	B	0.75	36	27.60	533	72	28.10	464
14	D	0.82	18	25.50	504	18	D	0.96	36	27.60	663	72	28.10	574
						20	B	0.80	48	36.70	758	96	37.50	675
16	B	0.70	24	34.00	594	20	D	1.03	48	36.70	964	96	37.50	858
16	D	0.89	24	34.00	750	24	B	0.89	60	45.90	1181	120	46.80	1072
						24	D	1.16	60	45.90	1515	120	46.80	1372
18	B	0.75	24	34.00	710	30	A	0.88	60	45.90	1475	120	46.80	1342
						30	B	1.03	60	45.90	1684	120	46.80	1523
18	D	0.96	24	34.00	888	30	C	1.20	60	45.90	1983	120	46.80	1800
						30	D	1.37	60	45.90	2291	120	46.80	2080
20	B	0.80	24	34.00	840									
						36	A	0.99	90	68.90	2472	180	70.20	2472
						36	B	1.15	90	68.90	2916	180	70.20	2916
						36	C	1.36	90	68.90	3430	180	70.20	3430
						36	D	1.58	90	68.90	4012	180	70.20	4012
20	D	1.03	24	34.00	1070									
						42	A	1.10	90	68.90	3286	180	70.20	3286
						42	B	1.28	90	68.90	3778	180	70.20	3778
						42	C	1.54	90	68.90	4600	180	70.20	4600
						42	D	1.78	90	68.90	5360	180	70.20	5360
24	B	0.89	30	42.40	1290									
						48	A	1.26	90	68.90	4230	180	70.20	4230
						48	B	1.42	90	68.90	4820	180	70.20	4820
						48	C	1.71	90	68.90	5796	180	70.20	5796
						48	D	1.96	90	68.90	6750	180	70.20	6750
24	D	1.16	30	42.40	1656									
						54	A	1.35	90	68.90	5180	180	70.20	5180
						54	B	1.55	90	68.90	5990	180	70.20	5990
						54	C	1.90	90	68.90	7330	180	70.20	7330
						54	D	2.23	90	68.90	8620	180	70.20	8620
30	A	0.88	36	50.90	1814									
						60	A	1.39	90	68.90	5990	180	70.20	5990
						60	B	1.67	90	68.90	7130	180	70.20	7130
						60	C	2.00	90	68.90	8590	180	70.20	8590
						60	D	2.38	90	68.90	10240	180	70.20	10240
30	B	1.03	36	50.90	2082									
30	C	1.20	36	50.90	2454									
30	D	1.37	36	50.90	2836									
36	A	0.99	48	67.90	2964									
36	B	1.15	48	67.90	3500									
36	C	1.36	48	67.90	4120									
36	D	1.58	48	67.90	4820									

S = 8 inches on sizes 4 and 6 inches. S = 10 inches on sizes 8 inches.
S = 12 inches on sizes 10 to 36 inches.
S = 6 inches on ⅛ Curves on sizes 4 to 30 inches inclusive.
S = 6 inches on $\frac{1}{16}$ Curves on sizes 4 to 12 inches inclusive.
All weights are approximate.

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33, 34.—Properties of Curves, Bell and Spigot.—Offsets. (A. W. W. A.)

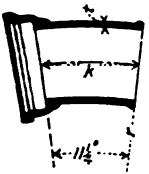

Fig. 33a.

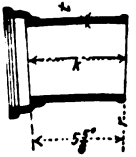

Fig. 33b.

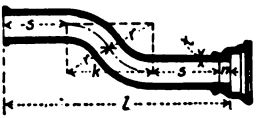

Fig. 34.

All dimensions are in inches.

Table 33.

Diam. Ins.	Class.	t	r	k	Approx. Weight. Pounds.	r	k	Approx. Weight. Pounds.
		1/32 Curves.				1/64 Curves.*		
[illegible]	D	0.52	120	23.52	66			
[illegible]	D	0.55	120	23.52	104			
[illegible]	D	0.60	120	23.52	150			
[illegible]	D	0.68	120	23.52	192			
[illegible]	D	0.75	120	23.52	250			
[illegible]	B	0.66	180	35.28	364			
[illegible]	D	0.82	180	35.28	450			
[illegible]	B	0.70	180	35.28	453			
[illegible]	D	0.89	180	35.28	570			
[illegible]	B	0.75	180	35.28	543			
[illegible]	D	0.96	180	35.28	674			
[illegible]	B	0.80	240	47.05	808	480	47.10	808
[illegible]	D	1.03	240	47.05	1028	480	47.10	1028
[illegible]	B	0.89	240	47.05	1080	480	47.10	1080
[illegible]	D	1.16	240	47.05	1380	480	47.10	1380
[illegible]	A	0.88	240	47.05	1350	480	47.10	1350
[illegible]	B	1.03	240	47.05	1540	480	47.10	1540
[illegible]	C	1.20	240	47.05	1810	480	47.10	1810
[illegible]	D	1.37	240	47.05	2090	480	47.10	2090
[illegible]	A	0.99	240	47.05	1790	480	47.10	1790
[illegible]	B	1.15	240	47.05	2100	480	47.10	2100
[illegible]	C	1.36	240	47.05	2470	480	47.10	2470
[illegible]	D	1.58	240	47.05	2880	480	47.10	2880
[illegible]	A	1.10	240	47.05	2380	480	47.10	2380
[illegible]	B	1.28	240	47.05	2720	480	47.10	2720
[illegible]	C	1.54	240	47.05	3310	480	47.10	3310
[illegible]	D	1.78	240	47.05	3850	480	47.10	3850
[illegible]	A	1.26	240	47.05	3150	480	47.10	3150
[illegible]	B	1.42	240	47.05	3480	480	47.10	3480
[illegible]	C	1.71	240	47.05	4170	480	47.10	4170
[illegible]	D	1.96	240	47.05	4860	480	47.10	4860
[illegible]	A	1.35	240	47.05	3750	480	47.10	3750
[illegible]	B	1.55	240	47.05	4330	480	47.10	4330
[illegible]	C	1.90	240	47.05	5290	480	47.10	5290
[illegible]	D	2.23	240	47.05	6220	480	47.10	6220
[illegible]	A	1.39	240	47.05	4340	480	47.10	4340
[illegible]	B	1.67	240	47.05	5140	480	47.10	5140
[illegible]	C	2.00	240	47.05	6200	480	47.10	6200
[illegible]	D	2.38	240	47.05	7400	480	47.10	7400

Table 34.

Offsets.				
Nominal Diam. Ins.	Class.	r	l	Approx. Wt. Lbs.
4	D	8	35.85	91
6	D	14	46.25	183
8	D	15	48.00	280
10	D	16	49.70	390
12	D	17	51.45	530
14	B	18	53.70	555
14	D	18	53.70	695
16	B	19	55.40	708
16	D	19	55.40	900

Nominal Diam. In.	Class.	t	k	s	n
4	D	0.52	13.85	10.00	2.00
6	D	0.55	24.25	10 00	2.00
8	D	0.60	26.00	10.00	2.00
10	D	0.68	27.70	10.00	2.00
12	D	0.75	29.45	10.00	2.00
14	B	0.66	31 20	10.00	2.50
14	D	0.82	31.20	10.00	2.50
16	B	0.70	32.90	10.00	2.50
16	D	0.89	32.90	10 00	2 50

*First three columns, for 20" to 60" diam., apply also to 1/64 curves.

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35.—Properties of Branches.—3-Way and 4-Way. (A. W. W. A.)

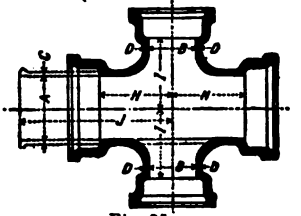

Fig. 35.

Nominal Diam. Inches.		Class.	Dimensions, Inches.			Approximate Weights, Pounds.			
						3-Way Branches		4-Way Branches	
A	B		H	J	I	2 Bells.	3 Bells.	3 Bells.	4 Bells.
4	3	D	11	23	11	121	120	153	153
4	4	D	11	23	11	125	128	164	166
6	3	D	12	24	12	173	170	207	204
6	4	D	12	24	12	185	183	223	221
6	6	D	12	24	12	203	200	259	257
8	4	D	13	25	13	262	255	301	294
8	6	D	13	25	13	278	270	333	325
8	8	D	13	25	13	301	294	378	372
10	4	D	14	26	14	356	338	395	377
10	6	D	14	26	14	371	352	424	406
10	8	D	14	26	14	389	371	461	443
10	10	D	14	26	14	414	395	511	493
12	4	D	15	27	15	473	445	514	486
12	6	D	15	27	15	486	458	540	512
12	8	D	15	27	15	502	474	573	545
12	10	D	15	27	15	519	491	605	577
12	12	D	15	27	15	540	512	651	623
14	4	B	16	28	16	485	480	535	530
14	4	D	16	28	16	614	588	666	641
14	6	B	16	28	16	500	495	560	555
14	6	D	16	28	16	634	608	730	700
14	8	B	16	28	16	515	510	600	595
14	8	D	16	28	16	662	636	787	761
14	10	B	16	28	16	535	525	635	625
14	10	D	16	28	16	679	653	822	796
14	12	B	16	28	16	560	550	680	670
14	12	D	16	28	16	698	672	860	834
14	14	B	16	28	16	575	569	723	715
14	14	D	16	28	16	750	734	938	963
16	4	B	17	29	17	615	610	675	670
16	4	D	17	29	17	783	760	864	841
16	6	B	17	29	17	630	625	695	690
16	6	D	17	29	17	802	779	902	879
16	8	B	17	29	17	645	640	730	725
16	8	D	17	29	17	831	808	961	933
16	10	B	17	29	17	660	655	760	755
16	10	D	17	29	17	872	849	1042	1019
16	12	B	17	29	17	685	680	805	800
16	12	D	17	29	17	884	861	1066	1042
16	14	B	17	29	17	695	690	825	820

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35.—BRANCHES.—3-WAY AND 4-WAY.—Continued.

Diam. ies.	Class.	Dimensions, Inches			Approximate Weights, Pounds.			
					3-Way Branches		4-Way Branches	
B		H	J	I	2 Bells	3 Bells.	3 Bells.	4 Bells.
14	D	17	29	17	903	880	1104	1082
16	B	17	29	17	729	727	904	901
16	D	17	29	17	991	969	1282	1259
4	B	18	30	18	755	750	820	815
4	D	18	30	18	953	927	1046	1020
6	B	18	30	18	765	760	840	835
6	D	18	30	18	968	942	1075	1049
8	B	18	30	18	780	775	870	865
8	D	18	30	18	1000	974	1140	1114
10	B	18	30	18	795	790	900	895
10	D	18	30	18	1038	1012	1216	1190
12	B	18	30	18	815	810	940	935
12	D	18	30	18	1075	1049	1290	1264
14	B	18	30	18	825	820	955	950
14	D	18	30	18	1083	1057	1306	1280
16	B	18	30	18	855	850	1020	1015
16	D	18	30	18	1108	1082	1356	1330
16	B	18	30	18	895	889	1101	1096
18	D	18	30	18	1170	1144	1480	1454
4	B	19	31	19	923	916	1006	999
4	D	19	31	19	1172	1148	1273	1248
6	B	19	31	19	930	920	1010	1000
6	D	19	31	19	1188	1164	1304	1280
8	B	19	31	19	945	935	1035	1025
8	D	19	31	19	1212	1188	1352	1328
10	B	19	31	19	955	945	1060	1050
10	D	19	31	19	1252	1227	1431	1407
12	B	19	31	19	975	965	1100	1090
12	D	19	31	19	1288	1263	1502	1479
14	B	19	31	19	980	970	1110	1100
14	D	19	31	19	1342	1318	1613	1588
16	B	19	31	19	1010	1000	1170	1160
16	D	19	31	19	1347	1323	1622	1597
18	B	19	31	19	1035	1025	1225	1215
18	D	19	31	19	1365	1341	1658	1634
20	B	19	31	19	1077	1070	1314	1307
20	D	19	31	19	1462	1438	1852	1828
6	B	21	33	21	1309	1289	1425	1405
6	D	21	33	21	1670	1637	1809	1775
8	B	21	33	21	1323	1303	1453	1433
8	D	21	33	21	1697	1664	1863	1830
10	B	21	33	21	1341	1321	1489	1469
10	D	21	33	21	1732	1699	1933	1900
12	B	21	33	21	1362	1342	1532	1511
12	D	21	33	21	1768	1735	2005	1972
14	B	21	33	21	1402	1381	1609	1589
14	D	21	33	21	1810	1777	2088	2055
16	B	21	33	21	1443	1423	1694	1673
16	D	21	33	21	1858	1825	2185	2151
18	B	21	33	21	1460	1440	1727	1706
18	D	21	33	21	1885	1852	2238	2205
20	B	21	33	21	1474	1454	1756	1736
20	D	21	33	21	2025	1991	2518	2484

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35.—BRANCHES.—3-WAY AND 4-WAY.—Continued.

Nominal Diam. Inches.		Class.	Dimensions, Inches			Approximate Weights, Pounds.			
						3-Way Branches		4-Way Branches	
A	B		H	J	I	2 Bells.	3 Bells.	3 Bells.	4 Bells.
24	24	B	21	33	21	1523	1503	1854	1834
24	24	D	21	33	21	2146	2113	2727	2694
30	6	A	13	25	24	1272	1300	1407	1434
30	6	B	13	25	24	1433	1417	1580	1563
30	6	C	13	25	24	1693	1673	1870	1850
30	6	D	13	25	24	1934	1920	2112	2099
30	8	A	14	26	24	1318	1346	1453	1481
30	8	B	14	26	24	1482	1466	1624	1609
30	8	C	14	26	24	1765	1745	1953	1934
30	8	D	14	26	24	2004	1990	2182	2168
30	10	A	15	27	24	1369	1396	1512	1540
30	10	B	15	27	24	1538	1521	1685	1668
30	10	C	15	27	24	1857	1837	2075	2056
30	10	D	15	27	24	2108	2094	2319	2306
30	12	A	15	27	24	1395	1420	1555	1580
30	12	B	15	27	24	1555	1540	1715	1700
30	12	C	15	27	24	1911	1891	2184	2164
30	12	D	15	27	24	2154	2140	2411	2398
30	14	A	18	30	26	1547	1575	1737	1764
30	14	B	18	30	26	1805	1789	2085	2069
30	14	C	18	30	26	2159	2140	2497	2477
30	14	D	18	30	26	2567	2553	3026	3012
30	16	A	19	31	26	1648	1675	1805	1832
30	16	B	19	31	26	1899	1883	2200	2184
30	16	C	19	31	26	2272	2253	2662	2642
30	16	D	19	31	26	2692	2678	3206	3192
30	18	A	20	34	26	1757	1741	2024	2007
30	18	B	20	34	26	2044	1976	2387	2318
30	18	C	20	34	26	2434	2353	2862	2781
30	18	D	20	34	26	2805	2791	3361	3348
30	20	A	21	36	26	1857	1818	2157	2118
30	20	B	21	36	26	2182	2088	2584	2490
30	20	C	21	36	26	2667	2555	3237	3126
30	20	D	21	36	26	3041	2921	3657	3538
30	24	A	23	38	26	1979	1940	2312	2274
30	24	B	23	38	26	2313	2219	2742	2648
30	24	C	23	38	26	2847	2736	3474	3362
30	24	D	23	38	26	3290	3170	4014	3895
30	30	A	26	43	26	2212	2129	2602	2520
30	30	B	26	43	26	2599	2453	3106	2960
30	30	C	26	43	26	3310	3137	4110	3937
30	30	D	26	43	26	3850	3660	4799	4609
36	8	A	14	26	27	1751	1777	1938	1963
36	8	B	14	26	27	2055	2073	2268	2287
36	8	C	14	26	27	2421	2433	2679	2691
36	8	D	14	26	27	2780	2780	3038	3039
36	10	A	15	27	27	1810	1835	1996	2021
36	10	B	15	27	27	2128	2147	2345	2364
36	10	C	15	27	27	2534	2546	2822	2834
36	10	D	15	27	27	2903	2902	3188	3188
36	12	A	16	28	27	1884	1909	2084	2109
36	12	B	16	28	27	2219	2238	2458	2477
36	12	C	16	28	27	2644	2656	2962	2973

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35.—BRANCHES.—3-WAY AND 4-WAY.—Continued.

Nominal Diam. Inches		Class	Dimensions, Inches			Approximate Weights, Pounds.			
						3-Way Branches		4-Way Branches	
A	B		H	J	I	2 Bells.	3 Bells.	3 Bells.	4 Bells.
36	12	D	16	28	27	3032	3033	3349	3350
36	14	A	18	30	29	2039	2065	2279	2304
36	14	B	18	30	29	2415	2433	2709	2728
36	14	C	18	30	29	2872	2883	3251	3263
36	14	D	18	30	29	3470	3470	4033	4033
36	16	A	19	31	29	2135	2160	2410	2436
36	16	B	19	31	29	2521	2540	2853	2872
36	16	C	19	31	29	3003	3014	3431	3442
36	16	D	19	31	29	3618	3617	4231	4230
36	18	A	20	34	29	2279	2246	2581	2548
36	18	B	20	34	29	2701	2650	3073	3022
36	18	C	20	34	29	3206	3136	3673	3604
36	18	D	20	34	29	3852	3755	4506	4409
36	20	A	21	36	29	2409	2346	2752	2689
36	20	B	21	36	29	2885	2800	3336	3251
36	20	C	21	36	29	3537	3426	4212	4101
36	20	D	21	36	29	4050	3905	4757	4612
36	24	A	23	38	29	2451	2513	2844	2907
36	24	B	23	38	29	3099	3014	2624	3539
36	24	C	23	38	29	3806	3695	4585	4474
36	24	D	23	38	29	4511	4366	5307	5161
36	30	A	26	43	29	2830	2708	3242	3120
36	30	B	26	43	29	3594	3438	4335	4179
36	30	C	26	43	29	4248	4055	5140	4947
36	30	D	26	43	29	5160	4918	6192	5950
36	36	A	29	46	29	3067	2946	3539	3418
36	36	B	29	46	29	4046	3891	4956	4800
36	36	C	29	46	29	4788	4595	5867	5673
36	36	D	29	46	29	5810	5567	7099	6857
42	12	A	16	28	30	2507	2577	3467	3537
42	12	B	16	28	30	2670	2889	3131	3170
42	12	C	16	28	30	3478	3507	3830	3860
42	12	D	16	28	30	3971	3989	4307	4325
42	14	A	18	30	32	2671	2739	2942	3010
42	14	B	18	30	32	3075	3114	3400	3440
42	14	C	18	30	32	3747	3776	4147	4177
42	14	D	18	30	32	4590	4609	5288	5306
42	16	A	19	31	32	2778	2846	3080	3148
42	16	B	19	31	32	3196	3235	3552	3592
42	16	C	19	31	32	3891	3920	4325	4354
42	16	D	19	31	32	4754	4772	5487	5506
42	18	A	20	34	32	2950	2941	3268	3258
42	18	B	20	34	32	3407	3357	3794	3744
42	18	C	20	34	32	4393	4312	5108	5028
42	18	D	20	34	32	5049	4939	5819	5709
42	20	A	21	36	32	3104	3056	3459	3411
42	20	B	21	36	32	3582	3486	4009	3913
42	20	C	21	36	32	4615	4479	5387	5251
42	20	D	21	36	32	5297	5123	6122	5948
42	24	A	23	38	32	3314	3266	3724	3676
42	24	B	23	38	32	3852	3756	4370	4274
42	24	C	23	38	32	4965	4829	5866	5730
42	24	D	23	38	32	5709	5535	6579	6405

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35.—Branches.—3-Way and 4-Way.—Concluded.

Nominal Diam. Inches.		Class.	Dimensions, Inches.			Approximate Weights, Pounds.			
						3-Way Branches		4-Way Branches	
A	B		H	J	I	2 Bells.	3 Bells.	3 Bells.	4 Bells.
42	30	A	26	43	32	3679	3553	4144	4018
42	30	B	26	43	32	4554	4370	5416	5230
42	30	C	26	43	32	5649	5402	6675	6428
42	30	D	26	43	32	6561	6258	7729	7436
42	36	A	29	46	32	4076	3950	4705	4579
42	36	B	29	46	32	4903	4718	5845	5659
42	36	C	29	46	32	6150	5904	7261	7015
42	36	D	29	46	32	7187	6884	8512	8209
42	42	A	32	49	32	4393	4267	5109	4983
42	42	B	32	49	32	5533	5348	6641	6455
42	42	C	32	49	32	7001	6755	8392	8146
42	42	D	32	49	32	8158	7855	9803	9500
48	12	A	17	29	33	3266	3319	3853	3707
48	12	B	17	29	33	3752	3804	4107	4160
48	12	C	17	29	33	4510	4576	4940	5007
48	12	D	17	29	33	5564	5634	6376	6436
48	14	A	18	30	35	3422	3476	3762	3815
48	14	B	18	30	35	4173	4226	4836	4889
48	14	C	18	30	35	4965	5030	5712	5778
48	14	D	18	30	35	5754	5815	6596	6656
48	16	A	19	31	35	3565	3619	3947	4001
48	16	B	19	31	35	4046	4098	4466	4519
48	16	C	19	31	35	5055	5121	5755	5821
48	16	D	19	31	35	5967	6028	6860	6921
48	18	A	20	34	35	3775	3729	4166	4120
48	18	B	20	34	35	4287	4225	4718	4655
48	18	C	20	34	35	5479	5407	6328	6256
48	18	D	20	34	35	6328	6227	7259	7188
48	20	A	21	36	35	3956	3860	4378	4283
48	20	B	21	36	35	4500	4380	4973	4853
48	20	C	21	36	35	5745	5604	6652	6511
48	20	D	21	36	35	6607	6425	7574	7392
48	24	A	23	38	35	4221	4125	4706	4609
48	24	B	23	38	35	5028	4908	5798	5678
48	24	C	23	38	35	6193	6052	7272	7131
48	24	D	23	38	35	7064	6882	7994	7812
48	30	A	26	43	35	4748	4553	5361	5166
48	30	B	26	43	35	5685	5451	6653	6418
48	30	C	26	43	35	7042	6762	8365	7985
48	30	D	26	43	35	8051	7708	9303	8960
48	36	A	29	46	35	5150	4953	5859	5662
48	36	B	29	46	35	6322	6088	7382	7148
48	36	C	29	46	35	7603	7323	8915	8635
48	36	D	29	46	35	8830	8487	10336	9993
48	42	A	32	49	35	5503	5307	6266	6069
48	42	B	32	49	35	6821	6587	7973	7739
48	42	C	32	49	35	8278	7999	9750	9470
48	42	D	32	49	35	9644	9301	11367	11024
48	48	A	35	52	35	6043	5846	7043	6846
48	48	B	35	52	35	7659	7424	9076	8841
48	48	C	35	52	35	9229	8950	11006	10726
48	48	D	35	52	35				

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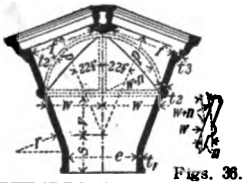
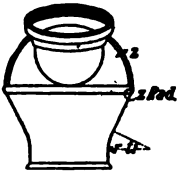
36.—Properties of Y Branches, Type 1.

(A. W. W. A.)





Figs. 36.

Nominal Diam. Ins. e	Nominal Diam. Ins. f	Class	s	p	v	w	n	r	Thickness, Ins. t_1	Thickness, Ins. t_2	Thickness, Ins. t_3	Approx. Weight. Pounds.
12	12	D	16 00	21.50	8.00	9.79	1.17	30	0.75	1.08	0.75	687
14	14	B	16.00	24.00	9.00	11.30	1.08	30	0.66	0.99	0.66	738
14	14	D	16.00	24.00	9.00	11.30	1.32	30	0.82	1.22	0.82	894
16	16	B	17.00	27.50	10.50	13.00	1.12	30	0.70	1.03	0.70	942
16	16	D	17 00	27.50	10.50	13.00	1.39	30	0.89	1.29	0.89	1275
18	18	B	18.00	30.00	12.00	14.70	1.17	30	0.75	1.08	0.75	1266
18	18	D	18.00	30.00	12.00	14.70	1.46	30	0.96	1.36	0.96	1607
20	20	B	18.00	34.00	13.50	16.40	1.26	30	0.80	1.16	0.80	1635
20	20	D	18.00	34.00	13.50	16.40	1.57	30	1.03	1.46	1.03	2296
24	20	B	12.00	34.00	13.50	16.40	1.26	30	0.89	1.16	0.80	1663
24	20	D	12.00	34.00	13.50	16.40	1.57	30	1.16	1.46	1.03	2393
24	24	B	18.00	38.00	15.25	19.30	1.36	30	0.89	1.26	0.89	2300
24	24	D	18.00	38.00	15.25	19.30	1.75	30	1.16	1.63	1.16	2957
30	24	A	12 00	38.00	15.25	19.30	1.36	30	0.88	1.26	0.89	2171
30	24	B	12.00	38.00	15.25	19.30	1.36	30	1.03	1.26	0.89	2217
30	24	C	12.00	38.00	15.25	19.30	1.75	30	1.20	1.63	1.16	2717
30	24	D	12.00	38.00	15.25	19.30	1.75	30	1.37	1.63	1.16	2811
30	30	A	18.00	48.00	18.00	23.70	1.32	30	0.88	1.22	0.88	3153
30	30	B	18.00	48.00	18.00	23.70	1.59	30	1.03	1.47	1.03	3687
30	30	C	18.00	48.00	18.00	23.70	1.88	30	1.20	1.74	1.20	4285
30	30	D	18.00	48.00	18.00	23.70	2.17	30	1.37	2.01	1.37	4941
36	30	A	10.00	48 00	18.00	23.70	1.32	30	0.99	1.22	0.88	3343
36	30	B	10.00	48.00	18.00	23.70	1.59	30	1.15	1.47	1.03	3874
36	30	C	10.00	48.00	18.00	23.70	1.88	30	1.36	1.74	1.20	4486
36	30	D	10.00	48.00	18.00	23.70	2.17	30	1.58	2.01	1.37	5189
36	36	A	18.00	56.00	21.00	28.20	1.50	24	0.99	1.39	0.99	4949
36	36	B	18.00	56.00	21.00	28.20	1.79	24	1.15	1.66	1.15	5858
36	36	C	18.00	56.00	21.00	28.20	2.13	24	1.36	1.98	1.36	6804
36	36	D	18.00	56.00	21.00	28.20	2 48	24	1.58	2.31	1.58	8062
42	30	A	6.00	48.00	18.00	23.70	1.32	30	1.10	1.22	0.88	3368
42	30	B	6.00	48.00	18.00	23.70	1.59	30	1.28	1.47	1.03	3890
42	30	C	6.00	48.00	18.00	23.70	1.88	30	1.54	1.74	1.20	4543
42	30	D	6.00	48.00	18.00	23.70	2.17	30	1.78	2.01	1.37	5241
42	36	A	10.00	56.00	21 00	28 20	1.50	24	1.10	1.39	0.99	4904
42	36	B	10 00	56.00	21.00	28.20	1.79	24	1.28	1.66	1.15	5789
42	36	C	10.00	56.00	21.00	28.20	2.13	24	1.54	1.98	1.36	6761
42	36	D	10.00	56.00	21 00	28.20	2.48	24	1.78	2.31	1.58	8025
42	42	A	18.00	66.00	25.00	33.10	1.72	24	1.10	1.60	1.10	7394
42	42	B	18.00	66 00	25.00	33.10	2.05	24	1.28	1.90	1.28	8417
42	42	C	18.00	66.00	25.00	33.10	2.46	24	1.54	2.28	1.54	10377
42	42	D	18 00	66 00	25.00	33.10	2.85	24	1.78	2.64	1.78	12072
48	36	A	2.00	56.00	21.00	28.20	1.50	24	1.26	1.39	0.99	4727
48	36	B	2.00	56.00	21 00	28.20	1.79	24	1.42	1.66	1.15	5584
48	36	C	2.00	56.00	21.00	28.20	2.13	24	1.71	1.98	1.36	6494
48	36	D	2.00	56.00	21.00	28.20	2.48	24	1.96	2.31	1.58	7731
48	42	A	10.00	66.00	25.00	33.10	1.72	24	1.26	1.60	1.10	7345
48	42	B	10.00	66.00	25.00	33.10	2.05	24	1.42	1.90	1.28	8338
48	42	C	10.00	66.00	25.00	33.10	2.46	24	1.71	2.28	1.54	10249
48	42	D	10.00	66.00	25.00	33.10	2.85	24	1.96	2.64	1 78	11924
48	48	A	18.00	76.00	28.00	37.60	1.99	24	1.26	1.86	1.26	10200
48	48	B	18.00	76.00	28.00	37.60	2.32	24	1.42	2.15	1.42	12132
48	48	C	18.00	76.00	28.00	37.60	2.78	24	1.71	2.57	1.71	14716
48	48	D	18.00	76.00	28.00	37.60	3.20	24	1.96	2.95	1.96	16965

Note.—All dimensions are in inches.

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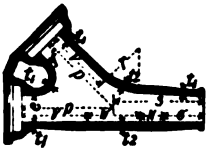
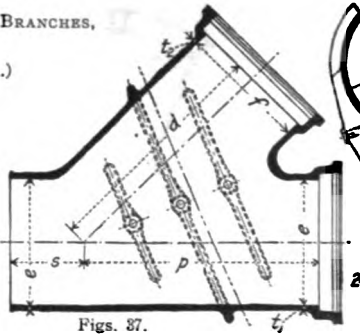
37.—Properties of Y Branches, Type 2.

(A. W. W. A.)





Figs. 37.

Nominal Diam. Ins. e	Nominal Diam. Ins. f	Class	s	p	v	w	n	r	Thickness. Inches. t_1	Thickness. Inches. t_2	Approx. Weight. Pounds.
4	4	D	11.50	10.50	7.18	6.64	2.18	6	0.52	0.64	183
6	6	D	13.00	12.00	9.27	7.46	3.27	6	0.55	0.67	181
8	8	D	14 00	16.00	11.85	8.30	3.85	6	0.60	0.72	294
10	10	D	15.50	18.50	13.94	9.12	4.94	6	0.68	0.83	434
12	12	D	15.50	21.50	16.54	9.93	4.54	6	0.75	0.93	632
14	14	B	16.00	24.00	18.62	10.76	4.62	6	0.66	0.84	690
14	14	D	16.00	24.00	18.62	10.76	4.62	6	0.82	1.00	985
16	16	B	17.50	31.00	25.20	11.60	5.70	6	0.70	1.03	967
16	16	D	17.50	31.00	25.20	11.60	5.70	6	0.89	1.29	1413
18	18	B	18.00	34.00	28.00	12.00	6.00	6	0.75	1.12	1258
18	18	D	18.00	34.00	28.00	12.00	6.00	6	0.96	1.44	1737
20	20	B	18.75	37.00	30.75	12.50	6.50	6	0.80	1.20	1725
20	20	D	18.75	37.00	30.75	12.50	6.50	6	1.03	1.50	2190
24	20	B	18.75	40.00	.....	.....	.....	6	0.89	0.89	2203
24	20	D	18.75	40.00	.....	.....	.....	6	1.16	1.03	3067
24	24	B	19.75	42.00	.....	.....	.....	6	0.89	0.89	2800
24	24	D	19.75	42.00	.....	.....	.....	6	1.16	1.16	3580
30	24	A	17.00	49.50	.....	.....	.....	6	0.88	0.89	3178
30	24	B	17.00	49.50	.....	.....	.....	6	1.03	0.89	3874
30	30	A	22.75	52.50	.....	.....	.....	6	0.88	0.88	3519
30	30	B	22.75	52.50	.....	.....	.....	6	1 03	1.03	4360
36	30	A	19.75	56.00	.....	.....	.....	6	0.99	0.88	4336
36	30	B	19.75	56.00	.....	.....	.....	6	1.15	1.03	4435
36	36	A	24.00	60.00	.....	.....	.....	6	0 99	0.99	4951
36	36	B	24.00	60.00	.....	.....	.....	6	1.15	1.15	6300
42	30	A	16.75	63.00	.....	.....	.....	6	1.10	0.88	5543
42	30	B	16.75	63 00	.....	.....	.....	6	1.28	1.03	6752
42	36	A	21.00	66.00	.....	.....	.....	6	1.10	0.99	6446
42	36	B	21.00	66.00	.....	.....	.....	6	1.28	1.15	7885
42	42	A	25.25	69.00	.....	.....	.....	6	1.10	1.10	7381
42	42	B	25.25	69.00	.....	.....	.....	6	1.28	1.28	9143
48	36	A	18.00	71.00	.....	.....	.....	6	1.26	0.99	7850
48	36	B	18.00	71.00	.....	.....	.....	6	1.42	1.15	9530
48	42	A	22.25	74.00	.....	.....	.....	6	1.26	1.10	9116
48	42	B	22.25	74.00	.....	.....	.....	6	1.42	1.28	10985
48	48	A	26.50	77.00	.....	.....	.....	6	1.26	1.26	10890
48	48	B	26.50	77.00	.....	.....	.....	6	1.42	1.42	13534

Note.—All dimensions are in inches.

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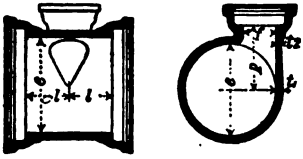
38.—Properties of Blow-off Branches.
(A. W. W. A.)



Figs. 38.

Nominal Diam. Inches. e	Nominal Diam. Inches. f	Class.	l	p	Thickness. Inches. t_1	Thickness. Inches. t_2	Approx. Weight. Pounds.	Nominal Diam. Inches. e	Nominal Diam. Inches. f	Class.	l	p	Thickness. Inches. t_1	Thickness. Inches. t_2	Approx. Weight. Pounds.
8	4	D	12	7	0.60	0.52	227	36	12	A	13	23	0.99	0.75	1702
10	4	D	12	8	0.68	0.52	286	36	12	B	13	23	1.15	0.75	1972
10	6	D	12	8	0.68	0.55	300	36	12	C	13	23	1.36	0.75	2285
12	4	D	12	10	0.75	0.52	365	36	12	D	13	23	1.58	0.75	2627
12	6	D	12	10	0.75	0.55	379	42	12	A	15	26	1.10	0.75	2432
14	4	B	12	11	0.66	0.52	400	42	12	B	15	26	1.28	0.75	2728
14	4	D	12	11	0.82	0.52	471	42	12	C	15	26	1.54	0.75	3271
14	6	B	12	11	0.66	0.55	415	42	12	D	15	26	1.78	0.75	3768
14	6	D	12	11	0.82	0.55	486	42	16	A	15	26	1.10	0.70	2489
16	4	B	12	12	0.70	0.52	497	42	16	B	15	26	1.28	0.70	2786
16	4	D	12	12	0.89	0.52	597	42	16	C	15	26	1.54	0.89	3365
16	6	B	12	12	0.70	0.55	513	42	16	D	15	26	1.78	0.89	3862
16	6	D	12	12	0.89	0.55	613	48	12	A	17	30	1.26	0.75	3274
18	4	B	12	13	0.75	0.52	586	48	12	B	17	30	1.42	0.75	3699
18	4	D	12	13	0.96	0.52	704	48	12	C	17	30	1.71	0.75	4417
18	6	B	12	13	0.75	0.55	603	48	12	D	17	30	1.96	0.75	5107
18	6	D	12	13	0.96	0.55	720	48	16	A	17	30	1.26	0.70	3337
20	4	B	12	14	0.80	0.52	687	48	16	B	17	30	1.42	0.70	3762
20	4	D	12	14	1.03	0.52	850	48	16	C	17	30	1.71	0.89	4523
20	6	B	12	14	0.80	0.55	705	48	16	D	17	30	1.96	0.89	5214
20	6	D	12	14	1.03	0.55	867	54	12	A	19	33	1.35	0.75	4287
24	6	B	12	16	0.89	0.55	916	54	12	B	19	33	1.55	0.75	4945
24	6	D	12	16	1.16	0.55	1149	54	12	C	19	33	1.90	0.75	5981
24	8	B	12	16	0.89	0.60	935	54	12	D	19	33	2.23	0.75	7002
24	8	D	12	16	1.16	0.60	1170	54	16	A	19	33	1.35	0.70	4355
30	8	A	13	20	0.88	0.60	1269	54	16	B	19	33	1.55	0.70	5013
30	8	B	13	20	1.03	0.60	1382	54	16	C	19	33	1.90	0.89	6096
30	8	C	13	20	1.20	0.60	1616	54	16	D	19	33	2.23	0.89	7126
30	8	D	13	20	1.37	0.60	1867	60	12	A	21	36	1.39	0.75	5263
30	12	A	13	20	0.88	0.75	1315	60	12	B	21	36	1.67	0.75	6159
30	12	B	13	20	1.03	0.75	1426	60	12	C	21	36	2.00	0.75	7418
30	12	C	13	20	1.20	0.75	1658	60	12	D	21	36	2.38	0.75	8798
30	12	D	13	20	1.37	0.75	1913	60	16	A	21	36	1.39	0.70	5336
36	8	A	13	23	0.99	0.60	1653	60	16	B	21	36	1.67	0.70	6233
36	8	B	13	23	1.15	0.60	1922	60	16	C	21	36	2.00	0.89	7542
36	8	C	13	23	1.36	0.60	2234	60	16	D	21	36	2.38	0.89	8927
36	8	D	13	23	1.58	0.60	2576								

Note.—All dimensions are in inches.

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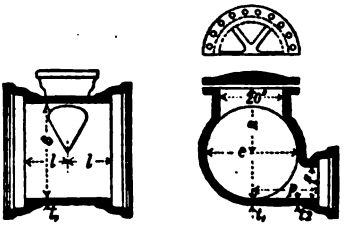
39.—Properties of Blow-off Branches with Manhole. (A. W. W. A.)

Approximate Weight of Cap, 290 Pounds.



Figs. 39.

Nomin'l Diam. Inches. e	Nomin'l Diam. Inches. f	Class.	l	p	n	Thickness, Inches. t_1	Thickness, Inches. t_2	Approx. W'ght, Lbs.	Nomin'l Diam. Inches. e	Nomin'l Diam. Inches. f	Class.	l	p	n	Thickness, Inches. t_1	Thickness, Inches. t_2	Approx. W'ght, Lbs.
30	8	A	17	20	21	0.88	0.60	1628	48	12	A	17	30	30	1.26	0.75	3391
30	8	B	17	20	21	1.03	0.60	1758	48	12	B	17	30	30	1.42	0.75	3803
30	8	C	17	20	21	1.20	0.60	2015	48	12	C	17	30	30	1.71	0.75	4487
30	8	D	17	20	21	1.37	0.60	2290	48	12	D	17	30	30	1.96	0.75	5167
30	12	A	17	20	21	0.88	0.75	1672	48	16	A	17	30	30	1.26	0.70	3454
30	12	B	17	20	21	1.03	0.75	1803	48	16	B	17	30	30	1.42	0.70	3866
30	12	C	17	20	21	1.20	0.75	2057	48	16	C	17	30	30	1.71	0.89	4604
30	12	D	17	20	21	1.37	0.75	2335	48	16	D	17	30	30	1.96	0.89	5274
36	8	A	17	23	24	0.99	0.60	2045	54	12	A	19	33	33	1.35	0.75	4390
36	8	B	17	23	24	1.15	0.60	2351	54	12	B	19	33	33	1.55	0.75	5033
36	8	C	17	23	24	1.36	0.60	2690	54	12	C	19	33	33	1.90	0.75	6039
36	8	D	17	23	24	1.58	0.60	3071	54	12	D	19	33	33	2.23	0.75	7033
36	12	A	17	23	24	0.99	0.75	2094	54	16	A	19	33	33	1.35	0.70	4458
36	12	B	17	23	24	1.15	0.75	2395	54	16	B	19	33	33	1.55	0.70	5100
36	12	C	17	23	24	1.36	0.75	2741	54	16	C	19	33	33	1.90	0.89	6154
36	12	D	17	23	24	1.58	0.75	3122	54	16	D	19	33	33	2.23	0.89	7157
42	12	A	17	26	27	1.10	0.75	2726	60	12	A	21	36	36	1.39	0.75	5357
42	12	B	17	26	27	1.28	0.75	3033	60	12	B	21	36	36	1.67	0.75	6230
42	12	C	17	26	27	1.54	0.75	3595	60	12	C	21	36	36	2.00	0.75	7462
42	12	D	17	26	27	1.78	0.75	4109	60	12	D	21	36	36	2.38	0.75	8810
42	16	A	17	26	27	1.10	0.70	2783	60	16	A	21	36	36	1.39	0.70	5429
42	16	B	17	26	27	1.28	0.70	3090	60	16	B	21	36	36	1.67	0.70	6304
42	16	C	17	26	27	1.54	0.89	3689	60	16	C	21	36	36	2.00	0.89	7587
42	16	D	17	26	27	1.78	0.89	4203	60	16	D	21	36	36	2.38	0.89	8939

Note.—All dimensions are in inches.

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40.—Properties of Manhole Pipe.
(A. W. W. A.)

Note.—All dimensions are in inches.

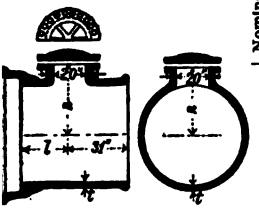


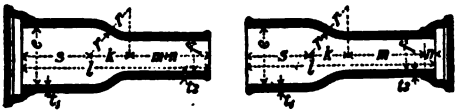
Fig.. 40

Approximate weight of cap, 290 Pounds.

Nominal Diam. Inches.	Class.	n	t	Weight, Pounds.	Nominal Diam. Inches.	Class.	n	t	Weight, Pounds.
30	A	21	0.88	1536	48	A	30	1.26	3194
30	B	21	1.03	1711	48	B	30	1.42	3610
30	C	21	1.20	1973	48	C	30	1.71	4292
30	D	21	1.37	2245	48	D	30	1.96	4968
36	A	24	0.99	1953	54	A	33	1.35	4006
36	B	24	1.15	2260	54	B	33	1.55	4598
36	C	24	1.36	2614	54	C	33	1.90	5578
36	D	24	1.58	3012	54	D	33	2.23	6522
42	A	27	1.10	2535	60	A	36	1.39	4750
42	B	27	1.28	2869	60	B	36	1.67	5606
42	C	27	1.54	3445	60	C	36	2.00	6720
42	D	27	1.78	3971	60	D	36	2.38	7959

l = 17 inches on 30 inches to 48 inches; 19 inches on 54 inches; 21 inches n 60 inches diameter.

41.—Properties of Reducers and Increasers, Type No. 1.
(A. W. W. A.)



Figs. 41.

Note.—All dimensions are in inches.

Diam., Inches.		k	m	r	Thickness, Inches.		Weights, Pounds.	
e	f				t_1	t_2	Large End Bell.	Small End Bell.
6	4	3.30	14 70	3	0.55	0.52	99	88
8	4	5.30	12.70	4	0.60	0.52	131	108
8	6	3.90	14.10	4	0.60	0.55	149	138
10	4	7 10	10.90	5	0.68	0.52	164	132
10	6	6.00	12.00	5	0.68	0.55	181	160
10	8	4.40	13.60	5	0.68	0.60	205	195
12	6	7.90	10.10	6	0.75	0.55	225	191
12	8	6.60	11 40	6	0 75	0.60	246	224
12	10	4.80	13.20	6	0.75	0.68	271	260

Class D. 6 x 4 inches to 12 x 10 inches. On all sizes n = 2 inches.
On all sizes l = 30 inches and s = 10 inches.

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42.—Properties of Reducers and Increasers, Type No. 2. (A. W. W. A.)

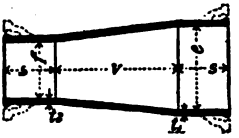

Fig. 42a.—6 x 4 inches to 60 x 54 inches.

Nominal Diam Inches.		v	Thickness, Inches.		Class.	Weights, Pounds.		
e	f	Ins.	t_1	t_2		Spigot Ends.	Large End Bell.	Small End Bell.
6	4	18	0.55	0.52	D	82	104	97
8	4	18	0.60	0.52	D	104	132	119
8	6	18	0.60	0.55	D	121	150	143
10	4	18	0.68	0.52	D	131	162	146
10	6	18	0.68	0.55	D	150	180	169
10	8	18	0.68	0.60	D	170	201	196
12	4	18	0.75	0.52	D	163	201	179
12	6	18	0.75	0.55	D	181	218	202
12	8	18	0.75	0.60	D	202	240	231
12	10	18	0.75	0.68	D	229	267	261
14	6	20	0.66	0.55	B	194	249	216
14	6	20	0.82	0.55	D	234	288	256
14	8	20	0.66	0.60	B	220	275	248
14	8	20	0.82	0.60	D	260	314	288
14	10	20	0.66	0.68	B	250	305	279
14	10	20	0.82	0.68	D	290	344	320
14	12	20	0.66	0.75	B	284	339	321
14	12	20	0.82	0.75	D	334	378	360
16	6	20	0.70	0.55	B	226	300	243
16	6	20	0.89	0.55	D	278	355	300
16	8	20	0.70	0.60	B	252	326	280
16	8	20	0.89	0.60	D	304	381	332
16	10	20	0.70	0.68	B	282	356	312
16	10	20	0.89	0.68	D	334	410	364
16	12	20	0.70	0.75	B	317	391	353
16	12	20	0.89	0.75	D	368	445	405
16	14	20	0.70	0.66	B	315	389	370
16	14	20	0.89	0.82	D	407	484	461
18	8	20	0.75	0.60	B	287	374	315
18	8	20	0.96	0.60	D	345	438	373
18	10	20	0.75	0.68	B	317	404	347
18	10	20	0.96	0.68	D	375	468	405
18	12	20	0.75	0.75	B	352	438	388
18	12	20	0.96	0.75	D	410	502	446
18	14	20	0.75	0.66	B	350	437	406
18	14	20	0.96	0.82	D	448	541	582
18	16	20	0.75	0.70	B	383	469	457
18	16	20	0.96	0.89	D	492	585	569
20	10	26	0.80	0.68	B	414	516	445
20	10	26	1.03	0.68	D	499	615	539

On all sizes $s = 8$ inches.

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42.—REDUCERS AND INCREASERS, TYPE No. 2.—Continued.
(See Fig. 42a, preceding page.)

inal Diam. Inches		V	Thickness, Inches.		Class.	Weights, Pounds.		
e	f	Ins.	t_1	t_2		Spigot Ends.	Large End Bell.	Small End Bell.
0	12	26	0 80	0.75	B	455	556	491
0	12	26	1 03	0.75	D	539	656	576
0	14	26	0.80	0.66	B	453	554	508
0	14	26	1.03	0.82	D	583	700	638
)	16	26	0.80	0.70	B	490	592	564
)	16	26	1.03	0.89	D	635	751	711
)	18	26	0 80	0.75	B	531	633	617
)	18	26	1.03	0.96	D	683	800	776
	14	26	0 89	0.66	B	552	680	607
	14	26	1.16	0.82	D	710	866	764
	16	26	0.89	0.70	B	589	717	663
	16	26	1.16	0.89	D	762	917	838
	18	26	0.89	0.75	B	630	758	717
	18	26	1.16	0.96	D	810	965	901
	20	26	0.89	0.80	B	675	803	776
	20	26	1.16	1.03	D	871	1027	987
	18	26	0.88	0.75	A	710	903	796
	18	26	1.03	0.75	B	791	969	878
	18	26	1.20	0.96	C	956	1166	1048
	18	26	1.37	0.96	D	1054	1305	1146
	20	26	0.88	0.80	A	754	947	856
	20	26	1.03	0.80	B	836	1014	937
	20	26	1.20	1.03	C	1018	1227	1134
	20	26	1.37	1.03	D	1115	1366	1232
	20	66	0.88	9.80	A	1468	1661	1569
	20	66	1.03	0.80	B	1626	1804	1728
	20	66	1.20	1.03	C	1981	2190	2098
	20	66	1.37	1.03	D	2172	2423	2289
	24	26	0.88	0.89	A	854	1047	981
	24	26	1.03	0.89	B	935	1113	1063
	24	26	1.20	1.16	C	1144	1354	1300
	24	26	1.37	1.16	D	1242	1493	1398
	24	66	0.88	0.89	A	1661	1921	1869
	24	66	1 03	0.89	B	1820	1998	1946
	24	66	1.20	1.16	C	2228	2438	2384
	24	66	1.37	1.16	D	2419	2670	2575
	20	32	0.99	0.80	A	1039	1286	1141
	20	32	1.15	0.80	B	1170	1450	1272
	20	32	1.36	1.03	C	1417	1739	1534
	20	32	1.58	1.03	D	1589	1951	1705
	20	66	0.99	0.80	A	1771	2018	1872
	20	66	1.15	0.80	B	1994	2274	2095
	20	66	1.36	1.03	C	2416	2738	2533
	20	66	1.58	1.03	D	2710	3072	2837
	24	32	0.99	0.89	A	1153	1339	1280
	24	32	1.15	0.89	B	1283	1564	1411
	24	32	1.36	1.16	C	1562	1884	1718
	24	32	1.58	1.16	D	1734	2096	1890

all sizes $s = 8$ inches.

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42.—Reducers and Increasers, Type No. 2.—Continued.

(See Fig. 42a, page 1260.)

Nominal Diam. Inches.		V	Thickness, Inches.		Class.	Weights, Pounds.		
e	f	Ins.	t_1	t_2		Spigot Ends.	Large End Bell.	Small End Bell.
36	24	66	0.99	0.89	A	1964	2211	2091
36	24	66	1.15	0.89	B	2188	2468	2314
36	24	66	1.36	1.16	C	2664	2985	2820
36	24	66	1.58	1.16	D	2957	3319	3113
36	30	32	0.99	0.88	A	1243	1490	1436
36	30	32	1.15	1.03	B	1467	1747	1645
36	30	32	1.36	1.20	C	1730	2051	1939
36	30	32	1.58	1 37	D	2013	2375	2264
36	30	66	0.99	0.89	A	2119	2366	2312
36	30	66	1.15	1.03	B	2502	2783	2680
36	30	66	1.36	1.20	C	2950	3271	3159
36	30	66	1.58	1.37	D	3434	3796	3684
42	20	32	1.10	0.80	A	1262	1602	1364
42	20	32	1.28	0.80	B	1413	1768	1515
42	20	32	1.54	1.03	C	1753	2168	1869
42	20	32	1.78	1.03	D	1975	2445	2092
42	20	66	1.10	0.80	A	2152	2491	2254
42	20	66	1.28	0.80	B	2410	2764	2511
42	20	66	1.54	1.03	C	2989	3405	3106
42	20	66	1.78	1.03	D	3369	3839	3486
42	24	32	1.10	0.89	A	1576	1715	1504
42	24	32	1.28	0.89	B	1527	1881	1654
42	24	32	1.54	1.16	C	1898	2313	2053
42	24	32	1.78	1.16	D	2120	2590	2276
42	24	66	1.10	0.89	A	2346	2685	2472
42	24	66	1.28	0.89	B	2603	2958	2730
42	24	66	1.54	1.16	C	3237	3652	3392
42	24	66	1.78	1.16	D	3616	4086	3772
42	30	32	1.10	0.88	A	1467	1806	1660
42	30	32	1.28	1.03	B	1711	2065	1889
42	30	32	1.54	1.20	C	2065	2480	2275
42	30	32	1.78	1.37	D	2399	2869	2605
42	30	66	1.10	0.88	A	2500	2839	2683
42	30	66	1.28	1.03	B	2917	3271	3095
42	30	66	1.54	1.20	C	3529	3938	3732
42	30	66	1.78	1.37	D	4093	4563	4344
42	36	32	1.10	0.99	A	1645	1984	1891
42	36	32	1.28	1.15	B	1926	2281	2207

On all sizes $s = 8$ inches.

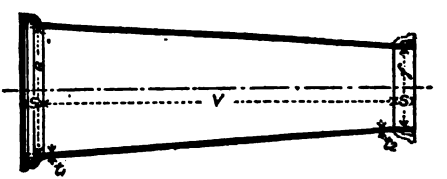

Fig. 42b.—Long Increaser. 48 to 30 inches x 132 inches v.

42.—REDUCERS AND INCREASERS, TYPE No. 2.—Continued.
(See Fig. 42a, page 1260.)

Nominal Diam. Inches.		V	Thickness, Inches		Class.	Weights, Pounds.		
e	f	Ins.	t_1	t_2		Spigot Ends.	Large End Bell.	Small End Bell
42	36	32	1.54	1.36	C	2320	2735	2642
42	36	32	1.78	1.58	D	2714	3184	3076
42	36	66	1.10	0.99	A	2803	3143	3050
42	36	66	1.28	1.15	B	3285	3639	3565
42	36	66	1.54	1.36	C	3958	4373	4279
42	36	66	1.78	1.58	D	4631	5101	4993
48	30	66	1.26	0.88	A	2975	3381	3168
48	30	66	1.42	1.03	B	3428	3883	3606
48	30	66	1.71	1.20	C	4092	4641	4301
48	30	66	1.96	1.37	D	4762	5388	5013
48	30	132	1.26	0.88	A	5363	5769	5556
48	30	132	1.42	1.03	B	6180	6635	6359
48	30	132	1.71	1.20	C	7379	7928	7588
48	30	132	1.96	1.37	D	8588	9214	8839
48	36	66	1.26	0.99	A	3278	3684	3525
48	36	66	1.42	1.15	B	3796	4252	4077
48	36	66	1.71	1.36	C	4527	5076	4849
48	36	66	1.96	1.58	D	5300	5925	5662
48	36	132	1.26	0.99	A	5909	6316	6156
48	36	132	1.42	1.15	B	6844	7299	7125
48	36	132	1.71	1.36	C	8164	8713	8485
48	36	132	1.96	1.58	D	9558	10184	9920
48	42	66	1.26	1.10	A	3659	4066	3998
48	42	66	1.42	1.28	B	4212	4667	4564
48	42	66	1.71	1.54	C	5100	5649	5516
48	42	66	1.96	1.78	D	5959	6585	6429
48	42	132	1.26	1.10	A	6597	7003	6936
48	42	132	1.42	1.28	B	7594	8049	7948
48	42	132	1.71	1.54	C	9197	9746	9612
48	42	132	1.96	1.78	D	10747	11373	11217
54	36	66	1.35	0.99	A	3722	4228	3969
54	36	66	1.55	1.15	B	4330	4925	4610
54	36	66	1.90	1.36	C	5259	5953	5580
54	36	66	2.23	1.58	D	6181	6995	6543
54	36	132	1.35	0.99	A	6710	7216	6957
54	36	132	1.55	1.15	B	7806	8401	8087
54	36	132	1.90	1.36	C	9484	10178	9805
54	36	132	2.23	1.58	D	11148	11962	11510

On all sizes $s = 8$ inches.

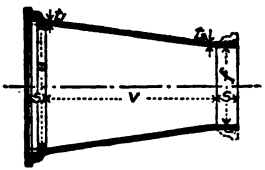

Fig. 42c.—Short Increaser. 48 to 30 x 66 inches V.

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42.—Reducers and Increasers, Type No. 2.—Concluded.

(See Fig. 42a, page 1260.)

Nominal Diam. Inches.		V	Thickness, Inches.		Class.	Weights, Pounds.		
e	f	Ins.	t_1	t_2		Spigot Ends.	Large-End Bell.	Small End Bell
54	42	66	1.35	1.10	A	4103	4609	4442
54	42	66	1.55	1.28	B	4745	5340	5109
54	42	66	1.90	1.54	C	5832	6526	6247
54	42	66	2.23	1.78	D	6841	7655	7310
54	42	132	1.35	1.10	A	7398	7903	7737
54	42	132	1.55	1.28	B	8556	9151	8910
54	42	132	1.90	1.54	C	10517	11211	10932
54	42	132	2.23	1.78	D	12338	13153	12897
54	48	66	1.35	1.26	A	4578	5083	4984
54	48	66	1.55	1.42	B	5256	5851	5711
54	48	66	1.90	1.71	C	6401	7095	6960
54	48	66	2.23	1.96	D	7512	8326	8137
54	48	132	1.35	1.26	A	8253	8759	8660
54	48	132	1.55	1.42	B	9478	10073	9933
54	48	132	1.90	1.71	C	11544	12239	12093
54	48	132	2.23	1.96	D	13550	14364	14175
60	36	66	1.39	0.99	A	4096	4711	4842
60	36	66	1.67	1.15	B	4906	5576	5196
60	36	66	2.00	1.36	C	5867	6692	6189
60	36	66	2.38	1.58	D	6960	7934	7332
60	36	132	1.39	0.99	A	7384	7999	7631
60	36	132	1.67	1.15	B	8846	9616	9126
60	36	132	2.00	1.36	C	10581	11405	10902
60	36	132	2.38	1.58	D	12554	13527	12916
60	42	66	1.39	1.10	A	4477	5092	4816
60	42	66	1.67	1.28	B	5321	5991	5676
60	42	66	2.00	1.54	C	6440	7264	6855
60	42	66	2.38	1.78	D	7619	8593	8069
60	42	132	1.39	1.10	A	8072	8687	8411
60	42	132	1.67	1.28	B	9595	10265	9960
60	42	132	2.00	1.54	C	11614	12439	12030
60	42	132	2.38	1.78	D	13743	14716	14213
60	48	66	1.39	1.26	A	4957	5572	5362
60	48	66	1.67	1.42	B	5832	6503	6287
60	48	66	2.00	1.71	C	7006	7830	7556
60	48	66	2.38	1.96	D	8285	9259	8910
60	48	132	1.39	1.26	A	8938	9553	9344
60	48	132	1.67	1.42	B	10517	11187	10972
60	48	132	2.00	1.71	C	12634	13458	13183
60	48	132	2.38	1.96	D	14943	15917	15568
60	54	66	1.39	1.35	A	5404	6019	5910
60	54	66	1.67	1.55	B	6348	7018	6961
60	54	66	2.00	1.90	C	7750	8574	8444
60	54	66	2.38	2.23	D	9178	10152	9992
60	54	132	1.39	1.35	A	9745	10360	10251
60	54	132	1.67	1.55	B	11462	12132	12075
60	54	132	2.00	1.90	C	13979	14803	14673
60	54	132	2.38	2.23	D	16557	17530	17371

On all sizes $s = 8$ inches.

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43.—Properties of Sleeves.
(A. W. W. A.)

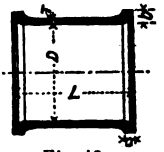

Fig. 43.
For dimensions *a* and *b* see Table No. 26.

Nominal Diameter. Inches.	Class.	D Ins.	L Ins.	T Ins.	Approx. Weight. Pounds.	Nominal Diameter. Inches.	Class.	D Ins.	L Ins.	T Ins.	Approx. Weight. Pounds.
4	D	5.80	10	0.65	47	36	B	39.40	15	1.40	943
4	D	5.80	15	0.65	61	36	C	39.80	15	1.60	1077
6	D	7.90	10	0.70	68	36	D	40.20	15	1.80	1217
6	D	7.90	15	0.70	87	36	A	39.00	24	1.25	1202
8	D	10.10	12	0.75	104	36	B	39.40	24	1.40	1362
8	D	10.10	15	0.75	119	36	C	39.80	24	1.60	1563
10	D	12.20	12	0.80	123	36	D	40.20	24	1.80	1772
10	D	12.20	18	0.80	176	42	A	45.30	15	1.40	1097
12	D	14.30	14	0.85	174	42	B	45.60	15	1.50	1184
12	D	14.30	18	0.85	223	42	C	46.20	15	1.75	1381
14	B	16.20	15	0.85	220	42	D	46.70	15	1.95	1561
14	B	16.20	18	0.85	249	42	A	45.30	24	1.40	1577
14	D	16.50	15	0.90	240	42	B	45.60	24	1.50	1702
14	D	16.50	18	0.90	280	42	C	46.20	24	1.75	1997
16	B	18.50	15	0.90	274	42	D	46.70	24	1.95	2262
16	B	18.50	24	0.90	391	48	A	51.60	15	1.50	1337
16	D	18.90	15	1.00	305	48	B	51.90	15	1.65	1481
16	D	18.90	24	1.00	443	48	C	52.50	15	1.95	1752
18	B	20.60	15	0.95	321	48	D	53.10	15	2.20	1986
18	B	20.60	24	0.95	462	48	A	51.60	24	1.50	1923
18	D	21.00	15	1.05	360	48	B	51.90	24	1.65	2129
18	D	21.00	24	1.05	518	48	C	52.50	24	1.95	2532
20	B	22.70	15	1.00	374	48	D	53.10	24	2.20	2879
20	B	22.70	24	1.00	532	54	A	57.70	15	1.60	1612
20	D	23.10	15	1.15	440	54	B	58.20	15	1.80	1835
20	D	23.10	24	1.15	625	54	C	58.90	15	2.15	2156
24	B	26.90	15	1.05	477	54	D	59.50	15	2.45	5450
24	B	26.90	24	1.05	680	54	A	57.70	24	1.60	2316
24	D	27.40	15	1.25	583	54	B	58.20	24	1.80	2634
24	D	27.40	24	1.25	821	54	C	58.90	24	2.15	3126
30	A	32.80	15	1.15	648	54	D	59.50	24	2.45	3571
30	B	33.10	15	1.15	652	60	A	63.90	15	1.70	1906
30	C	33.50	15	1.32	760	60	B	64.50	15	1.90	2127
30	D	33.80	15	1.50	876	60	C	65.30	15	2.25	2491
30	A	32.80	24	1.15	943	60	D	65.90	15	2.60	2895
30	B	33.10	24	1.15	949	60	A	63.90	24	1.70	2731
30	C	33.50	24	1.32	1088	60	B	64.50	24	1.90	3058
30	D	33.80	24	1.50	1262	60	C	65.30	24	2.25	3601
36	A	39.00	15	1.25	833	60	D	65.90	24	2.60	4231

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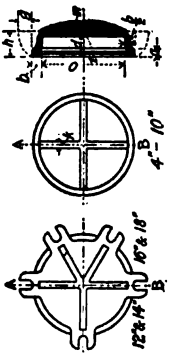
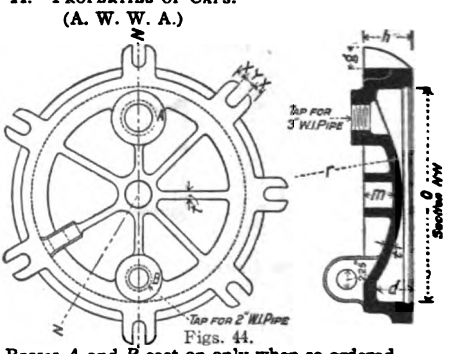
44.—Properties of Caps.
(A. W. W. A.)





Figs. 44.

Bosses *A* and *B* cast on only when so ordered.

Nom'l Diam. Inches.	Class.	d	o	l	t	m	k	r	Approx. Weight. Pounds.
4	D	4.00	5.70	.....	0.60	.....	.....	.....	26
6	D	4.00	7.80	.....	0.65	.....	.....	.....	40
8	D	4.00	10.00	.....	0.75	.....	.....	.....	59
10	D	4.00	12.10	.....	0.75	1.50	0.75	16.20	81
12	D	4.00	14.20	.....	0.75	1.75	0.75	18.70	104
14	B	4.00	16.10	.....	0.90	1.90	0.75	22.40	140
14	D	4.00	16.45	.....	0.90	1.90	0.75	22.40	149
16	B	4.00	18.40	.....	1.00	2.00	0.75	27.00	183
16	D	4.00	18.80	.....	1.00	2.00	0.75	27.00	198
18	B	4.00	20.50	.....	1.00	2.00	1.00	32.00	226
18	D	4.00	20.92	.....	1.00	2.00	1.00	32.90	242
20	B	4.00	22.60	.....	1.00	3.00	1.00	18.20	278
20	D	4.00	23.06	.....	1.00	3.00	1.00	18.20	308
24	B	4.00	26.80	2.50	1.05	3.50	1.00	23.50	392
24	D	4.00	27.32	2.50	1.05	3.50	1.00	23.50	443
30	A	4.50	32.74	2.62	1.15	3.50	1.15	34.80	589
30	B	4.50	33.00	2.62	1.15	3.50	1.15	34.80	596
30	C	4.50	33.40	2.62	1.15	3.50	1.15	34.80	647
30	D	4.50	33.74	2.62	1.15	3.50	1.15	34.80	704
36	A	4.50	38.96	3.12	1.25	4.00	1.25	44.00	849
36	B	4.50	39.30	3.12	1.30	3.95	1.25	44.00	918
36	C	4.50	39.70	3.12	1.35	3.90	1.25	44.00	996
36	D	4.50	40.16	3.12	1.40	3.85	1.25	44.00	1084
42	A	5.00	45.20	3.37	1.40	4.00	1.40	63.50	1380
42	B	5.00	45.50	3.37	1.50	3.90	1.40	63.50	1388
42	C	5.00	46.10	3.37	1.60	3.80	1.40	63.50	1539
42	D	5.00	46.58	3.37	1.70	3.70	1.40	63.50	1679
48	A	5.00	51.50	3.62	1.70	4.00	1.50	76.50	1772
48	B	5.00	51.80	3.62	1.90	3.80	1.50	76.50	1943
48	C	5.00	52.40	3.62	2.00	3.70	1.50	76.50	2144
48	D	5.00	52.98	3.62	2.10	3.60	1.50	76.50	2341
54	A	5.50	57.66	3.87	1.90	4.50	1.50	83.00	2329
54	B	5.50	58.10	3.87	2.00	4.40	1.50	82.00	2519
54	C	5.50	58.80	3.87	2.10	4.30	1.50	82.00	2770
54	D	5.50	59 40	3.87	2.20	4.20	1.50	82.00	3009
60	A	5.50	63.80	4.12	2.00	4.50	1.50	99.00	2868
60	B	5.50	64.40	4.12	2.10	4.40	1.50	99.00	3063
60	C	5.50	65.20	4.12	2.20	4.30	1.50	99.00	3383
60	D	5.50	65.82	4.12	2.30	4.20	1.50	99.00	3687

Note.—All dimensions are in inches.

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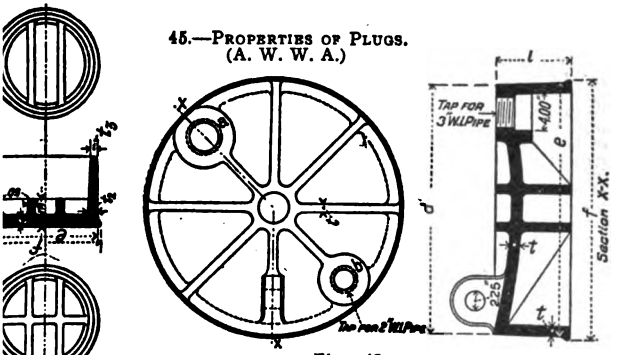
45.—Properties of Plugs. (A. W. W. A.)



Figs. 45.

20 ins. Bosses *a* and *b* cast on only when so ordered. 42 to 60 ins.

Allowed.	Class.	e	f	d	l	m	Thickness. Inches. t	t_2	t_3	Number of Ribs.	Approx. Weight. Pounds.
	D	4.90	5.28	.....	5.50	.....	0.50	0.40	0.20	.....	8
	D	7.00	7.38	.....	5.50	.....	0.60	0.40	0.20	.....	14
	D	9.15	9.65	.....	5.50	2.00	0.60	0.40	0.20	2	24
	D	11.20	11.70	.....	6.00	2.00	0.70	0.50	0.20	2	38
	D	13.30	13.80	.....	6.00	2.00	0.75	0.50	0.20	2	50
	B	15.30	15.80	.....	6.00	2.00	0.70	0.50	0.20	2	63
	D	15.65	16.15	.....	6.00	2.00	0.75	0.50	0.20	2	65
	B	17.40	17.90	.....	6.50	2.00	0.70	0.50	0.30	3	90
	D	17.80	18.30	.....	6.50	2.00	0.80	0.60	0.30	3	96
	B	19.50	20.00	.....	6.50	2.50	0.75	0.60	0.30	3	111
	D	19.92	20.42	.....	6.50	2.50	0.85	0.60	0.30	3	121
	B	21.60	22.10	.....	6.50	2.75	0.85	0.60	0.30	3	151
	D	22.06	22.56	.....	6.50	2.75	1.00	0.60	0.30	3	156
	B	25.92	26.30	25.68	8.00	.....	0.89	.....	.....	4	375
	D	26.44	26.82	26.20	8.00	.....	1.16	.....	.....	4	472
	A	31.86	32.24	31.62	8.00	.....	0.88	.....	.....	4	481
	B	32.12	32.50	31.88	8.00	.....	1.03	.....	.....	4	556
	C	32.52	32.90	32.28	8.00	.....	1.20	.....	.....	4	641
	D	32.86	33.24	32.62	8.00	.....	1.37	.....	.....	4	723
	A	38.08	38.46	37.84	8.00	.....	0.99	.....	.....	4	683
	B	38.42	38.80	38.18	8.00	.....	1.15	.....	.....	4	786
	C	38.82	39.20	38.58	8.00	.....	1.36	.....	.....	4	914
	D	39.28	39.66	39.04	8.00	.....	1.58	.....	.....	4	1050
	A	44.32	44.70	44.08	9.00	.....	1.10	.....	.....	4	991
	B	44.62	45.00	44.38	9.00	.....	1.28	.....	.....	4	1138
	C	45.32	45.60	44.98	9.00	.....	1.54	.....	.....	4	1353
	D	45.70	46.08	45.46	9.00	.....	1.78	.....	.....	4	1551
	A	50.62	51.00	50.38	9.00	.....	1.26	.....	.....	4	1349
	B	50.92	51.30	50.68	9.00	.....	1.42	.....	.....	4	1506
	C	51.52	51.90	51.28	9.00	.....	1.71	.....	.....	4	1800
	D	52.10	52.48	51.86	9.00	.....	1.96	.....	.....	4	2047
	A	56.78	57.16	56.54	9.00	.....	1.35	.....	.....	4	1697
	B	57.22	57.60	56.98	9.00	.....	1.55	.....	.....	4	1945
	C	57.92	58.30	57.68	9.00	.....	1.90	.....	.....	4	2356
	D	58.52	58.90	58.28	9.00	.....	2.23	.....	.....	4	2733
	A	62.92	63.30	62.68	9.00	.....	1.39	.....	.....	4	2045
	B	63.52	63.90	63.28	9.00	.....	1.67	.....	.....	4	2434
	C	64.32	64.70	64.08	9.00	.....	2.00	.....	.....	4	2904
	D	64.94	65.32	64.70	9.00	.....	2.38	.....	.....	4	3397

Note.—All dimensions are in inches.

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IIId.—WROUGHT IRON PIPE.

Wrought Iron Pipe corrodes more rapidly than cast iron, and less rapidly than steel. Its first cost, in the smaller sizes, is less than the former; and in all sizes it is greater than the latter. Between cast iron on the one hand and the steel on the other the use of wrought iron in pressure pipe lines has become limited, but it yet finds a demand in the smaller sizes of pipe for distributing systems. It remains to be seen whether the few remaining manufacturers of wrought iron pipe will be able to offer convincing proofs of the superior merit and ultimate economy of their article, as claimed. More attention is paid now to suitable coating for pipes than formerly, and a steel pipe well coated when laid should last from 25 to 40 years in ordinary soil, and if occasionally cleaned and painted it will last much longer. Wrought iron pipe may be manufactured in the same forms as steel pipe (IIIe), but it is usually lap-welded, seldom riveted.

IIIe.—STEEL PIPE.

Steel Pipe is usually designated by the kind of "seam" or "joint." The latter term is generally used with reference to the (end) connection between two pipes, and frequently also to the longitudinal seam at the joining of plates in the pipe itself. Seams or joints in a pipe may be lap-welded, spiral riveted, longitudinal riveted, or locking-bar. Riveted (longitudinal) joints may be lap- or butt-, and be (single-), double- or triple-riveted, with (single or) double straps. Single riveting or single straps are seldom used for longitudinal joints. The end joints for pipes may be screw-, bell-and-spigot- (rarely used), flange- (welded, riveted, or screw), single-riveted-, patent-locking-, etc. A few of these will be described as follows:

Riveted Steel Pipe is especially economical for large diameters, subjected to pressure head of 300 ft. or more. For heads between 200 and 300 ft. riveted steel has to compete with cast iron pipe; and for heads under 200 ft. both riveted steel and cast iron pipe have to compete with wood stave pipe, especially in the West where lumber is plentiful.

The thickness of steel plates is proportioned by the formula—

$$t = \frac{(p+p')d}{2se} + c \qquad (1)$$

where t = thickness of steel shell, in inches;

d = inside diameter of pipe, in inches;

h = pressure head in ft. ($= 2.304\,p$);

p = pressure of water in lbs. per square inch ($= 0.434\,h$);

p' = allowance for water ram, in lbs. per square inch;

s = allowable tensile stress of steel, in lbs. per square inch;

e = efficiency of riveted joint, say 0.50 to 0.80;

c = thickness to be added to plate for corrosion, etc.

Based on a safety factor of 4, s is assumed at 15,000 (to 16,000) lbs; and c may be assumed at say $\frac{1}{16}$ in.

The practical working formula will result as follows, assuming p' at 80 lbs. (see page 1215):

$$t = \frac{(p+p')d}{30000\,e} + \frac{1}{16} = \frac{(0.434\,h+80)d}{30000\,e} + 0.0625 \qquad (2)$$

If h = the static head in feet, we have, from (2),

$$h = \frac{69120\,e}{d}\left(t - \frac{1}{16}\right) - 184.32 \qquad (3)$$

In designing the riveting of longitudinal seams, we have, if F = total tension per lin. inch of seam, due to p and p',

$$F = \frac{d}{2}(p+p') = d(0.217\,h + 40) = 1500\,(t-c)e \qquad (4)$$

in which F should not exceed the bearing- nor the shearing resistance of the rivets. The bearing value may be assumed at 16,000 lbs. per square inch on the thickness of plate $t-c$, as it is assumed that c is the allowance for corrosion; while the shearing value may be assumed at say 10,000 lbs. per square inch of rivet section. The diameter of rivet is approximately twice the thickness of the plate.

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The following is a table of pitches and dimensions of rivets adopted in the manufacture of the 42-in. pipe for the Seattle (Wash.) Water-works, constructed in 1900:

Thickness of Steel, in inches.....	7/16	¼	7/16	⅜	7/16
Diameter of rivets, in inches.....	½	9/16	⅝	¾	13/16
Diameter of rivet holes, in inches.....	9/16	⅝	13/16	13/16	⅞
Pitch in double riveted seams, in ins..	1.75	2.00	2.25	2.50	2.50
Pitch in single riveted seams, in ins...	1.40	1.60	2.00	2.25	2.25
Distance between rows on double riveted seams, in inches.....	1.50	1.75	2.00	2.00	2.00
Lap from center of rivet to edge of plate, in inches.....	.750	.875	1.00	1.00	1.125

The edges of all plates are bevel sheared for caulking. Shop- riveting and caulking are invariably done by machine. For thick plates the rivets may be countersunk on inside of pipe. The field- riveting and caulking are usually done by hand, but machine riveters and caulkers are sometimes employed, not always to advantage. The field work consists in joining the pipes together in the trench and making them water-tight under pressure. If the end joints are lap-riveted there must necessarily be alternate courses with two separate diameters, but the internal diameters of the smaller course shall be "the diameter" of the pipe. The internal diameter of the larger course must equal exactly the external diameter of the smaller course. Pipes of ordinarily large diameter usually leave the shop made up in four courses, and in length from 20 to 30 ft., after being tested to a sufficient hydrostatic pressure, and suitably coated (see page 358, etc.).

In shipment it is well to bear in mind that a "full" carload cannot always be made up with pipes of the *same* diameter if large.

When laid in the trench the longitudinal seams should be at the top of the pipe, staggered not less than 6 inches.

* **Locking-Bar Joint Pipe** is designed to supplant the riveted joint. Fig. 46 shows a transverse section of the (longitudinal) bar and joint and also the (end) joint ring. In manufacture, the edges of the plate are upset and inserted in the grooves on either side of the locking bar. The bar is then subjected to a great hydraulic pressure, tightly locking the plates in a highly efficient manner. No riveted longitudinal joints are used. Sleeves or joint rings are used for end joint connections of pipes and are run with lead; or the end joints may be riveted.

Fig. 46.

Lap-Welded Pipe is seldom used in the pressure pipe line; it is better adapted to use in the distributing system (see page 1280). The four principal types used are those with screw ends or flange ends (Section 36), Converse Joint (p. 1282), and Matheson Joint (p. 1281).

Spiral Riveted Pipe (p.680) is used largely in hydraulic mining.

IIIf.—PRESSURE PIPE ATTACHMENTS.

Brief mention will be made of some of the more important items in connection with an ordinary pressure-pipe line. The varying conditions are so great in different lines that the merest hints only will be given.

* This form of joint was brought prominently to the attention of American engineers in 1898 by an article published in Eng. News, Vol. XXXIX, p. 373, describing its use in connection with the construction of a large pipe line in Australia.

Fig. 47 shows a **Sluice-Gate** for the inlet end of the pipe at the headworks, and Fig. 48 shows Stand and Wheel for operating same. Each may be fastened to either masonry (by stone-bolts) or to timber (by screw-bolts). This pattern is suitable for large inlets, 30 ins. or more in diameter. The gear-wheel is often horizontal instead of vertical.

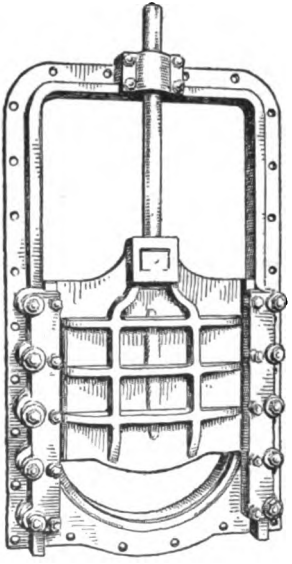

Fig. 47.—Sluice-Gate for Headworks.

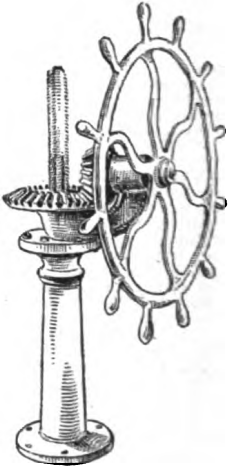

Fig. 48.—Stand and Wheel for Operating Sluice-Gate.

Air Relief Valves, or "Air Valves," are placed at summits of pipe lines to afford relief from air pressure or from vacuum. As the pipe fills with water the valve operates so as to allow the air to escape, but closes against the outlet of the water. As the water in the pipe subsides, the valve allows the air to enter the pipe and prevent collapse from vacuum. If air is allowed

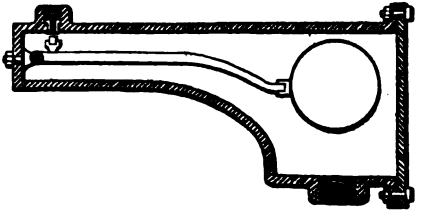

Fig. 49.

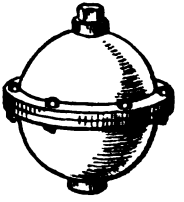

Fig. 50.

to collect at summits without air valves it is clearly seen that the water area of the pipe at those points, and hence the flow, will be decreased accordingly; and also that the pipe is more liable to leak and burst (from air-

from water pressure). Air valves may be single, or they may be ıged in clusters to operate consecutively. There is an advantage in ıg a stop-valve between the air valve and the top of the main pipe, so water can be shut off during repairs. Valves should be protected by 1- or metal casings and be out of reach of the frost.

Fig. 49 illustrates the Ludlow automatic lever and float air valve, and 50 the globe air valve.

Fig. 51 shows section and plan of Metropolitan Water Works air valve, table of dimensions.

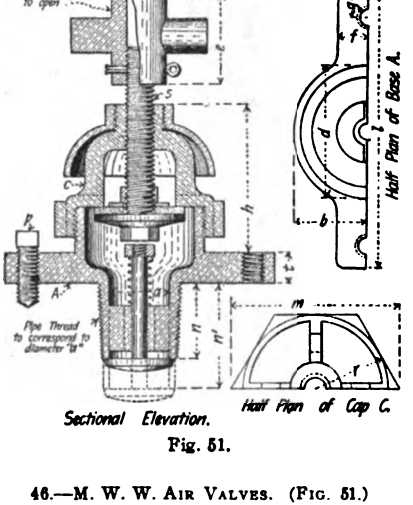

Fig. 51.

46.—M. W. W. Air Valves. (Fig. 51.)

Dimensions are in inches; weights, in pounds.

	b	m	d	r	l	f	g	e	h	t	n	n'	Stem s Diam.	Stem s Thds.	p Diam.	Wt. Lbs.
	1 9/32	2 13/16	2 3/16	1 7/32	4 1/2	1/2	3/8	1 1/2	2 7/16	1/2	1 1/4	1 3/4	1/2	12	3/8	3 1/4
1/2	1 9/16	3 5/8	2 3/4	1 9/16	5 1/16	9/16	3/8	1 1/2	2 3/4	1/2	1 3/4	...	5/8	12	3/8	5 1/4

Standpipes (small) are sometimes erected at the summits of pipe lines serve both as air valves, and also as piezometers for determining the ydraulic grade line (see p. 1159). They are often preferred to air valves here the static head is not great.

Gate Valves, stop valves, gates, or valves, as they are variously termed e of numerous designs, suited to special purposes. The best manufac- urers are beginning to standardize their product so that there is very little fference in quality of material and service. But there is a vast difference length of service between these standard makes and most of the cheaper lves thrown upon the market. Waterworks engineers, generally, fully alize the importance of securing at all times the best article even at the eater first cost. It is economy in the end. The best gates are bronze- ounted.

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Fig. 65, page 1285, shows a section of the Chapman valve with wedge shaped gate. Fig. 52, below, shows a section of a double gate valve of the Ludlow type; and Fig. 53, views of the gates and wedges. The pipe con

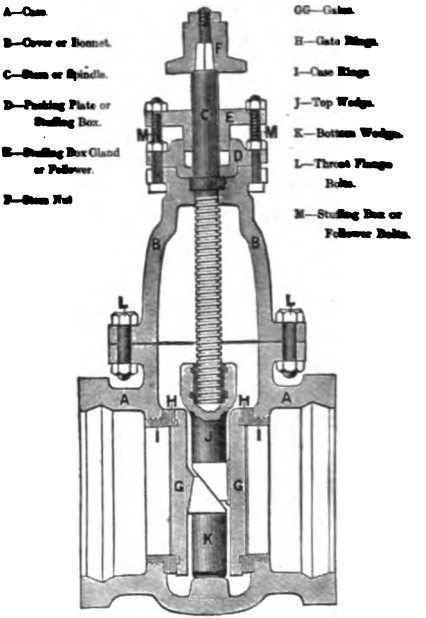

Fig. 52.—Ludlow Bronze Mounted Double Gate Valve With Bolted Stuffing Box.

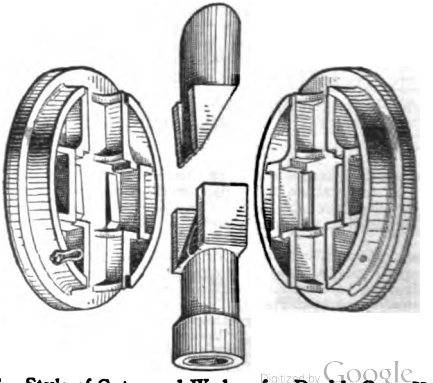

Fig. 53.—Style of Gates and Wedges for Double Gate Valves.

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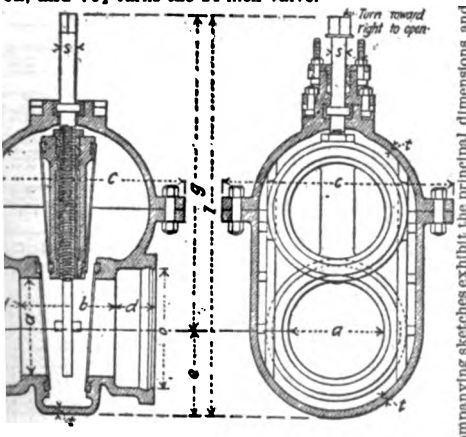
ns may be bell- (as shown in the illustrations), flange- or screw-, as ed. Gates are designed for either vertical or horizontal (the larger position, and for operating by hand or "power," with screw, piston, or gearing (gates under 16 ins. in size are seldom geared). They are manufactured with or without by-pass relief. The by-pass is useful in long pipe lines without impact, and in equalizing the pressure on both of a gate when being opened or closed.

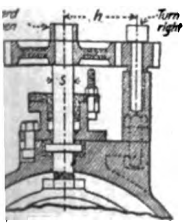
47.—M. W. W. Gate Valves.

Principal Dimensions, in Inches; Weights, in Pounds.

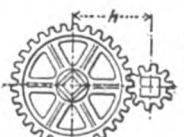
b	d	l	g	c	h	e	o	t	s	Pairs of Lugs.	Turns to Open.	Total Weight	Weight of Composit'n.
12	4	46 1/16	35 11/16	26	.....	10 3/8	14 1/4	7/8	1 3/4	2	58	1175	76
13	4 1/2	54 9/16	42 1/8	30 1/2	.....	12 7/16	18 5/8	15/16	2	2	74	1625	100
16	4 1/2	66 11/16	52 3/16	35	8	14 1/2	23	1	2 1/2	6	*203	2675	180
21	4 1/2	73 1/8	57 1/2	38	8	15 5/8	27	1 1/8	2 3/4	6	*230	3325	215
36	5	110 1/4	75 1/4	44 1/2	54 3/8	28 1/4	33	1 5/16	3 1/4	5	254	7000	600
40 5/8	5 1/2	125	87	53 1/2	66 3/8	32	39 1/4	1 3/8	3 1/2	0	306	10250	710

Using gears. With wrench on main stem, 67½ turns will open the ch, and 76½ turns the 24-inch valve.





ctional Elevation
s for 20" & 24" Valves.



Plan.

Commonwealth of Massachusetts
Metropolitan Water Works.

Vertical Gate Valves with & without Gears

Note.—The table above and the accompanying sketches exhibit the principal dimensions and data of the M. W. W. valves. All wrench nuts are 1¾" square by 2" deep. Turn wrench toward right hand to open valves, except on nut on main stem of a 20-inch or a 24-inch valve, require 21 turns to open. Others have a pair of lugs on each extremity of the horizontal axis; the 20-inch valve, in which case, the pair of lugs next to the 20-inch and 36-inch valves have 6-inch by-passes. The valves of these by-passes (if Chapman gonz) are no lugs on 36-inch valves. Others have a pair of lugs on each extremity of the horizontal axis; the 20-inch valve, in which case, the pair of lugs next to the dange is omitted, of each bell, except the 30-inch valve, in which case, the pair of lugs next to the are spaced equally around the circumference of the bell. The remaining pairs of lugs on the 20, 24 and 30-inch valves tion parts were computed from the neat dimensions of the drawings; the *rough* castings will weigh The total weights were obtained by weighing some of the valves. The weights of the composi-

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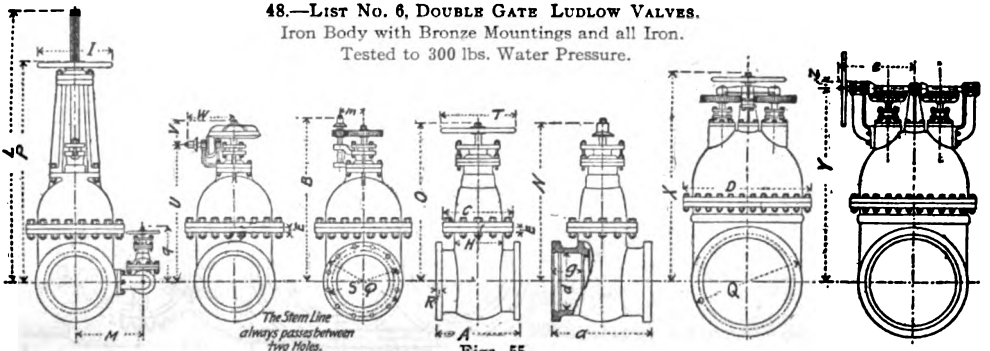
48.—List No. 6, Double Gate Ludlow Valves.

Iron Body with Bronze Mountings and all Iron.

Tested to 300 lbs. Water Pressure.



Figs. 55.

Note.—This Table gives dimensions, in inches, of various styles of the *larger sizes* of Ludlow valves. For smaller sizes see page 1286.

Size of Valve		14"	15"	16"	18"	20"	22"	24"	28"	30"	36"	40"	42"	48"
Face to Face of End Flanges without By-Pass	A	15¾	15¾	16	17	17¾	20¼	21	26½	26½	28	34½	34½	42½
Face to Face of End Flanges with By-Pass	A²	18½	18½	18½	22	22	22	26½	26½	26½	30	35¾	35¾	42½
Size of By-Pass		2	2	2	3	3	3	4	4	4	6	6	6	8
Diameter of End Flanges	Q	21	23¼	23½	25	27½	29	32	37	39	45	51	53	59
Thickness of End Flanges	R	1⅛	1⅛	1⅛	1¼	1¼	1 1/16	1⅜	1⅝	1⅝	1⅞	2⅛	2⅛	2⅜
Diameter of Bolt Circle	S	18¾	20	21¼	22¾	25	26⅞	29¾	34¼	36¼	42¾	48	50	56
Number and Size of End Flange Bolts		12–⅞	16–⅞	16–⅞	16–1	20–1	20–1	20–1	24–1	24–1	28–1	32–1⅛	36–1⅛	44–1¼
Width of Body	H	9	9⅛	9¼	11½	12¾	13¾	14⅝	18	19¾	21¼	23¾	23¾	29
Width of Throat Flange	C	13¼	13⅞	13¾	16⅛	16⅝	19¼	20⅜	24¼	26⅛	29¼	31¾	36¾	40
Length of Throat Flange	D	23⅜	24¾	25	28½	31¾	34	38¼	42⅜	46⅛	55	58	64½	71
Thickness of Throat Flange	E	1⅛	1 1/16	1¼	1 1/16	1¼	1¼	1¼	1⅝	1⅞	2	2¼	2⅜	3¼
Throat Flange Bolts		18–⅝	16–¾	18–⅝	20–¾	23–¾	22–¾	26–⅞	28–1	32–⅞	40–⅞	44–1⅛	40–1⅛	48–1¼
	F	1⅝	1¾	1¾	1⅞	2	2	2¼	2¾	2¾	3	3½	3½	4

48.—List No. 6, Double Gate Ludlow Valves.—Concluded.

Size of Valve		14"	15"	16"	18"	20"	22"	24"	28"	30"	36"	40"	42"	48"
Thickness of Metal in Body of Valve		⅞	1	1 1/32	1	1	1 1/16	1 1/16	1¼	1¼	1½	1¾	1¾	2
Diameter of Hand Wheel	T	15	18	18	21	24	24	24	.....	.....	.....	.....	.....	.....
Number of Turns to Open Valve without Gearing		45	48	51	57	63	69	75	.....	.....	.....	.....	.....	.....
Center of Port to Top of Hand Wheel	O	43	43	44¼	48¼	51	57	60½	.....	.....	.....	.....	.....	.....
Center of Port to Top of 2"x2" Nut	N	40	41	42½	47	50½	56½	60	.....	.....	.....	.....	.....	.....
End to End of Hub Valve without By-Pass	a	15¾	15½	15¾	18	19½	19½	22¼	24½	26	26⅞	33	33	39
End to End of Hub Valve with By-Pass	a^2	19¾	19¾	19¾	23⅛	23½	23½	26¾	32	32	33	33	33	39
Inside Diameter of Hub	d	16⅛	17½	18½	20⅜	22⅞	24½	26⅛	31½	33⅜	39⅛	44⅜	46⅛	53¼
Depth of Hub	g	4⅛	4	4⅛	5	5	5⅛	6¼	6½	6⅞	5¾	5½	5½	6¼
End to End of Spigot Valve		29¼	28¼	28¼	29¼	32¾	32¾	32¾	48	48	50¼	56	56	65¼
Diameter of Spigot		15½	16½	17⅜	19½	21¾	23⅜	25½	30	32	38¼	42½	44½	51
Center of Port to Top of Hand Wheel, when Closed (Outside Screw and Yoke Valves)	P	55	.....	58	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
Center of Port to Top of Stem when Open (Outside Screw and Yoke Valves)	L	71	.....	76	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
Diameter of O. S. and Y. Hand Wheel (Outside Screw and Yoke Valves)	I	15	18	18	21	24	24	24	.....	.....	.....	.....	.....	.....
Center of Valve to Center of By-Pass Valve	n	15⅛	16⅛	16¾	17½	18½	19¾	22⅝	25¼	26	30½	32½	33½	41¼
Center of By-Pass Valve to Top of Wheel	g	8¾	8¾	8¾	11½	11½	11½	16	16	16	21	21	21	26¼
Spur Gear Valve—Center of Port to Top of 2"x2" Nut (Single Stem Valves)	B	43	46	47	47	55⅝	56	58⅛	73	79	92¼	96¾	96¾	.....
Spur Gear Valve—Center of Stem to Center of Pinion (Single Stem Valves)	M	8¼	8¼	8¼	8¼	8¼	9½	9½	14	14	18¼	18¼	18¼	23½
Bevel Gear Valve—Center of Port to Center of Pinion (Single Stem Valves)	U	33	36	35½	38¼	43¾	48½	50	63⅛	67⅜	78⅞	.....	84	98½
Bevel Gear Valve—Center of Pinion to Top of Gear (Single Stem Valves)	V	7½	7½	7½	7½	7½	7½	7½	12¼	14	16¾	16¾	16¾	17¾
Bevel Gear Valve—Center of Stem to Top of 2"x2" Nut (Single Stem Valves)	W	14⅝	14⅝	14⅝	14⅝	14⅝	17¾	17¾	21¼	21¼	30	30	30	33
Ratio of Teeth in Spur Gearing (Single Stem Valves)		36–12	36–12	36–12	36–12	36–12	42–16	42–16	46–12	46–12	62–15	62–15	62–15	75–17
Ratio of Teeth in Bevel Gearing (Single Stem Valves)		32–16	32–16	32–16	32–16	32–16	43–16	43–16	46–12	46–12	72–18	72–18	72–18	75–17
Number of Turns to Open Valve, Spur Geared (Single Stem Valves)		135	144	153	171	189	181	197	335	360	471	348	363	441
Number of Turns to Open Valve, Bevel Geared (Single Stem Valves)		90	96	102	114	126	186	202	335	360	456	336	352	441
Spur Gear Valve—Center of Port to Top of Hand Wheel (Double Stem Valves)	X	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	99	106½
Bevel Gear Valve—Center of Port to Center of Pinion (Double Stem Valves)	Y	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	92	99½
Bevel Gear Valve—Center of Pinion to Top of Pinion (Double Stem Valves)	Z	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	5	5
Bevel Gear Valve—Center of Stem to Top of Hand Wheel (Double Stem Valves)	e	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	34	40
Ratio of Teeth in Spur Gearing (Double Stem Valves)		.....	.....	.....	.....	.....	.....	.....	.....	32–16	35–12	35–12	35–12	46–13
Ratio of Teeth in Bevel Gearing (Double Stem Valves)		.....	.....	.....	.....	.....	.....	.....	.....	32–16	39–15	39–15	39–15	46–12
Number of Turns to Open Valve, Spur Geared (Double Stem Valves)		.....	.....	.....	.....	.....	.....	.....	.....	188	333	368	385	530
Number of Turns to Open Valve, Bevel Geared (Double Stem Valves)		.....	.....	.....	.....	.....	.....	.....	.....	188	296	328	343	575

49.—Weights of Ludlow Gates and Valves, in Lbs.
(*Abbreviations.)

List No. 1. { 6-in. and under: T. P. W. = 150 lbs.; R. W. = 75 lbs.; R. S. = 50 lbs.
7in. and above: T. P. W. = 100 lbs.; R. W. = 50 lbs.; R. S. = 35 lbs.

Sizes	1"	1¼"	1½"	1¾"	2"	2½"	3"	3½"	4"	4½"	5"	6"	7"	8"	9"	10"	12"
Sc. Soc			8		13½	22½	29½	54	59		124	151	204	231		365	
Flg					22½	31½	42	68	73		129	158	206	234		358	452
Hub					23		65		78		141	173	208	234		361	458
Spig																	
O.S.& Y., ex.																	88
L. F., ex							25		28		35	40	50	65	85	100	130

List No. 2. { 20-in. and under: T. P. W. = 100 lbs.; R. W. = 50 lbs.; R. S. = 35 lbs.
24-in. and above: T. P. W. = 75 lbs.; R. W. = 40 lbs.; R. S. = 25 lbs.

Sizes	14"	15"	16"	17"	18"	20"	22"	24"	27"	30"	36"	40"	42"	48"	50"	60"	72"
Flg	558		680		830	905		1400		2950							
Hub	542		692		833	956		1305		2783	4100						
Spig																	
Sp. Gr., ex	105		105		105	105		67		160	160						
Bv. Gr., ex	114		144		114	114		68		146							
By-Ps., ex																	

List No. 3. { 20-in. and under: T. P. W. = 100 lbs.; R. W. = 60 lbs.; R. S. = 45 lbs.
24-in. and above: T. P. W. = 75 lbs.; R. W. = 50 lbs.; R. S. = 30 lbs.

Sizes	14"	15"	16"	17"	18"	20"	22"	24"	27"	30"	36"	With Gearing. 40"	42"	48"	50"	60"	72"
Flg	605	640	770		875	1057	1590	1712		3130	4530	6615					
Hub	578	675	740		875	1057	1600	1580		2900	4145	6850		11850			
Spig																	
Sp. Gr., ex	105	105	105		105	105		67		160							
Bv. Gr., ex	114	114	114		114	114		68		146							
By-Ps., ex																	

List No. 4. { 6-in. and under: T. P. W. = 300 lbs.; R.W. = 150 lbs.; R. S. = 85 lbs.
7-in. and above: T. P. W. = 300 lbs.; R.W. = 150 lbs.; R. S. = 70 lbs.

Sizes	½"	¾"	1"	1¼"	1½"	2"	2½"	3"	3½"	4"	4½"	5"	6"	7"	8"	9"	10"	12"
Sc. Soc					8	14	22½	29½	55	62	103	130	160	210	239		391	530
Flg					14	23	32	43	69½	76	111	131	166	220	236		406	513
Hub						25	28	67		80		142	180	208	244		384	490
Spig										80			180		251		420	530
O. S. & Y., ex										26		43	45		60		90	102
L. F., ex								25		28		35	40	50	65	85	100	130

* The following abbreviations are used in Table 49:

For connections, etc.: *Sc. Soc.* = Screw Socket; *Flg.* = Flanged; *Spig.* = Spigot; *O. S. and Y.*, *ex.* = Outside Screw and Yoke, extra; *L. F.*, *ex.* = Loose Flanges, extra; *Sp. Gr.*, *ex.* = Spur Gear, extra; *Bv. Gr.*, *ex.* = Bevel Gear, extra; *By-Ps.*, *ex.* = By-Pass, extra; *P. F.*, *R. O.* = Plain Frame, round opening; *P. F.*, *S. O.* = Plain Frame, square opening; *R. O. with Spig.* = Round Opening with Spigot; *R. O. with F. & N.* = Round Opening with Flange and Neck Piece; *E. F.*, *R. O.* = Extension Frame, round opening; *E. F.*, *S. O.* = Extension Frame, square opening; *S. O. with F. & N.* = Square Opening with Flange and Neck Piece.

For Test- and Working Pressures: *T. P. W.* = Test Pressure, Water; *R. W.* = Recommended for Water.—Working pressures not to exceed the number of pounds given in Table headings; *R. S.* = Recommended for Steam—Working Pressures not to exceed number of lbs. given in Table headings.

49.—Weights of Ludlow Gates and Valves, in Lbs.—Continued.

List No. 5. { 6-in. and under: *T. P. W.* = 350 lbs.; *R. W.* = 200 lbs.; *R. S.* = 100 lbs.
{ 7-in. and above: *T. P. W.* = 350 lbs.; *R. W.* = 200 lbs.; *R. S.* = 85 lbs.

Sizes	1½"	2"	2½"	3"	3½"	4"	4½"	5"	6"	7"	8"	9"	10"	12"
Sc. Soc	....	....	....	55	....	75	102	130	191	212	270	350	437	520
Flg	....	....	....	64	....	87	111	131	202	220	282	374	463	576
Hub	....	....	....	67	....	93	....	140	204	221	280	....	409	547
Spig	....	....	....	....	....	....	....	....	....	....	....	....	....	....
O. S. & Y., ex	....	....	....	....	....	....	....	43	45	....	60	....	90	102
L. F., ex	....	....	....	....	....	....	....	35	45	50	65	85	100	130

List No. 5¼. *T. P. W.* = 400 lbs.; *R. S.* = 150 lbs.

Sizes	1½"	2"	2½"	3"	3½"	4"	4½"	5"	6"	7"	8"	9"	10"	12"
Flanged	....	....	38	70	75	123	....	187	257	300	379	....	548	725
Screwed	....	....	....	64	....	107	....	185	237	....	370	....	....	....

List No. 5½. *T. P. W.* = 600 lbs.; *R. W.* = 400 lbs.; *R. S.* = 250 lbs.

Sizes	1½"	2"	2½"	3"	3½"	4"	4½"	5"	6"	7"	8"	9"	10"	12"
Sc. Soc	....	....	37	77	80	104	....	200	268	280	390	....	632	800
Flg	....	....	46	80	110	120	165	230	285	354	410	525	690	930
Hub	....	....	....	....	....	105	....	....	275	....	410	....	650	870
O. S. & Y., ex	....	....	12	17	....	30	....	50	59	90	90	....	115	130
L. F., ex	....	....	....	....	....	40	....	60	75	....	94	....	150	170
By-Ps., ex	....	....	....	....	....	....	....	....	....	....	....	....	....	....

Sizes	14"	15"	16"	18"	20"	24"	30"	36"	Remarks.
Flg	....	....	2050	....	3000	....	....	....	
Hub	....	....	....	....	....	5180	....	....	The 24" is bevel gear.
O. S. & Y., ex	....	....	....	....	....	....	....	....	
L. F., ex	....	....	....	....	320	....	....	....	
By-Ps., ex	....	....	....	....	....	....	....	....	

List No. 6. { 20-in and under: *T. P. W.* = 300 lbs.; *R. W.* = 200 lbs.; *R. S.* = 85 lbs.
{ 24-36-ins.: *T. P. W.* = 300 lbs.; *R. W.* = 160 lbs.; *R. S.* = 75 lbs.
{ 40-in. and above: *T. P. W.* = 300 lbs.; *R. W.* = 140 lbs.; *R. S.* = 65 lbs.

Sizes	14"	15"	16"	17"	18"	20"	22"	24"	30"	With Gearing.							
										36"	40"	42"	48"	50"	54"	60"	72"
Flg	770	890	963	....	1260	1650	....	2580	4260	5600	....	11650	18970	....	....	....	....
Hub	705	820	875	....	1203	1585	....	2461	4425	8500	11500	12160	18550	....	....	....	....
Spig	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
Sp. Gr., ex	105	105	105	....	67	67	....	103	200	....	....	....	....	....	....	....	....
Bv. Gr., ex	114	114	114	....	68	68	....	104	188	....	....	....	....	....	....	....	....
O. S. & Y., ex	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
L. F., ex	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
By-Ps., ex	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....

Notes.—16", Hub, #6, Bevel Gear, and 2" By-Pass, 1230 lbs.; 30", Hub, #6, Bevel Gear, and 6" By-Pass, DD Stem, 5650 lbs.; 36", Hub, #6, Bevel Gear, and By-Pass, 9500 lbs.

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49.—Weights of Ludlow Gates and Valves, in Lbs.—Continued.

List No. 7.
- 6-in. and under: *T. P. W.* = 300 lbs.; *R. W.* = 200 lbs.; *R. S.* (3-in.) = 150 lbs.
- 7-in. and above: *T. P. W.* = 250 lbs.; *R. W.* = 125 lbs.; *R. S.* (3½-6-in., inc.) = 125 lbs.
- 7-12-in., inc.: *R. S.* (7-12-in., inc.) = 85 lbs.

Sizes	¼"	⅜"	½"	¾"	1"	1¼"	1½"	1¾"	2"	2½"	3"	3½"	4"	4½"	5"	6"
Sc. Soc			1½	2⅝	3 5/16	4¾	5 9/16		9 3/16	15⅛	18⅞	30	44⅜		67	95
Flg			3¼	4½	5¾	7¾	10⅝		15½	22½	27	39½	58⅜	77½	88	118

Sizes	7"	8"	9"	10"	11"	12"	14"	16"	18"	20"	22"	24"	27"	30"
Sc. Soc														
Flg			290		351	583								

List No. 8. *T. P. W.* = 1500 lbs.; *R. W.* = 750 lbs.

Sizes	1"	1¼"	2"	2½"	3"	3½"	4"	4½"	5"	6"	7"	8"	9"	10"	12"
Sc. Soc			51		101		160			405					
Flg				124	139		198			435		815			
L. F., ex				14	30		61			95		160			

List No. 9. *T. P. W.* = 2000 lbs.; *R. W.* = 2000 lbs.; *R. S.* = 1200 lbs.

Sizes	1"	1½"	2"	2½"	3"	3½"	4"	4½"	5"	6"	7"	8"	9"	10"	12"
Sc. Soc				123	158		420								
Flg					205					1062					
L. F., ex										173					

List No. 10. Brass Hor. C'ks: *T. P. W.* = 300 lbs.; *R. W.* = 150 lbs.; *R. S.* = 100 lbs.

Sizes	¼"	½"	¾"	1"	1¼"	1½"	2"	2½"	3"	3½"	4"	5"	6"	7"	8"	9"	10"
Sc. Soc		1	1¼	1 11/16	2 11/16	3 3/16	5 3/16	8 3/16									
Flg																	

Horizontal Check Valves.

List No. 11.
- 6-in. and under: *T. P. W.* = 300 lbs.; *R. W.* = 150 lbs.; *R. S.* = 100 lbs.
- 7-in. and above: *T. P. W.* = 300 lbs.; *R. W.* = 150 lbs.; *R. S.* = 85 lbs.

List No. 12.
- 20-in. and under: *T. P. W.* = 300 lbs.; *R. W.* = 200 lbs. *R. S.* = 85 lbs.
- 24-36-in., inc.: *T. P. W.* = 300 lbs.; *R. W.* = 150 lbs.; *R. S.* = 75 lbs.
- 40-in. and above: *T. P. W.* = 300 lbs;. *R. W.* = 140 lbs.; *R. S.* = 65 lbs.

Sizes				2"	2½"	3"	3½"	4"	4½"	5"	6"	7"	8"	9"	10"	12"
Sc. Soc						45		80		120	133		225			650
Flg						47		92		133	150	230	242		490	620
Hub						55		95		150	160		245		453	630

Sizes	14"	15"	16"	18"	20"	22"	24"	28"	30"	36"	40"	42"	44"	48"	50"	60"
Flg	880		960	1200	1800		2700		7250							
Hub	885		990		1880		2710		7370	11360						

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49.—WEIGHTS OF LUDLOW GATES AND VALVES, IN LBS.—Concluded.

Vertical Check Valves. List No. 13. *T. P. W* = 300 lbs.; *R. W.* = 150 lbs.

Sizes	2"	2½"	3"	3½"	4"	4½"	5"	6"	7"	8"	9"	10"	12"
Sc. Soc	.....	.....	.....	.....	58	.....	.....	113	.....	155	.....	280	.....
Flg	.....	36	.....	.....	.....	.....	.....	152	.....	.....	.....	287	415
Hub	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....

Sizes	14"	15"	16"	18"	20"	24"	28"	30"	36"	40"	42"	44"	48"
Flg	685	.....	.....	.....	.....	.....	.....	7080	.....	.....	.....	.....	.....
Hub	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....

Vertical Foot Valves. List No. 14. *T. P. W.* = 200 lbs.; *R. W.* = 100 lbs.

Sizes	2"	2½"	3"	3½"	4"	4½"	5"	6"	7"	8"	9"	10"	12"
Sc. Soc	.....	.....	.....	.....	60	.....	.....	.....	.....	153	.....	.....	.....
Flg	.....	.....	.....	.....	58	.....	.....	105	.....	160	.....	240	340
Hub	.....	.....	.....	.....	.....	.....	.....	.....	.....	132	.....	275	410

Sizes	14"	15"	16"	18"	20"	24"	28"	30"	36"	40"	42"	44"	48"
Flg	519	.....	660	.....	875	.....	.....	3350	.....	.....	.....	.....	.....
Hub	562	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....

Flume Valves. List No. 15.
- 24-in. and under: *T. P. W.* = 40 lbs.; *R. W.* = 30 lbs.
- 30-in. and above: *T. P. W.* = 30 lbs.; *R. W.* = 25 lbs.

Sizes	14"	15"	16"	18"	20"	22"	24"	28"	30"	36"	40"	With Gearing. 42"	44"	48"	50"	54"	60"	72"
Flg	435	...	518	700	735	....	1100	....	1837	2700	....	.....	.....	.....	....	8890	.....	...
Hub	.....	...	492	...	730	....	1070	....	1790	2750	....	5900	....	6255	....	.....	.....	...
Sp. Gr., ex	.....	...	.....	...	.....	....	.....	....	.....	.....	....	.....	.....	.....	....	.....	.....	...
Bv. Gr., ex	.....	...	.....	...	.....	....	.....	....	.....	.....	....	.....	.....	.....	....	.....	.....	...

Sluice Gates. List No. 21.

Sizes	10"	12"	14"	15"	16"	18"	20"	24"	28"	30"	36"	40"	42"	44"	48"	Rem'ks
P. F., R. O.	.....	215	320	340	435	630	....	750	1080	1150	1535	2350	.....	.....	2764	Also in Sizes 50", 54", 60", 72", 80" and 90".
P. F., S. O.	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	
R. O. with Spig	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	
P. O. with F. & N.	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	
E. F., R. O.	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	
E. F., S. O	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	
S. O. with F. & N.	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	

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Blow-Offs are placed on the bottom of pipes at depressions in the pipe line for cleaning out or emptying the conduit. They consist essentially of a small pipe, say from 4 to 16 inches in diameter, leading to some suitable point where the waste can be discharged. A gate is inserted near the main pipe line. See Tables 16, 17, 38 and 39, preceding.

IIIg.—"SPECIALS."

"Specials" are usually of cast iron and include bends, reducers, tees, wyes, and in fact many other shapes that enter also into the distributing system (see preceding and following tables). Fig. 56 shows the method of connecting wood-stave pipe with a cast special. The socket or bell of the casting is usually about 6 inches deep, with offset equal to thickness of staves. Fig. 57 is a front half view of special casting for connecting a small

Fig. 56.

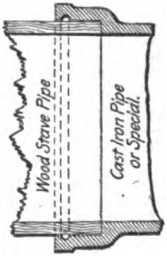

Fig. 57,

Fig. 58.

Fig. 59.

branch or a blow-off with a wood-stave pipe. Note the attached lugs (bosses) or shoes *s* for holding the ends of the steel bands which cinch around the opposite side of the pipe. These shoes are shown in side view by Fig. 58, and in the end view by Fig. 59, the band passing between the prongs.

E.—DISTRIBUTING SYSTEM.

Cast Iron Pipe is by far the best that can be used, although the first cost is greater than that of lap-welded pipe. For sizes, details and weights of cast iron pipe, see tables under IIIc, page 1214 and following.

It is to be noted especially that there should be no "dead ends" in a distributing system, i. e., cross connection should be made at terminal points, as at ends of streets.

Fig. 60 is a Portable **Lead-Melting Furnace** and ladle for pouring the joints. (See page 1215 for description and use of gasket.)

Matheson Patent Lock Joint Pipe (steel lap-welded) is manufactured in lengths up to about 20 feet (average about 17 to 18 feet over all, according to size) and is tested to 500 lbs. hydraulic pressure per square inch.

Tables 50-54, on following page, give sizes and weights of pipe and specials; also, lead required per joint.

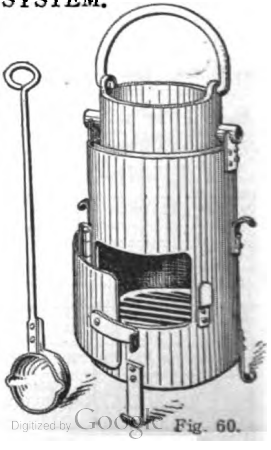

Fig. 60.

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50.—Matheson Pipe.

of Pipe.	Nearest Birmingham Wire Gauge.	Nominal Size of Rings.	Approximate Weights. Lead per Joint.	Approximate Weights. Complete per Foot
s.		Ins	Lbs.	Lbs.
	13	3/16 x ¾	1.07	1.93
	12	3/16 x ¾	1.77	3.36
	11	¼ x ¾	2.67	4.94
	10	¼ x ¾	3.50	6.56
	10	5/16 x 1	4.67	8.38
	9	5/16 x 1	5.62	10.32
	9	⅜ x 1	6.90	12.43
	8½	⅜ x 1	8.38	14.74
	8	7/16 x 1	9.82	17.26
	7	½ x 1¼	13.20	23.26
	6½	½ x 1¼	15.30	26.44
	6	⅝ x 1¼	17.20	30.07
	5½	⅝ x 1¼	19.20	33.81
	5	⅝ x 1¼	21.80	37.92
	4½	⅝ x 1¼	23.70	42.45
	3½	⅝ x 1¼	25.60	47.23
	3	¾ x 1¼	28.80	52.61
	2½	¾ x 1¼	31.10	58.34
	1	¾ x 1½	40.20	70.86
	.330"	¾ x 1½	48.10	84.88
	.362"	¾ x 1½	55.30	100.69
	.396"	1 x 1½	64.70	119.02
	.432"	1 x 1½	74.60	138.85

Furnished Asphalted only, and Kalamein and Asphalted.

51.—Tees.

Size. Ins.	Wt. Lbs.	Size. Ins.	Wt. Lbs.
2 x 2 x 2	11	6 x 6 x 4	96
3 x 3 x 3	19	6 x 6 x 3	93
3 x 3 x 4	35	6 x 4 x 4	100
4 x 4 x 4	35	6 x 3 x 6	90
4 x 4 x 4	39	7 x 7 x 7	....
4 x 4 x 3	35	7 x 7 x 6	115
4 x 4 x 3	35	8 x 8 x 8	159
4 x 4 x 2	37	8 x 8 x 6	173
4 x 4 x 2	36	8 x 8 x 4	172
4 x 4 x 1	34	8 x 6 x 8	176
4 x 4 x 6	98	9 x 9 x 9	....
4 x 3 x 4	35	10 x 10 x 10	256
5 x 5 x 5	41	10 x 10 x 8	270
5 x 5 x 4	58	10 x 10 x 6	268
5 x 5 x 4	58	10 x 10 x 4	285
5 x 3 x 5	56	11 x 11 x 11	353
6 x 6 x 6	91	12 x 12 x 12	....

Heavy-faced figures indicate openings tapped for Standard Pipe.

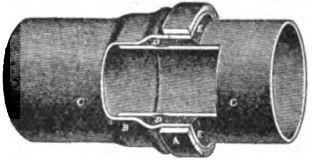

Fig. 61.—Matheson Joint.

52.—Plugs.

Size. Ins.	Wt., Lbs.	Size. Ins.	Wt., Lbs.	Size. Ins.	Wt., Lbs.
2	1	6	7	10	23
3	2	7	13	12	....
4	3	8	15	14	58
5	5	9	..	16	88

53.—Crosses.

Size. Ins.	Wt., Lbs.	Size. Ins.	Wt., Lbs.
2 x 2 x 2 x 2	13	6 x 4 x 3 x 3	125
3 x 3 x 3 x 3	28	7 x 7 x 7 x 7	135
4 x 4 x 4 x 4	42	7 x 7 x 6 x 6	153
4 x 4 x 4 x 3	43	8 x 8 x 8 x 8	200
4 x 4 x 3 x 3	46	8 x 8 x 8 x 4	229
4 x 4 x 2 x 2	45	8 x 8 x 8 x 6	230
4 x 4 x 2 x 2	43	8 x 8 x 4 x 4	209
4 x 3 x 3 x 3	45	8 x 8 x 14 x 16	1190
5 x 5 x 5 x 5	66	8 x 6 x 8 x 6	220
5 x 5 x 5 x 4	69	8 x 6 x 8 x 4	235
5 x 5 x 4 x 4	74	8 x 6 x 3 x 3	238
5 x 4 x 5 x 5	72	8 x 4 x 4 x 4	218
6 x 6 x 6 x 6	108	9 x 9 x 9 x 9	....
6 x 6 x 4 x 4	117	10 x 10 x 10 x 10	337
6 x 6 x 4 x 3	120	10 x 10 x 10 x 8	339
6 x 4 x 4 x 4	127	12 x 12 x 12 x 12	....

Heavy-faced figures indicate openings tapped for Standard Pipe.

54.—Reducers.

Size. Ins.	Wt., Lbs.	Size. Ins.	Wt., Lbs.	Size. Ins.	Wt., Lbs
3 x 2	.....	6 x 4	21	9 x 8	....
4 x 3	11	6 x 3	.....	9 x 7	....
4 x 3	14	6 x 3	25	9 x 6	....
4 x 2	12	7 x 6	.....	10 x 9	....
5 x 5	19	7 x 5	.....	10 x 8	50
5 x 4	17	8 x 7	.....	10 x 6	46
5 x 3	.....	8 x 6	39	10 x 4	52
6 x 5	.....	8 x 4	43	12 x 10	75
6 x 4	22				

Heavy-faced figures indicate openings tapped for Standard Pipe.

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Converse Patent Lock Joint Pipe (steel lap-welded) is manufactured in average lengths of about 18 ft. and tested to 500 lbs. per square inch. A hub is leaded to each length of pipe at mill to receive the spigot end of the adjoining pipe when laid.

55.—CONVERSE PIPE.

Size.		Approximate Weight.			
Nominal Outside Diameter of Pipe.	Nominal Thickness.	Plain Ends, Per Foot.	Hub.	Lead Required Per Joint.	Complete Per Foot.
Ins.	In.	Lbs.	Lbs.	Lbs.	Lbs.
2	.094	1.91	5.	1.00	2.24
3	.108	3.33	8.50	2.15	3.92
4	.118	4.89	12.50	2.75	5 73
5	.125	6.51	19.	3.00	7.72
6	.132	8.27	21.	3.75	9.64
7	.139	10.20	32.	5.50	12.26
8	.146	12.25	37.	6.50	14.66
9	.154	14.55	37.50	7.50	17.03
10	.162	17.02	41.	7.75	19.72
11	.171	19.78	50.	8.60	23.00
12	.181	22.85	58.	9.50	26.59
13	.190	26.00	65.	10.75	30.21
14	.200	29.48	73.	12.	34.16
15	.210	33.18	85.	15.	38.68
16	.221	37.85	132.	17.5	45.47
17	.233	41.73	140.	23.75	51.38
18	.245	46.46	149.	30.	56.31
19	.258	51.65	183.	34.	63.65
20	.272	57.32	217.	38.	71.35
22	.300	69.53	280.	50.	87.68
24	.330	83.43	342.	58.5	105.46
26	.362	99.13	380.	70.	123.88
28	.396	116.76	430.	85.	145.09
30	.432	136.44	475.	100.	168.07

Furnished Asphalted Only, and Kalamein and Asphalted.

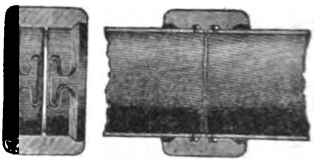

Fig. 62.—CONVERSE JOINT. (Cast Hub.)

For details of specials see National Tube Works catalog.

Pipe-Dipping Tank.—The writer has found it convenient in some cases to order the lap-welded pipe from the manufacturer uncoated, and to coat it in the field before laying. For this purpose a dipping tank is used as shown in Fig. 63. The tank should be about 20 ft. long, 2 ft. wide and 2 feet deep. Such a tank constructed of No. 12 gauge steel will weight about 700 lbs. It is set over an improvised brick furnace, cheaply constructed and provided with a smoke stack. Hard and liquid asphaltum are mixed in it in

Fig. 63.—Pipe-Dipping Tank.

the proper proportion and the pipe is dipped when it has reached the proper temperature. Approximately, the number of pounds of asphalt required per hundred feet of pipe is equal to 5.5 × diameter of pipe in inches.

Tapping Machines are employed for tapping mains for service connections. There are many styles, more or less expensive. The Mueller tap-

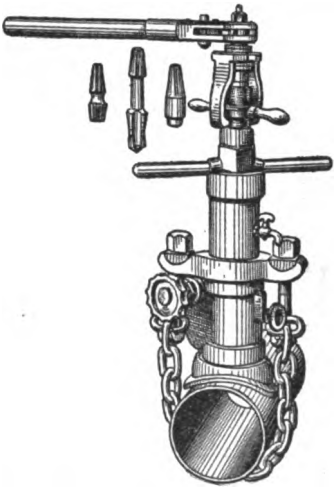

Fig. 64.—Mueller Tapping Machine.

ing machine is illustrated in Fig. 64. For water mains, the machine complete includes:

1 each, Combined Drill and Tap—½, ⅝, ¾ and 1 inch.
1 each, Screw or Hexagon, Plug—½, ⅝, ¾ and 1 inch.
4 Malleable Iron Saddles; any size.
1 Chain for any size of Pipe.

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Black or Galvanized Pipe of "standard" weight as manufactured by the National Tube Co. is shown in the following Table. For the "extra strong" pipe the 1-in. size is 0.182 in. thick, and 12-in. size 0.500 in.; while the "double extra" is 0.364 in. and 0.875 in. thick, respectively.

56.—DIMENSIONS, IN INCHES, AND WEIGHTS OF BLACK PIPE (STANDARD)

Diameter.			Thickness.	Transverse Areas.			Nom. Wt. per Foot, Pounds	Threads per In.	Wgt. of Coupling, Pounds
Nom.	Exter'l.	Inter'l.		Extern'l.	Internal.	Metal.			
⅛	0.405	0.269	0.068	0.1288	0.0568	0.0720	0.241	27	0.031
¼	0.540	0.364	0.088	0.2290	0.1041	0.1249	0.43	18	0.046
⅜	0.675	0.493	0.091	0.3578	0.1909	0.1669	0.559	18	0.078
½	0.840	0.622	0.109	0.5542	0.3039	0.2503	0.837	14	0.124
¾	1.050	0.824	0.113	0.8659	0.5333	0.3326	1.115	14	0.250
1	1.315	1.047	0.134	1.3581	0.8609	0.4972	1.668	11½	0.458
1¼	1.660	1.380	0.140	2.1642	1.4957	0.6685	2.244	11½	0.562
1½	1.900	1.610	0.145	2.8353	2.0358	0.7995	2.678	11½	0.800
2	2.375	2.067	0.154	4.4301	3.3556	1.074	3.609	11½	1.250
2½	2.875	2.467	0.204	6.4918	4.7800	1.712	5.739	8	1.757
3	3.500	3.066	0.217	9.6211	7.3837	2.238	7.536	8	2.625
3½	4.000	3.548	0.226	12.566	9.886	2.680	9.001	8	4.000
4	4.500	4.026	0.237	15.904	12.730	3.174	10.665	8	4.125
4½	5.000	4.508	0.246	19.635	15.960	3.675	12.34	8	4.875
5	5.563	5.045	0.259	24.306	19.985	4.321	14.502	8	8.437
6	6.625	6.065	0.280	34.472	28.886	5.586	18.762	8	10.625
7	7.625	7.023	0.301	45.664	38.743	6.921	23.271	8	11.270
8	8.625	7.981	0.322	58.426	50.021	8.405	28.177	8	15.150
9	9.625	8.937	0.344	72.760	62.722	10.04	33.701	8	17.820
10	10.750	10.018	0.366	90.763	78.822	11.94	40.065	8	27.700
11	11.750	11.000	0.375	108.43	95.034	13.40	45.95	8	33.250
12	12.750	12.000	0.375	127.68	113.09	14.59	48.985	8	43.187

57.—DISCHARGE IN GALLONS PER MINUTE THROUGH SMALL PIPES.
(Values are approximate.)

H = head in ft.; L = length of pipe in ft.; numbers in first column are values of $H + L$.
[Gallons per Minute.]

Head H.	Diameter of Pipe, in Inches.											
	½	⅝	¾	1	1¼	1½	2	2½	3	4	5	6
0.100L	2.0	3.5	5.4	11.2	19.5	30.8	63.2	110.4	174.5	358.1	624.7	985.5
0.111L	2.1	3.6	5.7	11.8	20.6	32.5	66.6	116.4	183.9	377.5	658.5	1039.
0.125L	2.2	3.9	6.1	12.5	21.8	34.4	70.7	123.5	195.1	399.0	698.5	1102.
0.143L	2.4	4.1	6.5	13.4	23.3	36.8	75.6	132.0	208.5	428.0	746.7	1178.
0.167L	2.6	4.4	7.0	14.4	25.2	39.8	81.6	142.6	225.2	463.2	806.5	1272.
0.200L	2.8	4.8	7.7	15.8	27.6	43.9	89.4	156.2	246.7	506.5	883.5	1384
0.250L	3.1	5.5	8.6	17.7	30.9	48.7	100.0	174.6	275.8	566.2	987.7	1558.
0.333L	3.6	6.3	9.9	20.4	35.6	56.2	115.4	201.6	317.6	653.8	1141.	1799.
0.500L	4.4	7.7	12.2	25.0	43.7	68.7	141.4	246.9	390.1	800.8	1394.	2204.
0.750L	5.4	9.5	14.9	30.6	53.5	84.3	173.1	302.4	477.1	979.3	1711.	2693.
L	6.3	10.9	17.2	35.3	61.7	97.4	199.9	349.2	555.5	1133.	1976.	3116.
1.25L	7.0	12.2	19.3	39.5	69.0	108.9	223.5	390.4	615.9	1264.	2209.	3484
1.50L	7.7	13.4	21.1	43.2	75.6	119.3	248.8	427.7	674.8	1385.	2420.	3817.
1.75L	8.3	14.4	22.8	46.8	81.6	128.8	264.4	462.0	728.8	1496.	2613.	4122.
2L	8.8	15.4	24.3	50.0	87.3	137.7	282.7	493.0	780.2	1602.	2791.	4407.
3L	10.8	18.9	29.8	61.2	106.9	168.7	346.3	604.9	955.5	1962.	3406.	5391.
4L	12.5	21.8	34.4	70.7	123.4	194.8	399.8	698.5	1103.	2265.	3951.	6233.
5L	14.0	24.4	38.5	79.0	138.0	217.7	447.0	780.9	1234.	2532.	4417.	6968.
6L	15.3	26.5	42.2	86.6	151.2	238.5	488.1	855.4	1351.	2774.	4839.	7633.
7L	16.5	28.9	45.6	93.5	163.3	257.6	528.9	924.0	1460.	2996.	5227.	8245.
8L	17.7	30.1	48.7	100.0	174.6	275.4	566.4	987.8	1560.	3203.	5588.	8814.
9L	18.7	32.7	51.7	106.0	185.2	292.1	599.7	1048.	1651.	3397	5928.	9349.
10L	19.8	34.5	54.4	111.8	195.2	308.0	632.2	1104.	1745.	3581.	6247.	9855.

Ex.—What is the capacity of a pipe 1¼ in. dia., and 150 ft. long, the head being 50 ft.? *Solution.*—$H = \frac{50}{150}L = .333L$; hence, for a 1¼ in. pipe the discharge is 35.6 gallons per min.

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Gate Valves should be of the best quality.

Fig. 65 shows a section of standard bronze mounted babbitt seat valve, up to 15 ins., as manufactured by the Chapman Valve Co. of Indian Orchard, Mass.

Fig. 66 shows a section of the Eddy gate valve.

Notation of Parts:—

A—Stem Nut.	G—Body.
B—Stem.	H—Body and Cover Bolts.
C—Follower.	I—Ball and Carrier.
D—Follower Bolts.	J—Gate.
E—Stuffing Box.	K—Gate Ring.
F—Cover.	L—Case Ring.

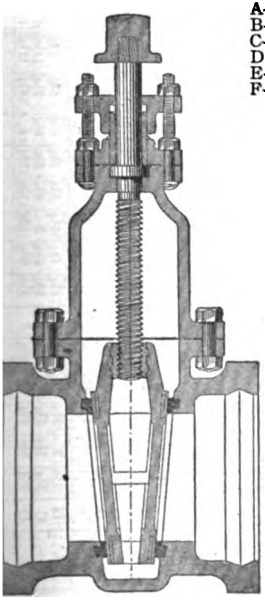

Fig. 65.—Chapman.

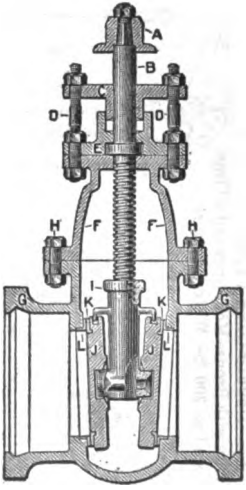

Fig. 66.—Eddy.

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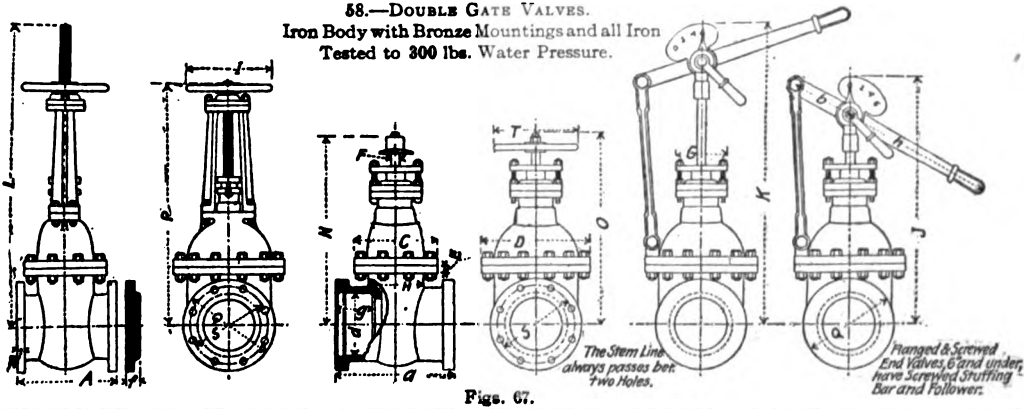
58.—Double Gate Valves.

Iron Body with Bronze Mountings and all Iron Tested to 300 lbs. Water Pressure.



Figs. 67.

Note.—This Table gives dimensions in inches, of various styles of the *smaller sizes* of Ludlow valves. For larger sizes see page 1274.

Size of Valve		1"	1¼"	1½"	2"	2½"	3"	3½"	4"	4½"	5"	6"	7"	8"	9"	B. Mtd. 10"	All Iron 10"	B. Mtd. 12"	All Iron 12"
Face to Face of End Flanges	A	5	5¼	5½	5⅝	6⅛	6⅞	8¼	8¾	9¾	10¾	11⅛	11¾	11	13	13¾	12⅜	14⅝	12⅞
Diameter of End Flanges	Q	4⅛	5	5½	6	7	7½	8½	9	9¼	10	11	12½	13⅛	15	16	16	19	19
Thickness of End Flanges	R	$\frac{9}{16}$	$\frac{9}{16}$	$\frac{9}{16}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{13}{16}$	$\frac{13}{16}$	$\frac{7}{8}$	1	1	1	1	1	1	1
Diameter of Bolt Circle	S	3¼	3¾	4¼	4¾	5½	6	7	7½	7¾	8½	9½	10¾	11¾	13¼	14¼	14¼	17	17
Number and Size of End Flange Bolts		4-½	4-½	4-½	4-½	4-½	4-½	4-½	4-⅝	8-⅝	8-⅝	8-⅝	8-⅝	8-⅝	12-⅝	12-¾	12-¾	12-¾	12-¾
Width of Body	H	2½	3	3½	3¾	3¼	3⅜	4¼	4¼	5¼	5½	5½	6¼	6⅝	7⅝	7⅞	6¾	8½	7¼
Width of Throat Flange	C	3¼	3⅛	3¾	5⅝	4⅞	5⅛	6¾	6⅞	8	9¾	8⅝	9¼	9⅛	11½	11¾	10¾	12⅝	10½
Length of Throat Flange	D	3¼	3⅛	3¾	5⅝	6¼	7	8⅞	9¼	10¾	11⅝	12¾	14	14¾	17⅛	16¾	17⅛	20⅛	19¼

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58.—Double Gate Valves.—Concluded.

Size o Valve		1"	1¼"	1½"	2"	2½"	3"	3½"	4"	4½"	5"	6"	7"	8"	9"	B. Mtd. 10"	All Iron 10"	B. Mtd. 12"	All Iron 12"
Thickness of Throat Flange	E	...	...	...	½	9/16	9/16	11/16	5/8	¾	¾	¾	13/16	¾	⅞	⅞	¾	⅞	⅞
Throat Flange Bolts		...	...	...	4-7/16	4-7/16	6-½	6-½	6-½	8-9/16	8-9/16	8-9/16	10-9/16	10-9/16	14-⅝	14-⅝	10-⅝	14-⅝	12-⅝
Diameter of Stem	F	⅝	⅝	⅝	⅝	¾	¾	1	1	1⅛	1⅛	1¼	1¼	1¼	1¼	1⅜	1⅜	1⅛	1⅜
Thickness of Metal in Body of Valve		¼	¼	⅜	⅜	½	½	⅝	9/16	9/16	⅝	⅝	11/16	11/16	13/16	¾	13/16	¾	¾
Diameter of Hand Wheel	T	3⅜	3⅝	3⅝	4½	5⅛	6	7½	7½	9	9	10	10	10	12	12	12	15	12
Number of Turns to Open Valve		3½	5	6	7½	12	12	12½	14	15	17	20	23	26	30	34	33	38	37
Diameter of Stuffing Box Flange	G	...	...	...	...	...	4¼	4¼	4¼	5	5	5½	5½	5½	6	6	6	6½	6
Center of Port to Top of Hand Wheel	O	7⅜	8	8	8¾	11	11½	15¾	16	19⅛	20	21	25¼	26¼	31¼	31½	31½	34¾	34¾
Center of Port to Top of 2"x2" Nut	N	...	...	8⅛	9⅜	11	11	15½	16⅜	19	19¾	22	24	25	29¾	29¼	30	33	32½
End to End of Hub Valve	a	...	...	8½	7	7¾	8⅝	9⅜	9¾	11	12	12½	13⅞	14⅛	15½	14⅝	14⅛	15	14¼
Inside Diameter of Hub	d	...	...	2 13/16	3⅜	3 13/16	4⅝	5 1/16	5 13/16	6 3/16	6 9/16	7⅝	8 9/16	10	11 1/16	12 1/16	12 7/16	14¼	14⅛
Depth of Hub	g	...	...	2¼	2¼	2¼	2½	2⅝	2¾	3¼	3½	3⅞	4	4	4¼	4	4	4	4
Face to Face of Screw End Valve		3¾	3¾	4	4⅞	5⅜	5½	7¼	7¼	9½	11	11¼	12	12⅝	13	13⅝	12⅝	13⅝	13¼
End to End of Spigot Valve		...	...	...	13¾	15⅜	20¼	20¼	21	22½	21½	24½	27	27	28½	29½	27½	30½	28
Diameter of Spigot		...	...	...	2¾	3¼	3⅞	4⅜	4⅞	5½	6	7	8⅛	9½	10⅜	11¼	11¼	13¼	13¼
Center of Port to Top of Hand Wheel When Closed — Outside Screw and Yoke Valves	P	...	...	...	12	13¼	13¼	19⅛	21	25	28¼	29⅝	31¼	33⅛	38	40	40	45	45
Center of Port to Top of Stem When Open	L	...	...	...	15½	17	17½	24¾	26¼	30¼	34	36¼	39¼	42¼	48¼	52	52	58	58
Diameter of O. S. and Y. Hand Wheel	I	...	...	...	6	6	6	9	10	10	10	12	12	12	14	14	14	15	15
Center of Port to Top of Stem When Closed — Side Stem and Lever Valves	J	...	...	...	11¼	14¼	15½	....	18¼	.....	....	23	26⅛	30⅜	.....	.....	35⅝	39⅛	39⅛
Center of Port to Top of Stem When Open	K	...	...	...	13½	17	19	....	22¾	.....	....	29¾	34⅛	39	.....	.....	46¾	52½	52½
Length from Standard to Stem	b	...	...	2¾	3	3⅛	3¾	4⅛	5⅛	5⅞	5⅞	6⅞	7½	8¼	8¼	9⅜	9⅜	11⅛	11⅛
Length from Stem to End of Lever	h	...	...	6½	6¾	9½	10¾	13¾	15½	18¼	18¼	20½	22⅝	24½	24½	27½	27½	27½	27½
Thickness of Loose Flange	f	...	...	...	...	...	1⅜	1⅜	1⅜	1⅜	1¾	1¾	1¾	1¾	2	2¼	2¼	2¼	2¼

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59.—Weights of Ludlow Gate- or Valve Boxes.
[Weight in Lbs.]

Size of Valve	2"	3"	4"	5"	6"	8"	10"	12"	14"	16"	18"	20"	24"	30"	36"
2½ ft. Trench															
3 " "		80	80		71	65	56								
3½ " "		85	85		80	80	71	56							
4 " "		105	105		100	80	80	65							
4½ " "		110	110		105	95	90	80	80	60		56			
5 " "		112	112		110	105	95	80	80	80		56			
5½ " "		121	121		119	113	103	95	85	80		74			
6 " "		126	126		121	119	113	103	95	85		80			
6½ " "															
7 " "															

Gate Boxes, Fig. 68, are telescopic casings of cast iron to place over the gate valves and protect them from surrounding earth or back-fill; and render them accessible to operate from the street in opening and closing. The cover is a circular cast plug which fits in the bell of the top casting or telescope portion of the box. The telescope is provided with a flange, placed at the middle or at the lower end to hold it at the proper elevation when placed. The main box is enlarged at the bottom to fit over the body of the gate. Caution must be used in designing gate boxes as the *outside dimensions* of gates by different makers vary considerably. Thus, a gate box which would fit a 12-in. gate manufactured by A, would perhaps be too small for the "same size" gate manufactured by B. See Table 59 for weights.

Fig. 68.

Check Valves, Fig. 69, are placed in pipe lines or mains (either in a vertical, inclined or horizontal position) to prevent back flow or excessive back pressure. They are usually located at or near the pumping station.

Pressure Relief Valves are specially designed to relieve excessive pressure within a main due to water hammer. They allow some of the water to escape, and operate automatically.

Hydrants or "fire plugs" are designated by the size of valve opening; by the number of nozzles, as single-, double,- etc.; and by the kind of discharge nozzles, as steamer-, or hose-. The 4, 4½, 5 and 6-inch (valve opening) fire hydrants are the most common. The essential principal of a hydrant is that the valve be placed below the reach of frost, and that when it is closed

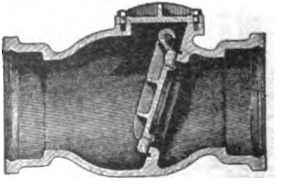

Fig. 69.—Check Valve.

the water remaining above it shall be allowed to "drip" out to prevent its freezing. For this reason the hydrants should be set on a bed of firm, loose rock to provide for the drip. Figs. 70 and 71 show sections of the Chapman and Ludlow types, respectively. Figs. 72 and 73 show sections of valves and seats for the Mathews' patent hydrant, the former being single-acting and the latter double-acting.

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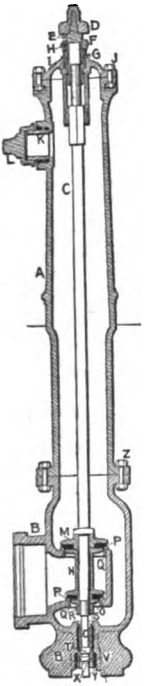
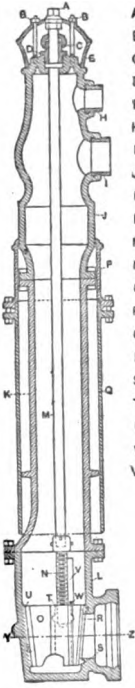
Hydrants are connected with transverse pipes leading from the street mains, and hence it will be seen that there is considerable hydrostatic pressure acting horizontally against them at the bottom. This should be resisted by a strong stone backing when the hydrant is set.

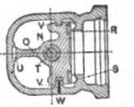


A—Hydrant Barrel or stand Pipe.
B—Bottom.
C—Stem.
D—Top Nut.
E—Bronze Sleeve.
F—Stuffing Box Nut.
G—Stuffing Box.
H—Follower.
I—Dome.
J—Dome Bolts.
K—Bronze Nozzle.
L—Nozzle Cap.
M—Upper Valve Plate.
N—Valve.
O—Lower Valve Plate.
P—Valve Rubbers.
Q—Bronze Seats.
R—Bronze Nut.
S—Bronze Lock Nut.
T—Bronze Corrugated Drip Piece.
V—Drip Rubber.
X—Bronze Drip Nut.
Y—Bronze Drip Cup.
Z—Flange Bolts.

Fig. 70.



A—Bronze Rod Nut
B—Bronze Tap Bolts
C—Bronze Packing Nut
D—Bronze Stuffing-box
E—Iron Post Cap
H—Bronze Hose Nozzle
I—Bronze Steamer Nozzle
J—Hydrant Post
K—Standpipe or Barrel
L—Hydrant Body
M—Iron Extension Rod
N—Bronze Spindle
O—Gate or Plug
P—Apron
Q—Frost Case
R—Bronze Face on Plug
S—Babbitt or Bronze Seat
T—Bronze Bushing in Plug
U—Babbitted Wedging Surface
V—Rib or Spline
W—Drip Hole



SECTION Y-Z.

Fig. 71.

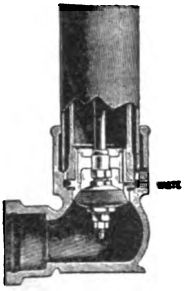


Fig. 72.

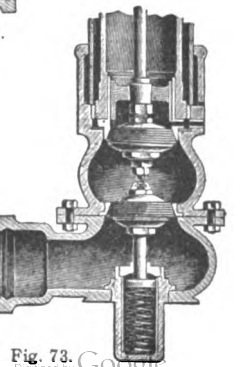


Fig. 73.

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60.—Weight of Ludlow Hydrants, in Lbs.

Yard Hydrants. List No. 25. } Test Pressure Water = 100 lbs.

Sizes	½"	¾"	1"
5-ft. Length	45	55	75
6-in. Length, extra	3	5	6

Wash Hydrants. List No. 25½. } Test Pressure Water = 100 lbs.

Sizes	½"	¾"	1"
5-ft. Length	....	40	50
6-in. Length, extra	....	5	6

Fire Hydrants. List No. 75. Test Pressure Water = 300 lbs.

Stand Pipe.	Seat Ring.	Pipe Connection.	Number and Size of Nozzles.	Wt. Standard Length 5 Feet.	Weight per Ft.	Frost Case for 5-Ft. Length.	Weight per Ft.
Ins.	Ins.	Ins.	Ins.	Lbs.	Lbs.	Lbs.	Lbs.
3	2	2 or 3	one 2	130	12	....	....
4⅝	3	3 or 4	one 2½	310 to 315	26	84	21
5¾	4	3 or 4	one 2½	365 to 370	30	96	24
5¾	4	4 or 6	two 2½	375 to 380	30	96	24
6¼	4½	5 or 6	two 2½	437 to 442	34	106	26
6¼	4½	5 or 6	three 2½	448 to 453	34	106	26
7	5	5 or 6	one steamer and two 2½	475 to 486	36	116	29
8	6	6 or 8	one steamer and two 2½	590 to 600	40	144	36
10	8	8 or 10	six 2½	1200 to 1220	73	219	....

Notes on List No. 75: Add for a 6-in. Hub or Bell over Hydrant with 4-in. Hub or Bell, 16 lbs.; add for each additional 2½-in. Nozzle, including Cap and Chain, 11 lbs.; add for each additional 4-in. Steamer Nozzle, including Cap and Chain, 25 lbs. The above weights are based on a length of 5 feet measuring from the surface of ground to bottom of connecting pipe.

Hydrants with Crane Attachment. List No. 76. Test Pressure Water = 300 lbs. Extra to add to weight of Hydrant: 300 lbs. for 3 in.; 310 lbs. for 4-in.

Water Cranes. List No. 77. Test Pressure Water = 300 lbs.
3-in. Length, 5 feet, 587 lbs. Each 6-ins. Stand Pipe and Box, 13 lbs.
4-in. Length, 5 feet, 600 lbs. Each 6-ins. Stand Pipe and Box, 13 lbs.

Flush Hydrants. List No. 78. Test Pressure Water = 300 lbs.

Seat Ring.	Stand Pipe.	Pipe Connection.	Nozzles.	Weight 5-ft. Length.	Weight per Ft.
Ins.	Ins.	Ins.	Ins.	Lbs.	Lbs.
2	3	2	one 2	90	12
3	4⅝	3 or 4	one 2½	290	26
4	5¾	4 or 6	one 2½	360	30

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EXCERPTS AND REFERENCES.

The New Water-Works Reservoir at Trenton, N. J. (By C. A. Hague. Eng. News, June 13, 1901).—Six illustrations: Section of reservoir, cross-section of wall and embankment; view of gate house; plans and sections of outlet pipe.

Iron or Wooden Lock Gates; Cost of Construction and Maintenance (Eng. News, Oct. 9, 1902).

A Stress Diagram for Water-Tank Hoops (By Ballinger & Perrot. Eng. News, Mar. 5, 1903).

Four Systems of Softening Water for Industrial Purposes (Eng. News, July 2, 1903).—Described and illustrated.

A Proposed High-Pressure Water-Supply System for Fire Protection in Chicago (Eng. News, Mar. 3, 1904).—Illustrations: Main conduit; details of steel and cast-iron pipe; details of fire-boat connection; section of main conduit lateral; maps of high-pressure water supply systems for fire protection in different cities.

Experience in Thawing Water Pipes By Electricity (Eng. News, Mar. 17, 1904).

The Five Dams and Wood-Stave Conduit of the Southern California Mountain Water Co. (Eng. News, April 7, 1904).—Fifteen illustrations; and 13 tables, including cost data.

The Philadelphia Filtration System (Eng. News, Dec. 8, 1904).—Numerous illustrations.

Cost of Laying a 12-in. Water Pipe Across a River (Eng. News, Mar. 2, 1905).—The labor cost of building 44 A-frames, placing and caulking the 516-ft. pipe line, was $122.

A Diagram for Estimating the Yardage of a Trench (Eng. News, July 6, 1905).—Quantities in cu. yds. for various widths in ft., up to 20 ft., and various depths in ft., up to 30 ft.

Purification of Water by Copper Sulphate (By D. D. Jackson. Eng. News, Sept. 21, 1905).—For other papers and discussions, see Eng. News of Nov. 30, 1905.

The Baiseleys, Springfield, Forest Stream, and Hempstead, Filter Plants, Borough of Brooklyn, New York (Eng. News, Aug. 23, 1906).—Table showing the net amount of water filtered at two mechanical and at two slow sand filter plants, during the year 1905; following totals are reproduced:

Baiseleys (mechanical)	1,435.5	million	gals.	@	$6 53	p. m. g.
Springfield (mechanical)	694.6	"	"	"	9.58	"
Hempstead (slow sand)	416.8	"	"	"	2.89	"
Forest Stream (slow sand)	1,075.3	"	"	"	2.28	"

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Disjointing Cast-Iron Water Mains (Eng. News, Oct. 10, 1907).—Methods used.

Repairing a Remarkable Leak in a Reservoir Embankment at Providence, R. I. (Eng. News, Oct. 17, 1907.)—Described and illustrated.

Diagrams for Computing Thickness of Steel Pipe Shells for Different Joint Efficiencies (By R Muller Eng. News, April 2, 1908).—The diagrams are based on the following assumptions: Tensile strength of plate, 50 000 lbs per sq. in of section; factor of safety, 4. Percentage of efficiency of riveted joints, assumed at: single riveted joint, 56%; double-riveted joint, 69%; triple riveted-joint, 75%; double-weld butt joint, 87%; quadruple-riveted joint, 95%

Method and Cost of Hauling a Water Main Across Channel at Vancouver, B. C. (Eng. News, May 14, 1909).—Illustrated: method, and section through flexible-joint.

Data on Silting Up of Reservoirs (By R. H. Bolster. Eng. News, July 30, 1908).

Repairs to a 72-in. Reinforced-Concrete Jacketed Steel Conduit Under 30-Ft. of Water (By A. W. Cuddeback. Paper, A. W. W. Assn., May, 1908; Eng. News, Aug. 6, 1908).—Illustrated.

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The Design of Elevated Tanks and Stand-Pipes (By C. W. Birch-Nord. Trans. A. S. C. E., Vol. LXIV., Sept., 1909).—Specifications.

Water-Works Valuation (By Leonard Metcalf. Trans. A. S. C. E., Vol. LXIV., Sept., 1909).—Numerous bond-, compound-interest-, and sinking-fund formulas, a bibliography of water-works valuation; various kinds of "values" discussed.

Cost of Clearing Water in Settling Basins (By S. Bent Russell. Paper, Central States W. W. Assn., Columbus, O., Sept. 28, 1909. Eng. News, Oct. 14, 1909).—Tables of cost of reservoirs and settling water.

The Purification of the Water Supply of Steelton, Pa. (By J. H. Fuertes. Trans. A. S. C. E., Vol. LXVI., Mar., 1910).—Cost data; illustrations; discussions.

The Use of Sulphate of Alumina and Hypochlorite of Lime in the Storage and Distributing Reservoir of the Nashville Water-Works (By George Reyer. Eng. News, Apr. 7, 1910).—The cost of sulphate of alumina (the ordinary is used) is $1.07½ per 100 lbs. and of the hypochlorite of lime (it contains about 36% of chlorine) is $1.50 per 100 lbs. The cost per 1,000,000 gals. of water treated is about $1.75 for sulphate of alumina and $1.05 for cost of hypochlorite of lime, making the combined cost of chemicals $2.80. The water comsumption for the year 1909 averaged about 14,000,000 gals., or some 107 gals. per capita.

This article is followed by five others, namely: A 20,000,000-Gal. Hypochlorite Water-Disinfecting Plant at Minneapolis, Minn. (By J. A. Jensen). The Use of Hypochlorite of Lime to Disinfect the Water-Supply of Montreal, P. Q. (Geo. Janin, C. E.). The Use of Hypochlorite of Lime in Connection with the Mechanical-Filtration Plant of Harrisburg, Pa. (G. C. Kennedy, Supt.). Hypochlorite of Lime as an Adjunct to Mechanical Water Filtration at Quincy, Ill. (By W. R. Gelston. Paper, Ill. Water Supply Assn., Mar. 8, 9, 1910).—Experiments with Hypochlorite of Lime as a Water Disinfectant at Hartford, Conn. (By Ermon M. Peck, Engr.).

The Groined Arch in Filter and Covered Reservoir Construction (By Thomas H. Wiggin. Paper, Nat'l Assn. Cement Users, Feb. 21-26, 1910; Eng. News, April 7, 1910).—**Discussions of:** Volumes of elliptical groined arch units; Methods of design; Computation of groined arch as cantilever; Effect of steel in groin arch; Shrinkage and temperature changes; Aid given by earth covering; Construction stresses; Floors of filters and reservoirs; Side walls and division walls; Comparison of groined arch roof with reinforced-concrete slab and beam construction. Illustrations. **Table** containing data on groined arch roofs for filters and reservoirs.

The Improved Water and Sewerage Works of Columbus, O. (By J. H. Gregory. Trans. A. S. C. E., Vol. LXVII., June, 1910).—**Illustrations:** Scioto river storage dam; pumping station; offices and laboratories; settling basins; details of filter gallery; details of filters; details of filtered-water reservoirs, etc.

Determination of the Resultant Angle in Laying Out Combined Bends for Pipe Lines (By C. A. Jackson. Eng. News, Aug. 11, 1910).—Graphical and analytical methods. Example given.

Concrete Tower Enclosing a Water-Works Tank at Gary, Ind. (Eng. News, Oct. 20, 1910).—Tower is octagonal in plan, 34 ft. diam., inside the faces, and 133 ft. high from the grade line. Fully described and illustrated.

Steel Pipes for Water-Works (By Emil Kuichling. Paper, Jour. Am. W. W. Assn., presented Sept. 21-23, 1910; Eng. News, Oct. 27, 1910).—Discussion of wrought-iron and of steel water mains and protective coatings, strength of pipes, vacuum relief valves, capacity of pipe. **Cost of steel and cast-iron pipe.**—"The fact that lock-bar and welded pipe can develop the full strength of the plate, while a riveted seam has only about 70% efficiency, influences the comparative costs. This difference is likely to be made up by a countervailing difference in unit-prices, however. Empirical formulas for cost of steel and cast-iron pipe have been devised by the author (Mr. Kuichling) as below. They assume (1) a water-pressure of 100 lbs., (2) an equivalent pressure, used in design, 50% greater than the static pressure to allow for water hammer and (3) an addendum of ¼" thickness for cast-iron pipe and 1-16" for steel pipe. Taking d as the nominal dia. of pipe, in ins., the cost in cents per lin. ft. is: Cost of steel pipe, $0.4125d^2$; cost of cast-iron pipe, $19+d\ (0.484\,5d-0.851)$. Subtracting the latter from the former gives the saving due to using steel: Steel pipe cheaper by $19+d\ (0.072d-$

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0.851), which in case of a 36-in. pipe, for example, amounts to 81.7 cts. per ft. For a complete comparison, however, the shorter life of the steel pipe must be taken into account."

Depreciation in Water-Works Operation and Accounting (By Leonard Metcalf. Paper read before the N. E. W. W. Assn., Sept. 22, 1910; Eng. News, Nov. 3, 1910).—Contains sinking-fund and depreciation diagrams and tables.

Pneumatic Caulking of Mains with Lead Wool (By C. C. Simpson, Jr. Paper read before the Am. Gas Institute; Eng. Rec., Nov. 12, 1910).—The Cons. Gas Co. of New York, in an effort to reduce the cost of caulking mains by hand, carried on extensive tests with pneumatic tools and lead wool, and the results were so satisfactory that the pneumatic method was finally adopted on a considerable portion of its work, which includes lines of 48, 36 and 30-in. mains. The article gives detailed costs.

Forms for Concrete (By J. D. Stevenson. Paper before Eng'rs Soc. of Western Penn., Sept. 20, 1910; Eng. Rec., Dec. 10, 1910).—Discussion of experience of three years in construction of the Pittsburg filtration works (placing about 334,000 cu. yds. of concrete) under the following heads:—Material, design of forms, support of forms, buoyancy of forms, joints, care while filling, and removing forms. For labor costs on forms, see Eng. Rec. of Dec. 17, 1910.

Waterproofing the New Ulm (Minn.) Reservoir (By H. F. Blomguist. Eng. Rec., Dec. 17, 1910).—Reservoir is 75 ft. in diam., 30 ft. deep, capacity 1,000,000 gals., built of reinforced concrete, with a conical reinforced concrete roof. Hard clay, well drained naturally, upon which a 12-in. layer of stone having the voids filled with a wet fine-grained concrete formed the foundation for the 10-in. floor slab proper, which was reinforced near the surface with 16-gauge, 3-in. mesh expanded metal. To insure water tightness, special care was taken during construction to grade the concrete aggregate. Pebbles varying in size from ¼ to 2½ in., screened from a gravel bank, were used in the upper floor and walls, as experiments had shown that these pebbles made a denser concrete than broken stone. To reduce the permeability of the concrete to a minimum 20 lbs. of hydrated lime was used to every barrel of cement, and after the forms were removed the walls were brushed and cleaned with steel brushes, and two coats of 1 : 2 cement plastering about ½ in. thick were applied. The mortar for the plastering contained the waterproofing compound to the extent of 10% of the cement used. A slush coat of 1 : 2 cement mortar was floated over the surface of the concrete floor at the time it was laid and a brush coat of cement grout applied to the slushed surface after it had set. After water was let in, some leakage took place and cracks developed. The reservoir then received further treatment, and was rendered watertight.

Illustrations of Important Works:—

Description.	Eng. News.
Intake caisson for Cincinnati Water works, showing break	May 30, 1901.
Intake tunnel of Cleveland water works, showing accident	Aug. 29, '01.
Water tank at Fairhaven, Mass., showing failure	Nov. 21, '01.
An 8" gate valve for River Rogue, Michigan	April 17, '02.
Concrete-lined reservoir with concave slopes, Aurora, Ill.	May 22, '02.
Slow sand filters of Hudson, N. Y., water works	Aug. 14, '02.
Cross-section of a fire hydrant with a balanced valve	Nov. 20, '02.
Water tank with hemispherical bottom, Chicago	Dec. 25, '02.
Plan of Klein's classifier for fine material	Jan. 8, '03.
Intake tunnel for the Champion Mill on Lake Superior	Oct. 1, '03.
Plan of scow for submarine pipe-laying	Dec. 24, '03.
New type of water tower, Victoria railways, Australia	Dec. 31, '03.
A tool for removing broken taps in water pipes, etc.	Jan. 7, '04.
Cast-iron pipe, 48" dia., flattened under back-fill	Dec. 15, '04.
Reinforced-concrete waterworks standpipe 81' high	Feb. 25, '04.
Sectional plan of 30" "Premier" water meter	May 19, '04.
Failure of a water tank designed by an architect	May 19, '04.
Sections of core walls for reservoir embankments	June 30, '04.
Laying submerged pipe at N. Tonawanda, N. Y.	Aug. 4, '04.
Details of coagulating plant, St. Louis settling basins	Oct. 27, '04.
New water tank and tower, E. Providence, R. I.	Nov. 10, '04.
Small brick reservoir, showing failure	Nov. 17, '04.

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Automatic regulating valve for reservoirs and standpipes	Mar. 9, '05.
The water softening plant at Oberlin, O.	Sept. 21, '05.
A 75 000-gal. rein.-conc. cistern for fire protection	Sept. 28, '05.
Details of elevated water tank and supporting tower	Oct. 26, '05.
Plans of rein.-conc. reservoir at Ft. Meade, So. Dak.	Dec. 28, '05.
Steel pipe line supported across gulch by arched girder	Feb. 8, '06.
Reinforced-concrete filter-bed walls and roofs	April 26, '06.
Plans of water purification plant, Paris, Ky.	May 3, '06.
Plans of a slow sand water filter for the home	Aug. 9, '06.
Reinforced-concrete mechanical filter plant, Germany	Sept. 6, '06.
Details of novel reinforced-concrete conduit, 42" dia.	Oct. 4, '06.
Apparatus for regulating discharge from reservoir	Oct. 25, '06.
Plans of slow sand filtration plant, Wash. D. C.	Nov. 8, '06.
Steel water tank (150' dia. x 20' high), vertical bracing	Nov. 15, '06.
Sand chutes for canal of Bijou Irrigation District	Jan. 3, '07.
Reinforced-concrete standpipe, Attleboro, Mass.	Feb. 21, '07.
A new design for balanced gate valve	Oct. 17, '07.
Forbes water-sterilizing apparatus	Oct. 31, '07.
Plan and section of gate house, reservoir, Portland, Ore.	Feb. 6, '07.
Automatic controlling valve for reservoirs, tanks, etc.	Mar. 12, '08.
Experimental water-filter plant, Oakland, Cal.	May 21, '08.
Reinforced-concrete reservoir, Indianapolis, Ind.	Oct. 15, '08.
Reinforced-concrete tank supported by hollow shaft	Jan. 7, '09.
Alternate designs of hand- and mach. washed slow sand filters	May 20, '09.
Fire-proof temporary crib, water tunnel, Chicago	June 17, '09.
Design of an 8-mile head hydraulic conduit	Aug. 5, '09.
Steel penstocks—expansion joints, elbow, saddle	Feb. 10, '10.
Brick water tower, 133 ft. high, 28 ft. diam.	Mar. 24, '10.
Forms for concrete vaulted roofs, water filters	Apr. 14, '10.
10,000,000-gal. pressure mechanical filter plant	Apr. 14, '10.
Sliding sluice-gates, Charles River Basin, Boston	May 26, '10.
Rein.-conc. pumping cistern, 42' high x 20' dia., sunk in ground	Aug. 25, '10.
A new filter-sand washing machine, in France	Oct. 13, '10.
Rein.-conc. water tank with dome-shaped bottom	Dec. 15, '10.
Large (102-in.) venturi meter for Montreal W. W.	Dec. 15, '10.
New filtration plant at Portsmouth, Eng.	Dec. 15, '10.
	Eng. Rec
An endless screen for a water-works intake	May 29, '09.
Rate controller and valve for Cincinnati filters	June 19, '09.
Plan of aerator, settling basin, filter, inlet, Dover, N. H.	June 19, '09.
Details pipe line construction, Canyon City water supply	Dec. 4, '09.
Collapsible form for concrete conduit, Los Angeles aqueduct	Jan. 1, '10.
10,500,000-gal. covered, rein.-conc. reservoir, Mexico	Aug. 6, '10.
Reinforced concrete water tower, Westerly, R. I.	Sept. 24, '10.
Details of concrete lining, Seattle reservoirs	Sept. 24, '10.
Ultra-violet ray sterilizer for sterilizing water; cost	Dec. 10, '10.
Illustrated details of the Toledo (O.) filtration plant	Nov. 26, '10.
Tall rein.-conc. block water tower (28,250 cu. ft. capac.) near Brussels	Dec. 10, '10.

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65.—SANITATION.

The Disposal of Refuse from buildings in cities and towns may be classified as follows:

- **Dry Refuse** ....
  - Paper. { In large cities, waste paper is collected and sold to paper mills.
  - Ashes. { Ashes are collected and used in extensive filling operations.
- **Garbage**.....
  - Dumped { In fill. Very objectionable—not sanitary. At sea. Expensive; not always sanitary.
  - Crematory plants. Sometimes self-supporting by reason of the residues of oil and grease.
- **Sewage (House-drainage)**
  - Cesspools. Economical for isolated buildings, as factories, and for suburban districts.
  - Sewers
    - Farm irrigation; largely employed in France.
    - Septic tanks; for inland towns.
    - Chemical process—sludge.
    - Rivers. Raw sewage emptying into streams is becoming prohibitive.
    - Ocean. Boston sewage reaches Moon Island by submarine tunnel where it is pumped into the Bay.

Storm water also becomes sewage when mixed with the latter.

House Drainage is effected by—

- **Waste Pipes** { Which receive and convey the waste water from baths, basins, sinks, wash tubs, etc., but no human excreta.
- **Soil Pipes** { Which receive and convey the human excreta, faeces (solid matter) and urine, from closets and urinals; *also*, general waste.

Drain pipes connect with the waste- and soil pipes at the building line, and convey the waste and soil to the cesspool or sewer.

Generally speaking, the waste- and soil pipes are, in the main, *vertical*, extending from top to bottom *inside* of the building, and connected by branch pipes with the various discharge fixtures; while the drain pipes are

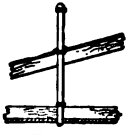

Fig. 1.

Fig. 2.

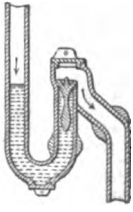

Fig. 3.

practically *horizontal* and *outside* of the building. Soil- and waste pipes should be extended vertically up through the roof of the building as shown in Fig. 1, to provide for the escape of sewer gas. In order to prevent the latter from finding its way into the rooms, a *trap* is placed on each

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pipe leading from the fixture to the soil- or waste pipe. These traps are designated by various letters of the alphabet, but the S-trap (Fig. 2) is perhaps the most common. Their efficiency is reduced when the water evaporates or becomes siphoned out of the trap. Fig. 3 shows a section of Waring's check valve, which remains efficient unless foreign matter lodges between the valve and its seat, or the bearings become too much worn.

Cesspools are excusable in sandy or gravelly soil in new additions to townsites where the water supply is not derived from wells in the immediate locality; or for isolated buildings where cost of sewer would be excessive. A notable example of the latter is the cesspool recently constructed for the discharge of soil and waste from the new boathouse pavilion in Prospect Park, Brooklyn. It is a large cylindrical well with a sand bottom and with sides lined with concrete.

Sewers.—Figs. 4 to 8, in Tables 4 to 8, illustrate five standard sections of sewers, namely, Circular, Catenary, Basket-Handle, Gothic, and Egg or Egg-shape. The proportional dimensions are based on the diametrical *height* of unity, in each case.

For the Circular section, Fig. 4, there are annexed diagrams of relative area a, velocity v, and discharge q, for relative depths of flow. Thus, if the conduit or sewer is full, a, v and q are assumed to be unity; if the depth is 0.8, $a=0.86$, $v=1.14$, $q=0.98$; if half full, $a=0.5$, $v=1.0$, $q=0.5$, etc. Note that v and q are based on the velocity v being proportional to $\sqrt{r}$, in which r = hydraulic radius in feet (see *Hydraulics*, page 1161). Similar diagrams of area, velocity and discharge may be drawn for the other sections—Catenary (Fig. 5), Basket-Handle (Fig. 6), Gothic (Fig. 7), and Egg (Fig. 8). The Circle, and the Horseshoe with invert, are the most common forms for tunnel construction.

The Egg-shape (Fig. 8) possesses the merit of maintaining a comparatively high value of r (and hence v) for small depths of flow; and it is to be noted that when flowing $\frac{3}{4}$ full depth, r (and v) are greater than when flowing full. (See Table 8.)

Table 1, following, gives the value of r, $\sqrt{r}$ and $a\sqrt{r}$ for Circular sections, advancing by inches up to 20 ft. 11 ins. dia. These properties are useful in designing, i. e., in connection with the use of Kutter's formula, being independent of the grade or slope s.

Table 2, used in connection with Table 1, enables us to find the values of r, $\sqrt{r}$ and $a\sqrt{r}$ for the Catenary, Basket-Handle, Gothic and Egg sections.

Table 3 gives the velocities in Circular brick sewers, unclean, using value of roughness $n=0.015$.

Tables 4, 5, 6, 7 and 8 show properties of Circular, Catenary, Basket-Handle, Gothic, and Egg sections respectively, with relative diameters, both vert. and hor., for equivalent areas. These tables will be found useful for comparison of the different sections in designing.

Table 9 gives the velocities in Egg-shaped sewers which have become somewhat fouled, using value of roughness $n=0.015$.

Size and Grade of Sewers.—The design of sewers may be by either of three methods, namely, (1) by formulas; (2) by tables; (3) by diagrams. For discussion of formulas, see *Hydraulics*, page 1167. In using Kutter's formula the value of c may be determined from $n=0.013$* for ordinary brick sewers, and $n=0.015$ for large sewers with unclean surfaces. As there is more or less sand or scouring matter, the velocity of flow should generally be not greater than 5 or 6 ft. per sec.

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* For ordinary brick sewers, new and well laid; but it is safer to use $n=0.014$. See, also, pages 1168 and 1188.

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LUES OF r, $\sqrt{r}$ AND $a\sqrt{r}$ FOR *CIRCULAR- CONDUITS OR SEWERS.

ulas:

aulic radius r = diam. in ft. ÷ 4.

ity v (in ft. per sec.) = $c\sqrt{rs} = c\sqrt{r}\sqrt{s}$.

arge q (in cu. ft. per sec.) = $av = c\,a\sqrt{r}\sqrt{s}$

Values of { (1) r in Feet. (2) $\sqrt{r}$ in Feet. (3) $a\sqrt{r}$ in Feet. }

Decimals of a Foot.

00 | .083 | .167 | .250 | .333 | .417 | .500 | .583 | .667 | .750 | .833 | .917

Inches.

0"	1"	2"	3"	4"	5"	6"	7"	8"	9"	10"	11"
000	0.021	0 042	0.063	0.083	0.104	0.125	0.146	0.167	0.188	0.208	0.229
000	**0.144**	**0.204**	**0.250**	**0.289**	**0.323**	**0.354**	**0.382**	**0.408**	**0.433**	**0.456**	**0.479**
000	.0008	.0045	.0123	.0252	.0440	.0694	.1021	.1425	.1913	.2489	.3159
250	0.271	0.292	0.313	0.333	0.354	0.375	0.396	0.417	0.438	0.458	0.479
500	**0.520**	**0.540**	**0.559**	**0.577**	**0.595**	**0.612**	**0.629**	**0.645**	**0.661**	**0 677**	**0.692**
927	.4797	.5773	.6860	.8061	.9381	1.082	1.239	1.408	1.591	1.787	1.997
500	0.521	0.542	0.563	0.583	0.604	0.625	0.646	0.667	0.688	0.708	0.729
707	**0.722**	**0.736**	**0.750**	**0.764**	**0.777**	**0.791**	**0.804**	**0.816**	**0.829**	**0.842**	**0.854**
221	2.460	2.714	2.982	3.266	3.565	3.881	4.212	4.560	4.925	5.306	5.705
750	0.771	0.792	0.813	0.833	0.854	0.875	0.896	0.917	0.938	0.958	0.979
866	**0.878**	**0.890**	**0.901**	**0.913**	**0.924**	**0.935**	**0.946**	**0.957**	**0.968**	**0.979**	**0.990**
122	6.556	7.008	7.478	7.966	8.474	9.000	9.545	10.11	10.69	11.30	11.92
000	1.021	1.042	1.063	1.083	1.104	1.125	1.146	1.167	1.188	1.208	1.229
000	**1.010**	**1.021**	**1.031**	**1.041**	**1.051**	**1.061**	**1.070**	**1.080**	**1.090**	**1.099**	**1.109**
,57	13.23	13.92	14.62	15.35	16.10	16.87	17.66	18.47	19.31	20.17	21.05
250	1.271	1.292	1.313	1.333	1.354	1.375	1.396	1.417	1.438	1.458	1.479
118	**1.127**	**1.137**	**1.146**	**1.155**	**1.164**	**1.173**	**1.181**	**1.190**	**1.199**	**1.208**	**1.216**
.95	22.88	23.83	24.80	25.80	26.82	27.86	28.93	30.02	31.13	32.27	33.44
500	1.521	1.542	1.563	1.583	1.604	1.625	1.646	1.667	1.688	1.708	1.729
225	**1.233**	**1.242**	**1.250**	**1.258**	**1.267**	**1.275**	**1.283**	**1.291**	**1.299**	**1.307**	**1.315**
,63	35.84	37.08	38.35	39.64	40.96	42.30	43.67	45.06	46.49	47.93	49.41
750	1.771	1.792	1.813	1.833	1.854	1.875	1.896	1.917	1.938	1.958	1.979
323	**1.331**	**1.339**	**1.346**	**1.354**	**1.362**	**1.369**	**1.377**	**1.384**	**1.392**	**1.399**	**1.407**
.91	52.44	53.99	55.58	57.19	58.83	60.49	62.19	63.91	65.66	67.44	69.25
000	2.021	2.042	2.063	2.083	2.104	2.125	2.146	2.167	2.188	2.208	2.229
414	**1.422**	**1.429**	**1.436**	**1.443**	**1.451**	**1.458**	**1.465**	**1.472**	**1.479**	**1.486**	**1.493**
.09	72.95	74.85	76.77	78.72	80.71	82.72	84.76	86.83	88.94	91.07	93.23
250	2.271	2.292	2.313	2.333	2.354	2.375	2.396	2.417	2.438	2.458	2.479
500	**1.507**	**1.514**	**1.521**	**1.528**	**1.534**	**1.541**	**1.548**	**1.555**	**1.561**	**1.568**	**1.575**
,43	97.65	99.91	102.2	104.5	106.9	109.2	111.6	114.1	116.6	119.1	121.6
500	2.521	2.542	2.563	2.583	2.604	2.625	2.646	2.667	2.688	2.708	2.729
581	**1.588**	**1.594**	**1.601**	**1.607**	**1.614**	**1.620**	**1.627**	**1.633**	**1.639**	**1.646**	**1.652**
4.2	126.8	129.4	132.1	134.8	137.5	140.3	143.1	145.9	148.8	151.7	154.6
750	2.771	2.792	2.813	2.833	2.854	2.875	2.896	2.917	2.938	2.958	2.979
658	**1.665**	**1.671**	**1.677**	**1.683**	**1.689**	**1.696**	**1.702**	**1.708**	**1.714**	**1.720**	**1.726**
7.6	160.6	163.6	166.7	169.8	172.9	176.1	179.3	182.6	185.8	189.2	192.5
000	3.021	3.042	3.063	3.083	3.104	3.125	3.146	3.167	3.188	3.208	3.229
732	**1.738**	**1.744**	**1.750**	**1.756**	**1.762**	**1.768**	**1.774**	**1.780**	**1.785**	**1.791**	**1.797**
5.9	199.3	202.8	206.3	209.8	213.3	216.9	220.6	224.2	227.9	231.7	235.5
250	3.271	3.292	3.313	3.333	3.354	3.375	3.396	3.417	3.438	3.458	3.479
803	**1.809**	**1.814**	**1.820**	**1.826**	**1.831**	**1.837**	**1.843**	**1.848**	**1.854**	**1.860**	**1.865**
9.3	243.1	247.0	251.0	254.9	258.9	263.0	267.0	271.2	275.3	279.5	283.7
500	3.521	3.542	3.563	3.583	3.604	3.625	3.646	3.667	3.688	3.708	3.729
871	**1.876**	**1.882**	**1.887**	**1.893**	**1.899**	**1.904**	**1.909**	**1.915**	**1.920**	**1.926**	**1.931**
8.0	292.3	296.6	301.0	305.4	309.9	314.4	318.9	323.5	328.1	332.8	337.5
750	3.771	3.792	3.813	3.833	3.854	3.875	3.896	3.917	3.938	3.958	3.979
936	**1.942**	**1.947**	**1.953**	**1.958**	**1.963**	**1.969**	**1.974**	**1.979**	**1.984**	**1.990**	**1.995**
2.2	347.0	351.8	356.6	361.5	366.5	371.4	376.5	381.5	386.6	391.7	396.9
000	4.021	4.042	4.063	4.083	4.104	4.125	4.146	4.167	4.188	4.208	4.229
000	**2.005**	**2.010**	**2.016**	**2.021**	**2.026**	**2.031**	**2.036**	**2.041**	**2.046**	**2.051**	**2.056**
2.1	407.4	412.7	418.0	423.4	428.8	434.3	439.8	445.3	450.9	456.5	462.2
250	4.271	4.292	4.313	4.333	4.354	4.375	4.396	4.417	4.438	4.458	4.479
062	**2.067**	**2.072**	**2.077**	**2.082**	**2.087**	**2.092**	**2.097**	**2.102**	**2.107**	**2.111**	**2.116**
7.9	473.7	479.5	485.3	491.2	497.1	503.1	509.1	515.2	521.3	527.4	533.6
500	4.521	4.542	4.563	4.583	4.604	4.625	4.646	4.667	4.688	4.708	4.729
121	**2.126**	**2.131**	**2.136**	**2.141**	**2.146**	**2.151**	**2.155**	**2.160**	**2.165**	**2.170**	**2.175**
9.8	546.1	552.4	558.7	565.1	571.6	578.1	584.6	591.2	597.8	604.5	611.2
750	4.771	4.792	4.813	4.833	4.854	4.875	4.896	4.917	4.938	4.958	4.979
179	**2.184**	**2.189**	**2.194**	**2.198**	**2.203**	**2.208**	**2.213**	**2.217**	**2.222**	**2.227**	**2.231**
7.9	624.7	631.6	638.5	645.4	652.4	659.4	666.5	673.6	680.7	687.9	695.2
000	5.021	5.042	5.063	5.083	5.104	5.125	5.146	5.167	5.188	5.208	5.229
236	**2.241**	**2.245**	**2.250**	**2.255**	**2.259**	**2.264**	**2.268**	**2.273**	**2.278**	**2.282**	**2.287**
2.5	709.8	717.2	724.6	732.1	739.6	747.2	754.8	762.5	770.2	778.0	785.8

*For a circular pipe 3′ 2″ (= 3.167 ft.) dia., the hydraulic radius $\sqrt{r} = 0.800$, $a\sqrt{r} = 7.008$ (a = area in sq. ft.; see table p. 1157.)

2.—Use of Table 1, preceding, for Other Sections than Circular.

(See also Tables 4, 5, 6, 7 and 8.)

Section.	Relation of Diameters.	To Find Value of—	Mult. Value of—			Multiplier.	Logarithm.
Catenary, Flowing Full Depth.	Vert. dia. = dia. circle.	r	r	in Table 1 by		0.92688	9.967 0235
	" = "	$\sqrt{r}$	$\sqrt{r}$	"	"	0.96275	9.983 5117
	" = "	$a\sqrt{r}$	$a\sqrt{r}$	"	"	0.86146	9.935 2358
	Hor. dia. = "	r	r	"	"	1.0427	0.018 1760
	" = "	$\sqrt{r}$	$\sqrt{r}$	"	"	1.0211	0.009 0880
	" = "	$a\sqrt{r}$	$a\sqrt{r}$	"	"	1.1564	0.063 1163
Basket-handle, Flowing Full Depth.	Vert. dia. = "	r	r	"	"	0.98572	9.993 7534
	" = "	$\sqrt{r}$	$\sqrt{r}$	"	"	0.99283	9.996 8768
	" = "	$a\sqrt{r}$	$a\sqrt{r}$	"	"	0.99386	9.997 3235
	Hor. dia. = "	r	r	"	"	1.0443	0.018 7816
	" = "	$\sqrt{r}$	$\sqrt{r}$	"	"	1.0219	0.009 3908
	" = "	$a\sqrt{r}$	$a\sqrt{r}$	"	"	1.1479	0.059 8955
Gothic, Flowing Full Depth.	Vert. dia. = "	r	r	"	"	0.90760	9.957 8945
	" = "	$\sqrt{r}$	$\sqrt{r}$	"	"	0.95268	9.978 9472
	" = "	$a\sqrt{r}$	$a\sqrt{r}$	"	"	0.79487	9.900 2975
	Hor. dia. = "	r	r	"	"	1.0948	0.039 3400
	" = "	$\sqrt{r}$	$\sqrt{r}$	"	"	1.0463	0.019 6700
	" = "	$a\sqrt{r}$	$a\sqrt{r}$	"	"	1.2703	0.103 9113
Egg-shape, Flowing Full Depth.	Vert. dia. = "	r	r	"	"	0.77253	9.887 9172
	" = "	$\sqrt{r}$	$\sqrt{r}$	"	"	0.87894	9.943 9586
	" = "	$a\sqrt{r}$	$a\sqrt{r}$	"	"	0.57125	9.756 8265
	Hor. dia. = "	r	r	"	"	1.1588	0.064 0085
	" = "	$\sqrt{r}$	$\sqrt{r}$	"	"	1.0765	0.032 0042
	" = "	$a\sqrt{r}$	$a\sqrt{r}$	"	"	1.5742	0.197 0547
Egg-shape, Flowing ⅞ Full Depth.	Vert. dia. = "	r	r	"	"	0.84187	9.925 2433
	" = "	$\sqrt{r}$	$\sqrt{r}$	"	"	0.91753	9.962 6216
	" = "	$a\sqrt{r}$	$a\sqrt{r}$	"	"	0.39244	9.593 7703
	Hor. dia. = "	r	r	"	"	1.2628	0.101 3346
	" = "	$\sqrt{r}$	$\sqrt{r}$	"	"	1.1237	0 050 6673
	" = "	$a\sqrt{r}$	$a\sqrt{r}$	"	"	1.0814	0.033 9986
Egg-shape, Flowing ½ Full Depth.	Vert. dia. = "	r	r	"	"	0.55093	9.741 0990
	" = "	$\sqrt{r}$	$\sqrt{r}$	"	"	0.74225	9.870 5496
	" = "	$a\sqrt{r}$	$a\sqrt{r}$	"	"	0.11929	9.076 5963
	Hor. dia. = "	r	r	"	"	0.82640	9.917 1903
	" = "	$\sqrt{r}$	$\sqrt{r}$	"	"	0.90907	9.958 5951
	" = "	$a\sqrt{r}$	$a\sqrt{r}$	"	"	0.32872	9.516 8236

Examples in Use of Table 2, above.

Ex. 1.—From Tables 2 and 1, find the hydraulic radius r, $\sqrt{r}$, and $a\sqrt{r}$ (a = area of section considered) for the Catenary, (1) whose vertical diameter is 8 ft. 3 ins., and (2) whose horizontal diameter is 7 ft 4 ins.?

Solution.—For (1), vert. dia. of 8 ft. 3 ins., we have, $r = 2.063 \times .9269 = 1.912$, $\sqrt{r} = 1.436 \times .9628 = 1.383$, $a\sqrt{r} = 76.77 \times .8615 = 66.14$; and for (2), hor. dia. of 7 ft. 4 ins., we have, $r = 1.833 \times 1.043 = 1.912$, $\sqrt{r} = 1.354 \times 1.0211 = 1.383$, $a\sqrt{r} = 57.19 \times 1.1564 = 66.13+$.

Comparing the above results it is found that they are equal, simply because 7 ft. 4 ins. (= 88 ins.) is the hor. dia. of a catenary whose vert. dia. is 8 ft. 3 ins. (= 99 ins.); i.e., the hor. dia. of catenary = ⅞ vert. dia. See Table 5 following.

Ex. 2.—In Kutter's formula, $v = c\sqrt{rs} = c\sqrt{r}\sqrt{s}$, it is required to find the velocity v of flow in an Egg-shaped sewer whose hor. dia. is 4 ft. 6 ins., flowing ⅞ full depth, and on a grade of $s = .0004$, assuming the value of coefficient $c = 100$?

Solution.—From Tables 1 and 2, preceding, the value of $\sqrt{r} = 1.061 \times 1.1237 = 1.192$; hence the velocity of flow is $v = 100 \times 1.192 \times .02 = 2.384$ ft. per sec.

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Ex. 3.—In Kutter's formula, $q = av = c \,.\, a\sqrt{r} \,.\, \sqrt{s}$, it is required to find the discharge from a Basket-Handle conduit whose vert. dia. is 8 ft., flowing full, and on a grade of .000225, assuming the value of roughness $n = .015$.

Solution.—$s = .000225$, and $\sqrt{s} = .015$. From Tables 1 and 2, $r = 2 \times .9857 = 1.9714$, and $a\sqrt{r} = 71.09 \times .9939 = 70.66$. The value of c is obtained from Table 8, page 1171, and $= 111$. Hence, the discharge $q = 111 \times 70.66 \times .015 = 117.6$ cu. ft. per sec.

3.—VELOCITIES IN FEET PER SECOND IN CIRCULAR BRICK SEWERS.*
By Kutter's Formula, using $n = 0.015$.
(Slope s = value in first column ÷ 100.)

Fall in Ft. per 100 Ft.	Diameter of Pipe or Sewer, in Feet.											$\sqrt{s}$
(100 s)	0.5	1	2	3	4	5	6	7	8	10	12	
.002	0.09	0.16	0.27	0.36	0.44	0.52	0.59	0.65	0.71	0.82	0.92	.00447
.004	0.13	0.22	0.38	0.51	0.63	0.73	0.83	0.92	1.00	1.16	1.31	.00632
.006	0.16	0.27	0.47	0.63	0.77	0.90	1.02	1.13	1.23	1.42	1.60	.00775
.008	0.18	0.32	0.54	0.72	0.89	1.03	1.17	1.30	1.42	1.64	1.85	.00894
.01	0.20	0.35	0.60	0.81	0.99	1.16	1.31	1.45	1.59	1.84	2.06	.01
.015	0.25	0.43	0.74	0.99	1.21	1.42	1.60	1.78	1.94	2.25	2.53	.01225
.02	0.29	0.50	0.85	1.14	1.40	1.64	1.85	2.05	2.24	2.60	2.92	.01414
.025	0.32	0.56	0.95	1.28	1.57	1.83	2.07	2.30	2.51	2.90	3.26	.01581
.03	0.35	0.61	1.04	1.40	1.72	2.00	2.27	2.52	2.75	3.18	3.58	.01732
.035	0.38	0.66	1.12	1.51	1.85	2.16	2.45	2.72	2.97	3.44	3.86	.01871
.04	0.40	0.71	1.20	1.62	1.98	2.31	2.62	2.91	3.17	3.67	4.13	.02
.045	0.43	0.75	1.27	1.71	2.10	2.45	2.78	3.08	3.37	3.90	4.38	.02121
.05	0.45	0.79	1.34	1.81	2.22	2.59	2.93	3.25	3.54	4.11	4.62	.02236
.06	0.50	0.87	1.47	1.98	2.43	2.83	3.21	3.56	3.89	4.50	5.06	.02449
.07	0.53	0.94	1.59	2.14	2.62	3.06	3.47	3.84	4.20	4.86	5.46	.02646
.08	0.57	1.00	1.70	2.28	2.80	3.27	3.70	4.11	4.49	5.20	5.84	.02828
.09	0.61	1.06	1.80	2.42	2.97	3.47	3.93	4.36	4.76	5.51	6.19	.03
.10	0.64	1.12	1.90	2.55	3.13	3.66	4.14	4.59	5.02	5.81	6.53	.03162
.12	0.70	1.23	2.08	2.80	3.43	4.01	4.54	5.03	5.50	6.36	7.15	.03464
.14	0.76	1.32	2.25	3.02	3.71	4.33	4.90	5.43	5.94	6.87	7.73	.03742
.16	0.81	1.42	2.40	3.23	3.96	4.63	5.24	5.81	6.35	7.35	8.26	.04
.18	0.86	1.50	2.55	3.43	4.20	4.91	5.56	6.17	6.73	7.79	8.76	.04243
.20	0.90	1.58	2.69	3.61	4.43	5.17	5.86	6.50	7.10	8.22	9.23	.04472
.25	1.01	1.77	3.00	4.04	4.96	5.79	6.55	7.27	7.94	9.19	10.3	.05
.30	1.11	1.94	3.29	4.42	5.43	6.34	7.17	7.96	8.69	10.1	11.3	.05477
.35	1.20	2.09	3.55	4.78	5.86	6.84	7.75	8.60	9.39	10.9	12.2	.05916
.40	1.28	2.24	3.80	5.11	6.27	7.32	8.29	9.19	10.0	11.6	13.1	.06325
.50	1.43	2.50	4.25	5.71	7.01	8.18	9.26	10.3	11.2	13.0	14.6	.07071
.60	1.57	2.74	4.65	6.26	7.68	8.96	10.1	11.3	12.3	14.2	16.0	.07746
70	1.69	2.96	5.03	6.76	8.29	9.68	11.0	12.2	13.3	15.4	17.3	.08367
.80	1.81	3.17	5.37	7.22	8.86	10.3	11.7	13.0	14.2	16.4	18.5	.08944
.90	1.92	3.36	5.70	7.66	9.40	11.0	12.4	13.8	15.1	17.4	19.6	.09487
1.00	2.03	3.54	6.00	8.08	9.91	11.6	13.1	14.5	15.9	18.4	20.6	.1
1.20	2.21	3.88	6.58	8.85	10.9	12.7	14.4	15.9	17.4	20.1	22.6	.10955
1.40	2.39	4.19	7.11	9.56	11.7	13.7	15.5	17.2	18.8	21.7	24.4	.11832
1.60	2.56	4.48	7.60	10.2	12.5	14.6	16.6	18.4	20.1	23.2	26.1	.12649
1.80	2.71	4.75	8.06	10.9	13.3	15.5	17.6	19.5	21.3	24.6	27.7	.13416
2.00	2.86	5.01	8.50	11.4	14.0	16.4	18.5	20.5	22.4	26.0	29.2	.14142
†a	0.196	0.785	3.142	7.068	12.566	19.635	28.274	38.485	50.266	78.540	113.10	
r	0.125	0.250	0.500	0.750	1.000	1.250	1.500	1.750	2.000	2.500	3.000	
$c\sqrt{r}$	20.21	35.40	60.08	80.77	99.10	115.7	131.0	145.3	158.7	183.7	206.5	
$ac\sqrt{r}$	3.9604	27.803	188.77	570.90	1245 3	2272.7	3702.3	5591.6	7978.3	14426	23352	

*For second-class brick, dressed stone, or tuberculated iron. (See Table 9.)

Note.—Velocities in table are equal to $c\sqrt{r} \times \sqrt{s}$; $c\sqrt{r}$ is in the next line to bottom for the particular diameter of pipe, and $\sqrt{s}$ is found in the last column.

†a = area of section in sq. ft.; r = hydraulic radius in ft.; c = coefficient in Kutter's formula, using mean value of $s = .001$. Velocity in ft. per sec. $= v = c\sqrt{rs} = c\sqrt{r}\,\sqrt{s}$. Discharge in cu. ft. per sec. $= q = av = ac\sqrt{rs} = ac\sqrt{r}\,\sqrt{s}$.

4.—Properties of Circular Conduits or Sewers.

(*a*) *General.*

When flowing full:

$$a = \frac{\pi d^2}{4},$$
$$= 0.7854d^2.$$
$$p = \pi d,$$
$$= 3.1416d.$$
$$r = \frac{d}{4}.$$

When flowing ½ full:

$$a = \frac{\pi d^2}{8},$$
$$= 0.3927d^2.$$
$$p = \frac{\pi d}{2},$$
$$= 1.5708d.$$
$$r = \frac{d}{4}.$$

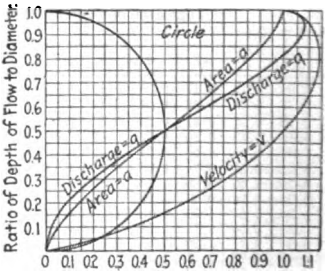

Fig. 4.—Ratios of a, v, and q for Filled Segments.
(a, v or q = unity when section is full.)

In which
a = area in sq. ft.
p = wetted perimeter in ft.
r = hydraulic radius in ft.
Log π = 0.4971499.

Log $\frac{\pi}{4}$ = 9.8950899.

(*b*) *Comparison of Circle with Other Sections of Equal Area.*
(See also Tables 5, 6, 7 and 8.)

					Logarithm.
d = dia. of Circle in ft.					
= 0.9407 × vert. dia. of Catenary of equiv. area					9.973 4667
= 1.0583 × hor. "	"	"	"		0.024 6192
= 1.001 × vert. "	Basket Handle	"	"		0.000 4345
= 1.060 × hor. "	"	"	"		0.025 4625
= 0.9045 × vert. "	Gothic	"	"		9.956 4020
= 1.091 × hor. "	"	"	"		0.037 8475
= 0.8062 × vert. "	Egg-shape	"	"		9.906 4340
= 1.2093 × hor. "	"	"	"		0.082 5253

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5.—Properties of Catenary- Conduits or Sewers.

(a) *General.*

	Logarithm.
a = area in sq. ft.	
$= 0.70277\ d_v^2$	9.846 8132
$= 0.88944\ d_h^2$	9.949 1182
p = wetted perimeter in ft.	
$= 3.03284\ d_v$	0.481 8497
$= 3.41195\ d_h$	0.533 0022
r = hydraulic radius in ft.	
$= 0.23172\ d_v$	9.364 9635
$= 0.260685\ d_h$	9.416 1160

Where d_v = vert. dia. in ft.
d_h = hor. dia. in ft.

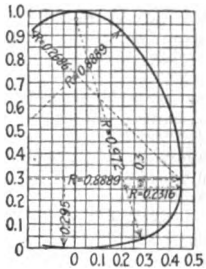

Fig. 5.—Catenary.

(b) *Comparison of Catenary with Other Sections of Equal Area.*
(See also Tables 4, 6, 7 and 8.)

	Logarithm.
d_v = vert. dia. of Catenary in ft.	
= 1.063 × dia. of Circle of equivalent area	0.026 5333
= 1.125 × hor. dia. of Catenary of equiv. area	0.051 1525
= 1.0641 × vert. " Basket-Handle " "	0.026 9678
= 1.1272 × hor. " " " "	0.051 9958
= 0.9615 × vert. " Gothic " "	9.982 9353
= 1.1598 × hor. " " " "	0.064 3808
= 0.8570 × vert. " Egg-shape " "	9.932 9673
= 1.2855 × hor. " " " "	0.109 0586
d_h = hor. dia. of Catenary in ft.	
= 0.9449 × dia. of Circle of equivalent area	9.975 3808
= 0.8889 × vert. dia. of Catenary of equiv. area	9.948 8475
= 0.9458 × vert. " Basket-Handle " "	9.975 8153
= 1.0019 × hor. " " " "	0.000 8433
= 0.8546 × vert. " Gothic " "	9.931 7828
= 1.0309 × hor. " " " "	0.013 2283
= 0.7618 × vert. " Egg-shape " "	9.881 8148
= 1.1426 × hor. " " " "	0.057 9061

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6.—Properties of Basket-Handle- Conduits or Sewers.

(*a*) *General.*

	Logarithm.
a = area in sq. ft.	
$= 0.78621\ d_v^2$	9.895 5386
$= 0.88226\ d_h^2$	9.945 5946
p = wetted perimeter in ft.	
$= 3.19040\ d_v$	0.503 8450
$= 3.37966\ d_h$	0.528 8730
r = hydraulic radius in ft.	
$= 0.24643\ d_v$	9.391 6936
$= 0.26105\ d_h$	9.416 7216

Where d_v = vert. dia. in ft.
d_h = hor. dia. in ft.

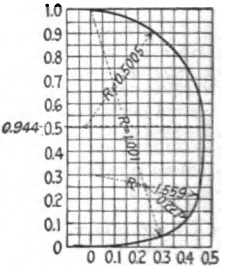

Fig. 6.—Basket-Handle.

(*b*) *Comparison of Basket-Handle with Other Sections of Equal Area.*
(See also Tables 4, 5, 7 and 8.)

d_v = vert. dia. of Basket-Handle in ft.					Logarithm.
= 0.999 × dia. of Circle of equivalent area					9.999 5655
= 0.9398 × vert. dia. of Catenary			of equiv.	area	9.973 0322
= 1.0573 × hor. "	"	"	"	"	0.024 1847
= 1.059 × " "	"	Basket-Handle	"	"	0.025 0280
= 0.9036 × vert. "	"	Gothic	"	"	9.955 9675
= 1.0900 × hor. "	"	"	"	"	0.037 4130
= 0.8054 × vert. "	"	Egg-shape	"	"	9.905 9995
= 1.2081 × hor. "	"	"	"	"	0.082 0908

d_h = hor. dia. of Basket-Handle in ft.					
= 0.943 × dia. of Circle of equivalent area					9.974 5375
= 0.8872 × vert. dia. of Catenary			of equiv.	area	9.948 0042
= 0.9981 × hor. "	"	"	"	"	9.999 1567
= 0.944 × vert. "	"	Basket-Handle	"	"	9.974 9720
= 0.8530 × " "	"	Gothic	"	"	9.930 9375
= 1.0289 × hor. "	"	"	"	"	0.012 3850
= 0.7603 × vert. "	"	Egg-shape	"	"	9.880 9715
= 1.1404 × hor. "	"	"	"	"	0.057 0628

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7.—Properties of Gothic- Conduits or Sewers.

(a) *General.*

	Logarithm.
a = area in sq. ft.	
$= 0.6553\, d_v^2$	9.816 4402
$= 0.9535\, d_h^2$	9.979 3312
p = wetted perimeter in ft.	
$= 2.8881\, d_v$	0.460 6057
$= 3.4838\, d_h$	0.542 0512
r = hydraulic radius in ft.	
$= 0.2269\, d_v$	9.355 8345
$= 0.2737\, d_h$	9.437 2800

Where d_v = vert. dia. in ft.
d_h = hor. dia. in ft.

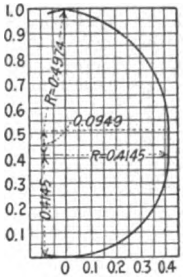

Fig. 7.—Gothic.

(b) *Comparison of Gothic with Other Sections of Equal Area.*

(See also Tables 4, 5, 6 and 8.)

	Logarithm.
d_v = vert. dia. of Gothic in ft.	
= 1.1056 × dia. of Circle of equivalent area	0.043 5980
= 1.0401 × vert. dia. of Catenary of equiv. area	0.017 0647
= 1.1701 × hor. " " " "	0.068 2172
= 1.1067 × vert. " Basket-Handle " "	0.044 0325
= 1.1724 × hor. " " " "	0.069 0605
= 1.2063 × " " Gothic " "	0.081 4455
= 0.8913 × vert. " Egg-shape " "	9.950 0320
= 1.3370 × hor. " " " "	0.126 1233
d_h = hor. dia. of Gothic in ft.	
= 0.9165 × dia. of Circle of equivalent area	9.962 1525
= 0.8622 × vert. dia. of Catenary of equiv. area	9.935 6192
= 0.9700 × hor. " " " "	9.986 7717
= 0.9175 × vert. " Basket-Handle " "	9.962 5870
= 0.9719 × hor. " " " "	9.987 6150
= 0.8290 × vert. " Gothic " "	9.918 5545
= 0.7389 × " " Egg-shape " "	9.868 5865
= 1.1084 × hor. " " " "	0.044 6778

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8.—Properties of Egg (-Shaped)- Conduits or Sewers.

(a) *General.*

Wetted Section.	Flowing Full Depth.	Flowing $\frac{3}{4}$ Full Depth.	Flowing $\frac{1}{2}$ Full Depth.
Area a.....	$0.510455d_v^2$	$0.335922d_v^2$	$0.126222d_v^2$
Log =	9.707 9578	9.526 2386	9.101 1357
Area a.....	$1.148525d_h^2$	$0.755825d_h^2$	$0.284d_h^2$
Log =	0.060 1404	9.878 4212	9.453 3183
Perimeter p	$2.6433d_v$	$1.59607d_v$	$0.91647d_v$
Log =	0.422 1409	0.203 0510	9.962 1166
Perimeter p	$3.9649d_h$	$2.3941d_h$	$1.3747d_h$
Log =	0.598 2322	0.379 1423	0.138 2079
Hyd. rad. r	$0.19313d_v$	$0.21047d_v$	$0.13773d$
Log =	9.285 8572	9.323 1833	9.139 0390
Hyd. rad. r	$0.2897d_h$	$0.3157d$	$0.2066d_h$
Log =	9.461 9485	9.499 2746	9.315 1303

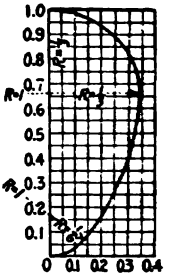

Fig. 8.—Egg

Where d_v = vert. dia. in ft.
d_h = hor. dia. in ft.

(b) *Comparison of Egg-Shape with Other Sections of Equal Area.*

(See also Tables 4, 5, 6 and 7.)

	Logarithm.
d_v = vert. dia. of Egg-shape in ft.	
= 1.2404 × dia. of Circle of equivalent area.....	0.093 5660
= 1.1669 × vert. dia. of Catenary of equiv. area.....	0.067 0327
= 1.3128 × hor. " " " "	0.118 1852
= 1.2417 × vert. " Basket-Handle " "	0.094 0005
= 1.3153 × hor. " " " "	0.119 0285
= 1.1219 × vert. " Gothic " "	0.049 9680
= 1.3534 × hor. " " " "	0.131 4135
= 1.5000 × hor. " Egg-shape " "	0.176 0913
d_h = hor. dia. of Egg-shape in ft.	
= 0.8269 × dia. of Circle of equivalent area.....	9.917 4747
= 0.7779 × vert. dia. of Catenary of equiv. area.....	9.890 9414
= 0.8752 × hor. " " " "	9.942 0939
= 0.8278 × vert. " Basket-Handle " "	9.917 9092
= 0.8769 × hor. " " " "	9.942 9372
= 0.7480 × vert. " Gothic " "	9.873 8767
= 0.9022 × hor. " " " "	9.955 3222
= 0.6667 × vert. " Egg-shape " "	9.823 9087

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9.—Velocities in Feet per Second in Egg-shaped Sewers.

By Kutter's Formula, using $n = 0.015$.
(Slope s = value in first column ÷ 100.)
[Velocities, v, in Feet per Second.]

	Fall in Ft. per 100 Ft. (100 s)	Greatest Transverse or Horizontal Diameter d_1 in Feet. 1	2	3	4	5	6	7	8	10	12	$\sqrt{s}$
Flowing full depth.	.01	0.40	0.67	0.90	1.10	1.28	1.45	1.60	1.75	2.02	2.27	.01
	.02	0.56	0.95	1.27	1.55	1.81	2.04	2.27	2.47	2.85	3.21	.01414
	.04	0.79	1.34	1.79	2.20	2.56	2.89	3.20	3.50	4.04	4.54	.02
	.06	0.97	1.64	2.20	2.69	3.13	3.54	3.92	4.28	4.95	5.56	.02449
	.10	1.25	2.12	2.84	3.48	4.05	4.57	5.07	5.53	6.38	7.17	.03162
	.14	1.48	2.50	3.36	4.11	4.79	5.41	5.99	6.54	7.56	8.49	.03742
	.20	1.77	2.99	4.01	4.91	5.72	6.47	7.16	7.82	9.03	10.1	.04472
	.40	2.51	4.23	5.67	6.95	8.10	9.15	10.1	11.1	12.8	14.3	.06325
	.60	3.07	5.18	6.95	8.51	9.91	11.2	12.4	13.5	15.6	17.6	.07746
	.80	3.54	5.99	8.02	9.83	11.4	12.9	14.3	15.6	18.1	20.3	.08944
	1.00	3.96	6.69	8.97	11.0	12.8	14.5	16.0	17.5	20.2	22.7	.1
	1.40	4.68	7.92	10.6	13.0	15.1	17.1	19.0	20.7	23.9	26.8	.11832
	2.00	5.60	9.46	12.7	15.5	18.1	20.4	22.7	24.7	28.5	32.1	.14142
	a	1.148	4.594	10.337	18.376	28.713	41.347	56.278	73.506	114.85	165.39	
	r	.2897	.5794	.8691	1.159	1.449	1.738	2.028	2.318	2.897	3.476	
	$c\sqrt{r}$	39.62	66.93	89.70	109.9	128.0	144.6	160.2	174.8	201.9	226.8	
Flowing ¾ full depth.	.01	0.42	0.71	0.95	1.16	1.35	1.53	1.70	1.85	2.13	2.39	.01
	.02	0.60	1.01	1.35	1.65	1.92	2.16	2.40	2.61	3.02	3.39	.01414
	.04	0.85	1.43	1.91	2.33	2.71	3.06	3.39	3.70	4.27	4.79	.02
	.06	1.04	1.75	2.34	2.85	3.32	3.75	4.16	4.53	5.23	5.86	.02449
	.10	1.34	2.26	3.01	3.68	4.28	4.84	5.36	5.85	6.74	7.57	.03162
	.14	1.59	2.67	3.57	4.36	5.07	5.73	6.35	6.92	7.98	8.96	.03742
	.20	1.90	3.19	4.26	5.21	6.06	6.85	7.58	8.27	9.54	10.7	.04472
	.40	2.68	4.52	6.03	7.37	8.57	9.68	10.7	11.7	13.5	15.1	.06325
	.60	3.28	5.53	7.38	9.02	10.5	11.9	13.1	14.3	16.5	18.5	.07746
	.80	3.79	6.39	8.53	10.4	12.1	13.7	15.2	16.5	19.1	21.4	.08944
	1.00	4.24	7.14	9.53	11.6	13.5	15.3	17.0	18.5	21.3	23.9	.1
	1.40	5.02	8.45	11.3	13.8	16.0	18.1	20.1	21.9	25.2	28.3	.11832
	2.00	6.00	10.1	13.5	16.5	19.2	21.6	24.0	26.1	30.2	33.9	.14142
	a	0.756	3.023	6.802	12.093	18.895	27.210	37.035	48.373	75.583	108.84	
	r	0.316	0.631	0.947	1.263	1.579	1.894	2.210	2.526	3.157	3.788	
	$c\sqrt{r}$	42.40	71.42	95.33	116.5	135.5	153.1	169.6	184.9	213.3	239.4	
Flowing ½ full depth.	.01	0.30	0.52	0.70	0.87	1.01	1.15	1.28	1.40	1.62	1.83	.01
	.02	0.43	0.74	1.00	1.22	1.43	1.63	1.81	1.98	2.29	2.59	.01414
	.04	0.61	1.04	1.41	1.73	2.03	2.30	2.56	2.80	3.24	3.66	.02
	.06	0.74	1.28	1.73	2.12	2.48	2.82	3.13	3.43	3.97	4.48	.02449
	.10	0.96	1.65	2.23	2.74	3.20	3.64	4.04	4.42	5.13	5.79	.03162
	.14	1.14	1.95	2.64	3.24	3.79	4.30	4.79	5.24	6.07	6.85	.03742
	.20	1.36	2.33	3.15	3.87	4.53	5.14	5.72	6.26	7.25	8.19	.04472
	.40	1.92	3.29	4.46	5.48	6.41	7.27	8.09	8.85	10.3	11.6	.06325
	.60	2.36	4.03	5.46	6.71	7.85	8.91	9.91	10.8	12.6	14.2	.07746
	.80	2.72	4.66	6.30	7.75	9.06	10.3	11.4	12.5	14.5	16.4	.08944
	1.00	3.04	5.21	7.05	8.66	10.1	11.5	12.8	14.0	16.2	18.3	.1
	1.40	3.60	6.16	8.34	10.2	12.0	13.6	15.1	16.6	19.3	21.7	.11832
	2.00	4.30	7.37	9.97	12.2	14.3	16.3	18.1	19.8	22.9	25.9	.14142
	a	0.284	1.136	2.556	4.544	7.100	10.224	13.916	18.176	28.400	40.892	
	r	0.207	0.413	0.620	0.826	1.033	1.240	1.446	1.653	2.066	2.479	
	$c\sqrt{r}$	30.41	52.09	70.48	86.61	101.3	115.0	127.9	139.9	162.1	183.1	

Notation.—a = sectional area of wetted section of sewer, in square feet; r = hydraulic mean radius; c = coefficient in Kutter's formula, using mean value of $s = .001$. Velocity $v = c\sqrt{r} \times \sqrt{s}$ for the particular diameter d_1 and the particular grade or slope s.

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Example in Use of Table 9, preceding.

Example.—What size of egg-shaped sewer, running two-thirds full depth, on grade of one-tenth of one per cent., will carry 16 cu. ft. per second, assuming $n = 0.015$?

Solution.—From the preceding table, for "Flowing two-thirds full depth" we select, if possible, the width d_h whose velocity multiplied by the corresponding area a will give the discharge q $(= av) = 16$, for a grade of 0.1 ft. per 100 ft. Now for $d_h = 2$ ft., $q = 3.023 \times 2.26 = 6.83$ cu. ft. per sec.; and for $d_h = 3$ ft., $q = 6.802 \times 3.01 = 20.47$ cu. ft. per sec. Hence, by proportion, for $q = 16$, $d_h = 2.67$ ft. or, say, 2 ft. 8 ins.; and the height $d_v = 1\frac{1}{2}d_h = 4$ ft. (See Table 8.) In practice it is customary to increase the calculated diameter by about 2 ins., more or less.

Thickness of Brick Sewer Walls.—No fixed rule can be used, but for brick sewers under 30 ins. in diameter a single 4-in. ring will generally suffice. Two rings (2×4 ins.) of brick are required for diameters from $2\frac{1}{2}$ to 6 ft.; and three rings from about 6 to 10 ft. Above 10 ft. diameter treat sewer above spring line as a masonry arch, and keep the line of resultant pressure well within the middle third. Baldwin Latham gives the following formula for thickness:

$$t = \frac{dr}{100} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where t = thickness of brick wall, in feet;
d = depth of excavation, in feet;
r = radius of sewer, in feet.

It is essential that sewer linings be water tight.

Sewer Foundations.—Many sewers are rendered leaky and unsanitary by reason of their having been constructed on improper foundations. An economical foundation for sewers projected throught soft, marshy ground consists of two-pile bents driven say 4 or 5 feet apart, capped with suitable cross timbers, on top of which are laid the thick longitudinal floor planking, forming a platform for the sewer proper. Sometimes the piling supporting the platform is omitted. Timber foundations should always be below the ground-water level to prevent decay.

10.—Some References to Illustrated Sewer Construction in "Engineering News."

June 6, 1901. Cast iron pipe outfall sewer, with frost casing, pile foundation, etc.

Oct. 10, 1901. Sewers in Brooklyn; circular, 13′ 6″ dia. with 16″ brick arch; deep manhole construction.

Jan. 1, 1903. Sewer construction in Flatbush, Brooklyn, showing various types, with foundations.

Nov. 19, 1903. Method and cost of constructing a 30″ concrete sewer with brick arch, at Medford, Mass.; illustrated, with forms.

Dec. 17, 1903. Pear-shaped concrete sewer with brick arch 12″ thick; vert. dia. = 72″, hor. dia. = 72″.

Feb. 4, 1904. Wear and repairs of inverts in St. Louis sewers; with diagram of sewer; vert. dia = 15′, hor. dia. = 20′; arch ring of cut stone.

Feb. 18, 1904. A novel form of centering for 5-ft. egg-shaped sewer at Washington, D. C.

Oct. 20, 1904. A steel center or form for constructing concrete sewers.

Jan. 5, 1905. A new form of reinforced concrete-block sewer construction; illustration of sewer 42″ dia. with 4″ thickness of shell.

Sept. 26, 1907. A 36″ steel lap-welded pipe, $\frac{5}{8}$″ metal, 18′ lengths, used for extending sewer outfalls at Blackpool, England.

Nov. 11, 1907. A new concrete-pipe joint; hot asphalt poured into grooves at ends of abutting cast concrete pipes.

Mar. 26, 1908. Intercepting and outfall sewer at Waterbury; vert. dia. 4′5″, hor. dia. 4′ 6″, crown thickness 6″; reinforced concrete.

July 30, 1908. Sewers at St. Louis, constructed with collapsible steel centering: (1) vert. dia. 18′ 6″, hor. dia. 29′ 0″, crown thickness about 18″; (2) vert. dia. about 17′ 0″, hor. dia. 25′ 0″, crown thickness about 13″; reinforced concrete.

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Sewer Pipe.—Vitrified clay pipe is admirably adapted for sewers. It comes in lengths of 2 ft. 6 ins. for the smaller sizes (8 to 18 ins.),

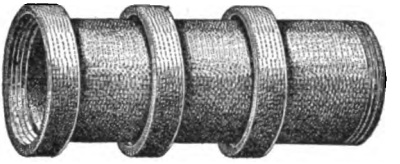

Fig. 9.

while the large diameters are made in 3-ft. lengths. The joints are cemented in place. Fig. 9 shows a section of three lengths.

11.—Standard Salt-Glazed Vitrified Pipe.
(Manufactured by Blackmer & Post Pipe Co., St. Louis.)

Inside Diameter.	Thickness of Shell.	Depth of Socket.	Annular Space.	Length of Sections.	Weight per Foot.	Car Load 14 Tons.	Price per Foot.	Branches Curves, etc. Each.
Ins.	Ins	Ins.	In.	Ft.	Lbs.	Ft.		
27	2⅛	4	¾	3	215	129	$3.25	$16.25
28	2 3-32	4	¾	3	230	120	3.50	17.50
30	2¼	4	⅞	3	270	108	4.00	20.00
33	2⅝	4½	1	3	320	90	5.00	25.00
36	2½	5	1	3	365	81	6.00	30.00

The above price list is subject to large discounts.

12.—Extra Heavy Sewer and Culvert Pipe.
(Manufactured by Evans & Howard, St. Louis.)

	Diameter of Pipe in Inches.							
	12	15	18	21	24	27	30	36
Weight, per foot	52	76	103	136	170	240	305	390
Thickness in inches	1⅛	1¼	1½	1¾	2	2¼	2½	2¾
Depth of Socket in inches	3	3½	3¾	4	4	4½	4½	5¼
No. of feet in carload of 15 tons	577	400	291	220	181	125	98	77
Price, per foot	$0.75	$1.00	$1.50	$2.00	$2.50	$3.25	$4.00	$6.00

Sizes 18-in. and under in 2½-foot lengths, sizes 21-in. and over in 3-foot lengths, improved corrugated deep sockets and corrugated ends.

Discount on application.

Kinds of Sewers.—Sewers may be classified according to the special purposes for which they are designed or according to the material entering into their construction. Thus we have branch sewers, main-line sewers, intercepting sewers, trunk-line sewers, and outfall sewers. Trunk-line sewers are often projected through several towns, the expense being borne proportionately. An outfall sewer is the main stem of discharge to the outlet. Small sewers are built of terra cotta pipe, cast iron pipe, concrete pipe, etc.; the larger sewers are constructed of brick, concrete, reinforced concrete, etc. Wood-stave pipe enters into the design of the outfall sewer of the City of Los Angeles, projected as follows: 2400 ft. of 52-in. circular brick conduit; 4,400 ft. of 40-in. circular brick conduit; 16,900 ft. of inverted syphon of 38-in. wood-stave pipe; 1,900 ft. of 40-in. circular brick

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conduit; 5,800 ft. of 6-ft. oval brick and concrete tunnel; 800 ft. of 40-in. circular brick conduit; 19,100 ft. of inverted syphon of 36-in. wood-stave pipe; 12,100 ft. of 40-in. circular brick conduit; 900 ft. of 6-ft. oval brick and concrete tunnel; 600 ft. of 40-in. circular brick conduit; 1,300 ft. of 6-ft. oval brick and concrete tunnel; 1,100 ft. of 24-in. cast iron pipe, to outlet in Pacific Ocean.

Location of Sewers.—Sewers are most commonly located under the *centers* of streets, or on either side of broad avenues. Some cities however prefer to build only one sewer near one curb, in streets of ordinary width. In cities where alleys exist, the latter are often selected for the sewer lines.

Manholes are street openings to sewers; they are built for inspection and cleaning of sewers, and for ventilation. Those systems of sewers which have been built without manholes are certainly at a disadvantage. For large sewers the manholes are built tangent to one side of sewer, in the form of a conic frustum; or, an entrance to side of sewer may be made by a slanting shaft with steps for descent, with a manhole entrance at top of steps. Manholes should be situated at bends of both alinement and grade of the sewer, so that a clean sight may be had through the sewer from one manhole to another. If this is impracticable by reason of the expense, *lamp-holes* may be substituted for them occasionally. These latter consist of a small circular shaft from the street to the top of sewer through which a lamp may be suspended to be sighted at from the nearest manhole in either direction. Fig. 10 shows a section of a circular manhole frame and cover, to be set flush with the street surface.

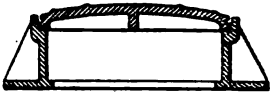

Fig. 10.—Manhole.

Catch Basins are placed along the gutters at the sides of streets, and especially at street corners, to act as settling basins for surface waters containing sand and dirt. The outlet from the catch basin to the sewer is near the top of the basin. They should be cleaned out before the deposit becomes too great. The styles used are innumerable. That shown in Fig. 11 is

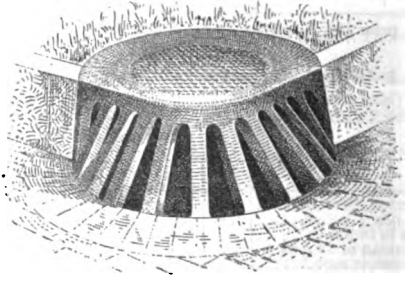

Fig. 11.—Catch Basin.

manufactured by James B. Clow & Sons, of Chicago, and is designed for street *corners*.

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EXCERPTS AND REFERENCES.

The Sanitary Protection of the Water Supply of Baltimore, Md. (Eng. News, Dec. 5, 1901).—Illustrations: Standard designs for cesspools, privies and catchbasins in the drainage area of the Baltimore water supply.

Electric Sewage Pumps, Septic Tanks and Contact Beds at Fond du Lac, Wis. (By G. S. Pierson. Eng. News, May 22, 1902).—Illustrations: General view of works; nearer view of pump house and main carrier; general plan of sewage disposal works and sections of contact filter beds; storm water overflows in intercepting sewers; grit and screen chamber near sewage pumping station; plan and section of pumping station; sewage distributing and effluent receiving chamber for contact beds.

Treatment of Sewage in a Large Open Septic Tank at Worcester, Mass. (Eng. News, May 29, 1902).—Table.

The Chicago Intercepting Sewer System (Eng. News, May 28, 1903). —Illustrations: Map of the system; driving sheet piling for trench of 16-ft. sewer; swinging-derrick with orange-peel bucket; bricklaying in 16-ft. sewer; trenching machine on intercepting sewer; train of dump cars on sewer work; shield and details of shield for 20-ft. sewer; plan of intake and pumping station; sections of channels at pumping station; details of screen chamber or sand catcher at intake.

The Sanitary Disposal of Municipal Refuse (Trans. A. S. C. E., Vol. L).

Sewage Disposal for Country Residences (Eng. News, July 2, 1903). —Illustration of septic tank, the estimated cost of which is: 3 casks at 75 cents each = $2.25; sewage siphon, $14.00; 100 ft. of farm tile, $1.00; pipe and plank, $2.50; labor and cement, $6.00; total, $25.75.

The Northwestern Ave. Sewer at Indianapolis, Ind. (By W. Buehler. Paper, Ind. Eng. Soc.; Eng. News, July 16, 1903).—Illustrations: Map; overflow section and junction with main sewer; dam in combined sewer for diverting sewage flow into separate sewer; construction of concrete floor and sides of brick and concrete overflow sewer; forms and I-beams in place for roof of concrete and steel rectangular sewer.

A Table Giving Quantities of Cement and Sand and of Cement Mortar for Sewer Pipe Joints (By J. N. Hazlehurst. Eng. News, Feb. 25, 1904).—

QUANTITIES OF CEMENT, SAND AND OF CEMENT MORTAR FOR SEWER PIPE JOINTS.

For Each 100 ft. of Sewer (With Portland Cement 375 lbs. net per bbl.)

Size of Pipe.	L'gth, ft.	Mortar cu. yds.	Proportions: 1 Cement to					
			1 Sand.			2 Sand.		
			Cement, bbls.	Sand, cu. yd.	No. ft. to bbl. Cemt	Cement, bbls.	Sand, cu. yd.	No. ft. to bbl. Cemt
6-in	2½	0.003	0.01248	0.00201	803	0.00855	0.00252	1,168
8-in	2½	0.038	0.15808	0.02546	633	0.10830	0.03192	923
10-in	2½	0.058	0.24128	0.03886	410	0.16530	0.04872	605
12-in	2½	0.089	0.37024	0.05963	270	0.25365	0.07476	394
15-in	2½	0.123	0.51268	0.08241	195	0.35055	0.10332	285
18-in	2½	0.167	0.69472	0.11189	144	0.47595	0.14018	210
20-in	2½	0.237	0.98592	0.15879	101	0.67545	0.19908	148
24-in	2½	0.299	1.24384	0.20033	80	0.85215	0.25116	117
27-in	3	0.492	2.04672	0.32964	49	1.40220	0.41328	71
30-in	3	0.548	2.27968	0.36716	44	1.56180	0.46032	64
36-in	3	0.849	3.53184	0.56883	29	2.41965	0.71316	41

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The Wear of Sewer Inverts (By E. A. Hermann. Eng. News, Feb. 4, 1904).—The materials most commonly used for sewer construction are vitrified clay pipe, vitrified brick, common building brick (more or less unburned), and concrete. In the sewers of St. Louis, which are on the combined system, the grades range from 0.2% to 2%, average about 0.5% for sewers more than 5 ft. in diameter and about 1% for the smaller sewers. The vitrified clay pipes show no appreciable wear after about 35 years' use; these sewers are mostly laterals, and have, of course, the smallest discharge, though some of those in the business and manufacturing sections of the city carry a constant stream from 1 to 3 ins. deep, containing more or less acids, scalding hot water and steam. The pipes vary from 12 to 24 ins. in dia., except a few lines, which are 30 to 36 ins. in dia. The vitrified brick (in use about 12 years) also shows no appreciable wear. The inverts of sewers built of common brick begin to show some wear after about 3 years of service, and after about 30 years' use the first ring of brick is worn away from 2 ins. to nearly its whole depth of 4 ins. This wear varies greatly in sewers of different size, grade, quantity and quality of sewage, and hardness of brick; the average life of such brick appears to be about 40 years. Illustrations: Method employed in repairing badly worn sewer inverts; section of large sewer at St. Louis, Mo.

A New Jointing Material—Sulphur and Sand—for Sewer Pipes (By Alex. Potter. Eng. News, Mar. 10, 1904).—The following table gives information concerning the cost and amount of material in making sulphur-sand joints:

APPROXIMATE COSTS.

Size of Pipe.	Amt. of Mixture Lbs. per Joint.	Mixture	Gasket.	Fuel.	Labor.	Total.	Per foot Lengths.	
							3-Ft.	2-Ft.
24-in.	10.0	.125	.02	.02	.13	.295	.10	.15
22-in.	9.0	.1125	.02	.02	.13	.282	.095	.14
20-in.	8.0	.10	.02	.02	.12	.260	.09	.13
18-in.	7.0	.087	.02	.02	.11	.247	.08	.12
15-in.	5.5	.069	.01	.01	.10	.187	.065	.095
12-in.	4.2	.052	.01	.01	.09	.162	.055	.08
10-in.	3.3	.041	.01	.01	.08	.141	.045	.07
8-in.	2.5	.031	.01	.01	.07	.121	.04	.06

The exclusion of ground water will increase the capacity of the sewer from 10% to 100% through territory subject to the admission of ground water, so that the increase in cost becomes a trifling matter, especially when it is considered that tree roots are kept out of the sewer.

Refuse Destructor Combined With Electric Light Plant at Westmount, P. Q. (Eng. News, May 24, 1906).—Table of estimated costs.

Specifications for Refuse Destructor, Borough of Richmond, New York City (Eng. News, Dec. 6, 1906).—The first thorough-going, if not absolutely the first, specifications and call for bids for a refuse destructor designed to produce heat for lighting or power purposes in the U. S.

Cost of Shallow and Deep Sewer Trenches (By J. G. Palmer. Eng. News, June 25, 1908).—Cost-data tables.

Principles of Sewage Purification on Land (By Rudolph Hering. Eng. News, May 27 and June 3, 1909).

Rainfall, and Run-Off in Storm-Water Sewers (By C. C. Gregory. Trans. A. S. C. E., Vol. LVIII).—Formulas, diagrams and tables. Table 1, Comparison of rainfall and discharge from the 6th Avenue sewer, N. Y. City, the drainage area of which is 221 acres; Table 2, Short-time storms; Table 3, Long-time storms.

Cost of a 66-Inch Brick Sewer at Gary, Ind. (By E. M. Scheflow. Eng. Rec., Jan. 2, 1909).—Total cost of sewer per lin. ft.: Excavation by machine: $0.503; excavation by hand, $1.25; pumping, $1.61; sheeting, $0.368.

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laying, $1.32; backfilling by machine, $0.406; backfilling by hand, $0.136; hauling material, $0.607; superintendence and general, $0.30; materials, $1.81; depreciation of machinery, repairs and the like (estimated), $1.50; total, $10.122.

Louisville Sewerage Improvements (By R. DeL. French. Eng. News, Oct. 14, 1909).—Illustrated:—Standard sections of circular concrete sewers, with table of dimensions and quantities, for hard and soft ground; Typical horseshoe section of reinforced-concrete sewer, 15 ft. high; Typical semi-elliptical section of reinforced concrete sewer, 10 ft. high; Typical junction chamber, reinforced-concrete sewers; Combined sewer and drain.

Aerial Distribution of Sewage over Percolating Filters (By Wm. Gavin Taylor. Eng. News, Nov. 11, 1909).—Illustrations of nozzle; diagrams of distribution effected by same; pressure undulating valves, described and illustrated.

The Improved Water and Sewerage Works of Columbus, O. (By J. H. Gregory. Trans. A. S. C. E., Vol. LXVII., June, 1910).—Illustrations: Lime saturators; mixing tanks; sewage pumping station; purification works; septic tanks; gate house; etc.

Modern Procedure in District Sewer Design (By W. W. Horner. Eng. News, Sept. 29, 1910).—**Diagrams:** (a) Rainfall curves for St. Louis; (b) Approximate curves for designing vitrified pipe and circular egg-shaped sewers under 60 ins. mean diameter; (c) Approximate curves for designing circular sewers 4 to 24 ft. in diameter; (d) Capacities of standard horse-shoe sewer. **Illustrations:** Details of sewer inlets. **Table:** Data for branch sewer.

The Design of Storm-Water Drains (Eng. Rec., Oct. 29, 1910).—Tables of heavy precipitations, and discussion of typical precipitation curves.

Illustrations of Interesting Designs:—

Description.	Eng. News.
Adjustable basin and manhole covers	July 11, 1901.
A centrifugal separator for shavings and dust	Sept. 26, '01.
Plan of small sewer system, at Lake Bluff, Ill.	June 6, '01.
Deep manhole.construction on 60th St. sewer tunnel, B'klyn	Oct. 10, '01.
Septic tank and double contact filter beds, Glencoe, Ill.	Oct. 24, '01.
Proposed light refuse crematory on dumping pier, N. Y. City	April 17, '02.
Steel plate mine ventilating fan, Modoc C. & M. Co., Ohio	June 19, '02.
Plan and details sewage purification works, Depew, N. Y.	June 26, '02.
Sewage disposal wks., filter beds and septic tk., Id. Pk. Resort	July 24, '02.
Method of anchoring a rein.-conc. lining for septic tank	Aug. 21, '02.
Couplings for rods used in cleaning sewers, conduits, etc.	Sept. 4, '02.
A new German automatic flush tank	Nov. 27, '02.
The 64th St. sewer tunnel and outlet tunnel, Brooklyn	Jan. 1, '03.
General arrangement of sewage sludge treatment apparatus, Cassel, Germany	Jan. 15, '03.
Cross-section of 72" concrete and brick sewer, Corning, N. Y.	Dec. 17, '03.
A novel center and form for concrete sewer work	Feb. 18, '04.
Catch pit, septic tank and automatic flush tank for a jail	Mar. 3, '04.
A steel form for concrete sewers	Oct. 20, '04.
A sewer pipe centering device	Feb. 8, '06.
Sprinkler motor for percolating sewage filter, England	May 2, '07.
Rein.-conc. intercepting and outfall sewer, Waterbury, Conn.	Mar. 26, '08.
Septic tank and percolating filters, Univ. of Minn.	June 25, '08.
Plans of Winston-Salem intercepting sewer, and details	June 25, '08.
Septic tanks and percolating filters, Washington, Pa.	July 16, '08.
Steel centering for Harlem Creek sewer	July 30, '08.
Adjustable metal forms for construction of large sewer	Oct. 8, '08.
Septic tank and filter bed for residence, Philippines	Oct. 8, '08.
A household cesspool and overflow, with details	Dec. 17, '08.
Reinforced-concrete outlet sewer, 10½ ft. dia.	Jan. 28, '09.
Details of 5½ and 6-ft. concrete-block sewers, Toledo, O.	Feb. 4, '09.
10-ft. sewers, rein.-conc., pile found., Kansas City	Jan. 27, '10.
Plans and cross-section of Milwaukee refuse incinerator	July 21, '10.

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Description.	Eng. Rec.
Section of Stony Brook conduit, and cableway carriage	Feb. 6, '09.
Design of a 15-ft. drop in a large sewer	Feb. 6, '09.
Sections of rein.-conc. sewers, Louisville, Ky.	May 1, '09.
Typical manholes, U. S. Military Academy	May 8, '09.
13.12-ft. dia., circular rein.-conc. conduit drain, Mexico	June 5, '09.
Ventilating manhole cover with dust-catching box	June 26, '09.
Molded concrete pipe and storm drains, Newark, N. J.	Nov. 6, '09.
Storm water drain, 3 ft. diam., with cost	Dec. 18, '09.
Details, septic tanks and contact beds, Grand Canyon	Jan. 29, '10.
Cost of catch basins in Boston; illustrated	Mar. 12, '10.
Cross-section rein.-conc. storm sewer (13 x 11 ft.), on piers	Mar. 19, '10.
Rein.-conc. sewers in soft ground, Seattle, Wash.	Oct. 22, '10.
Sewer sections, canal crossing and siphon shaft, N. trunk sewer, Seattle, Wash.	Dec. 10, '10.
New method of handling sewage sludge, in Germany	Dec. 10, '10.
Intercepting sewer and outfall at New Bedford	Dec. 17, '10.
Heavy plain concrete sewers (2½ ft. rad., 8" ring) at Albany	Dec. 17, '10.

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66.—IRRIGATION.

General Discussion.—Those who are specially interested in irrigation matters will find much valuable information in the Bulletins issued by the U. S. Department of Agriculture, and in the Water-Supply and Irrigation Papers, Annual Reports, etc., of the U. S. Geological Survey. Lists of the various Documents may be had on application, and the papers are generally for free distribution.

The Problems of Irrigation are broad and intricate, the local conditions of soil, water, climate, crops, etc., requiring much study, often extending over considerable periods of time. The amount of water required will depend upon the kind of crops, as well as upon the soil, and to a certain extent upon the climate; also upon the amount of seepage and evaporation before it reaches the land to be irrigated; and last of all upon the method of irrigation, which may be more or less wasteful. If there is any rainfall, the records of monthly precipitation, extending over say 15 to 20 years, should be examined for one-, two- and three- year periods of greatest drought, for a possible reduction in the artificial supply. (See Water Supply, page 1194.)

The Unit of Land Area in irrigation is the acre, equal to 43560 sq. ft. A square 1-acre tract is 208.71 ft. square; a square 2-acre tract is 295.16 ft. square; a square 3-acre tract is 361.50 ft. square; a square 4-acre tract is 417.42 ft. square; a square 5-acre tract is 466.69 ft. square; a square 10-acre tract is 660 ft. square = ¼ mile square; a square 20-acre tract is 933.38 ft. square; a square 40-acre tract is 1320 ft. square = ½ mile square; a square 80-acre tract is 1866.76 ft. square; a square 160-acre tract is 2640 ft. square = ½ mile square.

The Units of Flow of water are the "inch" (miner's inch) and the cubic foot per second.

The "Inch" is the volume of water (say in cubic feet) which will flow through a vertical standard orifice one-inch square (say in one minute) under a given pressure head (say 5 to 6½ inches, and fixed by State law). This amounts to about 1.5 cubic feet per minute (see Table 1). The "inch" is concenient in delivering water to small users because the quantity being delivered is apparent at a glance. Where the delivery calls for more than one "inch" a box is constructed with a long horizontal slot one inch or more in height and provided with a slide and scale.* By manipulating the slide, the slot may be elongated so as to deliver the number of inches required; but as the end contractions remain constant it is apparent that the rate of discharge increases faster than the scale readings indicate.

Table 1 is based on one "inch" discharging 1.5 cu. ft. per min.

1.—Equivalents of Discharge of 1 "Inch," at 1.5 Cu. Ft. per Min.

Duration of Flow	U. S. Gallons.	Cubic Feet.	Tons of 2000 Lbs.	Acre-Ins.	Acre-Ft.
1 Second	.187	0.025	.00078	.0000069	.00000057
1 Minute	11.221	1.5	.0469	.0004132	.00003444
1 Hour	673.25	90.	2.8125	.024793	.0020661
1 24-Hour Day	16157.9	2160.	67.5	.59504	.049587
1 30-Day Month	484738.	64800.	2025.	17.851	1.4876

Note.—1 cu. ft. = 7.48052 U. S. liquid gallons = .03125 tons (at 62.5 lbs. per cu. ft) = .000 275 4821 acre-in. = .000 022 956 84 acre-ft.

The Cubic Foot per Second is a unit rate of flow, used mainly in gaging streams and canals. It is definite, whereas the "Inch" is more or less indefinite. Note that both the "Inch" and the Cu. Ft. per Sec. are *rates* of

* See Foot-note on following page.

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*flow, and that where definite *quantities* are demanded the element of time should be included, as 10 "inches" for one month, or 0.25 cu. ft. per sec. for one month, etc. (See Tables 1 and 2.) Contracts for water based on any fixed number of cu. ft. per sec. throughout the season are unfair to the user, or wasteful, or both. There are many and various recording instruments on the market for determining the rates of discharge. They are self-registering and consist usually of a fluctuating line drawn on profile paper wound around a cylinder, by a pen- or pencil point traveling longitudinally to represent the time, and by the cylinder oscillating as the water rises or falls in the measuring chamber, in which a float is connected with the instrument.

2.—EQUIVALENTS OF DISCHARGE OF 1 CUBIC FOOT PER SECOND.
(1 cu. ft. per sec. = about 40 "inches.")

Duration of Flow.	U. S. Gallons.	Cubic Feet.	Tons of 2000 Lbs.	Acre-Ins.	Acre-Ft.
1 Second.....	7.48	1	.031	.00028	.000023
1 Minute.....	448.83	60	1.875	.01653	.001377
1 Hour.....	26 929.9	3 600	112.5	.99173	.08264
1 24-Hour Day.....	646 317.	86 400	2 700.	23.801	1.98347
1 30-Day Month.....	19 389 508.	2 592 000	81 000.	714.05	59.504

Note.—1. cu. ft. = 7.48052 U. S. liquid gallons = .03125 tons (at 62.5 lbs. per cu. ft.) = .000 275 4821 acre-in. = .0000 229 5684 acre-ft.

The Units of Volume of water are the Acre-Foot and the Acre-Inch, the latter being only occasionally used, and $\frac{1}{12}$ of the former.

The Acre-Foot is the volume of water which will cover an acre 1 ft. deep = 43560 cu. ft. Mr. Elwood Mead says of this unit:

It is a convenient unit for selling stored water, since the capacity of reservoirs can be measured by the same unit. Contracts in which the acre-foot is used provide for the delivery of water on the demand of the irrigator, or at intervals rather than in continuous flow, and canal companies have hesitated about adopting this unit because of a fear that satisfactory arrangements for delivery could not be made, but that more water would be called for at some time than the canal could supply, while at other times the entire volume would run to waste. Wherever the acre-foot has been adopted it has proved acceptable to irrigators, because they share in the benefit resulting from care and skill in distribution.

Table 3 gives the rates of flow equivalent to an acre-ft. in a 24-hour day, and Table 4 gives the rates of flow equivalent to an acre-ft. in a 30-day month.

3.—RATES OF FLOW FOR DISCHARGE OF 1 ACRE-FOOT IN 1 DAY (24 HOURS).
(For 1 acre-inch, divide by 12.)

Flow—	U. S. Gallons.	Cubic Feet.	Tons of 2000 Lbs.	Acre-Ins.	Acre-Ft.
Per Second.....	3.77	.504	.0158	.000139	.0000116
" Minute.....	226.29	30.25	.9453	.008333	.0004944
" Hour.....	13577.14	1815.	56.7188	.5	.04167
" 24-Hour Day.....	325851.5	43560.	1361.25	12.	1.
" 30-Day Month....	9775544.	1306800.	40837.5	360.	30.

* Water sold by the inch by any individual or corporation shall be measured as follows, to wit: Every inch shall be considered equal to an inch-square orifice under a five-inch pressure, and a five-inch pressure shall be from the top of the orifice of the box put into the banks of the ditch to the surface of the water. Said boxes or any slot or aperture through which such water shall be measured shall in all cases be six inches perpendicular, inside measurement, except boxes delivering less than twelve inches, which may be square, with or without slides. All slides for the same shall move horizontally, and not otherwise, and said box put into the banks of ditch shall have a descending grade from the water in ditch of not less than one-eight of an inch to the foot. (General Statutes of Colorado, sec. 3473.)

4.—RATES OF FLOW FOR DISCHARGE OF 1 ACRE-FOOT IN 1 MONTH (30 DAYS).

(For 1 acre-inch, divide by 12.)

Flow—	U. S. Gallons.	Cubic Feet.	Tons of 2000 Lbs.	Acre-Ins.	Acre-Ft.
Per Second.....	.1257	.0168	.000525	.00000463	.000000386
" Minute.....	7.543	1.0083	.03151	.0002778	.00002315
" Hour.....	452.57	60.5	1.8906	.016667	.0013889
" 24-Hour Day...	10861.7	1452.	45.375	.4	.033333
" 30-Day Month. .	325851.5	43560.	1361.25	12.	1.

The Duty of Water is the amount of land which a unit volume of water, properly applied, can irrigate; or, conversely, it is *measured* by the amount of water required to irrigate one acre of land. This amount of water may be measured by volume, as in storage, or by (average) flow, as in distribution.

The (miner's) "Inch" is often cited by many irrigators as the amount required for one acre, and by others as the amount required for two, or even three, acres of land. By referring to Table 1, perceding, it can readily be seen that a flow of 1 "inch" is equal to about 1.5 acre-ft. per month. If flowing constantly this would amount to about 6 acre-ft. for an irrigation period of 4 months (say from May 15 to September 15), and about 7.5 acre-ft. for an irrigation period of 5 months (say from May 1 to October 1). For shorter periods it would be proportionately less. In the Rocky Mountain regions the irrigation season ranges from 100 to 150 days; in parts of Southern California it is used throughout the year; while in the colder climates, as in some parts of Montana, the period is often shortened to two months.

Assuming that a continuous (average) flow of one "inch," throughout the irrigation season, will irrigate 1¾ acres, then from Table 1 this is equivalent to 1 cu. ft. per sec. for $\left(1\frac{3}{4} \times \frac{1}{.025} =\right)$ 70 acres of land. If the length of irrigation season is 120 days, this would amount to $\left(\frac{59.504 \times 4}{70} =\right)$ 3.4 acre-ft. per acre. Similarly, the equivalents in cu. ft. per sec. and in acre-ft. can be found for any other assumed duty in acreage per "inch."

The Cu. Ft. per Sec. is another unit of flow in measuring the duty of water. Assuming 1 cu. ft. per sec. will irrigate 60 acres, we see by Table 2 that this is equivalent to 59.5 acre-ft. per month, or about 1 acre-ft. per acre per month, or 4 acre-ft. per acre for 4-months period of irrigation. Again, assuming 1 cu. ft. per sec. for 80 acres, we would require $\frac{59.504}{80}$ or about ¾ acre-ft. per acre per month, or 3 acre-ft. per acre for 4-months period of irrigation.

The Acre-Foot is now generally conceded to be the best duty unit by most experts in irrigation, but it must be admitted that the "Inch" and the Cu. Ft. per Sec. have some merits as supplementary factors. The acre-ft. is a definite quantity both in volume (43560 cu. ft.) and in depth of equivalent precipitation (12 ins.); and by the use of the preceding tables, it is easily converted into other units. It is well to remember that 1 cu. ft. per sec. = about 40 "inches" = about 2 acre-ft. per 24-hour day = about 60 acre-ft. per month. One great advantage of the acre-ft. is that it is convenient to apply in measuring the storage or source of water supply, as well as the direct quantity required in irrigation, the contents of reservoirs being stated usually in acre-ft. = total storage capcaity in cu. ft. divided by 43560.

Tables 5, 6, 7 and 8, following, are from Experiment Stations Bulletin 86, U. S. Dept. of Agriculture. The first three tables show the duty of water under varying conditions, while Table 8 shows the length of the irrigation seasons.

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5.—Duty of Water where Measurements Were Made on Small Canals or Laterals.

Location.	Acre-Ft.	Location.	Acre-Ft.
Cronquist farm, Utah	2.60	Biles Lateral, Colo	1.82*
Long farm, Idaho	2.40	Middle Cr. Ditch, Mont	2.10
Gage Canal, Cal	2.24	Daggett farm, Nebr	2.47
Canal No. 2, Wyo	2.53		
Vance farm, Ariz	2.82	Mean of all above	2.37

* Low duty due in part to scanty supply of water.

6.—Duty of Water where Measurements Were Made at Margin of Fields.

Location.	Acre-Ft.
J lateral, Wyoming, oats	1.55
J lateral, Wyoming, corn	.70
Farm, Edgar Wilson, Idaho	1.48
Lowest division, Gage Canal	1.78
Mean of measurements at Bozeman, Mont., Exp. Sta	1.20

7.—Duty of Water where Losses in Main Canal Are Included.

Name of Canal.	Acre-Ft.	Name of Canal.	Acre-Ft.
Pecos Canal, New Mexico	6.61	Upper Canal, Utah	6.30
Mesa Canal, Ariz	3.81	Amity Canal, Colo	4.92
Butler Ditch, Utah	6.24	Rust Lateral, Idaho	5.06
Brown and Sandford Ditch, Utah	5.32	Average	5.47

8.—Duration of Irrigation Period on Some Main Canals.

State.	Canal.	Days.	State.	Canal.	Days.
Cal	Gage Canal	365	Utah	Lower Canal	160
Ariz	Mesa Canal	365	Idaho	Boise & Nampa Canal	155
N. Mex	Pecos Canal	175	Colo	Amity Canal	140
Utah	Brown & Sanford Ditch	175	Utah	Logan & Richmond Canal	125
Utah	Butler Ditch	175	Nebr	Gothenburg Canal	120
Utah	Upper Canal	165	Mont	Middle Creek Ditch	95
Utah	Big Ditch	165	Wyo	Canal No. 2, Wheatland	60
Utah	Green Ditch	165			

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Table 9, following, is from King's "Irrigation and Drainage," the second column having been adopted by King from Wilson's "Manual of Irrigation Engineering," and includes all losses from the head of the canals:

9.—DUTY OF 1 CU. FT. PER SEC., AND NO. OF "INCHES" IN 10 DAYS, IN VARIOUS COUNTRIES.

Name of Country.	No. of Acres per Second-Foot.	No. of Inches per 10 Days.
Northern India	60 to 150	3.967 to 1.587
Italy	65 to 70	3.661 to 3.4
Colorado	80 to 120	2.975 to 1.983
Utah	60 to 120	3.967 to 1.983
Montana	80 to 100	2.975 to 2.38
Wyoming	70 to 90	3.4 to 2.644
Idaho	60 to 80	3.967 to 2.975
New Mexico	60 to 80	3.967 to 2.975
Southern Arizona	100 to 150	2.38 to 1.587
San Joaquin Valley	100 to 150	2.38 to 1.587
*Southern California	150 to 300	1.587 to 0.793

* The comparitively high duty of water in California is due to the extra care taken in construction of conduits to prevent losses, and to economy of distribution.

Canals.—Canals in earth are usually made wide and shallow with side slopes varying from ½ to 2 horizontal on 1 vertical. It is best to select such side slopes, depending on the earthy material, as will be maintained during flow. The cross-section of a canal is determined often by the allowable velocity which may not exceed, ordinarily, about 2¾ feet per second in ordinary soils. If the soil is light 2 ft. is about the maximum, while if hard and gravelly 3 or 4 feet may be exceeded. The velocity of flow in a canal is dependent on the hydraulic radius, the grade or slope, and the roughness n of wetted surfaces (see Kutter's formula, page 1167). Canals with rubble-, concrete- or cement lining are often used where water is scarce and valuable.

In estimating the flow by Kutter's formula, use $n = .0225$ for ordinary earthen canal with clean bed, and $n = .035$ for bed in bad order having stones and weeds in great quantities. (See page 1168.)

Wilson, in his *Manual of Irrigation Engineering*, gives the following data relative to some great perennial canals:

10.—DATA ON SOME GREAT PERENNIAL CANALS.
(See Wilson's *Manual of Irrigation* for full and extended data.)

Name of Canal.	Locality	L'ngth Miles.	Capacity Sec.-Ft.	Grade.	Bed-Width Feet.	Depth Feet.
Bear River Canal	Utah	150	1000	1 in 5280	50	7
Idaho Min. & Irrig. Co. Canal	Idaho	70	2585	1 " 2640	40	10
Pecos Canal	N. Mex.	75	1100	1 " 6707	45	6
Turlock Canal	Cal.	93	1500	1 " 5280	70	7.5
King's R. & San Joaquin Canal	"	67	600	1 " 5280	32	4.5
Calloway Canal	"	32	700	1 " 6600	80	3.5
Arizona Canal	Ariz.	41	1000	1 " 2640	36	7.52
Highline Canal	Colo.	70	1184	1 " 3000	40	7
Del Monte Canal	"	50	2400	1 " 2112	65	5.5

Conduits and Flumes frequently take the place of canals even in localities other than at crossing of streams. The flumes are usually constructed of wood (sometimes of steel), generally rectangular in cross-section, and supported on trestle work of the same material as the flume. The V-shaped flume is seldom used in connection with large irrigation projects, unless combined with logging. The semi-circular flume, of wood-stave pipe construction, or of steel, has the merit of giving the maximum velocity of

flow for a given area of wetted cross-section. Pipe lines of wooden staves or of steel are frequently used as inverted syphons for conveying water across valleys where cost of flumes on high trestles would be very expensive. Wooden flumes may be made tight (1) by tangential compressive forces as with adjustable rods or wedges; (2) by caulking and tarring the joints; (3) by using two thicknesses of siding and bottom plank with tarred paper between. Steel flumes are caulked similarly to riveted steel pipe.

EXCERPTS AND REFERENCES.

Noteworthy Water Storage and Irrigation Works of Southern California (By R Fletcher. Eng. News, Aug. 22, 1901).—Descriptions of the following: System of the San Diego Land and Town Co.; Southern California Mountain Water Co.; The Morena Dam and Reservoir; The Barrett Dam and Reservoir; The Lower Otay Dam and Reservoir; The Upper Otay Dam and Reservoir.

Irrigation System of the Arkansas Valley Sugar Beet & Irrigated Land Co., Colo. (By W. P. Hardesty. Eng. News, Nov. 13, 1902).—Descriptions of Fort Lyon Canal System (canal, dam, head-gates, regulating gates, stream crossings, division gates); Kicking Bird Canal; Santava Canal; Lone Wolf Canal (reservoir system); Comanche Canal; Pawnee Canal; Amity Canal System (canal, dam, head-gate, secondary head-gate); Buffalo Canal. Ten illustrations of structures.

Establishing Irrigation Canal Tangents So Cut and Fill Will Balance (Eng. News Aug. 10, Sept. 21, 1896).—Illustrated.

A Diagram to Aid the Location of Small Irrigation Canals (By P. McGeehan. Eng. News, Feb. 1, 1906).—Also table for laying grade line on small irrigation canals.

An Underflow Canal Used for Irrigation at Ogalalla, Nebr. (By S. C. Slichter Eng. News, July 5, 1906).—Map. Table of flow.

The Truckee-Carson Project of the U. S. Reclamation Service (By W. P. Hardesty. Eng. News, Oct. 18, 1906).—Illustrations: Details of diversion dam and head-gates; lining in rock cut; junction of earth and concrete-lined tunnel; method of lining tunnel; details of waterway, with Taintor gates and mechanism for operation; a simple form of fall or drop in canal; details of headworks of distributing system; details of combined fall and waterway. Tables of cost data.

Effect of Changes in Canal Grades on Rate of Flow (By F. W. Hanna. Eng. News, Nov. 21, 1907).

Lining Ditches and Reservoirs to Prevent Seepage Losses (Eng. News. Dec. 5, 1907).—Illustrated methods of lining.

Cost of Canal Work on the Lower Yellowstone Project (Eng. News, July 16, 1908).

Depth of Water in Irrigation Canals (By C. E. Grunsky. Eng. News, Sept. 10, 1908).—Table 1: Effect of water depth upon required amount of excavation for irrigation canals. Table 2: Required fall for canals of varying capacity.

Cost Data of the Cold Springs Reservoir Construction, Oregon, U. S. Recl. Serv. (Eng. News, Nov. 12, 1908).

Earthwork Diagrams for Estimating Quantities in Small Irrigation Canals in Level Sections (Eng. News, June 10, 1909).

Cost of a Large Irrigation Canal (Eng. Rec., Jan. 2, 1909).—Table of unit costs on about 4,400 ft. of the south canal of the Uncompahgre Project of the U. S. Recl. Serv. The canal has a bottom width of 40 ft., side slopes of 2 to 1, and a depth of water of 8.3 ft.

Field Work in Locating Irrigation Ditch and Canal Lines (By A. B. Bartlett. Eng. News, May 26 1910).—**Small Ditches:** Ditches designed to irrigate 100 to 200 acres may be laid out by setting stakes on the lower side of the ditch, determined by the level only. Ditches 5 to 10 ft. on the bottom

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and 1 to 3 ft. deep require more care: the slope stakes on the lower side of the ditch are set first for about 1,500 ft., after which they are lined up by eye to tangents and curves nearly conforming to the contour which the slope stake on the lower side of the ditch would naturally follow, but improving on this alinement by increasing or decreasing the cut in order to have good curves and tangents, etc. **Ditches over 10 to 12 ft.** on the bottom generally require the methods of railway location and cross-sectioning. **Side Slopes.** Lower side of ditch, from 1 on 1½ to 1 on 3 in excavation, and from 1 on 2 to 1 on 4 for fill on lower bank. On side-hill work, cut on upper side of ditch is made from 1 on 1 to 1 on 2, for earth. **Greatest velocities**, in feet per second, permissible in different classes of material are as follows: Sandy soil, 1.3; loose gravelly soil, 2.5; firm soil and firm sandy loam, 3.0; gravel, 3.6; firm gravelly soil, 5.3; loose rock and shale, 6.0 to 7.0; solid rock, 7.0 to 15.0. **Kutter's formula:** Use $n = 0.03$ or 0.025 for clean ditches in ordinary earth.

Drainage of Irrigated Lands with Special Reference to Experiments in Utah (By C. F. Brown. Farmers' Bulletin No. 371, U. S. Dept. or Agric., Oct. 2, 1909; Eng. News, Oct. 13, 1910).—Illustrations: (1) Method of grading drainage trench; (2) Plan of drainage; (3) Box drains with and without bottoms; (4) Relief well and method of laying box drain through soft ground. The tract drained consisted of 31.5 acres. **Cost.**—Tile (1000′ of 10″ @ $0.18½ = $182.50, 1012′ of 8″ @ $0.12 = $121.14, 330′ of 6″ @ $0.06 = $19.80, 650′ of 5″ @ $0.058 = $37.70, 2 6″ on 8″ wyes @ $0.72 = $1.44) $362.88; Hauling (30 tons 1½ miles @ $0.30 per ton-mile) $13.50; Digging (134 rods @ $0.52 = $69.68, and 42 rods @ $0.40 = $16.80) $86.48; Laying tile, $18.75; Filling trench, $10.00. Total cost, $491.61 = $15.60 per acre. Labor, $2 per 10 hours; tile layers, $2.50; man with team, $3 per day.

Illustrations of Irrigation Structures:—

Description.	Eng. News.
Automatic drop shutters for irrigation dam, India	June 4, 1903.
Details of masonry and steel head-gate for irrigation canal	Aug. 13, '03.
Reinforced-concrete siphon on an irrigation canal, Spain	Aug. 1, '07.
Pumping plant of the Huntley irrigation project	Sept. 3, '08.
	Eng. Rec.
Construction of "cut-and-cover" canal, Mojave desert	Apr. 3, '09.
Economic sections for earth canals	July 31, '09.

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67.—WATERWAYS.*

The Suez Canal, connecting the Mediterranean and Red Seas, was begun in 1859 and completed in 1869. Its total length is 90 miles, of which about two-thirds is through shallow lakes. The material excavated was usually sand, though in some cases strata of solid rock from 2 to 3 feet in thickness were encountered. The total excavation was about 80,000,000 cubic yards under the original plan, which gave a depth of 25 feet. In 1895 the canal was so enlarged as to give a depth of 31 feet, a width at bottom of 108 feet and at the surface of 420 feet. The original cost was $95,000,000, and for the canal in its present form slightly in excess of $100,000,000. The canal is without locks, being at the sea level the entire distance. The length of time occupied in passing through the canal averages about 18 hours. By the use of electric lights throughout the entire length of the canal, passages are made at night with nearly equal facility to that of the day. The tolls charged are 8.50 francs per ton net register, "Danube measurement," which amounts to about $2 per ton United States measurement. Steam vessels passing through the canal are propelled by their own power. Since Jan. 1, 1902, the minimum draft of water has been raised to 26′ 3″ (8 meters).

Table 1, next page, shows the number and tonnage of vessels which passed through the Suez Canal, together with the transit receipts for same, and also the number of passengers carried, from its opening until 1903.

The Cronstadt and St. Petersburg Canal, connecting the bay of Cronstadt with St. Petersburg, is a work of great strategic and commercial importance to Russia. The canal and sailing course in the bay of Cronstadt are about 16 miles long, the canal proper being about 6 miles and the bay channel about 10 miles, and they together extend from Cronstadt, on the Gulf of Finland, to St. Pertersburg. The canal was opened in 1890 with a navigable depth of 20½ feet, the original depth having been about 9 feet. The width ranges from 220 to 350 feet. The total cost is estimated at about $10,000,000. There are no locks.

The Corinth Canal, connecting the Gulf of Corinth with the Gulf of Ægina, reduces the distance from Adriatic ports about 175 miles and from Mediterranean ports about 100 miles. Its length is about 4 miles, a part of which was cut through granitic soft rock and the remainder through soil. There are no locks, as is also the case in both the Suez and Cronstadt canals, described above. The width of the canal is 72 feet at bottom and the depth 26½ feet. The work was begun in 1884 and completed in 1893, at a cost of about $5,000,000. The average tolls are 18 cents per ton and 20 cents per passenger.

The Manchester Ship Canal, connecting Manchester, England, with the Mersey River, Liverpool, and the Atlantic Ocean, was opened for traffic January 1, 1894. The length of the canal is 35½ miles, the total rise from the water level to Manchester being 60 feet, which is divided between four sets of locks, giving an average to each of 15 feet. The minimum width is 120 feet at the bottom and averages 175 feet at the water level, though in places the width is extended to 230 feet. The minimum depth is 26 feet, and the time required for navigating the canal from 5 to 8 hours. The total amount of excavation in the canal and docks was about 45,000,000 cubic yards, of which about one-fourth was sandstone rock. The lock gates are operated by hydraulic power. Railways and bridges crossing the route of the canal have been raised to give a height of 75 feet to vessels traversing the canal, and an ordinary canal whose route it crosses is carried across by a springing aqueduct composed of an iron caisson resting upon a pivot pier. The total cost of the canal is given at $75,000,000.

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* Much of the information relating to large ship canals is from *Great Canals of the World*, 1905, Department of Commerce and Labor, Washington, D. C. Those who are particularly interested in this subject should consult the "List of Works Relating to Deep Waterways from the Great Lakes to the Atlantic Ocean with some other related works," published by Supt. of Documents, Washington, D. C.

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Years.	Vessels, Tonnage and Receipts.							Passengers.					
	Vessels. *Number.*	Gross Tonnage. *Tons.*	Net Tonnage. *Tons.*	Transit Receipts. *Fr.*	*c.*	Mean Net Tonnage per Vessel. *Tons.*	Military.	Civilians.	Pilgrims, emigrants, and convicts.	Total Passengers on Vessels.	Total Passengers on Boats.	Total.
1869...	10	10,557.61	6,576.00	54,460	80	657	.....	.....	.....	.....	.....	.....
1870...	486	654,915.02	436,609.37	5,159,327	22	898	.....	.....	.....	26,758	.....	.....
1871...	765	1,142,200.46	761,467.05	8,993,733	87	995	.....	.....	.....	48,422	.....	.....
1872...	1,082	1,744,481.33	1,160,743.54	16,407,591	42	1,071	.....	.....	.....	67,640	.....	.....
1873...	1,178	2,085,072.61	1,367,767.82	22,897,319	18	1,166	.....	.....	.....	68,030	.....	.....
1874...	1,264	2,423,672.22	1,631,650.14	24,859,383	00	1,290	.....	.....	.....	73,597	.....	.....
1875...	1,494	2,940,708.45	2,009,984.09	28,886,302	27	1,345	.....	.....	.....	84,446	.....	.....
1876...	1,457	3,072,107.01	2,096,771.61	29,974,998	74	1,439	a30,420	a20,832	a8,362	59,614	.....	.....
1877...	1,663	3,418,949.72	2,355,447.69	32,774,344	23	1,416	39,463	24,409	8,949	72,821	.....	.....
1878...	1,593	3,291,535.38	2,269,678.31	31,098,229	18	1,425	58,274	26,170	11,919	96,363	.....	.....
1879...	1,477	3,236,942.32	2,263,332.19	29,686,060	81	1,532	43,775	26,697	12,672	82,144	.....	.....
1880...	2,026	4,344,519.89	3,057,421.88	39,840,487	64	1,509	49,493	29,139	20,268	98,900	.....	.....
1881...	2,727	5,794,491.19	4,136,779.77	51,274,352	95	1,517	43,163	35,604	8,039	86,806	.....	.....
1882...	3,198	7,122,125.68	5,074,808.88	60,545,882	08	1,586	62,898	38,288	20,686	121,872	.....	.....
1883...	3,307	8,051,307.30	5,775,861.79	65,847,812	16	1,746	67,919	36,149	31,529	b115,597	3,579	119,176
1884...	3,284	8,319,967.36	5,871,500.92	62,378,115	54	1,787	74,318	39,987	34,993	148,298	3,618	151,916
1885...	3,624	8,985,411.80	6,335,752.98	62,207,439	21	1,748	112,230	47,068	42,473	201,771	4,178	205,949
1886...	3,100	8,183,313.15	5,767,655.84	56,527,390	58	1,860	84,593	55,320	27,709	167,622	3,788	171,410
1887...	3,137	8,430,043.20	5,903,024.09	57,862,370	71	1,881	91,966	53,414	33,405	178,785	4,211	182,996
1888...	3,440	9,437,957.32	6,640,834.44	64,832,273	20	1,930	87,131	63,103	29,231	179,465	4,430	183,895
1889...	3,425	9,605,745.48	6,783,187.12	66,167,579	14	1,951	75,445	72,192	27,868	175,505	5,087	180,592
1890...	3,389	9,749,129.09	6,890,094.41	66,984,000	22	2,033	67,767	69,479	18,430	155,676	5,676	161,352
1891...	4,207	12,217,986.41	8,698,777.36	83,422,101	24	2,067	93,780	73,562	20,784	188,126	6,347	194,473
1892...	3,559	10,866,401.46	7,712,028.61	74,452,436	03	2,166	91,743	70,239	21,929	183,912	5,907	189,820
1893...	3,341	10,753,798.15	7,559,068.10	70,667,361	43	2,292	71,880	67,262	41,290	180,432	6,066	186,498
1894...	3,352	11,283,854.72	8,039,175.27	73,776,827	68	2,398	73,428	71,996	20,562	165,986	17	166,003
1895...	3,434	11,833,637.33	8,448,383.01	78,103,717	56	2,460	118,635	74,878	23,423	216,936	4	216,940
1896...	3,409	12,039,858.79	8,560,283.60	79,569,994	31	2,511	198,520	78,652	31,055	308,227	13	308,241
1897...	2,986	11,123,403.27	7,899,373.84	72,830,545	43	2,645	92,639	83,833	14,743	191,215	9	191,224
1898...	3,503	12,962,631.81	9,238,603.38	85,294,769	88	2,637	122,052	79,836	17,783	219,671	58	219,729
1899...	3,607	13,815,991.72	9,895,630.04	91,318,772	07	2,743	108,552	88,616	24,179	221,347	1	221,348
1900...	3,441	13,699,237.80	9,738,152.18	90,623,608	08	2,830	154,249	102,415	25,530	282,194	9	282,203
1901...	3,699	15,163,233.00	10,823,840.00	100,386,397	00	2,926	136,514	92,046	41,661	270,221	.....	270,221
1902...	3,708	15,694,359.00	11,248,413.00	103,720,020	00	3,034	84,946	98,213	40,499	233,658	117	223,775
1903...	3,761	16,615,309.00	11,907,288.00	103,620,268	00	3,166	80,583	89,272	25,362	195,217	15	195,232

Footnotes:—*a* The distinction has only been made since 1876.
b Until 1883 the passengers on boats were included in the total of passengers on vessels.

The Kaiser Wilhelm Canal. Two canals connect the Baltic and North Seas through Germany; the first, known as the Kaiser Wilhelm Canal, having been begun in 1887 and completed in 1895 and constructed largely for military and naval purposes, but proving also of great value to general mercantile traffic. The length of the canal is 61 miles, the terminus in the Baltic Sea being at the harbor of Kiel. The depth is 29½ feet, the width at the bottom 72 feet, and the minimum width at the surface 190 feet. The route lies chiefly through marshes and shallow lakes and among river valleys. The total excavation amounted to about 100,000,000 cubic yards, and the cost to about $40,000,000.

The Elbe and Trave Canal, the second canal connecting the Baltic and North Seas through Germany, is smaller than the Kaiser Wilhelm Canal. It was opened in 1900, with a length of about 41 miles and a depth of about 10 feet. It is described in the International Yearbook, 1900, as follows:

"The Elbe and Trave Canal, in Germany, was opened by the Emperor of Germany on June 16, 1900. It has been under construction for five years, and has cost about $5,831,000, of which Prussia contributed $1,785,000 and the old Hanse town of Lubek $4,046,000. The length of the new canal is about 41 miles, and is the second to join the North Sea and the Baltic, following the Kaiser Wilhelm Canal (or Kiel Canal), built about five years ago at a cost of $37,128,000. The breadth of the new canal is 72 feet; breadth of the locks, 46 feet; length of locks, 261 feet; depth of locks, 8 feet 2 inches. It is crossed by 29 bridges, erected at a cost of $1,000,000. There are seven locks, five being between Lubek and the Mollner See (the summit point of the canal) and two between Mollner See and Fauenberg-on-the-Elbe. At this point it may be noted that the Germans began experiments during 1900 with electric towing on the Finow Canal between Berlin and Stettin. A track of 1-meter gauge was laid along the bank of the canal, having one 9-pound and one 18-pound rail laid partly on cross ties and partly on concrete blocks. The larger rail serves for the return current, and has bolted to it a rack which gears with a spur wheel on the locomotive. The locomotive is 6 feet 10 inches by 4 feet 10 inches, mounted on four wheels, with a wheel base of 3 feet 6 inches, and weighing 2 tons. It is fitted with a 12-horsepower motor, current for which is furnished by a 9-kilowatt dynamo, driven by a 15-horsepower engine. The current is 500 volts, and is transmitted by a wire carried on wooden poles 23 feet high and about 120 feet apart. The boats are about 132 feet long and 15 feet 6 inches beam, and carry from 150 to 175 tons on a draft of 4 feet 9 inches. During 1900 the Stettin-Swinemund Canal, with a length of 35 miles, has been dredged throughout, and is now open to steamers drawing 22 feet of water. Swinemund is on the Baltic Sea. Among the various projects for European canals may be mentioned one connecting the Danube a little below Vienna, Austria, with the Adriatic Sea at Trieste, a distance of about 319 miles. Herr Wagentahrer, of Vienna, is said to have the concession for this canal, the construction of which will cost some $120,000,000. Late in 1900 a canal from Liege to Antwerp, in Belgium, was being seriously discussed, in order to connect the prosperous city of Liege with the sea, and make it, like the city of Manchester, England, a seaport. The original promoter of the scheme was Mr. Joseph Redonti, who is now dead. Mr. Redonti's plans have recently been put in practical shape by Louis Hubin and Gaston Delville, who propose a canal 84 miles long, 200 feet wide, and 23 feet deep from Antwerp to Liege, with locks at Liege, Hasselt, Herenthals, and Antwerp. The difference in level to be overcome by locks would be 175 feet, and it is thought that thirteen single locks and one double lock would be sufficient. The total estimated cost of the work is $25,200,000."

The Welland Canal connects Lake Ontario and Lake Erie on the Canadian side of the river. It was constructed in 1833 and enlarged in 1871 and again in 1900. The length of the canal is 26¾ miles, the number of locks 25, the size of locks 270 by 45 feet, the total rise of lockage 326¾ feet, and the total cost about $25,000,000. The annual collection of tolls on freight, passengers, and vessels averaged about $225,000 and the canal is open on an average about 240 days in a year. By order in council dated April 27, 1903, the levy of tolls for passage through Dominion canals has been abolished for a period of two seasons of navigation.

The Sault Ste. Marie Canals, at Sault Ste. Marie, Mich., and Ontario are located adjacent to the falls of the St. Marys River, which connects Lake Superior with Lake Huron, and lower and raise vessels from one level to the other, a height of 17 to 20 feet.

The canal belonging to the United States was begun in 1853 by the State of Michigan and opened in 1855, the length of the canal being 5,674 feet, and provided with two tandem locks, each being 350 feet in length and 70 feet wide, and allowing passage of vessels drawing 12 feet, the original cost being $1,000,000. The United States Government, by consent of the State, began in 1870 to enlarge the canal, and by 1881 had increased

its length to 1.6 miles, its width to an average of 160 feet, and its depth to 16 feet; also had built a single lock 515 feet long and 80 feet wide, with a depth of 17 feet on the sills, which was located 100 feet south of the State locks. The State relinquished all control of the canal in March, 1882. In 1887 the State locks were torn down and replaced by a single lock 800 feet long, 100 feet wide, and a depth of 22 feet of water on the sills. This lock was put in commission in 1896. The canal was also deepened to 25 feet.

The canal on the Canadian side, on the north side of the river, is 1 miles long, 150 feet wide, and 22 feet deep, with lock 900 feet long, 60 feet wide, with 22 feet on the miter sills, and was built during the years 1888 to 1895.

Canadian Canal Systems.—The canal systems of the Dominion, under Government control, in connection with lakes and navigable rivers, are as follows:

First. The through route between Montreal and the head of Lake Superior, 14 feet navigation:

Name of Waterway.	Canal.	River.	Remarks.
	Miles.	*Miles.*	
1. Lachine Canal	8½	.....	
River St. Lawrence	.....	16	
2. Soulanges Canal	14	.....	
River St. Lawrence	.....	32¾	
3. Cornwall Canal	11	.....	
River St. Lawrence	.....	5	The Farrans Point, Rapide Plat, and Galops canals are collectively known as the Williamsburg canals.
4. Farrans Point Canal	1	.....	
River St. Lawrence	.....	10½	
5. Rapide Plat Canal	3⅝	.....	
River St. Lawrence	.....	4½	
6. Galops Canal	7½	.....	
River St. Lawrence and Lake Ontario	.....	236¾	
7. Welland Canal	26¾	.....	
Lake Erie, Detroit River, Lake St. Clair, and St. Marys River	.....	594	
8. Sault Ste. Marie Canal	1⅛	.....	
Lake Superior to Port Arthur	.....	266	
Lake Superior to Duluth, 390.			
Total	73⅜	1,165½	

Second. Ottawa to Lake Champlain: Greenville, Carillon, St. Annes, Chambly, St. Ours Canals.

Third. Ottawa to Kingston (and Perth): Rideau River and Canal. (Connects with Perth by Tay Canal.)

Fourth. Lake Ontario at Trenton to Lake Huron at mouth of river Severn: Trent Canal (not completed).

Fifth. Ocean to the Bras d'Or Lakes: St. Peters Canal.

The Lake Borgue, Louisiana, Canal was formerly opened in August, 1901. It opens continuous water communication with lakes Maurepas, Ponchartrain, and Borgue, the Mississippi Sound, Mobile, and the Alabama and Warrior Rivers, and the entire Mississippi River system, and has an important bearing as a regulator of freight rates between these sections. The effects of the canals may be briefly summed up as: Shortening the distance between New Orleans and the Gulf points east of the Mississippi; bringing shipments from the Gulf coast direct to the levees at New Orleans; saving the transshipment of through freights, with a consequent reduction in freight rates; enabling sea-going vessels, drawing 10 to 12 feet of water, to come within 20 miles of New Orleans, saving all such craft the cost of towage and shortening, by 60 miles, direct water communication between New Orleans and the deep water of the Gulf. In addition to these effects may be enumerated the cheapening of coal for consumption at New Orleans. Coal has hitherto been floated down the rivers from Pittsburg, a distance of 2100 miles. The canal opens up the coal fields in the interior of Alabama for New Orleans consumption and reduces coal prices considerably, giving

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an additional advantage to domestic industries and to steamers purchasing bunker coal. The canal is 7 miles long and from 150 to 200 feet wide. Bayou Dupree forms a portion of the canal. The lock chamber is 200 feet long, 50 feet wide and 25 feet deep, and connects the canal with the Mississippi River.

The Chicago Sanitary and Ship Canal connects Lake Michigan at Chicago with the Illinois River at Lockport, a distance of 34 miles. The canal was cut for the purpose of giving to the city of Chicago proper drainage facilities by reversing the movement of water, which formerly flowed into Lake Michigan through the Chicago River, and turning a current from Lake Michigan through the Chicago River to the Illinois River at Lockport, and thence down the Illinois River to the Mississippi. The minimum depth of the canal is 22 feet, its width at bottom 160 feet, and width at the top from 162 to 290 feet, according to the class of material through which it is cut. The work was begun September 3, 1892, and completed and the water turned into the channel January 2, 1900. The flow of water from Lake Michigan toward the gulf is now at the rate of 360,000 cubic feet per minute, and the channel is estimated to be capable of carrying nearly twice that amount. The total excavation in its construction included 28,500,000 cubic yards of glacial drift and 12,910,000 cubic yards of solid rock, an aggregate of 41,410,000 cubic yards. In addition to this the construction of a new channel for the Desplaines River became necessary in order to permit the canal to follow the bed of that river, and the material excavated in that work amounted to 2,068,659 cubic yards, making a grand total displacement in the work of 43,478,659 cubic yards of material which, according to a statement issued by the trustees of the sanitary district of Chicago, would, if deposited in Lake Michigan in 40 feet of water, form an island 1 mile square with its surface 12 feet above the water line.

All bridges along the canal are movable structures. The total cost of construction, including interest account, aggregated $34,000,000, of which $21,379,675 was for excavation and about $3,000,000 for rights of way and $4,000,000 for building railroad and highway bridges over the canal. The city and State authorities by whom the canal was constructed are now proposing to Congress to make this canal a commercial highway in case Congress will increase the depth of the Illinois and Mississippi Rivers to a depth of 14 feet, with locks for fleets of barges from Lockport, the terminus of the drainage canal, to St. Louis. This, it is argued, would give through water transportation from Lake Michigan to the Gulf by way of the drainage canal, the Illinois River, and the Mississippi River, and would enable the United States in case of war to quickly transport light-draft war vessels from the Gulf to the lakes. This work of deepening the Illinois River would also give through water connection from Rock Island, on the Upper Mississippi River, to Lake Michigan via the Illinois and Mississippi Canal, which extends from Rock Island, on the Mississippi River, to Hennepin, on the Illinois River. The estimate of the Chicago sanitary district trustees of the cost of deepening the Illinois and Mississippi Rivers from the terminus of the ship canal to St. Louis to a depth of 14 feet is $25,000,000, including five locks and dams.

Proposed American Isthmian Canal.—The construction of a waterway to connect the Atlantic and Pacific across the American isthmus has been a subject of consideration for nearly 400 years. Vasco Nunez de Balboa, governor of a province in Darien, in 1513 crossed the isthmus and discovered the Pacific, and from that time forward efforts were almost constantly made to discover a water connection between the oceans at this point, which it was hoped nature had supplied. When this hope was abandoned the construction of an artificial water route was immediately proposed, and Charles V. is said to have directed, as early as 1520, that the Isthmus of Panama be surveyed with the purpose of selecting a route for the construction of an artificial waterway to connect the two oceans. Another decree was issued in 1534, authorizing a careful examination by experienced men for this purpose; but the governor having reported that such work was impracticable, and that no king, however powerful, was capable of forming a junction of the two seas, and the suggestion having been made that the opening of a canal through the isthmus would be "in opposition to the will of the Almighty, who had placed this barrier in the way of navigation between the two oceans," the project was temporarily abandoned. Further examinations were made in 1771 in the hope of finding a continuous water-

way, as statements had been made that a river had been found flowing from ocean to ocean, but an examination proved the inaccuracy of this statement. In 1779 a survey was made to determine the practicability of connecting the oceans by way of Lake Nicaragua, but the report was not encouraging. From that date forward, however, numerous examinations and surveys of the isthmus were made at various points, the latest being that of the American Commission appointed in 1899, and which has recently presented its full report to Congress. That report concludes as follows:

"The investigations of this Commission have shown that the selection of 'the most feasible and practicable route' for an isthmian canal must be made between the Nicaragua and Panama locations. Furthermore, the complete problem involves both the sea-level plan of canal and that with locks. The Panama route alone is feasible for a sea-level canal, although both are entirely practicable and feasible for a canal with locks. The time required to complete a sea-level canal on the Panama route, probably more than twice that needed to build a canal with locks, excludes it from favorable consideration, aside from other serious features of its construction. It is the conclusion of this Commission, therefore, that a plan of canal with locks should be adopted.

"A comparison of the principal physical features, both natural and artificial, of the two routes reveals some points of similarity. Both routes cross the continental divide less than ten miles from the Pacific Ocean, the Panama summit being about double the height of that in Nicaragua. For more than half its length the location of each route on the Atlantic side is governed by the course of a river, the flow from whose drainage basin is the only source of water supply for the proposed canal; and the summit levels, differing but about 20 feet in elevation, Panama being the lower, are formed by lakes, natural in the one case and artificial in the other, requiring costly dams and wasteways for their regulation and for the impounding of surplus waters to reduce the effect of floods and to meet operating demands during low-water seasons.

"The investigations made in connection with the regulation of Lake Nicaragua have demonstrated that that lake affords an inexhaustible water supply for the canal by that route. The initial proposition, on the other hand, for the Panama route is to form Lake Bohio so as to yield a water supply for a traffic of 10,000,000 tons, which can be supplemented when needed by an amount sufficient for more than four times that traffic, by means of the Alhajuela reservoir. For all practical purposes this may be considered an unlimited supply for the Panama route. So far as the practical operation of a ship canal is concrened, therefore, the water supply features on both lines are satisfactory.

"The difficulties disclosed and likely to be encountered in the construction of the dams are less at Conchuda on the Nicaragua line than at Bohio on the Panama route. Both dams, however, are practicable, but the cost of that at Bohio is one-half more than at Conchuda. A less expensive dam at Bohio has been proposed, but through a portion of its length it would be underlaid by a deposit of sand and gravel pervious to water. The seepage might not prove dangerous, but the security of the canal is directly dependent upon this dam, and the policy of this Commission has been to select the more perfect structure even at a somewhat greater cost. The wasteways at both locations present no serious difficulties. The advantages in the design and construction of the dams are in favor of the Nicaragua route.

"The system of regulation at Lake Bohio consists only of the discharge of water over the crest of a weir, as the lake level rises under the influence of floods in the Chagres River. The plan of regulating the level of Lake Nicaragua is less simple, though perfectly practicable. It involves the operation of movable gates at such times and to such extent as the rainfall on the lake basin may require. The experience and judgment of the operator are essential elements in the effective regulation of this lake. The regulation of Lake Bohio is automatic.

"The only means of transportation now found on the Nicaragua route are the narrow-gauge Silico Lake Railroad, about 6 miles in length, and the limited navigation of the San Juan River and the lake, but the Nicaraguan Government is now building a railroad along the beach from Greytown to Monkey Point, about 45 miles to the northward, where it proposes to establish a commercial port. By means of a pier, in the area protected by the point, goods and material for canal purposes can readily be landed and transported by rail to Greytown. Such piers are in constant use on our Pacific coast. This railroad and port would be of great value during the

period of preparation and harbor construction, and should materially shorten that period. A well-equipped railroad is in operation along the entire length of the Panama route, and existing conditions there afford immediate accommodation for a large force of laborers.

"The Nicaragua route has no natural harbor at either end. At both the Atlantic and Pacific termini, however, satisfactory harbors may be created by the removal of material at low-unit prices, and by the construction of protective works of well-established design. An excellent roadstead, protected by islands, already exist at Panama, and no work need be done there for either harbor construction or maintenance. At Colon, the Atlantic terminus of the Panama route, a serviceable harbor already exists. It has afforded harbor accommodations for many years, but it is open to northers, which a few times in each year are liable to damage ships or force them to put to sea. Considerable work must be done there to create a suitable harbor at the entrace of the canal, which can easily be entered, and will give complete protection to shipping lying within it. The completion of the harbors as planned for both routes would yield but little advantage to either, but the balance of advantages, including those of maintenance and operation, is probably in favor of the Panama route.

"The existence of a harbor at each terminus of the Panama route, and a line of railroad across the Isthmus, will make it practicable to commence work there, after the concessions are acquired, as soon as the necessary plant can be collected and put in place, and the working force organized. This period of preparation is estimated at one year. In Nicaragua this period is estimated at two years, so as to include also the construction of working harbors and terminal and railroad facilities.

"The work of excavation on the Nicaragua route is distributed; it is heaviest near Conchuda, at Tamborcito, and in the divide west of the lake. On the Panama route it is largely concentrated in the Culebra and Emperador cuts, which are practically one. As a rule distributed work affords a greater number of available points of attack, contributing to a quicker completion; but in either of these cases such difficulties as may exist can be successfully met with suitable organization and efficient appliances.

"The time required for constructing the Nicaragua Canal will depend largely on the promptness with which the requisite force of laborers can be brought to Nicaragua, housed and organized at the locations of heaviest work along the route. The cut through the divide west of the lake will probably require the longest time of any single feature of construction. It contains about 18,000,000 cubic yards of earth and rock excavation, or a little less than 10 per cent of the total material of all classes to be removed. With adequate force and plant this Commission estimates that it can be completed in four years. This indicates, under reasonable allowance for ordinary delays, that if force and plant enough were available to secure a practically concurrent execution of all portions of work on the route, the completion of the entire work might be expected within six years after its beginning, exclusive of the two years estimated for the period of preparation.

"The securing and organizing of the great force of laborers needed, largely foreigners, so as to adjust the execution of the various portions of the work to such a definite programme of close-fitting parts in a practically unpopulated tropical country, involves unusual difficulties and would prolong the time required for completion.

"The greatest single feature of work on the Panama route is the excavation in the Culebra section, amounting to about 43,000,000 cubic yards of hard clay, much of which is classed as soft rock, or nearly 45 per cent of all classes of material to be removed. It is estimated that this cut can be completed in eight years, with allowance for ordinary delays, but exclusive of a two-year period for preparation and for unforseen delays, and that the remainder of the work can be finished within the same period. The great concentration of work on this route and its less amount will not require so great a force of laborers as on the Nicaragua route; hence the difficulties and delays involved in securing them will be correspondingly diminished.

"The total length of the Nicaragua route from sea to sea is 183.66 miles, while the total length of the Panama route is 49.09 miles. The length of standard canal section and in harbors and entrances is 73.78 miles for the Nicaragua route and 36.41 miles for the Panama route. The length of sailing line in Lake Nicaragua is 70.51 miles, while that in Lake Bohio is 12.68 miles. That portion of the Nicaragua route in the canalized San Juan is 39.37 miles.

"The preceding physical features of the two lines measure the magnitude

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of the work to be done in the construction of waterways along the two routes. The estimated cost of constructing the canal on the Nicaragua route is $45,630,704 more than that of completing the Panama Canal, omitting the cost of acquiring the latter property. This sum measures the difference in the magnitude of the obstacles to be overcome in the actual construction of the two canals and covers all physical considerations, such as the greater or less height of dams, the greater or less depth of cuts, the presence of absence of natural harbors, the presence or absence of a railroad, and the amount of work remaining to be done.

"The estimated annual cost of maintaining and operating the Nicaragua Canal is $1,300,000 greater than the corresponding charges for the Panama Canal.

"The Panama route would be 134.57 miles shorter from sea to sea than the Nicaragua route. It would have less summit elevation, fewer locks, 1,568 degrees and 26.44 miles less curvature. The estimated time for a deep-draft vessel to pass through is about 12 hours for Panama and thirty-three hours for Nicaragua. These periods are practically the measure of the relative advantages of the two canals as waterways connecting the two oceans, but not entirely, because the risks to vessels and the dangers of delay are greater in a canal than in the open sea.

"Except for the items of risks and delays, the time required to pass through the canals need be taken into account only as an element in the time required by vessels to make their voyages between terminal ports. Compared on this basis, the Nicaragua route is the more advantageous for all transisthmian commerce except that originating or ending on the west coast of South America. For the commerce in which the United States is most interested, that between our Pacific ports and Atlantic ports, European and American, the Nicaragua route is shorter by about 1 day. The same advantage exists between our Atlantic ports and the Orient. For our Gulf ports the advantage of the Nicaragua route is nearly two days. For commerce between North Atlantic ports and the west coast of South America the Panama route is shorter by about two days. Between Gulf ports and the west coast of South America the saving is about one day.

"The Nicaragua route would be the more favorable one for sailing vessels because of the uncertain winds in the Bay of Panama. This is not, however, a material matter, as sailing ships are being rapidly displaced by steamships.

"A canal by the Panama route will be simply a means of communication between the two oceans. That route has been a highway of commerce for more than three hundred years, and a railroad has been in operation there for nearly fifty years, but this has effected industrial changes of but little consequence, and the natural features of the country through which the route passes are such that no considerable development is likely to occur as a result of the construction and operation of a canal.

"In addition to its use as a means of communication between the two oceans, a canal by the Nicaragua route would bring Nicaragua and a large portion of Costa Rica and other Central American States into close and easy communication with the United States and with Europe. The intimate business relations that would be established with the people of the United States during the period of construction by the expenditure of vast sums of money in these States and the use of American products and manufactures would be likely to continue after the completion of the work, to the benefit of our manufacturing, agricultural, and other interests.

"The Nicaragua route lies in a region of sparse population and not in a pathway of much trade or movement of people: conditions productive of much sickness do not exist. On the other hand, a considerable population has long existed on the Panama route, and it lies on a pathway of comparatively large trade, along which currents of moving people from infected places sometimes converge, thus creating conditions favorable to epidemics. Existing conditions indicate hygienic advantages for the Nicaragua route, although it is probable that no less effective sanitary measures must be taken during construction in the one case than in the other"

The relative estimated cost of the Nicaragua and Panama canals is as follows: Nicaragua $189,864,962; completing the Panama canal $144,233,358+$40,000,000 for acquiring the rights and property of the New Panama Canal Co., making a total of $184,233,358. [Actual cost will be double.]

Estimates of annual cost of maintenance are: Nicaragua canal, $3,300,000; Panama canal $2,000,000.

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2.—Distances from American and European Atlantic Ports to Pacific Ports, via the Nicaragua and Panama Canals.
(Prof. Emory R. Johnson, in Annals of the Am. Acad. of Pol. and Soc. Science, January, 1902.)
[Distances in Miles.]

From—	Via—	To Port Townsend, via San Francisco.	To San Francisco.	To Guayaquil.	To Callao.	To Iquique.	To Valparaiso.	To Coronel.	To Yokohama, via San Francisco. a	To Shanghai, via San Francisco and Yokohama. a	To Manila, via San Francisco and Yokohama. a	To Sydney, via Tahiti. b	To Melbourne, via Tahiti c and Sydney. b	To Wellington, via Tahiti. d
New York	Nicaragua	5,696	4,921	3,246	3,751	4,393	4,928	5,161	9,457	10,507	11,207	9,676	10,251	8,716
	Panama	6,074	5,299	2,864	3,359	4,021	4,630	4,838	9,835	10,885	11,585	9,852	10,427	8,892
Norfolk	Nicaragua	5,485	4,710	3,035	3,540	4,182	4,767	4,950	9,247	10,297	10,997	9,466	10,041	8,505
	Panama	5,872	5,097	2,662	3,157	3,819	4,428	4,636	9,634	10,684	11,384	9,650	9,858	8,690
Charleston	Nicaragua	5,276	4,501	2,826	3,331	3,973	4,558	4,741	9,037	9,957	10,505	9,250	9,831	8,296
	Panama	5,673	4,898	2,463	2,958	3,638	4,229	4,437	9,344	10,367	10,809	9,451	10,006	8,491
Port Tampa	Nicaragua	4,753	3,978	2,303	2,808	3,450	4,035	4,218	8,514	9,564	10,264	8,733	9,308	7,773
	Panama	5,328	4,553	2,098	2,593	3,255	3,864	4,072	9,069	10,119	10,819	9,086	9,661	8,126
New Orleans	Nicaragua	4,893	4,118	2,443	2,948	3,590	4,175	4,358	8,654	9,704	10,404	8,873	9,448	7,913
	Panama	5,477	4,698	2,263	2,758	3,420	4,029	4,237	9,234	10,284	10,984	9,251	9,826	8,291
Galveston	Nicaragua	4,996	4,221	2,546	3,051	3,693	4,278	4,461	8,757	9,887	10,587	9,056	9,551	8,016
	Panama	5,574	4,799	2,364	2,858	3,520	4,129	4,338	9,335	10,385	11,085	9,352	9,927	8,392
Liverpool	Nicaragua	8,426	7,651	5,975	6,481	7,123	7,708	7,891	12,187	13,237	13,937	12,406	12,891	11,446
	Panama	8,813	8,038	5,603	6,098	6,760	7,369	7,577	12,574	13,624	14,324	12,591	13,166	11,631
Hamburg	Nicaragua	8,943	8,168	6,493	6,998	7,640	8,225	8,408	12,704	13,754	14,454	12,923	13,498	11,963
	Panama	9,242	8,467	6,032	6,527	7,189	7,798	8,006	13,003	14,053	14,753	13,020	13,595	12,060
Antwerp	Nicaragua	8,664	7,889	6,214	6,719	7,361	7,946	8,129	12,425	13,475	14,175	12,644	13,219	11,684
	Panama	8,963	8,188	5,753	6,248	6,910	7,519	7,727	12,724	13,774	14,474	12,741	13,316	11,781
Bordeaux	Nicaragua	8,414	7,639	5,964	6,469	7,111	7,696	7,879	12,175	13,225	13,925	12,394	12,969	11,434
	Panama	8,713	7,938	5,503	5,998	6,660	7,269	7,477	12,474	13,524	14,224	12,491	13,066	11,471
Gibraltar	Nicaragua	8,148	7,373	5,698	6,203	6,845	7,430	7,613	11,909	12,959	13,659	12,128	12,703	12,800
	Panama	8,447	7,672	5,237	5,723	6,394	7,003	7,211	12,208	13,258	13,958	12,225	11,168	11,265

a Via Honolulu adds 374 miles for Nicaragua and 252 for Panama.
b Omitting Tahiti reduces voyage from Brito by 52 miles.
c Voyage from Brito to Sydney by Wellington is 232 miles less than by way of Tahiti; from Panama it is 405 miles less.
d Voyage from Brito to Wellington direct is 185 miles shorter than via Tahiti, and from Panama it is 358 miles shorter.

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The Harlem River Ship Canal, connecting the Hudson River and Long Island Sound, by way of Spuyten Duyvil Creek and Harlem River, was opened for traffic on June 17, 1895, and cost about $2,700,000.

Cost of Maintenance and Operation of Canals.—In order to form an estimate of the cost of maintaining and operating the Isthmian Canal, the Isthmian Canal Commission obtained data bearing on this point from the Suez, Manchester, Kiel, and St. Marys Falls canals, as follows:

There are no locks on the Suez Canal, but the channel is through drifting sand for a great part of its length. The entrance to the harbor of Port Said on the Mediterranean intercepts the drift of sand discharged from the Nile and carried along the coast by the easterly current. The maintenance of the Suez Canal therefore requires a large amount of dredging and consists mainly of this class of work. The operating expenses are also large, the great traffic involving heavy costs for pilotage. The general expenses for administration have necessarily been greater for the Suez Canal than for the Kiel or Manchester Canals, on account of the distance of the work from the point of central control, a disadvantage which would also attend the operation of the Isthmian Canal. The annual cost of maintenance and operation of the Suez Canal is about $1,300,000, or about $13,000 per mile.

The annual cost of maintenance and operation of the Kiel Canal is $8,600 per mile. The cost of maintenance only of the Manchester Canal is $9,500 per mile. These canals have locks and other mechanical structures, and therefore might be expected to have a higher cost of maintenance than the Suez Canal, which has none, but this appears to be more than offset by reduced cost of maintaining the prism and more economical central control. The traffic being light on these canals, the cost of pilotage and port service is small. The mechanical structures are now nearly new, and will soon require larger annual outlays for maintenance, while, with the increase of traffic, operating expenses will become larger.

The St. Marys Falls Canal, when compared with those just mentioned, is remarkable by reason of its short length, large proportion of mechanical structures, and immense traffic. Its length is about 1½ miles. Its annual traffic, limited by the severity of the winter to a period of about eight months, is nearly three times that of the Suez Canal, eight times that of the Kiel Canal, and ten times that of the Manchester Canal. Both maintenance and operating expenses are therefore very large, amounting to from $70,000 to $90,000 per year, or $46,000 to $60,000 per mile. The annual cost per mile of maintenance and operation, however, for comparison with other canals, should be determined by considering the 18½ miles of dredged channel ways in St. Marys River as part of the canal. Then for the 20 miles of canal and canalized river the expenses per mile would be from $3,000 to $5,000 annually. Tolls were collected by the State from 1855 to 1881. Since its ownership by the government no tolls have been charged.

The Principal Commercial Canals of the U. S., showing cost of construction (which includes cost of improvements), date of completion, length, number of locks, and *navigable* depth, are given in Table 3, on following page, which is reproduced, by permission, from the New York World Almanac.

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Albemarle and Chesapeake	$1,641,363	1860	44	1	7½	Norfolk, Va., to Currituck Sound, N. C
Augusta	1,500,000	1847	9	.....	11	Savannah River, Ga., to Augusta, Ga.
Black River	3,581,954	1849	35	109	4	Rome, N. Y., to Lyons Falls, N. Y.
Cayuga and Seneca	2,232,632	1839	25	11	7	Montezuma, N. Y., to Cayuga and Seneca Lakes, N. Y.
Champlain	4,044,000	1822	81	32	6	Whitehall, N. Y., to West Troy, N. Y.
Chesapeake and Delaware	3,730,230	1829	14	3	9	Chesapeake City, Md., to Delaware City, Del.
Chesapeake and Ohio	11,290,327	1850	184	73	6	Cumberland, Md., to Washington, D. C.
Companys	90,000	1847	22	1	6	Mississippi River, La., to Bayou Black, La.
Delaware and Raritan	4,888,749	1838	66	14	7	New Brunswick, N. J., to Trenton, N. J.
Delaware Division	2,433,350	1830	60	33	6	Easton, Pa., to Bristol, Pa.
Des Moines Rapids	4,582,009	1877	7½	3	5	At Des Moines Rapids, Mississippi River.
Dismal Swamp	2,800,000	1822	22	7	6	Connects Chesapeake Bay with Albemarle Sound
Erie	52,540,800	1826	387	72	7	Albany, N. Y., to Buffalo, N. Y.
Fairfield	.....	.....	4½	None.	.....	Alligator River to Lake Mattimuskeet, N. C.
Galveston and Brozos	340,000	1851	38	.....	3½	Galveston, Tex., to Brazos River, Tex.
Hocking	975,481	1843	42	26	4	Carroll, O., to Nelsonville, O.
Illinois and Michigan	7,357,787	1848	102	15	6	Chicago, Ill., to La Salle, Ill.
Illinois and Mississippi	568,643	1895	4½	3	7	Around lower rapids of Rock Riv., Ill., connects with Miss. R.
Lehigh Coal and Navigation Co.	4,455,000	1821	108	57	6	Coalport, Pa., to Easton, Pa.
Louisville and Portland	5,578,631	1872	2½	2	.....	At Falls of Ohio River, Louisville, Ky.
Miami and Erie	8,062,680	1835	274	93	5½	Cincinnati, O., to Toledo, O.
Morris	6,000,000	1836	103	33	5	Easton, Pa., to Jersey City, N. J.
Muscle Shoals and Elk R. Shoals	3,156,919	1889	16	11	6	Big Muscle Shoals, Tenn., to Elk River Shoals, Tenn.
Newberne and Beaufort	.....	.....	3	None.	.....	Clubfoot Creek to Harlow Creek, N. C.
Ogeechee	407,810	1840	16	5	3	Savannah River, Ga., to Ogeechee River, Ga.
Ohio	4,695,204	1835	317	150	4	Cleveland, O., to Portsmouth, O.
Oswego	5,239,526	1828	38	18	7	Oswego, N. Y., to Syracuse, N. Y.
Pennsylvania	7,731,750	1839	193	71	6	Columbia, Northumberland, Wilkes-Barre, Huntingdon, Pa.
Portage Lake and Lake Superior	528,892	1873	25	None.	15	From Keweenaw Bay to Lake Superior.
Port Arthur	.....	1899	7	.....	26	Port Arthur, Tex., to Gulf of Mexico.
Santa Fe	70,000	1880	10	.....	5	Waldo, Fla., to Melrose, Fla.
Sault Ste. Marie (ship canal)	4,000,000	1895	3	2	18	Connects Lakes Superior and Huron at St. Mary's River.
Schuylkill Navigation Company	12,461,600	1826	108	71	6½	Mill Creek, Pa., to Philadelphia, Pa.
Sturgeon Bay and Lake Michigan	99,661	1881	1½	None.	15	Between Green Bay and Lake Michigan.
St. Mary's Falls	7,909,667	1896	1½	1	21	Connects Lakes Superior and Huron at Sault Ste. Marie, Mich.
Susquehanna and Tidewater	4,931,345	1840	45	32	5½	Columbia, Pa., to Havre de Grace, Md.
Walhonding	607,269	1843	25	11	4	Rochester O., to Roscoe, O.
Welland (ship canal)	28,796,352	.....	26½	55	14	Connects Lake Ontario and Lake Erie.

*And Improvements. †Navigable depth.

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EXCERPTS AND REFERENCES.

The Relation of Depth of Water to Speed and Power of Ships (Eng. News, Mar. 16, 1905).—Diagrams. Contains other references.

The Relation of the Depth of Harbor Channels to Modern Shipping (Eng. News, Dec. 27, 1906).—Table.

Construction and Unit Costs of Concrete Lock, Rough River, Ky. (Eng. News, Jan. 9, 1908).—Illustrated, with tables of costs.

The Design of Emergency Movable Dams for Canal Locks (Eng. News, June 24, 1909).—Illustrated.

Water Supply for the Lock Canal at Panama (By Julio F. Sorgano, Trans. A. S. C. E., Vol. LXVII., June, 1910).

The Siphon Lock on the New York State Barge Canal (By D. A. Watt, Eng. News, Nov. 17, 1910). Illustrated description of the general design and operation of the siphon lock at Oswego, N. Y.

Cost Data on the Panama Canal ("Canal Record," Nov. 9, 1910; Eng. News, Nov. 24, 1910).—General statement of construction expenditures, to Sept. 30, 1910; dam construction; lock and spillway construction.

Illustrations of Some Important Works:—

Description.	Eng. News.
Plan and section of concrete lock for small craft, Madison, Wis.	Dec. 10, 1903.
Suggested new type of lock, Sault Ste. Marie Canal	June 24, '09.
The lock gates of the Panama canal	Sept. 16, '09.
Typical sections and lock on N. Y. State barge canal	June 9, '10.
Design and cons. of movable dam and lock, Lockport	Oct. 6, '10.
Emergency gates on the Illinois and Mississippi Canal	Dec. 15, '10.
	Eng. Rec.
Plan, profile, section, Cape Cod ship canal, also breakwater	July 24, '09.
Siphon lock on barge canal at Oswego	July 30, '10.
Plans of new canal gates at Sault Ste. Marie	Dec. 10, '10.

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68.—WATER POWER.

Definitions and Formulas.—Power is the *rate* of work (see page 291); and Water Power is the rate of work available from stored or flowing water, through the agency of gravity. The unit of mechanical power is the Horse-Power ($H.\ P.$), equivalent to the energy expended in raising 550 lbs. one ft. high in one second, or to the energy of 550 lbs. falling one ft. in one second. Hence, the theoretic horse-power of a stream is—

$$H.\ P. = \frac{62.37\ Q\ h}{550} = 0.1134\ Q\ h \qquad (1)$$

where 62.37 = weight of a cu. ft. of water at 60°F., in lbs.;
Q = discharge of stream, in cu. ft. per sec.;
h = fall of water, or available head, in ft.

Whence
$$Q = \frac{8.82 \times H.\ P.}{h} \qquad (2)$$

and
$$h = \frac{8.82 \times H.\ P.}{Q} \qquad (3)$$

"Horse-Power Hours" is a term used to denote a certain *amount* of work or energy, and not the rate of work. It is the maintenance of one horse-power for one hour, or of ten horse-power for six minutes, etc.; and may be applied to definite quantities or volumes of stored or running water. Thus,

$$\text{No. of } H.\text{-}P.\text{ hours} = \frac{62.37\ V\ h}{550 \times 3600} = 0.0000315\ V\ h \qquad (4)$$

where V = volume of stored water, in cu. ft.;
3600 = No. of seconds in one hour.

Whence
$$V = \frac{31746 \times \text{No. of } H.\text{-}P.\text{ hours}}{h} \qquad (5)$$

and
$$h = \frac{31746 \times \text{No. of } H.\text{-}P.\text{ hours}}{V} \qquad (6)$$

At the Prime Motor the available horse-power is always less than the theoretic horse-power, the decrease being due to loss of head, leakage or waste, evaporation, etc., depending upon the character of conduit or canal. Equation (1) will give the available horse-power at the prime motor if—

Q = actual discharge, in cu. ft. per sec., at prime motor;
h = effective head, in ft., at prime motor.

Economic Design of Penstock.—For a high-pressure water-power pipe, the volume of discharge being assumed fixed or constant, the following economic relation holds true:

"That pipe fulfills the requirements of greatest economy wherein the value of the energy annually lost in frictional resistance equals four-tenths (0.4) of the annual cost of the pipe line."*

Thus, if L = value of energy annually lost in frictional resistance,
and C = annual cost of pipe line;
then $L = 0.4\,C$, for economic design of pipe.

* Mr. A. L. Adams, in Trans. Am. Soc. C. E., Vol. LIX, page 177.

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1.—Number of Horse-Power, Equivalent to Flow of 1 Cu. Ft. per Sec. Under Various Effective Heads. (Equa. 1.)
(To find the total *H. P.*, mult. values in Table by No. of cu. ft. per sec.)
[Horse-Power per Cu. Ft. per Sec. of Flow.]

*Head. Feet.	Units.									
	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
0-.	.0000	.1134	.2268	.3402	.4536	.5670	.6804	.7938	.9072	1.0206
1-.	1.1340	1.2474	1.3608	1.4742	1.5876	1.7010	1.8144	1.9278	2.0412	2.1546
2-.	2.2680	2.3814	2.4948	2.6082	2.7216	2.8350	2.9484	3.0618	3.1752	3.2886
3-.	3.4020	3.5154	3.6288	3.7422	3.8556	3.9690	4.0824	4.1958	4.3092	4.4226
4-.	4.5360	4.6494	4.7628	4.8762	4.9896	5.1030	5.2164	5.3298	5.4432	5.5566
5-.	5.6700	5.7834	5.8968	6.0102	6.1236	6.2370	6.3504	6.4638	6.5772	6 6906
6-.	6.8040	6.9174	7.0308	7.1442	7.2576	7 3710	7.4844	7.5978	7.7112	7.8246
7-.	7.9380	8.0514	8.1648	8.2782	8.3916	8.5050	8.6184	8.7318	8.8452	8.9586
8-.	9.0720	9.1854	9.2988	9.4122	9.5256	9.6390	9.7524	9.8658	9.9792	10.0926
9-	10.2060	10.3194	10.4328	10.5462	10.6596	10.7730	10.8864	10.9998	11.1132	11.2266
10-	11.3400	11.4534	11.5668	11.6802	11.7936	11.9070	12.0204	12.1338	12.2472	12.3606
11-	12.4740	12.5874	12.7008	12.8142	12.9276	13.0410	13.1544	13.2678	13.3812	13.4946
12-	13.6080	13.7214	13.8348	13.9482	14.0616	14.1750	14.2884	14.4018	14.5152	14.6286
13-	14.7420	14.8554	14.9688	15.0822	15.1956	15.3090	15.4224	15.5358	15.6492	15.7626
14-	15.8760	15.9894	16.1028	16.2162	16.3296	16.4430	16.5564	16.6698	16.7832	16.8966
15-	17.0100	17.1234	17.2368	17.3502	17.4636	17.5770	17.6904	17.8038	17.9172	18.0306
16-	18.1440	18.2574	18.3708	18.4842	18.5976	18.7110	18.8244	18.9378	19.0512	19.1646
17-	19.2780	19.3914	19.5048	19.6182	19.7316	19.8450	19.9584	20.0718	20.1852	20.2986
18-	20.4120	20.5254	20.6388	20.7522	20.8656	20.9790	21.0924	21.2058	21.3192	21.4326
19-	21.5460	21.6594	21.7728	21.8862	21.9996	22.1130	22.2264	22.3398	22.4532	22.5666
20-	22.6800	22.7934	22.9068	23.0202	23.1336	23.2470	23.3604	23.4738	23.5872	23.7006
21-	23.8140	23.9274	24.0408	24.1542	24.2676	24.3810	24.4944	24.6078	24.7212	24.8346
22-	24.9480	25.0614	25.1748	25.2882	25.4016	25.5150	25.6284	25.7418	25.8552	25.9686
23-	26.0820	26.1954	26.3088	26.4222	26.5356	26.6490	26.7624	26.8758	26.9892	27.1026
24-	27.2160	27.3294	27.4428	27.5562	27.6696	27.7830	27.8964	28.0098	28.1232	28.2366
25-	28.3509	28.4634	28.5768	28.6902	28 8036	28.9170	29.0304	29.1438	29.2572	29.3706
26-	29.4840	29.5974	29.7108	29.8242	29.9376	30.0510	30.1644	30.2778	30.3912	30.5046
27-	30.6180	30.7314	30.8448	30.9582	31.0716	31.1850	31.2984	31.4118	31.5252	31.6386
28-	31.7520	31.8654	31.9788	32.0922	32.2056	32.3190	32.4324	32.5458	32.6592	32.7726
29-	32.8860	32.9994	33.1128	33.2262	33.3396	33.4530	33.5664	33.6798	33.7932	33.9066
30-	34.0200	34.1334	34.2468	34.3602	34.4736	34.5870	34.7004	34.8138	34.9272	35.0406
31-	35.1540	35.2674	35.3808	35.4942	35.6076	35.7210	35.8344	35.9478	36.0612	36.1746
32-	36.2880	36.4014	36.5148	36.6282	36.7416	36.8550	36.9684	37.0818	37.1952	37.3086
33-	37.4220	37.5354	37.6488	37.7622	37.8756	37.9890	38.1024	38.2158	38.3292	38.4426
34-	38.5560	38.6694	38.7828	38.8962	39.0096	39.1230	39.2364	39.3498	39.4632	39.5766
35-	39.6900	39.8034	39.9168	40.0302	40.1436	40.2570	40.3704	40.4838	40.5972	40.7106
36-	40.8240	40.9374	41.0508	41.1642	41.2776	41.3910	41.5044	41.6178	41.7312	41.8446
37-	41.9580	42.0714	42.1848	42.2982	42.4116	42.5250	42.6384	42.7518	42.8652	42.9786
38-	43.0920	43.2054	43.3188	43.4322	43.5456	43.6590	43.7724	43.8858	43.9992	44.1126
39-	44.2260	44.3394	44.4528	44.5662	44.6796	44.7930	44.9064	45.0198	45.1332	45.2466
40-	45.3600	45.4734	45.5868	45.7002	45.8136	45.9270	46.0404	46.1538	46.2672	46.3806
41-	46.4940	46.6074	46.7208	46.8342	46.9476	47.0610	47.1744	47.2878	47.4012	47.5146
42-	47.6280	47.7414	47.8548	47.9682	48.0816	48.1950	48.3084	48.4218	48.5352	48.6486
43-	48.7620	48.8754	48.9888	49.1022	49.2156	49.3290	49.4424	49.5558	49.6692	49.7826
44-	49.8960	50.0094	50.1228	50.2362	50.3496	50.4630	50.5764	50.6898	50.8032	50.9166
45-	51.0300	51.1434	51.2568	51.3702	51.4836	51.5970	51.7104	51.8238	51.9372	52.0506
46-	52.1640	52.2774	52.3908	52.5042	52.6176	52.7310	52.8444	52.9578	53.0712	53.1846
47-	53.2980	53.4114	53.5248	53.6382	53.7516	53.8650	53.9784	54.0918	54.2052	54.3186
48-	54.4320	54.5454	54.6588	54.7722	54.8856	54.9990	55.1124	55.2258	55.3392	55.4526
49-	55.5660	55.6794	55.7928	55 9062	56.0196	56.1330	56.2464	56.3598	56.4732	56.5866
50-	56.7000	56.8134	56.9268	57.0402	57.1536	57.2670	57.3804	57.4938	57.6072	57.7206
51-	57.8340	57.9474	58.0608	58.1742	58.2876	58.4010	58.5144	58.6278	58.7412	58.8546
52-	58.9680	59.0814	59.1948	59.3082	59.4216	59 5350	59.6484	59 7618	59.8752	59.9886
53-	60.1020	60.2154	60.3288	60.4422	60.5556	60.6690	60.7824	60.8958	61.0092	61.1226
54-	61.2360	61.3494	61.4628	61.5762	61.6896	61.8030	61.9164	62.0298	62 1432	62.2566
55-	62.3700	62.4834	62.5968	62.7102	62.8236	62.9370	63.0504	63.1638	63.2772	63.3906
56-	63.5040	63.6174	63.7308	63.8442	63.9576	64.0710	64.1844	64.2978	64.4112	64.5246
57-	64.6380	64.7514	64.8648	64.9782	65.0916	65.2050	65.3184	65.4318	65.5452	65.6586
58-	65.7720	65.8854	65.9988	66.1122	66.2256	66.3390	66.4524	66.5658	66 6792	66.7926
59-	66.9060	67.0194	67.1328	67.2462	67.3596	67.4730	67.5864	67.6998	67.8132	67.9266
60-	68.0400	68.1534	68.2668	68.3802	68.4936	68.6070	68.7204	68.8338	68.9472	69.0606

Supply space occupied by dash, with unit figure at top of column.

* Effective head and actual horse power are proportional; hence the decimal point may be moved in each, a corresponding number of places to right or left.

Ex.—The equivalent to a flow of 1 cu. ft. per sec. under a head of 14 ft. is 1.5876 *H. P.*; under a 140-ft. head, 15.876 *H. P.*; under a 1400-ft. head, 158.76 *H. P.*

2.—Number of Horse-Power Hours, Equivalent to Storage of 1 000 000 Cu. Ft. Under Various Effective Heads. (Equa. 4.)

(To find total *H.-P. H.*, mult. values in Table by No. of millions of cu. ft. stored.)

[Horse-Power Hours per each million cu. ft.]

*Head. Feet.	Units.									
	0.	1.	2.	3.	4.	5	6.	7.	8.	9.
0-.	0	31.5	63	94.5	126	157.5	189	220.5	252	283.5
1-.	315	346.5	378	409.5	441	472.5	504	535.5	567	598.5
2-.	630	661.5	693	724.5	756	787.5	819	850.5	882	913.5
3-.	945	976.5	1008	1039.5	1071	1102.5	1134	1165.5	1197	1228.5
4-.	1260	1291.5	1323	1354.5	1386	1417.5	1449	1480.5	1512	1543.5
5-.	1575	1606.5	1638	1669.5	1701	1732.5	1764	1795.5	1827	1858.5
6-.	1890	1921.5	1953	1984.5	2016	2047.5	2079	2110.5	2142	2173.5
7-.	2205	2236.5	2268	2299.5	2331	2362.5	2394	2425.5	2457	2488.5
8-.	2520	2551.5	2583	2614.5	2646	2677.5	2709	2740.5	2772	2803.5
9-.	2835	2866.5	2898	2929.5	2961	2992.5	3024	3055.5	3087	3118.5
10-.	3150	3181.5	3213	3244.5	3276	3307.5	3339	3370.5	3402	3433.5
11-.	3465	3496.5	3528	3559.5	3591	3622.5	3654	3685.5	3717	3748.5
12-.	3780	3811.5	3843	3874.5	3906	3937.5	3969	4000.5	4032	4063.5
13-.	4095	4126.5	4158	4189.5	4221	4252.5	4284	4315.5	4347	4378.5
14-.	4410	4441.5	4473	4504.5	4536	4567.5	4599	4630.5	4662	4693.5
15-.	4725	4756.5	4788	4819.5	4851	4882.5	4914	4945.5	4977	5008.5
16-.	5040	5071.5	5103	5134.5	5166	5197.5	5229	5260.5	5292	5323.5
17-.	5355	5386.5	5418	5449.5	5481	5512.5	5544	5575.5	5607	5638.5
18-.	5670	5701.5	5733	5764.5	5796	5827.5	5859	5890.5	5922	5953.5
19-.	5985	6016.5	6048	6079.5	6111	6142.5	6174	6205.5	6237	6268.5
20-.	6300	6331.5	6363	6394.5	6426	6457.5	6489	6520.5	6552	6583.5
21-.	6615	6646.5	6678	6709.5	6741	6772.5	6804	6835.5	6867	6898.5
22-.	6930	6961.5	6993	7024.5	7056	7087.5	7119	7150.5	7182	7213.5
23-.	7245	7276.5	7308	7339.5	7371	7402.5	7434	7465.5	7497	7528.5
24-.	7560	7591.5	7623	7654.5	7686	7717.5	7749	7780.5	7812	7843.5
25-.	7875	7906.5	7938	7969.5	8001	8032.5	8064	8095.5	8127	8158.5
26-.	8190	8221.5	8253	8284.5	8316	8347.5	8379	8410.5	8442	8473.5
27-.	8505	8536.5	8568	8599.5	8631	8662.5	8694	8725.5	8757	8788.5
28-.	8820	8851.5	8883	8914.5	8946	8977.5	9009	9040.5	9072	9103.5
29-.	9135	9166.5	9198	9229.5	9261	9292.5	9324	9355.5	9387	9418.5
30-.	9450	9481.5	9513	9544.5	9576	9607.5	9639	9670.5	9702	9733.5
31-.	9765	9796.5	9828	9859.5	9891	9922.5	9954	9985.5	10017	10048.5
32-.	10080	10111.5	10143	10174.5	10206	10237.5	10269	10300.5	10332	10363.5
33-.	10395	10426.5	10458	10489.5	10521	10552.5	10584	10615.5	10647	10678.5
34-.	10710	10741.5	10773	10804.5	10836	10867.5	10899	10930.5	10962	10993.5
35-.	11025	11056.5	11088	11119.5	11151	11182.5	11214	11245.5	11277	11308.5
36-.	11340	11371.5	11403	11434.5	11466	11497.5	11529	11560.5	11592	11623.5
37-.	11655	11686.5	11718	11749.5	11781	11812.5	11844	11875.5	11907	11938.5
38-.	11970	12001.5	12033	12064.5	12096	12127.5	12159	12190.5	12222	12253.5
39-.	12285	12316.5	12348	12379.5	12411	12442.5	12474	12505.5	12537	12568.5
40-.	12600	12631.5	12663	12694.5	12726	12757.5	12789	12820.5	12852	12883.5
41-.	12915	12946.5	12978	13009.5	13041	13072.5	13104	13135.5	13167	13198.5
42-.	13230	13261.5	13293	13324.5	13356	13387.5	13419	13450.5	13482	13513.5
43-.	13545	13576.5	13608	13639.5	13671	13702.5	13734	13765.5	13797	13828.5
44-.	13860	13891.5	13923	13954.5	13986	14017.5	14049	14080.5	14112	14143.5
45-.	14175	14206.5	14238	14269.5	14301	14332.5	14364	14395.5	14427	14458.5
46-.	14490	14521.5	14553	14584.5	14616	14647.5	14679	14710.5	14742	14773.5
47-.	14805	14836.5	14868	14899.5	14931	14962.5	14994	15025.5	15057	15088.5
48-.	15120	15151.5	15183	15214.5	15246	15277.5	15309	15340.5	15372	15403.5
49-.	15435	15466.5	15498	15529.5	15561	15592.5	15624	15655.5	15687	15718.5
50-.	15750	15781.5	15813	15844.5	15876	15907.5	15939	15970.5	16002	16033.5
51-.	16065	16096.5	16128	16159.5	16191	16222.5	16254	16285.5	16317	16348.5
52-.	16380	16411.5	16443	16474.5	16506	16537.5	16569	16600.5	16632	16663.5
53-.	16695	16726.5	16758	16789.5	16821	16852.5	16884	16915.5	16947	16978.5
54-.	17010	17041.5	17073	17104.5	17136	17167.5	17199	17230.5	17262	17293.5
55-.	17325	17356.5	17388	17419.5	17451	17482.5	17514	17545.5	17577	17608.5
56-.	17640	17671.5	17703	17734.5	17766	17797.5	17829	17860.5	17892	17923.5
57-.	17955	17986.5	18018	18049.5	18081	18112.5	18144	18175.5	18207	18238.5
58-	18270	18301.5	18333	18364.5	18396	18427.5	18459	18490.5	18522	18553.5
59-.	18585	18616.5	18648	18679.5	18711	18742.5	18774	18805.5	18837	18868.5
60-.	18900	18931.5	18963	18994.5	19026	19057.5	19089	19120.5	19152	19183.5

Supply space occupied by dash, with unit figure at top of column.

* Effective head and actual horse-power hours are proportional; hence the decimal point may be moved in each, a corresponding number of places to right or left.

Ex.—The equivalent to a storage of 1 000 000 cu. ft. with a head of 14 ft. is 441 *H.-P. H*; with a 140-ft. head, 4410 *H.-P. H.*; with a 1400-ft. head, 44100 *H.-P. H.*

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3.—NUMBER OF HORSE-POWER HOURS, EQUIVALENT TO STORAGE OF 1 ACRE-FOOT. UNDER VARIOUS EFFECTIVE HEADS. (EQUA. 4.)

find the total *H.-P. H.*, mult. values in Table by No. of acre-feet stored.)

Note.—1 acre-foot = 43560 cubic feet.

[Horse-Power Hours per each acre-foot.]

	Units.									
	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
-.	0.0000	1.3721	2.7443	4.1164	5.4886	6.8607	8.2328	9.6050	10.977	12.349
-.	13.721	15.094	16.466	17.838	19.210	20.582	21.954	23.326	24.699	26.071
-.	27.443	28.815	30.187	31.559	32.931	34.304	35.676	37.048	38.420	39.792
-.	41 164	42.536	43.908	45.281	46.653	48.025	49.397	50.769	52.141	53.513
-.	54.886	56.258	57.630	59.002	60.374	61.746	63.118	64.491	65.863	67.235
-.	68.607	69.979	71.351	72.723	74.096	75.468	76.840	78.212	79.584	80.956
-.	82.328	83.701	85.073	86.445	87.817	89.189	90.561	91.933	93.306	94.678
-.	96.050	97.422	98.794	100.17	101.54	102.91	104.28	105.65	107.03	108.40
-.	109.77	111.14	112.52	113.89	115.26	116.63	118.00	119.38	120.75	122.12
-.	123.49	124.86	126.24	127.61	128.98	130.35	131.73	133.10	134.47	135.84
-.	137.21	138.59	139.96	141.33	142.70	144.07	145.45	146.82	148.19	149.56
-.	150.94	152.31	153.68	155.05	156.42	157.80	159.17	160.54	161.91	163.28
-.	164.66	166.03	167.40	168.77	170.15	171.52	172.89	174.26	175.63	177.01
-.	178.38	179.75	181.12	182.49	183.87	185.24	186.61	187.98	189.36	190.73
-.	192.10	193.47	194.85	196.22	197.59	198.96	200.33	201.70	203.08	204.45
-.	205.82	207.19	208.57	209.94	211.31	212.68	214.05	215.43	216.80	218.17
-.	219.54	220.91	222.29	223.66	225.03	226.40	227.78	229.15	230.52	231.89
-.	233.26	234.64	236.01	237.38	238.75	240.12	241.50	242.87	244.24	245.61
-.	246.99	248.36	249.73	251.10	252.47	253.85	255.22	256.59	257.96	259.33
-.	260.71	262.08	263.45	264.82	266.20	267.57	268.94	270.31	271.68	273.06
-.	274.43	275.80	277.17	278.54	279.92	281.29	282.66	284.03	285.41	286.78
-.	288.15	289.52	290.89	292.27	293.64	295.01	296.38	297.75	299.13	300.50
-.	301.87	303.24	304.62	305.99	307.36	308.73	310.10	311.48	312.85	314.22
-.	315.59	316.96	318.34	319.71	321.08	322.45	323.83	325.20	326.57	327.94
-.	329.31	330.69	332.06	333.43	334.80	336.17	337.55	338.92	340.29	341.66
-.	343.04	344.41	345.78	347.15	348.52	349.90	351.27	352.64	354.01	355.38
-.	356.76	358.13	359.50	360.87	362.24	363.62	364.99	366.36	367.73	369.11
-.	370.48	371.85	373.22	374.59	375.97	377.34	378.71	380.08	381.45	382.83
-.	384.20	385.57	386.94	388.32	389.69	391.06	392.43	393.80	395.18	396.55
-.	397.92	399.29	400.66	402.04	403.41	404.78	406.15	407.53	408.90	410.27
-.	411.64	413.01	414.39	415.76	417.13	418.50	419.87	421.25	422.62	423.99
-.	425.36	426.74	428.11	429.48	430.85	432.22	433.60	434.97	436.34	437.71
-.	439.08	440.46	441.83	443.20	444.57	445.95	447.32	448.69	450.06	451.43
-.	452.81	454.18	455.55	456.92	458.29	459.67	461.04	462.41	463.78	465.16
-.	466.53	467.90	469.27	470.64	472.02	473.39	474.76	476.13	477.50	478.88
-.	480.25	481.62	482.99	484.37	485.74	487.11	488.48	489.85	491.23	492.60
-.	493.97	495.34	496.71	498.09	499.46	500.83	502.20	503.58	504.95	506.32
-.	507.69	509.06	510.44	511.81	513.18	514.55	515.92	517.30	518.67	520.04
-.	521.41	522.79	524.16	525.53	526.90	528.27	529.65	531.02	532.39	533.76
-.	535.13	536.51	537.88	539.25	540.62	542.00	543.37	544.74	546.11	547.48
-.	548.86	550 23	551.60	552.97	554.34	555.72	557.09	558.46	559.83	561.21
-.	562.58	563.95	565.32	566.69	568.07	569.44	570.81	572.18	573.55	574.93
-.	576.30	577.67	579.04	580.42	581.79	583.16	584.53	585.90	587.28	588.65
-.	590.02	591.39	592.76	594.14	595.51	596.88	598.25	599.63	601.00	602.37
-.	603.74	605.11	606.49	607.86	609.23	610.60	611.97	613.35	614.72	616.09
-.	617.46	618.84	620.21	621.58	622.95	624.32	625.70	627.07	628.44	629.81
-.	631 18	632.56	633.93	635.30	636.67	638.05	639.42	640.79	642.16	643.53
-.	644.91	646.28	647.65	649.02	650.39	651.77	653.14	654.51	655.88	657.26
-.	658.63	660.00	661.37	662.74	664.12	665.49	666.86	668.23	669.60	670.98
-.	672.35	673.72	675.09	676.47	677.84	679.21	680.58	681.95	683.33	684.70
-.	686.07	687.44	688.81	690.19	691.56	692.93	694.30	695.67	697.05	698.72
-.	699.79	701.16	702.54	703.91	705.28	706.65	708.02	709.40	710.77	712.14
-.	713.51	714.88	716.26	717.63	719.00	720.37	721.75	723.12	724.49	725.86
-.	727.23	728.61	729.98	731.35	732.72	734.09	735.47	736.84	738.21	739.58
-.	740.96	742.33	743.70	745.07	746.44	747.82	749.19	750.56	751.93	753.30
-.	754.68	756.05	757.42	758.79	760.17	761.54	762.91	764.28	765.65	767.03
-.	768.40	769.77	771.14	772.51	773.89	775.26	776.63	778.00	779.38	780.75
-.	782.12	783.49	784.86	786.24	787.61	788.98	790.35	791.72	793.10	794.47
-.	795.84	797.21	798.59	799.96	801.33	802.70	804.07	805.45	806.82	808.19
-.	809.56	810.93	812.31	813.68	815.05	816.42	817.80	819.17	820.54	821.91
-.	823.28	824.66	826.03	827.40	828.77	830.14	831.52	832.89	834.26	835.63

* Effective head and actual horse-power hours are proportional; hence e decimal point may be moved in each, a corresponding number of places right or left.

Ex.—The equivalent to a storage of 1 acre-foot with a head of 14 ft. is .210 *H.-P. H.*; with a 140-ft.head, 192.10 *H.-P. H.*; with a 1400-ft. head, 21.0 *H.-P. H.*

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Water Motors.—The energy of flowing water, by virtue of its velocity, weight and pressure, all acting together to a greater or less extent, may be transmitted to mechanical motors and transformed into various kinds of forms of energy. The principal types of water motors comprise many and various forms of wheels, among which are the following:*

The Current Wheel is the simplest form. It consists essentially of a paddle wheel with blades, somewhat similar to the side wheel of a steamer, and is mounted on a horizontal shaft, supported over the current of water at the bank of the stream, and adjustable to height. The principal use of the current wheel in the West is for raising water for domestic supply and irrigation purposes. Attached to, or near, the periphery of the wheel are open metal buckets which scoop up the water from the stream and then deliver it into a flume whence it is conveyed into reservoirs, or directly on the land. A small wheel will irrigate several acres. Prof F. H. King, in describing the wheels used in Bavaria on the River Regnitz, a branch of the Main, where he counted no less than 20 in a distance of 1½ to 1¾ miles, says:†

These wheels have a diameter of 16 ft. and carry upon one or both sides a row of 24 churn-like buckets each lifting out of the stream, and to a height of 12 ft., not less than 3 galls. of water, from which it is conveyed to the bank through a conduit hewn from a log. The wheel under consideration was making at the time of the writer's visit four revolutions each minute, so that the water lifted was not less than 288 gallons per minute, and probably exceeded 300 gallons. Another wheel with a row of buckets on each side was making three revolutions and discharging not less than 450 gallons per minute. The first of these wheels was pumping water at a rate sufficient to irrigate, to a depth of 4 inches every 10 days, 38 acres, and the second 60 acres.

The ordinary current wheel is sometimes called an "undershot wheel," although this is a misnomer.

The Undershot Wheel of approved design has curved blades or buckets. The current is accelerated by the construction of a flume with a decided grade and perhaps also by inserting a headgate giving increased head. The principle is somewhat similar to that of the current wheel, the water flowing beneath it. The efficiency is very low, rarely exceeding 40 per cent.

The Breast Wheel has pronounced buckets and acts as a dam, backing the water up nearly to its top so that all the buckets on the up-stream side are filled with water. Being thus continually unbalanced it is revolved. The efficiency ranges at about 60 to 65 per cent.

The Overshot Wheel has a greater efficiency than either of those previously described, reaching in some cases 75 per cent. The water is conveyed by flume over the top of the wheel, filling the buckets on the down-stream side, producing an unbalanced force on the horizontal shaft, which is connected up with the working machinery.

Impulse Wheels and *Turbine Wheels* are the most common forms of water motors, and are discussed below.

Impulse Water Wheels.—This type is the most efficient and economical of any of the pure types of water wheel on the market. It also compares favorably with turbines for moderate heads of water, and surpasses them for high heads. In principle, the impulse water wheel consists essentially of a wheel mounted on a horizontal shaft. Attached to the circumference of the wheel are numerous double-cup-shaped buckets into which one or more tangential jets of water are played from nozzles. The greatest amount of energy of the jet has been imparted to the wheel when the velocity of the jet has been totally destroyed, and this is accomplished by the peculiar shape of the buckets which split the jet in the middle and reverse either half in direction by 180°. When the water simply falls from the buckets, by the action of gravity, the wheel is working with the greatest jet efficiency.

Fig. 1 illustrates the Pelton water wheel with needle and single deflecting nozzle, for economic regulation. The needle nozzle consists of a tapered needle which may be operated so as change the discharge area of the nozzle.

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* The hydraulic ram and the hydraulic pressure engine are not included in this discussion.

† See Farmer's Bulletin No. 46, U. S. Dept. of Agriculture.

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deflecting nozzle is simply a cast-iron nozzle provided with a ball and :et joint. It can be arranged automatically to raise and throw the jet :he buckets or to lower and throw it off. Intermediate positions are lated by change of "load."

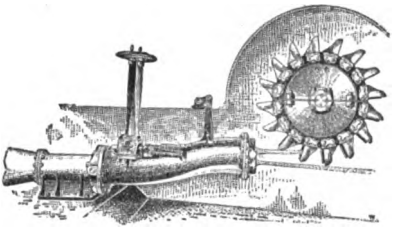

Fig. 1.

The number of nozzles (one, two, or more) which it is advisable to use wheel depends upon the head of water and also upon the amount of 'er required. As a general rule it may be stated that the power devel- d is proportional to the amount of water or number of nozzles used, head of water and size of nozzles remaining constant. Double nozzles often used on small wheels which are called for by certain speed require- ıts and where the single nozzle would not give the required power. quintex nozzle wheel is specially designed for very low head. (See ›le 5, page 1342.)

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4.—Single Nozzle Pelton Water Wheel Data.

(Suitable for high heads.)

Note.—Compare with Table 5 for low heads, making due allowance for 5 nozzles in that table.

The Calculations for Power in these Tables are based upon the application of one stream to the wheel and on *effective* heads. In using these tables liberal allowance should be made to cover the friction loss in pipe, elbows, gates, etc. The smaller figures under those denoting the various heads give the equivalent pressure in pounds per square inch, and spouting velocity of water in feet per minute. The water measurement is also based on the flow *per minute*.

*Head in Feet.	Size of Wheels.	6 Inch	12 Inch.	15 Inch.	18 Inch.	24 Inch.	3 Foot.	4 Foot.	5 Foot.	6 Foot.
20	Horse Power...	.05	.12	.20	.37	.66	1.50	2.64	4.18	6.00
8 Lbs.	Cubic Feet.....	1.67	3.91	6.62	11.72	20.83	46.93	83.32	130.60	187.73
2151.97	Miner's Inches..	1.04	2.44	4.00	7.82	13.88	31.28	55.53	86.90	125.12
	Revolutions....	684	342	274	228	171	114	85	70	57
30	Horse Power....	.10	.23	.38	.69	1.22	2.76	4.88	7.69	11.04
13 Lbs.	Cubic Feet.....	2.05	4.79	8.11	14.36	25.51	57.44	102.04	159.66	229.76
2635.62	Miner's Inches..	1.28	2.99	5.06	9.57	17.00	38.28	68.00	106.44	153.12
	Revolutions....	837	418	335	279	209	139	104	83	69
40	Horse Power....	.15	.35	.59	1.06	1.89	4.24	7.58	11.85	16.96
17 Lbs.	Cubic Feet.....	2.37	5.53	9.37	16.59	29.46	66.36	107.84	184.36	265.44
3043.39	Miner's Inches..	1.48	3.45	5.85	11.06	19.64	44.24	78.56	122.91	176.96
	Revolutions....	969	484	387	323	242	161	121	96	80
50	Horse Power....	.21	.49	.84	1.49	2.65	5.98	10.60	16.63	23.93
21 Lbs.	Cubic Feet.....	2.64	6.18	10.47	18.54	32.93	74.17	131.72	206.13	296.70
3402.61	Miner's Inches..	1.65	3.86	6.54	12.36	21.95	49.45	87.80	137.42	197.80
	Revolutions....	1083	541	433	361	270	180	135	108	90
60	Horse Power....	.28	.65	1.10	1.96	3.48	7.84	13.94	21.77	31.36
26 Lbs.	Cubic Feet	2.90	6.77	11.47	20.31	36.08	81.25	144.32	225.80	325.00
3727.37	Miner's Inches..	1.81	4.23	7.16	13.54	24.05	54.16	96.20	150.53	216.64
	Revolutions....	1185	592	473	395	296	197	148	118	98
70	Horse Power....	.35	.82	1.39	2.47	4.39	9.88	17.58	27.51	39.52
30 Lbs.	Cubic Feet.....	3.13	7.31	12.39	21.94	38.97	87.76	155.88	243.89	351.04
4026.00	Miner's Inches..	1.95	4.56	7.74	14.63	25.98	58.52	103.92	162.60	234.02
	Revolutions....	1281	640	512	427	320	213	160	130	106
80	Horse Power....	.43	1.00	1.70	3.01	5.36	12.04	21.44	33.54	48.16
34 Lbs.	Cubic Feet.....	3.35	7.82	13.25	23.46	41.66	93.84	166.64	260.73	375.36
4303.99	Miner's Inches..	2.09	4.88	8.28	15.64	27.77	62.56	111.08	173.82	250.24
	Revolutions....	1368	684	546	456	342	228	171	137	114
90	Horse Power....	.51	1.20	2.03	3.60	6.39	14.40	25.59	40.04	57.60
39 Lbs.	Cubic Feet.....	3.55	8.29	14.05	24.88	44.19	99.52	176.75	276.55	398.00
4565.04	Miner's Inches..	2.22	5.18	8.78	16.58	29.46	66.32	117.83	184.36	265.28
	Revolutions....	1452	726	581	484	363	242	181	145	121
100	Horse Power....	.60	1.40	2.32	4.21	7.49	16.84	29.93	46.85	67.36
43 Lbs.	Cubic Feet.....	3.74	8.74	14.81	26.22	46.58	104.88	186.32	291.51	419.52
4812.00	Miner's Inches..	2.33	5.46	9.25	17.48	31.05	69.93	124.21	194.34	279.72
	Revolutions....	1530	765	612	510	382	255	191	152	127
110	Horse Power....	.69	1.62	2.74	4.86	8.64	19.44	34.58	54.11	77.76
48 Lbs.	Cubic Feet.....	3.92	9.16	15.53	27.50	48.85	110.00	195.41	305.73	440.00
5046.87	Miner's Inches..	2.45	5.72	9.70	18.33	32.56	73.33	130.27	203.82	293.33
	Revolutions....	1605	802	642	535	401	267	200	160	133

* Theoretic head. The data in table, except in first column, are based on effective heads at 85 per cent of the theoretic heads; or in other words, the efficiency of wheel is assumed at 85 per cent.

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4.—SINGLE NOZZLE PELTON WATER WHEEL DATA.—Continued.

*Head in Feet.	Size of Wheels.	6 Inch	12 Inch.	15 Inch.	18 Inch.	24 Inch.	3 Foot.	4 Foot.	5 Foot.	6 Foot.
120 52 Lbs. 5271.30	Horse Power....	.79	1.84	3.12	5.54	9.85	22.18	39.41	61.66	88.75
	Cubic Feet.....	4.10	9.57	16.21	28.72	51.02	114.91	204.10	319.33	459.64
	Miner's Inches..	2.56	5.98	10.13	19.15	34.01	76.60	136.06	212.89	306.43
	Revolutions....	1677	838	671	559	419	279	209	167	139
130 56 Lbs. 5486.54	Horse Power....	.89	2.08	3.53	6.25	11.11	25.02	44.46	69.53	100.08
	Cubic Feet.....	4.27	9.96	16.89	29.90	53.10	119.60	212.43	332.37	478.41
	Miner's Inches..	2.66	6.22	10.55	19.93	35.40	79.73	141.62	221.58	318.94
	Revolutions....	1746	873	698	582	436	291	218	174	145
140 60 Lbs. 5693.65	Horse Power....	.99	2.33	3.94	6.99	12.41	27.96	49.64	77.71	111.85
	Cubic Feet.....	4.43	10.34	17.53	31.03	55.11	124.12	220.44	344.92	496.48
	Miner's Inches..	2.76	6.46	10.95	20.68	36.74	82.72	146.96	229.94	330.88
	Revolutions....	1812	906	725	604	453	302	226	181	151
150 65 Lbs. 5893.44	Horse Power....	1.10	2.58	4.37	7.75	13.77	31.01	55.08	86.22	124.04
	Cubic Feet.....	4.55	10.70	18.14	32.11	57.04	128.47	228.19	357.02	513.90
	Miner's Inches..	2.84	6.68	11.33	21.41	38.03	85.64	152.12	238.05	342.59
	Revolutions....	1875	937	750	625	468	312	234	187	156
160 69 Lbs. 6086.74	Horse Power....	1.22	2.84	4.82	8.54	15.17	34.16	60.68	94.94	136.65
	Cubic Feet.....	4.73	11.05	18.74	33.17	58.92	132.68	235.68	368.73	530.75
	Miner's Inches..	2.95	6.90	11.71	22.11	39.28	88.46	157.12	245.82	353.84
	Revolutions....	1938	969	775	646	484	323	242	193	161
180 78 Lbs. 6455.97	Horse Power....	1.45	3.39	5.75	10.19	18.10	40.77	72.41	113.30	163.08
	Cubic Feet.....	5.02	11.72	19.87	35.18	62.49	140.74	249.97	391.10	562.96
	Miner's Inches..	3.13	7.32	12.41	23.45	41.66	93.82	166.64	260.73	375.29
	Revolutions....	2049	1024	820	683	513	342	256	206	171
200 87 Lbs. 6805.17	Horse Power....	1.70	3.97	6.74	11.93	21.20	47.75	84.81	132.70	191.00
	Cubic Feet.....	5.29	12.36	20.94	37.08	65.87	148.35	263.49	412.25	593.40
	Miner's Inches..	3.30	7.72	13.08	24.72	43.91	98.90	175.66	274.83	395.60
	Revolutions....	2160	1080	864	720	540	360	270	216	180
220 95 Lbs. 7137.35	Horse Power....	1.96	4.59	7.77	13.77	24.46	55.09	97.85	153.10	220.36
	Cubic Feet.....	5.55	12.96	21.96	38.89	69.08	155.59	276.35	432.38	622.36
	Miner's Inches..	3.46	8.10	13.72	25.93	46.05	103.72	184.23	288.25	414.91
	Revolutions....	2268	1134	906	756	567	378	283	226	189
240 105 Lbs. 7454.70	Horse Power....	2.24	5.23	8.86	15.69	27.87	62.77	111.50	174.45	251.10
	Cubic Feet.....	5.80	13.54	22.93	40.62	72.16	162.50	288.64	451.60	650.03
	Miner's Inches..	3.62	8.46	14.33	27.08	48.10	108.34	192.43	301.07	433.36
	Revolutions....	2370	1185	948	790	592	395	296	237	197
260 113 Lbs. 7759.10	Horse Power....	2.52	5.89	10.05	17.69	31.43	70.78	125.72	196.71	283.15
	Cubic Feet.....	6.04	14.09	23.88	42.28	75.10	169.14	300.43	470.04	676.59
	Miner's Inches..	3.77	8.80	14.92	28.19	50.07	112.76	200.28	313.36	451.05
	Revolutions....	2466	1233	986	822	617	411	308	247	206
280 121 Lbs. 8052.01	Horse Power....	2.82	6.59	11.16	19.77	35.12	79.11	140.51	219.84	316.44
	Cubic Feet.....	6.26	14.62	24.79	43.88	77.94	175.53	311.77	487.79	702.12
	Miner's Inches.	3.91	9.13	15.49	29.25	51.29	117.02	205.18	325.19	468.06
	Revolutions....	2562	1281	1025	854	639	427	319	255	213
300 130 Lbs. 8334.62	Horse Power....	3.13	7.31	12.38	21.93	38.95	87.73	155.83	243.82	350.94
	Cubic Feet.....	6.48	15.13	25.66	45.42	80.67	181.69	322.71	504.91	726.76
	Miner's Inches..	4.05	9.45	16.03	30.28	53.78	121.12	215.14	336.60	484.51
	Revolutions....	2652	1326	1060	884	663	442	331	265	221
320 139 Lbs. 8607.94	Horse Power....	3.45	8.05	13.64	24.16	42.91	96.65	171.68	268.60	386.62
	Cubic Feet.....	6.70	15.63	26.50	46.91	83.32	187.65	333.29	521.46	750.60
	Miner's Inches..	4.18	9.76	16.56	31.27	55.55	125.10	222.19	347.64	500.40
	Revolutions....	2739	1369	1095	913	685	456	342	274	228

*See Note and Foot-note on preceding page.

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4.—SINGLE NOZZLE PELTON WATER WHEEL DATA.—Continued.

*Head in Feet.	Size of Wheels.	6 Inch	12 Inch.	15 Inch.	18 Inch.	24 Inch.	3 Foot.	4 Foot.	5 Foot.	6 Foot.
340	Horse Power....	3.78	8.82	14.94	26.46	47.00	105.86	188.02	294.18	423.44
147	Cubic Feet.....	6.90	16.12	27.31	48.35	85.88	193.42	343.55	537.51	773.71
Lbs.	Miner's Inches..	4.31	10.07	17.06	32.24	57.26	128.98	229.04	358.34	515.93
8872.89	Revolutions....	2823	1411	1130	941	706	470	353	282	235
360	Horse Power....	4.10	9.61	16.28	28.83	51.21	115.34	204.86	320.52	461.36
156	Cubic Feet.....	7.10	16.58	28.10	49.75	88.37	199.03	353.51	553.10	796.14
Lbs.	Miner's Inches..	4.43	10.36	17.56	33.17	58.91	132.68	235.64	368.73	530.75
9130.14	Revolutions....	2907	1453	1161	969	726	484	363	290	242
380	Horse Power....	4.46	10.42	17.66	31.27	55.54	125.08	222.16	347.60	500.33
165	Cubic Feet.....	7.30	17.04	28.88	51.12	90.80	204.48	363.20	568.25	817.95
Lbs.	Miner's Inches..	4.56	10.65	18.03	34.08	60.53	136.32	242.13	378.83	545.29
9380.32	Revolutions....	2985	1492	1194	995	746	497	373	298	248
400	Horse Power....	4.82	11.25	19.07	33.77	59.98	135.08	239.94	375.40	540.35
173	Cubic Feet.....	7.49	17.48	29.63	52.45	93.16	209.80	372.64	583.02	839.20
Lbs.	Miner's Inches..	4.68	10.92	18.51	34.96	62.10	139.84	248.40	388.68	559.35
9624.00	Revolutions....	3063	1531	1225	1021	765	510	382	306	253
420	Horse Power....	5.19	12.11	20.52	36.33	64.54	145.34	258.16	403.91	581.30
182	Cubic Feet.....	7.67	17.91	30.36	53.74	95.46	214.98	381.84	597.41	859.92
Lbs.	Miner's Inches..	4.79	11.19	18.93	35.83	63.64	143.32	254.56	398.28	573.28
9861.66	Revolutions....	3141	1570	1255	1047	785	523	392	313	261
440	Horse Power....	5.56	12.98	22.01	38.96	69.20	155.85	276.82	433.11	623.40
191	Cubic Feet.....	7.85	18.33	31.07	55.01	97.70	220.04	390.82	611.47	880.16
Lbs.	Miner's Inches..	4.90	11.45	19.41	36.66	65.13	146.64	260.53	407.65	586.56
10093.74	Revolutions....	3213	1606	1285	1071	803	535	401	320	267
460	Horse Power....	5.95	13.88	23.53	41.65	73.97	166.60	295.91	462.97	666.40
200	Cubic Feet.....	8.03	18.74	31.77	56.24	99.90	224.98	399.61	625.32	899.95
Lbs.	Miner's Inches..	5.01	11.71	19.79	37.50	66.60	150.00	266.40	416.88	600.00
10320.58	Revolutions....	3285	1642	1313	1095	821	547	410	337	273
480	Horse Power....	6.34	14.79	25.07	44.39	78.85	177.58	315.42	493.49	710.33
208 Lbs.	Cubic Feet.....	8.20	19.15	32.45	57.45	102.05	229.82	408.30	638.66	919.29
10542.56	Miner's Inches..	5.12	11.96	20.28	38.30	68.00	153.20	272.12	425.78	612.80
	Revolutions....	3357	1678	1343	1119	839	559	419	335	279
500	Horse Power....	6.74	15.73	26.66	47.20	83.83	188.80	335.34	524.66	755.28
217 Lbs.	Cubic Feet.....	8.37	19.54	33.12	58.64	104.15	234.56	416.62	651.83	938.25
10759.96	Miner's Inches..	5.23	12.21	20.72	39.09	69.41	156.36	277.64	434.56	625.44
	Revolutions....	3426	1713	1370	1142	856	571	428	342	285
520	Horse Power....	.....	.....	.....	.....	.....	200.22	355.62	556.39	800.88
226 Lbs.	Cubic Feet.....	.....	.....	.....	.....	.....	239.21	424.87	664.74	956.84
10973.04	Miner's Inches..	.....	.....	.....	.....	.....	159.47	283.24	443.16	637.88
	Revolutions....	.....	.....	.....	.....	.....	582	436	349	291
540	Horse Power....	.....	.....	.....	.....	.....	211.88	376.33	588.80	847.52
234 Lbs.	Cubic Feet.....	.....	.....	.....	.....	.....	243.76	432.96	677.41	975.07
11182.07	Miner's Inches..	.....	.....	.....	.....	.....	162.51	288.64	451.61	650.04
	Revolutions....	.....	.....	.....	.....	.....	593	445	356	296
560	Horse Power....	.....	.....	.....	.....	.....	223.76	397.43	621.82	895.04
243 Lbs.	Cubic Feet.....	.....	.....	.....	.....	.....	248.24	440.91	689.84	992.96
11387.26	Miner's Inches..	.....	.....	.....	.....	.....	165.49	293.94	459.89	661.96
	Revolutions....	.....	.....	.....	.....	.....	604	453	362	302
580	Horse Power....	.....	.....	.....	.....	.....	235.86	418.92	655.43	943.44
252 Lbs.	Cubic Feet.....	.....	.....	.....	.....	.....	252.63	448.71	702.04	1010.54
11588.83	Miner's Inches..	.....	.....	.....	.....	.....	168.42	299.14	468.03	673.69
	Revolutions....	.....	.....	.....	.....	.....	615	461	369	307

*See Note and Foot-note on second page preceding.

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4.—SINGLE NOZZLE PELTON WATER WHEEL DATA.—Concluded.

*Head in Feet.	Size of Wheels.	6 Inch	12 Inch.	15 Inch.	18 Inch.	24 Inch.	3 Foot.	4 Foot.	5 Foot.	6 Foot.
600	Horse Power....	.....	.....	.....	.....	.....	248.16	440.77	689.63	992.65
260 Lbs.	Cubic Feet.....	.....	.....	.....	.....	.....	256.95	456.38	714.05	1027.80
11786.94	Miner's Inches..	.....	.....	.....	.....	.....	171.30	304.24	476.03	685.20
	Revolutions....	.....	.....	.....	.....	.....	625	469	375	312
650	Horse Power....	.....	.....	.....	.....	.....	279.82	497.01	777.62	1119.29
282 Lbs.	Cubic Feet.....	.....	.....	.....	.....	.....	267.44	475.02	743.21	1069.77
12268.24	Miner's Inches..	.....	.....	.....	.....	.....	178.29	316.68	495.47	713.18
	Revolutions....	.....	.....	.....	.....	.....	651	488	390	325
700	Horse Power....	.....	.....	.....	.....	.....	312.73	555.46	869.06	1250.92
304 Lbs.	Cubic Feet.....	.....	.....	.....	.....	.....	277.54	492.95	771.26	1110.16
12731.34	Miner's Inches	.....	.....	.....	.....	.....	185.02	328.63	514.18	740.09
	Revolutions....	.....	.....	.....	.....	.....	675	506	405	337
750	Horse Power....	.....	.....	.....	.....	.....	346.83	616.03	963.82	1387.34
326 Lbs.	Cubic Feet.....	.....	.....	.....	.....	.....	287.28	510.35	798.33	1149.13
13178.19	Miner's Inches..	.....	.....	.....	.....	.....	191.52	340.16	532.22	766.09
	Revolutions ...	.....	.....	.....	.....	.....	699	524	419	349
800	Horse Power....	.....	.....	.....	.....	.....	382.09	678.66	1061.81	1528.36
348 Lbs.	Cubic Feet.....	.....	.....	.....	.....	.....	296.70	526.99	824.51	1186.81
13610.40	Miner's Inches	.....	.....	.....	.....	.....	197.80	351.32	549.68	791.21
	Revolutions....	.....	.....	.....	.....	.....	722	542	433	361
900	Horse Power....	.....	.....	.....	.....	.....	455.94	809.82	1267.02	1823 76
391 Lbs.	Cubic Feet.....	.....	.....	.....	.....	.....	314.70	558.96	874.53	1258.81
14436.00	Miner's Inches..	.....	.....	.....	.....	.....	209.80	372.64	583.02	839.20
	Revolutions....	.....	.....	.....	.....	.....	766	574	459	383
1000	Horse Power....	.....	.....	.....	.....	.....	534.01	948.48	1483.97	2136.04
434 Lbs.	Cubic Feet.....	.....	.....	.....	.....	.....	331.72	589.19	921.83	1326.91
15216.89	Miner's Inches..	.....	.....	.....	.....	.....	221.15	392.79	614.56	884.61
	Revolutions....	.....	.....	.....	.....	.....	807	605	484	403

*See Note and foot-note on third page preceding.

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5.—Quintex Nozzle Pelton Water Wheel Data.
(Five nozzles, suitable for low heads.)

Note.—Compare with Table 4, preceding, making due allowance for 5 nozzles in this table.

Revolutions are number *per minute.*

Head in Feet.	Size of Wheels.	18 In.	24 In.	36 In.	42 In.	48 In.	Head in Feet.	Size of Wheels.	18 In.	24 In.	36 In.	42 In.	48 In.
10	Horse Power	.91	1.33	2.95	5.17	8.17	60	Horse Power	13.3	19.3	43.5	77.4	120.
	Revolutions.	141	106	71	60	53		Revolutions.	345	259	173	148	129
15	Horse Power	1.68	2.42	5.43	9.68	15.0	65	Horse Power	15.1	21.9	49.1	86.3	136.
	Revolutions.	172	129	86	74	65		Revolutions.	359	269	180	154	135
20	Horse Power	2.58	3.72	8.35	14.9	23.1	70	Horse Power	16.9	24.4	54.8	97.4	152.
	Revolutions.	199	149	100	85	75		Revolutions.	373	279	186	160	140
25	Horse Power	3.60	5.20	11.7	20.8	32.3	75	Horse Power	18.7	26.3	60.7	108.	168.
	Revolutions.	223	167	111	95	84		Revolutions.	386	289	193	165	145
30	Horse Power	4.72	6.83	15.3	27.3	42.5	80	Horse Power	20.5	29.7	66.7	119.	184.
	Revolutions.	244	183	122	103	92		Revolutions.	399	299	199	171	149
35	Horse Power	5.90	8.62	19.3	34.4	53.5	85	Horse Power	22.6	32.5	73.3	130.	203.
	Revolutions.	263	197	132	113	99		Revolutions.	410	308	205	176	154
40	Horse Power	7.30	10.5	23.6	42.2	65.4	90	Horse Power	24.6	35.4	80.0	142.	221.
	Revolutions.	282	212	141	121	105		Revolutions.	423	317	211	181	158
45	Horse Power	8.68	12.6	28.2	50.2	78.0	95	Horse Power	26.8	38.5	86.7	154.	239.
	Revolutions.	299	224	179	128	112		Revolutions.	434	325	218	186	163
50	Horse Power	10.2	14.7	32.9	58.7	91.3	100	Horse Power	28.8	41.6	93.5	166.	258.
	Revolutions.	315	236	158	135	118		Revolutions.	446	334	223	191	167
55	Horse Power	11.7	16.9	38.2	67.7	105.		Weight, lbs..	750–1000	1000–1500	2400–3400	3500–4300	5000–6000
	Revolutions.	331	248	165	142	124							

Turbine Water Wheels.—Turbines differ from the wheels, described above, in this respect: That in the case of the wheels the water acts only upon a portion of their circumference at any one instant, while with the turbines the water acts symmetrically and uniformly upon the moving parts. Turbines consist essentially of two main parts, namely, (1) the wheel (runner) with vanes arranged around the circumference, and revolving on a vertical or horizontal shaft; and (2) the casing, provided with fixed guide vanes to give direction to the flow of water before it reaches the vanes of the wheel. Turbines are sometimes classified as "radial," "axial" and "combined or mixed;" and again, as *reaction-* and *impulse* turbines. In a radial turbine the water may flow from the circumference inward to the center, or the reverse; in an axial turbine, from the top downward or from the bottom upward; and in a combined turbine, inward and down or up, or outward and down or up. Turbines are most commonly mounted on vertical shafts with water flowing simply inward, outward or downward. The difference in principle between a reaction turbine and an impulse turbine is this: A reaction turbine is driven by the dynamic force of the flowing water augmented by static pressure to a greater or less extent; when there is no static pressure it becomes an impulse turbine. Standard makes of turbines will give an efficiency of from 65 to 85 per cent. An efficiency of 80 per cent can be counted upon for the best makes.

Nomenclature of Terms.—Mr. John W. Thurso suggests the following, for uniformity.

For "water wheel" say "turbine" whenever a turbine is meant.

For "Jonval" turbine say "reaction" turbine. For Girard, Pelton, impulse or free deviation turbine say "action" turbine. For Fourneyron or Boyden turbine say "radial outward flow reaction" turbine, or else simply "outflow reaction" turbine.

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For Francis turbine say "radial inward flow reaction" turbine, or simply "inflow reaction" turbine, as this type is usually understood by the name Francis turbine. While it is not likely that the term Francis turbine will go out of use, the term "inflow reaction" turbine should always be used where an exact expression is of importance, as in contracts, for the reason that Mr. Francis also designed outflow reaction turbines.

For "parallel flow" turbine say "axial" flow turbine. or axial turbine. For "segmental feed" turbine, say "partial feed" turbine. or "partial" turbine.

For one, two, three, four five or six turbines on one shaft say single, double, triple, quadruple, quintuple or sextuple turbine.

For "draft chest" or "camelback" say "draft-tee." For "butterfly gate" say "wing-gate." For "feeder-pipe" or "water feeder" say "penstock."

For open flume say "turbine-chamber" whenever a turbine chamber is meant, leaving the term "flume" to mean a water conductor only, built of wood, steel or masonry, and carrying water not under pressure.

For the distance to which a vertical draft tube reaches below the surface of the tailwater, say dip of draft tube.

For speed of water in the cylindrical part of the penstock say "penstock speed." For speed of water, while leaving or quitting lower end of draft tube, say "draft tube speed."

For the part of the head that is above the turbine say "pressure head." For the part of the head that is utilized by means of a draft tube, say "draft head."

The water available under the head utilized should be called "power water," corresponding in meaning to the term live steam. The water having descended either through the turbines or over the falls, should be called "tail-water," corresponding in meaning to the term exhaust steam.

The terms horizontal or vertical turbine should always mean a turbine on a horizontal or vertical shaft, and never one revolving in a horizontal or vertical plane, as it is now sometimes understood.

The area left open or clear by the regulating gate or gates for the passage of the water should be called "gate opening." At present the term "gate opening" or "gate" is nearly always used to mean the amount of water flowing through the gate opening, but this amount should be designated by discharge; for example, instead of saying: "This turbine, with five-eighths gate opening, gave an efficiency," etc., should be said: "This turbine, with five-eighths discharge, gave an efficiency," etc., whenever the discharge is meant.

The motor furnishing the power for actuating the regulating gates of a turbine, and usually controlled by a speed governor, should be called a relay. In Europe the term servomotor is used for such auxiliary machines.

Losses of Energy in Turbines.—These losses may be segregated as follows:

(1) The hydraulic loss in the casing, from the penstock flange to the entrance to the guides (guide vanes); this is dependent to a large extent on the velocity of flow in the casing, it being remembered that for a "given velocity" the percentage of loss decreases with the increase of head.

(2) The hydraulic loss in the guides and runners; this is affected by the type of runner (wheel) and the design of the guides, but the percentage of hydraulic loss remains practically constant for any one type if the speed is allowed to vary as $\sqrt{h}$, in which h = the head, in feet.

(3) The hydraulic loss due to the leakage around the runner; this, as well as the discharge, varies as the $\sqrt{h}$, hence the ratio of leakage to discharge remains constant.

(4) The hydraulic loss in the draft-tube.

(5) The mechanical loss due to the friction of the revolving parts; this increases with the head, h, but the percentage of mechanical loss decreases with the head.

Efficiencies of Turbines.—The term "efficiency" is often used loosely, and it is very important that in turbine tests, and in specifications and

contracts, the specific kind of efficiency which is meant should be stated There are four kinds of efficiencies which may be employed, namely:

(a) The hydraulic efficiency, embracing losses (1), (2) and (3), above.

(b) The mechanical efficiency, embracing less (5), above

(c) The efficiency of the turbine as a whole, being the product of the hydraulic and mechanical efficiencies, (a) and (b).

(d) The efficiency of the plant as a whole, comprising the turbine efficiency (c) and embracing the draft-tube loss (4) and the penstock loss.

Theoretic Horse-Power of Turbines.—The theoretic horse-power of a turbine varies as $h^{1.5}$, in which h = the head of water in feet. This is true since the horse-power is proportional to the pressure of the water (which in turn varies with h, being equal to about $62.37ah$, see Table 1, preceding), and to the velocity of flow per second (which in turn varies with $\sqrt{h}$, being equal to $v = 8.02\sqrt{h}$). Thus,

$$\text{Theoretic H. P.} = \frac{62.37ah \times 8.02\sqrt{h}}{550} = 0.909468\, ah^{1.5} \quad \ldots\ldots(7)$$

In which a = area of discharge in square feet;
h = head in feet.

Formula (1), preceding, is the same as formula (7), but expressed in terms of discharge Q instead of area a.

When the efficiency of the turbine (or plant) has been determined, the actual H. P. is obtained by multiplying the values of equations (1) or (7) by said efficiency, expressed in per cent.

The Transmission of Power from water motors to machinery is through the main shaft, and may be by belt, by gearing or by coupling. If the power is for electric transmission it is customary to connect the main shaft of the wheel or turbine directly with the electric dynamo or generator. Hence the term "direct-connected generator."

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EXCERPTS AND REFERENCES.

Modern Turbine Practice and the Development of Water Powers (By J. W. Thurso. Eng. News, Dec. 4, 1902 and Jan. 8, 1903).—Illustrations of turbines, hydraulic governors, and power-house installation.

An Analysis of the "Commercial" Value of Water Power per Horse-Power per Annum (By A. F. Nagle. Paper, Am. Soc. Mech. Engrs.; Eng. News, Jan. 22, 1903).—Discussion and tables of cost.

Water Power Development at Chaudiere Falls, P. Q. (Eng. News, May 7, 1903).—Illustrations: Sections of main and wing dams, bulkhead wall, steel framing for gates and screens, operating mechanism for gates and screens, supports and anchorages for penstocks on side hill, plan and section of power house, traveling platform for building main dam, etc.

The Niagara Power Plant of the Electrical Development Co., of Ont. (Eng. News, Nov. 9 and 30, 1905).—Fully illustrated with details.

Power Plant of the Chicago Drainage Canal (Eng. News, Jan. 18, 906).—Illustrated.

Theory of Determining the Principal Dimensions of Water-Turbine Runners (By S. J. Zowski. Eng. News, Jan. 6, 1910).—Formulas, illustrations and diagrams.

Characteristics of the Modern Hydraulic Turbines (By C. W. Larner. Trans. A. S. C. E., Vol. LXVI., Mar., 1910).—Tables of tests, of 28-in., 30-in., 31-in. and 32-in. turbines. Formulas for power, speed, etc.

Some Points in the Design of Impulse-Wheel Buckets (By Geo. M. Peck. Eng. News, May 5, 1910).—Illustrated.

Hydro-Electric Development of the Michoacan Power Co., Mexico (By I. C. McBride. Eng. Rec., Aug. 27, 1910).—Illustrations: Details of rating flume; penstock intake; plan and details of sand trap; plan and details of headgate structure, plan and section of power house substructure.

Important Designs for Reference:—

Description.	Eng. News.
3,000-H. P. turbine for a Niagara water-power plant	Nov. 14, 1901.
Concrete dam with automatic flashboards	June 12, '02.
Section through power house and wheel pit, Niagara Plant	July 3, '02.
A water-wheel governor of novel construction	Nov. 13, '02.
Designs of buckets for impulse water wheels	Oct. 8, '03.
Cross-section of power house, Puyallup power development	Sept. 29, '04.
Section of flume and power station, De Sabla, Cal.	Aug. 10, '05.
Turbines at Sewalls Falls, under low and variable heads	Jan. 18, '06.
10,000-H. P. single-wheel turbine, Snoqualmie Falls	Mar. 29, '06.
Section of flume and power house, Cazadero, Ore.	June 27, '07.
Plan and section of power house, High Falls, Ont.	July 18, '07.
Sections of canal and flume, Centerville, Cal.	Mar. 19, '08.
Herschel's "Fall-increaser" for utilizing waste water	July 11, '08.
Flumes, gates, power station, etc., Kern River, Cal.	Dec. 24, '08.
Comparison of Am. high-speed runners for turbines	Jan. 28, '09.
Designs of intakes for hydro-electric plants	April 8, '09.
	Eng. Rec.
Sections of dam, sluice gate, power house, hydro-elec. plant	Mar. 27, '09.
Expansion-joint details and reinforcement of concrete conduit	May 1, '09.
A low, 20-ft. head hydro-elec. development	May 15, '09.
Plans and sections of Hennepin power plant	May 29, '09.
Details turbine flumes, and their reinforcement, Schnec. Power Co.	July 24, '09.
Sections showing turbine settings, Cent. Ga. Power Co.	May 14, '10.
Details header pipe and upper end of penstock, Gt. W. Power Co.	June 18, '10.
Plan and section power house, Boulder hydro-elec. devel.	July 30, '10.

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69.—STEAM AND GAS POWER.

A.—HEAT.

Matter and Energy.—In the light of modern science, all natural phenomena are due to matter and energy.

Matter may be defined as anything capable of entering into chemical combination, and hence is made up of the chemical elements, of which the total known number, now about 90, may vary with future discoveries. By the Law of the Conservation of Matter, as deduced from chemistry, we learn that matter is indestructible: it may appear in various forms, yet the constituent elements are never actually lost or destroyed. Matter may exist in a solid, liquid, gaseous, or ethereal state.

Energy is matter in motion, and is measured by the formula,

$$\text{Energy} = \tfrac{1}{2}\ \text{mass} \times (\text{velocity})^2;\ \text{or}\ E = \tfrac{1}{2} M V^2 \ldots\ldots\ldots\ldots\ldots (1)$$

in which mass M = force $F \times$ acceleration a,

= weight W ÷ gravity acceleration g.

The Law of the Conservation of Energy, proven by various experiments, teaches us that energy is indestructible: it can appear and disappear in various forms, yet none of it is actually lost or destroyed.

From the foregoing laws of the conservation of matter and energy, and from equation (1), we deduce the following: That the summation of each individual, infinitesimal sub-atom of matter in the universe, multiplied by the square of its velocity, at any single instant, is equal to twice the total energy in the universe and is a constant. Or, applied to any independent system protected from outside influences, the same statement holds true; hence in any such system, we have,

$$\Sigma M V^2 = \text{a constant} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

without regard to any change in *form* of the matter or its motion.

Kinds of Energy.—There are two kinds of energy generally referred to, namely, Kinetic or *actual* ("moving") energy, and Potential or *stored* (static) energy. Strictly speaking, potential energy is simply a term used to express the amonut of kinetic energy which would be expended by a mass in changing itself from one state, condition or position which it has *assumed*, to another state, condition or position which is *predetermined*. The energy expended may be in one or more forms.

Forms of Energy.—There are four principal forms or outward manifestations of energy:

		Examples.
Mass (?) or Mechanical.	Potential:	A raised weight.
	Kinetic:	the falling of a raised weight.
Molecular (?) or Thermal.	Potential:	the latent heat of liquefaction.
	Kinetic:	the release of latent heat.
Atomic (?) or Chemical.	Potential:	the energy stored in dynamite.
	Kinetic:	the discharge of dynamite.
Ethereal (?) or Electrical.	Potential:	the electric charge in a leyden jar.
	Kinetic:	the discharge of a leyden jar.

Other manifestations of energy are light, sound, magnetism, etc.

Transformation of Energy.—Energy which is manifest in one form may appear in another forms:

		Examples.
Mechanical to	Thermal:	(1) Heat resulting from impact of a projectile
"	Chemical:	(2) [No direct transformation wholly kinetic?]
"	Electrical:	(3) Electric current generated from a dynamo.

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Thermal to	Chemical:	(4) Chemical building up of plant life by heat.
"	Electrical:	(5) Current generated by heating a thermo-electric pile.
"	Mechanical:	(6) The mechanical work of a steam engine.
Chemical to	Electrical:	(7) Current generated from a voltaic battery.
"	Mechanical:	(8) Energy of all animal and vegetable life.
"	Thermal:	(9) Heat from oxidation, as combustion of fuel
Electrical to	Mechanical:	(10) Energy of an electric motor or fan.
"	Thermal:	(11) Heat evolved by current through non-conductor.
"	Chemical:	(12) Electrolytic action, as electroplating, etc.

Thermal Energy.—Heat and its effects may be considered as related to molecular action; and the transference of heat, simply as the transference of molecular action from one body to another, or from one medium to another.

First Law of Thermodynamics: Heat and mechanical energy are mutually convertible; and heat requires for its production, and produces by its disappearance, a certain definite number of *units of work* for *each thermal unit*.

Thermal Units.—There are three units as follows:

The *British* unit of heat or British thermal unit (B. T. U.) is equivalent to 778 foot-pounds of work, and may be defined as the quantity of heat required to raise the temperature of one pound of pure water 1° Fahrenheit, at or near its maximum density at 39.1° F. (most authors); or at the more prevailing temperature of 62° F. (Professor Peabody).

The *French* thermal unit or calorie (Cal.) is equivalent to 426.84 meter-kilograms of work, and may be defined as the quantity of heat required to raise the temperature of one kilogram of water 1° centigrade, at about 4° C. (39.1° F.).

The British-French unit of heat or "pound-calorie" (Lb.-Cal.) is equivalent to 1400.4 foot-pounds of work, and may be defined as the quantity of heat required to raise the temperature of one pound of water 1° C., at about 4° C.

Mechanical Equivalent of Heat, *J*.—One Joule* $= J = 778$ ft.-lbs. $=$ 1 British thermal unit (*B. T. U.*), is generally used in the United States as the mechanical equivalent of heat (see above). Hence, $J =$ 1 *B. T. U.* $=$ 778 ft.-lbs. $= 1.41^{\vee}45$ horse-power seconds $= 0.023^{\vee}57$ h.-p. minute $= 0.00039^{\vee}29$ h.-p. hour; and $\frac{1}{J} = 1$ ft.-lb. $= 0.001285$ *B. T. U.* One horse-power hour $=$ 1,980,000 ft.-lbs. $=$ 2545 heat units. One horse-power $=$ 2545 heat units per hour $=$ 25,450 heat units per day of ten hours.

Tables 1 and 2, following, will be found useful in the conversion of mechanical and thermal power and work.

* Joule's original experiments gave $J = 772$ ft.-lbs.

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1.—Thermal-Unit Equivalents.

(See also Table 2, following.)

Heat Units.			Equivalent Mechanical Work.	
British Therm. Units. (1 Lb. 1° F.) B. T. U.	French Therm. Units. (1 Kg. 1° C.) Cal.	Br.-Fr. Therm. Units. (1 Lb. 1° C.) Lb.-Cal.	Foot-Pounds.	Meter-Kilograms.
1.	.252	.5556	778.	107.57
1.8	.4536	1.	1400.4	193.63
2.	.504	1.1111	1556.	215.14
3.	.756	1.6667	2334.	322.71
3.6	.9072	2.	2800.8	387.26
3.968	1.	2.2046	3087.	426.84
4.	1.008	2.2222	3112.	430.28
5.	1.260	2.7778	3890.	537.85
5.4	1.3608	3.	4201.2	580.89
6.	1.512	3.3333	4668.	645.42
7.	1.764	3.8889	5446.	752.99
7.2	1.8144	4.	5601.6	774.52
7.936	2.	4.4092	6174.	853.68
8.	2.016	4.4444	6224.	860.56
9.	2.268	5.	7002.	968.13
10.	2.520	5.5556	7780.	1075.70
10.8	2.7216	6.	8402.4	1161.78
11.904	3.	6.6138	9261.	1280.52
12.6	3.1752	7.	9802.8	1355.41
14.4	3.6288	8.	11203.2	1549.04
15.872	4.	8.8184	12348.	1707.36
16.2	4.0824	9.	12603.6	1742.67
18.0	4.5360	10.	14004.	1936.30
19.840	5.	11.0230	15435.	2134.20
23.808	6	13.2276	18522.	2561.04
27.776	7.	15.4322	21609.	2987.88
31.744	8.	17.6368	24696.	3414.72
35.712	9.	19.8414	27783.	3841.56
39.68	10.	22.046	30870.	4268.40

Ex.—9 B. T. U = 2.268 Cal. = 5 Lb.-Cal. = 7002 ft.-lbs. = 968.13 meter-kilograms.

Caution.—In the above Table it is to be noted that the equivalents are for the same amount of work in each case; thus, 1 B. T. U. and 0.252 Calorie and 0.5556 Lb.-Cal. are each equal to 778 ft.-lbs. of work. But using compound units we have:

1 B. T. U. per pound = $\frac{5}{9}$ Cal. per kilogram = $\frac{5}{9}$ Lb.-Cal. per pound.

1.8 B. T. U. per pound = 1 Cal. per kilogram = 1 Lb.-Cal. per pound.

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2.—Mechanical Equivalents of Heat (B. T. U.).

Thermal.	Mechanical.					
Heat Units (B. T. U.) Per Hour for 1 Hour.	Rate of Work in Foot-Pounds.			Amt. of Work in Horse-Power-		
	Per Min.	Per Hour.	Per Day. (24 Hours.)	Minutes.	Hours.	Days. (24 Hrs.)
Part 1.						
$.0_{4}53556$	.00069$^{\vee}$4	.041$^{\vee}$6	1.	$.0_{5}1^{\vee}26$	$.0_{7}21044$	$.0_{9}87682$
$.0_{3}10711$	.00138$^{\vee}$8	.083$^{\vee}$3	2.	$.0_{5}2^{\vee}52$	$.0_{7}42088$	$.0_{9}17536$
$.0_{3}16067$	.00208$^{\vee}$3	.125$^{\vee}$0	3.	$.0_{5}3^{\vee}78$	$.0_{7}63132$	$.0_{9}26305$
$.0_{3}21422$	.00277$^{\vee}$7	.166$^{\vee}$6	4.	$.0_{5}5^{\vee}05$	$.0_{7}84176$	$.0_{9}35073$
$.0_{3}26778$	.00347$^{\vee}$2	.208$^{\vee}$3	5.	$.0_{5}6^{\vee}31$	$.0_{8}10522$	$.0_{9}43841$
$.0_{3}32134$	.00416$^{\vee}$6	.250$^{\vee}$0	6.	$.0_{5}7^{\vee}57$	$.0_{8}12626$	$.0_{9}52609$
$.0_{3}37489$	.00486$^{\vee}$1	.291$^{\vee}$6	7.	$.0_{5}8^{\vee}83$	$.0_{8}14731$	$.0_{9}61378$
$.0_{3}42845$	.00555$^{\vee}$5	.333$^{\vee}$3	8.	$.0_{4}10^{\vee}10$	$.0_{8}16835$	$.0_{9}70146$
$.0_{3}48200$	.00625$^{\vee}$0	.375$^{\vee}$0	9.	$.0_{4}11^{\vee}36$	$.0_{8}18940$	$.0_{9}78914$
Part 2.						
.001285	.016$^{\vee}$6	1.	24.	.00003$^{\vee}$03	$.0_{6}5^{\vee}05$	$.0_{7}21044$
.002571	.033$^{\vee}$3	2.	48.	.00006$^{\vee}$06	$.0_{6}1^{\vee}01$	$.0_{7}42088$
.003856	.050$^{\vee}$0	3.	72.	.00009$^{\vee}$09	$.0_{6}1^{\vee}51$	$.0_{7}63132$
.005141	.066$^{\vee}$6	4.	96.	.00012$^{\vee}$12	$.0_{6}2^{\vee}02$	$.0_{7}84176$
.006427	.083$^{\vee}$3	5.	120.	.00015$^{\vee}$15	$.0_{6}2^{\vee}52$	$.0_{8}10522$
.007712	.100$^{\vee}$0	6.	144.	.00018$^{\vee}$18	$.0_{6}3^{\vee}03$	$.0_{8}12626$
.008997	.116$^{\vee}$6	7.	168.	.00021$^{\vee}$21	$.0_{6}3^{\vee}53$	$.0_{8}14731$
.010283	.133$^{\vee}$3	8.	192.	.00024$^{\vee}$24	$.0_{6}4^{\vee}04$	$.0_{8}16835$
.011568	.150$^{\vee}$0	9.	216.	.00027$^{\vee}$27	$.0_{6}4^{\vee}54$	$.0_{8}18940$
Part 3.						
.07712	1.	60.	1440	.001$^{\vee}$81	.00003$^{\vee}$03	$.0_{5}1^{\vee}26$
.15424	2.	120.	2880.	.003$^{\vee}$63	.00006$^{\vee}$06	$.0_{5}2^{\vee}52$
.23136	3.	180.	4320.	.005$^{\vee}$45	.00009$^{\vee}$09	$.0_{5}3^{\vee}78$
.30848	4	240.	5760.	.007$^{\vee}$27	.00012$^{\vee}$12	$.0_{5}5^{\vee}05$
.38560	5.	300	7200	.009$^{\vee}$09	.00015$^{\vee}$15	$.0_{5}6^{\vee}31$
.46273	6.	360.	8640.	.010$^{\vee}$90	.00018$^{\vee}$18	$.0_{5}7^{\vee}57$
.53985	7.	420.	10080.	.012$^{\vee}$72	.00021$^{\vee}$21	$.0_{5}8^{\vee}83$
.61697	8.	480.	11520.	.014$^{\vee}$54	.00024$^{\vee}$24	$.0_{4}10^{\vee}10$
.69409	9.	540.	12960.	.016$^{\vee}$36	.00027$^{\vee}$27	$.0_{4}11^{\vee}36$
Part 4.						
1.	12.96$^{\vee}$6	778.	18672.	.023$^{\vee}$57	.00039$^{\vee}$29	$.0_{4}1637$
2.	25.93$^{\vee}$3	1556.	37344.	.047$^{\vee}$15	.00078$^{\vee}$58	$.0_{4}3274$
3.	38.90$^{\vee}$0	2334.	56016.	.070$^{\vee}$72	.0011$^{\vee}$78	$.0_{4}4912$
4.	51.86$^{\vee}$6	3112.	74688.	.094$^{\vee}$30	.0015$^{\vee}$71	$.0_{4}6549$
5.	64.83$^{\vee}$3	3890.	93360.	.117$^{\vee}$87	.0019$^{\vee}$64	$.0_{4}8186$
6.	77.80$^{\vee}$0	4668.	112032.	.141$^{\vee}$45	.0023$^{\vee}$57	$.0_{4}9823$
7.	90.76$^{\vee}$6	5446.	130704.	.165$^{\vee}$03	.0027$^{\vee}$50	$.0_{3}1146$
8.	103.73$^{\vee}$3	6224.	149376.	.188$^{\vee}$60	.0031$^{\vee}$43	$.0_{3}1310$
9.	116.70$^{\vee}$0	7002.	168048.	.212$^{\vee}$18	.0035$^{\vee}$36	$.0_{3}1473$
Part 5.						
42.41$^{\vee}$6	550.	33000.	792000.	1.	.016$^{\vee}$6	.00069$^{\vee}$4
84.83$^{\vee}$3	1100.	66000.	1584000.	2.	.033$^{\vee}$3	.00138$^{\vee}$8
127.25$^{\vee}$0	1650.	99000.	2376000.	3.	.050$^{\vee}$0	.00208$^{\vee}$3
169.66$^{\vee}$6	2200.	132000.	3168000.	4.	.066$^{\vee}$6	.00277$^{\vee}$7
212.08$^{\vee}$3	2750.	165000.	3960000.	5.	.083$^{\vee}$3	.00347$^{\vee}$2
254.50$^{\vee}$0	3300.	198000.	4752000.	6.	.100$^{\vee}$0	.00416$^{\vee}$6
296.91$^{\vee}$6	3850.	231000.	5544000.	7.	.116$^{\vee}$6	.00486$^{\vee}$1
339.33$^{\vee}$3	4400.	264000.	6336000.	8.	.133$^{\vee}$3	.00555$^{\vee}$5
381.75$^{\vee}$0	4950.	297000.	7128000.	9.	.150$^{\vee}$0	.00625$^{\vee}$0
Part 6.						
2545.	33000.	1980000.	47520000.	60.	1.	.041$^{\vee}$6
5090.	66000.	3960000.	95040000.	120.	2.	.083$^{\vee}$3
7635.	99000.	5940000.	142560000.	180.	3.	.125$^{\vee}$0
10180.	132000.	7920000.	190080000.	240.	4.	.166$^{\vee}$6
12725.	165000.	9900000.	237600000.	300.	5.	.208$^{\vee}$3
15270.	198000.	11880000.	285120000.	360.	6.	.250$^{\vee}$0
17815.	231000.	13860000.	332640000.	420.	7.	.291$^{\vee}$6
20360.	264000.	15840000.	380160000.	480.	8.	.333$^{\vee}$3
22905.	297000.	17820000.	427680000.	540.	9.	.375$^{\vee}$0
Part 7.						
61080.	792000.	4752x10^{4}	114048x10^{4}	1440.	24.	1.
122160.	1584000.	9504x10^{4}	228096x10^{4}	2880.	48.	2.
183240.	2376000.	14256x10^{4}	342144x10^{4}	4320.	72.	3.
244320.	3168000.	19008x10^{4}	456192x10^{4}	5760.	96.	4.
305400.	3960000.	23760x10^{4}	570240x10^{4}	7200.	120.	5.
366480.	4752000.	28512x10^{4}	684288x10^{4}	8640.	144.	6.
427560.	5544000.	33264x10^{4}	798336x10^{4}	10080.	168.	7.
488640.	6336000.	38016x10^{4}	912384x10^{4}	11520.	192.	8.
549720.	7128000.	42768x10^{4}	1026432x10^{4}	12960.	216.	9

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Examples in the Use of Table 2, preceding.

Part 1.—1 000 000 ft.-lbs. per day is at the rate of 694 44 ft.-lbs. per min., and equivalent to 53.556 heat units per hour. 53.556 heat units represent an amount of work equal to 1.2626 horse-power min., or .021 h.-p hour, or .000877 h.-p. day.

Part 2.—5 000 000 ft.-lbs. per hour = 6427 heat units per hour. 5 000 000 ft.-lbs., or 6427 heat units, are equal to 2.5252 horse-power hours, or 1 horse-power for 2.5252 hours, or 2.5252 horse-power for 1 hour.

Part 3.—33 000 ft.-lbs. per min. = 2313.6 + 231.36 = 2545 heat units per hour.

Part 4.—One heat unit for any time T is equal to 778 ft.-lbs. in time T; hence Columns 1 and 3 may be used independently of time, throughout the Table, to give Equivalent Work in Ft.-Lbs. and Heat Units.

Part 5.— 60 horse-power min. = 2545 heat units.

Part 6.—357 horse-power hours = 763500 + 127250 + 17815 = 908565 heat units, and equivalent to 706 860 000 ($= 70686 \times 10^4$) ft.-lbs. (per hour for 1 hour.)

Part 7.—Six horse-power days = 366480 heat units; and 366480 heat units per hour represent work at the rate of 6 842 880 000 ft.-lbs. per day.

B.—FUEL.

Heating Power of Fuels.—The amount of heat energy, in *B. T. U.*, contained in coal, wood and other fuels may be determined by three methods: (*a*) by chemical analysis; (*b*) by burning the fuel in a calorimeter; (*c*) by practical test in connection with a steam boiler.

(*a*) *Chemical Analysis.*—There are two kinds of analysis of coal (or fuel), namely, "proximate analysis" and "ultimate analysis."

Proximate Analysis separates the coal into moisture, volatile matter, fixed carbon, and ash. In making this analysis a pulverized sample is weighed carefully and then heated to a temperature not to exceed 280° F. When, after repeated weighings, it ceases to lose more weight, the loss in weight by this heating is recorded as "moisture." The heating is then continued in a crucible with a lid cover, and after the temperature is raised gradually to a red heat and continued for a few minutes until gas ceases to be driven off, the crucible is cooled in a dessicator, to prevent absorption of moisture from the air, and weighed, the loss in weight by this heating being recorded as "volatile matter." The heating is then continued and the crucible raised to a white heat, the lid being partly open to admit air to burn the coal, and when the carbon is burned away, leaving nothing but the ash, the latter is weighed when cooled and the loss in weight by this heating is recorded as "fixed carbon." The weight of the ash is of course recorded as "ash". If the weighings are recorded in percentages the sum of the four constituent parts will add up to 100 per cent. Sometimes "sulphur" is included in the proximate analysis, either as part of the 100 per cent above mentioned or separately, the percentage of sulphur being determined by separate analysis, and 40 to 50% assumed to have escaped with the volatile matter and 60 to 50% with the fixed carbon.

The proximate analysis, above, indicates the character of the coal. Omitting the moisture and the ash, and letting the sum of the "volatile matter" and "fixed carbon" equal 100 per cent of the "combustible," we have the following classification:

3.—Coals Classified by Relative Percentages of Carbon and Volatiles.

Kind of Coal.	Per cent Fixed Carbon.	Per cent. Volatile Matter.	Heating Value. B. T. U. per Lb. Coal.	Relative Combustible Value.
Anthracite	100 to 92	0 to 8	14600 to 14800	93
Semi-Anthracite	92 to 87	8 to 13	14700 to 15000	94
Semi-Bituminous	87 to 75	13 to 25	15500 to 16000	100
Bituminous, Eastern	75 to 60	25 to 40	14800 to 15200	95
Bituminous, Western	65 to 50	35 to 50	13500 to 14800	90
Lignite	under 50	over 50	11000 to 13500	77

4.—Proximate Analysis and Heating Values of U. S. Coals.
Note.—See Table 5 for Ultimate Anaylses of Fuels.

Kind of Coal.	Moisture.	Volatile Matter.	Fixed Carbon.	Ash.	Sulphur.	Heating Value per lb. Coal. Heat Units.	Volatile Matter per cent. of combustible.	Heating Value per lb. Combustible. Heat Units.	Theoretical Evoporation lbs. Water from and at 212° per lb. Combustible.
Anthracite.									
Northern coal field	3.42	4.38	83.27	8.20	.73	13,160	5.00	14,900	15.42
East Middle coal field	3.71	3.08	86.40	6.22	.58	13,420	3.44	14,900	15.42
West Middle coal field	3.16	3.72	81.59	10.65	.50	12,840	4.36	14,900	15.42
Southern coal field	3.09	4.28	83.81	8.18	.64	13,220	4.85	14,900	15.42
Semi-Anthracite.									
Loyalsock field	1.30	8.10	83.34	6.23	1.63	13,920	8.86	15,500	16.05
Bernice basin	.65	9.40	83.69	5.34	.91	13,700	10.98	15,500	16.05
Semi-Bituminous.									
Broad Top, Pa.	.79	15.61	77.30	5.40	.90	14,820	17.60	15,800	16.36
Clearfield County, Pa.	.76	22.52	71.82	3.99	.91	14,950	24.60	15,700	16.25
Cambria County, Pa.	.94	19.20	71.12	7.04	1.70	14,450	22.71	15,700	16.25
Somerset County, Pa.	1.58	16.42	71.51	8.62	1.87	14,200	20.37	15,800	16.36
Cumberland, Md.	1.09	17.30	73.12	7.75	.74	14,400	19.79	15,800	16.36
Pocahontas, Va.	1.00	21.00	74.39	3.03	.58	15,070	22.50	15,700	16.25
New River, W. Va.	.85	17.88	77.64	3.36	.27	15,220	18.95	15,800	16.36
Bituminous.									
Connellsville, Pa.	1.26	30.12	59.61	8.23	.78	14,050	34.03	15,300	15.84
Youghiogheny, Pa.	1.03	36.50	59.05	2.61	.81	14,450	38.73	15,000	15.53
Pittsburg, Pa.	1.37	35.90	52.21	8.02	1.80	13,410	41.61	14,800	15.32
Jefferson County, Pa.	1.21	32.53	60.99	4.27	1.00	14,370	35.47	15,200	15.74
Middle Kittanning seam, Pa	1.81	35.33	53.70	7.18	1.98	13,200	40.27	14,500	15.01
Up. Freeport seam, Pa. and O	1.93	35.90	50.19	9.10	2.89	13,170	43.59	14,800	15.32
Thacker, W. Va.	1.38	35.04	56.03	6.27	1.28	14,040	39.33	15,200	15.74
Jackson County, O.	3.83	32.07	57.60	6.50	.....	13,090	35.76	14,600	15.11
Brier Hill, O.	4.80	34.60	56.30	4.30	.....	13,010	38.20	14,300	14.80
Hocking Valley, O.	6.59	34.97	48.85	8.00	1.59	12,130	42.81	14,200	14.70
Vanderpool, Ky.	4.00	34.10	54.60	7.30	.....	12,770	38.50	14,400	14.91
Muhlenberg County, Ky.	4.33	33.65	55.50	4.95	1.57	13,060	38.86	14,400?	14.91
Scott County, Tenn.	1.26	35.76	53.14	8.02	1.80	13,700	34.17	15,100?	15.63
Jefferson County, Ala.	1.55	34.44	59.77	2.62	1.42	13,770	37.63	14,400?	14.91
Big Muddy, Ill.	7.50	30.70	53.80	8.00	.....	12,420	36.30	14,700	15.22
Mt. Olive, Ill.	11.00	35.65	37.10	13.00	.....	10,490	47.00	13,800	14.29
Streator, Ill.	12.00	33.30	40.70	14.00	.....	10,580	45.00	14,300	14.80
Missouri	6.44	37.57	47.94	8.05	.....	12,230	43.94	14,300?	14.80
Lignites and Lignitic Coals.									
Iowa	8.45	37.09	35.60	18.86	.....	8,720	51.03	12,000?	12.42
Wyoming	8.19	38.72	41.83	11.26	.....	10,390	48.07	12,900?	13.35
Utah	9.29	41.97	44.37	3.20	1.18	11,030	48.60	12,600?	13.04
Oregon Lignite	15.25	42.98	33.32	7.11	1.66	8,540	54.95	11,000?	11.39

Note.—The following values are given for Anthracite coal from one mine: Egg coal (screen 2½"–1¾") 88.49% carbon, 5.66% ash; stove coal (screen 1¾"–1¼") 83.67% carbon, 10.17% ash; chestnut coal (screen 1¼"–¾") 80.72% carbon, 12.67% ash; pea coal (screen ¾"–½") 79.05% carbon, 14.66% ash; buckwheat coal (screen ½"–¼") 76.92% carbon, 16.62% ash.

Ultimate Analysis reduces the "combustible" constitutents of the fuel, *i. e.*, the "volatile matter" and "fixed carbon" (but not the moisture and ash) to the ultimate chemical elements.

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5.—Chemical Composition of Several Kinds of Solid Fuels.

(Ultimate Analyses.)

Note.—See Table 4 for Proximate Analyses of Coals.

Kind of Fuel.	Moisture.	Carbon.	Hydrogen	Oxygen.	Nitrogen.	Sulphur.	Ash.	B.T.U. per Lb. of Fuel.
Wood, dry, average	0	49.5	6.1	43.8	0.1	.....	0.5	8800
" " Ash	0	49.18	6.27	43.91	0 07	.....	0.57	8480
" " Beech	0	49.06	6.11	44.17	0.09	.....	0.57	8591
" " Birch	0	48.88	6.06	44.67	0.10	.....	0.29	8586
" " Elm	0	48.99	6.20	44.25	0.06	.....	0.50	8510
" " Fir	0	50.36	5.92	43.39	0.05	.....	0.28	9063
" " Oak	0	50.16	6.02	43.36	0.09	.....	0.37	9316
" " Pine	0	50.31	6.20	43.08	0.04	.....	0.37	9153
" 10% moisture, av	10.0	44.5	5.5	39.4	0.1	.....	0.5	.....
" 20% " "	20.0	39.6	4.9	35.0	0.1	.....	0.4	.....
" 40% " "	40.0	29.7	3.7	26.2	0.1	....	0.3	.....
Peat	30.0	40.6	4.2	21.7	.....	.....	3 5	.....
Charcoal	12.0	84	1.0	0.	.....	.....	3.	.....
Straw	16.0	36.	5.0	38.0	.....	.....	5.	.....
Coal, Anthracite	1.0	86.	1.0	1.0	0.5	0.5	10.	.....
" Semi-Bituminous	1.0	84.	4.2	3.4	0.8	0.6	6.	.....
" Bituminous Pitts'g	1.4	75.	5 0	8.0	1.0	1.6	8.	.....
" " Hocking Val., O	7.5	67.	4.8	10.0	1.2	1.5	8.	.....
" " Illinois	11.0	56.	5.0	11.0	1.0	3.0	13.	.....
Brown Coal, Pac. Coast	17.4	50.	4.0	14.0	1 0	.....	13.6	.....
Lignite, Pacific Coast	15 8	55.	4.5	16.9	1.1	1.1	5.6	.....

Green wood contains from 25 to 50 per cent moisture. Air-dried wood contains from 10 to 20 per cent moisture—usually 12 to 15 per cent.

Calculations of the Heat of Combustion of a fuel, based on the ultimate analysis, are usually performed by means of Dulong's formula, or by means of other formulas which resemble, or are more or less modifications of, Dulong's formula. Thus,

The total heat units (*B. T. U.*) per lb. of coal—

$$= 14650\,C + 62100\,(H - \tfrac{1}{8}\,O) \ldots\ldots \text{(Dulong)} \ldots\ldots\ldots (1)$$

$$= 14650\,C + 62100\,H - 5400\,(O + N) \text{ (Mahler)} \ldots\ldots\ldots (2)$$

In which C, H, O and N represent the relative parts of carbon, hydrogen, oxygen and nitrogen in the coal.

Example.—The ultimate analysis of a certain coal gave, in parts (percentages expressed in parts of a unit), carbon $(C) = 0.8566$, hydrogen $(H) = 0.0278$, nitrogen $(N) = 0.0077$, oxygen $(O) = 0.0287$, ash $= 0.0733$, volatile sulphur $= 0.0059$. How many heat units would one pound of this coal be expected to supply, or in other words, what is the total heat of combustion?

Solution.—From Dulong's formula, total heat $= 14650 \times .8566 + 62100\,(.0278 - \frac{1}{8} \times .0287) = 14052$ *B. T. U.*; and from Mahler's formula, total heat $= 14079$ *B. T. U.*

Mr. Henry J. Williams* gives the following values for heat of combustion, as calculated by Dulong's formula:—Anthracites: Lehigh, 13963 *B. T. U.*, Lykens Valley, 13954; Drifton, Pa., 14171. Semi-bituminous: Pocahontas, 14805; Georges Creek, 14484; Clearfield, Pa., 14448; New River, W. Va., 14607. Bituminous: Connellsville (coking), 14043; Big Muddy, Carterville, Ill., 12561; Dominion, Cape Breton, 13755; Pittsburg (steaming), 13719. But compare these heating values with those in Table 4, preceding.

(*b*) *Calorimeter.*—The "bomb calorimeter" is an instrument used to determine the actual heating value of coal. Mahler's instrument consists of a strong steel vessel immersed in water. One gram of the coal is placed in a platinum vessel within the bomb, oxygen gas is introduced, and the coal

* See page page 41, *Steam Boilers*, by Peabody and Miller; John Wiley & Sons, New York.

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ignited by an electric spark. The resultant heat of combustion is radiated into the surrounding water and is measured by the rise in temperature of the water, making the necessary corrections for absorption of heat by the instrument itself.

This method agrees usually within 2 per cent of the calculations from chemical analysis, when the tests are carefully made.

(*c*) *Practical* (Boiler) *Test.*—The following Table gives the summary of nine tests made on one 250 *H. P.* Cahall boiler at factory of the Armstrong Cork Co., Pittsburg, Pa., 1896:

6.—AVERAGE OF NINE BOILER TESTS—COAL AS FUEL.

Duration of test	hours	8¾
No. of boilers		1
Average Pressure of Steam in Boiler by Gage		98.8
Average Temperatures.		
Of feed-water entering boiler	deg. F.	62.6
Of steam in boiler	deg. F.	334.2
Fuel (Nut, Nut and Slack, Run of mine, etc.) *Coal.*		
Cost per ton of 2,000 pounds, delivered		$1.01
Calorific power by Calorimeter	B. T. U.	12,963.5
Total quantity consumed	lbs.	10,132.8
Total ash, clinkers and unburned coal	lbs.	975.06
Proportion of ash, etc., to coal	per cent	9.69
Total combustible burned	lbs.	8,961.94
Combustion per Hour.		
Coal actually consumed	lbs.	1,190.29
Combustible actually consumed	lbs.	1,082.79
Per sq. ft. grate surface—coal	lbs.	31.03
Per sq. ft. grate surface—combustible	lbs.	32.54
Per sq ft. heating surface—coal	lbs.	.47
Per sq. ft. heating surface—combustible	lbs.	.43
Water.		
Amount apparently evaporated	lbs.	86,199.08
Factor of evaporation		1.20
Equivalent evaporation into dry steam from and at 212°F.	lbs.	103,007.73
Economic Evaporation—per pound of coal.		
Water actually evaporated	bls.	8.735
Equivalent from and at 212° F.	lbs.	10.43
Per pound of combustible—water actually evaporated	lbs.	9.1106
Equivalent from and at 212° F.	lbs.	11.5869
Evaporation per Hour.		
Water actually evaporated	lbs.	10,448.57
Equivalent from and at 212° F.	lbs.	12,582.82
Per sq. ft. heating surface—water actually evaporated	lbs.	4.03
Equivalent from and at 212° F	lbs.	4.9571
Per sq. ft. grate surface—water actually evaporated	lbs.	298.53
Equivalent from and at 212° F.	lbs.	320.68
Efficiency.		
Percentage of total calorific power utilized, or efficiency	%	77.867
Water evaporated per $1.00 worth of fuel	lbs.	19,858.47
Cost of evaporating 1,000 lbs. or water	cents	4.98
Coal consumed per horse power per hour	lbs.	3.35
Cost of same	cents	0.1724
Horse Power.		
Actually developed on basis of 34½ lbs. of water evaporated per hour from and at 212° F.		364.708
Commercial rating		250.0
Proportion capacity developed is of commercial rating	%	145.859
Heating surface required to develop one horse power	sq. ft.	7.3601

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C.—STEAM.

General Discussion.—If one pound of ice at absolute zero (= − 273.1° C = − 460.66° F.) is gradually heated, its temperature will rise about directly in proportion to the amount of heat it receives (say approx. 1° C. per each Calorie, or 1° F. for each B. T. U.) until it reaches the melting point or fusion point (= 0° C. = + 32° F.); then its temperature will remain constant during melting or until it has received 143 additional B. T. U., making a total of about 460.66 + 32 + 143 = 635.66 B. T. U. to change one pound of ice at absolute zero into one pound of water at the temperature of freezing.*

The 143 B. T. U. required to change one pound of ice at + 32° F. into one pound of water at 32° F. is called the latent heat of fusion of ice, and represents the work required to change the molecular condition of ice to the molecular condition of water, since there is no rise in temperature. *Conversely*, one pound of water at 32° F. will *give out* 143 B. T. U. in changing to ice at 32° F.

If one pound of water is gradually heated in an open vessel, under an atmospheric pressure of 14.7 pounds per square inch, from freezing point at 32° F. up to the boiling point at 212° F., its temperature will rise almost directly in proportion to the amount of heat received, i. e., 1° F. for each B. T. U., or a total of 180 B. T. U.; then its temperature will remain constant during the process of boiling away, called vaporization. But if the atmospheric pressure on the surface of the water had been increased, its boiling point would have been raised. This is shown clearly in the first two columns of Tables 7 and 8, following. Thus, from Table 8, an *absolute* pressure of 90 pounds per square inch would raise the boiling point temperature to 320° F. and it would remain at that temperature until completely vaporized. In this case, the pound of water would become the receptacle of about 320 − 212 = 108 B. T. U. more than it could receive under ordinary atmospheric pressure. It is thus seen that the temperature under which steam is generated, depends upon the pressure of the liquid.

Starting with one pound of water at 32° F., and heating it, we find from Column 3, Tables 7 and 8, the number of heat units received at the various boiling-point temperatures, under corresponding pressures; and if the heating is continued at the boiling point, Column 5 shows the number of heat units required to convert the water into steam at the same temperature, called the heat of vaporization; and Column 4 shows the total heat in B. T. U., of the steam above that which the water contained at 32° F. Thus, for a temperature of 320° F., the total heat = 1179.5 = 290.0 + 889.5 = heat of the liquid + heat of vaporization.

It is to be remembered that the heat of vaporization, or latent heat of vaporization, represents the number of heat units consumed in changing one pound of water into steam at the same temperature. Now as the temperature is not raised, it is evident that some other work is being performed. In fact, the latent heat is converted into the work of separating the molecules of the water: (1) against molecular attraction, and (2) against external resistance or pressure. The 1st is called the heat equivalent of internal work (Table 7, Column 6), and the 2nd is called the heat equivalent of external work (Column 7). It is evident, then, that the value in Column 5 of Table 7, for any definite temperature or pressure, is equal to the sum of the corresponding values in Columns 6 and 7.

Steam generated in a closed vessel is necessarily in contact with the water and must have the same temperature and pressure as the water. In this condition it is called wet saturated steam or "wet steam." Its specific volume, or volume in cubic feet of one pound, and its density, or weight in pounds per cubic foot, are each constant for any definite temperature and pressure, as will be seen from the last two columns of Table 7. Just at the point when the water is completely evaporated the steam is known as dry saturated steam or "saturated steam." If further heated, it becomes superheated steam. Summarizing, we have wet steam from the time water begins to evaporate up to the time of complete evaporation; saturated steam at the moment of complete evaporation; and superheated steam when further heated.

Furthermore, *saturated* steam is dry steam of a temperature due to its pressure; that is, it contains no moisture, nor is it superheated. When it is heated above a temperature due to its pressure it becomes *superheated*

* This illustration is given simply to convey graphically to the mind the general effect of heat; the exact values are not important.

steam. *Dry* steam may be either saturated or superheated. If water is injected into superheated steam it will immediately vaporize, reducing the temperature of the latter; and if sufficient water is injected the steam will be reduced to saturated steam. If cold water is injected into saturated steam some of the steam will be condensed, and the temperature and pressure of the remainder will be lowered. *Wet* steam contains mist or globules of moisture, but its temperature and pressure bear the same relation as that for saturated steam. *Saturated* steam, then, can have but one temperature for any given pressure, while superheated steam can have any higher temperature.

Superheated Steam tends to follow the laws of perfect gases, the more nearly so the farther it is removed from the point of saturation From the laws of gases, we have,

$$pv = RT \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

In which p = pressure in lbs. per sq. in.;
v = volume in cu. ft.;
T = temp. in degrees F. above absolute zero;
R = a constant, depending upon the gas;
= foot-pounds of external work done in altering the temp. 1° under constant pressure;
= 53.2 for air;
= for any gas, 53.2 + the specific gravity of the gas, referred to air.

Saturated Steam.—The laws of saturated steam do not follow closely the laws of perfect gases, yet have a close analogy to them. The following notation and formulas are mainly those used in *Thermodynamics of the Steam Engine*,* by Prof C. H. Peabody (under whom the writer studied), and are used here with his permission. See also Tables 7 to 10.

Notation and Formulas.

F = fahrenheit (absolute zero = −460.66 F.);
C = centigrade (absolute zero = −273.7 C.);
t = temperature in degrees F. = $\frac{9}{5} t_0 + 32$;
t_0 = temperature in degrees C. = $\frac{5}{9}(t - 32)$;
T = absolute temperature in degrees F. = 460.66 + t;
T_0 = absolute temperature in degrees C. = 273.7 + t_0;

Observe the algebraic signs: + for above 0°; − for below 0°.

p = pressure (above vacuum) of saturated steam in lbs. per square inch.

$$\text{Log } p = A - \frac{B}{T} - \frac{D}{T^2}$$

In which $A = 6.1007$; $\log B = 3.43642$; $\log D = 5.59873$; and using $T = t + 461.2°$ F.

q = heat of the liquid; that is, the quantity of heat, in French units (number of heat units of 426.84 meter-kgs.), required to raise the temperature of one kilogram of water from freezing (0° C.) to a given temperature t_0;
$= t_0 + 0.000002\, t_0^2 + 0.0000003\, t_0^3$.
$= \int c\, dt$, in English units.

λ = total heat; that is, the quantity of heat (number of heat units of 778 ft.-lbs.) required to raise one pound of water from freezing (32° F.) to a given temperature t, and to entirely vaporize it under the pressure due to that temperature;
$= 1091.7 + 0.305\,(t - 32)$. But see Marks and Davis formula, page 1378.

r = heat of vaporization, which may be defined as the number of heat units, of 778 ft.-lbs., required to vaporize one pound of water at a given temperature under the corresponding pressure;
= total heat − heat of liquid = $\lambda - q$.

ρ = heat equivalent of internal work; that is, the internal latent heat or the heat units, of 778 ft.-lbs., required to do the disintegration work during the vaporization of one pound of water;
$= r - A\, p\, u = r - A\, p\, (s - \delta)$;
where A = reciprocal of mechanical equivalent of heat,

$$= \frac{1}{J} = \frac{1}{778} = 0.00128535, \text{ and}$$

* Published by John Wiley & Sons, New York.

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s = specific volume of saturated steam; that is, the volume in cubic feet of one pound of saturated steam.

δ = specific volume of the water.

$pu = p(s - \delta)$ = external work.

Apu = heat equivalent of external work; that is, the external latent heat or the heat units, of 778 ft.-lbs., required to overcome the external pressure, and do the work of increasing the volume from δ to s.

c = specific heat of the liquid $= \frac{dq}{dt}$.

$\int \frac{cdt}{T}$ = entropy of the liquid; the term entropy is used to denote a quality or condition of the liquid, increasing when heat is added, and decreasing when heat is subtracted; the entropy remains constant when no heat is added or subtracted; it is a very useful property in all thermodynamic calculations *

s = specific volume, or volume in cubic feet of one pound of steam.

r = density, or weight in pounds of one cubic foot of steam;

$= \frac{1}{s}$

SATURATED-STEAM TABLES.—Tables 7 and 8, following, are the old tables which have been used by engineers for the past ten years. These are followed by Tables 9 and 10, which are calculated from more recent experimental data. The older tables are reproduced here simply for methods of comparison, and Tables 9 and 10 should be used because the values are more correct.

The formulas given above are the old formulas. The new formula for heat of vaporization, in English units, is

$$r = 141.124\,(689 - t)^{0.3129}.$$

* Entropy Diagrams are very useful for this purpose. They represent, graphically, the successive thermal changes in a body due to the simultaneous variations of Temperature and Entropy—two of the coordinates characterizing the conditions of the body. In this connection it may be stated that *Total Heat* represents energy or *Work*. Now, *Work* is made up of two factors, *force* and *distance* ($W = fd$). Similarly, the *Total Heat* may be considered as made up of two factors, *temperature* and *entropy* ($H = T\theta$ or $\lambda = T\theta$); or, graphically, the total heat is the diagram area whose ordinate at the extreme right of the area considered is the temperature T, laid off from the abscissa whose value is the entropy θ; the total heat being the summation of the vertical strips.

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7.—Saturated Steam—English Units.*

(Condensed from Prof. Peabody's Tables.)

Note.—See preceding notation; also Table 8, following.

Temperature. Degrees Fahr. t	Pressure. Pounds per Square Inch. p	Heat of the Liquid. q	Total Heat. λ	Heat of Vaporization. r	Heat Equivalent of Internal Work. ρ	Heat Equivalent of External Work. Apu	Entropy of the Liquid. $\int \frac{cdt}{T}$	Specific Volume. s	Density. Weight in Pounds of One Cubic Foot. γ
32	0.0890	0.	1091.7	1091.7	1035.9	55.8	0.0000	3387	0.0002952
34	0.0963	2.01	1092.3	1090.3	1034.3	56 0	0.0041	3138	0.0003187
36	0.1042	4.03	1092.9	1088.9	1032.8	56.1	0.0081	2910	0.0003436
38	0.1126	6.04	1093.5	1087.5	1031.3	56.2	0.0122	2700	0.0003704
40	0.1216	8.06	1094.1	1086.0	1029.6	56.4	0.0162	2506	0.0003990
45	0.1471	13.08	1095.7	1082.6	1025.8	56.8	0.0262	2087	0.0004792
50	0.1773	18.10	1097.2	1079.1	1021.8	57.3	0.0361	1745	0.0005731
55	0.2128	23.11	1098 7	1075.6	1017.9	57.7	0.0459	1465	0.0006829
60	0.2545	28.12	1100 2	1072.1	1014.0	58.1	0.0555	1234	0.0008104
70	0.3602	38.11	1103.3	1065.2	1006.2	59.0	0.0745	885.0	0.001130
80	0.5027	48.09	1106.3	1058.2	998.3	59.9	0.0932	643.8	0.001553
90	0.6925	58.04	1109.4	1051.4	990.6	60.8	0 1114	474.6	0.002107
100	0.9421	68.01	1112 4	1044.4	982.7	61.7	0.1293	354.0	0.002824
110	1.2663	78.0	1115.5	1037.5	974.8	62.7	0.1470	267.5	0.003738
120	1.6828	88.1	1118.5	1030.4	966.7	63.7	0.1645	204 4	0.004892
130	2.2119	98.1	1121.6	1023.5	958.9	64.6	0.1817	157.8	0.006336
140	2.8774	108.2	1124.6	1016.4	950.8	65.6	0.1986	123.2	0.008120
150	3.7063	118.3	1127.7	1009.4	942.8	66.6	0.2152	97.03	0 01031
160	4.7292	128.4	1130.7	1002.3	934.8	67.5	0.2316	77.14	0.01296
170	5.981	138 5	1133.8	995.3	926.8	68.5	0.2477	61.85	0.01617
180	7.500	148.5	1136.8	988.3	918.9	69.4	0.2636	50 01	0.02000
190	9.330	158.6	1139.9	981.3	911.0	70.3	0.2792	40 73	0 02455
200	11.520	168.7	1142.9	974.2	903.0	71.2	0.2946	33.40	0.02994
210	14.123	178.8	1146.0	967.2	895.2	72.0	0.3097	27.57	0.03628
220	17.186	188.9	1149.0	960.1	887.1	73.0	0.3246	22.98	0.04352
230	20.783	198.9	1152.1	953.2	879.4	73.8	0.3393	19.20	0.05208
240	24.982	209.0	1155.1	946.1	871.6	74.5	0.3538	16.14	0.06195
250	29.856	219 1	1158.2	939.1	863.8	75.3	0.3681	13.65	0.07327
260	35.483	229.2	1161.2	932.0	855.9	76.1	0.3822	11.60	0.08619
270	41.945	239.3	1164.3	925.0	848.1	76.9	0.3961	9.918	0.1008
280	49.328	249.3	1167.3	918.0	840.4	77.6	0.4098	8.521	0.1173
290	57.72	259.4	1170.4	911.0	832.6	78.4	0.4233	7.356	0.1359
300	67.22	269.5	1173.4	903.9	824.7	79.2	0.4366	6.380	0.1567
310	77.93	279.6	1176.5	896.9	817.0	79.9	0.4498	5.558	0.1799
320	89.95	290.0	1179.5	889.5	808.8	80.7	0.4633	4.861	0.2058
330	103.38	300.5	1182.6	882.1	800.8	81.3	0.4766	4.267	0.2343
340	118.34	310.9	1185.6	874.7	792.7	82.0	0.4897	3.760	0.2660
350	134.95	321.4	1188.7	867.3	784.7	82.6	0.5027	3.324	0.3008
360	153.33	331.8	1191.7	859.9	776.7	83.2	0.5155	2.949	0.3391
370	173.60	342.3	1194.8	852.5	768.7	83.8	0.5282	2.623	0.3812
380	195.91	352.8	1197.8	845.0	760.8	84.2	0.5407	2.338	0.4276
390	220.39	363.2	1200.9	837.7	753.0	84.7	0.5531	2.092	0.4780
400	247.21	373.7	1203.9	830.2	745.2	85.0	0.5653	1.874	0.5336
410	276.54	384.1	1207.0	822.9	737.6	85.3	0.5774	1.682	0.5945
420	308.57	394.6	1210.0	815.4	730.0	85.4	0.5893	1.512	0.661
428	336 26	403.0	1212.5	809.5	724.0	85.5	0.5988	1.390	0.719

*See Revised Table, second page following.

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8.—Saturated Steam—English Units.*

(From Peabody, and Babock and Wilcox.)

Note.—See Notation and Table 7, preceding.

Pressure in Lbs. per Sqr. Inch above vacuum p	Temperature in Degrees, Fahrenheit. t	Heat in Liquid from 32° in Units. q	Total Heat in Heat Units From Water at 32°. λ	Heat of Vaporization, or Latent Heat in Heat Units. r	Factor of Equivalent Evaporation at 212°.	Volume of One Pound in Cubic Feet. s	Density, or Wt. of Cubic Foot in Pounds. γ	Total Pressure Above Vacuum. p
1	101.99	70.0	1113.1	1043.0	.9661	334.5	0.00299	1
2	126.27	94.4	1120.5	1026.1	.9738	173.6	0.00576	2
3	141.62	109.8	1125.1	1015.3	.9786	118.5	0.00844	3
4	153.09	121.4	1128.6	1007.2	.9822	90.33	0.01107	4
5	162.34	130.7	1131.5	1000.8	.9852	73.21	0.01366	5
6	170.14	138.6	1133.8	995.2	.9876	61.65	0.01622	6
7	176.90	145.4	1135.9	990.5	.9897	53.39	0.01874	7
8	182.92	151.5	1137.7	986.2	.9916	47.06	0.02125	8
9	188.33	156.9	1139.4	982.5	.9934	42.12	0.02374	9
10	193.25	161.9	1140.9	979.0	.9949	38.15	0.02621	10
15	213.03	181.8	1146.9	965.1	1.0003	26.14	0.03826	15
20	227.95	196.9	1151.5	954.6	1.0051	19.91	0.05023	20
25	240.04	209.1	1155.1	946.0	1.0099	16.13	0.06199	25
30	250.27	219.4	1158.3	938.9	1.0129	13.59	0.07360	30
35	259.19	228.4	1161.0	932.6	1.0157	11.75	0.08508	35
40	267.13	236.4	1163.4	927.0	1.0182	10.37	0.09644	40
45	274.29	243.6	1165.6	922.0	1.0205	9.285	0.1077	45
50	280.85	250.2	1167.6	917.4	1.0225	8.418	0.1188	50
55	286.89	256.3	1169.4	913.1	1.0245	7.698	0.1299	55
60	292.51	261.9	1171.2	909.3	1.0263	7.097	0.1409	60
65	297.77	267.2	1172.7	905.5	1.0280	6.583	0.1519	65
70	302.71	272.2	1174.3	902.1	1.0295	6.143	0.1628	70
75	307.38	276.9	1175.7	898.8	1.0309	5.760	0.1736	75
80	311.80	281.4	1177.0	895.6	1.0323	5.426	0.1843	80
85	316.02	285.8	1178.3	892.5	1.0337	5.126	0.1951	85
90	320.04	290.0	1179.6	889.6	1.0350	4.859	0.2058	90
95	323.89	294.0	1180.7	886.7	1.0362	4.619	0.2165	95
100	327.58	297.9	1181.9	884.0	1.0374	4.403	0.2271	100
105	331.13	301.6	1182.9	881.3	1.0385	4.205	0.2378	105
110	334.56	305.2	1184.0	878.8	1.0396	4.026	0.2484	110
115	337.86	308.7	1185.0	876.3	1.0406	3.862	0.2589	115
120	341.05	312.0	1186.0	874.0	1.0416	3.711	0.2695	120
125	344.13	315.2	1186.9	871.7	1.0426	3.571	0.2800	125
130	347.12	318.4	1187.8	869.4	1.0435	3.444	0.2904	130
140	352.85	324.4	1189.5	865.1	1.0453	3.212	0.3113	140
150	358.26	330.0	1191.2	861.2	1.0470	3.011	0.3321	150
160	363.40	335.4	1192.8	857.4	1.0486	2.833	0.3530	160
170	368.29	340.5	1194.3	853.8	1.0502	2.676	0.3737	170
180	372.97	345.4	1195.7	850.3	1.0517	2.535	0.3945	180
190	377.44	350.1	1197.1	847.0	1.0531	2.406	0.4153	190
200	381.73	354.6	1198.4	843.8	1.0545	2.294	0.4359	200
225	391.79	365.1	1201.4	836.3	1.0576	2.051	0.4876	225
250	400.99	374.7	1204.2	829.5	1.0605	1.854	0.5393	250
275	409.50	383.6	1206.8	823.2	1.0632	1.691	0.5913	275
300	417.42	391.9	1209.3	817.4	1.0657	1.553	0.644	300
325	424.82	399.6	1211.5	811.9	1.0680	1.437	0.696	325
350	431.90	406.9	1213.7	806.8	1.0703	1.337	0.748	350
375	438.46	414.2	1215.7	801.5	1.0724	1.250	0.800	375
400	445.15	421.4	1217.7	796.3	1.0745	1.172	0.853	400
500	466.57	444.3	1224.2	779.9	1.0812	.930	1.065	500

*See Revised Table, second page following.

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9.—Revised *Saturated Steam* Table—English Units.

(Condensed from Prof. Peabody's Tables.)

Note.—See also Table 10, following.

Degrees F. t	Pressure, Pounds per Square Inch. p	Heat of the Liquid. q	Heat of Vaporization. r	Heat Equivalent of Internal Work. ρ	Heat Equivalent of External Work. Apu	Entropy of the Liquid. θ	Entropy of Vaporization. $\frac{r}{T}$	Specific Volume, Cubic Feet per Pound. s	Density, Pounds per Cubic Foot. $\frac{1}{s}$
32	0.0886	0.0	1071.7	1017.5	54.2	0.0000	2.1804	3308.	0.000302
34	0.0960	2.0	1070.7	1016.3	54.4	0.0041	2.1695	3062.	0.000327
35	0.0999	3.0	1070.2	1015.6	54.6	0.0061	2.1642	2950.	0.000339
36	0.1040	4.0	1069.7	1015.0	54.7	0.0082	2.1588	2842	0.000352
38	0.1126	6.1	1068.7	1013.8	54.9	0.0122	2.1481	2634.	0.000379
40	0.1217	8.1	1067.6	1012.5	55.1	0.0163	2.1373	2446.	0.000409
45	0.1476	13.1	1065.0	1009.4	55.6	0.0262	2.1109	2035.	0.000491
50	0.1780	18.1	1062.3	1006.2	56.1	0.0361	2.0850	1703.	0.000587
55	0.2140	23.1	1059.7	1003.1	56.6	0.0459	2.0596	1429.	0.000700
60	0.2561	28.1	1057.0	998.8	57.2	0.0556	2.0347	1207.	0.000828
65	0.3054	33.1	1054.4	996.7	57.7	0.0652	2.0103	1021.	0.000979
70	0.3627	38.1	1051.8	993.6	58.2	0.0747	1.9863	868.	0.001152
75	0.4289	43.1	1049.2	990.5	58.7	0.0841	1.9629	739.	0.001353
80	0.5056	48.1	1046.5	987.2	59.3	0.0934	1.9398	634.	0.001577
90	0.6960	58.1	1041.2	980.9	60.3	0.1117	1.8948	469.2	0.002131
100	0.9461	68.0	1035.7	974.4	61.3	0.1297	1.8511	350.8	0.002851
110	1.271	78.0	1030.1	967.7	62.4	0.1473	1.8088	265.2	0.003771
120	1.689	88.0	1024.4	961.0	63.4	0.1647	1.7677	203.0	0.004926
130	2.220	98.0	1018.7	954.2	64.5	0.1818	1.7281	157.1	0.00637
140	2.885	108.0	1013.1	947.5	65.6	0.1986	1.6899	122.8	0.00814
150	3.715	118.0	1007.2	940.6	66.6	0.2151	1.6526	96.9	0.01032
160	4.738	128.0	1001.4	933.7	67.7	0.2314	1.6164	77.2	0.01296
170	5.990	138.0	995.5	926.8	68.7	0.2475	1.5814	62.0	0.01613
180	7.510	148.0	989.5	919.8	69.7	0.2633	1.5474	50.2	0.01993
190	9.339	158.1	983.4	912.7	70.7	0.2789	1.5141	40.92	0.02444
200	11.528	168.2	977.2	905.5	71.7	0.2943	1.4817	33.62	0.02974
210	14.125	178.3	970.9	898.3	72.6	0.3095	1.4502	27.80	0.03597
212	14.698	180.3	969.7	896.9	72.8	0.3125	1.4441	26.78	0.03734
220	17.188	188.4	964.6	891.0	73.6	0.3244	1.4196	23.14	0.04321
230	20.78	198.5	958.1	883.6	74.5	0.3392	1.3895	19.37	0.0516
240	24.97	208.6	951.4	876.0	75.4	0.3538	1.3602	16.31	0.0613
250	29.82	218.8	944.7	868.5	76.2	0.3683	1.3315	13.82	0.0724
260	35.42	229.0	937.8	860.7	77.1	0.3825	1.3034	11.75	0.0851
270	41.84	239.1	930.7	852.8	77.9	0.3965	1.2758	10.05	0.0995
280	49.19	249.4	923.6	844.9	78.7	0.4104	1.2489	8.639	0.1158
290	57.53	259.6	916.3	836.9	79.4	0.4242	1.2225	7.454	0.1349
300	66.98	269.8	908.9	828.8	80.1	0.4378	1.1967	6.462	0.1547
310	77.63	280.1	901.3	820.5	80.8	0.4512	1.1713	5.622	0.1779
320	89.59	290.4	893.7	812.3	81.4	0.4644	1.1465	4.907	0.2038
330	102.98	300.6	885.9	803.8	82.1	0.4775	1.1221	4.312	0.2319
340	117.91	310.9	878.0	795.3	82.7	0.4905	1.0982	3.784	0.2642
350	134.52	321.3	870.0	786.8	83.2	0.5034	1.0747	3.342	0.2992
360	152.89	331.6	861.8	778.1	83.7	0.5161	1.0516	2.960	0.3378
370	173.17	341.9	853.5	769.3	84.2	0.5286	1.0289	2.626	0.3808
380	195.52	352.3	845.1	760.5	84.6	0.5411	1.0066	2.339	0.4275
390	220.05	362.7	836.6	751.6	85.0	0.5534	0.9848	2.088	0.4789
400	246.9	373.1	827.9	742.6	85.3	0.5656	0.9633	1.868	0.535
410	276.3	383.5	819.1	733.6	85.5	0.5777	0.9420	1.673	0.598
420	308.5	394.0	810.1	724.5	85.6	0.5896	0.9211	1.499	0.667
428	336.2	402.3	802.9	717.3	85.6	0.5991	0.9047	1.376	0.727

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10.—Revised *Saturated Steam* Table—English Units.

(Condensed from Prof. Peabody's Tables.)

(Note.—See also Table 9, preceding.)

Pressure, Pounds per Square Inch. p	Temperature, Degree Fahrenheit. t	Heat of the Liquid. q	Heat of Vaporization. r	Heat Equivalent of Internal Work. ρ	Heat Equivalent of External Work. Apu	Entropy of the Liquid. θ	Entropy of Vaporization. $\frac{r}{T}$	Specific Volume, Cubic Feet per Pound. s	Density, Pounds per Cubic Foot. $\frac{1}{s}$
1	101.84	69.8	1034.7	973.1	61.6	0.1329	1.8433	333.1	0.00300
2	126.15	94.2	1021.9	957.8	64.1	0.1753	1.7432	173.1	0.00578
4	153.00	121.0	1005.5	938.6	66.9	0.2200	1.6416	90.4	0.01106
6	170.07	138.1	995.5	926.8	68.7	0.2476	1.5812	61.9	0.01616
8	182.86	150.9	987.8	917.8	70.0	0.2678	1.5378	47.26	0.02116
10	193.21	161.3	981.4	910.4	71.0	0.2838	1.5036	38.37	0.02606
12	201.95	170.1	976.0	904.1	71.9	0.2972	1.4756	32.40	0.03088
14	209.55	177.8	971.2	898.6	72.6	0.3088	1.4516	28.03	0.03567
14.7	212.00	180.3	969.7	896.9	72.8	0.3125	1.4441	26.78	0.03734
15	213.03	181.3	969.1	896.2	72.9	0.3140	1.4409	26.28	0.03805
20	227.95	196.4	959.4	885.1	74.3	0.3362	1.3957	20.09	0.04978
25	240.07	208.7	951.4	876.0	75.4	0.3593	1.3600	16.29	0.0614
30	250.34	219.1	944.4	868.2	76.2	0.3687	1.3305	13.74	0.0728
35	259.29	228.2	938.2	861.2	77.0	0.3815	1.3054	11.88	0.0842
40	267.26	236.4	932.6	855.0	77.6	0.3927	1.2833	10.49	0.0953
45	274.46	243.7	927.5	849.3	78.2	0.4027	1.2638	9.387	0.1065
50	281.03	250.4	922.8	844.1	78.7	0.4119	1.2462	8.507	0.1176
55	287.09	256.6	918.4	839.2	79.2	0.4202	1.2302	7.778	0.1286
60	292.74	262.4	914.3	834.7	79.6	0.4279	1.2154	7.166	0.1395
65	298.00	267.8	910.4	830.4	80.0	0.4351	1.2018	6.647	0.1504
70	302.96	272.9	906.6	826.3	80.3	0.4418	1.1892	6.199	0.1613
75	307.64	277.7	903.1	822.4	80.7	0.4480	1.1772	5.807	0.1772
80	312.08	282.2	899.8	818.9	80.9	0.4540	1.1661	5.466	0.1829
85	316.30	286.5	896.6	815.4	81.2	0.4595	1.1557	5.161	0.1938
90	320.32	290.7	893.5	812.1	81.4	0.4649	1.1457	4.886	0.2047
95	324.16	294.6	890.5	808.8	81.7	0.4699	1.1363	4.644	0.2153
100	327.86	298.5	887.6	805.7	81.9	0.4748	1.1273	4.432	0.2256
105	331.42	302.1	884.8	802.7	82.1	0.4794	1.1187	4.233	0.2362
110	334.83	305.6	882.1	799.7	82.4	0.4838	1.1105	4.047	0.2471
115	338.14	309 0	879.5	797.0	82.5	0.4881	1.1026	3 876	0.2580
120	341.31	312.3	876.9	794.2	82.7	0.4922	1.0951	3.723	0.2686
125	344.39	315.5	874.5	791.6	82.9	0.4962	1.0878	3.581	0.2792
130	347.38	318.6	872.1	789.0	83.1	0.5000	1.0808	3.451	0.2898
135	350.27	321.5	869.8	786.5	83.3	0.5037	1.0741	3.331	0.3003
140	353.09	324.4	867.4	784.0	83.4	0.5073	1.0675	3.220	0.3106
145	355.83	327.3	865.2	781.6	83.6	0.5108	1.0612	3.115	0.3210
150	358.50	330.0	863.0	779.3	83.7	0.5142	1.0551	3.014	0.3318
155	361.09	332.7	860.9	771.1	83.8	0.5175	1.0491	2.922	0.3422
160	363.62	335.3	858.8	774.9	83.9	0.5206	1.0434	2.834	0.3528
165	366.09	337.9	856.8	772.8	84.0	0.5237	1.0378	2.751	0.3635
170	368.50	340.4	854.8	770.6	84.2	0.5268	1.0324	2.673	0.3741
175	370.86	342.8	852.8	768.5	84.3	0.5297	1.0270	2.600	0.3846
180	373.16	345.2	850.9	766.5	84.4	0.5326	1.0219	2.531	0.3951
185	375.41	347.5	849.0	764.5	84.5	0.5354	1.0169	2.467	0.4054
190	377.61	349.8	847.1	762.6	84.5	0.5381	1.0121	2.403	0.4158
195	379.78	352.1	845.3	760.7	84.6	0.5408	1.0071	2.345	0.4264
200	381.89	354.3	843.5	758.8	84.7	0.5434	1.0025	2.288	0.4371
210	386.02	358.6	840.0	755.1	84.9	0.5485	0.9935	2.184	0.4579
220	389.98	362.7	836.6	751.6	85.0	0.5534	0.9848	2.088	0.4789
230	393.80	366.6	833.2	748.1	85.2	0.5580	0.9755	2.001	0.4997
240	397.50	370.5	830.1	744.8	85.3	0.5625	0.9686	1.921	0.521
250	401.10	374.2	826.9	741.5	85.4	0.5669	0.9609	1.845	0.542
260	404.55	377.8	823.9	738.5	85.4	0.5711	0.9535	1.775	0.563
270	407.90	381.3	820.9	735.4	85.5	0.5751	0.9464	1.711	0.584
280	411.19	384.8	818.0	732.5	85.5	0.5791	0.9395	1.652	0.605
290	414.35	388.1	815.2	729.6	85.6	0.5829	0.9329	1.595	0.627
300	417.45	391.3	812.4	726.8	85.6	0.5866	0.9264	1.542	0.649
310	420.45	394.4	809.7	724.1	85.6	0.5902	0.9202	1.492	0.670
320	423.40	397.5	807.1	721.5	85.6	0.5937	0.9141	1.446	0.692
330	426.26	400.5	804.5	718.9	85.6	0.5970	0.9083	1.402	0.713
338	427.94	402.2	803.0	717.4	85.6	0.5990	0.9048	1.377	0.726

Flow of Steam.—G. H. Babcock, in "Steam," gives the following formula:

Flow through pipes: $$W = \sqrt{\frac{\gamma(p_1 - p_2)\,d^3}{L\left(1 + \frac{3.6}{d}\right)}} \quad \ldots\ldots\ldots\ldots (1)$$

Where W = weight of steam which will flow per minute, in lbs.;
d = diameter of pipe, in ins.;
γ = density or weight per cubic foot for p_1, in lbs.;
p_1 = initial pressure, at entrance, in lbs. per sq. in.;
p_2 = pressure at end or exit of pipe, in lbs. per sq. in.;
L = length of pipe, in feet.

11.—Flow of Steam through Pipes.

Initial Pressure by Gage, Pounds per Sq. In.	Diameter of Pipe, in Inches. Length of Each = 240 Diameters.													
	¾	1	1½	2	2½	3	4	5	6	8	10	12	15	18
	Weight of Steam per Minute, in Pounds, with One Pound Loss of Pressure.													
1	1.12	2.05	5.65	10.15	15.26	25.12	46.27	76.1	111.6	209.1	336.3	495.3	792	1160
10	1.38	2.54	6.98	12.54	18.85	31.03	57.15	93.9	137.9	258.2	415.3	611.8	979	1433
20	1.62	2.97	8.18	14.70	22.09	36.36	66.97	110.1	161.6	302.6	486.7	716.9	1147	1679
30	1.82	3.34	9.21	16.54	24.86	40.92	75.37	123.9	181.8	340.6	547.8	806.9	1291	1889
40	2.01	3.68	10.12	18.18	27.34	44.99	82.87	136.3	199.9	374.5	602.3	887.2	1419	2078
50	2.17	3.98	10.95	19.67	29.57	48.67	89.64	147.4	206.3	404.9	651.5	959.7	1535	2248
60	2.32	4.25	11.71	21.04	31.63	52.06	95.89	157.7	231.3	433.3	696.9	1026.5	1642	2404
70	2.46	4.51	12.42	22.32	33.55	55.22	101.71	167.3	245.4	459.6	739.3	1088.9	1742	2550
80	2.59	4.75	13.09	23.52	35.36	58.19	107.18	176.3	258.6	484.3	778.9	1147.4	1836	2687
90	2.71	4.96	13.66	24.55	36.91	60.74	111.88	183.9	269.9	505.5	811.5	1197.8	1916	2805
100	2.84	5.20	14.32	25.73	38.71	63.66	117.25	192.8	282.9	529.8	852.2	1255.3	2008	2940
120	3.06	5.61	15.44	27.75	41.71	68.64	126.43	207.9	305.1	571.3	918.9	1353.6	2166	3170
150	3.37	6.16	16.97	30.49	45.83	75.42	138.91	228.4	335.2	627.7	1009.6	1487.2	2379	3483

Note.—"For sizes of pipe below 6-inch, the flow is calculated from the *actual* areas of 'standard' pipe of such nominal diameters. For horse-power, multiply the figures in the table by 2. For any other loss of pressure, multiply by the square root of the given loss. For any other length of pipe, *divide 240 by the given length expressed in diameters, and multiply the figures in the table by the square root of this quotient*, which will give the flow for 1 lb. loss of pressure. Conversely, dividing the given length by 240 will give the loss of pressure for the flow given in the table."

Steam Boilers.—*The Efficiency* of a steam boiler is the ratio of the heat utilized in heating the water and raising steam, to the total heat generated by the combustion of the fuel; and ranges from 50 to 75 per cent., the latter being rarely exceeded. Thus, if a pound of coal has a heating value of 14,000 *B. T. U.* it is theoretically capable of evaporating, "from and at"* 212°*F* (see Table 7, preceding), 14000 ÷ 965.8 = 14.5 lbs. of water; while if the efficiency of the boiler is 75 per cent., the amount actually evaporated by the pound of coal will be 14.5 × .75 = 10.87 lbs. of water.

One commercial horse power of a boiler is variously defined as follows:

(1) The evaporation of 30 lbs. of water per hour from a feed-water temperature of 100 *F* into steam at 70 lbs. gage pressure (above atmosphere); considered equivalent to
(2) 34.5 (34.488 exact) units of vaporization at 212° *F*; considered equal to
(3) 34.5 pounds of water evaporated from a feed-water temperature of 212° *F* into steam at the same temperature.
(5) From our steam table, 34.5 r = 34.5 × 965.8 = 33,320 *B. T. U.* per hour; but
(6) 33,305 British thermal units per hour is sometimes taken as a standard (= 34.488 r, using r at 965.7).

Conclusion: (1) is used in making actual tests of boilers; the others for purposes of calculation.

* "From and at" 212° *F*, means that the vaporization takes place at 212° *F* from feed water at the same temperature.

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Consumption of Coal per boiler horse-power hour may be determined from the above data. Assuming the boiler at 75 per cent efficiency, one pound of coal will generate say $14000 \times .75 = 10500$ British heat units (if feed water is at 212° *F*). Then (1) the amount of coal per boiler horse-power per hour $= \frac{33\ 320}{10\ 500} = 3.17$ lbs. This result is also obtained (2) by dividing 34.5 by 10.87; that is, the number of pounds of water evaporated per horse-power hour, divided by the number of pounds evaporated per pound of coal, as calculated above.

In practice, we generally assume that one pound of coal will generate about 10 000 heat units, in which case it would require 3.332 pounds of coal to develop one boiler horse-power per hour, which is very close to the average performance of the Cahall boiler, Table 6, preceding. Now a boiler horse-power per hour ($= 33\ 320$ heat units or $33\ 320 \times 778 = 25{,}922{,}960$ ft.-lbs.) is very different from an engine horse-power hour (see page 1363) which is equal to 1,980,000 ft.-lbs.

Thus, 1 boiler *H.-P.* hour = 13.0924 engine *H.-P.* hours,
or 1 engine *H.-P.* hour = 0.07638 boiler *H.-P.* hours.

Hence the number of pounds of coal consumption per *engine* horse-power, is approximately,

$$\frac{3.332 \times 0.07638 = 0.2545}{\text{Efficiency of the Engine}} \quad \text{(See also page 1363)} \ldots\ldots (1)$$

provided there is no loss *between* the boiler and the engine.

Kinds of Steam Boilers.—In the discussion of impulse water wheels, page 1363, it is stated that the greatest amount of the energy of the jet has been imparted to the wheel when the water loses its velocity on striking the buckets. The same principle holds good in the construction of boilers. They should be so designed that the feed water shall absorb the greatest amount practicable of the heat of combustion of the fuel, with the minimum waste of gases.

Ordinary steam boilers may be classified as water-tube boilers, fire-tube boilers and flue boilers:

(1) Water-tube boilers have a large number of tubes of moderate size connected together at their ends and also with a reservoir of water above; and are placed directly over the grate or in the path of the flames. The tubes may be horizontal, vertical, inclined, straight, curved, etc. There should be no "dead" ends. They are especially adapted to high steam-pressure. Notable examples: Babcock and Wilcox, Cahall, Thornycroft.

(2) Fire-tube boilers consist of a large number of small tubes acting as flues and *surrounded* by the water contained in an outer shell, which requires special design. The circular tubular boiler is of this type, including locomotive boilers.

(3) Flue-boilers differ from the fire-tube boilers in that the tubes are reduced to one or more in number and greatly increased in size. If the furnace is outside the flues it is "externally fired;" if inside the flues it is "internally fired." Of the latter class the Cornish boiler has one flue, while the Lancaster has two. The Scotch marine boiler is a flue- and fire-tube boiler combined.

Boiler Settings.—The following is from the notes of J. B. Stanwood:

The brick-work about a boiler should be thick to prevent loss by radiation—a 21″ wall should be used if possible. All flues and surfaces exposed to action of heat should be lined with best fire-brick. It is not a good plan to convey gases back over top of boiler, unless there is space enough for a man to enter and clean off soot. The distance from grate bars to lower portion of boiler shell should not be less than 24″; 26″ and 28″ are not too great, and in large shells 30″ can be employed.

The bridge-wall should curve to conform to shape of boiler shell. Ten inches makes a good space between wall and shell. Back of bridge-wall the surface should be paved with hard brick, the surface dipping down to a depth at rear end of boiler of about 18″ to 24″ according to size of shell. The distance between back tube sheet and back wall should be 18″ for a 48″ shell; 24″ for a 72″ shell.

Boiler walls will crack, and no form of construction seems to entirely prevent this. Walls with air spaces are as liable as those without, with the danger of leaking more air when they do crack. The best method to hold

boiler walls together is with "buck-staves" or "buck-bars." The best form is railway iron with ends mashed down under a hammer, to allow for drilling holes for tie-rod. Most builders do not supply "buck-staves" unless specially ordered. The cheapest form of fire-front is the so-called "half-arch," which does not cover any more of the front of the furnace than is absolutely decent. On small boilers it is employed as a support. For a good job a "full flush front" should be used with damper plate and damper.

Boilers, now-a-days, are not set in batteries, all to work together as a unit. They are and should be set, so that each boiler is independent of the other in the battery. In this way any one can be shut down for cleaning or for repairs. This arrangement does away with the old-fashioned steam and mud-drums, which connected the boilers of a battery together. Do not buy either a mud-drum or steam drum; they are a source of trouble, danger and expense. The trade usually includes with the boiler front, the grate bars, a bearing-bar to support same, a soot or ash-door with frame, a brick arch plate or supporting bars, and a boiler stand for small boilers.

Steam Engines are steam motors, or mechanisms for converting the thermal energy of steam into mechanical work. The efficiency of an engine is the ratio of the mechanical work done, to the energy of the steam consumed in doing it; or, the ratio of the heat changed into work, to the heat applied. A *perfect heat engine*,* working to absolute zero temperature, would of course have an efficiency of 100 per cent; that is, it would be able to convert *all* the applied heat into work. A "*perfect*" *steam engine*, from a mechanical standpoint, can never be expected to exceed an efficiency of about 25 *per cent*. The *best types of steam engines* have an efficiency in actual practice of only about 18 *per cent*. because they cannot utilize a greater ratio of the energy of the steam delivered to them. Such engines are of the condensing- and multiple-expansion type. Non-condensing engines have efficiencies ranging from 5 to 10 per cent. Recent engines of moderate power and high superheat have attained an efficiency of 22%.

Engine Horse-Power.—Steam engines are rated by the horse-power, the unit of work being the foot-pound. 1 *H.-P.* second = 550 ft.-lbs.; 1 *H.-P.* minute = 33,000 ft.-lbs.; 1 *H.-P.* hour = 1,980,000 ft.-lbs.

Problem.—A steam engine working at an efficiency of 8% uses 40 lbs. of coal per hour. Assuming that each pound of coal generates 10 000 British thermal units in the steam, what horse-power is realized?

Solution.— $\frac{40 \times 10\,000 \times 778 \times .08}{1\,980\,000} = 12.574$ *H.-P. Ans.*

In the above problem the coal consumed per horse-power per hour $= \frac{12.574}{40} = 3.18$ lbs. This result can also be obtained from equation (1); thus, $\frac{0.2545}{.08} = 3.18$ lbs.

Coal Consumption per horse-power per hour may vary from 2 lbs., for large efficient steam plants, to 4 lbs. for smaller plants, less efficient. It is well to estimate 6 or 7 lbs. for hoisting engines ordinarily used by contractors. Of course much depends upon the quality of the coal. If of poor quality the consumption will be greater.

Value of Wood as fuel.—Assuming the wood to be thoroughly air-dried, one ton of coal is equivalent to 1 cord of Douglas (Oregon or Washington) fir; 1.05 cords of hickory; 1.1 cord of maple; 1.2 cord of white oak; 1.5 cord of beech; 1.6 cord of black oak; 2.0 cords of elm; 2.1 cords of chestnut; 2.4 cords of pine. These values are necessarily approximate only. (See also page 1352.)

Principle of the Steam Engine.—An engine has one or more "cylinders." A cylinder is simply a cylindrical barrel, with closed ends, in which is fitted a piston *P* (Fig. 1) made to move back and forth by the pressure of steam admitted alternately on either side of it. The piston rod is connected, perhaps indirectly, with a crank or wheel, giving the latter a circular motion, thus producing continuous power in one direction.

* This is used in a distinct sense from the so-called "heat engines" which include gas- and oil engines. Such engines may have an efficiency of 20 per cent or more.

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Fig. 1 shows a longitudinal section of the cylinder of a Corliss engine. The steam enters at the top of the cylinder through a steam pipe leading directly from the boiler, and under a pressure of say 100 lbs. (per sq. in.) more or less. (The exact pressure is registered by a pressure gage or Indicator.) It enters the cylinder proper through one of the steam valves S or s,

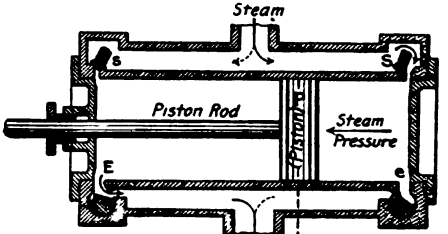

Fig. 1.

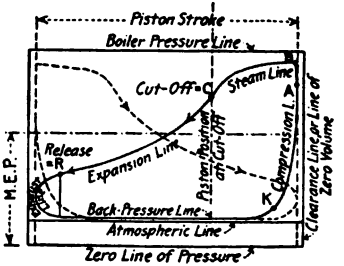

Fig. 2.

A = Admission
A-B = Admission Line
K = Compression

and after it has performed its work of pressure and expansion it escapes through the parts E and e, at the bottom of the cylinder, into the exhaust. It may either be wasted, or condensed into hot feed-water for the boiler.

Fig. 2 shows a typical double indicator diagram for a non-condensing engine, and illustrates graphically the action of the cylinder and the working pressure on the piston at every position. The full diagram is for the pressure on the right of the piston, and the dotted diagram for the pressure on the left; the two together illustrating a complete cycle, from the starting point at the right of the cylinder to its return to that position.

Starting with the piston at the right, Figs. 1 and 2, and considering the steam pressure at the right of the piston: A is the point of admission of steam; A–B the admission line; B the point of initial pressure (the distance between B and the boiler pressure line shows the loss in pressure from the boiler to the engine); B–C the steam line (the valve S being open from A to C); C the cut-off, at which point the valve S is closed and beyond which the steam works by expansion; C–R the expansion line, the pressure decreasing as the volume of steam increases (this curve is very nearly a hyperbola); R the point of release produced by the opening of the exhaust valve at e; R–X the exhaust line; X–K the back-pressure line; K the point of compression; K–A the compression line. The area of this curve, $A\ B\ C\ R\ X\ K\ A$, divided by the horizontal length of figure, practically the piston stroke, gives the mean effective pressure ($M.E.P.$) per sq. in. on the right-hand side of the piston during its cycle. Similarly, the $M.E.P.$ on the left-hand side of piston is obtained from the dotted diagram. Engine diagrams should be *started* at *admission* or during *exhaust*, otherwise they may not close.

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Mean Effective Pressure (*M. E. P.*)—The *M. E. P.* for any particular engine should be obtained by indicator diagrams as explained above. The following table gives average *M. E. P's.* for non-condensing engines:

12.—MEAN EFFECTIVE PRESSURES FOR VARYING CUT-OFFS AND INITIAL STEAM PRESSURES; NON-CONDENSING ENGINES.

Initial Pressure:	1/10 Cut-off.	18/100 Cut-off.	1/5 Cut-off.	1/4 Cut-off.	3/10 Cut-off.	35/100 Cut-off.	4/10 Cut-off.	1/2 Cut-off.
40	3.65	9.05	13.46	17.34	20.75	23.70	26.22	30.50
45	5.42	11.32	16.15	20.39	24.13	27.32	30.08	34.75
50	7.19	13.59	18.85	23.45	27.50	30.94	33.95	39.00
55	8.96	15.86	21.54	26.50	30.87	34.56	37.81	43.25
60	10.73	18.12	24.24	29.56	34.24	38.18	41.68	47.50
65	12.50	20.39	26.93	32.61	37.61	41.80	45.54	51.75
70	14.27	22.66	29.63	35.67	40.98	45.42	49.41	56.00
75	16.04	24.93	32.32	38.72	44.35	49.05	53.27	60.25
80	17.81	27.19	35.02	41.78	47.72	52.68	57.14	64.50
85	19.58	29.46	37.71	44.83	51.09	56.31	61.00	68.75
90	21.36	31.72	40.41	47.89	54.46	59.94	64.87	73.00
95	23.13	33.93	43.10	50.94	57.83	63.57	68.73	77.25
100	24.90	36.26	45.80	54.00	61.20	67.20	72.60	81.50

With throttling engines results are obtained dependent upon proportion of steam-ports, travel of valve, piston speed, size of governor, and design of engine.

The theoretic mean effective pressure may be calculated by means of a formula involving the use of a table of hyperbolic logarithms. This formula assumes that the "expansion line" (see Fig. 2) is a hyperbola. It takes into consideration the point of cut-off. It is used principally in the design of engines, and only about 70 per cent of the theoretic *M. E. P.* is attained in practice.

Horse-power from mean effective pressure (*M. E. P.*)—

NOTATION.

D = diameter of cylinder;

A = area of cylinder $= \frac{\pi D^2}{4} = 0.7854\ D^2$;

d = diameter of piston rod;

a = area of piston rod $= \frac{\pi d^2}{4} = 0.7854\ d^2$;

s = length of piston stroke;

n = number of revolutions per minute.

All dimensions in inches and square inches.

Then the horse-power is—

$$\frac{[(M.\ E.\ P.\ \text{for head end}) \times A + (M.\ E.\ P.\ \text{for crank end}) \times (A-a)]sn}{33\,000 \times 12}. \quad (2)$$

Problem.—A non-condensing engine is working at 3/10 cut-off under an initial pressure (at *B*—not boiler pressure) of 90 lbs., and at a speed of 130 revolutions per minute, with a stroke of 24 inches. The diameter of cylinder is 16 ins., and of piston rod 2.5 ins. What horse-power is realized?

Solution.—From Table 12, preceding, the *M. E. P.* (for both head- and crank end) is 54.46; hence, from equation (2), the horse power =

$$\frac{54.46 \times (2\,A - a)\,s\,n}{33\,000 \times 12} = 170\ H.\ P. \quad \text{Ans.}$$

Economic Performance of Steam Engines.—The following is from the notes of J. B. Stanwood:

NON-CONDENSING ENGINES.

Slide-Valve Engine.—75 to 80 lbs. of boiler pressure; stroke, long; mean effective pressure, 33 to 38 lbs. per sq. in; 25 to 100 H. P.; cut-off, 5/8 stroke; about 40 lbs. steam per indicated H. P. per hour. When valves and piston are tight this has been reduced to 33 lbs. of dry steam per indicated horse-power (I. H. P.) per hour by careful test.

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Automatic High-Speed Engines with single valves.—75 to 80 lbs. of boiler pressure; stroke, about equal to piston dia.; M. E. P., 40 lbs. per sq. in.; 50 to 150 H. P.; cut-off, ¼ stroke; about 40 lbs. steam per I. H. P. per hour. When valves and pistons are tight this has been reduced to 32 lbs. of dry steam per I. H. P. per hour. Valves difficult to keep tight.

Automatic High Speed Engines with double valves.—75 to 80 lbs. of boiler pressure; stroke 1½ to 2 times piston dia.; M. E. P., 40 lbs. per sq. in.; 50 to 150 H. P.; cut-off, ¼ stroke; about 35 lbs. steam per I. H. P. per hour. When valves and pistons are tight this has been reduced to 30 lbs. of dry steam per I. H. P. per hour by careful test.

Automatic Cut-off Engines, of the Corliss type.—Stroke, 2 to 3 times dia. of piston; 75 to 90 lbs. boiler pressure; M. E. P., 40 lbs. per sq. in.; under 200 H. P.; cut-off 1/5 to ¼ stroke, 29–30 lbs. steam per I. H. P. per hour; over 200 H. P., 27 lbs. steam per I. H. P per hour; when valves and piston are tight, this has been reduced to 23½ lbs. of dry steam per I. H. P. per hour by careful test.

Compound Engines.—High speed, automatic cut-off, short stroke; 110 to 120 lbs. boiler pressure; M. E. P., 25 to 27 lbs. per sq. in.; 6 expansions, 100 to 250 H. P.; 27 lbs. steam per I. H. P. per hour.

Condensing Engines.

Automatic Cut-Off Engines, of the Corliss type.—Stroke, 2 to 3 times piston dia.; 70 to 80 lbs. boiler pressure; M. E. P., 40 lbs. per sq. in.; over 200 H. P.; cut-off, 1/5 stroke, about 19–20 lbs. steam per I. H. P. per hour.

Compound Engines; high speed, automatic cut-off; short stroke; 110 to 120 lbs. boiler pressure; M. E. P., 27 to 30 lbs. per sq. in.; 9 expansions; 200 to 500 H. P; 17 to 19 lbs. steam per I. H. P. per hour.

Compound Automatic Cut-off Engines, of the Corliss type.—Stroke on high pressure cylinder, 2 to 3 times piston dia.; 135 to 110 lbs. boiler pressure; M. E. P., 24 to 14 lbs. per sq. in.; over 400 H. P.; 16 to 20 expansions; 14 to 17 lbs. of steam per I. H. P. per hour. One or two special cases, 13½ lbs. of steam per I. H. P. per hour has been obtained.

Compound Engines.

Compound Engines are devices by which high grades of expansion, and consequently high pressure of steam, can be successfully used; and the evils of leakage also can be reduced.

By expanding steam partially in one, then in a second, and perhaps a third cylinder, the internal condensation is kept small; for usually late cut-offs are employed, the advantage of which is that the surfaces of the cylinders presented for re-heating to the fresh charges of steam are small, compared with the volume of steam used; and the difference in temperature between fresh and exhausted steam is slight.

Leakage of steam with compound engines is not so serious a matter; the steam that leaks through one cylinder is caught by the second instead of being thrown away. As the cut-offs are frequently late, the slide-valve can be employed, which is the tightest of all valves.

The Effect of Load Upon Economy of Steam Engines.

A large engine working with an extremely light load is wasteful of fuel and steam; it is worse than a small business with a large staff of expensive officers, for it has more than the fixed charges always with it. A small engine overloaded may be an annoyance and care, but if in good condition it may be economical in fuel.

Steam Pumps.—Fig. 3 shows a longitudinal section of a Worthington direct-acting duplex steam pump. It is "direct-acting" because the plunger of the pump (on the right) is directly connected to the piston rod of the engine (on the left). It is "duplex" because two of the pumps are placed side by side, each one of which is connected by links and a rocker to the valve of the other. As the plunger works back and forth the water is sucked through the valves at the bottom, at either end, while the water previously sucked at the opposite end is at the same time forced through the upper valves into the discharge *D*.

Duplex Pumps are durable and easy acting, and are used for all classes of pumping.

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Centrifugal Pumps consist essentially of a pump wheel or impeller with curved vanes which move tightly in an outer casing. The wheel is fixed

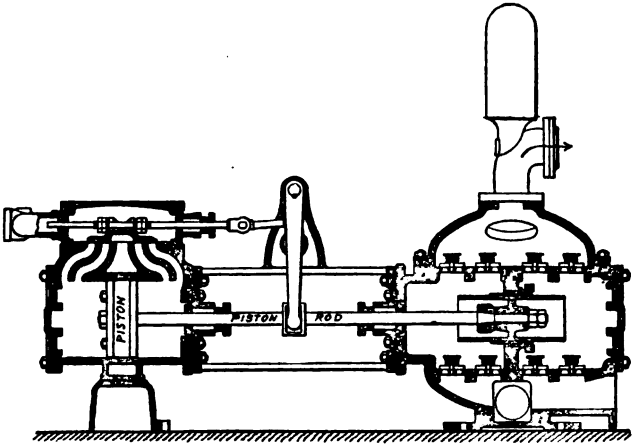

Fig. 3.—Worthington Steam Pump (p. 1366).

on a shaft rotated by a belt running over a pulley. In a side-suction pump the water enters on either side at the center of the wheel and is forced out at the periphery through a tangential outlet from the casing. Centrifugal pumps are economical in raising water against low heads,

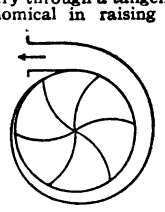

Fig. 4.—Centrifugal Pump.

say up to 30 ft., more of less; reciprocating pumps are more efficient for high heads.

Rotary Pumps are operated by two rotating valves in "gear," each moving tightly in an outer casing.

Duty of Pumps.—The duty of a pump was formerly tested and measured by the number of foot-pounds of work which it was capable of performing from the combustion of 100 pounds of coal used. As the quality of coal varies a new standard was proposed, in 1891, by a Committee of the A. S. C. E.:*

$$\text{Duty} = \frac{\text{No. of foot-pounds of work done} \times 1\,000\,000}{\text{Number of heat units consumed}}$$

This would be equivalent to the old rule, provided 100 lbs. of coal will generate 1 000 000 (100 × 10 000) heat units (see page 1362) to the water in the boiler, a result which can generally be obtained.

* See *Transactions A. S. C. E.*, Vol. XII., page 530.

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D.—HEAT (INTERNAL-COMBUSTION) ENGINES.

TESTS OF INTERNAL-COMBUSTION ENGINES ON ALCOHOL FUEL.

(Digest of Bulletin 191, U. S. Dept. of Agric.)

Introduction.—Recently in this country great interest has developed in the possibilities of alcohol as fuel, and the question of its being used as a substitute for the pertoleum fuels will become of increasing importance as time goes on. The supply of crude oil to be obtained in the United States must ultimately diminish, and the history of the past indicates that a constant increase in price of kerosene and gasoline may ultimately be expected. On the other hand, it is not improbable that the price of alcohol may fall, so that as regards cost alcohol may be used advantageously in comparison with the petroleum oils.

Specific objects of the investigation.—First, to determine whether the gasoline and kerosene engines at present on the American market can run on alcohol as fuel; the manipulation to be followed in making the engines run on alcohol; the measurement of the relative maximum powers of the engines when using alcohol and the fuels for which they were originally made; and the relative consumptions of the different fuels. Second, to determine as far as possible the improvements which might be desirable in the design of engines manufactured especially for alcohol.

Outline of the ground covered by the tests.—The engines used, and range of tests, were:

No. 1. Gasoline engine, 15 h. p. at 280 rev. per min.; 2 cyl., single-acting, vertical, 4 cycle, 6½" bore, 10" stroke. 15 tests reported, giving consumption of alcohol and gasoline under different brake loads and with different initial compressions. Lowest consumptions obtained were 0.71 lb. (0.12 gal.) of gasoline and 1.12 lb. (0.16 gal.) of alcohol per brake h. p. hour. The highest working m. e. p. obtained was about 90 lbs. with both gasoline and alcohol, but at best consumption the m. e. p. were considerably lower

No. 2. Gasoline engine, 6 h. p. at 300 rev. per min.; 1 cyl., water-cooled, horizontal, 4 cycle, 5½" bore, 9" stroke. 24 tests with gasoline and 30 with alcohol. Highest mechanical efficiencies were 86% for gasoline and 90% for alcohol.

No. 3. Gasoline engine, 6 h. p. at 340 rev. per min.; 1 cyl., horizontal, 4 cycle, 5½" bore, 10" stroke. 18 tests with gasoline and 19 with alcohol. Best consumption with gaosline was 0.85 lb. (0.14 gal.) per brake h. p. hour; and with alcohol, at 320 r. p. m., 1.25 lb. (0.18 gal.).

No. 4. Gasoline engine, 6 h. p. at 350 r. p. m., 1 cyl., vert., 4 cycle, 6" bore, 8" stroke. 11 tests on gasoline and 26 on alcohol; and also the effect of heating the air in advance of its entrance to the carburetter. Best alcohol consumption was 1.13 lb. (0.17 gal.) per brake h. p hour; and with air entering the carburetter heated to 125° F., the engine would self-ignite, the m. e. p. at best consumption being 93 lbs.

No. 5. Kerosene engine, 6 h. p. at 360 r. p. m., 1 cyl., horizontal, 2 cycle with crank case compression, cyl. dia. 7" with 8" stroke. The engine has no carburetter, but is fitted with a separate vaporizing chamber. Oil is supplied to a pump on top of the engine, which delivers it directly through a pipe to the vaporizer lip, and has a hand-operated handle to deliver oil in starting. Four tests were made with kerosene and 5 with alcohol. The best consumption with kerosene was 0.98 lb. (0.15 gal.), and with alcohol 1.60 lb. (0.23 gal.).

No. 6. Automobile gasoline engine, 40 h. p. at 900 r. p. m., 4 cycle, 4 cylinder, single acting, vertical, 4¾" bore, 5½" stroke. All valves are cam operated and the carburetter is of the constant level type. Two tests each were made with gasoline and alcohol. The result of these tests is as follows:

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No. of Test	Kind of Fuel.	Brake Load.			Revolutions per minute.	Brake horse-Power.	Duration of Test.	Fuel consumption			Vacuum in carburetter.
		W	w.	$W-w$				Per Hour	Per Horse-Power Hour		
		Lbs.	Lbs.	Lbs.			Min Sec	Lbs.	Lbs.	Gallon.	Inch. of Mercury.
153	Gasoline	215	25	190	660	26.3	2 19	24.2	0.92	0.16	$\frac{1}{4}$ to $\frac{1}{2}$
154	...do...	200	22	178	780	29.2	8 0	29.9	1.02	.17	$\frac{1}{4}$ to $\frac{5}{16}$
155	Alcohol	218	24	194	680	27.7	10 0	37.9	1.37	.20	$\frac{1}{4}$ to $\frac{11}{16}$
156	...do...	220	26	194	670	27.3	19 0	39.2	1.44	.21	$\frac{1}{2}$

in which $W-w$ is the force used in computing the brake horse-power.

No. 7. Automobile gasoline engine, 40 h. p. at 900 r. p. m., 4 cycle, 4 cylinder, $4\frac{1}{8}''$ bore, $5\frac{1}{16}''$ stroke. Twelve tests with gasoline and 7 with alcohol. The best consumption with gasoline was 0.69 lb. (0.12 gal.) per brake h. p. hour; with alcohol 1.30 lb. (0.19 gal.).

No. 8. Boat gasoline engine, 2 h. p. at 700 r. p. m., 1 cyl., vertical, 2 cycle, 4" bore, 4" stroke. Ten tests with gasoline and 7 with alcohol. Best consumption on gasoline was 1.36 lb. (0.23 gal.) per brake h. p. hour; and with alcohol 2.52 lb. (0.37 gal.).

Conclusions.—The following conclusions are drawn as a result of the investigations:

(1) Any gasoline engine of the ordinary types can be run on alcohol fuel without any material change in the construction of the engine. The only difficulties likely to be encountered are in starting and in supplying a sufficient quantity of fuel, a quantity which must be considerably greater than the quantity of gasoline required.

(2) When an engine is run on alcohol its operation is more noiseless than when run on gasoline, its maximum power is usually materially higher than it is on gasoline and there is no danger of any injurious hammering with alcohol such as may occur with gasoline.

(3) For automobile air-cooled engines alcohol seems to be especially adapted as a fuel, since the temperature of the engine cylinder may rise much higher before auto-ignition takes place than is possible with gasoline fuel; and if auto-ignition of the alcohol fuel does occur no injurious hammering can result.

(4) The consumption of fuel in lbs. per brake H. P., whether the fuel is gasoline or alcohol, depends chiefly upon the H. P. at which the engine is being run and upon the setting of the fuel supply valve. It is easily possible for the fuel consumption per H. P. hour to be increased to double the best value, either by running the engine on a load below its full power or by a poor setting of the fuel supply valve.

(5) These investigations also showed that the fuel consumption was affected by the time of ignition, by the speed, and by the initial compression of the fuel charge. No tests were made to determine the maximum possible change in fuel consumption that could be produced by changing the time of ignition, but when near the best fuel consumption it was shown to be important to have an early ignition. So far as tested the alcohol fuel consumption was better at low than at high speeds. So far as investigated, increasing the initial compression from 70 to 125 lbs. produced only a very slight improvement in the consumption of alcohol.

(6) It is probable that for any given engine the fuel consumption is also affected by the quantity and temp. of cooling water used and the nature of the cooling system, by the type of ignition apparatus, by the quantity and quality of lubricating oil, by the temp. and humidity of the atmosphere, and by the initial temp. of the fuel.

(7) It seems probable that all well-constructed engines of the same size will have approximately the same fuel consumption when working under the most advantageous conditions.

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(8) With any good small stationary engine as small a fuel consumption as 0.70 lb. of gasoline, or 1.16 lbs. of alcohol per brake H. P. hour may reasonably be expected under favorable conditions. These values correspond to 0.118 and 0.170 gallon respectively, or 0.95 pint of gasoline and 1.36 pints of alcohol. Based on the high calorific values of 21,120 British thermal units per pound of gasoline and 11,880 per pound of alcohol, these consumptions represent thermal efficiencies of 17.2% for gasoline and 18.5% for alcohol.

But calculated on the basis of the low calorific values of 19,660 B. T. U. per pound of gasoline and 10,620 for alcohol, the thermal efficiencies become 18.5 for the former fuel and 20.7 for alcohol. The ratio of the high calorific values used above is, gasoline to alcohol, 1.78. The corresponding ratio of the low calorific values is 1.85. The ratio of the consumptions mentioned above is, alcohol to gasoline, 1.66 by weight, or 1.44 by volume.

Properties of Liquid Fuels.—All liquid fuels available for commercial use are complicated mixtures of many different chemical substances, and hence are always liable to more or less change in chemical composition. Gasoline and kerosene are most easily examined by their specific gravities, but since each is a mixture of numerous lighter and heavier oils, a definite constant density is not a guarantee that the composition may not change sufficiently to affect the action of the fuel in an engine.

Commercially pure grain or ethyl alcohol is sensibly pure except for the water which may be mixed with it. In this country alcohol is described according to its strength by stating the percentage of absolutely pure alcohol, by volume, which exists in the mixture of alcohol and water. Thus, 90% alcohol contains 90% alcohol and 10% water by separate volume. Since alcohol is lighter than water, the stronger the alcohol the lighter the specific gravity, and 90% alcohol contains less than 90% of alcohol by weight. Moreover, since when pure alcohol and water are mixed the volume of the mixture is less than the sum of the volumes of the water and alcohol before mixing, 90% alcohol contains more than 10% of water by volume. A U. S. "proof" gallon contains 50% alcohol by volume, the remainder of the mixture being water; hence a quantity of alcohol when stated in proof gallons is expressed by a number just twice as large as it would be if stated in gallons of 100% alcohol.

The denatured alcohol which may be used in engines in the U. S. must be prepared as follows, according to the regulations of the Commissioner of Internal Revenue: To 100 volumes of ethyl or grain alcohol of a strength not less than 90% there must be added either 10 volumes of methyl or wood alcohol and ½ of 1 volume of benzine or 2 volumes of methyl alcohol and ½ of 1 volume of pyridin bases. The substances added to the grain alcohol will probably not be of uniform quality, and hence there will be some variability in the properties of the denatured alcohol which will affect its use as a fuel. The following figures are fair average values of the different fuels:

Substance.	Spec. Grav.	Lbs. per Gallon.	Substance.	Spec. Grav.	Lbs. per Gallon
Gasoline	0.71	5.9	95% ethyl alcohol	0.82	6.8
Kerosene	0.80	6.7	90% ethyl alcohol	0.83	6.9

The two most important properties of a liquid fuel, which determine its availability or adaptability for use in an engine, are its heat of combustion and its volatility.

Heat of Combustion.—For the various petroleum oils the heat of combustion varies between 19 000 and 21 000 B. T. U. per lb. of oil, and 20 000 is an average value; for pure alcohol, about 12 700 B. T. U. per lb.

All liquid fuels contain a considerable proportion of hydrogen, which when burned forms water in the condition of steam. When the fuel is burned in a calorimeter this steam is condensed by the cold water surrounding the calorimeter and in this condensation contributes a considerable amount of heat to the total amount absorbed by the cold water. When a fuel is burned in an internal-combustion engine the products of combustion

always leave the engine cylinder at a temperature much above the boiling point of water; hence the engine is unable to make use of the latent heat of condensation of the steam formed in combustion, although this latent heat is included in the heat measured by the calorimeter. On this account it is customary, in comparing fuels used in explosion engines, to calculate the heat of condensation of the steam in the products of combustion and to deduct this amount from the heat of combustion as measured in the calorimeter, giving what is called the low value of the heat of combustion. Correspondingly, the heat as measured by the calorimeter is called the high value of the heat of combustion.

Air Necessary for Combustion.—When a fuel has a definite chemical composition, the air necessary for its combustion can be exactly calculated. Hence, this calculation can be made for pure methyl or ethyl alcohol, but can be made only approximately for a fuel like gasoline, which is a mixture of variable proportions of a large number of different chemical substances. By the formula for ethyl alcohol, C_2H_5OH, its molecular weight is 46: carbon 24 $(=12\times2)$ + hydrogen 6 $(=1\times6)$ + oxygen 16 $(=16\times1)=46$. For the complete combustion of 1 molecule of alcohol the two atoms of carbon require 4 atoms of oxygen to form carbon dioxide, and the 6 atoms of hydrogen require 2 atoms of oxygen, in addition to the 1 atom present, to form steam, thus making 6 atoms of oxygen in all to be supplied. The weight of the 6 atoms is $6\times16=96$. Hence complete combustion of 1 lb. of C_2H_5OH requires $96\div46=2.086$ lbs. of oxygen. In one pound of pure dry air there is 0.230 lb. of oxygen, so that the combustion of 1 lb. of C_2H_5OH requires $2.086\div0.230=9.06$ lbs. of air, or about 119 cu. ft. of pure air at a temp. of 60° and at sea level. If the alcohol contains water, 1 lb. of the alcohol-water mixture requires less air than that stated. If the air is moist, 1 lb. of it contains slightly less than 0.230 lb. of oxygen and hence more air would be required.* In an actual engine the amount of air is proportioned to the amount of vapor, not by any exact measurement of either, but by trial to secure either the best results in maximum power or in minimum fuel consumption.

Vaporization of Fuel.—Before any liquid fuel can be used in the usual form of explosion engine, it must be vaporized, and this vapor must be mixed with air in proper proportions. Thus the preparation of the combustible mixture involves three steps: First, vaporization of the fuel; second, mixture of the fuel vapor and air; and, third, the proper adjustment of the proportions of fuel and air.

The differences in the devices used in engines to accomplish these objects constitute the widest variations in the detailed design of existing engines. In some of these devices the fuel is boiled in a separate chamber, called a vaporizer, from which the vapor flows into a stream of air entering the engine, the amount of vapor being regulated by a valve just as in the case of an engine using illuminating or producer gas.

In another type of vaporizer the fuel is dropped on a hot plate over which the air flows, the proportion of fuel being regulated by the amount of liquid fuel forced against the plate. Kerosene requires a high heat to vaporize it completely, since its boiling point is high, and hence it is much used with vaporizers of the hot-plate type. Alcohol will work satisfactorily with a vaporizer of this type if the temperature of the hot plate is properly regulated.

Gasoline is easily vaporized at ordinary atmospheric temperatures and hence requires no hot plate or heated vaporizing chamber. Usually the liquid gasoline is admitted directly into the air entering the engine through a device known as the carbureter, which is intended to regulate the proportion of fuel and to spray it uniformly through the mass of air so that as the liquid spray turns into vapor it will produce a homogeneous mixture of air and vapor. Alcohol can also be used in a gasoline carbureter.

As with all substances which liquefy at ordinary temperatures, there is a definite limit to the amount of alcohol vapor which can exist in a cubic foot of space at any given temperature. Assuming the laws for perfect gases to hold, at any given constant temperature the weight of alcohol vapor present in a cubic foot of space is proportional to its vapor pressure and is usually measured or represented by this vapor pressure. This may be

* Approx. results calculated in a similar manner for various petroleum fuels may be found in Sorel's *Alcohol Engines*.

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illustrated by imagining a cylinder provided with a tight piston and containing alcohol vapor at a pressure corresponding to 10 millimeters of mercury and kept constantly throughout the experiment at a temperature of 70° F. If now the vapor is compressed by the piston until its volume is reduced one-half, the vapor pressure will rise to 20, there being of course twice as much vapor per cubic foot of space occupied as there was originally. If the volume is again halved the pressure will rise to 40. But if the compression is continued until the vapor pressure rises to 47 millimeters of mercury a change takes place in the action. The pressure will not rise above 47 if the temp. is kept at 70°. If the piston is moved a further amount, so as to reduce the volume still more, part of the alcohol vapor will be condensed into liquid, but the vapor pressure will remain the same and the amount of vapor per cu. ft. of space will remain constant. Hence there is a definite maximum amount of alcohol vapor which can exist in a cu. ft. of space at any given temp. It is important to remember that the space may contain any smaller amount with a correspondingly lower vapor pressure, but can not contain a greater amount than the quantity corresponding to the saturated state.

The vapor pressure of saturation increases rapidly with the temp., and the values as determined by experiment for alcohol and some other substances at various temperatures are given in Table 13, next page.

When different gases or vapors exist simultaneously in the same space if they have no chemical action on each other, each one acts by itself just as though no other gas were present. Thus if air were also present in the cylinder used in the illustration above, the oxygen and nitrogen would not interefere at all with the action of the alcohol vapor. But it is to be noted that in such a case the pressure as measured by the barometer column or pressure gauge would be the sum of the separate pressures due to the air and due to the alcohol vapor. If moisture were presnet it would probably have some effect on the alcohol-vapor pressure, because water and alcohol have certain affinity for each other. On page 1371 it was shown that in a mixture of alcohol vapor and air the proportion for complete combustion of the alcohol should be 9.06 lbs. of air to 1 lb. of alcohol. If less air is present the alcohol cannot be completely consumed and the excess passes off as alcohol or some substance formed by the partial decomposition of the alcohol molecule. If more air is present than the required amount no harm is done provided the excess is not too great, and it is in fact better so far as economical consumption of fuel is concerned to have some excess of air present.

The vapor pressure of alcohol vapor in the theoretically best mixture of it with air may be calculated as follows: By Avagadro's law, for the same pressure and temp., the densities of gases are proportional to their molecular weights. Since the molecular weight of hydrogen is 2, the density of ethyl alcohol vapor is 46÷2, or 23, compared with hydrogen. Hence 1 lb. of alcohol vapor occupying any stated volume has a vapor pressure equal to $\frac{1}{23}$ of the vapor pressure of 1 lb. of hydrogen occupying the same volume. Likewise, since the density of air is 14.44 compared with hydrogen, 9.06 lbs. of air occupying the same stated volume has a vapor pressure equal to $\frac{9.06}{14.44}$ of the vapor pressure of 1 lb. of hydrogen occupying the same volume. Therefore the relative vapor pressures of the alcohol vapor and air are as $\frac{1}{23}$ and $\frac{9.06}{14.44}$, or as 0.0435 and 0.627, respectively. But 0.0435+0.627=0.670. Hence, of the total vapor pressure produced by the mixture, $\frac{435}{6700}$ or 6.5%, is due to alcohol vapor, and $\frac{627}{670}$, or 93.5%, is due to the air.

If the mixture is under the ordinary atmospheric pressure of 14.7 lbs. per sq. in., or under 760 mm. of mercury, the vapor pressure of the alcohol in the combustible mixture is 0.065 times 14.7 or 0.955 lb. per sq. in., or 0.065 times 760, or 49.4 mm. of mercury. Similarly, the pressure of the air is 0.935 times 14.7, or 13.74 lbs. per sq. in., or 0.935 times 760, or 711 mm of mercury.

Table 13, following, contains the vapor pressure of saturation, in mm or mercury, for pure ethyl or grain alcohol, pure methyl or wood alcohol, a sample of gasoline, and water, at various temperatures:

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13.—VAPOR PRESSURE OF SATURATION FOR VARIOUS LIQUIDS.*

Temperature.		Vapor Pressure of Saturation in Millimeters of Mercury			
		Pure Ethyl Alcohol.	Pure Methyl Alcohol.	Water.	Gasoline.
°C.	°F.				
0	32	12	30	5	99
5	41	17	40	7	115
10	50	24	54	9	133
15	59	32	71	13	154
20	68	44	94	17	179
25	77	59	123	24	210
30	86	78	159	32	251
35	95	103	204	42	301
40	104	134	259	55	360
45	113	172	327	71	422
50	122	220	409	92	493
55	131	279	508	117	561
60	140	350	624	149	648
65	149	437	761	187	739

* The values for ethyl and methyl alcohol are taken from the Smithsonian physical tables, the values for water from the steam tables in general use, and the values for gasoline, which were based upon tests of a sample of French commercial gasoline, are taken from Sorel's book on alcohol engines. Since gasoline is a variable substance, the values given for it are to be considered as only generally representative.

From the above table it is evident that ethyl alcohol can have a vapor pressure of 49 mm. only if its temp. is 72° F. or higher. Hence, a mixture of air and the alcohol vapor in the theoretical proportions for perfect combustion can not exist at a temp. below 72° F. A combustible mixture with some excess of air can exist at lower temperatures, as also a mixture in which a part of the alcohol is not in the form of vapor, but is carried with the air mechanically in the liquid form as a spray or fog.

The table shows that methyl alcohol vaporizes much more readily than ethyl alcohol, and gasoline much more readily than either. A mixture of different fuels will usually have a higher vapor pressure than either of the separate ingredients unless one is present in too small a quantity to produce saturation.

Since alcohol, as used commercially, is always mixed with some proportion of water, a combustible mixture formed by the vaporization of such alcohol may become saturated with the water vapor before it is saturated with alcohol, and this may retard the complete vaporization of the alcohol. Such a state is more likely to occur if the air originally contains a considerable amount of water vapor—that is, if the relative humidity is high. In such a case a temp. higher than 72° would be necessary to maintain the required amount of alcohol vapor in the mixture.

In order that alcohol may change from a liquid to a vapor it must receive a large amount of heat either from the air with which it mixes or from the metal parts of the carburetter with which it comes in contact. The hotter these parts the more quickly the alcohol can absorb the requisite amount of heat. But if the air is too hot there is danger that the mixture of air and alcohol vapor produced may be too rich in alcohol and some of the vapor must remain unburned. Still, if the air be moist, or the alcohol contain water, or the time allowed for vaporization be too short, the temp. of the air must be higher than 72° to form a proper explosive mixture.

Air at any temp. will take up some alcohol vapor, and the higher the temp. the quicker it will take up the amount necessary for the best explosive mixture. In the case of incomplete vaporization, some of the fuel may be carried along as spray, which may or may not be vaporized in the cylinder on the compression stroke. If not, it certainly will be vaporized after the explosion of the rest. It would seem desirable, therefore, to heat considerably the air supplied to an alcohol carburetter. But too much heating of the air will bring about a bad effect on the engine, because it will make the charge hotter at the end of compression, and thus decrease the weight of

the charge in the cylinder. The h. p. of the engine, other things being equal, will be decreased in direct proportion as the density of the charge is lowered by this heating, so that heating of the air before carburetting is good for complete vaporization, but bad if carried too far in its effects on power reduction.

Methods of Testing.—Each engine tested was fitted with a suitable brake for absorbing the power developed. No. 1 was provided with a special water-cooled pulley, attached to the fly wheel. The pulley had an internal rim for retaining the cooling water and an external rim for retaining the brake in place. The latter was formed of wooden blocks, attached to a belt whose length could be adjusted by a screw. A wooden arm, connected to the belt, rested upon platform scales. On all the slow-speed engines similar wooden block band brakes, bearing upon platform scales, were used.

Each automobile engine was fitted with a special water-cooled pulley, attached to its fly-wheel. The pulley was made by cutting out a disk of ⅛" boiler plate, which was bolted to the fly wheel. The outside face of the disk was grooved to receive the rim of a standard cast-iron belt pulley 5" wide. The other edge of the pulley rim was fitted to another ring of boiler plate in a similar manner. Bolts passed from plate to plate. A rope break was used on this pulley, and each end of the rope was fastened to a spring scale, which in turn was suspended from the arm of a beam pivoted to a standard. The other end of the beam was held by a chain block. A movement of the chain block increased or decreased the tension on the rope. The rotation of the engine tended to pull one side tighter than the other, just as in the case with a windlass. A heavy scale, capable of recording 400 lbs., was attached to the beam near its center and carried the tight side of the rope. The other end of the rope was attached to a lighter scale, capable of recording a maximum of 24 lbs. With this brake a 200-lb. pull was registered on the heavy scale, with only 8 lbs. on the smaller scale, and the tension could be quickly and rapidly varied by the chain block.

The speed of the engines was obtained by actual counting, using a stop watch, by a hand speed counter and by a tachometer. Usually the speed was determined by more than one observer, so as to reduce the chances of error.

The ordinary formula was used for computing the brake horse-power,

$$BHP = \frac{2 \pi RWN}{33,000},$$

in which R = the brake arm in feet, W = the brake load in pounds, and N = the number of revolutions per minute.

On all the slow-speed engines, indicator cards were taken not only for the purpose of determining indicated horse-power but also for studying the characteristics of the combustion, compression, and other conditions in the cylinder. For this use a new outside spring indicator was loaned for these tests by the manufacturers of the indicator.

It was difficult to determine the proper mean effective pressure to use in computing indicated horse-power, because successive strokes often gave indicator diagrams of very different size and shape. This was shown on some of the cards reproduced later. Hence it was impossible to determine exactly the average mean effective pressure for a stated period of time. This difficulty was avoided so far as possible by taking a large number of cards. When there were numerous differing cycles drawn on one sheet, usually the planimeter point was moved around on a line as near the average of the different cycles as could be determined by eye. This gave a sort of graphical average which seemed the best that could be done under the circumstances. In certain cases, explained in detail as they arise, cycles of different size were separately measured and the different areas were used in computing the indicated horse power. No attempt was made to measure the area of the lower or suction loop on the indicator cards with the planimeter; the mean effective pressure is based upon the measured area of the upper loop only.

The number of explosions per minute was determined in hit-and-miss governed engines both by counting the number of fuel admissions and also by listening to the exhausts. This dual method is necessary because, under certain circumstances, a charge may miss fire and an explosion may be recorded from observations of fuel admissions where one did not really occur.

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The indicated horse-power was computed by the usual formula,

$$IHP = \frac{PLAN}{33,000},$$

in which P = mean effective pressure in lbs. per sq. in., L = stroke of piston in feet, A = area of piston in sq. ins. and N = number of explosions per minute.

The engines were in all cases piped for alcohol fuel and also for either gasoline or kerosene, so that one could be switched on and the other off at any time. In addition a third connection was provided for the measuring tanks. Any variation in the adjustment of the engine was secured when using the permanent supply tanks before switching on the measuring tanks, and care was exercised to insure that there was no residue between the valve and the engine itself before measurements were taken.

Two methods of measuring fuel were used: (1) by noting the drop in level in a glass gage attached to a tank; and (2) by the use of a large glass beaker on a small spring platform scale, which could be read to tenths of an ounce.

Fuels used.—The alcohol used in tests was bought on competitive bids as 95% commerical alcohol and cost in barrels 38½ cents per wine gallon without the tax.

An ultimate analysis of the alcohol gives the following composition:

	Per cent.
Carbon	47.6
Hydrogen	12.7
Oxygen (by difference)	39.7

As received the alcohol had a spec. grav. of 0.82 at 60° F., corresponding to about 91.1% by weight or 94% by volume, according to the Smithsonian physical tables. It was tested in the chemical laboratory in the usual way to determine the percentage of alcohol present after treatment to eliminate impurities, as follows:

Twenty-six grams were diluted with water, redistilled, and the amount of alcohol in the distillate determined. From this the percentage by weight in the original sample was calculated. Using Richard's tables the result obtained was 93.1%. Using Morley's table, published in the journal of the American Chemical Society, October, 1904, the result obtained was 91.4%.

The ultimate analysis indicates a slightly smaller proportion of alcohol, since a strength of 91.3% would have the following composition:

	Per cent.
Carbon	47.6
Hydrogen	12.9
Oxygen	39.5

These discrepancies, illustrating the difficulties of accurate determinations, even with suitable facilities and skillful observers, show the impossibility of obtaining more than approximate results except under exceptionally favorably circumstances.

The percentage of alcohol found in a sample is always likely to be greater when determined chemically than when determined by the hydrometer, because the presence of impurities in the way of solids dissolved in the alcohol or as any of the series of higher alcohols tends to make the spec. grav. of the sample greater and hence indicate too low a percentage of alcohol.

The calorific power or heat of combustion of the alcohol used in these tests was found by calorimeter determinations to be 11,880 British thermal units per pound, high value. The corresponding low value is 10,620.

Using the customary values of calorific power of 14,500 for carbon and 62,000 for hydrogen, the high heat value of combustion for pure alcohol would be by calculation 12,950. The Smithsonian physical tables give 12,930 as the value for pure alcohol on the authority of Favre and Silbermann, which would correspond to 11,800 for 91.3% alcohol.

The gasoline used in the experiments, known as "motor gasoline," was bought in New York City at 15 cents a gallon by the barrel and had a spec. grav. of about 0.71 at 60° F. Its ultimate composition was as follows:

	Per cent.
Carbon	85.0
Hydrogen	14.8
Total	99.8

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Its heat of combustion, high value, as determined by the calorimeter, was 21,120 B. T. U. per pound. The corresponding low value is 19,660.

Using, as before, 14,500 for carbon and 62,000 for hydrogen, the calculated high heat value would be 21,500.

To show the nature of the complex mixture forming the gasoline, 150 cubic centimeters was distilled and collected in fractions of 10 cubic centimeters each. The temperatures required for the distillation of the successive fractions are shown in the following table:

14.—FRACTIONAL DISTILLATION OF GASOLINE.

Number of Fraction.	Temperatures.		Number of Fraction.	Temperatures.	
	°C.	°F.		°C.	°F.
1.....	46 to 60	114.8 to 140.0	8.....	97 to 100	206.6 to 212.0
2.....	64 to 75	147.2 to 167.0	9.....	100 to 104	212.0 to 219.2
3.....	75 to 80	167.0 to 176.0	10.....	104 to 108	219.2 to 226.4
4.....	80 constant	176 constant	11.....	108 to 112	226.4 to 233.6
			12.....	112 to 120	233.6 to 248.0
5.....	80 to 86	176.0 to 186.8	13.....	120 to 126	248.0 to 258.8
6.....	86 to 92	186.8 to 197.6	14.....	126 to 140	258.8 to 284.0
7.....	92 to 97	197.6 to 206.6	15.....	140 to 155	284.0 to 311.0

From this table it appears that when distillation began the vapor showed a temp. of 46° C. or 114.8° F. and that the distillation was stopped at a temp. of 155° C. or 311° F., and that at this temp. there was left 5 cu. cm. that had not yet been vaporized.

The kerosene used for testing had a spec. grav. of about 0.800 at 60° F. It was assumed to have the same heat of combustion as the gasoline used.

In making computations on the results of the tests, the following weights were used.

Substance.	Spec. Grav.	Lbs. per Gallon	Lbs. per Pint.	Pints per Lb.	Substance.	Spec. Grav.	Lbs. per Gallon	Lbs. per Pint.	Pints per Lb.
Water.....	1.000	8.33	1.04	0.96	Gasoline..	0.710	5.91	0.74	1.35
Alcohol....	0.820	6.83	0.85	1.17	Kerosene..	0.800	6.66	0.83	1.20

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EXCERPTS AND REFERENCES.

The Loomis Water-Gas and Producer-Gas Process (Eng. News, Sept. 2, 1901).—Described and illustrated.

Experiments on the Escape of Steam Through Orifices (By M. Rateau; aper, Inter. Eng. Cong. at Glascow; Eng. News, Sept. 19, 1901).—Sketch f apparatus used in the experiments; and diagram illustrating the experi- ıents.

Heat Resistance, the Reciprocal of Heat Conductivity (Eng. News,)ec. 4, 1902).—Includes the following table:

HEAT CONDUCTING AND RESISTING VALUES OF DIFFERENT INSULATING MATERIALS.

No.	Insulated Material.	Conductance B.T.U. per sq. ft. per day per degree of difference of temperature. 24 *K*.*	Coefficient of heat. resistance. *C*.*
1	⅛-in. oak board, 1-in. lampblack, ⅛-in. pine b'd (ordinary family refrigerator)	5.7	4.21
2	⅛-in. board, 1-in. pitch, ⅛-in. board	4.89	4.91
3	⅛-in. board, 2-in. pitch, ⅛-in. board	4.25	5.65
4	⅛-in. board, paper, 1-in. mineral wool, paper, ⅛-in. board	4.6	5.22
5	⅛-in. board, paper, 2½-in. mineral wool, paper, ⅛-in. board	3.62	6.63
6	⅛-in. board, paper, 2-in. calcined pumice, ⅛-in. board	3.38	7.10
7	Same as above, when wet	3.90	6.15
8	⅛-in. board, paper, 3-in. sheet cork, ⅛-in. board	2.10	11.43
9	Two ⅛-in. boards, paper, solid, no air space, paper, two ⅛-in. boards	4.28	5.61
10	Two ⅛-in. boards, paper, 1-in. air space, paper, two ⅛-in. boards	3.71	6.47
11	Two ⅛-in. boards, paper, 1-in. hair felt, paper, two ⅛-in. boards	3.32	7.23
12	Two ⅛-in. boards, paper, 8-in. mill shavings, paper, two ⅛-in. boards	1.35	17.78
13	The same, slightly moist	1.80	13.33
14	The same, damp	2.10	11.43
15	Two ⅛-in. boards, paper, 3-in. air space, 4-in. sheet cork, paper, two ⅛-in. boards	1.20	20.00
16	Same, with 5-in. sheet cork	0.90	26.67
17	Same, with 4-in. granulated cork	1.70	14.12
18	Same, with 1-in. sheet cork	3.30	7.27
19	Four double ⅛-in. boards (8 boards), with paper bet. three 8-in. air spaces	2.70	8.89
20	Four ⅛-in. boards, with 3 quilts of ⅛-in. hair bet. papers separating boards	2.52	9.52
21	⅛-in. board, 6½in. patented silicated strawboard finished inside with thin cement	2.48	9.68

*K=conductance per hour; $C=\frac{1}{K}$.

Gas Engine Principles and Management (By E. W. Roberts. Eng. News, Sept. 17, 1903).—Illustrated.

Notes on the Arrangement and Construction of Steam Pipes and Their Connections (By R. C. Monteagle. Paper, Soc. of Nav. Arch. and Mar. Engr.; Eng. News, Nov. 26, 1903).—Illustrated.

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Staybolts, Braces and Flat Surfaces in Boilers (By R. S. Hale. Paper. Am. Soc. M. E., Dec., 1904; Eng. News, Dec. 15, 1904).—Discussion and tables.

The Use of Superheated Steam in Locomotive Boilers (By H. H Vaughan. Paper, Am. Ry. M. M. Assn., June, 1905; Eng. News, June 22, 1905).

Modern Problems in Gas Engineering (By Fred B. Wheeler. "Wisconsin Engineer," Dec., 1905; Eng. News, Jan. 11, 1906).—Includes table of standard gases.

The Storage of Coal by Submergence in Salt Water (Eng. News, Aug. 23, 1906).

The Use of Oil Fuels for Locomotives (Com. Rept. Trav. Engrs. Assn., Aug., 1906; Eng. News, Jan. 10, 1907).—Illustrations of the Booth, Sheedy and Lassoe-Lovekin oil burners.

Comparative Cost of Gasoline, Gas, Steam and Electricity for Small Powers (By W. O. Webber. Eng. News, Aug. 15, 1907).—Diagram. See, also, in same issue, article by F. W. Ballard, entitled "Relative Economy of Steam and Gas Power Where Exhaust Steam is Used for Heating."

The Solution of Steam Problems by the Use of a Diagram (By Lionel S. Marks, "Steam Tables and Diagrams." Eng. News, Aug. 26, 1909).—The diagram is 12 x 15 ins. and may be used in solving steam problems, of which numerous examples are given. The old equation (Regnault's) for the total heat of dry and saturated steam is,

$$H = 1091.7 + 0.305\ (t - 32);$$

The Marks and Davis equation for the range of temperature from 212° to 400° F. is,

$$H = 1150.3 + 0.3745\ (t - 212) - 0.000550\ (t - 212)^2.$$

Below 212° the new values are well fixed by a number of individual determinations, but have not been represented by any simple equation.

Unique Direct-Acting Explosion Pump (Eng. News, Dec. 2, 1909).—Developed by H. A. Humphrey, an English engineer. This pump has neither piston nor cylinder, strictly speaking: as the water serves as the one and the waterway the other. The pumping is done with approx. 1 lb. of coal per water horse power in comparison with 2 lbs. for compound high-duty steam engines, 1.7 lbs. for triple-expansion engines, and 1.6 lbs. for a gas-engine driving a centrifugal pump. Described and illustrated, with tables of duty. It is possible that this type of pump might prove highly efficient in connection with hydraulic dredging.

Illustrations of Various Kinds:—

Description.	Eng. News.
Front and side views of Emerson steam vacuum pump	Oct. 31, 1901.
Plans of the Northern Power Sta. of St. Louis Transit Co.	April 10, '02.
Raw Peat briquetting press, and coking retort	June 12, '02.
The Francke single-rotary-valve engine	Feb. 5, '03.
An adjustable staybolt for locomotive boilers	July 23, '03.
Malleable packings of various designs	Aug. 6, '03.
Steam turbines (4 articles), well illustrated	June 6, '04.
Long. vert. section of Mietz & Weiss oil eng. with evap. jacket	Sept. 15, '04.
Section through lower stories of Phipps power building, Pitts.	Sept. 15, '04.
300-H. P. 2-stage centrifugal pump, turbine driven	Oct. 6, '04.
Street railway power house in Kansas City	Oct. 19, '05.
Construction details of a modern gas-holder	Oct. 26, '05.
Centrifugal cinder separator for power plant smoke	April 12, '06.
Structural details of Long Island Power Station	May 31, '06.
New designs in flexible stay-bolts	Jan. 21, '09.
General arrangement of suction ash-conveying plant	Aug. 8, '09.
Tarless oil-gas producers, with test tables	Dec. 9, '09.
A commercial fuel-briquette plant	Apr. 14, '10.
Smyth, and Humphrey, direct-acting explosion pumps	May 19, '10.
Test of locomotive using superheated steam, A. T. & S. F. Ry	June 2, '10.
The physical meaning of Entropy; illus.	Sept. 1, '10.

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70.—ELECTRIC POWER AND LIGHTING.

Electricity as a Form of Energy.—Under Steam and Gas Power (page 1347) we have seen that heat and mechanical work are mutually controvertible; that is, 1 heat unit (B. T. U.) is equivalent to 778 ft.-lbs. of work, or 1 ft.-lb. of work = 0.001285 heat unit. We will now show that mechanical-, thermal-, and electrical work- and power units are mutually convertible, or have definite relations to each other.

The Electric Power Unit is the *watt*, equivalent to a flow or "current" of 1 *ampere* under a "pressure" of 1 *volt*. Thus, 60 watts means 60 volt-amperes, or a current of 10 amperes at a pressure of 6 volts, or 15 amperes at 4 volts, etc. Watts (P) = volts (E) × amperes (C). Moreover, 1 watt = $\frac{1}{746}$ horse-power = 0.7373 ft.- lb. per second = 44.238 ft.-lbs. per minute = 265.428 ft.-lbs. per hour. Note that the watt does not represent work but the *rate* of work, or power. The watt- second, minute, or hour is equal to $\frac{1}{746}$ horse-power second, minute or hour, when applied to work. The watt unit, being small, is generally supplanted by the *kilowatt* in the design and rating of electric power plants and machines. (See Tables 34 and 35, Sec. 4, page 88, etc.; also Tables 1 and 2, Sec. 69, page 1348, etc.)

The Kilowatt, K.-W. (= 1000 watts) = 1000 × .7373 = 737.3 ft.-lbs. per second = 44,238 ft.-lbs. per minute = 265,428 ft.-lbs. per hour; which is equivalent to 1.3405 horse power. Conversely, 1 horse power = 0.746 kilowatt. Comparing with units of *mechanical work*, it is evident from the above that 1 kilowatt hour = 1.3405 horse-power hour; and that 1 horse-power hour = 0.746 kilowatt hour. Comparing with units of *thermal work*, 1 kilowatt hour = $\frac{265\ 428}{778}$ = 341.17 thermal units; and conversely, 1 thermal unit (B. T. U.) = 0.002931 kilowatt hour (= 2.931 watt hours), or 10,000 B. T. U. (= amount of heat that may be assumed to be generated in a boiler from the combustion of 1 lb. of coal) = 29.31 kilowatt hours.

A Kilowatt is an "electric horse-power."

Problem 1 (Steam-Electric).—What power can be expected from a dynamo (generator) operated by a steam engine, with a consumption of 500 lbs. of coal per hour (assuming that each 100 lbs. generates 1,000,000 B. T. U. in the steam); the engine having an efficiency of 10 per cent, and the dynamo 93 per cent?

Solution.—From the above we have 1,000,000 × 5 × 0.002 931 × 0.10 × 0.93 = 1362.92 kilowatts (or 1827 horse-power). *Ans.*

Problem 2 (Hydro-Electric).—A direct-connected dynamo is run by an impulse water-wheel under a static head of 200 feet, and using water at the rate of 48 cubic feet per minute. Assuming the loss of head in the pipe line (from the storage to the wheel) to be 20 ft., the efficiency of the wheel 85 per cent., and the generator 93 per cent*; what power will be furnished to the line?

Solution.—Theoretically, the water power = 200 × 48 × 62.5 = 600 000 ft.-lbs. per minute, or 18.182 horse-power. The efficiency of the pipe line is $\frac{200-20}{200}$ or 90 per cent. Hence, power delivered to line is 18.182 × 0.90 × 0.85 × 0.93 × 0.746 = 9.65 kilowatts (or 18.182 × 0.90 × 0.85 × 0.93 = 12.94 horse-power).

What are Electrical Machines?—Electrical machines are machines for generating, controlling, transforming or converting electrical energy.

Dynamos are machines for converting mechanical- into electrical power, that is, they generate electricity and hence the name "generator." An

* Generators are constructed with efficiencies as high as 96 per cent.

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electric "motor" differs from a generator in that it converts electrical- into mechanical power—a reverse operation. The essential principles of the motor are those of the dynamo. Dynamos and motors may be either alternate-current or continuous-current machines.

Transformers are machines placed at either end (or at any point) of a transmission line to change the potential of the current. For instance, if it is desired to carry the current over the line at a higher potential than it is practicable for the dynamos to generate, the *step-up* transformer is used at the power-house end of the line; while at the delivery end the *step-down* transformer is installed. Transformers are either air-cooled, water-cooled or oil-cooled. In the latter case the whole transformer is placed in oil which penetrates the pores and insulates the wires.

Converters, called rotary converters, are machines for converting alternate currents to direct currents, or vice versa; or they may be used for changing the voltage of continuous currents, or changing the voltage, phase or frequency of akernating currents.

Boosters are machines inserted at distant points in the line, as at the outer ends of street-railway circuits, to compensate the drop in voltage in direct-current mains. A booster is a combination motor-generator or motor-dynamo or series motor driving an armature placed as a shunt across the mains. It can be arranged to either raise or lower the voltage, being generally used for the former purpose.

Principles of Electricity and Magnetism.—Before proceeding immediately with the discussion of electrical machines let us first consider the question—

What is Electricity?—Scientists are pretty well agreed at the present day that *electricity* and *magnetism*, closely associated, are due to certain states of disturbance in the universal substance (?) called *ether*, which pervades all space and gross matter. Other states of disturbance are called *light* and *heat*. Indeed, by some it is considered not improbable that gross *matter* itself is ethereal and electrical by nature, thus accounting for the universal law of *gravitation*. It is not unlikely that the determination of the exact nature of either of the above phenomena will reveal the nature of all the others.

Ether, as a substance, necessarily has mass, and it has been estimated, from experiments made on the energy of waves of light, that a volume of ether the size of the earth will weigh as much as 4 cubic feet of water. It is considered to be perfectly elastic and in a continual state of unrest. It is not difficult to imagine that it may be a fourth state of "matter" in the rising graduation from solid, liquid and gaseous to the ethereal; the first three constituting *gross* matter, with definite chemical composition, and the last, radiant or transcendental matter, whose composition is as yet unknown.

Ether Waves vary greatly in length according to the phenomena produced. The shortest wave lengths of which we have any knowledge (by the action of the photographic plate) number about 300,000,000,000,000,000 vibrations per second (in a length of 186,500 miles or 11,816,640,000 inches) equivalent to less than one 25-millionth of an inch in length. These waves are able to penertate paper, wood and sheets of metal as ordinary light waves penetrate glass. They are far beyond the range of the eye as are also, but to a much less degree, the ultra-violet rays, estimated to be about one 70-thousandth of an inch in length. As the waves become longer they fall within the visible spectrum, the violet rays being about one 60-thousandth of an inch, and the red rays (the longest rays visible), about one 37-thousandth of an inch in length. Those rays from the violet to the red (including all the known colors) have the peculiar property, by virtue of the transverse vibrations, of acting on the retina of the eye (consisting of minute protuberances which are set in vibration) and producing the sensation of light. As the waves increase in length we get first the heat wave and finally the electric wave, the latter being the longest.

Electricity and Magnetism are mutually related, and it is owing to these peculiar properties or phenomena that we are enabled to generate electricity and use it, commercially.

Magnetic Field.—When a current of electricity flows "through" a wire in a closed circuit it *induces* around the wire a magnetic "field." Let Fig. 1 represent a section of such a circuit with the copper wire piercing a

small sheet of paper, and the current flowing downward as shown by the vertical arrows. If, now, a few iron filings are sprinkled on the paper and the paper tapped gently, it will be found that the filings will arrange themselves in concentric rings as shown. If, further, a compass is brought close to the wire, the needle will be found to swing in a position tangent to the filing rings, with the *N* pole pointing in the direction of the arrows shown on the paper. Upon reversal of the current through the wire the magnetic needle will also reverse in direction by 180°. We may say, then, that the "lines of force" in the induced magnetic field are right-handed or clockwise when looking in the direction of the circuit. The rapid reversal of the electric current in the circuit, producing what is called the alternating current (as against the continuous or direct current) induces, therefore, an alternating magnetic field. The "strength' of the field is directly proportional to the current strength, and decreases with the distance from the wire.

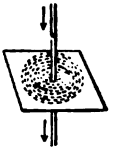

Fig. 1.

The Electro-Magnet.—The principle of the electro-magnet is based on the magnetic field, explained above. *A B*, in each of the following Figs., is the primary circuit with the current flowing in the direction *A B*, as shown

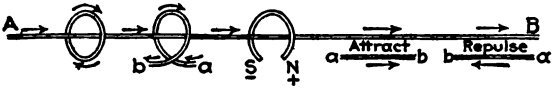

Fig. 2. Fig. 3. Fig. 4. Fig. 5. Fig. 6.

by the arrows. Now we have seen, Fig. 1, that when a current of electricity flows through the primary circuit, lines of force are set up, around the conductor, in a clock-wise or cock-screw direction. If the iron filings are replaced by a circular ring of metal, as in Fig. 2, lines of electro-magnetic force will be *induced* in the ring in the direction shown by the arrows. Fig. 3 shows the same phenomenon, but with the magnetic circuit forming merely a loop around the primary. Fig. 4 shows the circuit of the loop "broken" thus forming a primitive horse-shoe magnet, the + and − ends of which attract each other, tending to draw together and "close" the circuit. Figs. 5 and 6 show that two *parallel conductors attract* each other when the currents flow in the same direction, and *repel* each other when the currents flow in opposite directions. By reversing the direction of the current in *A B*, the direction of the magnetic circuit in Figs. 2 and 3 will also be reversed, and in Fig. 4 the poles of the magnet will change from + to −, and from − to +. Repulsion will now take place between the parallel conductors in Fig. 5, and attraction in Fig. 6, assuming that the currents *a b* remain unchanged. Now assume *a b*, Figs. 5 and 6, to be uncharged from any secondary batteries, and assume the current in the primary conductor to flow in the direction *A B*; then, if the current is continuous there will be no induced current in *a b*, but if it is alternating there will be, during the time the current is increasing in strength or "setting up of the field," a tendency toward generating currents in *a b*. Such currents are called induced currents, and the general phenomenon is termed Induction.

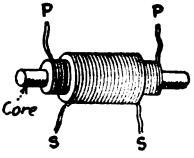

Fig. 7.

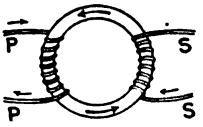

Fig. 8.

Induction can, perhaps, best be explained by the theory that magnetism is merely electricity in a whirl, rotating about a conductor in the surround-

ing ether. It is greatly assisted and intensified by the use of a soft-iron core around which a certain number of turns of each circuit is wound. Fig. 7 illustrates an induction coil or Transformer consisting essentially of a core around which is wound x turns of the primary circuit, P-P, and then y turns of the secondary circuit, S-S. From the number of turns of wire, we have, electro-motive force in S-$S = \frac{y}{x} \times$ (electro-motive force in P-P), therefore, by increasing the ratio of turns in secondary coil to those in primary coil the E. M. F.* is increased.

Faraday's ring, Fig. 8, consisted of a circular core with the primary and secondary coils on either side. The arrows on the ring indicate the direction of the electro-magnetic stresses. An alternating current in the primary coil P (Figs. 7 and 8) generates an alternating current in the secondary coil S; and again, S may be used as the primary coil and P as the secondary coil.

The Horse-Shoe Magnet is commonly made by bending a piece of steel in the form of a horse-shoe as shown in Fig. 9, winding it with fine copper wire (either partially as shown by the double lines or completely around the loop as shown by the single lines) and passing a current of electricity through the wire as shown by the arrows, "breaking" (shutting off) the current occasionally. The wire is now unwound and it is found that the bar of steel has become a *permanent* magnet, capable of attracting magnetic substances as iron or steel. If *soft iron* is used for the "field core" instead of steel it will retain the properties of a magnet only when the current of electricity is "exciting" the field. Such a magnet is called an *electro-magnet*. Note that the armature A when placed within the "influence" of the magnetic poles of the field core is attracted by the latter, that is, the armature tends to "close" the magnetic circuit. The telegraph "sounder" is founded on this principle.

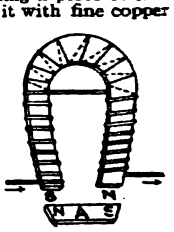

Fig. 9.

Electro-Magnets are made in various forms, depending on the kind of work they are intended to do. We stated in the last paragraph that the armature A (Fig. 9) is attracted by the poles of the field core and tends to "close" the magnetic circuit. This is only partly true. As a matter of fact the magnetic circuit in Fig. 9 is already complete without the presence of the armature, but it is comparatively feeble and grasps the armature to form a more "permeable" path for its circuit than the air offers, much the same as a man uses a plank to cross a small stream instead of wading. The "Theory of least work" applies to all electric and magnetic phenomena as it does to mechanics. It is simply another name for the "Conservation of Energy."

Principle of the Alternate-Current Dynamo.—A simple form of generator is illustrated in Fig. 10. The magnetic "field" between N and S is induced by either a permanent magnet or an electro-magnet (see Fig. 9). Between the poles of the magnet is shown a loop of copper wire which may be made to revolve on an axis $a\,b$; we have also to imagine that there are *lines*, tubes, or cylinders, *of force* stretching between the N and S poles of the magnet. Now when the copper loop is revolved, say right-handed, it cuts these lines of force and *alternate* currents of electricity are generated in the copper wire circuit, the N pole inducing a current in the nearest parallel portion of the loop, toward a, and the S pole toward b. Both currents therefore flow around the coil in the *same direction* for each half turn, and then reverse. It is to be noted, however, that the comparative strength of the current depends on the number of lines of force cut per second, that is, the rate of transverse cutting of the lines of force. By referring to the end view in Fig. 10 it will be seen that this rate is proportional to sin α; when $\alpha = 90°$ the copper loop is cutting the lines of force at a maximum rate, unity; when $\alpha = 45°$ the rate is 0.707; when $\alpha = 0°$ no lines

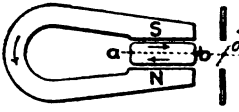

Fig. 10.

* Electro-motive force is voltage or difference of potential. E. M. F. is increased at the expense of current, where the power remains the same.

of force are being cut and hence no current is flowing in the circuit. This last position is also the point of alternation of the current, which continues in one direction for every *half* revolution of the coil and then reverses.

The electricity generated in the revolving armature, for one-half turn, is conducted to the revolving "collecting"* ring, *R* (Fig. 11), thence by the fixed "brush" *B* to the line *L*, where it is made to perform useful work such as running a motor, *M*. Passing through *M* it returns to the armature by way of *L'*, *B'* and *R'*, thus completing the circuit. For the next half turn it reverses in direction, thus alternating. If the circuit is disconnected or broken at any point no power can be furnished. The earth is sometimes used for the return current as in telegraph circuits. In the case of electric railways the rails are used. If they are "bonded" the "resistance" is greatly lessened.

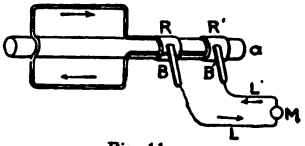

Fig. 11.

Now instead of having a "revolving armature" composed of only *one* copper wire loop as shown in Fig. 10, we may increase the current output of the machine by using a cylindrical armature composed of a *large number* of loops; also, the field-magnet with only two poles may be increased so as to contain a large number of pairs of poles.

The magnetic field of large commercial alternate-current dynamos is generally maintained by small constant-current generators called "exciters." One exciter will answer for one or more of the large alternators, and being continuous-current machines they are self-exciting (see page 1384).

Classification of Alternate-Current Dynamos.

(See also page 1384.)

A.—With respect to stationary and moving parts:
- (a) Stationary field magnets and rotating armatures;
- (b) Stationary armatures and rotating field-magnets;
- (c) Stationary field-magnets and armatures, and revolving inductors. (Not common. Ex.: Stanley-Kelly inductor alternator.)

B.—With respect to number of field-magnet poles:
- (a) 2-pole (bipolar), 4-pole, 6-pole, 8-pole machines;
- (b) Multipolar machines include all above the bipolar. (Multipolar machines, with many poles, used for high power.)

C.—With respect to form of armature:
- (a) Ring armature;
- (b) Drum armature;
- (c) Disk armature (core disks, sunk wound, are preferred to thin disks for high power machines, because of strength);
- (d) Pole armature.

D.—With respect to armature coils:
- (a) Open coil;
- (b) Closed coil;

} Also { (A) In parallel { For increase of current.
(B) In series { For increase of voltage.

E.—With respect to armature windings:
- (a) Spiral or ring winding (ring armatures);
- (b) Lap winding
- (c) Wave winding
- (d) Sunk winding.

} drum and disk armatures;

F.—With respect to field-magnet windings:
- (a) Spiral winding;
- (b) Lap winding;
- (c) Wave winding;
- (d) Sunk winding.

Also { (A) In parallel.
(B) In series.

* Collecting rings or simply collectors are used on alternate-current machines, while "commutators" (see page 1384) are used on continuous-current dynamos.

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G.—With respect to phase of machine:
(a) Single-phase, 2-phase, 3-phase machines;
(b) Polyphase machines include all above 2-phase.
A single-phase alternator is one in which the currents act in unison, rising and falling together; while in a polyphase alternator the coils are arranged so that the impulses are produced closer together, thereby maintaining a more uniform electro-motive force. 2-phase and 3-phase alternators are the most common.

Principle of the Continuous-Current Dynamo.—Many of the principles of the alternate-current dynamos, just described, apply equally as well to the continuous-current machines. We have the field-magnet and also the armature; but instead of using collector rings or collectors as for the alternators, we use *commutators*, which deliver to the external circuit a *continuous current*, that is, a current in one direction through the external circuit.

Fig. 12 shows a simple, two-part commutator, K and K', joining the ends of a single-loop armature revolving between the poles of a magnet on the axis $a\ b$, and delivering current to the brush B. Note that the brushes B and B' are fixed, and as the commutator revolves (with the armature) the K-part and the K'-part alternate in delivering current to B, and hence it always flows through the external circuit, from B to B', *in one direction*.

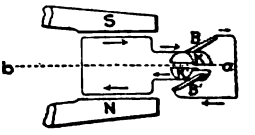

Fig. 12.

Continuous-current dynamos are made with two, four, six or eight poles, the two-pole machines being the most common. The armature winding, however, is very complex. Instead of one loop, as in Fig., 12 there are many loops; and the commutator, instead of being composed of two shells or bars, as above, may be composed of many bars (see Fig. 13). Hard-drawn copper is best for the commutator bars, and also for the brushes.* All field and armature windings, as well as commutator bars, should be insulated. The latter are insulated with mica.

Classification of Continuous-Current Dynamos.

(See also page 1383.)

I.—With respect to excitation of field-magnets:
(a) *Separately-excited* dynamo (Fig. 13), using a separate (small) dynamo as an exciter;
(b) *Series* dynamo (Fig. 14), using the full current of the external circuit as an exciter;
(c) *Shunt* dynamo (Fig. 15), using a small fraction of the external circuit (by a shunt circuit of thin wire) to excite the field;
(d) *Compound* dynamo (Fig. 16), using both shunt coils and series coils to excite the field.

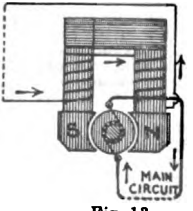

Fig. 13.

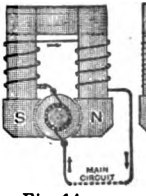

Fig. 14.

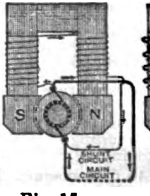

Fig. 15.

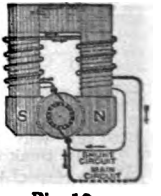

Fig. 16.

Note.—In addition to the above we have also the *Magneto* dynamo, in which the field-magnets are permanent steel magnets; and the *Separate-coil* dynamo, in which the *armature* is wound with a separate coil to act as an exciter.

* Carbon brushes are used for *motors*.

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II.—With respect to other characteristics see classification of alternate-current dynamos, page 1383, with the following comments for direct-current machines:

A.—(a) Stationary field-magnets and rotating armatures, is the prevailing type.

B.—Bipolar machines are generally used; although 4-, 6-, and 8-pole machines are not uncommon. There is another type, namely, the homopolar (unipolar or 1-pole) machine which is being developed with more or less success.

C.—(b) Drum armatures are the most efficient and are generally used where high potential is required. Ring armatures give lower E. M. F. and less current for the same number of revolutions.

D.—(b) Closed-coil armatures, generally speaking, give greater commercial efficiency than open-coil machines; but the latter (whether of the ring or drum type) are better adapted to high electro-motive forces because of the methods of insulation of commutator bars. The Brush and the Thomson-Houston open-coil dynamos are especially adapted to electric lighting, and it may be said that they practically monopolize the field for this purpose.

E.—(b) Lap winding is the most frequently used.

F.—Wave winding has certain disadvantages and has not come into general use.

G.—Phase applies to alternate-current machines.

Electric Transmission of Power.—Important considerations come up in connection with the projection of any power plant, especially for long-distance transmission; and usually the combined efforts of the civil-, electrical-, mechanical- and hydraulic engineer are required for its successful installation.

Uses of Electric Power.—Electric power can be used for every purpose for which power is required. Its practical utility as compared with the direct use of steam and water power, from which it is derived mainly, lies in the economic, convenient and sanitary distribution from a central station, where it can be generated economically.

Sources of Electric Power.—Entropy, in thermodynamics, has been defined by some scientists as the "available" energy in any system, while others define it as the "non-available" energy in any system. Electricity may similarly be defined. It is only a question of constructing our machines so that the kind of energy which is usually lost may be increased, captured and turned into useful work at some distant point. The dynamo, the transmission line and the motor are specially designed to accomplish this result economically.

The primary source of all electric- or other power is probably electricity itself. The immediate source appears to us in the forms of chemical and physical actions, the latter through the agencies of heat and gravitation, acting on gross matter, either solid, liquid or gaseous. Thus, we get the battery, thermopile, steam-engine, so-called heat engine (including the gas engine), and the water wheel. No method has yet been discovered by which electricity can be generated directly from heat on a large commercial scale. The present indirect use of heat of fuel means a loss of 90 per cent or more of heat energy.

For large electric plants the steam engine and the water wheel only can be considered as prime sources of power. The turbine wheel is best adapted to low heads, and the impulse (bucket) wheel for high heads. The exact dividing line between "low" and "high" heads is not well marked. They both overlap the 100-ft. head, sometimes to a considerable extent.

Steam and Water-Power Compared.—The cost of fuel on the one hand, and the length of transmission, on the other, are prime factors in the comparison. For example, the author has in mind a proposed change from steam- to water power in the generation of electric power for a street railway, demanding about 3000 horse-power. The fuel for the steam plant was very cheap, consisting of saw dust and slabs automatically fed from a neighboring saw-mill. This could have been supplanted by a transmission

30 miles long, and the installation of a power house with impulse wheels operating under a 600-ft. head. The low cost of fuel, however, favored the steam plant. With coal as a fuel the water-power proposition would have been cheaper.

Alternating- vs. Continuous Current.—Although alternate-current motors may be used to convert alternate-current directly into mechanical work, yet, owing to certain disadvantages they have not come into general use. Even in lighting, the arc lamp on a continuous current circuit gives a better illumination, and it is also free from the humming sound produced when the lamps are on alternate-current circuits. With the incandescent lamp, however, there is less contrast in candle power. But there are other considerations which sometimes outweigh the above, and we find both the alternating- and continuous current in distributing systems for motive power and lighting.

The electric transmission of power for *moderate distances* is almost always by *direct current*. This system involves line construction of the most simple character, namely, two lines of wire. The current is generated at the power station by continuous-current dynamos. Each machine may be designed to yield a voltage up to 2500 or even 4000 volts, although the latter is uncommon. In order to secure a high voltage in transmission (which means a saving in copper or aluminum wire) the dynamos are placed in "series." Thus, ten dynamos in series, each generating current at say 2000 volts, will deliver about $10 \times 2000 = 20{,}000$ volts to the line. At the distributing end of the line the total voltage of the motors receiving the energy at any one time must be equal to that of the generators, minus the "drop" in volts on the line. Continuous-current transmission may operate in three ways:

(1) Constant voltage and variable current;
(2) Constant current and variable voltage;
(3) Variable voltage and variable current.

Long-Distance Transmission may be accomplished by either alternating or continuous current. In Europe the latter has many adherents, while in America "long-distance" is almost synonymous with "alternate-current" transmission. The "first cost" of installation, however, is usually greater for alternate-current transmission.

Alternate-current transmission is carried out with three-phase three-wire, two-phase four-wire or single-phase two-wire circuits, the first named greatly predominating, and the last being seldom used. The number of wires varies from three, the lowest, to say six, a maximum per pole in good practice. High voltage for alternate-current transmission can be obtained only by step-up transformers installed at the power station because alternators cannot be connected up in series for higher voltage as can continuous-current dynamos. They may be designed, however, to deliver, individually, current at a much higher voltage, namely, up to 15,000 and possibly 20,000 volts. The step-down transformers at the sub-stations need not be the same in number as the step-up transformers at the generating stations. Transformers can be wound for any rating of voltage of primary to secondary coils, but there are certain economic ratios related more or less to generator capacity. Static transformers are often used to change alternating current from 3-phase to 2-phase, and vice versa.

The Transmission Line.—The material for line conductors has settled down to a choice mainly between aluminum and copper. In fact, the former metal has recently been used in many of the largest installations.* From an economic standpoint, considering the electrical and mechanical properties of the two metals, they possess equal merit when the price of aluminum wire, per pound, is about 2 to 2.1 times the price of copper; or when the price of copper wire is about 48 to 50 per cent of the price of aluminum. Hence, with copper at 22 cents per pound, aluminum would be selected if under 45 cents per pound†. Aluminum *cable* is substituted for *wire* if the size called for is large, say larger than No. 1 B. & S. gage, and sometimes even for smaller sizes. The cables are made up of wires ranging in size usually from 6 to 9 B. & S. gage, sometimes larger. Seven strands per cable are common, and as high as 37 have been used.

* The use of aluminum has not been confined to main transmission lines. It is being used also in distributing power to sub-stations of electric railways, and for city distribution of both light and power. †See page 496.

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The Size of Conductors which it is advisable to use in a transmission line cannot be determined by any simple rule. As a broad proposition it may be stated that the whole power plant, from reservoir (or other source of power) to distribution, should be so designed that, when furnishing the *required* power, it is found that *no slight increase or decrease of power could have been effected more economically at one part of the plant than at another.* But there are many practical considerations. The "required" power varies from hour to hour of the day, from season to season of the year, and also from year to year. We may perhaps assume the "required" power to be the maximum or "peak" load, either present or prospective; or we may assume it to range somewhere between the peak load and the average load, generally near the former and a little below it. If storage batteries are installed to take care of the peak load the "required" power for the line may be lowered accordingly, but if this is not done the peak loads must of course be carried by bringing into action additional (reserved) units at the power house. Such units consist each of say a water wheel (or an engine) with the connected generator or generators.

Let us assume that the "required" power, for which the transmission line is to be designed, has been determined as x watts ($=\frac{x}{1000}$ kilowatts) $=C$ (amperes) $\times E$ (volts). Now we have as limiting guides, in determining the size of the wire, the following, namely, (1) that it is inadvisable to use a single copper wire of a size less than No. 4 B. & S. gage, for reasons of strength and stiffness; (2) a tentative calculation of the line wire should show a loss of say from 5 to 10 per cent of the "required" power delivered to the line, less for short- and perhaps greater for long transmission. Then by applying the above rule (in italics) see whether or not an incremental increase or decrease of power can be effected in the line (by changing the size of wire) at less cost than could be effected at the generator station and at other points.

Transmission Line Problems.

Problem 1.—What size of copper wire is required in a 25-mile continuous-current transmission receiving 3,600 kilowatts at 30,000 volts pressure, allowing a "drop" in volts of 10 per cent on the line?

Solution.—3,600 kilowatts = 3,600,000 watts, hence the current in amperes = 120 amperes. With a 10 per cent drop in voltage the line loss = $30,000 \times 0.10 = 3,000$ volts. Now applying Ohm's law ($C=\frac{E}{R}$) we have, resistance in ohms $=\frac{E}{C}=\frac{3000}{120}=25$ ohms. As this resistance is distributed over 25×2 (2-wire circuit) = 50 miles, the resistance per mile $=\frac{25}{50}=0.50$ ohm, which, from table 1, page 1389 corresponds to a No. 0, B. & S. gage, copper wire. If aluminum is used it would require a No. 000. In general, there is a difference of two numbers between copper and aluminum wire for the same resistance. Note in the above that E = the *difference of potential* between the two ends of the line. Also that the resistance is taken at 75° F., and that the resistance increases with increase of temperature (see Table 1, page 1389).

The above method of solution applies to all continuous-current two-wire circuits, and can be used as well to find the weight of wire per unit of length, or the area in circular mils. It is a question simply of using different tables after the resistance has been obtained. Thus, a resistance of 0.5 ohm per mile is equal to 0.0947 ohm per 1,000 feet, or 10,560 feet per ohm; equivalent to 110,000 circular mils, or 3 feet per pound of wire, at 75° F.

For alternating-current circuits the above method of solution may be used by noting the following: (1) The "virtual volts" and "virtual amperes" of the alternating current* (and these are what are commonly meant), such as are recorded by volt- and ampere meters, must be used in the calculations. (2) There are two additional sources of loss in alternate-current lines that do not appear with the continuous current, namely, *inductance*

(Continued on page 1392.)

* In alternating currents the *maximum* volts and amperes rise to about 1.414 times the *virtual*, alternating between that and zero, the virtual being about 0.707 times the maximum.

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1.—Copper Wire Table of the—

(Supplement to Trans. of A. I. E. E., Oct., 1893.)

The data from which this table has been computed are as follows: Matthiessen's standard resistivity, Matthiessen's temperature coefficient, specific gravity of copper = 8.89. Resistance in terms of the international ohm.

Matthiessen's standard 1 meter-gram of hard drawn copper = 0.1469 B. A. U. at 0° C. Ratio of resistivity, hard or soft copper, 1.0226.

Matthiessen's standard 1 meter-gram of soft drawn copper = 0.14365 B. A. U. at 0° C. 1 B. A. U. = 0.9866 international ohm.

Matthiessen's standard 1 meter-gram of soft drawn copper = 9.141729 international ohms at 0° C.

Temperature coefficients of resistance* for 20° C., 50° C., and 80° C. are

(See Opposite Page for Areas in *Square* Mils.)

Gages.				Weight.				Length.			
					Pounds per Ohm.				Feet per Ohm.		
A.W.G. B.&S.	B.W.G. Stub's.	Diam. Ins.	Area Circ. Mils.	Lb. per Foot.	@ 20°C. = 68°F.	@ 50°C. = 122°F.	@ 80°C. = 176°F.	Ft. per Lb.	@ 20°C. = 68°F.	@ 50°C. = 122°F.	@ 80°C. = 176°F.
0000		.460	211600	.6405	13090	11720	10570	1.561	20440	18290	16510
	0000	.454	206100	.6239	12420	11120	10030	1.603	19910	17830	16080
	000	.425	180600	.5468	9538	8537	7704	1.829	17450	15620	14090
000		.4096	167800	.5080	8232	7369	6647	1.969	16210	14510	13090
	00	.380	144400	.4371	6096	5456	4924	2.288	13950	12480	11260
00		.3648	133100	.4028	5177	4634	4182	2.482	12850	11500	10380
	0	.340	115600	.3499	3907	3497	3155	2.858	11160	9993	9017
0		.3249	105500	.3195	3256	2914	2630	3.130	10190	9123	8232
	1	.3000	90000	2724	2368	2120	1913	3.671	8693	7780	7020
1		.2893	83690	.2533	2048	1833	1654	3.947	8083	7235	6528
	2	.2840	80660	.2441	1902	1702	1536	4.096	7790	6973	6292
	3	.2590	67080	.2031	1316	1178	1063	4.925	6479	5799	5233
2		.2576	66370	.2009	1288	1153	1040	4.977	6410	5738	5177
	4	.2380	56640	.1715	938.0	839.6	757.6	5.832	5471	4897	4419
3		.2294	52630	.1593	810.0	725.0	654.2	6.276	5084	4550	4106
	5	.2200	48400	.1465	684.9	613.0	553.1	6.826	4675	4184	3775
4		.2043	41740	.1264	509.4	455.9	411.4	7.914	4031	3608	3256
	6	.2030	41210	.1247	496.5	444.4	401.0	8.017	3980	3562	3215
5		.1819	33100	.1002	320.4	286.7	258.7	9.980	3197	2862	2582
	7	.1800	32400	.09808	306.9	274.7	247.9	10.20	3129	2801	2537
	8	.1650	27230	.08241	216.7	194.0	175.0	12.13	2629	2354	2124
6		.1620	26250	.07946	201.5	180.3	162.7	12.58	2535	2269	2048
	9	.1480	21900	.06630	140.3	125.6	113.3	15.08	2116	1894	1709
7		.1443	20820	.06302	126.7	113.4	102.3	15.87	2011	1800	1624
	10	.1340	17960	.05435	94.26	84.37	76.13	18.40	1734	1552	1401
8		.1285	16510	.04998	79.69	71.33	64.36	20.01	1595	1427	1288
	11	.1200	14400	.04359	60.62	54.26	48.96	23.94	1391	1245	1123
9		.1144	13090	.03963	50.12	44.86	40.48	25.23	1265	1132	1021
	12	.1090	11880	.03596	41.27	36.94	33.33	27.81	1147	1027	926.9
10		.1019	10380	.03143	31.52	28.21	25.46	31.82	1003	897.6	809.9
	13	.0950	9025	.02732	23.81	21.31	19.23	36.60	871.7	780.2	704.0
11		.09074	8234	.02493	19.82	17.74	16.01	40.12	795.3	711.8	642.3
	14	.08300	6889	.02085	13.87	12.42	11.21	47.95	665.4	595.5	537.4
12		.08081	6530	.01977	12.47	11.16	10.07	50.59	630.7	564.5	509.4
	15	.07200	5184	.01569	7.857	7.032	6.346	63.73	500.7	448.1	404.4
13		.07196	5178	.01568	7.840	7.017	6.332	63.79	500.1	447.7	404.0
	16	.06500	4225	.01279	5.219	4.671	4.215	78.19	408.1	365.2	329.6
14		.06408	4107	.01243	4.931	4.413	3.982	80.44	396.6	355.0	320.3
	17	.0580	3364	.01018	3.308	2.961	2.672	98.23	324.9	290.8	262.4

*Based on a resistance of unity at 0° C.

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—AM. INST. OF ELECTRICAL ENGINEERS.

1.07968, 1.20625, and 1.33681, respectively. 1 foot = 0.3048028 meter, 1 pound = 453.59256 grams.

Although the entries in the table are carried to the fourth significant digit, the computations have been carried to at least five figures. The last digit is therefore correct to within half a unit, representing an arithmetical degree of accuracy of at least one part in two thousand. The diameters of the B. & S. or A. W. G. wires are obtained from the geometrical series, in which No. 0000 = 0.46 inch and No. 36 = 0.005 inch, the nearest fourth significant digit being retained in the areas and diameters so deduced.

It is to be observed that while Matthiessen's standard of resistivity may be permanently recognized, the temperature coefficient of its variation which he introduced, and which is here used, may in future undergo slight revision.

(See Opposite Page for Areas in *Circular* Mils.)

Gages.					Resistance.					
					Ohms per Lb.			Ohms per Foot.		
A.W.G. B.&S.	B.W.G.	Stub's.	Diam. Ins.	Area in Sq. Mils.*	@ 20° C. = 68° F.	@ 50° C. = 122° F.	@ 80° C. = 176° F.	@ 20° C. = 68° F.	@ 50° C. = 122° F.	@ 80° C. = 176° F.
0000			.460	166190	.00007639	.00008535	.00009459	.00004893	.00005467	.00006058
	0000		.454	161883	.00008051	.00008996	.00009969	.00005023	.00005612	.00006220
	000		.425	141863	.0001048	.0001171	.0001298	.00005732	.00006404	.00007097
000			.4096	131790	.0001215	.0001357	.0001504	.00006170	.00006893	.00007640
	00		.380	113411	.0001640	.0001833	.0002031	.00007170	.00008011	.00008878
00			.3648	104518	.0001931	.0002158	.0002391	.00007780	.00008692	.00009633
	0		.340	90792	.0002560	.0002860	.0003169	.00008957	.0001001	.0001109
0			.3249	82887	.0003071	.0003431	.0003803	.00009811	.0001096	.0001215
	1		.3000	70686	.0004223	.0004718	.0005228	.0001150	.0001285	.0001424
1			.2893	65732	.0004883	.0005456	.0006046	.0001237	.0001382	.0001532
	2		.2840	63347	.0005258	.0005874	.0006510	.0001284	.0001434	.0001589
	3		.2590	52685	.0007691	.0008492	.0009412	.0001543	.0001724	.0001911
2			.2576	52128	.0007765	.0008675	.0009614	.0001560	.0001743	.0001932
	4		.2380	44488	.001066	.001191	.001320	.0001828	.0002042	.0002263
3			.2294	41339	.001235	.001379	.001529	.0001967	.0002196	.0002435
	5		.2200	38013	.001460	.001631	.001808	.0002139	.0002390	.0002649
4			.2043	32784	.001963	.002193	.002431	.0002480	.0002771	.0003071
	6		.2030	32365	.002014	.002250	.002494	.0002513	.0002807	.0003111
5			.1819	25999	.003122	.003487	.003865	.0003128	.0003495	.0003873
	7		.1800	25447	.003258	.003640	.004034	.0003196	.0003570	.0003957
	8		.1650	21382	.004615	.005156	.005714	.0003803	.0004249	.0004709
6			.1620	20618	.004963	.005545	.006145	.0003944	.0004406	.0004883
	9		.1480	17203	.007129	.007965	.008827	.0004727	.0005281	.0005853
7			.1443	16351	.007892	.008817	.009772	.0004973	.0005556	.0006158
	10		.1340	14103	.01061	.01185	.01314	.0005766	.0006442	.0007140
8			.1285	12967	.01255	.01402	.01554	.0006271	.0007007	.0007765
	11		.1200	11310	.01650	.01843	.02042	.0007190	.0008033	.0008903
9			.1144	10283	.01995	.02229	.02471	.0007908	.0008835	.0009791
	12		.1090	9331	.02423	.02707	.03000	.0008715	.0009736	.01079
10			.1019	8155	.03173	.03545	.03928	.0009972	.001114	.001235
	13		.0950	7088	.04199	.04692	.05200	.001147	.001282	.001420
11			.09074	6467	.05045	.05636	.06246	.001257	.001405	.001557
	14		.08300	5411	.07207	.08052	.08924	.001503	.001679	.001861
12			.08081	5129	.08022	.08962	.09932	.001586	.001771	.001963
	15		.07200	4072	.1273	.1422	1576	.001997	.002231	.002473
13			.07196	4067	.1276	.1425	.1579	.001999	.002234	.002476
	16		.06500	3318	.1916	.2141	.2373	.002451	.002738	.003034
14			.06408	3225	.2028	.2266	.2511	.002521	.002817	.003122
	17		.0580	2642	.3023	.3377	.3742	.003078	.003439	.003811

*To find area in square inches, divide by 1,000,000.

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1.—Copper Wire Table of the—
(See Opposite Page for Areas in *Square* Mils.)

Gages.				Weight.				Length.			
					Pounds per Ohm.				Feet per Ohm.		
A.W.G. B.&S.	B.W.G. Stub's.	Diam. Ins.	Area Circ. Mils.	Lb. per Foot.	@ 20° C. = 68° F.	@ 50° C. = 122° F.	@ 80° C. = 176° F.	Ft. per Lb.	@ 20° C. = 68° F.	@ 50° C. = 122° F.	@ 80° C. = 176° F.
15		.05707	3257	.009858	3.101	2.776	2.504	101.4	314.5	281.5	254.0
16		.05082	2583	.007818	1.950	1.746	1.575	127.9	249.4	223.3	201.5
	18	.049	2401	.007268	1.685	1.509	1.361	137.6	231.9	207.6	187.3
17		.04526	2048	.006200	1.226	1.098	.9906	161.3	197.8	177.1	159.8
	19	.042	1764	.005340	.9097	.8143	.7347	187.3	170.4	152.5	137.6
18		.0403	1624	.004917	.7713	.6904	.6230	203.4	156.9	140.4	126.7
19		.03589	1288	.003899	.4851	.4342	.3918	256.5	124.4	111.4	100.5
	20	.035	1225	.003708	.4387	.3927	.3543	269.7	118.3	105.9	95.56
	21	.032	1024	.003100	.3066	.2744	.2476	322.6	98.90	88.52	79.86
20		.03196	1022	.003092	.3051	.2731	.2464	323.4	98.66	88.31	79.68
21		.02846	810.1	.002452	.1919	.1717	.1550	407.8	78.24	70.03	63.19
	22	.028	784.0	.002373	.1797	.1608	.1451	421.4	75.72	67.78	61.16
22		.02535	642.4	.001945	.1207	.1080	.09746	514.2	62.05	55.54	50.11
	23	.025	625.0	.001892	.1142	.1022	.09224	528.6	60.36	54.03	48.75
23		.02257	509.5	.001542	.07589	.06793	.06129	648.4	49.21	44.04	39.74
24		.022	484.0	.001465	.06849	.06130	.05531	682.6	46.75	41.84	37.75
24		.0201	404.0	.001223	.04773	.04272	.03855	817.6	39.02	34.93	31.52
	25	.020	400.0	.001211	.04678	.04187	.03778	825.9	38.63	34.58	31.20
26		.018	324.0	.0009808	.03069	.02747	.02479	1020	31.29	28.01	25.27
25		.0179	320.4	.0009699	.03002	.02687	.02424	1031	30.95	27.70	24.99
	27	.016	256.0	.0007749	.01916	.01715	.01548	1290	24.73	22.13	19.97
26		.01594	254.1	.0007692	.01888	.01690	.01525	1300	24.54	21.97	19.82
27		.0142	201.5	.0006100	.01187	.01063	.009588	1639	19.46	17.42	15.72
	28	.014	196.0	.0005933	.01123	.01005	.009071	1685	18.93	16.94	15.29
29		.013	169.0	.0005116	.008350	.007474	.006744	1955	16.32	14.61	13.18
28		.01264	159.8	.0004837	.007466	.006683	.006030	2067	15.43	13.82	12.47
	30	.012	144.0	.0004359	.006062	.005426	.004896	2294	13.91	12.45	11.23
29		.01126	126.7	.0003836	.004696	.004203	.003792	2607	12.24	10.96	9.886
30		.01003	100.5	.0003042	.002953	.002643	.002385	3287	9.707	8.688	7.840
	31	.010	100.0	.0003027	.002924	.002617	.002361	3304	9.658	8.645	7.800
	32	.009	81.00	.0002452	.001918	.001717	.001549	4078	7.823	7.002	6.318
31		.008928	79.70	.0002413	.001857	.001662	.001500	4145	7.698	6.890	6.217
	33	.008	64.00	.0001937	.001197	.001072	.0009672	5162	6.181	5.533	4.992
32		.00795	63.21	.0001913	.001168	.001045	.0009436	5227	6.105	5.464	4.930
33		.00708	50.13	.0001517	.0007346	.0006575	.0005933	6591	4.841	4.333	3.910
	34	.007	49.00	.0001483	.0007019	.0006283	.0005669	6742	4.733	4.236	3.822
34		.006305	39.75	.0001203	.0004620	.0004135	.0003731	8311	3.839	3.436	3.101
35		.005615	31.52	.00009543	.0002905	.0002601	.0002347	10480	3.045	2.725	2.459
36	35	.005	25.00	.00007568	.0001827	.0001636	.0001476	13210	2.414	2.161	1.950
37		.004453	19.83	.00006001	.0001149	.0001029	.00009281	16660	1.915	1.714	1.547
	36	.004	16.00	.00004843	.00007484	.00006699	.00006045	20650	1.545	1.383	1.248
38		.003965	15.72	.00004759	.00007210	.00006454	.00005824	21010	1.519	1.359	1.226
39		.003531	12.47	.00003774	.00004545	.00004068	.00003671	26500	1.204	1.078	.9726
40		.003145	9.888	.00002993	.00002858	.00002559	.00002309	33410	.9560	.8548	.7713

Note.—Sq. mils = circ. mils × 0.785398; circ. mils = sq. mils × 1.273240.

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—Am. Inst. of Electrical Engineers.—Concluded.
(See Opposite Page for Areas in *Circular* Mils.)

Gages.		Resistance.					
		Ohms per Lb.			Ohms per Ft.		
Diam. Ins.	Area in Sq. Mils.*	@ 20°C. = 68°F.	@ 50°C. = 122°F.	@ 80°C. = 176°F.	@ 20°C. = 68°F.	@ 50°C. = 122°F.	@ 80°C. = 176°F.
.05707	2558	.3225	.3603	.3993	.003179	.003552	.003936
.05082	2029	.5128	.5729	.6349	.004009	.004479	.004964
.04900	1886	.5933	.6629	.7346	.004312	.004818	.005339
.04526	1609	.8153	.9109	1.010	.005055	.005648	.006259
.04200	1385	1.099	1.228	1.361	.005870	.006558	.007267
.04030	1276	1.296	1.448	1.605	.006374	.007122	.007892
.03589	1012	2.061	2.303	2.552	.008038	.008980	.009952
.03500	962.0	2.279	2.547	2.822	.008452	.009443	.01047
.03200	804 2	3.262	3.644	4.039	.01011	.01130	.01252
.03196	802.0	3.278	3.662	4.058	.01014	.01132	.01255
.02846	636.3	5.212	5.823	6.453	.01278	.01428	.01583
.02800	615.8	5.565	6.217	6.890	.01321	.01475	.01635
.02535	504.6	8.287	9.259	10.26	.01612	.01801	.01996
.0250	490.9	8.756	9.783	10.84	.01657	.01851	.02051
.02257	400.2	13 18	14.72	16.32	.02032	.02271	.02516
.0220	380.1	14.60	16.31	18.08	.02139	.02390	.02649
.02010	317.3	20 95	23.41	25.94	.02563	.02863	.03173
.0200	314.2	21.38	23.88	26.47	.02588	.02892	.03205
.018	254.5	32.58	36.40	40.34	.03196	.03570	.03957
.0179	251.7	33.32	37.22	41.25	.03231	.03610	.04001
.016	201 1	52.19	58.31	64.62	.04045	.04519	.05008
.01594	199.6	52.97	59.18	65.59	.04075	.04552	.05045
.0142	158.3	84.23	94.11	104.3	.05138	.05740	.06362
.014	153.9	89.04	99.48	110.2	.05283	.05902	.06541
.013	132.7	119.8	133.8	148.3	.06127	.06845	.07586
.01264	125.5	133.9	149.6	165.8	.06479	.07239	.08022
.012	113.1	165.0	184.3	204.2	.0719	.08033	.08903
.01126	99.53	213.0	237.9	263.7	.0817	.09128	.1012
.01003	78.94	338.6	378.3	419.2	.1030	.1151	.1276
.010	78.54	342.0	382.1	423.5	.1035	.1157	.1282
.009	63 62	521.3	582.5	645.5	.1278	.1428	.1583
.008928	62.60	538.4	601 6	666 7	.1299	.1451	.1608
.008	50.27	835.1	933.0	1034.	.1618	.1807	.2003
.00795	49.64	856.2	956.5	1060	.1638	.1830	.2028
.00708	39.37	1361.	1521.	1685.	.2066	.2308	.2558
.007	38.48	1425.	1592.	1764.	.2113	.2361	.2616
.006305	31.22	2165.	2418.	2680.	.2605	.2910	.3225
.005615	24.76	3441.	3845.	4262.	.3284	.3669	.4067
.005	19.64	5473.	6114.	6776.	.4142	.4627	.5129
.004453	15.57	8702.	9722.	10770.	.5222	.5835	.6466
.004	12.57	13360.	14930.	16540.	.6471	.7230	.8011
.003965	12.35	13870.	15490.	17170.	.6585	.7357	.8154
.003531	9.79	22000.	24580.	27240.	.8304	.9277	1.028
.003145	7.77	34980.	39080.	43320.	1.047	1.170	1.296

find area in square inches, divide by 1,000,000.

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and *impedance*. The former may be assumed to be no greater than the "drop" in volts on the line (and from that down to about one-third of the amount), while the latter is usually so small as to be practically negligible, say 5 per cent of the inductance. Hence from 5 to 10 per cent may be added for losses in alternate-current circuits over those for continuous; in other words the areas of the wires may be increased by this amount. (3) In a single-phase two-wire circuit the size of wires is the same as for continuous current *plus* 5 to 10 per cent for inductance. (4) In a two-phase four-wire circuit the area of each wire is one-half that for the continuous current circuit *plus* 5 to 10 per cent for inductance; that is, the same *weight* of wire is required as for the single-phase two-wire alternating circuit. (5) In the three-phase three-wire circuit the area of each wire is one-half that for the continuous current circuit *plus* 5 to 10 per cent for inductance; that is, only three-fourths the *weight* of wire is required as for the 1-phase 2-wire, and 2-phase 4-wire, circuits.

Problem 2.—In solving Problem 1 we find that the size of copper wire required is about 110,000 circular mills in area, corresponding to about No. 0 B. & S. gage. Now, from the foregoing discussion, what sizes of wire would probably be installed for the three types of alternating circuits? And what would be the theoretic relative weights of *total amount of copper* in the four systems?

Continuous current		2-wire;	each	110,000	circ. mills;	total weight,	1.00
Alternating Current	1-phase	2-wire;	"	120,000	"	" "	1.09
	2-phase	4-wire;	"	60,000	"	" "	1.09
	3-phase	3-wire;	"	60,000	"	" "	0.82

The 3-phase 3-wire system is the most commonly used.

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"NATIONAL ELECTRIC CODE."

Rules and Requirements of the National Board of Fire Underwriters for the Installation of Wiring and Apparatus (for Light, Heat and Power) as Recommended by the Underwriters' National Electric Association.
Edition of 1907.

The National Electric Code (originally drawn in 1897) is the result of the united efforts of the various electrical, architectural, insurance and other allied interests which, through the National Conference on Standard Electrical Rules, composed of delegates from various national associations, unanimously voted to recommend it to their respective associations for approval or adoption.

The following is a list of the associations composing the National Conference on Standard Electrical Rules:—American Institute of Architects, American Institute of Electrical Engineers, American Society of Mechanical Engineers, American Institute of Mining Engineers, American Street and Interurban Railway Association, Associated Factory Mutual Fire Ins. Co's., Association of Edison Illuminating Companies, International Association of Municipal Electricians, National Board of Fire Underwriters, National Electric Light Association, National Electric Contractors' Association, National Electric Inspectors' Association, Underwriters' National Electric Association.

GENERAL PLAN GOVERNING THE ARRANGEMENT OF RULES.*

Class A.—Stations and Dynamo Rooms. Includes Central Stations; Dynamo, Motor, and Storage-Battery Rooms; Transformer Substations, etc. Rules 1 to 11.

Class B.—Outside Work, all systems and voltages. Rules 12 to 13A.

Class C.—Inside Work. Rules 14 to 39. Subdivided as follows:
- General Rules, all systems and voltages. Rules 14 to 17.
- Constant-Current Systems. Rules 18 to 20.
- Constant-Potential Systems:—
  - General Rules, all voltages. Rules 21 to 23.
  - Low-Potential Systems, 550 volts or less. Rules 24 to 34.
  - High-Potential Systems, 550 to 3500 volts. Rules 35 to 37.
  - Extra-High-Potential Systems, over 3500 volts. Rules 38 to 39.

Class D.—Fittings, Materials, and Details of Construction, all systems and voltages. Rules 40 to 63.

Class E.—Miscellaneous. Rules 64 to 67.

Class F.—Marine Work. Rules 68 to 83.

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* *General Suggestions.*—In all electric work, conductors, however well insulated, should always be treated as bare to the end that under no conditions, existing or likely to exist, can a ground or short circuit occur, and so that all leakage from conductor to conductor, or between conductor and ground, may be reduced to a minimum.

In all wiring, special attention must be paid to the mechanical execution of the work. Careful and neat running, connecting, soldering, taping of conductors, and securing and attaching of fittings, are specially conducive to security and efficiency, and will be strongly insisted on.

In laying out an installation, except for constant current systems, every reasonable effort should be made to secure distribution centers located in easily accessible places, at which points the cut-outs and switches controlling the several branch circuits can be grouped for convenience and safety of operation. The load should be divided as evenly as possible among the branches, and all complicated and unnecessary wiring avoided.

The use of wire-ways for rendering concealed wiring permanently accessible is most heartily endorsed and recommended; and this method of accessible concealed construction is advised for general use.

Architects are urged, when drawing plans and specifications, to make provision for the channeling and pocketing of buildings for electric light or power wires, and also for telephone, district messenger and other signaling system wiring.

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Class A.—STATIONS AND DYNAMO ROOMS.

INCLUDES CENTRAL STATIONS, DYNAMO, MOTOR AND STORAGE-BATTERY ROOMS, TRANSFORMER, SUB-STATIONS, ETC.

1. **Generators.—a.** Must be located in a dry place.

It is recommended that water-proof covers be provided, which may be used in case of emergency.

b. Must never be placed in a room where any hazardous process is carried on, nor in places where they would be exposed to inflammable gases or flyings of combustible materials.

c. Must, when operating at a potential in excess of 550 volts, have their base frames permanently and effectively grounded.

Must, when operating at a potential of 550 volts or less, be thoroughly insulated from the ground wherever feasible. Wooden base frames used for this purpose, and wooden floors which are depended upon for insulation where, for any reason, it is necessary to omit the base frames, must be kept filled to prevent absorption of moisture, and must be kept clean and dry.

Where frame insulation is impracticable, the Inspection Department having jurisdiction may, in writing, permit its omission, in which case the frame must be permanently and effectively grounded.

A high potential machine should be surrounded by an insulated platform. This may be made of wood, mounted on insulating supports, and so arranged that a man must always stand upon it in order to touch any part of the machine.

In case of a machine having an insulated frame, if there is trouble from static electricity due to belt friction, it should be overcome by placing near the belt a metallic comb connected with the earth, or by grounding the frame through a resistance of not less than 300,000 ohms.

d. Constant potential generators, except alternating current machines and their exciters, must be protected from excessive current by safety fuses or equivalent devices of *approved* design.

For two-wire, direct-current generators, single pole protection will be considered as satisfying the above rule, provided the safety device is located in the lead not connected to the series winding. When supplying three-wire systems, the generators should be so arranged that these protective devices will come in the outside leads.

For three-wire, direct-current generators, a safety device must be placed in each armature, direct-current lead, or a double pole, double trip circuit breaker in each outside generator lead and corresponding equalizer connection.

In general, generators should preferably have no exposed live parts and the leads should be well insulated and thoroughly protected against mechanical injury. This protection of the bare live parts against accidental contact would apply also to any exposed, uninsulated conductors outside of the generator and not on the switchboard unless their potential is practically that of the ground.

Where the needs of the service make the above requirements impracticable, the Inspection Department having jurisdiction may, in writing, modify them.

e. Must each be provided with a name-plate, giving the maker's name, the capacity in volts and amperes, and the normal speed in revolutions per minute.

f. Terminal blocks when used on generators must be made of *approved* non-combustible, non-absorptive, insulating material, such as slate, marble or porcelain.

2. **Conductors.**—From generators to switchboards, rheostats or other instruments, and thence to oustide lines:—

a. Must be in plain sight or readily accessible.

Wires from generator to switchboard may, however, be placed in a conduit in the brick or cement pier on which the generator stands, provided that proper precautions are taken to protect them against moisture and to thoroughly insulate them from the pier. If lead-covered cable is used, no further protection will be required, but it should not be allowed to rest upon sharp edges which in time might cut into the lead sheath, especially if the cables were liable to vibration. A smooth runway is desired. If iron conduit is provided, double braided rubber-covered wire (see No. 47) will be satisfactory.

b. Must have an *approved* insulating covering as called for by rules in Class "C" for similar work, except that in central stations, on exposed circuits, the wire which is used must have a heavy braided, non-combustible outer covering.

Bus bars may be made of bare metal.

Rubber insulations ignite easily and burn freely. Where a number of wires are brought close together, as is generally the case in dynamo rooms, especially about the

switchboard, it is therefore necessary to surround this inflammable material with a tight, non-combustible outer cover. If this is not done, a fire once started at this point would spread rapidly along the wires, producing intense heat and a dense smoke Where the wires have such a covering and are well insulated and supported, using only non-combustible materials, it is believed that no appreciable fire hazard exists, even with a large group of wires.

Flame proofing should be stripped back on all cables a sufficient amount to give the necessary insulation distances for the voltage of the circuit on which the cable is used. The stripping back of the flame proofing is necessary on account of the poor insulating qualities of the flame proofing material now available. Flame proofing may be omitted where satisfactory fire-proofing is accomplished by other means, such as compartments, etc.

c. Must be kept so rigidly in place that they cannot come in contact.

d. Must in all other respects be installed with the same precautions as required by rules in Class "C" for wires carrying a current of the same volume and potential.

e. In wiring switchboards, the ground detector, voltmeter, pilot lights and potential transformers must be connected to a circuit of not less than No. 14 B. & S. gage wire that is protected by an *approved* fuse, this circuit is not to carry over 660 watts.

For the protection of instruments and pilot lights on switchboards, *approved* N. E. Code Standard Enclosed Fuses are preferred, but *approved* enclosed fuses of other designs of *not over two* (2) *amperes capacity*, may be used.

Voltmeter switches having concealed connections must be plainly marked, showing connections made.

3 **Switchboards.—a.** Must be so placed as to reduce to a minimum the danger of communicating fire to adjacent combustible material.

Special attention is called to the fact that switchboards should not be built down to the floor, nor up to the ceiling. A space of at least ten or twelve inches should be left between the floor and the board, except when the floor about the switchboard is of concrete or other fireproof construction, and a space of three feet, if possible, between the ceiling and the board, in order to prevent fire from communicating from the switchboard to the floor or ceiling, and also to prevent the forming of a partially concealed space very liable to be used for storage of rubbish and oily waste.

b. Must be made of non-combustible material or of hardwood in skeleton form, filled to prevent absorption of moisture.

If wood is used all wires and all current carrying parts of the apparatus on the switchboard must be separated therefrom by non-combustible, non-absorptive insulating material.

c. Must be accessible from all sides when the connections are on the back, but may be placed against a brick or stone wall when the wiring is entirely on the face.

If the wiring is on the back, there should be a clear space of at least eighteen inches between the wall and the apparatus on the board, and even if the wiring is entirely on the face, it is much better to have the board set out from the wall. The space back of the board should not be closed in, except by grating or netting either at the sides, top or bottom, as such an enclosure is almost sure to be used as a closet for clothing or for the storage of oil cans, rubbish, etc. An open space is much more likely to be kept clean, and is more convenient for making repairs, examinations, etc.

d. Must be kept free from moisture.

e. On switchboards the distances between bare live parts of opposite polarity must be made as great as practicable, and must not be less than those given for tablet-boards (see No. 53 A).

4. **Resistance Boxes and Equalizers.**—(For construction rules, see No. 60.) a. Must be placed on a switchboard, or if not thereon, at a distance of at least one foot from combustible material, or separated therefrom by a non-combustible, non-absortive insulating material such as slate or marble.

This will require the use of a slab or panel of non-combustible, non-absorptive insulating material such as slate or marble, somewhat larger than the rheostat, which shall be secured in position independently of the rheostat supports. Bolts for supporting the rheostat shall be counterrunk at least ½ inch below the surface at the back of the slab and filled. For proper mechanical strength, slab should be of a thickness consistent with the size and weight of the rheostat, and in no case to be less than ½ inch

If resistance devices are installed in rooms where dust or combustible flyings would be liable to accumulate on them, they should be equipped with a dust-proof face plate.

b. Where protective resistances are necessary in connection with automatic rheostats incandescent lamps may be used, provided that they do not

carry or control the main current nor constitute the regulating resistance of the device.

When so used, lamps must be mounted in porcelain receptables upon non-combustible supports, and must be so arranged that they cannot have impressed upon them a voltage greater than that for which they are rated. They must in all cases be provided with a name-plate, which shall be permanently attached beside the porcelain receptacle or receptacles and stamped with the candle-power and voltage of the lamp or lamps to be used in each receptacle.

c. Wherever insulated wire is used for connection between resistances and the contact plate of a rheostat, the insulation must be slow burning (see No. 43). For large field rheostats and similar resistances, where the contact plates are not mounted upon them, the connecting wires may be run together in groups so arranged that the maximum difference of potential between any two wires in a group shall not exceed 75 volts. Each group of wires must either be mounted on non-combustible, non-absorptive insulators giving at least ½ inch separation from surface wired over, or, where it is necessary to protect the wires from mechanical injury or moisture, be run in *approved* lined conduit or equivalent.

5. Lightning Arresters.—(For construction rules, see No. 63.)

a. Must be attached to each wire of every overhead circuit connected with the station.

It is recommended to all electric light and power companies that arresters be connected at intervals over systems in such numbers and so located as to prevent ordinary discharges entering (over the wires) buildings connected to the lines.

b. Must be located in readily accessible places away from combustible materials, and as near as practicable to the point where the wires enter the building.

In all cases, kinks, coils and sharp bends in the wires between the arresters and the outdoor lines must be avoided as far as possible.

The switchboard does not necessarily afford the only location meeting these requirements. In fact, if the arresters can be located in a safe and accessible place away from the board, this should be done, for, in case the arrester should fail or be seriously damaged there would then be less chance of starting arcs on the board.

c. Must be connected with a thoroughly good and permanent ground connection by metallic strips or wires having a conductivity not less than that of a No. 6 B. & S. gage copper wire, which must be run as nearly in a straight line as possible from the arresters to the ground connection.

Ground wires for lightning arresters must not be attached to gas pipes within the buildings.

It is often desirable to introduce a choke coil in circuit between the arresters and the dynamo. In no case should the ground wires from lightning arresters be put into iron pipes, as these would tend to impede the discharge.

d. All choke coils or other attachments, inherent to the lightning protection equipment, shall have an insulation from the ground or other conductors equal at least to the insulation demanded at other points of the circuit in the station.

6. Care and Attendance.—a. A competent man must be kept on duty where generators are operating. b. Oily waste must be kept in approved metal cans and removed daily.

Approved waste cans shall be made of metal, with legs raising can three inches from the floor, and with self-closing covers.

7. Testing and Insulation Resistance.—a. All circuits except such as are permanently grounded in accordance with No. 13 A must be provided with reliable ground detectors. Detectors which indicate continuously and give an instant and permanent indication of a ground are preferable. Ground wires from detectors must not be attached to gas pipes within the building.

b. Where continuously indicating detectors are not feasible, the circuits should be tested at least once a day, and preferably oftener.

c. Data obtained from all tests must be preserved for examination by the Inspection Department having jurisdiction.

These rules on testing to be applied at such places as may be designated by the Inspection Department having jurisdiction.

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8 **Motors.**—The use of motors operating at a potential in excess of 550 volts will only be approved when every practicable safeguard has been provided. Plans for such installations should be submitted to the Inspection Department having jurisdiction before any work is begun

a. Must, when operating at a potential in excess of 550 volts, have no exposed live metal parts, and have their base frames permanently and effectively grounded.

Motors operating at a potential of 550 volts or less must be thoroughly insulated from the ground wherever feasible Wooden base frames used for this purpose, and wooden floors, which are depended upon for insulation where, for any reason, it is necessary to omit the base frames, must be kept filled to prevent absorption of moisture, and must be kept clean and dry. Where frame insulation is impracticable, the Inspection Department having jurisdiction may, in writing, permit its omission, in which case the frame must be permanently and effectively grounded.

A high-potential machine should be surrounded with an insulated platform. This may be made of wood, mounted on insulating supports, and so arranged that a man must stand upon it in order to touch any part of the machine.

In case of a machine having an insulated frame, if there is trouble from static electricity due to belt friction, it should be overcome by placing near the belt a metallic comb connected to the earth, or by grounding the frame through a resistance of not less than 300,000 ohms.

b. Motors operating at a potential of 550 volts or less must be wired with the same precautions as required by rules in Class "C" for wires carrying a current of the same volume.

Motors operating at a potential between 550 and 3,500 volts must be wired with approved multiple conductor, metal sheathed cable in approved unlined metal conduit firmly secured in place. The metal sheath must be permanently and effectively grounded, and the construction and installation of the conduit must conform to rules for interior conduits (see No. 25 and No. 49 a, j, and k), except that at outlets approved outlet bushings shall be used.

The motor leads or branch circuits must be designed to carry a current at least 25 per cent greater than that for which the motor is rated, in order to provide for the inevitable occasional overloading of the motor and the increased current required in starting, without overfusing the wires; but where the wires under this rule would be overfused, in order to provide for the starting current, as in the case of many of the alternating current motors, the wires must be of such size as to be properly protected by these larger fuses.

The insulation of the several conductors for high potential motors, where leaving the metal sheath at outlets, must be thoroughly protected from moisture and mechanical injury. This may be accomplished by means of a pot head or some equivalent method. The conduit must be substantially bonded to the metal casings of all fittings and apparatus connected to the inside high tension circuit. It would be much preferable to make the conduit system continuous throughout by connecting the conduit to fittings and motors by means of screw joints, and this construction is strongly recommended wherever practicable.

High potential motors should preferably be so located that the amount of inside wiring will be reduced to a minimum. Inspection Department having jurisdiction may permit the wire for high potential motors to be installed according to the general rules for high potential systems when the outside wires directly enter a motor room (see Section f). Under these conditions there would generally be but a few feet of wire inside the building and none outside the motor room.

c. Each motor and resistance box must be protected by a cut-out and controlled by a switch (see No. 17 a), said switch plainly indicating whether "on" or "off." With motors of one-fourth horse power or less, on circuits where the voltage does not exceed 300, No. 21 d must be complied with, and single pole switches may be used as allowed in No. 22 c. The switch and rheostat must be located within sight of the motor, except in cases where special permission to locate them elsewhere is given, in writing, by the Inspection Department having jurisdiction.

The use of circuit-breakers with motors is recommended, and may be required by the Inspection Department having jurisdiction.

Where the circuit-breaking device on the motor-starting rheostat disconnects all wires of the circuit, the switch called for in this section may be omitted.

Overload-release devices on motor-starting rheostats will not be considered to take the place of the cut-out required by this section if they are inoperative during the starting of the motor

The switch is necessary for entirely disconnecting the motor when not in use, and the cut-out to protect the motor from excessive currents due to accidents or careless handling when starting. An automatic circuit-breaker disconnecting all wires of the circuit may, however, serve as both switch and cut-out.

In general, motors should preferably have no exposed live parts.

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d. Rheostats must be so installed as to comply with *all* the requirements of No. 4. Auto starters must comply with requirements of No. 4 c.

Starting rheostats and auto starters, unless equipped with tight casings enclosing all current-carrying parts, should be treated about the same as knife switches, and in all wet, dusty or linty places, should be enclosed in dust-tight, fireproof cabinets. If a special motor room is provided, the starting apparatus and safety devices should be included within it. Where there is any liability of short circuits across their exposed live parts being caused by accidental contacts, they should either be enclosed in cabinets, or else a railing should be erected around them to keep unauthorized persons away from their immediate vicinity.

e. Must not be run in series-multiple or multiple-series, except on constant potential systems, and then only by special permission of the Inspection Department having jurisdiction.

f. Must be covered with a waterproof cover when not in use, and, if deemed necessary by the Inspection Department having jurisdiction, must be enclosed in an *approved* case.

When it is necessary to locate a motor in the vicinity of combustibles or in wet or very dusty or dirty places, it is generally advisable to enclose it as above. Such enclosures should be readily accessible, dust proof and sufficiently ventilated to prevent an excessive rise of temperature. The sides should preferably be made largely of glass, so that the motor may be always plainly visible This lessens the chance of its being neglected, and allows any derangement to be at once noticed.

The use of enclosed type motor is recommended in dusty places, being preferable to wooden boxing. From the nature of the question the decision as to details of construction must be left to the Inspection Department having jurisdiction to determine in each instance.

g. Must, when combined with ceiling fans, be hung from insulated hooks, or else there must be an insulator interposed between the motor and its support.

h. Must each be provided with a name-plate, giving the makers' name, the capacity in volts and amperes, and the normal speed in revolutions per minute.

i. Terminal blocks when used on motors must be made of *approved* non-combustible, non-absorptive, insulating material such as slate, marble or porcelain.

j. Variable speed motors, unless of special and appropriate design, if controlled by means of field regulation, must be so arranged and connected that they cannot be started under weakened field.

9. **Railway Power Plants.**—a. Each feed wire before it leaves the station must be equipped with an *approved* automatic circuit-breaker (see No. 52) or other device, which will immediately cut off the current in case of an accidental ground. This device must be mounted on a fireproof base, and in full view and reach of the attendant.

10. **Storage or Primary Batteries.**—*a.* When current for light and power is taken from primary or secondary batteries, the same general regulations must be observed as apply to similar apparatus fed from dynamo generators delevoping the same difference of potential.

b. Storage battery rooms must be thoroughly ventilated.

c. Special attention is directed to the rules for wiring in rooms where acid fumes exist (see No. 24, i and j).

d. All secondary batteries must be mounted on non-absorptive, non-combustible insulators, such as glass or thoroughly vitrified and glazed porcelain.

e. The use of any metal liable to corrosion must be avoided in cell connections of secondary batteries.

11. **Transformers.**—(For construction rules, see No. 62.) (See also Nos. 13, 13A, 36.) a. In central or sub-stations the transformers must be so placed that smoke from the burning out of the coils or the boiling over of the oil (where oil filled cases are used) could do no harm.

If the insulation in a transformer breaks down, considerable heat is likely to be developed. This would cause a dense smoke, which might be mistaken for a fire and result in water being thrown into the building, and a heavy loss thereby entailed. Moreover, with oil-cooled transformers, especially if the cases are filled too full, the oil may become ignited and boil over, producing a very stubborn fire.

b. In central or sub-stations, casings of all transformers must be permanently and effectively grounded.

Transformers used exclusively to supply current to switchboard instruments need not be grounded, provided they are thoroughly insulated.

Class B.—OUTSIDE WORK.

(LIGHT, POWER AND HEAT. FOR SIGNALING SYSTEMS, SEE CLASS E.)

ALL SYSTEMS AND VOLTAGES.

12. Wires.—a. Line wires must have an *approved* weatherproof or rubber insulating covering (see No. 44 and No. 41). That portion of the service wires between the main cut-out and switch and the first support from the cut-out or switch on outside of the building must have an approved rubber insulating covering (see No. 41), but from the above-mentioned support to the line, may have an approved weatherproof insulating covering (see No. 44), if kept free from awnings, swinging signs, shutters, etc.

b. Must be so placed that moisture cannot form a cross connection between them, not less than a foot apart, and not in contact with any substance other than their insulating supports. Wooden blocks to which insulators are attached must be covered over their entire surface with at least two coats of waterproof paint.

c. Must be at least seven feet above the highest point of flat roofs, and at least one foot above the ridge of pitched roofs, over which they pass or to which they are attached.

Roof structures are frequently found which are too low or much too light for the work, or which have been carelessly put up. A structure which is to hold the wires a proper distance above the roof in all kinds of weather must not only be of sufficient height, but must be substantially constructed of strong material.

d. Must be protected by dead insulated guard irons or wires from possibility of contact with other conducting wires or substances to which current may leak. Special precautions of this kind must be taken where sharp angles occur, or where any wires might possibly come in contact with electric light or power wires.

Crosses, when unavoidable, should be made as nearly at right angles as possible.

e. Must be provided with petticoat insulators of glass or porcelain. Porcelain knobs or cleats and rubber hooks will not be approved.

f. Must be so spliced or joined as to be both mechanically and electrically secure without solder. The joints must then be soldered, to insure preservation, and covered with an insulation equal to that on the conductors.

All joints must be soldered, unless made with some form of *approved* splicing device. This ruling applies to joints and splices in all classes of wiring covered by these rules.

g. Must, where they enter buildings, have drip loops outside, and the holes through which the conductors pass must be bushed with non-combustible, non-absorptive, insulating tubes slanting upward toward the inside.

For low potential systems the service wires may be brought into buildings through a single iron conduit. The conduit to be curved downward at its outer end and carefully sealed or equipped with an approved service-head to prevent the entrance of moisture. The outer end must be at least one foot from any wood-work and the inner end must extend to the service cut-out, and if a cabinet is required by the Code must enter the cabinet in a manner similar to that described in fine print note under No. 250.

h. Electric light and power wires must not be placed on the same cross-arm with telegraph, telephone or similar wires, and when placed on the same pole with such wires the distance between the two inside pins of each cross-arm must not be less than twenty-six inches.

i. The metallic sheaths to cables must be permanently and effectively connected to "earth."

Trolley Wires.—j. Must not be smaller than No.0 B. & S. gage copper or No.4 B. & S. gage silicon bronze, and must readily stand the strain upon them when in use.

k. Must have a double insulation from the ground. In wooden pole construction the pole will be considered as one insulation.

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l. Must be capable of being disconnected at the power plant, or of being divided into sections, so that, in case of fire on the railway route, the current may be shut off from that particular section and not interfere with the work of the firemen. This rule also applies to feeders.

m. Must be safely protected against accidental contact where crossed by other conductors.

Guard wires should be insulated from the ground and should be electrically disconnected in sections of not more than 300 feet in length.

Ground Return Wires.—**n.** For the diminution of electrolytic corrosion of underground metal work, ground return wires must be so arranged that the difference of potential between the grounded dynamo terminal and any point on the return circuit will not exceed twenty-five volts.

It is suggested that the positive pole of the dynamo be connected to the trolley line, and that whenever pipes or other underground metal work are found to be electrically positive to the rails or surrounding earth, that they be connected by conductors arranged so as to prevent as far as possible current flow from the pipes into the ground.

12 A. Constant-Potential Pole Lines, Over 5,000 Volts.—(Overhead lines of this class unless properly arranged may increase the fire loss from the following causes:—Accidental crosses between such lines and low-potential lines may allow the high-voltage current to enter buildings over a large section of adjoining country. Moreover, such high-voltage lines, if carried close to buildings, hamper the work of firemen in case of fire in the building. The object of these rules is so to direct this class of construction that no increase in fire hazard will result, while at the same time care has been taken to avoid restrictions which would unreasonably impede progress in electrical development.

It is fully understood that it is impossible to frame rules which will cover all conceivable cases that may arise in construction work of such an extended and varied nature, and it is advised that the Inspection Department having jurisdiction be freely consulted as to any modification of the rules in particular cases.)

a. Every reasonable precaution must be taken in arranging routes so as to avoid exposure to contacts with other electric circuits. On existing lines where there is a liability to contact, the route should be changed by mutual agreement between the parties in interest wherever possible.

b. Such lines should not approach other pole lines nearer than a distance equal to the height of the taller pole line, and such lines should not be on the same poles with other wires, except that signaling wires used by the Company operating the high-pressure system, and which do not enter property other than that owned or occupied by such Company, may be carried over the same poles.

c. Where such lines must necessarily be carried nearer to other pole lines than is specified in Section *b* above, or where they must necessarily be carried on the same poles with other wires, extra precautions to reduce the liability of a breakdown to a minimum must be taken, such as the use of wires of ample mechanical strength, widely spaced cross-arms, short spans, double or extra heavy cross-arms, extra heavy pins, insulators, and poles thoroughly supported. If carried on the same poles with other wires, the high-pressure wires must be carried at least three feet above the other wires.

d. Where such lines cross other lines, the poles of both lines must be of heavy and substantial construction.

Whenever it is feasible, end-insulator guards should be placed on the cross-arms of the upper line. If the high-pressure wires cross below the other lines, the wires of the upper line should be dead-ended at each end of the span to double-grooved, or to standard transposition insulators, and the line completed by loops.

One of the following forms of construction must then be adopted:—

1. The height and length of the cross-over span may be made such that the shortest distance between the lower cross-arms of the upper line and any wire of the lower line will be greater than the length of the cross-over span, so that a wire breaking near one of the upper pins would not be long enough to reach any wire of the lower line. The high-pressure wires should preferably be above the other wires.

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2. A joint pole may be erected at the crossing point, the high-pressure wires being supported on this pole at least three feet above the other wires. Mechanical guards or supports must then be provided, so that in case of the breaking of any upper wire. it will be impossible for it to come into contact with any of the lower wires.

Such liability of contact may be prevented by the use of suspension wires, similar to those employed for suspending aerial telephone cables, which will prevent the high-pressure wires from falling, in case they break. The suspension wires should be supported on high-potential insulators, should have ample mechanical strength, and should be carried over the high-pressure wires for one span on each side of the joint pole, or where suspension wires are not desired guard wires may be carried above and below the lower wires for one span on each side of the joint pole, and so spread that a falling high-pressure wire would be held out of contact with the lower wires.

Such guard wires should be supported on high-potential insulators or should be grounded. When grounded, they must be of such size, and so connected and earthed, that they can surely carry to ground any current which may be delivered by any of the high-pressure wires. Further, the construction must be such that the guard wires will not be destroyed by any arcing at the point of contact likely to occur under the conditions existing.

3. Whenever neither of the above methods is feasible, a screen of wire should be interposed between the lines at the cross-over. This screen should be supported on high tension insulators or grounded, and should be of such construction and strength as to prevent the upper wires from coming into contact with the lower ones.

If the screen is grounded, each wire of the screen must be of such size and so connected and earthed that it can surely carry to ground any current which may be delivered by any of the high-pressure wires. Further, the construction must be such that the wires of screen will not be destroyed by any arcing at the point of contact likely to occur under the conditions existing.

e. When it is necessary to carry such lines near buildings, they must be at such height and distance from the building as not to interfere with firemen in event of fire; therefore, if within 25 feet of a building, they must be carried at a height not less than that of the front cornice, and the height must be greater than that of the cornice, as the wires come nearer to the building in accordance with the following table:—

Distance of wire from building.	Elevation of wire above cornice of building.	Distance of wire from building.	Elevation of wire above cornice of building.
Feet.	Feet.	Feet.	Feet.
25	0	10	6
20	2	5	8
15	4	2½	9

It is evident that where the roof of the building continues nearly in line with the walls, as in Mansard roofs, the height and distance of the line must be reckoned from some part of the roof instead of from the cornice.

13. Transformers.—(For construction rules, see No. 62.) (See also Nos. 11, 13 A and 36.) [Where transformers are to be connected to high-voltage circuits, it is necessary in many cases, for best protection to life and property, that the secondary system be permanently grounded, and provision should be made for it when the transformers are built.]

a. Must not be placed inside of any building, excepting central stations and sub-stations, unless by special permission of the Inspection Department having jurisdiction.

An outside location is always preferable; first, because it keeps the high-voltage primary wires entirely out of the building, and second, for the reasons given in the note to No. 11a.

b. Must not be attached to the outside walls of buildings, unless separated therefrom by substantial supports.

It is recommended that transformers be not attached to frame buildings when any other location is practicable.

13A. Grounding Low-Potential Circuits.—*The grounding of low-potential circuits under the following regulations is only allowed when such circuits are so arranged that under normal conditions of service there will be no passage of current over the ground wire.*

Direct-Current 3-Wire Systems.—a. Neutral wire may be grounded and when grounded the following rules must be complied with:—

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1. Must be grounded at the Central Station on a metal plate buried in coke beneath permanent moisture level, and also through all available underground water and gas pipe systems.
2. In underground systems the neutral wire must also be grounded at each distributing box through the box.
3. In overhead systems the neutral wire must be grounded every 500 feet, as provided in Sections c to g.

Inspection Departments having jurisdiction may *require* grounding if they deem it necessary.

Two-wire direct-current systems having no accessible neutral point are not to be grounded.

Alternating-Current Secondary Systems.—b. Transformer secondaries of distributing systems should preferably be grounded, and when grounded, the following rules must be complied with:—

1. The grounding must be made at the neutral point or wire, whenever a neutral point or wire is accessible.
2. When no neutral point or wire is accessible one side of the secondary circuit may be grounded, provided the maximum difference of potential between the grounded point and any other point in the circuit does not exceed 250 volts.
3. The ground connections must be at the transformer or on the individual service as provided in sections c to g, and when transformers feed systems with a neutral wire, the neutral wire must also be grounded at least every 250 feet for overhead systems, and every 500 feet for underground systems.

Inspection Departments having jurisdiction may *require* grounding if they deem it necessary.

Ground Connections.—c. When the ground connection is inside of any building, or the ground wire is inside of, or attached to any building (except Central or Sub-stations) the ground wire must be of copper and have an approved rubber insulating covering National Electric Code Standard, for from 0 to 600 volts. (See No. 41.)

d. The ground wire in direct-current 3-wire systems must not at Central Stations be smaller than the neutral wire and not smaller than No. 4 B. & S. gage elsewhere. The ground wire in alternating-current systems must never be less than No. 4 B. & S. gage.

On three-phase system, the ground wire must have a carrying capacity equal to that of any one of the three mains.

e. The ground wire should, except for Central Stations and transformer sub-stations, be kept outside of buildings as far as practicable, but may be directly attached to the building or pole by cleats or straps or on porcelain knobs. Staples must never be used. The wire must be carried in as nearly a straight line as practicable, avoiding kinks, coils and sharp bends, and must be protected when exposed to mechanical injury.

This protection can be secured by use of an approved moulding, and as a rule the ground wire on the outside of a building should be in moulding at all places where it is in within seven feet from the ground.

f. The ground connection for Central Stations, transformer sub-stations, and banks of transformers must be made through metal plates buried in coke below permanent moisture level, and connection should also be made to all available underground piping systems including the lead sheath of underground cables.

g. For individual transformers and building services, the ground connection may be made as in Section f, or may be made to water piping systems running into buildings. This connection may be made by carrying the ground wire into the cellar and connecting on the street side of meters, main cocks, etc.

Where it is necessary to run the ground wire through any part of a building it shall be protected by approved porcelain bushings through walls or partitions and shall be run in approved moulding, except that in basements it may be supported on porcelain.

In connecting a ground wire to a piping system, the wire should be sweat into a lug attached to an approved clamp, and the clamp firmly bolted to the water pipe after all rust and scale have been removed; or be soldered into a brass plug and the plug forcibly screwed into a pipe-fitting, or, where the pipes are cast iron, into a hole tapped into the pipe itself. For large stations, where connecting to underground pipes with bell and spigot joints, it is well to connect to several lengths, as the pipe joints may be of rather high resistance.

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Where ground plates are used, a No. 16 Stubbs' gage copper plate, about 3 x 6 feet in size, with about 2 feet of crushed coke or charcoal, about pea size, both under and over it, would make a ground of sufficient capacity for a moderate-sized station, and would probably answer for the ordinary sub-station or bank of transformers. For a large central station, a plate with considerable more area might be necessary, depending upon the other underground connections available. The ground wire should be riveted to the plate in a number of places, and soldered for its whole length. Perhaps even better than a copper plate is a cast-iron plate with projecting forks, the idea of the fork being to distribute the connection to the ground over a fairly broad area, and to give a large surface contact. The ground wire can probably best be connected to such a cast-iron plate by soldering it into brass plugs screwed into holes tapped in the plate. In all cases, the joint between the plate and the ground wire should be thoroughly protected against corrosion by painting it with waterproof paint or some equivalent.

CLASS C.—INSIDE WORK.

(LIGHT POWER AND HEAT. FOR SIGNALING SYSTEMS, SEE CLASS E.)

ALL SYSTEMS AND VOLTAGES

GENERAL RULES.

14. Wires.—(For special rules, see Nos. 16, 18, 24, 35, 38 and 39.)

a. Must not be of smaller size than No. 14 B. & S. gage, except as allowed under Nos. 24v and 45b.

b. Tie wires must have an insulation equal to that of the conductors they confine.

The use of some form of confining knob or insulator which will dispense with tie wires is recommended.

c. Must be so spliced or joined as to be both mechanically and electrically secure without solder. The joints must then be soldered to insure preservation, and covered with an insulation equal to that on the conductors.

Stranded wires must be soldered before being fastened under clamps or binding screws, and whether stranded or solid, when they have a conductivity greater than that of No. 8 B. & S. gage they must be soldered into lugs for all terminal connections.

All joints must be soldered unless made with some form of *approved* splicing device. This ruling applies to joints and splices in all classes of wiring covered by these rules.

d. Must be separated from contact with walls, floors, timbers or partitions through which they may pass by non-combustible, non-absorptive insulating tubes, such as glass or porcelain, except as provided in No. 24u.

Bushings must be long enough to bush the entire length of the hole in one continuous piece, or else the hole must first be bushed by a continuous waterproof tube This tube may be a conductor, such as iron pipe, but in that case an insulating bushing must be pushed into each end of it, extending far enough to keep the wire absolutely out of contact with the pipe.

e. Must be kept free from contact with gas, water or other metallic piping, or any other conductors or conducting material which they may cross, by some continuous and firmly fixed non-conductor, creating a permanent separation. Deviations from this rule may sometimes be allowed by special permission.

Where one wire crosses another wire the best and usual means of separating them is by a porcelain tube on one of the wires. The tubing must be prevented from moving out of place either by a cleat or knob on each end, or by taping it securely in place.

The same method may be adopted where wires pass close to iron pipes, beams, etc., or, where the wires are above the pipes, as is generally the case, ample protection can frequently be secured by supporting the wires with a porcelain cleat placed as nearly above the pipe as possible.

This rule must not be construed as in any way modifying No. 24, Sections h *and* j.

f. Must be so placed in wet places that an air space will be left between conductors and pipes in crossing, and the former must be run in such a way that they cannot come in contact with the pipe accidentally. Wires should be run over, rather than under, pipes upon which moisture is likely to gather or which, by leaking, might cause trouble on a circuit.

g. The installation of electrical conductors in wooden moulding, or on insulators, in elevator shafts will not be approved, but conductors may be installed in such shafts if encased in approved metal conduits.

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15. Underground Conductors.—**a.** Must be protected against moisture and mechanical injury where brought into a building, and all combustible material must be kept from the immediate vicinity.

b. Must not be so arranged as to shunt the current through a building around any catch-box.

c. Where underground service enters building through tubes, the tubes shall be tightly closed at outlets with asphaltum or other non-conductor, to prevent gases from entering the building through such channels.

d. No underground service from a subway to a building shall supply more than one building except by written permission from the Inspection Department having jurisdiction.

16. Table of Carrying Capacity of Wires.—**a.** The following table, showing the allowable carrying capacity of copper wires and cables of ninety-eight per cent conductivity, according to the standard adopted by the American Institute of Electrical Engineers, must be followed in placing interior conductors (See page 1388.)

For insulated aluminum wire the safe carrying capacity is eighty-four per cent of that given in the following tables for copper wire with the same kind of insulation.

B. & S. Gage.	Table A. Rubber Insulation. See No. 41. Amperes.	Table B. Other Insulations. See No. 42 to 44. Amperes.	Circular Mils.
18	3	5	1,624
16	6	8	2,583
14	12	16	4,107
12	17	23	6,530
10	24	32	10,380
8	33	46	16,510
6	46	65	26,250
5	54	77	33,100
4	65	92	41.740
3	76	110	52,630
2	90	131	66,370
1	107	156	83,690
0	127	185	105,500
00	150	220	133,100
000	177	262	167,800
0000	210	312	211,600

Circular Mils.	Table A. Concl'd. Amperes.	Table B Concl'd. Amperes.
200,000	200	300
300,000	270	400
400,000	330	500
500,000	390	590
600,000	450	680
700,000	500	760
800,000	550	840
900,000	600	920
1,000,000	650	1,000
1,100,000	690	1,080
1,200,000	730	1,150
1,300,000	770	1,220
1,400,000	810	1,290
1,500,000	850	1,360
1,600,000	890	1,430
1,700,000	930	1,490
1,800,000	970	1,550
1,900,000	1,010	1,610
2,000,000	1,050	1,670

The lower limit is specified for rubber-covered wires to prevent gradual deterioration of the high insulations by the heat of the wires, but not from fear of igniting the insulation. The question of drop is not taken into consideration in the above tables.

The carrying capacity of Nos. 16 and 18 B. & S. gage wire is given, but no smaller than No. 14 is to be used, except as allowed under Nos. 24v and 45b.

17. Switches, Cut-Outs, Circuit-Breakers, Etc.—(For construction rules, see Nos. 51, 52 and 53.) **a.** On constant potential circuits, all service switches and all switches controlling circuits supplying current to motors or heating devices, and all cut-outs, unless otherwise provided (for exceptions as to switches see Nos. 8c and 21a; for exceptions as to cut-outs see No. 21a and b) must be so arranged that the cut-outs will protect and the opening of the switch or circuit-breaker will disconnect all of the wires; that is, in the two-wire system the two wires, and the three-wire system the three wires, must be protected by the cut-out and disconnected by the operation of the switch or circuit-breaker.

This, of course, does not apply to the grounded circuit of street railway systems.

b. Must not be placed in the immediate vicinity of easily ignitable stuff or where exposed to inflammable gases or dust or to flyings of combustible material.

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When the occupancy of a building is such that switches, cut-outs, etc., cannot be located so as not to be exposed to dust or flyings of combustible material they must be enclosed in approved dust-proof cabinets with self-closing doors, except oil switches and circuit-breakers which have dust-tight casings.

c. Must, when exposed to dampness, either be enclosed in a moisture-proof box or mounted on porcelain knobs.

The cover of the box should be so made that no moisture which may collect on the top or sides of the box can enter it.

d. Time switches, sign flashers and similar appliances must be of approved design and enclosed in a steel box or cabinet lined with fire-resisting material.

The cover of the box should be so made that no moisture which may collect on the top or sides of the box can enter it.

If a steel box is used, the minimum thickness of the steel must be 0.128 of an inch (No. 8 B. & S. gage).

If a cabinet is used, it must be lined with marble or slate at least three-eighths of an inch thick, or with steel not less than 0.128 of an inch thick. Box or cabinet must be so constructed that when switch operates blade shall clear the door by at least one inch.

CONSTANT-CURRENT SYSTEMS.

Principally Series Arc Lighting.

18. Wires.—(See also Nos. 14, 15 and 16.) a. Must has an *approved* rubber insulating covering (see No. 41).

b. Must be arranged to enter and leave the building through an *approved* double-contact service switch (see No. 51b), mounted in a non-combustible case, kept free from moisture, and easy of access to police or firemen.

c. Must always be in plain sight, and never encased, except when *required* by the Inspection Department having jurisdiction.

d. Must be supported on glass or porcelain insulators, which separate the wire at least one inch from the surface wired over and must be kept *rigidly* at least 8 inches from each other, except within the structure of lamps, on hanger-boards or in cut-out boxes, or like places, where a less distance is necessary.

e. Must, on side walls be protected from mechanical injury by a substantial boxing, retaining an air space of one inch around the conductors, closed at the top (the wires passing through bushed holes), and extending not less than 7 feet from the floor. When crossing floor timbers in cellars, or in rooms where they might be exposed to injury, wires must be attached by their insulating supports to the under side of a wooden strip not less than one-half an inch in thickness. Instead of the running-boards, guard strips on each side of and close to the wires will be accepted. These strips to be not less than seven-eighths of an inch in thickness and at least as high as the insulators.

Except on joisted ceilings, a strip one-half of an inch thick is not considered sufficiently stiff and strong. For spans of say eight or ten feet, where there is but little vibration, one-inch stock is generally sufficiently stiff; but where the span is longer than this or there is considerable vibration, still heavier stock should be used.

19. Series Arc Lamps.—(For construction rules, see No. 57.)

a. Must be carefully isolated from inflammable material.

b. Must be provided at all times with a glass globe surrounding the arc, and securely fastened upon a closed base. Broken or cracked globes must not be used.

c. Must be provided with a wire netting (having a mesh not exceeding one and one-fourth inches) around the globe, and an *approved* spark arrester (see No. 58), when readily inflammable material is in the vicinity of the lamps, to prevent escape of sparks of carbon or melted copper. It is recommended that plain carbons, not copper-plated, be used for lamps in such places.

Outside arc lamps must be suspended at least eight feet above sidewalks. Inside arc lamps must be placed out of reach or suitably protected.

Arc lamps, when used in places where they are exposed to flyings of easily inflammable material, should have the carbons enclosed completely in a tight globe in such manner as to avoid the necessity for spark arresters.

"Enclosed arc" lamps, having tight inner globes, may be used, and the requirements of Sections b and c above would, of course, not apply to them, except that a

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wire netting around the inner globe may in some cases be required if the outer globe is omitted.

d. Where hanger-boards (see No. 56) are not used, lamps must be hung from insulating supports other than their conductors.

e. Lamps when arranged to be raised and lowered, either for carboning or other purposes, shall be connected up with stranded conductors from the last point of support to the lamp, when such conductor is larger than No. 14 B. & S. gage.

20. Incandescent Lamps in Series Circuits.—a. Must have the conductors installed as required in No. 18, and each lamp must be provided with an automatic cut-out.

b. Must have each lamp suspended from a hanger-board by means of rigid tube.

c. No electro-magnetic device for switches and no multiple-series or series-multiple system of lighting will be approved.

d. Must not under any circumstances be attached to gas fixtures.

CONSTANT-POTENTIAL SYSTEMS.

General Rules—All Voltages.

21. Automatic Cut-Outs (Fuses and Circuit-Breakers).—(See No. 17, and for construction, Nos. 52 and 53.) [Excepting on main switchboards, or where otherwise subject to expert supervision, circuit-breakers will not be accepted unless fuses are also provided.]

a. Must be placed on all service wires, either overhead or underground, as near as possible to the point where they enter the building and inside the walls, and arranged to cut off the entire current of the building.

Where the switch required by No. 22a is inside the building, the cut-out required by this section must be placed so as to protect it.

For three-wire (not three-phase) systems the fuse in the neutral wire may be omitted, *provided the neutral wire is of equal carrying capacity to the larger of the outside wires, and is grounded as provided for in No. 13A.*

In risks having private plants, the yard wires running from building to building are not generally considered as service wires, so that cut-outs would not be required where the wires enter buildings, provided that the next fuse back is small enough to properly protect the wires inside the building in question.

b. Must be placed at every point where a change is made in the size of wire [unless the cut-out in the larger wire will protect the smaller (see No. 16)].

For three-wire (not three-phase) systems the fuse in the neutral wire, except that called for under No. 21d, may be omitted, *provided the neutral wire is of equal carrying capacity to the larger of the outside wires, and is grounded as provided for in No. 13A.*

c. Must be in plain sight, or enclosed in an *approved* cabinet (see No. 54), and readily accessible. They must not be placed in the canopies or shells of fixtures.

The ordinary porcelain link fuse cut-out will not be approved. Link fuses may be used only when mounted on slate of marble bases conforming to No. 52 and must be enclosed in dust-tight, fireproofed cabinets, except on switchboards located well away from any combustible material, as in the ordinary engine and dynamo room and where these conditions will be maintained.

d. Must be so placed that no set of incandescent lamps requiring more than 660 watts, whether grouped on one fixture or on several fixtures or pendants, will be dependent upon one cut-out.

Special permission may be given in writing by the Inspection Department having jurisdiction, for departure from this rule, in the case of large chandeliers. (For exceptions, see No. 31 A, b, 3[*b*] and 4 [*b*] for border lights, see List of Fittings for rules for electric signs.) All branches or taps from any three-wire system which are directly connected to lamp sockets or other translating devices, must be run as two-wire circuits if the fuses are omitted in the neutral, or if the difference of potential between the two outside wires is over 250 volts, and both wires of such branch or tap circuits must be protected by proper fuses.

The above rule shall also apply to motors when more than one is dependent on a single cut-out.

The fuses in the branch cut-outs should not have a rated capacity greater than 6 amperes on 110 volt systems, and 3 amperes on 220 volt systems.

The idea is to have a small fuse to protect the lamp socket and the small wire used for fixtures, pendants, etc. It also lessens the chances of extinguishing a large number of lights if a short circuit occurs.

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On open work in large mills *approved* link fused rosettes may be used at a voltage of not over 125 and *approved* enclosed fused rosettes at a voltage of not over 250, the fuse in the rosettes not to exceed 3 amperes, and a fuse of over 25 amperes must not be used in the branch circuit.

e. The rated capacity of fuses must not exceed the allowable carrying capacity of the wire as given in No. 16. Circuit-breakers must not be set more than 30 per cent above the allowable carrying capacity of the wire, unless a fusible cut-out is also installed in the circuit.

In the arms of fixtures carrying a single socket a No. 18 B. & S. gage wire supplying only one socket will be considered as properly protected by a 6 ampere fuse.

22. Switches.—(See No. 17, and for construction, No. 51.)

a. Must be placed on all service wires, either overhead or underground, in a readily accessible place, as near as possible to the point where the wires enter the building, and arranged to cut off the entire current.

Service cut-out and switch must be arranged to cut off current from all devices including meters.

In risks having private plants the yard wires running from building to building are not generally considered as service wires, so that switches would not be required in each building if there are other switches conveniently located on the mains or if the generators are near at hand.

b. Must always be placed in dry, accessible places, and be grouped as far as possible. (See No. 17 c.) Single-throw knife switches must be so placed that gravity will tend to open rather than close them. Double-throw knife switches may be mounted so that the throw will be either vertical or horizontal as preferred.

When possible, switches should be so wired that blades will be "dead" when switch is open.

If switches are used in rooms where combustible flyings would be likely to accumulate around them, they should be enclosed in dust-tight cabinets. (See note under No. 17 b.) Even in rooms where there are no combustible materials it is better to put all knife switches in cabinets, in order to lessen the danger of accidental short circuits being made across their exposed metal parts by careless workmen.

Up to 250 volts and 30 amperes, *approved indicating snap* switches are advised in preference to knife switches on lighting circuits about the workrooms.

c. Single pole switches must never be used as service switches nor placed in the neutral wire of a three-wire system, except in the two-wire branch or tap circuit described in 21 d.

This, of course, does not apply to the grounded circuits of street railway systems.

Three-way switches are considered as single-pole switches and must be wired so that only one pole of the circuit is carried to either switch.

d. Where flush switches or receptacles are used, whether with conduit systems or not, they must be enclosed in boxes constructed of iron or steel. No push button for bells, gas-lighting circuits, or the like shall be placed in the same wall plate with switches controlling electric light or power wiring.

This requires an *approved* box in addition to the porcelain enclosure of the switch or receptacle.

e. Where possible, at all switch or fixture outlets, a ¾-inch block must be fastened between studs or floor timbers flush with the back of lathing to hold tubes, and to support switches or fixtures. When this cannot be done, wooden base blocks not less than ¾-inch in thickness, securely screwed to lathing, must be provided for switches, and also for fixtures which are not attached to gas pipes or conduit.

The above will not be necessary where outlet boxes are used which will give proper support for fixtures, etc.

f. Sub-bases of non-combustible, non-absorptive insulating material, which will separate the wires at least ½-inch from the surface wired over, must be installed under all snap switches used in exposed knob and cleat work. Sub-bases must also be used in moulding work, but they may be made of hardwood.

23. Electric Heaters.—It is often desirable to connect in multiple with the heaters and between the heater and the switch controlling same, an incandescent lamp of low candle power, as it shows at a glance whether or not the switch is open, and tends to prevent its being left closed through oversight. Inspection Departments having jurisdiction may require this provision to be carried out if they deem it necessary.

a. Must be protected by a cut-out and controlled by indicating switches. Switches must be double pole except when the device controlled does not require more than 660 watts of energy.

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b. Must never be concealed, but must at all times be in plain sight.

Special permission may be given in writing by the Inspection Department having jurisdiction for departure from this rule in certain cases.

c. Flexible conductors for smoothing irons and sad irons, and for all devices requiring over 250 watts must comply with No. 45 g.

d. For portable heating devices the flexible conductors must be connected to an *approved* plug device, so arranged that the plug will pull out and open the circuit in case any abnormal strain is put on the flexible conductor. This device may be stationary, or it may be placed in the cord itself. The cable or cord must be attached to the heating apparatus in such manner that it will be protected from kinking, chafing or like injury at or near the point of connection.

e. Smoothing irons, sad irons, and other heating appliances that are intended to be applied to inflammable articles, such as clothing, must conform to the above rules so far as they apply. They must also be provided with an approved stand, on which they should be placed when not in use.

An approved automatic attachment which will cut off the current when the iron is not on the stand or in actual use is desirable. Inspection Departments having jurisdiction may require this provision to be carried out if they deem it advisable.

f. Stationary electric heating apparatus, such as radiators, ranges, plate warmers, etc., must be placed in a safe location, isolated from inflammable materials, and be treated as sources of heat.

Devices of this description will often require a suitable heat-resisting material placed between the device and its surroundings. Such protection may best be secured by installing two or more plates of tin or sheet steel with a one-inch air space between, or by alternate layers of sheet steel and asbestos with a similar air space.

g. Must each be provided with name-plate, giving the maker's name and the normal capacity in volts and amperes.

CONSTANT-LOW-POTENTIAL SYSTEMS.

550 Volts or Less.

Any circuit attached to any machine, or combination of machines, which develops a difference of potential between any two wires, of over ten volts and less than 550 volts, shall be considered as a low-potential circuit, and as coming under this class, unless an approved transforming device is used, which cuts the difference of potential down to ten volts or less. The primary circuit not to exceed a potential of 3,500 volts unless the primary wires are installed in accordance with the requirements as given in No. 12 A, or are underground.

For 550 volt motor equipments a margin of ten per cent above the 550 volt limit will be allowed at the generator or transformer.

Before pressure is raised above 300 volts on any previously existing system of wiring, the whole must be strictly brought up to all of the requirements of the rules at date.

24. Wires.—General Rules. (See also Nos. 14, 15 and 16.)

a. *Must be so arranged that under no circumstances will there be a difference of potential of over 300 volts between any bare metal parts in any distributing switch or cut-out cabinet, or equivalent center of distribution.*

This rule is not intended to prohibit the placing of switches or single pole cut-outs for motor systems of voltages above 300 in cabinets, but would require that the cabinets be divided by *approved* barriers so arranged that no one section shall contain more than one switch nor more than one single pole cut-out.

b. Must not be laid in plaster, cement or similar finish, and must never be fastened with staples.

c. Must not be fished for any great distance, and only in places where the inspector can satisfy himself that the rules have been complied with.

d. Twin wires must never be used, except in conduits, or where flexible conductors are necessary.

e. Must be protected on side walls from mechanical injury. When crossing floor timbers in cellars, or in rooms where they might be exposed to injury, wires must be attached by their insulating supports to the under side of a wooden strip, not less than one-half inch in thickness and not less than three inches in width. Instead of the running-boards, guard strips on each

side of and close to the wires will be accepted. These strips to be not less than seven-eighths of an inch in thickness, and at least as high as the insulators.

Suitable protection on side walls should extend not less than five feet from the floor. This may be secured by substantial boxing, retaining an air space of one inch around the conductors, closed at the top (the wires passing through bushed holes) or by approved metal conduit, or pipe of equivalent strength.

When metal conduit or pipe is used, the insulation of each wire must be reinforced by *approved* flexible tubing extending from the insulator next below the pipe to the one next above it, unless the conduit is installed according to No. 25 (sections e and f excepted), and the wire used complies with No. 47. The two or more wires of a circuit *each* with its flexible tubing (when required), if carrying alternating current *must*, or if direct current, *may* be placed within the same pipe.

In damp places the wooden boxing may be preferable because of the precautions which would be necessary to secure proper insulation if the pipe were used. With this exception, however, iron piping is considered preferable to the wooden boxing, and its use is strongly urged. It is especially suitable for the protection of wires near belts, pulleys, etc.

f. When run in unfinished attics, will be considered as concealed, and when run in close proximity to water tanks or pipes, will be considered as exposed to moisture.

In unfinished attics wires are considered as exposed to mechanical injury, and must not be run on knobs on upper edge of joists.

Special Rules.

For Open Work—In dry places.

g. Must have an *approved* rubber, slow-burning weatherproof, or slow-burning insulation (see Nos. 41, 42 and 43).

A slow-burning covering, that is, one that will not carry fire, is considered good enough where the wires are entirely on insulating supports. Its main object is to prevent the copper conductors from coming accidentally into contact with each other or anything else

h. Must be rigidly supported on non-combustible, non-absorptive insulators, which will separate the wires from each other and from the surface wired over in accordance with the following table:—

Voltage.	Distance from Surface.	Distance between Wires.
0 to 300	½ inch	2½ inch
301 to 550	1 "	4 "

Rigid supporting requires under ordinary conditions, where wiring along flat surfaces, supports at least every four and one-half feet. If the wires are liable to be disturbed, the distance between supports should be shortened. In buildings of mill construction, mains of not less than No. 8 B. & S. gage, where not liable to be disturbed, may be separated about six inches, and run from timber to timber, not breaking around, and may be supported at each timber only.

This rule not to be interpreted to forbid the placing of the neutral of an Edison three-wire system in the center of a three-wire cleat where the difference of potential between the outside wires is not over 300 volts, provided the outside wires are separated two and one-half inches.

For Open Work—In damp places, or buildings specially subject to moisture or to acid or other fumes liable to injure the wires or their insulation.

i. Must have an *approved* insulating covering.

For protection against water, rubber insulation must be used. For protection against corrosive vapors, either weatherproof or rubber insulation must be used. (See Nos. 41 and 44.)

j. Must be rigidly supported on non-combustible, non-absorptive insulators, which separate the wire at least one inch from the surface wired over, and must be kept apart at least two and one-half inches for voltages up to 300, and four inches for higher voltages.

Rigid supporting requires under ordinary conditions, where wiring over flat surfaces, supports at least every four and one-half feet. If the wires are liable to be disturbed, the distance between supports should be shortened. In buildings of mill construction, mains of not less than No. 8 B. & S. gage, where not liable to be disturbed, may be separated about six inches, and run from timber to timber, not breaking around, and may be supported at each timber only.

For Moulding Work (Wooden and Metal). (For construction rules see No. 50. See also No. 25 A.)

k. Must have an *approved* rubber insulating covering. (For wooden moulding see No. 41, for metal moulding see No. 47.)

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l. Must never be placed in either metal or wooden moulding in concealed or damp places, or where the difference of potential between any two wires in the same moulding is over 300 volts. *Metal* mouldings must not be used for circuits requiring more than 660 watts of energy.

As a rule, wooden moulding should not be placed directly against a brick wall, as the wall is likely to "sweat" and thus introduce moisture back of the moulding.

m. Must, for alternating current systems if in metal moulding, have the two or more wires of a circuit installed in the same moulding.

It is advised that this be done for direct current systems also, so that they may be changed to alternating systems at any time, induction troubles preventing such a change if the wires are in separate mouldings.

For Conduit Work.

n. Must have an *approved* rubber insulating covering (see No. 47).

o. Must not be drawn in until all mechanical work on the building has been, as far as possible, completed.

Conductors in vertical conduit risers must be supported within the conduit system in accordance with the following table:—

No. 14 to 0 every 100 feet.
No. 00 to 0000 every 80 feet.
0000 to 350,000 C. M. every 60 feet.
350,000 C. M. to 500,000 C. M. every 50 feet.
500,000 C. M. to 750,000 C. M. every 40 feet.
750,000 C. M. every 35 feet.

A turn of 90 degrees in the conduit system will constitute a satisfactory support, as per above table.

The following methods of supporting cables are recommended:—

1. Junction boxes may be inserted in the conduit system at the required intervals, in which insulating supports of *approved* type must be installed and secured in a satisfactory manner so as to withstand the weight of the conductors attached thereto, the boxes to be provided with proper covers.
2. Cables may be supported in a*pproved* junction boxes on two or more insulating supports so placed that the conductors will be deflected at an angle of not less than 90 degrees, and carried a distance of not less than twice the diameter of the cable from its vertical position. Cables so suspended may be additionally secured to these insulators by tie wires.

Other methods, if used, must be approved by the Inspection Departments having jurisdiction.

p. Must, for alternating systems, have the two or more wires of a circuit drawn in the same conduit.

It is advised that this be done for direct current systems also, so that they may be changed to alternating systems at any time, induction troubles preventing such a change if the wires are in separate conduits.

The same conduit must never contain circuits of different systems, but may contain two or more circuits of the same system.

For Concealed "Knob and Tube" Work.

q. Must have an *approved* rubber insulating covering (see No. 41).

r. Must be rigidly supported on non-combustible, non-absorptive insulators which separate the wire at least one inch from the surface wired over. Should preferably be run singly on separate timbers, or studdings, and must be kept at least five inches apart.

Must be separated from contact with the walls, floor timbers and partitions through which they may pass by non-combustible, non-absorptive insulating tubes, such as glass or porcelain.

Rigid supporting requires under ordinary conditions, where wiring along flat surface, supports at least every four and one-half feet. If the wires are liable to be disturbed the distance between supports should be shortened.

At distributing centers, outlets or switches where space is limited and the five-inch separation cannot be maintained, each wire must be separately encased in a continuous length of approved flexible tubing.

Wires passing through timbers at the bottom of plastered partitions must be protected by an additional tube extending at least four inches above the timber.

s. When in a concealed knob and tube system, it is impracticable to place the whole of a circuit on non-combustible supports of glass or porcelain, that portion of the circuit which cannot be so supported must be installed with *approved* metal conduit, or *approved* armored cable (see No. 24 t), ex-

cept that if the difference of potential between the wires is not over 300 volts, and if the wires are not exposed to moisture, they may be fished if separately encased in *approved* flexible tubing, extending in continuous lengths from porcelain support to porcelain support, from porcelain support to outlet, or from outlet to outlet.

t. Mixed concealed knob and tube work as provided for in No. 24 s, must comply with requirements of No. 24 n to p, and No. 25, when conduit is used, and with requirements of No. 24 A, when armored cable is used.

u. Must at all outlets, except where conduit is used, be protected by *approved* flexible insulating tubing, extending in continuous lengths from the last porcelain support to at least one inch beyond the outlet. In the case of combination fixtures the tubes must extend at least flush with outer end of gas cap.

It is recommended but not required that *approved* outlet boxes or plates be installed at all outlets in concealed "knob and tube" work, the wires to be protected by *approved* flexible insulating tubing, extending in continuous lengths from the last porcelain support into the box.

For Fixture Work. v. Must have an *approved* rubber insulating covering (see No. 46), and be not less in size than No. 18 B. &. S. gage.

See No. 46 *e*, fine print note, for exceptions to the use of rubber-covered wire.

w. Supply conductors, and especially the splices to fixture wires, must be kept clear of the grounded part of gas pipes, and, where shells or outlets boxes are used, they must be made sufficiently large to allow the fulfillment of this requirement.

x. Must, when fixtures are wired outside, be so secured as not to be cut or abraded by the pressure of the fastenings or motion of the fixture.

y. Under no circumstances must there be a difference of potential of more than 300 volts between wires contained in or attached to the same fixture.

24 A. Armored Cables.—(For construction rules, see No. 48.)

a Must be continuous from outlet to outlet or to junction boxes, and the armor of the cable must properly enter and be secured to all fittings, and the entire system must be mechanically secured in position.

In case of underground service connections and main runs, this involves running such armored cable continuously into a main cut-out cabinet or gutter surrounding the panel board, as the case may be. (See No. 54.)

b. Must be equipped at every outlet with an *approved* outlet box or plate, as required in conduit work. (See No. 49 A.)

Outlet plates must not be used where it is practicable to install outlet boxes. The outlet box or plate shall be so installed that it will be flush with the finished surface, and if this surface is broken it shall be repaired so that it will not show any gaps or open spaces around the edge of the outlet box or plate.

In buildings already constructed where the conditions are such that neither outlet box nor plate can be installed, these appliances may be omitted by special permission of the Inspection Department having jurisdiction, provided the armored cable is firmly and rigidly secured in place.

c. Must have the metal armor of the cable permanently and effectively grounded.

It is essential that the metal armor of such systems be joined so as to afford electrical conductivity sufficient to allow the largest fuse of circuit-breaker in the circuit to operate before a dangerous rise in temperature in the system can occur. Armor of cables and gas pipes must be securely fastened in metal outlet boxes so as to secure good electrical connections. Where boxes used for centers of distribution do not afford good electrical connection the armor of the cables must be joined around them by suitable bond wires. Where sections of armored cable are installed without being fastened to the metal structure of buildings or grounded metal piping, they must be bonded together and joined to a permanent and efficient ground connection.

d. When installed in so-called fireproof buildings in course of construction or afterwards if concealed, or where it is exposed to the weather, or in damp places such as breweries, stables, etc., the cable must have a lead covering at least one thirty-second inch in thickness placed between the outer braid of the conductors and the steel armor.

e. Where entering junction boxes, and at all other outlets, etc., must be provided with *approved* terminal fittings which will protect the insulation of the conductors from abrasion, unless such junction or outlet boxes are specially designed and approved for use with the cable.

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f. Junction boxes must always be installed in such manner as to be accessible.

g. For alternating current systems must have the two or more conductors of the cable enclosed in one metal armor.

25. Interior Conduits.—(See also Nos. 24 n to p, and 49.)

The object of a tube or conduit is to facilitate the insertion or extraction of the conductors and to protect them from mechanical injury. Tubes or conduits are to be considered merely as raceways, and are not to be relied upon for insulation between wire and wire, or between the wire and the ground.

a. No conduit tube having an internal diameter of less than five-eighths of an inch shall be used. Measurements to be taken inside of metal conduits.

b. Must be continuous from outlet to outlet or to junction boxes, and the conduit must properly enter, and be secured to all fittings and the entire system must be mechanically secured in position.

In case of service connections and main runs, this involves running each conduit continuously into a main cut-out cabinet or gutter surrounding the panel board, as the case may be (see No. 54).

c. Must be first installed as a complete conduit system, without the conductors.

d. Must be equipped at every outlet with an *approved* outlet box or plate (see No. 49 A).

Outlet plates must not be used where it is practicable to install outlet boxes.

The outlet box or plate shall be so installed that it will be flush with the finished surface, and if this surface is broken it shall be repaired so that it will not show any gaps or open spaces around the edge of the outlet box or plate.

In buildings already constructed where the conditions are such that neither outlet box nor plate can be installed, these appliances may be omitted by special permission of the Inspection Department having jurisdiction, providing the conduit ends are bushed and secured.

e. Metal conduits where they enter junction boxes, and at all other outlets, etc., must be provided with *approved* bushings fitted so as to protect wire from abrasion, except when such protection is obtained by the use of *approved* nipples, properly fitted in boxes or devices.

f. Must have the metal of the conduits permanently and effectually grounded.

It is essential that the metal of conduit systems be joined so as to afford electrical conductivity sufficient to allow the largest fuse or circuit breaker in the circuit to operate before a dangerous rise in temperature in the conduit system can occur. Conduits and gas pipes must be securely fastened in metal outlet boxes so as to secure good electrical connection. Where boxes used for centers of distribution do not afford good electrical connection, the conduits must be joined around them by suitable bond wires. Where sections of metal conduit are installed without being fastened to the metal structure of buildings or grounded metal piping, they must be bonded together and joined to a permanent and efficient ground connection.

g. Junction boxes must always be installed in such manner as to be accessible.

h. All elbows or bends must be so made that the conduit or lining of same will not be injured. The radius of the curve of the inner edge of any elbow not to be less than three and one-half inches. Must have not more than the equivalent of four quarter bends from outlet to outlet, the bends at the outlets not being counted.

25 A. Metal Mouldings. (See also Nos. 24 k to m, and 50.)

a. Must be continuous from outlet to outlet, to junction boxes, or approved fittings designed especially for use with metal mouldings, and must at all outlets be provided with approved terminal fittings which will protect the insulation of conductors from abrasion, unless such protection is afforded by the construction of the boxes or fittings.

b. Such moulding where passing through a floor must be carried through an iron pipe extending from the ceiling below to a point five feet above the floor, which will serve as an additional mechanical protection and exclude the presence of moisture often prevalent in such locations.

In residences, office buildings and similar locations where appearance is an essential feature, and where the mechanical strength of the moulding itself is adequate, this ruling may be modified to require the protecting piping from the ceiling below to a point at least three inches above the flooring.

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. Backing must be secured in position by screws or bolts, the heads of
ı must be flush with the metal.

. The metal of the moulding must be permanently and effectively
ıded, and must be so installed that adjacent lengths of moulding will
echanically and electrically secured at all points.

; is essential that the metal of such systems be joined so as to afford electric
ctivity sufficient to allow the largest fuse in the circuit to operate before a
rous rise of temperature in the system can occur. Mouldings and gas pipes
be securely fastened in metal outlet boxes, so as to secure good electrical con-
n. Where boxes used for center of distribution do not afford good electrical
ction the metal moulding must be joined around them by suitable bond wires.
; sections are installed without being fastened to the metal structure of the
ng or grounded metal piping, they must be bonded together or joined to a
ment and effective ground connection.

Must be installed so that for alternating systems the two or more
of a circuit will be in the same metal moulding.

is advised that this be done for direct systems also, so that they may be changed
alternating system at any time, induction troubles preventing such change
wires must be in separate mouldings.

i. **Fixtures.**—(See also Nos. 22 e, 24 v to y.)

Must when supported from the gas piping or any grounded metal
of a building be insulated from such piping or metal work by means of
ved insulating joints (see No. 59) placed as close as possible to the ceil-
r walls.

ıs outlet pipes must be protected above the insulating joint by *approved* in-
ıg tubing, and where outlet tubes are used they must be of sufficient length to
below the insulating joint, and must be so secured that they will not be pushed
rhen the canopy is put in place.

here canopies are placed against plaster walls or ceilings in fireproof buildings,
inst metal walls or ceilings, or plaster walls or ceilings on metallic lathing in
ıss of buildings, they must be thoroughly and permanently insulated from such
ır ceilings.

Must have all burs or fins removed before the conductors are drawn
he fixture.

Must be tested for "contacts" between conductors and fixture, for
: circuits" and for ground connections before it is connected to its
/ conductors.

All fixture arms made of tubing smaller than ½-inch outside diameter,
ıe arms of all one-light brackets, must be secured after they are screwed
osition by the use of a set-screw properly placed, or by soldering or
ting or some equally good method to prevent the arms from becoming
ıwed. Arms must not be made of tubing lighter than No. 18 B. &
e, and must have at screw joints not less than five threads all engaging.
ule does not apply to fixtures or brackets with cast or heavy arms.

. **Sockets.**—(For construction rules, see No. 55.)

In rooms where inflammable gases may exist the incandescent lamp
cket must be enclosed in a vapor-tight globe, and supported on a
anger, wired with *approved* rubber-covered wire (see No. 41) soldered
y to the circuit.

y sockets contain a switch (*see* No. 17 b).

In damp or wet places "waterproof" sockets must be used. Unless
ıp on fixtures they must be hung by separate *stranded* rubber-covered
ıot smaller than No. 14 B. & S. gage, which should preferably be twisted
er when the pendant is over three feet long.

ese wires must be soldered direct to the circuit wires but supported in-
lently of them.

Key sockets will not be approved if installed over specially inflam-
stuff, or where exposed to flyings of combustible material.

Flexible Cord.—

Must have an *approved* insulation and covering (see No. 45).

Must not be used where the difference of potential between the two
ıs over 300 volts.

; above rule does not apply to the grounded circuits in street railway

Must not be used as a support for clusters.

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d. Must not be used except for pendants, wiring of fixtures, portable lamps or motors, and portable heating apparatus.

The practice of making the pendants unnecessarily long and then looping them up with cord adjusters is strongly advised against. It offers a temptation to carry about lamps which are intended to hang freely in the air, and the cord adjusters wear off the insulation very rapidly.

For all portable work, including those pendants which are liable to be moved about sufficiently to come in contact with surrounding objects, flexible wires and cables especially designed to withstand this severe service are on the market, and should be used. (See No. 45 f.)

The standard socket is threaded for one-eighth inch pipe, and if it is properly bushed, the reinforced flexible cord will not go into it, but this style of cord may be used with sockets threaded for three-eighths inch pipe, and provided with substantial insulating bushings. The cable to be supported independently of the overhead circuit by a single cleat, and the two conductors then separated and soldered to the overhead wires.

The bulb of an incandescent lamp frequently becomes hot enough to ignite paper, cotton and similar readily ignitible materials, and in order to prevent it from coming in contact with such materials, as well as to protect it from breakage, every portable lamp should be surrounded with a substantial wire guard.

e. Must not be used in show windows except when provided with an *approved* metal armor.

f. Must be protected by insulating bushings where the cord enters the socket.

g. Must be so suspended that the entire weight of the socket and lamp will be borne by some *approved* device under the bushing in the socket, and above the point where the cord comes through the ceiling block or rosette, in order that the strain may be taken from the joints and binding screws.

This is usually accomplished by knots in the cord inside the socket and rosette

29. Arc Lamps on Constant-Potential Circuits.—a. Must have a cut-out (see No. 17 a) for each lamp or each series of lamps.

The branch conductors should have a carrying capacity about fifty per cent in excess of the normal current required by the lamp, to provide for heavy current required when lamp is started, or when carbons become stuck without overfusing the wires.

b. Must only be furnished with such resistances or regulators as are enclosed in non-combustible material, such resistances being treated as sources of heat. Incandescent lamps must not be used for this purpose.

c. Must be supplied with globes and protected by spark arresters and wire netting around the globe, as in case of series arc lamps (see Nos. 19 and 58).

Outside arc lamps must be suspended at least eight feet above sidewalks. Inside arc lamps must be placed out of reach or suitably protected.

d. Lamps when arranged to be raised and lowered, either for carbonning or other purposes, shall be connected up with stranded conductors from the last point of support to the lamp, when such conductor is larger than No. 14 B. & S. gage.

30. Economy Coils.—a. Economy and compensator coils for arc lamps must be mounted on non-combustible, non-absorptive, insulating supports, such as glass or porcelain, allowing an air space of at least one inch between frame and support, and must in general be treated as sources of heat.

31. Decorative Lighting Systems.—a. Special permission may be given in writing by the Inspection Department having jurisdiction for the temporary installation of *approved* Systems of Decorative Lighting, provided the difference of potential between the wires of any circuit shall not be over 150 volts and also provided that no group of lamps requiring more than 1,320 watts shall be dependent on one cut-out.

No "System of Decorative Lighting" to be allowed under this rule which is not listed in the Supplement to the National Electrical Code containing list of approved fittings.

31 A. Theater Wiring.—(For rules governing Moving Picture Machine, see No. 65 A.)

All wiring, apparatus, etc., not specifically covered by special rules herein

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given must conform to the Standard Rules and Requirements of the National Electrical Code.

In so far as these Rules and Requirements are concerned, the term "theater" shall mean a building or part of a building in which it is designed to make a presentation of dramatic, operatic or other performances or shows for the entertainment of spectators which is capable of seating at least four hundred persons, and which has a stage for such performances that can be used for scenery and other stage appliances.

a. SERVICES.—1. Where source of supply is outside of building, there must be at least two separate and distinct services where practicable, fed from separate street mains, one service to be of sufficient capacity to supply current for the entire equipment of theater, while the other service must be at least of sufficient capacity to supply current for all emergency lights.

By "emergency lights" are meant exit lights and all lights in lobbies, stairways, corridors and other portions of theater to which the public have excess which are normally kept lighted during the performance.

2. Where source of supply is an isolated plant within same building, an auxiliary service of at least sufficient capacity to supply all emergency lights must be installed from some outside source, or a suitable storage battery within the premises may be considered the equivalent of such service.

b. STAGE.—1. All permanent construction on stage side of proscenium wall must be *approved* conduit, with the exception of border and switchboard wiring.

2. *Switchboards.*—Must be made of non-combustible, non-absorptive material, and where accessible from stage level must be protected by an *approved* guard rail to prevent accidental contact with live parts on the board.

3. *Footlights.*—*a.* Must be wired in *approved* conduit, each lamp receptacle being enclosed within an *approved* outlet box, the whole to be enclosed in a steel trough, metal to be of a thickness not less than No. 20 gage, or each lamp receptacle may be mounted on or in an iron or steel box so constructed as to enclose all the wires and live parts of receptacles.

b. Must be so wired that no set of lamps requiring more than 1,320 watts will be dependent on one cut-out.

4. *Borders.*—*a.* Must be constructed of steel of a thickness not less than No. 20 gage, treated to prevent oxidization, be suitably stayed and supported by a metal framework, and so designed that flanges of reflectors will protect lamps.

b. Must be so wired that no set of lamps requiring more than 1,320 watts will be dependent upon one cut-out.

c. Must be wired in *approved* conduit, each lamp receptable to be enclosed within an *approved* outlet box, the whole to be enclosed in a steel trough, or each lamp receptacle may be mounted on or in the cover of a steel box so constructed as to enclose all the wires and the live parts of receptacles, metal to be of a thickness not less than No. 20 gage.

d. Must be provided with suitable guards to prevent scenery or other combustible material coming in contact with lamps.

e. Cables must be continuous from stage switchboard to border; conduit construction must be used from switchboard to point where cables must be flexible to permit of the raising and lowering of border, and flexible portion must be enclosed in an *approved* fireproof hose or braid and be suitably supported.

Junction boxes will be allowed on fly floor and rigging loft in existing theaters where the wiring has been completed and approved by Inspection Department having jurisdiction.

f. For the wiring of the border proper, wire with slow burning insulation should be used.

g. Must be suspended with wire rope, same to be insulated from border by at least two *approved* strain insulators properly inserted.

5. *Stage Pockets.*—Must be of approved type controlled from switchboard, each receptacle to be of not less than fifty amperes rating, and each receptacle to be wired with a separate circuit to its full capacity.

6. *Proscenium Side Lights.*—Must be so installed that they cannot interfere with the operation of or come in contact with curtain.

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7. *Scene Docks.*—Where lamps are installed in Scene Docks, they must be so located and installed that they will not be liable to mechanical injury.

8. *Curtain Motors,*—Must be of ironclad type and installed so as to conform to the requirements of the National Electrical Code. (See No. 8.)

9. *Control for Stage Flues*:—

a. In cases where dampers are released by an electric device, the electric circuit operating same must be normally closed.

b. Magnet operating damper must be wound to take full voltage of circuit by which it is supplied, using no resistance device, and must not heat more than normal for apparatus of similar construction. It must be located in loft above scenery, and be installed in a suitable iron box with a tight self-closing door.

c. Such dampers must be controlled by at least two standard single pole switches mounted within *approved* iron boxes provided with self-closing doors without lock or latch, and located, one at the Electrician's station, and others as designated by the Inspection Department having jurisdiction.

c. Dressing Rooms.

1. Must be wired in *approved* conduit, except that in existing buildings where it is impracticable to install *approved* conduit, *approved* armored cable may be used, provided it is installed in accordance with No. 24 A.

2. All pendant lights must be equipped with *approved* reinforced cord or cable.

3. All lamps must be provided with *approved* guards.

d. Portable Equipments.

1. *Arc lamps* used for stage effects must conform to the following requirements:—

a. Must be constructed entirely of metal except where the use of *approved* insulating material is necessary.

b. Must be substantially constructed, and so designed as to provide for proper ventilation, and to prevent sparks being emitted from lamps when same is in operation, and mica must be used for frame insulation.

c. Front opening must be provided with a self-closing hinged door frame in which wire gauze or glass must be inserted, excepting lens lamps, where the front may be stationary, and solid door be provided on back or side.

d. Must be provided with a one-sixteenth-inch iron or steel guard having a mesh not larger than one inch, and be substantially placed over top and upper half of sides and back of lamp frame; this guard to be substantially riveted to frame of lamp, and to be placed at a distance of at least two inches from the lamp frame.

e. Switch on standard must be so constructed that accidental contact with any live portion of same will be impossible.

f. All stranded connections in lamp and at switch and rheostat must be provided with approved lugs.

g. Rheostat, if mounted on standard, must be raised to a height of at least three inches above floor line, and in addition to being properly enclosed must be surrounded with a substantially attached metal guard having a mesh not larger than one square inch, which guard is to be kept at least one inch from outside frame of rheostat.

h. A competent operator must be in charge of each arc lamp, except that one operator may have charge of two lamps when they are not more than ten feet apart, and are so located that he can properly watch and care for both lamps.

2. *Bunches*:—*a.* Must be substantially constructed of metal, and must not contain any exposed wiring.

b. The cable feeding same must be bushed in an approved manner where passing through the metal, and must be properly secured to prevent any mechanical strain from coming on the connection.

3. *Strips.*—*a.* Must be constructed of steel of a thickness not less than No. 20 gage, treated to prevent oxidization, and suitably stayed and supported by metal framework.

b. Cable feeding must be bushed in an *approved* manner where passing through the metal, and must be properly secured to prevent any mechanical strain from coming on the connections.

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4. *Portable Plugging Boxes.*—Must be constructed so that no current carrying part will be exposed, and each receptacle must be protected by *approved* fuses mounted on slate or marble bases and enclosed in a fireproof cabinet equipped with self-closing doors. Each receptacle must be constructed to carry thirty amperes without undue heating, and the bus-bars must have a carrying capacity equivalent to the current required for the total number of receptacles, allowing thirty amperes to each receptacle, and *approved* lugs must be provided for the connection of the master cable.

5. *Pin Plug Conductors—a.* When of *approved* type may be used to connect *approved* portable lights and appliances.

b. Must be so installed that the "female" part of plug will be on the live end of cable, and must be so constructed that tension on the cable will not cause any serious mechanical strain on the connections.

6. *Lights on Scenery.*—Where brackets are used they must be wired entirely on the inside, fixture stem must come through to the back of the scenery and end of stem be properly bushed.

7. *String or Festoon Lights.*—Wiring for same should be *approved* cable, joints where taps are taken from same for lights to be properly made, soldered and taped, and where lamps are used in lanterns or similar devices lamps must be provided with *approved* guards. Where taps are taken from cable, they should be so staggered that joints of different polarity will not come immediately opposite each other and must be properly protected from strain.

8. *Special Electrical Effects.*—Where devices are used for producing special effects such as lightning, waterfalls, etc., the apparatus must be so constructed and located that flames, sparks, etc., resulting from the operation cannot come in contact with combustible material.

e. AUDITORIUM.—1. All wiring must be installed in *approved* conduit, except that in existing buildings where it is impracticable to install *approved* conduit, *approved* armored cable may be used, provided it is installed in accordance with No. 24 A.

2. All fuses used in connection with lights illuminating all parts of the house used by the audience must be installed in fireproof enclosures so constructed that there will be a space of at least six inches between the fuses and the sides and face of enclosure.

3. Exit lights must not have more than one set of fuses between same and service fuses.

4. Exit lights and all lights in halls, corridors or any other part of the building used by the audience, except the general auditorium lighting, must be fed independently of the stage lighting, and must be controlled only from the lobby or other convenient place in front of the house.

5. Every portion of the theater devoted to the use or accommodation of the public, also all outlets leading to the streets and including all open courts, corridors, stairways, exits and emergency exit stairways, should be well and properly lighted during every performance, and the same should remain lighted until the entire audience has left the premises.

32. Car Wiring and Equipment of Cars.—a. *Protection of Car Body, etc.*—1. Under side of car bodies to be protected by *approved* fire-resisting, insulating material, not less than ⅛-inch in thickness, or by sheet iron or steel, not less than .04-inch in thickness, as specified in Section a, 2, 3 and 4. This protection to be provided over all electrical apparatus, such as motors with a capacity of over 75 H. P. each, resistances, contactors, lightning arresters, air-brake motors, etc., and also where wires are run, except that protection may be omitted over wires designed to carry 25 amperes or less if they are encased in metal conduit.

2. At motors of over 75 H. P. each, fire-resisting material or sheet iron or steel to extend not less than 8 inches beyond all edges of openings in motors, and not less than 6 inches beyond motor leads on all sides.

3. Over resistances, contactors, and lightning arresters, and other electrical apparatus, excepting when amply protected by their casing, fire-resisting material or sheet iron or steel to extend not less than 8 inches beyond all edges of the devices.

4. Over conductors, not encased in conduit, and conductors in conduit

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when designed to carry over 25 amperes, unless the conduit is so supported as to give not less than ½-inch clear air space between the conduit and the car, fire-resisting material or sheet iron or steel to extend at least 6 inches beyond conductors on either side.

The fire-resisting insulating material or sheet iron or steel may be omitted over cables made up of flameproof braided outer covering when surrounded by ½-inch flameproof covering, as called for by Section 1, 4.

5. In all cases fireproof material or sheet iron or steel to have joints well fitted, to be securely fastened to the sills, floor timbers and cross braces and to have the whole surface treated with a waterproof paint.

6. Cut-out and switch cabinets to be substantially made of hard wood. The entire inside of cabinet to be lined with not less than ½-inch fire-resisting insulating material which shall be securely fastened to the woodwork, and after the fire-resisting material is in place the inside of the cabinet shall be treated with a waterproof paint.

b. *Wires, Cables, etc.*—1. All conductors to be stranded, the allowable carrying capacity being determined by Table "A" of No. 16, except that motor, trolley and resistance leads shall not be less than No. 7 B. & S. gage, heater circuits not less than No. 12 B. & S. gage, and lighting and other auxiliary circuits not less than No. 14 B & S. gage.

The current used in determining the size of motor, trolley and resistance leads shall be the per cent of the full load current, based on one hour's run of motor, as given by the following table:—

Size of each Motor.	Motor Leads.	Trolley Leads.	Resistance Leads.
75 H. P. or less.....	50%	40%	15%
Over 75 H. P.....	45%	35%	15%

Fixture wire complying with No. 46 will be permitted for wiring *approved* cluster.

2. To have an insulation and braid as called for by No. 41 for wires carrying currents of the same potential.

3. When run in metal conduit, to be protected by an additional braid as called for by No. 47.

Where conductors are laid in conduit, not being drawn through, the additional braid will not be required

4. When not in conduit, in *approved* moulding, or in cables surrounded by ½-inch flameproof covering, must comply with the requirements of No. 41 (except that tape may be substituted for braid) and be protected by an additional flameproof braid, at least 1-32 of an inch in thickness, the outside being saturated with a preservative flameproof compound.

This rule will be interpreted to include the leads from the motors.

5. Must be so spliced or joined as to be both mechanically and electrically secure without solder. The joints must then be soldered and covered with an insulation equal to that on the conductors.

Joints made with *approved* splicing devices and those connecting the leads at motors, plows or third-rail shoes need not be soldered.

6. All connections of cables to cut-outs, switches and fittings, except those to controller connection boards, when designed to carry over 25 amperes, must be provided with lugs or terminals soldered to the cable, and securely fastened to the device, by bolts, screws or by clamping; or, the end of the cable, after the insulation is removed, shall be dipped in solder and be fastened into the device by at least two set screws having check nuts.

All connections for conductors to fittings, etc., designed to carry less than 25 amperes, must be provided with up-turned lugs that will grip the conductor between the screw and the lug, the screws being provided with flat washers; or by block terminals having two set screws, and the end of the conductors must be dipped in solder. Soldering, in addition to the connection of the binding screws, is strongly recommended, and will be insisted on when above requirements are not complied with.

This rule will not be construed to apply to circuits where the maximum potential is not over 25 volts, and current does not exceed 5 amperes.

c. *Cut-outs, Circuit-Breakers and Switches.*—1. All cut-outs and switches having exposed live metal parts to be located in cabinets. Cut-outs and switches, not in iron boxes or in cabinets, shall be mounted on not less than ½-inch fire-resisting insulating material, which shall project at least ½-inch beyond all sides of the cut-out or switch.

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2. Cut-outs to be of the *approved* cartridge or *approved* blow-out type.

3. All switches controlling circuits of over 5 ampere capacity shall be of *approved* single-pole, quick-break or *approved* magnetic blow-out type. Switches controlling circuits of 5 ampere or less capacity may be of the *approved* single-pole, double-break, snap type.

4. Circuit breakers to be of *approved* type.

5. Circuits must not be fused above their safe carrying capacity.

6. A cut-out must be placed as near as possible to the current collector, so that the opening of the fuse in this cut-out will cut off all current from the car.

When cars are operated by metallic return circuits, the circuit breakers connected to both sides of the circuit, no fuses in addition to the circuit breakers will be required.

d. *Conduit.*—[When from the nature of the case, or on account of the size of the conductors, the ordinary pipe and junction box construction is not permissible, a special form of conduit system may be used, provided the general requirements as given below are complied with.]

1. Metal conduits, outlet and junction boxes to be constructed in accordance with Nos. 49 and 49 A, except that conduit for lighting circuits need not be over 5/16 inch internal diameter and ½-inch external diameter, and for heating and air motor circuits need not be over ⅜-inch internal diameter and 9/16-inch external diameter, and all conduits where exposed to dampness must be water tight.

2. Must be continuous between and be firmly secured into all outlet or junction boxes and fittings, making a thorough mechanical and electrical connection between same.

3. Metal conduits, where they enter all outlet or junction boxes and fittings, must be provided with *approved* bushings fitted so as to protect cables from abrasion.

4. Except as noted in Section 1, 2, must have the metal of the conduit permanently and effectively grounded.

5. Junction and outlet boxes must be installed in such a manner as to be accessible.

6. All conduits, outlets or junction boxes and fittings to be firmly and substantially fastened to the framework of the car.

e. *Moulding.*—1. To consist of a backing and a capping and to be constructed of fire-resisting insulating material, except that it may be made of hard wood where the circuits which it is designed to support are normally not exposed to moisture.

2. When constructed of fire-resisting insulating material, the backing shall not be less than ¼-inch in thickness and be of a width sufficient to extend not less than 1 inch beyond conductors at sides.

The capping, to be not less than ⅛-inch in thickness shall cover and extend at least ⅜-inch beyond conductors on either side.

The joints in the moulding shall be mitered to fit close, the whole material being firmly secured in place by screws or nails, and treated on the inside and outside with a waterproof paint.

When fire-resisting moulding is used over surfaces already protected by ¼-inch fire-resisting insulating material no backing will be required.

3. Wooden mouldings must be so constructed as to thoroughly encase the wire and provide a thickness of not less than ⅜-inch at the sides and back of the conductors, the capping being not less than 3/16-inch in thickness. Must have both outside and inside two coats of waterproof paint.

The backing and the capping shall be secured in place by screws.

f. *Lighting and Lighting Circuits.*—1. Each outlet to be provided with an *approved* porcelain receptacle, or an *approved* cluster. No lamp of over 32 candle power to be used.

2. Circuits to be run in *approved* metal conduit, or *approved* moulding.

3. When metal conduit is used, except for sign lights, all outlets to be provided with *approved* outlet boxes.

4. At outlet boxes, except where *approved* clusters are used, porcelain receptacles to be fastened to the inside of the box, and the metal cover to have an insulating bushing around opening for the lamp.

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When *approved* clusters are used, the cluster shall be thoroughly insulated from the metal conduit, being mounted on a block of hard wood or fire-resisting insulating material.

5. Where conductors are run in moulding the porcelain receptacles or cluster to be mounted on blocks of hard wood or of fireproof insulating material.

g. *Heaters and Heating Circuits.*—1. Heaters to be of *approved* type.

2. Panel heaters to be so constructed and located that when heaters are in place all current-carrying parts will be at least 4 inches from all woodwork.

Heaters for cross seats to be so located that current-carrying parts will be at least 6 inches below under side of seat, unless under side of seat is protected by not less than ¼-inch fire-resisting insulating material, of .04 inch sheet metal with 1 inch air space over same, when the distance may be reduced to 3 inches.

2. Circuits to be run in *approved* metal conduit, or in *approved* moulding, or if the location of conductors is such as will permit an air space of not less than 2 inches on all sides except from the surface wired over, they may be supported on porcelain knobs or cleats, provided the knobs or cleats are mounted on not less than ¼-inch fire-resisting insulating material extending at least 3 inches beyond conductors at either side, the supports raising the conductors not less than ½-inch from the surface wired over, and being not over 12 inches apart.

h. *Air Pump Motor and Circuits.*—1. Circuits to be run in *approved* metal conduit or in *approved* moulding, except that when run below the floor of the car they may be supported on porcelain knobs or cleats, provided the supports raise the conductor at least ½-inch from the surface wired over and are not over 12 inches apart.

2. Automatic control to be enclosed in *approved* metal box. Air pump and motor, when enclosed, to be in *approved* metal box or a wooden box lined with metal of not less than 1/32 inch in thickness.

When conductors are run in metal conduit the boxes surrounding automatic control and air pump and motor may serve as outlet boxes.

i. *Main Motor Circuits and Devices.*—1. Conductors connecting between trolley stand and main cut-out or circuit breakers in hood to be protected where wires enter car to prevent ingress of moisture.

2. Conductors connecting between third rail shoes on same truck, to be supported in an *approved* fire-resisting insulating moulding, or in *approved* iron conduit supported by soft rubber or other *approved* insulating cleats.

3. Conductors on the under side of the car, except as noted in Section i, 4, to be supported in accordance with one of the following methods:—

a. To be run in *approved* metal conduit, junction boxes being provided where branches in conduit are made, and outlet boxes where conductors leave conduit.

b. To be run in *approved* fire-resisting insulating moulding.

c. To be supported by insulating cleats, the supports being not over 12 inches apart.

4. Conductors with flameproof braided outer covering, connecting between controllers at either end of car, or controllers and contactors, may be run as a cable, provided the cable where exposed to the weather is encased in a canvas hose or canvas tape, thoroughly taped or sewed at ends and where taps from the cable are made, and the hose or tape enters the controllers.

Conductors with or without flameproof braided outer covering connecting between controllers at either end of the car, or controllers and contactors, may be run as a cable, provided the cable throughout its entire length is surrounded by ½-inch flameproof covering, thoroughly taped or sewed at ends, or where taps from cable are made, and the flameproof covering enters the controllers.

Cables, where run below floor of car, may be supported by *approved* insulating straps or cleats. Where run above floor of car, to be in a metal conduit or wooden box painted on the inside with not less than two coats of flameproof paint, and where this box is so placed that it is exposed to water, as by washing of the car floor, attention should be given to making the box reasonably waterproof.

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Canvas hose or tape, or flameproof material surrounding cables after conductors are in same, to have not less than two coats of waterproof insulating material.

5. Motors to be so drilled that, on double truck cars, connecting cables can leave motor on side nearest to kingbolt.

6. Resistances to be so located that there will be at least 6-inch air space between resistances proper and fire-resisting material of the car. To be mounted on iron supports, being insulated by non-combustible bushings or washers, or the iron supports shall have at least 2 inches of insulating surface between them and metal work of car, or the resistances may be mounted on hard wood bars, supported by iron stirrups, which shall have not less than 2 inches of insulating surface between foot of resistance and metal stirrup, the entire surface of the bar being covered with at least ¼-inch fire-resisting insulating material.

The insulation of the conductor, for about 6 inches from terminal of the resistance, should be replaced, if any insulation is necessary, by a porcelain bushing or asbsetos sleeve.

7. Controllers to be raised above platform of car by not less than 1-inch hard wood block, the block being fitted and painted to prevent moisture working in between it and the platform.

j. *Lightning Arresters.*—1. To be preferably located to protect all auxiliary circuits in addition to main motor circuits.

2. The ground conductor shall be not less than No. 6 B. & S. gage, run with as few kinks and bends as possible, and be securely grounded.

k. *General Rules.*—1. When passing through floors, conductors or cables must be protected by *approved* insulating bushings, which shall fit the conductor or cable as closely as possible.

2. Moulding should never be concealed except where readily accessible. Conductors should never be tacked into moulding.

3. Short bends in conductors should be avoided where possible.

4. Sharp edges in conduit or in moulding must be smoothed to prevent injury to conductors.

33. **Car Houses.**—a. The trolley wires must be securely supported on insulating hangers.

b. The trolley hangers must be placed at such a distance apart that, in case of a break in the trolley wire, contact with the floor cannot be made.

c. Must have an emergency cut-out switch located at a proper place outside of the building, so that all the trolley wires in the building may be cut out at one point, and line insulators must be installed, so that when this emergency switch is open, the trolley wire will be dead at all points within 100 feet of the building. The current must be cut out of the building when not needed for use in the building.

This may be done by the emergency switch, or if preferred, a second switch may be used that will cut out all current from the building, but which need not cut out the trolley wire outside as would be the case with the emergency switch.

d. All lamps and stationary motors must be installed in such a way that one main switch may control the whole of each installation, lighting and power, independently of the main cut-out switch called for in Section c.

e. Where current for lighting and stationary motors is from a grounded trolley circuit, the following special rules to apply:—

1. Cut-outs must be placed between the non-grounded side and lights or motors they are to protect. No set or group of incandescent lamps requiring over 2,000 watts must be dependent upon one cut-out.
2. Switches must be placed between non-grounded side and lights and motors they are to protect.
3. Must have all rails bonded at each joint with a conductor having a carrying capacity at least equivalent to No. 00 B. & S. gage annealed copper wire, and all rails must be connected to the outside ground return circuit by not less than No. 00 B. & S. gage copper wire or by equivalent bonding through the track. All

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lighting and stationary motor circuits must be thoroughly and permanently connected to the rails or to the wire leading to the outside ground return circuit.

f. All pendant cords and portable conductors will be considered as subject to hard usage (see 45, f).

g. Must, except as provided in Section e, have all wiring and apparatus installed in accordance with the rules for constant potential systems.

h. Must not have any system of feeder distribution centering in the building.

i. Cars must not be left with the trolley in electrical connection with the trolley wire.

34. **Lighting and Power from Railway Wires.**—a. *Must not be permitted, under any pretense, in the same circuit with trolley wires with a ground return, except in electric railway cars, electric car houses and their power stations; nor shall the same dynamo be used for both purposes.*

CONSTANT-HIGH-POTENTIAL SYSTEMS.

550 TO 3,500 VOLTS.

Any circuit attached to any machine or combination of machines which develops a difference of potential between any two wires, of over 550 volts and less than 3,500 volts, shall be considered as a high-potential circuit, and as coming under that class, unless an approved transforming device is used, which cuts the difference of potential down to 550 volts or less.

(See note following first paragraph under Low-Potential systems, page 1408.)

35. **Wires.**—(See also Nos. 14, 15 and 16.) a. Must have an *approved* rubber-insulating covering (see No. 41).

b. Must be always in plain sight and never encased, except as provided for in No. 8 b, or where required by the Inspection Department having jurisdiction.

c. Must (except as provided for in No. 8 b), be rigidly supported on glass or porcelain insulators, which raise the wire at least 1 inch from the surface wired over, and must be kept about 8 inches apart.

Rigid supporting requires under ordinary conditions, where wiring along flat surfaces, supports at least about every 4½ feet. If the wires are unusually liable to be disturbed, the distance between supports should be shortened.

In buildings of mill construction, mains of not less than No. 8 B. & S. gage, where not liable to be disturbed, may be separated about ten inches and run from timber to timber, not breaking around, and may be supported at each timber only.

d. Must be protected on side walls from mechanical injury by a substantial boxing, retaining an air space of 1 inch around the conductors, closed at the top (the wires passing through bushed holes) and extending not less than 7 feet from the floor. When crossing floor timbers, in cellars, or in rooms where they might be exposed to injury, wires must be attached by their insulating supports to the under side of a wooden strip not less than ½-inch in thickness.

For general suggestions on protection, see note under No. 24 e. See also note under No. 18 e.

36. **Transformers.**—(When permitted inside buildings under No. 12.) (For construction rules see No. 62.) (See also Nos. 13 and 19 *A*.)

Transformers must not be placed inside of buildings without special permission from the Inspection Department having jurisdiction.

a. Must be located as near as possible to the point at which the primary wires enter the building.

b. Must be placed in an enclosure constructed of fire-resisting material; the enclosure to be used only for this purpose, and to be kept securely locked, and access to the same allowed only to responsible parties.

c. Must be thoroughly insulated from the ground, or permanently and effectually grounded, and the enclosure in which they are placed must be practically air-tight, except that it must be thoroughly ventilated to the outdoor air, if possible through a chimney or flue. There should be at least 6 inches air space on all sides of the transformer.

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37 Series Lamps.—a. No multiple-series or series-multiple system of lighting will be approved.

b. Must not, under any circumstances, be attached to gas fixtures.

CONSTANT EXTRA-HIGH-POTENTIAL SYSTEMS.

OVER 3,500 VOLTS.

Any circuit attached to any machine or combination of machines which develops a difference of potential, between any two wires, of over 3,500 volts, shall be considered as an extra-high-potential circuit, and as coming under that class, unless an approved transforming device is used, which cuts the difference of potential down to 3,500 volts or less.

38. Primary Wires.—a. Must not be brought into or over buildings, except power stations and sub-stations.

39. Secondary Wires.—a. Must be installed under rules for high-potential systems when their immediate primary wires carry a current at a potential of over 3,500 volts, unless the primary wires are installed in accordance with the requirements as given in No. 12 A or are entirely underground, within city, town and village limits.

Class D.—FITTINGS, MATERIALS AND DETAILS OF CONSTRUCTION.

(LIGHT, POWER AND HEAT. FOR SIGNALING SYSTEMS, SEE CLASS E.)

ALL SYSTEMS AND VOLTAGES.

The following rules are but a partial outline of requirements. Devices or materials which fulfill the conditions of these requirements and no more, will not necessarily be acceptable. All fittings and materials should be submitted for examination and test before being introduced for use.

INSULATED WIRES—RULES 40 TO 43.

40. General Rules.—a. Copper for insulated solid conductors of No. 4 B. & S. gage and smaller must not vary in diameter more than .002 of an inch from the standard. On solid sizes larger than No. 4 B. & S. gage the diameter shall not vary more than one per cent from the specified standard. The conductivity of solid conductors shall not be less than 97% of that of pure copper of the specified size.

In all stranded conductors the sum of the circular mils of the individual wires, shall not be less than the normal circular mils of the strand by more than one and one-half per cent The conductivity of the individual wires in a strand shall not be less than is given in the following table:—

Number.	Per cent.	Number.	Per cent.
14 and larger.	97.0	23	95.2
15	96.8	24	95.0
16	96.6	25	94.8
17	96.4	26	94.6
18	96.2	27	94.4
19	96.0	28	94.2
20	95.8	29	94.0
21	95.6	30	93.8
22	95.4		

The Standard for diameters and mileages shall be that adopted by the American Institute of Electrical Engineers.

b. Wires and cables of all kinds designed to meet the following specifications must have a distinctive marking the entire length of the coil so that they may be readily identified in the field. They must also be plainly tagged or marked as follows:—

1. The maximum voltage at which the wire is designed to be used.
2. The words "National Electrical Code Standard."

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3. Name of the manufacturing company and, if desired, trade name of the wire.
4. Month and year when manufactured.

Wires described under Nos. 42, 43 and 44 need not have the distinctive marking, but are to be tagged.

41. **Rubber-Covered Wire.—a.** Copper for conductors must be thoroughly tinned.

Insulation for Voltages, 0 to 600 inclusive.—**b.** Must be of rubber or other approved substances, homogeneous in character, adhering to the conductor and of a thickness not less than that given in the following table:—

B. & S. Gage.	Thickness.
18 to 16	1-32 inch.
15 to 8	3-64 "
7 to 2	1-16 "
1 to 0000	5-64 "
Circular Mils.	
250,000 to 500,000	3-32 "
500,000 to 1,000,000	7-64 "
Over 1,000,000	1-8 "

Measurements of insulating wall are to be made at the thinnest portion of the dielectric.

c. The completed coverings must show an insulation resistance of at least 100 megohms per mile during thirty days' immersion in water at 70° Fahrenheit (21° Centigrade).

d. Each foot of the completed covering must show a dielectric strength sufficient to resist throughout five minutes the application of an electromotive force proportionate to the thickness of insulation in accordance with the following table:—

Thickness in 64ths inches.	Breakdown Test on 1 foot.	Thickness in 64ths inches.	Breakdown Test on 1 foot.
1	3,000 Volts A. C.	7	16,500 Volts A. C.
2	6,000 " "	8	18,000 " "
3	9,000 " "	10	21,000 " "
4	11,000 " "	12	23,500 " "
5	13,000 " "	14	26,000 " "
6	15,000 " "	16	28,000 " "

The source of alternating electro-motive force shall be a transformer of at least one kilowatt capacity. The application of the electro-motive force shall first be made at 4,000 volts for five minutes, and then the voltage increased by steps of not over 3,000 volts, each held for five minutes, until the rupture of the insulation occurs. The tests for dielectric strength shall be made on a sample of wire which has been immersed in water for seventy-two hours. One foot of the wire under test is to be submerged in a conducting liquid held in a metal trough, one of the transformer terminals being connected to the copper of the wire and the other to the metal of the trough.

Insulations for Voltages, 601 to 3,500 inclusive.—**e.** The thickness of the insulating wall must not be less than that given in the following table:—

B. & S. Gage.	Thickness.	
14 to 1	3-32 inch.	
0 to 0000	3-32 "	covered by tape or braid.
Circular Mils.		
250,000 to 500,000	3-32 "	" " "
Over 500,000	1-8 "	" " "

f. The requirements as to insulation and breakdown resistance for wires for low-potential systems shall apply, with the exception that an insulation resistance of not less than 300 megohms per mile shall be required.

Insulations for Voltages Over 3,500.—**g.** Wire for arc light circuits exceeding 3,500 volts potential must have an insulating wall not less than $\frac{1}{16}$ of an inch in thickness, and shall withstand a breakdown test of at least 23,500 volts, and have an insulation of at least 500 megohms per mile.

The tests on this wire to be made under the same conditions as for low-potential wires.

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Specifications for insulations for alternating currents exceeding 3,500 volts have been considered, but on account of the somewhat complex conditions of such work, it has so far been deemed inexpedient to specify general insulations for this use.

General.—h. The rubber compound or other approved substance used as insulation must be sufficiently elastic to permit all wires smaller than No. 7 B. & S. gage and larger than No. 11 B. & S gage to be bent without injury to the insulation around a cylinder twice the diameter of the insulated wire measured over the outer covering. All wires No. 11 B. & S. gage and smaller to be bent without injury to the insulation around a cylinder equal to the diameter of the insulated wire measured over the outer covering.

i. All the above insulations must be protected by a substantial braided covering properly saturated with a preservative compound. This covering must be sufficiently strong to withstand all the abrasions likely to be met with in practice, and must substantially conform to approved samples submitted by the manufacturer.

42. Slow-burning Weatherproof Wire.—[This wire is not as burnable as "weatherproof" nor as subject to softening under heat. It is not suitable for outside work.]

a. The insulation must consist of two coatings, one to be fireproof in character and the other to be weatherproof. The fireproof coating must be on the outside and must comprise about 6/10 of the total thickness of the wall. The completed covering must be of a thickness not less than that given in the following table:—

B. & S. Gage.	Thickness.
14 to 8	3-64 inch.
7 to 2	1-16 "
1 to 0000	5-64 "
Circular Mils.	
250,000 to 500,000	3-32 "
500,000 to 1,000,000	7-64 "
Over 1,000,000	1-8 "

Measurements of insulating wall are to be made at the thinnest portion.

b. The fireproof coating shall be of the same kind as that required for "slow-burning wire," and must be finished with a hard, smooth surface.

c. The weatherproof coating shall consist of a stout braid, applied and treated as required for "weatherproof wire."

43. Slow-burning Wire.—a. The insulation must consist of three braids of cotton or other thread, all the interstices of which must be filled with the fireproofing compound or with material having equivalent resisting and insulating properties. The outer braid must be specially designed to withstand abrasion, and its surface must be finished smooth and hard. The completed covering must be of a thickness not less than that given in the table under No. 42 a.

The solid constituent of the fireproofing compound must not be susceptible to moisture, and must not burn even when ground in an oxidizable oil, making a compound, which, while proof against fire and moisture, at the same time has considerable elasticity, and which when dry will suffer no change at a temperature of 250° Fahrenheit (121° Centigrade), and which will not burn at even a higher temperature.

This is practically the old so-called "underwriters" insulation. It is especially useful in hot, dry places where ordinary insulations would perish, and where wires are bunched, as on the back of a large switchboard or in a wire tower, so that the accumulation of rubber insulation would result in an objectionably large mass of highly inflammable material.

44 **Weatherproof Wire.**—a. The insulating covering shall consist of at least three braids, all of which must be thoroughly saturated with a dense moisture-proof compound, applied in such a manner as to drive any atmospheric moisture from the cotton braiding, thereby securing a covering to a great degree waterproof and of high insulating power. This compound must retain its elasticity at 0° Fahrenheit (minus 18° Centigrade), and must not drip at 160° Fahrenheit (71° Centigrade). The thickness of insulation must not be less than that given in the table under No. 42 a, and the outer surface must be thoroughly slicked down.

This wire is for use outdoors, where moisture is certain and where fireproof qualities are not necessary.

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45. Flexible Cord.—(For installation rules, see No. 28). **a.** Must, except as required for portable heating apparatus (see Section g), be made of stranded copper conductors, each strand to be not larger than No. 26 or smaller than No. 30 B. & S. gage, and each stranded conductor must be covered by an *approved* insulation and protected from mechanical injury by a tough, braided outer covering.

For Pendant Lamps.—[In this class is to be included all flexible cord which, under usual conditions, hangs freely in air, and which is not likely to be moved sufficiently to come in contract with surrounding objects.

It should be noted that pendant lamps provided with long cords, so that they can be carried about or hung over nails, or on machinery, etc., are not included in this class, even though they are usually allowed to hang freely in air.]

b. Each stranded conductor must have a carrying capacity equivalent to not less than a No. 18 B. & S. gage wire.

c. The covering of each stranded conductor must be made up as follows:—

1. A tight, close wind of fine cotton.
2. The insulation proper, which shall be waterproof.
3. An outer cover of silk or cotton.

The wind of cotton tends to prevent a broken strand puncturing the insulation and causing a short circuit. It also keeps the rubber from corroding the copper.

d. The insulation must be solid, at least 1/32 of an inch thick, and must show an insulation resistance of 50 megohms per mile throughout two weeks immersion in water at 70° Fahrenheit (21° Centigrade), and stand the tests prescribed for low-tension wires as far as they apply.

e. The outer protecting braiding should be so put on and sealed in place that when cut it will not fray out, and where cotton is used it should be impregnated with a flameproof paint which will not have an injurious effect on the insulation.

For Portables.—[In this class is included all cord used on portable lamps, small portable motors, or any device which is liable to be carried about.]

f. Flexible cord for portable use, except in offices, dwellings or similar places, where cord is not liable to rough usage and where appearance is an essential feature, must meet all the requirements for flexible cord for "pendant lamps," both as to construction and thickness of insulation, and in addition must have a tough, braided cover over the whole. There must also be an extra layer of rubber between the outer cover and flexible cord, and in moist places the outer cover must be saturated with a moisture-proof compound, thoroughly slicked down, as required for "weatherproof wire." (See No. 44.) In offices, dwellings, or in similar places where cord is not liable to rough usage and where appearance is an essential feature, flexible cord for portable use must meet all of the requirements for flexible cord for "pendant lamps," both as to construction and thickness of insulation, and in addition must have a tough, braided cover over the whole, or, providing there is an extra layer of rubber between the flexible cord and the outer cover, the insulation proper on each stranded conductor of cord may be 1/64 of an inch in thickness instead of 1/32 of an inch as required for pendant cords

Flexible cord for portable use may, instead of the outer coverings described above, have an approved metal flexible armor.

For Portable Heating Apparatus.—[Applies to all smoothing and sad irons, and to any other device requiring over 250 watts.]

g. Must be made up as follows:—

1. Conductors must be of braided copper, each strand not to be larger than No. 30 or smaller than No. 36 B. & S. gage.

When conductors have a greater carrying capacity than No. 12 B. & S. gage they may be braided or stranded with each strand as large as No. 28 B. & S. gage. If stranded, there must be a tight, close wind of cotton between the conductor and the insulation.

2. An insulating covering of rubber or other *approved* material not less than 1/64 inch in thickness.
3. A braided covering of not less than 1/32 inch thick, composed of best quality long fiber asbestos, containing not over five per cent vegetable fiber.

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4. The several conductors comprising the cord to be enclosed by an outer reinforcing covering not less than 1/64 inch thick, especially designed to resist abrasion, and so treated as to prevent the cover from fraying.

46. Fixture Wire.—(For installation rules, see No. 24, v to y.) **a.** May be made of solid or stranded conductors, with no strands smaller than No. 30 B. & S. gage, and must have a carrying capacity not less than that of a No 18 B. & S. gage wire.

b. Solid conductors must be thoroughly tinned. If a stranded conductor is used, it must be covered by a tight, close wind of fine cotton.

c. Must have a solid rubber insulation of a thickness not less than 1/32 of an inch for Nos. 18 to 16 B. & S. gage, and 3/64 of an inch for Nos. 14 to 8 B. & S. gage, except that in arms of fixtures not exceeding 24 inches in length and used to supply not more than one 16-candle-power lamp or its equivalent, which are so constructed as to render impracticable the use of a wire with 1/32 of an inch in thickness of rubber insulation, a thickness of 1/64 of an inch will be permitted.

d. Must be protected with a covering at least 1/64 of an inch in thickness, sufficiently tenacious to withstand the abrasion of being pulled into the fixture, and sufficiently elastic to permit the wire to be bent around a cylinder with twice the diameter of the wire without injury to the braid.

e. Must successfully withstand the tests specified in Nos. 41 c and 41 d.

In wiring certain designs of show-case fixtures, ceiling bulls-eyes and similar appliances in which the wiring is exposed to temperatures in excess of 120° Fahrenheit (49° Centigrade), from the heat of lamps, slow-burning wire may be used (see No 43). All such forms of fixtures must be submitted for examination, test and approval before being introduced for use.

47. Conduit Wire.—(For installation rules, see No. 24 n to p.)

a. Single wire for lined conduits must comply with the requirements of No. 41. For unlined conduits it must comply with the same requirements (except that tape may be substituted for braid) and in addition there must be a second outer fibrous covering, at least 1/32 of an inch in thickness and sufficiently tenacious to withstand the abrasion of being hauled through the metal conduit.

b. For twin or duplex wires in lined conduit, each conductor must comply with the requirements of No. 41 (except that tape may be substituted for braid on the separate conductors), and must have a substantial braid covering the whole. For unlined conduit each conductor must comply with requirements of No. 41 (except that tape may be substituted for braid), and in addition must have a braid covering the whole, at least 1/32 of an inch in thickness and sufficiently tenacious to withstand the abrasion of being hauled through the metal conduit.

c. For concentric wire, the inner conductor must comply with the requirements of No. 41 (except that tape may be substituted for braid), and there must be outside of the outer conductor the same insulation as on the inner, the whole to be covered with a substantial braid, which, for unlined conduits, must be at least 1/32 of an inch in thickness, and sufficiently tenacious to withstand the abrasion of being hauled through the metal conduit.

The braid or tape required around each conductor in duplex, twin and concentric cables is to hold the rubber insulation in place and prevent jamming and flattening. All the braids specified in this rule must be properly saturated with a preservative compound.

48. Armored Cable.—(For installation rules, see No. 24 A). **a.** The armor of such cables must have at least as great strength to resist penetration of nails, etc., as is required for metal conduits (see No. 49 b), and its thickness must not be less than that specified in the following table:—

Nominal Internal Diameter. Inches.	Actual Internal Diameter. Inches.	Actual External Diameter. Inches.	Thickness of Wall. Inches.
1/8	.27	.40	.06
1/4	.36	.54	.08
3/8	.49	.67	.09
1/2	.62	.84	.10

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Nominal Internal Diameter. Inches.	Actual Internal Diameter. Inches.	Actual External Diameter. Inches.	Thickness of Wall. Inches.
¾	.82	1.05	.11
1	1.04	1.31	.13
1¼	1.38	1.66	.14
1½	1.61	1.90	.14
2	2.06	2.37	.15
2½	2.46	2.87	.20
3	3.06	3.50	.21
3½	3.54	4.00	.22
4	4.02	4.50	.23
4½	4.50	5.00	.24
5	5.04	5.56	.25
6	6.06	6.62	.28

An allowance of .02 of an inch for variation in manufacturing and loss of thickness by cleaning will be permitted.

b. The conductors in same, single wire or twin conductors, must have an insulating covering as required by No. 41; if any filler is used to secure a round exterior, it must be impregnated with a moisture repellent, and the whole bunch of conductors and fillers must have a separate exterior covering.

49. Interior Conduits.—(For installation rules, see Nos. 24 n to p and 25.) **a.** Each length of conduit, whether lined or unlined, must have the maker's name or initials stamped in the metal or attached thereto in a satisfactory manner, so that inspectors can readily see the same.

The use of paper stickers or tags cannot be considered satisfactory methods of marking, as they are readily loosened and lost off in the ordinary handling of the conduit.

Metal Conduits with Lining of Insulating Material.—**b.** The metal covering or pipe must be at least as strong as the ordinary commercial forms of gas pipe of the same size, and its thickness must be not less than that of standard gas pipe as specified in the table given in No. 48.

c. Must not be seriously affected externally by burning out a wire inside the tube when the iron pipe is connected to one side of the circuit.

d. Must have the insulating lining firmly secured to the pipe.

e. The insulating lining must not crack or break when a length of the conduit is uniformly bent at temperature of 212° Fahrenheit (100° Centigrade), to an angle of ninety degrees, with a curve having a radius of 15 ins., for pipes of one inch and less, and fifteen times the diameter of pipe for larger sizes.

f. The insulating lining must not soften injuriously at any temperature below 212° Fahrenhiet (100° Centigrade), and must leave water in which it is boiled practically neutral.

g. The insulating lining must be at least 1/32 of an inch in thickness. The materials of which it is composed must be of such a nature as will not have a deteriorating effect on the insulation of the conductor, and be sufficiently tough and tenacious to withstand the abrasion test of drawing long lengths of conductors in and out of same.

h. The insulating lining must not be mechanically weak after three days' immersion in water, and when removed from the pipe entire, must not absorb more than ten per cent of its weight of water during 100 hours of submersion.

i. All elbows or bends must be so made that the conduit or lining of same will not be injured. The radius of the curve of the inner edge of any elbow must not be less than 3½ inches.

Unlined Metal Conduits.—**j.** Pipe sizes to run as follows:—

Trade Size. Inches.	Approximate Internal Diameter. Inches.	Minimum Thickness of Wall. Inches.
½	.62	.100
¾	.82	.105
1	1.04	.125
1¼	1.38	.135

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Trade Size. Inches.	Approximate Internal Diameter. Inches.	Minimum Thickness of Wall. Inches.
1½	1.61	.140
2	2.06	.150
2½	2.46	.200
3	3.06	.210
3½	3.54	.220

At no point (except at screw thread) shall the thickness of wall of finished conduit be less than the minimum specified in last column of above table.

k. Pipe to be thoroughly cleaned to remove all scale. Pipe should be of sufficiently true circular section to admit of cutting true, clean threads, and should be very closely the same in wall thickness at all points with clean square weld.

l. Cleaned pipe to be protected against effects of oxidation, by baked enamel, zinc or other approved coating which will not soften at ordinary temperatures, and of sufficient weight and toughness to successfully withstand rough usuage likely to be received during shipment and installation; and of sufficient elasticity to prevent flaking when ½-inch conduit is bent in a curve the inner edge of which has a radius of 3½ inches.

m. All elbows or bends must be so made that the conduit will not be injured. The radius of the curve of the inner edge of any elbow not to be less than 3½ inches.

49 A. Switch and Outlet Boxes.— a. Must be of pressed steel having a wall thickness not less than .081 inch (No. 12 B. & S. gage), or of cast metal having a wall thickness not less than .128 inch (No. 8 B. & S. gage.)

b. Must be well galvanized, enameled or otherwise properly coated, inside and out, to prevent oxidation.

c. Must be so made that all openings not in use will be effectively closed by metal which will afford protection substantially equivalent to the walls of the box.

d. Must be plainly marked, where it may readily be seen when installed, with the name or trade-mark of the manufacturer.

e. Must be arranged to secure in position the conduit or flexible tubing protecting the wire.

This rule will be complied with if the conduit or tubing is firmly secured in position by means of some *approved* device which may or may not be a part of the box.

f. Boxes used with lined conduit must comply with the foregoing requirements, and in addition must have a tough and tenacious insulating lining at least 1/32 inch thick, firmly secured in position.

g. Switch and outlet boxes must be so arranged that they can be securely fastened in place independently of the support afforded by the conduit piping, except that when entirely exposed, *approved* boxes, which are threaded so as to be firmly supported by screwing on to the conduit pipe, may be used.

h. Switch boxes must completely enclose the switch on sides and back, and must provide a thoroughly substantial support for it. The retaining screws for the box must not be used to secure the switch in position.

i. Covers for outlet boxes must be of metal equal in thickness to that specified for the walls of the box, or must be of metal lined with an insulating material not less than 1/32 inch in thickness, firmly and permanently secured to the metal.

50. Mouldings.—(For wiring rules, see No. 24 k to m.)

Wooden Mouldings.—**a.** Must have, both outside and inside, at least two coats of waterproof material, or be impregnated with a moisture repellent.

b. Must be made in two pieces, a backing and a capping, and must afford suitable protection from abrasion. Must be so constructed as to thoroughly incase the wire, be provided with a tongue not less than ½ inch in thickness between the conductors, and have exterior walls which, under grooves,

shall not be less than 5/8 inch in thickness, and on the sides not less than 1/4 inch in thickness.

It is recommended that only hard-wood moulding be used.

Metal Mouldings.—(For wiring rules, see Nos. 24, k to m, and 25 A.)

c. Each length of such moulding must have maker's name or trade-mark stamped in the metal, or in some manner permanently attached thereto, in order that it may be readily identified in the field.

The use of paper stickers or tags cannot be considered satisfactory methods of marking, as they are readily loosened and lost off in ordinary handling of the moulding.

d. Must be constructed of iron or steel with backing at least .050 inch in thickness, and with capping not less than .040 inch in thickness, and so constructed that when in place the raceway will be entirely closed; must be thoroughly galvanized or coated with an approved rust preventative both inside and out to prevent oxidation.

e. Elbows, couplings and all other similar fittings must be constructed of at least the same thickness and quality of metal as the moulding itself, and so designed that they will both electrically and mechanically secure the different sections together and maintain the continuity of the raceway. The interior surfaces must be free from burrs or sharp corners which might cause abrasion of the wire coverings.

f. Must at all outlets be so arranged that the conductors cannot come in contact with the edges of the metal, either of capping or backing. Specially designed fittings which will interpose substantial barriers between conductors and the edges of metal are recommended.

g. When backing is secured in position by screws or bolts from the inside of the raceway, depressions must be provided to render the heads of the fastenings flush with the moulding.

h. Metal mouldings must be used for exposed work only and must be so constructed as to form an open raceway to be closed by the capping or cover after the wires are laid in.

50 A. Tubes and Bushings.—a. *Construction.*—Must be made straight and free from checks or rough projections, with ends smooth and rounded to facilitate the drawing in of the wire and prevent abrasion of its covering.

b. *Material and Test.*—Must be made of non-combustible insulating material, which, when broken and submerged for 100 hours in pure water at 70° Fahrenheit (21° Centigrade), will not absorb over one-half of one per cent of its weight.

c. *Marking.*—Must have the name, initials or trade-mark of the manufacturer stamped in the ware.

d. *Sizes.*—Dimensions of walls and heads must be at least as great as those given in the following table:—

Diameter of Hole. Inches.	External Diameter. Inches.	Thickness of Wall. Inches.	External Diameter of Head. Inches.	Length of Head. Inches.
5/16	9/16	1/8	11/16	1/2
3/8	11/16	3/16	13/16	1/2
1/2	13/16	3/16	1 1/16	1/2
5/8	15/16	3/16	1 1/16	1/2
3/4	1 1/16	3/16	1 1/16	5/8
1	1 7/16	3/16	1 9/16	5/8
1 1/4	1 13/16	3/16	2 1/16	5/8
1 1/2	2 3/16	3/16	2 7/16	3/4
1 3/4	2 9/16	3/16	3 1/16	3/4
2	2 13/16	3/16	3 7/16	3/4
2 1/4	3 3/16	3/16	3 13/16	1
2 1/2	3 11/16	3/16	4 3/16	1

An allowance of 1-64 of an inch for variation in manufacture will be permitted, except in the thickness of the wall.

50 B. Cleats.—a. *Construction.*—Must hold the wire firmly in place without injury to its covering.

Sharp edges which may cut the wire should be avoided.

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b. *Supports.*—Bearing points on the surface must be made by ridges or rings about the holes for supporting screws, in order to avoid cracking and breaking when screwed tight.

c. *Material and Test.*—Must be made of non-combustible insulating material, which, when broken and submerged for 100 hours in pure water at 70° Fahrenheit (21° Centigrade), will not absorb over one-half of one per cent of its weight.

d. *Marking.*—Must have the name, initials or trade-mark of the manufacturer stamped in the ware.

e. *Sizes.*—Must conform to the spacings given in the following table:—

Voltage.	Distance from Wire to Surface.	Distance Between Wires.
0-300	½ inch.	2½ inches.

This rule will not be interpreted to forbid the placing of the neutral of an Edison three-wire system in the center of a three-wire cleat where the difference of potential between the outside wires is not over 300 volts, provided the outside wires are separated 2½ inches.

50 C. Flexible Tubing.—a. Must have a sufficiently smooth interior surface to allow the ready introduction of the wire.

b. Must be constructed of or treated with materials which will serve as moisture repellents.

c. The tube must be so designed that it will withstand all the abrasion likely to be met with in practice.

d. The linings, if any, must not be removable in lengths of over 3 feet.

e. The ½-inch tube must be so flexible that it will not crack or break when bent in a circle with 6-inch radius at 50° Fahrenheit (10° Centigrade), and the covering must be thoroughly saturated with a dense moisture-proof compound which will not slide at 150° Fahrenheit (65° Centigrade). Other sizes must be as well made.

f. Must not convey fire on the application of a flame from Bunsen burner to the exterior of the tube when held in a vertical position.

g. Must be sufficiently tough and tenacious to withstand severe tension without injury; the interior diameter must not be diminished or the tube opened up at any point by the application of a reasonable stretching force.

h. Must not close to prevent the insertion of the wire after the tube has been kinked or flattened and straightened out.

51. Switches.—(For installation rules, see Nos. 17 and 22.)

GENERAL RULES.

a. Must, when used for service switches, indicate, on inspection, whether the current be "on" or "off."

b. Must, for constant-current systems, close the main circuit and disconnect the branch wires when turned "off;" must be so constructed that they shall be automatic in action, not stopping between points when started, and must prevent an arc between the points under all circumstances. They must indicate whether the current be "on" or "off."

KNIFE SWITCHES.

Knife switches must be made to comply with the following specifications, except in those few cases where peculiar design allows the switch to fulfil the general requirements in some other way, and where it can successfully withstand the test of Section 4. In such cases the switch should be submitted for special examination before being used.

c. *Base.*—Must be mounted on non-combustible, non-absorptive insulating bases, such as slate or porcelain. Bases with an area of over 25 square inches must have at least four supporting screws. Holes for the supporting screws must be so located or countersunk that there will be at least ½ an inch space, measured over the surface, between the head of the screw or washer and the nearest live metal part, and in all cases when between parts of opposite polarity must be countersunk.

d. *Mounting.*—Pieces carrying the contact jaws and hinge clips must be secured to the base by at least two screws, or else made with a square shoulder, or provided with dowel pins, to prevent possible turnings, and

the nuts or screw-heads on the under side of the base must be countersunk not less than ¼ inch and covered with a waterproof compound which will not melt below 150° Fahrenheit (65° Centigrade).

e. *Hinges.*—Hinges of knife switches must not be used to carry current unless they are equipped with spring washers, held by lock-nuts or pins, or their equivalent, so arranged that a firm and secure connection will be maintained at all positions of the switch blades.

Spring washers must be of sufficient strength to take up any wear in the hinge and maintain a good contact at all times.

f. *Metal.*—All switches must have ample metal for stiffness and to prevent rise in temperature of any part of over 50° Fahrenheit (28° Centigrade), at full load, the contacts being arranged so that a thoroughly good bearing at every point is obtained with contact surfaces advised for pure copper blades of about one square inch for each 75 amperes; the whole device must be mechanically well made throughout.

g. *Cross-Bars.*—All cross-bars less than 3 inches in length must be made of insulating material. Bars of 3 inches or over, which are made of metal to insure greater mechanical strength, must be sufficiently separated from the jaws of the switch to prevent arcs following from the contacts to the bar on the opening of the switch under any circumstances. Metal bars should preferably be covered with insulating material.

To prevent possible turning or twisting the cross-bar must be secured to each blade by two screws, or the joints made with square shoulders or provided with dowel-pins.

h. *Connections.*—Switches for currents of over 30 amperes must be equipped with lugs, firmly screwed or bolted to the switch, and into which the conducting wires shall be soldered. For the smaller sized switches simple clamps can be employed, provided they are heavy enough to stand considerable hard usage.

Where lugs are not provided, a rugged double-V groove clamp is advised. A set-screw gives a contact at only one point, is more likely to become loosened, and is almost sure to cut into the wire. For the smaller sizes, a screw and washer connection with up-turned lugs on the switch terminal gives a satisfactory contact.

i. *Test.*—Must operate successfully at 50 per cent overload in amperes and 25 per cent excess voltage, under the most severe conditions with which they are liable to meet in practice.

This test is designed to give a reasonable margin between the ordinary rating of the switch and the breaking-down point, thus securing a switch which can always safely handle its normal load. Moreover, there is enough leeway so that a moderate amount of overloading would not injure the switch.

j. *Marking.*—Must be plainly marked where it will be visible, when the switch is installed, with the name of the maker and the current and the voltage for which the switch is designed.

Switches designed for use on Edison three-wire systems must be marked with both voltages, that is, the voltage between the outside wires and the neutral, and also that between the outside wires, followed by the ampere rating and the words "three-wire." (For example, "125-250 v. 30 a., three-wire.")

k. *Spacings.*—Spacings must be at least as great as those given in the following table. The spacings specified are correct for switches to be used on direct-current systems, and can therefore be safely followed in devices designed for alternating currents.

125 Volts or Less:	Minimum Separation of Nearest Metal Parts of Opposite Polarity.	Minimum Break-Distance.
For Switchboards and Panel Boards:		
10 amperes or less.....	¾ inch	½ inch.
11-30 amperes.....	1 "	¾ "
31-50 "	1¼ "	1 "
For Individual Switches:		
10 amperes or less.....	1 inch	¾ inch.
11-30 amperes.....	1¼ "	1 "
31-100 "	1½ "	1¼ "
101-300 "	2¼ "	2 "
301-600 "	2¾ "	2½ "
601-1000 "	3 "	2¾ "

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126 TO 250 VOLTS:

For all Switches:	Min. Sep.	Min. B.-D.
10 amperes or less.....	1½ inch	1¼ inch.
11-30 amperes.....	1¾ "	1½ "
31-100 "	2¼ "	2 "
101-300 "	2½ "	2¼ "
301-600 "	2¾ "	2½ "
601-1000 "	3 "	2¾ "

For 100 ampere switches and larger, the above spacings for 250 volts direct current are also approved for 500 volts alternating current. Switches with these spacings intended for use on alternating-current systems with voltage above 250 volts must be stamped "250-volt D. C.," followed by the alternating-current voltage for which they are designed, and the letters "A. C."

251 TO 600 VOLTS:

For all Switches:		
10 amperes or less.....	3½ inch	3 inch.
11-35 amperes.....	4 "	3½ "
36-100 "	4½ "	4 "

Auxiliary breaks or the equivalent are recommended for switches designed for over 300 volts and less than 100 amperes, and will be required on switches designed *for use in breaking currents* greater than 100 amperes at a pressure of more than 300 volts.

For three-wire Edison systems the separations and break distances for plain three-pole knife switches must not be less than those required in the above table for switches designed for the voltage between the neutral and outside wires.

SNAP SWITCHES.

Flush, push-button, door, fixture and other snap switches used on constant-potential systems, must be constructed in accordance with the following specifications.

l. *Base.*—Current-carrying parts must be mounted on non-combustible, non-absorptive, insulating bases, such as slate or porcelain, and the holes for supporting screws should be countersunk not less than ⅛ of an inch. There must in no case be less than 3/64 of an inch space between supporting screws and current-carrying parts.

Sub-bases of non-combustible, non-absorptive insulating material, which will separate the wires at least ⅛ inch from the surface wired over, must be furnished with all snap switches used in exposed or moulding work.

m. *Mounting.*—Pieces carrying contact jaws must be secured to the base by at least two screws, or else made with a square shoulder, or provided with dowel-pins or otherwise arranged to prevent possible turnings; and the nuts or screw-heads on the under side of the base must be countersunk not less than ⅛ inch, and covered with a waterproof compound which will not melt below 150° Fahrenheit (65° Centigrade).

n. *Metal.*—All switches must have ample metal for stiffness and to prevent rise in temperature of any part of over 50° Fahrenheit (28° Centigrade) at full load, the contacts being arranged so that a thoroughly good bearing at every point is obtained. The whole device must be mechanically well made throughout.

In order to meet the above requirements on temperature rise without causing excessive friction and wear on current-carrying parts, contact surfaces of from 0.1 to 0.15 square inch for each 10 amperes will be required, depending upon the metal used and the form of construction adopted.

o. *Insulating Material.*—Any material used for insulating current-carrying parts must retain its insulating and mechanical strength when subject to continued use, and must not soften at a temperature of 212° Fahrenheit (100° Centigrade). It must also be non-absorptive.

p. *Binding Posts.*—Binding posts must be substantially made, and the screws must be of such size that the threads will not strip when set up tight.

A set-screw is likely to become loosened and is almost sure to cut into the wire. A binding screw under the head of which the wire may be clamped and a terminal plate provided with upturned lugs or some other equivalent arrangement, afford reliable contact. After July 1, 1908, switches with the set-screw form of contact will not be approved.

q. *Covers.*—Covers made of conducting material, except face plates for flush switches, must be lined on sides and top with insulating, tough and tenacious material at least 1/32 inch in thickness, firmly secured so that it

will not fall out with ordinary handling. The side lining must extend slightly beyond the lower edge of the cover.

r. *Handle or Button.*—The handle or button or any exposed parts must not be in electrical connection with the circuit.

s. *Test.*—Must "make" and "break" with a quick snap, and must not stop when motion has once been imparted by the button or handle.

Must operate successfully at 50 per cent overload in amperes and at 125 volt direct current, for all 125 volt or less switches, and at 250 volt direct current, for all 126 to 250 volt switches under the most severe conditions which they are liable to meet in practice.

When slowly turned "on" and "off" at the rate of about two or three times per minute, while carrying the rated current at rated voltage, must "make" and "break" the circuit 6,000 times before failing.

t. *Marking.*—Must be plainly marked, where it may be readily seen after the device is installed, with the name or trade-mark of the maker and the current and voltage for which the switch is designed.

On flush switches these markings may be placed on the back of the face plate or on the sub-plate. On other types they must be placed on the *front* of the cap, cover or plate.

Switches which indicate whether the current is "on" or "off" are recommended.

52. Cut-Outs and Circuit Breakers.—(For installation rules, see Nos. 17 and 21.) *These requirements do not apply to rosettes, attachment plugs, car lighting cut-outs and protective devices for signaling systems.*

GENERAL RULES.

a. Must be supported on bases of non-combustible, non-absorptive insulating material.

b. Cut-outs must be of plug or cartridge type, when not arranged in *approved* cabinets, so as to obviate any danger of the melted fuse metal coming in contact with any substance which might be ignited thereby.

c. Cut-outs must operate successfully on short-circuits, under the most severe conditions with which they are liable to meet in practice, at 25 per cent above their rated voltage, and for link fuse cut-outs with fuses rated at 50 per cent above the current for which the cut-out is designed, and for enclosed fuse cut-outs with the largest fuses for which the cut-out is designed.

With link fuse cut-outs there is always the possibility of a larger fuse being put into the cut-out than it was designed for, which is not true of enclosed fuse cut-outs classified as required under No. 52 q. Again, the voltage in most plants can, under some conditions, rise considerably above the normal. The need of some margin as a factor of safety to prevent the cut-outs from being ruined in ordinary service, is therefore evident.

The most severe service which can be required of a cut-out in practice is to open a "dead short-circuit" with only one fuse blowing, and it is with these conditions that all tests should be made. (See Section j.)

d. Circuit-breakers must operate successfully on short-circuits under the most severe conditions with which they are liable to meet in practice, at 25 per cent above their rated voltage and with the circuit-breaker set at the highest possible opening point.

For the same reason as in Section c.

e. Must be plainly marked where it will always be visible, with the name of the maker, and current and voltage for which the device is designed.

LINK-FUSE CUT-OUTS.

(*Cut-outs of porcelain are not approved for link fuses.*)

The following rules are intended to cover open link fuses mounted on slate or marble bases, including switchboards, tablet-boards and single fuse-blocks. They do not apply to fuses mounted on porcelain bases, to the ordinary porcelain cut-out blocks, enclosed fuses, or any special or covered type of fuse. When tablet-boards or single fuse-blocks with such open link fuses on them are used in general wiring, they must be enclosed in cabinet boxes made to meet the requirements of No. 54. This is necessary, because a severe flash may occur when such fuses melt, so that they would be dangerous if exposed in the neighborhood of any combustible material.

f. *Base.* Must be mounted on slate or marble bases. Bases with an area of over 25 square inches must have at least four supporting screws. Holes for supporting screws must be kept outside of the area included by

the outside edges of the fuse-block terminals, and must be so located or countersunk that there will be at least ½ inch space, measured over the surface, between the head of the screw or washer and the nearest live part.

g. *Mounting.*—Nuts or screw heads on the under side of the base must be countersunk not less than ⅛ inch, and covered with a waterproof compound which will not melt below 150° Fahrenheit (65° Centigrade).

h. *Metal.*—All fuse-block terminals must have ample metal for stiffness and to prevent rise in temperature of any part over 50° Fahrenheit (28° Centigrade) at full load. Terminals, as far as practicable, should be made of compact form instead of being rolled out in thin strips; and sharp edges or thin projecting pieces, as on wing thumb nuts and the like, should be avoided. Thin metal, sharp edges and projecting pieces are much more likely to cause an arc to start than a more solid mass of metal. It is a good plan to round all corners of the terminals and to chamfer the edges.

i. *Connections.*—Clamps for connecting the wires to the fuse-block terminals must be of solid, rugged construction, so as to insure a thoroughly good connection and to withstand considerable hard usage. For fuses rated at over 30 amperes, lugs firmly screwed or bolted to the terminals and into which the conducting wires are soldered must be used.

See note under No. 51 h.

j. *Test.*—Must operate successfully when blowing only one fuse at a time on short circuits with fuses rated at 50 per cent above and with a voltage 25 per cent above the current and voltage for which the cut-out is designed.

k. *Marking.*—Must be plainly marked, where it will be visible when the cut-out block is installed, with the name of the maker and the current and the voltage for which the block is designed.

l. *Spacings.*—Spacings must be at least as great as those given in the following table, which applies only to plain, open link-fuses mounted on slate or marble bases. The spaces given are correct for fuse-blocks to be used on direct-current systems, and can therefore be safely followed in devices designed for alternating currents. If the copper fuse-tips overhang the edges of the fuse-block terminals, the spacings should be measured between the nearest edges of the tips.

	Minimum Separation of Nearest Metal Parts of Opposite Polarity.	Minimum Break Distance.
125 Volts or Less:		
10 amperes or less.....	¾ inch	¾ inch.
11-100 amperes.....	1 "	¾ "
101-300 "	1 "	1 "
301-1000 "	1¼ "	1¼ "
126 to 250 Volts:		
10 amperes or less.....	1½ inch	1¼ inch.
11-100 amperes.....	1¾ "	1¼ "
101-300 "	2 "	1½ "
301-1000 "	2½ "	2 "

A space must be maintained between fuse terminals of the *same polarity* of at least ½ inch for voltages up to 125, and of at least ¾ inch for voltages from 126 to 250. This is the minimum distance allowable, and greater separation should be provided when practicable

For 250 volt boards or blocks with the ordinary front-connected terminals, except where these have a mass of compact form, equivalent to the back-connected terminals usually found in switchboard work, a substantial barrier of insulating material, not less than ⅛ of an inch in thickness, must be placed in the "break" gap—this barrier to extend out from the base at least ¼ of an inch farther than any bare live part of the fuse-block terminal, including binding screws, nuts and the like.

For three-wire systems cut-outs must have the break-distance required for circuits of the potential of the outside wires.

Enclosed-Fuse Cut-Outs—Plug and Cartridge Type.

m. *Base.*—Must be made of non-combustible, non-absorptive, insulating material. Blocks with an area of over 25 square inches must have at least four supporting screws. Holes for supporting screws must be so located or countersunk that there will be at least ¼ inch space, measured over the surface, between the screw-head or washer and the nearest live metal part, and in all cases when between parts of opposite polarity must be countersunk

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n. *Mounting.*—Nuts or screw-heads on the under side of the base must be countersunk at least ⅛ of an inch and covered with a waterproof compound which will not melt below 150° Fahrenheit (65° Centigrade).

o. *Terminals.*—Terminals must be of either the Edison plug, spring clip or knife-blade type, of *approved* design, to take the corresponding standard enclosed fuses. They must be secured to the base by two screws or the equivalent, so as to prevent them from turning, and must be so made as to secure a thoroughly good contact with the fuse. End stops must be provided to insure the proper location of the cartridge fuse in the cut-out.

p. *Connections.*—Clamps for connecting wires to the terminals must be of a design which will insure a thoroughly good connection, and must be sufficiently strong and heavy to withstand considerable hard usage. For fuses rated to carry over thirty amperes, lugs firmly screwed or bolted to the terminals and into which the connecting wires shall be soldered must be used.

q. *Classification.*—Must be classified as regards both current and voltage as given in the following table, and must be so designed that the bases of one class cannot be used with fuses of another class rated for a higher current or voltage.

0-250 Volts:	251-600 Volts:
0- 30 amperes.	0- 30 amperes.
31- 60 "	31- 60 "
61-100 "	61-100 "
101-200 "	101-200 "
201-400 "	201-400 "
401-600 "	

r. *Design.*—Must be of such a design that it will not be easy to form accidental short circuit across live metal parts of opposite polarity on the block or on the fuses in the block.

s. *Marking.*—Must be marked, where it will be plainly visible when the block is installed, with the name of the maker and the voltage and range of current for which it is designed.

53. Fuses.—(For installation rules, see Nos. 17 and 21.)

LINK FUSES.

a. *Terminals.*—Must have contact surfaces or tips of harder metal, having perfect electrical connections with the fusible part of the strip.

The use of the hard metal tip is to afford a strong mechanical bearing for the screws, clamps or other devices provided for holding the fuse.

b. *Rating.*—Must be stamped with about 80 per cent of the maximum current which they can carry indefinitely, thus allowing about 25 per cent overload before the fuse melts.

With naked open fuses, or ordinary shapes and with not over 500 amperes capacity, the *minimum* current which will melt them in about five minutes may be safely taken as the melting point, as the fuse practically reaches its maximum temperature in this time. With larger fuses a longer time is necessary. This data is given to facilitate testing.

c. *Marking.*—Fuse terminals must be stamped with the maker's name or initials, or with some known trade-mark.

ENCLOSED FUSES—PLUG AND CARTRIDGE TYPE.

These requirements do not apply to fuses for rosettes, attachment plugs, car lighting cut-outs and protective devices for signaling systems.

d. *Construction.*—The fuse plug or cartridge must be sufficiently dust-tight so that lint and dust cannot collect around the fusible wire and become ignited when the fuse is blown.

The fusible wire must be attached to the plug or cartridge terminals in such a way as to secure a thoroughly good connection and to make it difficult for it to be replaced when melted.

e. *Classification.*—Must be classified to correspond with the different classes of cut-out blocks, and must be so designed that it will be impossible to put any fuse of a given class into a cut-out block which is designed for a current or voltage lower than that of the class to which the fuse belongs.

f. *Terminals.*—The fuse terminals must be sufficiently heavy to insure mechanical strength and rigidity. The styles of terminals must be as follows:

0–250 VOLTS:

0–30 Amps.	*A* Cartridge fuse (ferrule contact)	to fit	*a*, spring clip terminals. *b*, Edison plug casings.
	B Approved plugs for Edison cut-outs.		
31–60 "	Cartridge fuse (ferrule contact)	to fit	*a*, spring clip terminals. *b*, Edison plug casings.
61–100 "	Cartridge fuse (knife-blade contact).		
101–200 "			
201–400 "			
401–600 "			

251–600 VOLTS:

0–30 Amps.	Cartridge fuse (ferrule contact).
31–60 "	
61–100 "	Cartridge fuse (knife-blade contact).
101–200 "	
201–400 "	

g. *Dimensions.*—Cartridge enclosed fuses and corresponding cut-out blocks must conform to the dimensions given in the Table, on next page.

h. *Rating.*—Fuses must be so constructed that with the surrounding atmosphere at a temperature of 75° Fahrenheit (24° Centigrade) they will carry indefinitely a current 10 per cent greater than that at which they are rated, and at a current 25 per cent greater than the rating, they will open the circuit without reaching a temperature which will injure the fuse tube or terminals of the fuse block. With a current 50 per cent greater than the rating and at room temperature of 75° Fahrenheit (24° Centigrade), the fuses starting cold, must blow within the time specified below:—

0– 30 amperes,	30 seconds.
31– 60 "	1 minute.
61–100 "	2 minutes.
101–200 "	4 "
201–400 "	8 "
401–600 "	10 "

i. *Marking.*—Must be marked, where it will be plainly visible, with the name or trade-mark of the maker, the voltage and current for which the fuse is designed, and the words "National Electrical Code Standard." Each fuse must have a label, the color of which must be green for 250-volt fuses and red for 600-volt fuses.

It will be satisfactory to abbreviate the above designation to "N. E. Code St'd" where space is necessarily limited.

j. *Temperature Rise.*—The temperature of the exterior of the fuse enclosure must not rise more than 125° Fahrenheit (70° Centigrade) above that of the surrounding air when the fuse is carrying the current for which it is rated.

k. *Test.*—Must not hold an arc or throw out melted metal or sufficient flame to ignite easily inflammable material on or near the cut-out when only one fuse is blown at a time on a short circuit on a system of the voltage for which the fuse is rated.

The normal capacity of the system must be in excess of the load on it just previous to the test by at least five times the rated capacity of the fuse under test.

The resistance of the circuit up to the cut-out terminals must be such that the impressed voltage at the terminals will be decreased not more than one per cent, when a current of 100 amperes is passed between them.

For convenience a current of different value may be used, in which case the per cent drop in voltage allowable would vary in direct proportion to the difference in current used.

The above requirement regarding the capacity of the testing circuit is to guard against making the test on a system of so small a capacity that the conditions would be sufficiently favorable to allow really poor fuses to stand the test acceptably. On the other hand, it must be remembered that if the test is made on a system of very large capacity, and especially if there is but little resistance between the generators and fuse, the conditions may be more severe than are liable to be met with in practice outside of the large power stations, the result being that fuses entirely safe for general use may be rejected if such test is insisted upon.

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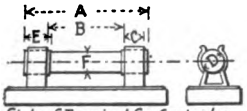


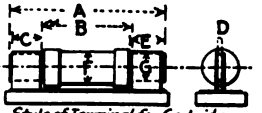
Style of Terminal for Cartridge Fuses, 0-60 Amperes.

Table of Dimensions of the National Electric Code Standard Cartridge Enclosed Fuse.

Form 1.—Cartridge Fuse (Ferrule Contact).

(For descriptive text, see pages 1436 and 1437.)

Form 2.—Cartridge Fuse (Knife-blade Contact).



Style of Terminal for Cartridge Fuses, 61-600 Amperes.

Voltage.	Rated Capacity. Amperes.		A Length Over Terminals. Inches.	B Distance Between Contact Clips. Inches.	C Width of Contact Clips. Inches.	D Diameter of Ferrules or Thickness of Terminal Blades. Inches.	E Min. Length of Ferrules or of Terminal Blades Outside of Tube. Inches.	F Diameter of Tube. Inches.	G Width of Terminal Blades. Inches.		Rated Capacity. Amperes.
0-250	0-30	Form 1	2	1	1/2	9/32	1/2	1/2		Form 1	0-30
	31-60		3	1 3/4	5/8	11/16	5/8	3/4			31-60
	61-100	Form 2	5 7/8	4	7/8	1/8	1	1	3/4	Form 2	61-100
	101-200		7 1/2	4 1/2	1 1/4	5/16	1 3/4	1 1/2	1 1/8		101-200
	201-400		8 5/8	5	1 3/4	1/4	1 7/8	2	1 5/8		201-400
	401-600		10 3/8	6	2 1/8	1/4	2 1/4	2 1/2	2		401-600
251-600	0-30	Form 1	5	4	1/2	11/16	1/2	3/4		Form 1	0-30
	31-60		5 1/2	4	5/8	1 1/16	5/8	1			31-60
	61-100	Form 2	7 7/8	6	7/8	1/8	1	1 1/4	3/4	Form 2	61-100
	101-200		9 3/4	7	1 1/4	5/16	1 3/4	1 5/8	1 1/8		101-200
	201-400		11 5/8	8	1 5/8	1/4	1 7/8	2 1/2	1 5/8		201-400

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53 A. Tablet and Panel Boards.—The following *minimum* distance between bare live metal parts (bus-bars, etc.) must be maintained:—

Between parts of opposite polarity, except at switches and link fuses,		Between parts of same polarity,
When mounted on the same surface.	When held free in the air.	At link fuses.
0–125 volts ¾ inch	½ inch	½ inch
126–250 volts 1¼ "	¾ "	¾ "

At switches or enclosed fuses, parts of the same polarity may be placed as close together as convenience in handling will allow.

It should be noted that the above distances are the *minimum* allowable, and it is urged that greater distances be adopted wherever the conditions will permit.

The spacings given in the first column apply to the branch conductors where inclosed fuses are used. Where link fuses or knife switches are used, the spacings must be at least as great as those required by Nos. 51 and 52.

The spacings given in the second column apply to the distance between the raised main bars, and between these bars and the branch bars over which they pass.

The spacings given in the third column are intended to prevent the melting of a link fuse by the blowing of an adjacent fuse of the same polarity.

54. Cut-Out Cabinets.—**a.** *Material.*—Cabinets must be substantially constructed of non-combustible, non-absorptive material, or of wood. When wood is used the inside of the cabinet must be completely lined with a non-combustible insulating material. Slate or marble at least ¼ inch in thickness is strongly recommended for such lining, but, except with metal conduit systems, asbestos board at least ⅛ inch in thickness may be used in dry places if firmly secured by shellac and tacks.

With metal conduit systems the lining of either the box or the gutter must be of 1/16 inch galvanized, painted or enameled steel, or preferably ¼ inch slate or marble.

The object of the lining of such cut-out cabinets or gutters is to render the same approximately fireproof in case of short circuit after the wires leave the protecting metal conduits.

Two thicknesses of 1-32 inch steel may be used instead of one 1-16 inch.

With wood cabinets the wood should be thoroughly filled and painted before the lining is put into place.

b. *Door.*—The door must close against a rabbet, so as to be perfectly dust-tight. Strong hinges and a strong hook or catch are required. Glass doors must be glazed with heavy glass, not less than ⅛ inch in thickness, and panes should not exceed 300 square inches in area. A space of at least two inches must be allowed between the fuses and the door. This is necessary to prevent cracking or breaking by the severe blow and intense heat which may be produced under some conditions.

A cabinet is of little use unless the door is kept tightly closed, and especial attention is therefore called to the importance of having a strong and reliable catch or other fastening. A spring catch is advised if a good one can be obtained, but most of those sold for use on cupboards, etc., are so small that they fail to catch when the door shrinks a little, or are so weak that they soon give out.

It is advised that the bottoms of cabinets be given a decided slant to prevent their use as a shelf, as well as the accumulation of dust, etc.

c. *Bushings.*—Bushings through which wires enter must fit tightly the holes in the box, and must be of *approved* construction. The wires should completely fill the holes in the bushings, using tape to build up the wire, if necessary, so as to keep out the dust.

54 A. Rosettes.—Ceiling rosettes, both fused and fuseless, must be constructed in accordance with the following specifications:—

a. *Base.*—Current-carrying parts must be mounted on non-combustible, non-absorptive, insulating bases. There should be no openings through the rosette base except those for the supporting screws and in the concealed type for the conductors also, and these openings should not be made any larger than necessary.

There must be at least ¼ inch space, measured over the surface, between supporting screws and current-carrying parts. The supporting screws must be so located or countersunk that the flexible cord cannot come in contact with them.

Bases for the knob and cleat type must have at least two holes for supporting screws; must be high enough to keep the wires and terminals at least ⅛ inch from the surface to which the rosette is attached, and must have a porcelain lug under each terminal to prevent the rosette from being

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placed over projections which would reduce the separation to less than ⅛ inch.

Bases for the moulding and conduit box types must be high enough to keep the wires and terminals at least ⅛ inch from the surface wired over.

b. *Mounting.*—Contact pieces and terminals must be secured in position by at least two screws, or made with a square shoulder, or otherwise arranged to prevent turning.

The nuts or screw head on the under side of the base must be countersunk not less than ⅛ inch and covered with a waterproof compound which will not melt below 150° Fahrenheit (65° Centigrade).

c. *Terminals.*—Line terminal plates must be at least .07 inch in thickness, and terminal screws must not be smaller than No. 6 standard screw with about 32 threads per inch.

Terminal plates for the flexible cord and for fuses must be at least .06 inch in thickness. The connection to these plates shall be by binding screws not smaller than No. 5 standard screw with about 40 threads per inch. At all binding screws for line wires and for flexible cord, up-turned lugs, or some equivalent arrangement, must be provided which will secure the wires being held under the screw heads.

d. *Cord Inlet.*—The diameter of the cord inlet hole should measure 13/32 inch in order that standard portable cord may be used.

e. *Knot Space.*—Ample space must be provided for a substantial knot tied in the cord as a whole.

All parts of the rosette upon which the knot is likely to bear must be smooth and well rounded.

f. *Cover.*—When the rosette is made in two parts, the cover must be secured to the base so that it will not work loose.

In fused rosettes, the cover must fit closely over the base so as to prevent the accumulation of dust or dirt on the inside, and also to prevent any flash or melted metal from being thrown out when the fuses melt.

g. *Markings.*—Must be plainly marked where it may readily be seen after the rosette has been installed, with the name or trade-mark of the manufacturer, and the rating in amperes and volts. Fuseless rosettes may be rated 3 amperes, 250 volts; fused rosettes, with link fuses, not over 2 amperes, 125 volts.

h. *Test.*—Fused rosettes must have a fuse in each pole and must operate successfully when short-circuited on the voltage for which they are designed, the test being made with the two fuses in circuit.

When link fuses are used the test shall be made with fuse wire which melts at about 7 amperes in one-inch lengths The larger fuse is specified for the test in order to more nearly approximate the severe conditions obtained when only one 2-ampere fuse (the rating of the rosette) is blown at a time.

Fused rosettes equipped with enclosed fuses are much preferable to the link fuse rosettes.

55. Sockets.—(For installation rules, see No. 27.) Sockets of all kinds, including wall receptacles, must be constructed in accordance with the following specifications:—

a. *Standard Sizes.*—The standard lamp socket must be suitable for use on any voltage not exceeding 250 and with any size lamp up to fifty candle-power. For lamps larger than fifty candle-power a standard keyless socket may be used, or if a key is required, a special socket designed for the current to be used must be made. Any special sockets must follow the general spirit of these specifications.

b. *Marking.*—All sockets must be marked with the manufacturer's name or trade-mark. The standard key socket must also be plainly marked 250 v. 50 c. p. Receptacles, keyless sockets and special sockets must be marked with the current and voltage for which they are designed.

c. *Shell.*—Metal used for shells must be moderately hard, but not hard enough to be brittle or so soft as to be easily dented or knocked out of shape. Brass shells must be at least thirteen one-thousandths of an inch in thickness, and shells of any other material must be thick enough to give the same stiffness and strength as the required thickness of brass.

d. *Lining.*—The inside of the shells must be lined with insulating material, which must absolutely prevent the shell from becoming a part of

the circuit, even though the wires inside the sockets should become loosened or detached from their position under the binding screws.

The material used for lining must be at least 1/32 of an inch in thickness, and must be tough and tenacious. It must not be injuriously affected by the heat from the largest lamp permitted in the socket, and must leave water in which it is boiled practically neutral. Is must be so firmly secured to the shell that it will not fall out with ordinary handling of the socket. It is preferable to have the lining in one piece.

The cap must also be lined, and this lining must comply with the requirements for shell linings.

The shell lining should extend beyond the shell far enough so that no part of the lamp base is exposed when a lamp is in the socket. The standard Edison lamp base measures 15-16 inches in a vertical plane from the bottom of the center contact to the upper edge of the screw-shell.

In sockets and receptacles of standard forms a ring of any material inserted between an outer metal shell of the device and the inner screw shell for insulating purposes and separable from the device as a whole, is considered an undesirable form of construction. This does not apply to the use of rings in lamp clusters or in devices where the outer shell is of porcelain, where such rings serve to hold the several porcelain parts together, and are thus a necessary part of the whole structure of the device.

e. *Cap.*—Caps, when of sheet brass, must be at least thirteen one-thousandths of an inch in thickness, and when cast or made of other metals must be of equivalent strength. The inlet piece, except for special sockets, must be tapped with a standard ¼ inch pipe thread. It must contain sufficient metal for a full, strong thread, and when not in one piece with the cap, must be joined to it in such a way as to give the strength of a single piece.

There must be sufficient room in the cap to enable the ordinary wireman to easily and quickly make a knot in the cord and to push it into place in the cap without crowding. All parts of the cap upon which the knot is likely to bear must be smooth and well insulated.

The cap lining called for in the note to Section *d* will provide a sufficiently smooth and well insulated surface for the knot to bear upon.

Sockets with an outlet threaded for ¼-inch pipe will, of course, be approved where circumstances demand their use. The size outlet is necessary with most stiff pendants and for the proper use of reinforced flexible cord, as explained in the note to No. 28 *d*.

f. *Frame and Screws.*—The frame which holds the moving parts must be sufficiently heavy to give ample strength and stiffness.

Brass pieces containing screw threads must be at least six one-hundredths of an inch in thickness.

Binding post screws must not be smaller than No. 5 standard screw with about 40 threads per inch.

g. *Spacing.*—Points of opposite polarity must everywhere be kept not less than 3/64 of an inch apart, unless separated by a reliable insulation.

h. *Connections.*—The connecting points for the flexible cord must be made to very securely grip a No. 16 or 18 B. & S. gage conductor. An upturned lug, arranged so that the cord may be gripped between the screw and the lug in such a way that it cannot possibly come out, is strongly advised.

i. *Lamp Holder.*—The socket must firmly hold the lamp in place so that it cannot be easily jarred out, and must provide a contact good enough to prevent undue heating with the maximum current allowed. The holding pieces, springs and the like, if a part of the circuit, must not be sufficiently exposed to allow them to be brought in contact with anything outside of the lamp and socket.

j. *Base.*—With the exception of the lining, all parts of insulating material inside the shell must be made of porcelain.

k. *Key.*—The socket key-handle must be of such a material that it will not soften from the heat of a fifty candle-power lamp hanging downwards from the socket in the air at 70° Fahrenheit (21° Centigrade), and must be securely, but not necessarily rigidly, attached to the metal spindle which it is designed to turn.

l. *Sealing.*—All screws in porcelain pieces, which can be firmly sealed in place, must be so sealed by a waterproof compound which will not melt below 200° Fahrenheit (93° Centigrade.)

m. *Putting Together.*—The socket as a whole must be so put together that it will not rattle to pieces. Bayonet joints or an equivalent are recommended.

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n. *Test.*—The socket, when slowly turned "on and off" at the rate of about two or three times per minute, while carrying a load of one ampere at 250 volts, must "make" and "break" the circuit 6,000 times before failing.

o. *Keyless Sockets.*—Keyless sockets of all kinds must comply with the requirements for key sockets as far as they apply.

p. *Sockets of Insulating Material.*—Sockets made of porcelain or other insulating material must conform to the above requirements as far as they apply, and all parts must be strong enough to withstand a moderate amount of hard usage without breaking.

Porcelain shell sockets being subject to breakage, and constituting a hazard when broken, will not be accepted for use in places where they would be exposed to hard usage.

q. *Inlet Bushing.*—When the socket is not attached to a fixture, the threaded inlet must be provided with a strong insulating bushing having a *smooth* hole at least 9/32 of an inch in diameter. The edges of the bushing must be rounded and all inside fins removed, so that in no place will the cord be subjected to the cutting or wearing action of a sharp edge.

Bushings for sockets having an outlet threaded for ⅜-inch pipe should have a hole 13-32 of an inch in diameter, so that they will accommodate *approved* reinforced flexible cord.

56. Hanger-boards for Series Arc Lamps.—**a.** Hanger-boards must be so constructed that all wires and current-carrying devices thereon will be exposed to view and thoroughly insulated by being mounted on a non-combustible, non-absorptive, insulating substance. All switches attached to the same must be so constructed that they shall be automatic in their action, cutting off both poles to the lamp, not stopping between points when started and preventing an arc between points under all circumstances.

57. Arc Lamps.—(For installation rules, see Nos. 19 and 29.) **a.** Must be provided with reliable stops to prevent carbons from falling out in case the clamps become loose.

b. All exposed parts must be carefully insulated from the circuit.

c. Must, for constant-current systems, be provided with an *approved* hand switch, and an automatic switch that will shunt the current around the carbons, should they fail to feed properly.

The hand switch to be approved, if placed anywhere except on the lamp itself, must comply with requirements for switches on hanger-boards as laid down in No. 56.

58. Spark Arresters.—(For installation rules, see Nos. 19 c and 29 c.)

a. Spark arresters must so close the upper orifice of the globe that it will be impossible for any sparks, thrown off by the carbons, to escape.

59. Insulating Joints.—(See No. 26 a.)—**a.** Must be entirely made of material that will resist the action of illuminating gases, and will not give way or soften under the heat of an ordinary gas flame or leak under a moderate pressure. Must be so arranged that a deposit of moisture will not destroy the insulating effect; must show a dielectric strength between gas-pipe attachments sufficient to resist throughout five minutes the application of an electro-motive force of 4,000 volts; and must be sufficiently strong to resist the strain to which they are liable to be subjected during installation.

Insulating joints having soft rubber in their construction will not be approved.

60. Rheostats.—(For installation rules, see Nos. 4 a and 8 c.)

a. *Materials.*—Must be made entirely of non-combustible materials, except such minor parts as handles, magnet insulation, etc. All segments, lever arms, etc., must be mounted on non-combustible, non-absorptive, insulating material.

Rheostats used in dusty or linty places or where exposed to flyings of combustible material, must be so constructed that even if the resistive conductor be fused by excessive current, the arc or any attendant flame will be quickly and safely extinguished. Rheostats used in places where the above conditions do not exist may be of any approved type.

b. *Construction.*—Must be so constructed that when mounted on a plane surface the casing will make contact with such surface only at the

points of support. An air space of at least ¼ inch between the rheostat casing and the supporting surface will be required.

The construction throughout must be heavy, rugged and thoroughly workmanlike.

c. *Connections.*—Clamps for connecting wires to the terminals must be of a design that will insure a thoroughly good connection, and must be sufficiently strong and heavy to withstand considerable hard usage. For currents above fifty amperes, lugs firmly screwed or bolted to the terminals, and into which the connecting wires shall be soldered, must be used.

Clamps or lugs will not be required when leads designed for soldered connections are provided.

d. *Marking.*—Must be plainly marked, where it may be readily seen after the device is installed, with the rating and the name of the maker; and the terminals of motor-starting rheostats must be marked to indicate to what part of the circuit each is to be connected, as "line," "armature" and "field."

e. *Contacts.*—The design of the fixed and movable contacts and the resistance in each section must be such as to secure the least tendency toward arcing and roughening of the contacts, even with careless handling or the presence of dirt.

In motor-starting rheostats, the contact at which the circuit is broken by the lever arm when moving from the running to the starting position, must be so designed that there will be no detrimental arcing. The final contact, if any, on which the arm is brought to rest in the starting position must have no electrical connection.

Experience has shown that sharp edges and segments of thin material help to maintain an arc, and it is recommended that these be avoided. Segments of heavy construction have a considerable cooling effect on the air, and rounded corners tend to spread it out and thus dissipate it.

It is recommended that the circuit-breaking contacts be so constructed as to "break" with a quick snap, independently of the slowness of movement of the operator's hand, or that a magnetic blowout or equivalent device be used. For dial type rheostats the movable contact should be flexible in a plane at right angle to the plane of its movement, and for medium and larger sizes the stationary contacts should be readily renewable.

f. *No-Voltage Release.*—Motor-starting rheostats must be so designed that the contact arm cannot be left on intermediate segments, and must be provided with an automatic device which will interrupt the supply circuit before the speed of the motor falls to less than one third of its normal value.

g. *Overload-Release.*—Overload-release devices which are inoperative during the process of starting a motor will not be approved unless other circuit-breakers or fuses are installed in connection with them.

If, for instance, the over-release device simply releases the starting arm and allows it to fly back and break the circuit, it is inoperative while the arm is being moved from the starting to the running position.

h. *Test.*—Must, after 100 operations under the most severe normal conditions for which the device is designed, show no serious burning of the contacts or other faults, and the release mechanism of motor-starting rheostats must not be impaired by such a test.

Field rheostats, or main-line regulators intended for continuous use, must not be burned out or depreciated by carrying the full normal current on any step for an indefinite period. Regulators intended for intermittent use (such as on electric cranes, elevators, etc.) must be able to carry their rated current on any step for as long a time as the character of the apparatus which they control will permit them to be used continuously.

61. Reactive Coils and Condensers.—**a.** Reactive coils must be made of non-combustible material, mounted on non-combustible bases and treated, in general, as sources of heat.

b. Condensers must be treated like other apparatus operating with equivalent voltage and currents. They must have non-combustible cases and supports, and must be isolated from all combustible material and, in general, treated as sources of heat.

62. Transformers.—(For installation rules, see Nos. 11, 13, 13 A and 36.)

a. Must not be placed in any but metallic or other non-combustible cases.

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On account of the possible dangers from burn-outs in the coils. (See note under No. 11 a.)

It is advised that every transformer be so designed and connected that the middle point of the secondary coil can be reached if, at any future time, it should be desired to ground it.

b. Must be constructed to comply with the following tests:—

1. Shall be run for eight consecutive hours at full load in watts under conditions of service, and at the end of that time the rise in temperature, as measured by the increase of resistance of the primary and secondary coils, shall not exceed 175° Fahrenheit (97° Centigrade).
2. The insulation of transformers when heated shall withstand continuously for five minutes a difference of potential of 10,000 volts (alternating) between primary and secondary coils and between the primary coils and core, and a no-load "run" at double voltage for thirty minutes.

63. Lightning Arresters.—(For installation rules, see No. 5.) a. Lightning arresters must be of *approved* construction. (See list of Electrical Fittings.)

Class E.—MISCELLANEOUS.

64. Signaling Systems.—*Governing wiring for telephone, telegraph, district messenger and call-bell circuits, fire and burglar alarms, and all similar systems which are hazardous only because of their liability to become crossed with electric light, heat or power circuits.*

a. Outside wires should be run in underground ducts or strung on poles, and, kept off the roofs of buildings, except by special permission of the Inspection Department having jurisdiction, and must not be placed on the same cross-arm with electric light or power wires. They should not occupy the same duct, manhole or handhole of conduit systems with electric light or power wires.

Single manholes, or handholes, may be separated into sections by means of partitions of brick or tile so as to be considered as conforming with the above rule.

The liability of accidental crossing of overhead signaling circuits with electric light and power circuits may be guarded against to a considerable extent by endeavoring to keep the two classes of circuits on different sides of the same street.

When the Entire Circuit from Central Station to Building is Run in Underground Conduits, Sections b to m Inclusive do Not Apply.

b. When outside wires are run on same pole with electric light or power wires, the distance between the two inside pins of each cross-arm must not be less than twenty-six inches.

Signaling wires being smaller and more liable to break and fall, should generally be placed on the lower cross-arms.

c. Where wires are attached to the outside walls of buildings they must have an *approved* rubber insulating covering (see No. 41), and on frame buildings or frame portions of other buildings shall be supported on glass or porcelain insulators, or knobs.

d. The wires from last outside support to the cut-outs or protectors must be of copper, and must have an *approved* rubber insulation (see No. 41); must be provided with drip loops immediately outside the building and at entrance; must be kept not less than 2½ inches apart, except when brought in through approved metal cables.

e. Wires must enter building through approved non-combustible, non-absorptive, insulating bushings sloping upward from the outside.

Installations where the Current-Carrying Parts of the Apparatus Installed are Capable of Carrying Indefinitely a Current of Ten Amperes.

f. An all-metallic circuit shall be provided, except in telegraph systems.

g. At the entrance of wires to buildings, *approved* single-pole cut-outs, designed for 251–600 volts potential and containing fuses rated at not over 10 amperes capacity, shall be provided for each wire. These cut-outs must not be placed in the immediate vicinity of easily ignitible stuff, or where

exposed to inflammable gases or dust or to flyings of combustible material.

h. The wires inside buildings shall be of copper not less than No. 16 B. & S. gage, and must have insulation and be supported, the same as would be required for an installation of electric light or power wiring, 0–600 volts potential.

i. The instruments shall be mounted on bases constructed of non-combustible, non-absorptive insulating material. Holes for the supporting screws must be so located, or countersunk, that there will be at least ½ inch space, measured over the surface, between the head of the screw and the nearest live metal part.

Installations where the Current-Carrying Parts of the Apparatus Installed are Not Capable of Carrying Indefinitely a Current of Ten Amperes.

j. Must be provided with an *approved* protective device located as near as possible to the entrance of wires to building. The protector must not be placed in the immediate vicinity of easily ignitible stuff, or where exposed to inflammable gases or dust or flyings of combustible materials.

k. Wires from entrance to building to protector must be supported on porcelain insulators, so that they will come in contact with nothing except their designed supports.

l. The ground wire of the protective device shall be run in accordance with the following requirements:—

1. Shall be of copper and not smaller than No. 18 B. & S. gage.
2. Must have an *approved* insulating covering as described in No. 41, for voltages from 0 to 600, except that the preservative compound specified in No. 41, Section h may be omitted.
3. Must run in as straight a line as possible to a good permanent ground.

   This may be obtained by connecting to a water or gas pipe connected to the street mains or to a ground rod or pipe driven in permanently damp earth. When connections are made to pipes, preference shall be given to water pipes. If attachment is made to gas pipe, the connection in all cases must be made between the meter and the street mains. In every case the connection shall be made as near as possible to the earth.

   When the ground wire is attached to water or gas pipes, these pipes shall be thoroughly cleaned and tinned with rosin flux solder, if such a method is practicable; the ground wire shall then be wrapped tightly around the pipe and thoroughly soldered to it.

   When the above method is impracticable, then, if there are fittings where a brass plug can be inserted, the ground wire shall be thoroughly soldered to it; if there are no such fittings, then the pipe shall be thoroughly cleaned and an approved ground clamp fastened to an exposed portion of the pipe and the ground wire well soldered to the ground clamp.

   When the ground wire is attached to a ground rod driven into the earth, the ground wire shall be soldered to the rod in a similar manner.

   Steam or hot water pipes must not be used for a protector ground.

m. The protector to be approved must comply with the following requirements:—

For Instrument Circuits of Telegraph Systems.—1. An *approved* single-pole cut-out, in each wire, designed for 2,000 volt potential, and containing fuses rated at not over one ampere capacity. When main line cut-outs are installed as called for in Section g, the instrument cut-outs may be placed between the switchboard and the instrument as near the switchboard as possible.

For All Other Systems.—1. Must be mounted on non-combustible, non-absorptive, insulating bases, so designed that when the protector is in place, all parts which may be alive will be thoroughly insulated from the wall to which the protector is attached.

2. Must have the following parts:—

   A lightning arrester which will operate with a difference of potential between wires of not over 500 volts, and so arranged that the chance of accidental grounding is reduced to a minimum.

A fuse designed to open the circuit in case the wires become crossed with light or power circuits. The fuse must be able to open the circuit without arcing or serious flashing when crossed with any ordinary commercial light or power circuit.

A heat coil, if the sensitiveness of the instrument demands it, which will operate before a sneak current can damage the instrument the protector is guarding.

Heat coils are necessary in all circuits normally closed through magnet windings, which cannot indefinitely carry a current of at least 5 amperes.

The heat coil is designed to warm up and melt out with a current large enough to endanger the instruments if continued for a long time, but so small that it would not blow the fuses ordinarily found necessary for such instruments. The smaller currents are often called "sneak" currents.

3. The fuses must be so placed as to protect the arrester and heat coils, and the protector terminals must be plainly marked "line," "instrument," "ground."

An easily read abbreviation of the above words will be allowed.

THE FOLLOWING RULES APPLY TO ALL SYSTEMS WHETHER THE WIRES FROM THE CENTRAL OFFICE TO THE BUILDING ARE OVERHEAD OR UNDERGROUND.

n. Wires beyond the protector, or wires inside buildings where no protector is used, must be neatly arranged and securely fastened in place in some convenient, workmanlike manner. They must not come nearer than 6 inches to any electric light or power wire in the building unless encased in *approved* tubing so secured as to prevent its slipping out of place.

The wires would ordinarily be insulated, but the kind of insulation is not specified, as the protector is relied upon to stop all dangerous currents. Porcelain tubing or *approved* flexible tubing may be used for encasing wires where required as above.

o. Wires where bunched together in a vertical run within any building must have a fire-resisting covering sufficient to prevent the wires from carrying fire from floor to floor unless they are run either in non-combustible tubing or in a fireproof shaft, which shaft shall be provided with fire stops at each floor.

Signaling wires and electric light or power wires may be run in the same shaft, provided that one of these classes of wires is run in non-combustible tubing, or provided that when run otherwise these two classes of wires shall be separated from each other by at least 2 inches.

In no case shall signaling wires be run in the same tube with electric light or power wires.

Ordinary rubber insulation is inflammable, and when a number of wires are contained in a shaft extending through a building they afford a ready means of carrying fire from floor to floor, unless they are covered with a fire-resisting material, or unless the shaft is provided with fire stops at each floor.

65. Electric Gas Lighting.—*a.* Electric gas lighting must not be used on the same fixture with the electric light.

The above rule does not apply to *frictional* systems of gas lighting.

65 A. Moving Picture Machines.—**a.** Arc lamp used as a part of moving picture machines must be constructed similar to arc lamps of theaters, and wiring of same must not be of less capacity than No. 6 B. & S. gage. (See No. 31 A d. [1].)

b. Rheostats must conform to rehostat requirements for theater arcs. (See No. 31 *A* d. [1].)

c. Top reel must be encased in a steel box with hole at the bottom only large enough for film to pass through, and cover so arranged that this hole can be instantly closed. No solder to be used in the construction of this box.

d. A steel box must be used, for receiving the film after being shown, with a hole in the top only large enough for the film to pass through freely, with a cover so arranged that this hole can be instantly closed. An opening may be placed at the side of the box to take the film out, with a door hung at the top, so arranged that it cannot be entirely opened, and provided with a spring-catch to lock it closed. No solder to be used in the construction of this box.

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e. The handle or crank used in operating the machine must be secured to the spindle or shaft, so that there will be no liability of its coming off and allowing the film to stop in front of lamp.

f. A shutter must be placed in front of the condenser, arranged so as to be readily closed.

g. Extra films must be kept in metal box with tight-fitting cover.

h. Machines must be operated by hand (motor-driven will not be permitted.)

i. Picture machine must be placed in an enclosure or house made of suitable fireproof material, be thoroughly ventilated and large enough for operator to walk freely on either side of or back of machine. All openings into this booth must be arranged so as to be entirely closed by doors or shutters constructed of the same or equally good fire-resisting material as the booth itself. Doors or covers must be arranged so as to be held normally closed by spring hinges or equivalent devices.

66. Insulation Resistance.—The wiring in any building must test free from grounds; *i. e.*, the complete installation must have an insulation between conductors and between all conductors and the ground (not including attachments, sockets, receptacles, etc.) not less than that given in the following table:—

Up to	5	amperes	4,000,000	ohms.
"	10	"	2,000,000	"
"	25	"	800,000	"
"	50	"	400,000	"
"	100	"	200,000	"
"	200	"	100,000	"
"	400	"	50,000	"
"	800	"	25,000	"
"	1,600	"	12,500	"

The test must be made with all cut-outs and safety devices in place. If the lamp sockets, receptacles, electroliers, etc., are also connected, only one-half of the resistances specified in the table will be required.

67. Soldering Fluid.—a. The following formula for soldering fluid is suggested:—

Saturated solution of zinc chloride	5 parts.
Alcohol	4 parts.
Glycerine	1 part.

Class F.—MARINE WORK.

68. Generators.—a. Must be located in a dry place.

b. Must have their frames insulated from their bed-plates.

c. Must each be proivded with a waterproof cover.

d. Must each be provided with a name-plate, giving the maker's name, the capacity in volts and amperes, and the normal speed in revolutions per minute.

69. Wires.—a. Must be supported in *approved* moulding or conduit, except at switchboards and for portables.

Special permission may be given for deviation from this rule in dynamo rooms.

b. Must have no single wire larger than No. 12 B. & S. gage. Wires to be stranded when greater carrying capacity is required. No single solid wire smaller than No. 14 B. & S. gage, except in fixture wiring, to be used.

Stranded wires must be soldered before being fastened under clamps or binding screws, and when they have a conductivity greater than that of No. 8 B. & S. gage copper wire they must be soldered into lugs.

c. Splices or taps in conductors must be avoided as far as possible. Where it is necessary to make them they must be so spliced or joined as to be both mechanically and electrically secure without solder. They must then be soldered, to insure preservation, covered with an insulating compound equal to the insulation of the wire, and further protected by a waterproof tape. The joint must then be coated or painted with a waterproof compound.

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All joints must be soldered unless made with some form of *approved* splicing device.

For Moulding Work.—**d.** Must have an *approved* insulating covering.

The insulation for conductors, to be approved, must be at least 3-32 of an inch in thickness and be covered with a substantial waterproof braid.

The physical characteristics shall not be affected by any change in temperature up to 200° Fahrenheit (93° Centigrade). After two weeks' submersion in salt water at 70° Fahrenheit (21° Centigrade), it must show an insulation resistance of 100 megohms per mile after three minutes' electrification with 550 volts.

e. Must have, when passing through water-tight bulkheads and through all decks, a metallic stuffing tube lined with hard rubber. In case of deck tubes, they must be boxed near deck to prevent mechanical injury.

f. Must be bushed with hard rubber tubing, ⅛ of an inch in thickness, when passing through beams and non-water-tight bulkheads.

For Conduit Work.—**g.** Must have an *approved* insulating covering.

The insulation for conductors, for use in lined conduits, to be approved, must be at least 3-32 of an inch in thickness and be covered with a substantial waterproof braid. The physical characteristics shall not be affected by any change in temperature up to 200° Fahrenheit (93° Centigrade).

After two weeks' submersion in salt water at 70° Fahrenheit (21° Centigrade), it must show an insulation resistance of 100 megohms per mile after three minutes' electrification with 550 volts.

For unlined metal conduits, conductors must conform to the specifications given for lined conduits, and in addition have a second outer fibrous covering at least 1/32 of an inch in thickness and sufficiently tenacious to withstand the abrasion of being hauled through the metal conduit.

h. Must not be drawn in until the mechanical work on the conduit is completed and same is in place.

i. Where run through coal bunkers, boiler rooms, and where they are exposed to severe mechanical injury, must be encased in *approved* conduit.

70. Portable Conductors.—**a.** Must be made of two stranded conductors each having a carrying capacity equivalent to not less than No. 14 B. & S. gage, and each covered with an *approved* insulation and covering.

Where not exposed to moisture or severe mechanical injury, each stranded conductor must have a solid insulation at least 1-32 of an inch in thickness, and must show an insulation resistance between conductors, and between either conductor and the ground, of at least 50 megohms per mile after two weeks' submersion in water at 70° Fahrenheit (21° Centigrade), and be protected by a slow-burning, tough-braided outer covering.

Where exposed to moisture and mechanical injury (as for use on decks, holds and fire-rooms), each stranded conductor shall have a solid insulation, to be approved, of at least 1-32 of an inch in thickness and protected by a tough braid. The two conductors shall then be stranded together, using a jute filling. The whole shall then be covered with a layer of flax, either woven or braided, at least 1-32 of an inch in thickness and treated with a non-inflammable, waterproof compound. After one week's submersion in water at 70° Fahrenheit (21° Centigrade), it must show an insulation between the two conductors, or between either conductor and the ground, of 50 megohms per mile.

71. Bell or Other Wires—**a.** Must never be run in same duct with lighting or power wires.

72. Table of Capacity of Wires.—

B. & S. G.	Area Actual C. M.	No. of Strands.	Size of Strands. B. & S. G.	Amperes.
19	1,288	..	..	..
18	1,624	..	..	3
17	2,048	..	..	..
16	2,583	..	..	6
15	3,257	..	..	..
14	4,107	..	..	12
12	6,530	..	..	17
..	9,016	7	19	21
..	11,368	7	18	25
..	14,336	7	17	30
..	18,081	7	16	35
..	22,799	7	15	40
..	30,856	19	18	50

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B. & S. G.	Area Actual C. M.	No. of Strands	Size of Strands B. & S. G.	Amperes.
..	38,912	19	17	60
..	49,077	19	16	70
..	60,088	37	18	85
..	75,776	37	17	100
..	99,064	61	18	120
..	124,928	61	17	145
..	157,563	61	16	170
..	198,677	61	15	200
..	250,527	61	14	235
..	296,387	91	15	270
..	373,737	91	14	320
..	413,639	127	15	340

When greater conducting area than that of No. 12 B. & S. gage is required, the conductor shall be stranded in a series of 7, 19, 37, 61, 91, or 127 wires, as may be required; the strand consisting of one central wire, the remainder laid around it concentrically, each layer to be twisted in the opposite direction from the preceding.

73. Switchboards.—**a.** Must be made of non-combustible, non-absorptive insulating material, such as marble or slate.

b. Must be kept free from moisture, and must be located so as to be accessible from all sides.

c. Must have a main switch, main cut-out and ammeter for each generator.

Must also have a voltmeter and ground detector.

d. Must have a cut-out and switch for each side of each current leading from board.

e. Must be wired with conductors having an insulation as required for moulding or conduit work and covered with a substantial flame-proof braid.

74. Resistance Boxes.—(For construction rules, see No. 60.) **a.** Must be located on switchboard or away from combustible material. When not placed on switchboard they must be mounted on non-inflammable, non-absorptive insulating material.

75. Switches.—(For construction rules, see No. 51) **a.** When exposed to dampness, they must be enclosed in a water-tight case.

b. Must be of the knife pattern when located on switchboard.

c. Must be provided so that each freight compartment may be separately controlled.

76. Cut-Outs.—(For construction rules, see No. 52.) **a.** Must be placed at every point where a change is made in the size of the wire (unless the cut-out in the larger wire will protect the smaller).

b. In such places as upper decks, holds, cargo spaces and fire-rooms, a water-tight and fire-proof cut-out may be used, connecting directly to mains when such cut-out supplies circuits requiring not more than 660 watts energy.

c. When placed anywhere except on switchboards and certain places, as cargo spaces, holds, fire-rooms, etc., where it is impossible to run from center of distribution, they must be in a cabinet lined with fire-resisting material.

d. Except for motors, searchlights and diving lamps must be so placed that no group of lamps, requiring a current of more than 660 watts, shall ultimately be dependent upon one cut-out.

77. Fixtures.—**a.** Must be mounted on blocks made from well-seasoned umber treated with two coats of white lead or shellac.

b. Where exposed to dampness, the lamp must be surrounded by a vapor-proof globe.

c. Where exposed to mechanical injury, the lamp must be surrounded by a globe protected by a stout wire guard.

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d. Must be wired with same grade of insulation as portable conductors which are not exposed to moisture or mechanical injury.

e. Ceiling fixtures over two feet in length must be provided with stay chains.

78. Sockets.—(For construction rules, see No. 55.)

79. Wooden Mouldings.—(For construction rules, see No. 50.)

a. Where moulding is run over rivets, beams, etc., a backing strip must first be put up and the moulding secured to this.

b. Capping must be secured by brass screws.

80. Interior Conduits.—(For installation rules, see No. 25.) (For construction rules, see No. 49.)

81. Signal Lights.—a. Must be provided with *approved* telltale board, located preferably in pilot house, which will immediately indicate a burned-out lamp.

82. Motors.—a. Must be wired under the same precautions as with a current of same volume and potential for lighting. The motor and resistance box must be protected by a double-pole cut-out and controlled by a double-pole switch, except in cases where one-quarter horse power or less is used.

The motor leads or branch circuits must be designed to carry a current at least 25 per cent greater than that for which the motor is rated, in order to provide for the inevitable occasional overloading of the motor, and the increased current required in starting, without overfusing the wires, but where the wires under this rule would be overfused, in order to provide for the starting current, as in the case of many of the alternating current motors, the wires must be of such size as to be properly protected by these larger fuses.

In general, motors should preferably have no exposed live parts.

b. Must be thoroughly insulated. Where possible, should be set on base frames made from filled, hard, dry wood and raised above surrounding deck. On hoists and winches they must be insulated from bed-plates by hard rubber, fiber or similar insulating material.

c. Must be covered with a waterproof cover when not in use.

d. Must each be provided with a name-plate, giving maker's name, the capacity in volts and amperes, and the normal speed in revolutions per minute.

83. Insulation Resistance.—The wiring in any vessel must test free from grounds; *i. e.*, the complete installation must have an insulation between conductors and between all conductors and the ground (not including attachments, sockets, receptacles, etc.), of not less than the following:—

Up to	25	amperes	800,000	ohms.
"	50	"	400,000	"
"	100	"	200,000	"
"	200	"	100,000	"
"	400	"	50,000	"
"	800	"	25,000	"
"	1,600	"	12,500	"

All cut-outs and safety devices in place in the above.

Where lamp sockets, receptacles and electroliers, etc., are connected, one-half of the above will be required.

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ELECTRICAL STANDARDIZATION.

STANDARDIZATION RULES OF THE AM. INST. OF ELEC. ENGRS.
Approved by the Board of Directors, June 21, 1907.
Accepted by the 24th Annual Convention, June 27, 1907.

I.—DEFINITIONS AND TECHNICAL DATA.

1 *Note.*—The following definitions and classifications are intended to be practically descriptive and not scientifically rigid.

A.—DEFINITIONS. CURRENTS.

2 A *Direct Current* is a unidirectional current.
3 A *Continuous Current* is a steady, or non-pulsating, direct current.
4 A *Pulsating Current* is a current equivalent to the superposition of an alternating current upon a continuous current.
5 An *Alternating Current* is a current which, when plotted, consists of half-waves of equal area in successively opposite directions from the zero line.
6 An *Oscillating Current* is a current alternating in direction, and of decreasing amplitude.

B.—DEFINITIONS. ROTATING MACHINES.

7 A *Generator* transforms mechanical power into electrical power.
8 A *Direct-Current Generator* produces a direct current that may or may not be continuous.
9 An *Alternator* or *Alternating-Current Generator* produces alternating current, either single-phase or polyphase.
10 A *Polyphase Generator* produces currents differing symmetrically in phase; such as two-phase currents, in which the terminal voltages on the two circuits differ in phase by 90°; or three-phase currents, in which the terminal voltages on the three circuits differ in phase by 120°.
11 A *Double-Current Generator* produces both direct and alternating currents.
12 A *Motor* transforms electrical into mechanical power.
13 A *Booster* is a machine inserted in series in a circuit to change its voltage. It may be driven by an electric motor (in which case it is termed a motor-booster) or otherwise.
14 A *Motor-Generator* is a transforming device consisting of a motor mechanically connected to one or more generators.
15 A *Dynamotor* is a transforming device combining both motor and generator action in one magnetic field, with two armatures; or with an armature having two separate windings and independent commutators.
16 A *Converter* is a machine employing mechanical rotation in changing electrical energy from one form to another. A converter may belong to either of several types, as follows:
17 *a.* A *Direct-Current Converter* converts from a direct current to a direct current.
18 *b.* A *Synchronous Converter* (commonly called a rotary converter) converts from an alternating to a direct current, or *vice versa*.
19 *c.* A *Motor-Converter* is a combination of an induction motor with a synchronous converter, the secondary of the former feeding the armature of the latter with current at some frequency other than the impressed frequency; *i. e.*, it is a synchronous converter concatenated with an induction motor.
20 *d.* A *Frequency-Converter* converts from an alternating-current system of one frequency to an alternating-current system of another frequency, with or without a change in the number of phases or in voltages.
21 *e.* A *Rotary Phase Converter* converts from an alternating-current system of one or more phases to an alternating-current system of a different number of phases, but of the same frequency.

C.—DEFINITIONS. STATIONARY INDUCTION APPARATUS.

22 *Stationary Induction Apparatus* change electric energy to electric energy through the medium of magnetic energy. They comprise several forms, distinguished as follows:

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23 *a.* In *Transformers* the primary and secondary windings are insulated from one another.

24 *b.* In *Auto-Transformers*, also called compensators, a part of the primary winding is used as a secondary winding, or conversely.

25 *c.* In *Potential Regulators* a coil is in shunt and a coil is in series with the circuit, so arranged that the ratio of transformation between them is variable at will. They are of the following three classes:

26 (*a*) *Compensator Potential Regulators* in which a number of turns of one of the coils are adjustable.

27 (*b*) *Induction Potential Regulators* in which the relative positions of the primary and secondary coils are adjustable.

28 (*c*) *Magneto Potential Regulators* in which the direction of the magnetic flux with respect to the coils is adjustable.

29 *d. Reactors*, or *Reactance Coils*, formerly called choking coils, are a form of stationary induction apparatus used to produce reactance or phase displacement.

D.—GENERAL CLASSIFICATION OF APPARATUS.

30 *Commutating Machines*. Under this head may be classed the following: Direct current generators; direct-current motors; direct-current boosters; motor-generators; dynamotors; converters, compensators or balancers; closed-coil arc machines, and alternating-current commutating motors.

31 Commutating machines may be further classified as follows:

32 *a. Direct-Current Commutating Machines*, which comprise a magnetic field of constant polarity, a closed-coil armature, and a multisegmental commutator connected therewith.

33 *b. Alternating-Current Commutating Machines*, which comprise a magnetic field of alternating polarity, a closed-coil armature, and a multisegmental commutator connected therewith.

34 *c. Synchronous Commutating Machines*, which comprise synchronous converters, motor converters and double-current generators.

35 *Synchronous Machines*, which comprise a constant magnetic field, and an armature receiving or delivering alternating-currents in synchronism with the motion of the machine; *i.e.*, having a frequency equal to the product of the number of pairs of poles and the speed of the machine in revolutions per second.

36 *Stationary Induction Apparatus*, which includes transformers, auto-transformers, potential regulators, and reactors or reactance coils.

37 *Rotary Induction Apparatus*, or *Induction Machines*, which include apparatus wherein the primary and secondary windings rotate with respect to each other; *i.e.*, induction motors, induction generators, frequency converters, and rotary phase converters.

38 *Unipolar* or *Acyclic Machines*, in which the voltage generated in the active conductors maintains the same direction with respect to those conductors.

39 *Rectifying Apparatus, Pulsating-Current Generators.*

40 *Electrostatic Apparatus*, such as condensers, etc.

41 *Electrochemical Apparatus*, such as batteries, etc.

42 *Electrothermal Apparatus*, such as rheostats, heaters, etc.

43 *Protective Apparatus*, such as fuses, lightning arresters, etc.

44 *Luminous Sources.*

E.—MOTORS. SPEED CLASSIFICATION.

45 *Motors* may, for convenience, be classified with reference to their speed characteristics as follows:

46 *a. Constant-Speed Motors*, in which the speed is either constant or does not materially vary; such as synchronous motors, induction motors with small slip, and ordinary direct-current shunt motors.

47 *b. Multispeed Motors* (two-speed, three-speed, etc.), which can be operated at any one of several distinct speeds, these speeds being practically independent of the load, such as motors with two armature windings.

48 *c. Adjustable-Speed Motors*, in which the speed can be varied gradually over a considerable range; but when once adjusted remains practically unaffected by the load, such as shunt motors designed for a considerable range of field variation.

49 *d. Varying-Speed Motors*, or motors in which the speed varies with the load, decreasing when the load increases; such as series motors.

F.—DEFINITION AND EXPLANATION OF TERMS.

(I) Load Factor.

50 The *Load Factor* of a machine, plant or system is the ratio of the average power to the maximum power during a certain period of time. The average power is taken over a certain interval of time, such as a day or a year, and the maximum is taken over a short interval of the maximum load within that interval.

51 In each case the interval of maximum load should be definitely specified. The proper interval is usually dependent upon local conditions and upon the purpose for which load factor is to be determined.

(II) Non-Inductive Load and Inductive load.

52 A non-inductive load is a load in which the current is in phase with the voltage across the load.

53 An inductive load is a load in which the current lags behind the voltage across the load. A load in which the current leads the voltage across the load is sometimes called in anti-inductive load.

(III) Power-Factor and Reactive-Factor.

54 The *Power-Factor* in alternating-current circuits or apparatus is the ratio of the electric power in watts to the apparent power in volt-amperes. It may be expressed as follows:

$$\frac{\text{true power}}{\text{apparent power}} = \frac{\text{watts}}{\text{volt-amperes}} = \frac{\text{energy current}}{\text{total current}} = \frac{\text{energy voltage}}{\text{total voltage}}$$

55 The *Reactive-Factor* is the ratio of the wattless volt-amperes (*i.e.*, the product of the wattless component of current by voltage, or wattless component of voltage by current) to the total amperes. It may be expressed as follows:

$$\frac{\text{wattless volt-amperes}}{\text{total volt-amperes}} = \frac{\text{wattless current}}{\text{total current}} = \frac{\text{wattless voltage}}{\text{total voltage}}.$$

56 *Power-Factor* and *Reactive-Factor* are related as follows:
If p = power-factor, q = reactive-factor, then with sine waves of voltage and current,

$$p^2 + q^2 = 1.$$

With distorted waves of voltage and current,

$$p^2 + q^2 = \text{or} < 1.$$

(IV) Saturation-Factor.

57 The *Saturation-Factor* of a machine is the ratio of a small percentage increase in field excitation to the corresponding percentage increase in voltage thereby produced. The saturation-factor is, therefore, a criterion of the degree of saturation attained in the magnetic circuits at any excitation selected. Unless otherwise specified, however, the saturation-factor of a machine refers to the excitation existing at normal rated speed and voltage. It is determined from measurements of saturation made on open circuit at rated speed.

58 The *Percentage of Saturation* of a machine at any excitation may be found from its saturation curve of generated voltage as ordinates, against excitation as abscissas, by drawing a tangent to the curve at the ordinate corresponding to the assigned excitation, and extending the tangent to intercept the axis of ordinates drawn through the origin. The ratio of the intercept on this axis to the ordinate at the assigned excitation, when expressed in percentage, is the percentage of saturation and is independent of the scale selected for excitation and voltage. This ratio is equal to the reciprocal of the saturation-factor at the same excitation, deducted from unity. Thus, if f be the saturation-factor and p the percentage of saturation ratio,

$$p = 1 - \frac{1}{f}.$$

(V) Variation and Pulsation.

59 The *Variation in Prime Movers* which do not give an absolutely uniform rate of rotation or speed, as in reciprocating steam engines, is the maximum angular displacement in position of the revolving member

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expressed in degrees, from the position it would occupy with uniform rotation, and with one revolution taken as 360°.

60 The *Pulsation in Prime Movers* is the ratio of the difference between the maximum and minimum velocities in an engine-cycle to the average velocity.

61 The *Variation in Alternators* or alternating-current circuits in general is the maximum difference in phase of the generated voltage wave from a wave of absolutely constant frequency, expressed in electrical degrees (one cycle equals 360°) and may be due to the variation of the prime mover.

62 The *Pulsation in Alternators* or alternating-current circuits, in general, is the ratio of the difference between maximum and minimum frequency during an engine cycle to the average frequency.

63 *Relation of Variation* in prime mover and alternator:

64 If n = number of pairs of poles, the variation of an alternator is n times the variation of its prime mover, if direct-connected, and n/p times the variation of the prime mover if rigidly connected thereto in the velocity ratio p.

II.—PERFORMANCE SPECIFICATIONS AND TESTS.

A.—RATING.

65 *Rating by Output.* All electrical apparatus should be rated by output and not by input. Generators, transformers, etc., should be rated by electrical output; motors by mechanical output.

66 *Rating in Kilowatts.* Electrical power should be expressed in kilowatts, except when otherwise specified.

67 *Apparent Power, Kilovolt-Amperes.* Apparent power in alternating-current circuits should be expressed in kilovolt-amperes as distinguished from real power in kilowatts. When the power factor is 100 per cent., the apparent power in kilovolt-amperes is equal to the kilowatts.

68 The *Rated (Full Load) Current* is that current which, with the rated terminal voltage, gives the rated kilowatts, or the rated kilovolt-amperes. In machines in which the rated voltage differs from the no-load voltage, the rated current should refer to the former.

69 *Determination of Rated Current.* The rated current may be determined as follows: If P = rating in watts, or apparent watts if the power factor be other than 100 per cent., and E = full-load terminal voltage, the rated current per terminal is:

70 $I = \frac{P}{E}$ in a direct-current machine or single-phase alternator.

71 $I = \frac{1}{\sqrt{3}} \frac{P}{E}$ in a three-phase alternator.

72 $I = \frac{1}{2} \frac{P}{E}$ in a two-phase alternator.

73 *Normal Conditions.* The rating of machines or apparatus should be based upon certain normal conditions to be assumed as standard, or to be specified. These conditions include voltage, current, power-factor, frequency, wave shape and speed; or such of them as may apply in each particular case. Performance tests should be made under these standard conditions unless otherwise specified.

74 *a. Power Factor.* Alternating-current apparatus should be rated in kilowatts, at 100 per cent power factor; *i.e.*, with current in phase with terminal voltage, unless a phase displacement is inherent in the apparatus or is specified. If a power factor other than 100 per cent is specified, the rating should be expressed in kilovolt-amperes and power factor, at rated load.

75 *b. Wave Shape.* In determining the rating of alternating-current machines or apparatus, a sine wave shape of alternating current and voltage is assumed, except where a distorted wave shape is inherent to the apparatus. See Secs. 79-83.

76 *Fuses.* The rating of a fuse should be tne maximum current which it will continuously carry.

77 *Circuit-Breakers.* The rating of a circuit-breaker should be the maximum current which it is designed to carry continuously.

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78 *a. Note.* In addition thereto, the maximum current and voltage at which a fuse or a circuit-breaker will open the circuit should be specified. It is to be noted that the behavior of fuses and of circuit-breakers is much influenced by the amount of electric power available on the circuit.

B.—WAVE SHAPE.

79 The *Sine Wave* should be considered as standard, except where a difference in the wave form from the sinusoidal is inherent in the operation of the apparatus.

80 A *Maximum Deviation* of the wave from sinusoidal shape not exceeding 10 per cent is permissible, except when otherwise specified.

81 The *Deviation* of wave form from the sinusoidal is measured by determining the form by oscillograph or wave meter, computing therefrom the equivalent sine wave of equal length, superposing the latter upon the observed wave in such a manner as to give least difference, and then dividing the maximum difference at any ordinate by the maximum value of the equivalent sine wave.

82 The *Equivalent Sine Wave* is a sine wave having the same frequency and the same effective or r.m.s. (root of mean square) value as the actual wave.

83 *Non-Sine Waves.* The phase displacement between two waves which are not sine waves, is that phase displacement between their equivalent sine waves which would give the same average product of instantaneous values as the actual waves; *i.e.*, the same electro-dynamometer reading.

C.—EFFICIENCY.

(I)—Definitions.

84 The *Efficiency* of an apparatus is the ratio of its net power output to its gross power input.

85 *a. Note.* An exception should be noted in the case of storage batteries or apparatus for storing energy in which the efficiency, unless otherwise qualified, should be understood as the ratio of the energy output to the energy intake in a normal cycle. An exception should also be noted in the case of luminous sources.

86 *Apparent Efficiency.* In apparatus in which a phase displacement is inherent to their operation, apparant efficiency should be understood as the ratio of the net power output to volt-ampere input.

87 *a. Note.* Such apparatus comprise induction motors, reactive synchronous converters, synchronous converters controlling the voltage of an alternating-current system, self-exciting synchronous motors, potential regulators and open magnetic circuit transformers, etc.

88 *b. Note.* Since the apparent efficiency of apparatus delivering electric power depends upon the power-factor of the load, the apparent efficiency, unless otherwise specified, should be referred to a load power-factor of unity.

(II)—Determination of Efficiency.

89 *Methods.* Efficiency may be determined by either of two methods. viz.: by measurement of input and output; or, by measurement of losses.

90 *a. Method of Input and Output.* The input and output may both be measured directly. The ratio of the latter to the former is the efficiency.

91 *b. Method by Losses.* The losses may be measured either collectively or individually. The total losses may be added to the output to derive the input, or subtracted from the input to derive the output.

92 *Comparison of Methods.* The output and input method is preferable with small machines. When, however, as in the case of large machines, it is impracticable to measure the output and input; or when the percentage of power loss is small and the efficiency is nearly unity, the method of determining efficiency by measuring the losses should be followed.

93 *Electric Power* should be measured at the terminals of the apparatus. In tests of polyphase machines, the measurement of power should not be confined to a single circuit but should be extended to all the circuits in order to avoid errors of unbalanced loading.

94 *Mechanical Power* in machines should be measured at the pulley, gearing, coupling, etc., thus excluding the loss of power in said pulley gearing or coupling, but including the bearing friction and windage. The

magnitude of bearing friction and windage may be considered, with constant speed, as independent of the load. The loss of power in the belt and the increase of bearing friction due to belt tension should be excluded. Where, however, a machine is mounted upon the shaft of a prime mover, in such a manner that it cannot be separated therefrom, the frictional losses in bearings and in windage, which ought, by definition, to be included in determining the efficiency, should be excluded, owing to the practical impossibility of determining them satisfactorily.

95 In *Auxiliary Apparatus*, such as an exciter, the power lost in the auxiliary apparatus should not be charged to the principal machine, but to the plant consisting of principal machine and auxiliary apparatus taken together. The plant efficiency in such cases should be distinguished from the machine efficiency.

96 *Normal Conditions.* Efficiency tests should be made under normal conditions herein set forth and which are to be assumed as standard. These conditions include voltage, current, power-factor, frequency, wave shape, speed and barometric pressure, temperature, or such of them as may apply in each particular case. Performance tests should be made under these standard conditions unless otherwire specified. See Secs. 73-75.

97 *a. Temperature.* The efficiency of all apparatus, except such as may be intended for intermittent service, should be either measured at, or reduced to, the temperature which the apparatus assumes under continuous operation at rated load, referred to a room temperature of 25° C. See Secs. 267-292.

98 With apparatus intended for intermittent service, the efficiency should be determined at the temperature assumed under specified conditions.

99 *b. Power Factor.* In determining the efficiency of alternating-current apparatus, the electric power should be measured when the current is in phase with the voltage, unless otherwise specified, except when a definite phase difference is inherent in the apparatus, as in induction motors, induction generators, frequency converters, etc.

100 *c. Wave Shape.* In electrical apparatus, the sine wave should be considered as standard, except where a difference in the wave form from the sinusoidal is inherent in the operation of the apparatus. See Secs. 79-83.

(III)—Measurement of Losses.

101 *Losses.* The usual sources of losses in electrical apparatus and the methods of determining these losses are as follows:

(*A*)—Bearing Friction and Windage.

102 The magnitude of bearing friction and windage (which may be considered as independent of the load) is conveniently measured by driving the machine from an independent motor, the output of which may be suitably determined. See Sec. 94.

(*B*)—Commutator Brush Friction.

103 The magnitude of the commutator brush friction (which may be considered as indepednent of the load) is determined by measuring the difference in power required for driving the machine with brushes on and with brushes off (the field being unexcited).

(*C*)—Collector-Ring Brush Friction.

104 Collector-ring brush friction may be determined in the same manner as commutator brush friction. It is usually negligible.

(*D*)—Molecular Magnetic Friction and Eddy Currents.

105 These losses include those due to molecular magnetic friction and eddy currents in iron and copper and other metallic parts, also the losses due to currents in the cross-connections of cross-connected armatures.

106 In *Machines* these losses should be determined on open circuit and at a voltage equal to the rated voltage $+ I r$ in a generator, and $- I r$ in a motor, where I denotes the current strength and r denotes the internal resistance of the machine. They should be measured at the correct speed and voltage, since they do not usually vary in any definite proportion to the speed or to the voltage.

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107 *Note.* The *Total Losses* in bearing friction and windage, brush friction, magnetic friction and eddy currents can, in general. be determined by a single measurement by driving the machine with the field excited, either as a motor, of by means of an independent motor.

108 *Retardation Method.* The no-load iron, friction, and windage losses may be segregated by the Retardation Method, in which the generator should be brought up to full speed (or, if possible, to about 10 per cent. above full speed) as a motor, and, after cutting off the driving power and excitation, frequent readings should be taken of speed and time, as the machine slows down, from which a speed-time curve can be plotted. A second curve should be taken in the same manner, but with full field excitation; from the second curve the iron losses may be found by subtracting the losses found in the first curve.

109 The speed-time curves can be plotted automatically by belting a small separately excited generator (say 1/10 kw.) to the generator shaft and connecting it to a recording voltmeter. When the retardation method is not feasible, the frictional losses in bearings and in windage, which ought, by definition, to be included in determining the efficiency, may be excluded; but this should be expressly stated.

(*E*)—Armature-Resistance Loss.

110 This loss may be expressed by $p I^2 r$; where r = resistance of one armature circuit or branch, I = the current in such armature circuit or branch, and p = the number of armature circuits or branches.

(*F*)—Commutator Brush and Brush-Contact Resistance Loss.

111 It is desirable to point out that with carbon brushes these losses may be considerable in low-voltage machines.

(*G*)—Collector-Ring and Brush-Contact Resistance Loss.

112 This loss is usually negligible, except in machines of extremely low voltage or in unipolar machines.

(*H*)—Field Excitation Loss.

113 With separately excited fields, the loss of power in the resistance of the field coils alone should be considered. With either shunt- or series-field windings, however, the loss of power in the accompanying rheostat should also be included, the said rheostat being considered as an essential part of the machine, and not as separate auxiliary apparatus.

(*I*)—Load Losses.

114 The load losses may be considered as the difference between the total losses under load and the sum of the losses above specified.

115 *a.* In *Commutating Machines* of small field distortion, the load losses are usually trivial and may, therefore, be neglected. When, however, the field distortion is large, as is shown, for instance, by the necessity for shifting the brushes between no load and full load, or with variations of load, these load losses may be considerable, and should be taken into account. In this case the efficiency may be determined either by input and output measurements, or the load losses may be estimated by the method of Sec. 116.

116 *b. Estimation of Load Losses.* While the load losses cannot well be determined individually, they may be considerable and, therefore, their joint influence should be determined by observation. This can be done by operating the machine on short-circuit and at full-load current, that is, by determining what may be called the "short-circuit core loss." With the low field intensity and great lag of current existing in this case, the load losses are usually greatly exaggerated.

117 One-third of the short-circuit core loss may, as an approximation, and in the absence of more accurate information, be assumed as the load loss.

(IV)—Efficiency of Different Types of Apparatus.

(A)—Direct-Current Commutating Machines.

118 In *Direct-Current Commutating Machines* the losses are:

119 *a. Bearing Friction and Windage.* See measurement of Losses (*A*), Sec. 102.

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120 *b. Molecular Magnetic Friction and Eddy Currents.* See measurement of Losses (*D*) Sec. 105.

121 *c. Armature Resistance Losses.* See Measuremeat of Losses (*E*), Sec. 110.

122 *d. Commutator Brush Friction.* See Measurement of Losses (*B*), Sec. 103.

123 *e. Commutator Brush and Brush Contact Resistance.* See Measurement of Losses (*F*), Sec. 111.

124 *f. Field Excitation Loss.* See Measurement of Losses (*H*), Sec. 113.

125 *g. Load Losses.* See Measurement of Losses (*I*), Sec, 114.

126 *Note. b* and *c* are losses in the armature or "armature losses"; *d* and *e* "commutator losses"; *f* "field losses."

(*B*)—ALTERNATING-CURRENT COMMUTATING MACHINES.

127 In *Alternating-Current Commutating Machines*, the losses are:

128 *a. Bearing Friction and Windage.* See Measurement of Losses (*A*), Sec. 102.

129 *b. Rotation Loss*, measured with the machine at open circuit, the brushes on the commutator, and the field excited by alternating current when driving the machine by a motor.

130 This loss includes molecular magnetic friction, and eddy currents, caused by rotation through the magnetic field, I^2r losses in cross-connections of cross-connected armatures, I^2r and other losses in armature-coils and armature-leads which are short-circuited by the brushes as far as these losses are due to rotation.

131 *c. Alternating* or *Transformer Loss.* These losses are measured by wattmeter in the field circuit, under the conditions of test *b*. They include molecular magnetic friction and eddy-currents due to the alternation of the magnetic field, I^2r losses in cross-connections of cross-connected armatures, I^2r and other losses in armature coil and commutator leads which are short-circuited by the brushes, as far as these losses are due to the alteration of the magnetic flux.

132 The losses in armature coils and commutator leads short-circuited by the brushes, can be separated in *b*, and *c*, from the other losses, by running the machine with and without brushes on the commutator.

133 *d. I^2r Loss, Other Load Losses* in armature and compensating winding and I^2r loss of brushes, measured by wattmeter connected across the armature and compensating winding.

134 *e. Field Excitation Loss.* See Measurement of Losses (*H*), Sec. 113.

135 *f. Commutator Brush-Friction.* See measurement of Losses (*B*), Sec. 103.

(*C*)—SYNCHRONOUS COMMUTATING MACHINES.

136 1. In *Double-Current Generators*, the efficiency of the machine should be determined as a direct-current generator, and also as an alternating-current generator. The two values of efficiency may be different, and should be clearly distinguished.

137 2. In *Converters* the losses should be determined when driving the machine by a motor. These losses are:

138 *a. Bearing Friction and Windage.* See Measurement of Losses (*A*). Sec. 102.

139 *b. Molecular Magnetic Friction and Eddy Currents.* See Measurement of Losses (*D*), Sec. 105.

140 *c. Armature Resistance Loss.* This loss in the armature is $q\,I^2r$, where I = direct current in armature, r = armature resistance and q, a factor which is equal to 1.47 in single-circuit single-phase, 1.15 in double-circuit single-phase, 0.59 in three-phase, 0.39 in two-phase, and 0.27 in six-phase converters.

141 *d. Commutator-Brush Friction.* See Measurement of Losses (*B*). Sec. 103.

142 *e. Collector-Ring Brush Friction.* See Measurement of Losses (*C*) Sec. 104.

143 *f. Commutator-Brush and Brush-Contact Resistance Loss.* See Measurement of Losses (*F*), Sec. 111.

144 *g. Collector-Ring Brush-Contact Resistance Loss.* See Measurement of Losses (*G*), Sec. 112.

145 *h. Field Excitation Loss.* See Measurement of Losses (*H*), Sec. 113

146 *i. Load Losses.* These can generally be neglected, owing to the absence of field distortion.

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147 3. *The Efficiency of Two Similar Converters* may be determined by operating one machine as a converter from direct to alternating, and the other as a converter from alternating to direct, connecting the alternating sides together, and measuring the difference between the direct-current input, and the direct-current output. This process may be modified by returning the output of the second machine through two boosters into the first machine and measuring the losses. Another modification is to supply the losses by an alternator between the two machines, using potential regulators.

(*D*)—SYNCHRONOUS MACHINES.

148 In *Synchronous Machines* the losses are:

149 *a. Bearing Friction and Windage.* See Measurement of Losses (*A*), Sec. 102.

150 *b. Molecular Magnetic Friction and Eddy Currents.* See Measurement of Losses (*D*), Sec. 105.

151 *c. Armature Resistance Loss.* See Measurement of Losses (*E*), Sec. 110.

152 *d. Collector-Ring Brush Friction.* See Measurement of Losses (*C*), Sec. 104.

153 *e. Collector-Ring Brush Contact Resistance Loss.* See Measurement of Losses (*G*), Sec. 112.

154 *f. Field Excitation Loss.* See Measurement of Losses (*H*), Sec. 113.

155 *g. Load Losses.* See Measurement of Losses (*I*), Sec. 114.

(*E*)—STATIONARY INDUCTION APPARATUS.

156 In *Stationary Induction Apparatus*, the losses are:

157 *a. Molecular Magnetic Friction and Eddy Currents* measured at open secondary circuit, rated frequency, and at rated voltage $-I\,r$, where I = rated current, r = resistance of primary circuit.

158 *b. Resistance Losses*, the sum of the I^2r losses in the primary and in the secondary windings of a transformer, or in the two sections of the coil in a compensator or auto-transformer, where I = rated current in the coil or section of coil, and r = resistance.

159 *c. Load Losses, i. e.*, eddy currents in the iron and especially in the copper conductors, caused by the current at rated load. For practical purposes they may be determined by short-circuiting the secondary of the transformer and impressing upon the primary a voltage sufficient to send rated load current through the transformer. The loss in the transformer under these conditions measured by wattmeter gives the load losses $+ I^2r$ losses in both primary and secondary coils.

160 In *Closed Magnetic Circuit Transformers*, either of the two circuits may be used as primary when determining the efficiency.

161 In *Potential Regulators*, the efficiency should be taken at the maximum voltage for which the apparatus is designed, and with non-inductive load, unless otherwise specified.

(*F*)—ROTARY INDUCTION APPARATUS, OR INDUCTION MACHINES.

162 In *Rotary Induction Apparatus*, the losses are:

163 *a. Bearing Friction and Windage.* See Measurement of Losses (*A*), Sec. 102.

164 *b. Molecular Magnetic Friction and Eddy Currents* in iron, copper and other metallic parts; also I^2r losses which may exist in multiple-circuit windings. *a* and *b* together are determined by running the motor without load at rated voltage, and measuring the power input.

165 *c. Primary I^2R Loss*, which may be determined by measurement of the current and the resistance.

166 *d. Secondary I^2R Loss*, which may be determined as in the primary, when feasible; otherwise, as in squirrel-cage secondaries, this loss is measured as part of *e*.

167 *e. Load Losses; i. e.*, molecular magnetic friction, and eddy currents in iron, copper, etc., caused by the stray field of primary and secondary currents, and secondary I^2R loss when undeterminable under (*d*). These losses may for practical purposes be determined by measuring the total power, with the rotor short-circuited at standstill and a current in the primary circuit equal to the primary energy current at full load. The loss in the motor under these conditions may be assumed to be equal to the load losses $+ I^2r$ losses in both primary and secondary coils.

(G)—UNIPOLAR OR ACYCLIC MACHINES.

168 In *Unipolar Machines*, the losses are:

169 (a) *Bearing Friction and Windage*. See Measurement of Losses (A), Sec. 102.

170 (b) *Molecular Magnetic Friction and Eddy Currents*. See Measurement of Losses (E), Sec. 106.

171 (c) *Armature Resistance Losses*. See Measurement of Losses (E), Sec. 110.

172 (d) *Collector Brush Friction*. See Measurement of Losses (C), Sec. 104.

173 (e) *Collector Brush Contact Resistance*. See Measurement of Losses (G), Sec. 112.

174 (f) *Field-Excitation* as in Sec. 113. See Measurement of Losses (H), Sec. 113.

175 (g) *Load Losses*. See Measurement of Losses (I), Sec. 114.

(H)—RECTIFYING APPARATUS, PULSATING-CURRENT GENERATORS.

176 *This Division Includes*: open-coil arc machines and mechanical and other rectifiers.

177 In *Rectifiers* the most satisfactory method of determining the efficiency is to measure both electric input and electric output by wattmeter. The input is usually inductive, owing to phase displacement and to wave distortion. For this reason the power factor and the apparent efficiency should also be considered, since the latter may be much lower than the true efficiency. The power consumed by auxiliary devices, such as the synchronous motor or cooling devices, should be included in the electric input.

178 In *Constant-Current Rectifiers*, transforming from constant potential alternating to constant direct current, by means of constant-current transforming devices and rectifying devices, the losses in the transforming devices are to be included in determining the efficiency and have to be measured when operating the rectifier, since in this case the losses may be greater than when feeding an alternating secondary circuit. In constant-current transforming devices, the load losses may be considerable, and, therefore, should not be neglected.

179 In *Open Coil Arc Machines*, the losses are essentially the same as in direct-current (closed coil) commutating machines. In this case, however, the load losses are usually greater, and the efficiency should preferably be measured by input- and output-test, using wattmeters for measuring the output. In alternating-current rectifiers, the output should, in general, be measured by wattmeter and not by voltmeter and ammeter, since owing to pulsation of current and voltage, a considerable discrepancy may exist between the watts and volt-amperes. If, however, a direct-current and an alternating-current meter in the rectified circuit (either a voltmeter or an ammeter) give the same reading, the output may be measured by direct-current voltmeter and ammeter. The type of alternating-current instrument here referred to should indicate the effective or root-of-mean-square value and the type of direct-current instrument the arithmetical mean value, which would be zero on an alternating-current circuit.

(I)—TRANSMISSION LINES.

180 The *Efficiency* of transmission lines should be measured with non-inductive load at the receiving end, with the rated receiving voltage and frequency, also with sinusoidal impressed wave form, except where expressly specified otherwise, and with the exclusion of transformers or other apparatus at the ends of the line.

(J)—PHASE-DISPLACING APPARATUS.

181 In *Apparatus Producing Phase Displacement* as, for example, synchronous compensators, exciters of induction generators, reactors, condensers, polarization cells, etc., the efficiency should be understood to be the ratio of the volt-amperes minus power loss to the volt-amperes.

182 The *Efficiency* may be calculated by determining the losses, subtracting them from the volt-amperes, and then dividing the difference by the volt-amperes.

183 In *Synchronous Compensators* and exciters of induction generators, the determination of losses is the same as in other synchronous machines.

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184 In *Reactors* the losses are molecular magnetic friction, eddy losses and I^2r loss. They should be measured by wattmeter. The efficiency of reactors should be determined with a sine wave of impressed voltage except where expressly specified otherwise.

185 In *Condensers*, the losses are due to dielectic hysteresis and leakage, and should be determined by wattmeter with a sine wave of voltage.

186 In *Polarization Cells*, the losses are those due to electric resistivity and a loss in the electrolyte of the nature of chemical hysteresis. These losses may be considerable. They depend upon the frequency, voltage and temperature, and should be determined with a sine wave of impressed voltage, except where expressly specified otherwise.

D.—REGULATION.

(I)—Definitions.

187 *Definition.* The regulation of a machine or apparatus in regard to some characteristic quantity (such as terminal voltage, current or speed) is the ratio of the deviation of that quantity from its normal value at rated load to the normal rated load value. The term "regulation," therefore, has the same meaning as the term "inherent regulation," occasionally used.

188 *Constant Standard.* If the characteristic quantity is intended to remain constant (*e. g.*, constant voltage, constant speed, etc.) between rated load and no load, the regulation is the ratio of the maximum variation from the rated-load value to the no-load value.

189 *Varying Standard.* If the characteristic quantity is intended to vary in a definite manner between rated load and no load, the regulation is the ratio of the maximum variation from the specified condition to the normal rated-load value.

190 (*a*) Note.—If the law of the variation (in voltage, current, speed, etc.) between rated-load and no-load is not specified, it should be assumed to be a simple linear relation; *i. e.*, one undergoing uniform variation between rated-load and no-load.

191 (*b*) Note.—The regulation of an apparatus may, therefore, differ according to its qualification for use. Thus, the regulation of a compound-wound generator specified as a constant-potential generator, will be different from that which it possesses when specified as an over-compounded generator.

192 In *Constant-Potential Machines*, the regulation is the ratio of the maximum difference of terminal voltage from the rated-load value (occurring within the range of rated load to open circuit) to the rated-load terminal voltage.

193 In *Constant-Current Machines*, the regulation is the ratio of the maximum difference of current from the rated-load value (occurring within the range from rated-load to short-circuit, or minimum limit of operation), to the rated-load current.

194 In *Constant-Power Apparatus*, the regulation is the ratio of maximum difference of power from the rated-load value (occurring within the range of operation specified) to the rated power.

195 In *Constant-Speed Direct-Current Motors* and *Induction Motors* the regulation is the ratio of the maximum variation of speed from its rated-load value (occurring within the range from rated-load to no-load) to the rated-load speed.

196 The regulation of an induction motor is, therefore, not identical with the slip of the motor, which is the ratio of the drop in speed from synchronism, to the synchronous speed.

197 In *Constant-Potential Transformers*, the regulation is the ratio of the rise of secondary terminal voltage from rated non-inductive load to no-load (at constant primary impressed terminal voltage) to the secondary terminal voltage at rated load.

198 In *Over-Compounded Machines*, the regulation is the ratio of the maximum difference in voltage from a straight line connecting the no-load and rated-load values of terminal voltage as function of the load current, to the rated-load terminal voltage.

199 In *Converters, Dynamotors, Motor-Generators and Frequency Converters*, the regulation is the ratio of the maximum difference of terminal voltage at the output side from the rated-load voltage, to the rated-load voltage on the output side.

200 In *Transmission Lines, Feeders, etc.*, the regulation is the ratio of

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the maximum voltage difference at the receiving end, between rated non-inductive load and no-load to the rated-load voltage at the receiving end (with constant voltage impressed upon the sending end).

201 In *Steam Engines*, the regulation is the ratio of the maximum variation of speed in passing slowly from rated-load to no-load (with constant steam pressure at the throttle) to the rated-load speed. For variation and pulsation, see Secs. 59–64.

202 In a *Hydraulic Turbine* or *Other Water-Motor*, the regulation is the ratio of the maximum variation of speed in passing slowly from rated-load to no-load (at constant head of water; *i. e.*, at constant difference of level between tail race and head race), to the rated-load speed. For variation and pulsation, see Secs. 59–64.

203 In a *Generator-Unit*, consisting of a generator united with a prime-mover, the regulation should be determined at constant conditions of the prime-mover; *i. e.*, constant steam pressure, head, etc. It includes the inherent speed variations of the prime-mover. For this reason the regulation of a generator-unit is to be distinguished from the regulation of either the prime-mover, or of the generator contained in it, when taken separately.

(II)—CONDITIONS FOR AND TESTS OF REGULATION.

204 *Speed.* The *Regulation of Generators* is to be determined at constant speed, and of alternating apparatus at constant impressed frequency.

205 *Non-Inductive Load.* In apparatus generating, transforming or transmitting alternating currents, regulation should be understood to refer to non-inductive load, that is, to a load in which the current is in phase with the e.m.f. at the output side of the apparatus, except where expressly specified otherwise.

206 *Wave Form.* In alternating apparatus receiving electric power, regulation should refer to a sine wave of e.m.f., except where expressly specified otherwise.

207 *Excitation.* In commutating machines, rectifying machines, and synchronous machines, such as direct-current generators and motors, alternating-current and polyphase generators, the regulation is to be determined under the following conditions:

(1) At constant excitation in separately excited fields.
(2) With constant resistance in shunt-field circuits, and
(3) With constant resistance shunting series-field circuits; *i. e.*, the field adjustment should remain constant, and should be so chosen as to give the required full-load voltage at full-load current.

208 *Impedance Ratio.* In alternating-current apparatus, in addition to the non-inductive regulation, the impedance ratio of the apparatus should be specified; *i. e.*, the ratio of the voltage consumed by the total internal impedance of the apparatus at full-load current, to its rated full-load voltage. As far as possible, a sinusoidal current should be used.

209 *Computation of Regulation.* When in synchronous machines the regulation is computed from the terminal voltage and impedance voltage, the exciting ampere-turns corresponding to terminal voltage plus armature-resistance-drop, and the ampere-turns at short-circuit corresponding to the armature-impedance-drop, should be combined vectorially to obtain the resultant ampere-turns, and the corresponding internal e. m. f. should be taken from the saturation curve.

E.—INSULATION.

(I)—INSULATION RESISTANCE.

210 *Insulation Resistance* is the ohmic resistance offered by an insulating coating, cover, material or support to an impressed voltage, tending to produce a leakage of current through the same.

211 *Ohmic Resistance and Dielectric Strength.* The ohmic resistance of the insulation is of secondary importance only, as compared with the dielectric strength, or resistance to rupture by high voltage. Since the ohmic resistance of the insulation can be very greatly increased by baking, but the dielectric strength is liable to be weakened thereby, it is preferable to specify a high dielectric strength rather than a high insulation resistance. The high-voltage test for dielectric strength should always be applied.

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212 *Recommended Value of Resistance.* The insulation resistance of complete apparatus should be such that the rated voltage of the apparatus which will not send more than $\frac{1}{1,000,000}$ of the rated-load current, at the rated terminal voltage, through the insulation. Where the value found in this way exceeds 1 megohm, it is usually sufficient.

213 *Insulation Resistance Tests* should, if possible, be made at the pressure for which the apparatus is designed.

(II)—DIELECTRIC STRENGTH.

(*A*)—TEST VOLTAGES.

214 *Definition.* The dielectric strength of an insulating wall, coating cover or path is measured by the voltage which must be applied to it in order to effect a disruptive discharge through the same.

215 *Basis for Determining Test Voltages.* The test voltage which should be applied to determine the suitability of insulation for commercial operation is dependent upon the kind and size of the apparatus and its normal operating voltage, upon the nature of the service in which it is to be used, and the severity of the mechanical and electrical stresses to which it may be subjected. The voltages and other conditions of test which are recommended have been determined as reasonable and proper for the great majority of cases and are proposed for general adoption, except when specific reasons make a modification desirable.

216 *Condition of Apparatus to be Tested.* Commercial tests should, in general, be made with the completely assembled apparatus and not with individual parts. The apparatus should be in good condition and high-voltage tests, unless otherwise specified, should be applied before the machine is put into commercial service, and should not be applied when the insulation resistance is low owing to dirt or moisture. High-voltage tests should, in general, be made at the temperature assumed under normal operation. High-voltage tests considerably in excess of the normal voltages to determine whether specifications are fulfilled are admissible on new machines only.

217 *Points of Application of Voltage.* The test voltage should be successively applied between each electric circuit and all other electric circuits including conducting material in the apparatus.

218 The *Frequency* of the alternating-current test voltage is, in general, immaterial within commercial ranges. When, however, the frequency has an appreciable effect, as in alternating-current apparatus of high voltage and considerable capacity, the rated frequency of the apparatus should be used.

219 *Table of Testing Voltages.* The following voltages are recommended for testing all apparatus, lines and cables, by a continued application for one minute. The test should be with alternating voltage having an effective value (or root mean square referred to a sine wave of voltage) given in the table and preferably for tests of alternating apparatus at the normal frequency of the apparatus.

	Rated Terminal Voltage of Circuit.	Rated Output.	Testing Voltage.
220	Not exceeding 400 volts	Under 10 kw.	1,000 volts.
	" " 400 "	10 kw. and over	1,500 "
	400 and over, but less than 800 volts.	Under 10 kw.	1,500 "
	400 " " 800 "	10 kw. and over	2,000 "
	800 " " 1,200 "	Any	3,500 "
	1,200 " " 2,500 "	Any	5,000 "
	2,500 "	Any	Double the normal rated Voltages.

221 *Exception.—Transformers.* Transformers having primary pressures of from 550 to 5,000 volts, the secondaries of which are directly connected to consumption circuits, should have a testing voltage of 10,000 volts, to be applied between the primary and secondary windings, and also between the primary winding and the core.

222 *Exception.—Field Windings.* The tests for field windings should be based on the rated voltage of the exciter and the rated output of the machine of which the coils are a part. Field windings of synchronous motors and converters, which are to be started by applying alternating

current to the armature when the field is not excited and a high voltage is induced in the field windings, should be tested at 5,000 volts.

223 *Rated Terminal Voltage.—Definition.* The rated terminal voltage of circuit in the above table, means the voltage between the conductors of the circuit to which the apparatus to be tested is to be connected,—for instance, in three-phase circuits the delta voltage should be taken. In the following specific cases, the rated terminal voltage of the circuit is to be determined as specified in ascertaining the testing voltage:

224 (*a*) *Transformers.* The test of the insulation between the primary and secondary windings of transformers, is to be the same as that between the high-voltage windings and core, and both tests should be made simultaneously by connecting the low-tension winding and core together during the test. If a voltage equal to the specified testing voltage be induced in the high-tension winding of a transformer it may be used for insulation tests instead of an independently induced voltage. These tests should be made first with one end and then with the other end of the high-tension winding connected to the low-tension winding and to the core.

225 (*b*) *Constant-Current Apparatus.* The testing voltage is to be based upon a rated terminal voltage equal to the maximum voltage which may exist at open or closed circuit.

226 (*c*) *Apparatus in Series.* For tests of machines or apparatus to be operated in series, so as to employ the sum of their separate voltages the testing voltage is to be based upon a rated terminal voltage equal to the sum of the separate voltages except where the frames of the machines are separately insulated, both from the ground and from each other, in which case the test for insulation between machines should be based upon the voltage of one machine, and the test between each machine and ground to be based upon the total voltage of the series.

(*B*)—METHODS OF TESTING.

227 *Classes of Tests.* Tests for dielectric strength cover such a wide range in voltage that the apparatus, methods and precautions which are essential in certain cases do not apply to others. For convenience, the tests will be separated into two classes:

228 *Class* 1. This class includes all apparatus for which the test voltage does not exceed 10 kilovolts, unless the apparatus is of very large static capacity, *e. g.*, a large cable system. This class also includes all apparatus of small static capacity, such as line insulators, switches and the like, for all test voltages.

229 *Method of Test for Class* 1. The test voltage is to be continuously applied for the prescribed interval,—(one minute, unless otherwise specified). The test voltage may be taken from a constant-potential source and applied directly to the apparatus to be tested, or it may be raised gradually as specified for tests under Class 2.

230 *Class* 2. This class includes all apparatus not included in Class 1.

231 *Method of Test for Class* 2. The test voltage is to be raised to the required value smoothly and without sudden large increments and is then to be continuously applied for the prescribed interval,—(one minute, unless otherwise specified), and then gradually decreased.

232 *Conditions and Precautions for Class* 1 and *Class* 2. The following apply to all tests:

233 The *Wave Shape* should be approximately sinusoidal and the apparatus in the testing circuits should not materially distort this wave.

234 The *Supply Circuit* should have ample current-supply capacity so that the charging current which may be taken by the apparatus under test will not materially alter the wave form nor materially affect the test voltage. The circuit should be free from accidental interruptions.

235 *Resistance or Inductance* in series with the primary of a raising transformer for the purpose of controlling its voltage is liable seriously to affect the wave form, thereby causing the maximum value of the voltage to bear a different and unknown ratio to the root mean square value. This method of voltage adjustment is, therefore, in general, undesirable. It may be noted that if a resistance or inductance is employed to limit the current when burning out a fault, such resistance or inductance should be short-circuited during the regular voltage test.

236 The *Insulation* under test should be in normal condition as to dryness and the temperature should, when possible, be that reached in normal service.

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237 *Additional Conditions and Precautions for Class 2.* The following conditions and precautions, in addition to the foregoing, apply to tests of apparatus included in Class 2.

238 *Sudden Increment of Testing Voltage* on the apparatus under test should be avoided, particularly at high voltages and with apparatus having considerable capacity, as a momentarily excessive rise in testing voltage will result.

239 *Sudden Variations in Testing Voltage* of the circuit supplying the voltage during the test should be avoided as they are likely to set up injurious oscillation.

240 *Good Connections* in the circuits supplying the test voltage are essential in order to prevent injurious high frequency disturbances from being set up. When a heavy current is carried by a small water rheostat, arcing may occur, causing high-frequency disturbances which should be carefully avoided.

241 *Transformer Coils.* In high-tension transformers, the low-tension coil should preferably be connected to the core and the ground when the high-tension test is being made, in order to avoid the stress from low-tension to core, which would otherwise result through condenser action. The various terminals of each winding of the high-tension transformer under test should be connected together during the test in order to prevent undue stress on the insulation between turns or sections of the winding in case the high-voltage test causes a breakdown.

(*C*)—METHODS FOR MEASURING THE TEST VOLTAGE.

242 *For Measuring the Test Voltage,* two instruments are in common use, (1) the spark gap, and (2) the voltmeter.

243 1. *The Spark Gap* is ordinarily adjusted so that it will break down with a certain predetermined voltage, and is connected in parallel with the insulation under test. It ensures that the voltage applied to the insulation is not greater than the break-down voltage of the spark gap. A given setting of the spark gap is a measure of one definite voltage, and, as its operation depends upon the maximum value of the voltage wave, it is independent of wave form and is a limit on the maximum stress to which the insulation is subjected. The spark gap is not conveniently adapted for comparatively low voltages.

244 In *Spark-Gap Measurements,* the spark gap may be set for the required voltage and the auxiliary apparatus adjusted to give a voltage at which this spark gap just breaks down. This spark gap should then be adjusted for, say, 10 per cent higher voltage, and the auxiliary apparatus again adjusted to give the voltage of the former breakdown, which is to be the assumed voltage for the test. This voltage is to be maintained for the required interval.

245 The *Spark Points* should consist of new sewing needles, supported axially at the ends of linear conductors which are each at least twice the length of the gap. There should be no extraneous body near the gap within a radius of twice its length. A table of approximate striking distances is given in Appendix D. This table should be used in connection with tests made by the spark-gap methods.

246 A *Non-Inductive Resistance* of about one-half ohm per volt should be inserted in series with each terminal of the gap so as to keep the discharge current between the limits of one-quarter ampere and 2 amperes. The purpose of the resistance is to limit the current in order to prevent the surges which might otherwise occur at the time of break-down.

247 2. The *Voltmeter* gives a direct reading, and the different values of the voltage can be read during the application and duration of the test. It is suitable for all voltages, and does not introduce disturbances into the test circuit.

248 In *Voltmeter Measurements,* the voltmeter should, in general, derive its voltage from the high-tension testing circuit either directly or through an auxiliary ratio transformer. It is permissible, however, to measure the voltage at other places,—for example, on the primary of the transformer, provided the ratio of transformation does not materially vary during the test; or that proper account is taken thereof.

249 *Spark Gap and Voltmeter.* The spark gap may be employed as a check upon the voltmeter used in high-tension tests in order to determine the transformation ratio of the transformer, the variation from the sine wave form and the like. It is also useful in conjunction with

voltmeter measurements to limit the stress applied to the insulating material.

(D)—APPARATUS FOR SUPPLYING TEST VOLTAGE.

250 The *Generator or Circuit* supplying voltage for the test should have ample current-carrying capacity, so that the current which may be taken for charging the apparatus to be tested will not materially alter the wave form nor otherwise materially change the voltage.

The *Testing Transformer* should be such that its ratio of transformation does not vary more than 10 per cent when delivering the charging current required by the apparatus under test. (This may be determined by short-circuiting the secondary or high voltage winding testing transformer and supplying 1/10 of the primary voltage to the primary under this condition. The primary current that flows under this condition is the maximum which should be permitted in regular dielectric tests.)

251 The *Voltage Control* may be secured in either of several ways, which, in order of preference, are as follows:

252 1. By generator field circuit.
253 2. By magnetic commutation.
254 3. By change in transformer ratio.
255 4. By resistance or choke coils.

256 In *Generator Voltage Control*, the voltage of the generator should preferably be about its approximate normal rated-load value when the full testing voltage is attained, which requires that the ratio of the raising transformer be such that the full testing voltage is reached when the generator voltage is normal. This avoids the instability in the generator which may occur if a considerable leading current is taken from it when it has low voltage and low field current.

257 In *Magnetic Commutation*, the control is effected by shunting the magnetic flux through a secondary coil so as to vary the induction through the coil and the voltage induced in it. The shunting should be effected smoothly, thus avoiding sudden changes in the induced voltage.

258 In *Transformer Voltage Control*, by change of ratio, it is necessary that the transition from one step to another be made without interruption of the test voltage, and by steps sufficiently small to prevent surges in the testing circuit. The necessity of this precaution is greater as the inductance or the static capacity of the apparatus in the testing circuit under test is greater.

259 When *Resistance Coils or Reactors* are used for voltage control, it is desirable that the testing voltage should be secured when the controlling resistance or reactance is very nearly or entirely out of circuit in order that the disturbing effect upon the wave form which results may be negligible at the highest volatge.

F.—CONDUCTIVITY.

260 *Copper*. The conductivity of copper in electric wires and cables should not be less than 98% of Matthiessen's standard of conductivity, as defined in the Copper Wire Table of the *American Institute of Electrical Engineers*. [See Table1, page 1388.]

G.—RISE OF TEMPERATURE.

(I)—MEASUREMENT OF TEMPERATURE.

(A)—METHODS.

261 There are two methods in common use for determining the rise in temperature, viz.: (1) by thermometer, and (2) by increase in resistance of an electric circuit.

262 1. By *Thermometer*. The following precautions should be observed in the use of thermometers:

263 *a. Protection*. The thermometers indicating the room temperature should be protected from thermal radiation emitted by heated bodies, or from draughts of air or from temporary fluctuations of temperature. Several room thermometers should be used. In using the thermometer by applying it to a heated part, care should be taken so to protect its bulb as to prevent radiation from it, and, at the same time, not to interfere seriously with the normal radiation from the part to which it is applied.

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264 *b. Bulb.* When a thermometer is applied to the free surface of a machine, it is desirable that the bulb of the thermometer should be covered by a pad of definite area. A convenient pad may be formed of cotton waste in a shallow circular box about one and a half inches in diameter, through a slot in the side in which the thermometer bulb is inserted. An unduly large pad over the thermometer tends to interfere with the natural liberation of heat from the surface to which the thermometer is applied.

265 2. By *Increase in Resistance.* The resistance may be measured either by Wheatstone bridge, or by drop-of potential method. A temperature coefficient of 0.42 per cent per degree C., from and at 0°C., may be assumed for copper.

The temperature-coefficients from and at each degree cent. between 0°C. and 50°C. are given in Appendix E. The temperature rise may be determined either (1) by dividing the percentage increase of initial resistance by the temperature-coefficient for the initial temperature expressed in per cent; or (2) by multiplying the increase in per cent of the initial resistance by 238.1 plus the initial temperature in degrees C., and then dividing the product by 100.

266 3. *Comparison of Methods.* In electrical conductors, the rise of temperature should be determined by their increase of resistance where practicable. Temperature elevations measured in this way are usually in excess of temperature elevations measured by thermometers. In very low resistance circuits, thermometer measurements are frequently more reliable than measurements by the resistance method. Where a thermometer applied to a coil or winding, indicates a higher temperature elevation than that shown by resistance measurement, the thermometer indication should be accepted.

(*B*)—Normal Conditions for Tests.

267 1. *Duration of Tests.* The temperature should be measured after a run of sufficient duration for the apparatus to reach a practically constant temperature. This is usually from 6 to 18 hours, according to the size and construction of the apparatus. It is permissible, however, to shorten the time of the test by running a lesser time on an overload in current and voltage, then reducing the load to normal, and maintaining it thus until the temperature has become constant.

268 2. *Room Temperature.* The rise of temperature thould be referred to the standard condition of a room temperature of 25°C.

269 *Temperature Correction.* If the room temperature during the test differs from 25°C., correction on account of difference in resistance should be made by changing the observed rise of temperature by one-half per cent for each degree C. Thus with a room temperature of 35°C., the observed rise of temperature has to be decreased by 5 per cent, and with a room temperature of 15°C., the observed rise of temperature has to be increased by 5 per cent. In certain cases, such as shunt-field circuit without rheostat, the current strength will be changed by a change of room temperature. The heat-production and dissipation may be thereby affected. Correction for this should be made by changing the observed rise in temperature in proportion as the I^2R loss in the resistance of the apparatus is altered owing to the difference in room temperature.

270 3. *Barometric Pressure. Ventilation.* A barometric pressure of 760 mm. and normal conditions of ventilation should be considered as standard, and the apparatus under test should neither be exposed to draught nor enclosed, except where expressly specified. The barometric pressure needs to be considered only when differing greatly from 760 mm.

271 *Barometric Pressure Correction.* When the barometric pressure differs greatly from the standard pressure of 760 mm. of mercury, as at high altitudes, a correction should be applied. In the absence of more accurate data, a correction of 1% of the observed rise in temperature for each 10 mm. deviation from the 760 mm. standard is recommended. For example, at a barometric pressure of 680 mm. the observed rise of temperature is to be reduced by $\frac{760-680}{10}=8\%$.

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(II)—LIMITING TEMPERATURE RISE.

272 *General.* The temperature of electrical machinery under regular service conditions, should never be allowed to remain at a point at which permanent deterioration of its insulating material takes place

273 *Limits Recommended.* It is recommended that the following maximum values of temperature elevation, referred to a standard room temperature of 25° centigrade, at rated load under normal conditions of ventilation or cooling, should not be exceeded.

(*A*)—MACHINES IN GENERAL.

274 In commutating machines, rectifying machines, pulsating-current generators, synchronous machines, synchronous commutating machines and unipolar machines, the temperature rise in the parts specified should not exceed the following:

275 Field and armature, 50°C.

276 Commutator and brushes, by thermometer, 55°C.

277 Collector rings, 65°C.

278 Bearings and other parts of machine, by thermometer, 40°C.

(*B*)—ROTARY INDUCTION APPARATUS.

279 The temperature rise should not exceed the following:

280 Electric circuit, 50°C., by resistance.

281 Bearings and other parts of the machine 40°C., by thermometer.

282 In squirrel-cage or short-circuited armatures, 55°C., by thermometer, may be allowed.

(*C*)—STATIONARY INDUCTION APPARATUS.

283 *a. Transformers for Continuous Service.* The temperature rise should not exceed 50° centigrade in electric circuits, by resistance; and in other parts, by thermometer.

284 *b. Transformers for Intermittent Service.* In the case of transformers intended for intermittent service, or not operationg continuously at rated load, but continuously in circuit, as in the ordinary case of lighting transformers, the temperature elevation above the surrounding air-temperature should not exceed 50°C., by resistance in electric circuits and by thermometer in other parts, after the period corresponding to the term of rated load. In this instance, the test load should not be applied until the transformer has been in circuit for a sufficient time to attain the temperature elevation due to core loss. With transformers for commercial lighting, the duration of the rated-load test may be taken as three hours, unless otherwise specified.

285 *c. Reactors,* induction- and magneto-regulators—electric circuts by resistance and other parts by thermometer, 50°C.

286 *a. Large Apparatus.* Large generators, motors, transformers, or other apparatus in which reliability and reserve overload capacity are important, are frequently specified not to rise in temperature more than 40° centigrade under rated load and 55° centigrade at rated overload. It is, however, ordinarily undersirable to specify lower temperature elevations than 40° centigrade at rated load, measured as above.

(*D*)—RHEOSTATS.

287 In *Rheostats, Heaters* and other electrothermal apparatus, no combustible or inflammable part or material, or portion liable to come in contact with such material, should rise more than 50°C. above the surrounding air under the service conditions for which it is designed.

288 *a. Parts of Rheostats.* Parts of rheostats and similar apparatus rising in temperature, under the specified service conditions, more than 50°C., should not contain any combustible material, and should be arranged or installed in such a manner that neither they, nor the hot air issuing from them, can come in contact with combustible material.

(*E*)—LIMITS RECOMMENDED IN SPECIAL CASES.

289 *a. Heat Resisting Insulation.* With apparatus in which the insulating materials have special heat-resisting qualities, a higher temperature elevation is permissible.

290 *b. High Air Temperature.* In apparatus intended for service in places of abnormally high temperature, a lower temperature elevation should be specified.

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291 *c. Apparatus Subject to Overload.* In apparatus which by the nature of its service may be exposed to overload, or is to be used in very high voltage circuits, a smaller rise of temperature is desirable than in apparatus not liable to overloads or in low-voltage apparatus. In apparatus built for conditions of limited space, as railway motors, a higher rise of temperature must be allowed.

292 *d. Apparatus for Intermittent Service.* In the case of apparatus intended for intermittent service, except railway motors, the temperature elevation which is attained at the end of the period corresponding to the term of rated load, should not exceed the values specified for machines in general. In such apparatus the temperature elevation, including railway motors, should be measured after operation, under as nearly as possible the conditions of service for which the apparatus is intended, and the conditions of the test should be specified.

H.—OVERLOAD CAPACITIES.

293 *Performance with Overload.* All apparatus should be able to carry the overload hereinafter specified without serious injury by heating, sparking, mechanical weakness, etc., and with an additional temperature rise not exceeding 15°C., above those specified for rated loads, the overload being applied after the apparatus has acquired the temperature corresponding to rated load continuous operation. Rheostats to which no temperature rise limits are attached are naturally exempt from this additional temperature rise of 15°C. under overload specified in these rules.

294 *Normal Conditions.* Overload guarantees should refer to normal conditions of operation reguarding speed, frequency, voltage, etc., and to non-inductive conditions in alternating apparatus, except where a phase displacement is inherent in the apparatus.

295 *Overload Capacities Recommended.* The following overload capacities are recommended:

296 *a. Generators.* Direct-current generators and alternating-current generators, 25 per cent for two hours.

297 *b. Motors.* Direct-current motors, induction motors and synchronous motors, not including railway and other motors intended for intermittent service, 25 per cent for two hours, and 50 per cent for one minute.

298 *c. Converters.* Synchronous converters, 25 per cent for two hours, 50 per cent for one-half hour.

299 *d. Transformers and Rectifiers.* Constant-potential transformers and rectifiers, 25 per cent for two hours; except in transformers connected to apparatus for which a different overload in guaranteed, in which case the same guarantees shall apply for the transformers as for the apparatus connected thereto.

300 *e. Exciters.* Exciters of alternators and other synchronous machines, 10 per cent more overload than is required for the excitation of the synchronous machine at its guaranteed overload, and for the same period of time. All exciters of alternating-current, single-phase or polyphase generators should be able to give at its rated speed, sufficient voltage and current to excite the alternator, at the rated speed, to the full-load terminal voltage, at the rated output in kilovolt-amperes and with 50 per cent power factor.

301 *f.* A *Continuous-Service Rheostat*, such as an armature- or field-regulating rheostat, should be capable of carrying without injury for two hours, a current 25 per cent greater than that at which it is rated. It should also be capable of carrying for one minute a current 50 per cent greater than its rated load current, without injury. This excess of capacity is intended for testing purposes only, and this margin of capacity should not be relied upon in the selection of the rheostat.

302 *g.* An *Intermittent Service or Motor-Starting Rheostat* is used for starting a motor from rest and accelerating it to rated speed. Under ordinary conditions of service, and unless expressly stated otherwise, a motor is assumed to start in fifteen seconds and with 150% of rated current strength. A motor-starter should be capable of starting the motor under these conditions once every four minutes for one hour.

303 (*a*) This *Test* may be carried out either by starting the motor at four-minute intervals, or by placing the starter at normal temperature across the maximum voltage for which it is marked, and moving the lever uniformly and gradually from the first to the last position during a

period of fifteen seconds, the current being maintained substantially constant at said 50% excess by introducing resistance in series or by other suitable means.

304 (*b*) *Other Rheostats for Intermittent-Service* are employed under such special and varied conditions, that no general rules are applicable to them.

III.—VOLTAGES AND FREQUENCIES.

A.—VOLTAGES.

305 *Direct-Current Generators.* In direct-current, low-voltage generators, the following average terminal voltages are in general use and are recommended:

125 volts. 250 volts. 550 to 600 volts.

306 *Low-Voltage Circuits.* In direct-current and alternating-current low-voltage circuits, the following average terminal voltages are in general use and are recommended:

110 volts. 220 volts.

307 *Primary Distribution Circuits.* In alternating-current, constant-potential, primary-distribution circuits, an average voltage of 2,200 volts, with step-down transformer ratios 1/10 and 1/20, is in general use, and is recommended.

308 *Transmission Circuits.* In alternating-current constant-potential transmission circuits, the following average voltages are recommended.

6,600 11,000 22,000 33,000 44,000 66,000 88,000

309 *Transformer Ratio.* It is recommended that the standard transformer ratios should be such as to transform between the standard voltages above named. The ratio will, therefore, usually be an exact multiple of 5 or 10, *e. g.*, 2,200 to 11,000; 2,200 to 44,000.

310 *Range in Voltage.* In alternating-current generators, or generating systems, a range of terminal voltage should be provided from rated voltage at no load to 10 per cent in excess thereof, to cover drop in transmission. If a greater range than ten per cent is specified, the generator should be considered as special.

B.—FREQUENCIES.

311 In *Alternating-Current Circuits*, the following frequencies are standard:

25~ 60~

312 These frequencies are already in extensive use and it is deemed advisable to adhere to them as closely as possible.

IV.—GENERAL RECOMMENDATIONS.

313 *Name Plates.* All electrical apparatus should be provided with a name plate giving the manufacturer's name, the voltage and the current in amperes for which it is designed. Where practicable, the kilowatt capacity, character of current, speed, frequency, type, designation and serial number should be added.

314 *Diagrams of Connections.* All electrical apparatus when leaving the factory should be accompanied by a diagram showing the electrical connections and the relation of the different parts in sufficient detail to give the necessary information for proper installation.

315 *Rheostat Data.* Every rheostat should be clearly and permanently marked with the voltage and amperes, or range of amperes, for which it is designed.

316 *Colored Indicating Lights.* When using colored indicating lights on switch-boards, red should denote danger such as "switch closed," or "circuit alive;" green should denote safety, such as "switch open," or "circuit dead."

317 When white lights are used a light turned on should denote danger, such as "switch closed" or "circuit alive;" while the light out should denote safety, such as "switch open," or "circuit dead." Low-efficiency lamps should be used.

318 The use of colored lights is recommended, as safer than white lights.

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319 *Grounding Metal Work.* It is desirable that all metal work near high potential circuits be grounded.

320 *Circuit Opening Devices.* The following definitions are recommended.

321 *a.* A *Circuit-Breaker* is an apparatus for breaking a circuit at the highest current which it may be called upon to carry.

322 *b.* A *Disconnecting Switch* is an apparatus designed to open a circuit only when carrying little or no current.

323 *c.* An *Automatic Circuit-Breaker* is an apparatus for breaking a circuit automatically under an excessive strength of current. It should be capable of breaking the circuit repeatedily at rated voltage and at the maximum current which it may be called upon to carry.

V.—APPENDICES AND TABULAR DATA.

APPENDIX A.—NOTATION.

The following notation is recommended:

324 E, e, voltage, e.m.f., potential difference
I, i, current
P, power
Φ, magnetic flux
B, B, magnetic density
R, r, resistance
x, reactance
Z, z, impedance
L, l, inductance
C, c, capacity
Y, y, admittance
b, susceptance
G, g, conductance
Vector quantities when used should be denoted by capital italics.

APPENDIX B.—RAILWAY MOTORS.

(I)—RATING.

325 *Introductory Note on Rating.* Railway motors usually operate in a service in which both the speed and the torque developed by the motor are varying almost continually. The average requirements, however, during successive hours in a given class of service are fairly uniform. On account of the wide variation of the instantaneous loads, it is impracticable to assign any simple and definite rating to a motor which will indicate accurately the absolute capacity of a given motor or the relative capacity of different motors under service conditions. It is also impracticable to select a motor for a particular service without much fuller data with regard both to the motor and to the service than is required, for example, in the case of stationary motors which run at constant speeds.

326 *Scope of Nominal Rating.* It is common usage to give railway motors a nominal rating in horse power on the basis of a one-hour test. As above explained, a simple rating of this kind is not a proper measure of service capacity. The nominal rating, however, indicates approximately the maximum output which the motor should ordinarily be called upon to develop during acceleration. Methods of determining the continuous capacity of a railway motor for service requirements are given under a subsequent heading.

327 The *Nominal Rating* of a railway motor is the horse-power output at the car-axle, that is, including gear and other transmission losses, which gives a rise of temperature above the surrounding air (referred to a room temperature of 25° Cent.) not exceeding 90° Cent. at the commutator and 75° Cent. at any other part after one hour's continuous run at its rated voltage (and frequency, in the case of an alternating-current motor) on a stand, with the motor-covers removed, and with natural ventilation. The rise in temperature is to be determined by thermometer, but the resistance of no electrical circuit in the motor shall increase more than 40% during the test.

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(II)—SELECTION OF MOTOR FOR SPECIFIED SERVICE.

328 *General Requirements.* The suitability of a railway motor for a specified service depends upon the following considerations:

329 *a.* Mechanical ability to develop the requisite torque and speeds as given by its speed-torque curve.

330 *b.* Ability to commutate successfully the current demanded.

331 *c.* Ability to operate in service without occasioning a temperature rise in any part which will endanger the life of the insulation.

332 *Operating Conditions, Typical Run.* The operating conditions which are important in the selection of a motor include the weight of load, the schedule speed, the distance between stops, the duration of stops, the rate of acceleration and of breaking retardation, the grades and the curves. With these data at hand, the outputs which are required of the motor may be determined, provided the service requirements are within the limits of the speed-torque curve of the motor. These outputs may be expressed in the form of curves giving the instantaneous values of current and of voltage which must be applied to the motor. Such curves may be laid out for the entire line, but they are usually constructed only for a certain average or typical run, which is fairly representative of the conditions of service. To determine whether the motor has sufficient capacity to perform the service safely, further tests or investigations must be made.

333 *Capacity Test of Railway Motor in Service.* The capacity of a railway motor to deliver the necessary output may be determined by measurement of its temperature after it has reached a maximum in service. If a running test cannot be made under the actual conditions of service, an equivalent test may be made in a typical run back and forth, under such conditions of schedule speed, length of run, rate of acceleration, etc., that the test cycle of motor losses and conditions of ventilation are essentially the same as would be obtained in the specified service.

334 *Methods of Comparing Motor Capacity with Service Requirements.* Where it is not convenient to test motors under actual service conditions or in an equivalent typical run, recourse may be had to one of the two following methods of determining temperature rise now in general use:

335 1. *Method by Losses and Thermal Capacity Curves.* The heat developed in a railway motor is carried partly by conduction through the several parts and partly by convection through the air to the motor-frame whence it is distributed to the outside air. As the temperature of the several parts is thus dependent not only upon their own internal losses but also upon the temperature of neighboring parts, it becomes necessary to determine accurately the actual value and distribution of losses in a railway motor for a given service and reproduce them in an equivalent test-run. The results of a series of typical runs expressed in the form of thermal capacity curves will give the relation between degrees rise per watt loss in the armature and in the field for all ratios of losses between them met with in the commercial application of a given motor.

336 This method consists, therefore, in calculating the several internal motor losses in a specified service and determining the temperature rise with these losses from thermal capacity curves giving the degrees rise per watt loss as obtained in experimental track tests made under the same conditions of ventilation.

337 The following motor losses cause its heating and should be carefully determined for a given service: I^2R in the field; I^2R in the armature; I^2R in the brush contacts, core loss and brush friction.

338 The loss in the bearings (in the case of geared motors) also adds somewhat to the motor-heating, but owing to the variable nature of such losses they are generally neglected in making calculations.

339 2. *Method by Continuous Capacity of Motor.* The essential losses in the motor, as found in the typical run, are in most cases those in the motor windings and in the core. The mean service conditions may be expressed in terms of the current which would produce the same losses in the motor windings and in the voltage which, with the current, would produce the same core losses as the average in service. The continuous capacity of the motor is given in terms of the amperes which it will carry when run on the testing stand—with covers on or off as specified—at different voltages, say, 40, 60, 80 and 100 per cent of

the rated voltage—with a temperature rise not exceeding 90° at the commutator and 75° at any other part, provided the resistance of no electric circuit in the motor increase more than 40 per cent. A comparison of the equivalent service conditions with the continuous capacity of the motor will determine whether the service requirements are within the safe capacity of the motor.

340 This method affords a ready means of determining whether a specified service is within the capacity of a given motor and it is also a convenient approximate method for comparing the service capacities of different motors.

APPENDIX C.—PHOTOMETRY AND LAMPS.

341 *Candle-Power.* The luminous intensity of sources of light is expressed in candle-power. The unit of candle-power should be derived from the standards maintained by the National Bureau of Standards at Washington, D. C., which standard unit of candle-power equals 100/88 of the Hefner unit under Reichsanstalt standard conditions for the Hefner. In practical measurements seasoned and carefully standardized incandescent lamps are more reliable and accurate than the primary standard.

342 *Candle-Lumen.* The total flux of light from a source is equal to its mean spherical intensity multiplied by 4π. The unit of flux is called the candle-lumen. A candle-lumen is the $\frac{1}{4\pi}$th part of the total flux of light emitted by a source having a mean spherical intensity of one candle-power.

343 *Candle-Meter.* The unit of illumination is the candle-meter. This is the normal illumination produced by one unit of candle-power at a distance of one metre.

344 *a. Candle-Foot.* Illumination is occasionally expressed in candle-feet. A candle-foot is the normal illumination produced by one unit of candle-power at a distance of one foot.

345 1 candle-foot = 10.764 candle-metres.

The use of the candle-metre unit is preferable and is recommended.

346 The *Efficiency of Electric Lamps* is properly stated in terms of mean spherical candle-power per watt at lamp terminals. This use of the term efficiency is to be considered as special, and not to be confused with the generally accepted definition of efficiency in Sec. 85.

347 *a. Efficiency, Auxiliary Devices.* In illuminants requiring auxiliary power-consuming devices outside of the luminous body, such as steadying resistances in constant potential arc lamps, a distinction should be made between the net efficiency of the luminous source and the gross efficiency of the lamp. This distinction should always be stated. The gross efficiency should include the power consumed in the auxiliary resistance, etc., The net efficiency should, however, include the power consumed in the controlling mechanism of the lamp itself. Comparison between such sources of light should be made on the basis of gross efficiency, since the power consumed in the auxiliary device is essential to the operation.

348 *b.* A *Standard Circuit Voltage* of 110 volts, or a multiple thereof may be assumed, except where expressly stated otherwise.

349 *Watts per Candle.* The specific consumption of an electric lamp is its watt consumption per mean spherical candle-power. "Watt per candle" is the term used commercially in connection with incandescent lamps, and denotes, watts per mean horizontal candle-power.

350 *Photometric Tests* in which the results are stated in candle-power should always be made at such a distance from the source of light that the latter may be regarded as practically a point. Where tests are made at shorter distances, as for example in the measurement of lamps with reflectors, the result thould always be given as "apparent candle-power" at the distance employed, which distance should always be specifically stated.

351 *Basis for Comparison.* Either the total flux of light in candle-lumens, or the mean spherical candle-power, should always be used as the basis for comparing various luminous sources with each other, unless there is a clear understanding or statement to the contrary.

352 *Incandescent Lamps, Rating.* It is customary to rate incandescent lamps on the basis of their mean horizontal candle-power; but in com-

paring incandescent lamps in which the relative distribution of luminous intensity differs, the comparison should be based on their total flux of light measured in lumens, or on their mean spherical candle-power.

353 The *Spherical Reduction-Factor* of a lamp

$$= \frac{\text{mean spherical candle-power}}{\text{mean horizontal candle-power}}.$$

354 The *Total Flux* of light in candle-lumens emitted by a lamp $= 4\pi \times$ mean horizontal candle-power $\times$ spherical reduction-factor.

355 The *Spherical Reduction-Factor* should only be used when properly determined for the particular type and characteristics of each lamp. The spherical reduction-factor permits of substantially accurate comparisons being made between the mean spherical candle-powers of different types of incandescent lamps, and may be used in the absence of proper facilities for direct measurement of mean spherical intensity.

356 "*Reading Distance*." Where standard photometric measurements are impracticable, approximate measurements of illuminants such as street lamps may be made by comparing their "reading distances;" *i. e.*, by determining alternately the distances at which an ordinary size of reading print can just be read, by the same person or persons, when all other light is screened. The angle below the horizontal at which the measurement is made should be specified when it exceeds 15°.

357 In *Comparing Different Luminous Sources* not only should their candle-power be compared, but also their relative form, intrinsic brilliancy, distribution of illumination and character of light.

APPENDIX D.—SPARKING DISTANCES.

358 Table of Sparking Distances in Air between Opposed Sharp Needle-Points, for Various Effective Sinusoidal Voltages, in inches and in centimeters. The table applies to the conditions specified in Secs. 240–246.

359

Kilovolts Sq. Root of Mean Square.	Distance. Inches.	Cms.	Kilovolts Sq. Root of Mean Square.	Distance. Inches.	Cms.
5	0.225	0.57	140	13.95	35.4
10	0.47	1.19	150	15.0	38.1
15	0.725	1.84	160	16.05	40.7
20	1.0	2.54	170	17.10	43.4
25	1.3	3.3	180	18.15	46.1
30	1.625	4.1	190	19.20	48.8
35	2.0	5.1	200	20.25	51.4
40	2.45	6.2	210	21.30	54.1
45	2.95	7.5	220	22.35	56.8
50	3.55	9.0	230	23.40	59.4
60	4.65	11.8	240	24.45	62.1
70	5.85	14.9	250	25.50	64.7
80	7.1	18.0	260	26.50	67.3
90	8.35	21.2	270	27.50	69.8
100	9.6	24.4	280	28.50	72.4
110	10.75	27.3	290	29.50	74.9
120	11.85	30.1	300	30.50	77.4
130	12.90	32.8			

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APPENDIX E.—TEMPERATURE COEFFICIENTS.

360 Table of Temperature Coefficients of Resistivity in Copper at Different Initial Temperatures Centigrade.

Initial temperature Cent. t	Temp. Coefficient in per cent per Degree Cent.	Initial temperature Cent. t	Temp. Coefficient in per cent per Degree Cent.
0	0.4200	26	0.3786
1	0.4182	27	0.3772
2	0.4165	28	0.3758
3	0.4148	29	0.3744
4	0.4131	30	0.3730
5	0.4114	31	0.3716
6	0.4097	32	0.3702
7	0.4080	33	0.3689
8	0.4063	34	0.3675
9	0.4047	35	0.3662
10	0.4031	36	0.3648
11	0.4015	37	0.3635
12	0.3999	38	0.3622
13	0.3983	39	0.3609
14	0.3967	40	0.3596
15	0.3951	41	0.3583
16	0.3936	42	0.3570
17	0.3920	43	0.3557
18	0.3905	44	0.3545
19	0.3890	45	0.3532
20	0.3875	46	0.3520
21	0.3860	47	0.3508
22	0.3845	48	0.3495
23	0.3830	49	0.3483
24	0.3815	50	0.3471
25	0.3801		

The fundamental relation between the increase of resistance in copper and the rise of temperature may be taken as

$$R_t = R_0\,(1 + 0.0042\,t)$$

where R_0 is the resistance of the copper conductor at 0° C. and R_t is the corresponding resistance at t° C. This is equivalent to taking a temperature coefficient of 0.42% per deg. C. temperature rise above 0° C. For initial temperatures other than 0° C., a similar formula may be used substituting the coefficients in the above table corresponding to the actual initial temperature. The formula thus becomes at 25° C.

$$R_{i+r} = R_i\left(1 + \frac{0.3801\,r}{100}\right)$$

where R_i is the initial resistance at 25° C. R_{i+r} the final resistance and r the temperature rise above 25° C.

In order to find the temperature rise in degrees Centigrade from the initial resistance R_i at the initial temperature i° C. and the final resistance R_{i+r} we may use the formula

$$r = (238.1 + i)\left(\frac{R_{i+r}}{R} - 1\right) \text{ degrees C.}$$

See Sec. 265.

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EXCERPTS AND REFERENCES.

Generators and Transformers for the Bay Counties Power Co., Cal. (By E. Heltmann and W. Currie. Eng. News, Nov. 21, 1901).—Illustrated.

Electric Switches and Fuses for Currents of Very High Voltage ("The Jl. of Elec., Power and Gas," June, 1901; Eng. News, Oct. 3, 1901).

The 50,000-Volt Transmission Plant of the Missouri River Power Co. in Mont. (By W. G. McConnon. Eng. News, June 5, 1902).—Details of insulators and spacing of poles.

Success in Long Distance Electric Power Transmission (By F. A. C. Perrine. "Technology Quarterly;" Eng. News, Aug. 21, 1902).—See, also, Eng. News, Sept. 29, 1904.

An Analytical Method of Determining Illumination (By Van R. Lansingh. Eng. News, Feb. 19, 1903).—Illustrated diagram for determining horizontal distribution of light from a given surface.

Long Spans for Electric Transmission Lines (By F. O. Blackwell. Paper, Am. Inst. E. E., June, 1904; Eng. News, July 7, 1904).—*Modulus of elasticity:* Copper hard-drawn wire, 19,500,000; aluminum hard-drawn wire, 10,200,000; iron telegraph wire, 24,000,000; copper hard-drawn wire cable, 16,300,000. *Coefficient of expansion* (*F.*): Copper, 0.0000096; aluminum, 0.000013; steel, 0.0000064.

The Kern River Company's Hydro-Electric Power Enterprise (By Burr Bassell. Eng. News, July 21, 1904).—Sixteen illustrations.

Medium-Span Electric Transmission Line Construction (By C. A. Copeland. Pac. Coast Elec. Transmission Assn., June, 1904; Eng. News, Aug. 18, 1904).—Illustration of steel pole construction for 300-ft. span; also an experimental medium-span construction.

The Design of Generators for Electric Power Transmission (By D. B. Rushmore. Eng. News, Oct 27, 1904).—Illustrated.

Storage Batteries for Block Signal Work (By E. L. Reynolds. Paper, Ry. Signal Assn., Jan., 1905; Eng. News, Jan. 19, 1905).—Tables: (1) First cost; (2) Maintenance cost.

Long Distance Electric Power Transmission Line in Nevada (By E. Prince. Eng. News, July 6, 1905).

Electric Light and Power Plant of Brigham City, Utah (By W. P. Hardesty. Eng. News, Sept. 7, 1905).—Illustrations: Power house; ironwork for gates for diverting dam; concrete anchorage for steel pressure supply pipe on slope; diverting dam and intake; details wood-stave pipe; cast-iron hub for connecting steel pipe with wood stave pipe.

Gas Engine Electric Plant as Auxiliary and Reserve for a Long-Distance Transmission System (Eng. News, Sept. 14, 1905).

A High Head Water Power Electric Plant on the Animas River, Colo. (By G. M. Peek. Eng. News, Jan. 4, 1906).—Illustrations: Cross-section of power house; reinforced-concrete pole with spread base, for single 3-phase power transmission line; four-post reinforced-concrete tower with prismatic base, for single 3-phase power transmission line.

Transmission and Distributing System, Long Island R. R (Eng. News, June 14, 1906).—Illustrations: Top of steel transmission line pole; strain insulators for transmission line cables; details of third-rail guard and supports; standard wooden side approach block construction; standard arrangement of third-rail connecting cables at public crossings.

Cost of Construction and Operating Expenses of the Municipal Electric Lighting Plant at Burlington, Vt. (Eng. News, May 30, 1907).—Six tables of costs.

Some New Methods in High Tension Line Construction (By H. W. Buck Eng. News, Aug. 8, 1907).—Illustrated. See, also, Paper by E. M. Hewlett, in same issue.

The McCall Ferry Hydro-Electric Power Plant on the Susquehanna

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River (Eng. News, Sept. 12, 1907).—Illustrations: Alternate dam sections, showing expansion joints and butt blocks for steel forms; details of steel forms; details of steel traveler; concrete mixer plant; section through power house.

Concrete Telegraph Poles (Eng. Rec., Dec. 17, 1910), 17½ ft. high above the roof have been erected on one of the buildings of the U. S. Aluminum Co., at Niagara Falls. They are 16" sq. at base, 10" sq. at top and reinforced with eight ½" bars bound with ¼" hoops 18" c.-c.

Cost of Constructing Steam-Driven Electric Power Plants (By F. Koester. Eng. News, Dec. 19, 1907).—*Superstructure.*—$15 to $25 for plants up to 5000 K W.; $15 where there is a compact arrangement with walls of common brick, wooden doors and window frames, steel roof trusses supported by the walls and a roof of the cheapest construction, such as corrugated iron, tin, etc.; about $20 to $25 per K W. for construction of higher grade masonry with fireproof windows and doors, roof trusses carried by steel columns which at the same time carry the crane runway, and the roof itself consisting of reinforced-concrete covered by tar and gravel. The cost of superstructure for large size plants usually runs from $10 to $20 per K W.; these are constructed of self-supporting steel skeleton and self-supporting walls. The superstructure at $20 per K W. may embrace multiple boiler floors, while those at $10 per K W. cover single boiler floor plants only. *Chimney.*—A radial brick chimney for large size power plants may be built for from $1.75 to $2.25 per K W. Reinforced-concrete chimneys and plate steel chimneys may cost from $1.50 to $2 per K W. *Coal and Ash Handling Systems.*—Experience shows that the figures for equipment for handling coal and ashes range from $1.50 to $3 per K W. *Boilers.*—The cost of water tube boilers ranges from $8 to $10 per K W., depending upon the square feet of heating surface in the boiler. These figures do not include mechanical stokers, for which from $2 to $3 may be assumed. Breeching, of course, is also a separate item and varies considerably as to cost per K W. The boiler setting is included in the above cost. *Blowers.*—In many of the modern power plants, especially plants for railway purposes, forced or induced draft is adopted. The blowers are usually steam-driven. The cost of such equipment is about $1 per K W. *Economizers.*—Where economizers are installed of sufficient capacity to heat the water to 200° or 220° F. such apparatus costs about $2 per K W., provided that there are not too many additional smoke flues necessary for by-passing, etc. *Boiler Feed Pumps.*—The cost of such pumps alone is some 50 cents per K W. When storage tanks are necessary the cost of the combined outfit amounts to 75 cents or $1, depending on the number and size of the tanks. *Piping.*—From $2 to $6 per K W. For plants of 10,000 to 20,000-K W. capacity, the piping system not being elaborate but sufficient for continuous operation, $2.50 to $2.75 has covered the cost. This includes a high grade of covering for steam piping valued at about 20 cts. per K W. *Prime Movers.*—A 5000-K W. turbo-generator should cost from $20 to $22 per K W. Reciprocating engines of this capacity are sold roughly at the same price, and about $10 per K W. needs to be added for the generator. The total cost for smaller units, 600 to 3000 K W. capacity, is from $20 to $25 per K W., whether they consist of turbine or reciprocating-engine apparatus. *Condensers.*—The cost of jet condenser equipment runs from $3 to $5 per K W., depending upon the type of pump used. The cost of surface condenser apparatus will vary from $5 to $8, depending partly upon the vacuum to be carried and whether the casing necessary forms part of the condenser equipment. *Exciters.*—A steam-driven exciter unit costs from 35 cts. to 40 cts. per K W. If a condenser should be installed in connection with it the cost may run as high at 70 cts. per K W., assuming that the exciter capacity is, approximately, 1% the total capacity of the plant. *Switchboards.*—For a high tension voltage the cost will run from $2 to $3.50, which for a low tension voltage (2,300 volts or lower) the switchboard equipment may be obtained for $1 to $2 per K W., depending largely upon the system of wiring adopted. *Miscellaneous.*—Traveling cranes, 25 or 50 cts. per K W.; smaller items, like house pumps, water meters, blow-off tanks, painting, supervision, etc, from $1 to $2 per K W. *Tabulations.*—A summary of the preceding figures is shown in the table below, to which should be added the engineering fee. All these figures represent costs (per K W.) of plants of large capacity, but some have cost as high as $125 or even, in an exceptional case, $150 per K W.

COST OF STEAM PLANTS OF LARGE CAPACITY.

Items.	Cost of Steam Turbine Plants, per K W.		Cost of Reciprocating Engine Plants per K W.	
Excavations and foundations	$ 2.00 to	$2.50	$ 3.00 to	$5.00
Building	10.00	15.00	10.00	20.00
Tunnels (condenser water conduit)	1.75	4.00	1.50	2.75
Flues and stacks	2.50	3.50	2.50	3.50
Boilers and stokers	8.50	12.00	8.50	12.00
Superheaters	2.00	2.50	1.75	2.25
Economizers	2.00	2.25	2.00	2.25
Coal and ash handling systems	1.50	3.00	1.50	3.00
Blowers and ducts	1.00	1.50	1.00	1.50
Pumps and tanks	1.00	1.25	1.00	1.25
Piping systems	2.25	4.50	2.50	5.00
Turbo-generators	22.00	25.00		
Engines			18.00	22.00
Condensers (surface)	5.00	8.00		
Condensers (jet)			3.00	5.00
Exciters	.75	1.00	.75	1.00
Generators			10.00	12.00
Crane	.25	.50	.25	.50
Switchboards	2.00	3.50	2.00	3.50
Plumbing, painting, labor, etc	1.00	2.00	1.00	2.00
Totals	$65.50	$92.00	$70.25	$104.50

Smaller plants of about 3000 K W. capacity have been erected in the West at from $120 to $130 per K W., which cost may be reduced if a simple combination of machines is provided.

Rates for Electric Current Furnished by the Municipal Plant of Pasadena, Cal. (Eng. News, Feb. 10 and Apr. 7, 1910).—Rates are shown in following table:

	Incandescent Lighting.	Arc Lighting.	Power.
Up to 100 KW.-hrs. per mo	7 cts.	6.75 cts.	4 cts.
100 to 500 " "	6 "	5.75 "	2.4 "
500 to 1000 " "	5 "	4.75 "	.....
1000 to 1500 " "	.....	.....	2 "
1000 to 2000 " "	4 "	.....	.....
Over 2000 " "	3 "	.....	.....
1500 to 5000 " "	.....	.....	1.9 "
Over 5000* " "	.....	.....	1.75 "
Over 5000† " "	.....	.....	1.5 "
Over 10000 " "	.....	.....	1.5 "

* If used between 5 p. m. and 10 p. m.
† If not used from 5 p. m. to 10 p. m.

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Cost of Overhead Trolley Systems (By A. D. Williams, Jr. Eng. News, Dec. 23, 1909).—Tables: (1) Cost per mile of overhead materials; (2) Bracket-arm construction (a—with 37-ft. poles placed in center; b—with 30-ft. poles placed in center); (3) Cross-span construction, with 37 and 30-ft. poles; (4) Transmission line, and feeder line; (5) Comparison of costs, per mile:—

	Bracket arm.	Cross span 30 and 37-ft. poles.	Cross span 30 and 30-ft. poles.
Trolley wire and poles	$2,136.05	$2,420.99	$2,264.99
Transmission line....	732.57	732.57	732.57
Feeder line.....	682.69	682.69	682.69
Total.....	$3,551.31	$3,836.25	$3,680.25

Illustrations of Electrical Works:—

Description.	Eng. News.
Transmission line for Bay Counties Power Co.	Oct. 3, 1901.
Electric conduit construction at Cincinnati, O.	Aug. 20, '03.
Section of subway for pipes and wires	Feb. 16, '05.
Line construction for high-pressure electric railroads	April 6, '05.
High-pressure line construction for alternating-current rail'ys	April 6, '05.
New high-duty electric storage battery plate	May 4, '05.
High voltage electrostatic voltmeter operating in oil	Jan. 11, '06.
Details of transmission line poles, N. Y. C. & H. R. R. R.	June 14, '06.
Carbon regulator for storage battery	Sept. 27, '06.
*Details of steel towers for high tension transmission	Mar. 12, '08.
Rein.-conc. conduit for electric cables, L. I. R. R.	July 23, '08.
New type of switchboard, Salt River Project	Aug. 27, '08.
Divided manhole for high and low-tension conduit lines	Sept. 24, '08.
Insulator for 15,000 volt trolley line, Switzerland	Nov. 11, '09.
Circuit breaker for 110,000-volt lines	Dec. 23, '09.
Underground conduit construction for large transmissions	Sept. 29, '10.
	Eng. Rec.
Hydro-elec. power development; power house, dam, etc.	Apr. 3, '09.
Power house, penstocks, dam, etc., Wis. Hydro-Elec. Power Co.	Sept. 4, '09.

* Failed. See Eng. News, May 13, 1908.

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71.—MISCELLANEOUS DATA AND ILLUSTRATIONS.

DERRICKS AND CRANES.

Description.	Eng. News.
A 30-ton gantry crane for C. & O. Ry. wharves	Sept. 19, 1901.
Modern types of cranes for shipyard service	May 28, '01.
A 30-ton-locomotive crane; a 5-ton derrick crane	Nov. 20, '02.
An electric traveling crane with transfer carriage	May 7, '03.
The 120-ton floating derrick for the Norfolk Navy Yard	June 25, '03.
Portable pneumatic revolving cranes	July 23, '03.
Electric pillar cranes for handling cupola charges	July 30, '03.
A gantry crane with double cantilever bridge	Nov. 26, '03.
A 10-ton stiff-leg der. with ball and socket foot-block bearing	Sept. 22, '04.
Ore-unlocking machines for use at receiving docks	Aug. 3, '05.
Recent heavy cranes in English shops	Aug. 29, '07.
A reversible hoist for elevators and derricks on construction	July 2, '08.
A combined gantry crane and coal elevator	Sept. 17, '08.
Traveling cranes equipped with scales	June 17, '09.
	Eng. Rec.
Steel guyed derrick (mast 70 ft.) for building erection	Mar. 27, '09.
Details of steel stiff-leg derrick	April 23, '10.
A detachable seat for a boom derrick	June 18, '10.
Derrick tower for erecting the Montana capitol	Aug. 13, '10.

CHIMNEYS.

	Eng. News.
Section and plan of concrete chimney in Switzerland	Sept. 19, '01.
Steel chimneys for power station of St. Louis Transit Co.	Dec. 19, '01.
The design of self-supporting steel chimneys	July 20, '05.
Details of 300-ft. chimney of reinforced-concrete	Aug. 3, '05.
A 350-ft. brick chimney for acid chemical gases	Feb. 15, '06.
The design of reinforced-concrete chimneys	Jan. 3, '07.
Method of building a steel chimney	Mar. 14, '07.
Lightning protection for power plant chimneys. U.S. Navy. Yds.	Aug. 22, '07.
Heat expansion stresses in chimneys. Not illustrated	Mar. 5, '08.
Wind stresses in reinforced-concrete chimneys. Diagram	Sept. 10, '08.
Largest Chimney in the world: 50′ x 506′	Nov. 26, '08.
Tapering concrete chimney, 258 ft. high	Jan. 13, '10.
	Eng. Rec.
Underpinning a leaning chimney, illustrated	July 3, '09.

MECHANISM AND GEARING.

	Eng. News.
High-speed toothed gearing	Feb. 28, '01.
Chains and chain gearing	Sept. 5, '01.
Two new power transmission devices	June 8, '05.
A mammoth sheave wood block: ht., 53″; wt,. 640 lbs.	Mar. 21, '07.
Friction coefficient of wire rope drives	June 27, '07.
The Schmidt silent drive chain	Nov. 28, '07.
The design of friction clutches (mainly for automobiles)	Oct. 1, '08.

MARINE ENGINEERING.

	Eng. News.
A comparison of typical marine engines	Aug. 29, '01.
Cunningham-Seaton system of coaling war vessels at sea	Jan. 21, '04.
Notes on design of steamships Minnesota and Dakota	Sept. 1, '04.
Half cross-section of car-ferry steamer "Detroit;" M. C. R. R.	May 4, '05.

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Description.	Eng. News.
Experience in the design of marine screw propellers	Nov. 2, '05.
A pneumatic submarine signalling bell	July 12, '06.
Ocean steamers with steam turbines	Aug. 23, '06.
The Cunard steamship "Mauretania"	Sept. 27, '06.
Steel barges for transporting steel products, O. and Miss. R.	June 9, '10.
Reinforced-concrete barges on the Panama Canal	July 28, '10.

CABLEWAYS AND CONVEYORS.

	Eng. News.
A cheap cableway: for building concrete piers	June 26, '02.
Lubricating the wheels of chain conveyors	July 10, '02.
Method of handling ore at Bingham Canon, Utah	July 24, '02.
Conveyor for loading iron ore in ships off rocky coast	Sept. 4, '02.
Cable haulage for transporting marl to cement mill	Jan. 21, '04.
Aerial tramways of the U. S. Mining Co., Bingham, Utah	Feb. 11, '04.
The Ridgeway "two-belt" conveyor	June 16, '04.
Side and end elevations of extensible 16" belt conveyor	June 21, '06.
Cableway for cars in filling; cost compared with trestle	Oct. 10, '07.
Formulas for the design of cableways	April 16, '08.
Traveling bridge suspended from cableway for making fills	April 22, '09.
A unique belt conveyor 2000-ft. long, with equalizer	May 13, '09.
The Wetterhorn cableway incline, illustrated details	July 22, '09.

REVETMENTS.

	Eng. News.
Bank Revetment on the Lower Mississippi River	Oct. 31, '01.
Recent experiments with bank protection works, Ark.	Jan. 28, '08.
The David Neale System of bank protection	Oct. 22, '08.
Du Muralt system of reinforced-concrete shore protection	Dec. 17, '08.
A wave-break added to a concrete sea wall	Aug. 18, '10.
Separately-molded sections for a concrete sea-wall	Sept. 29, '10.

WELL BORING.

	Eng. News.
An oil-well pump rod joint protector	July 31, '02.
Diamond drill work on the deep waterways survey; cost	July 23, '03.
Electric rock drills; by E. J. Munby. Not illustrated	Sept. 3, '03.
Competitive tests of rock drills for air consumption	July 16, '04.
The manufacture and use of diamond tools	Jan. 19, '05.
Recovery of a diamond-crown from a deep bore-hole	June 29, '05.
A rotary drill core from a steel I-beam	Nov. 30, '05.
The work of well-drilling machines on the P. R. R.	April 12, '06.
A direct-acting gasoline rock drill	Nov. 26, '08.
	Eng. Rec.
Double wells, and casings, for pumping diff. waters	Feb. 20, '09.

MACHINES.

	Eng. News.
Evolution of drop hammer for die forging	Jan. 1, '02.
Centrifugal machines and their uses	Dec. 11, '02.
Requirements of machine tool operation—motor drive	Jan. 8, '03.
Concrete mixing and handling machine—sea wall	Jan. 15, '03.
Machines for briquetting flue dust, fine ore and fuel	Feb. 12, '03.
A new pneumatic hammer	Mar. 14, '03.
Diagrams for estimating hydraulic machinery	Oct. 22, '03.
A trussed wagon for hauling heavy machinery	Jan. 21, '04.
A rock crusher of 800 tons per hour capacity	June 4, '08.
A molding machine for building cement sidewalks	June 18, '08.
A machine for handling coke in storage	July 16, '08.

BUNKERS AND BINS.

	Eng. News
Safe and proper design of grain storage elevators	Mar. 10, '04.
Grain pressures in deep bins. Tables and illustrations	Mar. 10, '04.

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Description.	Eng. News.
Hydraulic diaphragms and grain pressure tests	April 28, '04.
Design of reinforced-concrete grain elevator bins	June 23, '04.
Grain pressures in deep bins; strength of wooden bins	July 14, '04.
Tests of grain pressures in deep bins, Argentina	Dec. 15, '04.
A problem in detailing hopper work	Aug. 24, '05.
Large reinforced-concrete coal pocket at Charlestown, Mass.	Aug. 27, '08.
Disastrous grain elevator explosion	Oct. 22, '08.
Reinforced-concrete bins for storage of crushed stone	Nov. 26, '08.
Reinforced-concrete storage bins for crushed stone	Nov. 21, '09.
Reinforced-concrete cylindrical storage bins	Dec. 2, '09.
Rein.-conc. locomotive coaling station, 2,000 tons capacity	June 23, '10.
Rein.-conc. grain bins, Gt. Nor. Ry., Superior, Wis.	Aug. 4, '10.
	Eng. Rec.
Sections, coal and ash handling plant and coal bunker	May 1, '09.
Large, 10,000 ton, concrete and timber coal pocket	May 8, '09.
Plans of rein.-conc. coal bunkers, Annapolis, Md.	May 15, '09.

COMPRESSED AIR.

	Eng. News.
Use of comp. air for contractor's plant. 4-stage air comp'r	Nov. 26, '03.
Apparatus and methods for testing air motors and hammers	Dec. 10, '03.
Caisson illness and diver's palsy; experimental study	May 5, '04.
An ingenious and effective air-lift pump	Nov. 24, '04.
Ignitions and explosions in discharge pipes and receivers	Mar. 2, '05.
Specifications for an air compressor	Mar. 2, '05.
New method of pumping sand by means of compressed air	Dec. 7, '05.
A new positive-pressure blower; high pressure	Feb. 1, '06.
Compressed air plant used in boring the E. River tunnels	Aug. 2, '06.
Emergency air-lift equipment for deep wells	Dec. 13, '06.
Hydraulic compressed air plant, Victoria mines, Mich.	May 2, '07.
A high-speed oil engine air compressor	May 7, '08.
Effect of moisture in air on compressed air machinery	June 18, '08.
Experimental studies of air-lift pumps; tests	June 18, '08.
Controlling the output of the air compressor	Nov. 5, '08.
The Rateau centrifugal air compressors and blowers	Jan. 20, '10.

HEATING AND VENTILATION.

	Eng. News.
Some experiments with ventilating fans	Nov. 3, '04.
Recent tests of centrifugal mine-ventilating fans	Nov. 10, '04.
Problem of ventilating N. Y. subways and similar tunnels	June 22, '05.
A fan blower driven by a steam turbine	Nov. 9, '05.
A rapid current hot water heating system	Nov. 22, '06.
Data for the design of hot water heating systems	Jan. 30, '08.
Air washing and humidifying	Aug. 13, '08.

TELEPHONES.

	Eng. News.
Long spans in telephone work	Mar. 2, '05.
The development of telephony	April 8, '09.

MINING.

	Eng. News.
Methods of mining, hauling and screening, in Alabama	Aug. 30, '06.
New type of jig for separation of metallic ores	Dec. 7, '08.
Hammer drills for overhand stoping in gold mines	July 28, '10.

METAL SPRINGS.

	Eng. News.
New Helical Spring Formulas (following is excerpt)	Jan. 7, '09.

New Helical Spring Formulas have been arranged by Mr. Chester B. Albree and are explained in his paper "Spring Formulas Simplified," which

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is published in the November issue of the "Proceedings" of the Engineers' Society of Western Pennsylvania. These are not strictly new, being derived from the well-known Reuleaux' formulas:

$$P = S\frac{\pi d^3}{16R}$$

$$F = \frac{32}{\pi}\frac{PR^2L}{Gd^4}$$

in which R = radius of coil to center of wire,
L = uncoiled length of wire,
and the other symbols correspond to those used below. The process of simplification is explained by Mr. Albree, in part, as follows:

In comparing the various formulas, it was found that certain quantities could be combined giving formulas of much simpler character and yet equally exact. This was accomplished by cancellations and reductions, eliminating the third and fourth powers and replacing them with areas, diameters and constants. This is done with the intention of rendering the solution of helical spring problems easy for anyone having at hand standard tables of areas and decimal equivalents. The writer is not in the spring manufacturing business and is not an authority on the subject. The formulas derived with the terms used are given below:

$$P = \frac{adS}{2D}$$

$$\frac{P}{W} = \frac{f}{f_2} = \frac{F}{F_2}$$

$$f = \frac{D^2}{40\,d} \text{ for } S = 100{,}000 \text{ lbs}$$

$$= \frac{D^2}{50\,d} \text{ for } S = 80{,}000 \text{ lbs.}$$

$$= \frac{D^2}{67\,d} \text{ for } S = 60{,}000 \text{ lbs.}$$

$$n = \frac{H}{f+d} - 1$$

$$F_2 = n\frac{fW}{P} = n\frac{D^2W}{40\,d\,p}$$

in which
P = closing, or maximum permissible, load of spring.
S = torsional strain, outer fiber.
W = any load on spring.
f = deflection of one coil under closing load.
f_2 = deflection under any load.
F = total deflection under closing load.
F_2 = total deflection under any load.
d = diam. of bar of wire.
a = area of bar or wire.
D = diam. of coil, center to center of bar or wire.
H = free height of coiled spring.
n = number of free coils.
G = modulus of torsion.
= 12,500,000 lbs.

The formulas are based on a spring designed so that when it is closed under a certain load, the strain S, selected, will be reached. The deflection formulas give the pitch of coils to produce strain S, when closed. With what is known ordinarily as "spring steel," it is safe to use S = 100,000 lbs., which is the practice of the spring manufacturers of Pittsburg.

The value to be used for S should depend, of course, upon the nature of the work for which the spring is designed and the conditions under which it must operate. The value given above (S = 100,000 lbs. per sq. in.) is rather high for general conditions. The German engineer's pocketbook, "Hutte," gives S = 4,500 kg. per sq. cm. (which is closely equivalent to 64,000 lbs. per sq. in.) for spring steel with fairly constant loading. In springs designed for continual removal and application of load, such as inlet or exhaust-valve springs of gas engines, S should not exceed 45,000 lbs. per sq. in.

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Tests of Steel Springs (Proc. A. S. T. M., Vol. VIII., 1908).—The following table shows the effects of different methods of tempering on the elastic limit and modulus of elasticity of steel.

Annealed in lead at	Hardened in oil at	Hardened in water at	Drawn to	Elas. Limit. Lbs. per sq. in.	Mod. of Elas. Lbs. per sq. in.
1400° F.	.....	.....	.....	78 500	27 550 000
.....	1450° F.	.....	560° F.	137 500	28 700 000
.....	1450° F.	.....	560° F.	160 400	27 150 000
.....	1450° F.	.....	400° F.	177 600	29 000 000
.....	1450° F.	.....	.....	187 400	28 610 000
.....	.....	1425° F.	1050° F.	180 700	28 070 000
.....	.....	1425° F.	900° F.	233 900	28 860 000
.....	.....	1425° F.	750° F.	240 800	29 220 000 broke
.....	.....	1425° F.	600° F.	219 800	30 420 000 broke
.....	.....	1425° F.	.....	212 000	29 960 000 broke

METAL HOISTING CHAINS.

Iron, Oval, Open-Link Chains.—The following formulas give the dimensions, weights and strengths of iron chains with open, oval links:—

Let d = diam., in ins., of round iron used; then—
$1.5\,d$ = transverse inside diam. of oval link, in ins.,
$2.6\,d$ = longitudinal inside diam. of oval link, in ins.,
= effective length of each link of chain, in ins.,
l = length of round iron in each link, in ins.,
$= 9.475\,d_l$
w_0 = weight of each link, in lbs.,
$= 0.2225\,l\,d^2$,
$= 2.108\,d^2$;
W' = weight per lin. ft. of chain, in lbs.,
$= 9.73\,d^2$;
S = ultimate strength of chain, in lbs.,
= 1.625 × strength of single rod of diam. d.

SOLAR POWER.

	Eng. News.
Power from the sun's heat	May 13, '09.

EXPERT VALUATIONS AND REPORTS.

	Eng. News.
Mine valuation by mining experts	Oct. 30, '02.
Report on Chicago street railways and subways	July 19, '06.
Depreciation allowances for various public service industries	Jan. 23, '08.
Valuation and inspection work of Wis. Tax & R. R. Com.	Mar. 4, '09.
Provision for depreciation by public utility corporations	Mar. 4, '09.
Table of freight rates on coal, iron and cement	Mar. 11, '09.
Necessary elements for water works valuation.—Alvord	Mar. 10, '10.
Valuation of track of Detroit St. Ry. System	Sept. 8, '10.
	Eng. Rec.
Diagram illustrating method of estimating "Going Value"	June 19, '09.

CONTRACTS AND SPECIFICATIONS.

	Eng. News.
Schedule of 120 clauses as guide in drawing specifications	April 21, '04.

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GLOSSARY.

(See, also, Index, page 1539, etc.)

A.

Abaciscus.—A diminutive of abacus.

Abacus.—The flat slab (plinth) forming the upper member of the capital of a column to support the architrave.

Abscissa.—The horizontal or x distance, measured parallel with the horizontal axis X, from the vertical or inclined co-ordinate axis Y to any point on a curve whose ordinate is y; x and y are co-ordinates to any point on a curve, measured from the *origin* or from the axes Y and X. See Analytic Geometry, page 256.

Acclivity.—An upward slope or inclination of the ground; opposed to *declivity*, or a slope considered as descending.

Adit.—A "horizontal" excavation or drift, specially used to drain or operate a mine; it is not continuous as a *tunnel* proper, which strictly has two openings. Word *tunnel* often wrongly used for *adit*.

Adze.—A tool with a curved blade placed at right angle to the position of an axe blade, and used by ship- and bridge carpenters in dressing the top surfaces of timber, ties, etc.

Alternate.—To pass from one state, as motion or position, to a second, then back to the first, and so on in rotation.

Alternations.—A complete alternation is a change in the direction of a current in a circuit from its former direction back again to that direction; symbol $\sim$.

Alternator.—A common term for an alternate current dynamo.

Ammeter.—An instrument for measuring the amperes of a current.

Ampere.—The unit of electric current; symbol, C. $C = \frac{E}{R}$ in which E = electromotive force in volts, and R = resistance in ohms. Current flowing at the *rate* of one ampere transmits a *quantity* equal to one coulomb per second. One volt-ampere = one watt = $\frac{1}{746}$ horse-power.

Angle-bar.—A bar of angle-iron.

Angle-bead.—A plaster-bead or staff-bead used to protect plaster from injury.

Angle-beam.—A beam with flange set at an angle with the main portion.

Angle-bevel.—Bevel-square.

Angle-block.—In Howe trusses, a "triangular" block of cast-iron or wood set at the junction of the wooden chords and braces, and through which pass the vertical iron or steel rods called *ties*.

Angle-rafter.—A rafter joining the inclined planes of a hipped roof.

Angle-splice.—A splice for rails.

Anneal.—To remove the brittleness of metals, earthenware, glass, etc., by heating them and then allowing them to cool gradually. This toughens, but lowers the tensile strength.

Anticlinal.—The incline or dip of stratified rocks in an upward fold; opposed to *synclinal*.

Anticlinal Axis.—The ridge of an anticlinal.

Apex.—The top junction of two or more lines or members; as the apex of a roof.

A priori.—From that which precedes; from the former.

Apron.—Anything resembling a common apron in form or use. The bridge of a ferry dock. A platform or flooring at a dock entrance. A covering to protect anything, as a dam, from water flowing over it.

Arbor.—An axle or spindle of a pinion or wheel. A mandrel, in lathe turning.

Architrave.—The lower part of an entablature, which rests directly on the columns and supports those parts of the building above. The molding around the extrados of an arch. Sometimes applied to ornamental moldings on faces of jambs and lintels of doors, windows, etc.

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Archivolt.—Ornamental molding on extrados of arch.

Armature.—That part of a magnet, dynamo, or motor designed to act upon, or to be acted upon by, the *lines of force* set up by the poles of the field-magnet, in order to produce motion and power (when placed directly at the pole of a permanent magnet it is called a keeper).

Classification (1):

Polarized a.—One made of steel or of another electro magnet and having poles which act upon and are acted upon by the field magnet poles.

Non-polarized a.—One made of soft iron with coils of wire arranged in any form.

Classification (2):

Ring a.—Round and usually of circular cross-section.

Drum a.—An armature of cylindrical form.

Disc a.—

Pole (-or-radial) a.—

Spherical a.—Thomson-Houston type.

Classification (3):

Unipolar a.—One whose polarity is never reversed.

Bipolar a.—One whose polarity is reversed twice in every revolution through the field of the machine.

Multipolar a.—One whose polarity is reversed "multi" times (more than twice) in every revolution.

Armature, Flat-Ring.—A ring armature with a core shaped like a short cylinder.

Armature, Girder.—H-shaped core.

Armature, Toothed-Ring.—A ring armature with core provided with a number of teeth forming spaces between which the armature coils are placed.

Arrester, Lightning.—An apparatus for protecting an electric circuit from lightning.

Arrester, Lightning, Transformer.—A lightning arrester for protecting transformers.

Arris.—The edge-line of an exterior angle formed at the junction of two surfaces meeting.

Arris-gutter.—A V-gutter fixed to the eaves.

Ashlar.—Cut-stone masonry. See Masonry, page 432.

Astragal.—A small convex molding in the form of a string of beads.

Axis.—The imaginary line, relatively motionless, about which a rotating body turns.

Axle.—A shaft in the position of the axis of a rotating body.

Axle-box.—The box containing the bearings for the spindle of the axle.

Axle-guard.—The parts of a car in which the axle-box moves vertically when the springs yield.

Axle-seat.—The hole in the car wheel to receive the axle.

Axle-tree.—A fixed axle as for a carriage, the wheels revolving.

Azimuth.—An arc of the horizon intercepted between the meridian of a place and the vertical circle passing through the center of a celestial object. A star's azimuth and altitude determine its exact position.

B.

Backing.—The rough masonry of an arch, abutment or wall, faced with a better class of masonry. Of an arch, it is the course of masonry resting upon the extrados.

Balance-bar = balance-beam.—A long bar or beam attached to canal-lock gates and drawbridges, and used in opening and closing them (usually serving partly as counterbalances).

Balk.—A beam or timber of considerable size. In coal mining: the sudden contraction of a bed of coal, for a certain distance.

Ballast.—Broken stone, gravel, slag, sand, or other suitable material placed on the sub-grade of a roadbed to support the railroad ties, give them lateral stability, and decrease the dust due to passing trains.

Ball-cock.—A lever with a hollow metal ball attached to *one end* and floating in a tank, and operating (as the ball rises and falls with the water) a valve of the cistern at the *other end*.

Ball-valve.—A valve formed by a ball resting on a circular seat when valve is closed; but which is free to rise with the upward pressure of the water.

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Barge-board.—Gable-board of a house.
Bars, Bus.—Omnibus bars. (See Bars, Omnibus.)
Bars, Negative-Omnibus.—The bus-bars that are connected with the negative terminal of the dynamos.
Bars, Neutral-Omnibus.—The bus-bars that are connected with the neutral dynamo terminal in a three-wire system of distribution.
Bars, Omnibus.—Heavy bars of conducting material connected directly to the poles of dynamo-electric machines, in electric incandescent light or electric railway installation, and therefore receiving the entire current produced by the machine. . . . . Main conductors common to two or more dynamos in an electrical generating plant. (The terms "bus" and "omnibus" bars refer to the fact that the entire or whole current is carried by them.)
Bars, Positive-Omnibus.—The bus-bars that are connected with the positive terminal of the dynamos.
Bascule-bridge.—A counterpoised drawbridge, dating back to medieval times; as the leaf of the span rises, the counterpoise weights descend.
Batter (not **batir**).—The incline (as of a masonry wall) from the perpendicular; the ratio of horizontal to vertical distance, as 1 in 12 = 1 in. hor. to 12 ins. vert.
Bay.—A panel of one span; sometimes, one span of several in a bridge. The plain part of anything enclosed or bordered by features in relief.
Bead.—Any small projecting cylindrical, globular or annular body.
Bearing.—The direction of an object by the compass. In mining: the run, course or strike. In architecture: the clear, unsupported span of a beam or timber. In engineering: the actual surface of contact of and with something supported, as of a beam, girder, pivot, axle, etc. In shipbuilding: the widest part of a vessel below the plank-sheer; also, the line of flotation of a vessel when ready for sea; using in each case the plural, bearings.
Bed-molding = bedding-molding.—A molding of the cornice of an entablature, above the frieze and beneath the corona.
Bed-plate.—A plate (usually of iron or steel) laid on a foundation (say of masonry) and used to give direct support to something (as a machine or bridge) and distribute the stresses quite uniformly (above or below or both).
Beetle.—A heavy wooden mallet (maul) to drive wedges, with handle for swinging; a rammer, with handle set in middle of one head, used by pavers.
Bell-crank.—A right-angle lever pivoted at angle, for changing direction of motion, force, etc.
Bench-mark.—A permanent bench of known or determined elevation with reference to a datum plane, in a line of levels.
Berm (old form, berme) = **berm-bank.**—A terrace; a strip reserved between top of cut and the waste bank in excavation, or between the bottom of fill and the borrow-pit in embankment; the bank of a canal opposite the tow-path.
Bessemer steel.—Steel made by the pneumatic process, consisting in blowing air through molten pig-iron in a "converter" lined with a refractory material, decarbonizing the iron; later a certain amount of carbon is restored by introducing spiegeleisen or ferromanganese.
Beton.—Hydraulic-cement concrete.
Bevel.—An instrument with a blade and a handle or stock, movable on an adjustable pivot or joint, for including any angle.
Bevel-angle.—Any angle except a right angle.
Bevel-gear.—Toothed wheels which gear at an angle, most often at right angle.
Bilge.—The belly or widest part of a cask; the nearly horizontal part of a ship's bottom, adjacent to the keel.
Bilge-keelson.—A fore-and-aft timber placed inside the bilge to strengthen it.
Bit.—The biting or boring part of a tool. The boring-bit is held or turned by the brace or bit-stock. Old form, "bitt."
Blast-pipe.—The exhaust-pipe of a steam engine; in locomotives, it leads into the smoke stack to create a strong draft.
Board, Switch.—A board provided with a switch or switches, by means of which electric circuits connected therewith may be opened, closed, or interchanged.
Boaster = boasting-chisel.—A broad chisel for rough-hewing and dressing stone. The use of such a chisel is called "boasting."

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Body.—Consistency or density, as in paints, oils, etc.
Bolster.—A "pillow" of timber for various purposes; a short timber or cap-piece resting on a post or column to give more extended bearing to the string-piece or beam.
Bond.—The particular arrangement of brick, stone, or timber to give certain specified joints and courses. Friction-resistance of steel in concrete.
Bonnet.—A cast-iron plate for covering the opening in a pipe or the opening in the valve-chamber of a pump; the cap or lid of an iron pipe; a wire netting for the smoke stack of a locomotive, to serve as a spark arrester.
Bore.—The internal diameter or caliber of a hole, as in a pipe or hollow cylinder; need not have been "bored."
Borrow-pit.—The site where excavated material, as earth, is obtained for filling elsewhere.
Boss.—A projecting mass, as of stone, to be cut or carved later. The enlarged part in diam. of a shaft for keying a wheel or (if at end) making a coupling.
Box-drain.—A rectangular (or square) drain of masonry, or timber, etc., under an embankment (or under ground).
Brace.—A strut, or auxiliary compression member in a frame; a bit-stock, or curved handle for holding and turning boring-tools or bits.
Bracket.—A projecting piece of wood or metal, fastened to a wall, or ceiling, etc., and used as a support for some object; hence, wall-brackets, hanging-brackets, etc.
Bracket, Telegraphic.—A support or cross-piece placed on a telegraph pole for the support of the insulators, which are supported on either *arms* or *brackets*.
Brake.—Any mechanical device for retarding motion (as of a vehicle) by means of friction.
Brake-hanger.—A bar or link suspending brake-beams and accessories from the truck-frame or from the body of car.
Brake-head.—The brake-block (usually cast-iron) fastened to the brake-beam and bearing on the circumference of the car wheel, forming at the same time the brake-shoe.
Brake-shaft.—The shaft on which the chain operating a car-brake (by hand) is wound.
Brass.—A useful alloy of copper and zinc; harder than copper; malleable and ductile.
Braze.—To solder with "hard" solder, as an alloy of "brass and zinc" (=copper and zinc in different proportions than that composing brass).
Break.—A want of continuity in a circuit.
Breaker, Circuit.—A device for breaking a circuit.
Break joint.—To arrange parallel members so the joints will not be opposite; thus, with the splicing of the leaves composing the chord of a Howe truss, or the two rails composing a track, etc.
Breast-beam.—The transverse forward beam of a locomotive.
Breast-board.—The weighted board or sled used in rope walks to keep the yarns taut while being twisted into a strand.
Breast-drill.—A drill-stock having a breast piece against which the workman bears while operating the drill.
Breasting.—The (curved) channel in which a breast-wheel turns.
Breast-line.—The rope used to connect the pontoons of a floating bridge.
Breast-wall.—A (low) retaining wall at the bottom of a slope.
Breast-wheel.—A water-wheel with radial floats or buckets on its periphery, the water being confined to the floats by the "breasting" of timber or masonry, nearly touching the wheel.
Breech.—The hind part of anything; the angle of a knee-timber, opposite the "throat."
Brest-summer.—A lintel; a beam or summer placed to support an upper wall, as over a shop window or door.
Bricknog.—A timber framing, as a partition, filled with brickwork.
Brick-trimmer.—A brick arch to protect a wooden trimmer in front of a fireplace.
Bridge-bar.—The tension bar of a car-coupling.
Bridge-board = notch-board.—A notched board of a stair to receive the ends of the wooden steps (treads) and risers.
Bridge-pit.—The pit for the counterpoise of a bascule-bridge.
Bridges.—Heavy copper wires suitably shaped for connecting the dynamo-electric machines in an incandescent light station to the bus-rods or wires.

Bronze.—An alloy of 85% ± of copper and usually 15% ∓ of tin; variable; sometimes no tin; sometimes contains zinc.

Brush-Holders for Dynamo-Electric Machines.—Devices for supporting the collecting brushes of dynamo-electric machines. Brushes should be adjusted carefully to speed of machine and resistance of external circuit. Carbon brushes are plates of carbon for leading current to electric *motors*. Collecting brushes are conducting brushes which bear on the commutator cylinder, and takes off the current generated by the difference of potential in the armature coils. Copper is almost universally employed.

Bucket-engine.—An inprovised water-wheel for a high fall with scarcity of water, consisting of a series of buckets on an endless chain running over a pair of sprocket-wheels.

Bucket-lift.—In mining, a set of iron pipes attached to a lifting-pump.

Bucket-pitch.—The circular line intersecting the elbows of the buckets of an overshot water-wheel.

Bucket-valve.—The valve at top of the air-pump bucket in a steam engine.

Bucket-wheel.—A series of buckets arranged on an endless chain passing over a wheel, for raising water; or, the buckets may be attached to the rim of the wheel.

Bucking iron.—A tool for pulverizing ore on a plate called a "bucking-plate."

Buckram.—Coarse linen cloth stiffened with glue and used for binding books.

Buffer.—An apparatus for deadening the concussion of railway cars.

Bulkhead.—A partition; in a ship, to form separate apartments; in a tunnel, conduit or mine, to prevent the passage of water, mud and air. An improvised wharf, sometimes constructed of sunken cribs, to form a basin, or to run parallel with the shore.

Bull-pump.—A pumping-engine with piston-rod attached directly to the pumping-rod, the weight of the rods producing the down stroke.

Bumper-timber.—A timber to which the cow-catcher of a locomotive is fastened.

Bumping-post.—A fender or buffer at the end of a railroad track to stop the cars.

Bunker.—See "coal-bunker."

Bunker-plate.—An iron plate covering the hole in a ship's deck leading to the coal-bunker.

Buoy.—A floating body anchored in a harbor or stream to indicate the positions of objects beneath the surface, as rocks, shoals, etc., or to locate channels. Among the various kinds are spar-buoys, can-buoys, bell-buoys, whistling-buoys, etc. In the U. S., red buoys mark the right-hand, and black buoys the left-hand, side of channels coming into port. Buoys with black and red transverse stripes mark dangers in mid-channel; while buoys with black and white longitudinal stripes indicate fairway. Green buoys indicate sunken wrecks.

Burnish.—To polish by friction, as metals.

Bur-pump = burr-pump = bilge-pump.—Has a cup-shaped cone of leather fastened to end of pump-rod, the sides collapsing as the rod descends.

Bush = bushing.—A lining of harder material fitted into an orifice to reduce wear by friction; used in machinery of all kinds.

Bushel.—U. S. standard = 2150.42 cu. ins. British imperial bushel = 2218.192 cu. ins.

Butt.—A door-hinge. A large cask, containing 110 imperial gallons.

Butte.—A rising ground or mound.

Butterfly-valve = butterfly-cock.—Employed in lift-buckets of large water-pumps and for air-pump buckets of condensing steam-engines. It is a double clack-valve or wing-valve, the two wings being hinged to a cross-rib cast in the pump bucket.

Butt-joint = butting-joint.—Opposed to lap-joint. A joint formed by the two pieces of metal or timber abutting endwise, and usually spliced together with other pieces.

Buttress.—A prop or support of masonry. A bearing or thrusting structure built against a wall to give it stability.

Buzz-saw.—A circular saw; it creates a "buzzing" noise.

By-pass.—An extra gas- or water-pipe passing around a valve or chamber so as to give some flow when valve or chamber is closed.

By-wash = by-lead.—A channel for surplus water from a reservoir or aqueduct, to prevent overflow.

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Cableway.—A taut suspended cable for conveying loads suspended in a car and moved by a hauling-rope or other device.

Caisson.—A water-tight casing used in building the foundation of structures in water too deep for the coffer-dam. It is sunk by under-mining and the masonry is laid on top as it descends into the mud.

Calcination.—The process of expelling volatile matter from a substance by heat, and reducing it to a friable state: as carbonate of lime, to lime.

Caliber = calibre.—The diameter, especially the inner diameter or bore. In firearms, a 44 caliber rifle is one whose bore is 0.44 in. dia.

Caliper = calipers.—An instrument like a pair of dividers, but with curved legs, for measureng inside and outside diameters.

Calking = caulking.—The operation of filling seams of vessels with oakum to prevent leaks; or, the joints of cast iron pipes with oakum and lead, the oakum and lead being calked with a calking-iron or chisel, hammered with a calking-mallet.

Calorimeter.—An apparatus for measuring the quantity of heat given off by a body, etc.

Cam.—A device, usually upon a shaft, for converting a regular motion into an irregular, an alternating, or a reciprocal one. An eccentric. An elliptical lever for raising the ends of drawbridges, when closed, to a firm bearing.

Camber.—In a bridge, a slight upward curve in the span, to allow for settling when loaded, or for appearance. In laying cast-iron-pipe culverts under a railroad embankment, they should be jointed or laid in a vertical curve, convex upward, to allow for settlement in the middle.

Camel.—A water-tight apparatus which is filled with water, sunk, attached to vessel's bottom, water pumped out, and which then assists in floating the vessel over a shoal, or raising her from wreckage.

Candle-foot.—A unit of illumination equal to that produced by a standard candle at a distance of 1 foot. The power is inversely proportional to the square of the distance; thus, the illumination at 3 feet is one-ninth that at 1 foot.

Candle-power.—The standard is a spermaceti burning at the rate of 120 grains of sperm per hour.

Cantilever.—A block or bracket framed into the wall of the building, projecting from it, and used for supporting a molding, balcony, eaves, etc.

Capital.—The top of a column, pillar or pilaster.

Capstan.—An apparatus something on the principle of a windlass, but with a vertical axis. The man-power is applied to capstan-bars inserted horizontally in holes near top of windlass, a few turns of the rope, cable or chain being wound around the barrel of the latter. At the bottom of the barrel is a pawl-head with pawls to engage a ratchet-ring secured to the platform.

Carbonize.—To combine with carbon, as in the manufacture of steel by the cementation process. Hence the term *carbonisation.*

Carburize.—To combine with carbon or a carbon compound. (The vapors of volatile hydrocarbons are often mingled with combustible gases in order to produce higher illuminating power in the latter). The process is called *carburization.*

Case-hardening.—A quick process of cementation or converting the outer surface of iron into steel by heating it in contact with charcoal or some animal matter as bone, hoof parings, or leather.

Casemate.—The masonry vault in the rampart of a fortress, or the armored bulkhead in warships, pierced in front with embrasures or port-holes through which guns are fired.

Casting-pit.—The part of a foundry where molds are placed and castings made.

Causeway.—A raised path or road over bad ground. A raised sidewalk.

Cavetto.—A concave molding of at least ¼ circle used in cornices, etc. A recessed pattern; opposed to relief.

Cementation.—In metallurgy, a process of effecting a desired important chemical change in a substance when heated in contact with another substance. Bar-iron may be made into steel by heating it above redness while embedded in charcoal-powder. Such a process is termed carburization by cementation. Decarburization, or the converting of cast-iron into malleable-iron, is effected by embedding the casting in red-hematite powder and keeping it some time at a red heat.

Cement-copper.—Copper precipitated by the process of cementation.
Center-centering.—The frame supporting an arch during construction.
Center of oscillation.—Coincides with the center of percussion.
Center of percussion.—That point in a revolving or swinging body, or pendulum, at which if all the mass were concentrated the effect would remain unchanged; the point of greatest impact with another body, remaining immovable without rotation at time of contact if the opposing body is fixed.
Center-plate.—A plate which supports a car-body on the center of a truck.
Center-valve.—A four-way gas-cock.
Centrifugal.—Radiating or outward (force) from the center. Opposed to centripetal.
Cesspool.—A shallow well of large diameter for receiving sewerage from isolated buildings. Best constructed in sand or gravel, with linings, say of brick, for the side walls only. Covered top.
Chamfer.—To cut away the edge of a square corner so as to form a bevel edge, generally projecting.
Chamfered.—Anything in which the surface is beveled. See Side-hatchet.
Chase.—To decorate metal-work by tooling.
Check-nut.—A nut placed on a screw or bolt to prevent the main nut from turning when in place.
Check-stop.—In deep dredging, a device to prevent the dredge-line from breaking when the dredge fouls.
Check-valve.—A valve placed in a pipe to prevent the backward flow of the water, steam, or other fluid.
Cheek.—One of two symmetrical pieces enclosing something between them: one of the jaws of a vice, one of the walls of a vein of ore, one of the sides of a pillow-block, etc.
Chill.—A metal mold for making certain kinds or parts of iron castings; the surface in contact with the mold cools rapidly and hardens.
Chimney-cap.—A device on top of a chimney which is turned by the wind so the exit-aperture is always to leeward, thus helping the smoke to escape. A chimney jack.
Chimney-stack.—Several chimneys carried up together.
Chisel-draft.—The dressed edge of a stone, either complete or as a guide in dressing the stone.
Chock.—A piece of wood inserted to prevent movement, as the chock nailed on the cap of a trestle between the two lines of stringers.
Chuck.—A block or device in a lathe for holding anything to be turned.
Churn-drill.—A long stone-drill operated by hand: raised and let fall.
Cinder.—A mass of ashes, containing more or less unconsumed coal. Pig-iron slag from a blast-furnace.
Circuit, Closed-Magnetic.—A magnetic circuit which lies wholly in iron or other substance of high magnetic permeability. All lines of magnetic force form closed circuits. An iron ring forms a closed-magnetic circuit. Where an air gap is formed, as in the case of a horse-shoe magnet with its keeper, it is called an open-magnetic circuit. In other words a closed circuit is formed by a ring of high magnetic permeability.
Circuit, Electric.—The path in which electricity circulates or passes from a given point, around or through a conducting path, to its starting point.
Circuit, External.—That part of a circuit which is external to, or outside the electric source.
Circuit, Magnetic.—The path through which the lines of magnetic force pass. (They always form closed circuits.)
Circuit, Short.—Shunt circuit.
Clack-valve.—A hinged valve placed in a clack-box, and consisting of a plate of leather strengthened above by a plate of iron larger in dia. than the main pipe, and below by a plate of iron smaller in dia. than the main pipe. The dia. of box is about 1½ times dia. of pipe. Used in pumping. A flap-valve or clapper.
Clamp.—Any instrument used to hold anything or to hold two or more pieces together, by pressure. Various forms for different trades and uses.
Clapboards.—Long thin boards, thicker on one edge than on the other, and nailed horizontally on the sides of a house, lapping shingle-fashion.
Claw.—A part of a tool resembling a claw (hand).
Claw-hammer.—A hammer cleft for drawing nails.
Claw-wrench.—A common wrench with one jaw fixed and the other movable.

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Claying-bar.—In blasting, a rod for driving clay into crevices, to protect the charge.

Cleat.—Any piece, as wood or iron, for fastening a rope; or by nailing to other pieces to fasten them together.

Clevis.—An iron shaped like a horseshoe, or stirrup, or U, with provision for inserting a bolt across or between the ends in order to form a link.

Click = clicker.—A kind of rachet. A small bar pivoted at one end, free to move backward on a toothed rack, but in moving forward it engages one of the teeth and moves the object forward, leaving it at rest during the backward stroke.

Climbing-irons = climbers = creepers.—Iron frames with spikes, for climbing telegraph poles, trees, etc.

Clincher-built = clinker-built.—Composed of pieces overlapping one another, as in clincher-built boats where the boards overlap like clapboards.

Clinker.—The fused or melted ash formed by the combustion of coal; partially-vitrified bricks.

Clip.—A metal clasp, as for holding a bunch of paper, or a Y-level telescope, etc.

Clip-yoke.—The small plate through which the ends of a U-shaped clip pass, and serving as washer-plate for the nuts of the clip.

Cloister.—A covered walk or arched way around the walls of a building, the outer edge being supported on a series of arcades or arches resting on columns.

Close-hauled.—Sailing as close to the wind as possible.

Clutch.—A movable coupling, as for connecting or disconnecting the ends of two adjacent shafts to make them revolve together or to allow them to revolve separately. If operating by friction it is called a friction-clutch; by engaging prongs, a bayonet-clutch. The cross-head of a piston rod is a form of clutch.

Coal-breaker.—A person occupied in breaking the large masses of coal as they come from the mine; or who operates a machine for this purpose; or the machine itself; or the building or structure in which the breaking is done.

Coal-bunker.—A place for storing coal.

Coal-gas.—An illuminating-gas obtained by heating coal in closed iron vessels free from air; contains about 46% hydrogen, 35% marsh gas, 7% carbonic oxid, 4% olefiant gas, tetrylene, sulphureted hydrogen, nitrogen and carbonic acid and traces of other gases.

Coal-oil = petroleum = rock-oil.—By refining, we get kerosene, naphtha, etc.

Coaming = combing.—The raised borders or edges of the hatches, to prevent water on the deck of a vessel from running into the hold.

Coal-tar.—The thick, black liquid which condenses in pipes when gas is distilled from coal. Contains anthracene, benzol, carbolic acid, creosote, naphtha, naphthalin, paraffin, pitch, etc. Used for metal coatings, as for cast-iron pipe; in making asphalt for pavements, etc.

Cock.—A faucet, turn-valve, or valve, for regulating the flow of fluids; as air-cock, gage-cock, feed-cock, etc.

Cock-brass = cock-metal.—An alloy of two parts copper and one of lead; used for large vessels, cocks or taps.

Cock-water.—Water used to wash away sand from ores.

Coefficient.—A quantity, number or constant (as 1/10, 4, b, etc.) used as a multiplier into some algebraic expression or physical property of a substance or condition: The coefficient of friction is the tangent of the angle of repose of a body; the differential coefficient (in the Calculus) is the rate of change of a function; the coefficient c in Kutter's formula, $c = v\sqrt{rs}$, is the coefficient of velocity of flow for the particular conditions imposed, as roughness of surface n, hydraulic radius r, and hydraulic slope s; the coefficient of safety, as 5, means that the conditions allowed are 1/5 the ultimate. "Coefficient" is synonymous with "modulus" in many cases, as "Coef. of Elasticity" = "mod. of elas."

Coffer-dam.—A water-tight enclosure built in a body of water in order to exclude the water and maintain a dry space for the construction of foundations, bridge piers, etc. Sheet piling is useful for this purpose.

Cog.—The tooth, catch or projection, as of a cog-wheel.

Cog-rail.—A rack or rail with cogs, placed between the rails of a track to engage the cogged driving-gear of the locomotive in drawing trains on inclined railways, too steep for ordinary traction. The rack is composed of cogs fastened between two angle-irons.

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Coil, Choking.—A coil of wire so wound on a core of iron as to possess high self-induction. Such coils are used to obstruct or cut off an alternating current with a loss of power less than with the use of mere ohmic resistance.

Coil, Electric.—A convolution of insulated wire through which an electric current may be passed.

Coil, Impedance.—A term sometimes applied to a choking-coil. (Self-induction produces impedance.)

Coil, Induction.—An apparatus consisting of two parallel coils of insulated wire employed for the production of currents by mutual induction. It consists essentially of a *primary* coil, a *secondary* coil, and an iron core, usually laminated. The primary coil is wound around the core, and over that the secondary coil. The former is composed of thick wire, and the latter of thin wire. If a current is passed through the primary coil its voltage is *raised* in the secondary coil.

Coil, Induction, Inverted.—An induction coil in which the primary coil is made of a long, thin wire, and the secondary coil of a short thick wire. Hence, a current passing through the primary coil induces a current of *lower* potential in the secondary coil.

Coil, Magnet.—A coil ot insulated wire surrounding the core of an electro-magnet, and through which the magnetizing current is passed.

Coil, Primary.—That coil or conductor of an induction coil or transformer, through which the rapidly interrupted or alternate inducing currents are sent.

Coil, Secondary.—That coil or conductor of an induction coil or transformer, in which alternating currents are induced by the rapidly interrupted or alternating currents in the primary coil.

Coil, Shunt.—A coil placed in a derived or shunt circuit.

Coils, Armature, or Dynamo-Electric Machine.—The coils, strips or bars that are wound or placed on the armature core. The wire is as thick as possible consistent with the desired electromotive force without requiring excessive speed of rotation. The armature coils should enclose as many lines of force as possible. If rods or bars are used they should be laminated in planes parallel to the lines of force so as to avoid eddy currents. The coils of pole armatures should be wound near the poles rather than on the middle of the cores. Open-circuit coils or simply open coils are those which are independent of one another, either for a part or the whole revolution. Closed-circuit coils or simply closed coils are connected coils. In alternate current dynamos the separate coils that are used on the armature may be coupled either in *series* or in *multiple-arc* (multiple-arc or multiple circuit means a compound circuit in which all positive poles are joined to one end of a conductor, and all negative poles to the other end.) They are connected in *series* usually in alternate current machines of high electromotive force where the converter is at a considerable distance; and in *parallel* where low electromotive force is sufficient, as for incandescent lamps in multiple arc.

Coke.—A useful product of coal, for the manufacture of iron. Coke is the "charcoal" of coal.

Colatitude.—90° minus the Latitude of the place.

Cold-chisel.—A tempered steel chisel with a cutting edge for cutting metal.

Cold-shot = cold-shut.—In foundry work, small particles of iron found in chilled parts of a casting.

Collar.—A ring, or anything resembling a common collar; an enlarged part of a shaft, or the enlarged portion of a car-axle, etc.

Collar-beam = wind-beam.—A timber beam stretching horizontally between two rafters (forming a letter A) in order to prevent sagging, etc.

Collectors of Dynamo-Electric Machines.—In a restricted sense *collectors* are brushes or points used to carry off the current generated in alternate-current machines, being distinguished from *commutators* which carry off the current generated in continuous-current machines. In other words, commutators change alternate currents, generated on rotation of the armature, to continuous currents, while collectors do not. Nevertheless, the name "collectors" is sometimes used to embrace "commutators".

Column = pillar.—A vertical "shaft" set on a "base" and surmounted by a "capital." In classic architecture we have the Doric, Ionic and Corinthian.

Combustion.—A rapid oxidation of a combustible substance, caused by the chemical union with the oxygen of the air.

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Commutator.—A device for changing the direction of an electric current.

Commutator, Dynamo-Electric Machines.—That part of a dynamo-electric machine which is designed to cause alternating currents, produced in the armature, to flow in one and the same direction in the external circuit; that is, to change alternate to continuous currents.

Concrete.—The compact mass formed by an aggregate of broken stone or other coarse material mixed with a matrix of (hydraulic-cement) mortar. There are other kinds, as asphalt-concrete, etc.

Concrete rubble-masonry.—Rubble masonry in which concrete is used instead of the usual cement mortar: for economy of cement.

Condenser.—A device for accumulating or condensing, as electricity, water etc. The surface condenser in a steam-engine is one in which the exhaust-steam passes through a large number of pipes immersed in cold water, constantly renewed.

Conductivity.—The property which a body or substance has of conducting electricity, heat or sound. It vares with the temperature and the physical stress (as tension) of the substance. In electricity, it varies inversely with the resistance, as $C = E \div R$; and is defined as the reciprocal of electric resistance, R.

Conductor.—A substance which will permit the passage of an electric current. The term "conductor" is used in a relative sense, as we have no knowledge of any material that is absolutely a non-conductor.

Conductor, Anti-Induction.—A conductor so constructed as to avoid excessive inductive effects from neighboring circuits.

Conductor, Armored.—One provided with a covering of metal over the insulating covering for protection from abrasion.

Conductor, Potential of.—The relation existing between the quantity of electricity in a conductor and its capacity. For a given quantity: the smaller the wire, the higher the potential. For a given conductor: the greater the quantity the higher the potential.

Cone-gear.—Two cones transmitting motion by rolling friction.

Connecting-rod.—The "rod" which connects the crank of a body having a circular motion, in order to transmit motion or force to or from some other body, as the connecting-rod of a locomotive or of a beam-engine.

Console.—A bracket or corbel with ornamental carvings or shape like an S; used for support of cornice, etc. A wall-bracket for supporting machinery.

Contacts, Lamp.—Metallic plates or rings connected with the terminals of an incandescent lamp for ready connection with the line.

Contour-line.—A line on a topographical map joining points of equal elevation. A 10-ft.-contour map shows contour-lines marking surface elevations 10 ft. apart, vertically.

Controller.—Any device used to regulate the flow of air, water, electricity, etc.

Controlling-nozzle.—A device for regulating the size of stream issuing from a nozzle.

Convective, Electric.—The air particles, or air streams, which are thrown off from the pointed ends of a charged, insulated conductor. These streams act magnetically, and are themselves acted on by magnets.

Converter.—A vessel swung on an axis, lined with some refractory material, in which molten pig-iron is converted into Bessemer steel.

Coping.—The top or finishing course of a masonry wall, usually projecting a few inches beyond the line of neat-work of the facing, and beveled for appearance and to shed water.

Corbel.—A horizontal projecting piece, acting as a cantilever in assisting to support a beam or piece resting or partly resting upon it. In mill-building construction, the columns are often capped with corbels which project a short distance under the superimposed beams. Their utility is questioned by many engineers.

Corduroy-road.—A road constructed of logs laid transversely and close together, usually across muddy or marshy ground.

Core.—The inner portion or filling of a wall. The inner mold of a casting to make the hollow space, as of a pipe. The iron body of an electromagnet. The comparatively thin wall of masonry constructed in the heart of an earth dam to prevent leakage.

Core, Armature, Filamentous.—An armature core, the iron of which consists of wire.

Core, Armature, H.—An armature core in the shape of the letter H, generally known as the shuttle- or the girder armature. We also have the I armature.

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Core, Armature, Lamination of.—The subdivision of the core of the armature of a dynamo-electric machine into separate insulated plates or strips to prevent eddy currents.

Core, Armature, of Dynamo-Electric Machine.—The iron core, on, or around which, the armature coils of a dynamo-electric machine are wound or placed. It is laminated to prevent eddy currents. In *drum*, and in *ring* armatures the laminæ are in the form of thin insulated discs or soft-iron plates; in *pole* armatures, in bundles of insulated wires.

Core, Armature, Radially-Laminated.—An armature core, the iron of which consists of thin iron washers.

Core, Armature, Ribbed.—A cylindrical armature core provided with longitudinal projections or ribs that serve as spaced channels or grooves for the reception of the armature coils.

Cornice.—An ornamental or molded projection at the top of a building-wall, masonry wall, etc.

Corrugated.—Wrinkled in regular furrows, as corrugated iron.

Cotter = cotter-bolt = cotter-key.—A wedge inserted to fasten or tighten. A split bolt whose ends are spaced apart when inserted as a key.

Coulomb.—The unit of electrical *quantity*. That quantity of electricity that will pass in one second in a circuit whose resistance is one ohm, and electric-motive force one volt.

Coulomb-Volt.—Volt-coulomb or Joule = 0.7373 foot-pound.

Counterbrace.—In a frame, the brace which crosses the main brace and is designed to transmit the compressive stress in a panel due to negative shear, i. e., when the stress in the main brace would change from compression to tension.

Counterfort.—A buttress, or portion projecting from the face of a wall to stiffen it.

Counter-rod = counter.—In a frame, a rod which crosses the main diagonals and is designed to transmit the tensile stress in a panel due to negative shear, i. e., when the stress in the main diagonals would change from tension to compression.

Counter-shaft.—A secondary shaft running parallel with and driven by a main shaft.

Countersink.—A drill or bit for reaming or making a countersink (= countersunk hole). A hole enlarged at the top to receive the countersunk head of a bolt or rivet, so as to be flush with face of plate.

Counterweight.—A weight used to balance another; a counter-poise.

Coupling.—A device for connecting two shafts so they will act as one in running. Generally, anything that connects, as couplings for cars.

Coursing-joint.—A joint between two courses of masonry.

Cover.—The cap-head of an upright steam cylinder.

Crab.—A portable windlass for hoisting; used in building operation for hoisting bricks and mortar. A horizontal shaft with one or two cranks for turning by hand, and geared to a drum on which the hoisting rope winds. The whole thing set in a wooden or iron frame.

Cradle.—A frame placed under the bottom of a ship, on the "ways," to give support in launching or on marine railways. Any contrivance having a "cradle" form for enclosing or supporting.

Cramp = cramp-iron.—A piece of iron bent at the ends for holding together pieces of stone, timber, etc., in structures.

Crane.—A machine for moving heavy weights and placing them in any desirable position; hence there must be provision for motion in three directions, as vertical, longitudinal and lateral. The two latter may be combined in a circular motion by a *rotary crane*, consisting of a jib or swinging arm *rigidly attached* to a vertical post and rotating together about the axis of the post. In a *derrick-crane* the top of the post is held in position by guys or guy-ropes. A *traveling crane* is one in which the longitudinal motion is provided for by the whole crane traveling longitudinally on a track, as with the *locomotive crane* and various forms in heavy machine shops; the lateral motion being obtained by the movable hoisting carriage operating on the main transverse girder of the crane.

Crank.—A bent arm attached to a shaft or axle and forming a radial leverage for turning. A *single crank* is used only at the end of an axis, as in the common grindstone. A *double crank* is used in the middle of a shaft and has four bends, thus _|‾|_; or, sometimes used to denote two double cranks for reciprocating motion, thus —|‾|_|_|—. See also Bell-crank.

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Crematory furnace.—A furnace for burning garbage.
Crest.—The top, as of a dam.
Cross-cut.—In mining, a level driven across so as to connect two other levels.
Cross-hair = cross-wire.—A very fine strand of spider's web or metal wire stretched across the diameter of a telescope to mark the direction of sight on a distant object. Used in transits and levels. Two together, crossing each other at an angle of 90°, form cross-hairs. Two or three arranged horizontally and parallel in a transit are used as stadia wires for measuring distances instead of by chaining.
Cross-head.—The sliding bar at the end of the piston-rod of a steam engine.
Cross-cut saw.—A large saw operated by a man at each end, and used for sawing logs or large timbers *across* the grain. Opposed to *whip-saw* which is used to saw (also by hand) *with* the grain, as planking from logs.
Cross-section.—A section at right angle to the (longest) axis.
Cross-valve.—A valve placed at the junction of two or more pipes.
Crow = crowbar.—An iron bar with end pointed or sligthly bent, and used for prying, as a lever, etc.
Crowfoot.—A mark used by surveyors when chaining, consisting of a central line, marking the exact distance, and two flaring lines, forming a sort of arrow-head.
Crown-arch.—The arched plate which supports the crown-sheet of the fire-box of a boiler.
Crown-bar.—One of the bars on which the crown-sheet rests.
Crown-gate.—The head gate of a canal lock.
Crown-glass.—A good quality of common window-glass.
Crown-saw.—A cylindrical saw with teeth on edge of cylinder.
Crown-sheet.—The sheet forming the upper part of the fire-box of the furnace of a steam-boiler.
Crown-tile = hip-tile = ridge-tile.—A bent or curved tile used at the crown as a finish for pan-tile or flat-tile roofs.
Crown-wheel = contrate-wheel = face-wheel.—A wheel with teeth or cogs at right angles with its plane.
Crucible.—A pot for melting metals, ores, etc. The hollow at the bottom of a chemical furnace for collecting the molten metal.
Crucible steel.—Cast-steel.
Crypt.—The part of a church or cathedral below the main floor.
Cupola-furnace.—A furnace for remelting cast-iron.
Cup-valve.—A sort of semi-spherical valve, or balance-valve over an opening to which it fits when valve is closed.
Curb.—Anything used to curb or check. The outer casing of a turbine-wheel. The wall-plate at the bottom of a dome. The casing of masonry, wood or iron built inside a well that is being sunk. Stones or timber at edge of a well or of a street, etc.
Current, Alternating.—A current which flows alternately in opposite directions; that is, its direction is rapidly reversed.
Current, Assumed Direction of Flow.—The direction the current is assumed to take, i. e., from the positive pole of the source through the circuit to the negative pole of the source.
Current-breaker.—Any device for breaking the circuit of an electrical current.
Current, Constant.—A current that continues to flow in the same direction for some time without varying in strength.
Current, Continuous.—A current which flows in one and the same direction.
Current, Direct.—A continuous current.
Current, Electric.—The quantity of electricity which passes per second through any conductor or circuit; that is, the *rate* of flow. (See *Ampere*. *Coulomb*.)
Current, Induced.—The current produced in a conductor by cutting lines of force. It results from differences of potential produced by electro-dynamic induction.
Current-meter.—Any device for measuring the flow of water in streams.
Current-meter.—A form of galvanometer.
Current, Multi-phase.—A rotating current.
Current, Periodic.—A simple periodic current.
Current, Rotating.—A term applied to a current which results by combining a number of alternating currents, whose phases are displaced with respect to one another. A rotating current is sometimes called a *poly-*

phase or *multiple-phase current*, particularly if there are more currents combined. When three currents are combined the displacement between each set of phases is 120 degrees. A rotary current, unlike an alternating current, possesses, in a certain sense, a definite direction of flow. Its effect on a magnetic needle is to cause rotation.

Current strength.—The product obtained by dividing the electromotive force by the resistance. $C = \frac{E}{R}$. (See *Ampere*.)

Currents, Eddy.—Useless currents produced in the pole-pieces. armatures, field magnet cores of dynamo-electric machines or motors, or other metallic masses, either by their motion through magnetic fields, or by variation in the strength of electric currents flowing near them.

Current-wheel.—A wheel driven by the current of a stream.

Cut-off.—A device for automatically cutting off the steam from the steam-chest to the cylinder before the piston has made its full stroke, the balance of the stroke being made by the expansive force of the steam in the cylinder. A channel cut across a bend thereby shortening the main course of the river.

Cutwater.—The up-stream angle-edge of a bridge pier, designed to more effectively lessen the impact of moving water, ice, logs, etc.

Cyclopean Masonry.—Rubble concrete masonry. Massive concrete in which large rubble stone is added or filled in as the mass is built up; each piece of rubble stone must be thoroughly embedded in, and surrounded by, the concrete.

Cyma = cyme = cima.—A cornice molding having the profile of an ogee, letter S, or curve of contra-flexure.

D.

Dado.—The shaft of a pedestal, between cornice and base.

Damp = fire-damp.—A gas in coal-mines which explodes when mixed with air and ignited; very dangerous. Black-damp or choke-damp is a carbon-dioxide gas in collieries and differs from fire-damp but often found mixed with it. See *Davy*.

Damper.—A metal plate, slide or door, used to regulate the draft of or to a stove or furnace, in order to control the rate of combustion. Many are regulated automatically, as by heat or by steam.

Davy.—A safety-lamp for use in mines.

Dead-head = sinking-head = sprue.—The extra length of metal in a gun-casting, not used because of its inferior quality.

Dead load.—The "dead weight" or non-moving weight of a structure. Opposed to *live load* and *wind load*, but may include *snow load*. The dead load should always be specified in detail. The empty or non-paying rolling-stock of a train.

Dead-oil = heavy oil.—The oils obtained in the distillation of coal-tar above 340° F., and which are heavier than water.

Dead-point.—That position of the crank of an engine when the engine is on its *dead-center*, i. e., when the crank and connecting-rod are in a straight line.

Declination of a heavenly body is its angle north or south of the equator; i. e., it is its distance from the celestial equator measured on a great circle passing through the body and the pole.

Declivity.—A downward slope of ground.

Densimeter.—An apparatus for finding the specific gravity of a substance.

Dentil = dentel.—One of a series of small blocks, uniformly spaced, in a cornice.

Derrick.—A machine for lifting heavy weights, something similar to a crane but having the boom (corresponding to the jib of the crane) pivoted or hinged at its lower end (to the post). It is therefore more convenient than a crane, for use in general building operations. *Floating derricks* are large derricks erected on barges or vessels specially constructed.

Diaphragm.—A thin plate, serving as a partition, placed across a small opening or hollow tube, as the diaphragm of a telephone.

Die.—The cubical part of a pedestal between its cornice and base. An engraved steel for stamping a design. Pieces of hardened steel forming a female screw for cutting screw threads; they are fitted into a die-

stock and are adjustable for use in cutting threads of different diameters.

Dike (formerly *dyke*).—See *Dyke*.

Dip.—In geology, the angle which a stratum of rock makes with the horizontal. The *point of dip* is the direction of the compass to which the stratum inclines. The dip of a compass needle from a horizontal plane.

Disk = disc = discus.—A flat circular plate.

Disk-clutch.—A form of friction-clutch.

Dock.—An inclosed water-space for vessels while handling cargo; a space or structure for loading or unloading cargo, for repairs, etc. See "Wharves, Piers and Docks," page 802.

Dog = dog-iron.—An iron hook with one or more points at one end, to drive into timber for the purpose of moving it. Used largely in saw-mills. Has many forms and uses. A cramp (which see).

Donkey-engine.—A small engine used for performing light work, as pumping water into boilers, hoisting anchors, handling building material, etc.

Donkey-pump.—A feed-pump for boilers. An extra pump for special purposes.

Dormer-window.—A vertical window in the face of a projection built out from a sloping roof.

Doubling-frame.—A machine for winding double silk threads.

Dovetail.—One of a series of wedge-like projections or tenons and of corresponding mortises in boards or timbers for fastening them together.

Dowel.—A wooden or metallic pin inserted part way into two pieces of wood or stone to unite them.

Down-draft.—A downward draft of air in a mine, chimney, etc.

Draft.—The vertical depth of water which a vessel requires or "draws." The dressed edge of a stone. See *Chisel-draft*.

Draw-plate.—A drilled plate of hard steel, or a drilled ruby or diamond, for drawing wire to reduce its diameter, make it uniform, or shape it. The holes are somewhat conical. The wire may be drawn successively through holes of decreasing diameter.

Drift.—A nearly horizontal tunnel in a mine. Loose material, as timber, trees, etc., in a current. In geology, loose rocks, boulders, gravel, sand, etc., which have been deposited on bed rock; *glacial drift* if deposited by glacier.

Drift = drift-pin.—A long, round, tapering pin of steel, used in enlarging the punched holes in metal plates, or smoothing the inner edges.

Drift-bolt.—A steel bolt used in driving out other bolts. A round, steel pin for driving into auger-holes in timbers to fasten them together; bridge-stringers are thus drift-bolted to caps. When pointed it is called a pointed drift-bolt.

Drip.—Any small tube or channel to lead water from a structure and let it fall to the ground; as a projecting member of a cornice, or a small channel cut under the edge of a coping.

Drop.—One of a series of short cylinders or truncated cones placed in a row in cornices, as ornamental.

Drum.—A revolving cylinder around which ropes are wound in hoisting.

Dry-rot.—A rot in timber which has not been seasoned sufficiently. Thoroughly seasoned timber will not rot if protected from dampness; or if treated with a preservative the decay will be slow, even in damp places.

Ductility of a metal is that property which renders it capable of being extended by drawing, as through a draw-plate, with lessening diameter, and without fracture. Gold is the most ductile; then silver, platinum, iron, copper, palladium, aluminum, zinc, tin, lead.

Dyke (more modern spelling is *dike*).—A long bank of earth thrown up to prevent low lands from being overflowed. A levee. In geology, a fissure in rocks, filled with lava or other material while in a molten state.

Dynamics, Electro.—That branch of electric science which treats of the action of electric currents on one another and on themselves or on magnets.

Dynamo.—Dynamo-electric machine or generator.

Dyne.—The unit of force in the centimeter-gram-second system. It is about 1.02 times the weight of a milligram.

E.

Eccentric.—A sort of crank device for converting a regular circle motion into an irregular reciprocating straight-line motion. Thus, in the

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steam engine, it consists of a circular disk rigidly attached to a shaft, but not at center of disk, and revolving around with it; the circular disk being surrounded by a loose ring attached to the eccentric rod leading to the valve-gear of the cylinder, thereby regulating the cut-off and making the engine self-acting.

Electrolysis.—Chemical decomposition effected by means of an electric current. Water- and gas-pipes are affected by electrolysis when forming the return circuit of electric distribution as in street railways. The electrolytic action occurs when the current jumps *from* the pipe, carrying atoms of metal along with it.

Electrometer.—An apparatus for measuring differences of potential.

Entablature.—In architecture, a sort of lintel construction supported on columns and extending toward the roof, and comprises the architrave, frieze, and cornice.

Erg.—The unit of work or the work done when unit force is overcome through unit distance. A dyne-centimeter.

Escarpment.—The abrupt face of natural rock or soil in a cliff or high ridge. In fortifications, ground cut away forming a nearly vertical slope about a position to render it inaccessible.

Escutcheon.—The little plate for protecting the keyhole of a door; or the plate to which the handle is attached.

Expansion-drum.—A drum with adjustable diameter, used in connection with driving an endless cable.

Eye.—A circular hole in a plate, or formed by a loop of iron. The center hole of a wheel on a shaft.

Eye-bolt.—A bolt with an eye or ring at one end.

Eyepiece.—The lense or combination of lenses in an optical instrument to which the eye is applied.

F.

Face.—The front of anything. The face of a valve is the part of the surface which comes in contact with the seat.

Face-hammer.—One with a flat face. A hammer with a cutting and blunt end, used in preparing stone for finer tool-work.

Face-lathe.—A lathe for turning face-work.

Face-wheel.—See *Crown-wheel*.

Fall = fall-rope.—The fall of a tackle, or the rope used with pulleys in hoisting. "Fall and tackle" means "block and tackle."

False-work.—A temporary structure to aid in the erection of the permanent one.

Farad.—The practical unit of electric capacity.

Farad, Micro.—The millionth part of a farad.

Fascines.—Sticks or brush tied in bundles and used as a protection for river-banks; also used in the construction of sea-walls in connection with piling. Fascines are weighted with stone.

Fatigue.—The weakness of metal, as a bar, produced by repeated application of stress well within the breaking load.

Faucet.—A device in a pipe for regulating the flow of a liquid. The primitive form is a hollow plug in a cask, with a transverse hole near the outer end to be filled with a hollow plug when not in use.

Feather.—A thin rib cast on iron-framing to give it strength. A rib cast on a shaft to fit a corresponding groove in the eye of a wheel. A small steel slip inserted in a shaft and projecting so as to fit the groove in the eye of a wheel. One of two pieces of metal placed in a hole in a stone, which is to be split by driving a "plug" or steel wedge between them. The stone is said to be split by "plug and feather."

Feather-edge.—A very thin edge.

Feather-joint.—A joint between boards consisting of a small strip, bead or feather fitting into the opposite mortises on the edges of the boards.

Feeders.—In a system of distribution by constant potential, as in incandescent electric lighting, the conducting wires extending between the bus-wires or bars, and the junction boxes.

Feller = felling-machine.—A machine for cutting standing timber.

Felling-saw.—The saw in a felling-machine (See *Feller*.)

Felloe = felly.—The wooden rim of a cart wheel, into which the outer ends of the spokes are driven, and around the outer circumference of which the iron tire is fitted.

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Felt.—A coarse fabric of hair, wool, or wool and fur, matted together by moisture, heat or pressure, but not woven like cloth.

Fender.—A bundle of rope or piece of timber hung over the side of a vessel to protect it from injury by rubbing against another vessel or wharf, etc. A guard post at the edge of a pier or wharf.

Fender-pile.—One of a series of piles driven to protect a structure or work from injury by concussion resulting from moving bodies.

Ferro.—Relating to some compound of which iron is a constituent element.

Ferrule.—A metal ring around anything to prevent it from splitting or breaking. A bushing for expanding the end of a flue of a steam-boiler. Many things, in the nature of a ring for protection. A sleeve.

Ferry-bridge.—The landing-stage of a ferry.

Field, Alternating.—An electrostatic or magnetic field, the positive direction of the lines of force in which is alternately reversed or changed in direction.

Filler.—Anything to fill a space or void, as a long narrow plate between the web-plate of a girder and its vertical angle-iron stiffener, sometimes called *filler-plate*. Washers are often used as fillers. A separator, as one of the cast-iron spools near the ends of wooden bridge-stringers to space them one or two inches apart so the air can circulate and keep them seasoned. In painting, the prime coat for filling in between the fibers of the bare wood.

Fillet.—A small flat molding, as in a cornice.

Fire-damp.—The dangerous and explosive gas from coal in a mine. See *Davy*.

Fish.—A long piece of timber or iron secured alongside of another to strengthen it; or at a joint to give stiffness, as one of two fish-plates to stiffen a rail-joint.

Flag.—A broad flat stone (flagstone) used for paving. A flag-pole used by surveyors.

Flange—A projecting edge or rim, as the flange of a car-wheel, or of cast-iron flange-pipe (the flanges being bolted together when laid).

Flap.—A heavy valve to prevent back tidewater into a sewer, etc.

Flap-valve.—See *Clack-valve*.

Flashing.—Sheets of lead, copper, zinc, tin, etc., used on roofs and other places, at the junction of roof and chimney, and at corners, to prevent the rain from leaking through. At a chimney, the upper edge of the sheet of metal is inserted into the joints of the brickwork, so the rain cannot get beneath the flashing, and the rest of the sheet is flattened down against the chimney and passes between two courses of shingles.

Flask = molding-flask.—A wooden or iron mold used in foundries to hold the sand and patterns employed in molding and casting. May be in one or two parts, a lower and an upper.

Flatting-coat.—The last of four or five coats of paint prepared so as to dry without gloss; it is of pure white lead diluted with spirits of turpentine.

Flier.—One of several steps, called *fliers*, in a *straight* flight of stairs. Opposed to *winding stairs*.

Flint-glass.—Glass in which the silica is combined with oxid of lead in various proportions, and also containing potash. The lead gives it a higher specific gravity and refractive power, and greater brilliancy.

Flood-gate.—A gate designed to open on the rising tide to allow water to fill a basin, and to close at the flood tide to prevent it from flowing out at that point. A gate designed to allow water to escape at floods. Various uses.

Floor-hanger.—A bearing-bracket fastened to the floor and used for supporting shafts and countershafts.

Flume.—An artificial channel for a stream of water; used in gold-mining, logging, irrigation, etc.

Flush.—To drench plentifully with water, as flushing a sewer, gutter, etc.; having the idea of fullness. Even with the surface.

Flush-box = flush-tank.—A rectangular box in a water-closet for flushing out the bowl. The outlet-valve is opened by pulling a cord attached to a lever. As the water in the box lowers, the inlet-valve, consisting of a ball-valve or ball-and-lever valve, opens allowing the box to fill; and when full the valve closes automatically through the rise of the ball float.

Flux.—Any substance or mixture that will assist the welding or adhesion of two metals by preventing the formation of rust, which is very rapid at such times.

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Fly-wheel.—A wheel with a heavy rim placed on a revolving shaft of a machine for equalizing the motion of the machinery.
Follower.—Any cog-wheel, or other part of a machine, which is driven or which follows the motion of another part called the *leader*.
Foolscap.—A folded writing-paper 12 x 15 to 13 x 16 inches in size, making the sheet about 8 x 12¼.
Foot-board = foot-plate.—The platform on which the engineer and fireman of a locomotive stand.
Foot-poundal.—The unit of energy, equal to a foot-pound $+ g$ ($= 32.2 \pm$) = 421402 ergs.
Force, Electromotive.—The pressure which tends to move electricity from one place to another. The unit of E. M. F. is the volt, *V*.
Force, Lines of.—A term applied to the strength of a magnetic or electro-magnetic circuit. They reach between the opposite poles which produce them, and never intersect. They lie at right-angle with the direction of ether waves.
Forceps.—Tongs, pincers or pliers for seizing and manipulating things which it would be impracticable to handle with the fingers.
Forebay.—That part of a mill-race (channel where the water flows from the dam to the mill-wheel) where the water flows upon the wheel. The penstock.
Forge.—An open furnace provided with a bellows for heating metal to be hammered or forged into shape. Portable forges are used for heating rivets in bridge-erection. A hearth or furnace for making malleable iron by the "direct process." A forging-machine is called a drop-press and operates with a hammer, by power.
Forge-roll.—One of a series of rolls for rolling slabs or blooms into puddled bars.
Forging-machine.—A machine for forging metal, usually heated.
Foundry iron.—Iron containing sufficient carbon for casting.
Four-way cock.—A cock or valve with four passages: two in the plug and four for delivery.
Foxtail = fox-wedge.—A wedge inserted into the end of a pin or bolt so that when the latter is driven to the bottom of the hole the wedge will be forced into the pin, spreading the end of the pin and making it secure against withdrawal.
Frame.—Any construction composed of parts fitted together and designed to support itself or other things.
Friability.—The quality ot being friable, i. e., easily broken or crumbled.
Friction.—The resistance to the relative motion ot surfaces of bodies in contact: *sliding* friction if one body tends to slide on another; and *rolling* friction if the body is on wheels or rollers, as the rolling friction of a train is say 7 or 8 lbs. per ton. The angle of friction, called the *angle of repose*, is the angle of inclination (with the horizontal) of a surface at which a body will just tend to overcome the frictional resistance and begin to slide, by the force of gravity. The *coefficient of friction* is the tangent of the angle of repose. The friction of liquids is more or less associated with viscosity.
Friction-balls.—Balls used to reduce friction of moving parts, as in bicycles and some movable bridges; such bearings are called *ball-bearings*.
Friction-brake.—A brake acting by friction.
Friction-gearing.—A gearing of wheels imparting motion one to another by the friction of contact alone. They can be thrown in and out of contact readily; when in gear or contact they are called friction-tight.
Friction-rollers.—Cylinders used to reduce friction of moving parts, as the rollers of a movable land pile-driver.
Friction-wheels.—Wheels especially designed to reduce friction of moving parts; or to provide for excess of stress in machinery, as dredging, by allowing the outer rim ot wheel to give way to all stress in excess of the frictional resistance on the inner section of the rim.
Frieze.—In architecture, the decorative feature ot an entablature between the architrave and cornice; also similar decorative features elsewhere.
Frustum.—That part next to the base when the top is cut off, as of a cone.
Fulcrum.—The point of rest or support of a lever when lifting a body. The support itself.
Furring.—Strips nailed on to a wall for subsequent lathing and plastering; or to the bottoms of joists and rafters to bring them to a level surface. The placing of said strips.
Fuse = fuze.—A slow-burning tube-like or rope-like attachment to an

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explosive charge. A *time-fuse* is one which will explode the charge in a certain time. Electric fuses (fired by a spark caused by a break in the electric circuit) are most frequently used for blasting, in work of magnitude. *To fuse* is *to melt* and blend together.

Fusible-plug.—A plug of fusible metal placed in the shell of a boiler of a steam-engine, and intended to melt and allow the steam to escape when a dangerously high temperature is reached.

Fusion-point.—The temperature at which a substance melts.

G.

Gable.—The vertical end of a triangular- or pitched roof; the triangular canopy over a window. The gable-end of a house is the end-wall.

Gad.—A pointed steel bar or shorter tool for driving into anything and loosening it.

Gadding-machine = gadder.—In quarrying, a movable platform on which a steam-drill is mounted.

Gage = gauge.—An instrument for determining the dimensions, quantity, distance, force, capacity, etc., of anything; or the measurement itself. The gage of a wire is its diameter; in shingling, it is the exposed length of the slate, tile, etc., below the lap; in gunnery, the bore of a gun.

Gage-saw.—A saw with a gage-bar to determine the depth of kerf.

Gage-stuff = gaged-stuff.—In plastering, a plaster of Paris mixture for quick-setting, in making moldings, etc.

Gallon.—Four quarts. U. S. gallon = 231 cu. ins. = 3.7853 liters = capacity of a cylinder 7" dia. and 6" high; more accurately, such a cylinder has a capacity of 230.90706 cu. ins., or 0.9996 gallon, or almost exactly one part in 2310 too small.

Gallows-frame.—The frame for supporting the beam of a beam-engine. The structure for supporting the pulleys and cage in a mine shaft.

Gang-drill.—A machine containing a number of vertical drills in the same head.

Gang-plank = gang-board.—A plank with cleats nailed on transversely for steps and used as an inclined stair.

Gap-window.—A long narrow window.

Gas-compressor.—A pump for compressing coal-gas into reservoirs for railroad-cars, etc.

Gasket.—Any fibrous or soft substance used for packing, in machinery. The circular collar used when pouring lead around lead-pipe joints.

Gas-meter.—An apparatus for measuring the flow of illuminating gas in pipes, etc.

Gate.—A valve. When placed at the headworks of a water supply it is called a head-gate.

Gate-house.—A small house in which the gate of a reservoir is situated and operated.

Gauss.—The unit of intensity of magnetic field.

Generator, Dynamo-Electric.—An apparatus for producing electricity by the mechanical movement of conductors through a magnetic field, cutting lines of force.

Gear.—The connecting parts in machinery for transmitting motion.

Gearing.—A train of toothed wheels, or worms, belts, ropes, etc., in machinery.

Generator, Motor.—A generator driven by electricity instead of by steam-water-, or other power.

Gib.—A wooden support under the roof of a coal-mine. An iron clasp used in connection with a key for clasping pieces together. The arm of a crane. A fixed wedge used with the driving wedge to hold together the brasses at the end of a connecting-rod of an engine.

Gin.—One of the two main uprights of a pile-driver, between which the hammer operates. A machine for separating the seeds from cotton, also called a cotton-gin. A machine with a drum and winding rope, for various purposes as moving houses in streets, etc.

Gin-block.—A tackle-block over which a rope runs, and suspended by a hook attached to it.

Gin-tackle.—A double and a single block used as a system of pulleys for hoisting.

Girder.—A simple or composite beam of larger dimensions than an ordinary beam. Thus, *beam*-girder, *box*-girder, *plate*-girder, etc.

Glacis.—In fortifications, a gentle slope over which the advancing enemy is brought into a direct line of fire.
Gland = gland-box.—A stuffing box. A joint tightly packed and capable of retaining lubricants for a period of time.
Glaze.—A vitriable substance, as salt, applied to the surface of brick, tile, etc., and giving it a transparent coating. An enamel is an opaque coating.
Glue.—A substance having cement properties; the common gelatin boiled out of the hides and hoofs of animals.
Gooseneck.—A flexible coupling, or a pipe shaped like the letter *S*. A nozzle with a universal joint similar to that used on the stand-pipe of a fire-engine.
Governor.—An automatic regulator for controlling the supply of steam, water or gas. The compass-shaped two-ball apparatus on an engine; the supply-valve is connected to the levers which are operated by the radial movement of the balls, as the latter revolve faster or slower.
Grapnel = grapple.—One or more hooks in a cluster for grasping hold of things in deep water; a grappling-iron.
Graving-dock.—A dry-dock for graving or cleaning the bottoms of ships.
Grillage.—Two or more courses of heavy timbers laid parallel and at right angle (sometimes notched at their intersections) and usually drift-bolted together, to serve as a foundation resting on piles or on the bottom, and supporting a masonry pier or other structure.
Grille.—A grating or open work of metal, usually of wrought-iron, for ornamental work.
Groin.—The intersection of simple vaults or arches crossing each other at right angle. A breakwater constructed across a beach to form a protection from the waves and prevent the drifting and washing of sand and mud. Sometimes spelled *Groyne*.
Groove.—A long narrow channel as if made by a tool, for something to fit into.
Ground-ice = anchor-ice.—Ice formed at the river bottom, prior to surface-freezing.
Ground-swell.—A deep swell of the sea caused by a distant or late storm. The surface of a rolling country.
Grout.—Thin mortor poured or forced in joints of masonry.
Groyne.—See *Groin*.
Grubbing-hoe.—A long-handled instrument for digging up or cutting roots; used in "grubbing." A mattock.
Gudgeon.—The metal journal of a horizontal shaft, or that part which turns in the collar.
Guide-bar.—One of the two parallel sides fitted on the cross-head of a steam-engine, on which the cross-head slides.
Guncotton.—Cotton or other cellulose substances digested in a mixture of nitric and sulphuric acids, or in nitric acid alone. Explodes violently by percussion.
Gun-metal.—A bronze for making cannon; now supplanted by cast iron, and more frequently by steel.
Gun-pendulum.—An apparatus for determining the strength of gunpowder.
Gunwale.—The upper edge of a ship's side.
Gusset = gusset-plate.—A triangular or trapezoidal plate riveted to box-girders to stiffen them transversely; or used to connect the ends of steel floor-beams with the web of plate-girders, the gusset extending usually the full height of the girder or nearly so in order to give transverse stiffness. In general: a large steel connecting-plate.
Guy.—A rope, rod or chain fastened to anything to keep it from swinging, as the guy of a derrick.
Gyrate.—To whirl or revolve about a point or axis.

H.

Hacking.—In masonry, the cutting up of large courses into smaller ones for expediency when the stones run small.
Hack-iron.—A miners' pick or hack. A chisel for cutting nails.
Half-trap.—A sinking bend in a sewer-pipe.
Halving.—The notching of two timbers of equal thickness together, either crossing each other or at the ends, so the thickness of joint will be equal to that of one of the timbers.

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Hammer-beam.—A short beam projecting from the foot of a principal rafter, outward toward the center of the truss but not reaching a similar one from the opposite rafter: used in church roof-trusses.

Hammer-dressed.—In stone-cutting, dressed with a pick or pointed hammer.

Hand-lever.—In a steam-engine, the lever for starting, stopping or reversing the engine.

Handscrew.—A jack, or machine for raising heavy weights.

Handspike.—A wooden lever for raising weights or working a windlass or capstan.

Hand-wheel.—A form of circular crank, as the hand-wheel of a car-brake.

Hanger.—A bracket from the ceiling or wall, or a stand from the floor, with a box and oiling device, for supporting a line of shafting. The plates, straps or yokes at the ends of floor beams, for suspending them to bridge-trusses. *Yoke-hangers* are used in connection with wooden floor-beams and consist of square iron bent over the pin of lower truss, the ends passing down through the floor beam and an iron plate, using a nut and check-nut at each end of hanger. *Plate-hangers* are riveted directly to the ends of steel floor-beams, a hole being drilled in upper end for the truss-pin to pass through. A *hanger-board* is a board for supporting electric arc-lamps, making easy connection poles of lamp and line-circuit.

Hasp.—A metal clasp as for a door, with a slot for folding it over a staple, and fastened by a pin or padlock. A metal hook for the same purpose.

Hatch.—The opening in a ship's deck leading to the hold; usually termed *hatchway*. The cover of such an opening.

Haunch.—The part of one side of an arch between the crown and springing.

Headbay.—The water space just above a canal lock.

Head-block.—The forward carriage for supporting logs being sawed in a mill. Any block for supporting a pillow-block.

Header.—A stone or brick with its longest dimension at right angle to the face of the wall.

Heading.—A small passage or opening or driftway excavated in advance in the line of a tunnel to facilitate the work.

Head-valve.—The delivery valve in a steam-engine.

Headway = headroom.—The clear height of space overhead. In railroading, the clear height above rail to the lower part of an overhead bridge or other structure. Advance or progress in work.

Heart-cam = heart-wheel.—A cam-wheel with a heart-shaped channel on face of disk in which a guide-wheel travels at the end of an arm, and used for converting rotary into reciprocal motion.

Heart-shake.—Defects in timber, consisting in cracks or shakes extending from the center outward.

Heat-unit.—(B.T.U.) The amount of heat required to raise 1 lb. of water through 1° Fahr. See page 1347.

Hectrogam.—100 grams, equal to 1543.235 grains.

Hectoliter.—100 liters, equal to 26.4 U. S. gallons.

Hectometer.—100 meters, equal to 328′ 1″.

Helve.—The handle of an ax, hatchet or adz.

Helver.—The handle of a mining tool.

Hematite.—One of the most valuable of iron ores; red oxid of iron, Fe_2O_3. Found in large quantities in the Lake Superior region.

Hemp.—The fibre of a plant; used for making hemp rope.

Henry, A.—The practical unit of self-induction.

Herring-bone bridging = bridging.—The diagonal pieces nailed between the floor-joists to give stiffness, by distributing the resistance to the floor loads over several joists.

Highway.—A road or way of common right for all to pass.

Hinge.—A device for joining two pieces in such a manner that one may be turned or swung around or upon the other, as the hinge of a door or of a trunk. A common hinge consists of two straps or leaves, joined by the pin or pintle passing through the knuckle. A *rising hinge* is one which rises when the door opens, to clear the carpet, and usually closing itself. A *butt hinge* is a common door hinge where the leaves butt against each other.

Hinge-pin.—The pin or pintle of a hinge.

Hip.—The *external* angle or corner formed at the junction of two sloping roof faces, and supported by a *hip-rafter*. Opposed to *valley*, which is the *internal* angle, and supported by a *valley-rafter*.

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Hip-roof = hipped-roof.—A roof with four sloping faces, rising immediately from the wall-plates and with the same inclinations.
Hip-tile.—The tile which saddles the hip of a roof.
Hitch.—A kind of knot used in making one rope fast to another or to a spar, boom, post or timber. See p. 668.
Hoarding.—An English term for a fence for enclosing a building or materials while building work is in progress; a board fence used as an enclosure.
Hold-beam.—One of the transverse beams in the lowest tier of beams in a ship's hold.
Holding-plate = anchor-plate.—The plate at the anchorage of a cable or guy, through which the cable passes or is secured; the plates being backed with masonry, or loose rocks, earth, etc.
Hook.—A bent iron for holding a link, or suspending anything. A *pulley-suspension hook* is an S-hook which can be hung over a beam, and support a pulley from below.
Hook-block.—A pulley-block fitted with a hook for suspending it, or weights to it. The *standing part of the hook* is that part attached to the block.
Hook-bolt.—A bolt with one end in the form of a hook; used in fastening the wooden floor of a bridge to the iron stringers.
Hoop.—A circular band or clasp. Hoops around wooden tanks are often adjustable.
Horizon.—The *astronomical* or *celestial horizon* is the great circle of the celestial sphere whose plane is perpendicular to gravity at any station.
Horse.—A wooden frame with four legs for supporting staging; many similar things.
Horse-shovel.—A road scraper.
Horsing-iron = iron.—A long-handled calking-iron held by one man and driven by another.
Hour-circle.—A circle perpendicular to a north and south-axis, and graduated un-clockwise into 24 radial divisions. Any great circle which passes through the two poles. See *Hour Angle*, page 621.
Hour, Kilo-Watt.—A unit of electrical power equal to a kilo-watt maintained for one hour.
Hour, Watt.—A unit of electrical work—one watt for one hour.
Housing.—A niche in a wall for a statue. The *jaw* of a frame which holds the journal-box or housing-box.
Housing-frame.—The frame which holds the rollers, in a rolling-mill.
Hub.—The center of a wagon-wheel from which the spokes radiate and through which the axle-tree passes; or, in car-wheels, the metal part in the center to which the paper web is clamped. The bell-end of a pipe.
Hydrant = fire-plug.—An apparatus with a valve and with hose-connections for drawing water from a main.
Hydraulic balance.—A water-wheel regulator.
Hydraulic jack.—A jack operated by a plunger or piston against some liquid as oil.
Hydraulic main.—In gas-works, a large pipe containing water into which the raw gas is brought, and serving as a purifier and to convey the crude gas to the condenser.
Hydrocarbon.—A compound of hydrogen and carbon, alone.
Hydrometer.—An instrument for determining the specific gravity of fluids.
Hygrometer.—An instrument for determining the humidity of the atmosphere.
Hygroscope.—An instrument for determining the approximate humidity of the atmosphere.
Hysteresis.—Molecular friction due to magnetic change of stress.

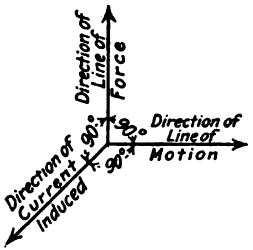
I.

Ice-breaker.—A structure built in the water to protect bridge-piers from moving ice.
Ice-machine.—A machine for producing ice. Anhydrous ammonia is the solution most used, and is most efficient.
Idle-wheel.—In toothed gearing, a wheel placed between two others to preserve the same direction of motion in both of them. In rope transmission of power, a wheel to make the cable sag and preserve its tension.

Impedance.—Opposition to current flow.

$$\text{Impedance} = \sqrt{(\text{Ohmic resistance})^2 + (\text{inductance resistance})^2}.$$

Impost.—The upper part of a wall or column from which an arch springs.
Inch.—2.54 centimeters. One meter = 39.37 inches.
Inductance.—The induction of a circuit on itself, or on other circuits. Self-induction. The practical unit is the *henry*. The coefficient of inductance is a constant quantity which, multiplied by the current strength passing in any coil or circuit will give the induction due to that current. The practical unit of inductance is 1,000,000,000 centimeters. (See *Ohm*.)
Induction, Electro-Dynamic.—Electromotive forces set up by induction in conductors which are either actually or practically moved so as to cut the lines of magnetic force. Flemings' rule:



Induction-pipe.—In a steam-engine, the pipe through which the live steam passes to the steam-chest. The *induction-port* is the opening from the steam-chest into the cylinder. The *induction-valve* is the valve controlling the steam into the cylinder.
Indurate.—To harden, as with *indurated clay*.
Infusorial earth.—Fine white earth composed of minute siliceous shells and resembling magnesia. Used as an absorbent in making dynamite (with nitroglycerin).
Ingot.—A cast of metal from a mold (ingot-mold), as pig iron.
Injector.—An apparatus for forcing water into a steam-boiler.
Inscribe.—In geometry, to draw within, as a square within a circle. Opposed to *circumscribe*.
Insulator, Oil.—A fluid insulator filled with oil.
Insulator, Single-Shed.—An insulator with a single inverted cup.
Insulator, Telegraphic or Telephonic.—A non-conducting support of telegraphic, telephonic, electric light or other wires. Insulators are generally made of glass, porcelain or hard rubber, and assume a variety of forms.
Interlocking system of signals.—In railroading, a system of operating switches and signals jointly by means of locking mechanism, operated from a central station, so trainmen can tell the position of the switches from a distance.
Interpolate.—To find the missing number of a series. In many mathematical tables it is desirable to find (usually by simple proportion, but not always) intermediate values to those given, and this is done by interpolation.
Intrados.—The inner line of an arch; the outer line is called the *extrados*.
Invert.—An inverted arch, as the floor of the lock-chamber of a canal, or the lower part of the brick sewer, or the inverted arches used in the foundation walls of buildings in order to distribute the pressure more uniformly.
Ion.—One of the elements of an electrolyte; *anions* are evolved at the anode, and *cations* at the cathode.
Isobar.—A contour-like line on a map, connecting places at which the barometric pressure is the same.
Isochime = isocheim.—A contour-like line on a map, connecting places having the same mean winter temperature.
Isoclinal.—In geology, strata having the same inclination or dip.
Isoclinal lines.—In magnetism, contour-like lines on a map, through points at which the dip of the needle is the same.

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Isometric.—In crystallography, that system which is characterized by three equal axes at right angles, and includes the cube.
Isotherm.—A contour-like line on a map, connecting points having the same mean temperature.

J.

Jack.—An instrument for raising weights: "jacking them up". A *screw-jack* consists of a screw or worm working in a thread; a *lever-jack* is a kind of a *ratched-jack*. Various forms. See Hydraulic jack.
Jack-engine.—A donkey-engine.
Jack-rafter.—A short rafter, usual in hip-roofs. A sort of sub-rafter or secondary rafter, parallel with the main rafters and supported on the main purlins; used for supporting directly the sub-purlins or sheathing of the roof.
Jack-rib.—A rib in a framed arch or dome shorter than the others.
Jack-timber.—A framing timber shorter than the others as in the floor of a bay.
Jad.—In quarrying, a long deep gash or hole made in quarrying soft rock, as a sort of heading for wedging or blasting the balance. In coal-mining, a "holing" or "benching" so the mass of coal may fall or be loosened by wedging or blasting.
Jadding-pick.—A sort of pick for cutting a jad in a quarry or coal-mine.
Jag-bolt.—A bolt with a barbed shank.
Jamb.—The vertical side of an opening or recess in a wall, as of a door or window, serving to support in part the weight above, as a lintel. A door-jamb, or window-jamb, or fireplace-jamb.
Jamb-post.—The upright post or timber at the side of an opening or jamb.
Jam-nut.—A sort of lock nut, or a nut screwed down on a bolt hard against another nut to prevent the latter from working loose. Used under wooden floor-beams of bridges when suspended by iron hangers.
Jaw.—Anything in the shape or use of a common jaw, as the jaws of a vice, or wrench, or stone-crusher.
Jaw-bit.—A bar under a journal-box for uniting the two pedestals in a car-truck.
Jaw-bolt.—A bolt with a U-shaped head perforated to carry a pin.
Jetty.—A sort of pier or arm constructed in the water to divert the current and protect banks from washing away, or to scour out the channel, or to cause slack-water and deposit of mud in any place. A pier in the ordinary sense of a wharf.
Jews'-harp.—A shackle, or partly-closed link in the shape of a horse-shoe and with eyes and bolt, for connecting the ring of an anchor with the chain or cable.
Jib.—The projecting arm of a crane.
Jig-pin.—In mining, a pin to prevent the turn-beams from turning.
Jig-saw.—A vertical reciprocating saw with a narrow blade for sawing scroll-work in boards.
Jimmy.—A short crow-bar.
Joggle.—A sub-tenon at the end of a framed timber to prevent it from moving laterally. A notch or mortise in a piece of stone or timber, or a key engaging such a notch, to prevent a corresponding piece or counterpart from movement.
Joggle-piece.—A piece like the king-post of a truss.
Jogglework.—In masonry, stones internotched or keyed together, as in light-house construction.
Joggling-table.—A table or machine for dressing or concentrating ore.
Joist.—One of the spaced beams supporting the boards of a floor, as floor-joist; or a ceiling, as ceiling-joist; or the floor of a bridge, as bridge-joists; etc. Where beams act singly or independently they are called girders, especially if larger than common joists.
Joule.—The unit of electric energy or work. A volt-coulomb. One joule = 0.7373 foot-pound. One joule per second = 1 watt.
Journal.—The part of an axle or shaft which rests in the bearings.
Journal-bearing.—The bearing-support of an axle or shaft. In general, it consists of the *brasses*, resting in the pillow-block and inclosed in the journal-box.
Journal-box = housing-box.—The box (cast-iron) which contains the journal (of the car-axle or shaft), the journal-bearing and key, and the oil-packing for lubricating the journal.

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Journal-brass.—The bearing of the journal. (Metals of different kinds while in rubbing contact will not wear as rapidly as those of the same kind.)
Jump.—A step in a masonry course to accommodate a rise or fall in ground level, or slope. Used in buildings.
Jump-coupling = thimble-coupling = ring-coupling.—A coupling with a coupling-box consisting of a ring or thimble over the two connected ends of the shaft, the connection being made by pins through thimble and shaft, or by parallel keys feather bedded.
Jumper.—A drill used for drilling holes in stone: a short drill worked by a hammer; a long drill, weighted, raised by hand and let fall, and not worked by a hammer.
Junction-box.—A chamber connecting lines of pipes or wires.
Jute.—A plant producing jute-fiber; swells with moisture and is inferior for rope.

K.

Kaolin.—A fine variety of clay or decomposed feldspar.
Kedge.—A small anchor with an iron stock.
Keeper.—A key which may be inserted in a stationary or sliding bolt or other piece, to keep it in place. An armature of a magnet, or a piece of soft iron across the poles of a magnet when not in use to maintain (or increase) the power.
Kerf.—A channel or cut made as by a saw. A saw-kerf.
Key.—Anything that locks or holds fast.
Key-bed = key-seat.—A groove for a key for locking, as a wheel to a shaft.
Key-bolt = cotter-bolt.—A bolt with a key or cotter, instead of a nut.
Keystone.—The center stone of an arch ring, at the apex or crown.
Kibble.—The bucket of a shaft or mine for hoisting material; the hoisting chain is called the *kibble-chain*.
Kiln.—A furnace or large oven for baking, burning or drying, as a brick kiln for burning or baking brick.
Kiln-dried.—Anything, as timber, deprived of moisture by treatment in a kiln or furnace.
Kilogram.—1,000 grams in weight, equal to 2.20462 lbs.
Kilogrammeter.—A unit of work, equal to 7.233 ft.-lbs.
Kiloliter.—A unit of capacity, equal to 1,000 liters.
Kilometer.—1000 meters.
Kilowatt.—1000 watts.
Kinetic energy.—Energy in some form of motion.
King-post = king-piece.—The vertical post in a truss with two sloping chords meeting the top of post and framed into it. The two chords may be rafters. The middle vertical member of a king-post truss.
King-rod.—A rod used in place of a king-post, in a king-truss.
King-truss = king-post truss.—A truss framed with a king-post.
Knee.—A piece of wood or metal having an angle and used to join two pieces together, giving support and stiffness; as the beam of a ship to a side timber.
Knee-strap.—An iron strap used in connection with a knee-timber.
Knot.—A fastening with a rope. See page 668.
Knuckle-joint.—A flexible joint, as with two adjoining links.
K. W.—A contraction for kilo-watt.

L.

Laboratory.—A place with suitable apparatus for conducting investigations or experiments.
Labyrinth.—A maze, or combination of passages making exit difficult.
Lacquer = lacker.—An opaque varnish containing lac.
Ladder-dredge.—A dredge with buckets carried on a ladder-like chain.
Lagging.—Falling behind. Narrow strips of wood or planking placed outside of and between the ribs of an arch or tunnel to give support to extraneous material. Used in tunnel construction. The outer wooden casing of boilers, or the wooden strips placed on the periphery of a winding drum. Also used in the sense of *sheeting*.
Lag-screw.—An iron bolt with a flat square or hexagonal head and with the other end sharp and threaded like a wood-screw; used in fastening small wooden guard-rails to the ties.

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Lamp, Arc.—An electric lamp, the source of whose light is the voltaic arc, formed between two or more carbon electrodes.

Lamp, Incandescent.—An electric lamp in which the light is produced by the electric incandescence of a strip or filament of some refractory substance, generally carbon.

Lancet-window.—A long narrow window crowned with an acutely-pointed arch.

Landing.—A resting-place or platform at the end of a flight of stairs, or interrupting a series of steps. A place on shore for discharging passengers or freight from water-craft, usually called *landing-place*.

Landlocked.—Protected from the wind and waves, as a small body of water nearly shut in by land.

Landmark.—A prominent object locating a line or corner of the boundary of a tract of land.

Lantern-wheel = lantern-pinion = trundle-wheel.—A sort of drum-like wheel with two parallel heads joined near their peripheries by parallel rods or spindles and so spaced as to engage the cogs of a spur-wheel.

Lap.—The length or width of exposed surface partly covered by another, as the lap of shingle or slate in roofing.

Lap-joint.—Opposed to *butt-joint*. A joint formed by the leaves overlapping as in the chord of a Howe-truss bridge. If each piece is composed of only one leaf, the ends are halved to form the lap-joint.

Lap-weld.—Opposed to *butt-weld*. A weld made by lapping two metals before hammering.

Larboard.—In navigation, the left-hand or port side. Opposed to *starboard* or right-hand side.

Latch.—A sort of self-locking device which may be disengaged usually without the use of a key.

Lattice-girder.—A girder with web consisting of diagonal pieces crossing like latticework.

Lead (pronounced leed).—Opposed to lag. In earthwork, the distance from c. of g. of cut to c. of g. of fill. The pay-lead may be less than this distance. In steam-engines, the advance of a valve or valves so that the steam is admitted in front of the piston, or allowed to escape behind it, before the end of the stroke.

Leader.—In mining, the leading vein. A pipe leading from the roof to conduct rain-water. The principal wheel in a mechanism, and giving motion to the *follower*.

Leading-block.—A block for simply keeping a rope in a certain lateral position, without transmitting any of its power.

Leading-wheel.—In a locomotive, one of the smaller wheels ahead of the driving-wheels.

Leaf.—A tooth of a small pinion.

Leaf-bridge.—A small draw with leaves swinging vertically.

Leaf-valve = flap-valve = clack-valve.—A hinged or pivoted valve in a pumping engine.

Lean-to.—A roof or building whose rafters pitch against another structure.

Ledge.—A shelf or something projecting, as a small horizontal molding, or the side of a rebate against which a window or door is stopped.

Lewis.—A contrivance for securing a hold in a vertical wedge-shaped hole in a block of stone, for hoisting it; consists of two side pieces of metal wedging a center piece (called lewis-bolt) firmly and to which the hoisting tackle is attached.

Lewis-bolt.—A wedge-shaped bolt fastened in a hole drilled in a stone, by pouring lead around it or by inserting metal wedges. If used for lifting the stones it is provided with an eye and is an eye-bolt.

Lewis-hole.—The hole drilled in a stone for a lewis.

Lift.—The rise in a canal-lock. The vertical distance from one level to another in a mine, or a set of pumps. A machine for lifting.

Lift-bridge.—A bridge which is raised to accommodate cross-traffic, as in a canal.

Lift-pump.—Opposed to *force-pump*.

Lift-wall.—The cross-wall of a canal-lock chamber.

Lighter.—A water-craft or barge used for unloading or loading cargoes of vessels while anchored in the harbor.

Light-ship.—A vessel at anchor used as a sort of light-house.

Linch-hoop.—A ring on a carriage-axle fastened by a linch-pin.

Linch-pin.—A pin on the end of a carriage-axle to hold the wheel on.

Link.—One of the rings or separate pieces of a chain. In surveying, the

hundredth part of a chain; equal to 12 inches in an engineer's chain, or 7.92 inches in a surveyor's or Gunter's chain. In a steam-engine, the links or parts forming the link-motion.

Link-lever.—In a steam-engine, the reversing lever controlling the link of the link-motion valve-gear.

Link-motion.—In a steam-engine, a system of levers for controlling the valves in starting and reversing the engine, and for cut-off.

Lintel.—A horizontal beam for supporting a wall over a door or window or other similar opening of moderate span; termed a *breast-summer* when the opening is large.

List.—In architecture, a square molding, fillet or *listel*. A narrow strip from the edge of a board. The first or thin coat of tin on iron plates, to be followed by a heavier coat. The tipping of a vessel due to unequal loading.

Liter.—A unit of capacity, equal to 1.056 U. S. quarts.

Litharge.—A protoxid of lead (PbO), used in the composition of flint-glass, varnishes and drying-oils.

Load.—In mechanics, the pressure upon any part of a structure.

Load-line.—The line around a vessel to show the allowable load: when she sinks in the water to that mark.

Loam.—In foundry-work, a mixture of clay, sand, sawdust, etc., used in making the molds for castings of iron and brass.

Lock.—A device for fastening doors, gates, etc.; composed of a bolt, wards (for guarding against the entrance of a key not of the right pattern), tumbler (to hold the bolt in position and render the operation of a wrong key difficult), and a spring. A mortise-lock is one concealed, as in a house-door. An enclosure or chamber in a canal with gates at each end, for allowing boats to pass from one level to another.

Lock-nut = check-nut = jam-nut = pinch-nut.—A nut screwed down on another to keep it in place.

Log-beam.—A traveling frame for supporting and feeding logs to the saw, in a saw-mill.

Log-scale.—A table showing the quantity of lumber, in B. M., procurable from a log when sawed; the length of log and its diam. beneath the bark being given.

Louver = louvre.—A long window, usually at tops of roofs of shops, depots, etc., with the opening traversed with broad slats sloping downward and outward, like the slats of a window-blind, to provide for ventilation and exclude rain.

Lozenge.—A plane figure shaped like a rhomb or diamond, having four equal sides with two acute and two obtuse angles. In art, the lozenge pattern is a pattern with diamond-shaped figures, or lines meeting or crossing each other at regular intervals but not at right angle.

Lug.—A short flange or projecting piece on anything, as in a casting, by or to which something is fastened or supported or kept rigid, etc.

Lug-bolt = strap-bolt.—A bolt terminating in a long flat extension or bar which takes the place of a head; made by welding a flat bar to a common bolt. The bar often contains holes for bolts or screws to fasten to timber. There are various forms.

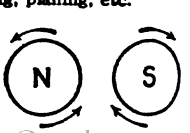
Lumber-car.—A railroad car for carrying lumber; usually 34 ft. long.

Lumber-kiln.—An artificially warmed chamber in which lumber is placed to deprive it of its moisture. The heat is often furnished by coils of steam-pipes, and the moisture in the air, from the wood, is condensed, as on cold-water pipes hung in the room, and the drippings conducted out of the chamber. The green lumber may be run into a kiln on cars and remain on them until dried.

Machine-bolt.—A threaded bolt with a square or hexagonal head.

Machine-tool = engine-tool.—A machine operated by power (water or steam, etc.) for performing operations which may be, or which formerly were, accomplished by the use of hand-tools, as drilling, planing, etc.

Magnet, Electro.—A magnet produced by a passage of an electric current through a coil of insulated wire surrounding a core of magnetizable material. The directions of the currents required to produce N and S poles, respectively, are shown in the accompanying illustrations. A magnetizing coil is called a helix or solenoid.



Main.—The chief pipe-line in a system of waterworks; any pipe-line in a

distributing system tapped for domestic supply. Water-main. Similarly with gas, as gas-main.

Main-link.—The bar that connects the piston-rod with the beam of the engine.

Malleability.—The property of being malleable, i. e., of being shaped, by hammering or rolling, without fracture, as malleable brass or iron.

Mallet.—A wooden hammer or small beetle used by stonecutters, carpenters, etc.

Mandrel.—A bar or spindle inserted in any work to hold it or shape it, as in a lathe. The spindle of a turning lathe; the arbor or axis of any tool, as a circular saw or cutter; a rod for shaping the inside or hollow of anything, as the plug-core of a metal casting.

Mandrel-collar.—A collar formed on the mandrel of a lathe, against which the chucks abut.

Mandrel-lathe.—A lathe for turning long hollow work; the material is clasped by a chuck on the end of the mandrel.

Manhole.—An opening in a sewer, culvert, drain, cesspool, steam-boiler, tank, etc., through which a man may enter for the purpose of inspecting, repairing, cleaning, etc.

Marble-saw.—One or more thin iron blades, set in a frame and reciprocated on a block of marble to be sawed, the kerfs being fed with sand and water.

Mark.—A German coin of the value of $0.238.

Marking-gage.—A small graduated rod with a steel point on one end for scratching a line on the wood, and with an adjustable block for gaging the line from the edge of the board.

Mari-brick = mari-stock.—A superior brick for the fronts of buildings and for arches.

Marlinespike.—A pointed iron tool used by riggers to separate the strands of rope in splicing.

Marsh-gas.—Carburetted hydrogen; a constituent of *fire-damp*.

Masonry.—Generally defined as anything constructed of the materials used by masons, as stonework, brickwork, tilework, etc.; but now it is considered as including concrete, cyclopean masonry (which see) or rubble-concrete masonry, and reinforced-concrete masonry. Dry masonry comprises masonry built without mortar. See subject of Masonry, Section 25.

Mass.—The weight of a body divided by the gravity acceleration at the place where the weight is measured.

Mass-center = center of mass.—A point through which if a plane is passed in any direction, the sum of the products of all the minute masses or particles on one side of the plane each multiplied by its respective distance from that plane, will be equal to the sum of similar products of particles and distances on the other side of the plane.

Mass, Magnetic.—A quantity of magnetism which at unit distance produces an action equal to unit force.

Master-wheel.—The chief wheel or the driving-wheel of a mechanism or machine.

Mat-boat = matting-boat.—A sort of framework on scows for making and launching mats to protect river-banks from scour.

Match-board.—A board with a tongue on one edge and a groove on the other for constructing partitions, floors, etc., of match-boarding or matched-boarding.

Match-plane.—One or two planes for preparing the edges of matched-boards.

Mathook.—A long pole with an iron hook used in the construction of mats for river-bank protection.

Mattock.—A kind of pick for digging, but having the edges broad instead of pointed.

Maul.—A heavy wooden hammer.

Maximum.—The greatest value or upper limit. The greatest of several maxima is termed the *absolute maximum*, or *maximum maximorum*. Opposed to *minimum*.

Mean.—An intermediate value in a series, from which it is derived. *Arithmetical mean* is the sum of n quantities divided by n. *Geometrical mean* is the square root of the product of two numbers. *Mean error* is the quadratic mean of the errors of observation.

Meander.—A series of transit or compass lines in a survey, with distances, angles or bearings, and perhaps levels.

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Meander-line.—A part or the whole of a *meander*.
Mean proportional.—Geometrical-mean. See *Mean*.
Megohm.—A large measure of electrical resistance: one million ohms.
Melt.—The charge of metal in a cupola or pot, for melting or after being melted.
Melting-pot.—A pot for melting. A crucible.
Member.—A subordinate part of a structure, as a post or a diagonal of a truss.
Mercury-furnace.—A furnace for roasting cinnabar so the mercurial fumes will arise, to be condensed in a series of vessels.
Meridian.—Noon. A north and south line, on the earth, or on the celestial sphere.
Meter.—A unit of length; 39.37 ins. by U. S. law.
Meter, Watt.—An instrument for measuring current flow in watts (volt-amperes).
Mica.—A mineral substance employed as an insulator, as for instance of commutator bars.
Micro.—The one-millionth.
Micrometer.—An instrument for measuring exceedingly small lengths and angles.
Mil.—The unit of length equal to the $\frac{1}{1000}$ of an inch, or .001 inch, used in measuring the diameter of wires.
Mil, Circular.—A unit of area employed in measuring the areas of cross-sections of wires; equal to 0.7854 square mil. One circular mil = .000000785 square inch.
Mill-furnace.—A furnace for re-heating metals to be rolled into shapes or welded under the hammer.
Milligram.—One thousandth of a gram, equal to about 1/65 of a grain.
Milliliter.—One thousandth of a liter, equal to about 0.061 cu. in.
Millimeter.—One thousandth of a meter, equal to 0.03937 inch.
Mil, Square.—A unit of area employed in measuring the areas of cross-sections of wires; equal to .001×.001 = .000001 square inch. One square mil = 1.2732 circular mil.
Miner's Inch.—See page 1313.
Minimum.—The smallest or least quantity. Pl., *minima*.
Miter = mitre = miter-joint.—A joint whose line makes an angle of 45° with each of the two pieces joined together at right angle. A *bevel-joint* is one whose line makes any angle greater or less than 45° with two pieces joined together at any angle.
Miter-sill.—A raised sill against which the bottom of the canal-lock gates shut on the floor of the lock-bay.
Miter-wheel.—One of two wheels of a mechanism whose teeth engage, the planes of the wheels being at right-angle with each other.
Modulus = coefficient.—A constant or positive number used as a measure of some function, as the modulus- or coefficient of elasticity of a material, the modulus depending upon the material and the method of testing, etc.
Moment.—The product of a force into its shortest leverage distance. See page 305.
Monkey = ram.—The hammer of a pile driver.
Monkey-engine.—A pile-driver.
Monkey-wrench.—A common wrench, with one jaw adjustable by a screw, for screwing on nuts, etc.
Monument.—An artificial landmark used by surveyors to fix a point or a corner on an instrument- or a property line. Granite monuments, 6″ square at top and 3 to 5 ft. long are serviceable. Small gas-pipes are frequently used in surburban districts for semi-permanent work.
Mooring.—That to which a ship or anything is secured.
Mooring-swivel = mooring-shackle.—A swivel used to connect two anchor-chains together just above the water, say at the forward end of the ship, when both anchors are out.
Mortise.—A part of a mortise-and-tenon joint; the hole in timber to receive the tenon, and made by a carpenter's mortise-chisel.
Motor, Electric.—A device for transforming electric power into mechanical power. Electric motors are specially designed for continuous current or for alternating current or for rotating current.
M-roof.—A double pitch-roof forming an inverted W.
Muck-bar.—An iron bar which has been passed through the muck-rolls only, of a rolling-mill.

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Muck-rolls.—The first pair of rolls for rolling iron; the bars are finished by passing them through the *merchant train* or *puddle-bar train* of rolls.
Mullion.—A vertical division between the lights of windows, screens, etc.
Munnion.—In ship-building, a vertical division or piece between the panels of framed bulkheads.
Muntin = munting.—The central vertical piece dividing the panels of a door.
Mutule.—A flat block projecting under the carona of the Doric cornice; a sort of modillion.

N.

Nadir.—The point in the heavens vertically below any station on the earth. Opposed to *zenith*, which is a point vertically above.
Nail-plate.—A metal plate of the proper thickness for cutting up into nails.
Naphtha.—A colorless liquid distilled from petroleum. Largely used in the manufacture of illuminating gas, and for light and power in general.
Nave.—The long, main, central interior part of a church, including the central aisle. The hub of a wheel.
Nave-box.—The metallic ring inserted in the nave or hub of a wheel, to reduce the wear.
Needle-beam.—The floor-beam of a bridge. A transverse bolster placed beneath the sills of a car and between the bolsters.
Nest.—A group of parallel steel rollers in a frame and used at the expansion end of a truss. A connected series of pulleys or cog-wheels.
Net-masonry.—Masonry with joints like the meshes of a net.
Neutral feeder.—The feeder that is connected with the neutral or intermediate terminal of the dynamos in a three-wire system of distribution.
Newel.—An upright pillar from which the steps of a winding stair radiate; or the large ornamental post supporting the hand-rail at the head or foot of a flight of stairs; or a round pillar at the end of the wing-wall of a bridge. Sometimes called *newel-post*.
Niche.—A nook or recess in a wall for a statue.
Nippers.—A tool like pincers or tongs for grasping hold of small objects; sometimes with cutting edges for cutting wire and small pieces of metal. In engineering, two toothed jaws attached to gearing, for cutting off piles under water.
Non-Conductors.—Substances that offer so great resistance to the passage of an electric current through their mass as to practically exclude a discharge passing through them. Insulators.
Normal.—Perpendicular; at right-angle to the tangent to a curve at the point of tangency. According to the rule or right principle.
Nose-piece.—The nozzle of a hose.
Nose-pipe.—The nozzle of a blast-pipe inside the twyer of a blast-furnace.
Nosing.—The projecting edge of a molding, or of a tread or step of a stair.
Nulled-work.—In wood-turning, pieces or wood turned to form a series of connected beads or knobs, as in the rounds of cheap chairs and bedsteads.
Nut.—A sort of adjustable head to a screw-bolt. A short piece of iron with a central female screw to fit the screw of a bolt.
Nut-coal.—Chestnut-coal.
Nut-lock = nut-fastening = jam-nut.—A device for fastening the nut on a bolt so it will not work loose.
Nut-machine.—A machine for making (cutting, stamping and swaging) iron nuts from a heated bar fed to it.

O.

Oakum.—The coarse part of hemp or flax, removed in combing or hackling.
Obelisk.—A large rectangular, monumental shaft, tapering from the base upward, and with a pointed top. Many abound in Egypt. One in Central Park, New York City, near Metropolitan Museum of Art.
Octastyle.—An architectural feature of a portico with eight columns in front.
Odometer.—An instrument to be attached to a wheeled vehicle for measuring distances traveled; useful in preliminary surveys in connection with the compass, and especially for making maps of country roads.
Ogee = O. G.—A reverse curve, as in a sectional outline of some molding, or of a cast-iron washer; hence *ogee washer* or *O. G. washer*. A cyma.

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Ohm.—The unit of electric resistance. It really expresses a velocity, namely. 1,000,000,000 centimeters per second. Thus, the formula for resistance in electro-magnetic units (see Units, Electro-Magnetic, Dimensions of) is $R = \frac{E}{C} = \frac{L}{T} = \frac{\text{Length}}{\text{Time}} = \text{Velocity}$.

Ohm, British Board of Trade.—The resistance of a column of mercury 106.3 centimeters in length and one square millimeter area of cross-section at 0° C.

Ohm, British Association.—The resistance of a column of mercury 104.9 centimeters in length and one square millimeter area of cross-section at 0° C. (Not used.)

Ohm, Legal.—The resistance of a column of mercury 106 centimeters in length and one square millimeter area of cross-section at 0° C. or 32° F. (Never legalized.) One ohm = 1.00112 B. A. Units.

Ohm, Standard.—A length of wire having a resistance of the value of the true or legal ohm, employed in standardizing resistance coils.

Ohmmeter.—A commercial galvanometer for measuring ohmic resistance.

Oil-cellar.—A metal box containing oil for oiling the crank-pin, and attached to the under side of the strap of the connecting-rod of the engine.

Oil-pump.—A pump for discharging oil upon a journal.

Oil-tempering.—The tempering of steel with oil (not water).

Oil-well.—A boring made for petroleum.

O. K.—Correct; all right. (Oll Korrect.)

Opaque.—Dark; shady; obscure; not transparent; impervious to light.

Open-hearth furnace.—A furnace used in making steel by the Siemens-Martin process, with certain late improvements.

Openwork = open-cast.—Relates to mining or quarrying done in the open air, i. e., not covered.

Ordinate.—An offset from a base line to any given point. In analytic geometry, one of the coordinates locating a point on a curve from two co-ordinate axes. See *Abscissa*.

Oscillator.—Anything which has or produces a rapid reciprocating motion within a limited range of distance, as a power-hammer, the shuttle of a sewing-machine, the piston of a steam-engine, etc.

Osier.—A specie of willow, much used in river-bank protection.

Outfall.—The discharge end of a river, sewer, culvert, drain, etc.

Outfall-sewer.—That portion of a large sewer which receives the sewage from one or more districts and discharges it, as into a river or ocean.

Out of wind.—Not winding; straight. Specifications calling for timber out of wind, mean that the faces shall not be warped.

Overshot-wheel.—A mill-wheel with blades or buckets around the periphery, and designed to operate by water shot over the top on the descent.

Ovolo = quarter-round.—A convex molding forming a quarter of a circle. In Greek architecture the moldings are a quarter-ellipse, like an egg, instead of a quadrant of a circle.

Ozone.—Modified oxygen; its density is one and one-half times that of oxygen. It exists in cold regions and in country districts. Can be produced by an electric spark passing through air or through oxygen. It is a great purifier and bleacher.

P.

Packing.—In machinery, material stuffed around moving parts, as in a stuffing-box, to prevent leakage of steam, water, etc.

Packing-ring.—A metal- or rubber-ring used as seat for a coupling-valve of a car, to make an air-tight joint.

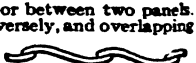
Pack-train.—Pack-animals with their loads.

Pallet-molding.—In brick-making, a process in which the mold is sanded after each using.

Panel.—Any area slightly sunk, or raised, or more or less distinct, as the panel of a door. The panel of a truss is the vertical area embraced between the chord-joints or floor-beams; the length of a panel times the number of panels is equal to the span.

Panel-strip.—A narrow strip on the edge of a panel or between two panels.

Pantile = pan-tile.—A tile with surfaces curved transversely, and overlapping and underlapping adjacent tiles, thus:



and laid on a roof shingle-fashion.

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Pantograph.—A lever-like instrument with arms for reproducing a sketch to the same scale or to another scale.

Paraffin = paraffine.—A substance obtained by the dry distillation of wood, wax, peat, bituminous coal, etc.

Parallax.—In an object glass, an apparent lateral movement of the cross-hairs as the eye changes position: occurs when the hairs are not coincident with the focal plane.

Parapet.—A top wall of a masonry structure, forming a sort of breastwork. The parapet of an abutment is the long transverse wall on the back edge of the coping, rising nearly to sub-grade, to protect the bridge-seat from the earth embankment of the roadbed.

Parcel.—To wind anything, as a rope, with strips of canvas.

Parget.—To cover, gloss over, or smooth over, the surface of anything with parget or plaster.

Parthenon.—The temple of Athene Parthenos, at Athens.

Party-wall.—A building-wall centrally located on a property-line or party-line, for joint use; it may belong to one or to both property owners.

Passimeter.—A watch-like pocket-odometer for registering walking- or running distance.

Patent hammer.—A hammer for dressing stone, and having sharp knife-like ridges on the face.

Pattern.—An original, or model, or mold for anything. Patterns for castings, as chord-blocks of Howe trusses and combination trusses, are made of wood, and due allowance must be made for shrinkage of metal.

Pawl.—A short iron bar or ratchet to engage the saw-like teeth of a ratchet-wheel to prevent a windlass or capstan from turning back.

Pay.—On a ship, to cover with a coat of tar or pitch, as a seam, or a rope.

Pay out.—To slacken rope, as a cable or main sheet.

Peak.—The upper corner of a sail. *Forepeak* is the forward extremity of a ship's hold, as opposed to *after-peak*. Peak load means maximum load.

Peat-charcoal.—Charcoal made by carbonizing peat.

Pediment.—The triangular front-end of a building included between the portico and the sloping edges of the roof.

Peen = pean.—The end of the head of a peen hammer (pean-hammer).

Peen-hammer.—A pean-hammer, or hammer with two opposite cutting edges for dressing stone. A hammer with a chisel edge, used for straightening iron plates.

Pendentive.—In architecture, a triangular segment of a hemispherical dome rising from four supports formed by two intersecting arches.

Penstock.—A channel or conduit supplying water from the race to the gate, through which the water flows to the wheel of the mill or power-plant.

Percussion-cap.—A small metal cap or cup containing fulminating (detonating or exploding) powder, for exploding dynamite or gunpowder.

Perihelion.—A point in the orbit of a planet or comet in which it is nearest to the sun. Opposed to *aphelion*, in which it is farthest from the sun.

Perimeter.—Circumference or outer boundary.

Periodicity.—The rate of change in the alternations or pulsations of an electric current.

Periphery.—Circumference of a circle; arc.

Peristyle.—In architecture, columns arranged around an enclosure, or any part of same, as a court, or cloister.

Permeability, Magnetic.—Conductibility for lines of magnetic forces. The ratio existing between the magnetization produced, and the magnetizing force producing it. Permeability, $M = \frac{B}{H}$.

Perron.—In architecture, a flight of steps to a building in which the principal floor is raised considerably above the ground level.

Pestle.—One of the vertical moving bars in a stamp-mill for crushing ore.

Petroleum-still.—A still for separating the hydro-carbons, in the order of their volatility, from crude petroleum.

Phase, Angle of Difference of, between Alternating Currents of Same Period.—The angle which measures the shifting of phase of a simple periodic current with respect to another due to lag or other cause.

Phase, Shifting of, of Alternating Current.—A change in phase of current due to magnetic lag or other causes.

Photometer.—An instrument for measuring the intensity of light, or comparing one with another.

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Pick.—A pointed instrument with a handle used in loosening any material by vertical swinging. The common pick for loosening earth. The stone-pick.

Pickax = pickaxe.—A sort of combination of pick and mattock; with one end pointed and the other flat.

Pier.—One of the supports of a bridge; generally, one of the central supports of two or more spans, the end supports being termed *abutments*. The solid support from which two or more arches spring. The support of a wall, between openings. A structure built out into the water, to support something, as freight, traffic, warehouses, etc. Usually conveys the ideas of *length* and *support*.

Pierre perdue.—Masses of stone thrown into the water to serve as a sub-foundation, as for a breakwater.

Pig.—A cast of metal in compact form, as iron from a blast-furnace.

Pigment.—Any substance used by painters to give the desired color.

Pilaster.—A sort of quarter- or half-pillar projecting from the face of a wall, and having the proportional parts of capital and base.

Pile.—Any long piece driven or planted in the ground to serve as a sub-foundation. May be of wood, iron, concrete, reinforced-concrete, etc.

Pile-hoop.—An iron ring driven over the head of a pile to prevent it from splitting, in driving.

Pile-plank.—Planks driven into the ground like piles, as the sheet-piling of a coffer-dam.

Pile-shoe.—An iron point fitted to the lower end of a pile so it will penetrate more easily and not burr at the end.

Pillar.—A column.

Pillow-block = plumber-block.—The metal case or support for the end of a revolving shaft or journal.

Pin, Insulator.—A bolt by means of which an insulator is attached to the telegraphic support or arm.

Pinch-bar = pinching-bar.—An iron bar with a small lever-like snout at the foot, for working heavy masses sideways.

Pinion.—A small cog-wheel or gear-wheel geared to a larger one, and usually *giving* it motion.

Pinnacle.—A relatively small structure rising from the roof or walls of a larger one.

Pin, Switch.—A metallic pin or plug provided for insertion in a telegraphic switchboard.

Pin-switch.—An electrical switchboard by which connections are made by means of pins inserted in holes between plates insulated from each other.

Pintle.—A pin or dowel or long bolt upon which anything turns or revolves, as the cylindrical pins on which a blind or a rudder swings, the bolt on which the forward axle of a carriage swings under the body, etc.

Pipe-coupling.—A sort of sleeve which screws on the ends of two abutting pipes.

Pipe-cutter.—A sort of chisel-tool for cutting iron pipes by forcing it down on the pipe and around it.

Pipe-line.—A pipe-conduit.

Pipe-tongs.—A pair of tongs, one projection being sharp for pushing and the other one hooked for pulling, used in screwing pipe together, or into their couplings.

Piston.—A movable piece operated reciprocally by the steam in the cylinder of an engine; consists of the piston-head which fits the inside diameter of the cylinder, and the piston-rod which connects with the mechanism outside.

Pitch.—Inclination. In mechanics, the distance center to center (c. to c., or c.–c.) of two adjacent teeth of a cog-wheel, or of rivets when the measurement is along some base line, or of the threads of a screw, etc. The residuum of tar. The sap from the bark of pines.

Pitch-board.—A guide or pattern for carpenters in framing the strings of stairs, the right-angled notches for the treads and risers.

Pitch-circle = pitch-line.—In toothed-wheels, a circle intersecting all the teeth near the middle of their length, and which is tangent to a similar circle of a wheel geared to it; outlines the theoretical size of a toothed-wheel.

Pitch-wheel.—One of two toothed-wheels geared together.

Pitman.—A connecting-rod between a rotary and a reciprocating part.

Pit-saw.—A large saw operated by two men one of whom is below in the "pit."

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Pivot.—That upon which something turns, as the center-pin or pivot-pin of a center-bearing drawbridge or turntable.

Place-brick = sandel = samel-brick.—In brickmaking, a soft brick, insufficiently burned.

Planimeter.—An instrument for measuring the plane area of any object or drawing of any irregular outline; it can be adjusted to the scale of the drawing. It is an integrator and is mathematically correct.

Planish.—To make smooth or polish. A *planisher* or flat-headed tool is used by tinners; a *planishing-hammer* is used by metal workers, also a *planishing-roller*.

Plant.—Machinery, tools, and general outfit used in any mechanical operation or construction work, etc.

Platband.—A wide fillet. A flat molding. A lintel formed with voussoirs but with intrados horizontal, as an arch with infinite radius.

Plate, Arrester, of Lightning Protector.—That plate of a lightning protector which is directly connected with the circuit to be protected, as distinguished from the plate that is connected with the ground.

Plate-girder.—A girder with a web composed of steel plate.

Plate, Ground, of Lightning Arrester.—That plate of a comb lightning arrester which is connected with the earth or ground.

Pliers.—Small pinchers with long jaws for handling and bending small pieces of metal.

Plinth.—The flat square member at the base of a column.

Plug.—A piece to stop a hole. A cast-iron cap leaded in the end of a cast-iron water main. A cap screwed into the end of a pipe.

Plug and feathers.—An iron wedge or *plug* inserted in one of a series of holes in a stone, and between two semi-cylindrical pieces of iron called *feathers*, all the holes being similarly treated, in order to split the stone on the line of the holes, by striking the plugs with a sledge-hammer.

Plumb.—**Vertical**, as in the direction of gravity.

Plumb-bob = plumb.—A top-shaped metallic instrument, with the lower end pointed, suspended by a cord or plumb-line and used in surveying, carpentry, mason-work, etc., to obtain vertical lines. There are various kinds and shapes, suited to different classes of work.

Plumber-block.—See *pillow-block*.

Plumb-joint.—A soldered lap-joint, the edges of the metal not being bent or seamed.

Plumb-level = pendulum-level.—A board with a line perpendicular to its edge, used in connection with a plumb for obtaining levels.

Plumb-rule.—A narrow board with parallel edges and with a straight line drawn through the middle, used in connection with a plumb, for obtaining verticals, in bricklaying, carpentry, etc.

Plummer-block = plumber-block = pillow-block.—See *Pillow-block*.

Plummet.—A plumb or plumb-bob used by carpenters, masons, etc.

Plummet-level = masons'-level.—Similar to *plumb-level*, preceding.

Plunger.—A solid piston, as that of a Cornish pump; one without a valve.

Pockets, Armature.—Spaces provided in an armature for the reception of the armature coils.

Point.—A pointed chisel for dressing stone. To point is to dress masonry with a point; or to finish the outer joints with mortar.

Pole-plate.—A small wall-plate resting on the ends of the tie-beams of a roof, for supporting the lower ends of the common- or jack-rafters.

Polygon.—A figure with numerous sides and angles.

Pontoon.—One of a series of flat-bottomed boats or floating structures, used in the construction of a temporary bridge across a stream, or for support of a pipe-line in hydraulic dredging, etc.

Pontoon-bridge.—A bridge or roadway supported on pontoons.

Port.—One of two passages leading from the steam-chest to the inside of the cylinder of an engine, above and below the piston, and controlled by valves so that the steam enters and exhausts at the proper time.

Portal.—A door, gate, opening, entrance, etc., to a passage, as to a tunnel or cathedral.

Post.—A compression member connecting the two chords of a truss. An *end*-post, or an *intermediate*-post, of a truss.

Potential, Alternating.—A potential, the sign or direction of which is alternately changing from positive to negative.

Potential, Constant.—A potential which remains constant under all conditions.

Potential, Difference of.—A term employed to denote that portion of the

electromotive force which exists between any two points in a circuit. In a battery or dynamo it is the total E. M. F. that is available. It may be measured by "method of weighing," by "use of electrometers," or by "use of galvanometers."

Potential, Drop of.—Fall of potential.

Potential, Electric.—The power of doing electric work. Comparing with flow of water, it is the "pressure" or "head."

Power.—The rate of work. (Energy is capacity for work.)

Power, Horse, Electric.—Such a rate of doing electric work as is equal to 746 watts or 746 volt-coulombs per second.

Pressure.—A force in equilibrium, i. e., opposed by an equal and opposite force. Pressure is usually stated in lbs. per sq. in. or in lbs. per sq. ft. The pressure on bridge masonry, under the pedestals, is usually limited to about 250 lbs per sq. in., depending upon the quantity of the stone, etc.

Prime.—First of anything, as the *prime* coat, in painting. To prime means to charge, as to pour water into a pump-tube to start suction.

Principal.—[This word may be used in various branches of mechanics to denote the *chief* or *main* of anything where there are also similar but subordinate parts, as *principal axis*, *principal rafter*, etc.]

Pronaos.—An open vestibule or portico.

Pro rata.—In proportion.

Proscenium.—That part of a theater between the drop-scene or curtain and the orchestra. Thus we have the proscenium-arch and the proscenium-box.

Pro tem. = pro tempore.—Temporary; for the time being.

Prow.—The bow of a vessel.

Prox. = proximo.—In or of the next or coming month.

Puddle-bar.—Bar-iron from the puddle-rolls of a mill.

Puddle-rolls.—Grooved iron-rollers for rolling iron as it comes from the puddling-furnace and forge.

Puddling.—The process of ramming plastic clay into a structure to prevent leakage, as in making the puddle-core of an earth-dam. Also, the converting of pig-iron (cast-iron) into wrought-iron in a puddling-furnace (reverberatory furnace).

Pugging.—Mixing clay for bricks. The *deadening* of sound through a floor or partition by interposing some composition or construction; the construction itself.

Pug-mill.—A machine for mixing and tempering clay for bricks, etc.

Pug-piling.—Dovetailed piling, the piles being mortised into one another with a dovetailed-joint.

Pulley.—A mechanism consisting of a *shell* (block) containing one or more *grooved wheels* (sheaves) over which a rope runs for hoisting. One or more pulleys together with the hoisting rope comprise what is called a *tackle*. Also, a sort of drum over which a belt or cable or rope runs, without winding around it.

Pulsometer.—A kind of pump without a piston; operates by steam-condensation and partial vacuum.

Punt.—A small boat, square-ended and with a flat bottom.

Puppet.—The head- or tail-stock of a lathe.

Puppet-valve.—A valve which lifts bodily from its seat when open.

Purlin = purline.—One of a series of parallel timbers laid horizontally on the main or principal rafters of a roof to support the common or jack-rafters. Pl.—Horizontal shapes, as tees, for supporting any roofing material, as tiling.

Put-log.—One of several pieces of timber used in forming the floor of a scaffold; one end being inserted into a *put-hole* in the side of the building and the other end supported by a horizontal string secured to poles erected from the ground.

Putty.—A mixture of soft carbonate of lime or whiting, with linseed-oil.

Pyx.—The metallic box in which the nautical compass-card is suspended.

Q.

Quadrangle.—A court, square or rectangular, nearly or quite surrounded by buildings.

Quadratic.—In algebra, an equation in which the highest power of the unknown quantity is the second power.

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Quadrature of the circle.—The *exact* determination in square measure of the area of a circle. Never been solved exactly, either arithmetically or geometrically, by any limited expression.

Quantity, Unit of Electric.—A definite amount or quantity of electricity called the coulomb.

Quay.—A landing place for vessels; a wharf.

Queen-post truss.—A truss with two upright intermediate posts meeting the end-posts at their tops. Used as roof-trusses. The posts are called queen-posts. Queen-post stays are the long rods running below the two queen-posts which support the body of the car between the trucks.

Quirked-molding = quick-molding.—A form of molding having an abrupt re-entrant angle at its extreme projection.

Quoin.—Blocks of stone at the corners of buildings and projecting somewhat from the face of the wall; the subordinate corners of the stones being chamfered off, usually. The recess into which the heel-post of a lock-gate is fitted.

Quoin-post.—The heel-post of a lock-gate, on which the latter turns.

R.

Rabbet.—A groove, channel, halving or other cut along the edge of a board to fit a corresponding cut on another board to fit it. A joint so formed by two boards is called a *rabbet-joint*. Rabbet-saws and rabbet-planes are used in preparing the cuts or grooves.

Race.—*Head-race* is a channel, canal or watercourse from a dam to a water-wheel; *tail-race* is such a water-course after it leaves the wheel.

Rack.—One or more long metal bars fitted end to end, forming either a straight or a circular piece, and having teeth on one of its sides or edges to engage or work into the teeth of a wheel, pinion or screw. If the rack is curved it is a *segment-rack;* if it is a circle it is called a *circular-rack* and is usually composed of segments, as the cast-iron rack in the turntable of a drawbridge.

Rack-and-Pinion.—A small cog-wheel or pinion geared to a rack.

Rack-and-pinion-jack.—A lifting-jack operated by a straight rack and pinion.

Rack-and-worm.—A rack geared to a worm or screw instead of to a pinion.

Rack-railway.—A railway operated by a gear-wheel of the car or locomotive engaging the teeth of a rack-rail—a rail laid along the track and provided with teeth or cogs. Used on inclined planes, as up the sides of mountains.

Rack-saw.—A saw with wide teeth.

Raft-dog.—An iron bar with ends pointed and bent at right-angle with body of bar; used for securing logs together in a raft.

Rag-bolt = barb-bolt = sprig-bolt.—A sort of jag-spike, or iron spike or pin with its shank barbed so as to make it difficult to withdraw after being driven.

Rag-wheel = chain-wheel = sprocket-wheel.—A wheel with teeth on the rim to engage the links of a chain.

Rail-bender = rail-bending machine.—A machine for applying lateral pressure on a rail supported against a bearer, for the purpose of straightening or curving it.

Rail-saw.—A portable saw for sawing steel rails.

Ram.—The hammer of a pile-driver. The steel hammer used in forming a bloom, in metal-working.

Rammer.—An instrument for ramming; thus, pavers' rammers, founders' rammers, gunners' rammers, etc.

Random stone.—Rip-rap; stone dumped but not evenly placed, as for slope protection or for the sub-foundation of a breakwater.

Random stonework = random work.—A masonry construction formed of stone laid in irregular courses.

Rasp.—A coarse file; various forms for the trades.

Ratchet = ratchet and pawl.—A bar or wheel furnished with teeth which engage the end of a pivoted bar or click when it tends to turn backward. Used for hoisting, etc. Hence, *ratchet-brace*, *ratchet-coupling*, *ratchet-drill*, *ratchet-jack*, *ratchet-lever*, *ratchet-wheel*, *ratchet-wrench*, etc.

Ratio, Velocity.—A ratio, in the nature of a velocity, that exists between the dimensions of the electro-static and the electro-magnetic units.

Ratline = ratlin = ratling = rattling.—One of the small horizontal ropes forming steps to the shrouds of a vessel, for going aloft.

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Reamer.—A tool with sharp lateral edges or fluted sides for smoothing punched holes in plates of metal, or for enlarging them. They may be reamed tapered by using a reamer with tapered flutes.

Réaumur's scale.—A thermometer with the freezing-point zero, and the boiling-point 80. Superseded by the centigrade scale, 0 and 100, respectively.

Rebate = rabbet = rabate.—See *Rabbet*.

Reciprocal.—The reciprocal of a number is 1 divided by that number, the result being expressed usually in decimal form. Reciprocal motion is alternating motion.

Reconnaissance = reconnoissance.—A critical examination of a country or territory prior to the preliminary survey.

Reducer.—A short pipe of variable diameter for connecting two pipes of different diameters. Also called an *increaser*.

Reëntering-angle = reëntrant angle.—An angle or corner pointing inward.

Reflux.—Flowing back. A *reflux-valve* is one designed to prevent back-flow; a back-pressure valve.

Refraction.—Deflection or change of direction of rays of light; due to the rays passing through a medium of varying density, as the air, or from one medium to another, as air to water or water to air. Witness the blade of an oar in the water; it *seems* to bend at the surface. When rays pass into a denser medium they are refracted toward the perpendicular to the surface, and *vice versa*.

Refrigerating-machine.—A machine for absorbing heat or converting it into work, and hence producing cold.

Reluctance, Magnetic.—Magnetic resistance.

$$\text{Reluctance} = \frac{\text{The magneto-motive force}}{\text{The magnetic flux}}.$$

Replacing-switch.—A device for replacing rolling-stock on the track.

Resistance, Electric.—Resistance in ohms $= R = \frac{E}{C} = \frac{\text{Electromotive force}}{\text{Current}} = \frac{\text{Volts}}{\text{Amperes}}$.

Retaining-wall = retain-wall = revetment.—A (steep) wall (of masonry) built to prevent a bank of earth from sliding or washing away. A structure designed to resist lateral pressure of loose material, as earth. See Retaining Walls, Section 48.

Revetment.—See *Retaining-wall* above, but applies particularly to fortifications, the protection of river-banks, etc.

Revolution.—Turning through 360°, or a complete circle; a cycle.

Rheostat.—An adjustable resistance. A rheostat enables the current to be brought to a standard, i. e., to a fixed value, by adjusting the resistance.

Rheostat, Dynamo-Balancing.—An adjustable resistance whose range is sufficient to balance the current of one dynamo against another with which it is required to run in parallel.

Rheostat, Water.—A rheostat the resistance of which is obtained by means of a mass of water of fixed dimensions.

Rib.—One of the curved pieces of iron or timber, as of a dome, arch, vault, vessel, etc., to which the outer shell is secured.

Ridge.—The highest part of a roof; the line of meeting of the upper ends of the rafters.

Ridge-pole = ridge-piece = ridge-plate.—The timber or iron piece along the ridge of the roof into which the rafters are secured.

Right and left.—In the frame of structures, certain members are *rights* and their counterparts are *lefts*, some small details being on opposite sides of the members otherwise alike, as the end-posts of a bridge. In such a case it is necessary to make only one drawing and call it the *right*, accompanied by explanatory notes describing the *left*. Similarly, we have right-and-left spring-frogs, switches, door-locks, screws, etc.

Right bank of a river is the bank on the right-hand side in descending a stream.

Right solid.—A solid with axis perpendicular to base.

Rim-saw.—A saw with a central circular disk over which is fitted a central band of teeth lying in the plane of the disk.

Ring-bolt = eye-bolt.—See *Eye-bolt*.

Ring-chuck.—A chuck to a lathe fitted with a ring over the end.

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Ring-dog.—Two iron dogs for driving into and hauling timber, and connected by a ring; called a *sling-dog* when connected by ropes or chains.
Riparian.—Marginal or bordering on, as relating to the shore of an ocean, a bay, or a stream.
Riparian owner = riparian proprietor.—One who has vested control in the soil to the thread of a stream or to some line in the water established by law.
Riparian rights.—The rights of a riparian owner, as fishing, ferriage, wharf or other construction, filling in, etc.
Riprap = rip-rap.—Broken stone loosely dumped; used for walls, foundation-beds, bank protection, etc.
Rip-saw = ripping-saw.—A saw used in sawing wood in the direction of the grain, by hand.
Rising-main.—The vertical column of pipe through which water is pumped from a mine.
Rising-rod.—The valve-rod of a Cornish pumping-engine.
Riveting-machine.—A power machine for driving and heading rivets; may be operated by steam, hydraulics, electricity, etc. Many are portable.
Road-machine.—A large scraper mounted on wheels and used for scraping, transporting and dumping earth; used in road- making, shaping and repairing.
Road-plow.—A strong plow for loosening earth, etc.
Road-roller.—A heavy roller for compacting the surfaces of roads; operated by steam-power usually.
Road-scraper.—A scraper for handling earth which is fairly loose or has previously been loosened.
Roadstead = road.—An unsheltered place where vessels can anchor; not a sheltered harbor.
Roadway.—The width of roadway in excavation or embankment is the width at sub-grade between edges of slopes.
Rock-crusher.—A machine for breaking rock into suitable sizes, as for concrete. A stone-breaker.
Rock-drill.—A machine-drill for quarries, mines, rock-excavation, etc.
Rocker, Brush.—In a dynamo-electric machine or electric motor, any device for shifting the position of the brushes on the commutator cylinder.
Rock-faced = quarry-faced.—The natural face of the stone, without dressing.
Rocking-bar.—A bar supporting a furnace-grate so that the grate can be tipped when desired.
Rocking-pier.—A bridge-pier hinged at the bottom, to accommodate the change in length of span, i. e., the expansion and contraction, due to temperature changes. Sometimes used in suspension bridges.
Rock-oil.—Petroleum.
Rock-shaft = rocker-shaft = rocking-shaft.—A shaft that rocks on its journals but does not revolve entirely.
Rockwork = quarry-faced masonry = rock-faced masonry.—See *Rock-faced.* Squared masonry with the face of stones left undressed.
Roll-joint.—A metal joint made by rolling one edge over the other and pressing the joint. Used in tinning roofs.
Roof-plate = wall-plate.—A plate on which the lower ends of the rafters of a roof rest.
Rope-clamp.—The metal attachment on the end of a rope or cable and forming a part of it.
Ropewalk.—A long shed in which rope is made.
Rosin (and Resin.)—The residuum of distilled turpentine. (See page 481.)
Rotary pump.—A pump having rotary parts as fans to force the liquid ahead. A centrifugal pump is a rotary pump.
Rubble.—Rough stones used in rubble-masonry or rubble-work; irregular, for common rubble-masonry, and squared for ranged rubble-work.
Rubble-Concrete masonry.—See *Cyclopean Masonry.*
Rundle.—The rung or round of a ladder.
Running-trap.—A depressed U-shaped section of a pipe, to contain water at all times and guard against the escape of gases.
Rustic.—Various classes of facings for masonry including rockwork.

S.

Sack-hoist.—An endless-chain device for hoisting filled sacks, as grain, cement, etc.

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Saddle.—Anything resembling a common saddle. A block resting on rollers on top of the pier of a suspension bridge to give the proper relief for expansion and contraction of cables; in such a case the resultant pressure is vertical. A chair for rails. The ridge-tile of a roof is often saddle-shaped and hence called saddle-tile.

Saddle-joint.—In tinning, etc., a joint made thus between two metal plates.

Saddle-plate = crown-sheet.—In locomotive boilers, the bent plate forming the arch of the furnace.

Safety-cage.—A mining cage or elevator car provided with a parachute or a safety clutch in case of breakage and too rapid descent.

Safety-catch = safety-stop.—A catch to hold an elevator in case of breakage of cable.

Safety-lamp. = A lamp used in coal mining and safe when even the inflammable coal-gas is present, from igniting the latter.

Safety-valve.—A relief valve in a steam-boiler.

Sag.—A downward curved bend, or depression.

Salient.—Projecting outward. A salient angle is one pointing outward, as in a common polygon; opposed to reentrant.

Saltern.—A salt-works, or place where salt is obtained by boiling or evaporation.

Saltpeter = saltpetre.—Potassium nitrate, or nitrate of potash. Common name, *niter*.

Sand-bag.—A bag of sand or earth, used for repairing leaks or breaks in foundation work under water.

Sand-blast.—A stream of sand driven through a tube onto iron-work for the purpose of removing paint, scale, rust, etc., preparatory to repainting or welding. Used in portable form in welding rail-joints. The sand is forced by a sand-blower.

Sand-pump.—A sludger used to remove the pulverized rock in rope-drilling in the oil regions. A sand-ejector used in caisson-work for bridge foundations.

Sand-trap.—A device consisting of a kind of pocket or chamber for collecting sand and the heavy sediment from water in pipes, etc.

Saw-set = saw-wrest.—A tool for springing the teeth of a saw alternately to right and left in order that the kerf will be wide enough not to bind the blade.

Scabble.—To dress off the rough projections of a stone with a broad chisel or stone-axe or heavy pointed pick preparatory to finer dressing.

Scale.—A flake or crust on iron due to oxidation, when hammered or rolled (milled); hence hammer-scale, mill-scale, etc. A flux is used to prevent scale.

Scaling-hammer.—A hammer used to remove scale.

Scantling.—A small stick of timber not over say 5 x 5 ins. in section.

Scarf = scarf-joint.—A joint for splicing the ends of two timbers together so as to make a continuous stick; may be reinforced by iron straps and bolts, and also be keyed. A usual form is:

Scarp.—In fortifications, the inner slope of the ditch. A slope.

Scoring.—In founding: The cracking of a casting when unequal cooling takes place; frequently happens to pipes and cylinders when the core does not give way to the contraction of the surrounding metal.

Scotia.—A receding or concave molding, as at the base of a column.

Scouter.—In stone-working, one who removes large projections by boring slanting or transverse holes and using the necessary tools for splitting, as wedges, etc.

Screed.—One of several strips of plaster 6 or 8 ins. wide and a few feet apart dividing a surface to be plastered, into bays; the screeds are flushed out to the same plane and serve as guides in bringing the whole surface to that plane.

Screw.—One of the six mechanical powers or simple machines, namely, *lever*, *wedge*, *wheel and axle*, *pulley*, *screw*, and *inclined plane*.

Screw-bolt.—A bolt headed at one end and provided with a screw-thread and nut at the other end.

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Screw-gear.—An endless screw working in the teeth of a pinion; a worm-screw working in a worm-wheel.
Screw-pile.—A pile with a screw at the lower end.
Screw-pin.—A cylindrical pin with a screw (and nut) at the end to hold it in position.
Screw-wheel.—A wheel which gears with an endless screw.
Screw-wrench.—A wrench having one or both jaws operated by a screw.
Scribe.—A pointed instrument for marking lines on wood, brick, metal, etc., as guides for cutting or scribing.
Scroll.—A convolved or spiral ornament resembling a partly unrolled sheet of paper or the letter S. The spiral ajutage around a reaction water-wheel.
Scupper = scupper-hole.—One of several openings in the side of a vessel at the deck level, for the escape of water.
Scupper-nail.—A short nail with a broad head, for nailing canvas, etc.
Scuttler.—A small hatchway in a vessel's deck; a hole in the side of a ship; a hole for sinking or scuttling a ship.
Sea-breeze.—A breeze from the sea toward the land.
Season.—To dry, as timber.
Seat-earth = seat-stone = under-clay.—In coal-mining, the bed of clay which characteristically underlies coal-seams.
Sea-wall.—A wall (generally artificial, but sometimes thrown up by the waves) to prevent encroachment of the sea.
Second, Ampere.—One ampere flowing for one second.
Second, Watt.—A unit of electrical work. A volt -coulomb.
Secret.—Covered up or hidden; thus, secret block, secret dovetail, secret nailing.
Section-liner.—An instrument for drawing parallel lines at certain distances apart, and consisting of triangle, straight-edge, scale and set-screw.
Sector.—A toothed gear comprising only an arc of a circle, for reciprocating motion; sector-gear. An astronomical instrument for measuring differences in declination. Sector of a circle is the area between two radii and the included arc.
Segment.—A distinct part of anything. Segment of a circle is the area included between arc and chord.
Segment-gear.—A gear extending over an arc only, for reciprocating motion.
Segment-rock.—A rock or cogged arc oscillating on a center.
Seismograph = seismometer.—An instrument for recording earthquake phenomena.
Seize.—To fasten together with small rope, cord or twine, by winding around it, as the end of a large rope, etc.
Semi-columnar.—Like a semi-column or half column appearing on the face of anything, as on a wall.
Semi-dome.—A half dome abutting a surface.
Semi-fused.—Half melted.
Septangular.—Having seven angles.
Serrated.—Notched or toothed, as a saw.
Serve.—To bind or wind around with twine, cord or marlin, as a rope, in order to protect it from rubbing and wearing.
Service-pipe.—A pipe for supplying water, or gas, etc., to a building, from a main.
Set.—The lateral bend of a saw-tooth. The last coat of plaster on a wall preparatory to papering.
Set-screw.—A screw for binding two or more things together, as in a cramp. A screw acting in a collar and with a point, for pressing into the metal of a shaft or other member to bind them together.
Sextant.—An instrument for measuring angular distances between objects. Used in navigation for obtaining latitude and longitude.
Shackle.—An unclosed link at the end of a chain, consisting of a U-shaped piece of iron fitted with a bolt (shackle-bolt) across the mouth, the shackle-bolt being held in place by a pin called a shackle-pin. A sort of clevis.
Shaft.—In mining and tunneling, a vertical hole, pit or well from the ground surface. The main body of a column. The interior of a blast-furnace above the hearth. In machinery, a large axle supporting something which revolves or oscillates.
Shank.—The long part of anything as the stem of a key or of an anchor, the shaft of a column, the holding part of a drill, the body of a bolt, etc.

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Shear.—The transverse stress in a girder at any cross-section, tending to make that part of the girder on the left (or right) of the section slide along the plane of the cross-section. A shear in any direction has an accompanying and equal shear at right angle to it; thus we have longitudinal shear in a beam. Rivets may be in single shear or double shear, the latter when connecting three bars the middle one of which is pulling in a direction opposite to that of the two outside bars.

Shears (formerly sheers).—Two or more poles fastened together near their tops, with their lower ends or legs spread apart as a base, and supporting hoisting tackle. Called *shear-legs*.

Shear-steel.—Blister-steel especially prepared and suitable for making shears, knives, etc. When re-worked, it is called *double shear steel*.

Sheathing.—A thin covering, as with plates, boards, etc.

Sheave.—A grooved wheel, as the wheel of a pulley.

Sheeting = sheet-piling = sheet-timbering.—Timber or metal piles, sheets or boarding driven to form a more or less water-tight lining, and used in connection with the construction of foundations under water, tunnel lining, etc.

Shellac.—A resinous substance possessing valuable insulating properties, which is exuded from the roots and branches of certain tropical plants.

Shim.—A sort of flat wedge to separate the surfaces of two adjacent bodies by wedging them or holding them apart. Wooden or iron shims are used in spacing rail-joints in track-laying. Heavy machinery is often shimmed up from the floor.

Shingle.—Stones on the sea-shore, a little coarser than gravel.

Ship-worm = teredo (*T. navalis*).—A worm that bores into and honeycombs timber and piling under water.

Shoe.—A metallic piece usually shaped to fit the end of various things, as the shoe or metallic point of a pile, or the malleable or cast fitting at the connecting ends of the bands or wood-stave pipe, etc.

Shore.—A prop or temporary support for bracing up anything, as a ship, or the sheeting in a sewer-trench, etc.

Shot.—A blast.

Shroud.—One of the ropes from the side of a ship to the mast-head, to support the mast.

Shunt.—To establish an additional path for the passage of an electrical current or discharge.

Shuttle.—A gate to allow water to flow on a water-wheel. A section of a shuttle-dam.

Side-beam.—One of the two beams of a side-beam engine.

Side-hatchet.—A hatchet with only one side of the blade chamfered.

Siding.—A short piece of railroad-track lying along the main track and used for passing of trains, etc. The boarding of, or for, the side of a building.

Silt.—An earthy sediment, or deposit of fine soft mud from standing water or a running stream.

Sink.—To excavate downward, as a shaft.

Sinusoid.—A curve of sines, the angles being laid off as abscissas, and the sines as ordinates.

Siphon.—A bent tube like an inverted V but with unequal legs, the shorter leg inserted in the basin of water. When the tube is thus placed and is filled with water, the water in the basin will be discharged through the tube unless air collects in the latter and stops the siphonic action.

Siphon, Electric.—A siphon in which the stoppage of flow, due to the gradual accumulation of air, is prevented by electrical means.

Sister-hooks = clip-hooks = clove-hooks.—A pair of hooks which close together like the jams of tongs and fit together side by side.

Size.—A gelatin wash used by painters.

S-joint.—A metal joint, thus

Skew.—Opposed to right-angled. Oblique. We have skew gearing, skew bridges, skew abutments, skew arches, etc.

Skewback.—The inclined stone or surface which takes the thrust of the arch.

Skid.—Any simple arrangement of one or more (generally two) poles or timbers placed on an incline (usually), for sliding or rolling freight upon in unloading from a car or vessel, etc.

Skirting-board = baseboard = mop-board = wash-board.—A narrow board placed around the walls of a room next the floor.

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Sledge.—A large heavy hammer; a sledge-hammer.
Sleeper.—A piece of wood or metal laid on the ground to support various classes of construction as rails of a track, floors of houses, etc. A tie.
Sleeve.—A hollow pipe, tube, or thimble, slid over the ends of two cylinders to be joined together.
Slide-bar.—A bar which is slid over the draft-opening of a furnace. One of the guides for the cross-heads of a piston-rod.
Slide-box.—The slide-valve chest of a steam engine.
Slide-rod.—The rod which operates the slide-valve of a steam-engine.
Slide-valve.—A valve which slides two and from its seat.
Slipe.—A sledge, or skip without wheels; used in coal-mining.
Sling.—A short rope or chain placed around anything to be hoisted.
Slip.—A docking place for vessels.
Slope-wall.—A wall built along a stream to prevent the bank from wash.
Sludge.—In mining, the finely powdered stone, mixed with water, in a drill-hole. Refuse from coal-washing and other operations in mining, and in the refining of crude petroleum. The deposited sediment in sewage tanks after the sewage is treated with chemicals.
Slug.—In mining, a loop in a rope through which a man puts his leg when being raised or lowered in a shaft.
Sluice.—A wooden trough which miners use in washing gold from gravel and sand. An artificial channel. A body of water held in check or flowing through a flood-gate or sluice-gate. The injection-valve to a condenser.
Sluice-gate = flood-gate.—The gate of a sluice.
Smoke-box.—A chamber through which smoke and gases pass from the furnace to the chimney.
Snap.—A tool used in riveting, to form the new head.
Snap-hook.—A metal hook with a spring-mousing to prevent the object hooked from slipping off.
Snatch-block.—A pulley-block with an opening on the side so that the bite of a rope can be passed over the sheave; when in use, the opening is covered by a strap.
Snip.—To cut off. In carpentry, to cut off nearly or quite at right-angle with the length of the timber. But see Snipe.
Snipe.—To cut off on a long bevel. In carpentry, to make long beveled cuts at the end of a timber, in framing the end to smaller section than the main timber.
Snub.—To check quite suddenly, as the speed of a boat; the checking is done by a snubbing-line, passed around a post called a snubbing-post.
Soaking-pit.—A pit in which cast ingots are placed so the mass may cool to a uniform temperature suitable for rolling.
Socket-bolt.—A bolt that passes through a thimble placed between the parts connected by the bolt.
Socket-chisel.—A form of heavy chisel for mortising.
Socket-drill.—A drill for enlarging or countersinking drilled holes.
Socket-joint.—An articulated-, flexible-, or ball-and-socket joint, as a flexible joint in a pipe-line. Flexible-joint pipes are used as water- and gas mains across streams; they are jointed together on a scow and sunk in a continuous line as fast as connected, the last joint being always out of water.
Soffit.—The lower surface of an arch; extended also to include the lower surface of a span over a door or window opening.
Solder.—A fusible alloy for uniting metal surfaces or joints. There are a great variety of solders for different metals. Hard solders, composed of copper and zinc, and called *spelter*, are used for uniting iron, copper and brass. Soft solders, composed of lead and tin, are used for uniting tin or lead.
Sole.—Anything resembling in function the sole of a shoe. The foundation-plate of a marine-engine. The lower edge of a turbine.
Solenoid.—A cylindrical coil of wire the convolutions of which are circular.
Sounding.—To measure the depth of water, soft mud, etc. In shallow water this may be done with a graduated rod or pole. In deep water, various kinds of apparatus are used; some of them, in addition to measuring the depth of water, are provided with a device for bringing to the surface samples of mud from the bottom of the sea.
Spall.—A piece chipped from a stone, as with a spalling-hammer.
Spandrel.—The triangular-like portion or space of an arch lying above or outside the extrados, and below the roadway.

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Spandrel-filling.—The filling of earth, or masonry, etc., in the spandrel of an arch.

Spandrel-wall.—A wall built on the extrados of an arch in the spandrel; used to give rigidity to the arch, or to support the roadway, etc.

Spanner.—A sort of lever-wrench with a hole or with movable jaws to fit over a nut for tightening it; or any instrument for clasping and tightening a nut or screw or wheel, etc. A rod connecting two parts having parallel motion.

Spar.—A round stick of timber used for various purposes, as the jib or boom of a derrick, the masts, booms and yards of a ship, the poles or common rafters of a roof, etc.

Spark-arrester.—A netting or device placed over or in the smoke-stack to prevent the escape of the sparks.

Spectroscope.—An instrument for producing a spectrum from rays of light, and for studying it.

Spectrum.—The continuous and successive colors in a band of light seen when rays of light are deflected through a prism.

Speculum-metal.—An alloy consisting of ten parts copper and one part tin; used as a mirror or speculum.

Speed-pulley.—A sort of cone-pulley, but stepped; different speeds are obtained by placing the belt over different steps or faces of the pulley.

Spelter.—Zinc. Spelter solder is hard solder composed of copper and zinc.

Spider, Armature.—A light framework or skeleton consisting of a central sleeve or hub keyed to the armature shaft, and provided with a number of radial spokes or arms for fixing or holding the armature core to the dynamo-electric machine.

Spiegeleisen = spiegel-iron—A pig-iron containing from $\frac{1}{12}$ to $\frac{1}{6}$ or more of manganese; used in the manufacture of Bessemer steel.

Spigot.—A plug with a hole in it and used as a faucet. The end of a cast-iron pipe which fits into the bell-end of an adjoining pipe; such a joint is called a *spigot-joint*.

Spike.—A large metal nail; may be pointed, chisel-pointed, barbed, grooved, or split, etc.; the head also may be variously shaped.

Spile.—A pile.

Spillway.—A sort of weir or gap or passage for surplus water from a reservoir; located near one end of the dam or at the dam itself.

Spindle.—A thin axle or shaft. A solid generated by a curve about its *chord;* may be circular, parabolic, elliptical, etc., depending upon the curve.

Splay.—The flaring or widening at the mouth of anything. The wings of a culvert when they spread back from the center line are said to be splayed.

Splice.—The joining of two pieces together by overlapping.

Spoil-bank = waste-bank.—A refuse bank in mining and in general excavation.

Springer.—The lowest voussoir or arch-stone of an arch.

Springing-line.—The lower face-line of the springers of an arch; the line from which the arch springs or rises or begins.

Spring-pole drilling.—In rock-boring, a simple method of using a spring-pole on the outer end of which is suspended the drill-rod. The spring of the pole raises the rod, the down motion being effected by hand.

Sprocket-wheel.—A *rag-wheel* or wheel with projections that engage the links of a chain passing over it. The projections may be pins, teeth or lugs, etc.

Spur-gear = spur-gearing.—Gearing in which spur-wheels are used.

Spur-wheel.—A common cog-wheel, the cogs being on the periphery of the wheel, and radial.

Square.—An instrument for laying off right angles. In roofing and flooring, an area equal to ten feet square = 100 sq. ft.

Square-headed.—Straight, as the upper edge of the opening of a door or window not arched or curved.

Stack.—One or more main flues, funnels, or chimneys grouped together for the passage of smoke. A smoke-stack.

Staff-angle.—In plastering, a square rod of wood flush with the wall at the external angle of a room, to protect the plastering.

Staging.—A temporary structure, including the flooring and supports, used in building operations.

Stamp-mill = stamping-mill—A mill for crushing ore by the use of vertically-acting stamps; may be operated by any kind of power.

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Stanchion.—A vertical support, as a column, post or strut used as a support to a vessel's deck, or part of a roof, etc.

Standing.—Anything quite permanent, as standing-rigging or the shrouds and stays of a ship. Anything rigidly fastened to and projecting from, as a standing-bolt, or stud-bolt.

Stand-pipe.—A vertical water-pipe used as a reservoir; or inserted in a main to show the hydraulic grade line or to act as an air-valve; and for other purposes as in a steam-engine.

Staple.—A U-shaped loop of metal driven into a door or other object, the projecting ends being bent or clinched on the inside, to receive a hook or hasp and form a kind of lock.

Starboard.—The right side of a vessel facing her bow. Opposed to *larboard* or *port*.

Starling.—A pile structure around, or up-stream or down-stream from, a bridge-pier for protection or support. One of such piles.

Static = statical.—Pertaining to weight, without motion; as static equilibrium, static load, static pressure, etc.

Stay.—A rope, tie, brace, or strut, etc., for keeping anything in place or making it "stay" in position.

Stay-bolt.—A bolt used to prevent two opposite plates or parts from being pressed or pulled apart further, as a stay-bolt in a steam-boiler.

Stay-rod.—Same as stay-bolt but longer, and used for various purposes as in building-construction.

Steam-box = steam-chest—A reservoir or chamber for steam to be used; situated above the boiler, in a locomotive. From this chamber the steam passes to the cylinders.

Steam-chest.—[The common name.] Same as *Steam-box*, above.

Steam-pipe.—A pipe which leads from the boiler to the engine or from the boiler to the steam-chest. Various pipes conveying steam.

Stem.—A projecting part, as the stem of a gate-valve.

Step.—In mechanics, anything resembling a step of a stair, as an offset on a cone-pulley. The foot or bearing of a vertical shaft.

Stile.—One of the main frames of a door to which the secondary or central frames are secured. The outer frame or main frame of anything.

Stilted-arch.—In architecture, an arch apparently raised above its springing as if stilted.

Stirrup.—In carpentry, an iron loop or strap for supporting one beam butting another, or a rafter, etc.

Stock and die.—A die and its holder, for cutting screws.

Stoker.—A fireman. A *mechanical* stoker is an automatic device for feeding fuel in a furnace and attending to the ashes, etc.

Stone-breaker.—See *Rock-crusher*.

Stone-saw.—A blade made to reciprocate in its kerf while sand is fed by a stream of water. Used for sawing marble, etc.

Stope.—An (horizontal) excavation in a mine, from a shaft or tunnel or drift, to remove the ore laid bare, or to pile the ore, or to receive refuse, etc.

S-trap.—An S-bend in a waste pipe to prevent gases from rising above the point of bend.

Strap.—A long narrow strap-like piece of metal, either straight or looped, and bolted to two pieces to hold them together, as in an iron strap used in a timber splice, etc.

Stratified.—Deposited in layers or strata. One of the layers is called a stratum. In geology, stratified rock or earth; bed rock.

Stress.—A force producing strain; measured in lbs. per sq. in. or per sq. ft. on the member to which it is applied. Within the elastic limit of the material, *stress is proportional to strain*—Hook's Law.

Stress-sheet = strain-sheet.—A diagram of a structure, as a bridge, giving stresses and sizes of members and other information which will determine the character of the structure to be built.

Stretcher.—In masonry, a brick or stone laid horizontally in the direction of the face of the wall; distinguished from header which heads onto the face of the wall.

Stretcher-bond.—Bricks or stones all laid as stretchers in continuous courses, no two joints being opposite transversely.

Stria.—A fillet between the flutes (base moldings) of columns.

Strike.—The direction, by the compass, of a stratified formation; at right angle to the *dip*.

Striking-plate.—In arch-centers, one of a series of compound wedges on which the centering rests while the masonry arch is being built; and

when the wedges are struck, the centering lowers and the arch becomes self-supporting.

String.—A string-course. A line of pieces used in construction, as timber, stone, etc.

String-course.—In architecture, a projecting molding or other prominent or distinct band.

Stringer.—A *string-board* or *string-piece* of a stair, or one of the slanting pieces supporting the treads and risers; or an ornamental piece fitted outside the supporting stringer. A longitudinal piece for supporting anything such as the ties or planking on a bridge.

Stripping.—In a quarry, mine or gravel-pit, the useless material stripped or removed preparatory to opening the quarry, mine or pit.

Strut.—A member acting in compression. Opposed to *tie* which acts in tension. A prop or brace.

Stub.—A short end or blunt end.

Stub-end.—The enlarged end of a connecting-rod or pitman, to which the strap is fastened.

Stud.—One of the vertical pieces of scantling in a partition and to which the laths are nailed.

Stud-bolt.—See *Standing-bolt.*

Studding.—In carpentry, material to be used as studs or joists.

Stuffing-box.—A sort of cast-iron box or chamber arranged around a movable rod to secure a tight joint against water, steam, or air, etc., passing along the rod through the wall which the rod pierces. The box is packed with greased hemp, or india-rubber, etc., and the ring-cap screwed or bolted on.

Suction.—The removal or the lessening of the atmospheric pressure on any part of a liquid so as to disturb its equilibrium and cause it to flow, the atmospheric pressure still remaining on some other surface of the liquid. The exhaustion of a gas or liquid from a chamber.

Suction-pipe.—A pipe connected with the bottom of a pump-barrel and leading down into a well or body of water to be raised. A pipe leading from beneath a water-wheel downward to the level of the tail-race in order to make the total head or fall available for power; the pipe must be air-tight.

Suction-pump.—A pump with a barrel or cylinder standing on and fastened to a suction-pipe leading to a well; at the junction of barrel and pipe is a flap-valve raising upward to allow water to enter from below, but which closes down tight against any back pressure; the barrel itself being fitted with a piston-rod operating a piston-head containing a flap-valve which also raises in the same manner as the other valve. At the down-stroke of the piston the lower valve closes and the upper one opens; at the up-stroke they reverse. Other kinds of valves may be used.

Suction-valve.—The lower valve of a suction-pump; the one below the piston. See *Suction-pump.*

Sump.—A depression in which water collects, as at the head of a land-slide; a kind of pool. A reservoir in a mine or other working into which drainage water is led and from which it is pumped out of the working; a *sump-pump* is used for this purpose, and the shaft through which it is pumped is called a *sump-shaft.*

Surbase.—An upper molding above a base, as that above the wainscoting or at the top of a pedestal.

Surd.—In mathematics, a quantity which cannot be expressed definitely, as $\sqrt{3}$.

Surface-condenser.—A condenser in which the exhaust-steam is condensed on metal surfaces which are cooled by flowing cold water in contact with the opposite faces.

Surface-tension.—The adhesion or tension of the surface-skin of a liquid, as when anything is made to touch it and is then raised. A kind of capillary attraction.

Swage.—A sort of die for giving shape to a piece of metal being hammered.

Swage-block.—A block with various holes, grooves, and other shapes for swaging or shaping pieces of metal, as for heading bolts, etc.

Sway-brace.—A brace used to cut a four-sided panel of a structure into rigid triangles. A sort of diagonal member or brace (strut or rod) between two opposite posts of bridge trusses.

Swing-saw.—A circular saw suspended from a frame and which can be used in sawing large bulky pieces at rest.

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Switch.—Any device for connecting or disconnecting, as a track, a current of electricity, etc.
Switchback.—A sort of zig-zag location of a railway, for gaining a gradual grade over a mountain.
Swivel.—A fastening comprising an axis which may be turned around freely in the other part, the end of the axis being headed like a bolt, to sustain tension.
Synchronize.—To cause to occur or act simultaneously.
Synclinal.—In geology, the dipping of strata toward each other and forming valley shapes. Opposed to *anticlinal*. A *synclinal axis* is a line following along the lowest points of the depression.
System, Three-Wire.—A system of electric distribution for lamps or other translating devices connected in multiple, in which three wires are used instead of two usually employed. In such a system two dynamos are usually employed, connected in series.

T.

T = tee.—Anything resembling the letter T, as a T-rail, T-square, T-iron, etc.
Table.—In mechanics, the table-like part of a machine on which the work is placed. In architecture, a horizontal molding or a projecting portion from a wall.
Tackle.—A rope operating in one or more pulleys for hoisting or hauling.
Tail-race.—A channel and stream of water leading from the water-wheel.
Talus = batter.—Slope, as of a parapet or rampart, in fortifications.
Tamp.—To force down with strokes or pressures, as tamping a charge of powder in a hole, or tamping earth, concrete, etc.
Tamping-bar = tamping-iron.—In blasting, an iron bar for forcing the tamping (material) on the charge of explosive in the hole.
Tamping-plug.—A form of cast-plug used instead of tamping material in a blast-hole.
Tangent-screw.—A sort of slow-motion screw for revolving circular disks slowly.
Tank-engine.—A locomotive that carries its coal and water without a tender.
Tank-iron.—Plate iron whose thickness is between that of boiler-plate and sheet-iron, the latter being the thinnest and about the same as stove-pipe iron.
Tap.—A tool with an external, tapered, and longitudinally-grooved screw for cutting the internal screws of nuts, etc. As a verb, to make a tap as in a pipe (pipe-tap); to bore or cut into; to pound as with a hammer in testing rivets or cast-iron pipe.
Tap-bolt = tap-screw.—A bolt screwed into a tapped hole, as in a plate, the plate acting as a nut.
Tapping-drill = tapping-machine.—A drill or machine for tapping holes in street-mains, or iron pipes.
Tappet.—A projection or arm on a revolving shaft, which strikes or taps something at each revolution.
Tee = T.—See T.
Teeth.—Plural of tooth; see *Tooth*.
Telford pavement = telford.—A road pavement consisting of a foundation of small stones laid by hand; on top of and in the crevices of these stones are packed smaller pieces; upon this, is broken stone. The whole mass is rolled until the surface becomes compact and smooth.
Temper.—To modify. To bring metal to a proper degree of hardness by first heating to a high temperature, and then suddenly cooling in a bath of oil, or water, etc. To mix mortar to the right consistency for bricklaying, etc.
Templet = template.—An outline of anything, used as a guide or model in shaping it, as the edge of a board cut to shape a molding, or a board or plate with holes spaced for punching other plates, etc.
Tender.—That which tenders or waits upon, as an engine tender, or a vessel supplying freight, provisions, etc, to another.
Tenon.—The framed projection at the end of a timber, to fit into a mortise; such a joint is called a *mortise and tenon* joint.
Teredo = ship-worm.—A sea-worm which bores into and honeycombs piling and timber.
Terne-plate.—An inferior tin-plate, or a plate of sheet-iron or sheet-steel coated with tin which is largely alloyed with lead.

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Terrace.—A sort of long horizontal step in an embankment; used on the sloping up-stream and down-stream faces or slopes of earthen dams. A kind of bench or level, on the side of a hill.

Terra-cotta.—A fine quality of clay baked very hard; used for brick, roofing tile, pipes, etc.

Test-pump.—A force-pump used for testing the strength or tightness of pipes, cylinders, etc. It is provided with a pressure-gage.

Theodolite.—A surveying instrument, something like a transit but whose telescope is not reversible.

Thermometer.—An instrument for measuring temperatures. *Centigrade thermometer* reads zero at freezing and +100° at boiling. *Deep-sea thermometer* used to record the temperature of the water at any depth in the sea. *Differential thermometer* is used to record small differences in temperature. *Fahrenheit thermometer* reads +32° at freezing and +212° at boiling. *Maximum thermometer* registers the maximum temperature. *Minimum thermometer* registers the minimum temperature. *Reamur thermometer* reads zero at freezing and +80° at boiling.

Thimble.—A sleeve or tube used to join the ends of pipes or rods. A circular ring, concave outside, to form the inside protection to a loop of rope when suspended on a hook or other contrivance.

Thimble-joint.—A sleeve or thimble slipped over a pipe-joint and packed to prevent leakage during expansion and contraction.

Thread.—The spiral ridge or worm of a screw.

Three-ply.—Three thicknesses, as three-ply roofing felt.

Throat.—The narrow part of an opening, not always near the end, as the throat of a venturi-meter.

Through-mortise.—A mortise cut entirely through a timber.

Through-stone = thorough-stone—A header extending entirely through a wall.

Throw.—The distance moved, as the throw of a railroad switch is from 5 to 6 inches. The extreme movement of a slide-valve. The double-radius of a crank.

Thrust.—The crushing of the pillars in a coal-mine.

Thrust-box.—A box-bearing sustaining a vertical shaft.

Tide-gate.—A gate through which water passes into a basin when the tide flows in, and which is shut to retain the water from flowing back at ebb.

Tie.—Any beam or rod used in construction to hold certain parts together by tension or pull. One of the sleepers or supports to the rails.

Tie-plate.—A metal plate resting on a tie and supporting the rail; used to prevent the flange of the rail from sinking into the wooden tie.

Tie-rod.—Any rod used as a tie to sustain tension or pull.

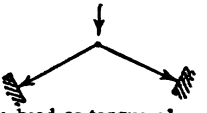
Tile.—A thin shape of baked clay, used for covering roofs, floors, walls, etc.; also for pipe, drains and sewers.

Tire.—An outside ring around the periphery of the wheel of a vehicle. In common wagons, the metal tire is heated so as to expand and then shrunk over the wooden felloe.

Tit.—A small projection, as like the end of a bolt on the surface of a casting.

Toe.—The sharp end, or front end, or moving end of many devices. The toe of a switch, or of a frog, etc.

Toggle-joint.—A joint formed by two bars hinged together at an obtuse angle so that when direct pressure or pull is applied at the joint the lateral force is greatly augmented.



Tongue.—Many things pointed or projecting, as the bead or tongue along one edge of a board to fit into the groove of an adjacent board; called *tongue and groove*, and used for flooring, etc. The pointed part of a crossing-frog.

Tooling.—In masonry, dressing with a chisel so the face shows the parallel marks of the tool with uniformity.

Tooth.—One of the teeth or cogs of a wheel. Also applies to other tooth-like projections as in a saw.

Torus.—A large convex molding, used at the base of a column. Opposed to *scotia*, a concave molding.

Tower, Electric.—A high tower provided for the support of a number of electric arc lamps, employed in systems of general illumination.

Tower, Electric-Transmission.—A tower on a transmission line, for supporting long-span wires.

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Traction-wheel.—A wheel which by friction on its circumference draws a vehicle, as the driver of a locomotive.
Traction-engine.—A steam-engine for drawing loads on common roads.
Trailing-wheel.—One of the wheels situated just behind the driving-wheels of a locomotive to support the rear weight.
Train.—A set of wheels, as cog-wheels, working in series, i. e., connected together in a train.
Trammel.—An instrument for drawing an ellipse; consists of a horizontal arm with a vertical pencil at one end, two points of the arm working along lines which cross each other at right angle.
Transformer.—An induction coil used for *raising* or *lowering* the electromotive force at any point of a circuit. The E. M. F. is raised by the step-up transformer, and lowered by the step-down transformer. Other terms are: Commuting-, Constant-current-, Core-, Hedgehog-, Lightning-arrester-, Multiple-, Oil-, Open-iron-circuit-, Pilot-, Rotary-current-, Rotary-phase-, Series-, etc.
Transom.—One of the horizontal framing timbers across the stern of a ship. A horizontal beam across the opening for a door. The opening above a door. A horizontal beam of timber or stone across a window.
Trap.—A pipe with a depressed bend to hold water and thus form a water-seal against the passage of gases; usually U- or S-shaped; used in closets in connection with soil-pipes, and elsewhere.
Trap-valve = clack-valve.—See *Clack-valve*.
Traverse.—In surveying, a polygonal base-line around a piece of land to be surveyed, showing distances and angles.
Tread.—The horizontal part of a step, the vertical part being the *riser*.
Treadle.—A foot-lever, for operating a sewing machine, grindstone, or lathe, etc.
Treenail.—A long wooden pin (hard wood) for fastening planks and timbers; used in ship-building. Auger-holes are first bored, and the pins driven in.
Trestlework.—A stilted framework for supporting the floor of a bridge or other structure.
Trimmer.—In carpentry, a short cross-joint inserted at right angle between two lines of joists and butting against them; and in turn supporting the ends of other joists parallel with the main ones. The ends may be supported by iron stirrups, or mortised into the other joists. Used at floor- and roof-openings, at chimneys and stairways.
Trip-gear.—A gear that trips the valve-closing mechanism in a steam-engine when the piston reaches a certain definite position.
Trip-hammer = tilt-hammer = tilting-hammer.—A hammer operated by a cam which trips a lever and allows the hammer to fall; used for heavy work.
Tripod.—A three-legged support, as for surveying-instruments, screw-jacks, drills, etc.
Tripper.—A part of a machine which causes another part to be released, as the tripper of a pile-driver hammer.
Trippet.—Any projecting part of a machine that trips.
Truck.—A framework with two or more pairs of wheels for supporting one end of a car or locomotive. A bogie-truck is used forward of the drivers and is carried by the bogie-wheels; this name also applies to street-car trucks. Also, a wagon with a solid platform for hauling heavy material.
Trundle-wheel = lantern-pinion = lantern-wheel = wallower.—See *Lantern-wheel*.
Trunk.—A long trough or box for conveying water, as from a race to a water-wheel; a penstock or flume.
Trunnion.—One of the cylindrical projections on the side of a cannon, which support it in its carriage and on which it revolves in a vertical plane. A hollow gudgeon on either side of an oscillating cylinder, for supporting the cylinder and through which steam enters and is exhausted.
Trunnion-valve.—A valve of an oscillating-cylinder, and operated by its oscillating motion.
Truss.—A framework acting as a beam or girder.
Truss-beam.—A simple or compound (wooden) beam reinforced by one or more tie-rods (or in some cases, A-shaped struts).
Tube, Crookes'.—A tube containing a high vacuum and adapted for showing any of the phenomena of the ultra-gaseous state of matter.
Tube-valve.—A tube pressed against a seat at outlet, the other end projecting above the surface of the water; operated in various ways.

Tudor style.—A style of English architecture of the 16th century.

Tumbler.—A kind of lever which drops into a notch at a certain definite position in the movement of a mechanism, and locks.

Tumbler-tank.—A tank which automatically discharges its contents when filled; if two tanks, they alternate in filling and emptying.

Tumbling-bay = waste-weir—That part of the outlet or waste from a canal or other body of water at which the water falls rapidly.

Tumbling-box.—A box pivoted at opposite ends or corners and made to revolve. A cubical concrete-mixer is a tumbling-box.

Tumbling-shaft.—A shaft used in stamping mills, in the link-motion of a locomotive, in thrashing-machines, etc.; a sort of a cam-shaft.

Turbine.—In the broadest sense: A motor with vanes (usually curved) and *acted upon* by the pressure or velocity (or both) of fluids, as air, gas, water, etc. (as ventilating fans, turbine water wheels, etc.); or *acting upon* fluids (as in the case of the centrifugal pump). In a restricted sense: A motor with curved vanes acted upon by fluids (water or steam), as the hydraulic turbine and steam turbine (see below).

Turbine, Hydraulic = Turbine Water Wheel = Turbine.—A turbine deriving its motive power from the pressure and velocity of water. See page 1342.

Turbine, Steam.—A turbine deriving its motive power from the velocity and expansive force of steam.

Turn, Ampere.—A single turn or winding in a coil of wire through which one ampere passes.

Turnbuckle.—A double-nut with right and left threads, used for connecting the ends of two rods or bars, making one adjustable member.

Turning-point.—In leveling, a temporary bench-mark.

Turn-table = turntable.—A sort of deck-drawbridge swinging in a horizontal circle, for turning locomotives; used at terminal points and at shops, round-houses, etc. There is a central pin or pivot-pin supporting the table, and the outer ends are steadied by wheels on a circular track.

Turpentine.—An oleoresinous substance obtained from the bark and wood of coniferous trees. The oil of turpentine is obtained by distillation.

Tuscan Order.—In architecture, one of the five Orders; similar to the Roman Doric.

Tusk-tenon.—A tenon stepped or shouldered into a mortise, to give additional strength to the connected beams.

Two-ply.—Having two thicknesses, or double thickness.

Twyer.—One of the tubes through which air enters a blast-furnace.

U.

U-bolt.—A U-shaped rod with nut and thread at each end.

Unctuous.—Oily; greasy, soapy. Having the nature of an unguent.

Underdrain.—A sub-drain, or drain under ground.

Undermine.—To render unstable by weakening the foundation. To excavate beneath, as by digging or washing.

Underpinning.—A support or new foundation placed under a wall not properly supported; it may be either temporary or permanent, and may consist in merely projecting the wall downward. The act of introducing such a support. The term *undersetting* may be used, especially with reference to machinery, pedestals, etc.

Undershot wheel.—A water-wheel operated by the force of the stream acting upon the blades or paddles as they fall below the level of the center or axis of the wheel.

Undertow.—A sub-surface current moving in a direction different from the surface-current.

Unguent.—Any soft substance or composition used for lubrication.

Uniclinal = monoclinal.—In geology, dipping in one direction: a monoclinal fold is half an anticlinal fold.

Unit.—A standard of quantity in any system, as unit of measure, capacity, weight, force, etc.

Upset.—The end of a rod, or bar, etc., which has been thickened by shortening its length, for the purpose of connecting it with some other member so that the joint will not be weaker than the body of the rod or bar; as for forming the head of an eye-bar, or the upset-end on a rod for cutting a screw-thread. The machine for making upsets, or upsetting, is called an *upsetting-machine*. The shaping of eye-bar heads is called *forging*; welding a separate head on the body of the bar is not allowable in a good class of structural work.

V.

Vacuum-brake.—A device for stopping a train by the operation of brakes through partial vacuum created in a pipe connected with the locomotive; the vacuum being created by a steam-jet escaping through an ejector, and the brakes being operated by the drawing of the brake-rods which are joined to collapsing bellows connected with the pipes.

Vacuum-gage.—A gage for indicating the pressure (or the amount of vacuum) in any chamber, as in the receiver of an air-pump, a steam-condenser, etc.

Vacuum-valve.—A safety-valve opening inward in a steam-boiler to give relief from collapse when the partial vacuum in the boiler reduces the internal pressure below the point of safety in resisting the external atmospheric pressure.

Valve.—Any device for controlling the flow of liquid, vapor or gas through a pipe or passage. *Valve-chest*, in a steam-engine, is the steam-chest; it is the chamber in which the valve works. *Valve face*, that part of the surface of the valve which comes in contact with the valve seat. *Valve gear* = valve motion, the system which gives motion to a valve, as the link motion of a locomotive for regulating the supply and exhaust of steam to and from the cylinder. *Valve seat*, the fixed surface or piece on which a valve presses and rests. *Valve stem*, a rod attached to a valve to operate it. *Valve yoke*, a strap to hold a valve.

Air valve; a valve to regulate flow of air, as in a steam-boiler.

Automatic valve; a valve that works automatically, as a clack-valve.

Back-pressure valve; a valve to prevent back-flow when the direction or pressure of fluid is reversed.

Balance-valve; a valve admitting fluid to both sides under nearly equal pressure.

Ball-cock valve; a sort of ball float valve, as used in water-closet tanks.

Ball valve; a valve formed by a ball resting upon a seat.

Blow-through valve; a valve situated in the opening through which steam enters a condensing steam-engine, for blowing through.

Brake-shoe valve; in an air- or vacuum-brake, a valve to relieve the excessive pressure upon the wheel.

Butterfly valve; a double clack-valve, used in pumps.

Check-valve; a valve placed in a pipe-line, or boiler, to prevent back-flow.

Clack valve; a trap-flap, or clapper, hinged to allow flow in one direction only.

Cone-valve; a valve with a conical face and seat.

Conical valve; a T-valve or puppet valve—a circular metal plate with beveled edge and seat.

Cup-valve; a valve cup-shaped, and becoming a balance-valve if consisting of two cups connected by a stem through the opening.

Double-beat valve; a double-seat valve.

D-valve; a valve resembling the letter D, used in the induction and eduction passages of a steam-engine cylinder.

Equilibrium valve; a balance valve (see above).

Flap-valve; a clack-valve (see above).

Globe-valve; a valve with a globular casing.

Hinged valve; a butterfly-valve or clack-valve (see above).

Key-valve; an air valve-plug.

Lifting valve; a ball-, cone-, poppet-, or safety-valve.

Long-slide valve; a long-valve, governing parts of both ends of a steam-engine cylinder, especially of the Cornish type of engine.

Long-valve; a long-slide valve (see above).

Low-water valve; a valve which allows the steam to escape when the water in the boiler is too low.

Oscillating valve; a valve that oscillates.

Piston-valve; a reciprocating valve, alternately opening and closing the port of a steam-engine cylinder.

Pocketed-valve; a valve fitted into the depression of a pocket.

Poppet-valve; a valve which lifts bodily from the seat.

Pot-lid valve; a cap valve used at the end of a pipe, or the cover of an air-pump of a steam-engine.

Puppet-valve; a conical valve or poppet valve (see above).

Regulator-valve; a throttle valve.

Relief-valve; a valve through which fluids escape at a certain determined high pressure.

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Reverse valve or *reversing valve;* the valve of a reversing cylinder, often a plain slide-valve.
Rotary valve; a rock valve which acts by partial rotation.
Safety-valve; a valve to relieve excessive pressure, as in a steam-boiler.
Screw-valve; a screw with a point forming a small valve, for regulating flow.
Slide-valve; a valve which slides over a seat.
Snifting-valve; the tail valve or blow valve in the cylinder of a steam-engine, for the escape or admission of air.
Spherical valve; a ball valve.
Throttle-valve; a valve in the steam pipe of a boiler for controlling the flow of steam, as to a cylinder.
Trap-valve; a clack-valve or flap-valve (see above).
Twin-valve; a double-connecton valve or gate.
Undershut valve; a valve beneath the sole-plate of a pump, etc., and which closes by upward pressure from below.

Vapor.—A gaseous form of a substance which ordinarily exists in solid or liquid form, and while it is in this gaseous form it is physically a real *gas*, which may be defined as a substance which at ordinary temperatures and pressures exists in the gaseous state; hence, all vapors are gases but all gases are not vapors. A *saturated vapor* is a vapor which is on the point of condensation. A *non-saturated vapor* is one which obeys the laws of gases, as superheated steam.

Variable gear.—Geared wheels or sectors which impart alternating changes in speed.

Vault.—A long arch (not in span but in the direction of the axis), or one whose length is great in proportion to its span. The space enclosed by or beneath such a vault.
Conical vault; a vault formed as upon part of the surface of a cone.
Compound vault; a vault composed of two or more simple vaults.
Cross vault; a vault which crosses another.
Cylindrical vault; a vault of cylindrical form.
Double vault; a vault placed above or enclosing another vault.
Elliptical vault; a vault of elliptical form.
Groined vault; a vault formed by two intersecting vaults.
Pointed vault; a vault pointed.
Rampant vault; a vault which springs from planes not horizontal.
Simple vault; an ordinary vault with one axis.
Single vault; one vault.
Spherical vault; a vault of spherical form.
Surbased vault; a circular vault whose height is less than half the span.
Surmounted vault; a circular vault whose height is greater than half the span.

Vaulting-shaft.—A shaft to receive the spring of a roof-vault rib; may extend downward to the floor or to the capital of a pier.

Vaulting-tile.—A tile used in vaulting; hollow and of various forms.

Vault-light.—A vault cover set with glass for the admission of light.

Vault-shell.—The masonry, skin, plate or thin filling between the ribs of a vault.

Veneer.—A thin layer of costly or ornamental wood glued over the surface of a cheaper variety composing the frame.

Venetian blind.—A hanging blind operated with cords, the slats being held together by some flexible material.

Vermicular work.—The surface of architectural stone so dressed or worked as to appear thickly covered or indented with worm tracks or shapes.

Vernier.—A small movable scale sliding parallel with a fixed scale, the number of subdivisions varying by 1, in order to obtain more exact readings; as the vernier of a transit instrument.

Viaduct.—A series of (masonry) arches supporting a roadway.

Viscosity.—Internal friction in the movement of liquids and gases. Viscous fluids are those in which viscosity is strongly present.

Vise = vice.—A tool with two gripping jaws that may be opened or closed by a screw worked with a lever; used by carpenters and machinists for gripping pieces to be worked.

Vitreous.—Resembling glass; glassy.

Vitrified.—The whole body, or the surface only, converted into glass; glazed, as vitrified- or glazed-brick, tile, terra cotta, pipes, etc. The vitrification is performed by the action of heat.

Volt.—The practical unit of electromotive force.

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Volt-Ammeter.—A watt-meter.
Volt-Ampere.—A watt.
Volt-Coulomb.—The unit of electrical work. The joule.
Voltage.—Electromotive force or differential of potential.
Voltmeter.—An instrument for measuring the difference of potential.

W.

Wainscot.—A wooden lining or paneling of the walls of a room, and reaching upward three feet or more above the floor.
Wale = wale-piece.—A longitudinal timber fastened to a row of piles to keep them more rigid and in position; or fastened along a ship, cofferdam, caisson, wharf, quay, or jetty, etc.
Wall.—In mining, one of the rock surfaces enclosing a vein or lode.
Wallow.—To wabble, as a water-wheel revolving unevenly on its journals.
Wallower = lantern-wheel.—See *Lantern-wheel*.
Wall-plate.—In building, a horizontal timber placed on top of the wall to bind it together and stiffen it; and to receive the ends of girders, joists, rafters, roof-trusses, etc., and distribute their pressures over the wall. In mining, the two long pieces of timbers of the four comprising a set of timbering in a shaft.
Warp.—A twist or bend as in a piece of timber which in drying has twisted. A rope smaller than a cable and used in towing or warping a vessel. Sediment.
Warped surface.—A surface which looks as though it had been twisted from a true plane; applies to the surface of boards, stones, soffits of arches, etc.
Warping-bank.—A ridge of earth raised around an area of land for holding water let in to enrich the land with warp or sediment.
Wash-board = mopboard = skirting board.—A board around the walls of a room next to the floor. A base-board.
Washer.—An annular piece of metal, leather, or rubber, etc., placed at a joint to prevent leakage, or under a nut to distribute pressure, etc. *Flat washers* are stamped out of plate (metal); *cast washers* are thicker and mostly of the O. G. pattern, a section of the edge describing a reversed curve.
Washout.—Excavation of a bank or hillside by the erosive action of water; the cutting or washing away of a road-bed by rains or floods; etc.
Waste-gate.—A gate placed at the waste-outlet of a reservoir, pond, or lake, etc. See *Tide gate*.
Waste-pipe.—A pipe for discharging waste water; an overflow-pipe.
Waste-trap.—A device in a pipe to allow the surplus water to escape and yet prevent gases from returning.
Waterway.—An opening or passage for waste water or overflow.
Waste-weir.—A cut in the side of a reservoir, pond, or canal, etc., for the discharge of surplus water.
Waste-well.—A well into which surplus water is discharged; should have a permeable bed, as gravel.
Water-butt.—A large cask used as a reservoir or tank for water.
Water-craft.—Vessels and boats in general.
Water-float.—A float placed in a tank, boiler, or cistern, etc., to control a valve.
Water-gage.—A device for indicating the height of water in tank, boiler, or reservoir, etc. A connecting glass tube may be used, or a float connected with an indicator, or a board with elevations marked upon it; various contrivances.
Water-gate.—A gateway, gate or valve for controlling the passage of water through an opening or pipe or channel.
Water-hammer.—The impact of water when its volume of flow is checked suddenly as by a gate in a water-pipe; large gates in long pipes are regulated by gearing to close slowly in cases where the combined volume and velocity would tend to produce considerable impact if checked suddenly, pipes often bursting from this cause.
Water-inch.—The quantity of water which will discharge through a circular hole one inch in diameter in 24 hours, the surface of the water being at top of hole; about 500 cubic feet.
Water-main.—One of the main pipes in a system of water-works; the main pipe in each street, or the main pipe supplying several streets, or the main pipe leading from the reservoir or headworks.

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Water-meter.—An instrument, device or apparatus for measuring the velocity or rate of discharge of water.

Water-motor.—A water-wheel or turbine. Any motor deriving its power from the pressure and flow of water, either for direct pumping or for the transmission of power to any kind of machinery.

Water-pillar = water-crane.—A vertical pipe with a swinging arm or goose-neck, acting as a sort of hydrant for supplying water to locomotives.

Water-plane.—A plane passing through the water line of a ship, i. e., the water-surface plane; hence the terms *light water-plane* and *load water-plane*.

Water-ram.—A hydraulic ram, for raising water.

Watershed.—The boundary of a *catchment area* or basin, i. e., the high ridge or *divide* which surrounds the area drained by a stream.

Water-table.—In architecture, a string-course around the base of a building, and projecting outward for ornament and as if to throw off water from the wall.

Water-tower.—A large standpipe.

Water-tube boiler.—A form of boiler containing pipes or tubes in which the water circulates, being heated by the surrounding flames

Watt.—The unit of electric power. The volt-ampere. One watt is equivalent to the work of 0.7373 foot-pound per second $= \frac{1}{746}$ horse power.

Watts = volt-amperes $= CE = C^2R = \frac{E^2}{R}$, in which C = current in amperes, E = electromotive force in volts, R = resistance in ohms.

Watt-Meter.—A galvanometer, for measuring simultaneously, the current and the difference of potential at any point of a circuit.

Watt-second.—A unit of electrical work.

Way-gate.—The tail-race of a mill.

Ways. = The inclined timbers on which a ship moves in launching

Weather-boarding.—A facing of boards on a building; the boards being either (1) clapboards, laid horizontally with feather-edge upward and overlapping each other like shingles, or (2) boards nailed vertically and having either tongued and grooved joints or else narrower boards nailed over the joints, or (3) ordinary shingling.

Weather-tile.—Tile used as a substitute for weather-boards.

Wedge-valve.—A wedged shaped valve operated by a screw.

Wedging.—The process of driving a wedge into a saw-kerf in the end of a tenon which just passes through a through-mortise, in order to expand the end of the tenon and make it bind firmly against the sides of the mortise, as in securing the helve or handle or an ax into the steel head.

Weir.—A dam across a stream, over which the water flows. A measuring weir, simply called a *weir* and more properly a *standard weir*, for measuring the flow of water.

Weld.—To unite metallic substances by hammering or compression, with or without previous heating; if heated to fusion, a flux is used to prevent oxidation or rapid rusting. Electric welding is accomplished by bringing the proposed joint into a circuit, the greater resistance at the joint causing the abutting surfaces to become intensely hot, and then applying great mechanical pressure.

Weld-iron.—Wrought-iron is weld-iron.

Weld-steel.—Puddled steel.

Well-boring.—The process of sinking or driving wells by drilling or boring through rock, often to great depths; percussion drills are most frequently employed. A form of *well-sinking*.

Well-trap = sink-trap.—A trap which allows water to pass down, but prevents air or gases from passing up; such as an S-trap

Welt.—A strip of metal riveted to two abutting plates, forming a butt-joint. Similarly in carpentry, a strip placed over a seam or joint to strengthen it.

Wharf.—A platform or depot for vessels. Plural is wharves or wharfs.

Wheel.—A circular body or frame revolving on an axis. The wheel and axle, lever, wedge, pulley, screw and inclined plane comprise the six simple machines or mechanical powers.

Wheel-base.—The distance between centers of the extreme front and rear wheels of a locomotive, or car, etc.

Wheel-window.—In architecture, a special circular window.

Whitewash.—(1) common, quicklime and water; (2) good, whiting, size and water.

Whiting.—Chalk specially prepared by drying, grinding, etc.

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Wicket.—A small gate, door or opening in a larger one; a small gate in the lock-gate of a canal, by which the chamber can be emptied; a small gate in a water-wheel chute, etc.

Winch.—An axle with one or two bent arms or cranks for turning, as a common windlass or a grindstone. The axis may be geared to a separate drum, thus giving more power for hoisting.

Wind [Pronounced with long i].—A turn, twist or bend. *Out of wind* means free from turns, twists, bends, etc.; used in specifications for timber, stone, etc.

Windage, of Dynamo.—A term proposed for the air gap between the armature and the pole- pieces of a dynamo.

Wind-beam = collar-beam.—A beam joining together the rafters of a pitched roof.

Wind-bore.—The end of the suction-pipe of a pump, and covered with a strainer to exclude foreign material.

Wind-bracing.—Any system of braces to stiffen a frame against the pressure of the wind.

Winder.—One of the steps in a stair where the staircase winds or turns.

Wind-gage = anemometer.—An instrument for determining the velocity and pressure of the wind.

Wind-hatch.—The opening where ore is taken out of a mine.

Winding-engine = drawing-engine = hoisting-engine.—An engine used in turning a drum around which is wound a hoisting- or winding-rope.

Windlass.—A large axle bulging into a drum at the center, or a modified wheel and axle, used with a winding rope in hoisting or hauling weight or loads, raising anchors, etc.; the ends of the axle are pierced with radial holes in which handspikes are inserted as levers or cranks in winding, and the drum is fitted with ratchet and pawls. Also operated by steam, as the *steam-windlass*.

Wing.—A prefix used with certain names of structures which flare out like a wing.

Wing-dam.—A dam projecting out from the shore so as to divert the current; used to deepen channels, protect river-banks from wash, etc.

Wing-gudgeon.—A short, winged metal-shaft used as a journal for wheels having wooden axles.

Wing-wall.—One of the lateral, flaring walls of an abutment, and acting as a sort of retaining-wall.

Winze.—In mining, a vertical or inclined shaft which does not reach the surface but usually connects one level with another, for ventilation, passages, etc.

Wiper.—In machinery, a sort of lever-cam attached to a (horizontal) shaft for the purpose of pressing against the toe or projection from another (vertical) shaft and raising it so it may fall again by its own weight; used in connection with marine-engines, stamp-mills, etc.

Wood-screw.—A common metal screw for fastening metal or wood to wood. But see Lag-screw.

Work, Unit of Electrical.—The erg.

Work, Units of.—(See pages 90, 91.)

Working-beam = walking-beam = beam.—The large beam of a steam-engine, usually the marine or pumping type, and used as an oscillating lever in connecting the piston-rod and crank-shaft or pump-rod.

Working-point.—The part of a tool or machine producing the desired effect or work.

Worm.—A shaft with a screw-thread which in revolving engages the teeth of a wheel and turns it; the worm is called an endless screw, and the wheel a worm-wheel.

Wrecking-pump.—A steam-pump of great capacity used in pumping water from wrecked or damaged vessels.

Wrench.—A tool with a lever arm and with jaws for holding or turning pipe, rods, bolts, heads, nuts, etc.

Wrought-iron.—Iron that has been forged or rolled, and may be forged, rolled or welded.

Y.

Yard.—A round spar with tapering end or ends. In railroading, the space set aside for handling and making up trains, and general switching; and by extension it includes the space, tracks, buildings, etc., at railway stations.

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Yard-limit.—The extreme end of a yard, at which a sign usually cautions extreme care or slower speed in running trains.

Y-level.—Common engineers' spirit-level.

Yoke.—Any sort of a U-shaped strap or coupling.

Yoke, Multiple-Pair Brush.—A device for holding a number of pairs of brushes of a dynamo electric machine in such a manner that they can readily be moved or rotated on the commutator cylinder. Also: Multiple-pair-brush-, Single-brush-, Single-pair-, Single-pair-brush-, etc.

Y-track.—A Y-shaped arrangement of tracks leading from another track, and often used instead of a turntable in turning engines, cars, or whole trains.

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